Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.2-Quartic-trinomial/114-1.2.2.1

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**344**]. This is test number [114].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (344)	0.00 (0)
Mathematica	100.00 (344)	0.00 (0)
Fricas	100.00 (344)	0.00 (0)
Maple	99.71 (343)	0.29 (1)
Mupad	10.17 (35)	89.83 (309)
Sympy	8.43 (29)	91.57 (315)
Giac	4.07 (14)	95.93 (330)
Reduce	4.07 (14)	95.93 (330)
Maxima	2.03 (7)	97.97 (337)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

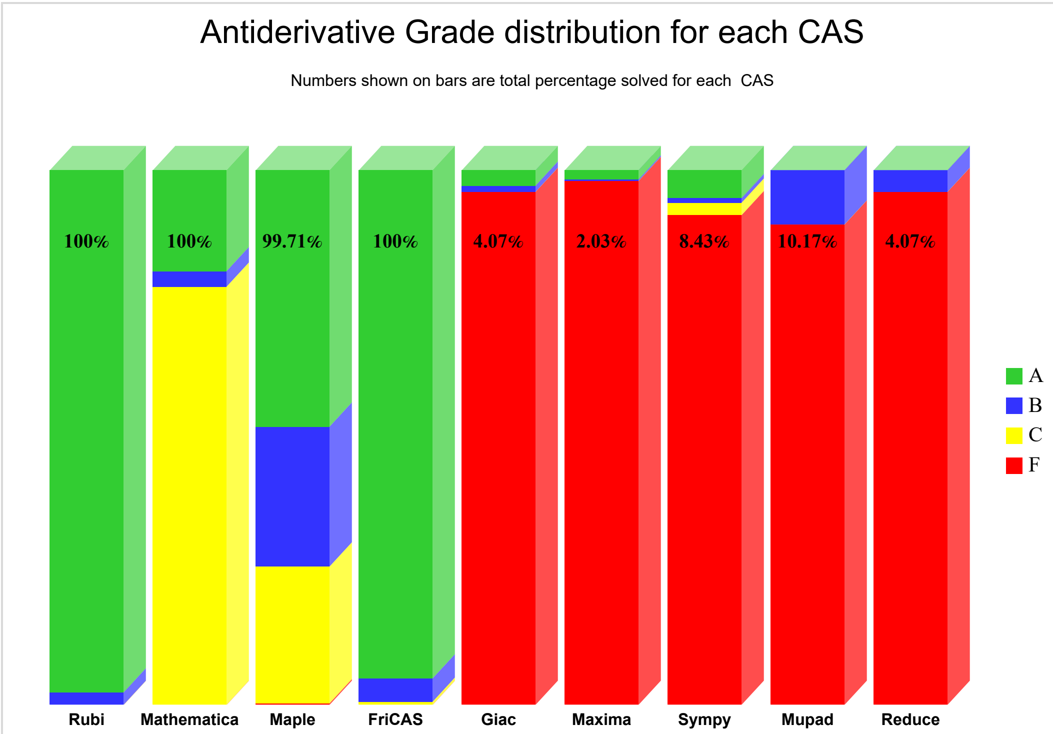
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

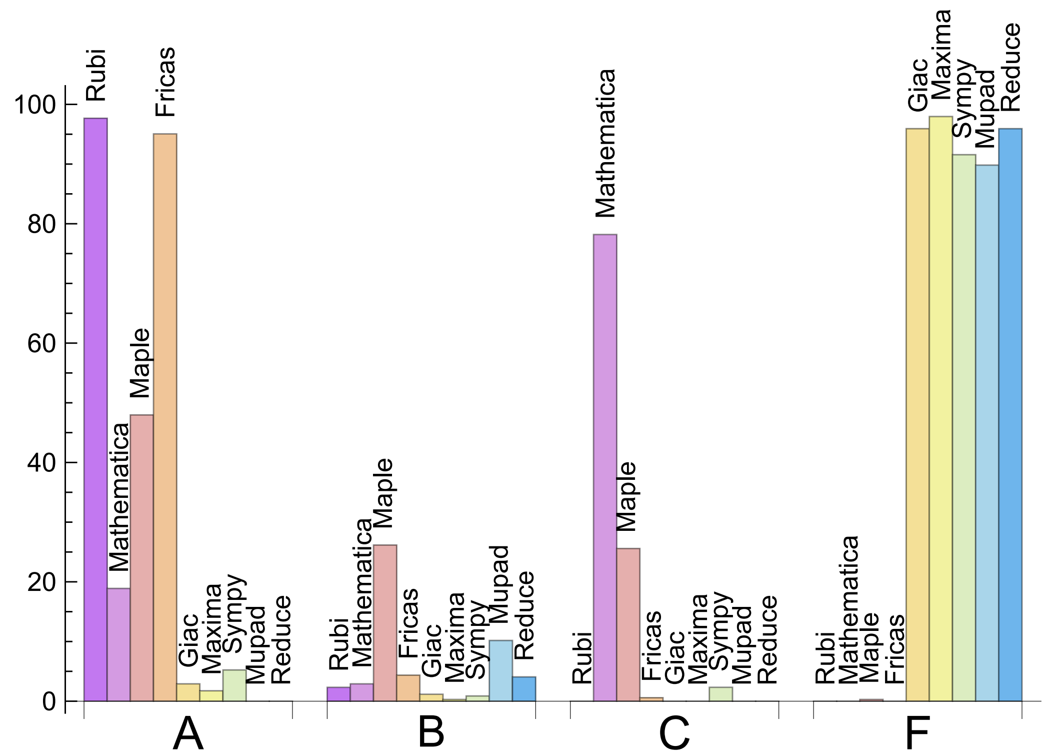
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.674	2.326	0.000	0.000
Fricas	95.058	4.360	0.581	0.000
Maple	47.965	26.163	25.581	0.291
Mathematica	18.895	2.907	78.198	0.000
Sympy	5.233	0.872	2.326	91.570
Giac	2.907	1.163	0.000	95.930
Maxima	1.744	0.291	0.000	97.965
Mupad	0.000	10.174	0.000	89.826
Reduce	0.000	4.070	0.000	95.930

