Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/88-1.2.1.1

Nasser M. Abbasi

May 17, 2024 Compiled on May 17, 2024 at 9:06pm

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**170**]. This is test number [88].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (170)	0.00 (0)
Mathematica	100.00 (170)	0.00 (0)
Maple	77.06 (131)	22.94 (39)
Fricas	77.06 (131)	22.94 (39)
Reduce	77.06 (131)	22.94 (39)
Giac	75.88 (129)	24.12 (41)
Mupad	75.29 (128)	24.71 (42)
Sympy	71.76 (122)	28.24 (48)
Maxima	70.00 (119)	30.00 (51)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

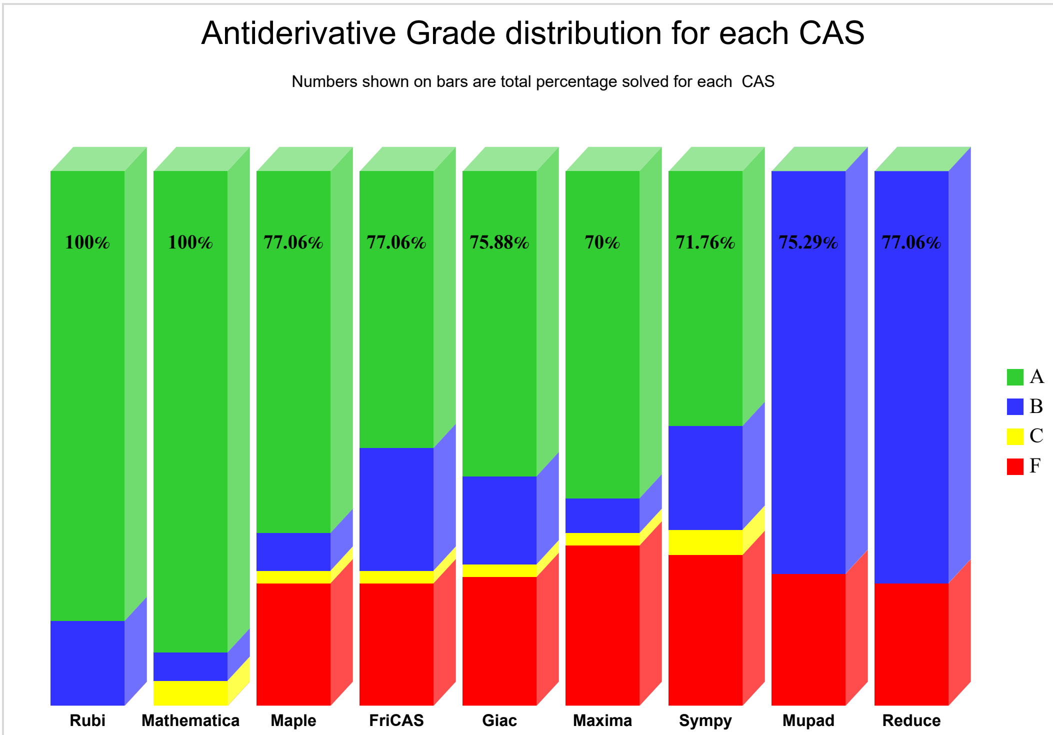
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

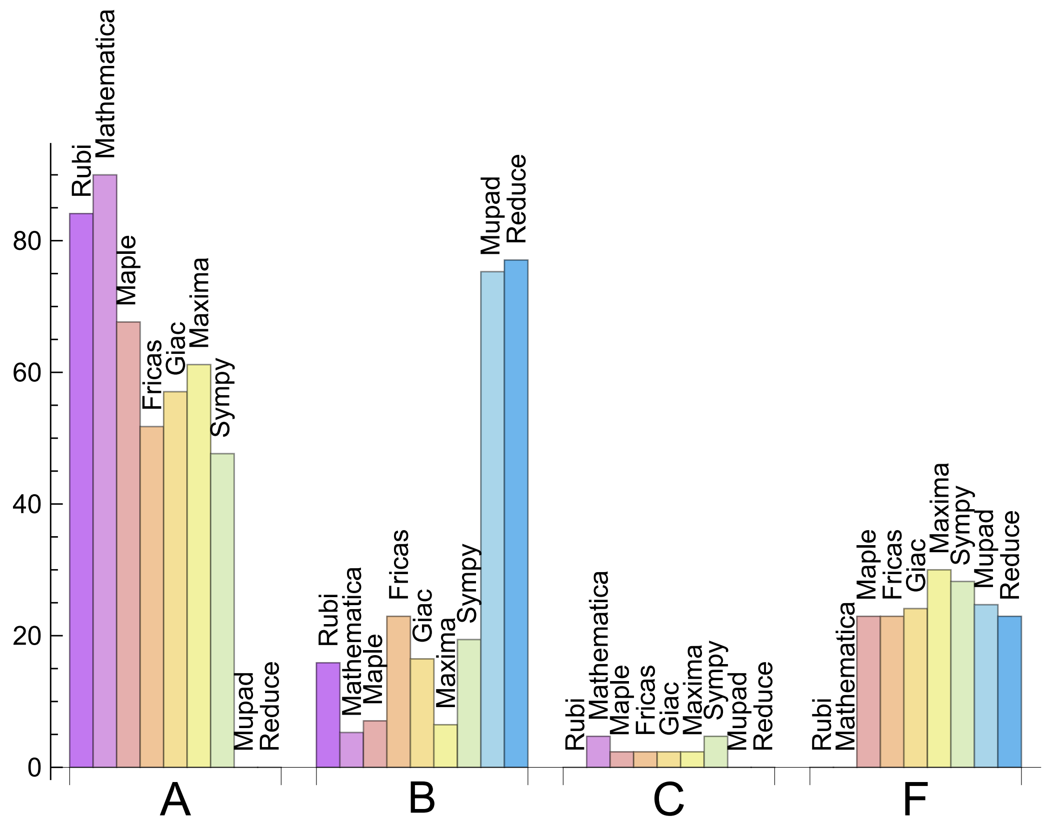
System	% A grade	% B grade	% C grade	% F grade
Mathematica	90.000	5.294	4.706	0.000
Rubi	84.118	15.882	0.000	0.000
Maple	67.647	7.059	2.353	22.941
Maxima	61.176	6.471	2.353	30.000
Giac	57.059	16.471	2.353	24.118
Fricas	51.765	22.941	2.353	22.941
Sympy	47.647	19.412	4.706	28.235
Mupad	0.000	75.294	0.000	24.706
Reduce	0.000	77.059	0.000	22.941

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	39	100.00	0.00	0.00
Maple	39	100.00	0.00	0.00
Reduce	39	100.00	0.00	0.00
Giac	41	100.00	0.00	0.00
Mupad	42	0.00	100.00	0.00
Sympy	48	100.00	0.00	0.00
Maxima	51	76.47	0.00	23.53

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Fricas	0.08
Giac	0.18
Reduce	0.23
Sympy	0.25
Rubi	0.35
Maple	0.64
Mathematica	1.05
Mupad	4.96

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	43.73	0.99	30.00	0.88
Mupad	46.59	1.02	30.00	0.84
Mathematica	50.01	1.20	40.00	1.00
Giac	57.40	1.29	39.00	1.00
Maxima	58.72	1.16	30.00	1.00
Fricas	86.34	1.72	41.00	1.25
Sympy	91.06	1.78	39.00	1.03
Rubi	91.81	1.98	43.00	1.07
Reduce	94.84	1.58	38.00	1.05

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

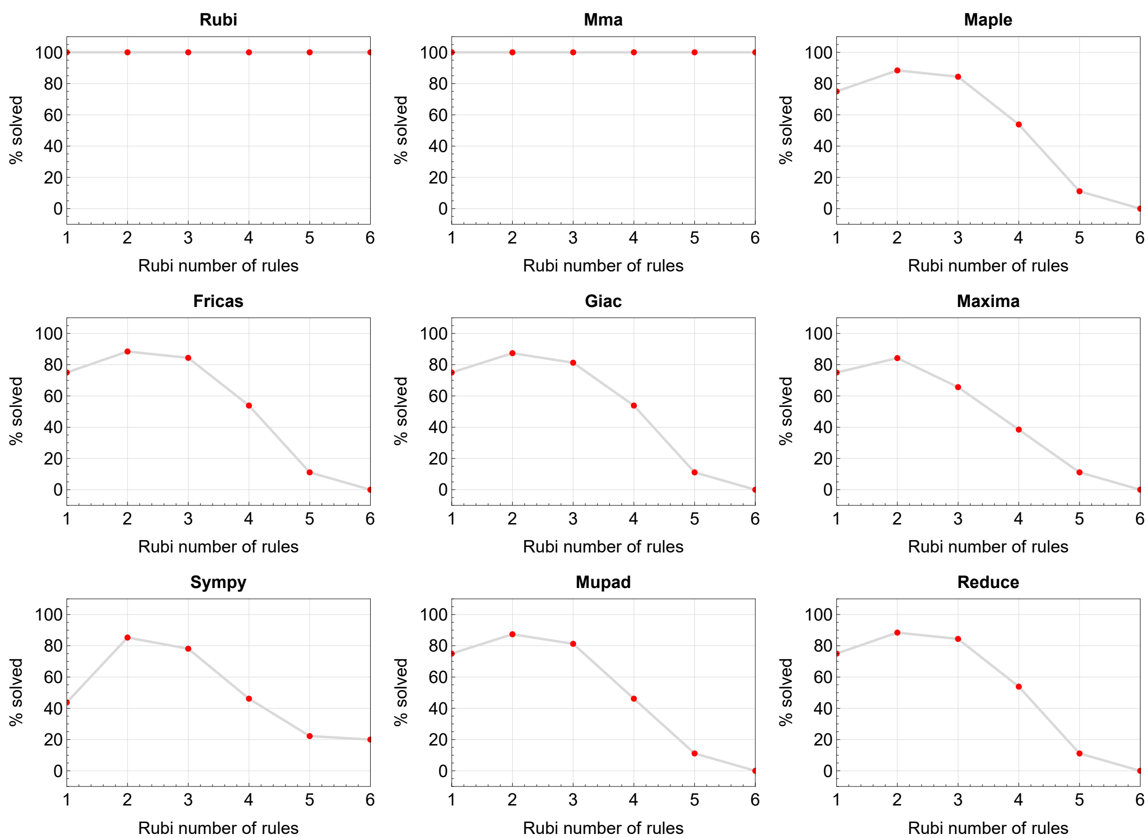


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

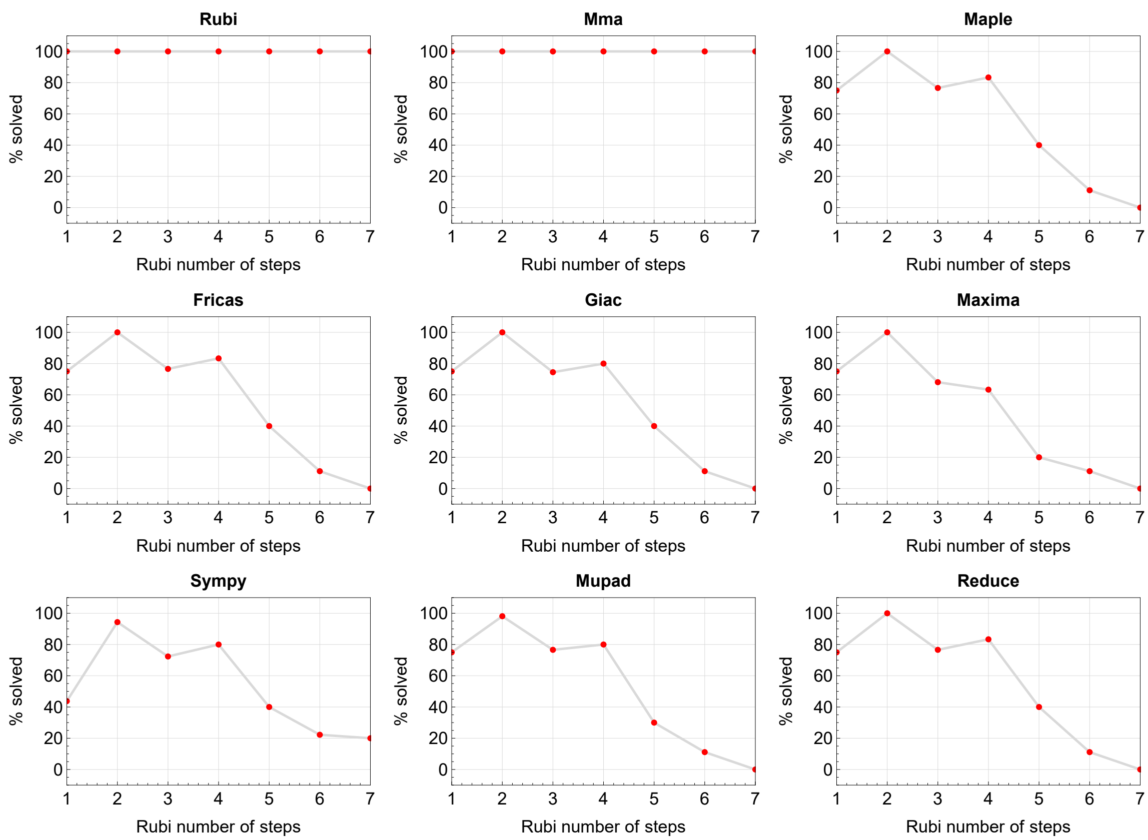


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

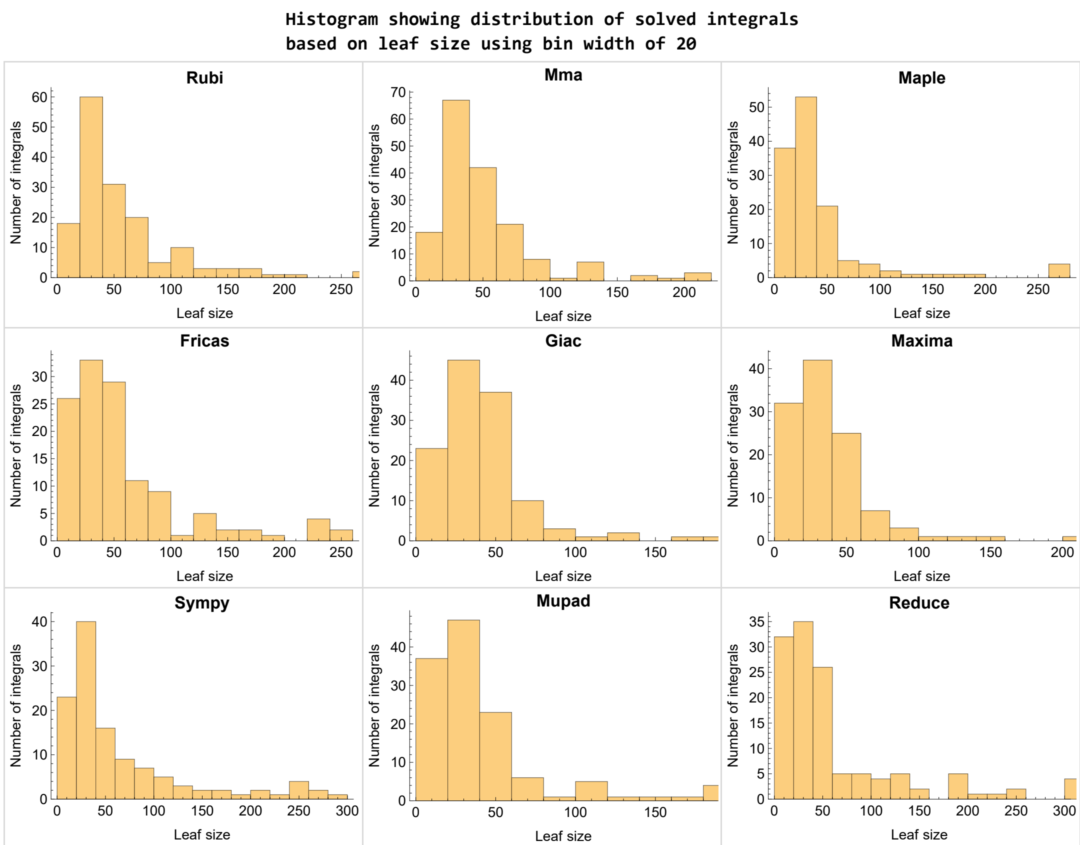


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

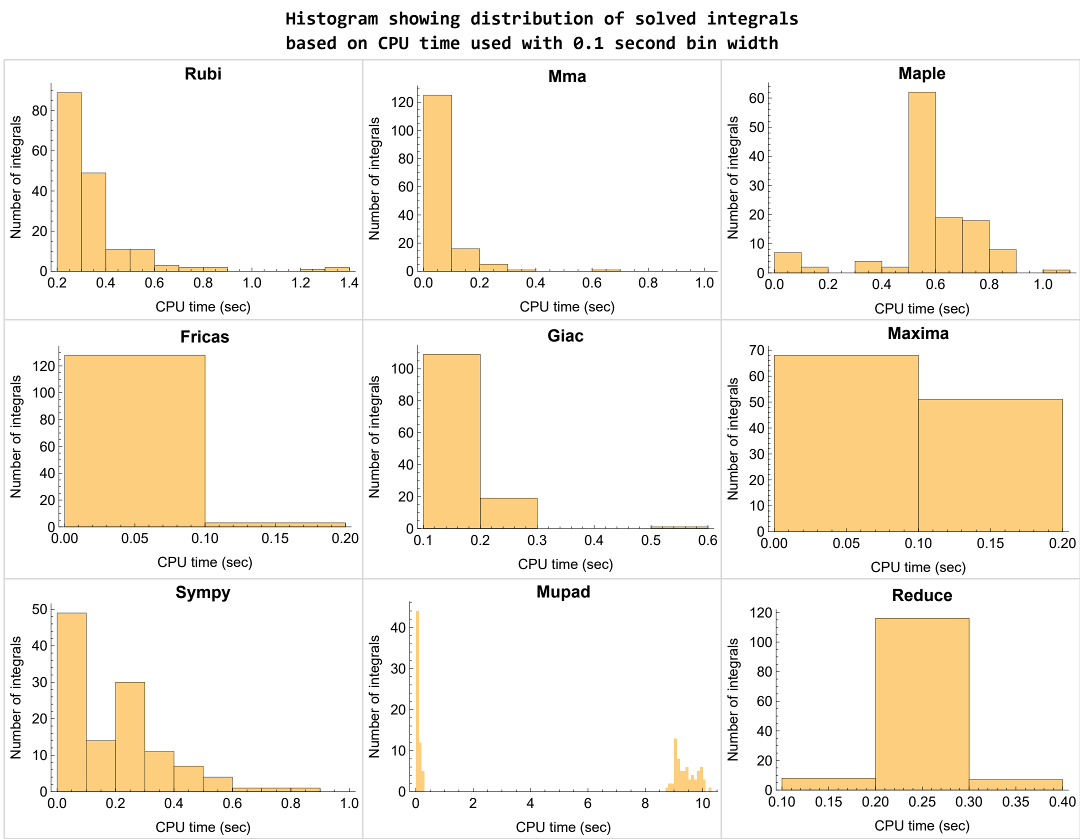


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

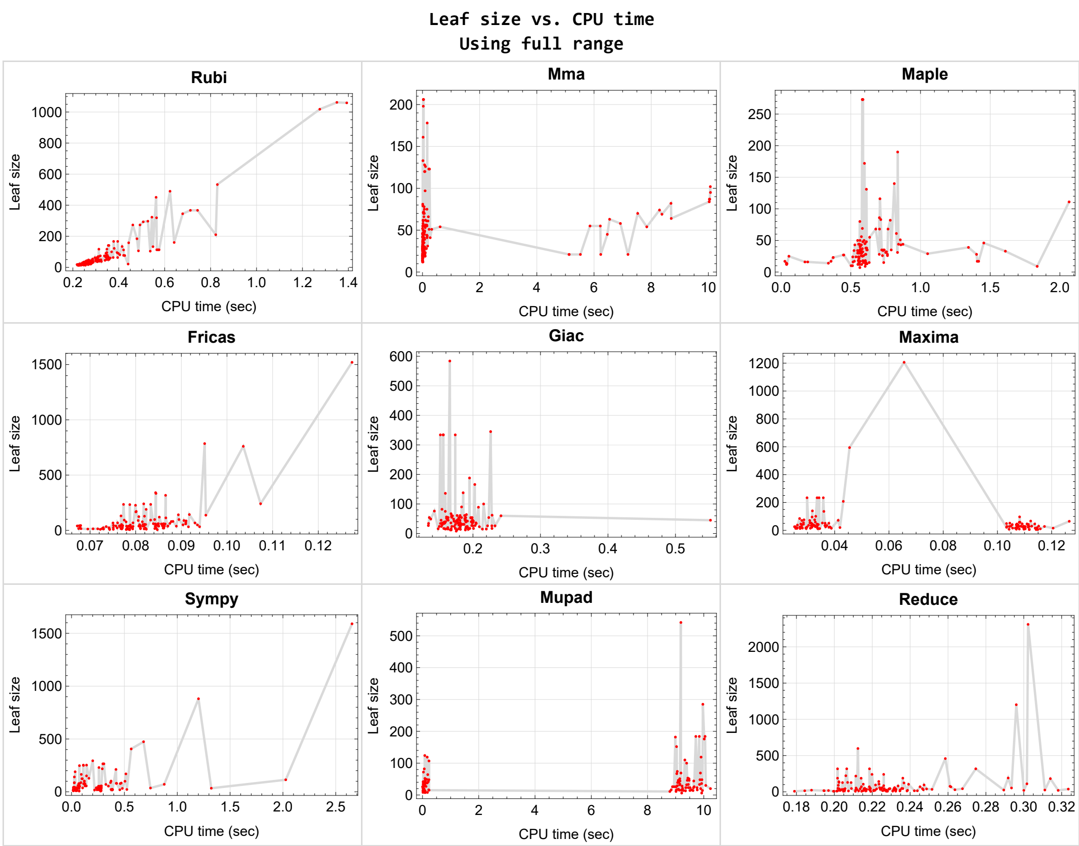


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 167, 169\}$

Mathematica {}

Maple $\{6, 9, 88, 89, 90, 91\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

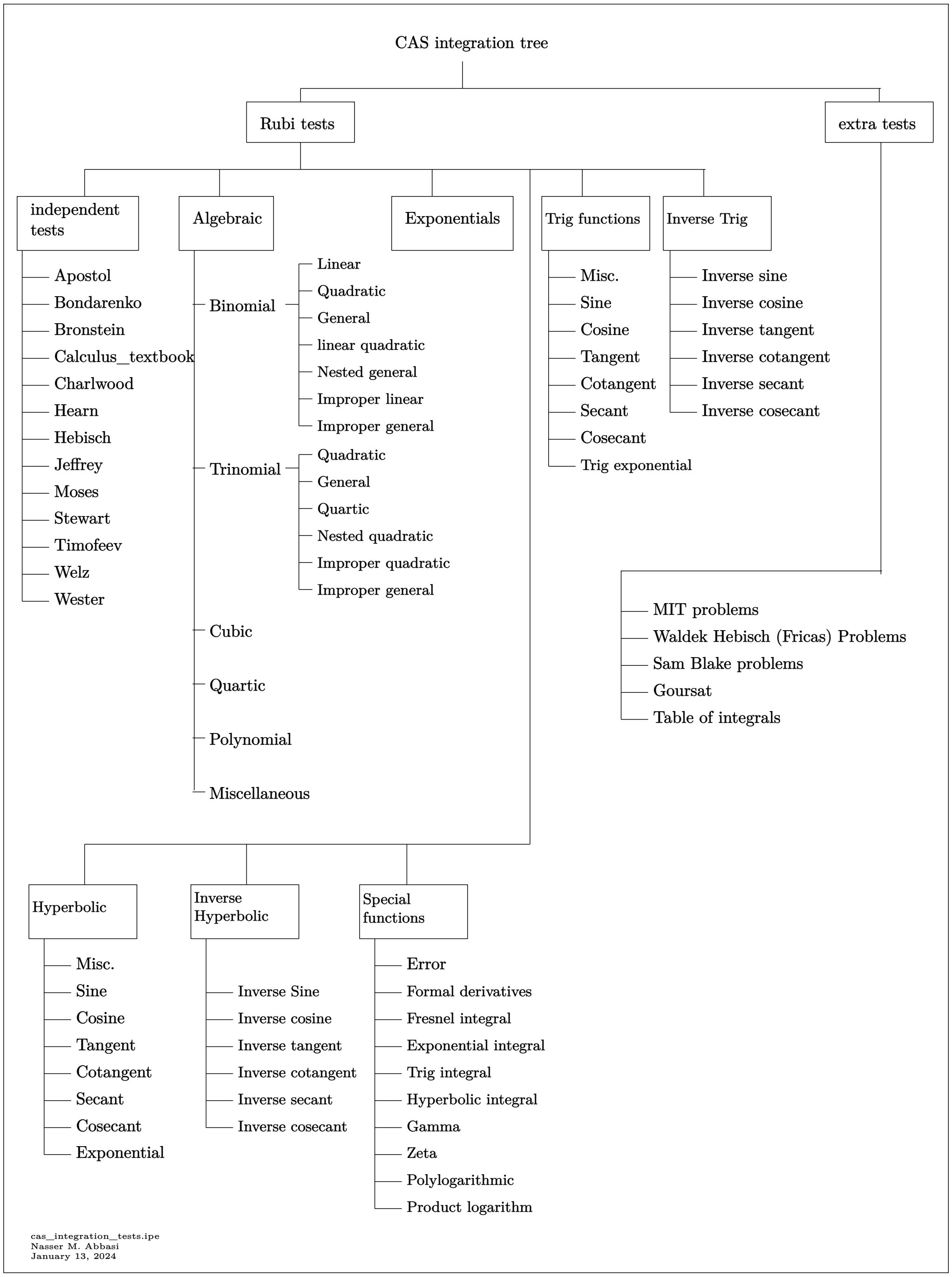
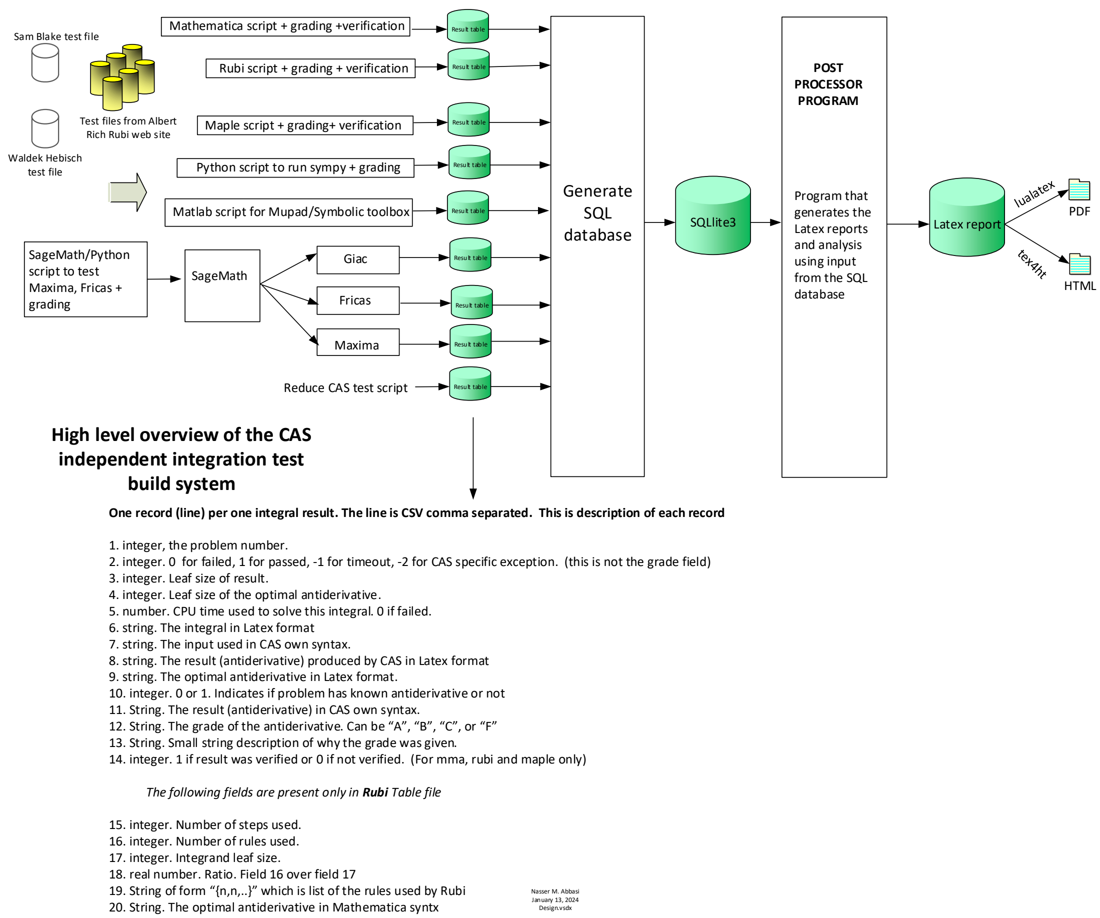


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167 }

B grade { 46, 49, 50, 52, 70, 71, 72, 74, 75, 76, 77, 78, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 168, 169, 170 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 127, 128, 129, 130, 131, 133, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 170 }

B grade { 50, 51, 52, 123, 126, 132, 134, 135, 144 }

C grade { 74, 75, 76, 77, 78, 167, 168, 169 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 2, 3, 4, 5, 7, 8, 10, 11, 13, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 81, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 136, 137, 138, 157, 163, 164, 165, 166 }

B grade { 1, 16, 50, 88, 89, 90, 91, 107, 108, 133, 134, 135 }

C grade { 6, 9, 12, 14 }

F normal fail { 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 167, 168, 169, 170 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 2, 3, 5, 6, 7, 8, 9, 10, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 47, 48, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 81, 82, 83, 84, 85, 92, 93, 94, 97, 99, 100, 102, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 124, 127, 129, 131, 138, 157, 163, 164, 165, 166 }

B grade { 1, 4, 20, 21, 22, 39, 46, 49, 50, 51, 52, 58, 65, 86, 87, 88, 89, 90, 91, 95, 96, 98, 101, 103, 104, 105, 112, 122, 123, 125, 126, 128, 130, 132, 133, 134, 135, 136, 137 }

C grade { 11, 12, 13, 14 }

F normal fail { 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 167, 168, 169, 170 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 2, 3, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 81, 82, 83, 84, 92, 93, 94, 95, 96, 97, 98, 101, 102, 105, 108, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 157, 165, 166 }

B grade { 1, 4, 20, 21, 22, 50, 88, 89, 90, 91, 107 }

C grade { 12, 14, 119, 130 }

F normal fail { 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 167, 168, 169, 170 }

F(-1) timedout fail { }

F(-2) exception fail { 85, 86, 87, 99, 100, 103, 104, 106, 109, 110, 163, 164 }

Giac

A grade { 2, 3, 4, 6, 7, 8, 9, 10, 15, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 81, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 124, 129, 136, 137, 138, 157, 163 }

B grade { 1, 5, 16, 20, 21, 22, 49, 50, 57, 88, 89, 90, 91, 107, 122, 123, 125, 126, 127, 128, 131, 132, 133, 134, 135, 164, 165, 166 }

C grade { 11, 12, 13, 14 }

F normal fail { 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 119, 130, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 167, 168, 169, 170 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 157, 165, 166 }

C grade { }

F normal fail { }

F(-1) timedout fail { 22, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 167, 168, 169, 170 }

F(-2) exception fail { }

Sympy

A grade { 3, 6, 7, 9, 10, 11, 12, 13, 14, 17, 18, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 56, 57, 62, 63, 64, 81, 82, 83, 84, 92, 93, 94, 102, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 157, 163, 165, 166 }

B grade { 1, 2, 4, 5, 15, 16, 19, 20, 21, 22, 50, 51, 52, 61, 85, 86, 87, 88, 89, 90, 91, 95, 96, 98, 99, 100, 101, 103, 104, 105, 106, 109, 110 }

C grade { 97, 107, 108, 119, 130, 164, 167, 169 }

F normal fail { 8, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 168, 170 }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 157, 163, 164, 165, 166 }

C grade { }

F normal fail { 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 167, 168, 169, 170 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	43	53	42	42	42	43	42
N.S.	1	1.00	1.00	3.07	3.79	3.00	3.00	3.00	3.07	3.00
time (sec)	N/A	0.220	0.002	0.529	0.035	0.077	0.021	0.192	0.229	0.028

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	22	13	20	20	19	20	21	20
N.S.	1	1.00	1.57	0.93	1.43	1.43	1.36	1.43	1.50	1.43
time (sec)	N/A	0.226	0.000	0.042	0.042	0.073	0.020	0.163	0.203	0.029

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	13	10	12	12	12
N.S.	1	1.00	1.00	1.08	1.08	1.08	0.83	1.00	1.00	1.00
time (sec)	N/A	0.226	0.002	0.606	0.030	0.079	0.049	0.176	0.235	0.037

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	35	35	37	12	34	37
N.S.	1	1.00	1.00	0.93	2.50	2.50	2.64	0.86	2.43	2.64
time (sec)	N/A	0.235	0.002	0.616	0.037	0.079	0.104	0.167	0.217	0.044

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	20	17	30	19	80	45	18	19
N.S.	1	1.00	0.80	0.68	1.20	0.76	3.20	1.80	0.72	0.76
time (sec)	N/A	0.242	0.007	0.526	0.109	0.077	0.427	0.552	0.197	0.052

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	16	30	9	19	26	8	13
N.S.	1	1.00	1.09	0.70	1.30	0.39	0.83	1.13	0.35	0.57
time (sec)	N/A	0.241	0.004	0.171	0.113	0.078	0.378	0.233	0.179	9.348

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	6	8	22	15	8	14
N.S.	1	1.00	0.90	0.79	0.21	0.28	0.76	0.52	0.28	0.48
time (sec)	N/A	0.241	0.004	0.373	0.114	0.074	0.374	0.220	0.200	9.507

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	20	16	9	14	0	17	14	21
N.S.	1	1.00	0.80	0.64	0.36	0.56	0.00	0.68	0.56	0.84
time (sec)	N/A	0.244	0.004	0.516	0.108	0.079	0.000	0.228	0.221	9.244

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	16	30	9	19	26	8	13
N.S.	1	1.00	1.09	0.70	1.30	0.39	0.83	1.13	0.35	0.57
time (sec)	N/A	0.241	0.008	0.191	0.105	0.075	0.457	0.222	0.205	0.092

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	6	8	22	15	8	14
N.S.	1	1.00	0.90	0.79	0.21	0.28	0.76	0.52	0.28	0.48
time (sec)	N/A	0.248	0.007	0.374	0.112	0.078	0.523	0.214	0.228	9.269

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	27	27	30	9	19	26	8	18
N.S.	1	1.00	1.17	1.17	1.30	0.39	0.83	1.13	0.35	0.78
time (sec)	N/A	0.239	0.005	0.447	0.110	0.072	0.387	0.228	0.200	9.099

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	10	6	6	24	23	8	15
N.S.	1	1.00	0.97	0.34	0.21	0.21	0.83	0.79	0.28	0.52
time (sec)	N/A	0.247	0.005	0.512	0.112	0.085	0.384	0.209	0.228	9.200

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	27	27	30	9	20	26	8	18
N.S.	1	1.00	1.17	1.17	1.30	0.39	0.87	1.13	0.35	0.78
time (sec)	N/A	0.243	0.004	0.450	0.115	0.078	0.388	0.200	0.211	0.059

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	10	6	6	24	23	8	15
N.S.	1	1.00	0.97	0.34	0.21	0.21	0.83	0.79	0.28	0.52
time (sec)	N/A	0.250	0.005	0.502	0.115	0.079	0.386	0.190	0.214	9.019

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	25	26	25	32	100	51	34	41
N.S.	1	1.12	0.74	0.76	0.74	0.94	2.94	1.50	1.00	1.21
time (sec)	N/A	0.270	0.011	0.763	0.037	0.092	0.506	0.188	0.226	9.091

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	161	172	140	167	190	188	189	152
N.S.	1	1.13	1.75	1.87	1.52	1.82	2.07	2.04	2.05	1.65
time (sec)	N/A	0.537	0.021	0.598	0.032	0.080	0.034	0.195	0.236	9.030

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	77	79	69	72	81	87	89	90	74
N.S.	1	1.18	1.22	1.06	1.11	1.25	1.34	1.37	1.38	1.14
time (sec)	N/A	0.421	0.011	0.583	0.041	0.088	0.027	0.208	0.202	9.058

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	32	25	24	24	26	24	25	25
N.S.	1	1.00	1.14	0.89	0.86	0.86	0.93	0.86	0.89	0.89
time (sec)	N/A	0.267	0.000	0.056	0.030	0.077	0.017	0.192	0.217	0.037

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	51	26	28	36	26	128	46	26	40
N.S.	1	1.42	0.72	0.78	1.00	0.72	3.56	1.28	0.72	1.11
time (sec)	N/A	0.358	0.009	0.763	0.030	0.079	0.164	0.189	0.235	0.077

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	106	66	82	208	241	406	166	459	182
N.S.	1	1.31	0.81	1.01	2.57	2.98	5.01	2.05	5.67	2.25
time (sec)	N/A	0.486	0.050	0.783	0.043	0.107	0.564	0.203	0.259	8.990

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	160	128	140	594	760	881	345	1202	542
N.S.	1	1.12	0.90	0.98	4.15	5.31	6.16	2.41	8.41	3.79
time (sec)	N/A	0.641	0.073	0.812	0.045	0.104	1.201	0.226	0.296	9.185

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	210	178	190	1206	1520	1590	584	2310	0
N.S.	1	1.09	0.92	0.98	6.25	7.88	8.24	3.03	11.97	0.00
time (sec)	N/A	0.822	0.164	0.836	0.066	0.127	2.654	0.166	0.302	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	49	42	34	34	34	39	34	33	34
N.S.	1	1.09	0.93	0.76	0.76	0.76	0.87	0.76	0.73	0.76
time (sec)	N/A	0.322	0.002	0.523	0.032	0.074	0.023	0.205	0.233	0.028

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	30	24	24	24	27	24	23	24
N.S.	1	1.12	0.88	0.71	0.71	0.71	0.79	0.71	0.68	0.71
time (sec)	N/A	0.309	0.001	0.514	0.038	0.073	0.020	0.169	0.205	0.018

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	12	12	13	13
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.86	0.86	0.93	0.93
time (sec)	N/A	0.247	0.000	0.039	0.034	0.073	0.016	0.168	0.227	0.022

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	21	13	12	13	13	10	15	13	8
N.S.	1	1.62	1.00	0.92	1.00	1.00	0.77	1.15	1.00	0.62
time (sec)	N/A	0.274	0.003	0.551	0.034	0.077	0.040	0.160	0.202	0.080

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	41	33	32	34	53	31	36	70	34
N.S.	1	1.32	1.06	1.03	1.10	1.71	1.00	1.16	2.26	1.10
time (sec)	N/A	0.300	0.009	0.575	0.033	0.077	0.058	0.155	0.245	0.084

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	53	55	44	54	93	49	46	128	51
N.S.	1	1.04	1.08	0.86	1.06	1.82	0.96	0.90	2.51	1.00
time (sec)	N/A	0.347	0.013	0.556	0.031	0.083	0.076	0.188	0.204	9.145

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	49	42	34	34	34	39	34	33	34
N.S.	1	1.09	0.93	0.76	0.76	0.76	0.87	0.76	0.73	0.76
time (sec)	N/A	0.354	0.002	0.548	0.032	0.078	0.021	0.179	0.204	0.028

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	30	24	24	24	27	24	23	24
N.S.	1	1.12	0.88	0.71	0.71	0.71	0.79	0.71	0.68	0.71
time (sec)	N/A	0.304	0.001	0.554	0.038	0.075	0.031	0.181	0.232	0.019

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	14	14	14	12	14	13	13
N.S.	1	1.00	1.00	0.88	0.88	0.88	0.75	0.88	0.81	0.81
time (sec)	N/A	0.249	0.000	0.039	0.026	0.072	0.019	0.193	0.213	0.024

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	23	21	14	15	15	14	17	15	8
N.S.	1	1.10	1.00	0.67	0.71	0.71	0.67	0.81	0.71	0.38
time (sec)	N/A	0.277	0.003	0.560	0.029	0.081	0.049	0.157	0.205	0.092

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	45	42	32	34	53	32	36	70	34
N.S.	1	1.05	0.98	0.74	0.79	1.23	0.74	0.84	1.63	0.79
time (sec)	N/A	0.307	0.012	0.566	0.036	0.082	0.072	0.177	0.218	0.080

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	50	44	54	93	53	46	128	51
N.S.	1	1.03	0.77	0.68	0.83	1.43	0.82	0.71	1.97	0.78
time (sec)	N/A	0.343	0.023	0.559	0.031	0.077	0.077	0.199	0.209	9.473

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	43	42	42	44	42	43	42
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.85	0.81	0.83	0.81
time (sec)	N/A	0.304	0.002	0.576	0.034	0.068	0.026	0.201	0.247	0.046

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	22	24	22	23	22
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.80	0.73	0.77	0.73
time (sec)	N/A	0.276	0.002	0.559	0.032	0.068	0.023	0.192	0.188	0.025

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	22	19	22	23	22
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.68	0.79	0.82	0.79
time (sec)	N/A	0.275	0.001	0.559	0.038	0.076	0.023	0.181	0.219	0.017

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	13	13
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81	0.81
time (sec)	N/A	0.244	0.000	0.039	0.034	0.070	0.019	0.189	0.238	0.023

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	38	32	17	27	34	39	29	25	16
N.S.	1	2.00	1.68	0.89	1.42	1.79	2.05	1.53	1.32	0.84
time (sec)	N/A	0.324	0.013	0.588	0.110	0.079	0.043	0.179	0.231	0.221

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	64	58	33	43	55	60	45	106	32
N.S.	1	1.49	1.35	0.77	1.00	1.28	1.40	1.05	2.47	0.74
time (sec)	N/A	0.354	0.018	0.572	0.111	0.082	0.061	0.181	0.224	9.388

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	90	71	48	65	89	78	55	188	53
N.S.	1	1.41	1.11	0.75	1.02	1.39	1.22	0.86	2.94	0.83
time (sec)	N/A	0.398	0.024	0.586	0.126	0.081	0.068	0.161	0.202	9.433

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	44	44	44	44	44	43	44
N.S.	1	1.00	1.00	0.96	0.96	0.96	0.96	0.96	0.93	0.96
time (sec)	N/A	0.322	0.002	0.563	0.028	0.068	0.025	0.176	0.232	9.642

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	34	34	34	34	33	34
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.94	0.94	0.92	0.94
time (sec)	N/A	0.307	0.002	0.560	0.031	0.076	0.021	0.157	0.218	0.024

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	24	24	26	24	23	24
N.S.	1	1.00	1.00	0.86	0.86	0.86	0.93	0.86	0.82	0.86
time (sec)	N/A	0.284	0.001	0.559	0.027	0.074	0.021	0.150	0.217	0.020

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	14	14	10	14	12	12
N.S.	1	1.00	1.00	0.86	1.00	1.00	0.71	1.00	0.86	0.86
time (sec)	N/A	0.255	0.000	0.039	0.026	0.071	0.017	0.181	0.207	0.023

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	45	34	17	27	39	39	31	25	15
N.S.	1	2.37	1.79	0.89	1.42	2.05	2.05	1.63	1.32	0.79
time (sec)	N/A	0.340	0.016	0.589	0.108	0.079	0.046	0.183	0.226	0.233

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	69	62	37	47	68	58	51	106	34
N.S.	1	1.60	1.44	0.86	1.09	1.58	1.35	1.19	2.47	0.79
time (sec)	N/A	0.353	0.018	0.583	0.109	0.079	0.064	0.161	0.215	9.150

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	95	70	56	67	98	78	61	188	52
N.S.	1	1.48	1.09	0.88	1.05	1.53	1.22	0.95	2.94	0.81
time (sec)	N/A	0.392	0.039	0.576	0.112	0.084	0.076	0.177	0.214	0.086

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	39	32	17	23	33	32	31	21	15
N.S.	1	2.29	1.88	1.00	1.35	1.94	1.88	1.82	1.24	0.88
time (sec)	N/A	0.316	0.012	0.568	0.111	0.094	0.039	0.187	0.221	9.128

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	17	17	14	13	13	12	15	13	6
N.S.	1	2.83	2.83	2.33	2.17	2.17	2.00	2.50	2.17	1.00
time (sec)	N/A	0.267	0.003	0.559	0.027	0.082	0.044	0.163	0.217	9.213

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	63	120	31	55	63	56	56	130	38
N.S.	1	1.91	3.64	0.94	1.67	1.91	1.70	1.70	3.94	1.15
time (sec)	N/A	0.395	0.086	0.835	0.028	0.087	0.140	0.161	0.207	0.171

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	63	120	31	51	59	56	54	130	38
N.S.	1	2.03	3.87	1.00	1.65	1.90	1.81	1.74	4.19	1.23
time (sec)	N/A	0.376	0.071	0.750	0.038	0.085	0.128	0.199	0.202	0.139

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	21	19	14	15	15	14	17	15	8
N.S.	1	1.11	1.00	0.74	0.79	0.79	0.74	0.89	0.79	0.42
time (sec)	N/A	0.278	0.003	0.557	0.039	0.074	0.050	0.194	0.242	0.100

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	21	19	14	15	15	14	17	15	8
N.S.	1	1.11	1.00	0.74	0.79	0.79	0.74	0.89	0.79	0.42
time (sec)	N/A	0.441	0.002	0.339	0.031	0.068	0.044	0.174	0.244	0.018

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	74	55	45	71	48	102	49	71	49
N.S.	1	1.12	0.83	0.68	1.08	0.73	1.55	0.74	1.08	0.74
time (sec)	N/A	0.320	0.155	0.602	0.028	0.079	0.374	0.137	0.214	0.150

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	48	45	35	46	38	39	39	43	33
N.S.	1	1.07	1.00	0.78	1.02	0.84	0.87	0.87	0.96	0.73
time (sec)	N/A	0.286	0.067	0.618	0.032	0.081	0.238	0.177	0.214	9.846

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	24	22	15	18	20	19	39	17	14
N.S.	1	1.14	1.05	0.71	0.86	0.95	0.90	1.86	0.81	0.67
time (sec)	N/A	0.242	0.061	0.609	0.029	0.084	0.241	0.157	0.225	9.952

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	26	38	0	17	45	15
N.S.	1	1.00	1.00	0.95	1.37	2.00	0.00	0.89	2.37	0.79
time (sec)	N/A	0.238	0.095	0.603	0.033	0.079	0.000	0.174	0.229	0.048

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	28	51	68	0	27	95	27
N.S.	1	1.00	0.95	0.65	1.19	1.58	0.00	0.63	2.21	0.63
time (sec)	N/A	0.274	0.157	0.605	0.036	0.091	0.000	0.134	0.222	0.034

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	69	51	38	76	98	0	37	143	69
N.S.	1	1.08	0.80	0.59	1.19	1.53	0.00	0.58	2.23	1.08
time (sec)	N/A	0.313	0.230	0.608	0.029	0.089	0.000	0.181	0.223	9.880

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	100	66	55	96	58	170	59	104	64
N.S.	1	1.11	0.73	0.61	1.07	0.64	1.89	0.66	1.16	0.71
time (sec)	N/A	0.369	0.205	0.635	0.032	0.086	0.514	0.168	0.239	0.187

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	56	45	71	48	88	49	76	49
N.S.	1	1.07	0.81	0.65	1.03	0.70	1.28	0.71	1.10	0.71
time (sec)	N/A	0.337	0.104	0.601	0.031	0.084	0.369	0.171	0.261	0.107

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	46	35	46	38	37	39	48	33
N.S.	1	1.00	0.96	0.73	0.96	0.79	0.77	0.81	1.00	0.69
time (sec)	N/A	0.285	0.052	0.614	0.027	0.082	0.233	0.177	0.216	0.044

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	15	18	20	17	39	22	14
N.S.	1	1.00	0.92	0.62	0.75	0.83	0.71	1.62	0.92	0.58
time (sec)	N/A	0.249	0.049	0.591	0.026	0.086	0.218	0.190	0.221	9.616

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	26	38	0	17	45	15
N.S.	1	1.00	1.00	0.86	1.24	1.81	0.00	0.81	2.14	0.71
time (sec)	N/A	0.236	0.081	0.605	0.025	0.082	0.000	0.201	0.231	0.040

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	28	51	68	0	27	95	27
N.S.	1	1.00	0.72	0.65	1.19	1.58	0.00	0.63	2.21	0.63
time (sec)	N/A	0.269	0.181	0.608	0.031	0.076	0.000	0.198	0.247	0.032

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	69	41	38	76	78	0	37	123	69
N.S.	1	1.08	0.64	0.59	1.19	1.22	0.00	0.58	1.92	1.08
time (sec)	N/A	0.310	0.270	0.614	0.033	0.081	0.000	0.200	0.237	9.187

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	95	51	48	101	128	0	47	191	67
N.S.	1	1.12	0.60	0.56	1.19	1.51	0.00	0.55	2.25	0.79
time (sec)	N/A	0.348	0.323	0.612	0.033	0.082	0.000	0.178	0.292	9.039

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	167	74	0	0	0	0	0	72	0
N.S.	1	1.86	0.82	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.395	8.290	0.000	0.000	0.000	0.000	0.000	0.366	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	141	63	0	0	0	0	0	42	0
N.S.	1	2.04	0.91	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.354	6.545	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	117	45	0	0	0	0	0	12	0
N.S.	1	2.60	1.00	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.313	6.465	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	141	54	0	0	0	0	0	44	0
N.S.	1	2.04	0.78	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.357	7.846	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	167	64	0	0	0	0	0	74	0
N.S.	1	1.86	0.71	0.00	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.378	8.701	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	323	82	0	0	0	0	0	72	0
N.S.	1	3.59	0.91	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.544	8.691	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	297	70	0	0	0	0	0	42	0
N.S.	1	4.30	1.01	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.527	7.527	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	273	55	0	0	0	0	0	12	0
N.S.	1	6.07	1.22	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.461	6.221	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	293	58	0	0	0	0	0	44	0
N.S.	1	4.51	0.89	0.00	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.507	6.923	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	319	69	0	0	0	0	0	74	0
N.S.	1	3.54	0.77	0.00	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.564	8.376	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	0	0	0	0	0	116	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	2.42	0.00
time (sec)	N/A	0.268	0.059	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	75	0	0	0	0	0	1347	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	14.80	0.00
time (sec)	N/A	0.344	0.064	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	133	131	136	131	141	138	140	124
N.S.	1	1.00	1.00	0.98	1.02	0.98	1.06	1.04	1.05	0.93
time (sec)	N/A	0.549	0.016	0.612	0.036	0.077	0.027	0.185	0.202	0.081

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	80	85	77	85	82	85	72
N.S.	1	1.00	1.00	0.99	1.05	0.95	1.05	1.01	1.05	0.89
time (sec)	N/A	0.405	0.009	0.565	0.027	0.078	0.026	0.154	0.226	0.035

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	45	40	42	42	44	41
N.S.	1	1.00	1.00	0.89	0.98	0.87	0.91	0.91	0.96	0.89
time (sec)	N/A	0.322	0.005	0.569	0.034	0.067	0.017	0.172	0.206	0.022

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	17	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.85	0.80
time (sec)	N/A	0.250	0.000	0.027	0.026	0.067	0.015	0.148	0.194	0.026

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	120	124	34	46	46
N.S.	1	1.00	1.12	1.03	0.00	3.53	3.65	1.00	1.35	1.35
time (sec)	N/A	0.279	0.010	0.722	0.000	0.085	0.098	0.134	0.209	9.413

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	341	265	76	241	119
N.S.	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	3.65	1.80
time (sec)	N/A	0.345	0.064	0.679	0.000	0.084	0.306	0.143	0.220	9.902

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	112	97	116	0	785	474	136	598	285
N.S.	1	1.11	0.96	1.15	0.00	7.77	4.69	1.35	5.92	2.82
time (sec)	N/A	0.421	0.092	0.709	0.000	0.095	0.681	0.159	0.212	9.966

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	109	113	206	273	234	233	253	334	318	184
N.S.	1	1.04	1.89	2.50	2.15	2.14	2.32	3.06	2.92	1.69
time (sec)	N/A	0.576	0.029	0.587	0.034	0.079	0.111	0.152	0.219	9.840

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	109	113	206	273	234	235	250	334	318	184
N.S.	1	1.04	1.89	2.50	2.15	2.16	2.29	3.06	2.92	1.69
time (sec)	N/A	0.569	0.036	0.582	0.036	0.077	0.113	0.174	0.202	10.045

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	109	113	198	273	234	227	253	334	318	176
N.S.	1	1.04	1.82	2.50	2.15	2.08	2.32	3.06	2.92	1.61
time (sec)	N/A	0.566	0.028	0.586	0.033	0.080	0.142	0.156	0.275	10.014

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	109	113	206	273	234	235	248	334	318	184
N.S.	1	1.04	1.89	2.50	2.15	2.16	2.28	3.06	2.92	1.69
time (sec)	N/A	0.574	0.034	0.584	0.030	0.083	0.075	0.156	0.207	9.716

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	15	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.79	0.84
time (sec)	N/A	0.251	0.007	1.417	0.107	0.073	0.049	0.179	0.195	9.567

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	21	18	17	16	16	22	16	15	16
N.S.	1	1.17	1.00	0.94	0.89	0.89	1.22	0.89	0.83	0.89
time (sec)	N/A	0.246	0.006	1.407	0.107	0.084	0.048	0.168	0.184	9.766

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	16	12	9	8	10	7	8	9	8
N.S.	1	1.33	1.00	0.75	0.67	0.83	0.58	0.67	0.75	0.67
time (sec)	N/A	0.245	0.016	1.837	0.110	0.082	0.072	0.175	0.232	0.176

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	38	50	76	40	34	23
N.S.	1	1.00	1.00	0.89	1.41	1.85	2.81	1.48	1.26	0.85
time (sec)	N/A	0.256	0.010	0.537	0.026	0.090	0.122	0.174	0.224	9.757

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	26	39	51	76	45	34	23
N.S.	1	1.00	1.07	0.96	1.44	1.89	2.81	1.67	1.26	0.85
time (sec)	N/A	0.255	0.012	0.575	0.032	0.081	0.114	0.163	0.216	9.360

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	27	41	87	27	34	23
N.S.	1	1.00	1.00	0.90	0.87	1.32	2.81	0.87	1.10	0.74
time (sec)	N/A	0.268	0.011	1.402	0.028	0.076	0.089	0.199	0.205	9.511

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	26	39	51	76	45	34	23
N.S.	1	1.00	1.07	0.96	1.44	1.89	2.81	1.67	1.26	0.85
time (sec)	N/A	0.260	0.012	0.592	0.028	0.086	0.127	0.153	0.252	9.449

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	0	113	124	34	46	46
N.S.	1	1.00	1.00	0.92	0.00	2.97	3.26	0.89	1.21	1.21
time (sec)	N/A	0.288	0.008	0.740	0.000	0.087	0.102	0.152	0.224	9.478

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	41	34	35	0	124	100	30	42	33
N.S.	1	1.17	0.97	1.00	0.00	3.54	2.86	0.86	1.20	0.94
time (sec)	N/A	0.279	0.008	0.734	0.000	0.081	0.121	0.155	0.216	9.448

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	37	41	32	49	67	102	55	40	28
N.S.	1	1.16	1.28	1.00	1.53	2.09	3.19	1.72	1.25	0.88
time (sec)	N/A	0.279	0.009	0.756	0.103	0.075	0.122	0.169	0.203	9.227

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	46	43	34	36	45	39	36	69	33
N.S.	1	1.07	1.00	0.79	0.84	1.05	0.91	0.84	1.60	0.77
time (sec)	N/A	0.283	0.019	0.731	0.107	0.075	0.065	0.159	0.262	0.041

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	68	0	334	265	67	241	119
N.S.	1	1.00	0.99	0.96	0.00	4.70	3.73	0.94	3.39	1.68
time (sec)	N/A	0.349	0.043	0.706	0.000	0.084	0.296	0.168	0.226	0.168

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	78	72	86	0	317	230	75	218	107
N.S.	1	1.08	1.00	1.19	0.00	4.40	3.19	1.04	3.03	1.49
time (sec)	N/A	0.350	0.038	0.705	0.000	0.087	0.259	0.159	0.211	0.231

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	78	83	97	171	218	90	222	100
N.S.	1	1.07	1.13	1.20	1.41	2.48	3.16	1.30	3.22	1.45
time (sec)	N/A	0.342	0.056	0.715	0.108	0.080	0.291	0.183	0.206	9.389

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	76	62	111	0	89	212	100	111	110
N.S.	1	1.23	1.00	1.79	0.00	1.44	3.42	1.61	1.79	1.77
time (sec)	N/A	0.426	0.104	2.066	0.000	0.091	0.420	0.216	0.302	9.326

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	24	17	33	27	15	165	27	36	27
N.S.	1	1.41	1.00	1.94	1.59	0.88	9.71	1.59	2.12	1.59
time (sec)	N/A	0.262	0.018	1.609	0.107	0.077	0.092	0.203	0.225	0.125

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	30	23	39	33	21	70	33	44	42
N.S.	1	1.30	1.00	1.70	1.43	0.91	3.04	1.43	1.91	1.83
time (sec)	N/A	0.274	0.032	1.344	0.108	0.082	0.325	0.181	0.219	0.207

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	50	47	43	0	190	151	46	58	42
N.S.	1	1.06	1.00	0.91	0.00	4.04	3.21	0.98	1.23	0.89
time (sec)	N/A	0.347	0.021	0.859	0.000	0.082	0.141	0.162	0.232	0.101

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	60	61	61	0	240	294	60	81	82
N.S.	1	1.05	1.07	1.07	0.00	4.21	5.16	1.05	1.42	1.44
time (sec)	N/A	0.363	0.022	0.827	0.000	0.082	0.201	0.170	0.234	0.064

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	48	29	38	40	32	40	49	39
N.S.	1	1.00	1.26	0.76	1.00	1.05	0.84	1.05	1.29	1.03
time (sec)	N/A	0.284	0.057	1.051	0.108	0.077	0.263	0.159	0.202	9.068

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	36	49	25	38	53	26	24	32	23
N.S.	1	1.20	1.63	0.83	1.27	1.77	0.87	0.80	1.07	0.77
time (sec)	N/A	0.286	0.074	0.583	0.111	0.082	0.278	0.172	0.212	9.634

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	49	40	52	40	44	41	47	39
N.S.	1	1.04	1.04	0.85	1.11	0.85	0.94	0.87	1.00	0.83
time (sec)	N/A	0.305	0.075	0.566	0.105	0.076	0.265	0.189	0.236	0.116

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	48	53	35	46	58	41	53	57	48
N.S.	1	1.07	1.18	0.78	1.02	1.29	0.91	1.18	1.27	1.07
time (sec)	N/A	0.303	0.094	0.790	0.114	0.084	0.282	0.197	0.224	9.933

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	48	61	35	46	60	42	36	43	35
N.S.	1	1.07	1.36	0.78	1.02	1.33	0.93	0.80	0.96	0.78
time (sec)	N/A	0.299	0.143	0.575	0.107	0.086	0.277	0.167	0.268	0.055

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	62	56	50	58	58	53	54	53	48
N.S.	1	1.09	0.98	0.88	1.02	1.02	0.93	0.95	0.93	0.84
time (sec)	N/A	0.303	0.146	0.549	0.108	0.091	0.255	0.163	0.294	0.101

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	56	32	41	60	37	31	38	30
N.S.	1	1.00	1.30	0.74	0.95	1.40	0.86	0.72	0.88	0.70
time (sec)	N/A	0.286	0.098	0.568	0.107	0.085	0.275	0.174	0.323	10.068

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	50	58	58	54	54	57	48
N.S.	1	1.00	1.03	0.85	0.98	0.98	0.92	0.92	0.97	0.81
time (sec)	N/A	0.304	0.192	0.595	0.108	0.086	0.271	0.223	0.225	9.864

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	46	46	86	56	0	44	36
N.S.	1	1.00	0.92	0.78	0.78	1.46	0.95	0.00	0.75	0.61
time (sec)	N/A	0.307	0.616	1.454	0.111	0.091	0.283	0.000	0.226	9.921

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	62	56	50	58	58	54	54	53	48
N.S.	1	1.09	0.98	0.88	1.02	1.02	0.95	0.95	0.93	0.84
time (sec)	N/A	0.296	0.151	0.566	0.108	0.086	0.349	0.183	0.211	0.228

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	56	32	41	60	32	31	38	30
N.S.	1	1.00	1.44	0.82	1.05	1.54	0.82	0.79	0.97	0.77
time (sec)	N/A	0.278	0.106	0.586	0.105	0.084	0.281	0.186	0.229	0.053

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	24	9	8	20	10	40	21	20
N.S.	1	1.00	1.71	0.64	0.57	1.43	0.71	2.86	1.50	1.43
time (sec)	N/A	0.235	0.046	0.603	0.104	0.081	0.279	0.197	0.239	10.237

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	14	25	7	8	33	7	24	6	6
N.S.	1	1.40	2.50	0.70	0.80	3.30	0.70	2.40	0.60	0.60
time (sec)	N/A	0.254	0.058	0.565	0.107	0.079	0.275	0.153	0.219	9.924

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	21	22	20	22	41	19	20
N.S.	1	1.00	0.96	0.84	0.88	0.80	0.88	1.64	0.76	0.80
time (sec)	N/A	0.270	0.050	0.558	0.103	0.086	0.250	0.175	0.203	9.887

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	21	27	15	16	38	20	53	29	26
N.S.	1	1.17	1.50	0.83	0.89	2.11	1.11	2.94	1.61	1.44
time (sec)	N/A	0.245	0.064	0.736	0.106	0.086	0.258	0.179	0.264	9.657

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	22	39	15	16	40	20	36	15	16
N.S.	1	1.16	2.05	0.79	0.84	2.11	1.05	1.89	0.79	0.84
time (sec)	N/A	0.244	0.076	0.632	0.114	0.087	0.254	0.199	0.215	8.839

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	30	30	28	38	32	54	25	26
N.S.	1	1.13	0.97	0.97	0.90	1.23	1.03	1.74	0.81	0.84
time (sec)	N/A	0.261	0.078	0.557	0.103	0.089	0.261	0.197	0.227	8.886

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	30	12	11	40	15	31	10	11
N.S.	1	1.00	1.76	0.71	0.65	2.35	0.88	1.82	0.59	0.65
time (sec)	N/A	0.243	0.071	0.564	0.116	0.081	0.271	0.185	0.201	8.793

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	27	30	28	37	32	54	29	26
N.S.	1	1.00	0.84	0.94	0.88	1.16	1.00	1.69	0.91	0.81
time (sec)	N/A	0.260	0.059	0.578	0.112	0.087	0.258	0.180	0.247	9.001

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	B	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	28	26	16	67	37	0	16	17
N.S.	1	1.00	0.85	0.79	0.48	2.03	1.12	0.00	0.48	0.52
time (sec)	N/A	0.257	0.138	0.727	0.121	0.093	0.276	0.000	0.216	9.046

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	30	30	28	38	32	54	25	26
N.S.	1	1.13	0.97	0.97	0.90	1.23	1.03	1.74	0.81	0.84
time (sec)	N/A	0.254	0.067	0.567	0.104	0.084	0.256	0.163	0.211	8.940

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	30	12	11	40	12	31	10	11
N.S.	1	1.00	2.31	0.92	0.85	3.08	0.92	2.38	0.77	0.85
time (sec)	N/A	0.228	0.068	0.578	0.110	0.081	0.279	0.183	0.231	9.012

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	44	51	16	137	80	60	40	40
N.S.	1	1.00	2.00	2.32	0.73	6.23	3.64	2.73	1.82	1.82
time (sec)	N/A	0.251	0.154	0.855	0.034	0.095	0.423	0.242	0.249	9.114

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	123	44	19	141	83	64	20	46
N.S.	1	1.00	5.35	1.91	0.83	6.13	3.61	2.78	0.87	2.00
time (sec)	N/A	0.251	0.245	0.874	0.110	0.089	0.470	0.190	0.226	9.092

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	123	44	19	143	78	62	19	44
N.S.	1	1.00	6.15	2.20	0.95	7.15	3.90	3.10	0.95	2.20
time (sec)	N/A	0.251	0.221	0.842	0.115	0.092	0.461	0.228	0.217	9.152

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	26	38	0	17	45	15
N.S.	1	1.00	1.00	0.95	1.37	2.00	0.00	0.89	2.37	0.79
time (sec)	N/A	0.223	0.091	0.553	0.025	0.085	0.000	0.195	0.236	0.055

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	30	41	0	17	50	17
N.S.	1	1.00	1.00	0.78	1.30	1.78	0.00	0.74	2.17	0.74
time (sec)	N/A	0.230	0.099	0.587	0.034	0.086	0.000	0.189	0.219	0.059

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	59	49	0	36	40	29
N.S.	1	1.00	1.00	0.84	1.37	1.14	0.00	0.84	0.93	0.67
time (sec)	N/A	0.261	0.166	0.568	0.033	0.094	0.000	0.204	0.222	9.058

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	184	21	0	0	0	0	0	82	0
N.S.	1	8.76	1.00	0.00	0.00	0.00	0.00	0.00	3.90	0.00
time (sec)	N/A	0.479	7.188	0.000	0.000	0.000	0.000	0.000	0.307	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	367	21	0	0	0	0	0	48	0
N.S.	1	17.48	1.00	0.00	0.00	0.00	0.00	0.00	2.29	0.00
time (sec)	N/A	0.712	6.235	0.000	0.000	0.000	0.000	0.000	0.285	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	158	21	0	0	0	0	0	48	0
N.S.	1	7.52	1.00	0.00	0.00	0.00	0.00	0.00	2.29	0.00
time (sec)	N/A	0.443	5.523	0.000	0.000	0.000	0.000	0.000	0.262	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	345	21	0	0	0	0	0	14	0
N.S.	1	16.43	1.00	0.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.678	5.126	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	134	21	0	0	0	0	0	14	0
N.S.	1	6.38	1.00	0.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.414	5.132	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	367	55	0	0	0	0	0	51	0
N.S.	1	17.48	2.62	0.00	0.00	0.00	0.00	0.00	2.43	0.00
time (sec)	N/A	0.743	5.859	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	533	95	0	0	0	0	0	14	0
N.S.	1	6.06	1.08	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.829	10.074	0.000	0.000	0.000	0.000	0.000	200.054	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	1059	87	0	0	0	0	0	78	0
N.S.	1	12.03	0.99	0.00	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	1.393	10.049	0.000	0.000	0.000	0.000	0.000	146.089	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	490	87	0	0	0	0	0	14	0
N.S.	1	5.57	0.99	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.623	10.035	0.000	0.000	0.000	0.000	0.000	200.064	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	1018	84	0	0	0	0	0	14	0
N.S.	1	11.98	0.99	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.275	10.030	0.000	0.000	0.000	0.000	0.000	0.326	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	451	84	0	0	0	0	0	14	0
N.S.	1	5.31	0.99	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.562	10.031	0.000	0.000	0.000	0.000	0.000	0.278	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	1062	102	0	0	0	0	0	51	0
N.S.	1	12.35	1.19	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	1.350	10.069	0.000	0.000	0.000	0.000	0.000	0.311	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	122	126	0	0	0	0	0	243	0
N.S.	1	1.44	1.48	0.00	0.00	0.00	0.00	0.00	2.86	0.00
time (sec)	N/A	0.380	0.104	0.000	0.000	0.000	0.000	0.000	0.275	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	40	37	0	0	0	0	0	128	0
N.S.	1	1.08	1.00	0.00	0.00	0.00	0.00	0.00	3.46	0.00
time (sec)	N/A	0.266	0.019	0.000	0.000	0.000	0.000	0.000	0.334	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	0	0	128	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	4.00	0.00
time (sec)	N/A	0.254	0.015	0.000	0.000	0.000	0.000	0.000	0.319	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	40	37	0	0	0	0	0	128	0
N.S.	1	1.08	1.00	0.00	0.00	0.00	0.00	0.00	3.46	0.00
time (sec)	N/A	0.266	0.017	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	30	21	0	0	0	0	0	128	0
N.S.	1	1.43	1.00	0.00	0.00	0.00	0.00	0.00	6.10	0.00
time (sec)	N/A	0.247	0.012	0.000	0.000	0.000	0.000	0.000	0.314	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	0	0	0	0	0	116	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	2.15	0.00
time (sec)	N/A	0.286	0.075	0.000	0.000	0.000	0.000	0.000	0.267	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	19	20	16	20	32
N.S.	1	1.00	1.00	0.94	0.89	1.06	1.11	0.89	1.11	1.78
time (sec)	N/A	0.218	0.003	0.358	0.028	0.078	0.019	0.158	0.318	9.144

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	33	26	0	0	0	0	0	124	0
N.S.	1	1.06	0.84	0.00	0.00	0.00	0.00	0.00	4.00	0.00
time (sec)	N/A	0.246	0.013	0.000	0.000	0.000	0.000	0.000	0.311	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	33	26	0	0	0	0	0	128	0
N.S.	1	1.06	0.84	0.00	0.00	0.00	0.00	0.00	4.13	0.00
time (sec)	N/A	0.253	0.014	0.000	0.000	0.000	0.000	0.000	0.330	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	37	0	0	0	0	0	128	0
N.S.	1	1.05	0.97	0.00	0.00	0.00	0.00	0.00	3.37	0.00
time (sec)	N/A	0.257	0.016	0.000	0.000	0.000	0.000	0.000	0.285	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	0	0	128	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	3.66	0.00
time (sec)	N/A	0.251	0.053	0.000	0.000	0.000	0.000	0.000	0.303	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	37	0	0	0	0	0	128	0
N.S.	1	1.05	0.97	0.00	0.00	0.00	0.00	0.00	3.37	0.00
time (sec)	N/A	0.258	0.022	0.000	0.000	0.000	0.000	0.000	0.294	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	75	68	0	80	114	61	182	0
N.S.	1	1.06	1.12	1.01	0.00	1.19	1.70	0.91	2.72	0.00
time (sec)	N/A	0.350	0.156	0.767	0.000	0.088	2.027	0.181	0.314	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	32	55	28	0	43	71	61	20	0
N.S.	1	1.14	1.96	1.00	0.00	1.54	2.54	2.18	0.71	0.00
time (sec)	N/A	0.288	0.039	0.704	0.000	0.085	0.877	0.188	0.300	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	30	30	28	38	32	54	25	26
N.S.	1	1.13	0.97	0.97	0.90	1.23	1.03	1.74	0.81	0.84
time (sec)	N/A	0.260	0.006	0.549	0.117	0.078	0.270	0.189	0.289	0.002

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	55	30	28	38	32	54	25	26
N.S.	1	1.13	1.77	0.97	0.90	1.23	1.03	1.74	0.81	0.84
time (sec)	N/A	0.282	0.053	0.559	0.109	0.078	0.481	0.135	0.311	9.018

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	76	26	0	0	0	36	0	17	0
N.S.	1	1.95	0.67	0.00	0.00	0.00	0.92	0.00	0.44	0.00
time (sec)	N/A	0.347	0.008	0.000	0.000	0.000	0.747	0.000	0.369	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	273	55	0	0	0	0	0	12	0
N.S.	1	6.07	1.22	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.492	0.007	0.000	0.000	0.000	0.000	0.000	0.349	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	49	24	0	0	0	34	0	17	0
N.S.	1	2.72	1.33	0.00	0.00	0.00	1.89	0.00	0.94	0.00
time (sec)	N/A	0.314	0.008	0.000	0.000	0.000	1.322	0.000	0.359	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	117	45	0	0	0	0	0	12	0
N.S.	1	2.60	1.00	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.348	0.005	0.000	0.000	0.000	0.000	0.000	0.341	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [74] had the largest ratio of [.5000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	18	0.111
2	A	2	2	1.00	16	0.125
3	A	2	2	1.00	18	0.111
4	A	2	2	1.00	18	0.111
5	A	2	2	1.00	14	0.143
6	A	2	2	1.00	14	0.143
7	A	2	2	1.00	14	0.143
8	A	1	1	1.00	14	0.071
9	A	2	2	1.00	14	0.143
10	A	2	2	1.00	14	0.143
11	A	2	2	1.00	14	0.143
12	A	2	2	1.00	14	0.143
13	A	2	2	1.00	14	0.143
14	A	2	2	1.00	14	0.143
15	A	2	2	1.12	18	0.111
16	A	2	2	1.13	21	0.095
17	A	2	2	1.18	21	0.095
18	A	1	1	1.00	19	0.053
19	A	2	2	1.42	21	0.095
20	A	2	2	1.31	21	0.095
21	A	2	2	1.12	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.09	21	0.095
23	A	2	2	1.09	12	0.167
24	A	2	2	1.12	12	0.167
25	A	1	1	1.00	10	0.100
26	A	2	2	1.62	12	0.167
27	A	2	2	1.32	12	0.167
28	A	2	2	1.04	12	0.167
29	A	2	2	1.09	12	0.167
30	A	2	2	1.12	12	0.167
31	A	1	1	1.00	10	0.100
32	A	2	2	1.10	12	0.167
33	A	2	2	1.05	12	0.167
34	A	2	2	1.03	12	0.167
35	A	2	2	1.00	12	0.167
36	A	2	2	1.00	12	0.167
37	A	2	2	1.00	12	0.167
38	A	1	1	1.00	10	0.100
39	A	2	2	2.00	12	0.167
40	A	3	3	1.49	12	0.250
41	A	4	4	1.41	12	0.333
42	A	2	2	1.00	12	0.167
43	A	2	2	1.00	12	0.167
44	A	2	2	1.00	12	0.167
45	A	1	1	1.00	10	0.100
46	B	2	2	2.37	12	0.167
47	A	3	3	1.60	12	0.250
48	A	4	4	1.48	12	0.333
49	B	2	2	2.29	10	0.200
50	B	2	2	2.83	10	0.200
51	A	2	2	1.91	30	0.067
52	B	2	2	2.03	31	0.065
53	A	2	2	1.11	12	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	3	3	1.11	22	0.136
55	A	5	4	1.12	12	0.333
56	A	4	3	1.07	12	0.250
57	A	3	2	1.14	12	0.167
58	A	1	1	1.00	12	0.083
59	A	2	2	1.00	12	0.167
60	A	3	3	1.08	12	0.250
61	A	6	5	1.11	12	0.417
62	A	5	4	1.07	12	0.333
63	A	4	3	1.00	12	0.250
64	A	3	2	1.00	12	0.167
65	A	1	1	1.00	12	0.083
66	A	2	2	1.00	12	0.167
67	A	3	3	1.08	12	0.250
68	A	4	4	1.12	12	0.333
69	A	5	4	1.86	12	0.333
70	B	4	3	2.04	12	0.250
71	B	3	2	2.60	12	0.167
72	B	4	3	2.04	12	0.250
73	A	5	4	1.86	12	0.333
74	B	7	6	3.59	12	0.500
75	B	6	5	4.30	12	0.417
76	B	5	4	6.07	12	0.333
77	B	6	5	4.51	12	0.417
78	B	7	6	3.54	12	0.500
79	A	1	1	1.00	10	0.100
80	A	1	1	1.00	21	0.048
81	A	2	2	1.00	12	0.167
82	A	2	2	1.00	12	0.167
83	A	2	2	1.00	12	0.167
84	A	1	1	1.00	10	0.100
85	A	3	2	1.00	12	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	3	1.00	12	0.250
87	A	5	4	1.11	12	0.333
88	A	2	2	1.04	23	0.087
89	A	2	2	1.04	23	0.087
90	A	2	2	1.04	23	0.087
91	A	2	2	1.04	23	0.087
92	A	3	2	1.00	10	0.200
93	A	3	2	1.17	12	0.167
94	A	3	2	1.33	15	0.133
95	A	3	2	1.00	12	0.167
96	A	3	2	1.00	12	0.167
97	A	3	2	1.00	12	0.167
98	A	3	2	1.00	12	0.167
99	A	3	2	1.00	12	0.167
100	A	3	2	1.17	13	0.154
101	A	3	2	1.16	14	0.143
102	A	4	3	1.07	12	0.250
103	A	4	3	1.00	12	0.250
104	A	4	3	1.08	13	0.231
105	A	4	3	1.07	14	0.214
106	A	3	2	1.23	40	0.050
107	A	3	2	1.41	14	0.143
108	A	3	2	1.30	16	0.125
109	A	4	3	1.06	15	0.200
110	A	4	3	1.05	16	0.188
111	A	4	3	1.00	14	0.214
112	A	4	3	1.20	14	0.214
113	A	4	3	1.04	14	0.214
114	A	4	3	1.07	14	0.214
115	A	4	3	1.07	14	0.214
116	A	4	3	1.09	14	0.214
117	A	4	3	1.00	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	4	3	1.00	14	0.214
119	A	4	3	1.00	14	0.214
120	A	4	3	1.09	14	0.214
121	A	4	3	1.00	14	0.214
122	A	3	2	1.00	14	0.143
123	A	3	2	1.40	14	0.143
124	A	3	2	1.00	14	0.143
125	A	3	2	1.17	14	0.143
126	A	3	2	1.16	14	0.143
127	A	3	2	1.13	14	0.143
128	A	3	2	1.00	14	0.143
129	A	3	2	1.00	14	0.143
130	A	3	2	1.00	14	0.143
131	A	3	2	1.13	14	0.143
132	A	3	2	1.00	14	0.143
133	A	3	2	1.00	27	0.074
134	A	3	2	1.00	30	0.067
135	A	3	2	1.00	28	0.071
136	A	1	1	1.00	12	0.083
137	A	1	1	1.00	14	0.071
138	A	2	2	1.00	14	0.143
139	B	6	5	8.76	14	0.357
140	B	7	6	17.48	14	0.429
141	B	5	4	7.52	14	0.286
142	B	6	5	16.43	14	0.357
143	B	4	3	6.38	14	0.214
144	B	7	6	17.48	14	0.429
145	B	5	4	6.06	14	0.286
146	B	6	5	12.03	14	0.357
147	B	4	3	5.57	14	0.214
148	B	5	4	11.98	14	0.286
149	B	3	2	5.31	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	B	6	5	12.35	14	0.357
151	A	1	1	1.44	12	0.083
152	A	3	2	1.08	12	0.167
153	A	3	2	1.00	12	0.167
154	A	3	2	1.08	12	0.167
155	A	3	2	1.43	12	0.167
156	A	1	1	1.00	10	0.100
157	A	1	1	1.00	7	0.143
158	A	3	2	1.06	12	0.167
159	A	3	2	1.06	12	0.167
160	A	3	2	1.05	12	0.167
161	A	3	2	1.00	12	0.167
162	A	3	2	1.05	12	0.167
163	A	5	4	1.06	15	0.267
164	A	4	3	1.14	15	0.200
165	A	3	2	1.13	14	0.143
166	A	4	3	1.13	13	0.231
167	A	7	6	1.95	19	0.316
168	B	6	5	6.07	15	0.333
169	B	6	5	2.72	19	0.263
170	B	4	3	2.60	15	0.200

CHAPTER 3

Listing of integrals

3.1 $\int (a^2+2abx+b^2x^2)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int (a^2+2abx+b^2x^2)^2\,dx=\frac{(a+bx)^5}{5b}$$

output
```
1/5*(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a^2+2abx+b^2x^2)^2\,dx=\frac{(a+bx)^5}{5b}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a + b*x)^5/(5*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)^2 dx$$

$$\Big\downarrow \text{1077}$$

$$\frac{\int \left(xb^2+ab\right)^4dx}{b^4}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bx)^5}{5b}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a + b*x)^5/(5*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1077
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int
[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] &&
 IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.53 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

method	result	size
default	$\frac{1}{5}b^4x^5 + a\,b^3x^4 + 2a^2b^2x^3 + 2a^3b\,x^2 + a^4x$	43
norman	$\frac{1}{5}b^4x^5 + a\,b^3x^4 + 2a^2b^2x^3 + 2a^3b\,x^2 + a^4x$	43
risch	$\frac{1}{5}b^4x^5 + a\,b^3x^4 + 2a^2b^2x^3 + 2a^3b\,x^2 + a^4x$	43
parallelrisch	$\frac{1}{5}b^4x^5 + a\,b^3x^4 + 2a^2b^2x^3 + 2a^3b\,x^2 + a^4x$	43
gosper	$\frac{x\left(b^4x^4+5a\,b^3x^3+10a^2b^2x^2+10a^3bx+5a^4\right)}{5}$	44
orering	$\frac{x\left(b^4x^4+5a\,b^3x^3+10a^2b^2x^2+10a^3bx+5a^4\right)\left(b^2x^2+2abx+a^2\right)^2}{5(bx+a)^4}$	69

input
```
int((b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^4*x^5+a*b^3*x^4+2*a^2*b^2*x^3+2*a^3*b*x^2+a^4*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \left(a^2 + 2abx + b^2x^2\right)^2 \, dx = \frac{1}{5}\,b^4x^5 + ab^3x^4 + 2\,a^2b^2x^3 + 2\,a^3bx^2 + a^4x$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \left(a^2+2abx+b^2x^2\right)^2\,dx = a^4x+2a^3bx^2+2a^2b^2x^3+ab^3x^4+\frac{b^4x^5}{5}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 + b**4*x**5/5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{1}{5}\,b^4x^5+ab^3x^4+\frac{4}{3}\,a^2b^2x^3+a^4x+\frac{2}{3}\,\left(b^2x^3+3\,abx^2\right)a^2$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/5*b^4*x^5 + a*b^3*x^4 + 4/3*a^2*b^2*x^3 + a^4*x + 2/3*(b^2*x^3 + 3*a*b*x
^2)*a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{1}{5}\,b^4x^5+ab^3x^4+2\,a^2b^2x^3+2\,a^3bx^2+a^4x$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \left(a^2+2abx+b^2x^2\right)^2\,dx=a^4\,x+2\,a^3\,b\,x^2+2\,a^2\,b^2\,x^3+a\,b^3\,x^4+\frac{b^4\,x^5}{5}$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
a^4*x + (b^4*x^5)/5 + 2*a^3*b*x^2 + a*b^3*x^4 + 2*a^2*b^2*x^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

$$\int \left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{x(b^4x^4+5a\,b^3x^3+10a^2b^2x^2+10a^3bx+5a^4)}{5}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(x*(5*a**4 + 10*a**3*b*x + 10*a**2*b**2*x**2 + 5*a*b**3*x**3 + b**4*x**4))
/5
```

3.2 $\int \left(a^2+2abx+b^2x^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \left(a^2+2abx+b^2x^2\right)\,dx=\frac{(a+bx)^3}{3b}$$

output
```
1/3*(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \left(a^2+2abx+b^2x^2\right)\,dx=a^2x+abx^2+\frac{b^2x^3}{3}$$

input
```
Integrate[a^2 + 2*a*b*x + b^2*x^2,x]
```

output
```
a^2*x + a*b*x^2 + (b^2*x^3)/3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right) \, dx$$

$$\Big\downarrow \text{1077}$$

$$\frac{\int \left(xb^2+ab\right)^2dx}{b^2}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bx)^3}{3b}$$

input
```
Int[a^2 + 2*a*b*x + b^2*x^2,x]
```

output
```
(a + b*x)^3/(3*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1077
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int
[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] &&
 IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(bx+a)^3}{3b}$	13
norman	$\frac{1}{3}b^2x^3 + ab\,x^2 + a^2x$	21
risch	$\frac{1}{3}b^2x^3 + ab\,x^2 + a^2x$	21
parallelrisch	$\frac{1}{3}b^2x^3 + ab\,x^2 + a^2x$	21
parts	$\frac{1}{3}b^2x^3 + ab\,x^2 + a^2x$	21
gosper	$\frac{x(b^2x^2+3abx+3a^2)}{3}$	22
orering	$\frac{x(b^2x^2+3abx+3a^2)(b^2x^2+2abx+a^2)}{3(bx+a)^2}$	45

input
```
int(b^2*x^2+2*a*b*x+a^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(a^2 + 2abx + b^2x^2\right)\,dx = \frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

input
```
integrate(b^2*x^2+2*a*b*x+a^2,x, algorithm="fricas")
```

output
```
1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \left(a^2+2abx+b^2x^2\right)\,dx=a^2x+abx^2+\frac{b^2x^3}{3}$$

input
```
integrate(b**2*x**2+2*a*b*x+a**2,x)
```

output
```
a**2*x + a*b*x**2 + b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{3}\,b^2x^3+abx^2+a^2x$$

input
```
integrate(b^2*x^2+2*a*b*x+a^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{3}\,b^2x^3+abx^2+a^2x$$

input
```
integrate(b^2*x^2+2*a*b*x+a^2,x, algorithm="giac")
```

output
```
1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(a^2+2abx+b^2x^2\right)\,dx=a^2\,x+a\,b\,x^2+\frac{b^2\,x^3}{3}$$

input
```
int(a^2 + b^2*x^2 + 2*a*b*x,x)
```

output
```
a^2*x + (b^2*x^3)/3 + a*b*x^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \left(a^2+2abx+b^2x^2\right)\,dx=\frac{x(b^2x^2+3abx+3a^2)}{3}$$

input
```
int(b^2*x^2+2*a*b*x+a^2,x)
```

output
```
(x*(3*a**2 + 3*a*b*x + b**2*x**2))/3
```

3.3 $\int \frac{1}{a^2+2abx+b^2x^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{1}{a^2+2abx+b^2x^2} \, dx = -\frac{1}{b(a+bx)}$$

output
```
-1/b/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+2abx+b^2x^2} \, dx = -\frac{1}{b(a+bx)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-1),x]
```

output
```
-(1/(b*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+2abx+b^2x^2}\,dx$$
$$\Big\downarrow \text{1077}$$
$$b^2\int \frac{1}{(xb^2+ab)^2}dx$$
$$\Big\downarrow \text{17}$$
$$-\frac{1}{b(a+bx)}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(-1),x]
```

output
```
-(1/(b*(a + b*x)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1077
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int
[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] &&
 IntegerQ[p]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13
parallelrisch	$-\frac{1}{b(bx+a)}$	13
orering	$-\frac{bx+a}{b(b^2x^2+2abx+a^2)}$	29

input
```
int(1/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{a^2 + 2abx + b^2x^2}\,dx = -\frac{1}{b^2x + ab}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
-1/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{a^2 + 2abx + b^2x^2} \, dx = -\frac{1}{ab + b^2x}$$

input
```
integrate(1/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
-1/(a*b + b**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{a^2 + 2abx + b^2x^2} \, dx = -\frac{1}{b^2x + ab}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-1/(b^2*x + a*b)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + 2abx + b^2x^2} \, dx = -\frac{1}{(bx + a)b}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
-1/((b*x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + 2abx + b^2x^2}\,dx = -\frac{1}{b\,(a + b\,x)}$$

input
```
int(1/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
-1/(b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + 2abx + b^2x^2}\,dx = \frac{x}{a\,(bx + a)}$$

input
```
int(1/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
x/(a*(a + b*x))
```

3.4 $\int \frac{1}{(a^2+2abx+b^2x^2)^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} \, dx = -\frac{1}{3b(a+bx)^3}$$

output
```
-1/3/b/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} \, dx = -\frac{1}{3b(a+bx)^3}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-2),x]
```

output
```
-1/3*1/(b*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx+b^2x^2)^2}dx$$

$$\Big\downarrow \text{1077}$$

$$b^4\int \frac{1}{(xb^2+ab)^4}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{3b(a+bx)^3}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(-2),x]
```

output
```
-1/3*1/(b*(a + b*x)^3)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1077
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int
[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] &&
 IntegerQ[p]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{3b(bx+a)^3}$	13
norman	$-\frac{1}{3b(bx+a)^3}$	13
risch	$-\frac{1}{3b(bx+a)^3}$	13
orering	$-\frac{bx+a}{3b(b^2x^2+2abx+a^2)^2}$	29
gosper	$-\frac{1}{3(bx+a)(b^2x^2+2abx+a^2)b}$	31
parallelrisch	$-\frac{1}{3(bx+a)(b^2x^2+2abx+a^2)b}$	31

input
```
int(1/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{1}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
-1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2}\,dx = -\frac{1}{3a^3b + 9a^2b^2x + 9ab^3x^2 + 3b^4x^3}$$

input
```
integrate(1/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
-1/(3*a**3*b + 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**4*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2}\,dx = -\frac{1}{3\,(b^4x^3 + 3\,ab^3x^2 + 3\,a^2b^2x + a^3b)}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2}\,dx = -\frac{1}{3\,(bx + a)^3b}$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
-1/3/((b*x + a)^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2}\,dx = -\frac{1}{3\,a^3\,b + 9\,a^2\,b^2\,x + 9\,a\,b^3\,x^2 + 3\,b^4\,x^3}$$

input
```
int(1/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
-1/(3*a^3*b + 3*b^4*x^3 + 9*a^2*b^2*x + 9*a*b^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2}\,dx = -\frac{1}{3b\,(b^3x^3 + 3a\,b^2x^2 + 3a^2bx + a^3)}$$

input
```
int(1/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
( - 1)/(3*b*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.5 $\int (4+12x+9x^2)^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx=\frac{1}{12}(2+3x)^3\sqrt{4+12x+9x^2}$$

output
```
1/12*(2+3*x)^3*((2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx=\frac{1}{12}(2+3x)\left((2+3x)^2\right)^{3/2}$$

input
```
Integrate[(4 + 12*x + 9*x^2)^(3/2),x]
```

output
```
((2 + 3*x)*((2 + 3*x)^2)^(3/2))/12
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(9x^2+12x+4\right)^{3/2} dx$$

$$\Big\downarrow \ 1079$$

$$\frac{\sqrt{9x^2+12x+4}\int (9x+6)^3dx}{27(3x+2)}$$

$$\Big\downarrow \ 17$$

$$\frac{1}{12}(3x+2)^3\sqrt{9x^2+12x+4}$$

input
```
Int[(4 + 12*x + 9*x^2)^(3/2),x]
```

output
```
((2 + 3*x)^3*Sqrt[4 + 12*x + 9*x^2])/12
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\left((3x+2)^{2}\right)^{\frac{3}{2}}(3x+2)}{12}$	17
risch	$\frac{(3x+2)^{3}\sqrt{(3x+2)^{2}}}{12}$	19
gosper	$\frac{x(27x^{3}+72x^{2}+72x+32)\left((3x+2)^{2}\right)^{\frac{3}{2}}}{4(3x+2)^{3}}$	35
orering	$\frac{x(27x^{3}+72x^{2}+72x+32)(9x^{2}+12x+4)^{\frac{3}{2}}}{4(3x+2)^{3}}$	38

input
```
int((9*x^2+12*x+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*((3*x+2)^2)^(3/2)*(3*x+2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx=\frac{27}{4}\,x^4+18\,x^3+18\,x^2+8\,x$$

input
```
integrate((9*x^2+12*x+4)^(3/2),x, algorithm="fricas")
```

output
```
27/4*x^4 + 18*x^3 + 18*x^2 + 8*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(17) = 34$.

Time = 0.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.20

$$\int (4+12x+9x^2)^{3/2}\,dx = 4\left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{9x^2+12x+4} + 12\left(\frac{x^2}{3}+\frac{x}{9}-\frac{2}{27}\right)\sqrt{9x^2+12x+4} + 9\sqrt{9x^2+12x+4}\left(\frac{x^3}{4}+\frac{x^2}{18}-\frac{x}{27}+\frac{2}{81}\right)$$

input
```
integrate((9*x**2+12*x+4)**(3/2),x)
```

output
```
4*(x/2 + 1/3)*sqrt(9*x**2 + 12*x + 4) + 12*(x**2/3 + x/9 - 2/27)*sqrt(9*x*
*2 + 12*x + 4) + 9*sqrt(9*x**2 + 12*x + 4)*(x**3/4 + x**2/18 - x/27 + 2/81
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int (4+12x+9x^2)^{3/2}\,dx = \frac{1}{4}\left(9\,x^2+12\,x+4\right)^{\frac{3}{2}}x + \frac{1}{6}\left(9\,x^2+12\,x+4\right)^{\frac{3}{2}}$$

input
```
integrate((9*x^2+12*x+4)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(9*x^2 + 12*x + 4)^(3/2)*x + 1/6*(9*x^2 + 12*x + 4)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.

Time = 0.55 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx = \frac{3}{4}\left(3\,x^2+4\,x\right)^2\mathrm{sgn}(3\,x+2) + 2\left(3\,x^2+4\,x\right)\mathrm{sgn}(3\,x+2) + \frac{4}{3}\,\mathrm{sgn}(3\,x+2)$$

input
```
integrate((9*x^2+12*x+4)^(3/2),x, algorithm="giac")
```

output
```
3/4*(3*x^2 + 4*x)^2*sgn(3*x + 2) + 2*(3*x^2 + 4*x)*sgn(3*x + 2) + 4/3*sgn(
3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx = \frac{(9\,x+6)\,\left(9\,x^2+12\,x+4\right)^{3/2}}{36}$$

input
```
int((12*x + 9*x^2 + 4)^(3/2),x)
```

output
```
((9*x + 6)*(12*x + 9*x^2 + 4)^(3/2))/36
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx = \frac{x(27x^3+72x^2+72x+32)}{4}$$

input
```
int((9*x^2+12*x+4)^(3/2),x)
```

output
```
(x*(27*x**3 + 72*x**2 + 72*x + 32))/4
```

3.6 $\int \sqrt{4+12x+9x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \sqrt{4+12x+9x^2}\,dx = \frac{1}{6}(2+3x)\sqrt{4+12x+9x^2}$$

output
```
1/6*(2+3*x)*((2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sqrt{4+12x+9x^2}\,dx = \frac{x\sqrt{(2+3x)^2}(4+3x)}{4+6x}$$

input
```
Integrate[Sqrt[4 + 12*x + 9*x^2],x]
```

output
```
(x*Sqrt[(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{9x^2+12x+4}\,dx$$

$$\Big\downarrow \text{1079}$$

$$\frac{\sqrt{9x^2+12x+4}\int (9x+6)dx}{3(3x+2)}$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{6}(3x+2)\sqrt{9x^2+12x+4}$$

input
```
Int[Sqrt[4 + 12*x + 9*x^2],x]
```

output
```
((2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])/6
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\operatorname{csgn}(3x+2)(3x+2)^2}{6}$	16
gosper	$\frac{x(3x+4)\sqrt{(3x+2)^2}}{6x+4}$	25
orering	$\frac{x(3x+4)\sqrt{(3x+2)^2}}{6x+4}$	25
risch	$\frac{3\sqrt{(3x+2)^2}\,x^2}{2(3x+2)}+\frac{2\sqrt{(3x+2)^2}\,x}{3x+2}$	42

input
```
int(((3*x+2)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*csgn(3*x+2)*(3*x+2)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.39

$$\int \sqrt{4+12x+9x^2}\,dx = \frac{3}{2}\,x^2+2\,x$$

input
```
integrate(((2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
3/2*x^2 + 2*x
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{4+12x+9x^2}\,dx = \left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{9x^2+12x+4}$$

input
```
integrate(((2+3*x)**2)**(1/2),x)
```

output
```
(x/2 + 1/3)*sqrt(9*x**2 + 12*x + 4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \sqrt{4+12x+9x^2}\,dx = \frac{1}{2}\sqrt{9\,x^2+12\,x+4}x+\frac{1}{3}\sqrt{9\,x^2+12\,x+4}$$

input
```
integrate(((2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(9*x^2 + 12*x + 4)*x + 1/3*sqrt(9*x^2 + 12*x + 4)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \sqrt{4+12x+9x^2}\,dx = \frac{1}{2}\left(3\,x^2+4\,x\right)\mathrm{sgn}(3\,x+2)+\frac{2}{3}\,\mathrm{sgn}(3\,x+2)$$

input
```
integrate(((2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(3*x^2 + 4*x)*sgn(3*x + 2) + 2/3*sgn(3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \sqrt{4+12x+9x^2}\,dx=\frac{|3\,x+2|\,\left(3\,x+2\right)}{6}$$

input
```
int(((3*x + 2)^2)^(1/2),x)
```

output
```
(abs(3*x + 2)*(3*x + 2))/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.35

$$\int \sqrt{4+12x+9x^2}\,dx=\frac{x(3x+4)}{2}$$

input
```
int(((2+3*x)^2)^(1/2),x)
```

output
```
(x*(3*x + 4))/2
```

3.7 $\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx=\frac{(2+3x)\log(2+3x)}{3\sqrt{4+12x+9x^2}}$$

output
```
1/3*(2+3*x)*ln(2+3*x)/((2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx=\frac{(2+3x)\log(2+3x)}{3\sqrt{(2+3x)^2}}$$

input
```
Integrate[1/Sqrt[4 + 12*x + 9*x^2],x]
```

output
```
((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[(2 + 3*x)^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{9x^2+12x+4}}dx$$
$$\Big\downarrow \text{1079}$$
$$\frac{3(3x+2)\int \frac{1}{9x+6}dx}{\sqrt{9x^2+12x+4}}$$
$$\Big\downarrow \text{16}$$
$$\frac{(3x+2)\log(3x+2)}{3\sqrt{9x^2+12x+4}}$$

input
```
Int[1/Sqrt[4 + 12*x + 9*x^2],x]
```

output
```
((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[4 + 12*x + 9*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{(3x+2)\ln(3x+2)}{3\sqrt{(3x+2)^{2}}}$	23
risch	$\frac{\sqrt{(3x+2)^{2}}\,\ln(3x+2)}{9x+6}$	25
meijerg	$\frac{x\ln\left(1+\frac{3x}{2}\right)}{\sqrt{(3x+2)^{2}}}+\frac{2\ln\left(1+\frac{3x}{2}\right)}{3\sqrt{(3x+2)^{2}}}$	36

input
```
int(1/((3*x+2)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*x+2)*ln(3*x+2)/((3*x+2)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx=\frac{1}{3}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/((2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx=\frac{\left(x+\frac{2}{3}\right)\log\left(x+\frac{2}{3}\right)}{3\sqrt{\left(x+\frac{2}{3}\right)^{2}}}$$

input
```
integrate(1/((2+3*x)**2)**(1/2),x)
```

output
```
(x + 2/3)*log(x + 2/3)/(3*sqrt((x + 2/3)**2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx = \frac{1}{3}\log\left(x+\frac{2}{3}\right)$$

input
```
integrate(1/((2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*log(x + 2/3)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx = \frac{1}{3}\log\left(|3x+2|\right)\operatorname{sgn}(3x+2)$$

input
```
integrate(1/((2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*log(abs(3*x + 2))*sgn(3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx = \frac{\ln\left(3x+2\right)\operatorname{sign}(3x+2)}{3}$$

input
```
int(1/((3*x + 2)^2)^(1/2),x)
```

output
```
(log(3*x + 2)*sign(3*x + 2))/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx = \frac{\log(3x+2)}{3}$$

input
```
int(1/((2+3*x)^2)^(1/2),x)
```

output
```
log(3*x + 2)/3
```

3.8 $\int \frac{1}{(4+12x+9x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{1}{(4+12x+9x^2)^{3/2}} \, dx = -\frac{1}{6(2+3x)\sqrt{4+12x+9x^2}}$$

output
```
-1/6/(2+3*x)/((2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(4+12x+9x^2)^{3/2}} \, dx = -\frac{2+3x}{6\left((2+3x)^2\right)^{3/2}}$$

input
```
Integrate[(4 + 12*x + 9*x^2)^(-3/2),x]
```

output
```
-1/6*(2 + 3*x)/((2 + 3*x)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(9x^2+12x+4)^{3/2}} \, dx$$

$$\Big\downarrow \, 1078$$

$$-\frac{1}{6(3x+2)\sqrt{9x^2+12x+4}}$$

input
```
Int[(4 + 12*x + 9*x^2)^(-3/2),x]
```

output
```
-1/6*1/((2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

method	result	size
meijerg	$\frac{x\left(\frac{3x}{2}+2\right)}{16\left(1+\frac{3x}{2}\right)^{2}}$	16
gosper	$-\frac{3x+2}{6\left(\left(3x+2\right)^{2}\right)^{\frac{3}{2}}}$	17
default	$-\frac{3x+2}{6\left(\left(3x+2\right)^{2}\right)^{\frac{3}{2}}}$	17
risch	$-\frac{\sqrt{\left(3x+2\right)^{2}}}{6\left(3x+2\right)^{3}}$	19
orering	$\frac{x(3x+4)(3x+2)}{8\left(9x^{2}+12x+4\right)^{\frac{3}{2}}}$	26

input
```
int(1/(9*x^2+12*x+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*x*(3/2*x+2)/(1+3/2*x)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = -\frac{1}{6\,(9\,x^2+12\,x+4)}$$

input
```
integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="fricas")
```

output
```
-1/6/(9*x^2 + 12*x + 4)
```

Sympy [F]

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = \int \frac{1}{(9x^2+12x+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(9*x**2+12*x+4)**(3/2),x)
```

output
```
Integral((9*x**2 + 12*x + 4)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.36

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = -\frac{1}{6\,(3\,x+2)^2}$$

input
```
integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="maxima")
```

output
```
-1/6/(3*x + 2)^2
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = -\frac{1}{6\,(3\,x+2)^2\mathrm{sgn}\,(3\,x+2)}$$

input
```
integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="giac")
```

output
```
-1/6/((3*x + 2)^2*sgn(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = -\frac{\sqrt{9\,x^2+12\,x+4}}{6\,(3\,x+2)^3}$$

input
```
int(1/(12*x + 9*x^2 + 4)^(3/2),x)
```

output
```
-(12*x + 9*x^2 + 4)^(1/2)/(6*(3*x + 2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = -\frac{1}{54x^2+72x+24}$$

input
```
int(1/(9*x^2+12*x+4)^(3/2),x)
```

output
```
( - 1)/(6*(9*x**2 + 12*x + 4))
```

3.9 $\int \sqrt{4-12x+9x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \sqrt{4-12x+9x^2}\,dx=-\frac{1}{6}(2-3x)\sqrt{4-12x+9x^2}$$

output
```
-1/6*(2-3*x)*((-2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sqrt{4-12x+9x^2}\,dx=\frac{\sqrt{(2-3x)^2}x(-4+3x)}{-4+6x}$$

input
```
Integrate[Sqrt[4 - 12*x + 9*x^2],x]
```

output
```
(Sqrt[(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{9x^2-12x+4}\,dx$$

$$\Big\downarrow \text{ 1079}$$

$$-\frac{\sqrt{9x^2-12x+4}\int (9x-6)dx}{3(2-3x)}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{1}{6}(2-3x)\sqrt{9x^2-12x+4}$$

input
```
Int[Sqrt[4 - 12*x + 9*x^2],x]
```

output
```
-1/6*((2 - 3*x)*Sqrt[4 - 12*x + 9*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\operatorname{csgn}(-2+3x)(-2+3x)^{2}}{6}$	16
gosper	$\frac{x(3x-4)\sqrt{(-2+3x)^{2}}}{-4+6x}$	25
orering	$\frac{x(3x-4)\sqrt{(-2+3x)^{2}}}{-4+6x}$	25
risch	$\frac{3\sqrt{(-2+3x)^{2}}\, x^{2}}{2(-2+3x)}-\frac{2\sqrt{(-2+3x)^{2}}\, x}{-2+3x}$	42

input
```
int(((-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*csgn(-2+3*x)*(-2+3*x)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.39

$$\int \sqrt{4-12x+9x^2}\, dx = \frac{3}{2}\, x^{2} - 2\, x$$

input
```
integrate(((-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
3/2*x^2 - 2*x
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{4-12x+9x^2}\,dx = \left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{9x^2-12x+4}$$

input
```
integrate(((-2+3*x)**2)**(1/2),x)
```

output
```
(x/2 - 1/3)*sqrt(9*x**2 - 12*x + 4)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \sqrt{4-12x+9x^2}\,dx = \frac{1}{2}\sqrt{9\,x^2-12\,x+4}x - \frac{1}{3}\sqrt{9\,x^2-12\,x+4}$$

input
```
integrate(((-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(9*x^2 - 12*x + 4)*x - 1/3*sqrt(9*x^2 - 12*x + 4)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \sqrt{4-12x+9x^2}\,dx = \frac{1}{2}\left(3\,x^2-4\,x\right)\mathrm{sgn}(3\,x-2) + \frac{2}{3}\,\mathrm{sgn}(3\,x-2)$$

input
```
integrate(((-2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(3*x^2 - 4*x)*sgn(3*x - 2) + 2/3*sgn(3*x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \sqrt{4-12x+9x^2}\,dx = \frac{|3\,x-2|\,\left(3\,x-2\right)}{6}$$

input
```
int(((3*x - 2)^2)^(1/2),x)
```

output
```
(abs(3*x - 2)*(3*x - 2))/6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.35

$$\int \sqrt{4-12x+9x^2}\,dx = \frac{x(3x-4)}{2}$$

input
```
int(((-2+3*x)^2)^(1/2),x)
```

output
```
(x*(3*x - 4))/2
```

3.10 $\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx=-\frac{(2-3x)\log(2-3x)}{3\sqrt{4-12x+9x^2}}$$

output
```
-1/3*(2-3*x)*ln(2-3*x)/((-2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx=-\frac{(2-3x)\log(2-3x)}{3\sqrt{(2-3x)^2}}$$

input
```
Integrate[1/Sqrt[4 - 12*x + 9*x^2],x]
```

output
```
-1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[(2 - 3*x)^2]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{9x^2-12x+4}}dx$$
$$\Big\downarrow \text{1079}$$
$$-\frac{3(2-3x)\int \frac{1}{9x-6}dx}{\sqrt{9x^2-12x+4}}$$
$$\Big\downarrow \text{16}$$
$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{9x^2-12x+4}}$$

input
```
Int[1/Sqrt[4 - 12*x + 9*x^2],x]
```

output
```
-1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[4 - 12*x + 9*x^2]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{(-2+3x)\ln(-2+3x)}{3\sqrt{(-2+3x)^{2}}}$	23
risch	$\frac{\sqrt{(-2+3x)^{2}}\,\ln(-2+3x)}{-6+9x}$	25
meijerg	$-\frac{2\ln\left(1-\frac{3x}{2}\right)}{3\sqrt{(-2+3x)^{2}}}+\frac{x\ln\left(1-\frac{3x}{2}\right)}{\sqrt{(-2+3x)^{2}}}$	36

input
```
int(1/((-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/((-2+3*x)^2)^(1/2)*(-2+3*x)*ln(-2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx = \frac{1}{3}\,\log\left(3\,x-2\right)$$

input
```
integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*log(3*x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx = \frac{\left(x-\frac{2}{3}\right)\log\left(x-\frac{2}{3}\right)}{3\sqrt{\left(x-\frac{2}{3}\right)^2}}$$

input
```
integrate(1/((-2+3*x)**2)**(1/2),x)
```

output
```
(x - 2/3)*log(x - 2/3)/(3*sqrt((x - 2/3)**2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx = \frac{1}{3}\log\left(x-\frac{2}{3}\right)$$

input
```
integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*log(x - 2/3)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx = \frac{1}{3}\log\left(|3\,x-2|\right)\operatorname{sgn}(3\,x-2)$$

input
```
integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*log(abs(3*x - 2))*sgn(3*x - 2)
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx = \frac{\ln\left(3\,x-2\right)\,\mathrm{sign}(3\,x-2)}{3}$$

input
```
int(1/((3*x - 2)^2)^(1/2),x)
```

output
```
(log(3*x - 2)*sign(3*x - 2))/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx=\frac{\log(3x-2)}{3}$$

input
```
int(1/((-2+3*x)^2)^(1/2),x)
```

output
```
log(3*x - 2)/3
```

3.11 $\int \sqrt{-4+12x-9x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \sqrt{-4+12x-9x^2}\,dx=-\frac{1}{6}(2-3x)\sqrt{-4+12x-9x^2}$$

output
```
-1/6*(2-3*x)*(-(-2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \sqrt{-4+12x-9x^2}\,dx=\frac{\sqrt{-(2-3x)^2}x(-4+3x)}{-4+6x}$$

input
```
Integrate[Sqrt[-4 + 12*x - 9*x^2],x]
```

output
```
(Sqrt[-(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-9x^2+12x-4}\,dx$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{\sqrt{-9x^2+12x-4}\int(6-9x)dx}{3(2-3x)}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{1}{6}(2-3x)\sqrt{-9x^2+12x-4}$$

input
```
Int[Sqrt[-4 + 12*x - 9*x^2],x]
```

output
```
-1/6*((2 - 3*x)*Sqrt[-4 + 12*x - 9*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

method	result	size
gosper	$\frac{x(3x-4)\sqrt{-(-2+3x)^{2}}}{-4+6x}$	27
default	$\frac{x(3x-4)\sqrt{-(-2+3x)^{2}}}{-4+6x}$	27
orering	$\frac{x(3x-4)\sqrt{-(-2+3x)^{2}}}{-4+6x}$	27
risch	$-\frac{2\sqrt{-(-2+3x)^{2}}\,x}{-2+3x}+\frac{3\sqrt{-(-2+3x)^{2}}\,x^{2}}{2(-2+3x)}$	46

input
```
int((-(-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(3*x-4)*(-(-2+3*x)^2)^(1/2)/(-2+3*x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.39

$$\int \sqrt{-4+12x-9x^2}\,dx = \frac{3}{2}i\,x^2 - 2i\,x$$

input
```
integrate((-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
3/2*I*x^2 - 2*I*x
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{-4+12x-9x^2}\,dx = \left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-9x^2+12x-4}$$

input
```
integrate((-(-2+3*x)**2)**(1/2),x)
```

output
```
(x/2 - 1/3)*sqrt(-9*x**2 + 12*x - 4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \sqrt{-4+12x-9x^2}\,dx = \frac{1}{2}\sqrt{-9\,x^2+12\,x-4}x-\frac{1}{3}\sqrt{-9\,x^2+12\,x-4}$$

input
```
integrate((-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-9*x^2 + 12*x - 4)*x - 1/3*sqrt(-9*x^2 + 12*x - 4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \sqrt{-4+12x-9x^2}\,dx = -\frac{1}{2}i\left(3\,x^2-4\,x\right)\mathrm{sgn}(-3\,x+2)-\frac{2}{3}i\,\mathrm{sgn}(-3\,x+2)$$

input
```
integrate((-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*I*(3*x^2 - 4*x)*sgn(-3*x + 2) - 2/3*I*sgn(-3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \sqrt{-4+12x-9x^2}\,dx = \frac{(3\,x-2)\,\sqrt{-{(3\,x-2)}^2}}{6}$$

input
```
int((-(3*x - 2)^2)^(1/2),x)
```

output
```
((3*x - 2)*(-(3*x - 2)^2)^(1/2))/6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.35

$$\int \sqrt{-4+12x-9x^2}\,dx = \frac{x(3x-4)}{2}$$

input
```
int((-(-2+3*x)^2)^(1/2),x)
```

output
```
(x*(3*x - 4))/2
```

3.12 $\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx=-\frac{(2-3x)\log(2-3x)}{3\sqrt{-4+12x-9x^2}}$$

output
```
-1/3*(2-3*x)*ln(2-3*x)/(-(-2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx=-\frac{(2-3x)\log(2-3x)}{3\sqrt{-(2-3x)^2}}$$

input
```
Integrate[1/Sqrt[-4 + 12*x - 9*x^2],x]
```

output
```
-1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[-(2 - 3*x)^2]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-9x^2+12x-4}}dx$$

$$\Big\downarrow \text{1079}$$

$$\frac{3(2-3x)\int \frac{1}{6-9x}dx}{\sqrt{-9x^2+12x-4}}$$

$$\Big\downarrow \text{16}$$

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-9x^2+12x-4}}$$

input
```
Int[1/Sqrt[-4 + 12*x - 9*x^2],x]
```

output
```
-1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[-4 + 12*x - 9*x^2]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.34

method	result	size
meijerg	$-\frac{i \ln\left(1-\frac{3x}{2}\right)}{3}$	10
default	$\frac{(-2+3x)\ln(-2+3x)}{3\sqrt{-(-2+3x)^2}}$	25
risch	$\frac{(-2+3x)\ln(-2+3x)}{3\sqrt{-(-2+3x)^2}}$	25

input
```
int(1/(-(-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*ln(1-3/2*x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx = -\frac{1}{3}i\,\log\left(x-\frac{2}{3}\right)$$

input
```
integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*I*log(x - 2/3)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx = \frac{\left(x-\frac{2}{3}\right)\log\left(x-\frac{2}{3}\right)}{3\sqrt{-\left(x-\frac{2}{3}\right)^2}}$$

input
```
integrate(1/(-(-2+3*x)**2)**(1/2),x)
```

output
```
(x - 2/3)*log(x - 2/3)/(3*sqrt(-(x - 2/3)**2))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx = \frac{1}{3}i\log\left(x-\frac{2}{3}\right)$$

input
```
integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*I*log(x - 2/3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx = \frac{i\log\left((-3i\,x+2i)\mathrm{sgn}(-3\,x+2)\right)}{3\,\mathrm{sgn}\left(-3\,x+2\right)}$$

input
```
integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*I*log((-3*I*x + 2*I)*sgn(-3*x + 2))/sgn(-3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx = -\frac{\ln(2-3\,x)\ \mathrm{sign}(3\,x-2)\ 1\mathrm{i}}{3}$$

input
```
int(1/(-(3*x - 2)^2)^(1/2),x)
```

output
```
-(log(2 - 3*x)*sign(3*x - 2)*1i)/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx = \frac{\log(3x-2)}{3}$$

input
```
int(1/(-(-2+3*x)^2)^(1/2),x)
```

output
```
log(3*x - 2)/3
```

3.13 $\int \sqrt{-4-12x-9x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \sqrt{-4-12x-9x^2}\,dx=\frac{1}{6}(2+3x)\sqrt{-4-12x-9x^2}$$

output
```
1/6*(2+3*x)*(-(2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \sqrt{-4-12x-9x^2}\,dx=\frac{x\sqrt{-(2+3x)^2}(4+3x)}{4+6x}$$

input
```
Integrate[Sqrt[-4 - 12*x - 9*x^2],x]
```

output
```
(x*Sqrt[-(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-9x^2-12x-4}\,dx$$

$$\Big\downarrow \text{ 1079}$$

$$-\frac{\sqrt{-9x^2-12x-4}\int(-9x-6)dx}{3(3x+2)}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{1}{6}(3x+2)\sqrt{-9x^2-12x-4}$$

input
```
Int[Sqrt[-4 - 12*x - 9*x^2],x]
```

output
```
((2 + 3*x)*Sqrt[-4 - 12*x - 9*x^2])/6
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

method	result	size
gosper	$\frac{x(3x+4)\sqrt{-(3x+2)^{2}}}{6x+4}$	27
default	$\frac{x(3x+4)\sqrt{-(3x+2)^{2}}}{6x+4}$	27
orering	$\frac{x(3x+4)\sqrt{-(3x+2)^{2}}}{6x+4}$	27
risch	$\frac{3\sqrt{-(3x+2)^{2}}\,x^{2}}{2(3x+2)}+\frac{2\sqrt{-(3x+2)^{2}}\,x}{3x+2}$	46

input
```
int((-(3*x+2)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(3*x+4)*(-(3*x+2)^2)^(1/2)/(3*x+2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.39

$$\int \sqrt{-4-12x-9x^2}\,dx = \frac{3}{2}i\,x^2 + 2i\,x$$

input
```
integrate((-(2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
3/2*I*x^2 + 2*I*x
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \sqrt{-4-12x-9x^2}\,dx = \left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{-9x^2-12x-4}$$

input
```
integrate((-(2+3*x)**2)**(1/2),x)
```

output
```
(x/2 + 1/3)*sqrt(-9*x**2 - 12*x - 4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \sqrt{-4-12x-9x^2}\,dx = \frac{1}{2}\sqrt{-9\,x^2-12\,x-4}x+\frac{1}{3}\sqrt{-9\,x^2-12\,x-4}$$

input
```
integrate((-(2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-9*x^2 - 12*x - 4)*x + 1/3*sqrt(-9*x^2 - 12*x - 4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \sqrt{-4-12x-9x^2}\,dx = -\frac{1}{2}i\left(3\,x^2+4\,x\right)\mathrm{sgn}(-3\,x-2)-\frac{2}{3}i\,\mathrm{sgn}(-3\,x-2)$$

input
```
integrate((-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*I*(3*x^2 + 4*x)*sgn(-3*x - 2) - 2/3*I*sgn(-3*x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \sqrt{-4-12x-9x^2}\,dx = \frac{(3\,x+2)\,\sqrt{-{(3\,x+2)}^2}}{6}$$

input
```
int((-(3*x + 2)^2)^(1/2),x)
```

output
```
((3*x + 2)*(-(3*x + 2)^2)^(1/2))/6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.35

$$\int \sqrt{-4-12x-9x^2}\,dx = \frac{x(3x+4)}{2}$$

input
```
int((-(2+3*x)^2)^(1/2),x)
```

output
```
(x*(3*x + 4))/2
```

3.14 $\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx=\frac{(2+3x)\log(2+3x)}{3\sqrt{-4-12x-9x^2}}$$

output
```
1/3*(2+3*x)*ln(2+3*x)/(-(2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx=\frac{(2+3x)\log(2+3x)}{3\sqrt{-(2+3x)^2}}$$

input
```
Integrate[1/Sqrt[-4 - 12*x - 9*x^2],x]
```

output
```
((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-9x^2-12x-4}}dx$$
$$\Big\downarrow \text{1079}$$
$$-\frac{3(3x+2)\int \frac{1}{-9x-6}dx}{\sqrt{-9x^2-12x-4}}$$
$$\Big\downarrow \text{16}$$
$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-9x^2-12x-4}}$$

input
```
Int[1/Sqrt[-4 - 12*x - 9*x^2],x]
```

output
```
((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-4 - 12*x - 9*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.34

method	result	size
meijerg	$-\frac{i \ln\left(1+\frac{3x}{2}\right)}{3}$	10
default	$\frac{(3x+2)\ln(3x+2)}{3\sqrt{-(3x+2)^2}}$	25
risch	$\frac{(3x+2)\ln(3x+2)}{3\sqrt{-(3x+2)^2}}$	25

input
```
int(1/(-(3*x+2)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*ln(1+3/2*x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx = -\frac{1}{3}i\,\log\left(x+\frac{2}{3}\right)$$

input
```
integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*I*log(x + 2/3)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx=\frac{\left(x+\frac{2}{3}\right)\log\left(x+\frac{2}{3}\right)}{3\sqrt{-\left(x+\frac{2}{3}\right)^2}}$$

input
```
integrate(1/(-(2+3*x)**2)**(1/2),x)
```

output
```
(x + 2/3)*log(x + 2/3)/(3*sqrt(-(x + 2/3)**2))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx=\frac{1}{3}i\log\left(x+\frac{2}{3}\right)$$

input
```
integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*I*log(x + 2/3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx=\frac{i\log\left((-3i\,x-2i)\mathrm{sgn}(-3\,x-2)\right)}{3\,\mathrm{sgn}\left(-3\,x-2\right)}$$

input
```
integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*I*log((-3*I*x - 2*I)*sgn(-3*x - 2))/sgn(-3*x - 2)
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx = -\frac{\ln\left(-3\,x-2\right)\,\mathrm{sign}(3\,x+2)\,\,1\mathrm{i}}{3}$$

input
```
int(1/(-(3*x + 2)^2)^(1/2),x)
```

output
```
-(log(- 3*x - 2)*sign(3*x + 2)*1i)/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx = \frac{\log(3x+2)}{3}$$

input
```
int(1/(-(2+3*x)^2)^(1/2),x)
```

output
```
log(3*x + 2)/3
```

3.15 $\int (a^2+2abx+b^2x^2)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 34

$$\int (a^2+2abx+b^2x^2)^p \, dx = \frac{(a+bx)(a^2+2abx+b^2x^2)^p}{b(1+2p)}$$

output
```
(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p/b/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int (a^2+2abx+b^2x^2)^p \, dx = \frac{(a+bx)((a+bx)^2)^p}{b(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output
```
((a + b*x)*((a + b*x)^2)^p)/(b*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{1079}$$

$$\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\int \left(xb^2+ab\right)^{2p}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(ab+b^2x\right)\left(a^2+2abx+b^2x^2\right)^p}{b^2(2p+1)}$$

input
```
Int[(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output
```
((a*b + b^2*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^2*(1 + 2*p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{(bx+a)\left((bx+a)^2\right)^p}{b(2p+1)}$	26
gosper	$\frac{(bx+a)(b^2x^2+2abx+a^2)^p}{b(2p+1)}$	35
orering	$\frac{(bx+a)(b^2x^2+2abx+a^2)^p}{b(2p+1)}$	35
parallelrisch	$\frac{x(b^2x^2+2abx+a^2)^p ab+(b^2x^2+2abx+a^2)^p a^2}{(2p+1)ab}$	60
norman	$\frac{x\,e^{p\ln\left(b^2x^2+2abx+a^2\right)}}{2p+1}+\frac{a\,e^{p\ln\left(b^2x^2+2abx+a^2\right)}}{b(2p+1)}$	63

input
```
int((b^2*x^2+2*a*b*x+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)/b/(2*p+1)*((b*x+a)^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int (a^2+2abx+b^2x^2)^p \, dx = \frac{(bx+a)(b^2x^2+2\,abx+a^2)^p}{2\,bp+b}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

output
```
(b*x + a)*(b^2*x^2 + 2*a*b*x + a^2)^p/(2*b*p + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(29) = 58$.

Time = 0.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.94

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx = \begin{cases} \frac{x}{\sqrt{a^2}} & \text{for } b=0 \wedge p=-\frac{1}{2} \\ x(a^2)^p & \text{for } b=0 \\ \frac{\left(\frac{a}{b}+x\right)\log\left(\frac{a}{b}+x\right)}{\sqrt{b^2\left(\frac{a}{b}+x\right)^2}} & \text{for } p=-\frac{1}{2} \\ \frac{a\left(a^2+2abx+b^2x^2\right)^p}{2bp+b}+\frac{bx\left(a^2+2abx+b^2x^2\right)^p}{2bp+b} & \text{otherwise} \end{cases}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2)**p,x)
```

output
```
Piecewise((x/sqrt(a**2), Eq(b, 0) & Eq(p, -1/2)), (x*(a**2)**p, Eq(b, 0)),
 ((a/b + x)*log(a/b + x)/sqrt(b**2*(a/b + x)**2), Eq(p, -1/2)), (a*(a**2 +
 2*a*b*x + b**2*x**2)**p/(2*b*p + b) + b*x*(a**2 + 2*a*b*x + b**2*x**2)**p
/(2*b*p + b), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx = \frac{(bx+a)(bx+a)^{2p}}{b(2p+1)}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

output
```
(b*x + a)*(b*x + a)^(2*p)/(b*(2*p + 1))
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx=\frac{(b^2x^2+2\,abx+a^2)^pbx+(b^2x^2+2\,abx+a^2)^pa}{2\,bp+b}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

output
```
((b^2*x^2 + 2*a*b*x + a^2)^p*b*x + (b^2*x^2 + 2*a*b*x + a^2)^p*a)/(2*b*p +
 b)
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx=\left(\frac{x}{2\,p+1}+\frac{a}{b\,\left(2\,p+1\right)}\right)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^p$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x)^p,x)
```

output
```
(x/(2*p + 1) + a/(b*(2*p + 1)))*(a^2 + b^2*x^2 + 2*a*b*x)^p
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx=\frac{(b^2x^2+2abx+a^2)^p\,(bx+a)}{b\,(2p+1)}$$

input
```
int((b^2*x^2+2*a*b*x+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x + b**2*x**2)**p*(a + b*x))/(b*(2*p + 1))
```

3.16 $\int (ac+(bc+ad)x+bdx^2)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int (ac+(bc+ad)x+bdx^2)^3\,dx = -\frac{(bc-ad)^3(c+dx)^4}{4d^4}+\frac{3b(bc-ad)^2(c+dx)^5}{5d^4} - \frac{b^2(bc-ad)(c+dx)^6}{2d^4}+\frac{b^3(c+dx)^7}{7d^4}$$

output
```
-1/4*(-a*d+b*c)^3*(d*x+c)^4/d^4+3/5*b*(-a*d+b*c)^2*(d*x+c)^5/d^4-1/2*b^2*(
-a*d+b*c)*(d*x+c)^6/d^4+1/7*b^3*(d*x+c)^7/d^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.75

$$\int (ac+(bc+ad)x+bdx^2)^3\,dx = a^3c^3x+\frac{3}{2}a^2c^2(bc+ad)x^2+ac\left(b^2c^2+3abcd+a^2d^2\right)x^3 + \frac{1}{4}\left(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3\right)x^4 + \frac{3}{5}bd\left(b^2c^2+3abcd+a^2d^2\right)x^5 + \frac{1}{2}b^2d^2(bc+ad)x^6+\frac{1}{7}b^3d^3x^7$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
a^3*c^3*x + (3*a^2*c^2*(b*c + a*d)*x^2)/2 + a*c*(b^2*c^2 + 3*a*b*c*d + a^2
*d^2)*x^3 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^4)/4 +
(3*b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5)/5 + (b^2*d^2*(b*c + a*d)*x^6)/
2 + (b^3*d^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x(ad+bc)+ac+bdx^2\right)^3 dx$$

$$\Big\downarrow \text{1084}$$

$$\frac{\int \left((bc+bdx)^6-3b^5(bc-ad)(c+dx)^5+3b^4(bc-ad)^2(c+dx)^4-b^3(bc-ad)^3(c+dx)^3\right)dx}{b^3d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^5(c+dx)^6(bc-ad)}{2d}+\frac{3b^4(c+dx)^5(bc-ad)^2}{5d}-\frac{b^3(c+dx)^4(bc-ad)^3}{4d}+\frac{b^6(c+dx)^7}{7d}}{b^3d^3}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
(-1/4*(b^3*(b*c - a*d)^3*(c + d*x)^4)/d + (3*b^4*(b*c - a*d)^2*(c + d*x)^5
)/(5*d) - (b^5*(b*c - a*d)*(c + d*x)^6)/(2*d) + (b^6*(c + d*x)^7)/(7*d))/(
b^3*d^3)
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(84) = 168$.

Time = 0.60 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.87

method	result
norman	$\frac{b^3d^3x^7}{7} + \left(\frac{1}{2}ab^2d^3 + \frac{1}{2}b^3cd^2\right)x^6 + \left(\frac{3}{5}a^2bd^3 + \frac{9}{5}ab^2cd^2 + \frac{3}{5}b^3c^2d\right)x^5 + (\frac{1}{4}a^3d^3 + \frac{9}{4}a^2bcd^2 +$
risch	$\frac{1}{7}b^3d^3x^7 + \frac{1}{2}ab^2d^3x^6 + \frac{1}{2}b^3cd^2x^6 + \frac{3}{5}a^2bd^3x^5 + \frac{9}{5}ab^2cd^2x^5 + \frac{3}{5}b^3c^2dx^5 + \frac{1}{4}x^4a^3d^3 + \frac{9}{4}a^2bc$
parallelrisch	$\frac{1}{7}b^3d^3x^7 + \frac{1}{2}ab^2d^3x^6 + \frac{1}{2}b^3cd^2x^6 + \frac{3}{5}a^2bd^3x^5 + \frac{9}{5}ab^2cd^2x^5 + \frac{3}{5}b^3c^2dx^5 + \frac{1}{4}x^4a^3d^3 + \frac{9}{4}a^2bc$
gosper	$\frac{x(20b^3d^3x^6+70ab^2d^3x^5+70b^3cd^2x^5+84a^2bd^3x^4+252ab^2cd^2x^4+84b^3c^2dx^4+35a^3d^3x^3+315a^2bcd^2x^3+315ab^2c^2dx^3+35}{140}$
default	$\frac{b^3d^3x^7}{7} + \frac{(ad+bc)b^2d^2x^6}{2} + \frac{\left(ab^2cd^2+2(ad+bc)^2bd+bd\left(2abcd+(ad+bc)^2\right)\right)x^5}{5} + \frac{(4ac(ad+bc)bd+(ad+bc)(2abcd+(a}{4}$
orering	$\frac{x(20b^3d^3x^6+70ab^2d^3x^5+70b^3cd^2x^5+84a^2bd^3x^4+252ab^2cd^2x^4+84b^3c^2dx^4+35a^3d^3x^3+315a^2bcd^2x^3+315ab^2c^2dx^3+35}{140(bx+a)^3(dx+}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^3*d^3*x^7+(1/2*a*b^2*d^3+1/2*b^3*c*d^2)*x^6+(3/5*a^2*b*d^3+9/5*a*b^2
*c*d^2+3/5*b^3*c^2*d)*x^5+(1/4*a^3*d^3+9/4*a^2*b*c*d^2+9/4*a*b^2*c^2*d+1/4
*b^3*c^3)*x^4+(a^3*c*d^2+3*a^2*b*c^2*d+a*b^2*c^3)*x^3+(3/2*a^3*c^2*d+3/2*a
^2*b*c^3)*x^2+a^3*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.82

$$\int \left(ac+(bc+ad)x+bdx^2\right)^3\,dx=\frac{1}{7}\,b^3d^3x^7+a^3c^3x+\frac{1}{2}\left(b^3cd^2+ab^2d^3\right)x^6+\frac{3}{5}\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)x^5+\frac{1}{4}\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)x^4+\left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right)x^3+\frac{3}{2}\left(a^2bc^3+a^3c^2d\right)x^2$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
1/7*b^3*d^3*x^7 + a^3*c^3*x + 1/2*(b^3*c*d^2 + a*b^2*d^3)*x^6 + 3/5*(b^3*c
^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^5 + 1/4*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*x^4 + (a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x^3 +
3/2*(a^2*b*c^3 + a^3*c^2*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(80) = 160$.

Time = 0.03 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.07

$$\int \left(ac+(bc+ad)x+bdx^2\right)^3\,dx=a^3c^3x+\frac{b^3d^3x^7}{7}+x^6\left(\frac{ab^2d^3}{2}+\frac{b^3cd^2}{2}\right)+x^5\cdot\left(\frac{3a^2bd^3}{5}+\frac{9ab^2cd^2}{5}+\frac{3b^3c^2d}{5}\right)+x^4\left(\frac{a^3d^3}{4}+\frac{9a^2bcd^2}{4}+\frac{9ab^2c^2d}{4}+\frac{b^3c^3}{4}\right)+x^3\left(a^3cd^2+3a^2bc^2d+ab^2c^3\right)+x^2\cdot\left(\frac{3a^3c^2d}{2}+\frac{3a^2bc^3}{2}\right)$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
a**3*c**3*x + b**3*d**3*x**7/7 + x**6*(a*b**2*d**3/2 + b**3*c*d**2/2) + x*
*5*(3*a**2*b*d**3/5 + 9*a*b**2*c*d**2/5 + 3*b**3*c**2*d/5) + x**4*(a**3*d*
*3/4 + 9*a**2*b*c*d**2/4 + 9*a*b**2*c**2*d/4 + b**3*c**3/4) + x**3*(a**3*c
*d**2 + 3*a**2*b*c**2*d + a*b**2*c**3) + x**2*(3*a**3*c**2*d/2 + 3*a**2*b*
c**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.52

$$\int (ac + (bc + ad)x + bdx^2)^3 \, dx = \frac{1}{7} b^3 d^3 x^7 + \frac{1}{2} (bc + ad) b^2 d^2 x^6 + \frac{3}{5} (bc + ad)^2 b d x^5 + a^3 c^3 x + \frac{1}{4} (bc + ad)^3 x^4 + \frac{1}{2} \left(2 \, b d x^3 + 3 \, (bc + ad) x^2\right) a^2 c^2 + \frac{1}{10} \left(6 \, b^2 d^2 x^5 + 15 \, (bc + ad) b d x^4 + 10 \, (bc + ad)^2 x^3\right) ac$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
1/7*b^3*d^3*x^7 + 1/2*(b*c + a*d)*b^2*d^2*x^6 + 3/5*(b*c + a*d)^2*b*d*x^5
+ a^3*c^3*x + 1/4*(b*c + a*d)^3*x^4 + 1/2*(2*b*d*x^3 + 3*(b*c + a*d)*x^2)*
a^2*c^2 + 1/10*(6*b^2*d^2*x^5 + 15*(b*c + a*d)*b*d*x^4 + 10*(b*c + a*d)^2*
x^3)*a*c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(84) = 168.

Time = 0.19 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.04

$$\int (ac + (bc + ad)x + bdx^2)^3 \, dx = \frac{1}{7} b^3 d^3 x^7 + \frac{1}{2} b^3 c d^2 x^6 + \frac{1}{2} a b^2 d^3 x^6 + \frac{3}{5} b^3 c^2 d x^5 + \frac{9}{5} a b^2 c d^2 x^5 + \frac{3}{5} a^2 b d^3 x^5 + \frac{1}{4} b^3 c^3 x^4 + \frac{9}{4} a b^2 c^2 d x^4 + \frac{9}{4} a^2 b c d^2 x^4 + \frac{1}{4} a^3 d^3 x^4 + a b^2 c^3 x^3 + 3 \, a^2 b c^2 d x^3 + a^3 c d^2 x^3 + \frac{3}{2} a^2 b c^3 x^2 + \frac{3}{2} a^3 c^2 d x^2 + a^3 c^3 x$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
1/7*b^3*d^3*x^7 + 1/2*b^3*c*d^2*x^6 + 1/2*a*b^2*d^3*x^6 + 3/5*b^3*c^2*d*x^
5 + 9/5*a*b^2*c*d^2*x^5 + 3/5*a^2*b*d^3*x^5 + 1/4*b^3*c^3*x^4 + 9/4*a*b^2*
c^2*d*x^4 + 9/4*a^2*b*c*d^2*x^4 + 1/4*a^3*d^3*x^4 + a*b^2*c^3*x^3 + 3*a^2*
b*c^2*d*x^3 + a^3*c*d^2*x^3 + 3/2*a^2*b*c^3*x^2 + 3/2*a^3*c^2*d*x^2 + a^3*
c^3*x
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.65

$$\int \left(ac+(bc+ad)x+bdx^2\right)^3\,dx = x^4\left(\frac{a^3\,d^3}{4}+\frac{9\,a^2\,b\,c\,d^2}{4}+\frac{9\,a\,b^2\,c^2\,d}{4}+\frac{b^3\,c^3}{4}\right)+a^3\,c^3\,x+\frac{b^3\,d^3\,x^7}{7}+a\,c\,x^3\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)+\frac{3\,b\,d\,x^5\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{5}+\frac{3\,a^2\,c^2\,x^2\,\left(a\,d+b\,c\right)}{2}+\frac{b^2\,d^2\,x^6\,\left(a\,d+b\,c\right)}{2}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
x^4*((a^3*d^3)/4 + (b^3*c^3)/4 + (9*a*b^2*c^2*d)/4 + (9*a^2*b*c*d^2)/4) +
a^3*c^3*x + (b^3*d^3*x^7)/7 + a*c*x^3*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d) + (3
*b*d*x^5*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/5 + (3*a^2*c^2*x^2*(a*d + b*c))/
2 + (b^2*d^2*x^6*(a*d + b*c))/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.05

$$\int \left(ac+(bc+ad)x+bdx^2\right)^3\,dx$$
$$= \frac{x(20b^3d^3x^6+70a\,b^2d^3x^5+70b^3c\,d^2x^5+84a^2b\,d^3x^4+252a\,b^2c\,d^2x^4+84b^3c^2d\,x^4+35a^3d^3x^3+315a^2bc\,d}{}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output
```
(x*(140*a**3*c**3 + 210*a**3*c**2*d*x + 140*a**3*c*d**2*x**2 + 35*a**3*d**
3*x**3 + 210*a**2*b*c**3*x + 420*a**2*b*c**2*d*x**2 + 315*a**2*b*c*d**2*x*
*3 + 84*a**2*b*d**3*x**4 + 140*a*b**2*c**3*x**2 + 315*a*b**2*c**2*d*x**3 +
 252*a*b**2*c*d**2*x**4 + 70*a*b**2*d**3*x**5 + 35*b**3*c**3*x**3 + 84*b**
3*c**2*d*x**4 + 70*b**3*c*d**2*x**5 + 20*b**3*d**3*x**6))/140
```

3.17 $\int (ac+(bc+ad)x+bdx^2)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int (ac+(bc+ad)x+bdx^2)^2 \, dx = \frac{(bc-ad)^2(c+dx)^3}{3d^3} - \frac{b(bc-ad)(c+dx)^4}{2d^3} + \frac{b^2(c+dx)^5}{5d^3}$$

output
```
1/3*(-a*d+b*c)^2*(d*x+c)^3/d^3-1/2*b*(-a*d+b*c)*(d*x+c)^4/d^3+1/5*b^2*(d*x
+c)^5/d^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int (ac+(bc+ad)x+bdx^2)^2 \, dx = a^2c^2x + ac(bc+ad)x^2 + \frac{1}{3}(b^2c^2+4abcd+a^2d^2)x^3 + \frac{1}{2}bd(bc+ad)x^4 + \frac{1}{5}b^2d^2x^5$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
a^2*c^2*x + a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3)/3
+ (b*d*(b*c + a*d)*x^4)/2 + (b^2*d^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x(ad+bc)+ac+bdx^2\right)^2 dx$$

$$\Big\downarrow \text{1084}$$

$$\frac{\int \left((bc+bdx)^4-2b^3(bc-ad)(c+dx)^3+b^2(bc-ad)^2(c+dx)^2\right)dx}{b^2d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^3(c+dx)^4(bc-ad)}{2d}+\frac{b^2(c+dx)^3(bc-ad)^2}{3d}+\frac{b^4(c+dx)^5}{5d}}{b^2d^2}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
((b^2*(b*c - a*d)^2*(c + d*x)^3)/(3*d) - (b^3*(b*c - a*d)*(c + d*x)^4)/(2*
d) + (b^4*(c + d*x)^5)/(5*d))/(b^2*d^2)
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

method	result
default	$\frac{b^2d^2x^5}{5}+\frac{(ad+bc)bd\,x^4}{2}+\frac{\left(2abcd+(ad+bc)^2\right)x^3}{3}+ac(ad+bc)\,x^2+a^2c^2x$
norman	$\frac{b^2d^2x^5}{5}+\left(\frac{1}{2}ab\,d^2+\frac{1}{2}cd\,b^2\right)x^4+\left(\frac{1}{3}a^2d^2+\frac{4}{3}abcd+\frac{1}{3}b^2c^2\right)x^3+\left(a^2cd+ab\,c^2\right)x^2+a^2c^2x$
risch	$\frac{1}{5}b^2d^2x^5+\frac{1}{2}ab\,d^2x^4+\frac{1}{2}b^2cd\,x^4+\frac{1}{3}a^2d^2x^3+\frac{4}{3}abcd\,x^3+\frac{1}{3}b^2c^2x^3+a^2cd\,x^2+ab\,c^2x^2+a^2c^2$
parallelrisch	$\frac{1}{5}b^2d^2x^5+\frac{1}{2}ab\,d^2x^4+\frac{1}{2}b^2cd\,x^4+\frac{1}{3}a^2d^2x^3+\frac{4}{3}abcd\,x^3+\frac{1}{3}b^2c^2x^3+a^2cd\,x^2+ab\,c^2x^2+a^2c^2$
gosper	$\frac{x\left(6b^2d^2x^4+15d^2x^3ab+15b^2cd\,x^3+10a^2d^2x^2+40abcd\,x^2+10b^2c^2x^2+30a^2cdx+30ab\,c^2x+30a^2c^2\right)}{30}$
orering	$\frac{x\left(6b^2d^2x^4+15d^2x^3ab+15b^2cd\,x^3+10a^2d^2x^2+40abcd\,x^2+10b^2c^2x^2+30a^2cdx+30ab\,c^2x+30a^2c^2\right)\left(ac+(ad+bc)x+bd\,x^2\right)^2}{30(dx+c)^2(bx+a)^2}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^2*d^2*x^5+1/2*(a*d+b*c)*b*d*x^4+1/3*(2*a*b*c*d+(a*d+b*c)^2)*x^3+a*c*
(a*d+b*c)*x^2+a^2*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int \left(ac+(bc+ad)x+bdx^2\right)^2\,dx=\frac{1}{5}\,b^2d^2x^5+a^2c^2x+\frac{1}{2}\left(b^2cd+abd^2\right)x^4+\frac{1}{3}\left(b^2c^2+4\,abcd+a^2d^2\right)x^3+\left(abc^2+a^2cd\right)x^2$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
1/5*b^2*d^2*x^5 + a^2*c^2*x + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/3*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^3 + (a*b*c^2 + a^2*c*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int \left(ac+(bc+ad)x+bdx^2\right)^2\,dx = a^2c^2x+\frac{b^2d^2x^5}{5}+x^4\left(\frac{abd^2}{2}+\frac{b^2cd}{2}\right)+x^3\left(\frac{a^2d^2}{3}+\frac{4abcd}{3}+\frac{b^2c^2}{3}\right)+x^2\left(a^2cd+abc^2\right)$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
a**2*c**2*x + b**2*d**2*x**5/5 + x**4*(a*b*d**2/2 + b**2*c*d/2) + x**3*(a*
*2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3) + x**2*(a**2*c*d + a*b*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \left(ac+(bc+ad)x+bdx^2\right)^2\,dx = \frac{1}{5}\,b^2d^2x^5+\frac{1}{2}\,(bc+ad)bdx^4+a^2c^2x+\frac{1}{3}\,(bc+ad)^2x^3+\frac{1}{3}\,\left(2\,bdx^3+3\,(bc+ad)x^2\right)ac$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
1/5*b^2*d^2*x^5 + 1/2*(b*c + a*d)*b*d*x^4 + a^2*c^2*x + 1/3*(b*c + a*d)^2*
x^3 + 1/3*(2*b*d*x^3 + 3*(b*c + a*d)*x^2)*a*c
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int \left(ac+(bc+ad)x+bdx^2\right)^2\,dx = \frac{1}{5}\,b^2d^2x^5+\frac{1}{2}\,b^2cdx^4+\frac{1}{2}\,abd^2x^4+\frac{1}{3}\,b^2c^2x^3+\frac{4}{3}\,abcdx^3+\frac{1}{3}\,a^2d^2x^3+abc^2x^2+a^2cdx^2+a^2c^2x$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
1/5*b^2*d^2*x^5 + 1/2*b^2*c*d*x^4 + 1/2*a*b*d^2*x^4 + 1/3*b^2*c^2*x^3 + 4/
3*a*b*c*d*x^3 + 1/3*a^2*d^2*x^3 + a*b*c^2*x^2 + a^2*c*d*x^2 + a^2*c^2*x
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int \left(ac+(bc+ad)x+bdx^2\right)^2\,dx = x^3\left(\frac{a^2\,d^2}{3}+\frac{4\,a\,b\,c\,d}{3}+\frac{b^2\,c^2}{3}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^5}{5}+a\,c\,x^2\,(a\,d+b\,c)+\frac{b\,d\,x^4\,(a\,d+b\,c)}{2}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^2,x)
```

output
```
x^3*((a^2*d^2)/3 + (b^2*c^2)/3 + (4*a*b*c*d)/3) + a^2*c^2*x + (b^2*d^2*x^5
)/5 + a*c*x^2*(a*d + b*c) + (b*d*x^4*(a*d + b*c))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.38

$$\int \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx$$
$$= \frac{x(6b^2d^2x^4+15ab\,d^2x^3+15b^2cd\,x^3+10a^2d^2x^2+40abcd\,x^2+10b^2c^2x^2+30a^2cdx+30ab\,c^2x+30a^2c^2)}{30}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
(x*(30*a**2*c**2 + 30*a**2*c*d*x + 10*a**2*d**2*x**2 + 30*a*b*c**2*x + 40*
a*b*c*d*x**2 + 15*a*b*d**2*x**3 + 10*b**2*c**2*x**2 + 15*b**2*c*d*x**3 + 6
*b**2*d**2*x**4))/30
```

3.18 $\int \left(ac+(bc+ad)x+bdx^2\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \left(ac+(bc+ad)x+bdx^2\right)\,dx=acx+\frac{1}{2}(bc+ad)x^2+\frac{1}{3}bdx^3$$

output
```
a*c*x+1/2*(a*d+b*c)*x^2+1/3*b*d*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \left(ac+(bc+ad)x+bdx^2\right)\,dx=acx+\frac{1}{2}bcx^2+\frac{1}{2}adx^2+\frac{1}{3}bdx^3$$

input
```
Integrate[a*c + (b*c + a*d)*x + b*d*x^2,x]
```

output
```
a*c*x + (b*c*x^2)/2 + (a*d*x^2)/2 + (b*d*x^3)/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x(ad+bc)+ac+bdx^2\right)\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}x^2(ad+bc)+acx+\frac{1}{3}bdx^3$$

input
```
Int[a*c + (b*c + a*d)*x + b*d*x^2,x]
```

output
```
a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^2}{2}+\frac{bd\,x^3}{3}$	25
gosper	$\frac{x(2bd\,x^2+3adx+3cbx+6ac)}{6}$	26
norman	$\frac{bd\,x^3}{3}+\left(\frac{ad}{2}+\frac{bc}{2}\right)x^2+acx$	26
risch	$acx+\frac{1}{2}ad\,x^2+\frac{1}{2}bc\,x^2+\frac{1}{3}bd\,x^3$	27
parallelrisch	$acx+\frac{1}{2}ad\,x^2+\frac{1}{2}bc\,x^2+\frac{1}{3}bd\,x^3$	27
parts	$acx+\frac{1}{2}ad\,x^2+\frac{1}{2}bc\,x^2+\frac{1}{3}bd\,x^3$	27
orering	$\frac{x(2bd\,x^2+3adx+3cbx+6ac)(ac+(ad+bc)x+bd\,x^2)}{6(bx+a)(dx+c)}$	59

input
```
int(a*c+(a*d+b*c)*x+b*d*x^2,x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/2*(a*d+b*c)*x^2+1/3*b*d*x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(ac+(bc+ad)x+bdx^2\right)\,dx=\frac{1}{3}\,bdx^3+acx+\frac{1}{2}\,(bc+ad)x^2$$

input
```
integrate(a*c+(a*d+b*c)*x+b*d*x^2,x, algorithm="fricas")
```

output
```
1/3*b*d*x^3 + a*c*x + 1/2*(b*c + a*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(ac+(bc+ad)x+bdx^2\right)\,dx=acx+\frac{bdx^3}{3}+x^2\left(\frac{ad}{2}+\frac{bc}{2}\right)$$

input
```
integrate(a*c+(a*d+b*c)*x+b*d*x**2,x)
```

output
```
a*c*x + b*d*x**3/3 + x**2*(a*d/2 + b*c/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(ac+(bc+ad)x+bdx^2\right)\,dx=\frac{1}{3}\,bdx^3+acx+\frac{1}{2}\,(bc+ad)x^2$$

input
```
integrate(a*c+(a*d+b*c)*x+b*d*x^2,x, algorithm="maxima")
```

output
```
1/3*b*d*x^3 + a*c*x + 1/2*(b*c + a*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(ac+(bc+ad)x+bdx^2\right)\,dx=\frac{1}{3}\,bdx^3+acx+\frac{1}{2}\,(bc+ad)x^2$$

input
```
integrate(a*c+(a*d+b*c)*x+b*d*x^2,x, algorithm="giac")
```

output
```
1/3*b*d*x^3 + a*c*x + 1/2*(b*c + a*d)*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \left(ac+(bc+ad)x+bdx^2\right)\,dx=\frac{b\,d\,x^3}{3}+\left(\frac{a\,d}{2}+\frac{b\,c}{2}\right)\,x^2+a\,c\,x$$

input
```
int(a*c + x*(a*d + b*c) + b*d*x^2,x)
```

output
```
x^2*((a*d)/2 + (b*c)/2) + a*c*x + (b*d*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \left(ac+(bc+ad)x+bdx^2\right)\,dx=\frac{x(2bd\,x^2+3adx+3bcx+6ac)}{6}$$

input
```
int(a*c+(a*d+b*c)*x+b*d*x^2,x)
```

output
```
(x*(6*a*c + 3*a*d*x + 3*b*c*x + 2*b*d*x**2))/6
```

3.19 $\int \frac{1}{ac+(bc+ad)x+bdx^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{1}{ac+(bc+ad)x+bdx^2} dx = \frac{\log(a+bx)}{bc-ad} - \frac{\log(c+dx)}{bc-ad}$$

output
```
ln(b*x+a)/(-a*d+b*c)-ln(d*x+c)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1}{ac+(bc+ad)x+bdx^2} dx = \frac{\log(a+bx)-\log(c+dx)}{bc-ad}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^(-1),x]
```

output
```
(Log[a + b*x] - Log[c + d*x])/(b*c - a*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(ad+bc)+ac+bdx^2}\,dx$$

$$\Big\downarrow \; 1081$$

$$bd\int \left(\frac{1}{d(bc-ad)(a+bx)}-\frac{1}{b(bc-ad)(c+dx)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$bd\left(\frac{\log(a+bx)}{bd(bc-ad)}-\frac{\log(c+dx)}{bd(bc-ad)}\right)$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^(-1),x]
```

output
```
b*d*(Log[a + b*x]/(b*d*(b*c - a*d)) - Log[c + d*x]/(b*d*(b*c - a*d)))
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$-\frac{\ln(bx+a)-\ln(dx+c)}{ad-bc}$	28
default	$-\frac{\ln(bx+a)}{ad-bc}+\frac{\ln(dx+c)}{ad-bc}$	37
norman	$-\frac{\ln(bx+a)}{ad-bc}+\frac{\ln(dx+c)}{ad-bc}$	37
risch	$-\frac{\ln(bx+a)}{ad-bc}+\frac{\ln(-dx-c)}{ad-bc}$	40

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
-(ln(b*x+a)-ln(d*x+c))/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log(bx+a)-\log(dx+c)}{bc-ad}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
(log(b*x + a) - log(d*x + c))/(b*c - a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(26) = 52$.

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.56

$$\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\log\left(x+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc} - \frac{\log\left(x+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**2/
(a*d - b*c) + b*c)/(2*b*d))/(a*d - b*c) - log(x + (a**2*d**2/(a*d - b*c) -
 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))/(a*d
- b*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx = \frac{b\log(|bx+a|)}{b^2c-abd} - \frac{d\log(|dx+c|)}{bcd-ad^2}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
b*log(abs(b*x + a))/(b^2*c - a*b*d) - d*log(abs(d*x + c))/(b*c*d - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{a\,d-b\,c}$$

input
```
int(1/(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

output
```
(atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(a*d - b*c)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx=\frac{-\log(bx+a)+\log(dx+c)}{ad-bc}$$

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
( - log(a + b*x) + log(c + d*x))/(a*d - b*c)
```

3.20 $\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{b}{(bc-ad)^2(a+bx)} - \frac{d}{(bc-ad)^2(c+dx)} - \frac{2bd\log(a+bx)}{(bc-ad)^3} + \frac{2bd\log(c+dx)}{(bc-ad)^3}$$

output
```
-b/(-a*d+b*c)^2/(b*x+a)-d/(-a*d+b*c)^2/(d*x+c)-2*b*d*ln(b*x+a)/(-a*d+b*c)^
3+2*b*d*ln(d*x+c)/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{\frac{b(-bc+ad)}{a+bx} + \frac{d(-bc+ad)}{c+dx} - 2bd\log(a+bx) + 2bd\log(c+dx)}{(bc-ad)^3}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^(-2),x]
```

output
```
((b*(-(b*c) + a*d))/(a + b*x) + (d*(-(b*c) + a*d))/(c + d*x) - 2*b*d*Log[a
 + b*x] + 2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{1084, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x(ad+bc)+ac+bdx^2)^2}dx$$

$$\Big\downarrow \text{1084}$$

$$b^2d^2\int \left(\frac{2}{b(bc-ad)^3(c+dx)}+\frac{1}{b^2(bc-ad)^2(c+dx)^2}-\frac{2}{d(bc-ad)^3(a+bx)}+\frac{1}{d^2(bc-ad)^2(a+bx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$b^2d^2\left(-\frac{1}{b^2d(c+dx)(bc-ad)^2}-\frac{1}{bd^2(a+bx)(bc-ad)^2}-\frac{2\log(a+bx)}{bd(bc-ad)^3}+\frac{2\log(c+dx)}{bd(bc-ad)^3}\right)$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^(-2),x]
```

output
```
b^2*d^2*(-(1/(b*d^2*(b*c - a*d)^2*(a + b*x))) - 1/(b^2*d*(b*c - a*d)^2*(c
+ d*x)) - (2*Log[a + b*x])/(b*d*(b*c - a*d)^3) + (2*Log[c + d*x])/(b*d*(b*
c - a*d)^3))
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

method	result
default	$-\frac{b}{(ad-bc)^2(bx+a)}+\frac{2bd\ln(bx+a)}{(ad-bc)^3}-\frac{d}{(ad-bc)^2(dx+c)}-\frac{2bd\ln(dx+c)}{(ad-bc)^3}$
risch	$\frac{-\frac{2bdx}{a^2d^2-2abcd+b^2c^2}-\frac{ad+bc}{a^2d^2-2abcd+b^2c^2}}{bd\,x^2+adx+cbx+ac}-\frac{2bd\ln(dx+c)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}+\frac{2bd\ln(-bx-a)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$
norman	$\frac{\frac{-ab\,d^2-cd\,b^2}{db\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{2bdx}{a^2d^2-2abcd+b^2c^2}}{(bx+a)(dx+c)}+\frac{2bd\ln(bx+a)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}-\frac{2bd\ln(dx+c)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$
parallelrisch	$\frac{2\ln(bx+a)x^2b^3d^3-2\ln(dx+c)x^2b^3d^3+2\ln(bx+a)xa\,b^2d^3+2\ln(bx+a)x\,b^3c\,d^2-2\ln(dx+c)xa\,b^2d^3-2\ln(dx+c)x\,b^3c\,d^2+2\ln}{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)(bd\,x^2+adx+cbx+ac)}$

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-b/(a*d-b*c)^2/(b*x+a)+2*b/(a*d-b*c)^3*d*ln(b*x+a)-d/(a*d-b*c)^2/(d*x+c)-2
*b/(a*d-b*c)^3*d*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(81) = 162.

Time = 0.11 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.98

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{b^2c^2-a^2d^2+2\,(b^2cd-abd^2)x+2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)\log{(bx+a)}-2\,(b^2d^2x^2+abcd-}{ab^3c^4-3\,a^2b^2c^3d+3\,a^3bc^2d^2-a^4cd^3+(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4)x^2+(b^4c^4-2\,ab}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
-(b^2*c^2 - a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2*x^2 + a*b*c*d +
 (b^2*c*d + a*b*d^2)*x)*log(b*x + a) - 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d
 + a*b*d^2)*x)*log(d*x + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^
2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4
)*x^2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(70) = 140$.

Time = 0.56 (sec) , antiderivative size = 406, normalized size of antiderivative = 5.01

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{2bd\log\left(x+\frac{-\frac{2a^4bd^5}{(ad-bc)^3}+\frac{8a^3b^2cd^4}{(ad-bc)^3}-\frac{12a^2b^3c^2d^3}{(ad-bc)^3}+\frac{8ab^4c^3d^2}{(ad-bc)^3}+2abd^2-\frac{2b^5c^4d}{(ad-bc)^3}+2b^2cd}{4b^2d^2}\right)}{(ad-bc)^3} + \frac{2bd\log\left(x+\frac{\frac{2a^4bd^5}{(ad-bc)^3}-\frac{8a^3b^2cd^4}{(ad-bc)^3}+\frac{12a^2b^3c^2d^3}{(ad-bc)^3}-\frac{8ab^4c^3d^2}{(ad-bc)^3}+2abd^2+\frac{2b^5c^4d}{(ad-bc)^3}+2b^2cd}{4b^2d^2}\right)}{(ad-bc)^3} + \frac{-ad-bc-2bdx}{a^3cd^2-2a^2bc^2d+ab^2c^3+x^2(a^2bd^3-2ab^2cd^2+b^3c^2d)+x(a^3d^3-a^2bcd^2-ab^2c^2d+b^3c^3)}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
-2*b*d*log(x + (-2*a**4*b*d**5/(a*d - b*c)**3 + 8*a**3*b**2*c*d**4/(a*d -
b*c)**3 - 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 8*a*b**4*c**3*d**2/(a*d
- b*c)**3 + 2*a*b*d**2 - 2*b**5*c**4*d/(a*d - b*c)**3 + 2*b**2*c*d)/(4*b**
2*d**2))/(a*d - b*c)**3 + 2*b*d*log(x + (2*a**4*b*d**5/(a*d - b*c)**3 - 8*
a**3*b**2*c*d**4/(a*d - b*c)**3 + 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 -
8*a*b**4*c**3*d**2/(a*d - b*c)**3 + 2*a*b*d**2 + 2*b**5*c**4*d/(a*d - b*c)
**3 + 2*b**2*c*d)/(4*b**2*d**2))/(a*d - b*c)**3 + (-a*d - b*c - 2*b*d*x)/(
a**3*c*d**2 - 2*a**2*b*c**2*d + a*b**2*c**3 + x**2*(a**2*b*d**3 - 2*a*b**2
*c*d**2 + b**3*c**2*d) + x*(a**3*d**3 - a**2*b*c*d**2 - a*b**2*c**2*d + b*
*3*c**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(81) = 162$.

Time = 0.04 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.57

$$\int \frac{1}{(ac + (bc + ad)x + bdx^2)^2} dx$$
$$= -\frac{2 bd \log(bx + a)}{b^3c^3 - 3 ab^2c^2d + 3 a^2bcd^2 - a^3d^3} + \frac{2 bd \log(dx + c)}{b^3c^3 - 3 ab^2c^2d + 3 a^2bcd^2 - a^3d^3}$$
$$- \frac{2 bdx + bc + ad}{ab^2c^3 - 2 a^2bc^2d + a^3cd^2 + (b^3c^2d - 2 ab^2cd^2 + a^2bd^3)x^2 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)x}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
-2*b*d*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) +
2*b*d*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - (
2*b*d*x + b*c + a*d)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d -
 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a
^3*d^3)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(81) = 162$.

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.05

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{2\,b^2d\log(|bx+a|)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3} + \frac{2\,bd^2\log(|dx+c|)}{b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4} - \frac{2\,bdx+bc+ad}{(b^2c^2-2\,abcd+a^2d^2)(bdx^2+bcx+adx+ac)}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
-2*b^2*d*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^
3*b*d^3) + 2*b*d^2*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*
b*c*d^3 - a^3*d^4) - (2*b*d*x + b*c + a*d)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*(b*d*x^2 + b*c*x + a*d*x + a*c))
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.25

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{4\,b\,d\,\mathrm{atanh}\left(\frac{a^3\,d^3-a^2\,b\,c\,d^2-a\,b^2\,c^2\,d+b^3\,c^3}{(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{(a\,d-b\,c)^3} - \frac{\frac{a\,d+b\,c}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}+\frac{2\,b\,d\,x}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{b\,d\,x^2+(a\,d+b\,c)\,x+a\,c}$$

input
```
int(1/(a*c + x*(a*d + b*c) + b*d*x^2)^2,x)
```

output
```
(4*b*d*atanh((a^3*d^3 + b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2)/(a*d - b*c)^3
 + (2*b*d*x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(a*d - b*c)^3
 - ((a*d + b*c)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + (2*b*d*x)/(a^2*d^2 + b^2
*c^2 - 2*a*b*c*d))/(a*c + x*(a*d + b*c) + b*d*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.67

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx$$
$$= \frac{2\log(bx+a)\,a^2bc\,d^2+2\log(bx+a)\,a^2b\,d^3x+2\log(bx+a)\,a\,b^2c^2d+4\log(bx+a)\,a\,b^2c\,d^2x+2\log(bx-$$

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
(2*log(a + b*x)*a**2*b*c*d**2 + 2*log(a + b*x)*a**2*b*d**3*x + 2*log(a + b
*x)*a*b**2*c**2*d + 4*log(a + b*x)*a*b**2*c*d**2*x + 2*log(a + b*x)*a*b**2
*d**3*x**2 + 2*log(a + b*x)*b**3*c**2*d*x + 2*log(a + b*x)*b**3*c*d**2*x**
2 - 2*log(c + d*x)*a**2*b*c*d**2 - 2*log(c + d*x)*a**2*b*d**3*x - 2*log(c
+ d*x)*a*b**2*c**2*d - 4*log(c + d*x)*a*b**2*c*d**2*x - 2*log(c + d*x)*a*b
**2*d**3*x**2 - 2*log(c + d*x)*b**3*c**2*d*x - 2*log(c + d*x)*b**3*c*d**2*
x**2 - a**3*d**3 + a**2*b*c*d**2 - a*b**2*c**2*d + 2*a*b**2*d**3*x**2 + b*
*3*c**3 - 2*b**3*c*d**2*x**2)/(a**5*c*d**4 + a**5*d**5*x - 2*a**4*b*c**2*d
**3 - a**4*b*c*d**4*x + a**4*b*d**5*x**2 - 2*a**3*b**2*c**2*d**3*x - 2*a**
3*b**2*c*d**4*x**2 + 2*a**2*b**3*c**4*d + 2*a**2*b**3*c**3*d**2*x - a*b**4
*c**5 + a*b**4*c**4*d*x + 2*a*b**4*c**3*d**2*x**2 - b**5*c**5*x - b**5*c**
4*d*x**2)
```

3.21 $\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3} dx = -\frac{b^2}{2(bc-ad)^3(a+bx)^2}+\frac{3b^2d}{(bc-ad)^4(a+bx)}+\frac{d^2}{2(bc-ad)^3(c+dx)^2}+\frac{3bd^2}{(bc-ad)^4(c+dx)}+\frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}-\frac{6b^2d^2\log(c+dx)}{(bc-ad)^5}$$

output
```
-1/2*b^2/(-a*d+b*c)^3/(b*x+a)^2+3*b^2*d/(-a*d+b*c)^4/(b*x+a)+1/2*d^2/(-a*d
+b*c)^3/(d*x+c)^2+3*b*d^2/(-a*d+b*c)^4/(d*x+c)+6*b^2*d^2*ln(b*x+a)/(-a*d+b
*c)^5-6*b^2*d^2*ln(d*x+c)/(-a*d+b*c)^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3} dx = \frac{-\frac{b^2(bc-ad)^2}{(a+bx)^2}+\frac{6b^2d(bc-ad)}{a+bx}+\frac{d^2(bc-ad)^2}{(c+dx)^2}+\frac{6bd^2(bc-ad)}{c+dx}+12b^2d^2\log(a+bx)-12b^2d^2\log(c+dx)}{2(bc-ad)^5}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^(-3),x]
```

output
```
(-((b^2*(b*c - a*d)^2)/(a + b*x)^2) + (6*b^2*d*(b*c - a*d))/(a + b*x) + (d
^2*(b*c - a*d)^2)/(c + d*x)^2 + (6*b*d^2*(b*c - a*d))/(c + d*x) + 12*b^2*d
^2*Log[a + b*x] - 12*b^2*d^2*Log[c + d*x])/(2*(b*c - a*d)^5)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x(ad+bc)+ac+bdx^2)^3}dx$$

$$\Big\downarrow \text{1084}$$

$$b^3d^3\int \left(-\frac{6}{b(bc-ad)^5(c+dx)}-\frac{3}{b^2(bc-ad)^4(c+dx)^2}-\frac{1}{b^3(bc-ad)^3(c+dx)^3}+\frac{6}{d(bc-ad)^5(a+bx)}-\frac{}{d^2(bc}\right.$$

$$\Big\downarrow \text{2009}$$

$$b^3d^3\left(\frac{1}{2b^3d(c+dx)^2(bc-ad)^3}+\frac{3}{b^2d(c+dx)(bc-ad)^4}-\frac{1}{2bd^3(a+bx)^2(bc-ad)^3}+\frac{3}{bd^2(a+bx)(bc-ad)^4}+\frac{6}{b}\right.$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^(-3),x]
```

output
```
b^3*d^3*(-1/2*1/(b*d^3*(b*c - a*d)^3*(a + b*x)^2) + 3/(b*d^2*(b*c - a*d)^4
*(a + b*x)) + 1/(2*b^3*d*(b*c - a*d)^3*(c + d*x)^2) + 3/(b^2*d*(b*c - a*d)
^4*(c + d*x)) + (6*Log[a + b*x])/(b*d*(b*c - a*d)^5) - (6*Log[c + d*x])/(b
*d*(b*c - a*d)^5))
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.98

method	result
default	$\frac{b^2}{2(ad-bc)^3(bx+a)^2}-\frac{6b^2d^2\ln(bx+a)}{(ad-bc)^5}+\frac{3b^2d}{(ad-bc)^4(bx+a)}-\frac{d^2}{2(ad-bc)^3(dx+c)^2}+\frac{6b^2d^2\ln(dx+c)}{(ad-bc)^5}+\frac{3d^2b}{(ad-bc)^4(dx+c}$
risch	$\frac{\frac{6b^3d^3x^3}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4}+\frac{9b^2d^2(ad+bc)x^2}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4}+\frac{2\left(a^2d^2+7abcd+b^2c^2\right)bdx}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4\cdots}}{(bd\,x^2+adx+cbx+ac)^2}$
norman	$\frac{\frac{\left(9a\,b^4d^5+9b^5c\,d^4\right)x^2}{d^2b^2\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4\right)}+\frac{-a^3d^5b^2+7a^2b^3c\,d^4+7d^3a\,c^2b^4-c^3d^2b^5}{2d^2b^2\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4\right)}+\frac{6b^3d^3x^3}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d\cdots}}{(dx+c)^2(bx+a)^2}$
parallelrisch	$-\frac{12\ln(bx+a)x^4b^6d^6-12\ln(dx+c)x^4b^6d^6-12x^3a\,b^5d^6+12x^3b^6c\,d^5-18x^2a^2b^4d^6+18x^2b^6c^2d^4-4x\,a^3b^3d^6+4x\,b^6c^3d^3+48\ln\cdots}{}$

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*b^2/(a*d-b*c)^5*d^2*ln(b*x+a)+3*b^2/(a*d-b
*c)^4*d/(b*x+a)-1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*b^2/(a*d-b*c)^5*d^2*ln(d*x
+c)+3*d^2/(a*d-b*c)^4*b/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(139) = 278$.

Time = 0.10 (sec) , antiderivative size = 760, normalized size of antiderivative = 5.31

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{b^4c^4-8\,ab^3c^3d+8\,a^3bcd^3-a^4d^4-12\,(b^4cd^3-ab^3d^4)x^3-18\,(b^4c^2d^2-a^2b^2d^4)x^2-4\,(b^4c^3d+}{2\,(a^2b^5c^7-5\,a^3b^4c^6d+10\,a^4b^3c^5d^2-10\,a^5b^2c^4d^3+5\,a^6bc^3d^4-a^7c^2d^5+(b^7c^5d^2-5\,ab^6c^4d^3+10\,a^2b^5}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^4*c^4 - 8*a*b^3*c^3*d + 8*a^3*b*c*d^3 - a^4*d^4 - 12*(b^4*c*d^3 -
a*b^3*d^4)*x^3 - 18*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 - 4*(b^4*c^3*d + 6*a*b
^3*c^2*d^2 - 6*a^2*b^2*c*d^3 - a^3*b*d^4)*x - 12*(b^4*d^4*x^4 + a^2*b^2*c^
2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2
*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a) + 12*(b^
4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2
 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)
*log(d*x + c))/(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^
5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5 + (b^7*c^5*d^2 - 5*a*b^6*c^4
*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2
*d^7)*x^4 + 2*(b^7*c^6*d - 4*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3
*c^2*d^5 + 4*a^5*b^2*c*d^6 - a^6*b*d^7)*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a
^2*b^5*c^5*d^2 + 25*a^3*b^4*c^4*d^3 - 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d
^5 + a^6*b*c*d^6 - a^7*d^7)*x^2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b
^4*c^5*d^2 - 5*a^5*b^2*c^3*d^4 + 4*a^6*b*c^2*d^5 - a^7*c*d^6)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(128) = 256$.

Time = 1.20 (sec) , antiderivative size = 881, normalized size of antiderivative = 6.16

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
6*b**2*d**2*log(x + (-6*a**6*b**2*d**8/(a*d - b*c)**5 + 36*a**5*b**3*c*d**
7/(a*d - b*c)**5 - 90*a**4*b**4*c**2*d**6/(a*d - b*c)**5 + 120*a**3*b**5*c
**3*d**5/(a*d - b*c)**5 - 90*a**2*b**6*c**4*d**4/(a*d - b*c)**5 + 36*a*b**
7*c**5*d**3/(a*d - b*c)**5 + 6*a*b**2*d**3 - 6*b**8*c**6*d**2/(a*d - b*c)*
*5 + 6*b**3*c*d**2)/(12*b**3*d**3))/(a*d - b*c)**5 - 6*b**2*d**2*log(x + (
6*a**6*b**2*d**8/(a*d - b*c)**5 - 36*a**5*b**3*c*d**7/(a*d - b*c)**5 + 90*
a**4*b**4*c**2*d**6/(a*d - b*c)**5 - 120*a**3*b**5*c**3*d**5/(a*d - b*c)**
5 + 90*a**2*b**6*c**4*d**4/(a*d - b*c)**5 - 36*a*b**7*c**5*d**3/(a*d - b*c
)**5 + 6*a*b**2*d**3 + 6*b**8*c**6*d**2/(a*d - b*c)**5 + 6*b**3*c*d**2)/(1
2*b**3*d**3))/(a*d - b*c)**5 + (-a**3*d**3 + 7*a**2*b*c*d**2 + 7*a*b**2*c*
*2*d - b**3*c**3 + 12*b**3*d**3*x**3 + x**2*(18*a*b**2*d**3 + 18*b**3*c*d*
*2) + x*(4*a**2*b*d**3 + 28*a*b**2*c*d**2 + 4*b**3*c**2*d))/(2*a**6*c**2*d
**4 - 8*a**5*b*c**3*d**3 + 12*a**4*b**2*c**4*d**2 - 8*a**3*b**3*c**5*d + 2
*a**2*b**4*c**6 + x**4*(2*a**4*b**2*d**6 - 8*a**3*b**3*c*d**5 + 12*a**2*b*
*4*c**2*d**4 - 8*a*b**5*c**3*d**3 + 2*b**6*c**4*d**2) + x**3*(4*a**5*b*d**
6 - 12*a**4*b**2*c*d**5 + 8*a**3*b**3*c**2*d**4 + 8*a**2*b**4*c**3*d**3 -
12*a*b**5*c**4*d**2 + 4*b**6*c**5*d) + x**2*(2*a**6*d**6 - 18*a**4*b**2*c*
*2*d**4 + 32*a**3*b**3*c**3*d**3 - 18*a**2*b**4*c**4*d**2 + 2*b**6*c**6) +
 x*(4*a**6*c*d**5 - 12*a**5*b*c**2*d**4 + 8*a**4*b**2*c**3*d**3 + 8*a**3*b
**3*c**4*d**2 - 12*a**2*b**4*c**5*d + 4*a*b**5*c**6))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(139) = 278$.

Time = 0.05 (sec) , antiderivative size = 594, normalized size of antiderivative = 4.15

$$\int \frac{1}{(ac + (bc + ad)x + bdx^2)^3}\, dx$$
$$= \frac{6\, b^2 d^2 \log (bx + a)}{b^5 c^5 - 5\, ab^4 c^4 d + 10\, a^2 b^3 c^3 d^2 - 10\, a^3 b^2 c^2 d^3 + 5\, a^4 bcd^4 - a^5 d^5}$$
$$- \frac{6\, b^2 d^2 \log (dx + c)}{b^5 c^5 - 5\, ab^4 c^4 d + 10\, a^2 b^3 c^3 d^2 - 10\, a^3 b^2 c^2 d^3 + 5\, a^4 bcd^4 - a^5 d^5}$$
$$+ \frac{}{2\,(a^2 b^4 c^6 - 4\, a^3 b^3 c^5 d + 6\, a^4 b^2 c^4 d^2 - 4\, a^5 bc^3 d^3 + a^6 c^2 d^4 + (b^6 c^4 d^2 - 4\, ab^5 c^3 d^3 + 6\, a^2 b^4 c^2 d^4 - 4\, a^3 b^3 cd^5}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
6*b^2*d^2*log(b*x + a)/(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*
a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) - 6*b^2*d^2*log(d*x + c)/(b^5*c
^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d
^4 - a^5*d^5) + 1/2*(12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*
d^2 - a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*
d^2 + a^2*b*d^3)*x)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4
*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*
c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*
d^2 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)
*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d
^4 + a^6*d^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2
*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(139) = 278$.

Time = 0.23 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.41

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{6\,b^3d^2\log\left(|bx+a|\right)}{b^6c^5-5\,ab^5c^4d+10\,a^2b^4c^3d^2-10\,a^3b^3c^2d^3+5\,a^4b^2cd^4-a^5bd^5} - \frac{6\,b^2d^3\log\left(|dx+c|\right)}{b^5c^5d-5\,ab^4c^4d^2+10\,a^2b^3c^3d^3-10\,a^3b^2c^2d^4+5\,a^4bcd^5-a^5d^6} + \frac{12\,b^3d^3x^3+18\,b^3cd^2x^2+18\,ab^2d^3x^2+4\,b^3c^2dx+28\,ab^2cd^2x+4\,a^2bd^3x-b^3c^3+7\,ab^2c^2d+7\,a^2bcd^2-}{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)(bdx^2+bcx+adx+ac)^2}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
6*b^3*d^2*log(abs(b*x + a))/(b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2
- 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5) - 6*b^2*d^3*log(abs(d*
x + c))/(b^5*c^5*d - 5*a*b^4*c^4*d^2 + 10*a^2*b^3*c^3*d^3 - 10*a^3*b^2*c^2
*d^4 + 5*a^4*b*c*d^5 - a^5*d^6) + 1/2*(12*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 +
 18*a*b^2*d^3*x^2 + 4*b^3*c^2*d*x + 28*a*b^2*c*d^2*x + 4*a^2*b*d^3*x - b^3
*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3)/((b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*d*x^2 + b*c*x + a*d*x +
a*c)^2)
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 542, normalized size of antiderivative = 3.79

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$=\frac{\frac{6\,b^3\,d^3\,x^3}{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}-\frac{a^3\,d^3-7\,a^2\,b\,c\,d^2-7\,a\,b^2\,c^2\,d+b^3\,c^3}{2\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}+\frac{9\,b\,d\,x^2\,(c\,b^2\,d+a\,b\,d^2}{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4}}{x\,\left(2\,d\,a^2\,c+2\,b\,a\,c^2\right)+x^2\,\left(a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)+x^3\,\left(2\,c\,b^2\,d+2\,a\,b\,d^2\right.}$$
$$-\frac{12\,b^2\,d^2\,\mathrm{atanh}\left(\frac{a^5\,d^5-3\,a^4\,b\,c\,d^4+2\,a^3\,b^2\,c^2\,d^3+2\,a^2\,b^3\,c^3\,d^2-3\,a\,b^4\,c^4\,d+b^5\,c^5}{(a\,d-b\,c)^5}+\frac{2\,b\,d\,x\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b}{(a\,d-b\,c)^5}\right.}{(a\,d-b\,c)^5}$$

input
```
int(1/(a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
((6*b^3*d^3*x^3)/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d -
4*a^3*b*c*d^3) - (a^3*d^3 + b^3*c^3 - 7*a*b^2*c^2*d - 7*a^2*b*c*d^2)/(2*(a
^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (
9*b*d*x^2*(a*b*d^2 + b^2*c*d))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*
a*b^3*c^3*d - 4*a^3*b*c*d^3) + (2*b*d*x*(a^2*d^2 + b^2*c^2 + 7*a*b*c*d))/(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(x
*(2*a*b*c^2 + 2*a^2*c*d) + x^2*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + x^3*(2*a*
b*d^2 + 2*b^2*c*d) + a^2*c^2 + b^2*d^2*x^4) - (12*b^2*d^2*atanh((a^5*d^5 +
 b^5*c^5 + 2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 - 3*a*b^4*c^4*d - 3*a^4*b
*c*d^4)/(a*d - b*c)^5 + (2*b*d*x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 -
4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(a*d - b*c)^5))/(a*d - b*c)^5
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1202, normalized size of antiderivative = 8.41

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output

```
( - 12*log(a + b*x)*a**3*b**2*c**2*d**3 - 24*log(a + b*x)*a**3*b**2*c*d**4
*x - 12*log(a + b*x)*a**3*b**2*d**5*x**2 - 12*log(a + b*x)*a**2*b**3*c**3*
d**2 - 48*log(a + b*x)*a**2*b**3*c**2*d**3*x - 60*log(a + b*x)*a**2*b**3*c
*d**4*x**2 - 24*log(a + b*x)*a**2*b**3*d**5*x**3 - 24*log(a + b*x)*a*b**4*
c**3*d**2*x - 60*log(a + b*x)*a*b**4*c**2*d**3*x**2 - 48*log(a + b*x)*a*b*
*4*c*d**4*x**3 - 12*log(a + b*x)*a*b**4*d**5*x**4 - 12*log(a + b*x)*b**5*c
**3*d**2*x**2 - 24*log(a + b*x)*b**5*c**2*d**3*x**3 - 12*log(a + b*x)*b**5
*c*d**4*x**4 + 12*log(c + d*x)*a**3*b**2*c**2*d**3 + 24*log(c + d*x)*a**3*
b**2*c*d**4*x + 12*log(c + d*x)*a**3*b**2*d**5*x**2 + 12*log(c + d*x)*a**2
*b**3*c**3*d**2 + 48*log(c + d*x)*a**2*b**3*c**2*d**3*x + 60*log(c + d*x)*
a**2*b**3*c*d**4*x**2 + 24*log(c + d*x)*a**2*b**3*d**5*x**3 + 24*log(c + d
*x)*a*b**4*c**3*d**2*x + 60*log(c + d*x)*a*b**4*c**2*d**3*x**2 + 48*log(c
+ d*x)*a*b**4*c*d**4*x**3 + 12*log(c + d*x)*a*b**4*d**5*x**4 + 12*log(c +
d*x)*b**5*c**3*d**2*x**2 + 24*log(c + d*x)*b**5*c**2*d**3*x**3 + 12*log(c
+ d*x)*b**5*c*d**4*x**4 - a**5*d**5 + 7*a**4*b*c*d**4 + 4*a**4*b*d**5*x +
2*a**3*b**2*c**2*d**3 + 16*a**3*b**2*c*d**4*x + 12*a**3*b**2*d**5*x**2 - 2
*a**2*b**3*c**3*d**2 - 7*a*b**4*c**4*d - 16*a*b**4*c**3*d**2*x - 6*a*b**4*
d**5*x**4 + b**5*c**5 - 4*b**5*c**4*d*x - 12*b**5*c**3*d**2*x**2 + 6*b**5*
c*d**4*x**4)/(2*(a**8*c**2*d**6 + 2*a**8*c*d**7*x + a**8*d**8*x**2 - 4*a**
7*b*c**3*d**5 - 6*a**7*b*c**2*d**6*x + 2*a**7*b*d**8*x**3 + 5*a**6*b**2...
```

3.22 $\int \frac{1}{(ac+(bc+ad)x+bdx^2)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 193

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^4}\,dx = -\frac{b^3}{3(bc-ad)^4(a+bx)^3}+\frac{2b^3d}{(bc-ad)^5(a+bx)^2} - \frac{10b^3d^2}{(bc-ad)^6(a+bx)}-\frac{d^3}{3(bc-ad)^4(c+dx)^3} - \frac{2bd^3}{(bc-ad)^5(c+dx)^2}-\frac{10b^2d^3}{(bc-ad)^6(c+dx)} - \frac{20b^3d^3\log(a+bx)}{(bc-ad)^7}+\frac{20b^3d^3\log(c+dx)}{(bc-ad)^7}$$

output
```
-1/3*b^3/(-a*d+b*c)^4/(b*x+a)^3+2*b^3*d/(-a*d+b*c)^5/(b*x+a)^2-10*b^3*d^2/
(-a*d+b*c)^6/(b*x+a)-1/3*d^3/(-a*d+b*c)^4/(d*x+c)^3-2*b*d^3/(-a*d+b*c)^5/(
d*x+c)^2-10*b^2*d^3/(-a*d+b*c)^6/(d*x+c)-20*b^3*d^3*ln(b*x+a)/(-a*d+b*c)^7
+20*b^3*d^3*ln(d*x+c)/(-a*d+b*c)^7
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^4}\,dx = -\frac{\frac{b^3(bc-ad)^3}{(a+bx)^3}-\frac{6b^3d(bc-ad)^2}{(a+bx)^2}+\frac{30b^3d^2(bc-ad)}{a+bx}-\frac{d^3(-bc+ad)^3}{(c+dx)^3}+\frac{6bd^3(bc-ad)^2}{(c+dx)^2}+\frac{30b^2d^3(bc-ad)}{c+dx}+60b^3d^3\log(a+bx)-}{3(bc-ad)^7}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^(-4),x]
```

output
```
-1/3*((b^3*(b*c - a*d)^3)/(a + b*x)^3 - (6*b^3*d*(b*c - a*d)^2)/(a + b*x)^
2 + (30*b^3*d^2*(b*c - a*d))/(a + b*x) - (d^3*(-(b*c) + a*d)^3)/(c + d*x)^
3 + (6*b*d^3*(b*c - a*d)^2)/(c + d*x)^2 + (30*b^2*d^3*(b*c - a*d))/(c + d*
x) + 60*b^3*d^3*Log[a + b*x] - 60*b^3*d^3*Log[c + d*x])/(b*c - a*d)^7
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x(ad+bc)+ac+bdx^2)^4}\,dx$$

$$\Big\downarrow \text{1084}$$

$$b^4d^4\int \left(\frac{20}{b(bc-ad)^7(c+dx)}+\frac{10}{b^2(bc-ad)^6(c+dx)^2}+\frac{4}{b^3(bc-ad)^5(c+dx)^3}+\frac{1}{b^4(bc-ad)^4(c+dx)^4}-\frac{}{d(bc}\right.$$

$$\Big\downarrow \text{2009}$$

$$b^4d^4\left(-\frac{1}{3b^4d(c+dx)^3(bc-ad)^4}-\frac{2}{b^3d(c+dx)^2(bc-ad)^5}-\frac{10}{b^2d(c+dx)(bc-ad)^6}-\frac{1}{3bd^4(a+bx)^3(bc-ad)^4}\right.$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^(-4),x]
```

output
```
b^4*d^4*(-1/3*1/(b*d^4*(b*c - a*d)^4*(a + b*x)^3) + 2/(b*d^3*(b*c - a*d)^5
*(a + b*x)^2) - 10/(b*d^2*(b*c - a*d)^6*(a + b*x)) - 1/(3*b^4*d*(b*c - a*d
)^4*(c + d*x)^3) - 2/(b^3*d*(b*c - a*d)^5*(c + d*x)^2) - 10/(b^2*d*(b*c -
a*d)^6*(c + d*x)) - (20*Log[a + b*x])/(b*d*(b*c - a*d)^7) + (20*Log[c + d*
x])/(b*d*(b*c - a*d)^7))
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98

method	result
default	$-\frac{b^3}{3(ad-bc)^4(bx+a)^3}+\frac{20b^3d^3\ln(bx+a)}{(ad-bc)^7}-\frac{10b^3d^2}{(ad-bc)^6(bx+a)}-\frac{2b^3d}{(ad-bc)^5(bx+a)^2}-\frac{d^3}{3(ad-bc)^4(dx+c)^3}-\frac{20b^3d^3\ln \cdots}{(ad- \cdots}$
risch	$-\frac{20b^5d^5x^5}{a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6a\,b^5c^5d+b^6c^6}-\frac{50b^4d^4(ad+bc)x^4}{a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6 \cdots}$
norman	$-\frac{20b^5d^5x^5}{a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6a\,b^5c^5d+b^6c^6}+\frac{\left(-50a\,b^6d^7-50b^7c\,d^6\right)x^4}{d^2b^2\left(a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4 \cdots\right.}$
parallelrisch	Expression too large to display

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^3/(a*d-b*c)^4/(b*x+a)^3+20*b^3/(a*d-b*c)^7*d^3*ln(b*x+a)-10*b^3/(a*
d-b*c)^6*d^2/(b*x+a)-2*b^3/(a*d-b*c)^5*d/(b*x+a)^2-1/3*d^3/(a*d-b*c)^4/(d*
x+c)^3-20*b^3/(a*d-b*c)^7*d^3*ln(d*x+c)-10*d^3/(a*d-b*c)^6*b^2/(d*x+c)+2*d
^3/(a*d-b*c)^5*b/(d*x+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1520 vs. $2(189) = 378$.

Time = 0.13 (sec) , antiderivative size = 1520, normalized size of antiderivative = 7.88

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^4,x, algorithm="fricas")
```

output
```
-1/3*(b^6*c^6 - 9*a*b^5*c^5*d + 45*a^2*b^4*c^4*d^2 - 45*a^4*b^2*c^2*d^4 +
9*a^5*b*c*d^5 - a^6*d^6 + 60*(b^6*c*d^5 - a*b^5*d^6)*x^5 + 150*(b^6*c^2*d^
4 - a^2*b^4*d^6)*x^4 + 10*(11*b^6*c^3*d^3 + 27*a*b^5*c^2*d^4 - 27*a^2*b^4*
c*d^5 - 11*a^3*b^3*d^6)*x^3 + 15*(b^6*c^4*d^2 + 18*a*b^5*c^3*d^3 - 18*a^3*
b^3*c*d^5 - a^4*b^2*d^6)*x^2 - 3*(b^6*c^5*d - 15*a*b^5*c^4*d^2 - 60*a^2*b^
4*c^3*d^3 + 60*a^3*b^3*c^2*d^4 + 15*a^4*b^2*c*d^5 - a^5*b*d^6)*x + 60*(b^6
*d^6*x^6 + a^3*b^3*c^3*d^3 + 3*(b^6*c*d^5 + a*b^5*d^6)*x^5 + 3*(b^6*c^2*d^
4 + 3*a*b^5*c*d^5 + a^2*b^4*d^6)*x^4 + (b^6*c^3*d^3 + 9*a*b^5*c^2*d^4 + 9*
a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^3 + 3*(a*b^5*c^3*d^3 + 3*a^2*b^4*c^2*d^4 +
a^3*b^3*c*d^5)*x^2 + 3*(a^2*b^4*c^3*d^3 + a^3*b^3*c^2*d^4)*x)*log(b*x + a)
 - 60*(b^6*d^6*x^6 + a^3*b^3*c^3*d^3 + 3*(b^6*c*d^5 + a*b^5*d^6)*x^5 + 3*(
b^6*c^2*d^4 + 3*a*b^5*c*d^5 + a^2*b^4*d^6)*x^4 + (b^6*c^3*d^3 + 9*a*b^5*c^
2*d^4 + 9*a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^3 + 3*(a*b^5*c^3*d^3 + 3*a^2*b^4*
c^2*d^4 + a^3*b^3*c*d^5)*x^2 + 3*(a^2*b^4*c^3*d^3 + a^3*b^3*c^2*d^4)*x)*lo
g(d*x + c))/(a^3*b^7*c^10 - 7*a^4*b^6*c^9*d + 21*a^5*b^5*c^8*d^2 - 35*a^6*
b^4*c^7*d^3 + 35*a^7*b^3*c^6*d^4 - 21*a^8*b^2*c^5*d^5 + 7*a^9*b*c^4*d^6 -
a^10*c^3*d^7 + (b^10*c^7*d^3 - 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 - 35*a
^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*d^7 - 21*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9
 - a^7*b^3*d^10)*x^6 + 3*(b^10*c^8*d^2 - 6*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*
d^4 - 14*a^3*b^7*c^5*d^5 + 14*a^5*b^5*c^3*d^7 - 14*a^6*b^4*c^2*d^8 + 6*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1590 vs. $2(175) = 350$.

Time = 2.65 (sec) , antiderivative size = 1590, normalized size of antiderivative = 8.24

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x**2)**4,x)
```

output
```
-20*b**3*d**3*log(x + (-20*a**8*b**3*d**11/(a*d - b*c)**7 + 160*a**7*b**4*
c*d**10/(a*d - b*c)**7 - 560*a**6*b**5*c**2*d**9/(a*d - b*c)**7 + 1120*a**
5*b**6*c**3*d**8/(a*d - b*c)**7 - 1400*a**4*b**7*c**4*d**7/(a*d - b*c)**7
+ 1120*a**3*b**8*c**5*d**6/(a*d - b*c)**7 - 560*a**2*b**9*c**6*d**5/(a*d -
 b*c)**7 + 160*a*b**10*c**7*d**4/(a*d - b*c)**7 + 20*a*b**3*d**4 - 20*b**1
1*c**8*d**3/(a*d - b*c)**7 + 20*b**4*c*d**3)/(40*b**4*d**4))/(a*d - b*c)**
7 + 20*b**3*d**3*log(x + (20*a**8*b**3*d**11/(a*d - b*c)**7 - 160*a**7*b**
4*c*d**10/(a*d - b*c)**7 + 560*a**6*b**5*c**2*d**9/(a*d - b*c)**7 - 1120*a
**5*b**6*c**3*d**8/(a*d - b*c)**7 + 1400*a**4*b**7*c**4*d**7/(a*d - b*c)**
7 - 1120*a**3*b**8*c**5*d**6/(a*d - b*c)**7 + 560*a**2*b**9*c**6*d**5/(a*d
 - b*c)**7 - 160*a*b**10*c**7*d**4/(a*d - b*c)**7 + 20*a*b**3*d**4 + 20*b*
*11*c**8*d**3/(a*d - b*c)**7 + 20*b**4*c*d**3)/(40*b**4*d**4))/(a*d - b*c)
**7 + (-a**5*d**5 + 8*a**4*b*c*d**4 - 37*a**3*b**2*c**2*d**3 - 37*a**2*b**
3*c**3*d**2 + 8*a*b**4*c**4*d - b**5*c**5 - 60*b**5*d**5*x**5 + x**4*(-150
*a*b**4*d**5 - 150*b**5*c*d**4) + x**3*(-110*a**2*b**3*d**5 - 380*a*b**4*c
*d**4 - 110*b**5*c**2*d**3) + x**2*(-15*a**3*b**2*d**5 - 285*a**2*b**3*c*d
**4 - 285*a*b**4*c**2*d**3 - 15*b**5*c**3*d**2) + x*(3*a**4*b*d**5 - 42*a*
*3*b**2*c*d**4 - 222*a**2*b**3*c**2*d**3 - 42*a*b**4*c**3*d**2 + 3*b**5*c*
*4*d))/(3*a**9*c**3*d**6 - 18*a**8*b*c**4*d**5 + 45*a**7*b**2*c**5*d**4 -
60*a**6*b**3*c**6*d**3 + 45*a**5*b**4*c**7*d**2 - 18*a**4*b**5*c**8*d +...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1206 vs. $2(189) = 378$.

Time = 0.07 (sec) , antiderivative size = 1206, normalized size of antiderivative = 6.25

$$\int \frac{1}{(ac + (bc + ad)x + bdx^2)^4}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^4,x, algorithm="maxima")
```

output
```
-20*b^3*d^3*log(b*x + a)/(b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 3
5*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^
6 - a^7*d^7) + 20*b^3*d^3*log(d*x + c)/(b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b
^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5
+ 7*a^6*b*c*d^6 - a^7*d^7) - 1/3*(60*b^5*d^5*x^5 + b^5*c^5 - 8*a*b^4*c^4*d
 + 37*a^2*b^3*c^3*d^2 + 37*a^3*b^2*c^2*d^3 - 8*a^4*b*c*d^4 + a^5*d^5 + 150
*(b^5*c*d^4 + a*b^4*d^5)*x^4 + 10*(11*b^5*c^2*d^3 + 38*a*b^4*c*d^4 + 11*a^
2*b^3*d^5)*x^3 + 15*(b^5*c^3*d^2 + 19*a*b^4*c^2*d^3 + 19*a^2*b^3*c*d^4 + a
^3*b^2*d^5)*x^2 - 3*(b^5*c^4*d - 14*a*b^4*c^3*d^2 - 74*a^2*b^3*c^2*d^3 - 1
4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/(a^3*b^6*c^9 - 6*a^4*b^5*c^8*d + 15*a^5*b^
4*c^7*d^2 - 20*a^6*b^3*c^6*d^3 + 15*a^7*b^2*c^5*d^4 - 6*a^8*b*c^4*d^5 + a^
9*c^3*d^6 + (b^9*c^6*d^3 - 6*a*b^8*c^5*d^4 + 15*a^2*b^7*c^4*d^5 - 20*a^3*b
^6*c^3*d^6 + 15*a^4*b^5*c^2*d^7 - 6*a^5*b^4*c*d^8 + a^6*b^3*d^9)*x^6 + 3*(
b^9*c^7*d^2 - 5*a*b^8*c^6*d^3 + 9*a^2*b^7*c^5*d^4 - 5*a^3*b^6*c^4*d^5 - 5*
a^4*b^5*c^3*d^6 + 9*a^5*b^4*c^2*d^7 - 5*a^6*b^3*c*d^8 + a^7*b^2*d^9)*x^5 +
 3*(b^9*c^8*d - 3*a*b^8*c^7*d^2 - 2*a^2*b^7*c^6*d^3 + 19*a^3*b^6*c^5*d^4 -
 30*a^4*b^5*c^4*d^5 + 19*a^5*b^4*c^3*d^6 - 2*a^6*b^3*c^2*d^7 - 3*a^7*b^2*c
*d^8 + a^8*b*d^9)*x^4 + (b^9*c^9 + 3*a*b^8*c^8*d - 30*a^2*b^7*c^7*d^2 + 62
*a^3*b^6*c^6*d^3 - 36*a^4*b^5*c^5*d^4 - 36*a^5*b^4*c^4*d^5 + 62*a^6*b^3*c^
3*d^6 - 30*a^7*b^2*c^2*d^7 + 3*a^8*b*c*d^8 + a^9*d^9)*x^3 + 3*(a*b^8*c^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(189) = 378$.

Time = 0.17 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.03

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^4}\,dx = -\frac{20\,b^4d^3\log(|bx+a|)}{b^8c^7-7\,ab^7c^6d+21\,a^2b^6c^5d^2-35\,a^3b^5c^4d^3+35\,a^4b^4c^3d^4-21\,a^5b^3c^2d^5+7\,a^6b^2cd^6-a^7bd^7} + \frac{20\,b^3d^4\log(|dx+c|)}{b^7c^7d-7\,ab^6c^6d^2+21\,a^2b^5c^5d^3-35\,a^3b^4c^4d^4+35\,a^4b^3c^3d^5-21\,a^5b^2c^2d^6+7\,a^6bcd^7-a^7d^8} - \frac{60\,b^5d^5x^5+150\,b^5cd^4x^4+150\,ab^4d^5x^4+110\,b^5c^2d^3x^3+380\,ab^4cd^4x^3+110\,a^2b^3d^5x^3+15\,b^5c^3d^2x^2+\cdots}{3\,(b^6c^6-6\,ab^5c \cdots}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^4,x, algorithm="giac")
```

output
```
-20*b^4*d^3*log(abs(b*x + a))/(b^8*c^7 - 7*a*b^7*c^6*d + 21*a^2*b^6*c^5*d^
2 - 35*a^3*b^5*c^4*d^3 + 35*a^4*b^4*c^3*d^4 - 21*a^5*b^3*c^2*d^5 + 7*a^6*b
^2*c*d^6 - a^7*b*d^7) + 20*b^3*d^4*log(abs(d*x + c))/(b^7*c^7*d - 7*a*b^6*
c^6*d^2 + 21*a^2*b^5*c^5*d^3 - 35*a^3*b^4*c^4*d^4 + 35*a^4*b^3*c^3*d^5 - 2
1*a^5*b^2*c^2*d^6 + 7*a^6*b*c*d^7 - a^7*d^8) - 1/3*(60*b^5*d^5*x^5 + 150*b
^5*c*d^4*x^4 + 150*a*b^4*d^5*x^4 + 110*b^5*c^2*d^3*x^3 + 380*a*b^4*c*d^4*x
^3 + 110*a^2*b^3*d^5*x^3 + 15*b^5*c^3*d^2*x^2 + 285*a*b^4*c^2*d^3*x^2 + 28
5*a^2*b^3*c*d^4*x^2 + 15*a^3*b^2*d^5*x^2 - 3*b^5*c^4*d*x + 42*a*b^4*c^3*d^
2*x + 222*a^2*b^3*c^2*d^3*x + 42*a^3*b^2*c*d^4*x - 3*a^4*b*d^5*x + b^5*c^5
 - 8*a*b^4*c^4*d + 37*a^2*b^3*c^3*d^2 + 37*a^3*b^2*c^2*d^3 - 8*a^4*b*c*d^4
 + a^5*d^5)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^
3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*(b*d*x^2 + b*c*x + a
*d*x + a*c)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ac + (bc + ad)x + bdx^2)^4}\,dx$$

$$= \begin{cases} -\frac{20\left(\frac{a\,d}{2}+\frac{b\,c}{2}+b\,d\,x\right)\left(\frac{b\,d}{30\left((a\,d+b\,c)^2-4\,a\,b\,c\,d\right)\left(b\,d\,x^2+(a\,d+b\,c)\,x+a\,c\right)^3}-\frac{b^2\,d^2}{6\left((a\,d+b\,c)^2-4\,a\,b\,c\,d\right)^2\left(b\,d\,x^2+(a\,d+b\,c)\,x+a\,c\right)^2}+\frac{}{\left((a\,d+b\,c)^2-4\,a\,b\,c\right.}\right.}{b\,d} \\ -\frac{20\left(\frac{a\,d}{2}+\frac{b\,c}{2}+b\,d\,x\right)\left(\frac{b\,d}{30\left((a\,d+b\,c)^2-4\,a\,b\,c\,d\right)\left(b\,d\,x^2+(a\,d+b\,c)\,x+a\,c\right)^3}-\frac{b^2\,d^2}{6\left((a\,d+b\,c)^2-4\,a\,b\,c\,d\right)^2\left(b\,d\,x^2+(a\,d+b\,c)\,x+a\,c\right)^2}+\frac{}{\left((a\,d+b\,c)^2-4\,a\right.}\right.}{b\,d} \\ \int \frac{1}{\left(b\,d\,x^2+(a\,d+b\,c)\,x+a\,c\right)^4}\,dx \end{cases}$$

input

```
int(1/(a*c + x*(a*d + b*c) + b*d*x^2)^4,x)
```

output

```
piecewise(0 < (a*d + b*c)^2 - 4*a*b*c*d, - (20*((a*d)/2 + (b*c)/2 + b*d*x)
*((b*d)/(30*((a*d + b*c)^2 - 4*a*b*c*d)*(a*c + x*(a*d + b*c) + b*d*x^2)^3)
 - (b^2*d^2)/(6*((a*d + b*c)^2 - 4*a*b*c*d)^2*(a*c + x*(a*d + b*c) + b*d*x
^2)^2) + (b^3*d^3)/(((a*d + b*c)^2 - 4*a*b*c*d)^3*(a*c + x*(a*d + b*c) + b
*d*x^2))))/(b*d) - (20*b^3*d^3*log((- ((a*d + b*c)^2/4 - a*b*c*d)^(1/2) +
(a*d)/2 + (b*c)/2 + b*d*x)/(((a*d + b*c)^2/4 - a*b*c*d)^(1/2) + (a*d)/2 +
(b*c)/2 + b*d*x)))/((a*d + b*c)^2 - 4*a*b*c*d)^(7/2), (a*d + b*c)^2 - 4*a*
b*c*d < 0, - (20*((a*d)/2 + (b*c)/2 + b*d*x)*((b*d)/(30*((a*d + b*c)^2 - 4
*a*b*c*d)*(a*c + x*(a*d + b*c) + b*d*x^2)^3) - (b^2*d^2)/(6*((a*d + b*c)^2
 - 4*a*b*c*d)^2*(a*c + x*(a*d + b*c) + b*d*x^2)^2) + (b^3*d^3)/(((a*d + b*
c)^2 - 4*a*b*c*d)^3*(a*c + x*(a*d + b*c) + b*d*x^2))))/(b*d) - (20*b^3*d^3
*atan(((a*d)/2 + (b*c)/2 + b*d*x)/(- (a*d + b*c)^2/4 + a*b*c*d)^(1/2)))/((
(a*d + b*c)^2 - 4*a*b*c*d)^3*(- (a*d + b*c)^2/4 + a*b*c*d)^(1/2)), ~in((a*
d + b*c)^2 - 4*a*b*c*d, 'real') | (a*d + b*c)^2 == 4*a*b*c*d, int(1/(a*c +
 x*(a*d + b*c) + b*d*x^2)^4, x))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 2310, normalized size of antiderivative = 11.97

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^4} \, dx = \text{Too large to display}$$

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2)^4,x)
```

output
```
(60*log(a + b*x)*a**4*b**3*c**3*d**4 + 180*log(a + b*x)*a**4*b**3*c**2*d**
5*x + 180*log(a + b*x)*a**4*b**3*c*d**6*x**2 + 60*log(a + b*x)*a**4*b**3*d
**7*x**3 + 60*log(a + b*x)*a**3*b**4*c**4*d**3 + 360*log(a + b*x)*a**3*b**
4*c**3*d**4*x + 720*log(a + b*x)*a**3*b**4*c**2*d**5*x**2 + 600*log(a + b*
x)*a**3*b**4*c*d**6*x**3 + 180*log(a + b*x)*a**3*b**4*d**7*x**4 + 180*log(
a + b*x)*a**2*b**5*c**4*d**3*x + 720*log(a + b*x)*a**2*b**5*c**3*d**4*x**2
 + 1080*log(a + b*x)*a**2*b**5*c**2*d**5*x**3 + 720*log(a + b*x)*a**2*b**5
*c*d**6*x**4 + 180*log(a + b*x)*a**2*b**5*d**7*x**5 + 180*log(a + b*x)*a*b
**6*c**4*d**3*x**2 + 600*log(a + b*x)*a*b**6*c**3*d**4*x**3 + 720*log(a +
b*x)*a*b**6*c**2*d**5*x**4 + 360*log(a + b*x)*a*b**6*c*d**6*x**5 + 60*log(
a + b*x)*a*b**6*d**7*x**6 + 60*log(a + b*x)*b**7*c**4*d**3*x**3 + 180*log(
a + b*x)*b**7*c**3*d**4*x**4 + 180*log(a + b*x)*b**7*c**2*d**5*x**5 + 60*l
og(a + b*x)*b**7*c*d**6*x**6 - 60*log(c + d*x)*a**4*b**3*c**3*d**4 - 180*l
og(c + d*x)*a**4*b**3*c**2*d**5*x - 180*log(c + d*x)*a**4*b**3*c*d**6*x**2
 - 60*log(c + d*x)*a**4*b**3*d**7*x**3 - 60*log(c + d*x)*a**3*b**4*c**4*d*
*3 - 360*log(c + d*x)*a**3*b**4*c**3*d**4*x - 720*log(c + d*x)*a**3*b**4*c
**2*d**5*x**2 - 600*log(c + d*x)*a**3*b**4*c*d**6*x**3 - 180*log(c + d*x)*
a**3*b**4*d**7*x**4 - 180*log(c + d*x)*a**2*b**5*c**4*d**3*x - 720*log(c +
 d*x)*a**2*b**5*c**3*d**4*x**2 - 1080*log(c + d*x)*a**2*b**5*c**2*d**5*x**
3 - 720*log(c + d*x)*a**2*b**5*c*d**6*x**4 - 180*log(c + d*x)*a**2*b**5...
```

3.23 $\int (2+5x+3x^2)^3 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int (2+5x+3x^2)^3 \, dx = \frac{1}{324}(2+3x)^4 + \frac{1}{135}(2+3x)^5 + \frac{1}{162}(2+3x)^6 + \frac{1}{567}(2+3x)^7$$

output
```
1/324*(2+3*x)^4+1/135*(2+3*x)^5+1/162*(2+3*x)^6+1/567*(2+3*x)^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int (2+5x+3x^2)^3 \, dx = 8x + 30x^2 + 62x^3 + \frac{305x^4}{4} + \frac{279x^5}{5} + \frac{45x^6}{2} + \frac{27x^7}{7}$$

input
```
Integrate[(2 + 5*x + 3*x^2)^3,x]
```

output
```
8*x + 30*x^2 + 62*x^3 + (305*x^4)/4 + (279*x^5)/5 + (45*x^6)/2 + (27*x^7)/
7
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^2+5x+2)^3\,dx$$

$$\Big\downarrow \text{1084}$$

$$\frac{1}{27}\int \left((3x+2)^6+3(3x+2)^5+3(3x+2)^4+(3x+2)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{27}\left(\frac{1}{21}(3x+2)^7+\frac{1}{6}(3x+2)^6+\frac{1}{5}(3x+2)^5+\frac{1}{12}(3x+2)^4\right)$$

input
```
Int[(2 + 5*x + 3*x^2)^3,x]
```

output
```
((2 + 3*x)^4/12 + (2 + 3*x)^5/5 + (2 + 3*x)^6/6 + (2 + 3*x)^7/21)/27
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{x(540x^6+3150x^5+7812x^4+10675x^3+8680x^2+4200x+1120)}{140}$	34
default	$\frac{27}{7}x^7+\frac{45}{2}x^6+\frac{279}{5}x^5+\frac{305}{4}x^4+62x^3+30x^2+8x$	35
norman	$\frac{27}{7}x^7+\frac{45}{2}x^6+\frac{279}{5}x^5+\frac{305}{4}x^4+62x^3+30x^2+8x$	35
risch	$\frac{27}{7}x^7+\frac{45}{2}x^6+\frac{279}{5}x^5+\frac{305}{4}x^4+62x^3+30x^2+8x$	35
parallelrisch	$\frac{27}{7}x^7+\frac{45}{2}x^6+\frac{279}{5}x^5+\frac{305}{4}x^4+62x^3+30x^2+8x$	35
orering	$\frac{x(540x^6+3150x^5+7812x^4+10675x^3+8680x^2+4200x+1120)(3x^2+5x+2)^3}{140(x+1)^3(3x+2)^3}$	58

input
```
int((3*x^2+5*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/140*x*(540*x^6+3150*x^5+7812*x^4+10675*x^3+8680*x^2+4200*x+1120)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int (2+5x+3x^2)^3\,dx=\frac{27}{7}\,x^7+\frac{45}{2}\,x^6+\frac{279}{5}\,x^5+\frac{305}{4}\,x^4+62\,x^3+30\,x^2+8\,x$$

input
```
integrate((3*x^2+5*x+2)^3,x, algorithm="fricas")
```

output
```
27/7*x^7 + 45/2*x^6 + 279/5*x^5 + 305/4*x^4 + 62*x^3 + 30*x^2 + 8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \left(2+5x+3x^2\right)^3 \, dx = \frac{27x^7}{7} + \frac{45x^6}{2} + \frac{279x^5}{5} + \frac{305x^4}{4} + 62x^3 + 30x^2 + 8x$$

input
```
integrate((3*x**2+5*x+2)**3,x)
```

output
```
27*x**7/7 + 45*x**6/2 + 279*x**5/5 + 305*x**4/4 + 62*x**3 + 30*x**2 + 8*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \left(2+5x+3x^2\right)^3 \, dx = \frac{27}{7}\,x^7 + \frac{45}{2}\,x^6 + \frac{279}{5}\,x^5 + \frac{305}{4}\,x^4 + 62\,x^3 + 30\,x^2 + 8\,x$$

input
```
integrate((3*x^2+5*x+2)^3,x, algorithm="maxima")
```

output
```
27/7*x^7 + 45/2*x^6 + 279/5*x^5 + 305/4*x^4 + 62*x^3 + 30*x^2 + 8*x
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \left(2+5x+3x^2\right)^3 \, dx = \frac{27}{7}\,x^7 + \frac{45}{2}\,x^6 + \frac{279}{5}\,x^5 + \frac{305}{4}\,x^4 + 62\,x^3 + 30\,x^2 + 8\,x$$

input
```
integrate((3*x^2+5*x+2)^3,x, algorithm="giac")
```

output
```
27/7*x^7 + 45/2*x^6 + 279/5*x^5 + 305/4*x^4 + 62*x^3 + 30*x^2 + 8*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int (2+5x+3x^2)^3\,dx = \frac{27\,x^7}{7} + \frac{45\,x^6}{2} + \frac{279\,x^5}{5} + \frac{305\,x^4}{4} + 62\,x^3 + 30\,x^2 + 8\,x$$

input
```
int((5*x + 3*x^2 + 2)^3,x)
```

output
```
8*x + 30*x^2 + 62*x^3 + (305*x^4)/4 + (279*x^5)/5 + (45*x^6)/2 + (27*x^7)/
7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int (2+5x+3x^2)^3\,dx = \frac{x(540x^6+3150x^5+7812x^4+10675x^3+8680x^2+4200x+1120)}{140}$$

input
```
int((3*x^2+5*x+2)^3,x)
```

output
```
(x*(540*x**6 + 3150*x**5 + 7812*x**4 + 10675*x**3 + 8680*x**2 + 4200*x + 1
120))/140
```

3.24 $\int (2+5x+3x^2)^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int (2+5x+3x^2)^2\,dx=\frac{1}{81}(2+3x)^3+\frac{1}{54}(2+3x)^4+\frac{1}{135}(2+3x)^5$$

output
```
1/81*(2+3*x)^3+1/54*(2+3*x)^4+1/135*(2+3*x)^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (2+5x+3x^2)^2\,dx=4x+10x^2+\frac{37x^3}{3}+\frac{15x^4}{2}+\frac{9x^5}{5}$$

input
```
Integrate[(2 + 5*x + 3*x^2)^2,x]
```

output
```
4*x + 10*x^2 + (37*x^3)/3 + (15*x^4)/2 + (9*x^5)/5
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^2+5x+2\right)^2 dx$$

$$\Big\downarrow \text{1084}$$

$$\frac{1}{9}\int \left((3x+2)^4+2(3x+2)^3+(3x+2)^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}\left(\frac{1}{15}(3x+2)^5+\frac{1}{6}(3x+2)^4+\frac{1}{9}(3x+2)^3\right)$$

input
```
Int[(2 + 5*x + 3*x^2)^2,x]
```

output
```
((2 + 3*x)^3/9 + (2 + 3*x)^4/6 + (2 + 3*x)^5/15)/9
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{x(54x^4+225x^3+370x^2+300x+120)}{30}$	24
default	$\frac{9}{5}x^5+\frac{15}{2}x^4+\frac{37}{3}x^3+10x^2+4x$	25
norman	$\frac{9}{5}x^5+\frac{15}{2}x^4+\frac{37}{3}x^3+10x^2+4x$	25
risch	$\frac{9}{5}x^5+\frac{15}{2}x^4+\frac{37}{3}x^3+10x^2+4x$	25
parallelrisch	$\frac{9}{5}x^5+\frac{15}{2}x^4+\frac{37}{3}x^3+10x^2+4x$	25
orering	$\frac{x(54x^4+225x^3+370x^2+300x+120)(3x^2+5x+2)^2}{30(3x+2)^2(x+1)^2}$	48

input
```
int((3*x^2+5*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*x*(54*x^4+225*x^3+370*x^2+300*x+120)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int (2+5x+3x^2)^2 \, dx = \frac{9}{5}\,x^5+\frac{15}{2}\,x^4+\frac{37}{3}\,x^3+10\,x^2+4\,x$$

input
```
integrate((3*x^2+5*x+2)^2,x, algorithm="fricas")
```

output
```
9/5*x^5 + 15/2*x^4 + 37/3*x^3 + 10*x^2 + 4*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int (2+5x+3x^2)^2 \, dx = \frac{9x^5}{5} + \frac{15x^4}{2} + \frac{37x^3}{3} + 10x^2 + 4x$$

input
```
integrate((3*x**2+5*x+2)**2,x)
```

output
```
9*x**5/5 + 15*x**4/2 + 37*x**3/3 + 10*x**2 + 4*x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int (2+5x+3x^2)^2 \, dx = \frac{9}{5}\,x^5 + \frac{15}{2}\,x^4 + \frac{37}{3}\,x^3 + 10\,x^2 + 4\,x$$

input
```
integrate((3*x^2+5*x+2)^2,x, algorithm="maxima")
```

output
```
9/5*x^5 + 15/2*x^4 + 37/3*x^3 + 10*x^2 + 4*x
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int (2+5x+3x^2)^2 \, dx = \frac{9}{5}\,x^5 + \frac{15}{2}\,x^4 + \frac{37}{3}\,x^3 + 10\,x^2 + 4\,x$$

input
```
integrate((3*x^2+5*x+2)^2,x, algorithm="giac")
```

output
```
9/5*x^5 + 15/2*x^4 + 37/3*x^3 + 10*x^2 + 4*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(2+5x+3x^2\right)^2 \, dx = \frac{9\,x^5}{5}+\frac{15\,x^4}{2}+\frac{37\,x^3}{3}+10\,x^2+4\,x$$

input
```
int((5*x + 3*x^2 + 2)^2,x)
```

output
```
4*x + 10*x^2 + (37*x^3)/3 + (15*x^4)/2 + (9*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \left(2+5x+3x^2\right)^2 \, dx = \frac{x(54x^4+225x^3+370x^2+300x+120)}{30}$$

input
```
int((3*x^2+5*x+2)^2,x)
```

output
```
(x*(54*x**4 + 225*x**3 + 370*x**2 + 300*x + 120))/30
```

3.25 $\int (2+5x+3x^2)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 14

$$\int (2+5x+3x^2)\, dx = 2x + \frac{5x^2}{2} + x^3$$

output
```
2*x+5/2*x^2+x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (2+5x+3x^2)\, dx = 2x + \frac{5x^2}{2} + x^3$$

input
```
Integrate[2 + 5*x + 3*x^2,x]
```

output
```
2*x + (5*x^2)/2 + x^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^2+5x+2\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$x^3+\frac{5x^2}{2}+2x$$

input
```
Int[2 + 5*x + 3*x^2,x]
```

output
```
2*x + (5*x^2)/2 + x^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$2x+\frac{5}{2}x^{2}+x^{3}$	13
norman	$2x+\frac{5}{2}x^{2}+x^{3}$	13
risch	$2x+\frac{5}{2}x^{2}+x^{3}$	13
parallelrisch	$2x+\frac{5}{2}x^{2}+x^{3}$	13
parts	$2x+\frac{5}{2}x^{2}+x^{3}$	13
gosper	$\frac{x\left(2x^{2}+5x+4\right)}{2}$	14
orering	$\frac{x\left(2x^{2}+5x+4\right)\left(3x^{2}+5x+2\right)}{2\left(x+1\right)\left(3x+2\right)}$	36

input
```
int(3*x^2+5*x+2,x,method=_RETURNVERBOSE)
```

output
```
2*x+5/2*x^2+x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(2+5x+3x^2\right)\,dx=x^{3}+\frac{5}{2}\,x^{2}+2\,x$$

input
```
integrate(3*x^2+5*x+2,x, algorithm="fricas")
```

output
```
x^3 + 5/2*x^2 + 2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (2+5x+3x^2)\ dx = x^3 + \frac{5x^2}{2} + 2x$$

input
```
integrate(3*x**2+5*x+2,x)
```

output
```
x**3 + 5*x**2/2 + 2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (2+5x+3x^2)\ dx = x^3 + \frac{5}{2}\,x^2 + 2\,x$$

input
```
integrate(3*x^2+5*x+2,x, algorithm="maxima")
```

output
```
x^3 + 5/2*x^2 + 2*x
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (2+5x+3x^2)\ dx = x^3 + \frac{5}{2}\,x^2 + 2\,x$$

input
```
integrate(3*x^2+5*x+2,x, algorithm="giac")
```

output
```
x^3 + 5/2*x^2 + 2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \left(2+5x+3x^2\right)\,dx=\frac{x\,\left(2\,x^2+5\,x+4\right)}{2}$$

input
```
int(5*x + 3*x^2 + 2,x)
```

output
```
(x*(5*x + 2*x^2 + 4))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \left(2+5x+3x^2\right)\,dx=\frac{x(2x^2+5x+4)}{2}$$

input
```
int(3*x^2+5*x+2,x)
```

output
```
(x*(2*x**2 + 5*x + 4))/2
```

3.26 $\int \frac{1}{2+5x+3x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{1}{2+5x+3x^2} dx = -\log(1+x) + \log(2+3x)$$

output
```
-ln(1+x)+ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+5x+3x^2} dx = -\log(1+x) + \log(2+3x)$$

input
```
Integrate[(2 + 5*x + 3*x^2)^(-1),x]
```

output
```
-Log[1 + x] + Log[2 + 3*x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^2+5x+2} \, dx$$

$$\Big\downarrow \text{1081}$$

$$3 \int \left(\frac{1}{3x+2} - \frac{1}{3(x+1)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$3 \left(\frac{1}{3} \log(3x+2) - \frac{1}{3} \log(x+1)\right)$$

input
```
Int[(2 + 5*x + 3*x^2)^(-1),x]
```

output
```
3*(-1/3*Log[1 + x] + Log[2 + 3*x]/3)
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$-\ln(x+1)+\ln\left(x+\frac{2}{3}\right)$	12
default	$-\ln(x+1)+\ln(3x+2)$	14
norman	$-\ln(x+1)+\ln(3x+2)$	14
risch	$-\ln(x+1)+\ln(3x+2)$	14

input
```
int(1/(3*x^2+5*x+2),x,method=_RETURNVERBOSE)
```

output
```
-ln(x+1)+ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+5x+3x^2}\,dx = \log(3\,x+2) - \log(x+1)$$

input
```
integrate(1/(3*x^2+5*x+2),x, algorithm="fricas")
```

output
```
log(3*x + 2) - log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{2+5x+3x^2}\,dx = \log\left(x+\frac{2}{3}\right) - \log(x+1)$$

input
```
integrate(1/(3*x**2+5*x+2),x)
```

output
```
log(x + 2/3) - log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+5x+3x^2} dx = \log(3x+2) - \log(x+1)$$

input
```
integrate(1/(3*x^2+5*x+2),x, algorithm="maxima")
```

output
```
log(3*x + 2) - log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{2+5x+3x^2} dx = \log(|3x+2|) - \log(|x+1|)$$

input
```
integrate(1/(3*x^2+5*x+2),x, algorithm="giac")
```

output
```
log(abs(3*x + 2)) - log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{2+5x+3x^2} dx = -2 \operatorname{atanh}(6x+5)$$

input
```
int(1/(5*x + 3*x^2 + 2),x)
```

output
```
-2*atanh(6*x + 5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+5x+3x^2}\,dx = \log(3x+2) - \log(x+1)$$

input
```
int(1/(3*x^2+5*x+2),x)
```

output
```
log(3*x + 2) - log(x + 1)
```

3.27 $\int \frac{1}{(2+5x+3x^2)^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{(2+5x+3x^2)^2} \, dx = -\frac{1}{1+x} - \frac{3}{2+3x} + 6\log(1+x) - 6\log(2+3x)$$

output
```
-1/(1+x)-3/(2+3*x)+6*ln(1+x)-6*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(2+5x+3x^2)^2} \, dx = \frac{-5-6x}{2+5x+3x^2} + 6\log(1+x) - 6\log(2+3x)$$

input
```
Integrate[(2 + 5*x + 3*x^2)^(-2),x]
```

output
```
(-5 - 6*x)/(2 + 5*x + 3*x^2) + 6*Log[1 + x] - 6*Log[2 + 3*x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2+5x+2)^2}dx$$

$$\Big\downarrow \text{1084}$$

$$9\int \left(-\frac{2}{3x+2}+\frac{1}{(3x+2)^2}+\frac{2}{3(x+1)}+\frac{1}{9(x+1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$9\left(-\frac{1}{9(x+1)}-\frac{1}{3(3x+2)}+\frac{2}{3}\log(x+1)-\frac{2}{3}\log(3x+2)\right)$$

input
```
Int[(2 + 5*x + 3*x^2)^(-2),x]
```

output
```
9*(-1/9*1/(1 + x) - 1/(3*(2 + 3*x)) + (2*Log[1 + x])/3 - (2*Log[2 + 3*x])/
3)
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{1}{x+1}-\frac{3}{3x+2}+6\ln\left(x+1\right)-6\ln\left(3x+2\right)$	32
risch	$\frac{-2x-\frac{5}{3}}{x^{2}+\frac{5}{3}x+\frac{2}{3}}+6\ln\left(x+1\right)-6\ln\left(3x+2\right)$	32
norman	$\frac{\frac{15}{2}x^{2}+\frac{13}{2}x}{3x^{2}+5x+2}+6\ln\left(x+1\right)-6\ln\left(3x+2\right)$	38
parallelrisch	$\frac{36\ln(x+1)x^{2}-36\ln\left(x+\frac{2}{3}\right)x^{2}+60\ln(x+1)x-60\ln\left(x+\frac{2}{3}\right)x+15x^{2}+24\ln(x+1)-24\ln\left(x+\frac{2}{3}\right)+13x}{6x^{2}+10x+4}$	68

input
```
int(1/(3*x^2+5*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(x+1)-3/(3*x+2)+6*ln(x+1)-6*ln(3*x+2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{1}{\left(2+5x+3x^2\right)^2}\,dx = -\frac{6\,(3\,x^2+5\,x+2)\log\left(3\,x+2\right)-6\,(3\,x^2+5\,x+2)\log\left(x+1\right)+6\,x+5}{3\,x^2+5\,x+2}$$

input
```
integrate(1/(3*x^2+5*x+2)^2,x, algorithm="fricas")
```

output
```
-(6*(3*x^2 + 5*x + 2)*log(3*x + 2) - 6*(3*x^2 + 5*x + 2)*log(x + 1) + 6*x
+ 5)/(3*x^2 + 5*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+5x+3x^2)^2}\,dx = \frac{-6x-5}{3x^2+5x+2} - 6\log\left(x+\frac{2}{3}\right) + 6\log(x+1)$$

input
```
integrate(1/(3*x**2+5*x+2)**2,x)
```

output
```
(-6*x - 5)/(3*x**2 + 5*x + 2) - 6*log(x + 2/3) + 6*log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1}{(2+5x+3x^2)^2}\,dx = -\frac{6\,x+5}{3\,x^2+5\,x+2} - 6\,\log(3\,x+2) + 6\,\log(x+1)$$

input
```
integrate(1/(3*x^2+5*x+2)^2,x, algorithm="maxima")
```

output
```
-(6*x + 5)/(3*x^2 + 5*x + 2) - 6*log(3*x + 2) + 6*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1}{(2+5x+3x^2)^2}\,dx = -\frac{6\,x+5}{3\,x^2+5\,x+2} - 6\,\log(|3\,x+2|) + 6\,\log(|x+1|)$$

input
```
integrate(1/(3*x^2+5*x+2)^2,x, algorithm="giac")
```

output
```
-(6*x + 5)/(3*x^2 + 5*x + 2) - 6*log(abs(3*x + 2)) + 6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1}{(2+5x+3x^2)^2}\,dx = -6\,\ln\left(\frac{3\,x+2}{x+1}\right) - \frac{2\,\left(3\,x+\frac{5}{2}\right)}{3\,x^2+5\,x+2}$$

input
```
int(1/(5*x + 3*x^2 + 2)^2,x)
```

output
```
- 6*log((3*x + 2)/(x + 1)) - (2*(3*x + 5/2))/(5*x + 3*x^2 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \frac{1}{(2+5x+3x^2)^2}\,dx$$
$$= \frac{-90\log(3x+2)\,x^2 - 150\log(3x+2)\,x - 60\log(3x+2) + 90\log(x+1)\,x^2 + 150\log(x+1)\,x + 60\log(}{15x^2+25x+10}$$

input
```
int(1/(3*x^2+5*x+2)^2,x)
```

output
```
( - 90*log(3*x + 2)*x**2 - 150*log(3*x + 2)*x - 60*log(3*x + 2) + 90*log(x
 + 1)*x**2 + 150*log(x + 1)*x + 60*log(x + 1) + 18*x**2 - 13)/(5*(3*x**2 +
 5*x + 2))
```

3.28 $\int \frac{1}{(2+5x+3x^2)^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{1}{(2+5x+3x^2)^3} dx = \frac{1}{2(1+x)^2} + \frac{9}{1+x} - \frac{9}{2(2+3x)^2} + \frac{27}{2+3x} - 54\log(1+x) + 54\log(2+3x)$$

output
```
1/2/(1+x)^2+9/(1+x)-9/2/(2+3*x)^2+27/(2+3*x)-54*ln(1+x)+54*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{1}{(2+5x+3x^2)^3} dx = \frac{-5-6x}{2(2+5x+3x^2)^2} + \frac{9(5+6x)}{2+5x+3x^2} - 54\log(1+x) + 54\log(2+3x)$$

input
```
Integrate[(2 + 5*x + 3*x^2)^(-3),x]
```

output
```
(-5 - 6*x)/(2*(2 + 5*x + 3*x^2)^2) + (9*(5 + 6*x))/(2 + 5*x + 3*x^2) - 54*
Log[1 + x] + 54*Log[2 + 3*x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2+5x+2)^3}dx$$

$$\Big\downarrow \text{1084}$$

$$27\int \left(\frac{6}{3x+2}-\frac{3}{(3x+2)^2}+\frac{1}{(3x+2)^3}-\frac{2}{x+1}-\frac{1}{3(x+1)^2}-\frac{1}{27(x+1)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$27\left(\frac{1}{3(x+1)}+\frac{1}{3x+2}+\frac{1}{54(x+1)^2}-\frac{1}{6(3x+2)^2}-2\log(x+1)+2\log(3x+2)\right)$$

input
```
Int[(2 + 5*x + 3*x^2)^(-3),x]
```

output
```
27*(1/(54*(1 + x)^2) + 1/(3*(1 + x)) - 1/(6*(2 + 3*x)^2) + (2 + 3*x)^(-1)
- 2*Log[1 + x] + 2*Log[2 + 3*x])
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result
norman	$\frac{162x^3+330x+405x^2+\frac{175}{2}}{(3x^2+5x+2)^2}-54\ln(x+1)+54\ln(3x+2)$
risch	$\frac{162x^3+330x+405x^2+\frac{175}{2}}{(3x^2+5x+2)^2}-54\ln(x+1)+54\ln(3x+2)$
default	$\frac{1}{2(x+1)^2}+\frac{9}{x+1}-\frac{9}{2(3x+2)^2}+\frac{27}{3x+2}-54\ln(x+1)+54\ln(3x+2)$
parallelrisch	$-\frac{3888\ln(x+1)x^4-3888\ln\left(x+\frac{2}{3}\right)x^4+12960\ln(x+1)x^3-12960\ln\left(x+\frac{2}{3}\right)x^3+1575x^4+15984\ln(x+1)x^2-15984\ln\left(x+\frac{2}{3}\right)x^2+}{8(3x^2+5x+2)^2}$

input
```
int(1/(3*x^2+5*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(162*x^3+330*x+405*x^2+175/2)/(3*x^2+5*x+2)^2-54*ln(x+1)+54*ln(3*x+2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.82

$$\int \frac{1}{(2+5x+3x^2)^3}\,dx = \frac{324\,x^3+810\,x^2+108\,(9\,x^4+30\,x^3+37\,x^2+20\,x+4)\log(3\,x+2)-108\,(9\,x^4+30\,x^3+37\,x^2+20\,x}{2\,(9\,x^4+30\,x^3+37\,x^2+20\,x+4)}$$

input
```
integrate(1/(3*x^2+5*x+2)^3,x, algorithm="fricas")
```

output
```
1/2*(324*x^3 + 810*x^2 + 108*(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)*log(3*x
+ 2) - 108*(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)*log(x + 1) + 660*x + 175)/
(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{1}{(2+5x+3x^2)^3}\,dx = \frac{324x^3+810x^2+660x+175}{18x^4+60x^3+74x^2+40x+8} + 54\log\left(x+\frac{2}{3}\right) - 54\log\left(x+1\right)$$

input
```
integrate(1/(3*x**2+5*x+2)**3,x)
```

output
```
(324*x**3 + 810*x**2 + 660*x + 175)/(18*x**4 + 60*x**3 + 74*x**2 + 40*x +
8) + 54*log(x + 2/3) - 54*log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{1}{(2+5x+3x^2)^3}\,dx = \frac{324\,x^3+810\,x^2+660\,x+175}{2\,(9\,x^4+30\,x^3+37\,x^2+20\,x+4)} + 54\,\log\left(3\,x+2\right) - 54\,\log\left(x+1\right)$$

input
```
integrate(1/(3*x^2+5*x+2)^3,x, algorithm="maxima")
```

output
```
1/2*(324*x^3 + 810*x^2 + 660*x + 175)/(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)
 + 54*log(3*x + 2) - 54*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{1}{(2+5x+3x^2)^3}\,dx = \frac{324\,x^3+810\,x^2+660\,x+175}{2\,(3\,x^2+5\,x+2)^2} + 54\,\log\left(|3\,x+2|\right) - 54\,\log\left(|x+1|\right)$$

input
```
integrate(1/(3*x^2+5*x+2)^3,x, algorithm="giac")
```

output
```
1/2*(324*x^3 + 810*x^2 + 660*x + 175)/(3*x^2 + 5*x + 2)^2 + 54*log(abs(3*x
 + 2)) - 54*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+5x+3x^2)^3}\,dx = 54\,\ln\left(\frac{3\,x+2}{x+1}\right) + 2\left(3\,x+\frac{5}{2}\right)\left(\frac{9}{3\,x^2+5\,x+2}-\frac{1}{2\,(3\,x^2+5\,x+2)^2}\right)$$

input
```
int(1/(5*x + 3*x^2 + 2)^3,x)
```

output
```
54*log((3*x + 2)/(x + 1)) + 2*(3*x + 5/2)*(9/(5*x + 3*x^2 + 2) - 1/(2*(5*x
 + 3*x^2 + 2)^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.51

$$\int \frac{1}{(2+5x+3x^2)^3}\,dx = \frac{4860\log(3x+2)\,x^4+16200\log(3x+2)\,x^3+19980\log(3x+2)\,x^2+10800\log(3x+2)\,x+2160\log(3x$$

input
```
int(1/(3*x^2+5*x+2)^3,x)
```

output
```
(4860*log(3*x + 2)*x**4 + 16200*log(3*x + 2)*x**3 + 19980*log(3*x + 2)*x**
2 + 10800*log(3*x + 2)*x + 2160*log(3*x + 2) - 4860*log(x + 1)*x**4 - 1620
0*log(x + 1)*x**3 - 19980*log(x + 1)*x**2 - 10800*log(x + 1)*x - 2160*log(
x + 1) - 486*x**4 + 2052*x**2 + 2220*x + 659)/(10*(9*x**4 + 30*x**3 + 37*x
**2 + 20*x + 4))
```

3.29 $\int (2+5x-3x^2)^3 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int (2+5x-3x^2)^3 \, dx = \frac{343}{324}(1+3x)^4 - \frac{49}{135}(1+3x)^5 + \frac{7}{162}(1+3x)^6 - \frac{1}{567}(1+3x)^7$$

output
```
343/324*(1+3*x)^4-49/135*(1+3*x)^5+7/162*(1+3*x)^6-1/567*(1+3*x)^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int (2+5x-3x^2)^3 \, dx = 8x + 30x^2 + 38x^3 - \frac{55x^4}{4} - \frac{171x^5}{5} + \frac{45x^6}{2} - \frac{27x^7}{7}$$

input
```
Integrate[(2 + 5*x - 3*x^2)^3,x]
```

output
```
8*x + 30*x^2 + 38*x^3 - (55*x^4)/4 - (171*x^5)/5 + (45*x^6)/2 - (27*x^7)/7
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-3x^2+5x+2\right)^3 \, dx$$

$$\Big\downarrow \text{1084}$$

$$-\frac{1}{27} \int \left((-3x-1)^6-21(3x+1)^5+147(3x+1)^4-343(3x+1)^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{27} \left(-\frac{1}{21}(3x+1)^7+\frac{7}{6}(3x+1)^6-\frac{49}{5}(3x+1)^5+\frac{343}{12}(3x+1)^4\right)$$

input
```
Int[(2 + 5*x - 3*x^2)^3,x]
```

output
```
((343*(1 + 3*x)^4)/12 - (49*(1 + 3*x)^5)/5 + (7*(1 + 3*x)^6)/6 - (1 + 3*x)
^7/21)/27
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{x(540x^6-3150x^5+4788x^4+1925x^3-5320x^2-4200x-1120)}{140}$	34
default	$-\frac{27}{7}x^7+\frac{45}{2}x^6-\frac{171}{5}x^5-\frac{55}{4}x^4+38x^3+30x^2+8x$	35
norman	$-\frac{27}{7}x^7+\frac{45}{2}x^6-\frac{171}{5}x^5-\frac{55}{4}x^4+38x^3+30x^2+8x$	35
risch	$-\frac{27}{7}x^7+\frac{45}{2}x^6-\frac{171}{5}x^5-\frac{55}{4}x^4+38x^3+30x^2+8x$	35
parallelrisch	$-\frac{27}{7}x^7+\frac{45}{2}x^6-\frac{171}{5}x^5-\frac{55}{4}x^4+38x^3+30x^2+8x$	35
orering	$\frac{x(540x^6-3150x^5+4788x^4+1925x^3-5320x^2-4200x-1120)\left(-3x^2+5x+2\right)^3}{140(3x+1)^3(x-2)^3}$	58

input
```
int((-3*x^2+5*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/140*x*(540*x^6-3150*x^5+4788*x^4+1925*x^3-5320*x^2-4200*x-1120)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \left(2+5x-3x^2\right)^3\,dx=-\frac{27}{7}\,x^7+\frac{45}{2}\,x^6-\frac{171}{5}\,x^5-\frac{55}{4}\,x^4+38\,x^3+30\,x^2+8\,x$$

input
```
integrate((-3*x^2+5*x+2)^3,x, algorithm="fricas")
```

output
```
-27/7*x^7 + 45/2*x^6 - 171/5*x^5 - 55/4*x^4 + 38*x^3 + 30*x^2 + 8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \left(2+5x-3x^2\right)^3 \, dx = -\frac{27x^7}{7} + \frac{45x^6}{2} - \frac{171x^5}{5} - \frac{55x^4}{4} + 38x^3 + 30x^2 + 8x$$

input
```
integrate((-3*x**2+5*x+2)**3,x)
```

output
```
-27*x**7/7 + 45*x**6/2 - 171*x**5/5 - 55*x**4/4 + 38*x**3 + 30*x**2 + 8*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \left(2+5x-3x^2\right)^3 \, dx = -\frac{27}{7}\,x^7 + \frac{45}{2}\,x^6 - \frac{171}{5}\,x^5 - \frac{55}{4}\,x^4 + 38\,x^3 + 30\,x^2 + 8\,x$$

input
```
integrate((-3*x^2+5*x+2)^3,x, algorithm="maxima")
```

output
```
-27/7*x^7 + 45/2*x^6 - 171/5*x^5 - 55/4*x^4 + 38*x^3 + 30*x^2 + 8*x
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \left(2+5x-3x^2\right)^3 \, dx = -\frac{27}{7}\,x^7 + \frac{45}{2}\,x^6 - \frac{171}{5}\,x^5 - \frac{55}{4}\,x^4 + 38\,x^3 + 30\,x^2 + 8\,x$$

input
```
integrate((-3*x^2+5*x+2)^3,x, algorithm="giac")
```

output
```
-27/7*x^7 + 45/2*x^6 - 171/5*x^5 - 55/4*x^4 + 38*x^3 + 30*x^2 + 8*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \left(2+5x-3x^2\right)^3\,dx=-\frac{27\,x^7}{7}+\frac{45\,x^6}{2}-\frac{171\,x^5}{5}-\frac{55\,x^4}{4}+38\,x^3+30\,x^2+8\,x$$

input
```
int((5*x - 3*x^2 + 2)^3,x)
```

output
```
8*x + 30*x^2 + 38*x^3 - (55*x^4)/4 - (171*x^5)/5 + (45*x^6)/2 - (27*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \left(2+5x-3x^2\right)^3\,dx$$
$$=\frac{x(-540x^6+3150x^5-4788x^4-1925x^3+5320x^2+4200x+1120)}{140}$$

input
```
int((-3*x^2+5*x+2)^3,x)
```

output
```
(x*( - 540*x**6 + 3150*x**5 - 4788*x**4 - 1925*x**3 + 5320*x**2 + 4200*x +
 1120))/140
```

3.30 $\int (2+5x-3x^2)^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int (2+5x-3x^2)^2 \, dx = \frac{49}{81}(1+3x)^3 - \frac{7}{54}(1+3x)^4 + \frac{1}{135}(1+3x)^5$$

output
```
49/81*(1+3*x)^3-7/54*(1+3*x)^4+1/135*(1+3*x)^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (2+5x-3x^2)^2 \, dx = 4x + 10x^2 + \frac{13x^3}{3} - \frac{15x^4}{2} + \frac{9x^5}{5}$$

input
```
Integrate[(2 + 5*x - 3*x^2)^2,x]
```

output
```
4*x + 10*x^2 + (13*x^3)/3 - (15*x^4)/2 + (9*x^5)/5
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-3x^2+5x+2\right)^2 \, dx$$

$$\Big\downarrow \text{1084}$$

$$\frac{1}{9}\int \left((-3x-1)^4-14(3x+1)^3+49(3x+1)^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}\left(\frac{1}{15}(3x+1)^5-\frac{7}{6}(3x+1)^4+\frac{49}{9}(3x+1)^3\right)$$

input
```
Int[(2 + 5*x - 3*x^2)^2,x]
```

output
```
((49*(1 + 3*x)^3)/9 - (7*(1 + 3*x)^4)/6 + (1 + 3*x)^5/15)/9
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{x(54x^4-225x^3+130x^2+300x+120)}{30}$	24
default	$\frac{9}{5}x^5-\frac{15}{2}x^4+\frac{13}{3}x^3+10x^2+4x$	25
norman	$\frac{9}{5}x^5-\frac{15}{2}x^4+\frac{13}{3}x^3+10x^2+4x$	25
risch	$\frac{9}{5}x^5-\frac{15}{2}x^4+\frac{13}{3}x^3+10x^2+4x$	25
parallelrisch	$\frac{9}{5}x^5-\frac{15}{2}x^4+\frac{13}{3}x^3+10x^2+4x$	25
orering	$\frac{x(54x^4-225x^3+130x^2+300x+120)(-3x^2+5x+2)^2}{30(x-2)^2(3x+1)^2}$	48

input
```
int((-3*x^2+5*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*x*(54*x^4-225*x^3+130*x^2+300*x+120)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int (2+5x-3x^2)^2 \, dx = \frac{9}{5}\, x^5 - \frac{15}{2}\, x^4 + \frac{13}{3}\, x^3 + 10\, x^2 + 4\, x$$

input
```
integrate((-3*x^2+5*x+2)^2,x, algorithm="fricas")
```

output
```
9/5*x^5 - 15/2*x^4 + 13/3*x^3 + 10*x^2 + 4*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \left(2+5x-3x^2\right)^2\,dx=\frac{9x^5}{5}-\frac{15x^4}{2}+\frac{13x^3}{3}+10x^2+4x$$

input
```
integrate((-3*x**2+5*x+2)**2,x)
```

output
```
9*x**5/5 - 15*x**4/2 + 13*x**3/3 + 10*x**2 + 4*x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(2+5x-3x^2\right)^2\,dx=\frac{9}{5}\,x^5-\frac{15}{2}\,x^4+\frac{13}{3}\,x^3+10\,x^2+4\,x$$

input
```
integrate((-3*x^2+5*x+2)^2,x, algorithm="maxima")
```

output
```
9/5*x^5 - 15/2*x^4 + 13/3*x^3 + 10*x^2 + 4*x
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(2+5x-3x^2\right)^2\,dx=\frac{9}{5}\,x^5-\frac{15}{2}\,x^4+\frac{13}{3}\,x^3+10\,x^2+4\,x$$

input
```
integrate((-3*x^2+5*x+2)^2,x, algorithm="giac")
```

output
```
9/5*x^5 - 15/2*x^4 + 13/3*x^3 + 10*x^2 + 4*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(2+5x-3x^2\right)^2\,dx=\frac{9\,x^5}{5}-\frac{15\,x^4}{2}+\frac{13\,x^3}{3}+10\,x^2+4\,x$$

input
```
int((5*x - 3*x^2 + 2)^2,x)
```

output
```
4*x + 10*x^2 + (13*x^3)/3 - (15*x^4)/2 + (9*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \left(2+5x-3x^2\right)^2\,dx=\frac{x(54x^4-225x^3+130x^2+300x+120)}{30}$$

input
```
int((-3*x^2+5*x+2)^2,x)
```

output
```
(x*(54*x**4 - 225*x**3 + 130*x**2 + 300*x + 120))/30
```

3.31 $\int (2+5x-3x^2) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int (2+5x-3x^2) \, dx = 2x + \frac{5x^2}{2} - x^3$$

output
```
2*x+5/2*x^2-x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (2+5x-3x^2) \, dx = 2x + \frac{5x^2}{2} - x^3$$

input
```
Integrate[2 + 5*x - 3*x^2,x]
```

output
```
2*x + (5*x^2)/2 - x^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-3x^2+5x+2\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$-x^3+\frac{5x^2}{2}+2x$$

input
```
Int[2 + 5*x - 3*x^2,x]
```

output
```
2*x + (5*x^2)/2 - x^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{x(2x^2-5x-4)}{2}$	14
default	$2x+\frac{5}{2}x^2-x^3$	15
norman	$2x+\frac{5}{2}x^2-x^3$	15
risch	$2x+\frac{5}{2}x^2-x^3$	15
parallelrisch	$2x+\frac{5}{2}x^2-x^3$	15
parts	$2x+\frac{5}{2}x^2-x^3$	15
orering	$\frac{x(2x^2-5x-4)(-3x^2+5x+2)}{2(3x+1)(x-2)}$	36

input
```
int(-3*x^2+5*x+2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(2*x^2-5*x-4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (2+5x-3x^2)\ dx = -x^3+\frac{5}{2}x^2+2x$$

input
```
integrate(-3*x^2+5*x+2,x, algorithm="fricas")
```

output
```
-x^3 + 5/2*x^2 + 2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (2+5x-3x^2)\ dx = -x^3 + \frac{5x^2}{2} + 2x$$

input
```
integrate(-3*x**2+5*x+2,x)
```

output
```
-x**3 + 5*x**2/2 + 2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (2+5x-3x^2)\ dx = -x^3 + \frac{5}{2}\,x^2 + 2\,x$$

input
```
integrate(-3*x^2+5*x+2,x, algorithm="maxima")
```

output
```
-x^3 + 5/2*x^2 + 2*x
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (2+5x-3x^2)\ dx = -x^3 + \frac{5}{2}\,x^2 + 2\,x$$

input
```
integrate(-3*x^2+5*x+2,x, algorithm="giac")
```

output
```
-x^3 + 5/2*x^2 + 2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \left(2+5x-3x^2\right)\,dx=\frac{x\,\left(-2\,x^2+5\,x+4\right)}{2}$$

input
```
int(5*x - 3*x^2 + 2,x)
```

output
```
(x*(5*x - 2*x^2 + 4))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \left(2+5x-3x^2\right)\,dx=\frac{x(-2x^2+5x+4)}{2}$$

input
```
int(-3*x^2+5*x+2,x)
```

output
```
(x*( - 2*x**2 + 5*x + 4))/2
```

3.32 $\int \frac{1}{2+5x-3x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{1}{2+5x-3x^2} \, dx = -\frac{1}{7} \log(2-x) + \frac{1}{7} \log(1+3x)$$

output
```
-1/7*ln(2-x)+1/7*ln(1+3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+5x-3x^2} \, dx = -\frac{1}{7} \log(2-x) + \frac{1}{7} \log(1+3x)$$

input
```
Integrate[(2 + 5*x - 3*x^2)^(-1),x]
```

output
```
-1/7*Log[2 - x] + Log[1 + 3*x]/7
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-3x^2+5x+2} \, dx$$
$$\Big\downarrow \text{1081}$$
$$-3 \int \left(-\frac{1}{7(3x+1)}-\frac{1}{21(2-x)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-3\left(\frac{1}{21}\log(2-x)-\frac{1}{21}\log(3x+1)\right)$$

input
```
Int[(2 + 5*x - 3*x^2)^(-1),x]
```

output
```
-3*(Log[2 - x]/21 - Log[1 + 3*x]/21)
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{\ln(x-2)}{7}+\frac{\ln\left(x+\frac{1}{3}\right)}{7}$	14
default	$-\frac{\ln(x-2)}{7}+\frac{\ln(3x+1)}{7}$	16
norman	$-\frac{\ln(x-2)}{7}+\frac{\ln(3x+1)}{7}$	16
risch	$-\frac{\ln(x-2)}{7}+\frac{\ln(3x+1)}{7}$	16

input
```
int(1/(-3*x^2+5*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*ln(x-2)+1/7*ln(x+1/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{2+5x-3x^2}\,dx = \frac{1}{7}\log\left(3\,x+1\right) - \frac{1}{7}\log\left(x-2\right)$$

input
```
integrate(1/(-3*x^2+5*x+2),x, algorithm="fricas")
```

output
```
1/7*log(3*x + 1) - 1/7*log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{1}{2+5x-3x^2}\,dx = -\frac{\log\left(x-2\right)}{7} + \frac{\log\left(x+\frac{1}{3}\right)}{7}$$

input
```
integrate(1/(-3*x**2+5*x+2),x)
```

output
```
-log(x - 2)/7 + log(x + 1/3)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{2+5x-3x^2}\,dx = \frac{1}{7}\log\left(3\,x+1\right) - \frac{1}{7}\log\left(x-2\right)$$

input
```
integrate(1/(-3*x^2+5*x+2),x, algorithm="maxima")
```

output
```
1/7*log(3*x + 1) - 1/7*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{2+5x-3x^2}\,dx = \frac{1}{7}\log\left(|3\,x+1|\right) - \frac{1}{7}\log\left(|x-2|\right)$$

input
```
integrate(1/(-3*x^2+5*x+2),x, algorithm="giac")
```

output
```
1/7*log(abs(3*x + 1)) - 1/7*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{1}{2+5x-3x^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{6\,x}{7}-\frac{5}{7}\right)}{7}$$

input
```
int(1/(5*x - 3*x^2 + 2),x)
```

output
```
(2*atanh((6*x)/7 - 5/7))/7
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{2+5x-3x^2}\,dx = \frac{\log(3x+1)}{7} - \frac{\log(x-2)}{7}$$

input
```
int(1/(-3*x^2+5*x+2),x)
```

output
```
(log(3*x + 1) - log(x - 2))/7
```

3.33 $\int \frac{1}{(2+5x-3x^2)^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{(2+5x-3x^2)^2} \, dx = \frac{1}{49(2-x)} - \frac{3}{49(1+3x)} - \frac{6}{343}\log(2-x) + \frac{6}{343}\log(1+3x)$$

output
```
1/(98-49*x)-3/(49+147*x)-6/343*ln(2-x)+6/343*ln(1+3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{(2+5x-3x^2)^2} \, dx = \frac{5-6x}{49(-2-5x+3x^2)} - \frac{6}{343}\log(2-x) + \frac{6}{343}\log(1+3x)$$

input
```
Integrate[(2 + 5*x - 3*x^2)^(-2),x]
```

output
```
(5 - 6*x)/(49*(-2 - 5*x + 3*x^2)) - (6*Log[2 - x])/343 + (6*Log[1 + 3*x])/
343
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^2+5x+2)^2} \, dx$$

$$\Big\downarrow \text{ 1084}$$

$$9 \int \left(\frac{2}{343(3x+1)} + \frac{1}{49(3x+1)^2} + \frac{2}{1029(2-x)} + \frac{1}{441(2-x)^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$9\left(\frac{1}{441(2-x)} - \frac{1}{147(3x+1)} - \frac{2\log(2-x)}{1029} + \frac{2\log(3x+1)}{1029}\right)$$

input
```
Int[(2 + 5*x - 3*x^2)^(-2),x]
```

output
```
9*(1/(441*(2 - x)) - 1/(147*(1 + 3*x)) - (2*Log[2 - x])/1029 + (2*Log[1 +
3*x])/1029)
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{1}{49(x-2)}-\frac{6\ln(x-2)}{343}-\frac{3}{49(3x+1)}+\frac{6\ln(3x+1)}{343}$	32
risch	$\frac{-\frac{2x}{49}+\frac{5}{147}}{x^2-\frac{5}{3}x-\frac{2}{3}}-\frac{6\ln(x-2)}{343}+\frac{6\ln(3x+1)}{343}$	32
norman	$\frac{\frac{15}{98}x^2-\frac{37}{98}x}{3x^2-5x-2}-\frac{6\ln(x-2)}{343}+\frac{6\ln(3x+1)}{343}$	38
parallelrisch	$-\frac{36\ln(x-2)x^2-36\ln\left(x+\frac{1}{3}\right)x^2-60\ln(x-2)x+60\ln\left(x+\frac{1}{3}\right)x-105x^2-24\ln(x-2)+24\ln\left(x+\frac{1}{3}\right)+259x}{686(3x^2-5x-2)}$	68

input
```
int(1/(-3*x^2+5*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/49/(x-2)-6/343*ln(x-2)-3/49/(3*x+1)+6/343*ln(3*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = \frac{6\,(3\,x^2-5\,x-2)\log(3\,x+1)-6\,(3\,x^2-5\,x-2)\log(x-2)-42\,x+35}{343\,(3\,x^2-5\,x-2)}$$

input
```
integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="fricas")
```

output
```
1/343*(6*(3*x^2 - 5*x - 2)*log(3*x + 1) - 6*(3*x^2 - 5*x - 2)*log(x - 2) -
 42*x + 35)/(3*x^2 - 5*x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = \frac{5-6x}{147x^2-245x-98} - \frac{6\log(x-2)}{343} + \frac{6\log\left(x+\frac{1}{3}\right)}{343}$$

input
```
integrate(1/(-3*x**2+5*x+2)**2,x)
```

output
```
(5 - 6*x)/(147*x**2 - 245*x - 98) - 6*log(x - 2)/343 + 6*log(x + 1/3)/343
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = -\frac{6\,x-5}{49\,(3\,x^2-5\,x-2)} + \frac{6}{343}\,\log(3\,x+1) - \frac{6}{343}\,\log(x-2)$$

input
```
integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="maxima")
```

output
```
-1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*log(3*x + 1) - 6/343*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = -\frac{6\,x-5}{49\,(3\,x^2-5\,x-2)} + \frac{6}{343}\,\log(|3\,x+1|) - \frac{6}{343}\,\log(|x-2|)$$

input
```
integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="giac")
```

output
```
-1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*log(abs(3*x + 1)) - 6/343*log(ab
s(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = \frac{6\,\ln\left(\frac{3\,x+1}{x-2}\right)}{343} + \frac{2\,\left(3\,x-\frac{5}{2}\right)}{49\,\left(-3\,x^2+5\,x+2\right)}$$

input
```
int(1/(5*x - 3*x^2 + 2)^2,x)
```

output
```
(6*log((3*x + 1)/(x - 2)))/343 + (2*(3*x - 5/2))/(49*(5*x - 3*x^2 + 2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = \frac{90\log(3x+1)\,x^2 - 150\log(3x+1)\,x - 60\log(3x+1) - 90\log(x-2)\,x^2 + 150\log(x-2)\,x + 60\log(x}{5145x^2 - 8575x - 3430}$$

input
```
int(1/(-3*x^2+5*x+2)^2,x)
```

output
```
(90*log(3*x + 1)*x**2 - 150*log(3*x + 1)*x - 60*log(3*x + 1) - 90*log(x -
2)*x**2 + 150*log(x - 2)*x + 60*log(x - 2) - 126*x**2 + 259)/(1715*(3*x**2
 - 5*x - 2))
```

3.34 $\int \frac{1}{(2+5x-3x^2)^3} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \frac{1}{(2+5x-3x^2)^3} \, dx = \frac{1}{686(2-x)^2} + \frac{9}{2401(2-x)} - \frac{9}{686(1+3x)^2} - \frac{27}{2401(1+3x)} - \frac{54\log(2-x)}{16807} + \frac{54\log(1+3x)}{16807}$$

output
```
1/686/(2-x)^2+9/(4802-2401*x)-9/686/(1+3*x)^2-27/(2401+7203*x)-54/16807*ln
(2-x)+54/16807*ln(1+3*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{1}{(2+5x-3x^2)^3} \, dx = \frac{-\frac{7(425-60x-810x^2+324x^3)}{(2+5x-3x^2)^2} - 108\log(2-x) + 108\log(1+3x)}{33614}$$

input
```
Integrate[(2 + 5*x - 3*x^2)^(-3),x]
```

output
```
((-7*(425 - 60*x - 810*x^2 + 324*x^3))/(2 + 5*x - 3*x^2)^2 - 108*Log[2 - x
] + 108*Log[1 + 3*x])/33614
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-3x^2+5x+2\right)^3}dx$$

$$\Big\downarrow \text{1084}$$

$$-27\int \left(-\frac{6}{16807(3x+1)}-\frac{3}{2401(3x+1)^2}-\frac{1}{343(3x+1)^3}-\frac{2}{16807(2-x)}-\frac{1}{7203(2-x)^2}-\frac{1}{9261(2-x)^3}\right)d$$

$$\Big\downarrow \text{2009}$$

$$-27\left(-\frac{1}{7203(2-x)}+\frac{1}{2401(3x+1)}-\frac{1}{18522(2-x)^2}+\frac{1}{2058(3x+1)^2}+\frac{2\log(2-x)}{16807}-\frac{2\log(3x+1)}{16807}\right)$$

input
```
Int[(2 + 5*x - 3*x^2)^(-3),x]
```

output
```
-27*(-1/18522*1/(2 - x)^2 - 1/(7203*(2 - x)) + 1/(2058*(1 + 3*x)^2) + 1/(2
401*(1 + 3*x)) + (2*Log[2 - x])/16807 - (2*Log[1 + 3*x])/16807)
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

method	result
norman	$\frac{-\frac{162}{2401}x^3+\frac{30}{2401}x+\frac{405}{2401}x^2-\frac{425}{4802}}{(3x^2-5x-2)^2}-\frac{54\ln(x-2)}{16807}+\frac{54\ln(3x+1)}{16807}$
risch	$\frac{-\frac{162}{2401}x^3+\frac{30}{2401}x+\frac{405}{2401}x^2-\frac{425}{4802}}{(3x^2-5x-2)^2}-\frac{54\ln(x-2)}{16807}+\frac{54\ln(3x+1)}{16807}$
default	$\frac{1}{686(x-2)^2}-\frac{9}{2401(x-2)}-\frac{54\ln(x-2)}{16807}-\frac{9}{686(3x+1)^2}-\frac{27}{2401(3x+1)}+\frac{54\ln(3x+1)}{16807}$
parallelrisch	$-\frac{3888\ln(x-2)x^4-3888\ln(x+\frac{1}{3})x^4-12960\ln(x-2)x^3+12960\ln(x+\frac{1}{3})x^3-26775x^4+5616\ln(x-2)x^2-5616\ln(x+\frac{1}{3})x^2+9}{134456(3x^2-5x-2)^2}$

input
```
int(1/(-3*x^2+5*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-162/2401*x^3+30/2401*x+405/2401*x^2-425/4802)/(3*x^2-5*x-2)^2-54/16807*l
n(x-2)+54/16807*ln(3*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{1}{(2+5x-3x^2)^3}\,dx = -\frac{2268\,x^3-5670\,x^2-108\,(9\,x^4-30\,x^3+13\,x^2+20\,x+4)\log(3\,x+1)+108\,(9\,x^4-30\,x^3+13\,x^2+2}{33614\,(9\,x^4-30\,x^3+13\,x^2+20\,x+4)}$$

input
```
integrate(1/(-3*x^2+5*x+2)^3,x, algorithm="fricas")
```

output
```
-1/33614*(2268*x^3 - 5670*x^2 - 108*(9*x^4 - 30*x^3 + 13*x^2 + 20*x + 4)*l
og(3*x + 1) + 108*(9*x^4 - 30*x^3 + 13*x^2 + 20*x + 4)*log(x - 2) - 420*x
+ 2975)/(9*x^4 - 30*x^3 + 13*x^2 + 20*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2+5x-3x^2)^3}\,dx = -\frac{324x^3-810x^2-60x+425}{43218x^4-144060x^3+62426x^2+96040x+19208} - \frac{54\log(x-2)}{16807} + \frac{54\log\left(x+\frac{1}{3}\right)}{16807}$$

input
```
integrate(1/(-3*x**2+5*x+2)**3,x)
```

output
```
-(324*x**3 - 810*x**2 - 60*x + 425)/(43218*x**4 - 144060*x**3 + 62426*x**2
 + 96040*x + 19208) - 54*log(x - 2)/16807 + 54*log(x + 1/3)/16807
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{1}{(2+5x-3x^2)^3}\,dx = -\frac{324\,x^3-810\,x^2-60\,x+425}{4802\,(9\,x^4-30\,x^3+13\,x^2+20\,x+4)} + \frac{54}{16807}\log(3\,x+1) - \frac{54}{16807}\log(x-2)$$

input
```
integrate(1/(-3*x^2+5*x+2)^3,x, algorithm="maxima")
```

output
```
-1/4802*(324*x^3 - 810*x^2 - 60*x + 425)/(9*x^4 - 30*x^3 + 13*x^2 + 20*x +
 4) + 54/16807*log(3*x + 1) - 54/16807*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{1}{(2+5x-3x^2)^3}\,dx = -\frac{324\,x^3-810\,x^2-60\,x+425}{4802\,(3\,x^2-5\,x-2)^2} + \frac{54}{16807}\,\log\left(|3\,x+1|\right) - \frac{54}{16807}\,\log\left(|x-2|\right)$$

input
```
integrate(1/(-3*x^2+5*x+2)^3,x, algorithm="giac")
```

output
```
-1/4802*(324*x^3 - 810*x^2 - 60*x + 425)/(3*x^2 - 5*x - 2)^2 + 54/16807*lo
g(abs(3*x + 1)) - 54/16807*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{1}{(2+5x-3x^2)^3}\,dx = \frac{54\,\ln\left(\frac{3\,x+1}{x-2}\right)}{16807} + 2\left(3\,x-\frac{5}{2}\right)\left(\frac{9}{2401\,(-3\,x^2+5\,x+2)} + \frac{1}{98\,(-3\,x^2+5\,x+2)^2}\right)$$

input
```
int(1/(5*x - 3*x^2 + 2)^3,x)
```

output
```
(54*log((3*x + 1)/(x - 2)))/16807 + 2*(3*x - 5/2)*(9/(2401*(5*x - 3*x^2 +
2)) + 1/(98*(5*x - 3*x^2 + 2)^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int \frac{1}{(2+5x-3x^2)^3}\,dx = \frac{4860\log(3x+1)\,x^4-16200\log(3x+1)\,x^3+7020\log(3x+1)\,x^2+10800\log(3x+1)\,x+2160\log(3x+}{1512630}$$

input
```
int(1/(-3*x^2+5*x+2)^3,x)
```

output
```
(4860*log(3*x + 1)*x**4 - 16200*log(3*x + 1)*x**3 + 7020*log(3*x + 1)*x**2
 + 10800*log(3*x + 1)*x + 2160*log(3*x + 1) - 4860*log(x - 2)*x**4 + 16200
*log(x - 2)*x**3 - 7020*log(x - 2)*x**2 - 10800*log(x - 2)*x - 2160*log(x
- 2) - 3402*x**4 + 23436*x**2 - 5460*x - 16387)/(168070*(9*x**4 - 30*x**3
+ 13*x**2 + 20*x + 4))
```

3.35 $\int (1-x-x^2)^4 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 52

$$\int (1-x-x^2)^4 \, dx = x-2x^2+\frac{2x^3}{3}+2x^4-x^5-\frac{4x^6}{3}+\frac{2x^7}{7}+\frac{x^8}{2}+\frac{x^9}{9}$$

output
```
x-2*x^2+2/3*x^3+2*x^4-x^5-4/3*x^6+2/7*x^7+1/2*x^8+1/9*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int (1-x-x^2)^4 \, dx = x-2x^2+\frac{2x^3}{3}+2x^4-x^5-\frac{4x^6}{3}+\frac{2x^7}{7}+\frac{x^8}{2}+\frac{x^9}{9}$$

input
```
Integrate[(1 - x - x^2)^4,x]
```

output
```
x - 2*x^2 + (2*x^3)/3 + 2*x^4 - x^5 - (4*x^6)/3 + (2*x^7)/7 + x^8/2 + x^9/
9
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-x^2-x+1\right)^4 dx$$

$$\Big\downarrow \text{ 1085}$$

$$\int \left(x^8+4x^7+2x^6-8x^5-5x^4+8x^3+2x^2-4x+1\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^9}{9}+\frac{x^8}{2}+\frac{2x^7}{7}-\frac{4x^6}{3}-x^5+2x^4+\frac{2x^3}{3}-2x^2+x$$

input
```
Int[(1 - x - x^2)^4,x]
```

output
```
x - 2*x^2 + (2*x^3)/3 + 2*x^4 - x^5 - (4*x^6)/3 + (2*x^7)/7 + x^8/2 + x^9/
9
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

method	result	size
default	$x-2x^2+\frac{2}{3}x^3+2x^4-x^5-\frac{4}{3}x^6+\frac{2}{7}x^7+\frac{1}{2}x^8+\frac{1}{9}x^9$	43
norman	$x-2x^2+\frac{2}{3}x^3+2x^4-x^5-\frac{4}{3}x^6+\frac{2}{7}x^7+\frac{1}{2}x^8+\frac{1}{9}x^9$	43
risch	$x-2x^2+\frac{2}{3}x^3+2x^4-x^5-\frac{4}{3}x^6+\frac{2}{7}x^7+\frac{1}{2}x^8+\frac{1}{9}x^9$	43
parallelrisch	$x-2x^2+\frac{2}{3}x^3+2x^4-x^5-\frac{4}{3}x^6+\frac{2}{7}x^7+\frac{1}{2}x^8+\frac{1}{9}x^9$	43
gosper	$\frac{x(14x^8+63x^7+36x^6-168x^5-126x^4+252x^3+84x^2-252x+126)}{126}$	44
orering	$\frac{x(14x^8+63x^7+36x^6-168x^5-126x^4+252x^3+84x^2-252x+126)(-x^2-x+1)^4}{126(x^2+x-1)^4}$	64

input
```
int((-x^2-x+1)^4,x,method=_RETURNVERBOSE)
```

output
```
x-2*x^2+2/3*x^3+2*x^4-x^5-4/3*x^6+2/7*x^7+1/2*x^8+1/9*x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \left(1-x-x^2\right)^4\,dx=\frac{1}{9}\,x^9+\frac{1}{2}\,x^8+\frac{2}{7}\,x^7-\frac{4}{3}\,x^6-x^5+2\,x^4+\frac{2}{3}\,x^3-2\,x^2+x$$

input
```
integrate((-x^2-x+1)^4,x, algorithm="fricas")
```

output
```
1/9*x^9 + 1/2*x^8 + 2/7*x^7 - 4/3*x^6 - x^5 + 2*x^4 + 2/3*x^3 - 2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int (1-x-x^2)^4 \, dx = \frac{x^9}{9} + \frac{x^8}{2} + \frac{2x^7}{7} - \frac{4x^6}{3} - x^5 + 2x^4 + \frac{2x^3}{3} - 2x^2 + x$$

input
```
integrate((-x**2-x+1)**4,x)
```

output
```
x**9/9 + x**8/2 + 2*x**7/7 - 4*x**6/3 - x**5 + 2*x**4 + 2*x**3/3 - 2*x**2
+ x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int (1-x-x^2)^4 \, dx = \frac{1}{9} x^9 + \frac{1}{2} x^8 + \frac{2}{7} x^7 - \frac{4}{3} x^6 - x^5 + 2 x^4 + \frac{2}{3} x^3 - 2 x^2 + x$$

input
```
integrate((-x^2-x+1)^4,x, algorithm="maxima")
```

output
```
1/9*x^9 + 1/2*x^8 + 2/7*x^7 - 4/3*x^6 - x^5 + 2*x^4 + 2/3*x^3 - 2*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int (1-x-x^2)^4 \, dx = \frac{1}{9} x^9 + \frac{1}{2} x^8 + \frac{2}{7} x^7 - \frac{4}{3} x^6 - x^5 + 2 x^4 + \frac{2}{3} x^3 - 2 x^2 + x$$

input
```
integrate((-x^2-x+1)^4,x, algorithm="giac")
```

output
```
1/9*x^9 + 1/2*x^8 + 2/7*x^7 - 4/3*x^6 - x^5 + 2*x^4 + 2/3*x^3 - 2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \left(1-x-x^2\right)^4\,dx=\frac{x^9}{9}+\frac{x^8}{2}+\frac{2\,x^7}{7}-\frac{4\,x^6}{3}-x^5+2\,x^4+\frac{2\,x^3}{3}-2\,x^2+x$$

input
```
int((x + x^2 - 1)^4,x)
```

output
```
x - 2*x^2 + (2*x^3)/3 + 2*x^4 - x^5 - (4*x^6)/3 + (2*x^7)/7 + x^8/2 + x^9/
9
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \left(1-x-x^2\right)^4\,dx$$
$$=\frac{x(14x^8+63x^7+36x^6-168x^5-126x^4+252x^3+84x^2-252x+126)}{126}$$

input
```
int((-x^2-x+1)^4,x)
```

output
```
(x*(14*x**8 + 63*x**7 + 36*x**6 - 168*x**5 - 126*x**4 + 252*x**3 + 84*x**2
 - 252*x + 126))/126
```

3.36 $\int (1-x-x^2)^3 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int (1-x-x^2)^3 \, dx = x - \frac{3x^2}{2} + \frac{5x^4}{4} - \frac{x^6}{2} - \frac{x^7}{7}$$

output
```
x-3/2*x^2+5/4*x^4-1/2*x^6-1/7*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (1-x-x^2)^3 \, dx = x - \frac{3x^2}{2} + \frac{5x^4}{4} - \frac{x^6}{2} - \frac{x^7}{7}$$

input
```
Integrate[(1 - x - x^2)^3,x]
```

output
```
x - (3*x^2)/2 + (5*x^4)/4 - x^6/2 - x^7/7
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-x^2-x+1\right)^3 dx$$

$$\Big\downarrow \text{1085}$$

$$\int \left(-x^6-3x^5+5x^3-3x+1\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x^7}{7}-\frac{x^6}{2}+\frac{5x^4}{4}-\frac{3x^2}{2}+x$$

input
```
Int[(1 - x - x^2)^3,x]
```

output
```
x - (3*x^2)/2 + (5*x^4)/4 - x^6/2 - x^7/7
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
default	$x-\frac{3}{2}x^{2}+\frac{5}{4}x^{4}-\frac{1}{2}x^{6}-\frac{1}{7}x^{7}$	23
norman	$x-\frac{3}{2}x^{2}+\frac{5}{4}x^{4}-\frac{1}{2}x^{6}-\frac{1}{7}x^{7}$	23
risch	$x-\frac{3}{2}x^{2}+\frac{5}{4}x^{4}-\frac{1}{2}x^{6}-\frac{1}{7}x^{7}$	23
parallelrisch	$x-\frac{3}{2}x^{2}+\frac{5}{4}x^{4}-\frac{1}{2}x^{6}-\frac{1}{7}x^{7}$	23
gosper	$-\frac{x(4x^{6}+14x^{5}-35x^{3}+42x-28)}{28}$	24
orering	$\frac{x(4x^{6}+14x^{5}-35x^{3}+42x-28)\left(-x^{2}-x+1\right)^{3}}{28(x^{2}+x-1)^{3}}$	44

input
```
int((-x^2-x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
x-3/2*x^2+5/4*x^4-1/2*x^6-1/7*x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1-x-x^{2}\right)^{3}\,dx=-\frac{1}{7}\,x^{7}-\frac{1}{2}\,x^{6}+\frac{5}{4}\,x^{4}-\frac{3}{2}\,x^{2}+x$$

input
```
integrate((-x^2-x+1)^3,x, algorithm="fricas")
```

output
```
-1/7*x^7 - 1/2*x^6 + 5/4*x^4 - 3/2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(1-x-x^2\right)^3 \, dx = -\frac{x^7}{7}-\frac{x^6}{2}+\frac{5x^4}{4}-\frac{3x^2}{2}+x$$

input
```
integrate((-x**2-x+1)**3,x)
```

output
```
-x**7/7 - x**6/2 + 5*x**4/4 - 3*x**2/2 + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1-x-x^2\right)^3 \, dx = -\frac{1}{7}\,x^7-\frac{1}{2}\,x^6+\frac{5}{4}\,x^4-\frac{3}{2}\,x^2+x$$

input
```
integrate((-x^2-x+1)^3,x, algorithm="maxima")
```

output
```
-1/7*x^7 - 1/2*x^6 + 5/4*x^4 - 3/2*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1-x-x^2\right)^3 \, dx = -\frac{1}{7}\,x^7-\frac{1}{2}\,x^6+\frac{5}{4}\,x^4-\frac{3}{2}\,x^2+x$$

input
```
integrate((-x^2-x+1)^3,x, algorithm="giac")
```

output
```
-1/7*x^7 - 1/2*x^6 + 5/4*x^4 - 3/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1-x-x^2\right)^3 \, dx = -\frac{x^7}{7} - \frac{x^6}{2} + \frac{5\,x^4}{4} - \frac{3\,x^2}{2} + x$$

input
```
int(-(x + x^2 - 1)^3,x)
```

output
```
x - (3*x^2)/2 + (5*x^4)/4 - x^6/2 - x^7/7
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \left(1-x-x^2\right)^3 \, dx = \frac{x(-4x^6 - 14x^5 + 35x^3 - 42x + 28)}{28}$$

input
```
int((-x^2-x+1)^3,x)
```

output
```
(x*( - 4*x**6 - 14*x**5 + 35*x**3 - 42*x + 28))/28
```

3.37 $\int (1-x-x^2)^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 28

$$\int \left(1-x-x^2\right)^2 \, dx = x-x^2-\frac{x^3}{3}+\frac{x^4}{2}+\frac{x^5}{5}$$

output
```
x-x^2-1/3*x^3+1/2*x^4+1/5*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(1-x-x^2\right)^2 \, dx = x-x^2-\frac{x^3}{3}+\frac{x^4}{2}+\frac{x^5}{5}$$

input
```
Integrate[(1 - x - x^2)^2,x]
```

output
```
x - x^2 - x^3/3 + x^4/2 + x^5/5
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-x^2-x+1\right)^2 dx$$

$$\Big\downarrow \text{ 1085}$$

$$\int \left(x^4+2x^3-x^2-2x+1\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^5}{5}+\frac{x^4}{2}-\frac{x^3}{3}-x^2+x$$

input
```
Int[(1 - x - x^2)^2,x]
```

output
```
x - x^2 - x^3/3 + x^4/2 + x^5/5
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$x-x^{2}-\frac{1}{3}x^{3}+\frac{1}{2}x^{4}+\frac{1}{5}x^{5}$	23
norman	$x-x^{2}-\frac{1}{3}x^{3}+\frac{1}{2}x^{4}+\frac{1}{5}x^{5}$	23
risch	$x-x^{2}-\frac{1}{3}x^{3}+\frac{1}{2}x^{4}+\frac{1}{5}x^{5}$	23
parallelrisch	$x-x^{2}-\frac{1}{3}x^{3}+\frac{1}{2}x^{4}+\frac{1}{5}x^{5}$	23
gosper	$\frac{x\left(6x^{4}+15x^{3}-10x^{2}-30x+30\right)}{30}$	24
orering	$\frac{x\left(6x^{4}+15x^{3}-10x^{2}-30x+30\right)\left(-x^{2}-x+1\right)^{2}}{30\left(x^{2}+x-1\right)^{2}}$	44

input
```
int((-x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x-x^2-1/3*x^3+1/2*x^4+1/5*x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \left(1-x-x^{2}\right)^{2}\,dx=\frac{1}{5}\,x^{5}+\frac{1}{2}\,x^{4}-\frac{1}{3}\,x^{3}-x^{2}+x$$

input
```
integrate((-x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/5*x^5 + 1/2*x^4 - 1/3*x^3 - x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \left(1-x-x^2\right)^2 \, dx = \frac{x^5}{5} + \frac{x^4}{2} - \frac{x^3}{3} - x^2 + x$$

input
```
integrate((-x**2-x+1)**2,x)
```

output
```
x**5/5 + x**4/2 - x**3/3 - x**2 + x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \left(1-x-x^2\right)^2 \, dx = \frac{1}{5}\,x^5 + \frac{1}{2}\,x^4 - \frac{1}{3}\,x^3 - x^2 + x$$

input
```
integrate((-x^2-x+1)^2,x, algorithm="maxima")
```

output
```
1/5*x^5 + 1/2*x^4 - 1/3*x^3 - x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \left(1-x-x^2\right)^2 \, dx = \frac{1}{5}\,x^5 + \frac{1}{2}\,x^4 - \frac{1}{3}\,x^3 - x^2 + x$$

input
```
integrate((-x^2-x+1)^2,x, algorithm="giac")
```

output
```
1/5*x^5 + 1/2*x^4 - 1/3*x^3 - x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int (1 - x - x^2)^2 \, dx = \frac{x^5}{5} + \frac{x^4}{2} - \frac{x^3}{3} - x^2 + x$$

input
```
int((x + x^2 - 1)^2,x)
```

output
```
x - x^2 - x^3/3 + x^4/2 + x^5/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int (1 - x - x^2)^2 \, dx = \frac{x(6x^4 + 15x^3 - 10x^2 - 30x + 30)}{30}$$

input
```
int((-x^2-x+1)^2,x)
```

output
```
(x*(6*x**4 + 15*x**3 - 10*x**2 - 30*x + 30))/30
```

3.38 $\int (1 - x - x^2) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int (1 - x - x^2) \, dx = x - \frac{x^2}{2} - \frac{x^3}{3}$$

output
```
x-1/2*x^2-1/3*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 - x - x^2) \, dx = x - \frac{x^2}{2} - \frac{x^3}{3}$$

input
```
Integrate[1 - x - x^2,x]
```

output
```
x - x^2/2 - x^3/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-x^2-x+1\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x^3}{3}-\frac{x^2}{2}+x$$

input
```
Int[1 - x - x^2,x]
```

output
```
x - x^2/2 - x^3/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$x-\frac{1}{2}x^{2}-\frac{1}{3}x^{3}$	13
norman	$x-\frac{1}{2}x^{2}-\frac{1}{3}x^{3}$	13
risch	$x-\frac{1}{2}x^{2}-\frac{1}{3}x^{3}$	13
parallelrisch	$x-\frac{1}{2}x^{2}-\frac{1}{3}x^{3}$	13
parts	$x-\frac{1}{2}x^{2}-\frac{1}{3}x^{3}$	13
gosper	$-\frac{x(2x^{2}+3x-6)}{6}$	14
orering	$\frac{x(2x^{2}+3x-6)(-x^{2}-x+1)}{6x^{2}+6x-6}$	32

input
```
int(-x^2-x+1,x,method=_RETURNVERBOSE)
```

output
```
x-1/2*x^2-1/3*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (1-x-x^2)\ dx=-\frac{1}{3}\,x^3-\frac{1}{2}\,x^2+x$$

input
```
integrate(-x^2-x+1,x, algorithm="fricas")
```

output
```
-1/3*x^3 - 1/2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \left(1-x-x^2\right) \, dx = -\frac{x^3}{3} - \frac{x^2}{2} + x$$

input
```
integrate(-x**2-x+1,x)
```

output
```
-x**3/3 - x**2/2 + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(1-x-x^2\right) \, dx = -\frac{1}{3}\,x^3 - \frac{1}{2}\,x^2 + x$$

input
```
integrate(-x^2-x+1,x, algorithm="maxima")
```

output
```
-1/3*x^3 - 1/2*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(1-x-x^2\right) \, dx = -\frac{1}{3}\,x^3 - \frac{1}{2}\,x^2 + x$$

input
```
integrate(-x^2-x+1,x, algorithm="giac")
```

output
```
-1/3*x^3 - 1/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \left(1-x-x^2\right) \, dx = -\frac{x\,\left(2\,x^2+3\,x-6\right)}{6}$$

input
```
int(1 - x^2 - x,x)
```

output
```
-(x*(3*x + 2*x^2 - 6))/6
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \left(1-x-x^2\right) \, dx = \frac{x(-2x^2-3x+6)}{6}$$

input
```
int(-x^2-x+1,x)
```

output
```
(x*( - 2*x**2 - 3*x + 6))/6
```

3.39 $\int \frac{1}{1-x-x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{1}{1-x-x^2} dx = \frac{2\text{arctanh}\left(\frac{1+2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

output
```
2/5*arctanh(1/5*(1+2*x)*5^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{1}{1-x-x^2} dx = -\frac{\log\left(-1+\sqrt{5}-2x\right)-\log\left(1+\sqrt{5}+2x\right)}{\sqrt{5}}$$

input
```
Integrate[(1 - x - x^2)^(-1),x]
```

output
```
-((Log[-1 + Sqrt[5] - 2*x] - Log[1 + Sqrt[5] + 2*x])/Sqrt[5])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-x^2-x+1} \, dx$$
$$\Big\downarrow \text{1081}$$
$$-\int \left(\frac{2}{\sqrt{5}\left(2x-\sqrt{5}+1\right)}-\frac{2}{2\sqrt{5}x+\sqrt{5}+5}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\log\left(2x+\sqrt{5}+1\right)}{\sqrt{5}}-\frac{\log\left(2x-\sqrt{5}+1\right)}{\sqrt{5}}$$

input
```
Int[(1 - x - x^2)^(-1),x]
```

output
```
-(Log[1 - Sqrt[5] + 2*x]/Sqrt[5]) + Log[1 + Sqrt[5] + 2*x]/Sqrt[5]
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)\sqrt{5}}{5}$	17
risch	$\frac{\sqrt{5}\,\ln\left(2x+1+\sqrt{5}\right)}{5}-\frac{\sqrt{5}\,\ln\left(2x-\sqrt{5}+1\right)}{5}$	32

input
```
int(1/(-x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
2/5*arctanh(1/5*(1+2*x)*5^(1/2))*5^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(16) = 32.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{1}{1-x-x^2}\,dx = \frac{1}{5}\sqrt{5}\log\left(\frac{2\,x^2+\sqrt{5}(2\,x+1)+2\,x+3}{x^2+x-1}\right)$$

input
```
integrate(1/(-x^2-x+1),x, algorithm="fricas")
```

output
```
1/5*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{1}{1-x-x^2}\,dx = \frac{\sqrt{5}\log\left(x+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)}{5}-\frac{\sqrt{5}\log\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)}{5}$$

input
```
integrate(1/(-x**2-x+1),x)
```

output
```
sqrt(5)*log(x + 1/2 + sqrt(5)/2)/5 - sqrt(5)*log(x - sqrt(5)/2 + 1/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{1}{1-x-x^2}\,dx = -\frac{1}{5}\,\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+1}{2\,x+\sqrt{5}+1}\right)$$

input
```
integrate(1/(-x^2-x+1),x, algorithm="maxima")
```

output
```
-1/5*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{1}{1-x-x^2}\,dx = -\frac{1}{5}\,\sqrt{5}\log\left(\frac{\left|2\,x-\sqrt{5}+1\right|}{\left|2\,x+\sqrt{5}+1\right|}\right)$$

input
```
integrate(1/(-x^2-x+1),x, algorithm="giac")
```

output
```
-1/5*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{1-x-x^2}\,dx = \frac{2\,\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,(2\,x+1)}{5}\right)}{5}$$

input
```
int(-1/(x + x^2 - 1),x)
```

output
```
(2*5^(1/2)*atanh((5^(1/2)*(2*x + 1))/5))/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{1}{1-x-x^2}\,dx = \frac{\sqrt{5}\left(-\log\left(-\sqrt{5}+2x+1\right)+\log\left(\sqrt{5}+2x+1\right)\right)}{5}$$

input
```
int(1/(-x^2-x+1),x)
```

output
```
(sqrt(5)*( - log( - sqrt(5) + 2*x + 1) + log(sqrt(5) + 2*x + 1)))/5
```

3.40 $\int \frac{1}{\left(1-x-x^2\right)^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{\left(1-x-x^2\right)^2} \, dx = \frac{1+2x}{5\left(1-x-x^2\right)} + \frac{4\text{arctanh}\left(\frac{1+2x}{\sqrt{5}}\right)}{5\sqrt{5}}$$

output
```
(1+2*x)/(-5*x^2-5*x+5)+4/25*arctanh(1/5*(1+2*x)*5^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(1-x-x^2\right)^2} \, dx = \frac{-1-2x}{5\left(-1+x+x^2\right)} - \frac{2\log\left(-1+\sqrt{5}-2x\right)}{5\sqrt{5}} + \frac{2\log\left(1+\sqrt{5}+2x\right)}{5\sqrt{5}}$$

input
```
Integrate[(1 - x - x^2)^(-2),x]
```

output
```
(-1 - 2*x)/(5*(-1 + x + x^2)) - (2*Log[-1 + Sqrt[5] - 2*x])/(5*Sqrt[5]) +
(2*Log[1 + Sqrt[5] + 2*x])/(5*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1086, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^2-x+1)^2}dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{2}{5}\int \frac{1}{-x^2-x+1}dx+\frac{2x+1}{5(-x^2-x+1)}$$

$$\Big\downarrow \text{1081}$$

$$\frac{2x+1}{5(-x^2-x+1)}-\frac{2}{5}\int \left(\frac{2}{\sqrt{5}\left(2x-\sqrt{5}+1\right)}-\frac{2}{2\sqrt{5}x+\sqrt{5}+5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x+1}{5(-x^2-x+1)}-\frac{2}{5}\left(\frac{\log\left(2x-\sqrt{5}+1\right)}{\sqrt{5}}-\frac{\log\left(2x+\sqrt{5}+1\right)}{\sqrt{5}}\right)$$

input
```
Int[(1 - x - x^2)^(-2),x]
```

output
```
(1 + 2*x)/(5*(1 - x - x^2)) - (2*(Log[1 - Sqrt[5] + 2*x]/Sqrt[5] - Log[1 +
 Sqrt[5] + 2*x]/Sqrt[5]))/5
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{1+2x}{5(x^2+x-1)}+\frac{4\,\mathrm{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)\sqrt{5}}{25}$	33
risch	$\frac{-\frac{2x}{5}-\frac{1}{5}}{x^2+x-1}+\frac{2\sqrt{5}\,\ln\left(2x+1+\sqrt{5}\right)}{25}-\frac{2\sqrt{5}\,\ln\left(2x-\sqrt{5}+1\right)}{25}$	46

input
```
int(1/(-x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(1+2*x)/(x^2+x-1)+4/25*arctanh(1/5*(1+2*x)*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int \frac{1}{(1-x-x^2)^2}\,dx=\frac{2\sqrt{5}(x^2+x-1)\log\left(\frac{2x^2+\sqrt{5}(2x+1)+2x+3}{x^2+x-1}\right)-10x-5}{25\,(x^2+x-1)}$$

input
```
integrate(1/(-x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/25*(2*sqrt(5)*(x^2 + x - 1)*log((2*x^2 + sqrt(5)*(2*x + 1) + 2*x + 3)/(x
^2 + x - 1)) - 10*x - 5)/(x^2 + x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \frac{1}{(1-x-x^2)^2}\,dx = \frac{-2x-1}{5x^2+5x-5} + \frac{2\sqrt{5}\log\left(x+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)}{25} - \frac{2\sqrt{5}\log\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)}{25}$$

input
```
integrate(1/(-x**2-x+1)**2,x)
```

output
```
(-2*x - 1)/(5*x**2 + 5*x - 5) + 2*sqrt(5)*log(x + 1/2 + sqrt(5)/2)/25 - 2*
sqrt(5)*log(x - sqrt(5)/2 + 1/2)/25
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-x-x^2)^2}\,dx = -\frac{2}{25}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+1}{2x+\sqrt{5}+1}\right) - \frac{2x+1}{5(x^2+x-1)}$$

input
```
integrate(1/(-x^2-x+1)^2,x, algorithm="maxima")
```

output
```
-2/25*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/5*(2*x + 1)
/(x^2 + x - 1)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-x-x^2)^2}\,dx = -\frac{2}{25}\sqrt{5}\log\left(\frac{|2x-\sqrt{5}+1|}{|2x+\sqrt{5}+1|}\right) - \frac{2x+1}{5(x^2+x-1)}$$

input
```
integrate(1/(-x^2-x+1)^2,x, algorithm="giac")
```

output
```
-2/25*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/5*(2*
x + 1)/(x^2 + x - 1)
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{1}{(1-x-x^2)^2} dx = \frac{4\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,(2\,x+1)}{5}\right)}{25} - \frac{\frac{2\,x}{5}+\frac{1}{5}}{x^2+x-1}$$

input
```
int(1/(x + x^2 - 1)^2,x)
```

output
```
(4*5^(1/2)*atanh((5^(1/2)*(2*x + 1))/5))/25 - ((2*x)/5 + 1/5)/(x + x^2 - 1
)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.47

$$\int \frac{1}{(1-x-x^2)^2} dx = \frac{-2\sqrt{5}\log\left(-\sqrt{5}+2x+1\right)x^2 - 2\sqrt{5}\log\left(-\sqrt{5}+2x+1\right)x + 2\sqrt{5}\log\left(-\sqrt{5}+2x+1\right) + 2\sqrt{5}\log\left(\sqrt{5}+\right.}{25x^2+25x-25}$$

input
```
int(1/(-x^2-x+1)^2,x)
```

output
```
( - 2*sqrt(5)*log( - sqrt(5) + 2*x + 1)*x**2 - 2*sqrt(5)*log( - sqrt(5) +
2*x + 1)*x + 2*sqrt(5)*log( - sqrt(5) + 2*x + 1) + 2*sqrt(5)*log(sqrt(5) +
 2*x + 1)*x**2 + 2*sqrt(5)*log(sqrt(5) + 2*x + 1)*x - 2*sqrt(5)*log(sqrt(5
) + 2*x + 1) + 10*x**2 - 15)/(25*(x**2 + x - 1))
```

3.41 $\int \frac{1}{(1-x-x^2)^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{1}{(1-x-x^2)^3} dx = \frac{1+2x}{10(1-x-x^2)^2} + \frac{3(1+2x)}{25(1-x-x^2)} + \frac{12\operatorname{arctanh}\left(\frac{1+2x}{\sqrt{5}}\right)}{25\sqrt{5}}$$

output
```
1/10*(1+2*x)/(-x^2-x+1)^2+3*(1+2*x)/(-25*x^2-25*x+25)+12/125*arctanh(1/5*(
1+2*x)*5^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{1}{(1-x-x^2)^3} dx = \frac{1}{250}\left(\frac{25(1+2x)}{(-1+x+x^2)^2} - \frac{30(1+2x)}{-1+x+x^2} - 12\sqrt{5}\log\left(-1+\sqrt{5}-2x\right) + 12\sqrt{5}\log\left(1+\sqrt{5}+2x\right)\right)$$

input
```
Integrate[(1 - x - x^2)^(-3),x]
```

output
```
((25*(1 + 2*x))/(-1 + x + x^2)^2 - (30*(1 + 2*x))/(-1 + x + x^2) - 12*Sqrt
[5]*Log[-1 + Sqrt[5] - 2*x] + 12*Sqrt[5]*Log[1 + Sqrt[5] + 2*x])/250
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.41, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1086, 1086, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^2-x+1)^3}dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{3}{5}\int \frac{1}{(-x^2-x+1)^2}dx+\frac{2x+1}{10(-x^2-x+1)^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{3}{5}\left(\frac{2}{5}\int \frac{1}{-x^2-x+1}dx+\frac{2x+1}{5(-x^2-x+1)}\right)+\frac{2x+1}{10(-x^2-x+1)^2}$$

$$\Big\downarrow \text{1081}$$

$$\frac{3}{5}\left(\frac{2x+1}{5(-x^2-x+1)}-\frac{2}{5}\int \left(\frac{2}{\sqrt{5}(2x-\sqrt{5}+1)}-\frac{2}{2\sqrt{5}x+\sqrt{5}+5}\right)dx\right)+\frac{2x+1}{10(-x^2-x+1)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x+1}{10(-x^2-x+1)^2}+\frac{3}{5}\left(\frac{2x+1}{5(-x^2-x+1)}-\frac{2}{5}\left(\frac{\log(2x-\sqrt{5}+1)}{\sqrt{5}}-\frac{\log(2x+\sqrt{5}+1)}{\sqrt{5}}\right)\right)$$

input
```
Int[(1 - x - x^2)^(-3),x]
```

output
```
(1 + 2*x)/(10*(1 - x - x^2)^2) + (3*((1 + 2*x)/(5*(1 - x - x^2)) - (2*(Log
[1 - Sqrt[5] + 2*x]/Sqrt[5] - Log[1 + Sqrt[5] + 2*x]/Sqrt[5]))/5))/5
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{1+2x}{10(x^2+x-1)^2}-\frac{3(1+2x)}{25(x^2+x-1)}+\frac{12\,\mathrm{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)\sqrt{5}}{125}$	48
risch	$\frac{-\frac{6}{25}x^3-\frac{9}{25}x^2+\frac{8}{25}x+\frac{11}{50}}{(x^2+x-1)^2}+\frac{6\sqrt{5}\,\ln\left(2x+1+\sqrt{5}\right)}{125}-\frac{6\sqrt{5}\,\ln\left(2x-\sqrt{5}+1\right)}{125}$	56

input
```
int(1/(-x^2-x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*(1+2*x)/(x^2+x-1)^2-3/25*(1+2*x)/(x^2+x-1)+12/125*arctanh(1/5*(1+2*x)
*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.39

$$\int \frac{1}{(1-x-x^2)^3}\,dx = -\frac{60\,x^3 - 12\,\sqrt{5}(x^4 + 2\,x^3 - x^2 - 2\,x + 1)\log\left(\frac{2\,x^2+\sqrt{5}(2\,x+1)+2\,x+3}{x^2+x-1}\right) + 90\,x^2 - 80\,x - 55}{250\,(x^4 + 2\,x^3 - x^2 - 2\,x + 1)}$$

input
```
integrate(1/(-x^2-x+1)^3,x, algorithm="fricas")
```

output
```
-1/250*(60*x^3 - 12*sqrt(5)*(x^4 + 2*x^3 - x^2 - 2*x + 1)*log((2*x^2 + sqr
t(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1)) + 90*x^2 - 80*x - 55)/(x^4 + 2*x^
3 - x^2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{1}{(1-x-x^2)^3}\,dx = -\frac{12x^3 + 18x^2 - 16x - 11}{50x^4 + 100x^3 - 50x^2 - 100x + 50} + \frac{6\sqrt{5}\log\left(x + \frac{1}{2} + \frac{\sqrt{5}}{2}\right)}{125} - \frac{6\sqrt{5}\log\left(x - \frac{\sqrt{5}}{2} + \frac{1}{2}\right)}{125}$$

input
```
integrate(1/(-x**2-x+1)**3,x)
```

output
```
-(12*x**3 + 18*x**2 - 16*x - 11)/(50*x**4 + 100*x**3 - 50*x**2 - 100*x + 5
0) + 6*sqrt(5)*log(x + 1/2 + sqrt(5)/2)/125 - 6*sqrt(5)*log(x - sqrt(5)/2
+ 1/2)/125
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{1}{(1-x-x^2)^3}\,dx = -\frac{6}{125}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+1}{2x+\sqrt{5}+1}\right) - \frac{12x^3+18x^2-16x-11}{50(x^4+2x^3-x^2-2x+1)}$$

input
```
integrate(1/(-x^2-x+1)^3,x, algorithm="maxima")
```

output
```
-6/125*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/50*(12*x^3
 + 18*x^2 - 16*x - 11)/(x^4 + 2*x^3 - x^2 - 2*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{1}{(1-x-x^2)^3}\,dx = -\frac{6}{125}\sqrt{5}\log\left(\frac{|2x-\sqrt{5}+1|}{|2x+\sqrt{5}+1|}\right) - \frac{12x^3+18x^2-16x-11}{50(x^2+x-1)^2}$$

input
```
integrate(1/(-x^2-x+1)^3,x, algorithm="giac")
```

output
```
-6/125*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/50*(
12*x^3 + 18*x^2 - 16*x - 11)/(x^2 + x - 1)^2
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{1}{(1-x-x^2)^3}\,dx = \frac{12\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}(2x+1)}{5}\right)}{125} + \frac{-\frac{6x^3}{25}-\frac{9x^2}{25}+\frac{8x}{25}+\frac{11}{50}}{x^4+2x^3-x^2-2x+1}$$

input
```
int(-1/(x + x^2 - 1)^3,x)
```

output
```
(12*5^(1/2)*atanh((5^(1/2)*(2*x + 1))/5))/125 + ((8*x)/25 - (9*x^2)/25 - (
6*x^3)/25 + 11/50)/(2*x^3 - x^2 - 2*x + x^4 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.94

$$\int \frac{1}{(1-x-x^2)^3}\,dx$$
$$= \frac{-12\sqrt{5}\log\left(-\sqrt{5}+2x+1\right)x^4-24\sqrt{5}\log\left(-\sqrt{5}+2x+1\right)x^3+12\sqrt{5}\log\left(-\sqrt{5}+2x+1\right)x^2+24\sqrt{5}\,\mathrm{lo}}{}$$

input
```
int(1/(-x^2-x+1)^3,x)
```

output
```
( - 12*sqrt(5)*log( - sqrt(5) + 2*x + 1)*x**4 - 24*sqrt(5)*log( - sqrt(5)
+ 2*x + 1)*x**3 + 12*sqrt(5)*log( - sqrt(5) + 2*x + 1)*x**2 + 24*sqrt(5)*l
og( - sqrt(5) + 2*x + 1)*x - 12*sqrt(5)*log( - sqrt(5) + 2*x + 1) + 12*sqr
t(5)*log(sqrt(5) + 2*x + 1)*x**4 + 24*sqrt(5)*log(sqrt(5) + 2*x + 1)*x**3
- 12*sqrt(5)*log(sqrt(5) + 2*x + 1)*x**2 - 24*sqrt(5)*log(sqrt(5) + 2*x +
1)*x + 12*sqrt(5)*log(sqrt(5) + 2*x + 1) + 30*x**4 - 120*x**2 + 20*x + 85)
/(250*(x**4 + 2*x**3 - x**2 - 2*x + 1))
```

3.42 $\int (2+4x-3x^2)^4 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int (2+4x-3x^2)^4 \, dx = 16x+64x^2+96x^3-16x^4-136x^5+16x^6+\frac{648x^7}{7}-54x^8+9x^9$$

output
```
16*x+64*x^2+96*x^3-16*x^4-136*x^5+16*x^6+648/7*x^7-54*x^8+9*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (2+4x-3x^2)^4 \, dx = 16x+64x^2+96x^3-16x^4-136x^5+16x^6+\frac{648x^7}{7}-54x^8+9x^9$$

input
```
Integrate[(2 + 4*x - 3*x^2)^4,x]
```

output
```
16*x + 64*x^2 + 96*x^3 - 16*x^4 - 136*x^5 + 16*x^6 + (648*x^7)/7 - 54*x^8
+ 9*x^9
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-3x^2+4x+2\right)^4 dx$$

$$\Big\downarrow \text{ 1085}$$

$$\int \left(81x^8-432x^7+648x^6+96x^5-680x^4-64x^3+288x^2+128x+16\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$9x^9-54x^8+\frac{648x^7}{7}+16x^6-136x^5-16x^4+96x^3+64x^2+16x$$

input
```
Int[(2 + 4*x - 3*x^2)^4,x]
```

output
```
16*x + 64*x^2 + 96*x^3 - 16*x^4 - 136*x^5 + 16*x^6 + (648*x^7)/7 - 54*x^8
+ 9*x^9
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x(63x^8-378x^7+648x^6+112x^5-952x^4-112x^3+672x^2+448x+112)}{7}$	44
default	$16x+64x^2+96x^3-16x^4-136x^5+16x^6+\frac{648}{7}x^7-54x^8+9x^9$	45
norman	$16x+64x^2+96x^3-16x^4-136x^5+16x^6+\frac{648}{7}x^7-54x^8+9x^9$	45
risch	$16x+64x^2+96x^3-16x^4-136x^5+16x^6+\frac{648}{7}x^7-54x^8+9x^9$	45
parallelrisch	$16x+64x^2+96x^3-16x^4-136x^5+16x^6+\frac{648}{7}x^7-54x^8+9x^9$	45
orering	$\frac{x(63x^8-378x^7+648x^6+112x^5-952x^4-112x^3+672x^2+448x+112)(-3x^2+4x+2)^4}{7(3x^2-4x-2)^4}$	68

input
```
int((-3*x^2+4*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/7*x*(63*x^8-378*x^7+648*x^6+112*x^5-952*x^4-112*x^3+672*x^2+448*x+112)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int (2+4x-3x^2)^4 \, dx = 9x^9-54x^8+\frac{648}{7}x^7+16x^6-136x^5-16x^4+96x^3+64x^2+16x$$

input
```
integrate((-3*x^2+4*x+2)^4,x, algorithm="fricas")
```

output
```
9*x^9 - 54*x^8 + 648/7*x^7 + 16*x^6 - 136*x^5 - 16*x^4 + 96*x^3 + 64*x^2 +
16*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \left(2+4x-3x^2\right)^4 \, dx = 9x^9 - 54x^8 + \frac{648x^7}{7} + 16x^6 - 136x^5 - 16x^4 + 96x^3 + 64x^2 + 16x$$

input
```
integrate((-3*x**2+4*x+2)**4,x)
```

output
```
9*x**9 - 54*x**8 + 648*x**7/7 + 16*x**6 - 136*x**5 - 16*x**4 + 96*x**3 + 6
4*x**2 + 16*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \left(2+4x-3x^2\right)^4 \, dx = 9\,x^9 - 54\,x^8 + \frac{648}{7}\,x^7 + 16\,x^6 - 136\,x^5 - 16\,x^4 + 96\,x^3 + 64\,x^2 + 16\,x$$

input
```
integrate((-3*x^2+4*x+2)^4,x, algorithm="maxima")
```

output
```
9*x^9 - 54*x^8 + 648/7*x^7 + 16*x^6 - 136*x^5 - 16*x^4 + 96*x^3 + 64*x^2 +
 16*x
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \left(2+4x-3x^2\right)^4 \, dx = 9\,x^9 - 54\,x^8 + \frac{648}{7}\,x^7 + 16\,x^6 - 136\,x^5 - 16\,x^4 + 96\,x^3 + 64\,x^2 + 16\,x$$

input
```
integrate((-3*x^2+4*x+2)^4,x, algorithm="giac")
```

output
```
9*x^9 - 54*x^8 + 648/7*x^7 + 16*x^6 - 136*x^5 - 16*x^4 + 96*x^3 + 64*x^2 +
 16*x
```

Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \left(2+4x-3x^2\right)^4 \, dx = 9\,x^9-54\,x^8+\frac{648\,x^7}{7}+16\,x^6-136\,x^5-16\,x^4+96\,x^3+64\,x^2+16\,x$$

input
```
int((4*x - 3*x^2 + 2)^4,x)
```

output
```
16*x + 64*x^2 + 96*x^3 - 16*x^4 - 136*x^5 + 16*x^6 + (648*x^7)/7 - 54*x^8
+ 9*x^9
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \left(2+4x-3x^2\right)^4 \, dx = \frac{x(63x^8-378x^7+648x^6+112x^5-952x^4-112x^3+672x^2+448x+112)}{7}$$

input
```
int((-3*x^2+4*x+2)^4,x)
```

output
```
(x*(63*x**8 - 378*x**7 + 648*x**6 + 112*x**5 - 952*x**4 - 112*x**3 + 672*x
**2 + 448*x + 112))/7
```

3.43 $\int (2+4x-3x^2)^3 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 36

$$\int (2+4x-3x^2)^3 \, dx = 8x+24x^2+20x^3-20x^4-18x^5+18x^6-\frac{27x^7}{7}$$

output
```
8*x+24*x^2+20*x^3-20*x^4-18*x^5+18*x^6-27/7*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (2+4x-3x^2)^3 \, dx = 8x+24x^2+20x^3-20x^4-18x^5+18x^6-\frac{27x^7}{7}$$

input
```
Integrate[(2 + 4*x - 3*x^2)^3,x]
```

output
```
8*x + 24*x^2 + 20*x^3 - 20*x^4 - 18*x^5 + 18*x^6 - (27*x^7)/7
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-3x^2+4x+2\right)^3 dx$$

$$\Big\downarrow \text{1085}$$

$$\int \left(-27x^6+108x^5-90x^4-80x^3+60x^2+48x+8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{27x^7}{7}+18x^6-18x^5-20x^4+20x^3+24x^2+8x$$

input
```
Int[(2 + 4*x - 3*x^2)^3,x]
```

output
```
8*x + 24*x^2 + 20*x^3 - 20*x^4 - 18*x^5 + 18*x^6 - (27*x^7)/7
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{x(27x^6-126x^5+126x^4+140x^3-140x^2-168x-56)}{7}$	34
default	$8x+24x^2+20x^3-20x^4-18x^5+18x^6-\frac{27}{7}x^7$	35
norman	$8x+24x^2+20x^3-20x^4-18x^5+18x^6-\frac{27}{7}x^7$	35
risch	$8x+24x^2+20x^3-20x^4-18x^5+18x^6-\frac{27}{7}x^7$	35
parallelrisch	$8x+24x^2+20x^3-20x^4-18x^5+18x^6-\frac{27}{7}x^7$	35
orering	$\frac{x(27x^6-126x^5+126x^4+140x^3-140x^2-168x-56)(-3x^2+4x+2)^3}{7(3x^2-4x-2)^3}$	58

input
```
int((-3*x^2+4*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/7*x*(27*x^6-126*x^5+126*x^4+140*x^3-140*x^2-168*x-56)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \left(2+4x-3x^2\right)^3 \, dx = -\frac{27}{7}\,x^7+18\,x^6-18\,x^5-20\,x^4+20\,x^3+24\,x^2+8\,x$$

input
```
integrate((-3*x^2+4*x+2)^3,x, algorithm="fricas")
```

output
```
-27/7*x^7 + 18*x^6 - 18*x^5 - 20*x^4 + 20*x^3 + 24*x^2 + 8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \left(2+4x-3x^2\right)^3\,dx=-\frac{27x^7}{7}+18x^6-18x^5-20x^4+20x^3+24x^2+8x$$

input
```
integrate((-3*x**2+4*x+2)**3,x)
```

output
```
-27*x**7/7 + 18*x**6 - 18*x**5 - 20*x**4 + 20*x**3 + 24*x**2 + 8*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \left(2+4x-3x^2\right)^3\,dx=-\frac{27}{7}\,x^7+18\,x^6-18\,x^5-20\,x^4+20\,x^3+24\,x^2+8\,x$$

input
```
integrate((-3*x^2+4*x+2)^3,x, algorithm="maxima")
```

output
```
-27/7*x^7 + 18*x^6 - 18*x^5 - 20*x^4 + 20*x^3 + 24*x^2 + 8*x
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \left(2+4x-3x^2\right)^3\,dx=-\frac{27}{7}\,x^7+18\,x^6-18\,x^5-20\,x^4+20\,x^3+24\,x^2+8\,x$$

input
```
integrate((-3*x^2+4*x+2)^3,x, algorithm="giac")
```

output
```
-27/7*x^7 + 18*x^6 - 18*x^5 - 20*x^4 + 20*x^3 + 24*x^2 + 8*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \left(2+4x-3x^2\right)^3 \, dx = -\frac{27\,x^7}{7} + 18\,x^6 - 18\,x^5 - 20\,x^4 + 20\,x^3 + 24\,x^2 + 8\,x$$

input
```
int((4*x - 3*x^2 + 2)^3,x)
```

output
```
8*x + 24*x^2 + 20*x^3 - 20*x^4 - 18*x^5 + 18*x^6 - (27*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \left(2+4x-3x^2\right)^3 \, dx = \frac{x(-27x^6+126x^5-126x^4-140x^3+140x^2+168x+56)}{7}$$

input
```
int((-3*x^2+4*x+2)^3,x)
```

output
```
(x*( - 27*x**6 + 126*x**5 - 126*x**4 - 140*x**3 + 140*x**2 + 168*x + 56))/
7
```

3.44 $\int (2+4x-3x^2)^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 28

$$\int (2+4x-3x^2)^2 \, dx = 4x+8x^2+\frac{4x^3}{3}-6x^4+\frac{9x^5}{5}$$

output
```
4*x+8*x^2+4/3*x^3-6*x^4+9/5*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (2+4x-3x^2)^2 \, dx = 4x+8x^2+\frac{4x^3}{3}-6x^4+\frac{9x^5}{5}$$

input
```
Integrate[(2 + 4*x - 3*x^2)^2,x]
```

output
```
4*x + 8*x^2 + (4*x^3)/3 - 6*x^4 + (9*x^5)/5
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-3x^2+4x+2\right)^2 dx$$

$$\Big\downarrow \text{ 1085}$$

$$\int \left(9x^4-24x^3+4x^2+16x+4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{9x^5}{5}-6x^4+\frac{4x^3}{3}+8x^2+4x$$

input
```
Int[(2 + 4*x - 3*x^2)^2,x]
```

output
```
4*x + 8*x^2 + (4*x^3)/3 - 6*x^4 + (9*x^5)/5
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{x(27x^4-90x^3+20x^2+120x+60)}{15}$	24
default	$4x+8x^2+\frac{4}{3}x^3-6x^4+\frac{9}{5}x^5$	25
norman	$4x+8x^2+\frac{4}{3}x^3-6x^4+\frac{9}{5}x^5$	25
risch	$4x+8x^2+\frac{4}{3}x^3-6x^4+\frac{9}{5}x^5$	25
parallelrisch	$4x+8x^2+\frac{4}{3}x^3-6x^4+\frac{9}{5}x^5$	25
orering	$\frac{x(27x^4-90x^3+20x^2+120x+60)\left(-3x^2+4x+2\right)^2}{15(3x^2-4x-2)^2}$	48

input
```
int((-3*x^2+4*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(27*x^4-90*x^3+20*x^2+120*x+60)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(2+4x-3x^2\right)^2\,dx=\frac{9}{5}\,x^5-6\,x^4+\frac{4}{3}\,x^3+8\,x^2+4\,x$$

input
```
integrate((-3*x^2+4*x+2)^2,x, algorithm="fricas")
```

output
```
9/5*x^5 - 6*x^4 + 4/3*x^3 + 8*x^2 + 4*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(2+4x-3x^2\right)^2\,dx=\frac{9x^5}{5}-6x^4+\frac{4x^3}{3}+8x^2+4x$$

input
```
integrate((-3*x**2+4*x+2)**2,x)
```

output
```
9*x**5/5 - 6*x**4 + 4*x**3/3 + 8*x**2 + 4*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(2+4x-3x^2\right)^2\,dx=\frac{9}{5}\,x^5-6\,x^4+\frac{4}{3}\,x^3+8\,x^2+4\,x$$

input
```
integrate((-3*x^2+4*x+2)^2,x, algorithm="maxima")
```

output
```
9/5*x^5 - 6*x^4 + 4/3*x^3 + 8*x^2 + 4*x
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(2+4x-3x^2\right)^2\,dx=\frac{9}{5}\,x^5-6\,x^4+\frac{4}{3}\,x^3+8\,x^2+4\,x$$

input
```
integrate((-3*x^2+4*x+2)^2,x, algorithm="giac")
```

output
```
9/5*x^5 - 6*x^4 + 4/3*x^3 + 8*x^2 + 4*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(2+4x-3x^2\right)^2\,dx=\frac{9\,x^5}{5}-6\,x^4+\frac{4\,x^3}{3}+8\,x^2+4\,x$$

input
```
int((4*x - 3*x^2 + 2)^2,x)
```

output
```
4*x + 8*x^2 + (4*x^3)/3 - 6*x^4 + (9*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \left(2+4x-3x^2\right)^2\,dx=\frac{x(27x^4-90x^3+20x^2+120x+60)}{15}$$

input
```
int((-3*x^2+4*x+2)^2,x)
```

output
```
(x*(27*x**4 - 90*x**3 + 20*x**2 + 120*x + 60))/15
```

3.45 $\int (2+4x-3x^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 14

$$\int (2+4x-3x^2)\,dx = 2x+2x^2-x^3$$

output
```
-x^3+2*x^2+2*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (2+4x-3x^2)\,dx = 2x+2x^2-x^3$$

input
```
Integrate[2 + 4*x - 3*x^2,x]
```

output
```
2*x + 2*x^2 - x^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-3x^2+4x+2\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$-x^3+2x^2+2x$$

input
```
Int[2 + 4*x - 3*x^2,x]
```

output
```
2*x + 2*x^2 - x^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

method	result	size
gosper	$-x(x^2-2x-2)$	12
default	$-x^3+2x^2+2x$	15
norman	$-x^3+2x^2+2x$	15
risch	$-x^3+2x^2+2x$	15
parallelrisch	$-x^3+2x^2+2x$	15
parts	$-x^3+2x^2+2x$	15
orering	$\frac{x(x^2-2x-2)(-3x^2+4x+2)}{3x^2-4x-2}$	33

input
```
int(-3*x^2+4*x+2,x,method=_RETURNVERBOSE)
```

output
```
-x*(x^2-2*x-2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(2+4x-3x^2\right)\,dx=-x^3+2\,x^2+2\,x$$

input
```
integrate(-3*x^2+4*x+2,x, algorithm="fricas")
```

output
```
-x^3 + 2*x^2 + 2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(2+4x-3x^2\right)\,dx=-x^3+2x^2+2x$$

input
```
integrate(-3*x**2+4*x+2,x)
```

output
```
-x**3 + 2*x**2 + 2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(2+4x-3x^2\right)\,dx=-x^3+2\,x^2+2\,x$$

input
```
integrate(-3*x^2+4*x+2,x, algorithm="maxima")
```

output
```
-x^3 + 2*x^2 + 2*x
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(2+4x-3x^2\right)\,dx=-x^3+2\,x^2+2\,x$$

input
```
integrate(-3*x^2+4*x+2,x, algorithm="giac")
```

output
```
-x^3 + 2*x^2 + 2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(2+4x-3x^2\right)\,dx=x\,\left(-x^2+2\,x+2\right)$$

input
```
int(4*x - 3*x^2 + 2,x)
```

output
```
x*(2*x - x^2 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (2+4x-3x^2) \, dx = x(-x^2+2x+2)$$

input
```
int(-3*x^2+4*x+2,x)
```

output
```
x*( - x**2 + 2*x + 2)
```

3.46 $\int \frac{1}{2+4x-3x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{1}{2+4x-3x^2}\,dx = -\frac{\text{arctanh}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}$$

output
```
-1/10*arctanh(1/10*(2-3*x)*10^(1/2))*10^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{1}{2+4x-3x^2}\,dx = \frac{-\log\left(2+\sqrt{10}-3x\right)+\log\left(-2+\sqrt{10}+3x\right)}{2\sqrt{10}}$$

input
```
Integrate[(2 + 4*x - 3*x^2)^(-1),x]
```

output
```
(-Log[2 + Sqrt[10] - 3*x] + Log[-2 + Sqrt[10] + 3*x])/(2*Sqrt[10])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. 2(19) = 38.

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-3x^2+4x+2}dx$$

$$\Big\downarrow \text{1081}$$

$$-3\int \left(\frac{1}{2\sqrt{10}\left(-3x-\sqrt{10}+2\right)}-\frac{1}{2\sqrt{10}\left(-3x+\sqrt{10}+2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-3\left(\frac{\log\left(-3x+\sqrt{10}+2\right)}{6\sqrt{10}}-\frac{\log\left(-3x-\sqrt{10}+2\right)}{6\sqrt{10}}\right)$$

input
```
Int[(2 + 4*x - 3*x^2)^(-1),x]
```

output
```
-3*(-1/6*Log[2 - Sqrt[10] - 3*x]/Sqrt[10] + Log[2 + Sqrt[10] - 3*x]/(6*Sqr
t[10]))
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\sqrt{10}\,\operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{10}$	17
risch	$\frac{\sqrt{10}\,\ln\left(3x-2+\sqrt{10}\right)}{20}-\frac{\sqrt{10}\,\ln\left(3x-2-\sqrt{10}\right)}{20}$	32

input
```
int(1/(-3*x^2+4*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/10*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(16) = 32$.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{1}{2+4x-3x^2}\,dx = \frac{1}{20}\sqrt{10}\log\left(\frac{9\,x^2+2\,\sqrt{10}(3\,x-2)-12\,x+14}{3\,x^2-4\,x-2}\right)$$

input
```
integrate(1/(-3*x^2+4*x+2),x, algorithm="fricas")
```

output
```
1/20*sqrt(10)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*x + 14)/(3*x^2 - 4*x
- 2))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{1}{2+4x-3x^2}\,dx = \frac{\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{20} - \frac{\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{20}$$

input
```
integrate(1/(-3*x**2+4*x+2),x)
```

output
```
sqrt(10)*log(x - 2/3 + sqrt(10)/3)/20 - sqrt(10)*log(x - sqrt(10)/3 - 2/3)
/20
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{1}{2+4x-3x^2}\,dx = -\frac{1}{20}\sqrt{10}\log\left(\frac{3x-\sqrt{10}-2}{3x+\sqrt{10}-2}\right)$$

input
```
integrate(1/(-3*x^2+4*x+2),x, algorithm="maxima")
```

output
```
-1/20*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{1}{2+4x-3x^2}\,dx = -\frac{1}{20}\sqrt{10}\log\left(\frac{|6x-2\sqrt{10}-4|}{|6x+2\sqrt{10}-4|}\right)$$

input
```
integrate(1/(-3*x^2+4*x+2),x, algorithm="giac")
```

output
```
-1/20*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{2+4x-3x^2}\,dx=\frac{\sqrt{10}\,\mathrm{atanh}\left(\sqrt{10}\,\left(\frac{3\,x}{10}-\frac{1}{5}\right)\right)}{10}$$

input
```
int(1/(4*x - 3*x^2 + 2),x)
```

output
```
(10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/10
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{1}{2+4x-3x^2}\,dx=\frac{\sqrt{10}\,\left(-\log\left(-\sqrt{10}+3x-2\right)+\log\left(\sqrt{10}+3x-2\right)\right)}{20}$$

input
```
int(1/(-3*x^2+4*x+2),x)
```

output
```
(sqrt(10)*( - log( - sqrt(10) + 3*x - 2) + log(sqrt(10) + 3*x - 2)))/20
```

3.47 $\int \frac{1}{(2+4x-3x^2)^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx = -\frac{2-3x}{20\,(2+4x-3x^2)} - \frac{3\text{arctanh}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

output
```
-1/20*(2-3*x)/(-3*x^2+4*x+2)-3/200*arctanh(1/10*(2-3*x)*10^(1/2))*10^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx = \frac{2-3x}{20\,(-2-4x+3x^2)} - \frac{3\log\left(2+\sqrt{10}-3x\right)}{40\sqrt{10}} + \frac{3\log\left(-2+\sqrt{10}+3x\right)}{40\sqrt{10}}$$

input
```
Integrate[(2 + 4*x - 3*x^2)^(-2),x]
```

output
```
(2 - 3*x)/(20*(-2 - 4*x + 3*x^2)) - (3*Log[2 + Sqrt[10] - 3*x])/(40*Sqrt[1
0]) + (3*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1086, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^2+4x+2)^2}\,dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{3}{20}\int \frac{1}{-3x^2+4x+2}dx-\frac{2-3x}{20(-3x^2+4x+2)}$$

$$\Big\downarrow \text{1081}$$

$$-\frac{9}{20}\int \left(\frac{1}{2\sqrt{10}\left(-3x-\sqrt{10}+2\right)}-\frac{1}{2\sqrt{10}\left(-3x+\sqrt{10}+2\right)}\right)dx-\frac{2-3x}{20(-3x^2+4x+2)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2-3x}{20(-3x^2+4x+2)}-\frac{9}{20}\left(\frac{\log\left(-3x+\sqrt{10}+2\right)}{6\sqrt{10}}-\frac{\log\left(-3x-\sqrt{10}+2\right)}{6\sqrt{10}}\right)$$

input
```
Int[(2 + 4*x - 3*x^2)^(-2),x]
```

output
```
-1/20*(2 - 3*x)/(2 + 4*x - 3*x^2) - (9*(-1/6*Log[2 - Sqrt[10] - 3*x]/Sqrt[
10] + Log[2 + Sqrt[10] - 3*x]/(6*Sqrt[10])))/20
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{6x-4}{40(3x^2-4x-2)}+\frac{3\sqrt{10}\,\operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{200}$	37
risch	$\frac{-\frac{x}{20}+\frac{1}{30}}{x^2-\frac{4}{3}x-\frac{2}{3}}+\frac{3\sqrt{10}\,\ln\left(3x-2+\sqrt{10}\right)}{400}-\frac{3\sqrt{10}\,\ln\left(3x-2-\sqrt{10}\right)}{400}$	48

input
```
int(1/(-3*x^2+4*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(6*x-4)/(3*x^2-4*x-2)+3/200*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx=\frac{3\sqrt{10}(3x^2-4x-2)\log\left(\frac{9x^2+2\sqrt{10}(3x-2)-12x+14}{3x^2-4x-2}\right)-60x+40}{400\,(3x^2-4x-2)}$$

input
```
integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="fricas")
```

output
```
1/400*(3*sqrt(10)*(3*x^2 - 4*x - 2)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12
*x + 14)/(3*x^2 - 4*x - 2)) - 60*x + 40)/(3*x^2 - 4*x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx = \frac{2-3x}{60x^2-80x-40} + \frac{3\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400} - \frac{3\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

input
```
integrate(1/(-3*x**2+4*x+2)**2,x)
```

output
```
(2 - 3*x)/(60*x**2 - 80*x - 40) + 3*sqrt(10)*log(x - 2/3 + sqrt(10)/3)/400
 - 3*sqrt(10)*log(x - sqrt(10)/3 - 2/3)/400
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx = -\frac{3}{400}\sqrt{10}\log\left(\frac{3x-\sqrt{10}-2}{3x+\sqrt{10}-2}\right) - \frac{3x-2}{20(3x^2-4x-2)}$$

input
```
integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="maxima")
```

output
```
-3/400*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2)) - 1/20*(3*x
 - 2)/(3*x^2 - 4*x - 2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx = -\frac{3}{400}\sqrt{10}\log\left(\frac{|6x-2\sqrt{10}-4|}{|6x+2\sqrt{10}-4|}\right) - \frac{3x-2}{20\,(3x^2-4x-2)}$$

input
```
integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="giac")
```

output
```
-3/400*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4)) -
 1/20*(3*x - 2)/(3*x^2 - 4*x - 2)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx = \frac{3\sqrt{10}\,\mathrm{atanh}\left(\sqrt{10}\left(\frac{3x}{10}-\frac{1}{5}\right)\right)}{200} + \frac{\frac{x}{20}-\frac{1}{30}}{-x^2+\frac{4x}{3}+\frac{2}{3}}$$

input
```
int(1/(4*x - 3*x^2 + 2)^2,x)
```

output
```
(3*10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/200 + (x/20 - 1/30)/((4*x)/3
 - x^2 + 2/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.47

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx = \frac{-9\sqrt{10}\log\left(-\sqrt{10}+3x-2\right)x^2+12\sqrt{10}\log\left(-\sqrt{10}+3x-2\right)x+6\sqrt{10}\log\left(-\sqrt{10}+3x-2\right)+9\sqrt{10}\,\mathrm{l}}{1200x^2-1600x}$$

input
```
int(1/(-3*x^2+4*x+2)^2,x)
```

output
```
( - 9*sqrt(10)*log( - sqrt(10) + 3*x - 2)*x**2 + 12*sqrt(10)*log( - sqrt(1
0) + 3*x - 2)*x + 6*sqrt(10)*log( - sqrt(10) + 3*x - 2) + 9*sqrt(10)*log(s
qrt(10) + 3*x - 2)*x**2 - 12*sqrt(10)*log(sqrt(10) + 3*x - 2)*x - 6*sqrt(1
0)*log(sqrt(10) + 3*x - 2) - 45*x**2 + 70)/(400*(3*x**2 - 4*x - 2))
```

3.48 $\int \frac{1}{(2+4x-3x^2)^3} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{1}{(2+4x-3x^2)^3} \, dx = -\frac{2-3x}{40(2+4x-3x^2)^2} - \frac{9(2-3x)}{800(2+4x-3x^2)} - \frac{27\text{arctanh}\left(\frac{2-3x}{\sqrt{10}}\right)}{800\sqrt{10}}$$

output
```
-1/40*(2-3*x)/(-3*x^2+4*x+2)^2-9*(2-3*x)/(-2400*x^2+3200*x+1600)-27/8000*a
rctanh(1/10*(2-3*x)*10^(1/2))*10^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{1}{(2+4x-3x^2)^3} \, dx = \frac{-\frac{20(76-42x-162x^2+81x^3)}{(2+4x-3x^2)^2} - 27\sqrt{10}\log\left(2+\sqrt{10}-3x\right) + 27\sqrt{10}\log\left(-2+\sqrt{10}+3x\right)}{16000}$$

input
```
Integrate[(2 + 4*x - 3*x^2)^(-3),x]
```

output
```
((-20*(76 - 42*x - 162*x^2 + 81*x^3))/(2 + 4*x - 3*x^2)^2 - 27*Sqrt[10]*Lo
g[2 + Sqrt[10] - 3*x] + 27*Sqrt[10]*Log[-2 + Sqrt[10] + 3*x])/16000
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1086, 1086, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-3x^2+4x+2\right)^3}dx$$

$$\Big\downarrow \text{ 1086}$$

$$\frac{9}{40}\int \frac{1}{\left(-3x^2+4x+2\right)^2}dx-\frac{2-3x}{40\left(-3x^2+4x+2\right)^2}$$

$$\Big\downarrow \text{ 1086}$$

$$\frac{9}{40}\left(\frac{3}{20}\int \frac{1}{-3x^2+4x+2}dx-\frac{2-3x}{20\left(-3x^2+4x+2\right)}\right)-\frac{2-3x}{40\left(-3x^2+4x+2\right)^2}$$

$$\Big\downarrow \text{ 1081}$$

$$\frac{9}{40}\left(-\frac{9}{20}\int \left(\frac{1}{2\sqrt{10}\left(-3x-\sqrt{10}+2\right)}-\frac{1}{2\sqrt{10}\left(-3x+\sqrt{10}+2\right)}\right)dx-\frac{2-3x}{20\left(-3x^2+4x+2\right)}\right)-\frac{2-3x}{40\left(-3x^2+4x+2\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{9}{40}\left(-\frac{2-3x}{20\left(-3x^2+4x+2\right)}-\frac{9}{20}\left(\frac{\log\left(-3x+\sqrt{10}+2\right)}{6\sqrt{10}}-\frac{\log\left(-3x-\sqrt{10}+2\right)}{6\sqrt{10}}\right)\right)-\frac{2-3x}{40\left(-3x^2+4x+2\right)^2}$$

input
```
Int[(2 + 4*x - 3*x^2)^(-3),x]
```

output
```
-1/40*(2 - 3*x)/(2 + 4*x - 3*x^2)^2 + (9*(-1/20*(2 - 3*x)/(2 + 4*x - 3*x^2
) - (9*(-1/6*Log[2 - Sqrt[10] - 3*x]/Sqrt[10] + Log[2 + Sqrt[10] - 3*x]/(6
*Sqrt[10])))/20))/40
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{6x-4}{80(3x^2-4x-2)^2}-\frac{9(6x-4)}{1600(3x^2-4x-2)}+\frac{27\sqrt{10}\,\operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{8000}$	56
risch	$\frac{-\frac{81}{800}x^3+\frac{81}{400}x^2+\frac{21}{400}x-\frac{19}{200}}{(3x^2-4x-2)^2}+\frac{27\sqrt{10}\,\ln\left(3x-2+\sqrt{10}\right)}{16000}-\frac{27\sqrt{10}\,\ln\left(3x-2-\sqrt{10}\right)}{16000}$	61

input
```
int(1/(-3*x^2+4*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/80*(6*x-4)/(3*x^2-4*x-2)^2-9/1600*(6*x-4)/(3*x^2-4*x-2)+27/8000*10^(1/2)
*arctanh(1/20*(6*x-4)*10^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

$$\int \frac{1}{(2+4x-3x^2)^3}\,dx =$$
$$-\frac{1620\,x^3 - 27\,\sqrt{10}(9\,x^4 - 24\,x^3 + 4\,x^2 + 16\,x + 4)\log\left(\frac{9\,x^2+2\,\sqrt{10}(3\,x-2)-12\,x+14}{3\,x^2-4\,x-2}\right) - 3240\,x^2 - 840\,x + 15}{16000\,(9\,x^4 - 24\,x^3 + 4\,x^2 + 16\,x + 4)}$$

input
```
integrate(1/(-3*x^2+4*x+2)^3,x, algorithm="fricas")
```

output
```
-1/16000*(1620*x^3 - 27*sqrt(10)*(9*x^4 - 24*x^3 + 4*x^2 + 16*x + 4)*log((
9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*x + 14)/(3*x^2 - 4*x - 2)) - 3240*x^2 -
840*x + 1520)/(9*x^4 - 24*x^3 + 4*x^2 + 16*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{1}{(2+4x-3x^2)^3}\,dx = -\frac{81x^3 - 162x^2 - 42x + 76}{7200x^4 - 19200x^3 + 3200x^2 + 12800x + 3200} + \frac{27\sqrt{10}\log\left(x - \frac{2}{3} + \frac{\sqrt{10}}{3}\right)}{16000} - \frac{27\sqrt{10}\log\left(x - \frac{\sqrt{10}}{3} - \frac{2}{3}\right)}{16000}$$

input
```
integrate(1/(-3*x**2+4*x+2)**3,x)
```

output
```
-(81*x**3 - 162*x**2 - 42*x + 76)/(7200*x**4 - 19200*x**3 + 3200*x**2 + 12
800*x + 3200) + 27*sqrt(10)*log(x - 2/3 + sqrt(10)/3)/16000 - 27*sqrt(10)*
log(x - sqrt(10)/3 - 2/3)/16000
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{1}{(2+4x-3x^2)^3}\,dx = -\frac{27}{16000}\sqrt{10}\log\left(\frac{3\,x-\sqrt{10}-2}{3\,x+\sqrt{10}-2}\right) - \frac{81\,x^3-162\,x^2-42\,x+76}{800\,(9\,x^4-24\,x^3+4\,x^2+16\,x+4)}$$

input
```
integrate(1/(-3*x^2+4*x+2)^3,x, algorithm="maxima")
```

output
```
-27/16000*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2)) - 1/800*
(81*x^3 - 162*x^2 - 42*x + 76)/(9*x^4 - 24*x^3 + 4*x^2 + 16*x + 4)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{1}{(2+4x-3x^2)^3}\,dx = -\frac{27}{16000}\sqrt{10}\log\left(\frac{|6\,x-2\,\sqrt{10}-4|}{|6\,x+2\,\sqrt{10}-4|}\right) - \frac{81\,x^3-162\,x^2-42\,x+76}{800\,(3\,x^2-4\,x-2)^2}$$

input
```
integrate(1/(-3*x^2+4*x+2)^3,x, algorithm="giac")
```

output
```
-27/16000*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4)
) - 1/800*(81*x^3 - 162*x^2 - 42*x + 76)/(3*x^2 - 4*x - 2)^2
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{1}{(2+4x-3x^2)^3}\,dx = \frac{27\sqrt{10}\,\mathrm{atanh}\left(\sqrt{10}\left(\frac{3x}{10}-\frac{1}{5}\right)\right)}{8000} + \frac{-\frac{9x^3}{800}+\frac{9x^2}{400}+\frac{7x}{1200}-\frac{19}{1800}}{x^4-\frac{8x^3}{3}+\frac{4x^2}{9}+\frac{16x}{9}+\frac{4}{9}}$$

input
```
int(1/(4*x - 3*x^2 + 2)^3,x)
```

output
```
(27*10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/8000 + ((7*x)/1200 + (9*x^2
)/400 - (9*x^3)/800 - 19/1800)/((16*x)/9 + (4*x^2)/9 - (8*x^3)/3 + x^4 + 4
/9)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.94

$$\int \frac{1}{(2+4x-3x^2)^3}\,dx = \frac{-486\sqrt{10}\log\left(-\sqrt{10}+3x-2\right)x^4+1296\sqrt{10}\log\left(-\sqrt{10}+3x-2\right)x^3-216\sqrt{10}\log\left(-\sqrt{10}+3x-2\right)x}{}$$

input
```
int(1/(-3*x^2+4*x+2)^3,x)
```

output
```
( - 486*sqrt(10)*log( - sqrt(10) + 3*x - 2)*x**4 + 1296*sqrt(10)*log( - sq
rt(10) + 3*x - 2)*x**3 - 216*sqrt(10)*log( - sqrt(10) + 3*x - 2)*x**2 - 86
4*sqrt(10)*log( - sqrt(10) + 3*x - 2)*x - 216*sqrt(10)*log( - sqrt(10) + 3
*x - 2) + 486*sqrt(10)*log(sqrt(10) + 3*x - 2)*x**4 - 1296*sqrt(10)*log(sq
rt(10) + 3*x - 2)*x**3 + 216*sqrt(10)*log(sqrt(10) + 3*x - 2)*x**2 + 864*s
qrt(10)*log(sqrt(10) + 3*x - 2)*x + 216*sqrt(10)*log(sqrt(10) + 3*x - 2) -
 1215*x**4 + 5940*x**2 - 480*x - 3580)/(32000*(9*x**4 - 24*x**3 + 4*x**2 +
 16*x + 4))
```

3.49 $\int \frac{1}{2+4x+x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 17

$$\int \frac{1}{2+4x+x^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2+x}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(1/2*(2+x)*2^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{1}{2+4x+x^2}\,dx = \frac{\log\left(-2+\sqrt{2}-x\right)-\log\left(2+\sqrt{2}+x\right)}{2\sqrt{2}}$$

input
```
Integrate[(2 + 4*x + x^2)^(-1),x]
```

output
```
(Log[-2 + Sqrt[2] - x] - Log[2 + Sqrt[2] + x])/(2*Sqrt[2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2+4x+2}\,dx$$

$$\Big\downarrow \text{ 1081}$$

$$\int \left(\frac{1}{2\sqrt{2}\left(x-\sqrt{2}+2\right)}-\frac{1}{2\sqrt{2}\left(x+\sqrt{2}+2\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\log\left(x-\sqrt{2}+2\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}+2\right)}{2\sqrt{2}}$$

input
```
Int[(2 + 4*x + x^2)^(-1),x]
```

output
```
Log[2 - Sqrt[2] + x]/(2*Sqrt[2]) - Log[2 + Sqrt[2] + x]/(2*Sqrt[2])
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+4)\sqrt{2}}{4}\right)}{2}$	17
risch	$\frac{\sqrt{2}\,\ln\left(x+2-\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(x+2+\sqrt{2}\right)}{4}$	28

input
```
int(1/(x^2+4*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)*arctanh(1/4*(2*x+4)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{1}{2+4x+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}(x+2)+4\,x+6}{x^2+4\,x+2}\right)$$

input
```
integrate(1/(x^2+4*x+2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 2) + 4*x + 6)/(x^2 + 4*x + 2))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{1}{2+4x+x^2}\,dx = \frac{\sqrt{2}\log\left(x-\sqrt{2}+2\right)}{4}-\frac{\sqrt{2}\log\left(x+\sqrt{2}+2\right)}{4}$$

input
```
integrate(1/(x**2+4*x+2),x)
```

output
```
sqrt(2)*log(x - sqrt(2) + 2)/4 - sqrt(2)*log(x + sqrt(2) + 2)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{1}{2+4x+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+2}{x+\sqrt{2}+2}\right)$$

input
```
integrate(1/(x^2+4*x+2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*log((x - sqrt(2) + 2)/(x + sqrt(2) + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{1}{2+4x+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}+4|}{|2x+2\sqrt{2}+4|}\right)$$

input
```
integrate(1/(x^2+4*x+2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 4)/abs(2*x + 2*sqrt(2) + 4))
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{2+4x+x^2}\,dx=-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\left(\frac{x}{2}+1\right)\right)}{2}$$

input
```
int(1/(4*x + x^2 + 2),x)
```

output
```
-(2^(1/2)*atanh(2^(1/2)*(x/2 + 1)))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{1}{2+4x+x^2}\,dx=\frac{\sqrt{2}\,\left(\log\left(-\sqrt{2}+x+2\right)-\log\left(\sqrt{2}+x+2\right)\right)}{4}$$

input
```
int(1/(x^2+4*x+2),x)
```

output
```
(sqrt(2)*(log( - sqrt(2) + x + 2) - log(sqrt(2) + x + 2)))/4
```

3.50 $\int \frac{1}{3+4x+x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 6

$$\int \frac{1}{3+4x+x^2}\,dx=-\operatorname{arctanh}(2+x)$$

output
```
-arctanh(2+x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{1}{3+4x+x^2}\,dx=\frac{1}{2}\log(1+x)-\frac{1}{2}\log(3+x)$$

input
```
Integrate[(3 + 4*x + x^2)^(-1),x]
```

output
```
Log[1 + x]/2 - Log[3 + x]/2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2+4x+3}\,dx$$
$$\Big\downarrow \text{1081}$$
$$\int \left(\frac{1}{2(x+1)}-\frac{1}{2(x+3)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x+3)$$

input
```
Int[(3 + 4*x + x^2)^(-1),x]
```

output
```
Log[1 + x]/2 - Log[3 + x]/2
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(6) = 12.

Time = 0.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

method	result	size
default	$\frac{\ln(x+1)}{2}-\frac{\ln(3+x)}{2}$	14
norman	$\frac{\ln(x+1)}{2}-\frac{\ln(3+x)}{2}$	14
risch	$\frac{\ln(x+1)}{2}-\frac{\ln(3+x)}{2}$	14
parallelrisch	$\frac{\ln(x+1)}{2}-\frac{\ln(3+x)}{2}$	14

input
```
int(1/(x^2+4*x+3),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x+1)-1/2*ln(3+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(6) = 12.

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 2.17

$$\int \frac{1}{3+4x+x^2}\,dx=-\frac{1}{2}\log{(x+3)}+\frac{1}{2}\log{(x+1)}$$

input
```
integrate(1/(x^2+4*x+3),x, algorithm="fricas")
```

output
```
-1/2*log(x + 3) + 1/2*log(x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(5) = 10.

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{1}{3+4x+x^2}\,dx = \frac{\log(x+1)}{2} - \frac{\log(x+3)}{2}$$

input
```
integrate(1/(x**2+4*x+3),x)
```

output
```
log(x + 1)/2 - log(x + 3)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(6) = 12.

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 2.17

$$\int \frac{1}{3+4x+x^2}\,dx = -\frac{1}{2}\log(x+3) + \frac{1}{2}\log(x+1)$$

input
```
integrate(1/(x^2+4*x+3),x, algorithm="maxima")
```

output
```
-1/2*log(x + 3) + 1/2*log(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{1}{3+4x+x^2}\,dx = -\frac{1}{2}\log(|x+3|) + \frac{1}{2}\log(|x+1|)$$

input
```
integrate(1/(x^2+4*x+3),x, algorithm="giac")
```

output
```
-1/2*log(abs(x + 3)) + 1/2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{3+4x+x^2}\,dx = -\operatorname{atanh}(x+2)$$

input
```
int(1/(4*x + x^2 + 3),x)
```

output
```
-atanh(x + 2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 2.17

$$\int \frac{1}{3+4x+x^2}\,dx = -\frac{\log(x+3)}{2} + \frac{\log(x+1)}{2}$$

input
```
int(1/(x^2+4*x+3),x)
```

output
```
( - log(x + 3) + log(x + 1))/2
```

3.51 $\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 33

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx=\frac{2\text{arctanh}\left(\frac{-\sqrt{b^2-4ab^3}+2b^2x}{b}\right)}{b}$$

output
```
2*arctanh((-(-4*a*b^3+b^2)^(1/2)+2*b^2*x)/b)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 120 vs. 2(33) = 66.

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.64

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx$$
$$=\frac{-2\sqrt{b^2-4ab^3}\arctan\left(\frac{-1+2bx}{\sqrt{-1+4ab}}\right)+2\sqrt{b^2-4ab^3}\arctan\left(\frac{1+2bx}{\sqrt{-1+4ab}}\right)+b\sqrt{-1+4ab}(\log\left(a+x+bx^2\right)-\text{lo}}{2b^2\sqrt{-1+4ab}}$$

input
```
Integrate[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

output
```
(-2*Sqrt[b^2 - 4*a*b^3]*ArcTan[(-1 + 2*b*x)/Sqrt[-1 + 4*a*b]] + 2*Sqrt[b^2
 - 4*a*b^3]*ArcTan[(1 + 2*b*x)/Sqrt[-1 + 4*a*b]] + b*Sqrt[-1 + 4*a*b]*(Log
[a + x + b*x^2] - Log[a + x*(-1 + b*x)]))/(2*b^2*Sqrt[-1 + 4*a*b])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{b^2-4ab^3}+ab-b^2x^2}dx$$

$$\Big\downarrow \text{1081}$$

$$-b^2\int \left(-\frac{2}{b\left(2xb^2+b-\sqrt{b^2-4ab^3}\right)}-\frac{2}{b\left(-2xb^2+b+\sqrt{b^2-4ab^3}\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-b^2\left(\frac{\log\left(\sqrt{b^2-4ab^3}-2b^2x+b\right)}{b^3}-\frac{\log\left(-\sqrt{b^2-4ab^3}+2b^2x+b\right)}{b^3}\right)$$

input
```
Int[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

output
```
-(b^2*(Log[b + Sqrt[b^2 - 4*a*b^3] - 2*b^2*x]/b^3 - Log[b - Sqrt[b^2 - 4*a
*b^3] + 2*b^2*x]/b^3))
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{-2b^2x+\sqrt{-b^2(4ab-1)}}{b}\right)}{b}$	31

input
```
int(1/(a*b+(-4*a*b^3+b^2)^(1/2)*x-b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
-2/b*arctanh((-2*b^2*x+(-b^2*(4*a*b-1))^(1/2))/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = \frac{\log\left(\frac{2\,b^2x+b-\sqrt{-4\,ab^3+b^2}}{b}\right)-\log\left(\frac{2\,b^2x-b-\sqrt{-4\,ab^3+b^2}}{b}\right)}{b}$$

input
```
integrate(1/(a*b+(-4*a*b^3+b^2)^(1/2)*x-b^2*x^2),x, algorithm="fricas")
```

output
```
(log((2*b^2*x + b - sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b - sqrt(-4*
a*b^3 + b^2))/b))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = -\frac{\log\left(x-\frac{1}{2b}-\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)-\log\left(x+\frac{1}{2b}-\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)}{b}$$

input
```
integrate(1/(a*b+(-4*a*b**3+b**2)**(1/2)*x-b**2*x**2),x)
```

output
```
-(log(x - 1/(2*b) - sqrt(-4*a*b**3 + b**2)/(2*b**2)) - log(x + 1/(2*b) - s
qrt(-4*a*b**3 + b**2)/(2*b**2)))/b
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = -\frac{\log\left(\frac{2\,b^2x-b-\sqrt{-4\,ab^3+b^2}}{2\,b^2x+b-\sqrt{-4\,ab^3+b^2}}\right)}{b}$$

input
```
integrate(1/(a*b+(-4*a*b^3+b^2)^(1/2)*x-b^2*x^2),x, algorithm="maxima")
```

output
```
-log((2*b^2*x - b - sqrt(-4*a*b^3 + b^2))/(2*b^2*x + b - sqrt(-4*a*b^3 + b
^2)))/b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = -\frac{\log\left(\frac{\left|2\,b^2x-\sqrt{-4\,ab+1}|b|-|b|\right|}{\left|2\,b^2x-\sqrt{-4\,ab+1}|b|+|b|\right|}\right)}{|b|}$$

input
```
integrate(1/(a*b+(-4*a*b^3+b^2)^(1/2)*x-b^2*x^2),x, algorithm="giac")
```

output
```
-log(abs(2*b^2*x - sqrt(-4*a*b + 1)*abs(b) - abs(b))/abs(2*b^2*x - sqrt(-4
*a*b + 1)*abs(b) + abs(b)))/abs(b)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b^2-4\,a\,b^3}}{\sqrt{b^2}}-\frac{2\,b^2\,x}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

input
```
int(1/(a*b + x*(b^2 - 4*a*b^3)^(1/2) - b^2*x^2),x)
```

output
```
-(2*atanh((b^2 - 4*a*b^3)^(1/2)/(b^2)^(1/2) - (2*b^2*x)/(b^2)^(1/2)))/(b^2
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.94

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx$$
$$= \frac{-2\sqrt{4ab-1}\,\sqrt{-4ab+1}\,atan\left(\frac{2bx-1}{\sqrt{4ab-1}}\right)+2\sqrt{4ab-1}\,\sqrt{-4ab+1}\,atan\left(\frac{2bx+1}{\sqrt{4ab-1}}\right)-4\log(b\,x^2+a-x)\,ab}{2b\,(4ab-1)}$$

input
```
int(1/(a*b+(-4*a*b^3+b^2)^(1/2)*x-b^2*x^2),x)
```

output
```
( - 2*sqrt(4*a*b - 1)*sqrt( - 4*a*b + 1)*atan((2*b*x - 1)/sqrt(4*a*b - 1))
 + 2*sqrt(4*a*b - 1)*sqrt( - 4*a*b + 1)*atan((2*b*x + 1)/sqrt(4*a*b - 1))
- 4*log(a + b*x**2 - x)*a*b + log(a + b*x**2 - x) + 4*log(a + b*x**2 + x)*
a*b - log(a + b*x**2 + x))/(2*b*(4*a*b - 1))
```

3.52 $\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b^2-4ab^3}+2b^2x}{b}\right)}{b}$$

output
```
2*arctanh(((-4*a*b^3+b^2)^(1/2)+2*b^2*x)/b)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 120 vs. $2(31) = 62$.

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.87

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx$$
$$= \frac{2\sqrt{b^2-4ab^3}\arctan\left(\frac{-1+2bx}{\sqrt{-1+4ab}}\right)-2\sqrt{b^2-4ab^3}\arctan\left(\frac{1+2bx}{\sqrt{-1+4ab}}\right)+b\sqrt{-1+4ab}(\log(a+x+bx^2)-\log}{2b^2\sqrt{-1+4ab}}$$

input
```
Integrate[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

output
```
(2*Sqrt[b^2 - 4*a*b^3]*ArcTan[(-1 + 2*b*x)/Sqrt[-1 + 4*a*b]] - 2*Sqrt[b^2
- 4*a*b^3]*ArcTan[(1 + 2*b*x)/Sqrt[-1 + 4*a*b]] + b*Sqrt[-1 + 4*a*b]*(Log[
a + x + b*x^2] - Log[a + x*(-1 + b*x)]))/(2*b^2*Sqrt[-1 + 4*a*b])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = $\{1081, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-x\sqrt{b^2-4ab^3}+ab-b^2x^2}dx$$

$$\Big\downarrow \text{1081}$$

$$-b^2\int \left(-\frac{2}{b\left(2xb^2+b+\sqrt{b^2-4ab^3}\right)}-\frac{2}{b\left(-2xb^2+b-\sqrt{b^2-4ab^3}\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-b^2\left(\frac{\log\left(-\sqrt{b^2-4ab^3}-2b^2x+b\right)}{b^3}-\frac{\log\left(\sqrt{b^2-4ab^3}+2b^2x+b\right)}{b^3}\right)$$

input
```
Int[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

output
```
-(b^2*(Log[b - Sqrt[b^2 - 4*a*b^3] - 2*b^2*x]/b^3 - Log[b + Sqrt[b^2 - 4*a
*b^3] + 2*b^2*x]/b^3))
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
 + c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2\,\operatorname{arctanh}\left(\frac{2b^2x+\sqrt{-b^2(4ab-1)}}{b}\right)}{b}$	31

input
```
int(1/(a*b-(-4*a*b^3+b^2)^(1/2)*x-b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/b*arctanh((2*b^2*x+(-b^2*(4*a*b-1))^(1/2))/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(29) = 58.

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx=\frac{\log\left(\frac{2\,b^2x+b+\sqrt{-4\,ab^3+b^2}}{b}\right)-\log\left(\frac{2\,b^2x-b+\sqrt{-4\,ab^3+b^2}}{b}\right)}{b}$$

input
```
integrate(1/(a*b-(-4*a*b^3+b^2)^(1/2)*x-b^2*x^2),x, algorithm="fricas")
```

output
```
(log((2*b^2*x + b + sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b + sqrt(-4*
a*b^3 + b^2))/b))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx=-\frac{\log\left(x-\frac{1}{2b}+\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)-\log\left(x+\frac{1}{2b}+\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)}{b}$$

input
```
integrate(1/(a*b-(-4*a*b**3+b**2)**(1/2)*x-b**2*x**2),x)
```

output
```
-(log(x - 1/(2*b) + sqrt(-4*a*b**3 + b**2)/(2*b**2)) - log(x + 1/(2*b) + s
qrt(-4*a*b**3 + b**2)/(2*b**2)))/b
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx=-\frac{\log\left(\frac{2\,b^2x-b+\sqrt{-4\,ab^3+b^2}}{2\,b^2x+b+\sqrt{-4\,ab^3+b^2}}\right)}{b}$$

input
```
integrate(1/(a*b-(-4*a*b^3+b^2)^(1/2)*x-b^2*x^2),x, algorithm="maxima")
```

output
```
-log((2*b^2*x - b + sqrt(-4*a*b^3 + b^2))/(2*b^2*x + b + sqrt(-4*a*b^3 + b
^2)))/b
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx=-\frac{\log\left(\frac{\left|2\,b^2x+\sqrt{-4\,ab+1}|b|-|b|\right|}{\left|2\,b^2x+\sqrt{-4\,ab+1}|b|+|b|\right|}\right)}{|b|}$$

input
```
integrate(1/(a*b-(-4*a*b^3+b^2)^(1/2)*x-b^2*x^2),x, algorithm="giac")
```

output
```
-log(abs(2*b^2*x + sqrt(-4*a*b + 1)*abs(b) - abs(b))/abs(2*b^2*x + sqrt(-4
*a*b + 1)*abs(b) + abs(b)))/abs(b)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx=\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b^2-4\,a\,b^3}}{\sqrt{b^2}}+\frac{2\,b^2\,x}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

input
```
int(-1/(x*(b^2 - 4*a*b^3)^(1/2) - a*b + b^2*x^2),x)
```

output
```
(2*atanh((b^2 - 4*a*b^3)^(1/2)/(b^2)^(1/2) + (2*b^2*x)/(b^2)^(1/2)))/(b^2)
^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.19

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx$$
$$=\frac{2\sqrt{4ab-1}\,\sqrt{-4ab+1}\,atan\left(\frac{2bx-1}{\sqrt{4ab-1}}\right)-2\sqrt{4ab-1}\,\sqrt{-4ab+1}\,atan\left(\frac{2bx+1}{\sqrt{4ab-1}}\right)-4\log\left(b\,x^2+a-x\right)ab+}{2b\,(4ab-1)}$$

input
```
int(1/(a*b-(-4*a*b^3+b^2)^(1/2)*x-b^2*x^2),x)
```

output
```
(2*sqrt(4*a*b - 1)*sqrt( - 4*a*b + 1)*atan((2*b*x - 1)/sqrt(4*a*b - 1)) -
2*sqrt(4*a*b - 1)*sqrt( - 4*a*b + 1)*atan((2*b*x + 1)/sqrt(4*a*b - 1)) - 4
*log(a + b*x**2 - x)*a*b + log(a + b*x**2 - x) + 4*log(a + b*x**2 + x)*a*b
 - log(a + b*x**2 + x))/(2*b*(4*a*b - 1))
```

3.53 $\int \frac{1}{3+10x+3x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{1}{3+10x+3x^2} \, dx = -\frac{1}{8}\log(3+x)+\frac{1}{8}\log(1+3x)$$

output
```
-1/8*ln(3+x)+1/8*ln(1+3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{3+10x+3x^2} \, dx = -\frac{1}{8}\log(3+x)+\frac{1}{8}\log(1+3x)$$

input
```
Integrate[(3 + 10*x + 3*x^2)^(-1),x]
```

output
```
-1/8*Log[3 + x] + Log[1 + 3*x]/8
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^2+10x+3}\,dx$$
$$\Big\downarrow \text{1081}$$
$$3\int \left(\frac{1}{8(3x+1)}-\frac{1}{24(x+3)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$3\left(\frac{1}{24}\log(3x+1)-\frac{1}{24}\log(x+3)\right)$$

input
```
Int[(3 + 10*x + 3*x^2)^(-1),x]
```

output
```
3*(-1/24*Log[3 + x] + Log[1 + 3*x]/24)
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$-\frac{\ln(3+x)}{8}+\frac{\ln(x+\frac{1}{3})}{8}$	14
default	$-\frac{\ln(3+x)}{8}+\frac{\ln(3x+1)}{8}$	16
norman	$-\frac{\ln(3+x)}{8}+\frac{\ln(3x+1)}{8}$	16
risch	$-\frac{\ln(3+x)}{8}+\frac{\ln(3x+1)}{8}$	16

input
```
int(1/(3*x^2+10*x+3),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(3+x)+1/8*ln(x+1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{3+10x+3x^2}\,dx=\frac{1}{8}\log\left(3\,x+1\right)-\frac{1}{8}\log\left(x+3\right)$$

input
```
integrate(1/(3*x^2+10*x+3),x, algorithm="fricas")
```

output
```
1/8*log(3*x + 1) - 1/8*log(x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{3+10x+3x^2}\,dx=\frac{\log\left(x+\frac{1}{3}\right)}{8}-\frac{\log\left(x+3\right)}{8}$$

input
```
integrate(1/(3*x**2+10*x+3),x)
```

output
```
log(x + 1/3)/8 - log(x + 3)/8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{3+10x+3x^2}\,dx = \frac{1}{8}\,\log\left(3\,x+1\right) - \frac{1}{8}\,\log\left(x+3\right)$$

input
```
integrate(1/(3*x^2+10*x+3),x, algorithm="maxima")
```

output
```
1/8*log(3*x + 1) - 1/8*log(x + 3)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{3+10x+3x^2}\,dx = \frac{1}{8}\,\log\left(|3\,x+1|\right) - \frac{1}{8}\,\log\left(|x+3|\right)$$

input
```
integrate(1/(3*x^2+10*x+3),x, algorithm="giac")
```

output
```
1/8*log(abs(3*x + 1)) - 1/8*log(abs(x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{1}{3+10x+3x^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{3\,x}{4}+\frac{5}{4}\right)}{4}$$

input
```
int(1/(10*x + 3*x^2 + 3),x)
```

output
```
-atanh((3*x)/4 + 5/4)/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{3+10x+3x^2}\,dx = \frac{\log(3x+1)}{8} - \frac{\log(x+3)}{8}$$

input
```
int(1/(3*x^2+10*x+3),x)
```

output
```
(log(3*x + 1) - log(x + 3))/8
```

3.54 $\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx$

Optimal result

Integrand size = 22, antiderivative size = 19

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx = -\frac{1}{8}\log(3+x)+\frac{1}{8}\log(1+3x)$$

output
```
-1/8*ln(3+x)+1/8*ln(1+3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx = -\frac{1}{8}\log(3+x)+\frac{1}{8}\log(1+3x)$$

input
```
Integrate[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
```

output
```
-1/8*Log[3 + x] + Log[1 + 3*x]/8
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {7239, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)\left(\frac{10x}{x^2+1}+3\right)}dx$$
$$\Big\downarrow \text{7239}$$
$$\int \frac{1}{3x^2+10x+3}dx$$
$$\Big\downarrow \text{1081}$$
$$3\int \left(\frac{1}{8(3x+1)}-\frac{1}{24(x+3)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$3\left(\frac{1}{24}\log(3x+1)-\frac{1}{24}\log(x+3)\right)$$

input
```
Int[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
```

output
```
3*(-1/24*Log[3 + x] + Log[1 + 3*x]/24)
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$-\frac{\ln(3+x)}{8}+\frac{\ln(x+\frac{1}{3})}{8}$	14
default	$-\frac{\ln(3+x)}{8}+\frac{\ln(3x+1)}{8}$	16
norman	$-\frac{\ln(3+x)}{8}+\frac{\ln(3x+1)}{8}$	16
risch	$-\frac{\ln(3+x)}{8}+\frac{\ln(3x+1)}{8}$	16

input
```
int(1/(x^2+1)/(3+10*x/(x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(3+x)+1/8*ln(x+1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \frac{1}{8}\,\log\left(3\,x+1\right)-\frac{1}{8}\,\log\left(x+3\right)$$

input
```
integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="fricas")
```

output
```
1/8*log(3*x + 1) - 1/8*log(x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \frac{\log\left(x+\frac{1}{3}\right)}{8} - \frac{\log(x+3)}{8}$$

input
```
integrate(1/(x**2+1)/(3+10*x/(x**2+1)),x)
```

output
```
log(x + 1/3)/8 - log(x + 3)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

input
```
integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="maxima")
```

output
```
1/8*log(3*x + 1) - 1/8*log(x + 3)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \frac{1}{8}\log(|3x+1|) - \frac{1}{8}\log(|x+3|)$$

input
```
integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="giac")
```

output
```
1/8*log(abs(3*x + 1)) - 1/8*log(abs(x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = -\frac{\operatorname{atanh}\left(\frac{3x}{4}+\frac{5}{4}\right)}{4}$$

input
```
int(1/((x^2 + 1)*((10*x)/(x^2 + 1) + 3)),x)
```

output
```
-atanh((3*x)/4 + 5/4)/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \frac{\log(3x+1)}{8} - \frac{\log(x+3)}{8}$$

input
```
int(1/(x^2+1)/(3+10*x/(x^2+1)),x)
```

output
```
(log(3*x + 1) - log(x + 3))/8
```

3.55 $\int (6-5x+x^2)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int (6-5x+x^2)^{3/2}\,dx = \frac{3}{64}(5-2x)\sqrt{6-5x+x^2} - \frac{1}{8}(5-2x)\left(6-5x+x^2\right)^{3/2} - \frac{3}{64}\operatorname{arctanh}\left(\frac{2-x}{\sqrt{6-5x+x^2}}\right)$$

output
```
3/64*(5-2*x)*(x^2-5*x+6)^(1/2)-1/8*(5-2*x)*(x^2-5*x+6)^(3/2)-3/64*arctanh(
(2-x)/(x^2-5*x+6)^(1/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int (6-5x+x^2)^{3/2}\,dx = \frac{1}{64}\sqrt{6-5x+x^2}\left(-225+290x-120x^2+16x^3\right) + \frac{3}{64}\operatorname{arctanh}\left(\frac{\sqrt{6-5x+x^2}}{-3+x}\right)$$

input
```
Integrate[(6 - 5*x + x^2)^(3/2),x]
```

output
```
(Sqrt[6 - 5*x + x^2]*(-225 + 290*x - 120*x^2 + 16*x^3))/64 + (3*ArcTanh[Sq
rt[6 - 5*x + x^2]/(-3 + x)])/64
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2-5x+6\right)^{3/2}dx$$

$$\Big\downarrow \text{1087}$$

$$-\frac{3}{16}\int \sqrt{x^2-5x+6}dx-\frac{1}{8}(5-2x)\left(x^2-5x+6\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$-\frac{3}{16}\left(-\frac{1}{8}\int \frac{1}{\sqrt{x^2-5x+6}}dx-\frac{1}{4}\sqrt{x^2-5x+6}(5-2x)\right)-\frac{1}{8}(5-2x)\left(x^2-5x+6\right)^{3/2}$$

$$\Big\downarrow \text{1092}$$

$$-\frac{3}{16}\left(-\frac{1}{4}\int \frac{1}{4-\frac{(5-2x)^2}{x^2-5x+6}}d\left(-\frac{5-2x}{\sqrt{x^2-5x+6}}\right)-\frac{1}{4}\sqrt{x^2-5x+6}(5-2x)\right)-\frac{1}{8}(5-2x)\left(x^2-5x+6\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3}{16}\left(\frac{1}{8}\text{arctanh}\left(\frac{5-2x}{2\sqrt{x^2-5x+6}}\right)-\frac{1}{4}(5-2x)\sqrt{x^2-5x+6}\right)-\frac{1}{8}(5-2x)\left(x^2-5x+6\right)^{3/2}$$

input
```
Int[(6 - 5*x + x^2)^(3/2),x]
```

output
```
-1/8*((5 - 2*x)*(6 - 5*x + x^2)^(3/2)) - (3*(-1/4*((5 - 2*x)*Sqrt[6 - 5*x
+ x^2]) + ArcTanh[(5 - 2*x)/(2*Sqrt[6 - 5*x + x^2])]/8))/16
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{(16x^3-120x^2+290x-225)\sqrt{x^2-5x+6}}{64}+\frac{3\ln\left(-\frac{5}{2}+x+\sqrt{x^2-5x+6}\right)}{128}$	45
trager	$\left(\frac{1}{4}x^3-\frac{15}{8}x^2+\frac{145}{32}x-\frac{225}{64}\right)\sqrt{x^2-5x+6}+\frac{3\ln\left(2x-5+2\sqrt{x^2-5x+6}\right)}{128}$	48
default	$\frac{(-5+2x)(x^2-5x+6)^{\frac{3}{2}}}{8}-\frac{3\sqrt{x^2-5x+6}\,(-5+2x)}{64}+\frac{3\ln\left(-\frac{5}{2}+x+\sqrt{x^2-5x+6}\right)}{128}$	52

input
```
int((x^2-5*x+6)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(16*x^3-120*x^2+290*x-225)*(x^2-5*x+6)^(1/2)+3/128*ln(-5/2+x+(x^2-5*x
+6)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int (6-5x+x^2)^{3/2}\,dx = \frac{1}{64}\left(16\,x^3-120\,x^2+290\,x-225\right)\sqrt{x^2-5\,x+6} - \frac{3}{128}\,\log\left(-2\,x+2\,\sqrt{x^2-5\,x+6}+5\right)$$

input
```
integrate((x^2-5*x+6)^(3/2),x, algorithm="fricas")
```

output
```
1/64*(16*x^3 - 120*x^2 + 290*x - 225)*sqrt(x^2 - 5*x + 6) - 3/128*log(-2*x
 + 2*sqrt(x^2 - 5*x + 6) + 5)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.55

$$\int (6-5x+x^2)^{3/2}\,dx = 6\left(\frac{x}{2}-\frac{5}{4}\right)\sqrt{x^2-5x+6}-5\left(\frac{x^2}{3}-\frac{5x}{12}-\frac{9}{8}\right)\sqrt{x^2-5x+6} + \sqrt{x^2-5x+6}\left(\frac{x^3}{4}-\frac{5x^2}{24}-\frac{53x}{96}-\frac{105}{64}\right)+\frac{3\log\left(2x+2\sqrt{x^2-5x+6}-5\right)}{128}$$

input
```
integrate((x**2-5*x+6)**(3/2),x)
```

output
```
6*(x/2 - 5/4)*sqrt(x**2 - 5*x + 6) - 5*(x**2/3 - 5*x/12 - 9/8)*sqrt(x**2 -
 5*x + 6) + sqrt(x**2 - 5*x + 6)*(x**3/4 - 5*x**2/24 - 53*x/96 - 105/64) +
 3*log(2*x + 2*sqrt(x**2 - 5*x + 6) - 5)/128
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \left(6-5x+x^2\right)^{3/2}\,dx=\frac{1}{4}\left(x^2-5\,x+6\right)^{\frac{3}{2}}x-\frac{5}{8}\left(x^2-5\,x+6\right)^{\frac{3}{2}}-\frac{3}{32}\sqrt{x^2-5\,x+6}x+\frac{15}{64}\sqrt{x^2-5\,x+6}+\frac{3}{128}\log\left(2\,x+2\sqrt{x^2-5\,x+6}-5\right)$$

input
```
integrate((x^2-5*x+6)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(x^2 - 5*x + 6)^(3/2)*x - 5/8*(x^2 - 5*x + 6)^(3/2) - 3/32*sqrt(x^2 -
5*x + 6)*x + 15/64*sqrt(x^2 - 5*x + 6) + 3/128*log(2*x + 2*sqrt(x^2 - 5*x
+ 6) - 5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \left(6-5x+x^2\right)^{3/2}\,dx=\frac{1}{64}\left(2\left(4\left(2\,x-15\right)x+145\right)x-225\right)\sqrt{x^2-5\,x+6}-\frac{3}{128}\log\left(\left|-2\,x+2\sqrt{x^2-5\,x+6}+5\right|\right)$$

input
```
integrate((x^2-5*x+6)^(3/2),x, algorithm="giac")
```

output
```
1/64*(2*(4*(2*x - 15)*x + 145)*x - 225)*sqrt(x^2 - 5*x + 6) - 3/128*log(ab
s(-2*x + 2*sqrt(x^2 - 5*x + 6) + 5))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \left(6-5x+x^2\right)^{3/2}\,dx=\frac{3\,\ln\left(x+\sqrt{x^2-5\,x+6}-\frac{5}{2}\right)}{128}-\frac{3\,\left(\frac{x}{2}-\frac{5}{4}\right)\,\sqrt{x^2-5\,x+6}}{16}+\frac{\left(x-\frac{5}{2}\right)\,\left(x^2-5\,x+6\right)^{3/2}}{4}$$

input
```
int((x^2 - 5*x + 6)^(3/2),x)
```

output
```
(3*log(x + (x^2 - 5*x + 6)^(1/2) - 5/2))/128 - (3*(x/2 - 5/4)*(x^2 - 5*x +
 6)^(1/2))/16 + ((x - 5/2)*(x^2 - 5*x + 6)^(3/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \left(6-5x+x^2\right)^{3/2}\,dx=\frac{\sqrt{x^2-5x+6}\,x^3}{4}-\frac{15\sqrt{x^2-5x+6}\,x^2}{8}+\frac{145\sqrt{x^2-5x+6}\,x}{32}-\frac{225\sqrt{x^2-5x+6}}{64}+\frac{3\log\left(2\sqrt{x^2-5x+6}+2x-5\right)}{128}$$

input
```
int((x^2-5*x+6)^(3/2),x)
```

output
```
(32*sqrt(x**2 - 5*x + 6)*x**3 - 240*sqrt(x**2 - 5*x + 6)*x**2 + 580*sqrt(x
**2 - 5*x + 6)*x - 450*sqrt(x**2 - 5*x + 6) + 3*log(2*sqrt(x**2 - 5*x + 6)
 + 2*x - 5))/128
```

3.56 $\int \sqrt{6-5x+x^2}\, dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \sqrt{6-5x+x^2}\, dx = -\frac{1}{4}(5-2x)\sqrt{6-5x+x^2}+\frac{1}{4}\text{arctanh}\left(\frac{2-x}{\sqrt{6-5x+x^2}}\right)$$

output
```
-1/4*(5-2*x)*(x^2-5*x+6)^(1/2)+1/4*arctanh((2-x)/(x^2-5*x+6)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \sqrt{6-5x+x^2}\, dx = \frac{1}{4}(-5+2x)\sqrt{6-5x+x^2}-\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{6-5x+x^2}}{-3+x}\right)$$

input
```
Integrate[Sqrt[6 - 5*x + x^2],x]
```

output
```
((-5 + 2*x)*Sqrt[6 - 5*x + x^2])/4 - ArcTanh[Sqrt[6 - 5*x + x^2]/(-3 + x)]
/4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2-5x+6}\,dx$$

$$\Big\downarrow \text{1087}$$

$$-\frac{1}{8}\int \frac{1}{\sqrt{x^2-5x+6}}dx-\frac{1}{4}\sqrt{x^2-5x+6}(5-2x)$$

$$\Big\downarrow \text{1092}$$

$$-\frac{1}{4}\int \frac{1}{4-\frac{(5-2x)^2}{x^2-5x+6}}d\left(-\frac{5-2x}{\sqrt{x^2-5x+6}}\right)-\frac{1}{4}\sqrt{x^2-5x+6}(5-2x)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{8}\text{arctanh}\left(\frac{5-2x}{2\sqrt{x^2-5x+6}}\right)-\frac{1}{4}(5-2x)\sqrt{x^2-5x+6}$$

input
```
Int[Sqrt[6 - 5*x + x^2],x]
```

output
```
-1/4*((5 - 2*x)*Sqrt[6 - 5*x + x^2]) + ArcTanh[(5 - 2*x)/(2*Sqrt[6 - 5*x +
 x^2])]/8
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\sqrt{x^2-5x+6}\,(-5+2x)}{4}-\frac{\ln\left(-\frac{5}{2}+x+\sqrt{x^2-5x+6}\right)}{8}$	35
risch	$\frac{\sqrt{x^2-5x+6}\,(-5+2x)}{4}-\frac{\ln\left(-\frac{5}{2}+x+\sqrt{x^2-5x+6}\right)}{8}$	35
trager	$\left(-\frac{5}{4}+\frac{x}{2}\right)\sqrt{x^2-5x+6}+\frac{\ln\left(2\sqrt{x^2-5x+6}+5-2x\right)}{8}$	38

input
```
int((x^2-5*x+6)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^2-5*x+6)^(1/2)*(-5+2*x)-1/8*ln(-5/2+x+(x^2-5*x+6)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int\sqrt{6-5x+x^2}\,dx=\frac{1}{4}\sqrt{x^2-5\,x+6}(2\,x-5)+\frac{1}{8}\,\log\left(-2\,x+2\,\sqrt{x^2-5\,x+6}+5\right)$$

input
```
integrate((x^2-5*x+6)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(x^2 - 5*x + 6)*(2*x - 5) + 1/8*log(-2*x + 2*sqrt(x^2 - 5*x + 6) +
 5)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \sqrt{6-5x+x^2}\,dx = \left(\frac{x}{2}-\frac{5}{4}\right)\sqrt{x^2-5x+6}-\frac{\log\left(2x+2\sqrt{x^2-5x+6}-5\right)}{8}$$

input
```
integrate((x**2-5*x+6)**(1/2),x)
```

output
```
(x/2 - 5/4)*sqrt(x**2 - 5*x + 6) - log(2*x + 2*sqrt(x**2 - 5*x + 6) - 5)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \sqrt{6-5x+x^2}\,dx = \frac{1}{2}\sqrt{x^2-5\,x+6}x-\frac{5}{4}\sqrt{x^2-5\,x+6} -\frac{1}{8}\log\left(2\,x+2\sqrt{x^2-5\,x+6}-5\right)$$

input
```
integrate((x^2-5*x+6)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^2 - 5*x + 6)*x - 5/4*sqrt(x^2 - 5*x + 6) - 1/8*log(2*x + 2*sqrt
(x^2 - 5*x + 6) - 5)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \sqrt{6-5x+x^2}\,dx = \frac{1}{4}\sqrt{x^2-5x+6}(2x-5)+\frac{1}{8}\log\left(\left|-2x+2\sqrt{x^2-5x+6}+5\right|\right)$$

input
```
integrate((x^2-5*x+6)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^2 - 5*x + 6)*(2*x - 5) + 1/8*log(abs(-2*x + 2*sqrt(x^2 - 5*x +
6) + 5))
```

Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \sqrt{6-5x+x^2}\,dx = \left(\frac{x}{2}-\frac{5}{4}\right)\sqrt{x^2-5x+6}-\frac{\ln\left(x+\sqrt{x^2-5x+6}-\frac{5}{2}\right)}{8}$$

input
```
int((x^2 - 5*x + 6)^(1/2),x)
```

output
```
(x/2 - 5/4)*(x^2 - 5*x + 6)^(1/2) - log(x + (x^2 - 5*x + 6)^(1/2) - 5/2)/8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \sqrt{6-5x+x^2}\,dx = \frac{\sqrt{x^2-5x+6}\,x}{2}-\frac{5\sqrt{x^2-5x+6}}{4}-\frac{\log\left(2\sqrt{x^2-5x+6}+2x-5\right)}{8}$$

input
```
int((x^2-5*x+6)^(1/2),x)
```

output
```
(4*sqrt(x**2 - 5*x + 6)*x - 10*sqrt(x**2 - 5*x + 6) - log(2*sqrt(x**2 - 5*
x + 6) + 2*x - 5))/8
```

3.57 $\int \frac{1}{\sqrt{6-5x+x^2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{1}{\sqrt{6-5x+x^2}}\,dx = -2\text{arctanh}\left(\frac{2-x}{\sqrt{6-5x+x^2}}\right)$$

output
```
-2*arctanh((2-x)/(x^2-5*x+6)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{6-5x+x^2}}\,dx = -\log\left(5-2x+2\sqrt{6-5x+x^2}\right)$$

input
```
Integrate[1/Sqrt[6 - 5*x + x^2],x]
```

output
```
-Log[5 - 2*x + 2*Sqrt[6 - 5*x + x^2]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-5x+6}}dx$$

$$\Big\downarrow \text{1092}$$

$$2\int \frac{1}{4-\frac{(5-2x)^2}{x^2-5x+6}}d\left(-\frac{5-2x}{\sqrt{x^2-5x+6}}\right)$$

$$\Big\downarrow \text{219}$$

$$-\text{arctanh}\left(\frac{5-2x}{2\sqrt{x^2-5x+6}}\right)$$

input
```
Int[1/Sqrt[6 - 5*x + x^2],x]
```

output
```
-ArcTanh[(5 - 2*x)/(2*Sqrt[6 - 5*x + x^2])]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

method	result	size
default	$\ln\left(-\frac{5}{2}+x+\sqrt{x^{2}-5x+6}\right)$	15
trager	$\ln\left(2x-5+2\sqrt{x^{2}-5x+6}\right)$	19

input
```
int(1/(x^2-5*x+6)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(-5/2+x+(x^2-5*x+6)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{6-5x+x^2}}\,dx = -\log\left(-2\,x+2\,\sqrt{x^2-5\,x+6}+5\right)$$

input
```
integrate(1/(x^2-5*x+6)^(1/2),x, algorithm="fricas")
```

output
```
-log(-2*x + 2*sqrt(x^2 - 5*x + 6) + 5)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{6-5x+x^2}}\,dx = \log\left(2x+2\sqrt{x^2-5x+6}-5\right)$$

input
```
integrate(1/(x**2-5*x+6)**(1/2),x)
```

output
```
log(2*x + 2*sqrt(x**2 - 5*x + 6) - 5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{6-5x+x^2}}\,dx = \log\left(2\,x + 2\,\sqrt{x^2-5\,x+6} - 5\right)$$

input
```
integrate(1/(x^2-5*x+6)^(1/2),x, algorithm="maxima")
```

output
```
log(2*x + 2*sqrt(x^2 - 5*x + 6) - 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(18) = 36.

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{1}{\sqrt{6-5x+x^2}}\,dx = \frac{1}{4}\,\sqrt{x^2-5\,x+6}(2\,x-5) + \frac{1}{8}\,\log\left(\left|-2\,x + 2\,\sqrt{x^2-5\,x+6} + 5\right|\right)$$

input
```
integrate(1/(x^2-5*x+6)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^2 - 5*x + 6)*(2*x - 5) + 1/8*log(abs(-2*x + 2*sqrt(x^2 - 5*x +
6) + 5))
```

Mupad [B] (verification not implemented)

Time = 9.95 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{6-5x+x^2}}\,dx = \ln\left(x + \sqrt{x^2-5\,x+6} - \frac{5}{2}\right)$$

input
```
int(1/(x^2 - 5*x + 6)^(1/2),x)
```

output
```
log(x + (x^2 - 5*x + 6)^(1/2) - 5/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{6-5x+x^2}}\,dx = \log\left(2\sqrt{x^2-5x+6}+2x-5\right)$$

input
```
int(1/(x^2-5*x+6)^(1/2),x)
```

output
```
log(2*sqrt(x**2 - 5*x + 6) + 2*x - 5)
```

3.58 $\int \frac{1}{(6-5x+x^2)^{3/2}} dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{1}{(6-5x+x^2)^{3/2}} dx = \frac{2(5-2x)}{\sqrt{6-5x+x^2}}$$

output
```
2*(5-2*x)/(x^2-5*x+6)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(6-5x+x^2)^{3/2}} dx = -\frac{2(-5+2x)}{\sqrt{6-5x+x^2}}$$

input
```
Integrate[(6 - 5*x + x^2)^(-3/2),x]
```

output
```
(-2*(-5 + 2*x))/Sqrt[6 - 5*x + x^2]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-5x+6)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{2(5-2x)}{\sqrt{x^2-5x+6}}$$

input
```
Int[(6 - 5*x + x^2)^(-3/2),x]
```

output
```
(2*(5 - 2*x))/Sqrt[6 - 5*x + x^2]
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{2(-5+2x)}{\sqrt{x^2-5x+6}}$	18
trager	$-\frac{2(-5+2x)}{\sqrt{x^2-5x+6}}$	18
risch	$-\frac{2(-5+2x)}{\sqrt{x^2-5x+6}}$	18
gosper	$-\frac{2(x-2)(-3+x)(-5+2x)}{(x^2-5x+6)^{\frac{3}{2}}}$	24
orering	$-\frac{2(x-2)(-3+x)(-5+2x)}{(x^2-5x+6)^{\frac{3}{2}}}$	24

input
```
int(1/(x^2-5*x+6)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(x^2-5*x+6)^(1/2)*(-5+2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(17) = 34.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \frac{1}{(6-5x+x^2)^{3/2}}\,dx = -\frac{2\left(2\,x^2+\sqrt{x^2-5\,x+6}(2\,x-5)-10\,x+12\right)}{x^2-5\,x+6}$$

input
```
integrate(1/(x^2-5*x+6)^(3/2),x, algorithm="fricas")
```

output
```
-2*(2*x^2 + sqrt(x^2 - 5*x + 6)*(2*x - 5) - 10*x + 12)/(x^2 - 5*x + 6)
```

Sympy [F]

$$\int \frac{1}{(6-5x+x^2)^{3/2}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x**2-5*x+6)**(3/2),x)
```

output
```
Integral((x**2 - 5*x + 6)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{(6-5x+x^2)^{3/2}}\,dx = -\frac{4\,x}{\sqrt{x^2-5\,x+6}} + \frac{10}{\sqrt{x^2-5\,x+6}}$$

input
```
integrate(1/(x^2-5*x+6)^(3/2),x, algorithm="maxima")
```

output
```
-4*x/sqrt(x^2 - 5*x + 6) + 10/sqrt(x^2 - 5*x + 6)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{(6-5x+x^2)^{3/2}}\,dx = -\frac{2\,(2\,x-5)}{\sqrt{x^2-5\,x+6}}$$

input
```
integrate(1/(x^2-5*x+6)^(3/2),x, algorithm="giac")
```

output
```
-2*(2*x - 5)/sqrt(x^2 - 5*x + 6)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(6-5x+x^2)^{3/2}}\,dx = -\frac{4\left(x-\frac{5}{2}\right)}{\sqrt{x^2-5\,x+6}}$$

input
```
int(1/(x^2 - 5*x + 6)^(3/2),x)
```

output
```
-(4*(x - 5/2))/(x^2 - 5*x + 6)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \frac{1}{(6-5x+x^2)^{3/2}}\,dx = \frac{-4\sqrt{x^2-5x+6}\,x+10\sqrt{x^2-5x+6}-4x^2+20x-24}{x^2-5x+6}$$

input
```
int(1/(x^2-5*x+6)^(3/2),x)
```

output
```
(2*( - 2*sqrt(x**2 - 5*x + 6)*x + 5*sqrt(x**2 - 5*x + 6) - 2*x**2 + 10*x -
 12))/(x**2 - 5*x + 6)
```

3.59 $\int \frac{1}{(6-5x+x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{(6-5x+x^2)^{5/2}}\,dx=\frac{2(5-2x)}{3\,(6-5x+x^2)^{3/2}}-\frac{16(5-2x)}{3\sqrt{6-5x+x^2}}$$

output
```
2/3*(5-2*x)/(x^2-5*x+6)^(3/2)-16/3*(5-2*x)/(x^2-5*x+6)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1}{(6-5x+x^2)^{5/2}}\,dx=\frac{2\sqrt{6-5x+x^2}(-235+294x-120x^2+16x^3)}{3(-3+x)^2(-2+x)^2}$$

input
```
Integrate[(6 - 5*x + x^2)^(-5/2),x]
```

output
```
(2*Sqrt[6 - 5*x + x^2]*(-235 + 294*x - 120*x^2 + 16*x^3))/(3*(-3 + x)^2*(-
2 + x)^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-5x+6)^{5/2}}dx$$

$$\Big\downarrow \text{ 1089}$$

$$\frac{2(5-2x)}{3(x^2-5x+6)^{3/2}}-\frac{8}{3}\int \frac{1}{(x^2-5x+6)^{3/2}}dx$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{2(5-2x)}{3(x^2-5x+6)^{3/2}}-\frac{16(5-2x)}{3\sqrt{x^2-5x+6}}$$

input
```
Int[(6 - 5*x + x^2)^(-5/2),x]
```

output
```
(2*(5 - 2*x))/(3*(6 - 5*x + x^2)^(3/2)) - (16*(5 - 2*x))/(3*Sqrt[6 - 5*x +
 x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

method	result	size
trager	$\frac{-\frac{470}{3}+196x+\frac{32}{3}x^3-80x^2}{(x^2-5x+6)^{\frac{3}{2}}}$	28
risch	$\frac{-\frac{470}{3}+196x+\frac{32}{3}x^3-80x^2}{(x^2-5x+6)^{\frac{3}{2}}}$	28
gosper	$\frac{2(x-2)(-3+x)(16x^3-120x^2+294x-235)}{3(x^2-5x+6)^{\frac{5}{2}}}$	34
orering	$\frac{2(x-2)(-3+x)(16x^3-120x^2+294x-235)}{3(x^2-5x+6)^{\frac{5}{2}}}$	34
default	$-\frac{2(-5+2x)}{3(x^2-5x+6)^{\frac{3}{2}}}+\frac{\frac{32x}{3}-\frac{80}{3}}{\sqrt{x^2-5x+6}}$	36

input
```
int(1/(x^2-5*x+6)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(16*x^3-120*x^2+294*x-235)/(x^2-5*x+6)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{1}{(6-5x+x^2)^{5/2}}\,dx = \frac{2\left(16\,x^4-160\,x^3+592\,x^2+(16\,x^3-120\,x^2+294\,x-235)\sqrt{x^2-5\,x+6}-960\right.}{3\,(x^4-10\,x^3+37\,x^2-60\,x+36)}$$

input
```
integrate(1/(x^2-5*x+6)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(16*x^4 - 160*x^3 + 592*x^2 + (16*x^3 - 120*x^2 + 294*x - 235)*sqrt(x^
2 - 5*x + 6) - 960*x + 576)/(x^4 - 10*x^3 + 37*x^2 - 60*x + 36)
```

Sympy [F]

$$\int \frac{1}{(6-5x+x^2)^{5/2}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(x**2-5*x+6)**(5/2),x)
```

output
```
Integral((x**2 - 5*x + 6)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{(6-5x+x^2)^{5/2}}\,dx = \frac{32\,x}{3\,\sqrt{x^2-5\,x+6}} - \frac{80}{3\,\sqrt{x^2-5\,x+6}} - \frac{4\,x}{3\,(x^2-5\,x+6)^{\frac{3}{2}}} + \frac{10}{3\,(x^2-5\,x+6)^{\frac{3}{2}}}$$

input
```
integrate(1/(x^2-5*x+6)^(5/2),x, algorithm="maxima")
```

output
```
32/3*x/sqrt(x^2 - 5*x + 6) - 80/3/sqrt(x^2 - 5*x + 6) - 4/3*x/(x^2 - 5*x +
 6)^(3/2) + 10/3/(x^2 - 5*x + 6)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1}{(6-5x+x^2)^{5/2}}\,dx = \frac{2\,(2\,(4\,(2\,x-15)x+147)x-235)}{3\,(x^2-5\,x+6)^{\frac{3}{2}}}$$

input
```
integrate(1/(x^2-5*x+6)^(5/2),x, algorithm="giac")
```

output
```
2/3*(2*(4*(2*x - 15)*x + 147)*x - 235)/(x^2 - 5*x + 6)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1}{(6-5x+x^2)^{5/2}}\,dx = \frac{(4\,x-10)\,\left(8\,x^2-40\,x+47\right)}{3\,\left(x^2-5\,x+6\right)^{3/2}}$$

input
```
int(1/(x^2 - 5*x + 6)^(5/2),x)
```

output
```
((4*x - 10)*(8*x^2 - 40*x + 47))/(3*(x^2 - 5*x + 6)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.21

$$\int \frac{1}{(6-5x+x^2)^{5/2}}\,dx = \frac{32\sqrt{x^2-5x+6}\,x^3-240\sqrt{x^2-5x+6}\,x^2+588\sqrt{x^2-5x+6}\,x-470\sqrt{x^2-5x}}{3x^4-30x^3+111x^2-180x+108}$$

input
```
int(1/(x^2-5*x+6)^(5/2),x)
```

output
```
(2*(16*sqrt(x**2 - 5*x + 6)*x**3 - 120*sqrt(x**2 - 5*x + 6)*x**2 + 294*sqr
t(x**2 - 5*x + 6)*x - 235*sqrt(x**2 - 5*x + 6) - 16*x**4 + 160*x**3 - 592*
x**2 + 960*x - 576))/(3*(x**4 - 10*x**3 + 37*x**2 - 60*x + 36))
```

3.60 $\int \frac{1}{(6-5x+x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{1}{(6-5x+x^2)^{7/2}}\,dx = \frac{2(5-2x)}{5\,(6-5x+x^2)^{5/2}} - \frac{32(5-2x)}{15\,(6-5x+x^2)^{3/2}} + \frac{256(5-2x)}{15\sqrt{6-5x+x^2}}$$

output
```
2/5*(5-2*x)/(x^2-5*x+6)^(5/2)-32/15*(5-2*x)/(x^2-5*x+6)^(3/2)+256/15*(5-2*
x)/(x^2-5*x+6)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \frac{1}{(6-5x+x^2)^{7/2}}\,dx = -\frac{2\sqrt{6-5x+x^2}(-22575+47030x-38800x^2+15840x^3-3200x^4+256x^5)}{15(-3+x)^3(-2+x)^3}$$

input
```
Integrate[(6 - 5*x + x^2)^(-7/2),x]
```

output
```
(-2*Sqrt[6 - 5*x + x^2]*(-22575 + 47030*x - 38800*x^2 + 15840*x^3 - 3200*x
^4 + 256*x^5))/(15*(-3 + x)^3*(-2 + x)^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-5x+6)^{7/2}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2(5-2x)}{5\,(x^2-5x+6)^{5/2}}-\frac{16}{5}\int \frac{1}{(x^2-5x+6)^{5/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2(5-2x)}{5\,(x^2-5x+6)^{5/2}}-\frac{16}{5}\left(\frac{2(5-2x)}{3\,(x^2-5x+6)^{3/2}}-\frac{8}{3}\int \frac{1}{(x^2-5x+6)^{3/2}}dx\right)$$

$$\Big\downarrow \text{1088}$$

$$\frac{2(5-2x)}{5\,(x^2-5x+6)^{5/2}}-\frac{16}{5}\left(\frac{2(5-2x)}{3\,(x^2-5x+6)^{3/2}}-\frac{16(5-2x)}{3\sqrt{x^2-5x+6}}\right)$$

input
```
Int[(6 - 5*x + x^2)^(-7/2),x]
```

output
```
(2*(5 - 2*x))/(5*(6 - 5*x + x^2)^(5/2)) - (16*((2*(5 - 2*x))/(3*(6 - 5*x +
 x^2)^(3/2)) - (16*(5 - 2*x))/(3*Sqrt[6 - 5*x + x^2])))/5
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.59

method	result	size
trager	$-\frac{2\left(256x^{5}-3200x^{4}+15840x^{3}-38800x^{2}+47030x-22575\right)}{15\left(x^{2}-5x+6\right)^{\frac{5}{2}}}$	38
risch	$-\frac{2\left(256x^{5}-3200x^{4}+15840x^{3}-38800x^{2}+47030x-22575\right)}{15\left(x^{2}-5x+6\right)^{\frac{5}{2}}}$	38
gosper	$-\frac{2(x-2)(-3+x)\left(256x^{5}-3200x^{4}+15840x^{3}-38800x^{2}+47030x-22575\right)}{15\left(x^{2}-5x+6\right)^{\frac{7}{2}}}$	44
orering	$-\frac{2(x-2)(-3+x)\left(256x^{5}-3200x^{4}+15840x^{3}-38800x^{2}+47030x-22575\right)}{15\left(x^{2}-5x+6\right)^{\frac{7}{2}}}$	44
default	$-\frac{2(-5+2x)}{5\left(x^{2}-5x+6\right)^{\frac{5}{2}}}+\frac{-\frac{32}{3}+\frac{64x}{15}}{\left(x^{2}-5x+6\right)^{\frac{3}{2}}}-\frac{256(-5+2x)}{15\sqrt{x^{2}-5x+6}}$	53

input
```
int(1/(x^2-5*x+6)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(256*x^5-3200*x^4+15840*x^3-38800*x^2+47030*x-22575)/(x^2-5*x+6)^(5/
2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

$$\int \frac{1}{(6-5x+x^2)^{7/2}}\,dx =$$
$$-\frac{2\,(256\,x^6 - 3840\,x^5 + 23808\,x^4 - 78080\,x^3 + 142848\,x^2 + (256\,x^5 - 3200\,x^4 + 15840\,x^3 - 38800\,x^2 + 47}{15\,(x^6 - 15\,x^5 + 93\,x^4 - 305\,x^3 + 558\,x^2 - 540\,x + 216}$$

input
```
integrate(1/(x^2-5*x+6)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(256*x^6 - 3840*x^5 + 23808*x^4 - 78080*x^3 + 142848*x^2 + (256*x^5
- 3200*x^4 + 15840*x^3 - 38800*x^2 + 47030*x - 22575)*sqrt(x^2 - 5*x + 6)
- 138240*x + 55296)/(x^6 - 15*x^5 + 93*x^4 - 305*x^3 + 558*x^2 - 540*x + 2
16)
```

Sympy [F]

$$\int \frac{1}{(6-5x+x^2)^{7/2}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(x**2-5*x+6)**(7/2),x)
```

output
```
Integral((x**2 - 5*x + 6)**(-7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \frac{1}{(6-5x+x^2)^{7/2}}\,dx = -\frac{512\,x}{15\,\sqrt{x^2-5\,x+6}} + \frac{256}{3\,\sqrt{x^2-5\,x+6}}$$
$$+\frac{64\,x}{15\,(x^2-5\,x+6)^{\frac{3}{2}}} - \frac{32}{3\,(x^2-5\,x+6)^{\frac{3}{2}}} - \frac{4\,x}{5\,(x^2-5\,x+6)^{\frac{5}{2}}} + \frac{2}{(x^2-5\,x+6)^{\frac{5}{2}}}$$

input
```
integrate(1/(x^2-5*x+6)^(7/2),x, algorithm="maxima")
```

output
```
-512/15*x/sqrt(x^2 - 5*x + 6) + 256/3/sqrt(x^2 - 5*x + 6) + 64/15*x/(x^2 -
 5*x + 6)^(3/2) - 32/3/(x^2 - 5*x + 6)^(3/2) - 4/5*x/(x^2 - 5*x + 6)^(5/2)
 + 2/(x^2 - 5*x + 6)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.58

$$\int \frac{1}{(6-5x+x^2)^{7/2}}\,dx = -\frac{2\,(2\,(8\,(2\,(4\,(2\,x-25)x+495)x-2425)x+23515)x-22575)}{15\,(x^2-5\,x+6)^{\frac{5}{2}}}$$

input
```
integrate(1/(x^2-5*x+6)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(2*(8*(2*(4*(2*x - 25)*x + 495)*x - 2425)*x + 23515)*x - 22575)/(x^2
 - 5*x + 6)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{1}{(6-5x+x^2)^{7/2}}\,dx = \frac{788\,x-512\,x\,(x^2-5\,x+6)^2+1280\,(x^2-5\,x+6)^2-160\,x^2+64\,x\,(x^2-5\,x+}{(x^2-5\,x+6)^{3/2}\,(15\,x^2-75\,x+90)}$$

input
```
int(1/(x^2 - 5*x + 6)^(7/2),x)
```

output
```
(788*x - 512*x*(x^2 - 5*x + 6)^2 + 1280*(x^2 - 5*x + 6)^2 - 160*x^2 + 64*x
*(x^2 - 5*x + 6) - 930)/((x^2 - 5*x + 6)^(3/2)*(15*x^2 - 75*x + 90))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.23

$$\int \frac{1}{(6-5x+x^2)^{7/2}}\,dx = \frac{-512\sqrt{x^2-5x+6}\,x^5+6400\sqrt{x^2-5x+6}\,x^4-31680\sqrt{x^2-5x+6}\,x^3+77600}{1}$$

input
```
int(1/(x^2-5*x+6)^(7/2),x)
```

output
```
(2*( - 256*sqrt(x**2 - 5*x + 6)*x**5 + 3200*sqrt(x**2 - 5*x + 6)*x**4 - 15
840*sqrt(x**2 - 5*x + 6)*x**3 + 38800*sqrt(x**2 - 5*x + 6)*x**2 - 47030*sq
rt(x**2 - 5*x + 6)*x + 22575*sqrt(x**2 - 5*x + 6) + 256*x**6 - 3840*x**5 +
 23808*x**4 - 78080*x**3 + 142848*x**2 - 138240*x + 55296))/(15*(x**6 - 15
*x**5 + 93*x**4 - 305*x**3 + 558*x**2 - 540*x + 216))
```

3.61 $\int (-1-x+x^2)^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int (-1-x+x^2)^{5/2}\,dx=-\frac{125}{512}(1-2x)\sqrt{-1-x+x^2}+\frac{25}{192}(1-2x)\left(-1-x+x^2\right)^{3/2}-\frac{1}{12}(1-2x)\left(-1-x+x^2\right)^{5/2}+\frac{625\text{arctanh}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right)}{1024}$$

output
```
-125/512*(1-2*x)*(x^2-x-1)^(1/2)+25/192*(1-2*x)*(x^2-x-1)^(3/2)-1/12*(1-2*
x)*(x^2-x-1)^(5/2)+625/1024*arctanh(1/2*(1-2*x)/(x^2-x-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int (-1-x+x^2)^{5/2}\,dx=\frac{\sqrt{-1-x+x^2}(-703+950x+1240x^2-400x^3-640x^4+256x^5)}{1536}+\frac{625\log\left(1-2x+2\sqrt{-1-x+x^2}\right)}{1024}$$

input
```
Integrate[(-1 - x + x^2)^(5/2),x]
```

output
```
(Sqrt[-1 - x + x^2]*(-703 + 950*x + 1240*x^2 - 400*x^3 - 640*x^4 + 256*x^5
))/1536 + (625*Log[1 - 2*x + 2*Sqrt[-1 - x + x^2]])/1024
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = $\{1087, 1087, 1087, 1092, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2-x-1\right)^{5/2} dx$$

$$\Big\downarrow \text{ 1087}$$

$$-\frac{25}{24} \int \left(x^2-x-1\right)^{3/2} dx-\frac{1}{12}(1-2x) \left(x^2-x-1\right)^{5/2}$$

$$\Big\downarrow \text{ 1087}$$

$$-\frac{25}{24} \left(-\frac{15}{16} \int \sqrt{x^2-x-1}dx-\frac{1}{8}(1-2x) \left(x^2-x-1\right)^{3/2}\right)-\frac{1}{12}(1-2x) \left(x^2-x-1\right)^{5/2}$$

$$\Big\downarrow \text{ 1087}$$

$$-\frac{25}{24} \left(-\frac{15}{16} \left(-\frac{5}{8} \int \frac{1}{\sqrt{x^2-x-1}}dx-\frac{1}{4}\sqrt{x^2-x-1}(1-2x)\right)-\frac{1}{8}(1-2x) \left(x^2-x-1\right)^{3/2}\right)-\frac{1}{12}(1-2x) \left(x^2-x-1\right)^{5/2}$$

$$\Big\downarrow \text{ 1092}$$

$$-\frac{25}{24} \left(-\frac{15}{16} \left(-\frac{5}{4} \int \frac{1}{4-\frac{(1-2x)^2}{x^2-x-1}}d\left(-\frac{1-2x}{\sqrt{x^2-x-1}}\right)-\frac{1}{4}\sqrt{x^2-x-1}(1-2x)\right)-\frac{1}{8}(1-2x) \left(x^2-x-1\right)^{3/2}\right)-\frac{1}{12}(1-2x) \left(x^2-x-1\right)^{5/2}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{25}{24}\left(-\frac{15}{16}\left(\frac{5}{8}\text{arctanh}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)-\frac{1}{4}(1-2x)\sqrt{x^2-x-1}\right)-\frac{1}{8}(1-2x)\left(x^2-x-1\right)^{3/2}\right)-\frac{1}{12}(1-2x)\left(x^2-x-1\right)^{5/2}$$

input
```
Int[(-1 - x + x^2)^(5/2),x]
```

output
```
-1/12*((1 - 2*x)*(-1 - x + x^2)^(5/2)) - (25*(-1/8*((1 - 2*x)*(-1 - x + x^
2)^(3/2)) - (15*(-1/4*((1 - 2*x)*Sqrt[-1 - x + x^2]) + (5*ArcTanh[(1 - 2*x
)/(2*Sqrt[-1 - x + x^2])])/8))/16))/24
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{(256x^5-640x^4-400x^3+1240x^2+950x-703)\sqrt{x^2-x-1}}{1536}-\frac{625\ln\left(x-\frac{1}{2}+\sqrt{x^2-x-1}\right)}{1024}$	55
trager	$\left(\frac{1}{6}x^5-\frac{5}{12}x^4-\frac{25}{96}x^3+\frac{155}{192}x^2+\frac{475}{768}x-\frac{703}{1536}\right)\sqrt{x^2-x-1}-\frac{625\ln\left(2\sqrt{x^2-x-1}-1+2x\right)}{1024}$	58
default	$\frac{(2x-1)(x^2-x-1)^{\frac{5}{2}}}{12}-\frac{25(x^2-x-1)^{\frac{3}{2}}(2x-1)}{192}+\frac{125(2x-1)\sqrt{x^2-x-1}}{512}-\frac{625\ln\left(x-\frac{1}{2}+\sqrt{x^2-x-1}\right)}{1024}$	69

input
```
int((x^2-x-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/1536*(256*x^5-640*x^4-400*x^3+1240*x^2+950*x-703)*(x^2-x-1)^(1/2)-625/10
24*ln(x-1/2+(x^2-x-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int (-1-x+x^2)^{5/2}\,dx=\frac{1}{1536}\left(256\,x^5-640\,x^4-400\,x^3+1240\,x^2+950\,x-703\right)\sqrt{x^2-x-1}+\frac{625}{1024}\log\left(-2\,x+2\,\sqrt{x^2-x-1}+1\right)$$

input
```
integrate((x^2-x-1)^(5/2),x, algorithm="fricas")
```

output
```
1/1536*(256*x^5 - 640*x^4 - 400*x^3 + 1240*x^2 + 950*x - 703)*sqrt(x^2 - x
 - 1) + 625/1024*log(-2*x + 2*sqrt(x^2 - x - 1) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(73) = 146$.

Time = 0.51 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.89

$$\int \left(-1-x+x^2\right)^{5/2} dx = \left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{x^2-x-1} + 2\left(\frac{x^2}{3}-\frac{x}{12}-\frac{11}{24}\right)\sqrt{x^2-x-1} - \sqrt{x^2-x-1}\left(\frac{x^3}{4}-\frac{x^2}{24}-\frac{17x}{96}-\frac{67}{192}\right) - 2\sqrt{x^2-x-1}\left(\frac{x^4}{5}-\frac{x^3}{40}-\frac{23x^2}{240}-\frac{151x}{960}-\frac{821}{1920}\right) + \sqrt{x^2-x-1}\left(\frac{x^5}{6}-\frac{x^4}{60}-\frac{29x^3}{480}-\frac{89x^2}{960}-\frac{793x}{3840}-\frac{3803}{7680}\right) - \frac{625\log\left(2x+2\sqrt{x^2-x-1}-1\right)}{1024}$$

input
```
integrate((x**2-x-1)**(5/2),x)
```

output
```
(x/2 - 1/4)*sqrt(x**2 - x - 1) + 2*(x**2/3 - x/12 - 11/24)*sqrt(x**2 - x -
 1) - sqrt(x**2 - x - 1)*(x**3/4 - x**2/24 - 17*x/96 - 67/192) - 2*sqrt(x*
*2 - x - 1)*(x**4/5 - x**3/40 - 23*x**2/240 - 151*x/960 - 821/1920) + sqrt
(x**2 - x - 1)*(x**5/6 - x**4/60 - 29*x**3/480 - 89*x**2/960 - 793*x/3840
- 3803/7680) - 625*log(2*x + 2*sqrt(x**2 - x - 1) - 1)/1024
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int \left(-1-x+x^2\right)^{5/2} dx = \frac{1}{6}\left(x^2-x-1\right)^{\frac{5}{2}}x - \frac{1}{12}\left(x^2-x-1\right)^{\frac{5}{2}} - \frac{25}{96}\left(x^2-x-1\right)^{\frac{3}{2}}x + \frac{25}{192}\left(x^2-x-1\right)^{\frac{3}{2}} + \frac{125}{256}\sqrt{x^2-x-1}x - \frac{125}{512}\sqrt{x^2-x-1} - \frac{625}{1024}\log\left(2\,x+2\,\sqrt{x^2-x-1}-1\right)$$

input
```
integrate((x^2-x-1)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(x^2 - x - 1)^(5/2)*x - 1/12*(x^2 - x - 1)^(5/2) - 25/96*(x^2 - x - 1)
^(3/2)*x + 25/192*(x^2 - x - 1)^(3/2) + 125/256*sqrt(x^2 - x - 1)*x - 125/
512*sqrt(x^2 - x - 1) - 625/1024*log(2*x + 2*sqrt(x^2 - x - 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

$$\int \left(-1-x+x^2\right)^{5/2}\,dx=\frac{1}{1536}\,\left(2\,\left(4\,\left(2\,\left(8\,\left(2\,x-5\right)x-25\right)x+155\right)x+475\right)x-703\right)\sqrt{x^2-x-1}+\frac{625}{1024}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x-1}+1\right|\right)$$

input
```
integrate((x^2-x-1)^(5/2),x, algorithm="giac")
```

output
```
1/1536*(2*(4*(2*(8*(2*x - 5)*x - 25)*x + 155)*x + 475)*x - 703)*sqrt(x^2 -
 x - 1) + 625/1024*log(abs(-2*x + 2*sqrt(x^2 - x - 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

$$\int \left(-1-x+x^2\right)^{5/2}\,dx=\frac{125\,\left(\frac{x}{2}-\frac{1}{4}\right)\,\sqrt{x^2-x-1}}{128}-\frac{625\,\ln\left(x+\sqrt{x^2-x-1}-\frac{1}{2}\right)}{1024}-\frac{25\,\left(x-\frac{1}{2}\right)\,\left(x^2-x-1\right)^{3/2}}{96}+\frac{\left(x-\frac{1}{2}\right)\,\left(x^2-x-1\right)^{5/2}}{6}$$

input
```
int((x^2 - x - 1)^(5/2),x)
```

output
```
(125*(x/2 - 1/4)*(x^2 - x - 1)^(1/2))/128 - (625*log(x + (x^2 - x - 1)^(1/
2) - 1/2))/1024 - (25*(x - 1/2)*(x^2 - x - 1)^(3/2))/96 + ((x - 1/2)*(x^2
- x - 1)^(5/2))/6
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16

$$\int \left(-1-x+x^2\right)^{5/2}\,dx=\frac{\sqrt{x^2-x-1}\,x^5}{6}-\frac{5\sqrt{x^2-x-1}\,x^4}{12}-\frac{25\sqrt{x^2-x-1}\,x^3}{96}+\frac{155\sqrt{x^2-x-1}\,x^2}{192}+\frac{475\sqrt{x^2-x-1}\,x}{768}-\frac{703\sqrt{x^2-x-1}}{1536}-\frac{625\log\left(\frac{2\sqrt{x^2-x-1}+2x-1}{\sqrt{5}}\right)}{1024}$$

input
```
int((x^2-x-1)^(5/2),x)
```

output
```
(512*sqrt(x**2 - x - 1)*x**5 - 1280*sqrt(x**2 - x - 1)*x**4 - 800*sqrt(x**
2 - x - 1)*x**3 + 2480*sqrt(x**2 - x - 1)*x**2 + 1900*sqrt(x**2 - x - 1)*x
 - 1406*sqrt(x**2 - x - 1) - 1875*log((2*sqrt(x**2 - x - 1) + 2*x - 1)/sqr
t(5)))/3072
```

3.62 $\int \left(-1-x+x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \left(-1-x+x^2\right)^{3/2} \, dx = \frac{15}{64}(1-2x)\sqrt{-1-x+x^2} - \frac{1}{8}(1-2x)\left(-1-x+x^2\right)^{3/2} - \frac{75}{128}\text{arctanh}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right)$$

output
```
15/64*(1-2*x)*(x^2-x-1)^(1/2)-1/8*(1-2*x)*(x^2-x-1)^(3/2)-75/128*arctanh(1
/2*(1-2*x)/(x^2-x-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \left(-1-x+x^2\right)^{3/2} \, dx = \frac{1}{64}\sqrt{-1-x+x^2}\left(23-38x-24x^2+16x^3\right) - \frac{75}{128}\log\left(1-2x+2\sqrt{-1-x+x^2}\right)$$

input
```
Integrate[(-1 - x + x^2)^(3/2),x]
```

output
```
(Sqrt[-1 - x + x^2]*(23 - 38*x - 24*x^2 + 16*x^3))/64 - (75*Log[1 - 2*x +
2*Sqrt[-1 - x + x^2]])/128
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2-x-1\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1087}$$

$$-\frac{15}{16}\int \sqrt{x^2-x-1}dx-\frac{1}{8}(1-2x)\left(x^2-x-1\right)^{3/2}$$

$$\Big\downarrow \text{ 1087}$$

$$-\frac{15}{16}\left(-\frac{5}{8}\int \frac{1}{\sqrt{x^2-x-1}}dx-\frac{1}{4}\sqrt{x^2-x-1}(1-2x)\right)-\frac{1}{8}(1-2x)\left(x^2-x-1\right)^{3/2}$$

$$\Big\downarrow \text{ 1092}$$

$$-\frac{15}{16}\left(-\frac{5}{4}\int \frac{1}{4-\frac{(1-2x)^2}{x^2-x-1}}d\left(-\frac{1-2x}{\sqrt{x^2-x-1}}\right)-\frac{1}{4}\sqrt{x^2-x-1}(1-2x)\right)-\frac{1}{8}(1-2x)\left(x^2-x-1\right)^{3/2}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{15}{16}\left(\frac{5}{8}\text{arctanh}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)-\frac{1}{4}(1-2x)\sqrt{x^2-x-1}\right)-\frac{1}{8}(1-2x)\left(x^2-x-1\right)^{3/2}$$

input
```
Int[(-1 - x + x^2)^(3/2),x]
```

output
```
-1/8*((1 - 2*x)*(-1 - x + x^2)^(3/2)) - (15*(-1/4*((1 - 2*x)*Sqrt[-1 - x +
 x^2]) + (5*ArcTanh[(1 - 2*x)/(2*Sqrt[-1 - x + x^2])])/8))/16
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{\left(16 x^{3}-24 x^{2}-38 x +23\right) \sqrt{x^{2}-x -1}}{64}+\frac{75 \ln \left(x -\frac{1}{2}+\sqrt{x^{2}-x -1}\right)}{128}$	45
trager	$\left(\frac{1}{4} x^{3}-\frac{3}{8} x^{2}-\frac{19}{32} x +\frac{23}{64}\right) \sqrt{x^{2}-x -1}+\frac{75 \ln \left(2 \sqrt{x^{2}-x -1}-1+2 x \right)}{128}$	48
default	$\frac{\left(x^{2}-x -1\right)^{\frac{3}{2}} \left(2 x -1\right)}{8}-\frac{15 \left(2 x -1\right) \sqrt{x^{2}-x -1}}{64}+\frac{75 \ln \left(x -\frac{1}{2}+\sqrt{x^{2}-x -1}\right)}{128}$	52

input
```
int((x^2-x-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(16*x^3-24*x^2-38*x+23)*(x^2-x-1)^(1/2)+75/128*ln(x-1/2+(x^2-x-1)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int \left(-1-x+x^2\right)^{3/2}\,dx=\frac{1}{64}\left(16\,x^3-24\,x^2-38\,x+23\right)\sqrt{x^2-x-1}-\frac{75}{128}\log\left(-2\,x+2\,\sqrt{x^2-x-1}+1\right)$$

input
```
integrate((x^2-x-1)^(3/2),x, algorithm="fricas")
```

output
```
1/64*(16*x^3 - 24*x^2 - 38*x + 23)*sqrt(x^2 - x - 1) - 75/128*log(-2*x + 2
*sqrt(x^2 - x - 1) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \left(-1-x+x^2\right)^{3/2}\,dx=-\left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{x^2-x-1}-\left(\frac{x^2}{3}-\frac{x}{12}-\frac{11}{24}\right)\sqrt{x^2-x-1}+\sqrt{x^2-x-1}\left(\frac{x^3}{4}-\frac{x^2}{24}-\frac{17x}{96}-\frac{67}{192}\right)+\frac{75\log\left(2x+2\sqrt{x^2-x-1}-1\right)}{128}$$

input
```
integrate((x**2-x-1)**(3/2),x)
```

output
```
-(x/2 - 1/4)*sqrt(x**2 - x - 1) - (x**2/3 - x/12 - 11/24)*sqrt(x**2 - x -
1) + sqrt(x**2 - x - 1)*(x**3/4 - x**2/24 - 17*x/96 - 67/192) + 75*log(2*x
 + 2*sqrt(x**2 - x - 1) - 1)/128
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \left(-1-x+x^2\right)^{3/2}\,dx=\frac{1}{4}\left(x^2-x-1\right)^{\frac{3}{2}}x-\frac{1}{8}\left(x^2-x-1\right)^{\frac{3}{2}}-\frac{15}{32}\sqrt{x^2-x-1}x+\frac{15}{64}\sqrt{x^2-x-1}+\frac{75}{128}\log\left(2\,x+2\,\sqrt{x^2-x-1}-1\right)$$

input
```
integrate((x^2-x-1)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(x^2 - x - 1)^(3/2)*x - 1/8*(x^2 - x - 1)^(3/2) - 15/32*sqrt(x^2 - x -
 1)*x + 15/64*sqrt(x^2 - x - 1) + 75/128*log(2*x + 2*sqrt(x^2 - x - 1) - 1
)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.71

$$\int \left(-1-x+x^2\right)^{3/2}\,dx=\frac{1}{64}\,(2\,(4\,(2\,x-3)x-19)x+23)\sqrt{x^2-x-1}-\frac{75}{128}\log\left(\left|-2\,x+2\,\sqrt{x^2-x-1}+1\right|\right)$$

input
```
integrate((x^2-x-1)^(3/2),x, algorithm="giac")
```

output
```
1/64*(2*(4*(2*x - 3)*x - 19)*x + 23)*sqrt(x^2 - x - 1) - 75/128*log(abs(-2
*x + 2*sqrt(x^2 - x - 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.71

$$\int \left(-1-x+x^2\right)^{3/2}\,dx=\frac{75\,\ln\left(x+\sqrt{x^2-x-1}-\frac{1}{2}\right)}{128}-\frac{15\,\left(\frac{x}{2}-\frac{1}{4}\right)\,\sqrt{x^2-x-1}}{16}+\frac{\left(x-\frac{1}{2}\right)\,\left(x^2-x-1\right)^{3/2}}{4}$$

input
```
int((x^2 - x - 1)^(3/2),x)
```

output
```
(75*log(x + (x^2 - x - 1)^(1/2) - 1/2))/128 - (15*(x/2 - 1/4)*(x^2 - x - 1
)^(1/2))/16 + ((x - 1/2)*(x^2 - x - 1)^(3/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \left(-1-x+x^2\right)^{3/2}\,dx=\frac{\sqrt{x^2-x-1}\,x^3}{4}-\frac{3\sqrt{x^2-x-1}\,x^2}{8}-\frac{19\sqrt{x^2-x-1}\,x}{32}+\frac{23\sqrt{x^2-x-1}}{64}+\frac{75\log\left(\frac{2\sqrt{x^2-x-1}+2x-1}{\sqrt{5}}\right)}{128}$$

input
```
int((x^2-x-1)^(3/2),x)
```

output
```
(32*sqrt(x**2 - x - 1)*x**3 - 48*sqrt(x**2 - x - 1)*x**2 - 76*sqrt(x**2 -
x - 1)*x + 46*sqrt(x**2 - x - 1) + 75*log((2*sqrt(x**2 - x - 1) + 2*x - 1)
/sqrt(5)))/128
```

3.63 $\int \sqrt{-1-x+x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 48

$$\int \sqrt{-1-x+x^2}\,dx=-\frac{1}{4}(1-2x)\sqrt{-1-x+x^2}+\frac{5}{8}\text{arctanh}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right)$$

output
```
-1/4*(1-2*x)*(x^2-x-1)^(1/2)+5/8*arctanh(1/2*(1-2*x)/(x^2-x-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \sqrt{-1-x+x^2}\,dx=\frac{1}{4}(-1+2x)\sqrt{-1-x+x^2}+\frac{5}{8}\log\left(1-2x+2\sqrt{-1-x+x^2}\right)$$

input
```
Integrate[Sqrt[-1 - x + x^2],x]
```

output
```
((-1 + 2*x)*Sqrt[-1 - x + x^2])/4 + (5*Log[1 - 2*x + 2*Sqrt[-1 - x + x^2]]
)/8
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2-x-1}\,dx$$

$$\Big\downarrow \text{1087}$$

$$-\frac{5}{8}\int \frac{1}{\sqrt{x^2-x-1}}dx-\frac{1}{4}\sqrt{x^2-x-1}(1-2x)$$

$$\Big\downarrow \text{1092}$$

$$-\frac{5}{4}\int \frac{1}{4-\frac{(1-2x)^2}{x^2-x-1}}d\left(-\frac{1-2x}{\sqrt{x^2-x-1}}\right)-\frac{1}{4}\sqrt{x^2-x-1}(1-2x)$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{8}\text{arctanh}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)-\frac{1}{4}(1-2x)\sqrt{x^2-x-1}$$

input
```
Int[Sqrt[-1 - x + x^2],x]
```

output
```
-1/4*((1 - 2*x)*Sqrt[-1 - x + x^2]) + (5*ArcTanh[(1 - 2*x)/(2*Sqrt[-1 - x
+ x^2])])/8
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{(2x-1)\sqrt{x^2-x-1}}{4}-\frac{5\ln\left(x-\frac{1}{2}+\sqrt{x^2-x-1}\right)}{8}$	35
risch	$\frac{(2x-1)\sqrt{x^2-x-1}}{4}-\frac{5\ln\left(x-\frac{1}{2}+\sqrt{x^2-x-1}\right)}{8}$	35
trager	$\left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{x^2-x-1}-\frac{5\ln\left(2\sqrt{x^2-x-1}-1+2x\right)}{8}$	38

input
```
int((x^2-x-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*x-1)*(x^2-x-1)^(1/2)-5/8*ln(x-1/2+(x^2-x-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int\sqrt{-1-x+x^2}\,dx=\frac{1}{4}\sqrt{x^2-x-1}(2x-1)+\frac{5}{8}\log\left(-2x+2\sqrt{x^2-x-1}+1\right)$$

input
```
integrate((x^2-x-1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(x^2 - x - 1)*(2*x - 1) + 5/8*log(-2*x + 2*sqrt(x^2 - x - 1) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \sqrt{-1-x+x^2}\,dx = \left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{x^2-x-1}-\frac{5\log\left(2x+2\sqrt{x^2-x-1}-1\right)}{8}$$

input
```
integrate((x**2-x-1)**(1/2),x)
```

output
```
(x/2 - 1/4)*sqrt(x**2 - x - 1) - 5*log(2*x + 2*sqrt(x**2 - x - 1) - 1)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \sqrt{-1-x+x^2}\,dx = \frac{1}{2}\sqrt{x^2-x-1}x-\frac{1}{4}\sqrt{x^2-x-1} - \frac{5}{8}\log\left(2\,x+2\sqrt{x^2-x-1}-1\right)$$

input
```
integrate((x^2-x-1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^2 - x - 1)*x - 1/4*sqrt(x^2 - x - 1) - 5/8*log(2*x + 2*sqrt(x^2
 - x - 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \sqrt{-1-x+x^2}\,dx = \frac{1}{4}\sqrt{x^2-x-1}(2\,x-1)+\frac{5}{8}\log\left(\left|-2\,x+2\sqrt{x^2-x-1}+1\right|\right)$$

input
```
integrate((x^2-x-1)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^2 - x - 1)*(2*x - 1) + 5/8*log(abs(-2*x + 2*sqrt(x^2 - x - 1) +
 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

$$\int \sqrt{-1-x+x^2}\,dx = \left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{x^2-x-1}-\frac{5\,\ln\left(x+\sqrt{x^2-x-1}-\frac{1}{2}\right)}{8}$$

input
```
int((x^2 - x - 1)^(1/2),x)
```

output
```
(x/2 - 1/4)*(x^2 - x - 1)^(1/2) - (5*log(x + (x^2 - x - 1)^(1/2) - 1/2))/8
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \sqrt{-1-x+x^2}\,dx = \frac{\sqrt{x^2-x-1}\,x}{2}-\frac{\sqrt{x^2-x-1}}{4}-\frac{5\log\left(\frac{2\sqrt{x^2-x-1}+2x-1}{\sqrt{5}}\right)}{8}$$

input
```
int((x^2-x-1)^(1/2),x)
```

output
```
(4*sqrt(x**2 - x - 1)*x - 2*sqrt(x**2 - x - 1) - 5*log((2*sqrt(x**2 - x -
1) + 2*x - 1)/sqrt(5)))/8
```

3.64 $\int \frac{1}{\sqrt{-1-x+x^2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 24

$$\int \frac{1}{\sqrt{-1-x+x^2}}\,dx = -\operatorname{arctanh}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right)$$

output
```
-arctanh(1/2*(1-2*x)/(x^2-x-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-1-x+x^2}}\,dx = -\log\left(1-2x+2\sqrt{-1-x+x^2}\right)$$

input
```
Integrate[1/Sqrt[-1 - x + x^2],x]
```

output
```
-Log[1 - 2*x + 2*Sqrt[-1 - x + x^2]]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-x-1}}dx$$

$$\Big\downarrow \text{1092}$$

$$2\int \frac{1}{4-\frac{(1-2x)^2}{x^2-x-1}}d\left(-\frac{1-2x}{\sqrt{x^2-x-1}}\right)$$

$$\Big\downarrow \text{219}$$

$$-\text{arctanh}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)$$

input
```
Int[1/Sqrt[-1 - x + x^2],x]
```

output
```
-ArcTanh[(1 - 2*x)/(2*Sqrt[-1 - x + x^2])]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

method	result	size
default	$\ln\left(x-\frac{1}{2}+\sqrt{x^2-x-1}\right)$	15
trager	$\ln\left(2\sqrt{x^2-x-1}-1+2x\right)$	19

input
```
int(1/(x^2-x-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(x-1/2+(x^2-x-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-1-x+x^2}}\,dx = -\log\left(-2\,x+2\,\sqrt{x^2-x-1}+1\right)$$

input
```
integrate(1/(x^2-x-1)^(1/2),x, algorithm="fricas")
```

output
```
-log(-2*x + 2*sqrt(x^2 - x - 1) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{-1-x+x^2}}\,dx = \log\left(2x+2\sqrt{x^2-x-1}-1\right)$$

input
```
integrate(1/(x**2-x-1)**(1/2),x)
```

output
```
log(2*x + 2*sqrt(x**2 - x - 1) - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{-1-x+x^2}}\,dx = \log\left(2\,x+2\,\sqrt{x^2-x-1}-1\right)$$

input
```
integrate(1/(x^2-x-1)^(1/2),x, algorithm="maxima")
```

output
```
log(2*x + 2*sqrt(x^2 - x - 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{-1-x+x^2}}\,dx = \frac{1}{4}\,\sqrt{x^2-x-1}(2\,x-1)+\frac{5}{8}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x-1}+1\right|\right)$$

input
```
integrate(1/(x^2-x-1)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^2 - x - 1)*(2*x - 1) + 5/8*log(abs(-2*x + 2*sqrt(x^2 - x - 1) +
 1))
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{-1-x+x^2}}\,dx = \ln\left(x+\sqrt{x^2-x-1}-\frac{1}{2}\right)$$

input
```
int(1/(x^2 - x - 1)^(1/2),x)
```

output
```
log(x + (x^2 - x - 1)^(1/2) - 1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-1-x+x^2}}\,dx = \log\left(\frac{2\sqrt{x^2-x-1}+2x-1}{\sqrt{5}}\right)$$

input
```
int(1/(x^2-x-1)^(1/2),x)
```

output
```
log((2*sqrt(x**2 - x - 1) + 2*x - 1)/sqrt(5))
```

3.65 $\int \frac{1}{(-1-x+x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{1}{(-1-x+x^2)^{3/2}} \, dx = \frac{2(1-2x)}{5\sqrt{-1-x+x^2}}$$

output
```
2/5*(1-2*x)/(x^2-x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-1-x+x^2)^{3/2}} \, dx = -\frac{2(-1+2x)}{5\sqrt{-1-x+x^2}}$$

input
```
Integrate[(-1 - x + x^2)^(-3/2),x]
```

output
```
(-2*(-1 + 2*x))/(5*Sqrt[-1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-x-1)^{3/2}}\,dx$$

$$\Big\downarrow \text{1088}$$

$$\frac{2(1-2x)}{5\sqrt{x^2-x-1}}$$

input
```
Int[(-1 - x + x^2)^(-3/2),x]
```

output
```
(2*(1 - 2*x))/(5*Sqrt[-1 - x + x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{2(2x-1)}{5\sqrt{x^2-x-1}}$	18
default	$-\frac{2(2x-1)}{5\sqrt{x^2-x-1}}$	18
trager	$-\frac{2(2x-1)}{5\sqrt{x^2-x-1}}$	18
risch	$-\frac{2(2x-1)}{5\sqrt{x^2-x-1}}$	18
orering	$-\frac{2(2x-1)}{5\sqrt{x^2-x-1}}$	18

input
```
int(1/(x^2-x-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(x^2-x-1)^(1/2)*(2*x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(17) = 34.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \frac{1}{(-1-x+x^2)^{3/2}}\,dx = -\frac{2\left(2\,x^2+\sqrt{x^2-x-1}(2\,x-1)-2\,x-2\right)}{5\,(x^2-x-1)}$$

input
```
integrate(1/(x^2-x-1)^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(2*x^2 + sqrt(x^2 - x - 1)*(2*x - 1) - 2*x - 2)/(x^2 - x - 1)
```

Sympy [F]

$$\int \frac{1}{(-1-x+x^2)^{3/2}}\,dx = \int \frac{1}{(x^2-x-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x**2-x-1)**(3/2),x)
```

output
```
Integral((x**2 - x - 1)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{(-1-x+x^2)^{3/2}}\,dx = -\frac{4\,x}{5\,\sqrt{x^2-x-1}} + \frac{2}{5\,\sqrt{x^2-x-1}}$$

input
```
integrate(1/(x^2-x-1)^(3/2),x, algorithm="maxima")
```

output
```
-4/5*x/sqrt(x^2 - x - 1) + 2/5/sqrt(x^2 - x - 1)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-1-x+x^2)^{3/2}}\,dx = -\frac{2\,(2\,x-1)}{5\,\sqrt{x^2-x-1}}$$

input
```
integrate(1/(x^2-x-1)^(3/2),x, algorithm="giac")
```

output
```
-2/5*(2*x - 1)/sqrt(x^2 - x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-1-x+x^2)^{3/2}}\,dx = -\frac{4\left(x-\frac{1}{2}\right)}{5\sqrt{x^2-x-1}}$$

input
```
int(1/(x^2 - x - 1)^(3/2),x)
```

output
```
-(4*(x - 1/2))/(5*(x^2 - x - 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.14

$$\int \frac{1}{(-1-x+x^2)^{3/2}}\,dx = \frac{-4\sqrt{x^2-x-1}\,x+2\sqrt{x^2-x-1}-4x^2+4x+4}{5x^2-5x-5}$$

input
```
int(1/(x^2-x-1)^(3/2),x)
```

output
```
(2*( - 2*sqrt(x**2 - x - 1)*x + sqrt(x**2 - x - 1) - 2*x**2 + 2*x + 2))/(5
*(x**2 - x - 1))
```

3.66 $\int \frac{1}{(-1-x+x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{(-1-x+x^2)^{5/2}}\,dx = \frac{2(1-2x)}{15\,(-1-x+x^2)^{3/2}} - \frac{16(1-2x)}{75\sqrt{-1-x+x^2}}$$

output
```
2/15*(1-2*x)/(x^2-x-1)^(3/2)-16/75*(1-2*x)/(x^2-x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{1}{(-1-x+x^2)^{5/2}}\,dx = \frac{2(-1+2x)\,(-13-8x+8x^2)}{75\,(-1-x+x^2)^{3/2}}$$

input
```
Integrate[(-1 - x + x^2)^(-5/2),x]
```

output
```
(2*(-1 + 2*x)*(-13 - 8*x + 8*x^2))/(75*(-1 - x + x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-x-1)^{5/2}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2(1-2x)}{15\,(x^2-x-1)^{3/2}}-\frac{8}{15}\int \frac{1}{(x^2-x-1)^{3/2}}dx$$

$$\Big\downarrow \text{1088}$$

$$\frac{2(1-2x)}{15\,(x^2-x-1)^{3/2}}-\frac{16(1-2x)}{75\sqrt{x^2-x-1}}$$

input
```
Int[(-1 - x + x^2)^(-5/2),x]
```

output
```
(2*(1 - 2*x))/(15*(-1 - x + x^2)^(3/2)) - (16*(1 - 2*x))/(75*Sqrt[-1 - x +
 x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{-\frac{12}{25}x+\frac{26}{75}+\frac{32}{75}x^3-\frac{16}{25}x^2}{(x^2-x-1)^{\frac{3}{2}}}$	28
trager	$\frac{-\frac{12}{25}x+\frac{26}{75}+\frac{32}{75}x^3-\frac{16}{25}x^2}{(x^2-x-1)^{\frac{3}{2}}}$	28
risch	$\frac{-\frac{12}{25}x+\frac{26}{75}+\frac{32}{75}x^3-\frac{16}{25}x^2}{(x^2-x-1)^{\frac{3}{2}}}$	28
orering	$\frac{-\frac{12}{25}x+\frac{26}{75}+\frac{32}{75}x^3-\frac{16}{25}x^2}{(x^2-x-1)^{\frac{3}{2}}}$	28
default	$-\frac{2(2x-1)}{15(x^2-x-1)^{\frac{3}{2}}}+\frac{\frac{32x}{75}-\frac{16}{75}}{\sqrt{x^2-x-1}}$	36

input
```
int(1/(x^2-x-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/75/(x^2-x-1)^(3/2)*(16*x^3-24*x^2-18*x+13)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{1}{(-1-x+x^2)^{5/2}}\,dx = \frac{2\left(16\,x^4-32\,x^3-16\,x^2+(16\,x^3-24\,x^2-18\,x+13)\sqrt{x^2-x-1}+32\,x+16\right)}{75\,(x^4-2\,x^3-x^2+2\,x+1)}$$

input
```
integrate(1/(x^2-x-1)^(5/2),x, algorithm="fricas")
```

output
```
2/75*(16*x^4 - 32*x^3 - 16*x^2 + (16*x^3 - 24*x^2 - 18*x + 13)*sqrt(x^2 -
x - 1) + 32*x + 16)/(x^4 - 2*x^3 - x^2 + 2*x + 1)
```

Sympy [F]

$$\int \frac{1}{(-1-x+x^2)^{5/2}}\,dx = \int \frac{1}{(x^2-x-1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(x**2-x-1)**(5/2),x)
```

output
```
Integral((x**2 - x - 1)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-1-x+x^2)^{5/2}}\,dx = \frac{32\,x}{75\,\sqrt{x^2-x-1}} - \frac{16}{75\,\sqrt{x^2-x-1}} - \frac{4\,x}{15\,(x^2-x-1)^{\frac{3}{2}}} + \frac{2}{15\,(x^2-x-1)^{\frac{3}{2}}}$$

input
```
integrate(1/(x^2-x-1)^(5/2),x, algorithm="maxima")
```

output
```
32/75*x/sqrt(x^2 - x - 1) - 16/75/sqrt(x^2 - x - 1) - 4/15*x/(x^2 - x - 1)
^(3/2) + 2/15/(x^2 - x - 1)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1}{(-1-x+x^2)^{5/2}}\,dx = \frac{2\,(2\,(4\,(2\,x-3)x-9)x+13)}{75\,(x^2-x-1)^{\frac{3}{2}}}$$

input
```
integrate(1/(x^2-x-1)^(5/2),x, algorithm="giac")
```

output
```
2/75*(2*(4*(2*x - 3)*x - 9)*x + 13)/(x^2 - x - 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1}{(-1-x+x^2)^{5/2}}\,dx = -\frac{(4\,x-2)\,(-8\,x^2+8\,x+13)}{75\,(x^2-x-1)^{3/2}}$$

input
```
int(1/(x^2 - x - 1)^(5/2),x)
```

output
```
-((4*x - 2)*(8*x - 8*x^2 + 13))/(75*(x^2 - x - 1)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.21

$$\int \frac{1}{(-1-x+x^2)^{5/2}}\,dx = \frac{32\sqrt{x^2-x-1}\,x^3-48\sqrt{x^2-x-1}\,x^2-36\sqrt{x^2-x-1}\,x+26\sqrt{x^2-x-1}-3}{75x^4-150x^3-75x^2+150x+75}$$

input
```
int(1/(x^2-x-1)^(5/2),x)
```

output
```
(2*(16*sqrt(x**2 - x - 1)*x**3 - 24*sqrt(x**2 - x - 1)*x**2 - 18*sqrt(x**2
 - x - 1)*x + 13*sqrt(x**2 - x - 1) - 16*x**4 + 32*x**3 + 16*x**2 - 32*x -
 16))/(75*(x**4 - 2*x**3 - x**2 + 2*x + 1))
```

3.67 $\int \frac{1}{(-1-x+x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{1}{(-1-x+x^2)^{7/2}}\,dx = \frac{2(1-2x)}{25\,(-1-x+x^2)^{5/2}} - \frac{32(1-2x)}{375\,(-1-x+x^2)^{3/2}} + \frac{256(1-2x)}{1875\sqrt{-1-x+x^2}}$$

output
```
2/25*(1-2*x)/(x^2-x-1)^(5/2)-32/375*(1-2*x)/(x^2-x-1)^(3/2)+256/1875*(1-2*
x)/(x^2-x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.64

$$\int \frac{1}{(-1-x+x^2)^{7/2}}\,dx = -\frac{2(-1+2x)\,(283+336x-208x^2-256x^3+128x^4)}{1875\,(-1-x+x^2)^{5/2}}$$

input
```
Integrate[(-1 - x + x^2)^(-7/2),x]
```

output
```
(-2*(-1 + 2*x)*(283 + 336*x - 208*x^2 - 256*x^3 + 128*x^4))/(1875*(-1 - x
+ x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-x-1)^{7/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2(1-2x)}{25(x^2-x-1)^{5/2}}-\frac{16}{25}\int \frac{1}{(x^2-x-1)^{5/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2(1-2x)}{25(x^2-x-1)^{5/2}}-\frac{16}{25}\left(\frac{2(1-2x)}{15(x^2-x-1)^{3/2}}-\frac{8}{15}\int \frac{1}{(x^2-x-1)^{3/2}}dx\right)$$

$$\Big\downarrow \text{1088}$$

$$\frac{2(1-2x)}{25(x^2-x-1)^{5/2}}-\frac{16}{25}\left(\frac{2(1-2x)}{15(x^2-x-1)^{3/2}}-\frac{16(1-2x)}{75\sqrt{x^2-x-1}}\right)$$

input
```
Int[(-1 - x + x^2)^(-7/2),x]
```

output
```
(2*(1 - 2*x))/(25*(-1 - x + x^2)^(5/2)) - (16*((2*(1 - 2*x))/(15*(-1 - x +
 x^2)^(3/2)) - (16*(1 - 2*x))/(75*Sqrt[-1 - x + x^2])))/25
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{2(256x^5-640x^4-160x^3+880x^2+230x-283)}{1875(x^2-x-1)^{\frac{5}{2}}}$	38
trager	$-\frac{2(256x^5-640x^4-160x^3+880x^2+230x-283)}{1875(x^2-x-1)^{\frac{5}{2}}}$	38
risch	$-\frac{2(256x^5-640x^4-160x^3+880x^2+230x-283)}{1875(x^2-x-1)^{\frac{5}{2}}}$	38
orering	$-\frac{2(256x^5-640x^4-160x^3+880x^2+230x-283)}{1875(x^2-x-1)^{\frac{5}{2}}}$	38
default	$-\frac{2(2x-1)}{25(x^2-x-1)^{\frac{5}{2}}}+\frac{\frac{64x}{375}-\frac{32}{375}}{(x^2-x-1)^{\frac{3}{2}}}-\frac{256(2x-1)}{1875\sqrt{x^2-x-1}}$	53

input
```
int(1/(x^2-x-1)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1875/(x^2-x-1)^(5/2)*(256*x^5-640*x^4-160*x^3+880*x^2+230*x-283)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{1}{(-1-x+x^2)^{7/2}}\,dx =$$
$$-\frac{2\left(256\,x^6-768\,x^5+1280\,x^3+\left(256\,x^5-640\,x^4-160\,x^3+880\,x^2+230\,x-283\right)\sqrt{x^2-x-1}-768\,x\right.}{1875\left(x^6-3\,x^5+5\,x^3-3\,x-1\right)}$$

input
```
integrate(1/(x^2-x-1)^(7/2),x, algorithm="fricas")
```

output
```
-2/1875*(256*x^6 - 768*x^5 + 1280*x^3 + (256*x^5 - 640*x^4 - 160*x^3 + 880
*x^2 + 230*x - 283)*sqrt(x^2 - x - 1) - 768*x - 256)/(x^6 - 3*x^5 + 5*x^3
- 3*x - 1)
```

Sympy [F]

$$\int \frac{1}{(-1-x+x^2)^{7/2}}\,dx = \int \frac{1}{(x^2-x-1)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(x**2-x-1)**(7/2),x)
```

output
```
Integral((x**2 - x - 1)**(-7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-1-x+x^2)^{7/2}}\,dx = -\frac{512\,x}{1875\,\sqrt{x^2-x-1}}+\frac{256}{1875\,\sqrt{x^2-x-1}}$$
$$+\frac{64\,x}{375\,(x^2-x-1)^{\frac{3}{2}}}-\frac{32}{375\,(x^2-x-1)^{\frac{3}{2}}}-\frac{4\,x}{25\,(x^2-x-1)^{\frac{5}{2}}}+\frac{2}{25\,(x^2-x-1)^{\frac{5}{2}}}$$

input
```
integrate(1/(x^2-x-1)^(7/2),x, algorithm="maxima")
```

output
```
-512/1875*x/sqrt(x^2 - x - 1) + 256/1875/sqrt(x^2 - x - 1) + 64/375*x/(x^2
 - x - 1)^(3/2) - 32/375/(x^2 - x - 1)^(3/2) - 4/25*x/(x^2 - x - 1)^(5/2)
+ 2/25/(x^2 - x - 1)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.58

$$\int \frac{1}{(-1-x+x^2)^{7/2}}\,dx = -\frac{2\,(2\,(8\,(2\,(4\,(2\,x-5)x-5)x+55)x+115)x-283)}{1875\,(x^2-x-1)^{\frac{5}{2}}}$$

input
```
integrate(1/(x^2-x-1)^(7/2),x, algorithm="giac")
```

output
```
-2/1875*(2*(8*(2*(4*(2*x - 5)*x - 5)*x + 55)*x + 115)*x - 283)/(x^2 - x -
1)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{1}{(-1-x+x^2)^{7/2}}\,dx = \frac{140\,x+512\,x\,(-x^2+x+1)^2-256\,(-x^2+x+1)^2+160\,x^2+320\,x\,(-x^2+x-}{(x^2-x-1)^{3/2}\,(-1875\,x^2+1875\,x+1875)}$$

input
```
int(1/(x^2 - x - 1)^(7/2),x)
```

output
```
(140*x + 512*x*(x - x^2 + 1)^2 - 256*(x - x^2 + 1)^2 + 160*x^2 + 320*x*(x
- x^2 + 1) - 310)/((x^2 - x - 1)^(3/2)*(1875*x - 1875*x^2 + 1875))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.92

$$\int \frac{1}{(-1-x+x^2)^{7/2}}\,dx = \frac{-512\sqrt{x^2-x-1}\,x^5+1280\sqrt{x^2-x-1}\,x^4+320\sqrt{x^2-x-1}\,x^3-1760\sqrt{x^2-}}{1875x^6-5625}$$

input
```
int(1/(x^2-x-1)^(7/2),x)
```

output
```
(2*( - 256*sqrt(x**2 - x - 1)*x**5 + 640*sqrt(x**2 - x - 1)*x**4 + 160*sqr
t(x**2 - x - 1)*x**3 - 880*sqrt(x**2 - x - 1)*x**2 - 230*sqrt(x**2 - x - 1
)*x + 283*sqrt(x**2 - x - 1) + 256*x**6 - 768*x**5 + 1280*x**3 - 768*x - 2
56))/(1875*(x**6 - 3*x**5 + 5*x**3 - 3*x - 1))
```

3.68 $\int \frac{1}{(-1-x+x^2)^{9/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 85

$$\int \frac{1}{(-1-x+x^2)^{9/2}}\,dx = \frac{2(1-2x)}{35(-1-x+x^2)^{7/2}} - \frac{48(1-2x)}{875(-1-x+x^2)^{5/2}} + \frac{256(1-2x)}{4375(-1-x+x^2)^{3/2}} - \frac{2048(1-2x)}{21875\sqrt{-1-x+x^2}}$$

output
```
2/35*(1-2*x)/(x^2-x-1)^(7/2)-48/875*(1-2*x)/(x^2-x-1)^(5/2)+256/4375*(1-2*
x)/(x^2-x-1)^(3/2)-2048/21875*(1-2*x)/(x^2-x-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\int \frac{1}{(-1-x+x^2)^{9/2}}\,dx = \frac{2(-1+2x)(-2889-4952x+1240x^2+6400x^3-640x^4-3072x^5+1024x^6)}{21875(-1-x+x^2)^{7/2}}$$

input
```
Integrate[(-1 - x + x^2)^(-9/2),x]
```

output
```
(2*(-1 + 2*x)*(-2889 - 4952*x + 1240*x^2 + 6400*x^3 - 640*x^4 - 3072*x^5 +
 1024*x^6))/(21875*(-1 - x + x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1089, 1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-x-1)^{9/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2(1-2x)}{35(x^2-x-1)^{7/2}}-\frac{24}{35}\int \frac{1}{(x^2-x-1)^{7/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2(1-2x)}{35(x^2-x-1)^{7/2}}-\frac{24}{35}\left(\frac{2(1-2x)}{25(x^2-x-1)^{5/2}}-\frac{16}{25}\int \frac{1}{(x^2-x-1)^{5/2}}dx\right)$$

$$\Big\downarrow \text{1089}$$

$$\frac{2(1-2x)}{35(x^2-x-1)^{7/2}}-\frac{24}{35}\left(\frac{2(1-2x)}{25(x^2-x-1)^{5/2}}-\frac{16}{25}\left(\frac{2(1-2x)}{15(x^2-x-1)^{3/2}}-\frac{8}{15}\int \frac{1}{(x^2-x-1)^{3/2}}dx\right)\right)$$

$$\Big\downarrow \text{1088}$$

$$\frac{2(1-2x)}{35(x^2-x-1)^{7/2}}-\frac{24}{35}\left(\frac{2(1-2x)}{25(x^2-x-1)^{5/2}}-\frac{16}{25}\left(\frac{2(1-2x)}{15(x^2-x-1)^{3/2}}-\frac{16(1-2x)}{75\sqrt{x^2-x-1}}\right)\right)$$

input
```
Int[(-1 - x + x^2)^(-9/2),x]
```

output
```
(2*(1 - 2*x))/(35*(-1 - x + x^2)^(7/2)) - (24*((2*(1 - 2*x))/(25*(-1 - x +
 x^2)^(5/2)) - (16*((2*(1 - 2*x))/(15*(-1 - x + x^2)^(3/2)) - (16*(1 - 2*x
))/(75*Sqrt[-1 - x + x^2])))/25))/35
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{-\frac{236}{3125}x+\frac{5778}{21875}-\frac{224}{625}x^3-\frac{3184}{3125}x^2+\frac{512}{3125}x^5+\frac{768}{625}x^4+\frac{4096}{21875}x^7-\frac{2048}{3125}x^6}{(x^2-x-1)^{\frac{7}{2}}}$	48
trager	$\frac{-\frac{236}{3125}x+\frac{5778}{21875}-\frac{224}{625}x^3-\frac{3184}{3125}x^2+\frac{512}{3125}x^5+\frac{768}{625}x^4+\frac{4096}{21875}x^7-\frac{2048}{3125}x^6}{(x^2-x-1)^{\frac{7}{2}}}$	48
risch	$\frac{-\frac{236}{3125}x+\frac{5778}{21875}-\frac{224}{625}x^3-\frac{3184}{3125}x^2+\frac{512}{3125}x^5+\frac{768}{625}x^4+\frac{4096}{21875}x^7-\frac{2048}{3125}x^6}{(x^2-x-1)^{\frac{7}{2}}}$	48
orering	$\frac{-\frac{236}{3125}x+\frac{5778}{21875}-\frac{224}{625}x^3-\frac{3184}{3125}x^2+\frac{512}{3125}x^5+\frac{768}{625}x^4+\frac{4096}{21875}x^7-\frac{2048}{3125}x^6}{(x^2-x-1)^{\frac{7}{2}}}$	48
default	$-\frac{2(2x-1)}{35(x^2-x-1)^{\frac{7}{2}}}+\frac{\frac{96x}{875}-\frac{48}{875}}{(x^2-x-1)^{\frac{5}{2}}}-\frac{256(2x-1)}{4375(x^2-x-1)^{\frac{3}{2}}}+\frac{\frac{4096x}{21875}-\frac{2048}{21875}}{\sqrt{x^2-x-1}}$	70

input
```
int(1/(x^2-x-1)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/21875/(x^2-x-1)^(7/2)*(2048*x^7-7168*x^6+1792*x^5+13440*x^4-3920*x^3-111
44*x^2-826*x+2889)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.51

$$\int \frac{1}{\left(-1-x+x^2\right)^{9/2}}\,dx = \frac{2\left(2048\,x^8-8192\,x^7+4096\,x^6+16384\,x^5-10240\,x^4-16384\,x^3+4096\,x^2+(\ldots\right.}{21875\left(x^8-4\,x^7\ldots\right.}$$

input
```
integrate(1/(x^2-x-1)^(9/2),x, algorithm="fricas")
```

output
```
2/21875*(2048*x^8 - 8192*x^7 + 4096*x^6 + 16384*x^5 - 10240*x^4 - 16384*x^
3 + 4096*x^2 + (2048*x^7 - 7168*x^6 + 1792*x^5 + 13440*x^4 - 3920*x^3 - 11
144*x^2 - 826*x + 2889)*sqrt(x^2 - x - 1) + 8192*x + 2048)/(x^8 - 4*x^7 +
2*x^6 + 8*x^5 - 5*x^4 - 8*x^3 + 2*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{1}{\left(-1-x+x^2\right)^{9/2}}\,dx = \int \frac{1}{\left(x^2-x-1\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(x**2-x-1)**(9/2),x)
```

output
```
Integral((x**2 - x - 1)**(-9/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{1}{\left(-1-x+x^2\right)^{9/2}}\,dx &= \frac{4096\,x}{21875\,\sqrt{x^2-x-1}} - \frac{2048}{21875\,\sqrt{x^2-x-1}} \\ &\quad - \frac{512\,x}{4375\,\left(x^2-x-1\right)^{\frac{3}{2}}} + \frac{256}{4375\,\left(x^2-x-1\right)^{\frac{3}{2}}} + \frac{96\,x}{875\,\left(x^2-x-1\right)^{\frac{5}{2}}} \\ &\quad - \frac{48}{875\,\left(x^2-x-1\right)^{\frac{5}{2}}} - \frac{4\,x}{35\,\left(x^2-x-1\right)^{\frac{7}{2}}} + \frac{2}{35\,\left(x^2-x-1\right)^{\frac{7}{2}}}\end{aligned}$$

input
```
integrate(1/(x^2-x-1)^(9/2),x, algorithm="maxima")
```

output
```
4096/21875*x/sqrt(x^2 - x - 1) - 2048/21875/sqrt(x^2 - x - 1) - 512/4375*x
/(x^2 - x - 1)^(3/2) + 256/4375/(x^2 - x - 1)^(3/2) + 96/875*x/(x^2 - x -
1)^(5/2) - 48/875/(x^2 - x - 1)^(5/2) - 4/35*x/(x^2 - x - 1)^(7/2) + 2/35/
(x^2 - x - 1)^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

$$\int \frac{1}{(-1-x+x^2)^{9/2}}\,dx = \frac{2\,(2\,(4\,(2\,(8\,(2\,(4\,(2\,x-7)x+7)x+105)x-245)x-1393)x-413)x+2889)}{21875\,(x^2-x-1)^{\frac{7}{2}}}$$

input
```
integrate(1/(x^2-x-1)^(9/2),x, algorithm="giac")
```

output
```
2/21875*(2*(4*(2*(8*(2*(4*(2*x - 7)*x + 7)*x + 105)*x - 245)*x - 1393)*x -
 413)*x + 2889)/(x^2 - x - 1)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.79

$$\int \frac{1}{(-1-x+x^2)^{9/2}}\,dx = \frac{\frac{96\,x}{875}-\frac{48}{875}}{(x^2-x-1)^{5/2}} - \frac{\frac{4\,x}{35}-\frac{2}{35}}{(x^2-x-1)^{7/2}} - \frac{\frac{512\,x}{4375}-\frac{256}{4375}}{(x^2-x-1)^{3/2}} + \frac{\frac{4096\,x}{21875}-\frac{2048}{21875}}{\sqrt{x^2-x-1}}$$

input
```
int(1/(x^2 - x - 1)^(9/2),x)
```

output
```
((96*x)/875 - 48/875)/(x^2 - x - 1)^(5/2) - ((4*x)/35 - 2/35)/(x^2 - x - 1
)^(7/2) - ((512*x)/4375 - 256/4375)/(x^2 - x - 1)^(3/2) + ((4096*x)/21875
- 2048/21875)/(x^2 - x - 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.25

$$\int \frac{1}{(-1-x+x^2)^{9/2}}\,dx = \frac{4096\sqrt{x^2-x-1}\,x^7-14336\sqrt{x^2-x-1}\,x^6+3584\sqrt{x^2-x-1}\,x^5+26880\sqrt{x^2}}{}$$

input
```
int(1/(x^2-x-1)^(9/2),x)
```

output
```
(2*(2048*sqrt(x**2 - x - 1)*x**7 - 7168*sqrt(x**2 - x - 1)*x**6 + 1792*sqr
t(x**2 - x - 1)*x**5 + 13440*sqrt(x**2 - x - 1)*x**4 - 3920*sqrt(x**2 - x
- 1)*x**3 - 11144*sqrt(x**2 - x - 1)*x**2 - 826*sqrt(x**2 - x - 1)*x + 288
9*sqrt(x**2 - x - 1) - 2048*x**8 + 8192*x**7 - 4096*x**6 - 16384*x**5 + 10
240*x**4 + 16384*x**3 - 4096*x**2 - 8192*x - 2048))/(21875*(x**8 - 4*x**7
+ 2*x**6 + 8*x**5 - 5*x**4 - 8*x**3 + 2*x**2 + 4*x + 1))
```

3.69 $\int (6-5x+x^2)^{5/4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int (6-5x+x^2)^{5/4}\,dx = \frac{5}{84}(5-2x)\sqrt[4]{6-5x+x^2}-\frac{1}{7}(5-2x)\left(6-5x+x^2\right)^{5/4} -\frac{5(-6+5x-x^2)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin(5-2x),2\right)}{84\sqrt{2}\left(6-5x+x^2\right)^{3/4}}$$

output
```
5/84*(5-2*x)*(x^2-5*x+6)^(1/4)-1/7*(5-2*x)*(x^2-5*x+6)^(5/4)+5/168*(-x^2+5
*x-6)^(3/4)*InverseJacobiAM(1/2*arcsin(-5+2*x),2^(1/2))*2^(1/2)/(x^2-5*x+6
)^(3/4)
```

Mathematica [A] (verified)

Time = 8.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\int (6-5x+x^2)^{5/4}\,dx = \frac{-4020+8558x-7170x^2+2956x^3-600x^4+48x^5-5\sqrt{2}(-6+5x-x^2)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\text{ar}\right.}{168\left(6-5x+x^2\right)^{3/4}}$$

input
```
Integrate[(6 - 5*x + x^2)^(5/4),x]
```

output
```
(-4020 + 8558*x - 7170*x^2 + 2956*x^3 - 600*x^4 + 48*x^5 - 5*Sqrt[2]*(-6 +
 5*x - x^2)^(3/4)*EllipticF[ArcSin[5 - 2*x]/2, 2])/(168*(6 - 5*x + x^2)^(3
/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1087, 1087, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2-5x+6\right)^{5/4}dx$$

$$\Big\downarrow \text{1087}$$

$$-\frac{5}{28}\int \sqrt[4]{x^2-5x+6}dx-\frac{1}{7}(5-2x)\left(x^2-5x+6\right)^{5/4}$$

$$\Big\downarrow \text{1087}$$

$$-\frac{5}{28}\left(-\frac{1}{12}\int \frac{1}{\left(x^2-5x+6\right)^{3/4}}dx-\frac{1}{3}\sqrt[4]{x^2-5x+6}(5-2x)\right)-\frac{1}{7}(5-2x)\left(x^2-5x+6\right)^{5/4}$$

$$\Big\downarrow \text{1094}$$

$$-\frac{5}{28}\left(\frac{\sqrt{(2x-5)^2}\int \frac{1}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}}{3(5-2x)}-\frac{1}{3}(5-2x)\sqrt[4]{x^2-5x+6}\right)-\frac{1}{7}(5-2x)\left(x^2-5x+6\right)^{5/4}$$

$$\Big\downarrow \text{761}$$

$$-\frac{5}{28}\left(\frac{\sqrt{(2x-5)^2}\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4(x^2-5x+6)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{6\sqrt{2}(5-2x)\sqrt{4\left(x^2-5x+6\right)+1}}-\frac{1}{3}(5- \right.$$
$$\frac{1}{7}(5-2x)\left(x^2-5x+6\right)^{5/4}$$

input
```
Int[(6 - 5*x + x^2)^(5/4),x]
```

output
```
-1/7*((5 - 2*x)*(6 - 5*x + x^2)^(5/4)) - (5*(-1/3*((5 - 2*x)*(6 - 5*x + x^
2)^(1/4)) + (Sqrt[(-5 + 2*x)^2]*(1 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[(1 + 4*(6
 - 5*x + x^2))/(1 + 2*Sqrt[6 - 5*x + x^2])^2]*EllipticF[2*ArcTan[Sqrt[2]*(
6 - 5*x + x^2)^(1/4)], 1/2])/(6*Sqrt[2]*(5 - 2*x)*Sqrt[1 + 4*(6 - 5*x + x^
2)])))/28
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

Maple [F]

$$\int (x^2 - 5x + 6)^{\frac{5}{4}} \, dx$$

input
```
int((x^2-5*x+6)^(5/4),x)
```

output
```
int((x^2-5*x+6)^(5/4),x)
```

Fricas [F]

$$\int \left(6-5x+x^2\right)^{5/4} dx = \int \left(x^2-5\,x+6\right)^{\frac{5}{4}} dx$$

input
```
integrate((x^2-5*x+6)^(5/4),x, algorithm="fricas")
```

output
```
integral((x^2 - 5*x + 6)^(5/4), x)
```

Sympy [F]

$$\int \left(6-5x+x^2\right)^{5/4} dx = \int \left(x^2-5x+6\right)^{\frac{5}{4}} dx$$

input
```
integrate((x**2-5*x+6)**(5/4),x)
```

output
```
Integral((x**2 - 5*x + 6)**(5/4), x)
```

Maxima [F]

$$\int \left(6-5x+x^2\right)^{5/4} dx = \int \left(x^2-5\,x+6\right)^{\frac{5}{4}} dx$$

input
```
integrate((x^2-5*x+6)^(5/4),x, algorithm="maxima")
```

output
```
integrate((x^2 - 5*x + 6)^(5/4), x)
```

Giac [F]

$$\int \left(6-5x+x^2\right)^{5/4}\,dx=\int \left(x^2-5\,x+6\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((x^2-5*x+6)^(5/4),x, algorithm="giac")
```

output
```
integrate((x^2 - 5*x + 6)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(6-5x+x^2\right)^{5/4}\,dx=\int \left(x^2-5\,x+6\right)^{5/4}\,dx$$

input
```
int((x^2 - 5*x + 6)^(5/4),x)
```

output
```
int((x^2 - 5*x + 6)^(5/4), x)
```

Reduce [F]

$$\int \left(6-5x+x^2\right)^{5/4}\,dx=\frac{2(x^2-5x+6)^{\frac{1}{4}}\,x^3}{7}-\frac{15(x^2-5x+6)^{\frac{1}{4}}\,x^2}{7}+\frac{31(x^2-5x+6)^{\frac{1}{4}}\,x}{6}-4\left(x^2-5x+6\right)^{\frac{1}{4}}+\frac{\left(\int \frac{x}{(x^2-5x+6)^{\frac{3}{4}}}dx\right)}{168}$$

input
```
int((x^2-5*x+6)^(5/4),x)
```

output
```
(48*(x**2 - 5*x + 6)**(1/4)*x**3 - 360*(x**2 - 5*x + 6)**(1/4)*x**2 + 868*
(x**2 - 5*x + 6)**(1/4)*x - 672*(x**2 - 5*x + 6)**(1/4) + int(((x**2 - 5*x
 + 6)**(1/4)*x)/(x**2 - 5*x + 6),x))/168
```

3.70 $\int \sqrt[4]{6-5x+x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \sqrt[4]{6-5x+x^2}\,dx=-\frac{1}{3}(5-2x)\sqrt[4]{6-5x+x^2}+\frac{\left(-6+5x-x^2\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin(5-2x),2\right)}{3\sqrt{2}\left(6-5x+x^2\right)^{3/4}}$$

output
```
-1/3*(5-2*x)*(x^2-5*x+6)^(1/4)-1/6*(-x^2+5*x-6)^(3/4)*InverseJacobiAM(1/2*
arcsin(-5+2*x),2^(1/2))*2^(1/2)/(x^2-5*x+6)^(3/4)
```

Mathematica [A] (verified)

Time = 6.54 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \sqrt[4]{6-5x+x^2}\,dx=\frac{-60+74x-30x^2+4x^3+\sqrt{2}\left(-6+5x-x^2\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin(5-2x),2\right)}{6\left(6-5x+x^2\right)^{3/4}}$$

input
```
Integrate[(6 - 5*x + x^2)^(1/4),x]
```

output
```
(-60 + 74*x - 30*x^2 + 4*x^3 + Sqrt[2]*(-6 + 5*x - x^2)^(3/4)*EllipticF[Ar
cSin[5 - 2*x]/2, 2])/(6*(6 - 5*x + x^2)^(3/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. $2(69) = 138$.

Time = 0.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1087, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{x^2-5x+6}\,dx$$

$$\Big\downarrow \text{1087}$$

$$-\frac{1}{12}\int \frac{1}{(x^2-5x+6)^{3/4}}dx-\frac{1}{3}\sqrt[4]{x^2-5x+6}(5-2x)$$

$$\Big\downarrow \text{1094}$$

$$\frac{\sqrt{(2x-5)^2}\int \frac{1}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}}{3(5-2x)}-\frac{1}{3}(5-2x)\sqrt[4]{x^2-5x+6}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt{(2x-5)^2}\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4(x^2-5x+6)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{6\sqrt{2}(5-2x)\sqrt{4\left(x^2-5x+6\right)+1}}-\frac{1}{3}(5-2x)\sqrt[4]{x^2-5x+6}$$

input
```
Int[(6 - 5*x + x^2)^(1/4),x]
```

output
```
-1/3*((5 - 2*x)*(6 - 5*x + x^2)^(1/4)) + (Sqrt[(-5 + 2*x)^2]*(1 + 2*Sqrt[6
 - 5*x + x^2])*Sqrt[(1 + 4*(6 - 5*x + x^2))/(1 + 2*Sqrt[6 - 5*x + x^2])^2]
*EllipticF[2*ArcTan[Sqrt[2]*(6 - 5*x + x^2)^(1/4)], 1/2])/(6*Sqrt[2]*(5 -
2*x)*Sqrt[1 + 4*(6 - 5*x + x^2)])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

Maple [F]

$$\int \left(x^{2}-5 x +6\right)^{\frac{1}{4}}d x$$

input
```
int((x^2-5*x+6)^(1/4),x)
```

output
```
int((x^2-5*x+6)^(1/4),x)
```

Fricas [F]

$$\int \sqrt[4]{6-5x+x^2}\,dx = \int (x^2-5\,x+6)^{\frac{1}{4}}\,dx$$

input
```
integrate((x^2-5*x+6)^(1/4),x, algorithm="fricas")
```

output
```
integral((x^2 - 5*x + 6)^(1/4), x)
```

Sympy [F]

$$\int \sqrt[4]{6-5x+x^2}\,dx = \int \sqrt[4]{x^2-5x+6}\,dx$$

input
```
integrate((x**2-5*x+6)**(1/4),x)
```

output
```
Integral((x**2 - 5*x + 6)**(1/4), x)
```

Maxima [F]

$$\int \sqrt[4]{6-5x+x^2}\,dx = \int (x^2-5\,x+6)^{\frac{1}{4}}\,dx$$

input
```
integrate((x^2-5*x+6)^(1/4),x, algorithm="maxima")
```

output
```
integrate((x^2 - 5*x + 6)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{6-5x+x^2}\,dx = \int (x^2-5\,x+6)^{\frac{1}{4}}\,dx$$

input
```
integrate((x^2-5*x+6)^(1/4),x, algorithm="giac")
```

output
```
integrate((x^2 - 5*x + 6)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[4]{6-5x+x^2}\,dx = \int (x^2-5\,x+6)^{1/4}\,dx$$

input
```
int((x^2 - 5*x + 6)^(1/4),x)
```

output
```
int((x^2 - 5*x + 6)^(1/4), x)
```

Reduce [F]

$$\int \sqrt[4]{6-5x+x^2}\,dx = \frac{2(x^2-5x+6)^{\frac{1}{4}}\,x}{3} - \frac{8(x^2-5x+6)^{\frac{1}{4}}}{5} - \frac{\left(\int \frac{x}{(x^2-5x+6)^{\frac{3}{4}}}dx\right)}{30}$$

input
```
int((x^2-5*x+6)^(1/4),x)
```

output
```
(20*(x**2 - 5*x + 6)**(1/4)*x - 48*(x**2 - 5*x + 6)**(1/4) - int(((x**2 -
5*x + 6)**(1/4)*x)/(x**2 - 5*x + 6),x))/30
```

3.71 $\int \frac{1}{\left(6-5x+x^2\right)^{3/4}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \frac{1}{\left(6-5x+x^2\right)^{3/4}}\,dx=-\frac{2\sqrt{2}\left(-6+5x-x^2\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin(5-2x),2\right)}{\left(6-5x+x^2\right)^{3/4}}$$

output
```
2*(-x^2+5*x-6)^(3/4)*InverseJacobiAM(1/2*arcsin(-5+2*x),2^(1/2))*2^(1/2)/(
x^2-5*x+6)^(3/4)
```

Mathematica [A] (verified)

Time = 6.47 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(6-5x+x^2\right)^{3/4}}\,dx=-\frac{2\sqrt{2}\left(-6+5x-x^2\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin(5-2x),2\right)}{\left(6-5x+x^2\right)^{3/4}}$$

input
```
Integrate[(6 - 5*x + x^2)^(-3/4),x]
```

output
```
(-2*Sqrt[2]*(-6 + 5*x - x^2)^(3/4)*EllipticF[ArcSin[5 - 2*x]/2, 2])/(6 - 5
*x + x^2)^(3/4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 117 vs. $2(45) = 90$.

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.60, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-5x+6)^{3/4}}\,dx$$

$$\Big\downarrow \ 1094$$

$$-\frac{4\sqrt{(2x-5)^2}\int \frac{1}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}}{5-2x}$$

$$\Big\downarrow \ 761$$

$$-\frac{\sqrt{2}\sqrt{(2x-5)^2}\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4(x^2-5x+6)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{(5-2x)\sqrt{4\left(x^2-5x+6\right)+1}}$$

input
```
Int[(6 - 5*x + x^2)^(-3/4),x]
```

output
```
-((Sqrt[2]*Sqrt[(-5 + 2*x)^2]*(1 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[(1 + 4*(6 -
 5*x + x^2))/(1 + 2*Sqrt[6 - 5*x + x^2])^2]*EllipticF[2*ArcTan[Sqrt[2]*(6
- 5*x + x^2)^(1/4)], 1/2])/((5 - 2*x)*Sqrt[1 + 4*(6 - 5*x + x^2)]))
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

Maple [F]

$$\int \frac{1}{(x^2-5x+6)^{\frac{3}{4}}}dx$$

input
```
int(1/(x^2-5*x+6)^(3/4),x)
```

output
```
int(1/(x^2-5*x+6)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(6-5x+x^2)^{3/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(3/4),x, algorithm="fricas")
```

output
```
integral((x^2 - 5*x + 6)^(-3/4), x)
```

Sympy [F]

$$\int \frac{1}{(6-5x+x^2)^{3/4}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(x**2-5*x+6)**(3/4),x)
```

output
```
Integral((x**2 - 5*x + 6)**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(6-5x+x^2)^{3/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(3/4),x, algorithm="maxima")
```

output
```
integrate((x^2 - 5*x + 6)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(6-5x+x^2)^{3/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(3/4),x, algorithm="giac")
```

output
```
integrate((x^2 - 5*x + 6)^(-3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(6-5x+x^2)^{3/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{3/4}}\,dx$$

input
```
int(1/(x^2 - 5*x + 6)^(3/4),x)
```

output
```
int(1/(x^2 - 5*x + 6)^(3/4), x)
```

Reduce [F]

$$\int \frac{1}{(6-5x+x^2)^{3/4}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{3}{4}}}dx$$

input
```
int(1/(x^2-5*x+6)^(3/4),x)
```

output
```
int(1/(x**2 - 5*x + 6)**(3/4),x)
```

3.72 $\int \frac{1}{(6-5x+x^2)^{7/4}} dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \frac{1}{(6-5x+x^2)^{7/4}} dx = \frac{4(5-2x)}{3(6-5x+x^2)^{3/4}} + \frac{8\sqrt{2}(-6+5x-x^2)^{3/4} \operatorname{EllipticF}\left(\frac{1}{2}\arcsin(5-2x),2\right)}{3(6-5x+x^2)^{3/4}}$$

output
```
4/3*(5-2*x)/(x^2-5*x+6)^(3/4)-8/3*(-x^2+5*x-6)^(3/4)*InverseJacobiAM(1/2*a
rcsin(-5+2*x),2^(1/2))*2^(1/2)/(x^2-5*x+6)^(3/4)
```

Mathematica [A] (verified)

Time = 7.85 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int \frac{1}{(6-5x+x^2)^{7/4}} dx = \frac{20-8x+8\sqrt{2}(-6+5x-x^2)^{3/4} \operatorname{EllipticF}\left(\frac{1}{2}\arcsin(5-2x),2\right)}{3(6-5x+x^2)^{3/4}}$$

input
```
Integrate[(6 - 5*x + x^2)^(-7/4),x]
```

output
```
(20 - 8*x + 8*Sqrt[2]*(-6 + 5*x - x^2)^(3/4)*EllipticF[ArcSin[5 - 2*x]/2,
2])/(3*(6 - 5*x + x^2)^(3/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. $2(69) = 138$.

Time = 0.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1089, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-5x+6)^{7/4}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{4(5-2x)}{3(x^2-5x+6)^{3/4}}-\frac{4}{3}\int \frac{1}{(x^2-5x+6)^{3/4}}dx$$

$$\Big\downarrow \text{1094}$$

$$\frac{16\sqrt{(2x-5)^2}\int \frac{1}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}}{3(5-2x)}+\frac{4(5-2x)}{3(x^2-5x+6)^{3/4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{4\sqrt{2}\sqrt{(2x-5)^2}\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4(x^2-5x+6)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{3\sqrt{4(x^2-5x+6)+1}(5-2x)}+\frac{4(5-2x)}{3(x^2-5x+6)^{3/4}}$$

input
```
Int[(6 - 5*x + x^2)^(-7/4),x]
```

output
```
(4*(5 - 2*x))/(3*(6 - 5*x + x^2)^(3/4)) + (4*Sqrt[2]*Sqrt[(-5 + 2*x)^2]*(1
 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[(1 + 4*(6 - 5*x + x^2))/(1 + 2*Sqrt[6 - 5*x
 + x^2])^2]*EllipticF[2*ArcTan[Sqrt[2]*(6 - 5*x + x^2)^(1/4)], 1/2])/(3*(5
 - 2*x)*Sqrt[1 + 4*(6 - 5*x + x^2)])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

Maple [F]

$$\int \frac{1}{(x^2 - 5x + 6)^{\frac{7}{4}}} dx$$

input
```
int(1/(x^2-5*x+6)^(7/4),x)
```

output
```
int(1/(x^2-5*x+6)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{(6-5x+x^2)^{7/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(7/4),x, algorithm="fricas")
```

output
```
integral((x^2 - 5*x + 6)^(1/4)/(x^4 - 10*x^3 + 37*x^2 - 60*x + 36), x)
```

Sympy [F]

$$\int \frac{1}{(6-5x+x^2)^{7/4}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(x**2-5*x+6)**(7/4),x)
```

output
```
Integral((x**2 - 5*x + 6)**(-7/4), x)
```

Maxima [F]

$$\int \frac{1}{(6-5x+x^2)^{7/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(7/4),x, algorithm="maxima")
```

output
```
integrate((x^2 - 5*x + 6)^(-7/4), x)
```

Giac [F]

$$\int \frac{1}{(6-5x+x^2)^{7/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(7/4),x, algorithm="giac")
```

output
```
integrate((x^2 - 5*x + 6)^(-7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(6-5x+x^2)^{7/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{7/4}}\,dx$$

input
```
int(1/(x^2 - 5*x + 6)^(7/4),x)
```

output
```
int(1/(x^2 - 5*x + 6)^(7/4), x)
```

Reduce [F]

$$\int \frac{1}{(6-5x+x^2)^{7/4}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{3}{4}}\,x^2-5\,(x^2-5x+6)^{\frac{3}{4}}\,x+6\,(x^2-5x+6)^{\frac{3}{4}}}dx$$

input
```
int(1/(x^2-5*x+6)^(7/4),x)
```

output
```
int(1/((x**2 - 5*x + 6)**(3/4)*x**2 - 5*(x**2 - 5*x + 6)**(3/4)*x + 6*(x**
2 - 5*x + 6)**(3/4)),x)
```

3.73 $\int \frac{1}{(6-5x+x^2)^{11/4}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(6-5x+x^2)^{11/4}}\,dx = \frac{4(5-2x)}{7(6-5x+x^2)^{7/4}} - \frac{80(5-2x)}{21(6-5x+x^2)^{3/4}} - \frac{160\sqrt{2}(-6+5x-x^2)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin(5-2x),2\right)}{21(6-5x+x^2)^{3/4}}$$

output
```
4/7*(5-2*x)/(x^2-5*x+6)^(7/4)-80/21*(5-2*x)/(x^2-5*x+6)^(3/4)+160/21*(-x^2
+5*x-6)^(3/4)*InverseJacobiAM(1/2*arcsin(-5+2*x),2^(1/2))*2^(1/2)/(x^2-5*x
+6)^(3/4)
```

Mathematica [A] (verified)

Time = 8.70 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

$$\int \frac{1}{(6-5x+x^2)^{11/4}}\,dx = \frac{4\left(-585+734x-300x^2+40x^3+40\sqrt{2}(-6+5x-x^2)^{7/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin(5\right.\right.}{21(6-5x+x^2)^{7/4}}$$

input
```
Integrate[(6 - 5*x + x^2)^(-11/4),x]
```

output
```
(4*(-585 + 734*x - 300*x^2 + 40*x^3 + 40*Sqrt[2]*(-6 + 5*x - x^2)^(7/4)*El
lipticF[ArcSin[5 - 2*x]/2, 2]))/(21*(6 - 5*x + x^2)^(7/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1089, 1089, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-5x+6)^{11/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{4(5-2x)}{7(x^2-5x+6)^{7/4}}-\frac{20}{7}\int \frac{1}{(x^2-5x+6)^{7/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{4(5-2x)}{7(x^2-5x+6)^{7/4}}-\frac{20}{7}\left(\frac{4(5-2x)}{3(x^2-5x+6)^{3/4}}-\frac{4}{3}\int \frac{1}{(x^2-5x+6)^{3/4}}dx\right)$$

$$\Big\downarrow \text{1094}$$

$$\frac{4(5-2x)}{7(x^2-5x+6)^{7/4}}-\frac{20}{7}\left(\frac{16\sqrt{(2x-5)^2}\int \frac{1}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}}{3(5-2x)}+\frac{4(5-2x)}{3(x^2-5x+6)^{3/4}}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{4(5-2x)}{7(x^2-5x+6)^{7/4}}-\frac{20}{7}\left(\frac{4\sqrt{2}\sqrt{(2x-5)^2}\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4(x^2-5x+6)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{3\sqrt{4(x^2-5x+6)+1}(5-2x)}+\frac{}{3(}\right.$$

input
```
Int[(6 - 5*x + x^2)^(-11/4),x]
```

output
```
(4*(5 - 2*x))/(7*(6 - 5*x + x^2)^(7/4)) - (20*((4*(5 - 2*x))/(3*(6 - 5*x +
 x^2)^(3/4)) + (4*Sqrt[2]*Sqrt[(-5 + 2*x)^2]*(1 + 2*Sqrt[6 - 5*x + x^2])*S
qrt[(1 + 4*(6 - 5*x + x^2))/(1 + 2*Sqrt[6 - 5*x + x^2])^2]*EllipticF[2*Arc
Tan[Sqrt[2]*(6 - 5*x + x^2)^(1/4)], 1/2])/(3*(5 - 2*x)*Sqrt[1 + 4*(6 - 5*x
 + x^2)])))/7
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

Maple [F]

$$\int \frac{1}{(x^2-5x+6)^{\frac{11}{4}}}dx$$

input
```
int(1/(x^2-5*x+6)^(11/4),x)
```

output
```
int(1/(x^2-5*x+6)^(11/4),x)
```

Fricas [F]

$$\int \frac{1}{(6-5x+x^2)^{11/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(11/4),x, algorithm="fricas")
```

output
```
integral((x^2 - 5*x + 6)^(1/4)/(x^6 - 15*x^5 + 93*x^4 - 305*x^3 + 558*x^2
- 540*x + 216), x)
```

Sympy [F]

$$\int \frac{1}{(6-5x+x^2)^{11/4}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(x**2-5*x+6)**(11/4),x)
```

output
```
Integral((x**2 - 5*x + 6)**(-11/4), x)
```

Maxima [F]

$$\int \frac{1}{(6-5x+x^2)^{11/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(11/4),x, algorithm="maxima")
```

output
```
integrate((x^2 - 5*x + 6)^(-11/4), x)
```

Giac [F]

$$\int \frac{1}{(6-5x+x^2)^{11/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(11/4),x, algorithm="giac")
```

output
```
integrate((x^2 - 5*x + 6)^(-11/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(6-5x+x^2)^{11/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{11/4}}\,dx$$

input
```
int(1/(x^2 - 5*x + 6)^(11/4),x)
```

output
```
int(1/(x^2 - 5*x + 6)^(11/4), x)
```

Reduce [F]

$$\int \frac{1}{(6-5x+x^2)^{11/4}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{3}{4}}\,x^4-10\,(x^2-5x+6)^{\frac{3}{4}}\,x^3+37\,(x^2-5x+6)^{\frac{3}{4}}\,x^2-60\,(x^2-5}$$

input
```
int(1/(x^2-5*x+6)^(11/4),x)
```

output
```
int(1/((x**2 - 5*x + 6)**(3/4)*x**4 - 10*(x**2 - 5*x + 6)**(3/4)*x**3 + 37
*(x**2 - 5*x + 6)**(3/4)*x**2 - 60*(x**2 - 5*x + 6)**(3/4)*x + 36*(x**2 -
5*x + 6)**(3/4)),x)
```

3.74 $\int (6-5x+x^2)^{7/4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int (6-5x+x^2)^{7/4}\,dx = \frac{7}{180}(5-2x)\left(6-5x+x^2\right)^{3/4} - \frac{1}{9}(5-2x)\left(6-5x+x^2\right)^{7/4} - \frac{7\sqrt[4]{-6+5x-x^2}\,E\left(\frac{1}{2}\arcsin(5-2x)\middle|\,2\right)}{120\sqrt{2}\sqrt[4]{6-5x+x^2}}$$

output
```
7/180*(5-2*x)*(x^2-5*x+6)^(3/4)-1/9*(5-2*x)*(x^2-5*x+6)^(7/4)+7/240*(-x^2+
5*x-6)^(1/4)*EllipticE(sin(1/2*arcsin(-5+2*x)),2^(1/2))*2^(1/2)/(x^2-5*x+6
)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.69 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int (6-5x+x^2)^{7/4}\,dx = \frac{(-5+2x)\left(8(678-1165x+733x^2-200x^3+20x^4)+21\sqrt{2}\sqrt[4]{-6+5x-x^2}\,\text{Hypergeometr}\right.}{1440\sqrt[4]{6-5x+x^2}}$$

input
```
Integrate[(6 - 5*x + x^2)^(7/4),x]
```

output
```
((-5 + 2*x)*(8*(678 - 1165*x + 733*x^2 - 200*x^3 + 20*x^4) + 21*Sqrt[2]*(-
6 + 5*x - x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (5 - 2*x)^2]))/(1440
*(6 - 5*x + x^2)^(1/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 323 vs. $2(90) = 180$.

Time = 0.54 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.59, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1087, 1087, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2-5x+6\right)^{7/4}dx$$

$$\Big\downarrow \text{ 1087}$$

$$-\frac{7}{36}\int \left(x^2-5x+6\right)^{3/4}dx-\frac{1}{9}(5-2x)\left(x^2-5x+6\right)^{7/4}$$

$$\Big\downarrow \text{ 1087}$$

$$-\frac{7}{36}\left(-\frac{3}{20}\int \frac{1}{\sqrt[4]{x^2-5x+6}}dx-\frac{1}{5}\left(x^2-5x+6\right)^{3/4}(5-2x)\right)-\frac{1}{9}(5-2x)\left(x^2-5x+6\right)^{7/4}$$

$$\Big\downarrow \text{ 1094}$$

$$-\frac{7}{36}\left(\frac{3\sqrt{(2x-5)^2}\int \frac{\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}}{5(5-2x)}-\frac{1}{5}(5-2x)\left(x^2-5x+6\right)^{3/4}\right)-\frac{1}{9}(5-2x)\left(x^2-5x+6\right)^{7/4}$$

$$\Big\downarrow \text{ 834}$$

$$-\frac{7}{36}\left(\frac{3\sqrt{(2x-5)^2}\left(\frac{1}{2}\int\frac{1}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}-\frac{1}{2}\int\frac{1-2\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}\right)}{5(5-2x)}-\frac{1}{5}(5-2x)\left(x^2-\right.\right.$$

$$\frac{1}{9}(5-2x)\left(x^2-5x+6\right)^{7/4}$$

$\downarrow$ 761

$$-\frac{7}{36}\left(\frac{3\sqrt{(2x-5)^2}\left(\frac{\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4\left(x^2-5x+6\right)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt{4(x^2-5x+6)+1}}-\frac{1}{2}\int\frac{1-2\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}\right.}{5(5-2x)}\right.$$

$$\frac{1}{9}(5-2x)\left(x^2-5x+6\right)^{7/4}$$

$\downarrow$ 1510

$$-\frac{7}{36}\left(\frac{3\sqrt{(2x-5)^2}\left(\frac{\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4\left(x^2-5x+6\right)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt{4(x^2-5x+6)+1}}+\frac{1}{2}\left(\frac{\sqrt[4]{x^2-5x+6}}{2\sqrt{x^2-5}}\right.\right.}{5(5-2x)}\right.$$

$$\frac{1}{9}(5-2x)\left(x^2-5x+6\right)^{7/4}$$

input
```
Int[(6 - 5*x + x^2)^(7/4),x]
```

output
```
-1/9*((5 - 2*x)*(6 - 5*x + x^2)^(7/4)) - (7*(-1/5*((5 - 2*x)*(6 - 5*x + x^
2)^(3/4)) + (3*Sqrt[(-5 + 2*x)^2]*((((6 - 5*x + x^2)^(1/4)*Sqrt[1 + 4*(6 -
 5*x + x^2)])/(1 + 2*Sqrt[6 - 5*x + x^2]) - ((1 + 2*Sqrt[6 - 5*x + x^2])*S
qrt[(1 + 4*(6 - 5*x + x^2))/(1 + 2*Sqrt[6 - 5*x + x^2])^2]*EllipticE[2*Arc
Tan[Sqrt[2]*(6 - 5*x + x^2)^(1/4)], 1/2])/(Sqrt[2]*Sqrt[1 + 4*(6 - 5*x + x
^2)]))/2 + ((1 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[(1 + 4*(6 - 5*x + x^2))/(1 +
2*Sqrt[6 - 5*x + x^2])^2]*EllipticF[2*ArcTan[Sqrt[2]*(6 - 5*x + x^2)^(1/4)
], 1/2])/(4*Sqrt[2]*Sqrt[1 + 4*(6 - 5*x + x^2)])))/(5*(5 - 2*x))))/36
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \left(x^2-5x+6\right)^{\frac{7}{4}}dx$$

input
```
int((x^2-5*x+6)^(7/4),x)
```

output
```
int((x^2-5*x+6)^(7/4),x)
```

Fricas [F]

$$\int \left(6-5x+x^2\right)^{7/4}\,dx=\int \left(x^2-5\,x+6\right)^{\frac{7}{4}}\,dx$$

input
```
integrate((x^2-5*x+6)^(7/4),x, algorithm="fricas")
```

output
```
integral((x^2 - 5*x + 6)^(7/4), x)
```

Sympy [F]

$$\int \left(6-5x+x^2\right)^{7/4}\,dx=\int \left(x^2-5x+6\right)^{\frac{7}{4}}\,dx$$

input
```
integrate((x**2-5*x+6)**(7/4),x)
```

output
```
Integral((x**2 - 5*x + 6)**(7/4), x)
```

Maxima [F]

$$\int \left(6-5x+x^2\right)^{7/4} \, dx = \int \left(x^2-5\,x+6\right)^{\frac{7}{4}} \, dx$$

input
```
integrate((x^2-5*x+6)^(7/4),x, algorithm="maxima")
```

output
```
integrate((x^2 - 5*x + 6)^(7/4), x)
```

Giac [F]

$$\int \left(6-5x+x^2\right)^{7/4} \, dx = \int \left(x^2-5\,x+6\right)^{\frac{7}{4}} \, dx$$

input
```
integrate((x^2-5*x+6)^(7/4),x, algorithm="giac")
```

output
```
integrate((x^2 - 5*x + 6)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(6-5x+x^2\right)^{7/4} \, dx = \int \left(x^2-5\,x+6\right)^{7/4} \, dx$$

input
```
int((x^2 - 5*x + 6)^(7/4),x)
```

output
```
int((x^2 - 5*x + 6)^(7/4), x)
```

Reduce [F]

$$\int \left(6-5x+x^2\right)^{7/4} dx = \frac{2(x^2-5x+6)^{\frac{3}{4}} x^3}{9} - \frac{5(x^2-5x+6)^{\frac{3}{4}} x^2}{3} + \frac{121(x^2-5x+6)^{\frac{3}{4}} x}{30} - \frac{236(x^2-5x+6)^{\frac{3}{4}}}{75} + \frac{7\left(\int \frac{x}{(x^2-5x+6)^{\frac{1}{4}}} dx\right)}{600}$$

input

```
int((x^2-5*x+6)^(7/4),x)
```

output

```
(400*(x**2 - 5*x + 6)**(3/4)*x**3 - 3000*(x**2 - 5*x + 6)**(3/4)*x**2 + 72
60*(x**2 - 5*x + 6)**(3/4)*x - 5664*(x**2 - 5*x + 6)**(3/4) + 21*int(((x**
2 - 5*x + 6)**(3/4)*x)/(x**2 - 5*x + 6),x))/1800
```

3.75 $\int (6-5x+x^2)^{3/4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int (6-5x+x^2)^{3/4}\,dx = -\frac{1}{5}(5-2x)\left(6-5x+x^2\right)^{3/4}+\frac{3\sqrt[4]{-6+5x-x^2}E\left(\frac{1}{2}\arcsin(5-2x)\middle|2\right)}{10\sqrt{2}\sqrt[4]{6-5x+x^2}}$$

output
```
-1/5*(5-2*x)*(x^2-5*x+6)^(3/4)-3/20*(-x^2+5*x-6)^(1/4)*EllipticE(sin(1/2*a
rcsin(-5+2*x)),2^(1/2))*2^(1/2)/(x^2-5*x+6)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int (6-5x+x^2)^{3/4}\,dx = \frac{(-5+2x)\left(8(6-5x+x^2)-3\sqrt{2}\sqrt[4]{-6+5x-x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},(5-2x)^2\right)\right)}{40\sqrt[4]{6-5x+x^2}}$$

input
```
Integrate[(6 - 5*x + x^2)^(3/4),x]
```

output
```
((-5 + 2*x)*(8*(6 - 5*x + x^2) - 3*Sqrt[2]*(-6 + 5*x - x^2)^(1/4)*Hypergeo
metric2F1[1/4, 1/2, 3/2, (5 - 2*x)^2]))/(40*(6 - 5*x + x^2)^(1/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 297 vs. 2(69) = 138.

Time = 0.53 (sec) , antiderivative size = 297, normalized size of antiderivative = 4.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1087, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2-5x+6\right)^{3/4}dx$$

$$\Big\downarrow \text{1087}$$

$$-\frac{3}{20}\int \frac{1}{\sqrt[4]{x^2-5x+6}}dx-\frac{1}{5}\left(x^2-5x+6\right)^{3/4}(5-2x)$$

$$\Big\downarrow \text{1094}$$

$$\frac{3\sqrt{(2x-5)^2}\int \frac{\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}}{5(5-2x)}-\frac{1}{5}(5-2x)\left(x^2-5x+6\right)^{3/4}$$

$$\Big\downarrow \text{834}$$

$$\frac{3\sqrt{(2x-5)^2}\left(\frac{1}{2}\int \frac{1}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}-\frac{1}{2}\int \frac{1-2\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}\right)}{5(5-2x)}-$$
$$\frac{1}{5}(5-2x)\left(x^2-5x+6\right)^{3/4}$$

$$\Big\downarrow \text{761}$$

$$\frac{3\sqrt{(2x-5)^2}\left(\frac{\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4\left(x^2-5x+6\right)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt{4(x^2-5x+6)+1}}-\frac{1}{2}\int \frac{1-2\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2}\right.}{5(5-2x)}$$
$$\frac{1}{5}(5-2x)\left(x^2-5x+6\right)^{3/4}$$

$$\Big\downarrow \text{1510}$$

$$\frac{3\sqrt{(2x-5)^2}\left(\frac{\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4\left(x^2-5x+6\right)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt{4(x^2-5x+6)+1}}+\frac{1}{2}\left(\frac{\sqrt[4]{x^2-5x+6}\sqrt{4(x^2-}}{2\sqrt{x^2-5x+6}+1}\right.\right.}{5(5-2x)}$$

$$\frac{1}{5}(5-2x)\left(x^2-5x+6\right)^{3/4}$$

input
```
Int[(6 - 5*x + x^2)^(3/4),x]
```

output
```
-1/5*((5 - 2*x)*(6 - 5*x + x^2)^(3/4)) + (3*Sqrt[(-5 + 2*x)^2]*((((6 - 5*x
 + x^2)^(1/4)*Sqrt[1 + 4*(6 - 5*x + x^2)])/(1 + 2*Sqrt[6 - 5*x + x^2]) - (
(1 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[(1 + 4*(6 - 5*x + x^2))/(1 + 2*Sqrt[6 - 5
*x + x^2])^2]*EllipticE[2*ArcTan[Sqrt[2]*(6 - 5*x + x^2)^(1/4)], 1/2])/(Sq
rt[2]*Sqrt[1 + 4*(6 - 5*x + x^2)]))/2 + ((1 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[
(1 + 4*(6 - 5*x + x^2))/(1 + 2*Sqrt[6 - 5*x + x^2])^2]*EllipticF[2*ArcTan[
Sqrt[2]*(6 - 5*x + x^2)^(1/4)], 1/2])/(4*Sqrt[2]*Sqrt[1 + 4*(6 - 5*x + x^2
)])))/(5*(5 - 2*x))
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \left(x^2-5x+6\right)^{\frac{3}{4}}dx$$

input
```
int((x^2-5*x+6)^(3/4),x)
```

output
```
int((x^2-5*x+6)^(3/4),x)
```

Fricas [F]

$$\int \left(6-5x+x^2\right)^{3/4} dx = \int \left(x^2-5\,x+6\right)^{\frac{3}{4}}dx$$

input
```
integrate((x^2-5*x+6)^(3/4),x, algorithm="fricas")
```

output
```
integral((x^2 - 5*x + 6)^(3/4), x)
```

Sympy [F]

$$\int \left(6-5x+x^2\right)^{3/4}\,dx=\int \left(x^2-5x+6\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((x**2-5*x+6)**(3/4),x)
```

output
```
Integral((x**2 - 5*x + 6)**(3/4), x)
```

Maxima [F]

$$\int \left(6-5x+x^2\right)^{3/4}\,dx=\int \left(x^2-5\,x+6\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((x^2-5*x+6)^(3/4),x, algorithm="maxima")
```

output
```
integrate((x^2 - 5*x + 6)^(3/4), x)
```

Giac [F]

$$\int \left(6-5x+x^2\right)^{3/4}\,dx=\int \left(x^2-5\,x+6\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((x^2-5*x+6)^(3/4),x, algorithm="giac")
```

output
```
integrate((x^2 - 5*x + 6)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(6-5x+x^2\right)^{3/4}\,dx=\int \left(x^2-5\,x+6\right)^{3/4}\,dx$$

input
```
int((x^2 - 5*x + 6)^(3/4),x)
```

output
```
int((x^2 - 5*x + 6)^(3/4), x)
```

Reduce [F]

$$\int \left(6-5x+x^2\right)^{3/4}\,dx=\frac{2(x^2-5x+6)^{\frac{3}{4}}x}{5}-\frac{24(x^2-5x+6)^{\frac{3}{4}}}{25}-\frac{3\left(\int \frac{x}{(x^2-5x+6)^{\frac{1}{4}}}dx\right)}{50}$$

input
```
int((x^2-5*x+6)^(3/4),x)
```

output
```
(20*(x**2 - 5*x + 6)**(3/4)*x - 48*(x**2 - 5*x + 6)**(3/4) - 3*int((((x**2
- 5*x + 6)**(3/4)*x)/(x**2 - 5*x + 6),x))/50
```

3.76 $\int \frac{1}{\sqrt[4]{6-5x+x^2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \frac{1}{\sqrt[4]{6-5x+x^2}}\,dx=-\frac{\sqrt{2}\sqrt[4]{-6+5x-x^2}E\left(\frac{1}{2}\arcsin(5-2x)\middle|2\right)}{\sqrt[4]{6-5x+x^2}}$$

output
```
(-x^2+5*x-6)^(1/4)*EllipticE(sin(1/2*arcsin(-5+2*x)),2^(1/2))*2^(1/2)/(x^2
-5*x+6)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt[4]{6-5x+x^2}}\,dx=-\frac{(5-2x)\sqrt[4]{-6+5x-x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},(5-2x)^2\right)}{\sqrt{2}\sqrt[4]{6-5x+x^2}}$$

input
```
Integrate[(6 - 5*x + x^2)^(-1/4),x]
```

output
```
-(((5 - 2*x)*(-6 + 5*x - x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (5 -
2*x)^2])/(Sqrt[2]*(6 - 5*x + x^2)^(1/4)))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 273 vs. $2(45) = 90$.

Time = 0.46 (sec) , antiderivative size = 273, normalized size of antiderivative = 6.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{x^2-5x+6}}dx$$

$$\Big\downarrow \text{1094}$$

$$-\frac{4\sqrt{(2x-5)^2}\int \frac{\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}}{5-2x}$$

$$\Big\downarrow \text{834}$$

$$-\frac{4\sqrt{(2x-5)^2}\left(\frac{1}{2}\int \frac{1}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}-\frac{1}{2}\int \frac{1-2\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}\right)}{5-2x}$$

$$\Big\downarrow \text{761}$$

$$-\frac{4\sqrt{(2x-5)^2}\left(\frac{\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4\left(x^2-5x+6\right)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt{4(x^2-5x+6)+1}}-\frac{1}{2}\int \frac{1-2\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x$$

$$\Big\downarrow \text{1510}$$

$$-\frac{4\sqrt{(2x-5)^2}\left(\frac{\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4\left(x^2-5x+6\right)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt{4(x^2-5x+6)+1}}+\frac{1}{2}\left(\frac{\sqrt[4]{x^2-5x+6}\sqrt{4(x^2}{2\sqrt{x^2-5x+6}+}\right.\right.}{5-2x}$$

input
```
Int[(6 - 5*x + x^2)^(-1/4),x]
```

output
```
(-4*Sqrt[(-5 + 2*x)^2]*((((6 - 5*x + x^2)^(1/4)*Sqrt[1 + 4*(6 - 5*x + x^2)
])/(1 + 2*Sqrt[6 - 5*x + x^2]) - ((1 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[(1 + 4*
(6 - 5*x + x^2))/(1 + 2*Sqrt[6 - 5*x + x^2])^2]*EllipticE[2*ArcTan[Sqrt[2]
*(6 - 5*x + x^2)^(1/4)], 1/2])/(Sqrt[2]*Sqrt[1 + 4*(6 - 5*x + x^2)]))/2 +
((1 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[(1 + 4*(6 - 5*x + x^2))/(1 + 2*Sqrt[6 -
5*x + x^2])^2]*EllipticF[2*ArcTan[Sqrt[2]*(6 - 5*x + x^2)^(1/4)], 1/2])/(4
*Sqrt[2]*Sqrt[1 + 4*(6 - 5*x + x^2)])))/(5 - 2*x)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{(x^2-5x+6)^{\frac{1}{4}}}dx$$

input
```
int(1/(x^2-5*x+6)^(1/4),x)
```

output
```
int(1/(x^2-5*x+6)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{6-5x+x^2}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(1/4),x, algorithm="fricas")
```

output
```
integral((x^2 - 5*x + 6)^(-1/4), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{6-5x+x^2}}\,dx = \int \frac{1}{\sqrt[4]{x^2-5x+6}}\,dx$$

input
```
integrate(1/(x**2-5*x+6)**(1/4),x)
```

output
```
Integral((x**2 - 5*x + 6)**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{6-5x+x^2}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(1/4),x, algorithm="maxima")
```

output
```
integrate((x^2 - 5*x + 6)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{6-5x+x^2}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(1/4),x, algorithm="giac")
```

output
```
integrate((x^2 - 5*x + 6)^(-1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{6-5x+x^2}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{1/4}}\,dx$$

input
```
int(1/(x^2 - 5*x + 6)^(1/4),x)
```

output
```
int(1/(x^2 - 5*x + 6)^(1/4), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{6-5x+x^2}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{1}{4}}}dx$$

input
```
int(1/(x^2-5*x+6)^(1/4),x)
```

output
```
int(1/(x**2 - 5*x + 6)**(1/4),x)
```

3.77 $\int \frac{1}{(6-5x+x^2)^{5/4}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \frac{1}{(6-5x+x^2)^{5/4}}\,dx=\frac{4(5-2x)}{\sqrt[4]{6-5x+x^2}}-\frac{4\sqrt{2}\sqrt[4]{-6+5x-x^2}\,E\left(\frac{1}{2}\arcsin(5-2x)\middle|2\right)}{\sqrt[4]{6-5x+x^2}}$$

output
```
4*(5-2*x)/(x^2-5*x+6)^(1/4)+4*(-x^2+5*x-6)^(1/4)*EllipticE(sin(1/2*arcsin(
-5+2*x)),2^(1/2))*2^(1/2)/(x^2-5*x+6)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.92 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{1}{(6-5x+x^2)^{5/4}}\,dx=\frac{2(-5+2x)\left(-2+\sqrt{2}\sqrt[4]{-6+5x-x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},(5-2x)^2\right)\right)}{\sqrt[4]{6-5x+x^2}}$$

input
```
Integrate[(6 - 5*x + x^2)^(-5/4),x]
```

output
```
(2*(-5 + 2*x)*(-2 + Sqrt[2]*(-6 + 5*x - x^2)^(1/4)*Hypergeometric2F1[1/4,
1/2, 3/2, (5 - 2*x)^2]))/(6 - 5*x + x^2)^(1/4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 293 vs. 2(65) = 130.

Time = 0.51 (sec) , antiderivative size = 293, normalized size of antiderivative = 4.51, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1089, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-5x+6)^{5/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$4\int \frac{1}{\sqrt[4]{x^2-5x+6}}dx+\frac{4(5-2x)}{\sqrt[4]{x^2-5x+6}}$$

$$\Big\downarrow \text{1094}$$

$$\frac{4(5-2x)}{\sqrt[4]{x^2-5x+6}}-\frac{16\sqrt{(2x-5)^2}\int \frac{\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}}{5-2x}$$

$$\Big\downarrow \text{834}$$

$$\frac{4(5-2x)}{\sqrt[4]{x^2-5x+6}}-\frac{16\sqrt{(2x-5)^2}\left(\frac{1}{2}\int \frac{1}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}-\frac{1}{2}\int \frac{1-2\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}\right)}{5-2x}$$

$$\Big\downarrow \text{761}$$

$$\frac{4(5-2x)}{\sqrt[4]{x^2-5x+6}}-\frac{16\sqrt{(2x-5)^2}\left(\frac{\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4\left(x^2-5x+6\right)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt{4(x^2-5x+6)+1}}-\frac{1}{2}\int \frac{1-2\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2 \cdots\right)}{5-2x}$$

$$\Big\downarrow \text{1510}$$

$$\frac{4(5-2x)}{\sqrt[4]{x^2-5x+6}}-\frac{16\sqrt{(2x-5)^2}\left(\frac{\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4\left(x^2-5x+6\right)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt{4(x^2-5x+6)+1}}+\frac{1}{2}\left(\frac{\sqrt[4]{x^2-5x+6}\sqrt{4(x^2\ldots}}{2\sqrt{x^2-5x+6}+1\ldots}\right.\right.}{5-2x}$$

input
```
Int[(6 - 5*x + x^2)^(-5/4),x]
```

output
```
(4*(5 - 2*x))/(6 - 5*x + x^2)^(1/4) - (16*Sqrt[(-5 + 2*x)^2]*((((6 - 5*x +
 x^2)^(1/4)*Sqrt[1 + 4*(6 - 5*x + x^2)])/(1 + 2*Sqrt[6 - 5*x + x^2]) - ((1
 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[(1 + 4*(6 - 5*x + x^2))/(1 + 2*Sqrt[6 - 5*x
 + x^2])^2]*EllipticE[2*ArcTan[Sqrt[2]*(6 - 5*x + x^2)^(1/4)], 1/2])/(Sqrt
[2]*Sqrt[1 + 4*(6 - 5*x + x^2)]))/2 + ((1 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[(1
 + 4*(6 - 5*x + x^2))/(1 + 2*Sqrt[6 - 5*x + x^2])^2]*EllipticF[2*ArcTan[Sq
rt[2]*(6 - 5*x + x^2)^(1/4)], 1/2])/(4*Sqrt[2]*Sqrt[1 + 4*(6 - 5*x + x^2)]
)))/(5 - 2*x)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{(x^2-5x+6)^{\frac{5}{4}}}dx$$

input
```
int(1/(x^2-5*x+6)^(5/4),x)
```

output
```
int(1/(x^2-5*x+6)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(6-5x+x^2)^{5/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(5/4),x, algorithm="fricas")
```

output
```
integral((x^2 - 5*x + 6)^(3/4)/(x^4 - 10*x^3 + 37*x^2 - 60*x + 36), x)
```

Sympy [F]

$$\int \frac{1}{(6-5x+x^2)^{5/4}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(x**2-5*x+6)**(5/4),x)
```

output
```
Integral((x**2 - 5*x + 6)**(-5/4), x)
```

Maxima [F]

$$\int \frac{1}{(6-5x+x^2)^{5/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(5/4),x, algorithm="maxima")
```

output
```
integrate((x^2 - 5*x + 6)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(6-5x+x^2)^{5/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(5/4),x, algorithm="giac")
```

output
```
integrate((x^2 - 5*x + 6)^(-5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(6-5x+x^2)^{5/4}}\,dx=\int \frac{1}{{\left(x^2-5\,x+6\right)}^{5/4}}\,dx$$

input
```
int(1/(x^2 - 5*x + 6)^(5/4),x)
```

output
```
int(1/(x^2 - 5*x + 6)^(5/4), x)
```

Reduce [F]

$$\int \frac{1}{(6-5x+x^2)^{5/4}}\,dx=\int \frac{1}{(x^2-5x+6)^{\frac{1}{4}}\,x^2-5\,(x^2-5x+6)^{\frac{1}{4}}\,x+6\,(x^2-5x+6)^{\frac{1}{4}}}dx$$

input
```
int(1/(x^2-5*x+6)^(5/4),x)
```

output
```
int(1/((x**2 - 5*x + 6)**(1/4)*x**2 - 5*(x**2 - 5*x + 6)**(1/4)*x + 6*(x**
2 - 5*x + 6)**(1/4)),x)
```

3.78 $\int \frac{1}{(6-5x+x^2)^{9/4}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(6-5x+x^2)^{9/4}}\,dx = \frac{4(5-2x)}{5(6-5x+x^2)^{5/4}} - \frac{48(5-2x)}{5\sqrt[4]{6-5x+x^2}} + \frac{48\sqrt{2}\sqrt[4]{-6+5x-x^2}E\left(\frac{1}{2}\arcsin(5-2x)\middle|2\right)}{5\sqrt[4]{6-5x+x^2}}$$

output
```
4/5*(5-2*x)/(x^2-5*x+6)^(5/4)-48/5*(5-2*x)/(x^2-5*x+6)^(1/4)-48/5*(-x^2+5*
x-6)^(1/4)*EllipticE(sin(1/2*arcsin(-5+2*x)),2^(1/2))*2^(1/2)/(x^2-5*x+6)^
(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

$$\int \frac{1}{(6-5x+x^2)^{9/4}}\,dx = -\frac{4(-5+2x)\left(-71+60x-12x^2-6\sqrt{2}(-6+5x-x^2)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},(5-2x)^2\right)\right)}{5(6-5x+x^2)^{5/4}}$$

input
```
Integrate[(6 - 5*x + x^2)^(-9/4),x]
```

output
```
(-4*(-5 + 2*x)*(-71 + 60*x - 12*x^2 - 6*Sqrt[2]*(-6 + 5*x - x^2)^(5/4)*Hyp
ergeometric2F1[1/4, 1/2, 3/2, (5 - 2*x)^2]))/(5*(6 - 5*x + x^2)^(5/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 319 vs. $2(90) = 180$.

Time = 0.56 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.54, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1089, 1089, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-5x+6)^{9/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{4(5-2x)}{5(x^2-5x+6)^{5/4}}-\frac{12}{5}\int \frac{1}{(x^2-5x+6)^{5/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{4(5-2x)}{5(x^2-5x+6)^{5/4}}-\frac{12}{5}\left(4\int \frac{1}{\sqrt[4]{x^2-5x+6}}dx+\frac{4(5-2x)}{\sqrt[4]{x^2-5x+6}}\right)$$

$$\Big\downarrow \text{1094}$$

$$\frac{4(5-2x)}{5(x^2-5x+6)^{5/4}}-\frac{12}{5}\left(\frac{4(5-2x)}{\sqrt[4]{x^2-5x+6}}-\frac{16\sqrt{(2x-5)^2}\int \frac{\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}}{5-2x}\right)$$

$$\Big\downarrow \text{834}$$

$$\frac{4(5-2x)}{5(x^2-5x+6)^{5/4}}-\frac{12}{5}\left(\frac{4(5-2x)}{\sqrt[4]{x^2-5x+6}}-\frac{16\sqrt{(2x-5)^2}\left(\frac{1}{2}\int \frac{1}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}-\frac{1}{2}\int \frac{1-2\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}\right)}{5-2x}\right)$$

$\Big\downarrow$ 761

$$\frac{4(5-2x)}{5\left(x^2-5x+6\right)^{5/4}}-\frac{12}{5}\left(\frac{4(5-2x)}{\sqrt[4]{x^2-5x+6}}-\frac{16\sqrt{(2x-5)^2}\left(\frac{\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4\left(x^2-5x+6\right)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt{4(x^2-5x+6)+1}}-\frac{1}{2}\right.}{5-2x}\right.$$

$\Big\downarrow$ 1510

$$\frac{4(5-2x)}{5\left(x^2-5x+6\right)^{5/4}}-\frac{12}{5}\left(\frac{4(5-2x)}{\sqrt[4]{x^2-5x+6}}-\frac{16\sqrt{(2x-5)^2}\left(\frac{\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4\left(x^2-5x+6\right)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt{4(x^2-5x+6)+1}}+\frac{1}{2}\right.}{}\right.$$

input
```
Int[(6 - 5*x + x^2)^(-9/4),x]
```

output
```
(4*(5 - 2*x))/(5*(6 - 5*x + x^2)^(5/4)) - (12*((4*(5 - 2*x))/(6 - 5*x + x^
2)^(1/4) - (16*Sqrt[(-5 + 2*x)^2]*((((6 - 5*x + x^2)^(1/4)*Sqrt[1 + 4*(6 -
 5*x + x^2)])/(1 + 2*Sqrt[6 - 5*x + x^2]) - ((1 + 2*Sqrt[6 - 5*x + x^2])*S
qrt[(1 + 4*(6 - 5*x + x^2))/(1 + 2*Sqrt[6 - 5*x + x^2])^2]*EllipticE[2*Arc
Tan[Sqrt[2]*(6 - 5*x + x^2)^(1/4)], 1/2])/(Sqrt[2]*Sqrt[1 + 4*(6 - 5*x + x
^2)]))/2 + ((1 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[(1 + 4*(6 - 5*x + x^2))/(1 +
2*Sqrt[6 - 5*x + x^2])^2]*EllipticF[2*ArcTan[Sqrt[2]*(6 - 5*x + x^2)^(1/4)
], 1/2])/(4*Sqrt[2]*Sqrt[1 + 4*(6 - 5*x + x^2)])))/(5 - 2*x)))/5
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{(x^2-5x+6)^{\frac{9}{4}}}dx$$

input
```
int(1/(x^2-5*x+6)^(9/4),x)
```

output
```
int(1/(x^2-5*x+6)^(9/4),x)
```

Fricas [F]

$$\int \frac{1}{(6-5x+x^2)^{9/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(9/4),x, algorithm="fricas")
```

output
```
integral((x^2 - 5*x + 6)^(3/4)/(x^6 - 15*x^5 + 93*x^4 - 305*x^3 + 558*x^2
- 540*x + 216), x)
```

Sympy [F]

$$\int \frac{1}{(6-5x+x^2)^{9/4}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(x**2-5*x+6)**(9/4),x)
```

output
```
Integral((x**2 - 5*x + 6)**(-9/4), x)
```

Maxima [F]

$$\int \frac{1}{(6-5x+x^2)^{9/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(9/4),x, algorithm="maxima")
```

output
```
integrate((x^2 - 5*x + 6)^(-9/4), x)
```

Giac [F]

$$\int \frac{1}{(6-5x+x^2)^{9/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(x^2-5*x+6)^(9/4),x, algorithm="giac")
```

output
```
integrate((x^2 - 5*x + 6)^(-9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(6-5x+x^2)^{9/4}}\,dx = \int \frac{1}{(x^2-5\,x+6)^{9/4}}\,dx$$

input
```
int(1/(x^2 - 5*x + 6)^(9/4),x)
```

output
```
int(1/(x^2 - 5*x + 6)^(9/4), x)
```

Reduce [F]

$$\int \frac{1}{(6-5x+x^2)^{9/4}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{1}{4}}\,x^4-10\,(x^2-5x+6)^{\frac{1}{4}}\,x^3+37\,(x^2-5x+6)^{\frac{1}{4}}\,x^2-60\,(x^2-5$$

input
```
int(1/(x^2-5*x+6)^(9/4),x)
```

output
```
int(1/((x**2 - 5*x + 6)**(1/4)*x**4 - 10*(x**2 - 5*x + 6)**(1/4)*x**3 + 37
*(x**2 - 5*x + 6)**(1/4)*x**2 - 60*(x**2 - 5*x + 6)**(1/4)*x + 36*(x**2 -
5*x + 6)**(1/4)),x)
```

3.79 $\int (6-5x+x^2)^p \, dx$

Optimal result

Integrand size = 10, antiderivative size = 48

$$\int (6-5x+x^2)^p \, dx = -\frac{2^{1+p}(6-2x)^{-1-p}(6-5x+x^2)^{1+p}\,\text{Hypergeometric2F1}(-p,1+p,2+p,-2+x)}{1+p}$$

output
```
-2^(p+1)*(6-2*x)^(-1-p)*(x^2-5*x+6)^(p+1)*hypergeom([-p, p+1],[2+p],-2+x)/
(p+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int (6-5x+x^2)^p \, dx = \frac{(-3+x)(-2+x)^{-p}(6-5x+x^2)^p\,\text{Hypergeometric2F1}(-p,1+p,2+p,3-x)}{1+p}$$

input
```
Integrate[(6 - 5*x + x^2)^p,x]
```

output
```
((-3 + x)*(6 - 5*x + x^2)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, 3 - x])/((
1 + p)*(-2 + x)^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2-5x+6\right)^p\,dx$$

$$\Big\downarrow \text{1096}$$

$$-\frac{2^{p+1}(6-2x)^{-p-1}\left(x^2-5x+6\right)^{p+1}\text{Hypergeometric2F1}(-p,p+1,p+2,x-2)}{p+1}$$

input
```
Int[(6 - 5*x + x^2)^p,x]
```

output
```
-((2^(1 + p)*(6 - 2*x)^(-1 - p)*(6 - 5*x + x^2)^(1 + p)*Hypergeometric2F1[
-p, 1 + p, 2 + p, -2 + x])/(1 + p))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

Maple [F]

$$\int (x^2 - 5x + 6)^p \, dx$$

input
```
int((x^2-5*x+6)^p,x)
```

output
```
int((x^2-5*x+6)^p,x)
```

Fricas [F]

$$\int (6 - 5x + x^2)^p \, dx = \int (x^2 - 5\,x + 6)^p \, dx$$

input
```
integrate((x^2-5*x+6)^p,x, algorithm="fricas")
```

output
```
integral((x^2 - 5*x + 6)^p, x)
```

Sympy [F]

$$\int (6 - 5x + x^2)^p \, dx = \int (x^2 - 5x + 6)^p \, dx$$

input
```
integrate((x**2-5*x+6)**p,x)
```

output
```
Integral((x**2 - 5*x + 6)**p, x)
```

Maxima [F]

$$\int (6-5x+x^2)^p \, dx = \int (x^2-5x+6)^p \, dx$$

input
```
integrate((x^2-5*x+6)^p,x, algorithm="maxima")
```

output
```
integrate((x^2 - 5*x + 6)^p, x)
```

Giac [F]

$$\int (6-5x+x^2)^p \, dx = \int (x^2-5x+6)^p \, dx$$

input
```
integrate((x^2-5*x+6)^p,x, algorithm="giac")
```

output
```
integrate((x^2 - 5*x + 6)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (6-5x+x^2)^p \, dx = \int (x^2-5x+6)^p \, dx$$

input
```
int((x^2 - 5*x + 6)^p,x)
```

output
```
int((x^2 - 5*x + 6)^p, x)
```

Reduce [F]

$$\int \left(6-5x+x^2\right)^p \, dx$$
$$= \frac{5(x^2-5x+6)^p x - 12(x^2-5x+6)^p - 2\left(\int \frac{(x^2-5x+6)^p x}{2p\,x^2-10px+x^2+12p-5x+6}dx\right)p^2 - \left(\int \frac{(x^2-5x+6)^p x}{2p\,x^2-10px+x^2+12p-5x+6}dx\right.}{10p+5}$$

input
```
int((x^2-5*x+6)^p,x)
```

output
```
(5*(x**2 - 5*x + 6)**p*x - 12*(x**2 - 5*x + 6)**p - 2*int(((x**2 - 5*x + 6
)**p*x)/(2*p*x**2 - 10*p*x + 12*p + x**2 - 5*x + 6),x)*p**2 - int(((x**2 -
 5*x + 6)**p*x)/(2*p*x**2 - 10*p*x + 12*p + x**2 - 5*x + 6),x)*p)/(5*(2*p
+ 1))
```

3.80 $\int (ac+(bc+ad)x+bdx^2)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int (ac+(bc+ad)x+bdx^2)^p \, dx = -\frac{\left(-\frac{d(a+bx)}{bc-ad}\right)^{-1-p}(ac+(bc+ad)x+bdx^2)^{1+p}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)(1+p)}$$

output
```
-(-d*(b*x+a)/(-a*d+b*c))^(-1-p)*(a*c+(a*d+b*c)*x+b*d*x^2)^(p+1)*hypergeom(
[-p, p+1],[2+p],b*(d*x+c)/(-a*d+b*c))/(-a*d+b*c)/(p+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int (ac+(bc+ad)x+bdx^2)^p \, dx = \frac{(a+bx)\left(\frac{b(c+dx)}{bc-ad}\right)^{-p}((a+bx)(c+dx))^p\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{d(a+bx)}{-bc+ad}\right)}{b(1+p)}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^p,x]
```

output
```
((a + b*x)*((a + b*x)*(c + d*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(1 + p)*((b*(c + d*x))/(b*c - a*d))^p)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{1096\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x(ad+bc)+ac+bdx^2\right)^p\,dx$$

$$\Big\downarrow \; 1096$$

$$-\frac{\left(-\frac{d(a+bx)}{bc-ad}\right)^{-p-1}\left(x(ad+bc)+ac+bdx^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b(c+dx)}{bc-ad}\right)}{(p+1)(bc-ad)}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^p,x]
```

output
```
-(((-((d*(a + b*x))/(b*c - a*d)))^(-1 - p)*(a*c + (b*c + a*d)*x + b*d*x^2)
^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b*(c + d*x))/(b*c - a*d)])/(
(b*c - a*d)*(1 + p)))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

Maple [F]

$$\int \left(ac+(ad+bc)\,x+bd\,x^2\right)^p\,dx$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

output
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

Fricas [F]

$$\int \left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\int \left(bdx^2+ac+(bc+ad)x\right)^p\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="fricas")
```

output
```
integral((b*d*x^2 + a*c + (b*c + a*d)*x)^p, x)
```

Sympy [F]

$$\int \left(ac+(bc+ad)x+bdx^2\right)^p\,dx=\int \left(ac+bdx^2+x(ad+bc)\right)^p\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**p,x)
```

output
```
Integral((a*c + b*d*x**2 + x*(a*d + b*c))**p, x)
```

Maxima [F]

$$\int \left(ac+(bc+ad)x+bdx^2\right)^p \, dx=\int \left(bdx^2+ac+(bc+ad)x\right)^p dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p, x)
```

Giac [F]

$$\int \left(ac+(bc+ad)x+bdx^2\right)^p \, dx=\int \left(bdx^2+ac+(bc+ad)x\right)^p dx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^p,x, algorithm="giac")
```

output
```
integrate((b*d*x^2 + a*c + (b*c + a*d)*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(ac+(bc+ad)x+bdx^2\right)^p \, dx=\int \left(b\,d\,x^2+(a\,d+b\,c)\,x+a\,c\right)^p dx$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^p,x)
```

output
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^p, x)
```

Reduce [F]

$$\int \left(ac + (bc + ad)x + bdx^2\right)^p \, dx = \text{Too large to display}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^p,x)
```

output
```
(2*(a*c + a*d*x + b*c*x + b*d*x**2)**p*a*c + (a*c + a*d*x + b*c*x + b*d*x*
*2)**p*a*d*x + (a*c + a*d*x + b*c*x + b*d*x**2)**p*b*c*x + 2*int(((a*c + a
*d*x + b*c*x + b*d*x**2)**p*x)/(2*a**2*c*d*p + a**2*c*d + 2*a**2*d**2*p*x
+ a**2*d**2*x + 2*a*b*c**2*p + a*b*c**2 + 4*a*b*c*d*p*x + 2*a*b*c*d*x + 2*
a*b*d**2*p*x**2 + a*b*d**2*x**2 + 2*b**2*c**2*p*x + b**2*c**2*x + 2*b**2*c
*d*p*x**2 + b**2*c*d*x**2),x)*a**3*d**3*p**2 + int(((a*c + a*d*x + b*c*x +
 b*d*x**2)**p*x)/(2*a**2*c*d*p + a**2*c*d + 2*a**2*d**2*p*x + a**2*d**2*x
+ 2*a*b*c**2*p + a*b*c**2 + 4*a*b*c*d*p*x + 2*a*b*c*d*x + 2*a*b*d**2*p*x**
2 + a*b*d**2*x**2 + 2*b**2*c**2*p*x + b**2*c**2*x + 2*b**2*c*d*p*x**2 + b*
*2*c*d*x**2),x)*a**3*d**3*p - 2*int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x
)/(2*a**2*c*d*p + a**2*c*d + 2*a**2*d**2*p*x + a**2*d**2*x + 2*a*b*c**2*p
+ a*b*c**2 + 4*a*b*c*d*p*x + 2*a*b*c*d*x + 2*a*b*d**2*p*x**2 + a*b*d**2*x*
*2 + 2*b**2*c**2*p*x + b**2*c**2*x + 2*b**2*c*d*p*x**2 + b**2*c*d*x**2),x)
*a**2*b*c*d**2*p**2 - int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(2*a**2*
c*d*p + a**2*c*d + 2*a**2*d**2*p*x + a**2*d**2*x + 2*a*b*c**2*p + a*b*c**2
 + 4*a*b*c*d*p*x + 2*a*b*c*d*x + 2*a*b*d**2*p*x**2 + a*b*d**2*x**2 + 2*b**
2*c**2*p*x + b**2*c**2*x + 2*b**2*c*d*p*x**2 + b**2*c*d*x**2),x)*a**2*b*c*
d**2*p - 2*int(((a*c + a*d*x + b*c*x + b*d*x**2)**p*x)/(2*a**2*c*d*p + a**
2*c*d + 2*a**2*d**2*p*x + a**2*d**2*x + 2*a*b*c**2*p + a*b*c**2 + 4*a*b*c*
d*p*x + 2*a*b*c*d*x + 2*a*b*d**2*p*x**2 + a*b*d**2*x**2 + 2*b**2*c**2*p...
```

3.81 $\int (a+bx+cx^2)^4 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 133

$$\int (a+bx+cx^2)^4 \, dx = a^4x+2a^3bx^2+\frac{2}{3}a^2(3b^2+2ac)\,x^3+ab(b^2+3ac)\,x^4 +\frac{1}{5}(b^4+12ab^2c+6a^2c^2)\,x^5+\frac{2}{3}bc(b^2+3ac)\,x^6 +\frac{2}{7}c^2(3b^2+2ac)\,x^7+\frac{1}{2}bc^3x^8+\frac{c^4x^9}{9}$$

output
```
a^4*x+2*a^3*b*x^2+2/3*a^2*(2*a*c+3*b^2)*x^3+a*b*(3*a*c+b^2)*x^4+1/5*(6*a^2
*c^2+12*a*b^2*c+b^4)*x^5+2/3*b*c*(3*a*c+b^2)*x^6+2/7*c^2*(2*a*c+3*b^2)*x^7
+1/2*b*c^3*x^8+1/9*c^4*x^9
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2)^4 \, dx = a^4x+2a^3bx^2+\frac{2}{3}a^2(3b^2+2ac)\,x^3+ab(b^2+3ac)\,x^4 +\frac{1}{5}(b^4+12ab^2c+6a^2c^2)\,x^5+\frac{2}{3}bc(b^2+3ac)\,x^6 +\frac{2}{7}c^2(3b^2+2ac)\,x^7+\frac{1}{2}bc^3x^8+\frac{c^4x^9}{9}$$

input
```
Integrate[(a + b*x + c*x^2)^4,x]
```

output
```
a^4*x + 2*a^3*b*x^2 + (2*a^2*(3*b^2 + 2*a*c)*x^3)/3 + a*b*(b^2 + 3*a*c)*x^
4 + ((b^4 + 12*a*b^2*c + 6*a^2*c^2)*x^5)/5 + (2*b*c*(b^2 + 3*a*c)*x^6)/3 +
 (2*c^2*(3*b^2 + 2*a*c)*x^7)/7 + (b*c^3*x^8)/2 + (c^4*x^9)/9
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^4 dx$$

$$\Big\downarrow \text{1085}$$

$$\int \left(a^4+4a^3bx+6a^2b^2x^2\left(\frac{2ac}{3b^2}+1\right)+6b^2c^2x^6\left(\frac{2ac}{3b^2}+1\right)+b^4x^4\left(\frac{6ac(ac+2b^2)}{b^4}+1\right)+4b^3cx^5\left(\frac{3ac}{b^2}+1\right)+\right.$$

$$\Big\downarrow \text{2009}$$

$$a^4x+2a^3bx^2+\frac{2}{3}a^2x^3\left(2ac+3b^2\right)+\frac{1}{5}x^5\left(6a^2c^2+12ab^2c+b^4\right)+\frac{2}{7}c^2x^7\left(2ac+3b^2\right)+\frac{2}{3}bcx^6\left(3ac+b^2\right)+abx^4\left(3ac+b^2\right)+\frac{1}{2}bc^3x^8+\frac{c^4x^9}{9}$$

input
```
Int[(a + b*x + c*x^2)^4,x]
```

output
```
a^4*x + 2*a^3*b*x^2 + (2*a^2*(3*b^2 + 2*a*c)*x^3)/3 + a*b*(b^2 + 3*a*c)*x^
4 + ((b^4 + 12*a*b^2*c + 6*a^2*c^2)*x^5)/5 + (2*b*c*(b^2 + 3*a*c)*x^6)/3 +
 (2*c^2*(3*b^2 + 2*a*c)*x^7)/7 + (b*c^3*x^8)/2 + (c^4*x^9)/9
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

method	result
norman	$\frac{c^4 x^9}{9}+\frac{b\,c^3 x^8}{2}+\left(\frac{4}{7}c^3 a+\frac{6}{7}b^2 c^2\right)x^7+\left(2ab\,c^2+\frac{2}{3}b^3 c\right)x^6+\left(\frac{6}{5}a^2 c^2+\frac{12}{5}ca\,b^2+\frac{1}{5}b^4\right)x^5+(3c\,a$
gosper	$\frac{1}{9}c^4 x^9+\frac{1}{2}b\,c^3 x^8+\frac{4}{7}x^7 c^3 a+\frac{6}{7}b^2 c^2 x^7+2x^6 ab\,c^2+\frac{2}{3}b^3 c\,x^6+\frac{6}{5}x^5 a^2 c^2+\frac{12}{5}a\,b^2 c\,x^5+\frac{1}{5}b^4 x^5+$
risch	$\frac{1}{9}c^4 x^9+\frac{1}{2}b\,c^3 x^8+\frac{4}{7}x^7 c^3 a+\frac{6}{7}b^2 c^2 x^7+2x^6 ab\,c^2+\frac{2}{3}b^3 c\,x^6+\frac{6}{5}x^5 a^2 c^2+\frac{12}{5}a\,b^2 c\,x^5+\frac{1}{5}b^4 x^5+$
parallelrisch	$\frac{1}{9}c^4 x^9+\frac{1}{2}b\,c^3 x^8+\frac{4}{7}x^7 c^3 a+\frac{6}{7}b^2 c^2 x^7+2x^6 ab\,c^2+\frac{2}{3}b^3 c\,x^6+\frac{6}{5}x^5 a^2 c^2+\frac{12}{5}a\,b^2 c\,x^5+\frac{1}{5}b^4 x^5+$
orering	$\frac{x(70c^4 x^8+315b\,c^3 x^7+360a\,c^3 x^6+540b^2 c^2 x^6+1260ab\,c^2 x^5+420b^3 c\,x^5+756a^2 c^2 x^4+1512a\,b^2 c\,x^4+126b^4 x^4+1890a^2 bc\,x^3+6}{630}$
default	$\frac{c^4 x^9}{9}+\frac{b\,c^3 x^8}{2}+\frac{(2(2ac+b^2)c^2+4b^2c^2)x^7}{7}+\frac{(4ab\,c^2+4(2ac+b^2)bc)x^6}{6}+\frac{\left(2a^2c^2+8ca\,b^2+(2ac+b^2)^2\right)x^5}{5}+\frac{(4c\,a^2b-$

input
```
int((c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/9*c^4*x^9+1/2*b*c^3*x^8+(4/7*c^3*a+6/7*b^2*c^2)*x^7+(2*a*b*c^2+2/3*b^3*c
)*x^6+(6/5*a^2*c^2+12/5*c*a*b^2+1/5*b^4)*x^5+(3*a^2*b*c+a*b^3)*x^4+(4/3*c*
a^3+2*a^2*b^2)*x^3+2*a^3*b*x^2+a^4*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int \left(a+bx+cx^2\right)^4 \, dx = \frac{1}{9}\,c^4x^9 + \frac{1}{2}\,bc^3x^8 + \frac{2}{7}\,\left(3\,b^2c^2 + 2\,ac^3\right)x^7 + \frac{2}{3}\,\left(b^3c + 3\,abc^2\right)x^6 + 2\,a^3bx^2 + \frac{1}{5}\,\left(b^4 + 12\,ab^2c + 6\,a^2c^2\right)x^5 + a^4x + \left(ab^3 + 3\,a^2bc\right)x^4 + \frac{2}{3}\,\left(3\,a^2b^2 + 2\,a^3c\right)x^3$$

input
```
integrate((c*x^2+b*x+a)^4,x, algorithm="fricas")
```

output
```
1/9*c^4*x^9 + 1/2*b*c^3*x^8 + 2/7*(3*b^2*c^2 + 2*a*c^3)*x^7 + 2/3*(b^3*c +
 3*a*b*c^2)*x^6 + 2*a^3*b*x^2 + 1/5*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*x^5 + a
^4*x + (a*b^3 + 3*a^2*b*c)*x^4 + 2/3*(3*a^2*b^2 + 2*a^3*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.06

$$\int \left(a+bx+cx^2\right)^4 \, dx = a^4x + 2a^3bx^2 + \frac{bc^3x^8}{2} + \frac{c^4x^9}{9} + x^7 \cdot \left(\frac{4ac^3}{7} + \frac{6b^2c^2}{7}\right) + x^6 \cdot \left(2abc^2 + \frac{2b^3c}{3}\right) + x^5 \cdot \left(\frac{6a^2c^2}{5} + \frac{12ab^2c}{5} + \frac{b^4}{5}\right) + x^4 \cdot \left(3a^2bc + ab^3\right) + x^3 \cdot \left(\frac{4a^3c}{3} + 2a^2b^2\right)$$

input
```
integrate((c*x**2+b*x+a)**4,x)
```

output
```
a**4*x + 2*a**3*b*x**2 + b*c**3*x**8/2 + c**4*x**9/9 + x**7*(4*a*c**3/7 +
6*b**2*c**2/7) + x**6*(2*a*b*c**2 + 2*b**3*c/3) + x**5*(6*a**2*c**2/5 + 12
*a*b**2*c/5 + b**4/5) + x**4*(3*a**2*b*c + a*b**3) + x**3*(4*a**3*c/3 + 2*
a**2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.02

$$\int (a+bx+cx^2)^4\,dx = \frac{1}{9}c^4x^9 + \frac{1}{2}bc^3x^8 + \frac{6}{7}b^2c^2x^7 + \frac{2}{3}b^3cx^6 + \frac{1}{5}b^4x^5 + a^4x + \frac{2}{3}(2cx^3+3bx^2)a^3 + \frac{1}{5}(6c^2x^5+15bcx^4+10b^2x^3)a^2 + \frac{1}{35}(20c^3x^7+70bc^2x^6+84b^2cx^5+35b^3x^4)a$$

input
```
integrate((c*x^2+b*x+a)^4,x, algorithm="maxima")
```

output
```
1/9*c^4*x^9 + 1/2*b*c^3*x^8 + 6/7*b^2*c^2*x^7 + 2/3*b^3*c*x^6 + 1/5*b^4*x^
5 + a^4*x + 2/3*(2*c*x^3 + 3*b*x^2)*a^3 + 1/5*(6*c^2*x^5 + 15*b*c*x^4 + 10
*b^2*x^3)*a^2 + 1/35*(20*c^3*x^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^
4)*a
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.04

$$\int (a+bx+cx^2)^4\,dx = \frac{1}{9}c^4x^9 + \frac{1}{2}bc^3x^8 + \frac{6}{7}b^2c^2x^7 + \frac{4}{7}ac^3x^7 + \frac{2}{3}b^3cx^6 + 2abc^2x^6 + \frac{1}{5}b^4x^5 + \frac{12}{5}ab^2cx^5 + \frac{6}{5}a^2c^2x^5 + ab^3x^4 + 3a^2bcx^4 + 2a^2b^2x^3 + \frac{4}{3}a^3cx^3 + 2a^3bx^2 + a^4x$$

input
```
integrate((c*x^2+b*x+a)^4,x, algorithm="giac")
```

output
```
1/9*c^4*x^9 + 1/2*b*c^3*x^8 + 6/7*b^2*c^2*x^7 + 4/7*a*c^3*x^7 + 2/3*b^3*c*
x^6 + 2*a*b*c^2*x^6 + 1/5*b^4*x^5 + 12/5*a*b^2*c*x^5 + 6/5*a^2*c^2*x^5 + a
*b^3*x^4 + 3*a^2*b*c*x^4 + 2*a^2*b^2*x^3 + 4/3*a^3*c*x^3 + 2*a^3*b*x^2 + a
^4*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \left(a+bx+cx^2\right)^4\,dx = x^5\left(\frac{6\,a^2\,c^2}{5}+\frac{12\,a\,b^2\,c}{5}+\frac{b^4}{5}\right)+a^4\,x+\frac{c^4\,x^9}{9}+x^3\left(\frac{4\,c\,a^3}{3}+2\,a^2\,b^2\right)+x^7\left(\frac{6\,b^2\,c^2}{7}+\frac{4\,a\,c^3}{7}\right)+2\,a^3\,b\,x^2+\frac{b\,c^3\,x^8}{2}+a\,b\,x^4\,\left(b^2+3\,a\,c\right)+\frac{2\,b\,c\,x^6\,\left(b^2+3\,a\,c\right)}{3}$$

input
```
int((a + b*x + c*x^2)^4,x)
```

output
```
x^5*(b^4/5 + (6*a^2*c^2)/5 + (12*a*b^2*c)/5) + a^4*x + (c^4*x^9)/9 + x^3*(
(4*a^3*c)/3 + 2*a^2*b^2) + x^7*((4*a*c^3)/7 + (6*b^2*c^2)/7) + 2*a^3*b*x^2
 + (b*c^3*x^8)/2 + a*b*x^4*(3*a*c + b^2) + (2*b*c*x^6*(3*a*c + b^2))/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.05

$$\int \left(a+bx+cx^2\right)^4\,dx$$

$$= \frac{x(70c^4x^8+315b\,c^3x^7+360a\,c^3x^6+540b^2c^2x^6+1260ab\,c^2x^5+420b^3c\,x^5+756a^2c^2x^4+1512a\,b^2c\,x^4+}{630}$$

input
```
int((c*x^2+b*x+a)^4,x)
```

output
```
(x*(630*a**4 + 1260*a**3*b*x + 840*a**3*c*x**2 + 1260*a**2*b**2*x**2 + 189
0*a**2*b*c*x**3 + 756*a**2*c**2*x**4 + 630*a*b**3*x**3 + 1512*a*b**2*c*x**
4 + 1260*a*b*c**2*x**5 + 360*a*c**3*x**6 + 126*b**4*x**4 + 420*b**3*c*x**5
 + 540*b**2*c**2*x**6 + 315*b*c**3*x**7 + 70*c**4*x**8))/630
```

3.82 $\int (a+bx+cx^2)^3\, dx$

Optimal result

Integrand size = 12, antiderivative size = 81

$$\int (a+bx+cx^2)^3\, dx = a^3x+\frac{3}{2}a^2bx^2+a(b^2+ac)\,x^3+\frac{1}{4}b(b^2+6ac)\,x^4 + \frac{3}{5}c(b^2+ac)\,x^5+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}$$

output
```
a^3*x+3/2*a^2*b*x^2+a*(a*c+b^2)*x^3+1/4*b*(6*a*c+b^2)*x^4+3/5*c*(a*c+b^2)*
x^5+1/2*b*c^2*x^6+1/7*c^3*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2)^3\, dx = a^3x+\frac{3}{2}a^2bx^2+a(b^2+ac)\,x^3+\frac{1}{4}b(b^2+6ac)\,x^4 + \frac{3}{5}c(b^2+ac)\,x^5+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}$$

input
```
Integrate[(a + b*x + c*x^2)^3,x]
```

output
```
a^3*x + (3*a^2*b*x^2)/2 + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^4)/4 + (3
*c*(b^2 + a*c)*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^3 \, dx$$

$$\Big\downarrow \text{1085}$$

$$\int \left(a^3+3a^2bx+3b^2cx^4\left(\frac{ac}{b^2}+1\right)+3ab^2x^2\left(\frac{ac}{b^2}+1\right)+b^3x^3\left(\frac{6ac}{b^2}+1\right)+3bc^2x^5+c^3x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^3x+\frac{3}{2}a^2bx^2+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}$$

input
```
Int[(a + b*x + c*x^2)^3,x]
```

output
```
a^3*x + (3*a^2*b*x^2)/2 + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^4)/4 + (3
*c*(b^2 + a*c)*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

method	result
norman	$\frac{c^3x^7}{7}+\frac{b\,c^2x^6}{2}+\left(\frac{3}{5}a\,c^2+\frac{3}{5}b^2c\right)x^5+\left(\frac{3}{2}abc+\frac{1}{4}b^3\right)x^4+\left(a^2c+a\,b^2\right)x^3+\frac{3a^2b\,x^2}{2}+a^3x$
gosper	$\frac{1}{7}c^3x^7+\frac{1}{2}b\,c^2x^6+\frac{3}{5}x^5a\,c^2+\frac{3}{5}x^5b^2c+\frac{3}{2}abc\,x^4+\frac{1}{4}b^3x^4+a^2c\,x^3+a\,b^2x^3+\frac{3}{2}a^2b\,x^2+a^3x$
risch	$\frac{1}{7}c^3x^7+\frac{1}{2}b\,c^2x^6+\frac{3}{5}x^5a\,c^2+\frac{3}{5}x^5b^2c+\frac{3}{2}abc\,x^4+\frac{1}{4}b^3x^4+a^2c\,x^3+a\,b^2x^3+\frac{3}{2}a^2b\,x^2+a^3x$
parallelrisch	$\frac{1}{7}c^3x^7+\frac{1}{2}b\,c^2x^6+\frac{3}{5}x^5a\,c^2+\frac{3}{5}x^5b^2c+\frac{3}{2}abc\,x^4+\frac{1}{4}b^3x^4+a^2c\,x^3+a\,b^2x^3+\frac{3}{2}a^2b\,x^2+a^3x$
orering	$\frac{x\left(20c^3x^6+70b\,c^2x^5+84a\,c^2x^4+84b^2c\,x^4+210abc\,x^3+35b^3x^3+140a^2c\,x^2+140a\,b^2x^2+210a^2bx+140a^3\right)}{140}$
default	$\frac{c^3x^7}{7}+\frac{b\,c^2x^6}{2}+\frac{\left(a\,c^2+2b^2c+c\left(2ac+b^2\right)\right)x^5}{5}+\frac{\left(4abc+b\left(2ac+b^2\right)\right)x^4}{4}+\frac{\left(a\left(2ac+b^2\right)+2a\,b^2+a^2c\right)x^3}{3}+\frac{3a^2b\,x^2}{2}+a$

input
```
int((c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*c^3*x^7+1/2*b*c^2*x^6+(3/5*a*c^2+3/5*b^2*c)*x^5+(3/2*a*b*c+1/4*b^3)*x^
4+(a^2*c+a*b^2)*x^3+3/2*a^2*b*x^2+a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \left(a+bx+cx^2\right)^3\,dx=\frac{1}{7}\,c^3x^7+\frac{1}{2}\,bc^2x^6+\frac{3}{5}\left(b^2c+ac^2\right)x^5+\frac{3}{2}\,a^2bx^2+\frac{1}{4}\left(b^3+6\,abc\right)x^4+a^3x+\left(ab^2+a^2c\right)x^3$$

input
```
integrate((c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*(b^2*c + a*c^2)*x^5 + 3/2*a^2*b*x^2 + 1/
4*(b^3 + 6*a*b*c)*x^4 + a^3*x + (a*b^2 + a^2*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \left(a+bx+cx^2\right)^3\,dx=a^3x+\frac{3a^2bx^2}{2}+\frac{bc^2x^6}{2}+\frac{c^3x^7}{7}+x^5\cdot\left(\frac{3ac^2}{5}+\frac{3b^2c}{5}\right)+x^4\cdot\left(\frac{3abc}{2}+\frac{b^3}{4}\right)+x^3\left(a^2c+ab^2\right)$$

input
```
integrate((c*x**2+b*x+a)**3,x)
```

output
```
a**3*x + 3*a**2*b*x**2/2 + b*c**2*x**6/2 + c**3*x**7/7 + x**5*(3*a*c**2/5
+ 3*b**2*c/5) + x**4*(3*a*b*c/2 + b**3/4) + x**3*(a**2*c + a*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \left(a+bx+cx^2\right)^3\,dx=\frac{1}{7}\,c^3x^7+\frac{1}{2}\,bc^2x^6+\frac{3}{5}\,b^2cx^5+\frac{1}{4}\,b^3x^4+a^3x+\frac{1}{2}\left(2\,cx^3+3\,bx^2\right)a^2+\frac{1}{10}\left(6\,c^2x^5+15\,bcx^4+10\,b^2x^3\right)a$$

input
```
integrate((c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 1/4*b^3*x^4 + a^3*x + 1/2*(2
*c*x^3 + 3*b*x^2)*a^2 + 1/10*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*a
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \left(a+bx+cx^2\right)^3 \, dx = \frac{1}{7}c^3x^7 + \frac{1}{2}bc^2x^6 + \frac{3}{5}b^2cx^5 + \frac{3}{5}ac^2x^5 + \frac{1}{4}b^3x^4 + \frac{3}{2}abcx^4 + ab^2x^3 + a^2cx^3 + \frac{3}{2}a^2bx^2 + a^3x$$

input
```
integrate((c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 3/5*a*c^2*x^5 + 1/4*b^3*x^4
+ 3/2*a*b*c*x^4 + a*b^2*x^3 + a^2*c*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \left(a+bx+cx^2\right)^3 \, dx = x^4 \left(\frac{b^3}{4} + \frac{3\,a\,c\,b}{2}\right) + a^3\,x + \frac{c^3\,x^7}{7} + \frac{3\,a^2\,b\,x^2}{2} + \frac{b\,c^2\,x^6}{2} + a\,x^3\,\left(b^2+a\,c\right) + \frac{3\,c\,x^5\,\left(b^2+a\,c\right)}{5}$$

input
```
int((a + b*x + c*x^2)^3,x)
```

output
```
x^4*(b^3/4 + (3*a*b*c)/2) + a^3*x + (c^3*x^7)/7 + (3*a^2*b*x^2)/2 + (b*c^2
*x^6)/2 + a*x^3*(a*c + b^2) + (3*c*x^5*(a*c + b^2))/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \left(a+bx+cx^2\right)^3\,dx$$
$$=\frac{x(20c^3x^6+70b\,c^2x^5+84a\,c^2x^4+84b^2c\,x^4+210abc\,x^3+35b^3x^3+140a^2c\,x^2+140a\,b^2x^2+210a^2bx+1}{140}$$

input
```
int((c*x^2+b*x+a)^3,x)
```

output
```
(x*(140*a**3 + 210*a**2*b*x + 140*a**2*c*x**2 + 140*a*b**2*x**2 + 210*a*b*
c*x**3 + 84*a*c**2*x**4 + 35*b**3*x**3 + 84*b**2*c*x**4 + 70*b*c**2*x**5 +
 20*c**3*x**6))/140
```

3.83 $\int (a+bx+cx^2)^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int (a+bx+cx^2)^2 \, dx = a^2x + abx^2 + \frac{1}{3}(b^2+2ac)\,x^3 + \frac{1}{2}bcx^4 + \frac{c^2x^5}{5}$$

output
```
a^2*x+a*b*x^2+1/3*(2*a*c+b^2)*x^3+1/2*b*c*x^4+1/5*c^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2)^2 \, dx = a^2x + abx^2 + \frac{1}{3}(b^2+2ac)\,x^3 + \frac{1}{2}bcx^4 + \frac{c^2x^5}{5}$$

input
```
Integrate[(a + b*x + c*x^2)^2,x]
```

output
```
a^2*x + a*b*x^2 + ((b^2 + 2*a*c)*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^2 \, dx$$

$$\Big\downarrow \text{1085}$$

$$\int \left(a^2+b^2x^2\left(\frac{2ac}{b^2}+1\right)+2abx+2bcx^3+c^2x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2x+\frac{1}{3}x^3\left(2ac+b^2\right)+abx^2+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

input
```
Int[(a + b*x + c*x^2)^2,x]
```

output
```
a^2*x + a*b*x^2 + ((b^2 + 2*a*c)*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result	size
default	$a^{2}x+ab\,x^{2}+\frac{(2ac+b^{2})x^{3}}{3}+\frac{bc\,x^{4}}{2}+\frac{c^{2}x^{5}}{5}$	41
norman	$\frac{c^{2}x^{5}}{5}+\frac{bc\,x^{4}}{2}+\left(\frac{2ac}{3}+\frac{b^{2}}{3}\right)x^{3}+ab\,x^{2}+a^{2}x$	42
gosper	$\frac{1}{5}c^{2}x^{5}+\frac{1}{2}bc\,x^{4}+\frac{2}{3}ac\,x^{3}+\frac{1}{3}b^{2}x^{3}+ab\,x^{2}+a^{2}x$	43
risch	$\frac{1}{5}c^{2}x^{5}+\frac{1}{2}bc\,x^{4}+\frac{2}{3}ac\,x^{3}+\frac{1}{3}b^{2}x^{3}+ab\,x^{2}+a^{2}x$	43
parallelrisch	$\frac{1}{5}c^{2}x^{5}+\frac{1}{2}bc\,x^{4}+\frac{2}{3}ac\,x^{3}+\frac{1}{3}b^{2}x^{3}+ab\,x^{2}+a^{2}x$	43
orering	$\frac{x(6c^{2}x^{4}+15bc\,x^{3}+20ac\,x^{2}+10b^{2}x^{2}+30abx+30a^{2})}{30}$	45

input
```
int((c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+a*b*x^2+1/3*(2*a*c+b^2)*x^3+1/2*b*c*x^4+1/5*c^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int (a+bx+cx^2)^2\,dx=\frac{1}{5}\,c^2x^5+\frac{1}{2}\,bcx^4+abx^2+\frac{1}{3}\,(b^2+2\,ac)x^3+a^2x$$

input
```
integrate((c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + a*b*x^2 + 1/3*(b^2 + 2*a*c)*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a+bx+cx^2\right)^2\,dx=a^2x+abx^2+\frac{bcx^4}{2}+\frac{c^2x^5}{5}+x^3\cdot\left(\frac{2ac}{3}+\frac{b^2}{3}\right)$$

input
```
integrate((c*x**2+b*x+a)**2,x)
```

output
```
a**2*x + a*b*x**2 + b*c*x**4/2 + c**2*x**5/5 + x**3*(2*a*c/3 + b**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \left(a+bx+cx^2\right)^2\,dx=\frac{1}{5}\,c^2x^5+\frac{1}{2}\,bcx^4+\frac{1}{3}\,b^2x^3+a^2x+\frac{1}{3}\,\left(2\,cx^3+3\,bx^2\right)a$$

input
```
integrate((c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3 + a^2*x + 1/3*(2*c*x^3 + 3*b*x^2)*
a
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a+bx+cx^2\right)^2\,dx=\frac{1}{5}\,c^2x^5+\frac{1}{2}\,bcx^4+\frac{1}{3}\,b^2x^3+\frac{2}{3}\,acx^3+abx^2+a^2x$$

input
```
integrate((c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3 + 2/3*a*c*x^3 + a*b*x^2 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \left(a+bx+cx^2\right)^2\,dx=a^2\,x+x^3\,\left(\frac{b^2}{3}+\frac{2\,a\,c}{3}\right)+\frac{c^2\,x^5}{5}+a\,b\,x^2+\frac{b\,c\,x^4}{2}$$

input
```
int((a + b*x + c*x^2)^2,x)
```

output
```
a^2*x + x^3*((2*a*c)/3 + b^2/3) + (c^2*x^5)/5 + a*b*x^2 + (b*c*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \left(a+bx+cx^2\right)^2\,dx=\frac{x(6c^2x^4+15bc\,x^3+20ac\,x^2+10b^2x^2+30abx+30a^2)}{30}$$

input
```
int((c*x^2+b*x+a)^2,x)
```

output
```
(x*(30*a**2 + 30*a*b*x + 20*a*c*x**2 + 10*b**2*x**2 + 15*b*c*x**3 + 6*c**2
*x**4))/30
```

3.84 $\int (a+bx+cx^2) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int (a+bx+cx^2) \, dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

output
```
a*x+1/2*b*x^2+1/3*c*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2) \, dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Integrate[a + b*x + c*x^2,x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Int[a + b*x + c*x^2,x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
default	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
norman	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
risch	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
parallelrisch	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
parts	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
orering	$\frac{x\left(2c\,x^{2}+3bx+6a\right)}{6}$	18

input
```
int(c*x^2+b*x+a,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b*x^2+1/3*c*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a+bx+cx^2\right)\,dx=\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

input
```
integrate(c*x^2+b*x+a,x, algorithm="fricas")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \left(a + bx + cx^2\right) dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

input
```
integrate(c*x**2+b*x+a,x)
```

output
```
a*x + b*x**2/2 + c*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a + bx + cx^2\right) dx = \frac{1}{3} cx^3 + \frac{1}{2} bx^2 + ax$$

input
```
integrate(c*x^2+b*x+a,x, algorithm="maxima")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a + bx + cx^2\right) dx = \frac{1}{3} cx^3 + \frac{1}{2} bx^2 + ax$$

input
```
integrate(c*x^2+b*x+a,x, algorithm="giac")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (a + bx + cx^2) \, dx = \frac{c\,x^3}{3} + \frac{b\,x^2}{2} + a\,x$$

input
```
int(a + b*x + c*x^2,x)
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int (a + bx + cx^2) \, dx = \frac{x(2c\,x^2 + 3bx + 6a)}{6}$$

input
```
int(c*x^2+b*x+a,x)
```

output
```
(x*(6*a + 3*b*x + 2*c*x**2))/6
```

3.85 $\int \frac{1}{a+bx+cx^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int \frac{1}{a+bx+cx^2} \, dx = -\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
-2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{1}{a+bx+cx^2} \, dx = \frac{2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

input
```
Integrate[(a + b*x + c*x^2)^(-1),x]
```

output
```
(2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx+cx^2}dx$$
$$\Big\downarrow \text{1083}$$
$$-2\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)$$
$$\Big\downarrow \text{219}$$
$$-\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(a + b*x + c*x^2)^(-1),x]
```

output
```
(-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	35
risch	$-\frac{\ln\left(2cx+\sqrt{-4ac+b^2}+b\right)}{\sqrt{-4ac+b^2}}+\frac{\ln\left(-2cx+\sqrt{-4ac+b^2}-b\right)}{\sqrt{-4ac+b^2}}$	61

input
```
int(1/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.53

$$\int \frac{1}{a+bx+cx^2}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}},\ -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

input
```
integrate(1/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(
c*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(34) = 68$.

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.65

$$\int \frac{1}{a+bx+cx^2}\,dx = -\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right) + \sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

input
```
integrate(1/(c*x**2+b*x+a),x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sq
rt(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c
*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx+cx^2}\,dx = \frac{2\,\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

input
```
integrate(1/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{a+bx+cx^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

input
```
int(1/(a + b*x + c*x^2),x)
```

output
```
(2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{a+bx+cx^2}\,dx = \frac{2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{4ac-b^2}$$

input
```
int(1/(c*x^2+b*x+a),x)
```

output
```
(2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)))/(4*a*c - b**2)
```

3.86 $\int \frac{1}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx = -\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)} + \frac{4c\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
-(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx = -\frac{\frac{b+2cx}{a+x(b+cx)} + \frac{4c\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{b^2-4ac}$$

input
```
Integrate[(a + b*x + c*x^2)^(-2),x]
```

output
```
-(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a
*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx+cx^2)^2} \, dx$$

$$\Big\downarrow \text{1086}$$

$$-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{4c\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[(a + b*x + c*x^2)^(-2),x]
```

output
```
-((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result	si
default	$\frac{2cx+b}{(4ac-b^2)(c\,x^2+bx+a)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	68
risch	$\frac{\frac{2cx}{4ac-b^2}+\frac{b}{4ac-b^2}}{c\,x^2+bx+a}+\frac{2c\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{2c\ln\left(\left(8a\,c^2-2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$	14

input
```
int(1/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(62) = 124$.

Time = 0.08 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.17

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx$$
$$= \left[-\frac{b^3-4\,abc+2\,(c^2x^2+bcx+ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x},\right.$$
$$\left.-\frac{b^3-4\,abc-4\,(c^2x^2+bcx+ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x}\right]$$

input
```
integrate(1/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
+ a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b
^3 - 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-
b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 -
 8*a*b^3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(61) = 122$.

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.02

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx = -2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right) +2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right) + \frac{b+2cx}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x(4abc-b^3)}$$

input
```
integrate(1/(c*x**2+b*x+a)**2,x)
```

output
```
-2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*
a*c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x
+ (32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c
 - b**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b
 + 2*c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**
3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx = -\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,cx+b}{(cx^2+bx+a)(b^2-4\,ac)}$$

input
```
integrate(1/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a
*c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.80

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx = \frac{\frac{b}{4\,a\,c-b^2}+\frac{2\,c\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a} - \frac{4\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,c^2\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,c}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

input
```
int(1/(a + b*x + c*x^2)^2,x)
```

output
```
(b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan(((
(2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))
*(4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.65

$$\int \frac{1}{(a+bx+cx^2)^2}\,dx$$
$$= \frac{4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abc+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2cx+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b\,c^2x^2-}{b\,(16a^2c^3x^2-8a\,b^2c^2x^2+b^4c\,x^2+16a^2b\,c^2x-8a\,b^3cx+b^5x+16a^3c^2-8a^2}$$

input
```
int(1/(c*x^2+b*x+a)^2,x)
```

output
```
(4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c + 4*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*x + 4*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**2*x**2 - 8*a**2*c**2 +
6*a*b**2*c - 8*a*c**3*x**2 - b**4 + 2*b**2*c**2*x**2)/(b*(16*a**3*c**2 - 8
*a**2*b**2*c + 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*
x - 8*a*b**2*c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.87 $\int \frac{1}{(a+bx+cx^2)^3} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{1}{(a+bx+cx^2)^3} \, dx = -\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{3c(b+2cx)}{(b^2-4ac)^2(a+bx+cx^2)} - \frac{12c^2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
-1/2*(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+3*c*(2*c*x+b)/(-4*a*c+b^2)^2/(
c*x^2+b*x+a)-12*c^2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/
2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+bx+cx^2)^3} \, dx = \frac{-\frac{(b+2cx)(b^2-6bcx-2c(5a+3cx^2))}{(a+x(b+cx))^2} + \frac{24c^2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2-4ac)^2}$$

input
```
Integrate[(a + b*x + c*x^2)^(-3),x]
```

output
```
(-(((b + 2*c*x)*(b^2 - 6*b*c*x - 2*c*(5*a + 3*c*x^2)))/(a + x*(b + c*x))^2
) + (24*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2
*(b^2 - 4*a*c)^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx+cx^2)^3}\,dx$$

$$\Big\downarrow \; 1086$$

$$-\frac{3c\int \frac{1}{(cx^2+bx+a)^2}dx}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \; 1086$$

$$-\frac{3c\left(-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \; 1083$$

$$-\frac{3c\left(\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

input
```
Int[(a + b*x + c*x^2)^(-3),x]
```

output
```
-1/2*(b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*c*(-((b + 2*c*x)
/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15

method	result
default	$\frac{2cx+b}{2(4ac-b^2)(c\,x^2+bx+a)^2}+\frac{3c\left(\frac{2cx+b}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}\right)}{4ac-b^2}$
risch	$\frac{\frac{6c^3x^3}{16a^2c^2-8ca\,b^2+b^4}+\frac{9b\,c^2x^2}{16a^2c^2-8ca\,b^2+b^4}+\frac{2\left(5ac+b^2\right)cx}{16a^2c^2-8ca\,b^2+b^4}+\frac{b\left(10ac-b^2\right)}{32a^2c^2-16ca\,b^2+2b^4}}{(c\,x^2+bx+a)^2}-\frac{6c^2\ln\left(\left(32a^2c^3-16a\,c^2b^2+2b^4c\right)x+\left(-4ac+b^2\right.\right.}{(-4ac+b^2)^{\frac{5}{2}}}$

input
```
int(1/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+3*c/(4*a*c-b^2)*((2*c*x+b)/(4*a*
c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(95) = 190$.

Time = 0.10 (sec) , antiderivative size = 785, normalized size of antiderivative = 7.77

$$\int \frac{1}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(
b^3*c^2 - 4*a*b*c^3)*x^2 - 12*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c
^2 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x +
 b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 4*(b^4*
c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 +
2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a
*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(b^5 - 14*a*b^3*c +
40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b^3*c^2 - 4*a*b*c^3)*x^2 +
 24*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^2 + 2*a*c^3)*x
^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) - 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3
*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 +
 (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 +
2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(95) = 190$.

Time = 0.68 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.69

$$\int \frac{1}{(a+bx+cx^2)^3}\,dx =$$

$$-6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+}{12c^3}\right.$$

$$+6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-6}{12c^3}\right.$$

$$+\frac{10abc-b^3+18bc^2x^2+12c^3x^3+x(20ac^2+4b^2c}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(6}$$

input
```
integrate(1/(c*x**2+b*x+a)**3,x)
```

output
```
-6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**3*c**5*sqrt(-1/(4*a*c
- b**2)**5) + 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 72*a*b**4*c*
*3*sqrt(-1/(4*a*c - b**2)**5) + 6*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 6
*b*c**2)/(12*c**3)) + 6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**3*
c**5*sqrt(-1/(4*a*c - b**2)**5) - 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2
)**5) + 72*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 6*b**6*c**2*sqrt(-1/(4
*a*c - b**2)**5) + 6*b*c**2)/(12*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**
2 + 12*c**3*x**3 + x*(20*a*c**2 + 4*b**2*c))/(32*a**4*c**2 - 16*a**3*b**2*
c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**
3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a
*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a+bx+cx^2)^3}\,dx = \frac{12\,c^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx+20\,ac^2x-b^3+10\,abc}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

input
```
integrate(1/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
12*c^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*x^3 + 18*b*c^2*x^2 + 4*b^2*c*x + 20*
a*c^2*x - b^3 + 10*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a
)^2)
```

Mupad [B] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.82

$$\int \frac{1}{(a+bx+cx^2)^3}\,dx = \frac{\frac{6\,c^3\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{b^3-10\,a\,b\,c}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{9\,b\,c^2\,x^2}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{2\,c\,x\,(b^2+5\,a\,c)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} + \frac{12\,c^2\,\mathrm{atan}\left(\frac{\left(\frac{12\,c^3\,x}{(4\,a\,c-b^2)^{5/2}}+\frac{6\,c^2\,(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5)}{(4\,a\,c-b^2)^{5/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}\right)(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}{6\,c^2}\right)}{(4\,a\,c-b^2)^{5/2}}$$

input
```
int(1/(a + b*x + c*x^2)^3,x)
```

output
```
((6*c^3*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3 - 10*a*b*c)/(2*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*x^2)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (
2*c*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2) +
a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (12*c^2*atan((((12*c^3*x)/(4*a*c -
b^2)^(5/2) + (6*c^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2))
)/(4*a*c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 598, normalized size of antiderivative = 5.92

$$\int \frac{1}{(a+bx+cx^2)^3}\,dx = \frac{24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2b\,c^2+48\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2c^2x+48\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)c}{2b\,(64a^3c^5x^4-48a^2b^2c^4x^4+12a\,b^4c^3x^4-b^6c^2x^4+1}$$

input
```
int(1/(c*x^2+b*x+a)^3,x)
```

output
```
(24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2 +
48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c**2*x +
 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**3*x**2
+ 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c**2*x**
2 + 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c**3*x
**3 + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**4*x*
*4 - 24*a**3*c**3 + 46*a**2*b**2*c**2 + 32*a**2*b*c**3*x - 48*a**2*c**4*x*
*2 - 14*a*b**4*c + 8*a*b**3*c**2*x + 60*a*b**2*c**3*x**2 - 24*a*c**5*x**4
+ b**6 - 4*b**5*c*x - 12*b**4*c**2*x**2 + 6*b**2*c**4*x**4)/(2*b*(64*a**5*
c**3 - 48*a**4*b**2*c**2 + 128*a**4*b*c**3*x + 128*a**4*c**4*x**2 + 12*a**
3*b**4*c - 96*a**3*b**3*c**2*x - 32*a**3*b**2*c**3*x**2 + 128*a**3*b*c**4*
x**3 + 64*a**3*c**5*x**4 - a**2*b**6 + 24*a**2*b**5*c*x - 24*a**2*b**4*c**
2*x**2 - 96*a**2*b**3*c**3*x**3 - 48*a**2*b**2*c**4*x**4 - 2*a*b**7*x + 10
*a*b**6*c*x**2 + 24*a*b**5*c**2*x**3 + 12*a*b**4*c**3*x**4 - b**8*x**2 - 2
*b**7*c*x**3 - b**6*c**2*x**4))
```

3.88 $\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 \, dx = \frac{(1-b-2cx)^6}{384c^6}-\frac{5(1-b-2cx)^7}{896c^6}+\frac{5(1-b-2cx)^8}{1024c^6}-\frac{5(1-b-2cx)^9}{2304c^6}+\frac{(1-b-2cx)^{10}}{2048c^6}-\frac{(1-b-2cx)^{11}}{22528c^6}$$

output
```
1/384*(-2*c*x-b+1)^6/c^6-5/896*(-2*c*x-b+1)^7/c^6+5/1024*(-2*c*x-b+1)^8/c^
6-5/2304*(-2*c*x-b+1)^9/c^6+1/2048*(-2*c*x-b+1)^10/c^6-1/22528*(-2*c*x-b+1
)^11/c^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.89

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{(-1+b^2)^5 x}{1024c^5}+\frac{5b(-1+b^2)^4 x^2}{512c^4} + \frac{5(-1+b^2)^3(-1+9b^2)x^3}{768c^3} + \frac{5b(-1+b^2)^2(-1+3b^2)x^4}{64c^2} + \frac{(-1+b^2)(1-14b^2+21b^4)x^5}{32c} + \frac{1}{48}b(15-70b^2+63b^4)x^6 + \frac{5}{56}(1-14b^2+21b^4)cx^7+\frac{5}{8}b(-1+3b^2)c^2x^8 + \frac{5}{36}(-1+9b^2)c^3x^9+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

input
```
Integrate[((-1 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((-1 + b^2)^5*x)/(1024*c^5) + (5*b*(-1 + b^2)^4*x^2)/(512*c^4) + (5*(-1 +
b^2)^3*(-1 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-1 + b^2)^2*(-1 + 3*b^2)*x^4)/(
64*c^2) + ((-1 + b^2)*(1 - 14*b^2 + 21*b^4)*x^5)/(32*c) + (b*(15 - 70*b^2
+ 63*b^4)*x^6)/48 + (5*(1 - 14*b^2 + 21*b^4)*c*x^7)/56 + (5*b*(-1 + 3*b^2)
*c^2*x^8)/8 + (5*(-1 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2-1}{4c}+bx+cx^2\right)^5 dx$$

$$\Big\downarrow \text{1084}$$

$$\frac{\int \left(\left(\frac{b-1}{2}+cx\right)^{10}+\left(\frac{b-1}{2}+cx\right)^5-\frac{5}{512}(-b-2cx+1)^9+\frac{5}{128}(-b-2cx+1)^8-\frac{5}{64}(-b-2cx+1)^7+\frac{5}{64}(-b-2cx\right.}{c^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(-b-2cx+1)^{11}}{22528c}+\frac{(-b-2cx+1)^{10}}{2048c}-\frac{5(-b-2cx+1)^9}{2304c}+\frac{5(-b-2cx+1)^8}{1024c}-\frac{5(-b-2cx+1)^7}{896c}+\frac{(-b-2cx+1)^6}{384c}}{c^5}$$

input
```
Int[((-1 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((1 - b - 2*c*x)^6/(384*c) - (5*(1 - b - 2*c*x)^7)/(896*c) + (5*(1 - b - 2
*c*x)^8)/(1024*c) - (5*(1 - b - 2*c*x)^9)/(2304*c) + (1 - b - 2*c*x)^10/(2
048*c) - (1 - b - 2*c*x)^11/(22528*c))/c^5
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(97) = 194$.

Time = 0.59 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.50

method	result
norman	$\left(\frac{5}{4}b^2c^7-\frac{5}{36}c^7\right)x^9+\left(\frac{15}{8}b^3c^6-\frac{5}{8}b\,c^6\right)x^8+\left(\frac{15}{8}b^4c^5-\frac{5}{4}c^5b^2+\frac{5}{56}c^5\right)x^7+\left(\frac{21}{16}b^5c^4-\frac{35}{24}b^3c^4+\frac{5}{16}b\,c^4\right)x^6+\left(\frac{21}{32}b^6c^3-\frac{35}{32}b^4c^3+\frac{15}{32}c^3b^2\right.$
gosper	$x\left(64512c^{10}x^{10}+354816c^9b\,x^9+887040x^8b^2c^8+1330560b^3c^7x^7+1330560x^6b^4c^6-98560x^8c^8+931392x^5b^5c^5-443520b\,c^7x^7+\right.$
parallelrisch	$64512c^{10}x^{11}+354816c^9b\,x^{10}+887040x^9b^2c^8+1330560b^3c^7x^8+1330560x^7b^4c^6-98560x^9c^8+931392x^6b^5c^5-443520b\,c^7x^8+4$
risch	$\frac{5b\,x^6}{16}-\frac{35b^3x^6}{24}-\frac{5b^2c\,x^7}{4}+\frac{15x^3b^8}{256c^3}-\frac{35x^3b^6}{192c^3}+\frac{15c^2b^3x^8}{8}-\frac{5b^3x^2}{128c^4}+\frac{5b^9x^2}{512c^4}-\frac{5c^2b\,x^8}{8}+\frac{21b^6x^5}{32c}+\frac{15b^2x^5}{32c}$
orering	$x\left(64512c^{10}x^{10}+354816c^9b\,x^9+887040x^8b^2c^8+1330560b^3c^7x^7+1330560x^6b^4c^6-98560x^8c^8+931392x^5b^5c^5-443520b\,c^7x^7+\right.$
default	$\frac{c^5x^{11}}{11}+\frac{b\,c^4x^{10}}{2}+\frac{\left(256(b^2-1)c^3+4096c^3b^2+4c\left(32(24b^2-8)c^2+1024b^2c^2\right)\right)x^9}{9216}+\frac{\left(1024(b^2-1)c^2b+4b\left(32(24b^2-8)c^2+\right.\right.}{}$

input
```
int((1/4*(b^2-1)/c+b*x+c*x^2)^5,x,method=_RETURNVERBOSE)
```

output
```
((5/4*b^2*c^7-5/36*c^7)*x^9+(15/8*b^3*c^6-5/8*b*c^6)*x^8+(15/8*b^4*c^5-5/4
*c^5*b^2+5/56*c^5)*x^7+(21/16*b^5*c^4-35/24*b^3*c^4+5/16*b*c^4)*x^6+(21/32
*b^6*c^3-35/32*b^4*c^3+15/32*c^3*b^2-1/32*c^3)*x^5+(15/64*b^7*c^2-35/64*b^
5*c^2+25/64*b^3*c^2-5/64*b*c^2)*x^4+(5/512*b^9-5/128*b^7+15/256*b^5-5/128*
b^3+5/512*b)*x^2+(15/256*b^8*c-35/192*b^6*c+25/128*b^4*c-5/64*b^2*c+5/768*
c)*x^3+1/11*c^9*x^11+1/2*b*c^8*x^10+1/1024*(b^10-5*b^8+10*b^6-10*b^4+5*b^2
-1)/c*x)/c^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(85) = 170$.

Time = 0.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.14

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5\,dx$$
$$=\frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+98560\,(9\,b^2-1)c^8x^9+443520\,(3\,b^3-b)c^7x^8+63360\,(21\,b^4-14\,b^2+1)c}{}$$

input
```
integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

output
```
1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 1)*c^8*x^9
+ 443520*(3*b^3 - b)*c^7*x^8 + 63360*(21*b^4 - 14*b^2 + 1)*c^6*x^7 + 14784
*(63*b^5 - 70*b^3 + 15*b)*c^5*x^6 + 22176*(21*b^6 - 35*b^4 + 15*b^2 - 1)*c
^4*x^5 + 55440*(3*b^7 - 7*b^5 + 5*b^3 - b)*c^3*x^4 + 4620*(9*b^8 - 28*b^6
+ 30*b^4 - 12*b^2 + 1)*c^2*x^3 + 6930*(b^9 - 4*b^7 + 6*b^5 - 4*b^3 + b)*c*
x^2 + 693*(b^10 - 5*b^8 + 10*b^6 - 10*b^4 + 5*b^2 - 1)*x)/c^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(95) = 190$.

Time = 0.11 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.32

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{bc^4x^{10}}{2}+\frac{c^5x^{11}}{11}+x^9\cdot\left(\frac{5b^2c^3}{4}-\frac{5c^3}{36}\right)+x^8\cdot\left(\frac{15b^3c^2}{8}-\frac{5bc^2}{8}\right)+x^7\cdot\left(\frac{15b^4c}{8}-\frac{5b^2c}{4}+\frac{5c}{56}\right)+x^6\cdot\left(\frac{21b^5}{16}-\frac{35b^3}{24}+\frac{5b}{16}\right)+\frac{x^5\cdot(21b^6-35b^4+15b^2-1)}{32c}+\frac{x^4\cdot(15b^7-35b^5+25b^3-5b)}{64c^2}+\frac{x^3\cdot(45b^8-140b^6+150b^4-60b^2+5)}{768c^3}+\frac{x^2\cdot(5b^9-20b^7+30b^5-20b^3+5b)}{512c^4}+\frac{x(b^{10}-5b^8+10b^6-10b^4+5b^2-1)}{1024c^5}$$

input
```
integrate((1/4*(b**2-1)/c+b*x+c*x**2)**5,x)
```

output
```
b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/36) + x**8*(
15*b**3*c**2/8 - 5*b*c**2/8) + x**7*(15*b**4*c/8 - 5*b**2*c/4 + 5*c/56) +
x**6*(21*b**5/16 - 35*b**3/24 + 5*b/16) + x**5*(21*b**6 - 35*b**4 + 15*b**
2 - 1)/(32*c) + x**4*(15*b**7 - 35*b**5 + 25*b**3 - 5*b)/(64*c**2) + x**3*
(45*b**8 - 140*b**6 + 150*b**4 - 60*b**2 + 5)/(768*c**3) + x**2*(5*b**9 -
20*b**7 + 30*b**5 - 20*b**3 + 5*b)/(512*c**4) + x*(b**10 - 5*b**8 + 10*b**
6 - 10*b**4 + 5*b**2 - 1)/(1024*c**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(85) = 170$.

Time = 0.03 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.15

$$\begin{aligned}
&\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx \\
&= \frac{1}{11}\,c^5x^{11}+\frac{1}{2}\,bc^4x^{10}+\frac{10}{9}\,b^2c^3x^9+\frac{5}{4}\,b^3c^2x^8+\frac{5}{7}\,b^4cx^7+\frac{1}{6}\,b^5x^6 \\
&\quad+\frac{5\,(2\,cx^3+3\,bx^2)(b^2-1)^4}{1536\,c^4}+\frac{(6\,c^2x^5+15\,bcx^4+10\,b^2x^3)(b^2-1)^3}{192\,c^3} \\
&\quad+\frac{(20\,c^3x^7+70\,bc^2x^6+84\,b^2cx^5+35\,b^3x^4)(b^2-1)^2}{224\,c^2} \\
&\quad+\frac{(70\,c^4x^9+315\,bc^3x^8+540\,b^2c^2x^7+420\,b^3cx^6+126\,b^4x^5)(b^2-1)}{504\,c}+\frac{(b^2-1)^5x}{1024\,c^5}
\end{aligned}$$

input
```
integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

output
```
1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*
b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 1)^4/c^4 + 1/1
92*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 1)^3/c^3 + 1/224*(20*c^3*x
^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 1)^2/c^2 + 1/504*(70
*c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*
(b^2 - 1)/c + 1/1024*(b^2 - 1)^5*x/c^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(85) = 170$.

Time = 0.15 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.06

$$\begin{aligned}
&\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx \\
&= \frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+887040\,b^2c^8x^9+1330560\,b^3c^7x^8+1330560\,b^4c^6x^7-98560\,c^8x^9+93139}{}
\end{aligned}$$

input
```
integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

output
```
1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 13305
60*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 98560*c^8*x^9 + 931392*b^5*c^5*x^6
- 443520*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 887040*b^2*c^6*x^7 + 166320*b^7*
c^3*x^4 - 1034880*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 776160*b^4*c^4*x^5 + 6
3360*c^6*x^7 + 6930*b^9*c*x^2 - 388080*b^5*c^3*x^4 + 221760*b*c^5*x^6 + 69
3*b^10*x - 129360*b^6*c^2*x^3 + 332640*b^2*c^4*x^5 - 27720*b^7*c*x^2 + 277
200*b^3*c^3*x^4 - 3465*b^8*x + 138600*b^4*c^2*x^3 - 22176*c^4*x^5 + 41580*
b^5*c*x^2 - 55440*b*c^3*x^4 + 6930*b^6*x - 55440*b^2*c^2*x^3 - 27720*b^3*c
*x^2 - 6930*b^4*x + 4620*c^2*x^3 + 6930*b*c*x^2 + 3465*b^2*x - 693*x)/c^5
```

Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.69

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{c^5 x^{11}}{11} + \frac{x\,(b^2-1)^5}{1024\,c^5} + \frac{b\,x^6\,(63\,b^4-70\,b^2+15)}{48} + \frac{5\,c\,x^7\,(21\,b^4-14\,b^2+1)}{56} + \frac{b\,c^4\,x^{10}}{2} + \frac{5\,c^3\,x^9\,(9\,b^2-1)}{36} + \frac{x^5\,(21\,b^6-35\,b^4+15\,b^2-1)}{32\,c} + \frac{5\,b\,c^2\,x^8\,(3\,b^2-1)}{8} + \frac{5\,b\,x^2\,(b^2-1)^4}{512\,c^4} + \frac{5\,x^3\,(b^2-1)^3\,(9\,b^2-1)}{768\,c^3} + \frac{5\,b\,x^4\,(b^2-1)^2\,(3\,b^2-1)}{64\,c^2}$$

input
```
int((b*x + c*x^2 + (b^2/4 - 1/4)/c)^5,x)
```

output
```
(c^5*x^11)/11 + (x*(b^2 - 1)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 70*b^2 + 15)
)/48 + (5*c*x^7*(21*b^4 - 14*b^2 + 1))/56 + (b*c^4*x^10)/2 + (5*c^3*x^9*(9
*b^2 - 1))/36 + (x^5*(15*b^2 - 35*b^4 + 21*b^6 - 1))/(32*c) + (5*b*c^2*x^8
*(3*b^2 - 1))/8 + (5*b*x^2*(b^2 - 1)^4)/(512*c^4) + (5*x^3*(b^2 - 1)^3*(9*
b^2 - 1))/(768*c^3) + (5*b*x^4*(b^2 - 1)^2*(3*b^2 - 1))/(64*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.92

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{x(64512c^{10}x^{10}+354816b\,c^9x^9+887040b^2c^8x^8+1330560b^3c^7x^7+1330560b^4c^6x^6-98560c^8x^8+931392}{}$$

input
```
int((1/4*(b^2-1)/c+b*x+c*x^2)^5,x)
```

output
```
(x*(693*b**10 + 6930*b**9*c*x + 41580*b**8*c**2*x**2 - 3465*b**8 + 166320*
b**7*c**3*x**3 - 27720*b**7*c*x + 465696*b**6*c**4*x**4 - 129360*b**6*c**2
*x**2 + 6930*b**6 + 931392*b**5*c**5*x**5 - 388080*b**5*c**3*x**3 + 41580*
b**5*c*x + 1330560*b**4*c**6*x**6 - 776160*b**4*c**4*x**4 + 138600*b**4*c*
*2*x**2 - 6930*b**4 + 1330560*b**3*c**7*x**7 - 1034880*b**3*c**5*x**5 + 27
7200*b**3*c**3*x**3 - 27720*b**3*c*x + 887040*b**2*c**8*x**8 - 887040*b**2
*c**6*x**6 + 332640*b**2*c**4*x**4 - 55440*b**2*c**2*x**2 + 3465*b**2 + 35
4816*b*c**9*x**9 - 443520*b*c**7*x**7 + 221760*b*c**5*x**5 - 55440*b*c**3*
x**3 + 6930*b*c*x + 64512*c**10*x**10 - 98560*c**8*x**8 + 63360*c**6*x**6
- 22176*c**4*x**4 + 4620*c**2*x**2 - 693))/(709632*c**5)
```

3.89 $\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 \, dx = \frac{(2-b-2cx)^6}{12c^6}-\frac{5(2-b-2cx)^7}{56c^6}+\frac{5(2-b-2cx)^8}{128c^6} -\frac{5(2-b-2cx)^9}{576c^6}+\frac{(2-b-2cx)^{10}}{1024c^6}-\frac{(2-b-2cx)^{11}}{22528c^6}$$

output
```
1/12*(-2*c*x-b+2)^6/c^6-5/56*(-2*c*x-b+2)^7/c^6+5/128*(-2*c*x-b+2)^8/c^6-5
/576*(-2*c*x-b+2)^9/c^6+1/1024*(-2*c*x-b+2)^10/c^6-1/22528*(-2*c*x-b+2)^11
/c^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.89

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{(-4+b^2)^5 x}{1024c^5}+\frac{5b(-4+b^2)^4 x^2}{512c^4}+\frac{5(-4+b^2)^3 (-4+9b^2) x^3}{768c^3}+\frac{5b(-4+b^2)^2 (-4+3b^2) x^4}{64c^2}+\frac{(-4+b^2) (16-56b^2+21b^4) x^5}{32c}+\frac{1}{48}b(240-280b^2+63b^4) x^6+\frac{5}{56}(16-56b^2+21b^4) cx^7+\frac{5}{8}b(-4+3b^2) c^2x^8+\frac{5}{36}(-4+9b^2) c^3x^9+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

input
```
Integrate[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((-4 + b^2)^5*x)/(1024*c^5) + (5*b*(-4 + b^2)^4*x^2)/(512*c^4) + (5*(-4 +
b^2)^3*(-4 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-4 + b^2)^2*(-4 + 3*b^2)*x^4)/(
64*c^2) + ((-4 + b^2)*(16 - 56*b^2 + 21*b^4)*x^5)/(32*c) + (b*(240 - 280*b
^2 + 63*b^4)*x^6)/48 + (5*(16 - 56*b^2 + 21*b^4)*c*x^7)/56 + (5*b*(-4 + 3*
b^2)*c^2*x^8)/8 + (5*(-4 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11
)/11
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2-4}{4c}+bx+cx^2\right)^5 dx$$

$\big\downarrow$ 1084

$$\frac{\int \left(\left(\frac{b-2}{2}+cx\right)^{10}-\frac{5}{256}(-b-2cx+2)^9+\frac{5}{32}(-b-2cx+2)^8-\frac{5}{8}(-b-2cx+2)^7+\frac{5}{4}(-b-2cx+2)^6+(b+2cx\right.}{c^5}$$

$\big\downarrow$ 2009

$$\frac{-\frac{(-b-2cx+2)^{11}}{22528c}+\frac{(-b-2cx+2)^{10}}{1024c}-\frac{5(-b-2cx+2)^9}{576c}+\frac{5(-b-2cx+2)^8}{128c}-\frac{5(-b-2cx+2)^7}{56c}+\frac{(-b-2cx+2)^6}{12c}}{c^5}$$

input
```
Int[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((2 - b - 2*c*x)^6/(12*c) - (5*(2 - b - 2*c*x)^7)/(56*c) + (5*(2 - b - 2*c
*x)^8)/(128*c) - (5*(2 - b - 2*c*x)^9)/(576*c) + (2 - b - 2*c*x)^10/(1024*
c) - (2 - b - 2*c*x)^11/(22528*c))/c^5
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(97) = 194$.

Time = 0.58 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.50

method	result
norman	$\frac{(\frac{5}{4}b^2c^7-\frac{5}{9}c^7)x^9+(\frac{15}{8}b^3c^6-\frac{5}{2}bc^6)x^8+(\frac{15}{8}b^4c^5-5c^5b^2+\frac{10}{7}c^5)x^7+(\frac{21}{16}b^5c^4-\frac{35}{6}b^3c^4+5bc^4)x^6+(\frac{21}{32}b^6c^3-\frac{35}{8}b^4c^3+\frac{15}{2}c^3b^2-2}{}$
gosper	$\frac{x(64512c^{10}x^{10}+354816c^9bx^9+887040x^8b^2c^8+1330560b^3c^7x^7+1330560x^6b^4c^6-394240x^8c^8+931392x^5b^5c^5-1774080bc^7x}{}$
parallelrisch	$\frac{64512c^{10}x^{11}+354816c^9bx^{10}+887040x^9b^2c^8+1330560b^3c^7x^8+1330560x^7b^4c^6-394240x^9c^8+931392x^6b^5c^5-1774080bc^7x^8}{}$
risch	$5bx^6-\frac{35b^3x^6}{6}-5b^2cx^7+\frac{15x^3b^8}{256c^3}-\frac{35x^3b^6}{48c^3}+\frac{15c^2b^3x^8}{8}-\frac{5b^3x^2}{2c^4}+\frac{5b^9x^2}{512c^4}-\frac{5c^2bx^8}{2}+\frac{21b^6x^5}{32c}+\frac{15b^2x}{2c}$
orering	$\frac{x(64512c^{10}x^{10}+354816c^9bx^9+887040x^8b^2c^8+1330560b^3c^7x^7+1330560x^6b^4c^6-394240x^8c^8+931392x^5b^5c^5-1774080bc^7x}{}$
default	$\frac{c^5x^{11}}{11}+\frac{bc^4x^{10}}{2}+\frac{(256(b^2-4)c^3+4096c^3b^2+4c(32(24b^2-32)c^2+1024b^2c^2))x^9}{9216}+\frac{(1024(b^2-4)c^2b+4b(32(24b^2-32)c}{}$

input
```
int((1/4*(b^2-4)/c+b*x+c*x^2)^5,x,method=_RETURNVERBOSE)
```

output
```
((5/4*b^2*c^7-5/9*c^7)*x^9+(15/8*b^3*c^6-5/2*b*c^6)*x^8+(15/8*b^4*c^5-5*c^
5*b^2+10/7*c^5)*x^7+(21/16*b^5*c^4-35/6*b^3*c^4+5*b*c^4)*x^6+(21/32*b^6*c^
3-35/8*b^4*c^3+15/2*c^3*b^2-2*c^3)*x^5+(15/64*b^7*c^2-35/16*b^5*c^2+25/4*b
^3*c^2-5*b*c^2)*x^4+(5/512*b^9-5/32*b^7+15/16*b^5-5/2*b^3+5/2*b)*x^2+(15/2
56*b^8*c-35/48*b^6*c+25/8*b^4*c-5*b^2*c+5/3*c)*x^3+1/11*c^9*x^11+1/2*b*c^8
*x^10+1/1024*(b^10-20*b^8+160*b^6-640*b^4+1280*b^2-1024)/c*x)/c^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(85) = 170$.

Time = 0.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.16

$$\int\left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5dx$$
$$=\frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+98560\,(9\,b^2-4)c^8x^9+443520\,(3\,b^3-4\,b)c^7x^8+63360\,(21\,b^4-56\,b^2+1}{}$$

input
```
integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

output
```
1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 4)*c^8*x^9
+ 443520*(3*b^3 - 4*b)*c^7*x^8 + 63360*(21*b^4 - 56*b^2 + 16)*c^6*x^7 + 14
784*(63*b^5 - 280*b^3 + 240*b)*c^5*x^6 + 22176*(21*b^6 - 140*b^4 + 240*b^2
 - 64)*c^4*x^5 + 55440*(3*b^7 - 28*b^5 + 80*b^3 - 64*b)*c^3*x^4 + 4620*(9*
b^8 - 112*b^6 + 480*b^4 - 768*b^2 + 256)*c^2*x^3 + 6930*(b^9 - 16*b^7 + 96
*b^5 - 256*b^3 + 256*b)*c*x^2 + 693*(b^10 - 20*b^8 + 160*b^6 - 640*b^4 + 1
280*b^2 - 1024)*x)/c^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(95) = 190$.

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.29

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{bc^4x^{10}}{2} + \frac{c^5x^{11}}{11} + x^9 \cdot \left(\frac{5b^2c^3}{4} - \frac{5c^3}{9}\right) + x^8 \cdot \left(\frac{15b^3c^2}{8} - \frac{5bc^2}{2}\right) + x^7 \cdot \left(\frac{15b^4c}{8} - 5b^2c + \frac{10c}{7}\right) + x^6 \cdot \left(\frac{21b^5}{16} - \frac{35b^3}{6} + 5b\right) + \frac{x^5 \cdot (21b^6 - 140b^4 + 240b^2 - 64)}{32c} + \frac{x^4 \cdot (15b^7 - 140b^5 + 400b^3 - 320b)}{64c^2} + \frac{x^3 \cdot (45b^8 - 560b^6 + 2400b^4 - 3840b^2 + 1280)}{768c^3} + \frac{x^2 \cdot (5b^9 - 80b^7 + 480b^5 - 1280b^3 + 1280b)}{512c^4} + \frac{x(b^{10} - 20b^8 + 160b^6 - 640b^4 + 1280b^2 - 1024)}{1024c^5}$$

input
```
integrate((1/4*(b**2-4)/c+b*x+c*x**2)**5,x)
```

output
```
b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/9) + x**8*(1
5*b**3*c**2/8 - 5*b*c**2/2) + x**7*(15*b**4*c/8 - 5*b**2*c + 10*c/7) + x**
6*(21*b**5/16 - 35*b**3/6 + 5*b) + x**5*(21*b**6 - 140*b**4 + 240*b**2 - 6
4)/(32*c) + x**4*(15*b**7 - 140*b**5 + 400*b**3 - 320*b)/(64*c**2) + x**3*
(45*b**8 - 560*b**6 + 2400*b**4 - 3840*b**2 + 1280)/(768*c**3) + x**2*(5*b
**9 - 80*b**7 + 480*b**5 - 1280*b**3 + 1280*b)/(512*c**4) + x*(b**10 - 20*
b**8 + 160*b**6 - 640*b**4 + 1280*b**2 - 1024)/(1024*c**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(85) = 170$.

Time = 0.04 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.15

$$\begin{aligned}&\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx\\&=\frac{1}{11}\,c^5x^{11}+\frac{1}{2}\,bc^4x^{10}+\frac{10}{9}\,b^2c^3x^9+\frac{5}{4}\,b^3c^2x^8+\frac{5}{7}\,b^4cx^7+\frac{1}{6}\,b^5x^6\\&\quad+\frac{5\,(2\,cx^3+3\,bx^2)(b^2-4)^4}{1536\,c^4}+\frac{(6\,c^2x^5+15\,bcx^4+10\,b^2x^3)(b^2-4)^3}{192\,c^3}\\&\quad+\frac{(20\,c^3x^7+70\,bc^2x^6+84\,b^2cx^5+35\,b^3x^4)(b^2-4)^2}{224\,c^2}\\&\quad+\frac{(70\,c^4x^9+315\,bc^3x^8+540\,b^2c^2x^7+420\,b^3cx^6+126\,b^4x^5)(b^2-4)}{504\,c}+\frac{(b^2-4)^5x}{1024\,c^5}\end{aligned}$$

input
```
integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

output
```
1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*
b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 4)^4/c^4 + 1/1
92*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 4)^3/c^3 + 1/224*(20*c^3*x
^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 4)^2/c^2 + 1/504*(70
*c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*
(b^2 - 4)/c + 1/1024*(b^2 - 4)^5*x/c^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(85) = 170$.

Time = 0.17 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.06

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+887040\,b^2c^8x^9+1330560\,b^3c^7x^8+1330560\,b^4c^6x^7-394240\,c^8x^9+9313}{}$$

input
```
integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

output
```
1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 13305
60*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 394240*c^8*x^9 + 931392*b^5*c^5*x^6
 - 1774080*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 3548160*b^2*c^6*x^7 + 166320*b
^7*c^3*x^4 - 4139520*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 3104640*b^4*c^4*x^5
 + 1013760*c^6*x^7 + 6930*b^9*c*x^2 - 1552320*b^5*c^3*x^4 + 3548160*b*c^5*
x^6 + 693*b^10*x - 517440*b^6*c^2*x^3 + 5322240*b^2*c^4*x^5 - 110880*b^7*c
*x^2 + 4435200*b^3*c^3*x^4 - 13860*b^8*x + 2217600*b^4*c^2*x^3 - 1419264*c
^4*x^5 + 665280*b^5*c*x^2 - 3548160*b*c^3*x^4 + 110880*b^6*x - 3548160*b^2
*c^2*x^3 - 1774080*b^3*c*x^2 - 443520*b^4*x + 1182720*c^2*x^3 + 1774080*b*
c*x^2 + 887040*b^2*x - 709632*x)/c^5
```

Mupad [B] (verification not implemented)

Time = 10.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.69

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{c^5\,x^{11}}{11}+\frac{x\,\left(b^2-4\right)^5}{1024\,c^5}+\frac{b\,x^6\,\left(63\,b^4-280\,b^2+240\right)}{48}+\frac{5\,c\,x^7\,\left(21\,b^4-56\,b^2+16\right)}{56}+\frac{b\,c^4\,x^{10}}{2}+\frac{5\,c^3\,x^9\,\left(9\,b^2-4\right)}{36}+\frac{x^5\,\left(21\,b^6-140\,b^4+240\,b^2-64\right)}{32\,c}+\frac{5\,b\,c^2\,x^8\,\left(3\,b^2-4\right)}{8}+\frac{5\,b\,x^2\,\left(b^2-4\right)^4}{512\,c^4}+\frac{5\,x^3\,\left(b^2-4\right)^3\,\left(9\,b^2-4\right)}{768\,c^3}+\frac{5\,b\,x^4\,\left(b^2-4\right)^2\,\left(3\,b^2-4\right)}{64\,c^2}$$

input
```
int((b*x + c*x^2 + (b^2/4 - 1)/c)^5,x)
```

output
```
(c^5*x^11)/11 + (x*(b^2 - 4)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 280*b^2 + 24
0))/48 + (5*c*x^7*(21*b^4 - 56*b^2 + 16))/56 + (b*c^4*x^10)/2 + (5*c^3*x^9
*(9*b^2 - 4))/36 + (x^5*(240*b^2 - 140*b^4 + 21*b^6 - 64))/(32*c) + (5*b*c
^2*x^8*(3*b^2 - 4))/8 + (5*b*x^2*(b^2 - 4)^4)/(512*c^4) + (5*x^3*(b^2 - 4)
^3*(9*b^2 - 4))/(768*c^3) + (5*b*x^4*(b^2 - 4)^2*(3*b^2 - 4))/(64*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.92

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{x(64512c^{10}x^{10}+354816b\,c^9x^9+887040b^2c^8x^8+1330560b^3c^7x^7+1330560b^4c^6x^6-394240c^8x^8+93139}{}$$

input
```
int((1/4*(b^2-4)/c+b*x+c*x^2)^5,x)
```

output
```
(x*(693*b**10 + 6930*b**9*c*x + 41580*b**8*c**2*x**2 - 13860*b**8 + 166320
*b**7*c**3*x**3 - 110880*b**7*c*x + 465696*b**6*c**4*x**4 - 517440*b**6*c*
*2*x**2 + 110880*b**6 + 931392*b**5*c**5*x**5 - 1552320*b**5*c**3*x**3 + 6
65280*b**5*c*x + 1330560*b**4*c**6*x**6 - 3104640*b**4*c**4*x**4 + 2217600
*b**4*c**2*x**2 - 443520*b**4 + 1330560*b**3*c**7*x**7 - 4139520*b**3*c**5
*x**5 + 4435200*b**3*c**3*x**3 - 1774080*b**3*c*x + 887040*b**2*c**8*x**8
- 3548160*b**2*c**6*x**6 + 5322240*b**2*c**4*x**4 - 3548160*b**2*c**2*x**2
 + 887040*b**2 + 354816*b*c**9*x**9 - 1774080*b*c**7*x**7 + 3548160*b*c**5
*x**5 - 3548160*b*c**3*x**3 + 1774080*b*c*x + 64512*c**10*x**10 - 394240*c
**8*x**8 + 1013760*c**6*x**6 - 1419264*c**4*x**4 + 1182720*c**2*x**2 - 709
632))/(709632*c**5)
```

3.90 $\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 \, dx = \frac{81(3-b-2cx)^6}{128c^6}-\frac{405(3-b-2cx)^7}{896c^6}+\frac{135(3-b-2cx)^8}{1024c^6}-\frac{5(3-b-2cx)^9}{256c^6}+\frac{3(3-b-2cx)^{10}}{2048c^6}-\frac{(3-b-2cx)^{11}}{22528c^6}$$

output
```
81/128*(-2*c*x-b+3)^6/c^6-405/896*(-2*c*x-b+3)^7/c^6+135/1024*(-2*c*x-b+3)
^8/c^6-5/256*(-2*c*x-b+3)^9/c^6+3/2048*(-2*c*x-b+3)^10/c^6-1/22528*(-2*c*x
-b+3)^11/c^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.82

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{(-9+b^2)^5 x}{1024c^5}+\frac{5b(-9+b^2)^4 x^2}{512c^4} + \frac{15(-9+b^2)^3(-1+b^2)x^3}{256c^3} + \frac{15b(-9+b^2)^2(-3+b^2)x^4}{64c^2} + \frac{3(-9+b^2)(27-42b^2+7b^4)x^5}{32c} + \frac{3}{16}b(135-70b^2+7b^4)x^6 + \frac{15}{56}(27-42b^2+7b^4)cx^7+\frac{15}{8}b(-3+b^2)c^2x^8 + \frac{5}{4}(-1+b^2)c^3x^9+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

input
```
Integrate[((-9 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((-9 + b^2)^5*x)/(1024*c^5) + (5*b*(-9 + b^2)^4*x^2)/(512*c^4) + (15*(-9 +
 b^2)^3*(-1 + b^2)*x^3)/(256*c^3) + (15*b*(-9 + b^2)^2*(-3 + b^2)*x^4)/(64
*c^2) + (3*(-9 + b^2)*(27 - 42*b^2 + 7*b^4)*x^5)/(32*c) + (3*b*(135 - 70*b
^2 + 7*b^4)*x^6)/16 + (15*(27 - 42*b^2 + 7*b^4)*c*x^7)/56 + (15*b*(-3 + b^
2)*c^2*x^8)/8 + (5*(-1 + b^2)*c^3*x^9)/4 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2-9}{4c}+bx+cx^2\right)^5 dx$$

$$\Big\downarrow \text{1084}$$

$$\frac{\int \left(\left(\frac{b-3}{2}+cx\right)^{10}-\frac{15}{512}(-b-2cx+3)^9+\frac{45}{128}(-b-2cx+3)^8-\frac{135}{64}(-b-2cx+3)^7+\frac{405}{64}(-b-2cx+3)^6-\frac{243}{32}\right.}{c^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(-b-2cx+3)^{11}}{22528c}+\frac{3(-b-2cx+3)^{10}}{2048c}-\frac{5(-b-2cx+3)^9}{256c}+\frac{135(-b-2cx+3)^8}{1024c}-\frac{405(-b-2cx+3)^7}{896c}+\frac{81(-b-2cx+3)^6}{128c}}{c^5}$$

input
```
Int[((-9 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((81*(3 - b - 2*c*x)^6)/(128*c) - (405*(3 - b - 2*c*x)^7)/(896*c) + (135*(
3 - b - 2*c*x)^8)/(1024*c) - (5*(3 - b - 2*c*x)^9)/(256*c) + (3*(3 - b - 2
*c*x)^10)/(2048*c) - (3 - b - 2*c*x)^11/(22528*c))/c^5
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(97) = 194$.

Time = 0.59 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.50

method	result
norman	$\frac{(\frac{5}{4}b^2c^7-\frac{5}{4}c^7)x^9+(\frac{15}{8}b^3c^6-\frac{45}{8}bc^6)x^8+(\frac{15}{8}b^4c^5-\frac{45}{4}c^5b^2+\frac{405}{56}c^5)x^7+(\frac{21}{16}b^5c^4-\frac{105}{8}b^3c^4+\frac{405}{16}bc^4)x^6+(\frac{21}{32}b^6c^3-\frac{315}{32}b^4c^3+\frac{1}{}}{}$
gosper	$\frac{x(7168c^{10}x^{10}+39424c^9bx^9+98560x^8b^2c^8+147840b^3c^7x^7+147840x^6b^4c^6-98560x^8c^8+103488x^5b^5c^5-443520bc^7x^7+51744}{}$
parallelrisch	$\frac{7168c^{10}x^{11}+39424c^9bx^{10}+98560x^9b^2c^8+147840b^3c^7x^8+147840x^7b^4c^6-98560x^9c^8+103488x^6b^5c^5-443520bc^7x^8+51744b}{}$
risch	$\frac{405bx^6}{16}-\frac{105b^3x^6}{8}-\frac{45b^2cx^7}{4}+\frac{15x^3b^8}{256c^3}-\frac{105x^3b^6}{64c^3}+\frac{15c^2b^3x^8}{8}-\frac{3645b^3x^2}{128c^4}+\frac{5b^9x^2}{512c^4}-\frac{45c^2bx^8}{8}+\frac{21b^6x^5}{32c}$
orering	$\frac{x(7168c^{10}x^{10}+39424c^9bx^9+98560x^8b^2c^8+147840b^3c^7x^7+147840x^6b^4c^6-98560x^8c^8+103488x^5b^5c^5-443520bc^7x^7+51744}{}$
default	$\frac{c^5x^{11}}{11}+\frac{bc^4x^{10}}{2}+\frac{(256(b^2-9)c^3+4096c^3b^2+4c(32(24b^2-72)c^2+1024b^2c^2))x^9}{9216}+\frac{(1024(b^2-9)c^2b+4b(32(24b^2-72)c}{}$

input
```
int((1/4*(b^2-9)/c+b*x+c*x^2)^5,x,method=_RETURNVERBOSE)
```

output
```
((5/4*b^2*c^7-5/4*c^7)*x^9+(15/8*b^3*c^6-45/8*b*c^6)*x^8+(15/8*b^4*c^5-45/
4*c^5*b^2+405/56*c^5)*x^7+(21/16*b^5*c^4-105/8*b^3*c^4+405/16*b*c^4)*x^6+(
21/32*b^6*c^3-315/32*b^4*c^3+1215/32*c^3*b^2-729/32*c^3)*x^5+(15/64*b^7*c^
2-315/64*b^5*c^2+2025/64*b^3*c^2-3645/64*b*c^2)*x^4+(5/512*b^9-45/128*b^7+
1215/256*b^5-3645/128*b^3+32805/512*b)*x^2+(15/256*b^8*c-105/64*b^6*c+2025
/128*b^4*c-3645/64*b^2*c+10935/256*c)*x^3+1/11*c^9*x^11+1/2*b*c^8*x^10+1/1
024*(b^10-45*b^8+810*b^6-7290*b^4+32805*b^2-59049)/c*x)/c^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(85) = 170$.

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.08

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{7168\,c^{10}x^{11}+39424\,bc^9x^{10}+98560\,(b^2-1)c^8x^9+147840\,(b^3-3\,b)c^7x^8+21120\,(7\,b^4-42\,b^2+27)c^6x^7}{}$$

input
```
integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

output
```
1/78848*(7168*c^10*x^11 + 39424*b*c^9*x^10 + 98560*(b^2 - 1)*c^8*x^9 + 147
840*(b^3 - 3*b)*c^7*x^8 + 21120*(7*b^4 - 42*b^2 + 27)*c^6*x^7 + 14784*(7*b
^5 - 70*b^3 + 135*b)*c^5*x^6 + 7392*(7*b^6 - 105*b^4 + 405*b^2 - 243)*c^4*
x^5 + 18480*(b^7 - 21*b^5 + 135*b^3 - 243*b)*c^3*x^4 + 4620*(b^8 - 28*b^6
+ 270*b^4 - 972*b^2 + 729)*c^2*x^3 + 770*(b^9 - 36*b^7 + 486*b^5 - 2916*b^
3 + 6561*b)*c*x^2 + 77*(b^10 - 45*b^8 + 810*b^6 - 7290*b^4 + 32805*b^2 - 5
9049)*x)/c^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(99) = 198$.

Time = 0.14 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.32

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{bc^4x^{10}}{2}+\frac{c^5x^{11}}{11}+x^9\cdot\left(\frac{5b^2c^3}{4}-\frac{5c^3}{4}\right)+x^8\cdot\left(\frac{15b^3c^2}{8}-\frac{45bc^2}{8}\right)+x^7\cdot\left(\frac{15b^4c}{8}-\frac{45b^2c}{4}+\frac{405c}{56}\right)+x^6\cdot\left(\frac{21b^5}{16}-\frac{105b^3}{8}+\frac{405b}{16}\right)+\frac{x^5\cdot(21b^6-315b^4+1215b^2-729)}{32c}+\frac{x^4\cdot(15b^7-315b^5+2025b^3-3645b)}{64c^2}+\frac{x^3\cdot(15b^8-420b^6+4050b^4-14580b^2+10935)}{256c^3}+\frac{x^2\cdot(5b^9-180b^7+2430b^5-14580b^3+32805b)}{512c^4}+\frac{x(b^{10}-45b^8+810b^6-7290b^4+32805b^2-59049)}{1024c^5}$$

input
```
integrate((1/4*(b**2-9)/c+b*x+c*x**2)**5,x)
```

output
```
b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/4) + x**8*(1
5*b**3*c**2/8 - 45*b*c**2/8) + x**7*(15*b**4*c/8 - 45*b**2*c/4 + 405*c/56)
 + x**6*(21*b**5/16 - 105*b**3/8 + 405*b/16) + x**5*(21*b**6 - 315*b**4 +
1215*b**2 - 729)/(32*c) + x**4*(15*b**7 - 315*b**5 + 2025*b**3 - 3645*b)/(
64*c**2) + x**3*(15*b**8 - 420*b**6 + 4050*b**4 - 14580*b**2 + 10935)/(256
*c**3) + x**2*(5*b**9 - 180*b**7 + 2430*b**5 - 14580*b**3 + 32805*b)/(512*
c**4) + x*(b**10 - 45*b**8 + 810*b**6 - 7290*b**4 + 32805*b**2 - 59049)/(1
024*c**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(85) = 170$.

Time = 0.03 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.15

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{1}{11}\,c^5x^{11}+\frac{1}{2}\,bc^4x^{10}+\frac{10}{9}\,b^2c^3x^9+\frac{5}{4}\,b^3c^2x^8+\frac{5}{7}\,b^4cx^7+\frac{1}{6}\,b^5x^6 + \frac{5\,(2\,cx^3+3\,bx^2)(b^2-9)^4}{1536\,c^4}+\frac{(6\,c^2x^5+15\,bcx^4+10\,b^2x^3)(b^2-9)^3}{192\,c^3} + \frac{(20\,c^3x^7+70\,bc^2x^6+84\,b^2cx^5+35\,b^3x^4)(b^2-9)^2}{224\,c^2} + \frac{(70\,c^4x^9+315\,bc^3x^8+540\,b^2c^2x^7+420\,b^3cx^6+126\,b^4x^5)(b^2-9)}{504\,c}+\frac{(b^2-9)^5x}{1024\,c^5}$$

input
```
integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

output
```
1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*
b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 9)^4/c^4 + 1/1
92*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 9)^3/c^3 + 1/224*(20*c^3*x
^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 9)^2/c^2 + 1/504*(70
*c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*
(b^2 - 9)/c + 1/1024*(b^2 - 9)^5*x/c^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(85) = 170$.

Time = 0.16 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.06

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{7168\,c^{10}x^{11} + 39424\,bc^9x^{10} + 98560\,b^2c^8x^9 + 147840\,b^3c^7x^8 + 147840\,b^4c^6x^7 - 98560\,c^8x^9 + 103488\,b^5c^5x}{}$$

input
```
integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

output
```
1/78848*(7168*c^10*x^11 + 39424*b*c^9*x^10 + 98560*b^2*c^8*x^9 + 147840*b^
3*c^7*x^8 + 147840*b^4*c^6*x^7 - 98560*c^8*x^9 + 103488*b^5*c^5*x^6 - 4435
20*b*c^7*x^8 + 51744*b^6*c^4*x^5 - 887040*b^2*c^6*x^7 + 18480*b^7*c^3*x^4
- 1034880*b^3*c^5*x^6 + 4620*b^8*c^2*x^3 - 776160*b^4*c^4*x^5 + 570240*c^6
*x^7 + 770*b^9*c*x^2 - 388080*b^5*c^3*x^4 + 1995840*b*c^5*x^6 + 77*b^10*x
- 129360*b^6*c^2*x^3 + 2993760*b^2*c^4*x^5 - 27720*b^7*c*x^2 + 2494800*b^3
*c^3*x^4 - 3465*b^8*x + 1247400*b^4*c^2*x^3 - 1796256*c^4*x^5 + 374220*b^5
*c*x^2 - 4490640*b*c^3*x^4 + 62370*b^6*x - 4490640*b^2*c^2*x^3 - 2245320*b
^3*c*x^2 - 561330*b^4*x + 3367980*c^2*x^3 + 5051970*b*c*x^2 + 2525985*b^2*
x - 4546773*x)/c^5
```

Mupad [B] (verification not implemented)

Time = 10.01 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.61

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{c^5\,x^{11}}{11} + \frac{5\,c^3\,x^9\,(b^2-1)}{4} + \frac{x\,(b^2-9)^5}{1024\,c^5} + \frac{3\,b\,x^6\,(7\,b^4-70\,b^2+135)}{16} + \frac{15\,c\,x^7\,(7\,b^4-42\,b^2+27)}{56} + \frac{b\,c^4\,x^{10}}{2} + \frac{3\,x^5\,(7\,b^6-105\,b^4+405\,b^2-243)}{32\,c} + \frac{15\,b\,c^2\,x^8\,(b^2-3)}{8} + \frac{15\,x^3\,(b^2-1)\,(b^2-9)^3}{256\,c^3} + \frac{5\,b\,x^2\,(b^2-9)^4}{512\,c^4} + \frac{15\,b\,x^4\,(b^2-3)\,(b^2-9)^2}{64\,c^2}$$

input
```
int((b*x + c*x^2 + (b^2/4 - 9/4)/c)^5,x)
```

output
```
(c^5*x^11)/11 + (5*c^3*x^9*(b^2 - 1))/4 + (x*(b^2 - 9)^5)/(1024*c^5) + (3*
b*x^6*(7*b^4 - 70*b^2 + 135))/16 + (15*c*x^7*(7*b^4 - 42*b^2 + 27))/56 + (
b*c^4*x^10)/2 + (3*x^5*(405*b^2 - 105*b^4 + 7*b^6 - 243))/(32*c) + (15*b*c
^2*x^8*(b^2 - 3))/8 + (15*x^3*(b^2 - 1)*(b^2 - 9)^3)/(256*c^3) + (5*b*x^2*
(b^2 - 9)^4)/(512*c^4) + (15*b*x^4*(b^2 - 3)*(b^2 - 9)^2)/(64*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.92

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{x(7168c^{10}x^{10}+39424b\,c^9x^9+98560b^2c^8x^8+147840b^3c^7x^7+147840b^4c^6x^6-98560c^8x^8+103488b^5c^5x^5}{}$$

input
```
int((1/4*(b^2-9)/c+b*x+c*x^2)^5,x)
```

output
```
(x*(77*b**10 + 770*b**9*c*x + 4620*b**8*c**2*x**2 - 3465*b**8 + 18480*b**7
*c**3*x**3 - 27720*b**7*c*x + 51744*b**6*c**4*x**4 - 129360*b**6*c**2*x**2
 + 62370*b**6 + 103488*b**5*c**5*x**5 - 388080*b**5*c**3*x**3 + 374220*b**
5*c*x + 147840*b**4*c**6*x**6 - 776160*b**4*c**4*x**4 + 1247400*b**4*c**2*
x**2 - 561330*b**4 + 147840*b**3*c**7*x**7 - 1034880*b**3*c**5*x**5 + 2494
800*b**3*c**3*x**3 - 2245320*b**3*c*x + 98560*b**2*c**8*x**8 - 887040*b**2
*c**6*x**6 + 2993760*b**2*c**4*x**4 - 4490640*b**2*c**2*x**2 + 2525985*b**
2 + 39424*b*c**9*x**9 - 443520*b*c**7*x**7 + 1995840*b*c**5*x**5 - 4490640
*b*c**3*x**3 + 5051970*b*c*x + 7168*c**10*x**10 - 98560*c**8*x**8 + 570240
*c**6*x**6 - 1796256*c**4*x**4 + 3367980*c**2*x**2 - 4546773))/(78848*c**5
)
```

3.91 $\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{8(4-b-2cx)^6}{3c^6}-\frac{10(4-b-2cx)^7}{7c^6}+\frac{5(4-b-2cx)^8}{16c^6}-\frac{5(4-b-2cx)^9}{144c^6}+\frac{(4-b-2cx)^{10}}{512c^6}-\frac{(4-b-2cx)^{11}}{22528c^6}$$

output
```
8/3*(-2*c*x-b+4)^6/c^6-10/7*(-2*c*x-b+4)^7/c^6+5/16*(-2*c*x-b+4)^8/c^6-5/1
44*(-2*c*x-b+4)^9/c^6+1/512*(-2*c*x-b+4)^10/c^6-1/22528*(-2*c*x-b+4)^11/c^
6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.89

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{(-16+b^2)^5 x}{1024c^5}+\frac{5b(-16+b^2)^4 x^2}{512c^4} + \frac{5(-16+b^2)^3 (-16+9b^2) x^3}{768c^3} + \frac{5b(-16+b^2)^2 (-16+3b^2) x^4}{64c^2} + \frac{(-16+b^2)(256-224b^2+21b^4) x^5}{32c} + \frac{1}{48}b(3840-1120b^2+63b^4) x^6 + \frac{5}{56}(256-224b^2+21b^4) cx^7+\frac{5}{8}b(-16+3b^2) c^2x^8 + \frac{5}{36}(-16+9b^2) c^3x^9+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

input
```
Integrate[((-16 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((-16 + b^2)^5*x)/(1024*c^5) + (5*b*(-16 + b^2)^4*x^2)/(512*c^4) + (5*(-16
 + b^2)^3*(-16 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-16 + b^2)^2*(-16 + 3*b^2)*
x^4)/(64*c^2) + ((-16 + b^2)*(256 - 224*b^2 + 21*b^4)*x^5)/(32*c) + (b*(38
40 - 1120*b^2 + 63*b^4)*x^6)/48 + (5*(256 - 224*b^2 + 21*b^4)*c*x^7)/56 +
(5*b*(-16 + 3*b^2)*c^2*x^8)/8 + (5*(-16 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10
)/2 + (c^5*x^11)/11
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2-16}{4c}+bx+cx^2\right)^5 dx$$

$$\Big\downarrow \text{1084}$$

$$\frac{\int \left(\left(\frac{b-4}{2}+cx\right)^{10}-\frac{5}{128}(-b-2cx+4)^9+\frac{5}{8}(-b-2cx+4)^8-5(-b-2cx+4)^7+20(-b-2cx+4)^6-32(-b-\right.}{c^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(-b-2cx+4)^{11}}{22528c}+\frac{(-b-2cx+4)^{10}}{512c}-\frac{5(-b-2cx+4)^9}{144c}+\frac{5(-b-2cx+4)^8}{16c}-\frac{10(-b-2cx+4)^7}{7c}+\frac{8(-b-2cx+4)^6}{3c}}{c^5}$$

input
```
Int[((-16 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

output
```
((8*(4 - b - 2*c*x)^6)/(3*c) - (10*(4 - b - 2*c*x)^7)/(7*c) + (5*(4 - b -
2*c*x)^8)/(16*c) - (5*(4 - b - 2*c*x)^9)/(144*c) + (4 - b - 2*c*x)^10/(512
*c) - (4 - b - 2*c*x)^11/(22528*c))/c^5
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(97) = 194$.

Time = 0.58 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.50

method	result
norman	$\frac{(\frac{5}{4}b^2c^7-\frac{20}{9}c^7)x^9+(\frac{15}{8}b^3c^6-10bc^6)x^8+(\frac{15}{8}b^4c^5-20c^5b^2+\frac{160}{7}c^5)x^7+(\frac{21}{16}b^5c^4-\frac{70}{3}b^3c^4+80bc^4)x^6+(\frac{21}{32}b^6c^3-\frac{35}{2}b^4c^3+120c}{}$
gosper	$\frac{x(64512c^{10}x^{10}+354816c^9bx^9+887040x^8b^2c^8+1330560b^3c^7x^7+1330560x^6b^4c^6-1576960x^8c^8+931392x^5b^5c^5-7096320bc^7}{}$
parallelrisch	$\frac{64512c^{10}x^{11}+354816c^9bx^{10}+887040x^9b^2c^8+1330560b^3c^7x^8+1330560x^7b^4c^6-1576960x^9c^8+931392x^6b^5c^5-7096320bc^7x}{}$
risch	$80bx^6-\frac{70b^3x^6}{3}-20b^2cx^7+\frac{15x^3b^8}{256c^3}-\frac{35x^3b^6}{12c^3}+\frac{15c^2b^3x^8}{8}-\frac{160b^3x^2}{c^4}+\frac{5b^9x^2}{512c^4}-10c^2bx^8+\frac{21b^6x^5}{32c}$
orering	$\frac{x(64512c^{10}x^{10}+354816c^9bx^9+887040x^8b^2c^8+1330560b^3c^7x^7+1330560x^6b^4c^6-1576960x^8c^8+931392x^5b^5c^5-7096320bc^7}{}$
default	$\frac{c^5x^{11}}{11}+\frac{bc^4x^{10}}{2}+\frac{(256(b^2-16)c^3+4096c^3b^2+4c(32(24b^2-128)c^2+1024b^2c^2))x^9}{9216}+\frac{(1024(b^2-16)c^2b+4b(32(24b^2-1}{}$

input
```
int((1/4*(b^2-16)/c+b*x+c*x^2)^5,x,method=_RETURNVERBOSE)
```

output
```
((5/4*b^2*c^7-20/9*c^7)*x^9+(15/8*b^3*c^6-10*b*c^6)*x^8+(15/8*b^4*c^5-20*c
^5*b^2+160/7*c^5)*x^7+(21/16*b^5*c^4-70/3*b^3*c^4+80*b*c^4)*x^6+(21/32*b^6
*c^3-35/2*b^4*c^3+120*c^3*b^2-128*c^3)*x^5+(15/64*b^7*c^2-35/4*b^5*c^2+100
*b^3*c^2-320*b*c^2)*x^4+(5/512*b^9-5/8*b^7+15*b^5-160*b^3+640*b)*x^2+(15/2
56*b^8*c-35/12*b^6*c+50*b^4*c-320*b^2*c+1280/3*c)*x^3+1/11*c^9*x^11+1/2*b*
c^8*x^10+1/1024*(b^10-80*b^8+2560*b^6-40960*b^4+327680*b^2-1048576)/c*x)/c
^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(85) = 170$.

Time = 0.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.16

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+98560\,(9\,b^2-16)c^8x^9+443520\,(3\,b^3-16\,b)c^7x^8+63360\,(21\,b^4-224\,b^2}{}$$

input
```
integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

output
```
1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 16)*c^8*x^9
 + 443520*(3*b^3 - 16*b)*c^7*x^8 + 63360*(21*b^4 - 224*b^2 + 256)*c^6*x^7
+ 14784*(63*b^5 - 1120*b^3 + 3840*b)*c^5*x^6 + 22176*(21*b^6 - 560*b^4 + 3
840*b^2 - 4096)*c^4*x^5 + 55440*(3*b^7 - 112*b^5 + 1280*b^3 - 4096*b)*c^3*
x^4 + 4620*(9*b^8 - 448*b^6 + 7680*b^4 - 49152*b^2 + 65536)*c^2*x^3 + 6930
*(b^9 - 64*b^7 + 1536*b^5 - 16384*b^3 + 65536*b)*c*x^2 + 693*(b^10 - 80*b^
8 + 2560*b^6 - 40960*b^4 + 327680*b^2 - 1048576)*x)/c^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(97) = 194$.

Time = 0.07 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.28

$$\begin{aligned}
&\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx \\
&= \frac{bc^4x^{10}}{2}+\frac{c^5x^{11}}{11}+x^9\cdot\left(\frac{5b^2c^3}{4}-\frac{20c^3}{9}\right)+x^8\cdot\left(\frac{15b^3c^2}{8}-10bc^2\right) \\
&\quad+x^7\cdot\left(\frac{15b^4c}{8}-20b^2c+\frac{160c}{7}\right)+x^6\cdot\left(\frac{21b^5}{16}-\frac{70b^3}{3}+80b\right) \\
&\quad+\frac{x^5\cdot(21b^6-560b^4+3840b^2-4096)}{32c}+\frac{x^4\cdot(15b^7-560b^5+6400b^3-20480b)}{64c^2} \\
&\quad+\frac{x^3\cdot(45b^8-2240b^6+38400b^4-245760b^2+327680)}{768c^3} \\
&\quad+\frac{x^2\cdot(5b^9-320b^7+7680b^5-81920b^3+327680b)}{512c^4} \\
&\quad+\frac{x(b^{10}-80b^8+2560b^6-40960b^4+327680b^2-1048576)}{1024c^5}
\end{aligned}$$

input
```
integrate((1/4*(b**2-16)/c+b*x+c*x**2)**5,x)
```

output
```
b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 20*c**3/9) + x**8*(
15*b**3*c**2/8 - 10*b*c**2) + x**7*(15*b**4*c/8 - 20*b**2*c + 160*c/7) + x
**6*(21*b**5/16 - 70*b**3/3 + 80*b) + x**5*(21*b**6 - 560*b**4 + 3840*b**2
 - 4096)/(32*c) + x**4*(15*b**7 - 560*b**5 + 6400*b**3 - 20480*b)/(64*c**2
) + x**3*(45*b**8 - 2240*b**6 + 38400*b**4 - 245760*b**2 + 327680)/(768*c*
*3) + x**2*(5*b**9 - 320*b**7 + 7680*b**5 - 81920*b**3 + 327680*b)/(512*c*
*4) + x*(b**10 - 80*b**8 + 2560*b**6 - 40960*b**4 + 327680*b**2 - 1048576)
/(1024*c**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(85) = 170$.

Time = 0.03 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.15

$$\begin{aligned}
\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx
&= \frac{1}{11}\,c^5x^{11}+\frac{1}{2}\,bc^4x^{10}+\frac{10}{9}\,b^2c^3x^9+\frac{5}{4}\,b^3c^2x^8+\frac{5}{7}\,b^4cx^7+\frac{1}{6}\,b^5x^6 \\
&\quad+\frac{5\,(2\,cx^3+3\,bx^2)(b^2-16)^4}{1536\,c^4}+\frac{(6\,c^2x^5+15\,bcx^4+10\,b^2x^3)(b^2-16)^3}{192\,c^3} \\
&\quad+\frac{(20\,c^3x^7+70\,bc^2x^6+84\,b^2cx^5+35\,b^3x^4)(b^2-16)^2}{224\,c^2} \\
&\quad+\frac{(70\,c^4x^9+315\,bc^3x^8+540\,b^2c^2x^7+420\,b^3cx^6+126\,b^4x^5)(b^2-16)}{504\,c}+\frac{(b^2-16)^5x}{1024\,c^5}
\end{aligned}$$

input
```
integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

output
```
1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*
b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 16)^4/c^4 + 1/
192*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 16)^3/c^3 + 1/224*(20*c^3
*x^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 16)^2/c^2 + 1/504*
(70*c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^
5)*(b^2 - 16)/c + 1/1024*(b^2 - 16)^5*x/c^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(85) = 170$.

Time = 0.16 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.06

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx$$
$$= \frac{64512\,c^{10}x^{11} + 354816\,bc^9x^{10} + 887040\,b^2c^8x^9 + 1330560\,b^3c^7x^8 + 1330560\,b^4c^6x^7 - 1576960\,c^8x^9 + 931\ldots}{}$$

input
```
integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

output
```
1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 13305
60*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 1576960*c^8*x^9 + 931392*b^5*c^5*x^
6 - 7096320*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 14192640*b^2*c^6*x^7 + 166320
*b^7*c^3*x^4 - 16558080*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 12418560*b^4*c^4
*x^5 + 16220160*c^6*x^7 + 6930*b^9*c*x^2 - 6209280*b^5*c^3*x^4 + 56770560*
b*c^5*x^6 + 693*b^10*x - 2069760*b^6*c^2*x^3 + 85155840*b^2*c^4*x^5 - 4435
20*b^7*c*x^2 + 70963200*b^3*c^3*x^4 - 55440*b^8*x + 35481600*b^4*c^2*x^3 -
 90832896*c^4*x^5 + 10644480*b^5*c*x^2 - 227082240*b*c^3*x^4 + 1774080*b^6
*x - 227082240*b^2*c^2*x^3 - 113541120*b^3*c*x^2 - 28385280*b^4*x + 302776
320*c^2*x^3 + 454164480*b*c*x^2 + 227082240*b^2*x - 726663168*x)/c^5
```

Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.69

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{c^5 x^{11}}{11}+\frac{x\left(b^2-16\right)^5}{1024\,c^5}+\frac{b\,x^6\left(63\,b^4-1120\,b^2+3840\right)}{48}+\frac{5\,c\,x^7\left(21\,b^4-224\,b^2+256\right)}{56}+\frac{b\,c^4\,x^{10}}{2}+\frac{5\,c^3\,x^9\left(9\,b^2-16\right)}{36}+\frac{x^5\left(21\,b^6-560\,b^4+3840\,b^2-4096\right)}{32\,c}+\frac{5\,b\,c^2\,x^8\left(3\,b^2-16\right)}{8}+\frac{5\,b\,x^2\left(b^2-16\right)^4}{512\,c^4}+\frac{5\,x^3\left(b^2-16\right)^3\left(9\,b^2-16\right)}{768\,c^3}+\frac{5\,b\,x^4\left(b^2-16\right)^2\left(3\,b^2-16\right)}{64\,c^2}$$

input
```
int((b*x + c*x^2 + (b^2/4 - 4)/c)^5,x)
```

output
```
(c^5*x^11)/11 + (x*(b^2 - 16)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 1120*b^2 +
3840))/48 + (5*c*x^7*(21*b^4 - 224*b^2 + 256))/56 + (b*c^4*x^10)/2 + (5*c^
3*x^9*(9*b^2 - 16))/36 + (x^5*(3840*b^2 - 560*b^4 + 21*b^6 - 4096))/(32*c)
 + (5*b*c^2*x^8*(3*b^2 - 16))/8 + (5*b*x^2*(b^2 - 16)^4)/(512*c^4) + (5*x^
3*(b^2 - 16)^3*(9*b^2 - 16))/(768*c^3) + (5*b*x^4*(b^2 - 16)^2*(3*b^2 - 16
))/(64*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.92

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{x(64512c^{10}x^{10}+354816b\,c^9x^9+887040b^2c^8x^8+1330560b^3c^7x^7+1330560b^4c^6x^6-1576960c^8x^8+9313}{}$$

input
```
int((1/4*(b^2-16)/c+b*x+c*x^2)^5,x)
```

output

```
(x*(693*b**10 + 6930*b**9*c*x + 41580*b**8*c**2*x**2 - 55440*b**8 + 166320
*b**7*c**3*x**3 - 443520*b**7*c*x + 465696*b**6*c**4*x**4 - 2069760*b**6*c
**2*x**2 + 1774080*b**6 + 931392*b**5*c**5*x**5 - 6209280*b**5*c**3*x**3 +
 10644480*b**5*c*x + 1330560*b**4*c**6*x**6 - 12418560*b**4*c**4*x**4 + 35
481600*b**4*c**2*x**2 - 28385280*b**4 + 1330560*b**3*c**7*x**7 - 16558080*
b**3*c**5*x**5 + 70963200*b**3*c**3*x**3 - 113541120*b**3*c*x + 887040*b**
2*c**8*x**8 - 14192640*b**2*c**6*x**6 + 85155840*b**2*c**4*x**4 - 22708224
0*b**2*c**2*x**2 + 227082240*b**2 + 354816*b*c**9*x**9 - 7096320*b*c**7*x*
*7 + 56770560*b*c**5*x**5 - 227082240*b*c**3*x**3 + 454164480*b*c*x + 6451
2*c**10*x**10 - 1576960*c**8*x**8 + 16220160*c**6*x**6 - 90832896*c**4*x**
4 + 302776320*c**2*x**2 - 726663168))/(709632*c**5)
```

3.92 $\int \frac{1}{2-x+x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{2-x+x^2} dx = -\frac{2\arctan\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

output
```
-2/7*arctan(1/7*(1-2*x)*7^(1/2))*7^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{2-x+x^2} dx = \frac{2\arctan\left(\frac{-1+2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

input
```
Integrate[(2 - x + x^2)^(-1),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2-x+2}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-(2x-1)^2-7}d(2x-1)$$

$$\Big\downarrow \text{217}$$

$$\frac{2\arctan\left(\frac{2x-1}{\sqrt{7}}\right)}{\sqrt{7}}$$

input
```
Int[(2 - x + x^2)^(-1),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2\sqrt{7}\,\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17
risch	$\frac{2\sqrt{7}\,\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17

input
```
int(1/(x^2-x+2),x,method=_RETURNVERBOSE)
```

output
```
2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2\,x-1)\right)$$

input
```
integrate(1/(x^2-x+2),x, algorithm="fricas")
```

output
```
2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

input
```
integrate(1/(x**2-x+2),x)
```

output
```
2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

input
```
integrate(1/(x^2-x+2),x, algorithm="maxima")
```

output
```
2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

input
```
integrate(1/(x^2-x+2),x, algorithm="giac")
```

output
```
2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{\sqrt{7}\,(2x-1)}{7}\right)}{7}$$

input
```
int(1/(x^2 - x + 2),x)
```

output
```
(2*7^(1/2)*atan((7^(1/2)*(2*x - 1))/7))/7
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2\sqrt{7}\,atan\left(\frac{2x-1}{\sqrt{7}}\right)}{7}$$

input
```
int(1/(x^2-x+2),x)
```

output
```
(2*sqrt(7)*atan((2*x - 1)/sqrt(7)))/7
```

3.93 $\int \frac{1}{2+4x+3x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 18

$$\int \frac{1}{2+4x+3x^2} \, dx = \frac{\arctan\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(1/2*(2+3*x)*2^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+4x+3x^2} \, dx = \frac{\arctan\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[(2 + 4*x + 3*x^2)^(-1),x]
```

output
```
ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^2+4x+2}\,dx$$

$$\Big\downarrow \text{ 1083}$$

$$-2\int \frac{1}{-(6x+4)^2-8}d(6x+4)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\arctan\left(\frac{6x+4}{2\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Int[(2 + 4*x + 3*x^2)^(-1),x]
```

output
```
ArcTan[(4 + 6*x)/(2*Sqrt[2])]/Sqrt[2]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{(6x+4)\sqrt{2}}{4}\right)}{2}$	17
risch	$\frac{\arctan\left(\frac{(3x+2)\sqrt{2}}{2}\right)\sqrt{2}}{2}$	17

input
```
int(1/(3*x^2+4*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctan(1/4*(6*x+4)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{2+4x+3x^2}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x+2)\right)$$

input
```
integrate(1/(3*x^2+4*x+2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{2+4x+3x^2}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{3\sqrt{2}x}{2}+\sqrt{2}\right)}{2}$$

input
```
integrate(1/(3*x**2+4*x+2),x)
```

output
```
sqrt(2)*atan(3*sqrt(2)*x/2 + sqrt(2))/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{2+4x+3x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right)$$

input
```
integrate(1/(3*x^2+4*x+2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{2+4x+3x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right)$$

input
```
integrate(1/(3*x^2+4*x+2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{2+4x+3x^2}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(3x+2)}{2}\right)}{2}$$

input
```
int(1/(4*x + 3*x^2 + 2),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*(3*x + 2))/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{2+4x+3x^2}\,dx = \frac{\sqrt{2}\,atan\left(\frac{3x+2}{\sqrt{2}}\right)}{2}$$

input
```
int(1/(3*x^2+4*x+2),x)
```

output
```
(sqrt(2)*atan((3*x + 2)/sqrt(2)))/2
```

3.94 $\int \frac{1}{4-2\sqrt{3}x+x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{1}{4-2\sqrt{3}x+x^2} \, dx = -\arctan\left(\sqrt{3}-x\right)$$

output
```
arctan(-3^(1/2)+x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{4-2\sqrt{3}x+x^2} \, dx = -\arctan\left(\sqrt{3}-x\right)$$

input
```
Integrate[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]
```

output
```
-ArcTan[Sqrt[3] - x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2-2\sqrt{3}x+4}\,dx$$

$$\Big\downarrow \ 1083$$

$$-2\int \frac{1}{-\left(2x-2\sqrt{3}\right)^2-4}d\left(2x-2\sqrt{3}\right)$$

$$\Big\downarrow \ 217$$

$$\arctan\left(\frac{1}{2}\left(2x-2\sqrt{3}\right)\right)$$

input
```
Int[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]
```

output
```
ArcTan[(-2*Sqrt[3] + 2*x)/2]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$\arctan\left(-\sqrt{3}+x\right)$	9
risch	$\arctan\left(-\sqrt{3}+x\right)$	9
parallelrisch	$\frac{i\ln\left(x+i-\sqrt{3}\right)}{2}-\frac{i\ln\left(x-\sqrt{3}-i\right)}{2}$	28

input
```
int(1/(4-2*x*3^(1/2)+x^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(-3^(1/2)+x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx=-\arctan\left(-x+\sqrt{3}\right)$$

input
```
integrate(1/(4-2*x*3^(1/2)+x^2),x, algorithm="fricas")
```

output
```
-arctan(-x + sqrt(3))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx=\operatorname{atan}\left(x-\sqrt{3}\right)$$

input
```
integrate(1/(4-2*x*3**(1/2)+x**2),x)
```

output
```
atan(x - sqrt(3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx = \arctan\left(x-\sqrt{3}\right)$$

input
```
integrate(1/(4-2*x*3^(1/2)+x^2),x, algorithm="maxima")
```

output
```
arctan(x - sqrt(3))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx = \arctan\left(x-\sqrt{3}\right)$$

input
```
integrate(1/(4-2*x*3^(1/2)+x^2),x, algorithm="giac")
```

output
```
arctan(x - sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx = \mathrm{atan}\left(x-\sqrt{3}\right)$$

input
```
int(1/(x^2 - 2*3^(1/2)*x + 4),x)
```

output
```
atan(x - 3^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx = -atan\left(\sqrt{3}-x\right)$$

input
```
int(1/(4-2*x*3^(1/2)+x^2),x)
```

output
```
 - atan(sqrt(3) - x)
```

3.95 $\int \frac{1}{1+\pi x+2x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{1}{1+\pi x+2x^2} dx = -\frac{2\text{arctanh}\left(\frac{\pi+4x}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

output
```
-2*arctanh((Pi+4*x)/(Pi^2-8)^(1/2))/(Pi^2-8)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\pi x+2x^2} dx = -\frac{2\text{arctanh}\left(\frac{\pi+4x}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

input
```
Integrate[(1 + Pi*x + 2*x^2)^(-1),x]
```

output
```
(-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2x^2+\pi x+1}dx$$

$$\Big\downarrow \ 1083$$

$$-2\int \frac{1}{-(4x+\pi)^2+\pi^2-8}d(4x+\pi)$$

$$\Big\downarrow \ 219$$

$$-\frac{2\text{arctanh}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

input
```
Int[(1 + Pi*x + 2*x^2)^(-1),x]
```

output
```
(-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\pi+4x}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$	24
risch	$\frac{\ln\left(-\pi^2+\pi\sqrt{\pi^2-8}+4x\sqrt{\pi^2-8}+8\right)}{\sqrt{\pi^2-8}}-\frac{\ln\left(\pi^2+\pi\sqrt{\pi^2-8}+4x\sqrt{\pi^2-8}-8\right)}{\sqrt{\pi^2-8}}$	71

input
```
int(1/(Pi*x+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((Pi+4*x)/(Pi^2-8)^(1/2))/(Pi^2-8)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(23) = 46.

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \frac{1}{1+\pi x+2x^2}\,dx = \frac{\log\left(\frac{\pi^2+4\,\pi x+8\,x^2-(\pi+4\,x)\sqrt{\pi^2-8}-4}{\pi x+2\,x^2+1}\right)}{\sqrt{\pi^2-8}}$$

input
```
integrate(1/(pi*x+2*x^2+1),x, algorithm="fricas")
```

output
```
log((pi^2 + 4*pi*x + 8*x^2 - (pi + 4*x)*sqrt(pi^2 - 8) - 4)/(pi*x + 2*x^2
+ 1))/sqrt(pi^2 - 8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.81

$$\int \frac{1}{1+\pi x+2x^2}\,dx = \frac{\log\left(x-\frac{\pi^2}{4\sqrt{-8+\pi^2}}+\frac{\pi}{4}+\frac{2}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}} - \frac{\log\left(x-\frac{2}{\sqrt{-8+\pi^2}}+\frac{\pi}{4}+\frac{\pi^2}{4\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

input
```
integrate(1/(pi*x+2*x**2+1),x)
```

output
```
log(x - pi**2/(4*sqrt(-8 + pi**2)) + pi/4 + 2/sqrt(-8 + pi**2))/sqrt(-8 +
pi**2) - log(x - 2/sqrt(-8 + pi**2) + pi/4 + pi**2/(4*sqrt(-8 + pi**2)))/s
qrt(-8 + pi**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{1}{1+\pi x+2x^2}\,dx = \frac{\log\left(\frac{\pi+4x-\sqrt{\pi^2-8}}{\pi+4x+\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

input
```
integrate(1/(pi*x+2*x^2+1),x, algorithm="maxima")
```

output
```
log((pi + 4*x - sqrt(pi^2 - 8))/(pi + 4*x + sqrt(pi^2 - 8)))/sqrt(pi^2 - 8
)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{1}{1+\pi x+2x^2}\,dx = \frac{\log\left(\frac{\left|\pi+4\,x-\sqrt{\pi^2-8}\right|}{\left|\pi+4\,x+\sqrt{\pi^2-8}\right|}\right)}{\sqrt{\pi^2-8}}$$

input
```
integrate(1/(pi*x+2*x^2+1),x, algorithm="giac")
```

output
```
log(abs(pi + 4*x - sqrt(pi^2 - 8))/abs(pi + 4*x + sqrt(pi^2 - 8)))/sqrt(pi
^2 - 8)
```

Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+\pi x+2x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\Pi+4\,x}{\sqrt{\Pi^2-8}}\right)}{\sqrt{\Pi^2-8}}$$

input
```
int(1/(Pi*x + 2*x^2 + 1),x)
```

output
```
-(2*atanh((Pi + 4*x)/(Pi^2 - 8)^(1/2)))/(Pi^2 - 8)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{1}{1+\pi x+2x^2}\,dx = -\frac{2\sqrt{-\pi^2+8}\,atan\left(\frac{\pi+4x}{\sqrt{-\pi^2+8}}\right)}{\pi^2-8}$$

input
```
int(1/(Pi*x+2*x^2+1),x)
```

output
```
( - 2*sqrt( - pi**2 + 8)*atan((pi + 4*x)/sqrt( - pi**2 + 8)))/(pi**2 - 8)
```

3.96 $\int \frac{1}{1+\pi x-2x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{1}{1+\pi x-2x^2} \, dx = -\frac{2\text{arctanh}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

output
```
-2*arctanh((Pi-4*x)/(Pi^2+8)^(1/2))/(Pi^2+8)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{1+\pi x-2x^2} \, dx = \frac{2\text{arctanh}\left(\frac{-\pi+4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

input
```
Integrate[(1 + Pi*x - 2*x^2)^(-1),x]
```

output
```
(2*ArcTanh[(-Pi + 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-2x^2+\pi x+1} \, dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-(\pi-4x)^2+\pi^2+8}d(\pi-4x)$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

input
```
Int[(1 + Pi*x - 2*x^2)^(-1),x]
```

output
```
(-2*ArcTanh[(Pi - 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{-\pi+4x}{\sqrt{\pi^2+8}}\right)}{\sqrt{\pi^2+8}}$	26
risch	$\frac{\ln\left(\pi^2-\pi\sqrt{\pi^2+8}+4x\sqrt{\pi^2+8}+8\right)}{\sqrt{\pi^2+8}}-\frac{\ln\left(-\pi^2-\pi\sqrt{\pi^2+8}+4x\sqrt{\pi^2+8}-8\right)}{\sqrt{\pi^2+8}}$	73

input
```
int(1/(Pi*x-2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2/(Pi^2+8)^(1/2)*arctanh((-Pi+4*x)/(Pi^2+8)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(23) = 46.

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int \frac{1}{1+\pi x-2x^2}\,dx=\frac{\log\left(-\frac{\pi^2-4\,\pi x+8\,x^2-(\pi-4\,x)\sqrt{\pi^2+8}+4}{\pi x-2\,x^2+1}\right)}{\sqrt{\pi^2+8}}$$

input
```
integrate(1/(pi*x-2*x^2+1),x, algorithm="fricas")
```

output
```
log(-(pi^2 - 4*pi*x + 8*x^2 - (pi - 4*x)*sqrt(pi^2 + 8) + 4)/(pi*x - 2*x^2
 + 1))/sqrt(pi^2 + 8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.81

$$\int \frac{1}{1+\pi x-2x^2}\,dx = -\frac{\log\left(x-\frac{\pi}{4}-\frac{\pi^2}{4\sqrt{8+\pi^2}}-\frac{2}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}+\frac{\log\left(x-\frac{\pi}{4}+\frac{2}{\sqrt{8+\pi^2}}+\frac{\pi^2}{4\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

input
```
integrate(1/(pi*x-2*x**2+1),x)
```

output
```
-log(x - pi/4 - pi**2/(4*sqrt(8 + pi**2)) - 2/sqrt(8 + pi**2))/sqrt(8 + pi
**2) + log(x - pi/4 + 2/sqrt(8 + pi**2) + pi**2/(4*sqrt(8 + pi**2)))/sqrt(
8 + pi**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{1}{1+\pi x-2x^2}\,dx = -\frac{\log\left(\frac{\pi-4\,x+\sqrt{\pi^2+8}}{\pi-4\,x-\sqrt{\pi^2+8}}\right)}{\sqrt{\pi^2+8}}$$

input
```
integrate(1/(pi*x-2*x^2+1),x, algorithm="maxima")
```

output
```
-log((pi - 4*x + sqrt(pi^2 + 8))/(pi - 4*x - sqrt(pi^2 + 8)))/sqrt(pi^2 +
8)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \frac{1}{1+\pi x-2x^2}\,dx = -\frac{\log\left(\frac{\left|-\pi+4\,x-\sqrt{\pi^2+8}\right|}{\left|-\pi+4\,x+\sqrt{\pi^2+8}\right|}\right)}{\sqrt{\pi^2+8}}$$

input
```
integrate(1/(pi*x-2*x^2+1),x, algorithm="giac")
```

output
```
-log(abs(-pi + 4*x - sqrt(pi^2 + 8))/abs(-pi + 4*x + sqrt(pi^2 + 8)))/sqrt
(pi^2 + 8)
```

Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+\pi x-2x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\Pi-4\,x}{\sqrt{\Pi^2+8}}\right)}{\sqrt{\Pi^2+8}}$$

input
```
int(1/(Pi*x - 2*x^2 + 1),x)
```

output
```
-(2*atanh((Pi - 4*x)/(Pi^2 + 8)^(1/2)))/(Pi^2 + 8)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{1}{1+\pi x-2x^2}\,dx = -\frac{2\sqrt{-\pi^2-8}\,atan\left(\frac{\pi-4x}{\sqrt{-\pi^2-8}}\right)}{\pi^2+8}$$

input
```
int(1/(Pi*x-2*x^2+1),x)
```

output
```
( - 2*sqrt( - pi**2 - 8)*atan((pi - 4*x)/sqrt( - pi**2 - 8)))/(pi**2 + 8)
```

3.97 $\int \frac{1}{1+\pi x+3x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{1+\pi x+3x^2}\,dx=\frac{2\arctan\left(\frac{\pi+6x}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

output
```
2*arctan((Pi+6*x)/(-Pi^2+12)^(1/2))/(-Pi^2+12)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\pi x+3x^2}\,dx=\frac{2\arctan\left(\frac{\pi+6x}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

input
```
Integrate[(1 + Pi*x + 3*x^2)^(-1),x]
```

output
```
(2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^2+\pi x+1} \, dx$$

$$\Big\downarrow \; 1083$$

$$-2 \int \frac{1}{-(6x+\pi)^2+\pi^2-12} d(6x+\pi)$$

$$\Big\downarrow \; 217$$

$$\frac{2 \arctan \left(\frac{6x+\pi}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

input
```
Int[(1 + Pi*x + 3*x^2)^(-1),x]
```

output
```
(2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2\arctan\left(\frac{\pi+6x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$	28
risch	$\frac{\ln\left(-\pi^2+\pi\sqrt{\pi^2-12}+6x\sqrt{\pi^2-12}+12\right)}{\sqrt{\pi^2-12}}-\frac{\ln\left(\pi^2+\pi\sqrt{\pi^2-12}+6x\sqrt{\pi^2-12}-12\right)}{\sqrt{\pi^2-12}}$	71

input
```
int(1/(Pi*x+3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2*arctan((Pi+6*x)/(-Pi^2+12)^(1/2))/(-Pi^2+12)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{1}{1+\pi x+3x^2}\,dx=\frac{2\sqrt{-\pi^2+12}\arctan\left(\frac{(\pi+6x)\sqrt{-\pi^2+12}}{\pi^2-12}\right)}{\pi^2-12}$$

input
```
integrate(1/(pi*x+3*x^2+1),x, algorithm="fricas")
```

output
```
2*sqrt(-pi^2 + 12)*arctan((pi + 6*x)*sqrt(-pi^2 + 12)/(pi^2 - 12))/(pi^2 -
 12)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.81

$$\int \frac{1}{1+\pi x+3x^2}\,dx=-\frac{i\log\left(x+\frac{\pi}{6}-\frac{2i}{\sqrt{12-\pi^2}}+\frac{i\pi^2}{6\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}+\frac{i\log\left(x+\frac{\pi}{6}-\frac{i\pi^2}{6\sqrt{12-\pi^2}}+\frac{2i}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

input
```
integrate(1/(pi*x+3*x**2+1),x)
```

output
```
-I*log(x + pi/6 - 2*I/sqrt(12 - pi**2) + I*pi**2/(6*sqrt(12 - pi**2)))/sqr
t(12 - pi**2) + I*log(x + pi/6 - I*pi**2/(6*sqrt(12 - pi**2)) + 2*I/sqrt(1
2 - pi**2))/sqrt(12 - pi**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{1+\pi x+3x^2}\,dx=\frac{2\arctan\left(\frac{\pi+6\,x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

input
```
integrate(1/(pi*x+3*x^2+1),x, algorithm="maxima")
```

output
```
2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{1+\pi x+3x^2}\,dx=\frac{2\arctan\left(\frac{\pi+6\,x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

input
```
integrate(1/(pi*x+3*x^2+1),x, algorithm="giac")
```

output
```
2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{1+\pi x+3x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\Pi+6\,x}{\sqrt{\Pi^2-12}}\right)}{\sqrt{\Pi^2-12}}$$

input
```
int(1/(Pi*x + 3*x^2 + 1),x)
```

output
```
-(2*atanh((Pi + 6*x)/(Pi^2 - 12)^(1/2)))/(Pi^2 - 12)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1}{1+\pi x+3x^2}\,dx = -\frac{2\sqrt{-\pi^2+12}\,atan\left(\frac{\pi+6x}{\sqrt{-\pi^2+12}}\right)}{\pi^2-12}$$

input
```
int(1/(Pi*x+3*x^2+1),x)
```

output
```
( - 2*sqrt( - pi**2 + 12)*atan((pi + 6*x)/sqrt( - pi**2 + 12)))/(pi**2 - 1
2)
```

3.98 $\int \frac{1}{1+\pi x-3x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{1}{1+\pi x-3x^2} \, dx = -\frac{2\operatorname{arctanh}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

output
```
-2*arctanh((Pi-6*x)/(Pi^2+12)^(1/2))/(Pi^2+12)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{1+\pi x-3x^2} \, dx = \frac{2\operatorname{arctanh}\left(\frac{-\pi+6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

input
```
Integrate[(1 + Pi*x - 3*x^2)^(-1),x]
```

output
```
(2*ArcTanh[(-Pi + 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-3x^2+\pi x+1}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-(\pi-6x)^2+\pi^2+12}d(\pi-6x)$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

input
```
Int[(1 + Pi*x - 3*x^2)^(-1),x]
```

output
```
(-2*ArcTanh[(Pi - 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{-\pi+6x}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$	26
risch	$\frac{\ln\left(\pi^2-\pi\sqrt{\pi^2+12}+6x\sqrt{\pi^2+12}+12\right)}{\sqrt{\pi^2+12}}-\frac{\ln\left(-\pi^2-\pi\sqrt{\pi^2+12}+6x\sqrt{\pi^2+12}-12\right)}{\sqrt{\pi^2+12}}$	73

input
```
int(1/(Pi*x-3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2/(Pi^2+12)^(1/2)*arctanh((-Pi+6*x)/(Pi^2+12)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(23) = 46$.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int \frac{1}{1+\pi x-3x^2}\,dx=\frac{\log\left(-\frac{\pi^2-6\,\pi x+18\,x^2-(\pi-6\,x)\sqrt{\pi^2+12}+6}{\pi x-3\,x^2+1}\right)}{\sqrt{\pi^2+12}}$$

input
```
integrate(1/(pi*x-3*x^2+1),x, algorithm="fricas")
```

output
```
log(-(pi^2 - 6*pi*x + 18*x^2 - (pi - 6*x)*sqrt(pi^2 + 12) + 6)/(pi*x - 3*x
^2 + 1))/sqrt(pi^2 + 12)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.81

$$\int \frac{1}{1+\pi x-3x^2}\,dx$$
$$=\frac{\log\left(x-\frac{\pi}{6}+\frac{\pi^2}{6\sqrt{\pi^2+12}}+\frac{2}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}-\frac{\log\left(x-\frac{\pi}{6}-\frac{2}{\sqrt{\pi^2+12}}-\frac{\pi^2}{6\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

input
```
integrate(1/(pi*x-3*x**2+1),x)
```

output
```
log(x - pi/6 + pi**2/(6*sqrt(pi**2 + 12)) + 2/sqrt(pi**2 + 12))/sqrt(pi**2
 + 12) - log(x - pi/6 - 2/sqrt(pi**2 + 12) - pi**2/(6*sqrt(pi**2 + 12)))/s
qrt(pi**2 + 12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{1}{1+\pi x-3x^2}\,dx=-\frac{\log\left(\frac{\pi-6\,x+\sqrt{\pi^2+12}}{\pi-6\,x-\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

input
```
integrate(1/(pi*x-3*x^2+1),x, algorithm="maxima")
```

output
```
-log((pi - 6*x + sqrt(pi^2 + 12))/(pi - 6*x - sqrt(pi^2 + 12)))/sqrt(pi^2
+ 12)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \frac{1}{1+\pi x-3x^2}\,dx = -\frac{\log\left(\frac{\left|-\pi+6\,x-\sqrt{\pi^2+12}\right|}{\left|-\pi+6\,x+\sqrt{\pi^2+12}\right|}\right)}{\sqrt{\pi^2+12}}$$

input
```
integrate(1/(pi*x-3*x^2+1),x, algorithm="giac")
```

output
```
-log(abs(-pi + 6*x - sqrt(pi^2 + 12))/abs(-pi + 6*x + sqrt(pi^2 + 12)))/sq
rt(pi^2 + 12)
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+\pi x-3x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\Pi-6\,x}{\sqrt{\Pi^2+12}}\right)}{\sqrt{\Pi^2+12}}$$

input
```
int(1/(Pi*x - 3*x^2 + 1),x)
```

output
```
-(2*atanh((Pi - 6*x)/(Pi^2 + 12)^(1/2)))/(Pi^2 + 12)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{1}{1+\pi x-3x^2}\,dx = -\frac{2\sqrt{-\pi^2-12}\,atan\left(\frac{\pi-6x}{\sqrt{-\pi^2-12}}\right)}{\pi^2+12}$$

input
```
int(1/(Pi*x-3*x^2+1),x)
```

output

```
( - 2*sqrt( - pi**2 - 12)*atan((pi - 6*x)/sqrt( - pi**2 - 12)))/(pi**2 + 1
2)
```

3.99 $\int \frac{1}{a+cx+bx^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \frac{1}{a+cx+bx^2} \, dx = \frac{2\arctan\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

output
```
2*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))/(4*a*b-c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+cx+bx^2} \, dx = \frac{2\arctan\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

input
```
Integrate[(a + c*x + b*x^2)^(-1),x]
```

output
```
(2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^2+cx}\,dx$$
$$\Big\downarrow \text{ 1083}$$
$$-2\int \frac{1}{c^2-(c+2bx)^2-4ab}d(c+2bx)$$
$$\Big\downarrow \text{ 217}$$
$$\frac{2\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

input
```
Int[(a + c*x + b*x^2)^(-1),x]
```

output
```
(2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{2\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$	35
risch	$-\frac{\ln\left(2bx+\sqrt{-4ab+c^2}+c\right)}{\sqrt{-4ab+c^2}}+\frac{\ln\left(-2bx+\sqrt{-4ab+c^2}-c\right)}{\sqrt{-4ab+c^2}}$	61

input
```
int(1/(b*x^2+c*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))/(4*a*b-c^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.97

$$\int \frac{1}{a+cx+bx^2}\,dx=\left[-\frac{\sqrt{-4\,ab+c^2}\log\left(\frac{2\,b^2x^2+2\,bcx-2\,ab+c^2-\sqrt{-4\,ab+c^2}(2\,bx+c)}{bx^2+cx+a}\right)}{4\,ab-c^2},\ -\frac{2\,\arctan\left(-\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{\sqrt{4\,ab-c^2}}\right]$$

input
```
integrate(1/(b*x^2+c*x+a),x, algorithm="fricas")
```

output
```
[-sqrt(-4*a*b + c^2)*log((2*b^2*x^2 + 2*b*c*x - 2*a*b + c^2 - sqrt(-4*a*b
+ c^2)*(2*b*x + c))/(b*x^2 + c*x + a))/(4*a*b - c^2), -2*arctan(-(2*b*x +
c)/sqrt(4*a*b - c^2))/sqrt(4*a*b - c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.26

$$\int \frac{1}{a+cx+bx^2}\,dx = -\sqrt{-\frac{1}{4ab-c^2}}\log\left(x+\frac{-4ab\sqrt{-\frac{1}{4ab-c^2}}+c^2\sqrt{-\frac{1}{4ab-c^2}}+c}{2b}\right) + \sqrt{-\frac{1}{4ab-c^2}}\log\left(x+\frac{4ab\sqrt{-\frac{1}{4ab-c^2}}-c^2\sqrt{-\frac{1}{4ab-c^2}}+c}{2b}\right)$$

input
```
integrate(1/(b*x**2+c*x+a),x)
```

output
```
-sqrt(-1/(4*a*b - c**2))*log(x + (-4*a*b*sqrt(-1/(4*a*b - c**2)) + c**2*sq
rt(-1/(4*a*b - c**2)) + c)/(2*b)) + sqrt(-1/(4*a*b - c**2))*log(x + (4*a*b
*sqrt(-1/(4*a*b - c**2)) - c**2*sqrt(-1/(4*a*b - c**2)) + c)/(2*b))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+cx+bx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+c*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-4*a*b>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{a+cx+bx^2}\,dx = \frac{2\arctan\left(\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{\sqrt{4\,ab-c^2}}$$

input
```
integrate(1/(b*x^2+c*x+a),x, algorithm="giac")
```

output
```
2*arctan((2*b*x + c)/sqrt(4*a*b - c^2))/sqrt(4*a*b - c^2)
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{1}{a+cx+bx^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{c}{\sqrt{4\,a\,b-c^2}}+\frac{2\,b\,x}{\sqrt{4\,a\,b-c^2}}\right)}{\sqrt{4\,a\,b-c^2}}$$

input
```
int(1/(a + c*x + b*x^2),x)
```

output
```
(2*atan(c/(4*a*b - c^2)^(1/2) + (2*b*x)/(4*a*b - c^2)^(1/2)))/(4*a*b - c^2
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{1}{a+cx+bx^2}\,dx = \frac{2\sqrt{4ab-c^2}\,atan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{4ab-c^2}$$

input
```
int(1/(b*x^2+c*x+a),x)
```

output
```
(2*sqrt(4*a*b - c**2)*atan((2*b*x + c)/sqrt(4*a*b - c**2)))/(4*a*b - c**2)
```

3.100 $\int \frac{1}{b+2ax+bx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1}{b+2ax+bx^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

output
```
-arctanh((b*x+a)/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{1}{b+2ax+bx^2}\,dx = \frac{\arctan\left(\frac{a+bx}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

input
```
Integrate[(b + 2*a*x + b*x^2)^(-1),x]
```

output
```
ArcTan[(a + b*x)/Sqrt[-a^2 + b^2]]/Sqrt[-a^2 + b^2]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2ax+bx^2+b}\,dx$$

$$\Big\downarrow \; 1083$$

$$-2\int \frac{1}{4\left(a^2-b^2\right)-(2a+2bx)^2}d(2a+2bx)$$

$$\Big\downarrow \; 219$$

$$-\frac{\text{arctanh}\left(\frac{2a+2bx}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
Int[(b + 2*a*x + b*x^2)^(-1),x]
```

output
```
-(ArcTanh[(2*a + 2*b*x)/(2*Sqrt[a^2 - b^2])]/Sqrt[a^2 - b^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2bx+2a}{2\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	35
risch	$\frac{\ln\left(-bx+\sqrt{a^2-b^2}-a\right)}{2\sqrt{a^2-b^2}}-\frac{\ln\left(bx+\sqrt{a^2-b^2}+a\right)}{2\sqrt{a^2-b^2}}$	65

input
```
int(1/(b*x^2+2*a*x+b),x,method=_RETURNVERBOSE)
```

output
```
1/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.54

$$\int \frac{1}{b+2ax+bx^2}\,dx = \left[\frac{\log\left(\frac{b^2x^2+2\,abx+2\,a^2-b^2-2\,\sqrt{a^2-b^2}(bx+a)}{bx^2+2\,ax+b}\right)}{2\,\sqrt{a^2-b^2}}, -\frac{\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(bx+a)}{a^2-b^2}\right)}{a^2-b^2}\right]$$

input
```
integrate(1/(b*x^2+2*a*x+b),x, algorithm="fricas")
```

output
```
[1/2*log((b^2*x^2 + 2*a*b*x + 2*a^2 - b^2 - 2*sqrt(a^2 - b^2)*(b*x + a))/(
b*x^2 + 2*a*x + b))/sqrt(a^2 - b^2), -sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
 b^2)*(b*x + a)/(a^2 - b^2))/(a^2 - b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(27) = 54$.

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.86

$$\int \frac{1}{b+2ax+bx^2}\,dx = \frac{\sqrt{\frac{1}{(a-b)(a+b)}}\log\left(x+\frac{-a^2\sqrt{\frac{1}{(a-b)(a+b)}}+a+b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2} - \frac{\sqrt{\frac{1}{(a-b)(a+b)}}\log\left(x+\frac{a^2\sqrt{\frac{1}{(a-b)(a+b)}}+a-b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2}$$

input
```
integrate(1/(b*x**2+2*a*x+b),x)
```

output
```
sqrt(1/((a - b)*(a + b)))*log(x + (-a**2*sqrt(1/((a - b)*(a + b))) + a + b
**2*sqrt(1/((a - b)*(a + b))))/b)/2 - sqrt(1/((a - b)*(a + b)))*log(x + (a
**2*sqrt(1/((a - b)*(a + b))) + a - b**2*sqrt(1/((a - b)*(a + b))))/b)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{b+2ax+bx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+2*a*x+b),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{1}{b+2ax+bx^2}\,dx = \frac{\arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

input
```
integrate(1/(b*x^2+2*a*x+b),x, algorithm="giac")
```

output
```
arctan((b*x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{b+2ax+bx^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{a+b\,x}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(1/(b + 2*a*x + b*x^2),x)
```

output
```
-atanh((a + b*x)/((a + b)^(1/2)*(a - b)^(1/2)))/((a + b)^(1/2)*(a - b)^(1/
2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{1}{b+2ax+bx^2}\,dx = -\frac{\sqrt{-a^2+b^2}\,atan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{a^2-b^2}$$

input
```
int(1/(b*x^2+2*a*x+b),x)
```

output
```
( - sqrt( - a**2 + b**2)*atan((a + b*x)/sqrt( - a**2 + b**2)))/(a**2 - b**
2)
```

3.101 $\int \frac{1}{b+2ax-bx^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \frac{1}{b+2ax-bx^2} \, dx = -\frac{\operatorname{arctanh}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

output
```
-arctanh((-b*x+a)/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{1}{b+2ax-bx^2} \, dx = -\frac{\arctan\left(\frac{-a+bx}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

input
```
Integrate[(b + 2*a*x - b*x^2)^(-1),x]
```

output
```
-(ArcTan[(-a + b*x)/Sqrt[-a^2 - b^2]]/Sqrt[-a^2 - b^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2ax - bx^2 + b} \, dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{4(a^2+b^2) - (2a-2bx)^2} d(2a - 2bx)$$

$$\Big\downarrow \text{219}$$

$$-\frac{\operatorname{arctanh}\left(\frac{2a-2bx}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
Int[(b + 2*a*x - b*x^2)^(-1),x]
```

output
```
-(ArcTanh[(2*a - 2*b*x)/(2*Sqrt[a^2 + b^2])]/Sqrt[a^2 + b^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{-2bx+2a}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$	32
risch	$\frac{\ln\left(bx+\sqrt{a^2+b^2}-a\right)}{2\sqrt{a^2+b^2}}-\frac{\ln\left(-bx+\sqrt{a^2+b^2}+a\right)}{2\sqrt{a^2+b^2}}$	57

input
```
int(1/(-b*x^2+2*a*x+b),x,method=_RETURNVERBOSE)
```

output
```
-1/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*x+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.09

$$\int \frac{1}{b+2ax-bx^2}\,dx=\frac{\log\left(\frac{b^2x^2-2\,abx+2\,a^2+b^2+2\,\sqrt{a^2+b^2}(bx-a)}{bx^2-2\,ax-b}\right)}{2\,\sqrt{a^2+b^2}}$$

input
```
integrate(1/(-b*x^2+2*a*x+b),x, algorithm="fricas")
```

output
```
1/2*log((b^2*x^2 - 2*a*b*x + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*(b*x - a))/(b
*x^2 - 2*a*x - b))/sqrt(a^2 + b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(27) = 54$.

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.19

$$\int \frac{1}{b+2ax-bx^2}\,dx = -\frac{\sqrt{\frac{1}{a^2+b^2}}\log\left(x+\frac{-a^2\sqrt{\frac{1}{a^2+b^2}}-a-b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2} + \frac{\sqrt{\frac{1}{a^2+b^2}}\log\left(x+\frac{a^2\sqrt{\frac{1}{a^2+b^2}}-a+b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2}$$

input
```
integrate(1/(-b*x**2+2*a*x+b),x)
```

output
```
-sqrt(1/(a**2 + b**2))*log(x + (-a**2*sqrt(1/(a**2 + b**2)) - a - b**2*sqr
t(1/(a**2 + b**2)))/b)/2 + sqrt(1/(a**2 + b**2))*log(x + (a**2*sqrt(1/(a**
2 + b**2)) - a + b**2*sqrt(1/(a**2 + b**2)))/b)/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.53

$$\int \frac{1}{b+2ax-bx^2}\,dx = -\frac{\log\left(\frac{bx-a-\sqrt{a^2+b^2}}{bx-a+\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}$$

input
```
integrate(1/(-b*x^2+2*a*x+b),x, algorithm="maxima")
```

output
```
-1/2*log((b*x - a - sqrt(a^2 + b^2))/(b*x - a + sqrt(a^2 + b^2)))/sqrt(a^2
 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{1}{b+2ax-bx^2}\,dx = -\frac{\log\left(\frac{\left|2\,bx-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,bx-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,\sqrt{a^2+b^2}}$$

input
```
integrate(1/(-b*x^2+2*a*x+b),x, algorithm="giac")
```

output
```
-1/2*log(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a + 2*sqrt(a^2
 + b^2)))/sqrt(a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{b+2ax-bx^2}\,dx = -\frac{\mathrm{atanh}\left(\frac{a-b\,x}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
int(1/(b + 2*a*x - b*x^2),x)
```

output
```
-atanh((a - b*x)/(a^2 + b^2)^(1/2))/(a^2 + b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{1}{b+2ax-bx^2}\,dx = \frac{\sqrt{a^2+b^2}\,atan\left(\frac{-bix+ai}{\sqrt{a^2+b^2}}\right)i}{a^2+b^2}$$

input
```
int(1/(-b*x^2+2*a*x+b),x)
```

output
```
(sqrt(a**2 + b**2)*atan((a*i - b*i*x)/sqrt(a**2 + b**2))*i)/(a**2 + b**2)
```

3.102 $\int \frac{1}{\left(2+4x+3x^2\right)^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{\left(2+4x+3x^2\right)^2}\,dx=\frac{2+3x}{4\left(2+4x+3x^2\right)}+\frac{3\arctan\left(\frac{2+3x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

output
```
(2+3*x)/(12*x^2+16*x+8)+3/8*arctan(1/2*(2+3*x)*2^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(2+4x+3x^2\right)^2}\,dx=\frac{2+3x}{4\left(2+4x+3x^2\right)}+\frac{3\arctan\left(\frac{2+3x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

input
```
Integrate[(2 + 4*x + 3*x^2)^(-2),x]
```

output
```
(2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(4*Sqrt[2]
)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2+4x+2)^2}dx$$

$$\Big\downarrow \ 1086$$

$$\frac{3}{4}\int \frac{1}{3x^2+4x+2}dx+\frac{3x+2}{4(3x^2+4x+2)}$$

$$\Big\downarrow \ 1083$$

$$\frac{3x+2}{4(3x^2+4x+2)}-\frac{3}{2}\int \frac{1}{-(6x+4)^2-8}d(6x+4)$$

$$\Big\downarrow \ 217$$

$$\frac{3\arctan\left(\frac{6x+4}{2\sqrt{2}}\right)}{4\sqrt{2}}+\frac{3x+2}{4(3x^2+4x+2)}$$

input
```
Int[(2 + 4*x + 3*x^2)^(-2),x]
```

output
```
(2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(4 + 6*x)/(2*Sqrt[2])])/(4*Sqr
t[2])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{\frac{x}{4}+\frac{1}{6}}{x^2+\frac{4}{3}x+\frac{2}{3}}+\frac{3\arctan\left(\frac{(3x+2)\sqrt{2}}{2}\right)\sqrt{2}}{8}$	34
default	$\frac{6x+4}{24x^2+32x+16}+\frac{3\sqrt{2}\,\arctan\left(\frac{(6x+4)\sqrt{2}}{4}\right)}{8}$	37

input
```
int(1/(3*x^2+4*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*x+1/6)/(x^2+4/3*x+2/3)+3/8*arctan(1/2*(3*x+2)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx = \frac{3\,\sqrt{2}(3\,x^2+4\,x+2)\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x+2)\right)+6\,x+4}{8\,(3\,x^2+4\,x+2)}$$

input
```
integrate(1/(3*x^2+4*x+2)^2,x, algorithm="fricas")
```

output
```
1/8*(3*sqrt(2)*(3*x^2 + 4*x + 2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 6*x + 4)/
(3*x^2 + 4*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx = \frac{3x+2}{12x^2+16x+8} + \frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{3\sqrt{2}x}{2}+\sqrt{2}\right)}{8}$$

input
```
integrate(1/(3*x**2+4*x+2)**2,x)
```

output
```
(3*x + 2)/(12*x**2 + 16*x + 8) + 3*sqrt(2)*atan(3*sqrt(2)*x/2 + sqrt(2))/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx = \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right) + \frac{3x+2}{4(3x^2+4x+2)}$$

input
```
integrate(1/(3*x^2+4*x+2)^2,x, algorithm="maxima")
```

output
```
3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2 + 4*x + 2
)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx = \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right) + \frac{3x+2}{4(3x^2+4x+2)}$$

input
```
integrate(1/(3*x^2+4*x+2)^2,x, algorithm="giac")
```

output
```
3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2 + 4*x + 2
)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx = \frac{\frac{x}{4}+\frac{1}{6}}{x^2+\frac{4x}{3}+\frac{2}{3}} + \frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{3\sqrt{2}\,x}{2}+\sqrt{2}\right)}{8}$$

input
```
int(1/(4*x + 3*x^2 + 2)^2,x)
```

output
```
(x/4 + 1/6)/((4*x)/3 + x^2 + 2/3) + (3*2^(1/2)*atan((3*2^(1/2)*x)/2 + 2^(1
/2)))/8
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx = \frac{18\sqrt{2}\,atan\left(\frac{3x+2}{\sqrt{2}}\right)x^2+24\sqrt{2}\,atan\left(\frac{3x+2}{\sqrt{2}}\right)x+12\sqrt{2}\,atan\left(\frac{3x+2}{\sqrt{2}}\right)-9x^2+2}{48x^2+64x+32}$$

input
```
int(1/(3*x^2+4*x+2)^2,x)
```

output
```
(18*sqrt(2)*atan((3*x + 2)/sqrt(2))*x**2 + 24*sqrt(2)*atan((3*x + 2)/sqrt(
2))*x + 12*sqrt(2)*atan((3*x + 2)/sqrt(2)) - 9*x**2 + 2)/(16*(3*x**2 + 4*x
 + 2))
```

3.103 $\int \frac{1}{(a+cx+bx^2)^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 71

$$\int \frac{1}{(a+cx+bx^2)^2} \, dx = \frac{c+2bx}{(4ab-c^2)(a+cx+bx^2)} + \frac{4b \arctan\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}$$

output
```
(2*b*x+c)/(4*a*b-c^2)/(b*x^2+c*x+a)+4*b*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2)
)/(4*a*b-c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+cx+bx^2)^2} \, dx = \frac{c+2bx}{(4ab-c^2)(a+x(c+bx))} + \frac{4b \arctan\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}$$

input
```
Integrate[(a + c*x + b*x^2)^(-2),x]
```

output
```
(c + 2*b*x)/((4*a*b - c^2)*(a + x*(c + b*x))) + (4*b*ArcTan[(c + 2*b*x)/Sq
rt[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2+cx)^2}dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{2b\int \frac{1}{bx^2+cx+a}dx}{4ab-c^2}+\frac{2bx+c}{(4ab-c^2)(a+bx^2+cx)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{2bx+c}{(4ab-c^2)(a+bx^2+cx)}-\frac{4b\int \frac{1}{c^2-(c+2bx)^2-4ab}d(c+2bx)}{4ab-c^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{4b\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}+\frac{2bx+c}{(4ab-c^2)(a+bx^2+cx)}$$

input
```
Int[(a + c*x + b*x^2)^(-2),x]
```

output
```
(c + 2*b*x)/((4*a*b - c^2)*(a + c*x + b*x^2)) + (4*b*ArcTan[(c + 2*b*x)/Sq
rt[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

method	result	s
default	$\frac{2bx+c}{(4ab-c^2)(b\,x^2+cx+a)}+\frac{4b\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{\frac{3}{2}}}$	6
risch	$\frac{\frac{2bx}{4ab-c^2}+\frac{c}{4ab-c^2}}{b\,x^2+cx+a}+\frac{2b\ln\left(\left(-8a\,b^2+2b\,c^2\right)x+\left(-4ab+c^2\right)^{\frac{3}{2}}-4abc+c^3\right)}{\left(-4ab+c^2\right)^{\frac{3}{2}}}-\frac{2b\ln\left(\left(8a\,b^2-2b\,c^2\right)x+\left(-4ab+c^2\right)^{\frac{3}{2}}+4abc-c^3\right)}{\left(-4ab+c^2\right)^{\frac{3}{2}}}$	1

input
```
int(1/(b*x^2+c*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(2*b*x+c)/(4*a*b-c^2)/(b*x^2+c*x+a)+4*b*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2)
)/(4*a*b-c^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(67) = 134$.

Time = 0.08 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.70

$$\int \frac{1}{(a+cx+bx^2)^2}\,dx$$
$$= \left[\frac{4\,abc - c^3 + 2\,(b^2x^2 + bcx + ab)\sqrt{-4\,ab + c^2}\log\left(\frac{2\,b^2x^2 + 2\,bcx - 2\,ab + c^2 + \sqrt{-4\,ab + c^2}(2\,bx + c)}{bx^2 + cx + a}\right) + 2\,(4\,ab^2 - bc^2)x}{16\,a^3b^2 - 8\,a^2bc^2 + ac^4 + (16\,a^2b^3 - 8\,ab^2c^2 + bc^4)x^2 + (16\,a^2b^2c - 8\,abc^3 + c^5)x} \right.$$

input
```
integrate(1/(b*x^2+c*x+a)^2,x, algorithm="fricas")
```

output
```
[(4*a*b*c - c^3 + 2*(b^2*x^2 + b*c*x + a*b)*sqrt(-4*a*b + c^2)*log((2*b^2*
x^2 + 2*b*c*x - 2*a*b + c^2 + sqrt(-4*a*b + c^2)*(2*b*x + c))/(b*x^2 + c*x
 + a)) + 2*(4*a*b^2 - b*c^2)*x)/(16*a^3*b^2 - 8*a^2*b*c^2 + a*c^4 + (16*a^
2*b^3 - 8*a*b^2*c^2 + b*c^4)*x^2 + (16*a^2*b^2*c - 8*a*b*c^3 + c^5)*x), (4
*a*b*c - c^3 - 4*(b^2*x^2 + b*c*x + a*b)*sqrt(4*a*b - c^2)*arctan(-(2*b*x
+ c)/sqrt(4*a*b - c^2)) + 2*(4*a*b^2 - b*c^2)*x)/(16*a^3*b^2 - 8*a^2*b*c^2
 + a*c^4 + (16*a^2*b^3 - 8*a*b^2*c^2 + b*c^4)*x^2 + (16*a^2*b^2*c - 8*a*b*
c^3 + c^5)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(60) = 120$.

Time = 0.30 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.73

$$\int \frac{1}{(a+cx+bx^2)^2}\,dx =$$
$$-2b\sqrt{-\frac{1}{(4ab-c^2)^3}}\log\left(x + \frac{-32a^2b^3\sqrt{-\frac{1}{(4ab-c^2)^3}} + 16ab^2c^2\sqrt{-\frac{1}{(4ab-c^2)^3}} - 2bc^4\sqrt{-\frac{1}{(4ab-c^2)^3}} + 2bc}{4b^2}\right)$$
$$+2b\sqrt{-\frac{1}{(4ab-c^2)^3}}\log\left(x + \frac{32a^2b^3\sqrt{-\frac{1}{(4ab-c^2)^3}} - 16ab^2c^2\sqrt{-\frac{1}{(4ab-c^2)^3}} + 2bc^4\sqrt{-\frac{1}{(4ab-c^2)^3}} + 2bc}{4b^2}\right)$$
$$+ \frac{2bx + c}{4a^2b - ac^2 + x^2\cdot(4ab^2 - bc^2) + x\,(4abc - c^3)}$$

input
```
integrate(1/(b*x**2+c*x+a)**2,x)
```

output
```
-2*b*sqrt(-1/(4*a*b - c**2)**3)*log(x + (-32*a**2*b**3*sqrt(-1/(4*a*b - c*
*2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*b - c**2)**3) - 2*b*c**4*sqrt(-1/(4*
a*b - c**2)**3) + 2*b*c)/(4*b**2)) + 2*b*sqrt(-1/(4*a*b - c**2)**3)*log(x
+ (32*a**2*b**3*sqrt(-1/(4*a*b - c**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*b
 - c**2)**3) + 2*b*c**4*sqrt(-1/(4*a*b - c**2)**3) + 2*b*c)/(4*b**2)) + (2
*b*x + c)/(4*a**2*b - a*c**2 + x**2*(4*a*b**2 - b*c**2) + x*(4*a*b*c - c**
3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+cx+bx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+c*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-4*a*b>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+cx+bx^2)^2}\,dx = \frac{4\,b\arctan\left(\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{(4\,ab-c^2)^{\frac{3}{2}}} + \frac{2\,bx+c}{(bx^2+cx+a)(4\,ab-c^2)}$$

input
```
integrate(1/(b*x^2+c*x+a)^2,x, algorithm="giac")
```

output
```
4*b*arctan((2*b*x + c)/sqrt(4*a*b - c^2))/(4*a*b - c^2)^(3/2) + (2*b*x + c
)/((b*x^2 + c*x + a)*(4*a*b - c^2))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.68

$$\int \frac{1}{(a+cx+bx^2)^2}\,dx = \frac{\frac{c}{4\,a\,b-c^2}+\frac{2\,b\,x}{4\,a\,b-c^2}}{b\,x^2+c\,x+a}-\frac{4\,b\,\mathrm{atan}\left(\frac{\left(\frac{2\,b\,\left(c^3-4\,a\,b\,c\right)}{{\left(4\,a\,b-c^2\right)}^{5/2}}-\frac{4\,b^2\,x}{{\left(4\,a\,b-c^2\right)}^{3/2}}\right)\,\left(4\,a\,b-c^2\right)}{2\,b}\right)}{{\left(4\,a\,b-c^2\right)}^{3/2}}$$

input
```
int(1/(a + c*x + b*x^2)^2,x)
```

output
```
(c/(4*a*b - c^2) + (2*b*x)/(4*a*b - c^2))/(a + c*x + b*x^2) - (4*b*atan(((
(2*b*(c^3 - 4*a*b*c))/(4*a*b - c^2)^(5/2) - (4*b^2*x)/(4*a*b - c^2)^(3/2))
*(4*a*b - c^2))/(2*b)))/(4*a*b - c^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.39

$$\int \frac{1}{(a+cx+bx^2)^2}\,dx = \frac{4\sqrt{4ab-c^2}\,atan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)abc+4\sqrt{4ab-c^2}\,atan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)b^2c\,x^2+4\sqrt{4ab-c^2}\,atan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)b\,c^2x-}{c\,(16a^2b^3x^2-8a\,b^2c^2x^2+b\,c^4x^2+16a^2b^2cx-8ab\,c^3x+c^5x+16a^3b^2-8a^2}$$

input
```
int(1/(b*x^2+c*x+a)^2,x)
```

output

```
(4*sqrt(4*a*b - c**2)*atan((2*b*x + c)/sqrt(4*a*b - c**2))*a*b*c + 4*sqrt(
4*a*b - c**2)*atan((2*b*x + c)/sqrt(4*a*b - c**2))*b**2*c*x**2 + 4*sqrt(4*
a*b - c**2)*atan((2*b*x + c)/sqrt(4*a*b - c**2))*b*c**2*x - 8*a**2*b**2 -
8*a*b**3*x**2 + 6*a*b*c**2 + 2*b**2*c**2*x**2 - c**4)/(c*(16*a**3*b**2 + 1
6*a**2*b**3*x**2 + 16*a**2*b**2*c*x - 8*a**2*b*c**2 - 8*a*b**2*c**2*x**2 -
 8*a*b*c**3*x + a*c**4 + b*c**4*x**2 + c**5*x))
```

3.104 $\int \frac{1}{(b+2ax+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{1}{(b+2ax+bx^2)^2} dx = -\frac{a+bx}{2(a^2-b^2)(b+2ax+bx^2)} + \frac{b\,\text{arctanh}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}}$$

output
```
-1/2*(b*x+a)/(a^2-b^2)/(b*x^2+2*a*x+b)+1/2*b*arctanh((b*x+a)/(a^2-b^2)^(1/
2))/(a^2-b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{1}{(b+2ax+bx^2)^2} dx = \frac{a+bx}{2(-a^2+b^2)(b+2ax+bx^2)} + \frac{b\arctan\left(\frac{a+bx}{\sqrt{-a^2+b^2}}\right)}{2(-a^2+b^2)^{3/2}}$$

input
```
Integrate[(b + 2*a*x + b*x^2)^(-2),x]
```

output
```
(a + b*x)/(2*(-a^2 + b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTan[(a + b*x)/Sqrt[
-a^2 + b^2]])/(2*(-a^2 + b^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2ax+bx^2+b)^2}dx$$

$$\Big\downarrow \text{ 1086}$$

$$-\frac{b\int \frac{1}{bx^2+2ax+b}dx}{2(a^2-b^2)}-\frac{a+bx}{2(a^2-b^2)(2ax+bx^2+b)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{b\int \frac{1}{4(a^2-b^2)-(2a+2bx)^2}d(2a+2bx)}{a^2-b^2}-\frac{a+bx}{2(a^2-b^2)(2ax+bx^2+b)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{b\text{arctanh}\left(\frac{2a+2bx}{2\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}}-\frac{a+bx}{2(a^2-b^2)(2ax+bx^2+b)}$$

input
```
Int[(b + 2*a*x + b*x^2)^(-2),x]
```

output
```
-1/2*(a + b*x)/((a^2 - b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTanh[(2*a + 2*b*x
)/(2*Sqrt[a^2 - b^2])])/(2*(a^2 - b^2)^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{2bx+2a}{(-4a^2+4b^2)(b\,x^2+2ax+b)}+\frac{2b\arctan\left(\frac{2bx+2a}{2\sqrt{-a^2+b^2}}\right)}{(-4a^2+4b^2)\sqrt{-a^2+b^2}}$	86
risch	$\frac{-\frac{bx}{4\left(a^2-b^2\right)}-\frac{a}{4\left(a^2-b^2\right)}}{\frac{1}{2}b\,x^2+ax+\frac{1}{2}b}+\frac{b\ln\left(\left(-a^2b+b^3\right)x-\left(a^2-b^2\right)^{\frac{3}{2}}-a^3+a\,b^2\right)}{4\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{b\ln\left(\left(a^2b-b^3\right)x-\left(a^2-b^2\right)^{\frac{3}{2}}+a^3-a\,b^2\right)}{4\left(a^2-b^2\right)^{\frac{3}{2}}}$	150

input
```
int(1/(b*x^2+2*a*x+b)^2,x,method=_RETURNVERBOSE)
```

output
```
(2*b*x+2*a)/(-4*a^2+4*b^2)/(b*x^2+2*a*x+b)+2*b/(-4*a^2+4*b^2)/(-a^2+b^2)^(
1/2)*arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(64) = 128$.

Time = 0.09 (sec) , antiderivative size = 317, normalized size of antiderivative = 4.40

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx$$
$$= \left[-\frac{2\,a^3-2\,ab^2+(b^2x^2+2\,abx+b^2)\sqrt{a^2-b^2}\log\left(\frac{b^2x^2+2\,abx+2\,a^2-b^2-2\,\sqrt{a^2-b^2}(bx+a)}{bx^2+2\,ax+b}\right)+2\,(a^2b-b^3)x}{4\,(a^4b-2\,a^2b^3+b^5+(a^4b-2\,a^2b^3+b^5)x^2+2\,(a^5-2\,a^3b^2+ab^4)x)},\right.$$
$$\left.-\frac{a^3-ab^2-(b^2x^2+2\,abx+b^2)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(bx+a)}{a^2-b^2}\right)+(a^2b-b^3)x}{2\,(a^4b-2\,a^2b^3+b^5+(a^4b-2\,a^2b^3+b^5)x^2+2\,(a^5-2\,a^3b^2+ab^4)x)}\right]$$

input
```
integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a^3 - 2*a*b^2 + (b^2*x^2 + 2*a*b*x + b^2)*sqrt(a^2 - b^2)*log((b^
2*x^2 + 2*a*b*x + 2*a^2 - b^2 - 2*sqrt(a^2 - b^2)*(b*x + a))/(b*x^2 + 2*a*
x + b)) + 2*(a^2*b - b^3)*x)/(a^4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3
 + b^5)*x^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x), -1/2*(a^3 - a*b^2 - (b^2*x^2
 + 2*a*b*x + b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*x + a)/(a^2
 - b^2)) + (a^2*b - b^3)*x)/(a^4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3
+ b^5)*x^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(58) = 116$.

Time = 0.26 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.19

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx = -\frac{b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}\log\left(x+\frac{-a^4b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+2a^2b^3\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+ab-b^5\sqrt{\frac{1}{(a-b)^3(a+b)^3}}}{b^2}\right)}{4} + \frac{b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}\log\left(x+\frac{a^4b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}-2a^2b^3\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+ab+b^5\sqrt{\frac{1}{(a-b)^3(a+b)^3}}}{b^2}\right)}{4} + \frac{-a-bx}{2a^2b-2b^3+x^2\cdot(2a^2b-2b^3)+x(4a^3-4ab^2)}$$

input
```
integrate(1/(b*x**2+2*a*x+b)**2,x)
```

output
```
-b*sqrt(1/((a - b)**3*(a + b)**3))*log(x + (-a**4*b*sqrt(1/((a - b)**3*(a
+ b)**3)) + 2*a**2*b**3*sqrt(1/((a - b)**3*(a + b)**3)) + a*b - b**5*sqrt(
1/((a - b)**3*(a + b)**3)))/b**2)/4 + b*sqrt(1/((a - b)**3*(a + b)**3))*lo
g(x + (a**4*b*sqrt(1/((a - b)**3*(a + b)**3)) - 2*a**2*b**3*sqrt(1/((a - b
)**3*(a + b)**3)) + a*b + b**5*sqrt(1/((a - b)**3*(a + b)**3)))/b**2)/4 +
(-a - b*x)/(2*a**2*b - 2*b**3 + x**2*(2*a**2*b - 2*b**3) + x*(4*a**3 - 4*a
*b**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx = -\frac{b\arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{2\,(a^2-b^2)\sqrt{-a^2+b^2}} - \frac{bx+a}{2\,(bx^2+2\,ax+b)(a^2-b^2)}$$

input
```
integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="giac")
```

output
```
-1/2*b*arctan((b*x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2)) -
 1/2*(b*x + a)/((b*x^2 + 2*a*x + b)*(a^2 - b^2))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.49

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx = -\frac{\frac{a}{2\,(a^2-b^2)}+\frac{b\,x}{2\,(a^2-b^2)}}{b\,x^2+2\,a\,x+b} + \frac{b\,\mathrm{atan}\left(\frac{-a^3\,1\mathrm{i}-1\mathrm{i}\,x\,a^2\,b+a\,b^2\,1\mathrm{i}+1\mathrm{i}\,x\,b^3}{(a+b)^{3/2}\,(a-b)^{3/2}}\right)\,1\mathrm{i}}{2\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int(1/(b + 2*a*x + b*x^2)^2,x)
```

output
```
(b*atan((a*b^2*1i + b^3*x*1i - a^3*1i - a^2*b*x*1i)/((a + b)^(3/2)*(a - b)
^(3/2)))*1i)/(2*(a + b)^(3/2)*(a - b)^(3/2)) - (a/(2*(a^2 - b^2)) + (b*x)/
(2*(a^2 - b^2)))/(b + 2*a*x + b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.03

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx$$
$$= \frac{4\sqrt{-a^2+b^2}\,atan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)a^2bx+2\sqrt{-a^2+b^2}\,atan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)a\,b^2x^2+2\sqrt{-a^2+b^2}\,atan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)a\,b}{4a\left(a^4b\,x^2-2a^2b^3x^2+b^5x^2+2a^5x-4a^3b^2x+2a\,b^4x+a^4b-2a^2b^3+\right.}$$

input
```
int(1/(b*x^2+2*a*x+b)^2,x)
```

output
```
(4*sqrt( - a**2 + b**2)*atan((a + b*x)/sqrt( - a**2 + b**2))*a**2*b*x + 2*
sqrt( - a**2 + b**2)*atan((a + b*x)/sqrt( - a**2 + b**2))*a*b**2*x**2 + 2*
sqrt( - a**2 + b**2)*atan((a + b*x)/sqrt( - a**2 + b**2))*a*b**2 - 2*a**4
+ a**2*b**2*x**2 + 3*a**2*b**2 - b**4*x**2 - b**4)/(4*a*(2*a**5*x + a**4*b
*x**2 + a**4*b - 4*a**3*b**2*x - 2*a**2*b**3*x**2 - 2*a**2*b**3 + 2*a*b**4
*x + b**5*x**2 + b**5))
```

3.105 $\int \frac{1}{(b+2ax-bx^2)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx = -\frac{a-bx}{2(a^2+b^2)(b+2ax-bx^2)} - \frac{b\text{arctanh}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}$$

output
```
-1/2*(-b*x+a)/(a^2+b^2)/(-b*x^2+2*a*x+b)-1/2*b*arctanh((-b*x+a)/(a^2+b^2)^
(1/2))/(a^2+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx = \frac{\frac{-a+bx}{b+2ax-bx^2} - \frac{b\arctan\left(\frac{-a+bx}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{2(a^2+b^2)}$$

input
```
Integrate[(b + 2*a*x - b*x^2)^(-2),x]
```

output
```
((-a + b*x)/(b + 2*a*x - b*x^2) - (b*ArcTan[(-a + b*x)/Sqrt[-a^2 - b^2]])/
Sqrt[-a^2 - b^2])/(2*(a^2 + b^2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2ax-bx^2+b)^2}dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{b\int \frac{1}{-bx^2+2ax+b}dx}{2(a^2+b^2)}-\frac{a-bx}{2(a^2+b^2)(2ax-bx^2+b)}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{b\int \frac{1}{4(a^2+b^2)-(2a-2bx)^2}d(2a-2bx)}{a^2+b^2}-\frac{a-bx}{2(a^2+b^2)(2ax-bx^2+b)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{b\text{arctanh}\left(\frac{2a-2bx}{2\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{a-bx}{2(a^2+b^2)(2ax-bx^2+b)}$$

input
```
Int[(b + 2*a*x - b*x^2)^(-2),x]
```

output
```
-1/2*(a - b*x)/((a^2 + b^2)*(b + 2*a*x - b*x^2)) - (b*ArcTanh[(2*a - 2*b*x
)/(2*Sqrt[a^2 + b^2])])/(2*(a^2 + b^2)^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{-2bx+2a}{\left(-4a^{2}-4b^{2}\right)\left(-b\,x^{2}+2ax+b\right)}+\frac{2b\,\operatorname{arctanh}\left(\frac{-2bx+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(-4a^{2}-4b^{2}\right)\sqrt{a^{2}+b^{2}}}$	83
risch	$\frac{\frac{bx}{4a^{2}+4b^{2}}-\frac{a}{4\left(a^{2}+b^{2}\right)}}{-\frac{1}{2}b\,x^{2}+ax+\frac{1}{2}b}+\frac{b\ln\left(\left(a^{2}b+b^{3}\right)x+\left(a^{2}+b^{2}\right)^{\frac{3}{2}}-a^{3}-a\,b^{2}\right)}{4\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}-\frac{b\ln\left(\left(-a^{2}b-b^{3}\right)x+\left(a^{2}+b^{2}\right)^{\frac{3}{2}}+a^{3}+a\,b^{2}\right)}{4\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}$	134

input
```
int(1/(-b*x^2+2*a*x+b)^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*b*x+2*a)/(-4*a^2-4*b^2)/(-b*x^2+2*a*x+b)+2*b/(-4*a^2-4*b^2)/(a^2+b^2)^
(1/2)*arctanh(1/2*(-2*b*x+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(65) = 130$.

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.48

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx = -\frac{2\,a^3+2\,ab^2+(b^2x^2-2\,abx-b^2)\sqrt{a^2+b^2}\log\left(\frac{b^2x^2-2\,abx+2\,a^2+b^2+2\,\sqrt{a^2+b^2}(bx-a)}{bx^2-2\,ax-b}\right)-2\,(a^2b+b^3)x}{4\,(a^4b+2\,a^2b^3+b^5-(a^4b+2\,a^2b^3+b^5)x^2+2\,(a^5+2\,a^3b^2+ab^4)x)}$$

input
```
integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*a^3 + 2*a*b^2 + (b^2*x^2 - 2*a*b*x - b^2)*sqrt(a^2 + b^2)*log((b^2
*x^2 - 2*a*b*x + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*(b*x - a))/(b*x^2 - 2*a*x
 - b)) - 2*(a^2*b + b^3)*x)/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3
+ b^5)*x^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(56) = 112$.

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.16

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx = -\frac{b\sqrt{\frac{1}{(a^2+b^2)^3}}\log\left(x+\frac{-a^4b\sqrt{\frac{1}{(a^2+b^2)^3}}-2a^2b^3\sqrt{\frac{1}{(a^2+b^2)^3}}-ab-b^5\sqrt{\frac{1}{(a^2+b^2)^3}}}{b^2}\right)}{4} + \frac{b\sqrt{\frac{1}{(a^2+b^2)^3}}\log\left(x+\frac{a^4b\sqrt{\frac{1}{(a^2+b^2)^3}}+2a^2b^3\sqrt{\frac{1}{(a^2+b^2)^3}}-ab+b^5\sqrt{\frac{1}{(a^2+b^2)^3}}}{b^2}\right)}{4} + \frac{a-bx}{-2a^2b-2b^3+x^2\cdot(2a^2b+2b^3)+x\,(-4a^3-4ab^2)}$$

input
```
integrate(1/(-b*x**2+2*a*x+b)**2,x)
```

output
```
-b*sqrt((a**2 + b**2)**(-3))*log(x + (-a**4*b*sqrt((a**2 + b**2)**(-3)) -
2*a**2*b**3*sqrt((a**2 + b**2)**(-3)) - a*b - b**5*sqrt((a**2 + b**2)**(-3
)))/b**2)/4 + b*sqrt((a**2 + b**2)**(-3))*log(x + (a**4*b*sqrt((a**2 + b**
2)**(-3)) + 2*a**2*b**3*sqrt((a**2 + b**2)**(-3)) - a*b + b**5*sqrt((a**2
+ b**2)**(-3)))/b**2)/4 + (a - b*x)/(-2*a**2*b - 2*b**3 + x**2*(2*a**2*b +
 2*b**3) + x*(-4*a**3 - 4*a*b**2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx = -\frac{b\log\left(\frac{bx-a-\sqrt{a^2+b^2}}{bx-a+\sqrt{a^2+b^2}}\right)}{4\,(a^2+b^2)^{\frac{3}{2}}} + \frac{bx-a}{2\,(a^2b+b^3-(a^2b+b^3)x^2+2\,(a^3+ab^2)x)}$$

input
```
integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="maxima")
```

output
```
-1/4*b*log((b*x - a - sqrt(a^2 + b^2))/(b*x - a + sqrt(a^2 + b^2)))/(a^2 +
 b^2)^(3/2) + 1/2*(b*x - a)/(a^2*b + b^3 - (a^2*b + b^3)*x^2 + 2*(a^3 + a*
b^2)*x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx = -\frac{b\log\left(\frac{\left|2\,bx-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,bx-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{4\,(a^2+b^2)^{\frac{3}{2}}} - \frac{bx-a}{2\,(bx^2-2\,ax-b)(a^2+b^2)}$$

input
```
integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="giac")
```

output
```
-1/4*b*log(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a + 2*sqrt(a
^2 + b^2)))/(a^2 + b^2)^(3/2) - 1/2*(b*x - a)/((b*x^2 - 2*a*x - b)*(a^2 +
b^2))
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.45

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx = -\frac{\frac{a}{2\,(a^2+b^2)}-\frac{b\,x}{2\,(a^2+b^2)}}{-b\,x^2+2\,a\,x+b}+\frac{b\,\mathrm{atan}\left(\frac{a\,b^2\,1\mathrm{i}+a^3\,1\mathrm{i}-b\,x\,(a^2+b^2)\,1\mathrm{i}}{(a^2+b^2)^{3/2}}\right)\,1\mathrm{i}}{2\,(a^2+b^2)^{3/2}}$$

input
```
int(1/(b + 2*a*x - b*x^2)^2,x)
```

output
```
(b*atan((a*b^2*1i + a^3*1i - b*x*(a^2 + b^2)*1i)/(a^2 + b^2)^(3/2))*1i)/(2
*(a^2 + b^2)^(3/2)) - (a/(2*(a^2 + b^2)) - (b*x)/(2*(a^2 + b^2)))/(b + 2*a
*x - b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.22

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx$$
$$= \frac{4\sqrt{a^2+b^2}\,atan\left(\frac{-bix+ai}{\sqrt{a^2+b^2}}\right)a^2bix-2\sqrt{a^2+b^2}\,atan\left(\frac{-bix+ai}{\sqrt{a^2+b^2}}\right)a\,b^2i\,x^2+2\sqrt{a^2+b^2}\,atan\left(\frac{-bix+ai}{\sqrt{a^2+b^2}}\right)a\,b^2i-2a}{4a\,(-a^4b\,x^2-2a^2b^3x^2-b^5x^2+2a^5x+4a^3b^2x+2a\,b^4x+a^4b+2a^2b^3+b}$$

input
```
int(1/(-b*x^2+2*a*x+b)^2,x)
```

output
```
(4*sqrt(a**2 + b**2)*atan((a*i - b*i*x)/sqrt(a**2 + b**2))*a**2*b*i*x - 2*
sqrt(a**2 + b**2)*atan((a*i - b*i*x)/sqrt(a**2 + b**2))*a*b**2*i*x**2 + 2*
sqrt(a**2 + b**2)*atan((a*i - b*i*x)/sqrt(a**2 + b**2))*a*b**2*i - 2*a**4
+ a**2*b**2*x**2 - 3*a**2*b**2 + b**4*x**2 - b**4)/(4*a*(2*a**5*x - a**4*b
*x**2 + a**4*b + 4*a**3*b**2*x - 2*a**2*b**3*x**2 + 2*a**2*b**3 + 2*a*b**4
*x - b**5*x**2 + b**5))
```

3.106 $\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 62

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx = -\left(\frac{a}{b}\right)^{-1/n}\arctan\left(\cot\left(\frac{\pi-2k\pi}{n}\right)-\left(\frac{a}{b}\right)^{-1/n}x\csc\left(\frac{\pi-2k\pi}{n}\right)\right)\csc\left(\frac{\pi-2k\pi}{n}\right)$$

output
```
arctan(-cot((-2*Pi*k+Pi)/n)+x*csc((-2*Pi*k+Pi)/n)/((a/b)^(1/n)))*csc((-2*P
i*k+Pi)/n)/((a/b)^(1/n))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx = -\left(\frac{a}{b}\right)^{-1/n}\arctan\left(\cot\left(\frac{\pi-2k\pi}{n}\right)-\left(\frac{a}{b}\right)^{-1/n}x\csc\left(\frac{\pi-2k\pi}{n}\right)\right)\csc\left(\frac{\pi-2k\pi}{n}\right)$$

input
```
Integrate[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-1)
,x]
```

output
```
-((ArcTan[Cot[(Pi - 2*k*Pi)/n] - (x*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1)]*Cs
c[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-2x\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi-2\pi k}{n}\right)+\left(\frac{a}{b}\right)^{2/n}+x^2}dx$$

$$\Big\downarrow \text{ 1083}$$

$$-2\int \frac{1}{-4\left(1-\cos^2\left(\frac{\pi-2k\pi}{n}\right)\right)\left(\frac{a}{b}\right)^{2/n}-\left(2x-2\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi-2k\pi}{n}\right)\right)^2}d\left(2x-2\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi-2k\pi}{n}\right)\right)$$

$$\Big\downarrow \text{ 217}$$

$$\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\arctan\left(\frac{1}{2}\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\left(2x-2\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi-2\pi k}{n}\right)\right)\right)$$

input
```
Int[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-1),x]
```

output
```
(ArcTan[((2*x - 2*(a/b)^n^(-1)*Cos[(Pi - 2*k*Pi)/n])*Csc[(Pi - 2*k*Pi)/n])
/(2*(a/b)^n^(-1))]*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.79

method	result	size
default	$\frac{\arctan\left(\frac{2x-2\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi(2k-1)}{n}\right)}{2\sqrt{-\left(\frac{a}{b}\right)^{\frac{2}{n}}\cos\left(\frac{\pi(2k-1)}{n}\right)^{2}+\left(\frac{a}{b}\right)^{\frac{2}{n}}}}\right)}{\sqrt{-\left(\frac{a}{b}\right)^{\frac{2}{n}}\cos\left(\frac{\pi(2k-1)}{n}\right)^{2}+\left(\frac{a}{b}\right)^{\frac{2}{n}}}}$	111
risch	Expression too large to display	1200

input
```
int(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*Pi*k+Pi)/n)),x,method=_RETU
RNVERBOSE)
```

output
```
1/(-((a/b)^(1/n))^2*cos(Pi*(2*k-1)/n)^2+(a/b)^(2/n))^(1/2)*arctan(1/2*(2*x
-2*(a/b)^(1/n)*cos(Pi*(2*k-1)/n))/(-((a/b)^(1/n))^2*cos(Pi*(2*k-1)/n)^2+(a
/b)^(2/n))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.44

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx=-\frac{\arctan\left(\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-x}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\sin\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)}\right)}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\sin\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)}$$

input
```
integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algor
ithm="fricas")
```

output
```
-arctan(((a/b)^(1/n)*cos(2*pi*k/n - pi/n) - x)/((a/b)^(1/n)*sin(2*pi*k/n -
 pi/n)))/((a/b)^(1/n)*sin(2*pi*k/n - pi/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(46) = 92.

Time = 0.42 (sec) , antiderivative size = 212, normalized size of antiderivative = 3.42

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx=$$

$$-\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}}\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}}\left(-2\left(\frac{a}{b}\right)^{\frac{2}{n}}\cos^2\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+2\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)}{2}\right)}{2}$$

$$+\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}}\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}}\left(-2\left(\frac{a}{b}\right)^{\frac{2}{n}}\cos^2\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+2\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)}{2}\right)}{2}$$

input
```
integrate(1/((a/b)**(2/n)+x**2-2*(a/b)**(1/n)*x*cos((-2*pi*k+pi)/n)),x)
```

output
```
-sqrt(1/((a/b)**(2/n)*(cos(pi*(2*k - 1)/n)**2 - 1)))*log(x - (a/b)**(1/n)*
cos(2*pi*k/n - pi/n) - sqrt(1/((a/b)**(2/n)*(cos(pi*(2*k - 1)/n)**2 - 1)))
*(-2*(a/b)**(2/n)*cos(2*pi*k/n - pi/n)**2 + 2*(a/b)**(2/n))/2)/2 + sqrt(1/
((a/b)**(2/n)*(cos(pi*(2*k - 1)/n)**2 - 1)))*log(x - (a/b)**(1/n)*cos(2*pi
*k/n - pi/n) + sqrt(1/((a/b)**(2/n)*(cos(pi*(2*k - 1)/n)**2 - 1)))*(-2*(a/
b)**(2/n)*cos(2*pi*k/n - pi/n)**2 + 2*(a/b)**(2/n))/2)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1>0)', see `assume?` for more de
tails)Is 1
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx = \frac{\arctan\left(-\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(-\frac{2\pi k}{n}+\frac{\pi}{n}\right)-x}{\sqrt{-\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^2+1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}\right)}{\sqrt{-\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^2+1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algor
ithm="giac")
```

output
```
arctan(-((a/b)^(1/n)*cos(-2*pi*k/n + pi/n) - x)/(sqrt(-cos(2*pi*k/n - pi/n
)^2 + 1)*(a/b)^(1/n)))/(sqrt(-cos(2*pi*k/n - pi/n)^2 + 1)*(a/b)^(1/n))
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx = -\frac{\operatorname{atanh}\left(\frac{x-\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)\left(\frac{a}{b}\right)^{1/n}}{\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)-1}\,\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)+1}\,\left(\frac{a}{b}\right)^{1/n}}\right)}{\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)-1}\,\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)+1}\,\left(\frac{a}{b}\right)^{1/n}}$$

input
```
int(1/((a/b)^(2/n) + x^2 - 2*x*cos((Pi - 2*Pi*k)/n)*(a/b)^(1/n)),x)
```

output
```
-atanh((x - cos((Pi*(2*k - 1))/n)*(a/b)^(1/n))/((cos((Pi*(2*k - 1))/n) - 1
)^(1/2)*(cos((Pi*(2*k - 1))/n) + 1)^(1/2)*(a/b)^(1/n)))/((cos((Pi*(2*k - 1
))/n) - 1)^(1/2)*(cos((Pi*(2*k - 1))/n) + 1)^(1/2)*(a/b)^(1/n))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx = \frac{b^{\frac{1}{n}}\sqrt{-\cos\left(\frac{2k\pi-\pi}{n}\right)^2+1}\,atan\left(\frac{a^{\frac{1}{n}}\cos\left(\frac{2k\pi-\pi}{n}\right)-b^{\frac{1}{n}}x}{a^{\frac{1}{n}}\sqrt{-\cos\left(\frac{2k\pi-\pi}{n}\right)^2+1}}\right)}{a^{\frac{1}{n}}\left(\cos\left(\frac{2k\pi-\pi}{n}\right)^2-1\right)}$$

input
```
int(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*Pi*k+Pi)/n)),x)
```

output
```
(b**(1/n)*sqrt( - cos((2*k*pi - pi)/n)**2 + 1)*atan((a**(1/n)*cos((2*k*pi
- pi)/n) - b**(1/n)*x)/(a**(1/n)*sqrt( - cos((2*k*pi - pi)/n)**2 + 1))))/(
a**(1/n)*(cos((2*k*pi - pi)/n)**2 - 1))
```

3.107 $\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)} dx$

Optimal result

Integrand size = 14, antiderivative size = 17

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)} dx = \arctan\left(\left(x+\cos\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)$$

output
```
arctan((x+cos(1/7))*csc(1/7))*csc(1/7)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)} dx = \arctan\left(\cot\left(\frac{1}{7}\right)+x\csc\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)$$

input
```
Integrate[(1 + x^2 + 2*x*Cos[1/7])^(-1),x]
```

output
```
ArcTan[Cot[1/7] + x*Csc[1/7]]*Csc[1/7]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2+2x\cos\left(\frac{1}{7}\right)+1}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-\left(2x+2\cos\left(\frac{1}{7}\right)\right)^2-4\sin^2\left(\frac{1}{7}\right)}d\left(2x+2\cos\left(\frac{1}{7}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\csc\left(\frac{1}{7}\right)\arctan\left(\frac{1}{2}\csc\left(\frac{1}{7}\right)\left(2x+2\cos\left(\frac{1}{7}\right)\right)\right)$$

input
```
Int[(1 + x^2 + 2*x*Cos[1/7])^(-1),x]
```

output
```
ArcTan[((2*x + 2*Cos[1/7])*Csc[1/7])/2]*Csc[1/7]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(11) = 22.

Time = 1.61 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

method	result	size
default	$\frac{\arctan\left(\frac{2x+2\cos\left(\frac{1}{7}\right)}{2\sqrt{-\cos\left(\frac{1}{7}\right)^{2}+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^{2}+1}}$	33
risch	Expression too large to display	3085

input
```
int(1/(1+x^2+2*x*cos(1/7)),x,method=_RETURNVERBOSE)
```

output
```
1/(-cos(1/7)^2+1)^(1/2)*arctan(1/2*(2*x+2*cos(1/7))/(-cos(1/7)^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx=\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sin\left(\frac{1}{7}\right)}\right)}{\sin\left(\frac{1}{7}\right)}$$

input
```
integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="fricas")
```

output
```
arctan((x + cos(1/7))/sin(1/7))/sin(1/7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 9.71

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx = -\frac{i\log\left(x+\cos\left(\frac{1}{7}\right)-\frac{i}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i\cos^2\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}\right)}{2\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}} + \frac{i\log\left(x+\cos\left(\frac{1}{7}\right)-\frac{i\cos^2\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}\right)}{2\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}$$

input
```
integrate(1/(1+x**2+2*x*cos(1/7)),x)
```

output
```
-I*log(x + cos(1/7) - I/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)) + I*cos(1/
7)**2/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)))/(2*sqrt(1 - cos(1/7))*sqrt(
cos(1/7) + 1)) + I*log(x + cos(1/7) - I*cos(1/7)**2/(sqrt(1 - cos(1/7))*sq
rt(cos(1/7) + 1)) + I/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)))/(2*sqrt(1 -
 cos(1/7))*sqrt(cos(1/7) + 1))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx = \frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}$$

input
```
integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="maxima")
```

output
```
arctan((x + cos(1/7))/sqrt(-cos(1/7)^2 + 1))/sqrt(-cos(1/7)^2 + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx = \frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}$$

input
```
integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="giac")
```

output
```
arctan((x + cos(1/7))/sqrt(-cos(1/7)^2 + 1))/sqrt(-cos(1/7)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx = \frac{\mathrm{atan}\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)^2}}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)^2}}$$

input
```
int(1/(2*x*cos(1/7) + x^2 + 1),x)
```

output
```
atan((x + cos(1/7))/(1 - cos(1/7)^2)^(1/2))/(1 - cos(1/7)^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx = -\frac{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}\,atan\left(\frac{\cos\left(\frac{1}{7}\right)+x}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}\right)}{\cos\left(\frac{1}{7}\right)^2-1}$$

input
```
int(1/(1+x^2+2*x*cos(1/7)),x)
```

output
```
( - sqrt( - cos(1/7)**2 + 1)*atan((cos(1/7) + x)/sqrt( - cos(1/7)**2 + 1))
)/(cos(1/7)**2 - 1)
```

3.108 $\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx=\arctan\left(\left(x+\cos\left(\frac{\pi}{7}\right)\right)\csc\left(\frac{\pi}{7}\right)\right)\csc\left(\frac{\pi}{7}\right)$$

output
```
arctan((x+cos(1/7*Pi))*csc(1/7*Pi))*csc(1/7*Pi)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx=\arctan\left(\cot\left(\frac{\pi}{7}\right)+x\csc\left(\frac{\pi}{7}\right)\right)\csc\left(\frac{\pi}{7}\right)$$

input
```
Integrate[(1 + x^2 + 2*x*Cos[Pi/7])^(-1),x]
```

output
```
ArcTan[Cot[Pi/7] + x*Csc[Pi/7]]*Csc[Pi/7]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2+2x\cos\left(\frac{\pi}{7}\right)+1}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-\left(2x+2\cos\left(\frac{\pi}{7}\right)\right)^2-4\sin^2\left(\frac{\pi}{7}\right)}d\left(2x+2\cos\left(\frac{\pi}{7}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\csc\left(\frac{\pi}{7}\right)\arctan\left(\frac{1}{2}\csc\left(\frac{\pi}{7}\right)\left(2x+2\cos\left(\frac{\pi}{7}\right)\right)\right)$$

input
```
Int[(1 + x^2 + 2*x*Cos[Pi/7])^(-1),x]
```

output
```
ArcTan[((2*x + 2*Cos[Pi/7])*Csc[Pi/7])/2]*Csc[Pi/7]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(17) = 34.

Time = 1.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

method	result
default	$\frac{\arctan\left(\frac{2x+2\cos\left(\frac{\pi}{7}\right)}{2\sqrt{-\cos\left(\frac{\pi}{7}\right)^{2}+1}}\right)}{\sqrt{-\cos\left(\frac{\pi}{7}\right)^{2}+1}}$
norman	$\left(-\frac{4(-1)^{\frac{5}{7}}}{7}+\frac{(-1)^{\frac{4}{7}}}{7}-\frac{5(-1)^{\frac{3}{7}}}{7}+\frac{2(-1)^{\frac{2}{7}}}{7}-\frac{6(-1)^{\frac{1}{7}}}{7}+\frac{3}{7}\right)\ln\left((-1)^{\frac{5}{7}}-(-1)^{\frac{4}{7}}+(-1)^{\frac{3}{7}}-(-1)^{\frac{2}{7}}+(-\right.$
risch	$-\frac{4\ln\left(-(-1)^{\frac{5}{7}}+(-1)^{\frac{4}{7}}-(-1)^{\frac{3}{7}}+(-1)^{\frac{2}{7}}-(-1)^{\frac{1}{7}}+x+1\right)(-1)^{\frac{5}{7}}}{7}+\frac{\ln\left(-(-1)^{\frac{5}{7}}+(-1)^{\frac{4}{7}}-(-1)^{\frac{3}{7}}+(-1)^{\frac{2}{7}}-(-1)^{\frac{1}{7}}+x+1\right)(-1)^{\frac{4}{7}}}{7}$

input
```
int(1/(1+x^2+2*x*cos(1/7*Pi)),x,method=_RETURNVERBOSE)
```

output
```
1/(-cos(1/7*Pi)^2+1)^(1/2)*arctan(1/2*(2*x+2*cos(1/7*Pi))/(-cos(1/7*Pi)^2+
1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx=\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\,\pi\right)}{\sin\left(\frac{1}{7}\,\pi\right)}\right)}{\sin\left(\frac{1}{7}\,\pi\right)}$$

input
```
integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="fricas")
```

output
```
arctan((x + cos(1/7*pi))/sin(1/7*pi))/sin(1/7*pi)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.04

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx = -\frac{i\log\left(x+\cos\left(\frac{\pi}{7}\right)-\frac{i\left(2-2\cos^2\left(\frac{\pi}{7}\right)\right)}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)} + \frac{i\log\left(x+\cos\left(\frac{\pi}{7}\right)+\frac{i\left(2-2\cos^2\left(\frac{\pi}{7}\right)\right)}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)}$$

input
```
integrate(1/(1+x**2+2*x*cos(1/7*pi)),x)
```

output
```
-I*log(x + cos(pi/7) - I*(2 - 2*cos(pi/7)**2)/(2*sin(pi/7)))/(2*sin(pi/7))
 + I*log(x + cos(pi/7) + I*(2 - 2*cos(pi/7)**2)/(2*sin(pi/7)))/(2*sin(pi/7
))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx = \frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}$$

input
```
integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="maxima")
```

output
```
arctan((x + cos(1/7*pi))/sqrt(-cos(1/7*pi)^2 + 1))/sqrt(-cos(1/7*pi)^2 + 1
)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx = \frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}$$

input
```
integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="giac")
```

output
```
arctan((x + cos(1/7*pi))/sqrt(-cos(1/7*pi)^2 + 1))/sqrt(-cos(1/7*pi)^2 + 1
)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx = -\frac{\operatorname{atanh}\left(\frac{x+\cos\left(\frac{\Pi}{7}\right)}{\sqrt{\cos\left(\frac{\Pi}{7}\right)-1}\,\sqrt{\cos\left(\frac{\Pi}{7}\right)+1}}\right)}{\sqrt{\cos\left(\frac{\Pi}{7}\right)-1}\,\sqrt{\cos\left(\frac{\Pi}{7}\right)+1}}$$

input
```
int(1/(x^2 + 2*x*cos(Pi/7) + 1),x)
```

output
```
-atanh((x + cos(Pi/7))/((cos(Pi/7) - 1)^(1/2)*(cos(Pi/7) + 1)^(1/2)))/((co
s(Pi/7) - 1)^(1/2)*(cos(Pi/7) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx = -\frac{\sqrt{-\cos\left(\frac{\pi}{7}\right)^2+1}\,atan\left(\frac{\cos\left(\frac{\pi}{7}\right)+x}{\sqrt{-\cos\left(\frac{\pi}{7}\right)^2+1}}\right)}{\cos\left(\frac{\pi}{7}\right)^2-1}$$

input
```
int(1/(1+x^2+2*x*cos(1/7*Pi)),x)
```

output
```
( - sqrt( - cos(pi/7)**2 + 1)*atan((cos(pi/7) + x)/sqrt( - cos(pi/7)**2 +
1)))/(cos(pi/7)**2 - 1)
```

3.109 $\int \frac{1}{bx+c(d+ex)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{1}{bx + c(d + ex)^2} dx = -\frac{2\text{arctanh}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b + 4cde}}$$

output
```
-2*arctanh((b+2*c*e*(e*x+d))/b^(1/2)/(4*c*d*e+b)^(1/2))/b^(1/2)/(4*c*d*e+b
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{bx + c(d + ex)^2} dx = -\frac{2\text{arctanh}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b + 4cde}}$$

input
```
Integrate[(b*x + c*(d + e*x)^2)^(-1),x]
```

output
```
(-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*S
qrt[b + 4*c*d*e])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2080, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx+c(d+ex)^2}dx$$

$$\Big\downarrow \text{2080}$$

$$\int \frac{1}{x(b+2cde)+cd^2+ce^2x^2}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{b(b+4cde)-(2cxe^2+2cde+b)^2}d\left(2cxe^2+2cde+b\right)$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{b+2cde+2ce^2x}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

input
```
Int[(b*x + c*(d + e*x)^2)^(-1),x]
```

output
```
(-2*ArcTanh[(b + 2*c*d*e + 2*c*e^2*x)/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[
b]*Sqrt[b + 4*c*d*e])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2080
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{2c\,e^{2}x+2dec+b}{\sqrt{4bcde+b^{2}}}\right)}{\sqrt{4bcde+b^{2}}}$	43
risch	$\frac{\ln\left(-2c\,e^{2}x-2dec+\sqrt{b(4dec+b)}-b\right)}{\sqrt{b(4dec+b)}}-\frac{\ln\left(2c\,e^{2}x+2dec+\sqrt{b(4dec+b)}+b\right)}{\sqrt{b(4dec+b)}}$	81

input
```
int(1/(b*x+c*(e*x+d)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/(4*b*c*d*e+b^2)^(1/2)*arctanh((2*c*e^2*x+2*c*d*e+b)/(4*b*c*d*e+b^2)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 4.04

$$\int \frac{1}{bx+c(d+ex)^2}\,dx = \left[\frac{\log\left(\frac{2\,c^2e^4x^2+2\,c^2d^2e^2+4\,bcde+b^2+2\,(2\,c^2de^3+bce^2)x-\sqrt{4\,bcde+b^2}\,(2\,ce^2x+2\,cde+b)}{ce^2x^2+cd^2+(2\,cde+b)x}\right)}{\sqrt{4\,bcde+b^2}}, \frac{2\sqrt{-4\,bcde-b^2}\arctan\left(\frac{\sqrt{-4\,bcde-b}}{4}\right.}{4\,bcde+b^2}\right.$$

input
```
integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

output
```
[log((2*c^2*e^4*x^2 + 2*c^2*d^2*e^2 + 4*b*c*d*e + b^2 + 2*(2*c^2*d*e^3 + b
*c*e^2)*x - sqrt(4*b*c*d*e + b^2)*(2*c*e^2*x + 2*c*d*e + b))/(c*e^2*x^2 +
c*d^2 + (2*c*d*e + b)*x))/sqrt(4*b*c*d*e + b^2), 2*sqrt(-4*b*c*d*e - b^2)*
arctan(sqrt(-4*b*c*d*e - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e + b^2))
/(4*b*c*d*e + b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(49) = 98$.

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.21

$$\int \frac{1}{bx + c(d + ex)^2}\,dx = \sqrt{\frac{1}{b(b + 4cde)}}\log\left(x + \frac{-b^2\sqrt{\frac{1}{b(b+4cde)}} - 4bcde\sqrt{\frac{1}{b(b+4cde)}} + b + 2cde}{2ce^2}\right) - \sqrt{\frac{1}{b(b + 4cde)}}\log\left(x + \frac{b^2\sqrt{\frac{1}{b(b+4cde)}} + 4bcde\sqrt{\frac{1}{b(b+4cde)}} + b + 2cde}{2ce^2}\right)$$

input
```
integrate(1/(b*x+c*(e*x+d)**2),x)
```

output
```
sqrt(1/(b*(b + 4*c*d*e)))*log(x + (-b**2*sqrt(1/(b*(b + 4*c*d*e))) - 4*b*c
*d*e*sqrt(1/(b*(b + 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2)) - sqrt(1/(b*(b +
 4*c*d*e)))*log(x + (b**2*sqrt(1/(b*(b + 4*c*d*e))) + 4*b*c*d*e*sqrt(1/(b*
(b + 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{bx + c(d + ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c*d*e+b>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{1}{bx + c(d+ex)^2} \, dx = \frac{2 \arctan\left(\frac{2 ce^2x + 2 cde + b}{\sqrt{-4 bcde - b^2}}\right)}{\sqrt{-4 bcde - b^2}}$$

input
```
integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="giac")
```

output
```
2*arctan((2*c*e^2*x + 2*c*d*e + b)/sqrt(-4*b*c*d*e - b^2))/sqrt(-4*b*c*d*e
 - b^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{1}{bx + c(d+ex)^2} \, dx = -\frac{2 \operatorname{atanh}\left(\frac{2 c x e^2 + 2 c d e + b}{\sqrt{b} \sqrt{b + 4 c d e}}\right)}{\sqrt{b} \sqrt{b + 4 c d e}}$$

input
```
int(1/(c*(d + e*x)^2 + b*x),x)
```

output
```
-(2*atanh((b + 2*c*d*e + 2*c*e^2*x)/(b^(1/2)*(b + 4*c*d*e)^(1/2))))/(b^(1/
2)*(b + 4*c*d*e)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{1}{bx + c(d + ex)^2}\, dx = -\frac{2\sqrt{b}\,\sqrt{-4cde - b}\, atan\left(\frac{2c\,e^2x + 2cde + b}{\sqrt{b}\,\sqrt{-4cde - b}}\right)}{b\,(4cde + b)}$$

input
```
int(1/(b*x+c*(e*x+d)^2),x)
```

output
```
( - 2*sqrt(b)*sqrt( - b - 4*c*d*e)*atan((b + 2*c*d*e + 2*c*e**2*x)/(sqrt(b
)*sqrt( - b - 4*c*d*e))))/(b*(b + 4*c*d*e))
```

3.110 $\int \frac{1}{a+bx+c(d+ex)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{a+bx+c(d+ex)^2} \, dx = -\frac{2\text{arctanh}\left(\frac{b+2ce(d+ex)}{\sqrt{b^2+4bcde-4ace^2}}\right)}{\sqrt{b^2+4bcde-4ace^2}}$$

output
```
-2*arctanh((b+2*c*e*(e*x+d))/(-4*a*c*e^2+4*b*c*d*e+b^2)^(1/2))/(-4*a*c*e^2
+4*b*c*d*e+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{1}{a+bx+c(d+ex)^2} \, dx = \frac{2\arctan\left(\frac{b+2ce(d+ex)}{\sqrt{-b^2-4bcde+4ace^2}}\right)}{\sqrt{-b^2-4bcde+4ace^2}}$$

input
```
Integrate[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

output
```
(2*ArcTan[(b + 2*c*e*(d + e*x))/Sqrt[-b^2 - 4*b*c*d*e + 4*a*c*e^2]])/Sqrt[
-b^2 - 4*b*c*d*e + 4*a*c*e^2]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2080, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx+c(d+ex)^2}dx$$

$$\Big\downarrow \text{ 2080}$$

$$\int \frac{1}{a+x(b+2cde)+cd^2+ce^2x^2}dx$$

$$\Big\downarrow \text{ 1083}$$

$$-2\int \frac{1}{b^2+4cdeb-4ace^2-(2cxe^2+2cde+b)^2}d(2cxe^2+2cde+b)$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{2\text{arctanh}\left(\frac{b+2cde+2ce^2x}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

input
```
Int[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

output
```
(-2*ArcTanh[(b + 2*c*d*e + 2*c*e^2*x)/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]])/
Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2080
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{2\arctan\left(\frac{2c\,e^{2}x+2dec+b}{\sqrt{4ac\,e^{2}-4bcde-b^{2}}}\right)}{\sqrt{4ac\,e^{2}-4bcde-b^{2}}}$	61
risch	$-\frac{\ln\left(2c\,e^{2}x+2dec+\sqrt{-4ac\,e^{2}+4bcde+b^{2}}+b\right)}{\sqrt{-4ac\,e^{2}+4bcde+b^{2}}}+\frac{\ln\left(-2c\,e^{2}x-2dec+\sqrt{-4ac\,e^{2}+4bcde+b^{2}}-b\right)}{\sqrt{-4ac\,e^{2}+4bcde+b^{2}}}$	113

input
```
int(1/(a+b*x+c*(e*x+d)^2),x,method=_RETURNVERBOSE)
```

output
```
2/(4*a*c*e^2-4*b*c*d*e-b^2)^(1/2)*arctan((2*c*e^2*x+2*c*d*e+b)/(4*a*c*e^2-
4*b*c*d*e-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.21

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx = \left[\frac{\log\left(\frac{2\,c^{2}e^{4}x^{2}+4\,bcde+2\,\left(c^{2}d^{2}-ac\right)e^{2}+b^{2}+2\,\left(2\,c^{2}de^{3}+bce^{2}\right)x-\sqrt{4\,bcde-4\,ace^{2}+b^{2}}\left(2\,ce^{2}x+2\,cde+b\right)}{ce^{2}x^{2}+cd^{2}+\left(2\,cde+b\right)x+a}\right)}{\sqrt{4\,bcde-4\,ace^{2}+b^{2}}},\ -\frac{2\sqrt{-4\,bcde+4\,ace^{2}-b^{2}}\arctan\left(-\frac{\sqrt{-4\,bcde+4\,ace^{2}-b^{2}}\left(2\,ce^{2}x+2\,cde+b\right)}{4\,bcde-4\,ace^{2}+b^{2}}\right)}{4\,bcde-4\,ace^{2}+b^{2}}\right]$$

input
```
integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

output
```
[log((2*c^2*e^4*x^2 + 4*b*c*d*e + 2*(c^2*d^2 - a*c)*e^2 + b^2 + 2*(2*c^2*d
*e^3 + b*c*e^2)*x - sqrt(4*b*c*d*e - 4*a*c*e^2 + b^2)*(2*c*e^2*x + 2*c*d*e
 + b))/(c*e^2*x^2 + c*d^2 + (2*c*d*e + b)*x + a))/sqrt(4*b*c*d*e - 4*a*c*e
^2 + b^2), -2*sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)*arctan(-sqrt(-4*b*c*d*e +
 4*a*c*e^2 - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e - 4*a*c*e^2 + b^2))
/(4*b*c*d*e - 4*a*c*e^2 + b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(63) = 126$.

Time = 0.20 (sec) , antiderivative size = 294, normalized size of antiderivative = 5.16

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx =$$
$$-\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{-4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+4bcde\sqrt{-\frac{1}{4ace^2-b^2-}}}{2ce^2}\right.$$
$$+\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}-b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}-4bcde\sqrt{-\frac{1}{4ace^2-b^2-4b}}}{2ce^2}\right.$$

input
```
integrate(1/(a+b*x+c*(e*x+d)**2),x)
```

output
```
-sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (-4*a*c*e**2*sqrt(-1/(4*
a*c*e**2 - b**2 - 4*b*c*d*e)) + b**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*
e)) + 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e)/(2
*c*e**2)) + sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (4*a*c*e**2*s
qrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) - b**2*sqrt(-1/(4*a*c*e**2 - b**2
- 4*b*c*d*e)) - 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2
*c*d*e)/(2*c*e**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c*e^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx = \frac{2\arctan\left(\frac{2\,ce^2x+2\,cde+b}{\sqrt{-4\,bcde+4\,ace^2-b^2}}\right)}{\sqrt{-4\,bcde+4\,ace^2-b^2}}$$

input
```
integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="giac")
```

output
```
2*arctan((2*c*e^2*x + 2*c*d*e + b)/sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2))/sqr
t(-4*b*c*d*e + 4*a*c*e^2 - b^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b+2\,c\,d\,e}{\sqrt{-b^2-4\,c\,d\,b\,e+4\,a\,c\,e^2}}+\frac{2\,c\,e^2\,x}{\sqrt{-b^2-4\,c\,d\,b\,e+4\,a\,c\,e^2}}\right)}{\sqrt{-b^2-4\,c\,d\,b\,e+4\,a\,c\,e^2}}$$

input
```
int(1/(a + c*(d + e*x)^2 + b*x),x)
```

output
```
(2*atan((b + 2*c*d*e)/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2) + (2*c*e^2*x)/(4
*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2)))/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.42

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx = \frac{2\sqrt{4ac\,e^2-4bcde-b^2}\,atan\left(\frac{2c\,e^2x+2cde+b}{\sqrt{4ac\,e^2-4bcde-b^2}}\right)}{4ac\,e^2-4bcde-b^2}$$

input
```
int(1/(a+b*x+c*(e*x+d)^2),x)
```

output
```
(2*sqrt(4*a*c*e**2 - b**2 - 4*b*c*d*e)*atan((b + 2*c*d*e + 2*c*e**2*x)/sqr
t(4*a*c*e**2 - b**2 - 4*b*c*d*e)))/(4*a*c*e**2 - b**2 - 4*b*c*d*e)
```

3.111 $\int \sqrt{5-6x+9x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 38

$$\int \sqrt{5-6x+9x^2}\,dx = -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2}+\frac{2}{3}\text{arcsinh}\left(\frac{1}{2}(-1+3x)\right)$$

output
```
-1/6*(1-3*x)*(9*x^2-6*x+5)^(1/2)+2/3*arcsinh(-1/2+3/2*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \sqrt{5-6x+9x^2}\,dx = \frac{1}{6}(-1+3x)\sqrt{5-6x+9x^2}-\frac{2}{3}\log\left(1-3x+\sqrt{5-6x+9x^2}\right)$$

input
```
Integrate[Sqrt[5 - 6*x + 9*x^2],x]
```

output
```
((-1 + 3*x)*Sqrt[5 - 6*x + 9*x^2])/6 - (2*Log[1 - 3*x + Sqrt[5 - 6*x + 9*x
^2]])/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{9x^2-6x+5}\,dx$$

$$\Big\downarrow \ 1087$$

$$2\int \frac{1}{\sqrt{9x^2-6x+5}}dx-\frac{1}{6}(1-3x)\sqrt{9x^2-6x+5}$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{18}\int \frac{1}{\sqrt{\frac{1}{144}(18x-6)^2+1}}d(18x-6)-\frac{1}{6}(1-3x)\sqrt{9x^2-6x+5}$$

$$\Big\downarrow \ 222$$

$$\frac{2}{3}\text{arcsinh}\left(\frac{1}{12}(18x-6)\right)-\frac{1}{6}(1-3x)\sqrt{9x^2-6x+5}$$

input
```
Int[Sqrt[5 - 6*x + 9*x^2],x]
```

output
```
-1/6*((1 - 3*x)*Sqrt[5 - 6*x + 9*x^2]) + (2*ArcSinh[(-6 + 18*x)/12])/3
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{(18x-6)\sqrt{9x^2-6x+5}}{36}+\frac{2\,\mathrm{arcsinh}\left(-\frac{1}{2}+\frac{3x}{2}\right)}{3}$	29
risch	$\frac{\sqrt{9x^2-6x+5}\,(3x-1)}{6}+\frac{2\,\mathrm{arcsinh}\left(-\frac{1}{2}+\frac{3x}{2}\right)}{3}$	29
trager	$\left(\frac{x}{2}-\frac{1}{6}\right)\sqrt{9x^2-6x+5}+\frac{2\ln\left(-1+3x+\sqrt{9x^2-6x+5}\right)}{3}$	40

input
```
int((9*x^2-6*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/36*(18*x-6)*(9*x^2-6*x+5)^(1/2)+2/3*arcsinh(-1/2+3/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int\sqrt{5-6x+9x^2}\,dx=\frac{1}{6}\sqrt{9\,x^2-6\,x+5}(3\,x-1)-\frac{2}{3}\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

input
```
integrate((9*x^2-6*x+5)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5)
 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \sqrt{5-6x+9x^2}\,dx = \left(\frac{x}{2}-\frac{1}{6}\right)\sqrt{9x^2-6x+5}+\frac{2\,\mathrm{asinh}\left(\frac{3x}{2}-\frac{1}{2}\right)}{3}$$

input
```
integrate((9*x**2-6*x+5)**(1/2),x)
```

output
```
(x/2 - 1/6)*sqrt(9*x**2 - 6*x + 5) + 2*asinh(3*x/2 - 1/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sqrt{5-6x+9x^2}\,dx = \frac{1}{2}\sqrt{9\,x^2-6\,x+5}x-\frac{1}{6}\sqrt{9\,x^2-6\,x+5}+\frac{2}{3}\,\mathrm{arsinh}\left(\frac{3}{2}\,x-\frac{1}{2}\right)$$

input
```
integrate((9*x^2-6*x+5)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(9*x^2 - 6*x + 5)*x - 1/6*sqrt(9*x^2 - 6*x + 5) + 2/3*arcsinh(3/2*
x - 1/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \sqrt{5-6x+9x^2}\,dx = \frac{1}{6}\sqrt{9\,x^2-6\,x+5}(3\,x-1)-\frac{2}{3}\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

input
```
integrate((9*x^2-6*x+5)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5)
 + 1)
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \sqrt{5-6x+9x^2}\,dx = \frac{2\,\ln\left(3\,x+\sqrt{9\,x^2-6\,x+5}-1\right)}{3}+\left(\frac{x}{2}-\frac{1}{6}\right)\sqrt{9\,x^2-6\,x+5}$$

input
```
int((9*x^2 - 6*x + 5)^(1/2),x)
```

output
```
(2*log(3*x + (9*x^2 - 6*x + 5)^(1/2) - 1))/3 + (x/2 - 1/6)*(9*x^2 - 6*x +
5)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \sqrt{5-6x+9x^2}\,dx = \frac{\sqrt{9x^2-6x+5}\,x}{2}-\frac{\sqrt{9x^2-6x+5}}{6}+\frac{2\log\left(\frac{\sqrt{9x^2-6x+5}}{2}+\frac{3x}{2}-\frac{1}{2}\right)}{3}$$

input
```
int((9*x^2-6*x+5)^(1/2),x)
```

output
```
(3*sqrt(9*x**2 - 6*x + 5)*x - sqrt(9*x**2 - 6*x + 5) + 4*log((sqrt(9*x**2
- 6*x + 5) + 3*x - 1)/2))/6
```

3.112 $\int \sqrt{3-4x-4x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \sqrt{3-4x-4x^2}\,dx=\frac{1}{4}(1+2x)\sqrt{3-4x-4x^2}+\arcsin\left(\frac{1}{2}+x\right)$$

output
```
1/4*(1+2*x)*(-4*x^2-4*x+3)^(1/2)+arcsin(1/2+x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

$$\int \sqrt{3-4x-4x^2}\,dx=\frac{1}{4}(1+2x)\sqrt{3-4x-4x^2}-2\arctan\left(\frac{\sqrt{3-4x-4x^2}}{3+2x}\right)$$

input
```
Integrate[Sqrt[3 - 4*x - 4*x^2],x]
```

output
```
((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 - 2*ArcTan[Sqrt[3 - 4*x - 4*x^2]/(3 + 
2*x)]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-4x^2-4x+3}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$2\int \frac{1}{\sqrt{-4x^2-4x+3}}dx+\frac{1}{4}\sqrt{-4x^2-4x+3}(2x+1)$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{4}(2x+1)\sqrt{-4x^2-4x+3}-\frac{1}{8}\int \frac{1}{\sqrt{1-\frac{1}{64}(-8x-4)^2}}d(-8x-4)$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{4}(2x+1)\sqrt{-4x^2-4x+3}-\arcsin\left(\frac{1}{8}(-8x-4)\right)$$

input
```
Int[Sqrt[3 - 4*x - 4*x^2],x]
```

output
```
((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 - ArcSin[(-4 - 8*x)/8]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result
default	$-\frac{(-8x-4)\sqrt{-4x^2-4x+3}}{16}+\arcsin\left(\frac{1}{2}+x\right)$
risch	$-\frac{(4x^2+4x-3)(1+2x)}{4\sqrt{-4x^2-4x+3}}+\arcsin\left(\frac{1}{2}+x\right)$
trager	$\left(\frac{1}{4}+\frac{x}{2}\right)\sqrt{-4x^2-4x+3}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-2\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-4x^2-4x+3}\right.$

input
```
int((-4*x^2-4*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-8*x-4)*(-4*x^2-4*x+3)^(1/2)+arcsin(1/2+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int\sqrt{3-4x-4x^2}\,dx=\frac{1}{4}\sqrt{-4\,x^2-4\,x+3}(2\,x+1)-\arctan\left(\frac{\sqrt{-4\,x^2-4\,x+3}(2\,x+1)}{4\,x^2+4\,x-3}\right)$$

input
```
integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) - arctan(sqrt(-4*x^2 - 4*x + 3)*(2*x
+ 1)/(4*x^2 + 4*x - 3))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \sqrt{3-4x-4x^2}\,dx = \left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{-4x^2-4x+3}+\operatorname{asin}\left(x+\frac{1}{2}\right)$$

input
```
integrate((-4*x**2-4*x+3)**(1/2),x)
```

output
```
(x/2 + 1/4)*sqrt(-4*x**2 - 4*x + 3) + asin(x + 1/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int \sqrt{3-4x-4x^2}\,dx = \frac{1}{2}\sqrt{-4\,x^2-4\,x+3}x+\frac{1}{4}\sqrt{-4\,x^2-4\,x+3}-\arcsin\left(-x-\frac{1}{2}\right)$$

input
```
integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-4*x^2 - 4*x + 3)*x + 1/4*sqrt(-4*x^2 - 4*x + 3) - arcsin(-x - 1/
2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \sqrt{3-4x-4x^2}\,dx = \frac{1}{4}\sqrt{-4\,x^2-4\,x+3}(2\,x+1)+\arcsin\left(x+\frac{1}{2}\right)$$

input
```
integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) + arcsin(x + 1/2)
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \sqrt{3-4x-4x^2}\,dx = \operatorname{asin}\left(x+\frac{1}{2}\right)+\left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{-4\,x^2-4\,x+3}$$

input
```
int((3 - 4*x^2 - 4*x)^(1/2),x)
```

output
```
asin(x + 1/2) + (x/2 + 1/4)*(3 - 4*x^2 - 4*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \sqrt{3-4x-4x^2}\,dx = asin\left(x+\frac{1}{2}\right)+\frac{\sqrt{-4x^2-4x+3}\,x}{2}+\frac{\sqrt{-4x^2-4x+3}}{4}$$

input
```
int((-4*x^2-4*x+3)^(1/2),x)
```

output
```
(4*asin((2*x + 1)/2) + 2*sqrt( - 4*x**2 - 4*x + 3)*x + sqrt( - 4*x**2 - 4*
x + 3))/4
```

3.113 $\int \sqrt{-8+6x+9x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \sqrt{-8+6x+9x^2}\,dx = \frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2} + 3\text{arctanh}\left(\frac{2-3x}{\sqrt{-8+6x+9x^2}}\right)$$

output
```
1/6*(1+3*x)*(9*x^2+6*x-8)^(1/2)+3*arctanh((2-3*x)/(9*x^2+6*x-8)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \sqrt{-8+6x+9x^2}\,dx = \frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2} - 3\text{arctanh}\left(\frac{\sqrt{-8+6x+9x^2}}{-2+3x}\right)$$

input
```
Integrate[Sqrt[-8 + 6*x + 9*x^2],x]
```

output
```
((1 + 3*x)*Sqrt[-8 + 6*x + 9*x^2])/6 - 3*ArcTanh[Sqrt[-8 + 6*x + 9*x^2]/(-
2 + 3*x)]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{9x^2+6x-8}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8}-\frac{9}{2}\int \frac{1}{\sqrt{9x^2+6x-8}}dx$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8}-9\int \frac{1}{36-\frac{36(3x+1)^2}{9x^2+6x-8}}d\frac{6(3x+1)}{\sqrt{9x^2+6x-8}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8}-\frac{3}{2}\text{arctanh}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

input
```
Int[Sqrt[-8 + 6*x + 9*x^2],x]
```

output
```
((1 + 3*x)*Sqrt[-8 + 6*x + 9*x^2])/6 - (3*ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x
+ 9*x^2]])/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
trager	$\left(\frac{x}{2}+\frac{1}{6}\right)\sqrt{9x^2+6x-8}-\frac{3\ln\left(\sqrt{9x^2+6x-8}+1+3x\right)}{2}$	40
default	$\frac{(18x+6)\sqrt{9x^2+6x-8}}{36}-\frac{\ln\left(\frac{(9x+3)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)\sqrt{9}}{2}$	50
risch	$\frac{(3x+1)\sqrt{9x^2+6x-8}}{6}-\frac{\ln\left(\frac{(9x+3)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)\sqrt{9}}{2}$	50

input
```
int((9*x^2+6*x-8)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/2*x+1/6)*(9*x^2+6*x-8)^(1/2)-3/2*ln((9*x^2+6*x-8)^(1/2)+1+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int\sqrt{-8+6x+9x^2}\,dx=\frac{1}{6}\sqrt{9\,x^2+6\,x-8}(3\,x+1)+\frac{3}{2}\log\left(-3\,x+\sqrt{9\,x^2+6\,x-8}-1\right)$$

input
```
integrate((9*x^2+6*x-8)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(-3*x + sqrt(9*x^2 + 6*x - 8)
 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \sqrt{-8+6x+9x^2}\,dx = \left(\frac{x}{2}+\frac{1}{6}\right)\sqrt{9x^2+6x-8} - \frac{3\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)}{2}$$

input
```
integrate((9*x**2+6*x-8)**(1/2),x)
```

output
```
(x/2 + 1/6)*sqrt(9*x**2 + 6*x - 8) - 3*log(18*x + 6*sqrt(9*x**2 + 6*x - 8)
 + 6)/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \sqrt{-8+6x+9x^2}\,dx = \frac{1}{2}\sqrt{9x^2+6x-8}x + \frac{1}{6}\sqrt{9x^2+6x-8} - \frac{3}{2}\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)$$

input
```
integrate((9*x^2+6*x-8)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(9*x^2 + 6*x - 8)*x + 1/6*sqrt(9*x^2 + 6*x - 8) - 3/2*log(18*x + 6
*sqrt(9*x^2 + 6*x - 8) + 6)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \sqrt{-8+6x+9x^2}\,dx = \frac{1}{6}\sqrt{9\,x^2+6\,x-8}(3\,x+1) + \frac{3}{2}\log\left(\left|-3\,x+\sqrt{9\,x^2+6\,x-8}-1\right|\right)$$

input
```
integrate((9*x^2+6*x-8)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(abs(-3*x + sqrt(9*x^2 + 6*x
- 8) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \sqrt{-8+6x+9x^2}\,dx = \left(\frac{x}{2}+\frac{1}{6}\right)\sqrt{9\,x^2+6\,x-8} - \frac{3\,\ln\left(3\,x+\sqrt{9\,x^2+6\,x-8}+1\right)}{2}$$

input
```
int((6*x + 9*x^2 - 8)^(1/2),x)
```

output
```
(x/2 + 1/6)*(6*x + 9*x^2 - 8)^(1/2) - (3*log(3*x + (6*x + 9*x^2 - 8)^(1/2)
 + 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \sqrt{-8+6x+9x^2}\,dx = \frac{\sqrt{9x^2+6x-8}\,x}{2} + \frac{\sqrt{9x^2+6x-8}}{6} - \frac{3\log\left(\frac{\sqrt{9x^2+6x-8}}{3}+x+\frac{1}{3}\right)}{2}$$

input
```
int((9*x^2+6*x-8)^(1/2),x)
```

output
```
(3*sqrt(9*x**2 + 6*x - 8)*x + sqrt(9*x**2 + 6*x - 8) - 9*log((sqrt(9*x**2
+ 6*x - 8) + 3*x + 1)/3))/6
```

3.114 $\int \sqrt{2+4x+3x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \sqrt{2+4x+3x^2}\,dx=\frac{1}{6}(2+3x)\sqrt{2+4x+3x^2}+\frac{\operatorname{arcsinh}\left(\frac{2+3x}{\sqrt{2}}\right)}{3\sqrt{3}}$$

output
```
1/6*(2+3*x)*(3*x^2+4*x+2)^(1/2)+1/9*arcsinh(1/2*(2+3*x)*2^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \sqrt{2+4x+3x^2}\,dx=\frac{1}{6}(2+3x)\sqrt{2+4x+3x^2}-\frac{\log\left(-2-3x+\sqrt{6+12x+9x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[Sqrt[2 + 4*x + 3*x^2],x]
```

output
```
((2 + 3*x)*Sqrt[2 + 4*x + 3*x^2])/6 - Log[-2 - 3*x + Sqrt[6 + 12*x + 9*x^2
]]/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x^2+4x+2}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{3x^2+4x+2}}dx+\frac{1}{6}\sqrt{3x^2+4x+2}(3x+2)$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{\int \frac{1}{\sqrt{\frac{1}{8}(6x+4)^2+1}}d(6x+4)}{6\sqrt{6}}+\frac{1}{6}\sqrt{3x^2+4x+2}(3x+2)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{\text{arcsinh}\left(\frac{6x+4}{2\sqrt{2}}\right)}{3\sqrt{3}}+\frac{1}{6}\sqrt{3x^2+4x+2}(3x+2)$$

input
```
Int[Sqrt[2 + 4*x + 3*x^2],x]
```

output
```
((2 + 3*x)*Sqrt[2 + 4*x + 3*x^2])/6 + ArcSinh[(4 + 6*x)/(2*Sqrt[2])]/(3*Sq
rt[3])
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{(6x+4)\sqrt{3x^2+4x+2}}{12}+\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{3\sqrt{2}\left(x+\frac{2}{3}\right)}{2}\right)}{9}$	35
risch	$\frac{(3x+2)\sqrt{3x^2+4x+2}}{6}+\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{3\sqrt{2}\left(x+\frac{2}{3}\right)}{2}\right)}{9}$	35
trager	$\left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{3x^2+4x+2}+\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^2-3\right)x+2\operatorname{RootOf}\left(\_Z^2-3\right)+3\sqrt{3x^2+4x+2}\right)}{9}$	61

input
```
int((3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(6*x+4)*(3*x^2+4*x+2)^(1/2)+1/9*3^(1/2)*arcsinh(3/2*2^(1/2)*(x+2/3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int\sqrt{2+4x+3x^2}\,dx=\frac{1}{6}\sqrt{3\,x^2+4\,x+2}(3\,x+2)+\frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+4\,x+2}(3\,x+2)-9\,x^2-12\,x-5\right)$$

input
```
integrate((3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) + 1/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2
 + 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x - 5)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \sqrt{2+4x+3x^2}\,dx = \left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{3x^2+4x+2}+\frac{\sqrt{3}\,\mathrm{asinh}\left(\frac{3\sqrt{2}\left(x+\frac{2}{3}\right)}{2}\right)}{9}$$

input
```
integrate((3*x**2+4*x+2)**(1/2),x)
```

output
```
(x/2 + 1/3)*sqrt(3*x**2 + 4*x + 2) + sqrt(3)*asinh(3*sqrt(2)*(x + 2/3)/2)/
9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \sqrt{2+4x+3x^2}\,dx = \frac{1}{2}\sqrt{3\,x^2+4\,x+2}x+\frac{1}{9}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{2}(3\,x+2)\right)+\frac{1}{3}\sqrt{3\,x^2+4\,x+2}$$

input
```
integrate((3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(3*x^2 + 4*x + 2)*x + 1/9*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2)) +
 1/3*sqrt(3*x^2 + 4*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \sqrt{2+4x+3x^2}\,dx = \frac{1}{6}\sqrt{3\,x^2+4\,x+2}(3\,x+2) - \frac{1}{9}\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+4\,x+2}\right)-2\right)$$

input
```
integrate((3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 1/9*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x
- sqrt(3*x^2 + 4*x + 2)) - 2)
```

Mupad [B] (verification not implemented)

Time = 9.93 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \sqrt{2+4x+3x^2}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3\,x^2+4\,x+2}+\frac{\sqrt{3}\,(3\,x+2)}{3}\right)}{9} + \left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{3\,x^2+4\,x+2}$$

input
```
int((4*x + 3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*log((4*x + 3*x^2 + 2)^(1/2) + (3^(1/2)*(3*x + 2))/3))/9 + (x/2 +
1/3)*(4*x + 3*x^2 + 2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \sqrt{2+4x+3x^2}\,dx = \frac{\sqrt{3x^2+4x+2}\,x}{2} + \frac{\sqrt{3x^2+4x+2}}{3} + \frac{\sqrt{3}\log\left(\frac{\sqrt{3x^2+4x+2}\,\sqrt{3}+3x+2}{\sqrt{2}}\right)}{9}$$

input
```
int((3*x^2+4*x+2)^(1/2),x)
```

output
```
(9*sqrt(3*x**2 + 4*x + 2)*x + 6*sqrt(3*x**2 + 4*x + 2) + 2*sqrt(3)*log((sq
rt(3*x**2 + 4*x + 2)*sqrt(3) + 3*x + 2)/sqrt(2)))/18
```

3.115 $\int \sqrt{2+4x-3x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \sqrt{2+4x-3x^2}\,dx=-\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2}-\frac{5\arcsin\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

output
```
-1/6*(2-3*x)*(-3*x^2+4*x+2)^(1/2)-5/9*arcsin(1/10*(2-3*x)*10^(1/2))*3^(1/2
)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \sqrt{2+4x-3x^2}\,dx=\frac{1}{6}(-2+3x)\sqrt{2+4x-3x^2}+\frac{10\arctan\left(\frac{-2-\sqrt{10}+3x}{\sqrt{6+12x-9x^2}}\right)}{3\sqrt{3}}$$

input
```
Integrate[Sqrt[2 + 4*x - 3*x^2],x]
```

output
```
((-2 + 3*x)*Sqrt[2 + 4*x - 3*x^2])/6 + (10*ArcTan[(-2 - Sqrt[10] + 3*x)/Sq
rt[6 + 12*x - 9*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-3x^2+4x+2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{5}{3}\int \frac{1}{\sqrt{-3x^2+4x+2}}dx-\frac{1}{6}(2-3x)\sqrt{-3x^2+4x+2}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{6}\sqrt{\frac{5}{6}}\int \frac{1}{\sqrt{1-\frac{1}{40}(4-6x)^2}}d(4-6x)-\frac{1}{6}\sqrt{-3x^2+4x+2}(2-3x)$$

$$\Big\downarrow \text{223}$$

$$-\frac{5\arcsin\left(\frac{4-6x}{2\sqrt{10}}\right)}{3\sqrt{3}}-\frac{1}{6}\sqrt{-3x^2+4x+2}(2-3x)$$

input
```
Int[Sqrt[2 + 4*x - 3*x^2],x]
```

output
```
-1/6*((2 - 3*x)*Sqrt[2 + 4*x - 3*x^2]) - (5*ArcSin[(4 - 6*x)/(2*Sqrt[10])]
)/(3*Sqrt[3])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result
default	$-\frac{(-6x+4)\sqrt{-3x^2+4x+2}}{12}+\frac{5\sqrt{3}\,\arcsin\left(\frac{3\sqrt{10}\,\left(x-\frac{2}{3}\right)}{10}\right)}{9}$
risch	$-\frac{(3x^2-4x-2)(-2+3x)}{6\sqrt{-3x^2+4x+2}}+\frac{5\sqrt{3}\,\arcsin\left(\frac{3\sqrt{10}\,\left(x-\frac{2}{3}\right)}{10}\right)}{9}$
trager	$\left(-\frac{1}{3}+\frac{x}{2}\right)\sqrt{-3x^2+4x+2}-\frac{5\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(3\,\mathrm{RootOf}\left(\_Z^2+3\right)x+3\sqrt{-3x^2+4x+2}-2\,\mathrm{RootOf}\left(\_Z^2+3\right)\right)}{9}$

input
```
int((-3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-6*x+4)*(-3*x^2+4*x+2)^(1/2)+5/9*3^(1/2)*arcsin(3/10*10^(1/2)*(x-2/
3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \sqrt{2+4x-3x^2}\,dx=\frac{1}{6}\sqrt{-3\,x^2+4\,x+2}(3\,x-2)-\frac{5}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+4\,x+2}(3\,x-2)}{3\,(3\,x^2-4\,x-2)}\right)$$

input
```
integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) - 5/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt
(-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 - 4*x - 2))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \sqrt{2+4x-3x^2}\,dx = \left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-3x^2+4x+2}+\frac{5\sqrt{3}\operatorname{asin}\left(\frac{3\sqrt{10}\left(x-\frac{2}{3}\right)}{10}\right)}{9}$$

input
```
integrate((-3*x**2+4*x+2)**(1/2),x)
```

output
```
(x/2 - 1/3)*sqrt(-3*x**2 + 4*x + 2) + 5*sqrt(3)*asin(3*sqrt(10)*(x - 2/3)/
10)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \sqrt{2+4x-3x^2}\,dx = \frac{1}{2}\sqrt{-3\,x^2+4\,x+2}x-\frac{5}{9}\sqrt{3}\arcsin\left(-\frac{1}{10}\sqrt{10}(3\,x-2)\right) - \frac{1}{3}\sqrt{-3\,x^2+4\,x+2}$$

input
```
integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-3*x^2 + 4*x + 2)*x - 5/9*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2)
) - 1/3*sqrt(-3*x^2 + 4*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \sqrt{2+4x-3x^2}\,dx = \frac{1}{6}\sqrt{-3\,x^2+4\,x+2}(3\,x-2)+\frac{5}{9}\sqrt{3}\arcsin\left(\frac{1}{10}\sqrt{10}(3\,x-2)\right)$$

input
```
integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) + 5/9*sqrt(3)*arcsin(1/10*sqrt(10)*(3
*x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \sqrt{2+4x-3x^2}\,dx = \frac{5\sqrt{3}\,\mathrm{asin}\left(\frac{\sqrt{10}\,(3\,x-2)}{10}\right)}{9}+\left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-3\,x^2+4\,x+2}$$

input
```
int((4*x - 3*x^2 + 2)^(1/2),x)
```

output
```
(5*3^(1/2)*asin((10^(1/2)*(3*x - 2))/10))/9 + (x/2 - 1/3)*(4*x - 3*x^2 + 2
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \sqrt{2+4x-3x^2}\,dx = \frac{5\sqrt{3}\,asin\left(\frac{3x-2}{\sqrt{10}}\right)}{9}+\frac{\sqrt{-3x^2+4x+2}\,x}{2}-\frac{\sqrt{-3x^2+4x+2}}{3}$$

input
```
int((-3*x^2+4*x+2)^(1/2),x)
```

output
```
(10*sqrt(3)*asin((3*x - 2)/sqrt(10)) + 9*sqrt( - 3*x**2 + 4*x + 2)*x - 6*s
qrt( - 3*x**2 + 4*x + 2))/18
```

3.116 $\int \sqrt{2+5x+3x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \sqrt{2+5x+3x^2}\,dx = \frac{1}{12}(5+6x)\sqrt{2+5x+3x^2} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}(1+x)}{\sqrt{2+5x+3x^2}}\right)}{12\sqrt{3}}$$

output
```
1/12*(5+6*x)*(3*x^2+5*x+2)^(1/2)-1/36*arctanh(3^(1/2)*(1+x)/(3*x^2+5*x+2)^
(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \sqrt{2+5x+3x^2}\,dx = \frac{1}{36}\left(3(5+6x)\sqrt{2+5x+3x^2} - \sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}+\frac{5x}{3}+x^2}}{1+x}\right)\right)$$

input
```
Integrate[Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(3*(5 + 6*x)*Sqrt[2 + 5*x + 3*x^2] - Sqrt[3]*ArcTanh[Sqrt[2/3 + (5*x)/3 +
x^2]/(1 + x)])/36
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x^2+5x+2}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2}-\frac{1}{24}\int \frac{1}{\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2}-\frac{1}{12}\int \frac{1}{12-\frac{(6x+5)^2}{3x^2+5x+2}}d\frac{6x+5}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2}-\frac{\operatorname{arctanh}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{24\sqrt{3}}$$

input
```
Int[Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
((5 + 6*x)*Sqrt[2 + 5*x + 3*x^2])/12 - ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2
 + 5*x + 3*x^2])]/(24*Sqrt[3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

method	result	si
default	$\frac{(6x+5)\sqrt{3x^2+5x+2}}{12}-\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}}{72}$	50
risch	$\frac{(6x+5)\sqrt{3x^2+5x+2}}{12}-\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}}{72}$	50
trager	$\left(\frac{x}{2}+\frac{5}{12}\right)\sqrt{3x^2+5x+2}+\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-6\operatorname{RootOf}\left(\_Z^2-3\right)x-5\operatorname{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2+5x+2}\right)}{72}$	6

input
```
int((3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(6*x+5)*(3*x^2+5*x+2)^(1/2)-1/72*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+
2)^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int\sqrt{2+5x+3x^2}\,dx=\frac{1}{12}\sqrt{3x^2+5x+2}(6x+5)+\frac{1}{144}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2+5x+2}(6x+5)+72x^2+120x+49\right)$$

input
```
integrate((3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/144*sqrt(3)*log(-4*sqrt(3)*sqrt(3
*x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x + 49)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \sqrt{2+5x+3x^2}\,dx$$
$$=\left(\frac{x}{2}+\frac{5}{12}\right)\sqrt{3x^2+5x+2}-\frac{\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+5x+2}+5\right)}{72}$$

input
```
integrate((3*x**2+5*x+2)**(1/2),x)
```

output
```
(x/2 + 5/12)*sqrt(3*x**2 + 5*x + 2) - sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x
**2 + 5*x + 2) + 5)/72
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \sqrt{2+5x+3x^2}\,dx=\frac{1}{2}\sqrt{3\,x^2+5\,x+2}x-\frac{1}{72}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3\,x^2+5\,x+2}+6\,x+5\right)+\frac{5}{12}\sqrt{3\,x^2+5\,x+2}$$

input
```
integrate((3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(3*x^2 + 5*x + 2)*x - 1/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x
+ 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \sqrt{2+5x+3x^2}\,dx = \frac{1}{12}\sqrt{3\,x^2+5\,x+2}(6\,x+5) + \frac{1}{72}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+5\,x+2}\right)-5\right|\right)$$

input
```
integrate((3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/72*sqrt(3)*log(abs(-2*sqrt(3)*(sq
rt(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \sqrt{2+5x+3x^2}\,dx = \left(\frac{x}{2}+\frac{5}{12}\right)\sqrt{3\,x^2+5\,x+2} - \frac{\sqrt{3}\,\ln\left(\sqrt{3\,x^2+5\,x+2}+\frac{\sqrt{3}\,\left(3\,x+\frac{5}{2}\right)}{3}\right)}{72}$$

input
```
int((5*x + 3*x^2 + 2)^(1/2),x)
```

output
```
(x/2 + 5/12)*(5*x + 3*x^2 + 2)^(1/2) - (3^(1/2)*log((5*x + 3*x^2 + 2)^(1/2
) + (3^(1/2)*(3*x + 5/2))/3))/72
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \sqrt{2+5x+3x^2}\,dx = \frac{\sqrt{3x^2+5x+2}\,x}{2} + \frac{5\sqrt{3x^2+5x+2}}{12} - \frac{\sqrt{3}\log\left(2\sqrt{3x^2+5x+2}\,\sqrt{3}+6x+5\right)}{72}$$

input
```
int((3*x^2+5*x+2)^(1/2),x)
```

output
```
(36*sqrt(3*x**2 + 5*x + 2)*x + 30*sqrt(3*x**2 + 5*x + 2) - sqrt(3)*log(2*s
qrt(3*x**2 + 5*x + 2)*sqrt(3) + 6*x + 5))/72
```

3.117 $\int \sqrt{2+5x-3x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \sqrt{2+5x-3x^2}\,dx = -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2} - \frac{49\arcsin\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

output
```
-1/12*(5-6*x)*(-3*x^2+5*x+2)^(1/2)+49/72*arcsin(-5/7+6/7*x)*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \sqrt{2+5x-3x^2}\,dx = \frac{1}{36}\left(3(-5+6x)\sqrt{2+5x-3x^2} - 49\sqrt{3}\arctan\left(\frac{\sqrt{6+15x-9x^2}}{1+3x}\right)\right)$$

input
```
Integrate[Sqrt[2 + 5*x - 3*x^2],x]
```

output
```
(3*(-5 + 6*x)*Sqrt[2 + 5*x - 3*x^2] - 49*Sqrt[3]*ArcTan[Sqrt[6 + 15*x - 9*
x^2]/(1 + 3*x)])/36
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-3x^2+5x+2}\,dx$$

$$\Big\downarrow \ 1087$$

$$\frac{49}{24}\int \frac{1}{\sqrt{-3x^2+5x+2}}dx-\frac{1}{12}(5-6x)\sqrt{-3x^2+5x+2}$$

$$\Big\downarrow \ 1090$$

$$-\frac{7\int \frac{1}{\sqrt{1-\frac{1}{49}(5-6x)^2}}d(5-6x)}{24\sqrt{3}}-\frac{1}{12}\sqrt{-3x^2+5x+2}(5-6x)$$

$$\Big\downarrow \ 223$$

$$-\frac{49\arcsin\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}-\frac{1}{12}\sqrt{-3x^2+5x+2}(5-6x)$$

input
```
Int[Sqrt[2 + 5*x - 3*x^2],x]
```

output
```
-1/12*((5 - 6*x)*Sqrt[2 + 5*x - 3*x^2]) - (49*ArcSin[(5 - 6*x)/7])/(24*Sqr
t[3])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result
default	$-\frac{(5-6x)\sqrt{-3x^2+5x+2}}{12}+\frac{49\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)\sqrt{3}}{72}$
risch	$-\frac{(3x^2-5x-2)(-5+6x)}{12\sqrt{-3x^2+5x+2}}+\frac{49\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)\sqrt{3}}{72}$
trager	$\left(-\frac{5}{12}+\frac{x}{2}\right)\sqrt{-3x^2+5x+2}-\frac{49\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2+3\right)x-5\,\mathrm{RootOf}\left(\_Z^2+3\right)+6\sqrt{-3x^2+5x+2}\right.}{72}$

input
```
int((-3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(5-6*x)*(-3*x^2+5*x+2)^(1/2)+49/72*arcsin(-5/7+6/7*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \sqrt{2+5x-3x^2}\,dx=\frac{1}{12}\sqrt{-3\,x^2+5\,x+2}(6\,x-5)-\frac{49}{72}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+5\,x+2}(6\,x-5)}{6\,(3\,x^2-5\,x-2)}\right)$$

input
```
integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) - 49/72*sqrt(3)*arctan(1/6*sqrt(3)*s
qrt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 - 5*x - 2))
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \sqrt{2+5x-3x^2}\,dx=\left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3x^2+5x+2}+\frac{49\sqrt{3}\,\mathrm{asin}\left(\frac{6x}{7}-\frac{5}{7}\right)}{72}$$

input
```
integrate((-3*x**2+5*x+2)**(1/2),x)
```

output
```
(x/2 - 5/12)*sqrt(-3*x**2 + 5*x + 2) + 49*sqrt(3)*asin(6*x/7 - 5/7)/72
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \sqrt{2+5x-3x^2}\,dx=\frac{1}{2}\sqrt{-3\,x^2+5\,x+2}x-\frac{49}{72}\sqrt{3}\arcsin\left(-\frac{6}{7}\,x+\frac{5}{7}\right)-\frac{5}{12}\sqrt{-3\,x^2+5\,x+2}$$

input
```
integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-3*x^2 + 5*x + 2)*x - 49/72*sqrt(3)*arcsin(-6/7*x + 5/7) - 5/12*s
qrt(-3*x^2 + 5*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \sqrt{2+5x-3x^2}\,dx = \frac{1}{12}\sqrt{-3\,x^2+5\,x+2}(6\,x-5)+\frac{49}{72}\sqrt{3}\arcsin\left(\frac{6}{7}\,x-\frac{5}{7}\right)$$

input
```
integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) + 49/72*sqrt(3)*arcsin(6/7*x - 5/7)
```

Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \sqrt{2+5x-3x^2}\,dx = \frac{49\,\sqrt{3}\,\mathrm{asin}\left(\frac{6\,x}{7}-\frac{5}{7}\right)}{72}+\left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3\,x^2+5\,x+2}$$

input
```
int((5*x - 3*x^2 + 2)^(1/2),x)
```

output
```
(49*3^(1/2)*asin((6*x)/7 - 5/7))/72 + (x/2 - 5/12)*(5*x - 3*x^2 + 2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \sqrt{2+5x-3x^2}\,dx = \frac{49\sqrt{3}\,asin\left(\frac{6x}{7}-\frac{5}{7}\right)}{72}+\frac{\sqrt{-3x^2+5x+2}\,x}{2}-\frac{5\sqrt{-3x^2+5x+2}}{12}$$

input
```
int((-3*x^2+5*x+2)^(1/2),x)
```

output
```
(49*sqrt(3)*asin((6*x - 5)/7) + 36*sqrt( - 3*x**2 + 5*x + 2)*x - 30*sqrt(
- 3*x**2 + 5*x + 2))/72
```

3.118 $\int \sqrt{-2+4x+3x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \sqrt{-2+4x+3x^2}\,dx=\frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2}-\frac{5\text{arctanh}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{3\sqrt{3}}$$

output
```
1/6*(2+3*x)*(3*x^2+4*x-2)^(1/2)-5/9*arctanh(1/3*(2+3*x)*3^(1/2)/(3*x^2+4*x
-2)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \sqrt{-2+4x+3x^2}\,dx=\frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2}-\frac{10\text{arctanh}\left(\frac{\sqrt{-6+12x+9x^2}}{2+\sqrt{10}+3x}\right)}{3\sqrt{3}}$$

input
```
Integrate[Sqrt[-2 + 4*x + 3*x^2],x]
```

output
```
((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (10*ArcTanh[Sqrt[-6 + 12*x + 9*x^2]
/(2 + Sqrt[10] + 3*x)])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x^2+4x-2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{5}{3}\int \frac{1}{\sqrt{3x^2+4x-2}}dx$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{10}{3}\int \frac{1}{12-\frac{4(3x+2)^2}{3x^2+4x-2}}d\frac{2(3x+2)}{\sqrt{3x^2+4x-2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{5\text{arctanh}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

input
```
Int[Sqrt[-2 + 4*x + 3*x^2],x]
```

output
```
((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (5*ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[
-2 + 4*x + 3*x^2])])/(3*Sqrt[3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

method	result	si
default	$\frac{(6x+4)\sqrt{3x^2+4x-2}}{12}-\frac{5\ln\left(\frac{(3x+2)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)\sqrt{3}}{9}$	5
risch	$\frac{(3x+2)\sqrt{3x^2+4x-2}}{6}-\frac{5\ln\left(\frac{(3x+2)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)\sqrt{3}}{9}$	5
trager	$\left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{3x^2+4x-2}+\frac{5\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-3\,\mathrm{RootOf}\left(\_Z^2-3\right)x-2\,\mathrm{RootOf}\left(\_Z^2-3\right)+3\sqrt{3x^2+4x-2}\right)}{9}$	6

input
```
int((3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(6*x+4)*(3*x^2+4*x-2)^(1/2)-5/9*ln(1/3*(3*x+2)*3^(1/2)+(3*x^2+4*x-2)^
(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \sqrt{-2+4x+3x^2}\,dx = \frac{1}{6}\,\sqrt{3\,x^2+4\,x-2}(3\,x+2) + \frac{5}{18}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+4\,x-2}(3\,x+2)+9\,x^2+12\,x-1\right)$$

input
```
integrate((3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2
 + 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \sqrt{-2+4x+3x^2}\,dx = \left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{3x^2+4x-2} - \frac{5\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+4x-2}+4\right)}{9}$$

input
```
integrate((3*x**2+4*x-2)**(1/2),x)
```

output
```
(x/2 + 1/3)*sqrt(3*x**2 + 4*x - 2) - 5*sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*
x**2 + 4*x - 2) + 4)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \sqrt{-2+4x+3x^2}\,dx = \frac{1}{2}\sqrt{3x^2+4x-2}\,x - \frac{5}{9}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+4x-2}+6x+4\right) + \frac{1}{3}\sqrt{3x^2+4x-2}$$

input
```
integrate((3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(3*x^2 + 4*x - 2)*x - 5/9*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x -
 2) + 6*x + 4) + 1/3*sqrt(3*x^2 + 4*x - 2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \sqrt{-2+4x+3x^2}\,dx = \frac{1}{6}\sqrt{3\,x^2+4\,x-2}(3\,x+2) + \frac{5}{9}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+4\,x-2}\right)-2\right|\right)$$

input
```
integrate((3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3
)*x - sqrt(3*x^2 + 4*x - 2)) - 2))
```

Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \sqrt{-2+4x+3x^2}\,dx = \left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{3\,x^2+4\,x-2} - \frac{5\sqrt{3}\,\ln\left(\sqrt{3\,x^2+4\,x-2}+\frac{\sqrt{3}\,(3\,x+2)}{3}\right)}{9}$$

input
```
int((4*x + 3*x^2 - 2)^(1/2),x)
```

output
```
(x/2 + 1/3)*(4*x + 3*x^2 - 2)^(1/2) - (5*3^(1/2)*log((4*x + 3*x^2 - 2)^(1/
2) + (3^(1/2)*(3*x + 2))/3))/9
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \sqrt{-2+4x+3x^2}\,dx = \frac{\sqrt{3x^2+4x-2}\,x}{2} + \frac{\sqrt{3x^2+4x-2}}{3} - \frac{5\sqrt{3}\log\left(\frac{\sqrt{3x^2+4x-2}\,\sqrt{3}+3x+2}{\sqrt{10}}\right)}{9}$$

input
```
int((3*x^2+4*x-2)^(1/2),x)
```

output
```
(9*sqrt(3*x**2 + 4*x - 2)*x + 6*sqrt(3*x**2 + 4*x - 2) - 10*sqrt(3)*log((s
qrt(3*x**2 + 4*x - 2)*sqrt(3) + 3*x + 2)/sqrt(10)))/18
```

3.119 $\int \sqrt{-2+4x-3x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \sqrt{-2+4x-3x^2}\,dx=-\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2}+\frac{\arctan\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{3\sqrt{3}}$$

output
```
-1/6*(2-3*x)*(-3*x^2+4*x-2)^(1/2)+1/9*arctan(1/3*(2-3*x)*3^(1/2)/(-3*x^2+4
*x-2)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \sqrt{-2+4x-3x^2}\,dx=\frac{1}{6}(-2+3x)\sqrt{-2+4x-3x^2}-\frac{\arctan\left(\frac{-2+3x}{\sqrt{-6+12x-9x^2}}\right)}{3\sqrt{3}}$$

input
```
Integrate[Sqrt[-2 + 4*x - 3*x^2],x]
```

output
```
((-2 + 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 - ArcTan[(-2 + 3*x)/Sqrt[-6 + 12*x -
 9*x^2]]/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-3x^2+4x-2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$-\frac{1}{3}\int \frac{1}{\sqrt{-3x^2+4x-2}}dx-\frac{1}{6}\sqrt{-3x^2+4x-2}(2-3x)$$

$$\Big\downarrow \text{1092}$$

$$-\frac{2}{3}\int \frac{1}{-\frac{4(2-3x)^2}{-3x^2+4x-2}-12}d\frac{2(2-3x)}{\sqrt{-3x^2+4x-2}}-\frac{1}{6}\sqrt{-3x^2+4x-2}(2-3x)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}}-\frac{1}{6}(2-3x)\sqrt{-3x^2+4x-2}$$

input
```
Int[Sqrt[-2 + 4*x - 3*x^2],x]
```

output
```
-1/6*((2 - 3*x)*Sqrt[-2 + 4*x - 3*x^2]) + ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-
2 + 4*x - 3*x^2])]/(3*Sqrt[3])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

method	result
default	$-\frac{(-6x+4)\sqrt{-3x^2+4x-2}}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x-\frac{2}{3}\right)}{\sqrt{-3x^2+4x-2}}\right)}{9}$
risch	$-\frac{(3x^2-4x+2)(-2+3x)}{6\sqrt{-3x^2+4x-2}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x-\frac{2}{3}\right)}{\sqrt{-3x^2+4x-2}}\right)}{9}$
trager	$\left(-\frac{1}{3}+\frac{x}{2}\right)\sqrt{-3x^2+4x-2}-\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(-3\operatorname{RootOf}\left(\_Z^2+3\right)x+2\operatorname{RootOf}\left(\_Z^2+3\right)+3\sqrt{-3x^2+4x-2}\right)}{9}$

input
```
int((-3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-6*x+4)*(-3*x^2+4*x-2)^(1/2)-1/9*3^(1/2)*arctan(3^(1/2)*(x-2/3)/(-3
*x^2+4*x-2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int \sqrt{-2+4x-3x^2}\,dx = \frac{1}{6}\sqrt{-3\,x^2+4\,x-2}(3\,x-2) - \frac{1}{6}\sqrt{-\frac{1}{3}}\log\left(-\frac{2\left(3\sqrt{-\frac{1}{3}}\sqrt{-3\,x^2+4\,x-2}+3\,x-2\right)}{x}\right) + \frac{1}{6}\sqrt{-\frac{1}{3}}\log\left(\frac{2\left(3\sqrt{-\frac{1}{3}}\sqrt{-3\,x^2+4\,x-2}-3\,x+2\right)}{x}\right)$$

input
```
integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(-3*x^2 + 4*x - 2)*(3*x - 2) - 1/6*sqrt(-1/3)*log(-2*(3*sqrt(-1/3)
*sqrt(-3*x^2 + 4*x - 2) + 3*x - 2)/x) + 1/6*sqrt(-1/3)*log(2*(3*sqrt(-1/3)
*sqrt(-3*x^2 + 4*x - 2) - 3*x + 2)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \sqrt{-2+4x-3x^2}\,dx = \left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-3x^2+4x-2} + \frac{\sqrt{3}i\log\left(-6x+2\sqrt{3}i\sqrt{-3x^2+4x-2}+4\right)}{9}$$

input
```
integrate((-3*x**2+4*x-2)**(1/2),x)
```

output
```
(x/2 - 1/3)*sqrt(-3*x**2 + 4*x - 2) + sqrt(3)*I*log(-6*x + 2*sqrt(3)*I*sqr
t(-3*x**2 + 4*x - 2) + 4)/9
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \sqrt{-2+4x-3x^2}\,dx = \frac{1}{2}\sqrt{-3\,x^2+4\,x-2}x + \frac{1}{9}i\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{2}(3\,x-2)\right) - \frac{1}{3}\sqrt{-3\,x^2+4\,x-2}$$

input
```
integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-3*x^2 + 4*x - 2)*x + 1/9*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2)
) - 1/3*sqrt(-3*x^2 + 4*x - 2)
```

Giac [F]

$$\int \sqrt{-2+4x-3x^2}\,dx = \int \sqrt{-3\,x^2+4\,x-2}\,dx$$

input
```
integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-3*x^2 + 4*x - 2), x)
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \sqrt{-2+4x-3x^2}\,dx = \frac{\sqrt{3}\,\mathrm{asin}\left(\frac{\sqrt{2}\,(3\,x-2)\,1\mathrm{i}}{2}\right)}{9} + \left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-3\,x^2+4\,x-2}$$

input
```
int((4*x - 3*x^2 - 2)^(1/2),x)
```

output
```
(3^(1/2)*asin((2^(1/2)*(3*x - 2)*1i)/2))/9 + (x/2 - 1/3)*(4*x - 3*x^2 - 2)
^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \sqrt{-2+4x-3x^2}\,dx = \frac{\sqrt{3}\,asinh\left(\frac{3x-2}{\sqrt{2}}\right)i}{9} - \frac{\sqrt{-3x^2+4x-2}\,x}{2} + \frac{\sqrt{-3x^2+4x-2}}{3}$$

input
```
int((-3*x^2+4*x-2)^(1/2),x)
```

output
```
(2*sqrt(3)*asinh((3*x - 2)/sqrt(2))*i - 9*sqrt( - 3*x**2 + 4*x - 2)*x + 6*
sqrt( - 3*x**2 + 4*x - 2))/18
```

3.120 $\int \sqrt{-2+5x+3x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \sqrt{-2+5x+3x^2}\,dx=\frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2}-\frac{49\text{arctanh}\left(\frac{\sqrt{3}(2+x)}{\sqrt{-2+5x+3x^2}}\right)}{12\sqrt{3}}$$

output
```
1/12*(5+6*x)*(3*x^2+5*x-2)^(1/2)-49/36*arctanh(3^(1/2)*(2+x)/(3*x^2+5*x-2)
^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \sqrt{-2+5x+3x^2}\,dx=\frac{1}{36}\left(3(5+6x)\sqrt{-2+5x+3x^2}-49\sqrt{3}\text{arctanh}\left(\frac{\sqrt{-\frac{2}{3}+\frac{5x}{3}+x^2}}{2+x}\right)\right)$$

input
```
Integrate[Sqrt[-2 + 5*x + 3*x^2],x]
```

output
```
(3*(5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2] - 49*Sqrt[3]*ArcTanh[Sqrt[-2/3 + (5*x)
/3 + x^2]/(2 + x)])/36
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x^2+5x-2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2}-\frac{49}{24}\int \frac{1}{\sqrt{3x^2+5x-2}}dx$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2}-\frac{49}{12}\int \frac{1}{12-\frac{(6x+5)^2}{3x^2+5x-2}}d\frac{6x+5}{\sqrt{3x^2+5x-2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2}-\frac{49\text{arctanh}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{24\sqrt{3}}$$

input
```
Int[Sqrt[-2 + 5*x + 3*x^2],x]
```

output
```
((5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2])/12 - (49*ArcTanh[(5 + 6*x)/(2*Sqrt[3]*S
qrt[-2 + 5*x + 3*x^2])])/(24*Sqrt[3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

method	result	s
default	$\frac{(6x+5)\sqrt{3x^2+5x-2}}{12}-\frac{49\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x-2}\right)\sqrt{3}}{72}$	5
risch	$\frac{(6x+5)\sqrt{3x^2+5x-2}}{12}-\frac{49\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x-2}\right)\sqrt{3}}{72}$	5
trager	$\left(\frac{x}{2}+\frac{5}{12}\right)\sqrt{3x^2+5x-2}-\frac{49\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+5\,\mathrm{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2+5x-2}\right)}{72}$	6

input
```
int((3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(6*x+5)*(3*x^2+5*x-2)^(1/2)-49/72*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x
-2)^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \sqrt{-2+5x+3x^2}\,dx = \frac{1}{12}\sqrt{3\,x^2+5\,x-2}(6\,x+5) + \frac{49}{144}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2+5\,x-2}(6\,x+5)+72\,x^2+120\,x+1\right)$$

input
```
integrate((3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/144*sqrt(3)*log(-4*sqrt(3)*sqrt(
3*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \sqrt{-2+5x+3x^2}\,dx = \left(\frac{x}{2}+\frac{5}{12}\right)\sqrt{3x^2+5x-2} - \frac{49\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+5x-2}+5\right)}{72}$$

input
```
integrate((3*x**2+5*x-2)**(1/2),x)
```

output
```
(x/2 + 5/12)*sqrt(3*x**2 + 5*x - 2) - 49*sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(
3*x**2 + 5*x - 2) + 5)/72
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \sqrt{-2+5x+3x^2}\,dx = \frac{1}{2}\sqrt{3x^2+5x-2}x - \frac{49}{72}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x-2}+6x+5\right) + \frac{5}{12}\sqrt{3x^2+5x-2}$$

input
```
integrate((3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(3*x^2 + 5*x - 2)*x - 49/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x
 - 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x - 2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \sqrt{-2+5x+3x^2}\,dx = \frac{1}{12}\sqrt{3x^2+5x-2}(6x+5) + \frac{49}{72}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x-2}\right)-5\right|\right)$$

input
```
integrate((3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/72*sqrt(3)*log(abs(-2*sqrt(3)*(s
qrt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \sqrt{-2+5x+3x^2}\,dx = \left(\frac{x}{2}+\frac{5}{12}\right)\sqrt{3\,x^2+5\,x-2} - \frac{49\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2+5\,x-2}+\frac{\sqrt{3}\,\left(3\,x+\frac{5}{2}\right)}{3}\right)}{72}$$

input
```
int((5*x + 3*x^2 - 2)^(1/2),x)
```

output
```
(x/2 + 5/12)*(5*x + 3*x^2 - 2)^(1/2) - (49*3^(1/2)*log((5*x + 3*x^2 - 2)^(
1/2) + (3^(1/2)*(3*x + 5/2))/3))/72
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \sqrt{-2+5x+3x^2}\,dx = \frac{\sqrt{3x^2+5x-2}\,x}{2} + \frac{5\sqrt{3x^2+5x-2}}{12} - \frac{49\sqrt{3}\log\left(\frac{2\sqrt{3x^2+5x-2}\,\sqrt{3}}{7}+\frac{6x}{7}+\frac{5}{7}\right)}{72}$$

input
```
int((3*x^2+5*x-2)^(1/2),x)
```

output
```
(36*sqrt(3*x**2 + 5*x - 2)*x + 30*sqrt(3*x**2 + 5*x - 2) - 49*sqrt(3)*log(
(2*sqrt(3*x**2 + 5*x - 2)*sqrt(3) + 6*x + 5)/7))/72
```

3.121 $\int \sqrt{-2+5x-3x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \sqrt{-2+5x-3x^2}\,dx = -\frac{1}{12}(5-6x)\sqrt{-2+5x-3x^2} - \frac{\arcsin(5-6x)}{24\sqrt{3}}$$

output
```
-1/12*(5-6*x)*(-3*x^2+5*x-2)^(1/2)+1/72*arcsin(-5+6*x)*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \sqrt{-2+5x-3x^2}\,dx = \frac{1}{36}\left(3(-5+6x)\sqrt{-2+5x-3x^2} - \sqrt{3}\arctan\left(\frac{\sqrt{-6+15x-9x^2}}{-2+3x}\right)\right)$$

input
```
Integrate[Sqrt[-2 + 5*x - 3*x^2],x]
```

output
```
(3*(-5 + 6*x)*Sqrt[-2 + 5*x - 3*x^2] - Sqrt[3]*ArcTan[Sqrt[-6 + 15*x - 9*x
^2]/(-2 + 3*x)])/36
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-3x^2+5x-2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{24}\int \frac{1}{\sqrt{-3x^2+5x-2}}dx-\frac{1}{12}(5-6x)\sqrt{-3x^2+5x-2}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\int \frac{1}{\sqrt{1-(5-6x)^2}}d(5-6x)}{24\sqrt{3}}-\frac{1}{12}\sqrt{-3x^2+5x-2}(5-6x)$$

$$\Big\downarrow \text{223}$$

$$-\frac{\arcsin(5-6x)}{24\sqrt{3}}-\frac{1}{12}\sqrt{-3x^2+5x-2}(5-6x)$$

input
```
Int[Sqrt[-2 + 5*x - 3*x^2],x]
```

output
```
-1/12*((5 - 6*x)*Sqrt[-2 + 5*x - 3*x^2]) - ArcSin[5 - 6*x]/(24*Sqrt[3])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result
default	$-\frac{(5-6x)\sqrt{-3x^2+5x-2}}{12}+\frac{\arcsin(-5+6x)\sqrt{3}}{72}$
risch	$-\frac{(3x^2-5x+2)(-5+6x)}{12\sqrt{-3x^2+5x-2}}+\frac{\arcsin(-5+6x)\sqrt{3}}{72}$
trager	$\left(-\frac{5}{12}+\frac{x}{2}\right)\sqrt{-3x^2+5x-2}+\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2+3\right)x+5\,\mathrm{RootOf}\left(\_Z^2+3\right)+6\sqrt{-3x^2+5x-2}\right.}{72}$

input
```
int((-3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(5-6*x)*(-3*x^2+5*x-2)^(1/2)+1/72*arcsin(-5+6*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \sqrt{-2+5x-3x^2}\,dx = \frac{1}{12}\sqrt{-3\,x^2+5\,x-2}(6\,x-5) - \frac{1}{72}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+5\,x-2}(6\,x-5)}{6\,(3\,x^2-5\,x+2)}\right)$$

input
```
integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) - 1/72*sqrt(3)*arctan(1/6*sqrt(3)*sq
rt(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 - 5*x + 2))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \sqrt{-2+5x-3x^2}\,dx = \left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3x^2+5x-2}+\frac{\sqrt{3}\,\mathrm{asin}\,(6x-5)}{72}$$

input
```
integrate((-3*x**2+5*x-2)**(1/2),x)
```

output
```
(x/2 - 5/12)*sqrt(-3*x**2 + 5*x - 2) + sqrt(3)*asin(6*x - 5)/72
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \sqrt{-2+5x-3x^2}\,dx = \frac{1}{2}\sqrt{-3\,x^2+5\,x-2}x+\frac{1}{72}\sqrt{3}\arcsin\left(6\,x-5\right) - \frac{5}{12}\sqrt{-3\,x^2+5\,x-2}$$

input
```
integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-3*x^2 + 5*x - 2)*x + 1/72*sqrt(3)*arcsin(6*x - 5) - 5/12*sqrt(-3
*x^2 + 5*x - 2)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \sqrt{-2+5x-3x^2}\,dx = \frac{1}{12}\sqrt{-3\,x^2+5\,x-2}(6\,x-5)+\frac{1}{72}\sqrt{3}\arcsin\left(6\,x-5\right)$$

input
```
integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) + 1/72*sqrt(3)*arcsin(6*x - 5)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \sqrt{-2+5x-3x^2}\,dx = \frac{\sqrt{3}\,\mathrm{asin}(6\,x-5)}{72}+\left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3\,x^2+5\,x-2}$$

input
```
int((5*x - 3*x^2 - 2)^(1/2),x)
```

output
```
(3^(1/2)*asin(6*x - 5))/72 + (x/2 - 5/12)*(5*x - 3*x^2 - 2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \sqrt{-2+5x-3x^2}\,dx = \frac{\sqrt{3}\,asin(6x-5)}{72}+\frac{\sqrt{-3x^2+5x-2}\,x}{2}-\frac{5\sqrt{-3x^2+5x-2}}{12}$$

input
```
int((-3*x^2+5*x-2)^(1/2),x)
```

output
```
(sqrt(3)*asin(6*x - 5) + 36*sqrt( - 3*x**2 + 5*x - 2)*x - 30*sqrt( - 3*x**
2 + 5*x - 2))/72
```

3.122 $\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx=\frac{1}{3}\text{arcsinh}\left(\frac{1}{2}(-1+3x)\right)$$

output
```
1/3*arcsinh(-1/2+3/2*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx=-\frac{1}{3}\log\left(1-3x+\sqrt{5-6x+9x^2}\right)$$

input
```
Integrate[1/Sqrt[5 - 6*x + 9*x^2],x]
```

output
```
-1/3*Log[1 - 3*x + Sqrt[5 - 6*x + 9*x^2]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{9x^2-6x+5}}dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{36}\int \frac{1}{\sqrt{\frac{1}{144}(18x-6)^2+1}}d(18x-6)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{3}\text{arcsinh}\left(\frac{1}{12}(18x-6)\right)$$

input
```
Int[1/Sqrt[5 - 6*x + 9*x^2],x]
```

output
```
ArcSinh[(-6 + 18*x)/12]/3
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\operatorname{arcsinh}\left(-\frac{1}{2}+\frac{3x}{2}\right)}{3}$	9
trager	$\frac{\ln\left(-1+3x+\sqrt{9x^2-6x+5}\right)}{3}$	21

input
```
int(1/(9*x^2-6*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsinh(-1/2+3/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx = -\frac{1}{3}\,\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

input
```
integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*log(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{3x}{2}-\frac{1}{2}\right)}{3}$$

input
```
integrate(1/(9*x**2-6*x+5)**(1/2),x)
```

output
```
asinh(3*x/2 - 1/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx = \frac{1}{3}\operatorname{arsinh}\left(\frac{3}{2}x-\frac{1}{2}\right)$$

input
```
integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="maxima")
```

output
```
1/3*arcsinh(3/2*x - 1/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(8) = 16$.

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.86

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx = \frac{1}{6}\sqrt{9x^2-6x+5}(3x-1)-\frac{2}{3}\log\left(-3x+\sqrt{9x^2-6x+5}+1\right)$$

input
```
integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5)
 + 1)
```

Mupad [B] (verification not implemented)

Time = 10.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx = \frac{\ln\left(3\,x+\sqrt{9\,x^2-6\,x+5}-1\right)}{3}$$

input
```
int(1/(9*x^2 - 6*x + 5)^(1/2),x)
```

output
```
log(3*x + (9*x^2 - 6*x + 5)^(1/2) - 1)/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx = \frac{\log\left(\frac{\sqrt{9x^2-6x+5}}{2}+\frac{3x}{2}-\frac{1}{2}\right)}{3}$$

input
```
int(1/(9*x^2-6*x+5)^(1/2),x)
```

output
```
log((sqrt(9*x**2 - 6*x + 5) + 3*x - 1)/2)/3
```

3.123 $\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 10

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx=\frac{1}{2}\arcsin\left(\frac{1}{2}+x\right)$$

output
```
1/2*arcsin(1/2+x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx=-\arctan\left(\frac{\sqrt{3-4x-4x^2}}{3+2x}\right)$$

input
```
Integrate[1/Sqrt[3 - 4*x - 4*x^2],x]
```

output
```
-ArcTan[Sqrt[3 - 4*x - 4*x^2]/(3 + 2*x)]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-4x^2-4x+3}}dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{16}\int \frac{1}{\sqrt{1-\frac{1}{64}(-8x-4)^2}}d(-8x-4)$$

$$\Big\downarrow \text{223}$$

$$-\frac{1}{2}\arcsin\left(\frac{1}{8}(-8x-4)\right)$$

input
```
Int[1/Sqrt[3 - 4*x - 4*x^2],x]
```

output
```
-1/2*ArcSin[(-4 - 8*x)/8]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arcsin\left(\frac{1}{2}+x\right)}{2}$	7
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right) \ln\left(2 \operatorname{RootOf}\left(\_Z^2+1\right) x+\sqrt{-4x^2-4x+3}+\operatorname{RootOf}\left(\_Z^2+1\right)\right)}{2}$	38

input
```
int(1/(-4*x^2-4*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(1/2+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(6) = 12.

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 3.30

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx = -\frac{1}{2}\arctan\left(\frac{\sqrt{-4\,x^2-4\,x+3}(2\,x+1)}{4\,x^2+4\,x-3}\right)$$

input
```
integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*arctan(sqrt(-4*x^2 - 4*x + 3)*(2*x + 1)/(4*x^2 + 4*x - 3))
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx = \frac{\operatorname{asin}\left(x+\frac{1}{2}\right)}{2}$$

input
```
integrate(1/(-4*x**2-4*x+3)**(1/2),x)
```

output
```
asin(x + 1/2)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx = -\frac{1}{2}\arcsin\left(-x-\frac{1}{2}\right)$$

input
```
integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arcsin(-x - 1/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(6) = 12.

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx = \frac{1}{4}\sqrt{-4x^2-4x+3}(2x+1)+\arcsin\left(x+\frac{1}{2}\right)$$

input
```
integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) + arcsin(x + 1/2)
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx = \frac{\operatorname{asin}\left(x+\frac{1}{2}\right)}{2}$$

input
```
int(1/(3 - 4*x^2 - 4*x)^(1/2),x)
```

output
```
asin(x + 1/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx = \frac{asin\left(x+\frac{1}{2}\right)}{2}$$

input
```
int(1/(-4*x^2-4*x+3)^(1/2),x)
```

output
```
asin((2*x + 1)/2)/2
```

3.124 $\int \frac{1}{\sqrt{-8+6x+9x^2}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{1}{\sqrt{-8+6x+9x^2}} \, dx = -\frac{2}{3}\text{arctanh}\left(\frac{2-3x}{\sqrt{-8+6x+9x^2}}\right)$$

output
```
-2/3*arctanh((2-3*x)/(9*x^2+6*x-8)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-8+6x+9x^2}} \, dx = -\frac{1}{3}\log\left(-1-3x+\sqrt{-8+6x+9x^2}\right)$$

input
```
Integrate[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

output
```
-1/3*Log[-1 - 3*x + Sqrt[-8 + 6*x + 9*x^2]]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{9x^2+6x-8}}dx$$

$$\Big\downarrow \ 1092$$

$$2\int \frac{1}{36-\frac{36(3x+1)^2}{9x^2+6x-8}}d\frac{6(3x+1)}{\sqrt{9x^2+6x-8}}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{3}\text{arctanh}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

input
```
Int[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

output
```
ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
trager	$-\frac{\ln\left(-1-3x+\sqrt{9x^2+6x-8}\right)}{3}$	21
default	$\frac{\ln\left(\frac{(9x+3)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)\sqrt{9}}{9}$	30

input
```
int(1/(9*x^2+6*x-8)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(-1-3*x+(9*x^2+6*x-8)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = -\frac{1}{3}\log\left(-3\,x+\sqrt{9\,x^2+6\,x-8}-1\right)$$

input
```
integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*log(-3*x + sqrt(9*x^2 + 6*x - 8) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)}{3}$$

input
```
integrate(1/(9*x**2+6*x-8)**(1/2),x)
```

output
```
log(18*x + 6*sqrt(9*x**2 + 6*x - 8) + 6)/3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{1}{3}\log\left(18\,x + 6\sqrt{9\,x^2+6\,x-8} + 6\right)$$

input
```
integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="maxima")
```

output
```
1/3*log(18*x + 6*sqrt(9*x^2 + 6*x - 8) + 6)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{1}{6}\sqrt{9\,x^2+6\,x-8}(3\,x+1) + \frac{3}{2}\log\left(\left|-3\,x + \sqrt{9\,x^2+6\,x-8} - 1\right|\right)$$

input
```
integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(abs(-3*x + sqrt(9*x^2 + 6*x
- 8) - 1))
```

Mupad [B] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{\ln\left(3\,x + \sqrt{9\,x^2+6\,x-8} + 1\right)}{3}$$

input
```
int(1/(6*x + 9*x^2 - 8)^(1/2),x)
```

output
```
log(3*x + (6*x + 9*x^2 - 8)^(1/2) + 1)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{\log\left(\frac{\sqrt{9x^2+6x-8}}{3}+x+\frac{1}{3}\right)}{3}$$

input
```
int(1/(9*x^2+6*x-8)^(1/2),x)
```

output
```
log((sqrt(9*x**2 + 6*x - 8) + 3*x + 1)/3)/3
```

3.125 $\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 18

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx=\frac{\operatorname{arcsinh}\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{3}}$$

output
```
1/3*arcsinh(1/2*(2+3*x)*2^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx=-\frac{\log\left(-2-3x+\sqrt{6+12x+9x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[2 + 4*x + 3*x^2],x]
```

output
```
-(Log[-2 - 3*x + Sqrt[6 + 12*x + 9*x^2]]/Sqrt[3])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^2+4x+2}}dx$$
$$\Big\downarrow \text{1090}$$
$$\frac{\int \frac{1}{\sqrt{\frac{1}{8}(6x+4)^2+1}}d(6x+4)}{2\sqrt{6}}$$
$$\Big\downarrow \text{222}$$
$$\frac{\operatorname{arcsinh}\left(\frac{6x+4}{2\sqrt{2}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[2 + 4*x + 3*x^2],x]
```

output
```
ArcSinh[(4 + 6*x)/(2*Sqrt[2])]/Sqrt[3]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{3\sqrt{2}\,\left(x+\frac{2}{3}\right)}{2}\right)}{3}$	15
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^2-3\right)x+2\operatorname{RootOf}\left(\_Z^2-3\right)+3\sqrt{3x^2+4x+2}\right)}{3}$	42

input
```
int(1/(3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arcsinh(3/2*2^(1/2)*(x+2/3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(16) = 32.

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+4\,x+2}(3\,x+2)-9\,x^2-12\,x-5\right)$$

input
```
integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x -
5)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx = \frac{\sqrt{3}\operatorname{asinh}\left(\frac{3\sqrt{2}\left(x+\frac{2}{3}\right)}{2}\right)}{3}$$

input
```
integrate(1/(3*x**2+4*x+2)**(1/2),x)
```

output
```
sqrt(3)*asinh(3*sqrt(2)*(x + 2/3)/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx = \frac{1}{3}\,\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{2}(3\,x+2)\right)$$

input
```
integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(16) = 32.

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx = \frac{1}{6}\,\sqrt{3\,x^2+4\,x+2}(3\,x+2) - \frac{1}{9}\,\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+4\,x+2}\right)-2\right)$$

input
```
integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 1/9*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x
- sqrt(3*x^2 + 4*x + 2)) - 2)
```

Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3}\,\left(x+\frac{2}{3}\right)+\sqrt{3\,x^2+4\,x+2}\right)}{3}$$

input
```
int(1/(4*x + 3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x + 2/3) + (4*x + 3*x^2 + 2)^(1/2)))/3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(\frac{\sqrt{3x^2+4x+2}\,\sqrt{3}+3x+2}{\sqrt{2}}\right)}{3}$$

input
```
int(1/(3*x^2+4*x+2)^(1/2),x)
```

output
```
(sqrt(3)*log((sqrt(3*x**2 + 4*x + 2)*sqrt(3) + 3*x + 2)/sqrt(2)))/3
```

3.126 $\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx = -\frac{\arcsin\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

output
```
-1/3*arcsin(1/10*(2-3*x)*10^(1/2))*3^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{3}x}{\sqrt{2}-\sqrt{2+4x-3x^2}}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[2 + 4*x - 3*x^2],x]
```

output
```
(-2*ArcTan[(Sqrt[3]*x)/(Sqrt[2] - Sqrt[2 + 4*x - 3*x^2])])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^2+4x+2}}dx$$
$$\Big\downarrow \text{ 1090}$$
$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{40}(4-6x)^2}}d(4-6x)}{2\sqrt{30}}$$
$$\Big\downarrow \text{ 223}$$
$$-\frac{\arcsin\left(\frac{4-6x}{2\sqrt{10}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[2 + 4*x - 3*x^2],x]
```

output
```
-(ArcSin[(4 - 6*x)/(2*Sqrt[10])]/Sqrt[3])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\sqrt{3}\,\arcsin\left(\frac{3\sqrt{10}\,\left(x-\frac{2}{3}\right)}{10}\right)}{3}$	15
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^2+3\right)x+3\sqrt{-3x^2+4x+2}-2\operatorname{RootOf}\left(\_Z^2+3\right)\right)}{3}$	42

input
```
int(1/(-3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arcsin(3/10*10^(1/2)*(x-2/3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(16) = 32.

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+4\,x+2}(3\,x-2)}{3\,(3\,x^2-4\,x-2)}\right)$$

input
```
integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 -
4*x - 2))
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}\left(\frac{3\sqrt{10}\left(x-\frac{2}{3}\right)}{10}\right)}{3}$$

input
```
integrate(1/(-3*x**2+4*x+2)**(1/2),x)
```

output
```
sqrt(3)*asin(3*sqrt(10)*(x - 2/3)/10)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx = -\frac{1}{3}\sqrt{3}\arcsin\left(-\frac{1}{10}\sqrt{10}(3x-2)\right)$$

input
```
integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(16) = 32.

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx = \frac{1}{6}\sqrt{-3x^2+4x+2}(3x-2)+\frac{5}{9}\sqrt{3}\arcsin\left(\frac{1}{10}\sqrt{10}(3x-2)\right)$$

input
```
integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) + 5/9*sqrt(3)*arcsin(1/10*sqrt(10)*(3
*x - 2))
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}\left(\frac{\sqrt{40}\,(6\,x-4)}{40}\right)}{3}$$

input
```
int(1/(4*x - 3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*asin((40^(1/2)*(6*x - 4))/40))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx = \frac{\sqrt{3}\,asin\left(\frac{3x-2}{\sqrt{10}}\right)}{3}$$

input
```
int(1/(-3*x^2+4*x+2)^(1/2),x)
```

output
```
(sqrt(3)*asin((3*x - 2)/sqrt(10)))/3
```

3.127 $\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}(1+x)}{\sqrt{2+5x+3x^2}}\right)}{\sqrt{3}}$$

output
```
2/3*arctanh(3^(1/2)*(1+x)/(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}+\frac{5x}{3}+x^2}}{1+x}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(2*ArcTanh[Sqrt[2/3 + (5*x)/3 + x^2]/(1 + x)])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{1092}$$

$$2\int \frac{1}{12-\frac{(6x+5)^2}{3x^2+5x+2}}d\frac{6x+5}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[3]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^{2}+5x+2}\right)\sqrt{3}}{3}$	30
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(-6\operatorname{RootOf}\left(\_Z^{2}-3\right)x-5\operatorname{RootOf}\left(\_Z^{2}-3\right)+6\sqrt{3x^{2}+5x+2}\right)}{3}$	42

input
```
int(1/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{1}{6}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^{2}+5x+2}(6x+5)+72x^{2}+120x+49\right)$$

input
```
integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x
 + 49)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^{2}+5x+2}+5\right)}{3}$$

input
```
integrate(1/(3*x**2+5*x+2)**(1/2),x)
```

output
```
sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x**2 + 5*x + 2) + 5)/3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3\,x^2+5\,x+2}+6\,x+5\right)$$

input
```
integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{1}{12}\sqrt{3\,x^2+5\,x+2}(6\,x+5) + \frac{1}{72}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+5\,x+2}\right)-5\right|\right)$$

input
```
integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/72*sqrt(3)*log(abs(-2*sqrt(3)*(sq
rt(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3}\left(x+\frac{5}{6}\right)+\sqrt{3\,x^2+5\,x+2}\right)}{3}$$

input
```
int(1/(5*x + 3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x + 5/6) + (5*x + 3*x^2 + 2)^(1/2)))/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(2\sqrt{3x^2+5x+2}\,\sqrt{3}+6x+5\right)}{3}$$

input
```
int(1/(3*x^2+5*x+2)^(1/2),x)
```

output
```
(sqrt(3)*log(2*sqrt(3*x**2 + 5*x + 2)*sqrt(3) + 6*x + 5))/3
```

3.128 $\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 17

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx = -\frac{\arcsin\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

output
```
1/3*arcsin(-5/7+6/7*x)*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{6+15x-9x^2}}{1+3x}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[2 + 5*x - 3*x^2],x]
```

output
```
(-2*ArcTan[Sqrt[6 + 15*x - 9*x^2]/(1 + 3*x)])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^2+5x+2}}dx$$
$$\Big\downarrow \text{1090}$$
$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{49}(5-6x)^2}}d(5-6x)}{7\sqrt{3}}$$
$$\Big\downarrow \text{223}$$
$$-\frac{\arcsin\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[2 + 5*x - 3*x^2],x]
```

output
```
-(ArcSin[(5 - 6*x)/7]/Sqrt[3])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)\sqrt{3}}{3}$	12
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(-6\operatorname{RootOf}\left(\_Z^2+3\right)x+6\sqrt{-3x^2+5x+2}+5\operatorname{RootOf}\left(\_Z^2+3\right)\right)}{3}$	42

input
```
int(1/(-3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsin(-5/7+6/7*x)*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(11) = 22$.

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.35

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+5\,x+2}(6\,x-5)}{6\,(3\,x^2-5\,x-2)}\right)$$

input
```
integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 -
5*x - 2))
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}\left(\frac{6x}{7}-\frac{5}{7}\right)}{3}$$

input
```
integrate(1/(-3*x**2+5*x+2)**(1/2),x)
```

output
```
sqrt(3)*asin(6*x/7 - 5/7)/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx = -\frac{1}{3}\,\sqrt{3}\arcsin\left(-\frac{6}{7}\,x+\frac{5}{7}\right)$$

input
```
integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arcsin(-6/7*x + 5/7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx = \frac{1}{12}\,\sqrt{-3\,x^2+5\,x+2}(6\,x-5)+\frac{49}{72}\,\sqrt{3}\arcsin\left(\frac{6}{7}\,x-\frac{5}{7}\right)$$

input
```
integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) + 49/72*sqrt(3)*arcsin(6/7*x - 5/7)
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}\left(\frac{6\,x}{7}-\frac{5}{7}\right)}{3}$$

input
```
int(1/(5*x - 3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*asin((6*x)/7 - 5/7))/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx = \frac{\sqrt{3}\,asin\left(\frac{6x}{7}-\frac{5}{7}\right)}{3}$$

input
```
int(1/(-3*x^2+5*x+2)^(1/2),x)
```

output
```
(sqrt(3)*asin((6*x - 5)/7))/3
```

3.129 $\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{\sqrt{3}}$$

output
```
1/3*arctanh(1/3*(2+3*x)*3^(1/2)/(3*x^2+4*x-2)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = -\frac{\log\left(-2-3x+\sqrt{-6+12x+9x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[-2 + 4*x + 3*x^2],x]
```

output
```
-(Log[-2 - 3*x + Sqrt[-6 + 12*x + 9*x^2]]/Sqrt[3])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^2+4x-2}}dx$$
$$\Big\downarrow \text{1092}$$
$$2\int \frac{1}{12-\frac{4(3x+2)^2}{3x^2+4x-2}}d\frac{2(3x+2)}{\sqrt{3x^2+4x-2}}$$
$$\Big\downarrow \text{219}$$
$$\frac{\text{arctanh}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[-2 + 4*x + 3*x^2],x]
```

output
```
ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x + 3*x^2])]/Sqrt[3]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\ln\left(\frac{(3x+2)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)\sqrt{3}}{3}$	30
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^2-3\right)x+2\operatorname{RootOf}\left(\_Z^2-3\right)+3\sqrt{3x^2+4x-2}\right)}{3}$	42

input
```
int(1/(3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(1/3*(3*x+2)*3^(1/2)+(3*x^2+4*x-2)^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+4\,x-2}(3\,x+2)+9\,x^2+12\,x-1\right)$$

input
```
integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(sqrt(3)*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x - 1
)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+4x-2}+4\right)}{3}$$

input
```
integrate(1/(3*x**2+4*x-2)**(1/2),x)
```

output
```
sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x**2 + 4*x - 2) + 4)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = \frac{1}{3}\,\sqrt{3}\log\left(2\,\sqrt{3}\sqrt{3\,x^2+4\,x-2}+6\,x+4\right)$$

input
```
integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x - 2) + 6*x + 4)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = \frac{1}{6}\,\sqrt{3\,x^2+4\,x-2}(3\,x+2) + \frac{5}{9}\,\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+4\,x-2}\right)-2\right|\right)$$

input
```
integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3
)*x - sqrt(3*x^2 + 4*x - 2)) - 2))
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3}\left(x+\frac{2}{3}\right)+\sqrt{3\,x^2+4\,x-2}\right)}{3}$$

input
```
int(1/(4*x + 3*x^2 - 2)^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x + 2/3) + (4*x + 3*x^2 - 2)^(1/2)))/3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(\frac{\sqrt{3x^2+4x-2}\,\sqrt{3}+3x+2}{\sqrt{10}}\right)}{3}$$

input
```
int(1/(3*x^2+4*x-2)^(1/2),x)
```

output
```
(sqrt(3)*log((sqrt(3*x**2 + 4*x - 2)*sqrt(3) + 3*x + 2)/sqrt(10)))/3
```

3.130 $\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{\arctan\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{\sqrt{3}}$$

output
```
-1/3*arctan(1/3*(2-3*x)*3^(1/2)/(-3*x^2+4*x-2)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{\arctan\left(\frac{2-3x}{\sqrt{-6+12x-9x^2}}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[-2 + 4*x - 3*x^2],x]
```

output
```
-(ArcTan[(2 - 3*x)/Sqrt[-6 + 12*x - 9*x^2]]/Sqrt[3])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^2+4x-2}}dx$$

$$\Big\downarrow \text{1092}$$

$$2\int \frac{1}{-\frac{4(2-3x)^2}{-3x^2+4x-2}-12}d\frac{2(2-3x)}{\sqrt{-3x^2+4x-2}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[-2 + 4*x - 3*x^2],x]
```

output
```
-(ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x - 3*x^2])]/Sqrt[3])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x-\frac{2}{3}\right)}{\sqrt{-3x^{2}+4x-2}}\right)}{3}$	26
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^{2}+3\right)x+3\sqrt{-3x^{2}+4x-2}-2\operatorname{RootOf}\left(\_Z^{2}+3\right)\right)}{3}$	42

input
```
int(1/(-3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arctan(3^(1/2)*(x-2/3)/(-3*x^2+4*x-2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(28) = 56.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.03

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = \frac{1}{2}\sqrt{-\frac{1}{3}}\log\left(-\frac{2\left(3\sqrt{-\frac{1}{3}}\sqrt{-3x^2+4x-2}+3x-2\right)}{x}\right) - \frac{1}{2}\sqrt{-\frac{1}{3}}\log\left(\frac{2\left(3\sqrt{-\frac{1}{3}}\sqrt{-3x^2+4x-2}-3x+2\right)}{x}\right)$$

input
```
integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-1/3)*log(-2*(3*sqrt(-1/3)*sqrt(-3*x^2 + 4*x - 2) + 3*x - 2)/x) -
 1/2*sqrt(-1/3)*log(2*(3*sqrt(-1/3)*sqrt(-3*x^2 + 4*x - 2) - 3*x + 2)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{\sqrt{3}i\log\left(-6x+2\sqrt{3}i\sqrt{-3x^2+4x-2}+4\right)}{3}$$

input
```
integrate(1/(-3*x**2+4*x-2)**(1/2),x)
```

output
```
-sqrt(3)*I*log(-6*x + 2*sqrt(3)*I*sqrt(-3*x**2 + 4*x - 2) + 4)/3
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{1}{3}i\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{2}(3\,x-2)\right)$$

input
```
integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2))
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = \int \frac{1}{\sqrt{-3\,x^2+4\,x-2}}\,dx$$

input
```
integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^2 + 4*x - 2), x)
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{\sqrt{3}\,\mathrm{asin}\left(\sqrt{2}\,\left(\frac{3\,x}{2}-1\right)\,1\mathrm{i}\right)}{3}$$

input
```
int(1/(4*x - 3*x^2 - 2)^(1/2),x)
```

output
```
-(3^(1/2)*asin(2^(1/2)*((3*x)/2 - 1)*1i))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = -\frac{\sqrt{3}\,asinh\left(\frac{3x-2}{\sqrt{2}}\right)i}{3}$$

input
```
int(1/(-3*x^2+4*x-2)^(1/2),x)
```

output
```
( - sqrt(3)*asinh((3*x - 2)/sqrt(2))*i)/3
```

3.131 $\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}(2+x)}{\sqrt{-2+5x+3x^2}}\right)}{\sqrt{3}}$$

output
```
2/3*arctanh(3^(1/2)*(2+x)/(3*x^2+5*x-2)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2}{3}+\frac{5x}{3}+x^2}}{2+x}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[-2 + 5*x + 3*x^2],x]
```

output
```
(2*ArcTanh[Sqrt[-2/3 + (5*x)/3 + x^2]/(2 + x)])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^2+5x-2}}dx$$
$$\Big\downarrow \text{1092}$$
$$2\int \frac{1}{12-\frac{(6x+5)^2}{3x^2+5x-2}}d\frac{6x+5}{\sqrt{3x^2+5x-2}}$$
$$\Big\downarrow \text{219}$$
$$\frac{\text{arctanh}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[-2 + 5*x + 3*x^2],x]
```

output
```
ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[-2 + 5*x + 3*x^2])]/Sqrt[3]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x-2}\right)\sqrt{3}}{3}$	30
trager	$\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+5\,\mathrm{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2+5x-2}\right)}{3}$	42

input
```
int(1/(3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x-2)^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^2+5\,x-2}(6\,x+5)+72\,x^2+120\,x+1\right)$$

input
```
integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120*x
 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+5x-2}+5\right)}{3}$$

input
```
integrate(1/(3*x**2+5*x-2)**(1/2),x)
```

output
```
sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x**2 + 5*x - 2) + 5)/3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = \frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x-2}+6x+5\right)$$

input
```
integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x - 2) + 6*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = \frac{1}{12}\sqrt{3x^2+5x-2}(6x+5) + \frac{49}{72}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x-2}\right)-5\right|\right)$$

input
```
integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/72*sqrt(3)*log(abs(-2*sqrt(3)*(s
qrt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = \frac{\sqrt{3}\ln\left(\sqrt{3}\left(x+\frac{5}{6}\right)+\sqrt{3\,x^2+5\,x-2}\right)}{3}$$

input
```
int(1/(5*x + 3*x^2 - 2)^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x + 5/6) + (5*x + 3*x^2 - 2)^(1/2)))/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(\frac{2\sqrt{3x^2+5x-2}\,\sqrt{3}}{7}+\frac{6x}{7}+\frac{5}{7}\right)}{3}$$

input
```
int(1/(3*x^2+5*x-2)^(1/2),x)
```

output
```
(sqrt(3)*log((2*sqrt(3*x**2 + 5*x - 2)*sqrt(3) + 6*x + 5)/7))/3
```

3.132 $\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx=-\frac{\arcsin(5-6x)}{\sqrt{3}}$$

output
```
1/3*arcsin(-5+6*x)*3^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.31

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{-6+15x-9x^2}}{-2+3x}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[-2 + 5*x - 3*x^2],x]
```

output
```
(-2*ArcTan[Sqrt[-6 + 15*x - 9*x^2]/(-2 + 3*x)])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^2+5x-2}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{\int \frac{1}{\sqrt{1-(5-6x)^2}}d(5-6x)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{\arcsin(5-6x)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[-2 + 5*x - 3*x^2],x]
```

output
```
-(ArcSin[5 - 6*x]/Sqrt[3])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\arcsin(-5+6x)\sqrt{3}}{3}$	12
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(6\operatorname{RootOf}\left(\_Z^2+3\right)x+6\sqrt{-3x^2+5x-2}-5\operatorname{RootOf}\left(\_Z^2+3\right)\right)}{3}$	42

input
```
int(1/(-3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsin(-5+6*x)*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(11) = 22.

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.08

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+5\,x-2}(6\,x-5)}{6\,(3\,x^2-5\,x+2)}\right)$$

input
```
integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 -
5*x + 2))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx = \frac{\sqrt{3}\operatorname{asin}(6x-5)}{3}$$

input
```
integrate(1/(-3*x**2+5*x-2)**(1/2),x)
```

output
```
sqrt(3)*asin(6*x - 5)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx = \frac{1}{3}\sqrt{3}\arcsin(6x-5)$$

input
```
integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arcsin(6*x - 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx = \frac{1}{12}\sqrt{-3x^2+5x-2}(6x-5) + \frac{1}{72}\sqrt{3}\arcsin(6x-5)$$

input
```
integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) + 1/72*sqrt(3)*arcsin(6*x - 5)
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}(6\,x-5)}{3}$$

input
```
int(1/(5*x - 3*x^2 - 2)^(1/2),x)
```

output
```
(3^(1/2)*asin(6*x - 5))/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx = \frac{\sqrt{3}\,asin(6x-5)}{3}$$

input
```
int(1/(-3*x^2+5*x-2)^(1/2),x)
```

output
```
(sqrt(3)*asin(6*x - 5))/3
```

3.133 $\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 22

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx=\frac{\operatorname{arcsinh}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

output
```
arcsinh(1/2*(2*c*x+b)/c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx=-\frac{\log\left(b+2cx-\sqrt{c}\sqrt{4+\frac{b^2}{c}+4bx+4cx^2}\right)}{\sqrt{c}}$$

input
```
Integrate[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2],x]
```

output
```
-(Log[b + 2*c*x - Sqrt[c]*Sqrt[4 + b^2/c + 4*b*x + 4*c*x^2]]/Sqrt[c])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{\int \frac{1}{\sqrt{\frac{(b+2cx)^2}{4c}+1}}d(b+2cx)}{2c}$$

$$\Big\downarrow \text{222}$$

$$\frac{\text{arcsinh}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
Int[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2],x]
```

output
```
ArcSinh[(b + 2*c*x)/(2*Sqrt[c])]/Sqrt[c]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(16) = 32$.

Time = 0.86 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

method	result	size
default	$\frac{\ln\left(\frac{(4cx+2b)\sqrt{4}}{4\sqrt{c}}+\sqrt{\frac{b^2+4c}{c}+4bx+4c\,x^2}\right)\sqrt{4}}{2\sqrt{c}}$	51

input
```
int(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(1/4*(4*c*x+2*b)*4^(1/2)/c^(1/2)+((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2))*
4^(1/2)/c^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(16) = 32$.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 6.23

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx = \left[\frac{\log\left(-4\,c^2x^2-4\,bcx-b^2-(2\,cx+b)\sqrt{c}\sqrt{\frac{4\,c^2x^2+4\,bcx+b^2+4\,c}{c}}-2\,c\right)}{2\,\sqrt{c}},\ -\frac{\sqrt{-c}\arctan\left(\frac{(2\,cx+b)\sqrt{-c}\sqrt{\frac{4\,c^2x^2+4\,bcx+b^2+4\,c}{c}}}{4\,c^2x^2+4\,bcx+b^2+4\,c}\right)}{c}\right]$$

input
```
integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-4*c^2*x^2 - 4*b*c*x - b^2 - (2*c*x + b)*sqrt(c)*sqrt((4*c^2*x^2
+ 4*b*c*x + b^2 + 4*c)/c) - 2*c)/sqrt(c), -sqrt(-c)*arctan((2*c*x + b)*sqr
t(-c)*sqrt((4*c^2*x^2 + 4*b*c*x + b^2 + 4*c)/c)/(4*c^2*x^2 + 4*b*c*x + b^2
 + 4*c))/c]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.64

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx = 2\left(\begin{cases} \frac{\log\left(4b+4\sqrt{c}\sqrt{4bx+4cx^2+\frac{b^2+4c}{c}}+8cx\right)}{2\sqrt{c}} & \text{for}\ c\neq 0\\ \frac{\sqrt{4bx+\frac{b^2+4c}{c}}}{2b} & \text{for}\ b\neq 0\\ \frac{x}{\sqrt{\frac{b^2+4c}{c}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(2/((b**2+4*c)/c+4*b*x+4*c*x**2)**(1/2),x)
```

output
```
2*Piecewise((log(4*b + 4*sqrt(c)*sqrt(4*b*x + 4*c*x**2 + (b**2 + 4*c)/c) +
 8*c*x)/(2*sqrt(c)), Ne(c, 0)), (sqrt(4*b*x + (b**2 + 4*c)/c)/(2*b), Ne(b,
 0)), (x/sqrt((b**2 + 4*c)/c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx = \frac{\operatorname{arsinh}\left(\frac{2\,cx+b}{2\,\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(1/2*(2*c*x + b)/sqrt(c))/sqrt(c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(16) = 32$.

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx = -\frac{\log\left(\left|-bc^2-\left(2\sqrt{c^3}x-\sqrt{4\,c^3x^2+4\,bc^2x+b^2c+4\,c^2}\right)\sqrt{c}|c|\right|\right)}{\sqrt{c}}$$

input
```
integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(-b*c^2 - (2*sqrt(c^3)*x - sqrt(4*c^3*x^2 + 4*b*c^2*x + b^2*c + 4*
c^2))*sqrt(c)*abs(c)))/sqrt(c)
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx = \frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+\sqrt{\frac{b^2+4\,c}{c}+4\,b\,x+4\,c\,x^2}\right)}{\sqrt{c}}$$

input
```
int(2/((4*c + b^2)/c + 4*b*x + 4*c*x^2)^(1/2),x)
```

output
```
log((b + 2*c*x)/c^(1/2) + ((4*c + b^2)/c + 4*b*x + 4*c*x^2)^(1/2))/c^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx = \frac{\sqrt{c}\log\left(\frac{\sqrt{4c^2x^2+4bcx+b^2+4c}+b+2cx}{2\sqrt{c}}\right)}{c}$$

input
```
int(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*log((sqrt(b**2 + 4*b*c*x + 4*c**2*x**2 + 4*c) + b + 2*c*x)/(2*sqr
t(c))))/c
```

3.134 $\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 23

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx=-\frac{\arcsin\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

output
```
-arcsin(1/2*(-2*c*x+b)/c^(1/2))/c^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 123 vs. $2(23) = 46$.

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 5.35

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx = 2\left(-\frac{\arctan\left(\frac{2\sqrt{-c^2}x-\sqrt{c}\sqrt{4-\frac{b^2}{c}+4bx-4cx^2}}{b}\right)}{2\sqrt{c}} - \frac{\log\left(2c^2x^2+c\left(-1-bx+\sqrt{-c}x\sqrt{4-\frac{b^2}{c}+4bx-4cx^2}\right)\right)}{4\sqrt{-c}}\right)$$

input
```
Integrate[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2],x]
```

output
```
2*(-1/2*ArcTan[(2*Sqrt[-c^2]*x - Sqrt[c]*Sqrt[4 - b^2/c + 4*b*x - 4*c*x^2]
)/b]/Sqrt[c] - Log[2*c^2*x^2 + c*(-1 - b*x + Sqrt[-c]*x*Sqrt[4 - b^2/c + 4
*b*x - 4*c*x^2])]/(4*Sqrt[-c]))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{1090, 223\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{4c-b^2}{4c}+bx-cx^2}}\,dx$$

$$\Big\downarrow \; 1090$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{(b-2cx)^2}{4c}}}d(b-2cx)}{2c}$$

$$\Big\downarrow \; 223$$

$$-\frac{\arcsin\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
Int[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2],x]
```

output
```
-(ArcSin[(b - 2*c*x)/(2*Sqrt[c])]/Sqrt[c])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(17) = 34$.

Time = 0.87 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

method	result	size
default	$\frac{\arctan\left(\frac{2\sqrt{c}\left(x-\frac{b}{2c}\right)}{\sqrt{-4c\,x^2+4bx-\frac{b^2-4c}{c}}}\right)}{\sqrt{c}}$	44

input
```
int(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^(1/2)*arctan(2*c^(1/2)*(x-1/2*b/c)/(-4*c*x^2+4*b*x-(b^2-4*c)/c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(19) = 38$.

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 6.13

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c} + bx - cx^2}}\,dx = \left[-\frac{\sqrt{-c}\log\left(4\,c^2x^2 - 4\,bcx + b^2 - (2\,cx - b)\sqrt{-c}\sqrt{-\frac{4\,c^2x^2 - 4\,bcx + b^2 - 4\,c}{c}} - 2\,c\right)}{2\,c}, -\frac{\arctan\left(\frac{(2\,cx - b)\sqrt{c}\sqrt{-\frac{4\,c^2x^2 - 4\,bcx + b^2 - 4\,c}{c}}}{4\,c^2x^2 - 4\,bcx + b^2 - 4\,c}\right)}{\sqrt{c}}\right]$$

input
```
integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-c)*log(4*c^2*x^2 - 4*b*c*x + b^2 - (2*c*x - b)*sqrt(-c)*sqrt(-
(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c) - 2*c)/c, -arctan((2*c*x - b)*sqrt(c)
*sqrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c)/(4*c^2*x^2 - 4*b*c*x + b^2 - 4
*c))/sqrt(c)]
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.61

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c} + bx - cx^2}}\,dx = 2\left(\begin{cases} \frac{\log\left(4b - 8cx + 4\sqrt{-c}\sqrt{4bx - 4cx^2 + \frac{-b^2+4c}{c}}\right)}{2\sqrt{-c}} & \text{for}\ c \neq 0 \\ \frac{\sqrt{4bx + \frac{-b^2+4c}{c}}}{2b} & \text{for}\ b \neq 0 \\ \frac{x}{\sqrt{\frac{-b^2+4c}{c}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(2/((-b**2+4*c)/c+4*b*x-4*c*x**2)**(1/2),x)
```

output
```
2*Piecewise((log(4*b - 8*c*x + 4*sqrt(-c)*sqrt(4*b*x - 4*c*x**2 + (-b**2 +
 4*c)/c))/(2*sqrt(-c)), Ne(c, 0)), (sqrt(4*b*x + (-b**2 + 4*c)/c)/(2*b), N
e(b, 0)), (x/sqrt((-b**2 + 4*c)/c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx=-\frac{\arcsin\left(-\frac{2\,cx-b}{2\,\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-1/2*(2*c*x - b)/sqrt(c))/sqrt(c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(19) = 38$.

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\begin{aligned}&\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx\\&=-\frac{\log\left(\left|b\sqrt{-c}c-\left(2\,\sqrt{-c^3}x-\sqrt{-4\,c^3x^2+4\,bc^2x-b^2c+4\,c^2}\right)|c|\right|\right)}{\sqrt{-c}}\end{aligned}$$

input
```
integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(b*sqrt(-c)*c - (2*sqrt(-c^3)*x - sqrt(-4*c^3*x^2 + 4*b*c^2*x - b^
2*c + 4*c^2))*abs(c)))/sqrt(-c)
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx=\frac{\ln\left(\frac{b-2\,c\,x}{\sqrt{-c}}+\sqrt{4\,b\,x+\frac{4\,c-b^2}{c}-4\,c\,x^2}\right)}{\sqrt{-c}}$$

input
```
int(2/(4*b*x + (4*c - b^2)/c - 4*c*x^2)^(1/2),x)
```

output
```
log((b - 2*c*x)/(-c)^(1/2) + (4*b*x + (4*c - b^2)/c - 4*c*x^2)^(1/2))/(-c)
^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx=-\frac{\sqrt{c}\,asin\left(\frac{-2cx+b}{2\sqrt{c}}\right)}{c}$$

input
```
int(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x)
```

output
```
( - sqrt(c)*asin((b - 2*c*x)/(2*sqrt(c))))/c
```

3.135 $\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 20

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx=-\frac{\arcsin\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

output
```
-arcsin((-2*c*x+b)/c^(1/2))/c^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 123 vs. $2(20)=40$.

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 6.15

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx = -\frac{-2\sqrt{-c}\arctan\left(\frac{\sqrt{c}\left(-2\sqrt{-c}x+\sqrt{1-\frac{b^2}{c}+4bx-4cx^2}\right)}{b}\right)+\sqrt{c}\log\left(c\left(-1-4bx+8cx^2+4\sqrt{-c}x\sqrt{1-\frac{b^2}{c}+4\right.\right.}{2\sqrt{-c^2}}$$

input
```
Integrate[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2],x]
```

output
```
-1/2*(-2*Sqrt[-c]*ArcTan[(Sqrt[c]*(-2*Sqrt[-c]*x + Sqrt[1 - b^2/c + 4*b*x
- 4*c*x^2]))/b] + Sqrt[c]*Log[c*(-1 - 4*b*x + 8*c*x^2 + 4*Sqrt[-c]*x*Sqrt[
1 - b^2/c + 4*b*x - 4*c*x^2])])/Sqrt[-c^2]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{c-b^2}{4c}+bx-cx^2}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{(b-2cx)^2}{c}}}d(b-2cx)}{c}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{\arcsin\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
Int[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2],x]
```

output
```
-(ArcSin[(b - 2*c*x)/Sqrt[c]]/Sqrt[c])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(16) = 32$.

Time = 0.84 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

method	result	size
default	$\frac{\arctan\left(\frac{2\sqrt{c}\left(x-\frac{b}{2c}\right)}{\sqrt{-4c\,x^2+4bx-\frac{b^2-c}{c}}}\right)}{\sqrt{c}}$	44

input
```
int(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^(1/2)*arctan(2*c^(1/2)*(x-1/2*b/c)/(-4*c*x^2+4*b*x-(b^2-c)/c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(19) = 38$.

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 7.15

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx = \left[-\frac{\sqrt{-c}\log\left(8\,c^2x^2-8\,bcx+2\,b^2-2\,(2\,cx-b)\sqrt{-c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-c}{c}}-c\right)}{2\,c}, -\frac{\arctan\left(\frac{(2\,cx-b)\sqrt{c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-c}{c}}}{4\,c^2x^2-4\,bcx+b^2-c}\right)}{\sqrt{c}}\right]$$

input
```
integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-c)*log(8*c^2*x^2 - 8*b*c*x + 2*b^2 - 2*(2*c*x - b)*sqrt(-c)*sq
rt(-(4*c^2*x^2 - 4*b*c*x + b^2 - c)/c) - c)/c, -arctan((2*c*x - b)*sqrt(c)
*sqrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - c)/c)/(4*c^2*x^2 - 4*b*c*x + b^2 - c))
/sqrt(c)]
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.90

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx = 2\left(\begin{cases} \frac{\log\left(4b-8cx+4\sqrt{-c}\sqrt{4bx-4cx^2+\frac{-b^2+c}{c}}\right)}{2\sqrt{-c}} & \text{for}\ c\neq 0 \\ \frac{\sqrt{4bx+\frac{-b^2+c}{c}}}{2b} & \text{for}\ b\neq 0 \\ \frac{x}{\sqrt{\frac{-b^2+c}{c}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(2/((-b**2+c)/c+4*b*x-4*c*x**2)**(1/2),x)
```

output
```
2*Piecewise((log(4*b - 8*c*x + 4*sqrt(-c)*sqrt(4*b*x - 4*c*x**2 + (-b**2 +
 c)/c))/(2*sqrt(-c)), Ne(c, 0)), (sqrt(4*b*x + (-b**2 + c)/c)/(2*b), Ne(b,
 0)), (x/sqrt((-b**2 + c)/c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx=-\frac{\arcsin\left(-\frac{2\,cx-b}{\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-(2*c*x - b)/sqrt(c))/sqrt(c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(19) = 38$.

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.10

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx = -\frac{\log\left(\left|b\sqrt{-c}c-\left(2\sqrt{-c^3}x-\sqrt{-4\,c^3x^2+4\,bc^2x-b^2c+c^2}\right)|c|\right|\right)}{\sqrt{-c}}$$

input
```
integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(b*sqrt(-c)*c - (2*sqrt(-c^3)*x - sqrt(-4*c^3*x^2 + 4*b*c^2*x - b^
2*c + c^2))*abs(c)))/sqrt(-c)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx=\frac{\ln\left(\frac{b-2\,c\,x}{\sqrt{-c}}+\sqrt{\frac{c-b^2}{c}+4\,b\,x-4\,c\,x^2}\right)}{\sqrt{-c}}$$

input
```
int(2/((c - b^2)/c + 4*b*x - 4*c*x^2)^(1/2),x)
```

output
```
log((b - 2*c*x)/(-c)^(1/2) + ((c - b^2)/c + 4*b*x - 4*c*x^2)^(1/2))/(-c)^(
1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx=-\frac{\sqrt{c}\,asin\left(\frac{-2cx+b}{\sqrt{c}}\right)}{c}$$

input
```
int(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x)
```

output
```
( - sqrt(c)*asin((b - 2*c*x)/sqrt(c)))/c
```

3.136 $\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = -\frac{2(3+2x)}{\sqrt{2+3x+x^2}}$$

output
```
(-6-4*x)/(x^2+3*x+2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = -\frac{2(3+2x)}{\sqrt{2+3x+x^2}}$$

input
```
Integrate[(2 + 3*x + x^2)^(-3/2),x]
```

output
```
(-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+3x+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1088}$$

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

input
```
Int[(2 + 3*x + x^2)^(-3/2),x]
```

output
```
(-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$	18
trager	$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$	18
risch	$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$	18
gosper	$-\frac{2(2+x)(x+1)(2x+3)}{(x^2+3x+2)^{\frac{3}{2}}}$	24
orering	$-\frac{2(2+x)(x+1)(2x+3)}{(x^2+3x+2)^{\frac{3}{2}}}$	24

input
```
int(1/(x^2+3*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2*x+3)/(x^2+3*x+2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(17) = 34.

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = -\frac{2\left(2\,x^2+\sqrt{x^2+3\,x+2}(2\,x+3)+6\,x+4\right)}{x^2+3\,x+2}$$

input
```
integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="fricas")
```

output
```
-2*(2*x^2 + sqrt(x^2 + 3*x + 2)*(2*x + 3) + 6*x + 4)/(x^2 + 3*x + 2)
```

Sympy [F]

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = \int \frac{1}{(x^2+3x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x**2+3*x+2)**(3/2),x)
```

output
```
Integral((x**2 + 3*x + 2)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = -\frac{4\,x}{\sqrt{x^2+3\,x+2}} - \frac{6}{\sqrt{x^2+3\,x+2}}$$

input
```
integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-4*x/sqrt(x^2 + 3*x + 2) - 6/sqrt(x^2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = -\frac{2\,(2\,x+3)}{\sqrt{x^2+3\,x+2}}$$

input
```
integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="giac")
```

output
```
-2*(2*x + 3)/sqrt(x^2 + 3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = -\frac{4\left(x+\frac{3}{2}\right)}{\sqrt{x^2+3\,x+2}}$$

input
```
int(1/(3*x + x^2 + 2)^(3/2),x)
```

output
```
-(4*(x + 3/2))/(3*x + x^2 + 2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = \frac{-4\sqrt{x^2+3x+2}\,x-6\sqrt{x^2+3x+2}-4x^2-12x-8}{x^2+3x+2}$$

input
```
int(1/(x^2+3*x+2)^(3/2),x)
```

output
```
(2*( - 2*sqrt(x**2 + 3*x + 2)*x - 3*sqrt(x**2 + 3*x + 2) - 2*x**2 - 6*x -
4))/(x**2 + 3*x + 2)
```

3.137 $\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = \frac{3-x}{9\sqrt{27-24x+4x^2}}$$

output
```
1/9*(3-x)/(4*x^2-24*x+27)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = \frac{3-x}{9\sqrt{27-24x+4x^2}}$$

input
```
Integrate[(27 - 24*x + 4*x^2)^(-3/2),x]
```

output
```
(3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4x^2-24x+27)^{3/2}} \, dx$$

$$\Big\downarrow \text{1088}$$

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

input
```
Int[(27 - 24*x + 4*x^2)^(-3/2),x]
```

output
```
(3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
trager	$-\frac{-3+x}{9\sqrt{4x^2-24x+27}}$	18
risch	$-\frac{-3+x}{9\sqrt{4x^2-24x+27}}$	18
default	$-\frac{8x-24}{72\sqrt{4x^2-24x+27}}$	20
gosper	$-\frac{(2x-3)(2x-9)(-3+x)}{9(4x^2-24x+27)^{\frac{3}{2}}}$	28
orering	$-\frac{(2x-3)(2x-9)(-3+x)}{9(4x^2-24x+27)^{\frac{3}{2}}}$	28

input
```
int(1/(4*x^2-24*x+27)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-3+x)/(4*x^2-24*x+27)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(17) = 34.

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = -\frac{4\,x^2+2\,\sqrt{4\,x^2-24\,x+27}(x-3)-24\,x+27}{18\,(4\,x^2-24\,x+27)}$$

input
```
integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="fricas")
```

output
```
-1/18*(4*x^2 + 2*sqrt(4*x^2 - 24*x + 27)*(x - 3) - 24*x + 27)/(4*x^2 - 24*
x + 27)
```

Sympy [F]

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = \int \frac{1}{(4x^2-24x+27)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4*x**2-24*x+27)**(3/2),x)
```

output
```
Integral((4*x**2 - 24*x + 27)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = -\frac{x}{9\sqrt{4x^2-24x+27}} + \frac{1}{3\sqrt{4x^2-24x+27}}$$

input
```
integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="maxima")
```

output
```
-1/9*x/sqrt(4*x^2 - 24*x + 27) + 1/3/sqrt(4*x^2 - 24*x + 27)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = -\frac{x-3}{9\sqrt{4x^2-24x+27}}$$

input
```
integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="giac")
```

output
```
-1/9*(x - 3)/sqrt(4*x^2 - 24*x + 27)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = -\frac{x-3}{9\sqrt{4\,x^2-24\,x+27}}$$

input
```
int(1/(4*x^2 - 24*x + 27)^(3/2),x)
```

output
```
-(x - 3)/(9*(4*x^2 - 24*x + 27)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = \frac{-2\sqrt{4x^2-24x+27}\,x+6\sqrt{4x^2-24x+27}-4x^2+24x-27}{72x^2-432x+486}$$

input
```
int(1/(4*x^2-24*x+27)^(3/2),x)
```

output
```
( - 2*sqrt(4*x**2 - 24*x + 27)*x + 6*sqrt(4*x**2 - 24*x + 27) - 4*x**2 + 2
4*x - 27)/(18*(4*x**2 - 24*x + 27))
```

3.138 $\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx=\frac{2+x}{27\left(5-4x-x^2\right)^{3/2}}+\frac{2(2+x)}{243\sqrt{5-4x-x^2}}$$

output
```
1/27*(2+x)/(-x^2-4*x+5)^(3/2)+2/243*(2+x)/(-x^2-4*x+5)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx=\frac{\sqrt{5-4x-x^2}(38+3x-12x^2-2x^3)}{243(-1+x)^2(5+x)^2}$$

input
```
Integrate[(5 - 4*x - x^2)^(-5/2),x]
```

output
```
(Sqrt[5 - 4*x - x^2]*(38 + 3*x - 12*x^2 - 2*x^3))/(243*(-1 + x)^2*(5 + x)^
2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^2-4x+5)^{5/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2}{27}\int \frac{1}{(-x^2-4x+5)^{3/2}}dx+\frac{x+2}{27(-x^2-4x+5)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27(-x^2-4x+5)^{3/2}}$$

input
```
Int[(5 - 4*x - x^2)^(-5/2),x]
```

output
```
(2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*x - x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && 
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p + 
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{(x+5)(x-1)(2x^3+12x^2-3x-38)}{243(-x^2-4x+5)^{\frac{5}{2}}}$	36
orering	$\frac{(x+5)(x-1)(2x^3+12x^2-3x-38)}{243(-x^2-4x+5)^{\frac{5}{2}}}$	36
default	$-\frac{-2x-4}{54(-x^2-4x+5)^{\frac{3}{2}}}-\frac{-2x-4}{243\sqrt{-x^2-4x+5}}$	40
trager	$-\frac{(2x^3+12x^2-3x-38)\sqrt{-x^2-4x+5}}{243(x^2+4x-5)^2}$	40
risch	$\frac{2x^3+12x^2-3x-38}{243(x^2+4x-5)\sqrt{-x^2-4x+5}}$	40

input
```
int(1/(-x^2-4*x+5)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/243*(x+5)*(x-1)*(2*x^3+12*x^2-3*x-38)/(-x^2-4*x+5)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = -\frac{(2\,x^3+12\,x^2-3\,x-38)\sqrt{-x^2-4\,x+5}}{243\,(x^4+8\,x^3+6\,x^2-40\,x+25)}$$

input
```
integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="fricas")
```

output
```
-1/243*(2*x^3 + 12*x^2 - 3*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^4 + 8*x^3 + 6*x
^2 - 40*x + 25)
```

Sympy [F]

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = \int \frac{1}{(-x^2-4x+5)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x**2-4*x+5)**(5/2),x)
```

output
```
Integral((-x**2 - 4*x + 5)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = \frac{2\,x}{243\,\sqrt{-x^2-4\,x+5}} + \frac{4}{243\,\sqrt{-x^2-4\,x+5}} + \frac{x}{27\,(-x^2-4\,x+5)^{\frac{3}{2}}} + \frac{2}{27\,(-x^2-4\,x+5)^{\frac{3}{2}}}$$

input
```
integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="maxima")
```

output
```
2/243*x/sqrt(-x^2 - 4*x + 5) + 4/243/sqrt(-x^2 - 4*x + 5) + 1/27*x/(-x^2 -
 4*x + 5)^(3/2) + 2/27/(-x^2 - 4*x + 5)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = -\frac{((2\,(x+6)x-3)x-38)\sqrt{-x^2-4\,x+5}}{243\,(x^2+4\,x-5)^2}$$

input
```
integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="giac")
```

output
```
-1/243*((2*(x + 6)*x - 3)*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^2 + 4*x - 5)^2
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = -\frac{(4\,x+8)\,\left(8\,x^2+32\,x-76\right)}{3888\,{\left(-x^2-4\,x+5\right)}^{3/2}}$$

input
```
int(1/(5 - x^2 - 4*x)^(5/2),x)
```

output
```
-((4*x + 8)*(32*x + 8*x^2 - 76))/(3888*(5 - x^2 - 4*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = \frac{2x^3+12x^2-3x-38}{243\sqrt{-x^2-4x+5}\,\left(x^2+4x-5\right)}$$

input
```
int(1/(-x^2-4*x+5)^(5/2),x)
```

output
```
(2*x**3 + 12*x**2 - 3*x - 38)/(243*sqrt( - x**2 - 4*x + 5)*(x**2 + 4*x - 5
))
```

3.139 $\int (3+4x+2x^2)^{4/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \left(3+4x+2x^2\right)^{4/3}\,dx=(1+x)\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},\frac{3}{2},-2(1+x)^2\right)$$

output
```
(1+x)*hypergeom([-4/3, 1/2],[3/2],-2*(1+x)^2)
```

Mathematica [A] (verified)

Time = 7.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(3+4x+2x^2\right)^{4/3}\,dx=(1+x)\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},\frac{3}{2},-2(1+x)^2\right)$$

input
```
Integrate[(3 + 4*x + 2*x^2)^(4/3),x]
```

output
```
(1 + x)*Hypergeometric2F1[-4/3, 1/2, 3/2, -2*(1 + x)^2]
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 184 vs. $2(21) = 42$.

Time = 0.48 (sec) , antiderivative size = 184, normalized size of antiderivative = 8.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1087, 1087, 1090, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2+4x+3\right)^{4/3} dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{8}{11}\int \sqrt[3]{2x^2+4x+3}dx+\frac{3}{11}(x+1)\left(2x^2+4x+3\right)^{4/3}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{8}{11}\left(\frac{2}{5}\int \frac{1}{\left(2x^2+4x+3\right)^{2/3}}dx+\frac{3}{5}\sqrt[3]{2x^2+4x+3}(x+1)\right)+\frac{3}{11}(x+1)\left(2x^2+4x+3\right)^{4/3}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{8}{11}\left(\frac{1}{10}\int \frac{1}{\left(\frac{1}{8}(4x+4)^2+1\right)^{2/3}}d(4x+4)+\frac{3}{5}\sqrt[3]{2x^2+4x+3}(x+1)\right)+\frac{3}{11}(x+1)\left(2x^2+4x+3\right)^{4/3}$$

$$\Big\downarrow \text{ 234}$$

$$\frac{8}{11}\left(\frac{3\sqrt{(4x+4)^2}\int \frac{2\sqrt{2}}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}}{5\sqrt{2}(4x+4)}+\frac{3}{5}\sqrt[3]{2x^2+4x+3}(x+1)\right)+\frac{3}{11}(x+1)\left(2x^2+4x+3\right)^{4/3}$$

$$\Big\downarrow \text{ 760}$$

$$\frac{8}{11}\left(\frac{3}{5}(x+1)\sqrt[3]{2x^2+4x+3}-\frac{4\,3^{3/4}\sqrt{2-\sqrt{3}}(-4x-3)\sqrt{\frac{4x+\left(\frac{1}{8}(4x+4)^2+1\right)^{2/3}+5}{\left(-4x-\sqrt{3}-3\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-4x+\sqrt{3}-3}{-4x-\sqrt{3}-3}\right)\right.}{5\sqrt{-\frac{-4x-3}{\left(-4x-\sqrt{3}-3\right)^2}}(4x+4)}\right.$$

$$\frac{3}{11}(x+1)\left(2x^2+4x+3\right)^{4/3}$$

input
```
Int[(3 + 4*x + 2*x^2)^(4/3),x]
```

output
```
(3*(1 + x)*(3 + 4*x + 2*x^2)^(4/3))/11 + (8*((3*(1 + x)*(3 + 4*x + 2*x^2)^
(1/3))/5 - (4*3^(3/4)*Sqrt[2 - Sqrt[3]]*(-3 - 4*x)*Sqrt[(5 + 4*x + (1 + (4
 + 4*x)^2/8)^(2/3))/(-3 - Sqrt[3] - 4*x)^2]*EllipticF[ArcSin[(-3 + Sqrt[3]
 - 4*x)/(-3 - Sqrt[3] - 4*x)], -7 + 4*Sqrt[3]])/(5*Sqrt[-((-3 - 4*x)/(-3 -
 Sqrt[3] - 4*x)^2)]*(4 + 4*x))))/11
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int \left(2x^2+4x+3\right)^{\frac{4}{3}} dx$$

input
```
int((2*x^2+4*x+3)^(4/3),x)
```

output
```
int((2*x^2+4*x+3)^(4/3),x)
```

Fricas [F]

$$\int \left(3+4x+2x^2\right)^{4/3} dx = \int \left(2\,x^2+4\,x+3\right)^{\frac{4}{3}} dx$$

input
```
integrate((2*x^2+4*x+3)^(4/3),x, algorithm="fricas")
```

output
```
integral((2*x^2 + 4*x + 3)^(4/3), x)
```

Sympy [F]

$$\int \left(3+4x+2x^2\right)^{4/3} dx = \int \left(2x^2+4x+3\right)^{\frac{4}{3}} dx$$

input
```
integrate((2*x**2+4*x+3)**(4/3),x)
```

output
```
Integral((2*x**2 + 4*x + 3)**(4/3), x)
```

Maxima [F]

$$\int \left(3+4x+2x^2\right)^{4/3}\,dx=\int \left(2\,x^2+4\,x+3\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((2*x^2+4*x+3)^(4/3),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 4*x + 3)^(4/3), x)
```

Giac [F]

$$\int \left(3+4x+2x^2\right)^{4/3}\,dx=\int \left(2\,x^2+4\,x+3\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((2*x^2+4*x+3)^(4/3),x, algorithm="giac")
```

output
```
integrate((2*x^2 + 4*x + 3)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3+4x+2x^2\right)^{4/3}\,dx=\int \left(2\,x^2+4\,x+3\right)^{4/3}\,dx$$

input
```
int((4*x + 2*x^2 + 3)^(4/3),x)
```

output
```
int((4*x + 2*x^2 + 3)^(4/3), x)
```

Reduce [F]

$$\int (3+4x+2x^2)^{4/3}\,dx = \frac{6(2x^2+4x+3)^{\frac{1}{3}}x^3}{11} + \frac{18(2x^2+4x+3)^{\frac{1}{3}}x^2}{11} + \frac{129(2x^2+4x+3)^{\frac{1}{3}}x}{55} + \frac{81(2x^2+4x+3)^{\frac{1}{3}}}{55} - \frac{16\left(\int \frac{x}{(2x^2+4x+3)^{\frac{2}{3}}}dx\right)}{55}$$

input
```
int((2*x^2+4*x+3)^(4/3),x)
```

output
```
(30*(2*x**2 + 4*x + 3)**(1/3)*x**3 + 90*(2*x**2 + 4*x + 3)**(1/3)*x**2 + 1
29*(2*x**2 + 4*x + 3)**(1/3)*x + 81*(2*x**2 + 4*x + 3)**(1/3) - 16*int(((2
*x**2 + 4*x + 3)**(1/3)*x)/(2*x**2 + 4*x + 3),x))/55
```

3.140 $\int (3+4x+2x^2)^{2/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \left(3+4x+2x^2\right)^{2/3}\,dx=(1+x)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},-2(1+x)^2\right)$$

output
```
(1+x)*hypergeom([-2/3, 1/2],[3/2],-2*(1+x)^2)
```

Mathematica [A] (verified)

Time = 6.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(3+4x+2x^2\right)^{2/3}\,dx=(1+x)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},-2(1+x)^2\right)$$

input
```
Integrate[(3 + 4*x + 2*x^2)^(2/3),x]
```

output
```
(1 + x)*Hypergeometric2F1[-2/3, 1/2, 3/2, -2*(1 + x)^2]
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 367 vs. $2(21) = 42$.

Time = 0.71 (sec) , antiderivative size = 367, normalized size of antiderivative = 17.48, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1087, 1090, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2+4x+3\right)^{2/3} dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{4}{7}\int \frac{1}{\sqrt[3]{2x^2+4x+3}}dx+\frac{3}{7}\left(2x^2+4x+3\right)^{2/3}(x+1)$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{7}\int \frac{1}{\sqrt[3]{\frac{1}{8}(4x+4)^2+1}}d(4x+4)+\frac{3}{7}\left(2x^2+4x+3\right)^{2/3}(x+1)$$

$$\Big\downarrow \text{233}$$

$$\frac{3\sqrt{2}\sqrt{(4x+4)^2}\int \frac{2\sqrt{2}\sqrt[3]{\frac{1}{8}(4x+4)^2+1}}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}}{7(4x+4)}+\frac{3}{7}\left(2x^2+4x+3\right)^{2/3}(x+1)$$

$$\Big\downarrow \text{833}$$

$$\frac{3\sqrt{2}\sqrt{(4x+4)^2}\left(\left(1+\sqrt{3}\right)\int \frac{2\sqrt{2}}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}-\int \frac{2\sqrt{2}\left(-4x+\sqrt{3}-3\right)}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}\right)}{7(4x+4)}+\frac{3}{7}\left(2x^2+4x+3\right)^{2/3}(x+1)$$

$$\Big\downarrow \text{760}$$

$$\frac{3\sqrt{2}\sqrt{(4x+4)^2}\left(-\int \frac{2\sqrt{2}\left(-4x+\sqrt{3}-3\right)}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}-\frac{4\sqrt{2\left(2-\sqrt{3}\right)}\left(1+\sqrt{3}\right)(-4x-3)\sqrt{\frac{4x+\left(\frac{1}{8}(4x+4)^2+1\right)^{2/3}+5}{\left(-4x-\sqrt{3}-3\right)^2}}\,\text{Ellipticl}}{\sqrt[4]{3}\sqrt{-\frac{-4x-3}{\left(-4x-\sqrt{3}-3\right)^2}}\sqrt{(4x+4)^2}}}{7(4x+4)}$$
$$\frac{3}{7}\left(2x^2+4x+3\right)^{2/3}(x+1)$$

$$\Big\downarrow \text{2418}$$

$$\frac{3\sqrt{2}\sqrt{(4x+4)^2}\left(-\frac{4\sqrt{2\left(2-\sqrt{3}\right)}\left(1+\sqrt{3}\right)(-4x-3)\sqrt{\frac{4x+\left(\frac{1}{8}(4x+4)^2+1\right)^{2/3}+5}{\left(-4x-\sqrt{3}-3\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-4x+\sqrt{3}-3}{-4x-\sqrt{3}-3}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{-4x-3}{\left(-4x-\sqrt{3}-3\right)^2}}\sqrt{(4x+4)^2}}+\frac{2\sqrt[4]{3}\sqrt{2(2+}}{}\right.}{7(4x+4)}$$
$$\frac{3}{7}\left(2x^2+4x+3\right)^{2/3}(x+1)$$

input
```
Int[(3 + 4*x + 2*x^2)^(2/3),x]
```

output
```
(3*(1 + x)*(3 + 4*x + 2*x^2)^(2/3))/7 + (3*Sqrt[2]*Sqrt[(4 + 4*x)^2]*(-(Sq
rt[(4 + 4*x)^2]/(Sqrt[2]*(-3 - Sqrt[3] - 4*x))) + (2*3^(1/4)*Sqrt[2*(2 + S
qrt[3])]*(-3 - 4*x)*Sqrt[(5 + 4*x + (1 + (4 + 4*x)^2/8)^(2/3))/(-3 - Sqrt[
3] - 4*x)^2]*EllipticE[ArcSin[(-3 + Sqrt[3] - 4*x)/(-3 - Sqrt[3] - 4*x)],
-7 + 4*Sqrt[3]])/(Sqrt[-((-3 - 4*x)/(-3 - Sqrt[3] - 4*x)^2)]*Sqrt[(4 + 4*x
)^2]) - (4*Sqrt[2*(2 - Sqrt[3])]*(1 + Sqrt[3])*(-3 - 4*x)*Sqrt[(5 + 4*x +
(1 + (4 + 4*x)^2/8)^(2/3))/(-3 - Sqrt[3] - 4*x)^2]*EllipticF[ArcSin[(-3 +
Sqrt[3] - 4*x)/(-3 - Sqrt[3] - 4*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((-3
 - 4*x)/(-3 - Sqrt[3] - 4*x)^2)]*Sqrt[(4 + 4*x)^2])))/(7*(4 + 4*x))
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1087

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2418

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \left(2x^2+4x+3\right)^{\frac{2}{3}} dx$$

input
```
int((2*x^2+4*x+3)^(2/3),x)
```

output
```
int((2*x^2+4*x+3)^(2/3),x)
```

Fricas [F]

$$\int \left(3+4x+2x^2\right)^{2/3} dx = \int \left(2\,x^2+4\,x+3\right)^{\frac{2}{3}} dx$$

input
```
integrate((2*x^2+4*x+3)^(2/3),x, algorithm="fricas")
```

output
```
integral((2*x^2 + 4*x + 3)^(2/3), x)
```

Sympy [F]

$$\int \left(3+4x+2x^2\right)^{2/3} dx = \int \left(2x^2+4x+3\right)^{\frac{2}{3}} dx$$

input
```
integrate((2*x**2+4*x+3)**(2/3),x)
```

output
```
Integral((2*x**2 + 4*x + 3)**(2/3), x)
```

Maxima [F]

$$\int \left(3+4x+2x^2\right)^{2/3}\,dx=\int \left(2\,x^2+4\,x+3\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((2*x^2+4*x+3)^(2/3),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 4*x + 3)^(2/3), x)
```

Giac [F]

$$\int \left(3+4x+2x^2\right)^{2/3}\,dx=\int \left(2\,x^2+4\,x+3\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((2*x^2+4*x+3)^(2/3),x, algorithm="giac")
```

output
```
integrate((2*x^2 + 4*x + 3)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3+4x+2x^2\right)^{2/3}\,dx=\int \left(2\,x^2+4\,x+3\right)^{2/3}\,dx$$

input
```
int((4*x + 2*x^2 + 3)^(2/3),x)
```

output
```
int((4*x + 2*x^2 + 3)^(2/3), x)
```

Reduce [F]

$$\int \left(3+4x+2x^2\right)^{2/3} \, dx = \frac{3(2x^2+4x+3)^{\frac{2}{3}} x}{7} + \frac{9(2x^2+4x+3)^{\frac{2}{3}}}{14} - \frac{4\left(\int \frac{x}{(2x^2+4x+3)^{\frac{1}{3}}} dx\right)}{7}$$

input
```
int((2*x^2+4*x+3)^(2/3),x)
```

output
```
(6*(2*x**2 + 4*x + 3)**(2/3)*x + 9*(2*x**2 + 4*x + 3)**(2/3) - 8*int(((2*x
**2 + 4*x + 3)**(2/3)*x)/(2*x**2 + 4*x + 3),x))/14
```

3.141 $\int \sqrt[3]{3+4x+2x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \sqrt[3]{3+4x+2x^2}\,dx=(1+x)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},-2(1+x)^2\right)$$

output
```
(1+x)*hypergeom([-1/3, 1/2],[3/2],-2*(1+x)^2)
```

Mathematica [A] (verified)

Time = 5.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{3+4x+2x^2}\,dx=(1+x)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},-2(1+x)^2\right)$$

input
```
Integrate[(3 + 4*x + 2*x^2)^(1/3),x]
```

output
```
(1 + x)*Hypergeometric2F1[-1/3, 1/2, 3/2, -2*(1 + x)^2]
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 158 vs. $2(21) = 42$.

Time = 0.44 (sec) , antiderivative size = 158, normalized size of antiderivative = 7.52, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1087, 1090, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{2x^2+4x+3}\,dx$$

$$\Big\downarrow \ 1087$$

$$\frac{2}{5}\int \frac{1}{(2x^2+4x+3)^{2/3}}dx+\frac{3}{5}\sqrt[3]{2x^2+4x+3}(x+1)$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{10}\int \frac{1}{\left(\frac{1}{8}(4x+4)^2+1\right)^{2/3}}d(4x+4)+\frac{3}{5}\sqrt[3]{2x^2+4x+3}(x+1)$$

$$\Big\downarrow \ 234$$

$$\frac{3\sqrt{(4x+4)^2}\int \frac{2\sqrt{2}}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}}{5\sqrt{2}(4x+4)}+\frac{3}{5}\sqrt[3]{2x^2+4x+3}(x+1)$$

$$\Big\downarrow \ 760$$

$$\frac{3}{5}(x+1)\sqrt[3]{2x^2+4x+3}-\frac{4\,3^{3/4}\sqrt{2-\sqrt{3}}(-4x-3)\sqrt{\frac{4x+\left(\frac{1}{8}(4x+4)^2+1\right)^{2/3}+5}{\left(-4x-\sqrt{3}-3\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-4x+\sqrt{3}-3}{-4x-\sqrt{3}-3}\right),-7+4\sqrt{3}\right)}{5\sqrt{-\frac{-4x-3}{\left(-4x-\sqrt{3}-3\right)^2}}(4x+4)}$$

input

```
Int[(3 + 4*x + 2*x^2)^(1/3),x]
```

output
```
(3*(1 + x)*(3 + 4*x + 2*x^2)^(1/3))/5 - (4*3^(3/4)*Sqrt[2 - Sqrt[3]]*(-3 -
 4*x)*Sqrt[(5 + 4*x + (1 + (4 + 4*x)^2/8)^(2/3))/(-3 - Sqrt[3] - 4*x)^2]*E
llipticF[ArcSin[(-3 + Sqrt[3] - 4*x)/(-3 - Sqrt[3] - 4*x)], -7 + 4*Sqrt[3]
])/(5*Sqrt[-((-3 - 4*x)/(-3 - Sqrt[3] - 4*x)^2)]*(4 + 4*x))
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int \left(2x^{2}+4x+3\right)^{\frac{1}{3}}dx$$

input
```
int((2*x^2+4*x+3)^(1/3),x)
```

output
```
int((2*x^2+4*x+3)^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{3+4x+2x^2}\,dx=\int \left(2\,x^{2}+4\,x+3\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((2*x^2+4*x+3)^(1/3),x, algorithm="fricas")
```

output
```
integral((2*x^2 + 4*x + 3)^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{3+4x+2x^2}\,dx=\int \sqrt[3]{2x^{2}+4x+3}\,dx$$

input
```
integrate((2*x**2+4*x+3)**(1/3),x)
```

output
```
Integral((2*x**2 + 4*x + 3)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{3+4x+2x^2}\,dx = \int \left(2\,x^2+4\,x+3\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((2*x^2+4*x+3)^(1/3),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 4*x + 3)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{3+4x+2x^2}\,dx = \int \left(2\,x^2+4\,x+3\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((2*x^2+4*x+3)^(1/3),x, algorithm="giac")
```

output
```
integrate((2*x^2 + 4*x + 3)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{3+4x+2x^2}\,dx = \int \left(2\,x^2+4\,x+3\right)^{1/3}\,dx$$

input
```
int((4*x + 2*x^2 + 3)^(1/3),x)
```

output
```
int((4*x + 2*x^2 + 3)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{3+4x+2x^2}\,dx = \frac{3(2x^2+4x+3)^{\frac{1}{3}}\,x}{5} + \frac{9(2x^2+4x+3)^{\frac{1}{3}}}{10} - \frac{2\left(\int \frac{x}{(2x^2+4x+3)^{\frac{2}{3}}}dx\right)}{5}$$

input
```
int((2*x^2+4*x+3)^(1/3),x)
```

output
```
(6*(2*x**2 + 4*x + 3)**(1/3)*x + 9*(2*x**2 + 4*x + 3)**(1/3) - 4*int(((2*x
**2 + 4*x + 3)**(1/3)*x)/(2*x**2 + 4*x + 3),x))/10
```

3.142 $\int \frac{1}{\sqrt[3]{3+4x+2x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{1}{\sqrt[3]{3+4x+2x^2}}\,dx = (1+x)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-2(1+x)^2\right)$$

output
```
(1+x)*hypergeom([1/3, 1/2],[3/2],-2*(1+x)^2)
```

Mathematica [A] (verified)

Time = 5.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{3+4x+2x^2}}\,dx = (1+x)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-2(1+x)^2\right)$$

input
```
Integrate[(3 + 4*x + 2*x^2)^(-1/3),x]
```

output
```
(1 + x)*Hypergeometric2F1[1/3, 1/2, 3/2, -2*(1 + x)^2]
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 345 vs. $2(21) = 42$.

Time = 0.68 (sec) , antiderivative size = 345, normalized size of antiderivative = 16.43, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1090, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{2x^2+4x+3}}\,dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{4}\int \frac{1}{\sqrt[3]{\frac{1}{8}(4x+4)^2+1}}d(4x+4)$$

$$\Big\downarrow \text{233}$$

$$\frac{3\sqrt{(4x+4)^2}\int \frac{2\sqrt{2}\sqrt[3]{\frac{1}{8}(4x+4)^2+1}}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}}{2\sqrt{2}(4x+4)}$$

$$\Big\downarrow \text{833}$$

$$\frac{3\sqrt{(4x+4)^2}\left(\left(1+\sqrt{3}\right)\int \frac{2\sqrt{2}}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}-\int \frac{2\sqrt{2}\left(-4x+\sqrt{3}-3\right)}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}\right)}{2\sqrt{2}(4x+4)}$$

$$\Big\downarrow \text{760}$$

$$\frac{3\sqrt{(4x+4)^2}\left(-\int \frac{2\sqrt{2}\left(-4x+\sqrt{3}-3\right)}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}-\frac{4\sqrt{2\left(2-\sqrt{3}\right)}\left(1+\sqrt{3}\right)(-4x-3)\sqrt{\frac{4x+\left(\frac{1}{8}(4x+4)^2+1\right)^{2/3}+5}{\left(-4x-\sqrt{3}-3\right)^2}}\text{EllipticF}\left(\text{ar}\right.}{\sqrt[4]{3}\sqrt{-\frac{-4x-3}{\left(-4x-\sqrt{3}-3\right)^2}}\sqrt{(4x+4)^2}}\right.}{2\sqrt{2}(4x+4)}$$

$$\Big\downarrow \text{2418}$$

$$\frac{3\sqrt{(4x+4)^2}\left(-\frac{4\sqrt{2\left(2-\sqrt{3}\right)}\left(1+\sqrt{3}\right)(-4x-3)\sqrt{\frac{4x+\left(\frac{1}{8}(4x+4)^2+1\right)^{2/3}+5}{\left(-4x-\sqrt{3}-3\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-4x+\sqrt{3}-3}{-4x-\sqrt{3}-3}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{-4x-3}{\left(-4x-\sqrt{3}-3\right)^2}}\sqrt{(4x+4)^2}}+\frac{2\sqrt[4]{3}\sqrt{2\left(2+\sqrt{3}\right.}}{}\right.}{2\sqrt{2}(4x+4)}$$

input
```
Int[(3 + 4*x + 2*x^2)^(-1/3),x]
```

output
```
(3*Sqrt[(4 + 4*x)^2]*(-(Sqrt[(4 + 4*x)^2]/(Sqrt[2]*(-3 - Sqrt[3] - 4*x)))
+ (2*3^(1/4)*Sqrt[2*(2 + Sqrt[3])]*(-3 - 4*x)*Sqrt[(5 + 4*x + (1 + (4 + 4*
x)^2/8)^(2/3))/(-3 - Sqrt[3] - 4*x)^2]*EllipticE[ArcSin[(-3 + Sqrt[3] - 4*
x)/(-3 - Sqrt[3] - 4*x)], -7 + 4*Sqrt[3]])/(Sqrt[-((-3 - 4*x)/(-3 - Sqrt[3
] - 4*x)^2)]*Sqrt[(4 + 4*x)^2]) - (4*Sqrt[2*(2 - Sqrt[3])]*(1 + Sqrt[3])*(
-3 - 4*x)*Sqrt[(5 + 4*x + (1 + (4 + 4*x)^2/8)^(2/3))/(-3 - Sqrt[3] - 4*x)^
2]*EllipticF[ArcSin[(-3 + Sqrt[3] - 4*x)/(-3 - Sqrt[3] - 4*x)], -7 + 4*Sqr
t[3]])/(3^(1/4)*Sqrt[-((-3 - 4*x)/(-3 - Sqrt[3] - 4*x)^2)]*Sqrt[(4 + 4*x)^
2])))/(2*Sqrt[2]*(4 + 4*x))
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(2x^2+4x+3)^{\frac{1}{3}}}dx$$

input
```
int(1/(2*x^2+4*x+3)^(1/3),x)
```

output
```
int(1/(2*x^2+4*x+3)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{3+4x+2x^2}}\,dx=\int \frac{1}{\left(2\,x^{2}+4\,x+3\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(2*x^2+4*x+3)^(1/3),x, algorithm="fricas")
```

output
```
integral((2*x^2 + 4*x + 3)^(-1/3), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{3+4x+2x^2}}\,dx=\int \frac{1}{\sqrt[3]{2x^{2}+4x+3}}\,dx$$

input
```
integrate(1/(2*x**2+4*x+3)**(1/3),x)
```

output
```
Integral((2*x**2 + 4*x + 3)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{3+4x+2x^2}}\,dx=\int \frac{1}{\left(2\,x^{2}+4\,x+3\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(2*x^2+4*x+3)^(1/3),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 4*x + 3)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{3+4x+2x^2}}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(2*x^2+4*x+3)^(1/3),x, algorithm="giac")
```

output
```
integrate((2*x^2 + 4*x + 3)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{3+4x+2x^2}}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)^{1/3}}\,dx$$

input
```
int(1/(4*x + 2*x^2 + 3)^(1/3),x)
```

output
```
int(1/(4*x + 2*x^2 + 3)^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{3+4x+2x^2}}\,dx = \int \frac{1}{(2x^2+4x+3)^{\frac{1}{3}}}dx$$

input
```
int(1/(2*x^2+4*x+3)^(1/3),x)
```

output
```
int(1/(2*x**2 + 4*x + 3)**(1/3),x)
```

3.143 $\int \frac{1}{(3+4x+2x^2)^{2/3}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{1}{(3+4x+2x^2)^{2/3}} \, dx = (1+x) \, \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-2(1+x)^2\right)$$

output
```
(1+x)*hypergeom([1/2, 2/3],[3/2],-2*(1+x)^2)
```

Mathematica [A] (verified)

Time = 5.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3+4x+2x^2)^{2/3}} \, dx = (1+x) \, \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-2(1+x)^2\right)$$

input
```
Integrate[(3 + 4*x + 2*x^2)^(-2/3),x]
```

output
```
(1 + x)*Hypergeometric2F1[1/2, 2/3, 3/2, -2*(1 + x)^2]
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 134 vs. $2(21) = 42$.

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 6.38, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1090, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2+4x+3)^{2/3}}dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{4}\int \frac{1}{\left(\frac{1}{8}(4x+4)^2+1\right)^{2/3}}d(4x+4)$$

$$\Big\downarrow \text{234}$$

$$\frac{3\sqrt{(4x+4)^2}\int \frac{2\sqrt{2}}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}}{2\sqrt{2}(4x+4)}$$

$$\Big\downarrow \text{760}$$

$$-\frac{2\ 3^{3/4}\sqrt{2-\sqrt{3}}(-4x-3)\sqrt{\frac{4x+\left(\frac{1}{8}(4x+4)^2+1\right)^{2/3}+5}{\left(-4x-\sqrt{3}-3\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-4x+\sqrt{3}-3}{-4x-\sqrt{3}-3}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{-4x-3}{\left(-4x-\sqrt{3}-3\right)^2}}(4x+4)}$$

input
```
Int[(3 + 4*x + 2*x^2)^(-2/3),x]
```

output
```
(-2*3^(3/4)*Sqrt[2 - Sqrt[3]]*(-3 - 4*x)*Sqrt[(5 + 4*x + (1 + (4 + 4*x)^2/
8)^(2/3))/(-3 - Sqrt[3] - 4*x)^2]*EllipticF[ArcSin[(-3 + Sqrt[3] - 4*x)/(-
3 - Sqrt[3] - 4*x)], -7 + 4*Sqrt[3]])/(Sqrt[-((-3 - 4*x)/(-3 - Sqrt[3] - 4
*x)^2)]*(4 + 4*x))
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int \frac{1}{(2x^2+4x+3)^{\frac{2}{3}}}dx$$

input
```
int(1/(2*x^2+4*x+3)^(2/3),x)
```

output
```
int(1/(2*x^2+4*x+3)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(3+4x+2x^2)^{2/3}}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(2*x^2+4*x+3)^(2/3),x, algorithm="fricas")
```

output
```
integral((2*x^2 + 4*x + 3)^(-2/3), x)
```

Sympy [F]

$$\int \frac{1}{(3+4x+2x^2)^{2/3}}\,dx = \int \frac{1}{(2x^2+4x+3)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(2*x**2+4*x+3)**(2/3),x)
```

output
```
Integral((2*x**2 + 4*x + 3)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(3+4x+2x^2)^{2/3}}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(2*x^2+4*x+3)^(2/3),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 4*x + 3)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(3+4x+2x^2)^{2/3}}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(2*x^2+4*x+3)^(2/3),x, algorithm="giac")
```

output
```
integrate((2*x^2 + 4*x + 3)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+4x+2x^2)^{2/3}}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)^{2/3}}\,dx$$

input
```
int(1/(4*x + 2*x^2 + 3)^(2/3),x)
```

output
```
int(1/(4*x + 2*x^2 + 3)^(2/3), x)
```

Reduce [F]

$$\int \frac{1}{(3+4x+2x^2)^{2/3}}\,dx = \int \frac{1}{(2x^2+4x+3)^{\frac{2}{3}}}dx$$

input
```
int(1/(2*x^2+4*x+3)^(2/3),x)
```

output
```
int(1/(2*x**2 + 4*x + 3)**(2/3),x)
```

3.144 $\int \frac{1}{(3+4x+2x^2)^{4/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{1}{(3+4x+2x^2)^{4/3}}\,dx = (1+x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{3}{2},-2(1+x)^2\right)$$

output
```
(1+x)*hypergeom([1/2, 4/3],[3/2],-2*(1+x)^2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.

Time = 5.86 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.62

$$\int \frac{1}{(3+4x+2x^2)^{4/3}}\,dx = -\frac{(1+x)\left(-3+\sqrt[3]{3+4x+2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-2(1+x)^2\right)\right)}{2\sqrt[3]{3+4x+2x^2}}$$

input
```
Integrate[(3 + 4*x + 2*x^2)^(-4/3),x]
```

output
```
-1/2*((1 + x)*(-3 + (3 + 4*x + 2*x^2)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/
2, -2*(1 + x)^2]))/(3 + 4*x + 2*x^2)^(1/3)
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 367 vs. $2(21) = 42$.

Time = 0.74 (sec) , antiderivative size = 367, normalized size of antiderivative = 17.48, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1089, 1090, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2+4x+3)^{4/3}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{3(x+1)}{2\sqrt[3]{2x^2+4x+3}}-\frac{1}{2}\int \frac{1}{\sqrt[3]{2x^2+4x+3}}dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{3(x+1)}{2\sqrt[3]{2x^2+4x+3}}-\frac{1}{8}\int \frac{1}{\sqrt[3]{\frac{1}{8}(4x+4)^2+1}}d(4x+4)$$

$$\Big\downarrow \text{233}$$

$$\frac{3(x+1)}{2\sqrt[3]{2x^2+4x+3}}-\frac{3\sqrt{(4x+4)^2}\int \frac{2\sqrt{2}\sqrt[3]{\frac{1}{8}(4x+4)^2+1}}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}}{4\sqrt{2}(4x+4)}$$

$$\Big\downarrow \text{833}$$

$$\frac{3(x+1)}{2\sqrt[3]{2x^2+4x+3}}-\frac{3\sqrt{(4x+4)^2}\left(\left(1+\sqrt{3}\right)\int \frac{2\sqrt{2}}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}-\int \frac{2\sqrt{2}\left(-4x+\sqrt{3}-3\right)}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}\right)}{4\sqrt{2}(4x+4)}$$

$$\Big\downarrow \text{760}$$

$$\frac{3(x+1)}{2\sqrt[3]{2x^2+4x+3}}-\frac{3\sqrt{(4x+4)^2}\left(-\int\frac{2\sqrt{2}\left(-4x+\sqrt{3}-3\right)}{\sqrt{(4x+4)^2}}d\sqrt[3]{\frac{1}{8}(4x+4)^2+1}-\frac{4\sqrt{2\left(2-\sqrt{3}\right)}\left(1+\sqrt{3}\right)(-4x-3)\sqrt{\frac{4x+\left(\frac{1}{8}(4x+4)^2+1\right)^{2/3}+5}{\left(-4x-\sqrt{3}-3\right)^2}}\text{EllipticF}\left(\text{ar}\ldots\right.}{\sqrt[4]{3}\sqrt{-\frac{-4x-3}{\left(-4x-\sqrt{3}-3\right)^2}}\sqrt{(4x+4)^2}}\ldots\right.}{4\sqrt{2}(4x+4)}$$

$\Big\downarrow$ 2418

$$\frac{3(x+1)}{2\sqrt[3]{2x^2+4x+3}}-\frac{3\sqrt{(4x+4)^2}\left(-\frac{4\sqrt{2\left(2-\sqrt{3}\right)}\left(1+\sqrt{3}\right)(-4x-3)\sqrt{\frac{4x+\left(\frac{1}{8}(4x+4)^2+1\right)^{2/3}+5}{\left(-4x-\sqrt{3}-3\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-4x+\sqrt{3}-3}{-4x-\sqrt{3}-3}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{-4x-3}{\left(-4x-\sqrt{3}-3\right)^2}}\sqrt{(4x+4)^2}}+\frac{2\sqrt[4]{3}\sqrt{2\left(2+\sqrt{3}\ldots\right.}}{\ldots}\ldots\right.}{4\sqrt{2}(4x+4)}$$

input
```
Int[(3 + 4*x + 2*x^2)^(-4/3),x]
```

output
```
(3*(1 + x))/(2*(3 + 4*x + 2*x^2)^(1/3)) - (3*Sqrt[(4 + 4*x)^2]*(-(Sqrt[(4
+ 4*x)^2]/(Sqrt[2]*(-3 - Sqrt[3] - 4*x))) + (2*3^(1/4)*Sqrt[2*(2 + Sqrt[3]
)]*(-3 - 4*x)*Sqrt[(5 + 4*x + (1 + (4 + 4*x)^2/8)^(2/3))/(-3 - Sqrt[3] - 4
*x)^2]*EllipticE[ArcSin[(-3 + Sqrt[3] - 4*x)/(-3 - Sqrt[3] - 4*x)], -7 + 4
*Sqrt[3]])/(Sqrt[-((-3 - 4*x)/(-3 - Sqrt[3] - 4*x)^2)]*Sqrt[(4 + 4*x)^2])
- (4*Sqrt[2*(2 - Sqrt[3])]*(1 + Sqrt[3])*(-3 - 4*x)*Sqrt[(5 + 4*x + (1 + (
4 + 4*x)^2/8)^(2/3))/(-3 - Sqrt[3] - 4*x)^2]*EllipticF[ArcSin[(-3 + Sqrt[3
] - 4*x)/(-3 - Sqrt[3] - 4*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((-3 - 4*x
)/(-3 - Sqrt[3] - 4*x)^2)]*Sqrt[(4 + 4*x)^2])))/(4*Sqrt[2]*(4 + 4*x))
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(2x^2+4x+3)^{\frac{4}{3}}}dx$$

input
```
int(1/(2*x^2+4*x+3)^(4/3),x)
```

output
```
int(1/(2*x^2+4*x+3)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{(3+4x+2x^2)^{4/3}}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(2*x^2+4*x+3)^(4/3),x, algorithm="fricas")
```

output
```
integral((2*x^2 + 4*x + 3)^(2/3)/(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9), x)
```

Sympy [F]

$$\int \frac{1}{(3+4x+2x^2)^{4/3}}\,dx = \int \frac{1}{(2x^2+4x+3)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(2*x**2+4*x+3)**(4/3),x)
```

output
```
Integral((2*x**2 + 4*x + 3)**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(3+4x+2x^2)^{4/3}}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(2*x^2+4*x+3)^(4/3),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 4*x + 3)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(3+4x+2x^2)^{4/3}}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(2*x^2+4*x+3)^(4/3),x, algorithm="giac")
```

output
```
integrate((2*x^2 + 4*x + 3)^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+4x+2x^2)^{4/3}}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)^{4/3}}\,dx$$

input
```
int(1/(4*x + 2*x^2 + 3)^(4/3),x)
```

output
```
int(1/(4*x + 2*x^2 + 3)^(4/3), x)
```

Reduce [F]

$$\int \frac{1}{(3+4x+2x^2)^{4/3}}\,dx = \int \frac{1}{2\left(2x^2+4x+3\right)^{\frac{1}{3}}x^2+4\left(2x^2+4x+3\right)^{\frac{1}{3}}x+3\left(2x^2+4x+3\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(2*x^2+4*x+3)^(4/3),x)
```

output
```
int(1/(2*(2*x**2 + 4*x + 3)**(1/3)*x**2 + 4*(2*x**2 + 4*x + 3)**(1/3)*x +
3*(2*x**2 + 4*x + 3)**(1/3)),x)
```

3.145 $\int (a+bx+cx^2)^{4/3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int (a+bx+cx^2)^{4/3} \, dx = \frac{(b+2cx)(a+bx+cx^2)^{4/3} \text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{8\ 2^{2/3} c \left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{4/3}}$$

output
```
1/16*(2*c*x+b)*(c*x^2+b*x+a)^(4/3)*hypergeom([-4/3, 1/2],[3/2],(2*c*x+b)^2
/(-4*a*c+b^2))*2^(1/3)/c/(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(4/3)
```

Mathematica [A] (verified)

Time = 10.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int (a+bx+cx^2)^{4/3} \, dx = -\frac{(b^2-4ac)(b+2cx)\sqrt[3]{a+x(b+cx)} \text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{8\ 2^{2/3} c^2 \sqrt[3]{\frac{c(a+x(b+cx))}{-b^2+4ac}}}$$

input
```
Integrate[(a + b*x + c*x^2)^(4/3),x]
```

output
```
-1/8*((b^2 - 4*a*c)*(b + 2*c*x)*(a + x*(b + c*x))^(1/3)*Hypergeometric2F1[
-4/3, 1/2, 3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(2^(2/3)*c^2*((c*(a + x*(b +
 c*x)))/(-b^2 + 4*a*c))^(1/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 533 vs. $2(88) = 176$.

Time = 0.83 (sec) , antiderivative size = 533, normalized size of antiderivative = 6.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1087, 1087, 1095, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^{4/3}dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{3(b+2cx)\left(a+bx+cx^2\right)^{4/3}}{22c}-\frac{2\left(b^2-4ac\right)\int \sqrt[3]{cx^2+bx+a}dx}{11c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{3(b+2cx)\left(a+bx+cx^2\right)^{4/3}}{22c}-\frac{2\left(b^2-4ac\right)\left(\frac{3(b+2cx)\sqrt[3]{a+bx+cx^2}}{10c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\left(cx^2+bx+a\right)^{2/3}}dx}{10c}\right)}{11c}$$

$$\Big\downarrow \text{1095}$$

$$\frac{3(b+2cx)\left(a+bx+cx^2\right)^{4/3}}{22c}-\frac{2\left(b^2-4ac\right)\left(\frac{3(b+2cx)\sqrt[3]{a+bx+cx^2}}{10c}-\frac{3\left(b^2-4ac\right)\sqrt{(b+2cx)^2}\int \frac{1}{\sqrt{b^2-4ac+4c\left(cx^2+bx+a\right)}}d\sqrt[3]{cx^2+bx+a}}{10c(b+2cx)}\right)}{11c}$$

$\downarrow$ 759

$$\frac{3(b+2cx)\left(a+bx+cx^2\right)^{4/3}}{22c}-\frac{2\left(b^2-4ac\right)\left(\frac{3(b+2cx)\sqrt[3]{a+bx+cx^2}}{10c}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(b^2-4ac\right)\sqrt{(b+2cx)^2}\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)\sqrt{\frac{-2^{2/3}\sqrt[3]{\cdots}}{\cdots}}\cdots}{5\ 2^{2/3}c^{4/3}(b+2cx)\cdots}\right)}{\cdots}$$

input
```
Int[(a + b*x + c*x^2)^(4/3),x]
```

output
```
(3*(b + 2*c*x)*(a + b*x + c*x^2)^(4/3))/(22*c) - (2*(b^2 - 4*a*c)*((3*(b +
 2*c*x)*(a + b*x + c*x^2)^(1/3))/(10*c) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b^2
- 4*a*c)*Sqrt[(b + 2*c*x)^2]*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b
*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 - 4*a*
c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^(2/
3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)
^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c
^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/
3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(5*2^(2/3)*c^(4/3)*
(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/
3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*
c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*Sqrt[b^2 - 4*a*c + 4*c*(a + b*x + c*x^
2)])))/(11*c)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1095
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[3*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(3*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^3], x], x, (a + b*x + c*x^2)^(1/3)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[3*p]
```

Maple [F]

$$\int \left(c\,x^{2}+b x +a \right)^{\frac{4}{3}}d x$$

input
```
int((c*x^2+b*x+a)^(4/3),x)
```

output
```
int((c*x^2+b*x+a)^(4/3),x)
```

Fricas [F]

$$\int \left(a+bx+cx^2\right)^{4/3}\,dx=\int \left(cx^{2}+bx+a\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^(4/3), x)
```

Sympy [F]

$$\int \left(a+bx+cx^2\right)^{4/3}\,dx=\int \left(a+bx+cx^2\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(4/3),x)
```

output
```
Integral((a + b*x + c*x**2)**(4/3), x)
```

Maxima [F]

$$\int \left(a+bx+cx^2\right)^{4/3}\,dx=\int \left(cx^2+bx+a\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(4/3), x)
```

Giac [F]

$$\int \left(a+bx+cx^2\right)^{4/3}\,dx=\int \left(cx^2+bx+a\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^{4/3} \, dx = \int \left(c\,x^2+b\,x+a\right)^{4/3} \,dx$$

input
```
int((a + b*x + c*x^2)^(4/3),x)
```

output
```
int((a + b*x + c*x^2)^(4/3), x)
```

Reduce [F]

$$\int \left(a+bx+cx^2\right)^{4/3} \, dx = \int \left(c\,x^2+bx+a\right)^{\frac{4}{3}}dx$$

input
```
int((c*x^2+b*x+a)^(4/3),x)
```

output
```
int((c*x^2+b*x+a)^(4/3),x)
```

3.146 $\int (a+bx+cx^2)^{2/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int (a+bx+cx^2)^{2/3}\,dx=\frac{(b+2cx)\,(a+bx+cx^2)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{4\sqrt[3]{2}c\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{2/3}}$$

output
```
1/8*(2*c*x+b)*(c*x^2+b*x+a)^(2/3)*hypergeom([-2/3, 1/2],[3/2],(2*c*x+b)^2/
(-4*a*c+b^2))*2^(2/3)/c/(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(2/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int (a+bx+cx^2)^{2/3}\,dx=\frac{(b+2cx)(a+x(b+cx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{4\sqrt[3]{2}c\left(\frac{c(a+x(b+cx))}{-b^2+4ac}\right)^{2/3}}$$

input
```
Integrate[(a + b*x + c*x^2)^(2/3),x]
```

output
```
((b + 2*c*x)*(a + x*(b + c*x))^(2/3)*Hypergeometric2F1[-2/3, 1/2, 3/2, (b
+ 2*c*x)^2/(b^2 - 4*a*c)])/(4*2^(1/3)*c*((c*(a + x*(b + c*x)))/(-b^2 + 4*a
*c))^(2/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1059 vs. $2(88) = 176$.

Time = 1.39 (sec) , antiderivative size = 1059, normalized size of antiderivative = 12.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1087, 1095, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^{2/3} dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{3(b+2cx)\left(a+bx+cx^2\right)^{2/3}}{14c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt[3]{cx^2+bx+a}}dx}{7c}$$

$$\Big\downarrow \text{ 1095}$$

$$\frac{3(b+2cx)\left(a+bx+cx^2\right)^{2/3}}{14c}-\frac{3\left(b^2-4ac\right)\sqrt{(b+2cx)^2}\int \frac{\sqrt[3]{cx^2+bx+a}}{\sqrt{b^2-4ac+4c(cx^2+bx+a)}}d\sqrt[3]{cx^2+bx+a}}{7c(b+2cx)}$$

$$\Big\downarrow \text{ 832}$$

$$\frac{3(b+2cx)\left(a+bx+cx^2\right)^{2/3}}{14c}-\frac{3\left(b^2-4ac\right)\sqrt{(b+2cx)^2}\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}}{\sqrt{b^2-4ac+4c\left(cx^2+bx+a\right)}}d\sqrt[3]{cx^2+bx+a}}{2^{2/3}\sqrt[3]{c}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b^2-4ac}\int \frac{\cdots}{\sqrt{b^2\cdots}}}{\cdots}\cdots\right.}{7c(b+2cx)}$$

$\big\downarrow$ 759

$$\frac{3(b+2cx)\left(a+bx+cx^2\right)^{2/3}}{14c}-\frac{3\left(b^2-4ac\right)\sqrt{(b+2cx)^2}\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}}{\sqrt{b^2-4ac+4c\left(cx^2+bx+a\right)}}d\sqrt[3]{cx^2+bx+a}}{2^{2/3}\sqrt[3]{c}}-\frac{\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{b^2-4a\cdots}}{\cdots}\cdots\right.}{\cdots}$$

$\big\downarrow$ 2416

$$\frac{3(b+2cx)\left(cx^2+bx+a\right)^{2/3}}{14c}-\frac{3\left(b^2-4ac\right)\sqrt{(b+2cx)^2}\left(\frac{\frac{\sqrt[3]{2}\sqrt{b^2-4ac+4c\left(cx^2+bx+a\right)}}{\sqrt[3]{c}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b^2-4ac}\left(\sqrt[3]{b^2-4ac}+2^{2/3}\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}$$

input
```
Int[(a + b*x + c*x^2)^(2/3),x]
```

output
```
(3*(b + 2*c*x)*(a + b*x + c*x^2)^(2/3))/(14*c) - (3*(b^2 - 4*a*c)*Sqrt[(b
+ 2*c*x)^2]*(((2^(1/3)*Sqrt[b^2 - 4*a*c + 4*c*(a + b*x + c*x^2)])/(c^(1/3)
*((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1
/3))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3
) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2
^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(
2/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)
*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(b^2
- 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(
b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqr
t[3]])/(2^(2/3)*c^(1/3)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2
^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3
) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*Sqrt[b^2 - 4*a*c + 4*c*(a
+ b*x + c*x^2)])))/(2^(2/3)*c^(1/3)) - ((1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(b^
2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^
(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a
+ b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sq
rt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a +
b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3...
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1095
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[3*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(3*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^3], x], x, (a + b*x + c*x^2)^(1/3)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[3*p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \left(c\,x^{2}+bx+a\right)^{\frac{2}{3}}dx$$

input
```
int((c*x^2+b*x+a)^(2/3),x)
```

output
```
int((c*x^2+b*x+a)^(2/3),x)
```

Fricas [F]

$$\int \left(a+bx+cx^2\right)^{2/3} dx = \int \left(cx^2+bx+a\right)^{\frac{2}{3}} dx$$

input
```
integrate((c*x^2+b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^(2/3), x)
```

Sympy [F]

$$\int \left(a+bx+cx^2\right)^{2/3} dx = \int \left(a+bx+cx^2\right)^{\frac{2}{3}} dx$$

input
```
integrate((c*x**2+b*x+a)**(2/3),x)
```

output
```
Integral((a + b*x + c*x**2)**(2/3), x)
```

Maxima [F]

$$\int \left(a+bx+cx^2\right)^{2/3} dx = \int \left(cx^2+bx+a\right)^{\frac{2}{3}} dx$$

input
```
integrate((c*x^2+b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(2/3), x)
```

Giac [F]

$$\int \left(a+bx+cx^2\right)^{2/3}\,dx=\int \left(cx^2+bx+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^{2/3}\,dx=\int {\left(c\,x^2+b\,x+a\right)}^{2/3}\,dx$$

input
```
int((a + b*x + c*x^2)^(2/3),x)
```

output
```
int((a + b*x + c*x^2)^(2/3), x)
```

Reduce [F]

$$\int (a+bx+cx^2)^{2/3}\,dx=\frac{6(c\,x^2+bx+a)^{\frac{2}{3}}\,a+3(c\,x^2+bx+a)^{\frac{2}{3}}\,bx-8\left(\int \frac{x}{(c\,x^2+bx+a)^{\frac{1}{3}}}dx\right)ac+2\left(\int \frac{x}{(c\,x^2+bx+a)^{\frac{1}{3}}}dx\right)}{7b}$$

input
```
int((c*x^2+b*x+a)^(2/3),x)
```

output
```
(6*(a + b*x + c*x**2)**(2/3)*a + 3*(a + b*x + c*x**2)**(2/3)*b*x - 8*int((
(a + b*x + c*x**2)**(2/3)*x)/(a + b*x + c*x**2),x)*a*c + 2*int(((a + b*x +
 c*x**2)**(2/3)*x)/(a + b*x + c*x**2),x)*b**2)/(7*b)
```

3.147 $\int \sqrt[3]{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \sqrt[3]{a+bx+cx^2}\,dx=\frac{(b+2cx)\sqrt[3]{a+bx+cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{2\,2^{2/3}c\sqrt[3]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}}$$

output
```
1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/3)*hypergeom([-1/3, 1/2],[3/2],(2*c*x+b)^2/
(-4*a*c+b^2))*2^(1/3)/c/(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/3)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int \sqrt[3]{a+bx+cx^2}\,dx=\frac{(b+2cx)\sqrt[3]{a+x(b+cx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{2\,2^{2/3}c\sqrt[3]{\frac{c(a+x(b+cx))}{-b^2+4ac}}}$$

input
```
Integrate[(a + b*x + c*x^2)^(1/3),x]
```

output
```
((b + 2*c*x)*(a + x*(b + c*x))^(1/3)*Hypergeometric2F1[-1/3, 1/2, 3/2, (b
+ 2*c*x)^2/(b^2 - 4*a*c)])/(2*2^(2/3)*c*((c*(a + x*(b + c*x)))/(-b^2 + 4*a
*c))^(1/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 490 vs. $2(88) = 176$.

Time = 0.62 (sec) , antiderivative size = 490, normalized size of antiderivative = 5.57, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1087, 1095, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx+cx^2}\,dx$$

$$\Big\downarrow \ 1087$$

$$\frac{3(b+2cx)\sqrt[3]{a+bx+cx^2}}{10c}-\frac{(b^2-4ac)\int \frac{1}{(cx^2+bx+a)^{2/3}}dx}{10c}$$

$$\Big\downarrow \ 1095$$

$$\frac{3(b+2cx)\sqrt[3]{a+bx+cx^2}}{10c}-\frac{3(b^2-4ac)\sqrt{(b+2cx)^2}\int \frac{1}{\sqrt{b^2-4ac+4c(cx^2+bx+a)}}d\sqrt[3]{cx^2+bx+a}}{10c(b+2cx)}$$

$$\Big\downarrow \ 759$$

$$\frac{3(b+2cx)\sqrt[3]{a+bx+cx^2}}{10c}-$$

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}(b^2-4ac)\sqrt{(b+2cx)^2}\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)\sqrt{\frac{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b^2-4ac}\sqrt[3]{a+bx+cx^2}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\cdots\right)}}}{5\,2^{2/3}c^{4/3}(b+2cx)\sqrt{\frac{\sqrt[3]{b^2-4ac}\left(\sqrt[3]{b^2-4ac}+2^{2/3}\cdots\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\cdots\right)}}}$$

input
```
Int[(a + b*x + c*x^2)^(1/3),x]
```

output
```
(3*(b + 2*c*x)*(a + b*x + c*x^2)^(1/3))/(10*c) - (3^(3/4)*Sqrt[2 + Sqrt[3]
]*(b^2 - 4*a*c)*Sqrt[(b + 2*c*x)^2]*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)
*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2
 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x + c*x
^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x +
 c*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^
(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3)
+ 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(5*2^(2/3)*c
^(4/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3
)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2
^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*Sqrt[b^2 - 4*a*c + 4*c*(a + b*x
 + c*x^2)])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1095
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[3*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(3*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^3], x], x, (a + b*x + c*x^2)^(1/3)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[3*p]
```

Maple [F]

$$\int \left(c\,x^{2}+b x +a \right)^{\frac{1}{3}}d x$$

input
```
int((c*x^2+b*x+a)^(1/3),x)
```

output
```
int((c*x^2+b*x+a)^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx+cx^2}\,dx = \int \left(cx^2+bx+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{a+bx+cx^2}\,dx = \int \sqrt[3]{a+bx+cx^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/3),x)
```

output
```
Integral((a + b*x + c*x**2)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{a+bx+cx^2}\,dx = \int (cx^2+bx+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx+cx^2}\,dx = \int (cx^2+bx+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx+cx^2}\,dx = \int (c\,x^2+b\,x+a)^{1/3}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/3),x)
```

output
```
int((a + b*x + c*x^2)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx+cx^2}\,dx = \int \left(c\,x^2+bx+a\right)^{\frac{1}{3}}dx$$

input
```
int((c*x^2+b*x+a)^(1/3),x)
```

output
```
int((c*x^2+b*x+a)^(1/3),x)
```

3.148 $\int \frac{1}{\sqrt[3]{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \frac{1}{\sqrt[3]{a+bx+cx^2}}\,dx=\frac{(b+2cx)\sqrt[3]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{\sqrt[3]{2}c\sqrt[3]{a+bx+cx^2}}$$

output
```
1/2*(2*c*x+b)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/3)*hypergeom([1/3, 1/2],[
3/2],(2*c*x+b)^2/(-4*a*c+b^2))*2^(2/3)/c/(c*x^2+b*x+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[3]{a+bx+cx^2}}\,dx =\frac{(b+2cx)\sqrt[3]{\frac{c(a+x(b+cx))}{-b^2+4ac}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{\sqrt[3]{2}c\sqrt[3]{a+x(b+cx)}}$$

input
```
Integrate[(a + b*x + c*x^2)^(-1/3),x]
```

output
```
((b + 2*c*x)*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/3)*Hypergeometric2F
1[1/3, 1/2, 3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(2^(1/3)*c*(a + x*(b + c*x)
)^(1/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1018 vs. 2(85) = 170.

Time = 1.28 (sec) , antiderivative size = 1018, normalized size of antiderivative = 11.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1095, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx+cx^2}}dx$$

$$\Big\downarrow \; 1095$$

$$\frac{3\sqrt{(b+2cx)^2}\int \frac{\sqrt[3]{cx^2+bx+a}}{\sqrt{b^2-4ac+4c(cx^2+bx+a)}}d\sqrt[3]{cx^2+bx+a}}{b+2cx}$$

$$\Big\downarrow \; 832$$

$$\frac{3\sqrt{(b+2cx)^2}\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}}{\sqrt{b^2-4ac+4c\left(cx^2+bx+a\right)}}d\sqrt[3]{cx^2+bx+a}}{2^{2/3}\sqrt[3]{c}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b^2-4ac}\int \frac{1}{\sqrt{b^2-4ac+4c\left(cx^2+\right.}}}{2^{2/3}\sqrt[3]{c}}\right.}{b+2cx}$$

$$\Big\downarrow \; 759$$

$$\frac{3\sqrt{(b+2cx)^2}\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}}{\sqrt{b^2-4ac+4c\left(cx^2+bx+a\right)}}d\sqrt[3]{cx^2+bx+a}}{2^{2/3}\sqrt[3]{c}}-\frac{\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{b^2-4ac}\left(\sqrt[3]{b^2-\right.}{}\right.}{}$$

$$\Big\downarrow \text{2416}$$

$$3\sqrt{(b+2cx)^2}\left(\frac{\frac{\sqrt[3]{2}\sqrt{b^2-4ac+4c\left(cx^2+bx+a\right)}}{\sqrt[3]{c}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b^2-4ac}\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2-\cdots}\right.}{\cdots}}{\cdots}\right.$$

input
```
Int[(a + b*x + c*x^2)^(-1/3),x]
```

output
```
(3*Sqrt[(b + 2*c*x)^2]*(((2^(1/3)*Sqrt[b^2 - 4*a*c + 4*c*(a + b*x + c*x^2)
])/(c^(1/3)*((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x
+ c*x^2)^(1/3))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(b^2 - 4*a*c)^(1/3)*((b^2 -
4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c
)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*
2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3
) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 +
 Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))],
 -7 - 4*Sqrt[3]])/(2^(2/3)*c^(1/3)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c
)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2 -
4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*Sqrt[b^2 - 4*a*
c + 4*c*(a + b*x + c*x^2)]))/(2^(2/3)*c^(1/3)) - ((1 - Sqrt[3])*Sqrt[2 + S
qrt[3]]*(b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*
x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c
)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^(2/3
))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^
(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^
(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3
)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(2^(1/3)*3^(1/4)*...
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1095
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[3*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(3*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^3], x], x, (a + b*x + c*x^2)^(1/3)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[3*p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(c\,x^2+bx+a)^{\frac{1}{3}}}dx$$

input
```
int(1/(c*x^2+b*x+a)^(1/3),x)
```

output
```
int(1/(c*x^2+b*x+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{a+bx+cx^2}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^(-1/3), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt[3]{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/3),x)
```

output
```
Integral((a + b*x + c*x**2)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx+cx^2}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx+cx^2}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx+cx^2}}\,dx = \int \frac{1}{(c\,x^2+b\,x+a)^{1/3}}\,dx$$

input
```
int(1/(a + b*x + c*x^2)^(1/3),x)
```

output
```
int(1/(a + b*x + c*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx+cx^2}}\,dx = \int \frac{1}{(c\,x^2+bx+a)^{\frac{1}{3}}}dx$$

input
```
int(1/(c*x^2+b*x+a)^(1/3),x)
```

output
```
int(1/(a + b*x + c*x**2)**(1/3),x)
```

3.149 $\int \frac{1}{(a+bx+cx^2)^{2/3}} dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \frac{1}{(a+bx+cx^2)^{2/3}}\,dx=\frac{\sqrt[3]{2}(b+2cx)\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{c\left(a+bx+cx^2\right)^{2/3}}$$

output
```
2^(1/3)*(2*c*x+b)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(2/3)*hypergeom([1/2, 2/
3],[3/2],(2*c*x+b)^2/(-4*a*c+b^2))/c/(c*x^2+b*x+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+bx+cx^2)^{2/3}}\,dx=\frac{\sqrt[3]{2}(b+2cx)\left(\frac{c(a+x(b+cx))}{-b^2+4ac}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{c(a+x(b+cx))^{2/3}}$$

input
```
Integrate[(a + b*x + c*x^2)^(-2/3),x]
```

output
```
(2^(1/3)*(b + 2*c*x)*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(2/3)*Hypergeo
metric2F1[1/2, 2/3, 3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(c*(a + x*(b + c*x)
)^(2/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 451 vs. $2(85) = 170$.

Time = 0.56 (sec) , antiderivative size = 451, normalized size of antiderivative = 5.31, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1095, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx+cx^2)^{2/3}} \, dx$$

$$\Big\downarrow \text{ 1095}$$

$$\frac{3\sqrt{(b+2cx)^2}\int \frac{1}{\sqrt{b^2-4ac+4c(cx^2+bx+a)}}d\sqrt[3]{cx^2+bx+a}}{b+2cx}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt[3]{2}3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{(b+2cx)^2}\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)\sqrt{\frac{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b^2-4ac}\sqrt[3]{a+bx+cx^2}+(b^2-4\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{\ldots}\right.}}}{\sqrt[3]{c}(b+2cx)\sqrt{\frac{\sqrt[3]{b^2-4ac}\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a\ldots}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+\ldots}\right.}}}$$

input
```
Int[(a + b*x + c*x^2)^(-2/3),x]
```

output
```
(2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*Sqrt[(b + 2*c*x)^2]*((b^2 - 4*a*c)^(1/3
) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2
^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(
2/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)
*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2
- 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(
b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqr
t[3]])/(c^(1/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3)
 + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2 - 4*a*c)^
(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*Sqrt[b^2 - 4*a*c + 4*c
*(a + b*x + c*x^2)])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1095
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[3*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(3*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^3], x], x, (a + b*x + c*x^2)^(1/3)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[3*p]
```

Maple [F]

$$\int \frac{1}{(c\,x^2 + bx + a)^{\frac{2}{3}}} dx$$

input
```
int(1/(c*x^2+b*x+a)^(2/3),x)
```

output
```
int(1/(c*x^2+b*x+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx+cx^2)^{2/3}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^(-2/3), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx+cx^2)^{2/3}}\,dx = \int \frac{1}{(a+bx+cx^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(2/3),x)
```

output
```
Integral((a + b*x + c*x**2)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx+cx^2)^{2/3}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx+cx^2)^{2/3}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{2/3}}\,dx = \int \frac{1}{(c\,x^2+b\,x+a)^{2/3}}\,dx$$

input
```
int(1/(a + b*x + c*x^2)^(2/3),x)
```

output
```
int(1/(a + b*x + c*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx+cx^2)^{2/3}}\,dx = \int \frac{1}{(c\,x^2+bx+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(c*x^2+b*x+a)^(2/3),x)
```

output
```
int(1/(a + b*x + c*x**2)**(2/3),x)
```

3.150 $\int \frac{1}{(a+bx+cx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 86

$$\int \frac{1}{(a+bx+cx^2)^{4/3}}\,dx = \frac{2\,2^{2/3}(b+2cx)\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{c\,(a+bx+cx^2)^{4/3}}$$

output
```
2*2^(2/3)*(2*c*x+b)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(4/3)*hypergeom([1/2,
4/3],[3/2],(2*c*x+b)^2/(-4*a*c+b^2))/c/(c*x^2+b*x+a)^(4/3)
```

Mathematica [A] (verified)

Time = 10.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.19

$$\int \frac{1}{(a+bx+cx^2)^{4/3}}\,dx = -\frac{(b+2cx)\left(3\sqrt[3]{2}-2\sqrt[3]{\frac{c(a+x(b+cx))}{-b^2+4ac}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)\right)}{\sqrt[3]{2}\,(b^2-4ac)\sqrt[3]{a+x(b+cx)}}$$

input
```
Integrate[(a + b*x + c*x^2)^(-4/3),x]
```

output
```
-(((b + 2*c*x)*(3*2^(1/3) - 2*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/3)
*Hypergeometric2F1[1/3, 1/2, 3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(2^(1/3)*
(b^2 - 4*a*c)*(a + x*(b + c*x))^(1/3)))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1062 vs. $2(86) = 172$.

Time = 1.35 (sec) , antiderivative size = 1062, normalized size of antiderivative = 12.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1089, 1095, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx+cx^2)^{4/3}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2c\int \frac{1}{\sqrt[3]{cx^2+bx+a}}dx}{b^2-4ac}-\frac{3(b+2cx)}{(b^2-4ac)\sqrt[3]{a+bx+cx^2}}$$

$$\Big\downarrow \text{1095}$$

$$\frac{6c\sqrt{(b+2cx)^2}\int \frac{\sqrt[3]{cx^2+bx+a}}{\sqrt{b^2-4ac+4c(cx^2+bx+a)}}d\sqrt[3]{cx^2+bx+a}}{(b^2-4ac)(b+2cx)}-\frac{3(b+2cx)}{(b^2-4ac)\sqrt[3]{a+bx+cx^2}}$$

$$\Big\downarrow \text{832}$$

$$\frac{6c\sqrt{(b+2cx)^2}\left(\frac{\int \frac{(1-\sqrt{3})\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}}{\sqrt{b^2-4ac+4c(cx^2+bx+a)}}d\sqrt[3]{cx^2+bx+a}}{2^{2/3}\sqrt[3]{c}}-\frac{(1-\sqrt{3})\sqrt[3]{b^2-4ac}\int \frac{1}{\sqrt{b^2-4ac+4c(cx^2}}}{2^{2/3}\sqrt[3]{c}}\right.}{(b^2-4ac)(b+2cx)}$$

$$\frac{3(b+2cx)}{(b^2-4ac)\sqrt[3]{a+bx+cx^2}}$$

$$\Big\downarrow \text{759}$$

$$\frac{6c\sqrt{(b+2cx)^2}\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}}{\sqrt{b^2-4ac+4c\left(cx^2+bx+a\right)}}d\sqrt[3]{cx^2+bx+a}}{2^{2/3}\sqrt[3]{c}}-\frac{\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{b^2-4ac}\left(\sqrt[3]{b^2-\right.}{}\right.}{}}{}$$

$$\frac{3(b+2cx)}{(b^2-4ac)\sqrt[3]{a+bx+cx^2}}$$

$\downarrow$ 2416

$$\frac{6c\sqrt{(b+2cx)^2}\left(\frac{\sqrt[3]{2}\sqrt{b^2-4ac+4c\left(cx^2+bx+a\right)}}{\sqrt[3]{c}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b^2-4ac}\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2\right.}{}\right.}{}}{}$$

$$\frac{3(b+2cx)}{(b^2-4ac)\sqrt[3]{cx^2+bx+a}}$$

input
```
Int[(a + b*x + c*x^2)^(-4/3),x]
```

output
```
(-3*(b + 2*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^(1/3)) + (6*c*Sqrt[(b +
2*c*x)^2]*(((2^(1/3)*Sqrt[b^2 - 4*a*c + 4*c*(a + b*x + c*x^2)])/(c^(1/3)*(
(1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3
))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3)
+ 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(
2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/
3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c
^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(b^2 -
4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^
2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[
3]])/(2^(2/3)*c^(1/3)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(
2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3)
+ 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*Sqrt[b^2 - 4*a*c + 4*c*(a +
b*x + c*x^2)])))/(2^(2/3)*c^(1/3)) - ((1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(b^2
- 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1
/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a +
b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt
[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*
x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*...
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1095
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[3*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(3*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^3], x], x, (a + b*x + c*x^2)^(1/3)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[3*p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{\left(c\,x^{2}+bx+a\right)^{\frac{4}{3}}}dx$$

input
```
int(1/(c*x^2+b*x+a)^(4/3),x)
```

output
```
int(1/(c*x^2+b*x+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx+cx^2)^{4/3}}\,dx=\int \frac{1}{(cx^2+bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^(2/3)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2
*a*c)*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx+cx^2)^{4/3}}\,dx=\int \frac{1}{(a+bx+cx^2)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(4/3),x)
```

output
```
Integral((a + b*x + c*x**2)**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx+cx^2)^{4/3}}\,dx=\int \frac{1}{(cx^2+bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx+cx^2)^{4/3}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{4/3}}\,dx = \int \frac{1}{(c\,x^2+b\,x+a)^{4/3}}\,dx$$

input
```
int(1/(a + b*x + c*x^2)^(4/3),x)
```

output
```
int(1/(a + b*x + c*x^2)^(4/3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx+cx^2)^{4/3}}\,dx = \int \frac{1}{(c\,x^2+bx+a)^{\frac{1}{3}}\,a+(c\,x^2+bx+a)^{\frac{1}{3}}\,bx+(c\,x^2+bx+a)^{\frac{1}{3}}\,c\,x^2}dx$$

input
```
int(1/(c*x^2+b*x+a)^(4/3),x)
```

output
```
int(1/((a + b*x + c*x**2)**(1/3)*a + (a + b*x + c*x**2)**(1/3)*b*x + (a +
b*x + c*x**2)**(1/3)*c*x**2),x)
```

3.151 $\int (a+bx+cx^2)^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 85

$$\int (a+bx+cx^2)^p \, dx = \frac{2^{-1-2p}(b+2cx)\left(a+bx+cx^2\right)^p \left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{c}$$

output
```
2^(-1-2*p)*(2*c*x+b)*(c*x^2+b*x+a)^p*hypergeom([1/2, -p],[3/2],(2*c*x+b)^2
/(-4*a*c+b^2))/c/((-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.48

$$\int (a+bx+cx^2)^p \, dx = \frac{2^{-1+p}\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{-p}(a+x(b+cx))^p \text{Hypergeometric2F1}\left(-p,1+p,2+p,\right.}{c(1+p)}$$

input
```
Integrate[(a + b*x + c*x^2)^p,x]
```

output
```
(2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(a + x*(b + c*x))^p*Hypergeome
tric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4
*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c])^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1096\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 1096}$$

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

input
```
Int[(a + b*x + c*x^2)^p,x]
```

output
```
-((2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 -
p)*(a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p))
)
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

Maple [F]

$$\int \left(c\,x^{2}+b x +a \right)^{p}d x$$

input
```
int((c*x^2+b*x+a)^p,x)
```

output
```
int((c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx+cx^2\right)^p\,dx=\int \left(cx^2+bx+a\right)^p\,dx$$

input
```
integrate((c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p, x)
```

Sympy [F]

$$\int \left(a+bx+cx^2\right)^p\,dx=\int \left(a+bx+cx^2\right)^p\,dx$$

input
```
integrate((c*x**2+b*x+a)**p,x)
```

output
```
Integral((a + b*x + c*x**2)**p, x)
```

Maxima [F]

$$\int \left(a+bx+cx^2\right)^p\,dx=\int \left(cx^2+bx+a\right)^p\,dx$$

input
```
integrate((c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p, x)
```

Giac [F]

$$\int \left(a+bx+cx^2\right)^p\,dx=\int \left(cx^2+bx+a\right)^p\,dx$$

input
```
integrate((c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^p\,dx=\int \left(c\,x^2+b\,x+a\right)^p\,dx$$

input
```
int((a + b*x + c*x^2)^p,x)
```

output
```
int((a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int (a + bx + cx^2)^p \, dx$$

$$= \frac{2(c\,x^2 + bx + a)^p \, a + (c\,x^2 + bx + a)^p \, bx - 8\left(\int \frac{(c\,x^2+bx+a)^p x}{2cp\,x^2+2bpx+c\,x^2+2ap+bx+a}dx\right) ac\,p^2 - 4\left(\int \frac{(c\,x^2+bx+a)^p}{2cp\,x^2+2bpx+c\,x^2+2}\right.}{b\,(2p+1)}$$

input
```
int((c*x^2+b*x+a)^p,x)
```

output
```
(2*(a + b*x + c*x**2)**p*a + (a + b*x + c*x**2)**p*b*x - 8*int(((a + b*x +
 c*x**2)**p*x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*a*c*p*
*2 - 4*int(((a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x*
*2 + c*x**2),x)*a*c*p + 2*int(((a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p
*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**2*p**2 + int(((a + b*x + c*x**2)**p*
x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**2*p)/(b*(2*p +
1))
```

3.152 $\int (3+4x+5x^2)^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int (3+4x+5x^2)^p \, dx = 5^{-1-p}11^p(2+5x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{11}(2+5x)^2\right)$$

output
```
5^(-1-p)*11^p*(2+5*x)*hypergeom([1/2, -p],[3/2],-1/11*(2+5*x)^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (3+4x+5x^2)^p \, dx = 5^{-1-p}11^p(2+5x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{11}(2+5x)^2\right)$$

input
```
Integrate[(3 + 4*x + 5*x^2)^p,x]
```

output
```
5^(-1 - p)*11^p*(2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/11*(2 + 5*x)^
2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+4x+3)^p\,dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}5^{-p-1}11^p\int \left(\frac{1}{44}(10x+4)^2+1\right)^p d(10x+4)$$

$$\Big\downarrow \text{237}$$

$$\frac{1}{2}5^{-p-1}11^p(10x+4)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{44}(10x+4)^2\right)$$

input
```
Int[(3 + 4*x + 5*x^2)^p,x]
```

output
```
(5^(-1 - p)*11^p*(4 + 10*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/44*(4 + 10*
x)^2])/2
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int \left(5x^2 + 4x + 3\right)^p dx$$

input
```
int((5*x^2+4*x+3)^p,x)
```

output
```
int((5*x^2+4*x+3)^p,x)
```

Fricas [F]

$$\int \left(3 + 4x + 5x^2\right)^p \, dx = \int \left(5\,x^2 + 4\,x + 3\right)^p dx$$

input
```
integrate((5*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((5*x^2 + 4*x + 3)^p, x)
```

Sympy [F]

$$\int \left(3 + 4x + 5x^2\right)^p \, dx = \int \left(5x^2 + 4x + 3\right)^p \, dx$$

input
```
integrate((5*x**2+4*x+3)**p,x)
```

output
```
Integral((5*x**2 + 4*x + 3)**p, x)
```

Maxima [F]

$$\int (3+4x+5x^2)^p \, dx = \int (5\,x^2+4\,x+3)^p \, dx$$

input
```
integrate((5*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 4*x + 3)^p, x)
```

Giac [F]

$$\int (3+4x+5x^2)^p \, dx = \int (5\,x^2+4\,x+3)^p \, dx$$

input
```
integrate((5*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((5*x^2 + 4*x + 3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (3+4x+5x^2)^p \, dx = \int (5\,x^2+4\,x+3)^p \, dx$$

input
```
int((4*x + 5*x^2 + 3)^p,x)
```

output
```
int((4*x + 5*x^2 + 3)^p, x)
```

Reduce [F]

$$\int (3+4x+5x^2)^p \, dx$$
$$= \frac{2(5x^2+4x+3)^p x + 3(5x^2+4x+3)^p - 44\left(\int \frac{(5x^2+4x+3)^p x}{10p\,x^2+8px+5x^2+6p+4x+3}dx\right)p^2 - 22\left(\int \frac{(5x^2+4x+3)^p x}{10p\,x^2+8px+5x^2+6p+4x+ }\right.}{4p+2}$$

input
```
int((5*x^2+4*x+3)^p,x)
```

output
```
(2*(5*x**2 + 4*x + 3)**p*x + 3*(5*x**2 + 4*x + 3)**p - 44*int(((5*x**2 + 4
*x + 3)**p*x)/(10*p*x**2 + 8*p*x + 6*p + 5*x**2 + 4*x + 3),x)*p**2 - 22*in
t(((5*x**2 + 4*x + 3)**p*x)/(10*p*x**2 + 8*p*x + 6*p + 5*x**2 + 4*x + 3),x
)*p)/(2*(2*p + 1))
```

3.153 $\int (3+4x+4x^2)^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int (3+4x+4x^2)^p \, dx = 2^{-1+p}(1+2x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{2}(1+2x)^2\right)$$

output
```
2^(-1+p)*(1+2*x)*hypergeom([1/2, -p],[3/2],-1/2*(1+2*x)^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (3+4x+4x^2)^p \, dx = 2^{-3+p}(4+8x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{32}(4+8x)^2\right)$$

input
```
Integrate[(3 + 4*x + 4*x^2)^p,x]
```

output
```
2^(-3 + p)*(4 + 8*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/32*(4 + 8*x)^2]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (4x^2+4x+3)^p \, dx$$

$$\Big\downarrow \text{ 1090}$$

$$2^{p-3} \int \left(\frac{1}{32}(8x+4)^2+1\right)^p d(8x+4)$$

$$\Big\downarrow \text{ 237}$$

$$2^{p-3}(8x+4) \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{32}(8x+4)^2\right)$$

input
```
Int[(3 + 4*x + 4*x^2)^p,x]
```

output
```
2^(-3 + p)*(4 + 8*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/32*(4 + 8*x)^2]
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int (4x^2 + 4x + 3)^p \, dx$$

input
```
int((4*x^2+4*x+3)^p,x)
```

output
```
int((4*x^2+4*x+3)^p,x)
```

Fricas [F]

$$\int \left(3 + 4x + 4x^2\right)^p \, dx = \int (4\,x^2 + 4\,x + 3)^p \, dx$$

input
```
integrate((4*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((4*x^2 + 4*x + 3)^p, x)
```

Sympy [F]

$$\int \left(3 + 4x + 4x^2\right)^p \, dx = \int \left(4x^2 + 4x + 3\right)^p \, dx$$

input
```
integrate((4*x**2+4*x+3)**p,x)
```

output
```
Integral((4*x**2 + 4*x + 3)**p, x)
```

Maxima [F]

$$\int \left(3+4x+4x^2\right)^p \, dx = \int \left(4\,x^2+4\,x+3\right)^p \, dx$$

input
```
integrate((4*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((4*x^2 + 4*x + 3)^p, x)
```

Giac [F]

$$\int \left(3+4x+4x^2\right)^p \, dx = \int \left(4\,x^2+4\,x+3\right)^p \, dx$$

input
```
integrate((4*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((4*x^2 + 4*x + 3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3+4x+4x^2\right)^p \, dx = \int \left(4\,x^2+4\,x+3\right)^p \, dx$$

input
```
int((4*x + 4*x^2 + 3)^p,x)
```

output
```
int((4*x + 4*x^2 + 3)^p, x)
```

Reduce [F]

$$\int \left(3+4x+4x^2\right)^p\,dx$$
$$=\frac{2(4x^2+4x+3)^p x+3(4x^2+4x+3)^p-32\left(\int \frac{(4x^2+4x+3)^p x}{8p\,x^2+8px+4x^2+6p+4x+3}dx\right)p^2-16\left(\int \frac{(4x^2+4x+3)^p x}{8p\,x^2+8px+4x^2+6p+4x+3}\right.}{4p+2}$$

input
```
int((4*x^2+4*x+3)^p,x)
```

output
```
(2*(4*x**2 + 4*x + 3)**p*x + 3*(4*x**2 + 4*x + 3)**p - 32*int(((4*x**2 + 4
*x + 3)**p*x)/(8*p*x**2 + 8*p*x + 6*p + 4*x**2 + 4*x + 3),x)*p**2 - 16*int
(((4*x**2 + 4*x + 3)**p*x)/(8*p*x**2 + 8*p*x + 6*p + 4*x**2 + 4*x + 3),x)*
p)/(2*(2*p + 1))
```

3.154 $\int (3+4x+3x^2)^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int (3+4x+3x^2)^p \, dx = 3^{-1-p}5^p(2+3x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{5}(2+3x)^2\right)$$

output
```
3^(-1-p)*5^p*(2+3*x)*hypergeom([1/2, -p],[3/2],-1/5*(2+3*x)^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (3+4x+3x^2)^p \, dx = 3^{-1-p}5^p(2+3x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{5}(2+3x)^2\right)$$

input
```
Integrate[(3 + 4*x + 3*x^2)^p,x]
```

output
```
3^(-1 - p)*5^p*(2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/5*(2 + 3*x)^2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^2+4x+3\right)^p \, dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}3^{-p-1}5^p \int \left(\frac{1}{20}(6x+4)^2+1\right)^p d(6x+4)$$

$$\Big\downarrow \text{237}$$

$$\frac{1}{2}3^{-p-1}5^p(6x+4)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{20}(6x+4)^2\right)$$

input
```
Int[(3 + 4*x + 3*x^2)^p,x]
```

output
```
(3^(-1 - p)*5^p*(4 + 6*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/20*(4 + 6*x)^
2])/2
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int (3x^2 + 4x + 3)^p \, dx$$

input
```
int((3*x^2+4*x+3)^p,x)
```

output
```
int((3*x^2+4*x+3)^p,x)
```

Fricas [F]

$$\int (3 + 4x + 3x^2)^p \, dx = \int (3\,x^2 + 4\,x + 3)^p \, dx$$

input
```
integrate((3*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((3*x^2 + 4*x + 3)^p, x)
```

Sympy [F]

$$\int (3 + 4x + 3x^2)^p \, dx = \int (3x^2 + 4x + 3)^p \, dx$$

input
```
integrate((3*x**2+4*x+3)**p,x)
```

output
```
Integral((3*x**2 + 4*x + 3)**p, x)
```

Maxima [F]

$$\int \left(3+4x+3x^2\right)^p \, dx = \int \left(3\,x^2+4\,x+3\right)^p \, dx$$

input
```
integrate((3*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 4*x + 3)^p, x)
```

Giac [F]

$$\int \left(3+4x+3x^2\right)^p \, dx = \int \left(3\,x^2+4\,x+3\right)^p \, dx$$

input
```
integrate((3*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((3*x^2 + 4*x + 3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3+4x+3x^2\right)^p \, dx = \int \left(3\,x^2+4\,x+3\right)^p \, dx$$

input
```
int((4*x + 3*x^2 + 3)^p,x)
```

output
```
int((4*x + 3*x^2 + 3)^p, x)
```

Reduce [F]

$$\int \left(3+4x+3x^2\right)^p\,dx$$
$$= \frac{2(3x^2+4x+3)^p x+3(3x^2+4x+3)^p-20\left(\int \frac{(3x^2+4x+3)^p x}{6p\,x^2+8px+3x^2+6p+4x+3}dx\right)p^2-10\left(\int \frac{(3x^2+4x+3)^p x}{6p\,x^2+8px+3x^2+6p+4x+3}\right.}{4p+2}$$

input
```
int((3*x^2+4*x+3)^p,x)
```

output
```
(2*(3*x**2 + 4*x + 3)**p*x + 3*(3*x**2 + 4*x + 3)**p - 20*int(((3*x**2 + 4
*x + 3)**p*x)/(6*p*x**2 + 8*p*x + 6*p + 3*x**2 + 4*x + 3),x)*p**2 - 10*int
(((3*x**2 + 4*x + 3)**p*x)/(6*p*x**2 + 8*p*x + 6*p + 3*x**2 + 4*x + 3),x)*
p)/(2*(2*p + 1))
```

3.155 $\int (3+4x+2x^2)^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int (3+4x+2x^2)^p \, dx = (1+x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-2(1+x)^2\right)$$

output
```
(1+x)*hypergeom([1/2, -p],[3/2],-2*(1+x)^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (3+4x+2x^2)^p \, dx = (1+x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-2(1+x)^2\right)$$

input
```
Integrate[(3 + 4*x + 2*x^2)^p,x]
```

output
```
(1 + x)*Hypergeometric2F1[1/2, -p, 3/2, -2*(1 + x)^2]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2+4x+3\right)^p \, dx$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{4}\int \left(\frac{1}{8}(4x+4)^2+1\right)^p d(4x+4)$$

$$\Big\downarrow \text{ 237}$$

$$\frac{1}{4}(4x+4)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{1}{8}(4x+4)^2\right)$$

input
```
Int[(3 + 4*x + 2*x^2)^p,x]
```

output
```
((4 + 4*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/8*(4 + 4*x)^2])/4
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int \left(2x^2+4x+3\right)^p dx$$

input
```
int((2*x^2+4*x+3)^p,x)
```

output
```
int((2*x^2+4*x+3)^p,x)
```

Fricas [F]

$$\int \left(3+4x+2x^2\right)^p \, dx = \int \left(2\,x^2+4\,x+3\right)^p dx$$

input
```
integrate((2*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((2*x^2 + 4*x + 3)^p, x)
```

Sympy [F]

$$\int \left(3+4x+2x^2\right)^p \, dx = \int \left(2x^2+4x+3\right)^p \, dx$$

input
```
integrate((2*x**2+4*x+3)**p,x)
```

output
```
Integral((2*x**2 + 4*x + 3)**p, x)
```

Maxima [F]

$$\int (3+4x+2x^2)^p \, dx = \int (2\,x^2+4\,x+3)^p \, dx$$

input
```
integrate((2*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 4*x + 3)^p, x)
```

Giac [F]

$$\int (3+4x+2x^2)^p \, dx = \int (2\,x^2+4\,x+3)^p \, dx$$

input
```
integrate((2*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((2*x^2 + 4*x + 3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (3+4x+2x^2)^p \, dx = \int (2\,x^2+4\,x+3)^p \, dx$$

input
```
int((4*x + 2*x^2 + 3)^p,x)
```

output
```
int((4*x + 2*x^2 + 3)^p, x)
```

Reduce [F]

$$\int \left(3+4x+2x^2\right)^p\,dx$$
$$=\frac{2(2x^2+4x+3)^p x+3(2x^2+4x+3)^p-8\left(\int \frac{(2x^2+4x+3)^p x}{4p\,x^2+8px+2x^2+6p+4x+3}dx\right)p^2-4\left(\int \frac{(2x^2+4x+3)^p x}{4p\,x^2+8px+2x^2+6p+4x+3}dx\right.}{4p+2}$$

input
```
int((2*x^2+4*x+3)^p,x)
```

output
```
(2*(2*x**2 + 4*x + 3)**p*x + 3*(2*x**2 + 4*x + 3)**p - 8*int(((2*x**2 + 4*
x + 3)**p*x)/(4*p*x**2 + 8*p*x + 6*p + 2*x**2 + 4*x + 3),x)*p**2 - 4*int((
(2*x**2 + 4*x + 3)**p*x)/(4*p*x**2 + 8*p*x + 6*p + 2*x**2 + 4*x + 3),x)*p)
/(2*(2*p + 1))
```

3.156 $\int (3+4x+x^2)^p \, dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int (3+4x+x^2)^p \, dx = -\frac{2^{1+2p}(-2-2x)^{-1-p}(3+4x+x^2)^{1+p}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{3+x}{2}\right)}{1+p}$$

output
```
-2^(1+2*p)*(-2-2*x)^(-1-p)*(x^2+4*x+3)^(p+1)*hypergeom([-p, p+1],[2+p],3/2
+1/2*x)/(p+1)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int (3+4x+x^2)^p \, dx = \frac{2^p(1+x)(3+x)^{-p}(3+4x+x^2)^p\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{1}{2}(-1-x)\right)}{1+p}$$

input
```
Integrate[(3 + 4*x + x^2)^p,x]
```

output
```
(2^p*(1 + x)*(3 + 4*x + x^2)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (-1 - x
)/2])/((1 + p)*(3 + x)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^2+4x+3)^p\,dx$$

$$\Big\downarrow \; 1096$$

$$-\frac{2^{2p+1}(-2x-2)^{-p-1}\left(x^2+4x+3\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{x+3}{2}\right)}{p+1}$$

input
```
Int[(3 + 4*x + x^2)^p,x]
```

output
```
-((2^(1 + 2*p)*(-2 - 2*x)^(-1 - p)*(3 + 4*x + x^2)^(1 + p)*Hypergeometric2
F1[-p, 1 + p, 2 + p, (3 + x)/2])/(1 + p))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

Maple [F]

$$\int \left(x^2+4x+3\right)^p dx$$

input
```
int((x^2+4*x+3)^p,x)
```

output
```
int((x^2+4*x+3)^p,x)
```

Fricas [F]

$$\int \left(3+4x+x^2\right)^p \, dx = \int \left(x^2+4\,x+3\right)^p dx$$

input
```
integrate((x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((x^2 + 4*x + 3)^p, x)
```

Sympy [F]

$$\int \left(3+4x+x^2\right)^p \, dx = \int \left(x^2+4x+3\right)^p \, dx$$

input
```
integrate((x**2+4*x+3)**p,x)
```

output
```
Integral((x**2 + 4*x + 3)**p, x)
```

Maxima [F]

$$\int \left(3+4x+x^2\right)^p\,dx=\int \left(x^2+4\,x+3\right)^p\,dx$$

input
```
integrate((x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((x^2 + 4*x + 3)^p, x)
```

Giac [F]

$$\int \left(3+4x+x^2\right)^p\,dx=\int \left(x^2+4\,x+3\right)^p\,dx$$

input
```
integrate((x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((x^2 + 4*x + 3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3+4x+x^2\right)^p\,dx=\int \left(x^2+4\,x+3\right)^p\,dx$$

input
```
int((4*x + x^2 + 3)^p,x)
```

output
```
int((4*x + x^2 + 3)^p, x)
```

Reduce [F]

$$\int \left(3+4x+x^2\right)^p\,dx$$
$$=\frac{2(x^2+4x+3)^p\,x+3(x^2+4x+3)^p+4\left(\int \frac{(x^2+4x+3)^p x}{2p\,x^2+8px+x^2+6p+4x+3}dx\right)p^2+2\left(\int \frac{(x^2+4x+3)^p x}{2p\,x^2+8px+x^2+6p+4x+3}dx\right)p}{4p+2}$$

input
```
int((x^2+4*x+3)^p,x)
```

output
```
(2*(x**2 + 4*x + 3)**p*x + 3*(x**2 + 4*x + 3)**p + 4*int(((x**2 + 4*x + 3)
**p*x)/(2*p*x**2 + 8*p*x + 6*p + x**2 + 4*x + 3),x)*p**2 + 2*int(((x**2 +
4*x + 3)**p*x)/(2*p*x**2 + 8*p*x + 6*p + x**2 + 4*x + 3),x)*p)/(2*(2*p + 1
))
```

3.157 $\int (3+4x)^p \, dx$

Optimal result

Integrand size = 7, antiderivative size = 18

$$\int (3+4x)^p \, dx = \frac{(3+4x)^{1+p}}{4(1+p)}$$

output
```
(3+4*x)^(p+1)/(4*p+4)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (3+4x)^p \, dx = \frac{(3+4x)^{1+p}}{4(1+p)}$$

input
```
Integrate[(3 + 4*x)^p,x]
```

output
```
(3 + 4*x)^(1 + p)/(4*(1 + p))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (4x+3)^p\,dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(4x+3)^{p+1}}{4(p+1)}$$

input
```
Int[(3 + 4*x)^p,x]
```

output
```
(3 + 4*x)^(1 + p)/(4*(1 + p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{(3+4x)^{p+1}}{4p+4}$	17
default	$\frac{(3+4x)^{p+1}}{4p+4}$	17
meijerg	$3^{p}x\,\text{hypergeom}\left(\left[1,-p\right],\left[2\right],-\frac{4x}{3}\right)$	17
risch	$\frac{(3+4x)(3+4x)^{p}}{4p+4}$	20
orering	$\frac{(3+4x)(3+4x)^{p}}{4p+4}$	20
parallelrisch	$\frac{12(3+4x)^{p}x+9(3+4x)^{p}}{12p+12}$	28
norman	$\frac{x\,\mathrm{e}^{p\ln(3+4x)}}{p+1}+\frac{3\,\mathrm{e}^{p\ln(3+4x)}}{4(p+1)}$	34

input
```
int((3+4*x)^p,x,method=_RETURNVERBOSE)
```

output
```
1/4*(3+4*x)^(p+1)/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int (3+4x)^p\,dx=\frac{(4\,x+3)^{p}(4\,x+3)}{4\,(p+1)}$$

input
```
integrate((3+4*x)^p,x, algorithm="fricas")
```

output
```
1/4*(4*x + 3)^p*(4*x + 3)/(p + 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (3+4x)^p\,dx = \frac{\begin{cases} \frac{(4x+3)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(4x+3) & \text{otherwise} \end{cases}}{4}$$

input
```
integrate((3+4*x)**p,x)
```

output
```
Piecewise(((4*x + 3)**(p + 1)/(p + 1), Ne(p, -1)), (log(4*x + 3), True))/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (3+4x)^p\,dx = \frac{(4\,x+3)^{p+1}}{4\,(p+1)}$$

input
```
integrate((3+4*x)^p,x, algorithm="maxima")
```

output
```
1/4*(4*x + 3)^(p + 1)/(p + 1)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (3+4x)^p\,dx = \frac{(4\,x+3)^{p+1}}{4\,(p+1)}$$

input
```
integrate((3+4*x)^p,x, algorithm="giac")
```

output
```
1/4*(4*x + 3)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int (3+4x)^p\,dx = \begin{cases} \frac{\ln(4\,x+3)}{4} & \text{if } p=-1 \\ \frac{(4\,x+3)^{p+1}}{4\,(p+1)} & \text{if } p\neq -1 \end{cases}$$

input
```
int((4*x + 3)^p,x)
```

output
```
piecewise(p == -1, log(4*x + 3)/4, p ~= -1, (4*x + 3)^(p + 1)/(4*(p + 1)))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (3+4x)^p\,dx = \frac{(4x+3)^p\,(4x+3)}{4p+4}$$

input
```
int((3+4*x)^p,x)
```

output
```
((4*x + 3)**p*(4*x + 3))/(4*(p + 1))
```

3.158 $\int (3+4x-x^2)^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int (3+4x-x^2)^p \, dx = -7^p(2-x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{7}(2-x)^2\right)$$

output
```
-7^p*(2-x)*hypergeom([1/2, -p],[3/2],1/7*(2-x)^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (3+4x-x^2)^p \, dx = 7^p(-2+x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{7}(-2+x)^2\right)$$

input
```
Integrate[(3 + 4*x - x^2)^p,x]
```

output
```
7^p*(-2 + x)*Hypergeometric2F1[1/2, -p, 3/2, (-2 + x)^2/7]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-x^2+4x+3\right)^p \, dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{1}{2}7^p \int \left(1-\frac{1}{28}(4-2x)^2\right)^p d(4-2x)$$

$$\Big\downarrow \text{ 237}$$

$$-\frac{1}{2}7^p(4-2x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{28}(4-2x)^2\right)$$

input
```
Int[(3 + 4*x - x^2)^p,x]
```

output
```
-1/2*(7^p*(4 - 2*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 2*x)^2/28])
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int \left(-x^2+4x+3\right)^p dx$$

input
```
int((-x^2+4*x+3)^p,x)
```

output
```
int((-x^2+4*x+3)^p,x)
```

Fricas [F]

$$\int \left(3+4x-x^2\right)^p \, dx = \int \left(-x^2+4\,x+3\right)^p dx$$

input
```
integrate((-x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((-x^2 + 4*x + 3)^p, x)
```

Sympy [F]

$$\int \left(3+4x-x^2\right)^p \, dx = \int \left(-x^2+4x+3\right)^p \, dx$$

input
```
integrate((-x**2+4*x+3)**p,x)
```

output
```
Integral((-x**2 + 4*x + 3)**p, x)
```

Maxima [F]

$$\int \left(3+4x-x^2\right)^p\,dx=\int \left(-x^2+4\,x+3\right)^p\,dx$$

input
```
integrate((-x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((-x^2 + 4*x + 3)^p, x)
```

Giac [F]

$$\int \left(3+4x-x^2\right)^p\,dx=\int \left(-x^2+4\,x+3\right)^p\,dx$$

input
```
integrate((-x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((-x^2 + 4*x + 3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3+4x-x^2\right)^p\,dx=\int \left(-x^2+4\,x+3\right)^p\,dx$$

input
```
int((4*x - x^2 + 3)^p,x)
```

output
```
int((4*x - x^2 + 3)^p, x)
```

Reduce [F]

$$\int \left(3+4x-x^2\right)^p\,dx$$

$$= \frac{2(-x^2+4x+3)^p x+3(-x^2+4x+3)^p-28\left(\int \frac{(-x^2+4x+3)^p x}{2p\,x^2-8px+x^2-6p-4x-3}dx\right)p^2-14\left(\int \frac{(-x^2+4x+3)^p x}{2p\,x^2-8px+x^2-6p-4x-3}\right.}{4p+2}$$

input
```
int((-x^2+4*x+3)^p,x)
```

output
```
(2*( - x**2 + 4*x + 3)**p*x + 3*( - x**2 + 4*x + 3)**p - 28*int((( - x**2
+ 4*x + 3)**p*x)/(2*p*x**2 - 8*p*x - 6*p + x**2 - 4*x - 3),x)*p**2 - 14*in
t((( - x**2 + 4*x + 3)**p*x)/(2*p*x**2 - 8*p*x - 6*p + x**2 - 4*x - 3),x)*
p)/(2*(2*p + 1))
```

3.159 $\int (3+4x-2x^2)^p\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int (3+4x-2x^2)^p\,dx = -5^p(1-x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{2}{5}(1-x)^2\right)$$

output
```
-5^p*(1-x)*hypergeom([1/2, -p],[3/2],2/5*(1-x)^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (3+4x-2x^2)^p\,dx = 5^p(-1+x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{2}{5}(-1+x)^2\right)$$

input
```
Integrate[(3 + 4*x - 2*x^2)^p,x]
```

output
```
5^p*(-1 + x)*Hypergeometric2F1[1/2, -p, 3/2, (2*(-1 + x)^2)/5]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-2x^2+4x+3\right)^p \, dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{1}{4}5^p \int \left(1-\frac{1}{40}(4-4x)^2\right)^p d(4-4x)$$

$$\Big\downarrow \text{ 237}$$

$$-\frac{1}{4}5^p(4-4x) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{40}(4-4x)^2\right)$$

input
```
Int[(3 + 4*x - 2*x^2)^p,x]
```

output
```
-1/4*(5^p*(4 - 4*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 4*x)^2/40])
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int \left(-2x^2+4x+3\right)^p dx$$

input
```
int((-2*x^2+4*x+3)^p,x)
```

output
```
int((-2*x^2+4*x+3)^p,x)
```

Fricas [F]

$$\int \left(3+4x-2x^2\right)^p \, dx = \int \left(-2\,x^2+4\,x+3\right)^p dx$$

input
```
integrate((-2*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((-2*x^2 + 4*x + 3)^p, x)
```

Sympy [F]

$$\int \left(3+4x-2x^2\right)^p \, dx = \int \left(-2x^2+4x+3\right)^p \, dx$$

input
```
integrate((-2*x**2+4*x+3)**p,x)
```

output
```
Integral((-2*x**2 + 4*x + 3)**p, x)
```

Maxima [F]

$$\int (3+4x-2x^2)^p \, dx = \int (-2x^2+4x+3)^p \, dx$$

input
```
integrate((-2*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((-2*x^2 + 4*x + 3)^p, x)
```

Giac [F]

$$\int (3+4x-2x^2)^p \, dx = \int (-2x^2+4x+3)^p \, dx$$

input
```
integrate((-2*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((-2*x^2 + 4*x + 3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (3+4x-2x^2)^p \, dx = \int (-2x^2+4x+3)^p \, dx$$

input
```
int((4*x - 2*x^2 + 3)^p,x)
```

output
```
int((4*x - 2*x^2 + 3)^p, x)
```

Reduce [F]

$$\int \left(3+4x-2x^2\right)^p\,dx$$
$$= \frac{2(-2x^2+4x+3)^p x + 3(-2x^2+4x+3)^p - 40\left(\int \frac{(-2x^2+4x+3)^p x}{4p\,x^2-8px+2x^2-6p-4x-3}dx\right)p^2 - 20\left(\int \frac{(-2x^2+4x+3)^p x}{4p\,x^2-8px+2x^2-6p-\ldots}\right.}{4p+2}$$

input
```
int((-2*x^2+4*x+3)^p,x)
```

output
```
(2*( - 2*x**2 + 4*x + 3)**p*x + 3*( - 2*x**2 + 4*x + 3)**p - 40*int((( - 2
*x**2 + 4*x + 3)**p*x)/(4*p*x**2 - 8*p*x - 6*p + 2*x**2 - 4*x - 3),x)*p**2
 - 20*int((( - 2*x**2 + 4*x + 3)**p*x)/(4*p*x**2 - 8*p*x - 6*p + 2*x**2 -
4*x - 3),x)*p)/(2*(2*p + 1))
```

3.160 $\int (3+4x-3x^2)^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \left(3+4x-3x^2\right)^p \, dx = -3^{-1-p}13^p(2-3x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{13}(2-3x)^2\right)$$

output
```
-3^(-1-p)*13^p*(2-3*x)*hypergeom([1/2, -p],[3/2],1/13*(2-3*x)^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \left(3+4x-3x^2\right)^p \, dx = 3^{-1-p}13^p(-2+3x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{13}(2-3x)^2\right)$$

input
```
Integrate[(3 + 4*x - 3*x^2)^p,x]
```

output
```
3^(-1 - p)*13^p*(-2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 3*x)^2/13]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-3x^2+4x+3\right)^p\,dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{1}{2}3^{-p-1}13^p\int \left(1-\frac{1}{52}(4-6x)^2\right)^p d(4-6x)$$

$$\Big\downarrow \text{ 237}$$

$$-\frac{1}{2}3^{-p-1}13^p(4-6x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{52}(4-6x)^2\right)$$

input
```
Int[(3 + 4*x - 3*x^2)^p,x]
```

output
```
-1/2*(3^(-1 - p)*13^p*(4 - 6*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 6*x)^
2/52])
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int \left(-3x^2+4x+3\right)^p dx$$

input
```
int((-3*x^2+4*x+3)^p,x)
```

output
```
int((-3*x^2+4*x+3)^p,x)
```

Fricas [F]

$$\int \left(3+4x-3x^2\right)^p \, dx = \int \left(-3\,x^2+4\,x+3\right)^p dx$$

input
```
integrate((-3*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 4*x + 3)^p, x)
```

Sympy [F]

$$\int \left(3+4x-3x^2\right)^p \, dx = \int \left(-3x^2+4x+3\right)^p \, dx$$

input
```
integrate((-3*x**2+4*x+3)**p,x)
```

output
```
Integral((-3*x**2 + 4*x + 3)**p, x)
```

Maxima [F]

$$\int \left(3+4x-3x^2\right)^p \, dx = \int \left(-3\,x^2+4\,x+3\right)^p \, dx$$

input
```
integrate((-3*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 4*x + 3)^p, x)
```

Giac [F]

$$\int \left(3+4x-3x^2\right)^p \, dx = \int \left(-3\,x^2+4\,x+3\right)^p \, dx$$

input
```
integrate((-3*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 4*x + 3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3+4x-3x^2\right)^p \, dx = \int \left(-3\,x^2+4\,x+3\right)^p \, dx$$

input
```
int((4*x - 3*x^2 + 3)^p,x)
```

output
```
int((4*x - 3*x^2 + 3)^p, x)
```

Reduce [F]

$$\int (3+4x-3x^2)^p \, dx$$
$$= \frac{2(-3x^2+4x+3)^p x + 3(-3x^2+4x+3)^p - 52\left(\int \frac{(-3x^2+4x+3)^p x}{6p\,x^2-8px+3x^2-6p-4x-3}dx\right)p^2 - 26\left(\int \frac{(-3x^2+4x+3)^p x}{6p\,x^2-8px+3x^2-6p-}\right.}{4p+2}$$

input
```
int((-3*x^2+4*x+3)^p,x)
```

output
```
(2*( - 3*x**2 + 4*x + 3)**p*x + 3*( - 3*x**2 + 4*x + 3)**p - 52*int((( - 3
*x**2 + 4*x + 3)**p*x)/(6*p*x**2 - 8*p*x - 6*p + 3*x**2 - 4*x - 3),x)*p**2
 - 26*int((( - 3*x**2 + 4*x + 3)**p*x)/(6*p*x**2 - 8*p*x - 6*p + 3*x**2 -
4*x - 3),x)*p)/(2*(2*p + 1))
```

3.161 $\int (3+4x-4x^2)^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 35

$$\int (3+4x-4x^2)^p \, dx = -2^{-1+2p}(1-2x) \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, \frac{1}{4}(1-2x)^2\right)$$

output
```
-2^(-1+2*p)*(1-2*x)*hypergeom([1/2, -p],[3/2],1/4*(1-2*x)^2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (3+4x-4x^2)^p \, dx = -2^{-3+2p}(4-8x) \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, \frac{1}{64}(4-8x)^2\right)$$

input
```
Integrate[(3 + 4*x - 4*x^2)^p,x]
```

output
```
-(2^(-3 + 2*p)*(4 - 8*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 8*x)^2/64])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-4x^2+4x+3)^p \, dx$$

$$\Big\downarrow \ 1090$$

$$-2^{2p-3}\int \left(1-\frac{1}{64}(4-8x)^2\right)^p d(4-8x)$$

$$\Big\downarrow \ 237$$

$$-2^{2p-3}(4-8x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{64}(4-8x)^2\right)$$

input

```
Int[(3 + 4*x - 4*x^2)^p,x]
```

output

```
-(2^(-3 + 2*p)*(4 - 8*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 8*x)^2/64])
```

Defintions of rubi rules used

rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int \left(-4x^2+4x+3\right)^p dx$$

input
```
int((-4*x^2+4*x+3)^p,x)
```

output
```
int((-4*x^2+4*x+3)^p,x)
```

Fricas [F]

$$\int \left(3+4x-4x^2\right)^p \, dx = \int \left(-4\,x^2+4\,x+3\right)^p dx$$

input
```
integrate((-4*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((-4*x^2 + 4*x + 3)^p, x)
```

Sympy [F]

$$\int \left(3+4x-4x^2\right)^p \, dx = \int \left(-4x^2+4x+3\right)^p \, dx$$

input
```
integrate((-4*x**2+4*x+3)**p,x)
```

output
```
Integral((-4*x**2 + 4*x + 3)**p, x)
```

Maxima [F]

$$\int \left(3+4x-4x^2\right)^p \, dx = \int \left(-4\,x^2+4\,x+3\right)^p \, dx$$

input
```
integrate((-4*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((-4*x^2 + 4*x + 3)^p, x)
```

Giac [F]

$$\int \left(3+4x-4x^2\right)^p \, dx = \int \left(-4\,x^2+4\,x+3\right)^p \, dx$$

input
```
integrate((-4*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((-4*x^2 + 4*x + 3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3+4x-4x^2\right)^p \, dx = \int \left(-4\,x^2+4\,x+3\right)^p \, dx$$

input
```
int((4*x - 4*x^2 + 3)^p,x)
```

output
```
int((4*x - 4*x^2 + 3)^p, x)
```

Reduce [F]

$$\int \left(3+4x-4x^2\right)^p \, dx$$
$$= \frac{2(-4x^2+4x+3)^p x + 3(-4x^2+4x+3)^p - 64\left(\int \frac{(-4x^2+4x+3)^p x}{8p\,x^2-8px+4x^2-6p-4x-3} dx\right) p^2 - 32\left(\int \frac{(-4x^2+4x+3)^p x}{8p\,x^2-8px+4x^2-6p-}\right.}{4p+2}$$

input
```
int((-4*x^2+4*x+3)^p,x)
```

output
```
(2*( - 4*x**2 + 4*x + 3)**p*x + 3*( - 4*x**2 + 4*x + 3)**p - 64*int((( - 4
*x**2 + 4*x + 3)**p*x)/(8*p*x**2 - 8*p*x - 6*p + 4*x**2 - 4*x - 3),x)*p**2
 - 32*int((( - 4*x**2 + 4*x + 3)**p*x)/(8*p*x**2 - 8*p*x - 6*p + 4*x**2 -
4*x - 3),x)*p)/(2*(2*p + 1))
```

3.162 $\int (3+4x-5x^2)^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int (3+4x-5x^2)^p \, dx = -5^{-1-p}19^p(2-5x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{19}(2-5x)^2\right)$$

output
```
-5^(-1-p)*19^p*(2-5*x)*hypergeom([1/2, -p],[3/2],1/19*(2-5*x)^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int (3+4x-5x^2)^p \, dx = 5^{-1-p}19^p(-2+5x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{19}(2-5x)^2\right)$$

input
```
Integrate[(3 + 4*x - 5*x^2)^p,x]
```

output
```
5^(-1 - p)*19^p*(-2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 5*x)^2/19]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-5x^2+4x+3\right)^p \, dx$$

$$\Big\downarrow \ 1090$$

$$-\frac{1}{2}5^{-p-1}19^p \int \left(1-\frac{1}{76}(4-10x)^2\right)^p d(4-10x)$$

$$\Big\downarrow \ 237$$

$$-\frac{1}{2}5^{-p-1}19^p(4-10x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{76}(4-10x)^2\right)$$

input
```
Int[(3 + 4*x - 5*x^2)^p,x]
```

output
```
-1/2*(5^(-1 - p)*19^p*(4 - 10*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 10*x
)^2/76])
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int \left(-5x^2+4x+3\right)^p dx$$

input
```
int((-5*x^2+4*x+3)^p,x)
```

output
```
int((-5*x^2+4*x+3)^p,x)
```

Fricas [F]

$$\int \left(3+4x-5x^2\right)^p dx = \int \left(-5\,x^2+4\,x+3\right)^p dx$$

input
```
integrate((-5*x^2+4*x+3)^p,x, algorithm="fricas")
```

output
```
integral((-5*x^2 + 4*x + 3)^p, x)
```

Sympy [F]

$$\int \left(3+4x-5x^2\right)^p dx = \int \left(-5x^2+4x+3\right)^p dx$$

input
```
integrate((-5*x**2+4*x+3)**p,x)
```

output
```
Integral((-5*x**2 + 4*x + 3)**p, x)
```

Maxima [F]

$$\int \left(3+4x-5x^2\right)^p\,dx=\int \left(-5\,x^2+4\,x+3\right)^p\,dx$$

input
```
integrate((-5*x^2+4*x+3)^p,x, algorithm="maxima")
```

output
```
integrate((-5*x^2 + 4*x + 3)^p, x)
```

Giac [F]

$$\int \left(3+4x-5x^2\right)^p\,dx=\int \left(-5\,x^2+4\,x+3\right)^p\,dx$$

input
```
integrate((-5*x^2+4*x+3)^p,x, algorithm="giac")
```

output
```
integrate((-5*x^2 + 4*x + 3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3+4x-5x^2\right)^p\,dx=\int \left(-5\,x^2+4\,x+3\right)^p\,dx$$

input
```
int((4*x - 5*x^2 + 3)^p,x)
```

output
```
int((4*x - 5*x^2 + 3)^p, x)
```

Reduce [F]

$$\int \left(3+4x-5x^2\right)^p \, dx$$
$$= \frac{2(-5x^2+4x+3)^p x + 3(-5x^2+4x+3)^p - 76\left(\int \frac{(-5x^2+4x+3)^p x}{10p\,x^2-8px+5x^2-6p-4x-3}dx\right)p^2 - 38\left(\int \frac{(-5x^2+4x+3)^p}{10p\,x^2-8px+5x^2-6p}\right.}{4p+2}$$

input
```
int((-5*x^2+4*x+3)^p,x)
```

output
```
(2*( - 5*x**2 + 4*x + 3)**p*x + 3*( - 5*x**2 + 4*x + 3)**p - 76*int((( - 5
*x**2 + 4*x + 3)**p*x)/(10*p*x**2 - 8*p*x - 6*p + 5*x**2 - 4*x - 3),x)*p**
2 - 38*int((( - 5*x**2 + 4*x + 3)**p*x)/(10*p*x**2 - 8*p*x - 6*p + 5*x**2
- 4*x - 3),x)*p)/(2*(2*p + 1))
```

3.163 $\int \sqrt{(b-x)(-a+x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \sqrt{(b-x)(-a+x)}\,dx = -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} - \frac{1}{4}(a-b)^2\arctan\left(\frac{a-x}{\sqrt{-ab+(a+b)x-x^2}}\right)$$

output
```
-1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/4*(a-b)^2*arctan((a-x)/(-a*b+(a+
b)*x-x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \sqrt{(b-x)(-a+x)}\,dx = \frac{1}{4}\sqrt{(a-x)(-b+x)}\left(-a-b+2x+\frac{(a-b)^2\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{b-x}}\right)}{\sqrt{b-x}\sqrt{-a+x}}\right)$$

input
```
Integrate[Sqrt[(b - x)*(-a + x)],x]
```

output
```
(Sqrt[(a - x)*(-b + x)]*(-a - b + 2*x + ((a - b)^2*ArcTan[Sqrt[-a + x]/Sqr
t[b - x]])/(Sqrt[b - x]*Sqrt[-a + x])))/4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2048, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{(x-a)(b-x)}\,dx$$

$$\Big\downarrow \text{2048}$$

$$\int \sqrt{x(a+b)-ab-x^2}dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{8}(a-b)^2\int \frac{1}{\sqrt{-x^2+(a+b)x-ab}}dx-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{4}(a-b)^2\int \frac{1}{-\frac{(a+b-2x)^2}{-x^2+(a+b)x-ab}-4}d\frac{a+b-2x}{\sqrt{-x^2+(a+b)x-ab}}-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{8}(a-b)^2\arctan\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}$$

input
```
Int[Sqrt[(b - x)*(-a + x)],x]
```

output
```
-1/4*((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2]) - ((a - b)^2*ArcTan[(a
 + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

method	result	size
default	$-\frac{(a+b-2x)\sqrt{-ab+(b+a)x-x^2}}{4}-\frac{\left(4ab-(b+a)^2\right)\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(b+a)x-x^2}}\right)}{8}$	68
risch	$\frac{(b-x)(a-x)(a+b-2x)}{4\sqrt{-(-b+x)(-a+x)}}-\left(\frac{1}{4}ab-\frac{1}{8}b^2-\frac{1}{8}a^2\right)\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(b+a)x-x^2}}\right)$	78

input
```
int(((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(a+b-2*x)*(-a*b+(b+a)*x-x^2)^(1/2)-1/8*(4*a*b-(b+a)^2)*arctan((x-1/2*
b-1/2*a)/(-a*b+(b+a)*x-x^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \sqrt{(b-x)(-a+x)}\,dx$$
$$= -\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)}{2\left(ab-(a+b)x+x^2\right)}\right)$$
$$-\frac{1}{4}\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)$$

input
```
integrate(((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
 2*x)/(a*b - (a + b)*x + x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
 2*x)
```

Sympy [A] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \sqrt{(b-x)(-a+x)}\,dx = \left(-\frac{ab}{2}\right.$$
$$\left.+\frac{\left(\frac{a}{4}+\frac{b}{4}\right)(a+b)}{2}\right)\left(\begin{cases}-i\log\left(a+b-2x+2i\sqrt{-ab-x^2+x\,(a+b)}\right) & \text{for } ab-\frac{(a+b)^2}{4}\neq 0\\ \frac{\left(-\frac{a}{2}-\frac{b}{2}+x\right)\log\left(-\frac{a}{2}-\frac{b}{2}+x\right)}{\sqrt{-\left(-\frac{a}{2}-\frac{b}{2}+x\right)^2}} & \text{otherwise}\end{cases}\right)$$
$$+\left(-\frac{a}{4}-\frac{b}{4}+\frac{x}{2}\right)\sqrt{-ab-x^2+x\,(a+b)}$$

input
```
integrate(((b-x)*(-a+x))**(1/2),x)
```

output
```
(-a*b/2 + (a/4 + b/4)*(a + b)/2)*Piecewise((-I*log(a + b - 2*x + 2*I*sqrt(
-a*b - x**2 + x*(a + b))), Ne(a*b - (a + b)**2/4, 0)), ((-a/2 - b/2 + x)*l
og(-a/2 - b/2 + x)/sqrt(-(-a/2 - b/2 + x)**2), True)) + (-a/4 - b/4 + x/2)
*sqrt(-a*b - x**2 + x*(a + b))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{(b-x)(-a+x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \sqrt{(b-x)(-a+x)}\,dx = \frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\operatorname{sgn}(-a+b) - \frac{1}{4}\sqrt{-ab+ax+bx-x^2}(a+b-2\,x)$$

input
```
integrate(((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

output
```
1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sq
rt(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{(b-x)(-a+x)}\,dx = \int \sqrt{-(a-x)\,(b-x)}\,dx$$

input
```
int((-(a - x)*(b - x))^(1/2),x)
```

output
```
int((-(a - x)*(b - x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.72

$$\int \sqrt{(b-x)(-a+x)}\,dx = \frac{i\left(-asinh\left(\frac{\sqrt{-a+x}\,i}{\sqrt{-a+b}}\right)a^3+3asinh\left(\frac{\sqrt{-a+x}\,i}{\sqrt{-a+b}}\right)a^2b-3asinh\left(\frac{\sqrt{-a+x}\,i}{\sqrt{-a+b}}\right)a\,b^2+asinh\left(\frac{\sqrt{-a+x}\,i}{\sqrt{-a+b}}\right)b^3-\sqrt{b-x}\,\sqrt{}\right.}{4a}$$

input
```
int(((b-x)*(-a+x))^(1/2),x)
```

output
```
(i*( - asinh((sqrt( - a + x)*i)/sqrt( - a + b))*a**3 + 3*asinh((sqrt( - a
+ x)*i)/sqrt( - a + b))*a**2*b - 3*asinh((sqrt( - a + x)*i)/sqrt( - a + b)
)*a*b**2 + asinh((sqrt( - a + x)*i)/sqrt( - a + b))*b**3 - sqrt(b - x)*sqr
t(a - b)*sqrt( - a + x)*sqrt( - a + b)*a - sqrt(b - x)*sqrt(a - b)*sqrt( -
 a + x)*sqrt( - a + b)*b + 2*sqrt(b - x)*sqrt(a - b)*sqrt( - a + x)*sqrt(
- a + b)*x))/(4*(a - b))
```

3.164 $\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = -2\arctan\left(\frac{a-x}{\sqrt{-ab+(a+b)x-x^2}}\right)$$

output
```
-2*arctan((a-x)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \frac{2\sqrt{b-x}\sqrt{-a+x}\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{b-x}}\right)}{\sqrt{(a-x)(-b+x)}}$$

input
```
Integrate[1/Sqrt[(b - x)*(-a + x)],x]
```

output
```
(2*Sqrt[b - x]*Sqrt[-a + x]*ArcTan[Sqrt[-a + x]/Sqrt[b - x]])/Sqrt[(a - x)
*(-b + x)]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2048, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(x-a)(b-x)}}\,dx$$

$$\Big\downarrow \ 2048$$

$$\int \frac{1}{\sqrt{x(a+b)-ab-x^2}}dx$$

$$\Big\downarrow \ 1092$$

$$2\int \frac{1}{-\frac{(a+b-2x)^2}{-x^2+(a+b)x-ab}-4}d\frac{a+b-2x}{\sqrt{-x^2+(a+b)x-ab}}$$

$$\Big\downarrow \ 217$$

$$-\arctan\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

input
```
Int[1/Sqrt[(b - x)*(-a + x)],x]
```

output
```
-ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

method	result	size
default	$\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(b+a)x-x^2}}\right)$	28

input
```
int(1/((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan((x-1/2*b-1/2*a)/(-a*b+(b+a)*x-x^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = -\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)}{2\,(ab-(a+b)x+x^2)}\right)$$

input
```
integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

output
```
-arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)/(a*b - (a + b)*x +
 x^2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \begin{cases} -i\log\left(a+b-2x+2i\sqrt{-ab-x^2+x\,(a+b)}\right) & \text{for } ab-\frac{(a+b)^2}{4}\neq 0 \\ \frac{\left(-\frac{a}{2}-\frac{b}{2}+x\right)\log\left(-\frac{a}{2}-\frac{b}{2}+x\right)}{\sqrt{-\left(-\frac{a}{2}-\frac{b}{2}+x\right)^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/((b-x)*(-a+x))**(1/2),x)
```

output
```
Piecewise((-I*log(a + b - 2*x + 2*I*sqrt(-a*b - x**2 + x*(a + b))), Ne(a*b
 - (a + b)**2/4, 0)), ((-a/2 - b/2 + x)*log(-a/2 - b/2 + x)/sqrt(-(-a/2 -
b/2 + x)**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(26) = 52$.

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sgn}(-a+b) - \frac{1}{4}\sqrt{-ab+ax+bx-x^2}(a+b-2\,x)$$

input
```
integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

output
```
1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sq
rt(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \int \frac{1}{\sqrt{-(a-x)\,(b-x)}}\,dx$$

input
```
int(1/(-(a - x)*(b - x))^(1/2),x)
```

output
```
int(1/(-(a - x)*(b - x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = -2asinh\left(\frac{\sqrt{-a+x}\,i}{\sqrt{-a+b}}\right)i$$

input
```
int(1/((b-x)*(-a+x))^(1/2),x)
```

output

```
- 2*asinh((sqrt( - a + x)*i)/sqrt( - a + b))*i
```

3.165 $\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}(1+x)}{\sqrt{2+5x+3x^2}}\right)}{\sqrt{3}}$$

output
```
2/3*arctanh(3^(1/2)*(1+x)/(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}+\frac{5x}{3}+x^2}}{1+x}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(2*ArcTanh[Sqrt[2/3 + (5*x)/3 + x^2]/(1 + x)])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^2+5x+2}}dx$$
$$\Big\downarrow \text{ 1092}$$
$$2\int \frac{1}{12-\frac{(6x+5)^2}{3x^2+5x+2}}d\frac{6x+5}{\sqrt{3x^2+5x+2}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{\text{arctanh}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[3]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}}{3}$	30
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-6\operatorname{RootOf}\left(\_Z^2-3\right)x-5\operatorname{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2+5x+2}\right)}{3}$	42

input
```
int(1/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{1}{6}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2+5\,x+2}(6\,x+5)+72\,x^2+120\,x+49\right)$$

input
```
integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x
 + 49)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+5x+2}+5\right)}{3}$$

input
```
integrate(1/(3*x**2+5*x+2)**(1/2),x)
```

output
```
sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x**2 + 5*x + 2) + 5)/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{1}{3}\,\sqrt{3}\log\left(2\,\sqrt{3}\sqrt{3\,x^2+5\,x+2}+6\,x+5\right)$$

input
```
integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{1}{12}\,\sqrt{3\,x^2+5\,x+2}(6\,x+5) + \frac{1}{72}\,\sqrt{3}\log\left(\left|-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+5\,x+2}\right)-5\right|\right)$$

input
```
integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/72*sqrt(3)*log(abs(-2*sqrt(3)*(sq
rt(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3}\,\left(x+\frac{5}{6}\right)+\sqrt{3\,x^2+5\,x+2}\right)}{3}$$

input
```
int(1/(5*x + 3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x + 5/6) + (5*x + 3*x^2 + 2)^(1/2)))/3
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(2\sqrt{3x^2+5x+2}\,\sqrt{3}+6x+5\right)}{3}$$

input
```
int(1/(3*x^2+5*x+2)^(1/2),x)
```

output
```
(sqrt(3)*log(2*sqrt(3*x**2 + 5*x + 2)*sqrt(3) + 6*x + 5))/3
```

3.166 $\int \frac{1}{\sqrt{(1+x)(2+3x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{\sqrt{(1+x)(2+3x)}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}(1+x)}{\sqrt{2+5x+3x^2}}\right)}{\sqrt{3}}$$

output
```
2/3*arctanh(3^(1/2)*(1+x)/(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{(1+x)(2+3x)}}\,dx=\frac{2\sqrt{1+x}\sqrt{2+3x}\operatorname{arctanh}\left(\frac{\sqrt{2+3x}}{\sqrt{3}\sqrt{1+x}}\right)}{\sqrt{6+15x+9x^2}}$$

input
```
Integrate[1/Sqrt[(1 + x)*(2 + 3*x)],x]
```

output
```
(2*Sqrt[1 + x]*Sqrt[2 + 3*x]*ArcTanh[Sqrt[2 + 3*x]/(Sqrt[3]*Sqrt[1 + x])])
/Sqrt[6 + 15*x + 9*x^2]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2048, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(x+1)(3x+2)}}dx$$

$$\Big\downarrow \text{ 2048}$$

$$\int \frac{1}{\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{ 1092}$$

$$2\int \frac{1}{12-\frac{(6x+5)^2}{3x^2+5x+2}}d\frac{6x+5}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[(1 + x)*(2 + 3*x)],x]
```

output
```
ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[3]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^{2}+5x+2}\right)\sqrt{3}}{3}$	30
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(6\operatorname{RootOf}\left(\_Z^{2}-3\right)x+5\operatorname{RootOf}\left(\_Z^{2}-3\right)+6\sqrt{3x^{2}+5x+2}\right)}{3}$	42

input
```
int(1/((x+1)*(3*x+2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{(1+x)(2+3x)}}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^{2}+5\,x+2}(6\,x+5)+72\,x^{2}+120\,x+49\right)$$

input
```
integrate(1/((1+x)*(2+3*x))^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x
 + 49)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{(1+x)(2+3x)}}\,dx = \frac{\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+5x+2}+5\right)}{3}$$

input
```
integrate(1/((1+x)*(2+3*x))**(1/2),x)
```

output
```
sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x**2 + 5*x + 2) + 5)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{(1+x)(2+3x)}}\,dx = \frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)$$

input
```
integrate(1/((1+x)*(2+3*x))^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{(1+x)(2+3x)}}\,dx = \frac{1}{12}\sqrt{3x^2+5x+2}(6x+5) + \frac{1}{72}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x+2}\right)-5\right|\right)$$

input
```
integrate(1/((1+x)*(2+3*x))^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/72*sqrt(3)*log(abs(-2*sqrt(3)*(sq
rt(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{(1+x)(2+3x)}}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3}\left(x+\frac{5}{6}\right)+\sqrt{3\,x^2+5\,x+2}\right)}{3}$$

input
```
int(1/((3*x + 2)*(x + 1))^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x + 5/6) + (5*x + 3*x^2 + 2)^(1/2)))/3
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{(1+x)(2+3x)}}\,dx = \frac{\sqrt{3}\log\left(2\sqrt{3x^2+5x+2}\,\sqrt{3}+6x+5\right)}{3}$$

input
```
int(1/((1+x)*(2+3*x))^(1/2),x)
```

output
```
(sqrt(3)*log(2*sqrt(3*x**2 + 5*x + 2)*sqrt(3) + 6*x + 5))/3
```

3.167 $\int \frac{1}{\sqrt[4]{2-x}\sqrt[4]{3-x}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \frac{1}{\sqrt[4]{2-x}\sqrt[4]{3-x}}\,dx=-\frac{2(2-x)^{3/4}}{\sqrt[4]{3-x}}-2E\left(\frac{1}{2}\arcsin\left(\frac{1}{\sqrt{3-x}}\right)\middle|2\right)$$

output
```
-2*(2-x)^(3/4)/(3-x)^(1/4)-2*EllipticE(sin(1/2*arcsin(1/(3-x)^(1/2))),2^(1
/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[4]{2-x}\sqrt[4]{3-x}}\,dx=-\frac{4}{3}(2-x)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-2+x\right)$$

input
```
Integrate[1/((2 - x)^(1/4)*(3 - x)^(1/4)),x]
```

output
```
(-4*(2 - x)^(3/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -2 + x])/3
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{2-x}\sqrt[4]{3-x}}dx$$

$$\Big\downarrow \text{73}$$

$$-4\int \frac{\sqrt{2-x}}{\sqrt[4]{3-x}}d\sqrt[4]{2-x}$$

$$\Big\downarrow \text{839}$$

$$-4\left(\frac{(2-x)^{3/4}}{2\sqrt[4]{3-x}}-\frac{1}{2}\int \frac{\sqrt{2-x}}{(3-x)^{5/4}}d\sqrt[4]{2-x}\right)$$

$$\Big\downarrow \text{813}$$

$$-4\left(\frac{(2-x)^{3/4}}{2\sqrt[4]{3-x}}-\frac{\sqrt[4]{\frac{1}{2-x}+1}\sqrt[4]{2-x}\int \frac{1}{\left(1+\frac{1}{2-x}\right)^{5/4}(2-x)^{3/4}}d\sqrt[4]{2-x}}{2\sqrt[4]{3-x}}\right)$$

$$\Big\downarrow \text{858}$$

$$-4\left(\frac{\sqrt[4]{\frac{1}{2-x}+1}\sqrt[4]{2-x}\int \frac{1}{\sqrt[4]{2-x}(3-x)^{5/4}}d\frac{1}{\sqrt[4]{2-x}}}{2\sqrt[4]{3-x}}+\frac{(2-x)^{3/4}}{2\sqrt[4]{3-x}}\right)$$

$$\Big\downarrow \text{807}$$

$$-4\left(\frac{\sqrt[4]{\frac{1}{2-x}+1}\sqrt[4]{2-x}\int \frac{1}{\left(\sqrt{2-x}+1\right)^{5/4}}d\sqrt{2-x}}{4\sqrt[4]{3-x}}+\frac{(2-x)^{3/4}}{2\sqrt[4]{3-x}}\right)$$

$$\Big\downarrow \text{212}$$

$$-4\left(\frac{\sqrt[4]{\frac{1}{2-x}+1}\sqrt[4]{2-x}E\left(\frac{1}{2}\arctan\left(\sqrt{2-x}\right)\middle|2\right)}{2\sqrt[4]{3-x}}+\frac{(2-x)^{3/4}}{2\sqrt[4]{3-x}}\right)$$

input
```
Int[1/((2 - x)^(1/4)*(3 - x)^(1/4)),x]
```

output
```
-4*((2 - x)^(3/4)/(2*(3 - x)^(1/4)) + ((1 + (2 - x)^(-1))^(1/4)*(2 - x)^(1
/4)*EllipticE[ArcTan[Sqrt[2 - x]]/2, 2])/(2*(3 - x)^(1/4)))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(2-x)^{\frac{1}{4}}(3-x)^{\frac{1}{4}}}dx$$

input
```
int(1/(2-x)^(1/4)/(3-x)^(1/4),x)
```

output
```
int(1/(2-x)^(1/4)/(3-x)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{2-x}\sqrt[4]{3-x}}\,dx = \int \frac{1}{(-x+3)^{\frac{1}{4}}(-x+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(2-x)^(1/4)/(3-x)^(1/4),x, algorithm="fricas")
```

output
```
integral((-x + 3)^(3/4)*(-x + 2)^(3/4)/(x^2 - 5*x + 6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[4]{2-x}\sqrt[4]{3-x}}\,dx = -2\sqrt[4]{-1}\sqrt{x-2}e^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{1}{x-2}\right)$$

input
```
integrate(1/(2-x)**(1/4)/(3-x)**(1/4),x)
```

output
```
-2*(-1)**(1/4)*sqrt(x - 2)*exp(I*pi/4)*hyper((-1/2, 1/4), (1/2,), 1/(x - 2
))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{2-x}\sqrt[4]{3-x}}\,dx = \int \frac{1}{(-x+3)^{\frac{1}{4}}(-x+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(2-x)^(1/4)/(3-x)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-x + 3)^(1/4)*(-x + 2)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{2-x}\sqrt[4]{3-x}}\,dx = \int \frac{1}{(-x+3)^{\frac{1}{4}}(-x+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(2-x)^(1/4)/(3-x)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-x + 3)^(1/4)*(-x + 2)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{2-x}\sqrt[4]{3-x}}\,dx = \int \frac{1}{(2-x)^{1/4}\,(3-x)^{1/4}}\,dx$$

input
```
int(1/((2 - x)^(1/4)*(3 - x)^(1/4)),x)
```

output
```
int(1/((2 - x)^(1/4)*(3 - x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{2-x}\sqrt[4]{3-x}}\,dx = \int \frac{1}{(-x+2)^{\frac{1}{4}}\,(-x+3)^{\frac{1}{4}}}dx$$

input
```
int(1/(2-x)^(1/4)/(3-x)^(1/4),x)
```

output
```
int(1/(( - x + 2)**(1/4)*( - x + 3)**(1/4)),x)
```

3.168 $\int \frac{1}{\sqrt[4]{(2-x)(3-x)}} dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{1}{\sqrt[4]{(2-x)(3-x)}} dx = -\frac{\sqrt{2}\sqrt[4]{-6+5x-x^2}E\left(\frac{1}{2}\arcsin(5-2x)\middle|2\right)}{\sqrt[4]{6-5x+x^2}}$$

output
```
(-x^2+5*x-6)^(1/4)*EllipticE(sin(1/2*arcsin(-5+2*x)),2^(1/2))*2^(1/2)/(x^2
-5*x+6)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt[4]{(2-x)(3-x)}} dx = -\frac{(5-2x)\sqrt[4]{-6+5x-x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},(5-2x)^2\right)}{\sqrt{2}\sqrt[4]{6-5x+x^2}}$$

input
```
Integrate[((2 - x)*(3 - x))^(-1/4),x]
```

output
```
-(((5 - 2*x)*(-6 + 5*x - x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (5 -
2*x)^2])/(Sqrt[2]*(6 - 5*x + x^2)^(1/4)))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 273 vs. $2(45) = 90$.

Time = 0.49 (sec) , antiderivative size = 273, normalized size of antiderivative = 6.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2048, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{(2-x)(3-x)}} \, dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{1}{\sqrt[4]{x^2-5x+6}}dx$$

$$\Big\downarrow \text{1094}$$

$$-\frac{4\sqrt{(2x-5)^2}\int \frac{\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}}{5-2x}$$

$$\Big\downarrow \text{834}$$

$$-\frac{4\sqrt{(2x-5)^2}\left(\frac{1}{2}\int \frac{1}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}-\frac{1}{2}\int \frac{1-2\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}\right)}{5-2x}$$

$$\Big\downarrow \text{761}$$

$$-\frac{4\sqrt{(2x-5)^2}\left(\frac{\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4\left(x^2-5x+6\right)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt{4(x^2-5x+6)+1}}-\frac{1}{2}\int \frac{1-2\sqrt{x^2-5x+6}}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x ...}{5-2x}$$

$$\Big\downarrow \text{1510}$$

$$-\frac{4\sqrt{(2x-5)^2}\left(\frac{\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4\left(x^2-5x+6\right)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt{4(x^2-5x+6)+1}}+\frac{1}{2}\left(\frac{\sqrt[4]{x^2-5x+6}\sqrt{4(x^2}{2\sqrt{x^2-5x+6}+}\right.\right.}{5-2x}$$

input
```
Int[((2 - x)*(3 - x))^(-1/4),x]
```

output
```
(-4*Sqrt[(-5 + 2*x)^2]*((((6 - 5*x + x^2)^(1/4)*Sqrt[1 + 4*(6 - 5*x + x^2)
])/(1 + 2*Sqrt[6 - 5*x + x^2]) - ((1 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[(1 + 4*
(6 - 5*x + x^2))/(1 + 2*Sqrt[6 - 5*x + x^2])^2]*EllipticE[2*ArcTan[Sqrt[2]
*(6 - 5*x + x^2)^(1/4)], 1/2])/(Sqrt[2]*Sqrt[1 + 4*(6 - 5*x + x^2)]))/2 +
((1 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[(1 + 4*(6 - 5*x + x^2))/(1 + 2*Sqrt[6 -
5*x + x^2])^2]*EllipticF[2*ArcTan[Sqrt[2]*(6 - 5*x + x^2)^(1/4)], 1/2])/(4
*Sqrt[2]*Sqrt[1 + 4*(6 - 5*x + x^2)])))/(5 - 2*x)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [F]

$$\int \frac{1}{\left(\left(2-x\right)\left(3-x\right)\right)^{\frac{1}{4}}}dx$$

input
```
int(1/((2-x)*(3-x))^(1/4),x)
```

output
```
int(1/((2-x)*(3-x))^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{(2-x)(3-x)}}\,dx=\int \frac{1}{\left((x-2)(x-3)\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((2-x)*(3-x))^(1/4),x, algorithm="fricas")
```

output
```
integral((x^2 - 5*x + 6)^(-1/4), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{(2-x)(3-x)}}\,dx = \int \frac{1}{\sqrt[4]{(2-x)\,(3-x)}}\,dx$$

input
```
integrate(1/((2-x)*(3-x))**(1/4),x)
```

output
```
Integral(((2 - x)*(3 - x))**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{(2-x)(3-x)}}\,dx = \int \frac{1}{((x-2)(x-3))^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((2-x)*(3-x))^(1/4),x, algorithm="maxima")
```

output
```
integrate(((x - 2)*(x - 3))^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{(2-x)(3-x)}}\,dx = \int \frac{1}{((x-2)(x-3))^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((2-x)*(3-x))^(1/4),x, algorithm="giac")
```

output
```
integrate(((x - 2)*(x - 3))^(-1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{(2-x)(3-x)}}\,dx = \int \frac{1}{\left((x-2)\,(x-3)\right)^{1/4}}\,dx$$

input
```
int(1/((x - 2)*(x - 3))^(1/4),x)
```

output
```
int(1/((x - 2)*(x - 3))^(1/4), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{(2-x)(3-x)}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{1}{4}}}dx$$

input
```
int(1/((2-x)*(3-x))^(1/4),x)
```

output
```
int(1/(x**2 - 5*x + 6)**(1/4),x)
```

3.169 $\int \frac{1}{(2-x)^{3/4}(3-x)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{1}{(2-x)^{3/4}(3-x)^{3/4}}\,dx = 4\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{1}{\sqrt{3-x}}\right),2\right)$$

output
```
4*InverseJacobiAM(1/2*arcsin(1/(3-x)^(1/2)),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{1}{(2-x)^{3/4}(3-x)^{3/4}}\,dx = -4\sqrt[4]{2-x}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-2+x\right)$$

input
```
Integrate[1/((2 - x)^(3/4)*(3 - x)^(3/4)),x]
```

output
```
-4*(2 - x)^(1/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -2 + x]
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 49 vs. $2(18) = 36$.

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-x)^{3/4}(3-x)^{3/4}}dx$$

$$\Big\downarrow \; 73$$

$$-4\int \frac{1}{(3-x)^{3/4}}d\sqrt[4]{2-x}$$

$$\Big\downarrow \; 768$$

$$-\frac{4\left(\frac{1}{2-x}+1\right)^{3/4}(2-x)^{3/4}\int \frac{1}{\left(1+\frac{1}{2-x}\right)^{3/4}(2-x)^{3/4}}d\sqrt[4]{2-x}}{(3-x)^{3/4}}$$

$$\Big\downarrow \; 858$$

$$\frac{4\left(\frac{1}{2-x}+1\right)^{3/4}(2-x)^{3/4}\int \frac{1}{\sqrt[4]{2-x}(3-x)^{3/4}}d\frac{1}{\sqrt[4]{2-x}}}{(3-x)^{3/4}}$$

$$\Big\downarrow \; 807$$

$$\frac{2\left(\frac{1}{2-x}+1\right)^{3/4}(2-x)^{3/4}\int \frac{1}{\left(\sqrt{2-x}+1\right)^{3/4}}d\sqrt{2-x}}{(3-x)^{3/4}}$$

$$\Big\downarrow \; 229$$

$$\frac{4\left(\frac{1}{2-x}+1\right)^{3/4}(2-x)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{2-x}\right),2\right)}{(3-x)^{3/4}}$$

input
```
Int[1/((2 - x)^(3/4)*(3 - x)^(3/4)),x]
```

output
```
(4*(1 + (2 - x)^(-1))^(3/4)*(2 - x)^(3/4)*EllipticF[ArcTan[Sqrt[2 - x]]/2,
 2])/(3 - x)^(3/4)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(2-x)^{\frac{3}{4}} (3-x)^{\frac{3}{4}}}dx$$

input
```
int(1/(2-x)^(3/4)/(3-x)^(3/4),x)
```

output
```
int(1/(2-x)^(3/4)/(3-x)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(2-x)^{3/4}(3-x)^{3/4}}\,dx = \int \frac{1}{(-x+3)^{\frac{3}{4}}(-x+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(2-x)^(3/4)/(3-x)^(3/4),x, algorithm="fricas")
```

output
```
integral((-x + 3)^(1/4)*(-x + 2)^(1/4)/(x^2 - 5*x + 6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{1}{(2-x)^{3/4}(3-x)^{3/4}}\,dx = \frac{2(-1)^{\frac{3}{4}} e^{\frac{3i\pi}{4}} {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{1}{x-2}\right)}{\sqrt{x-2}}$$

input
```
integrate(1/(2-x)**(3/4)/(3-x)**(3/4),x)
```

output
```
2*(-1)**(3/4)*exp(3*I*pi/4)*hyper((1/2, 3/4), (3/2,), 1/(x - 2))/sqrt(x -
2)
```

Maxima [F]

$$\int \frac{1}{(2-x)^{3/4}(3-x)^{3/4}}\,dx = \int \frac{1}{(-x+3)^{\frac{3}{4}}(-x+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(2-x)^(3/4)/(3-x)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-x + 3)^(3/4)*(-x + 2)^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{(2-x)^{3/4}(3-x)^{3/4}}\,dx = \int \frac{1}{(-x+3)^{\frac{3}{4}}(-x+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(2-x)^(3/4)/(3-x)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-x + 3)^(3/4)*(-x + 2)^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-x)^{3/4}(3-x)^{3/4}}\,dx = \int \frac{1}{(2-x)^{3/4}\,(3-x)^{3/4}}\,dx$$

input
```
int(1/((2 - x)^(3/4)*(3 - x)^(3/4)),x)
```

output
```
int(1/((2 - x)^(3/4)*(3 - x)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(2-x)^{3/4}(3-x)^{3/4}}\,dx = \int \frac{1}{(-x+2)^{\frac{3}{4}}\,(-x+3)^{\frac{3}{4}}}dx$$

input
```
int(1/(2-x)^(3/4)/(3-x)^(3/4),x)
```

output
```
int(1/(( - x + 2)**(3/4)*( - x + 3)**(3/4)),x)
```

3.170 $\int \frac{1}{((2-x)(3-x))^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{1}{((2-x)(3-x))^{3/4}}\,dx=-\frac{2\sqrt{2}(-6+5x-x^2)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin(5-2x),2\right)}{(6-5x+x^2)^{3/4}}$$

output
```
2*(-x^2+5*x-6)^(3/4)*InverseJacobiAM(1/2*arcsin(-5+2*x),2^(1/2))*2^(1/2)/(
x^2-5*x+6)^(3/4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1}{((2-x)(3-x))^{3/4}}\,dx=-\frac{2\sqrt{2}(-6+5x-x^2)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin(5-2x),2\right)}{(6-5x+x^2)^{3/4}}$$

input
```
Integrate[((2 - x)*(3 - x))^(-3/4),x]
```

output
```
(-2*Sqrt[2]*(-6 + 5*x - x^2)^(3/4)*EllipticF[ArcSin[5 - 2*x]/2, 2])/(6 - 5
*x + x^2)^(3/4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 117 vs. 2(45) = 90.

Time = 0.35 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.60, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2048, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{((2-x)(3-x))^{3/4}} \, dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{1}{(x^2-5x+6)^{3/4}}dx$$

$$\Big\downarrow \text{1094}$$

$$-\frac{4\sqrt{(2x-5)^2}\int \frac{1}{\sqrt{4(x^2-5x+6)+1}}d\sqrt[4]{x^2-5x+6}}{5-2x}$$

$$\Big\downarrow \text{761}$$

$$-\frac{\sqrt{2}\sqrt{(2x-5)^2}\left(2\sqrt{x^2-5x+6}+1\right)\sqrt{\frac{4(x^2-5x+6)+1}{\left(2\sqrt{x^2-5x+6}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{2}\sqrt[4]{x^2-5x+6}\right),\frac{1}{2}\right)}{(5-2x)\sqrt{4(x^2-5x+6)+1}}$$

input
```
Int[((2 - x)*(3 - x))^(-3/4),x]
```

output
```
-((Sqrt[2]*Sqrt[(-5 + 2*x)^2]*(1 + 2*Sqrt[6 - 5*x + x^2])*Sqrt[(1 + 4*(6 -
 5*x + x^2))/(1 + 2*Sqrt[6 - 5*x + x^2])^2]*EllipticF[2*ArcTan[Sqrt[2]*(6
- 5*x + x^2)^(1/4)], 1/2])/((5 - 2*x)*Sqrt[1 + 4*(6 - 5*x + x^2)]))
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [F]

$$\int \frac{1}{\left(\left(2-x\right)\left(3-x\right)\right)^{\frac{3}{4}}}dx$$

input
```
int(1/((2-x)*(3-x))^(3/4),x)
```

output
```
int(1/((2-x)*(3-x))^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{((2-x)(3-x))^{3/4}}\,dx=\int \frac{1}{\left((x-2)(x-3)\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((2-x)*(3-x))^(3/4),x, algorithm="fricas")
```

output
```
integral((x^2 - 5*x + 6)^(-3/4), x)
```

Sympy [F]

$$\int \frac{1}{((2-x)(3-x))^{3/4}}\,dx = \int \frac{1}{\left((2-x)\,(3-x)\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((2-x)*(3-x))**(3/4),x)
```

output
```
Integral(((2 - x)*(3 - x))**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{((2-x)(3-x))^{3/4}}\,dx = \int \frac{1}{\left((x-2)(x-3)\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((2-x)*(3-x))^(3/4),x, algorithm="maxima")
```

output
```
integrate(((x - 2)*(x - 3))^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{((2-x)(3-x))^{3/4}}\,dx = \int \frac{1}{\left((x-2)(x-3)\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((2-x)*(3-x))^(3/4),x, algorithm="giac")
```

output
```
integrate(((x - 2)*(x - 3))^(-3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{((2-x)(3-x))^{3/4}}\,dx = \int \frac{1}{((x-2)\ (x-3))^{3/4}}\,dx$$

input
```
int(1/((x - 2)*(x - 3))^(3/4),x)
```

output
```
int(1/((x - 2)*(x - 3))^(3/4), x)
```

Reduce [F]

$$\int \frac{1}{((2-x)(3-x))^{3/4}}\,dx = \int \frac{1}{(x^2-5x+6)^{\frac{3}{4}}}dx$$

input
```
int(1/((2-x)*(3-x))^(3/4),x)
```

output
```
int(1/(x**2 - 5*x + 6)**(3/4),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
           return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
           if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
              return 1
           else:
              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
           return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file