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	1	100.00	0.00	0.00
Mupad	309	0.00	100.00	0.00
Sympy	315	98.73	1.27	0.00
Giac	330	99.39	0.30	0.30
Reduce	330	100.00	0.00	0.00
Maxima	337	99.70	0.00	0.30

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.05
Fricas	0.09
Giac	0.22
Reduce	0.45
Rubi	0.49
Maple	1.25
Sympy	3.87
Mathematica	7.58
Mupad	11.62

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	34.57	0.98	25.00	0.93
Sympy	89.31	1.16	37.00	0.75
Mathematica	147.55	1.28	123.00	1.16
Maple	156.65	1.30	112.00	1.00
Rubi	165.48	1.20	124.50	1.05
Fricas	179.45	1.01	81.00	0.75
Giac	497.50	2.48	56.00	0.96
Reduce	802.29	3.31	78.50	1.48
Mupad	1309.29	5.18	34.00	0.78

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

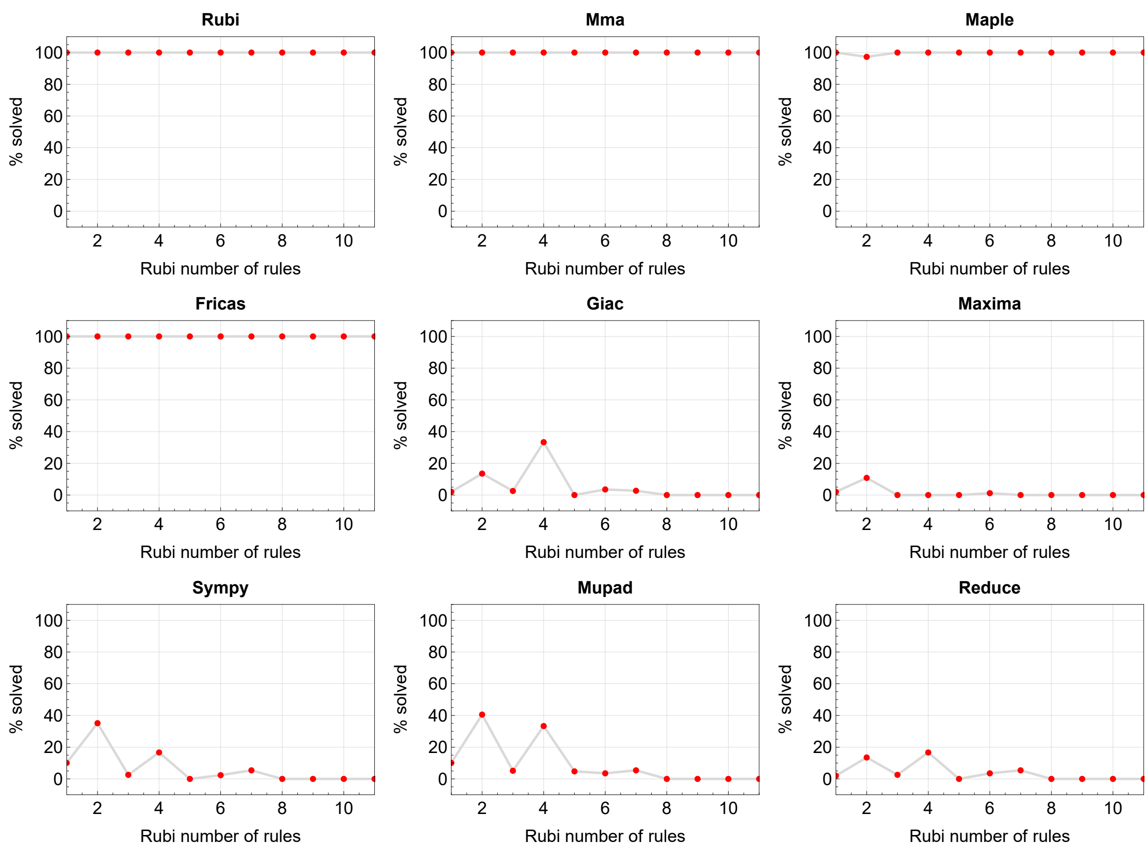


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

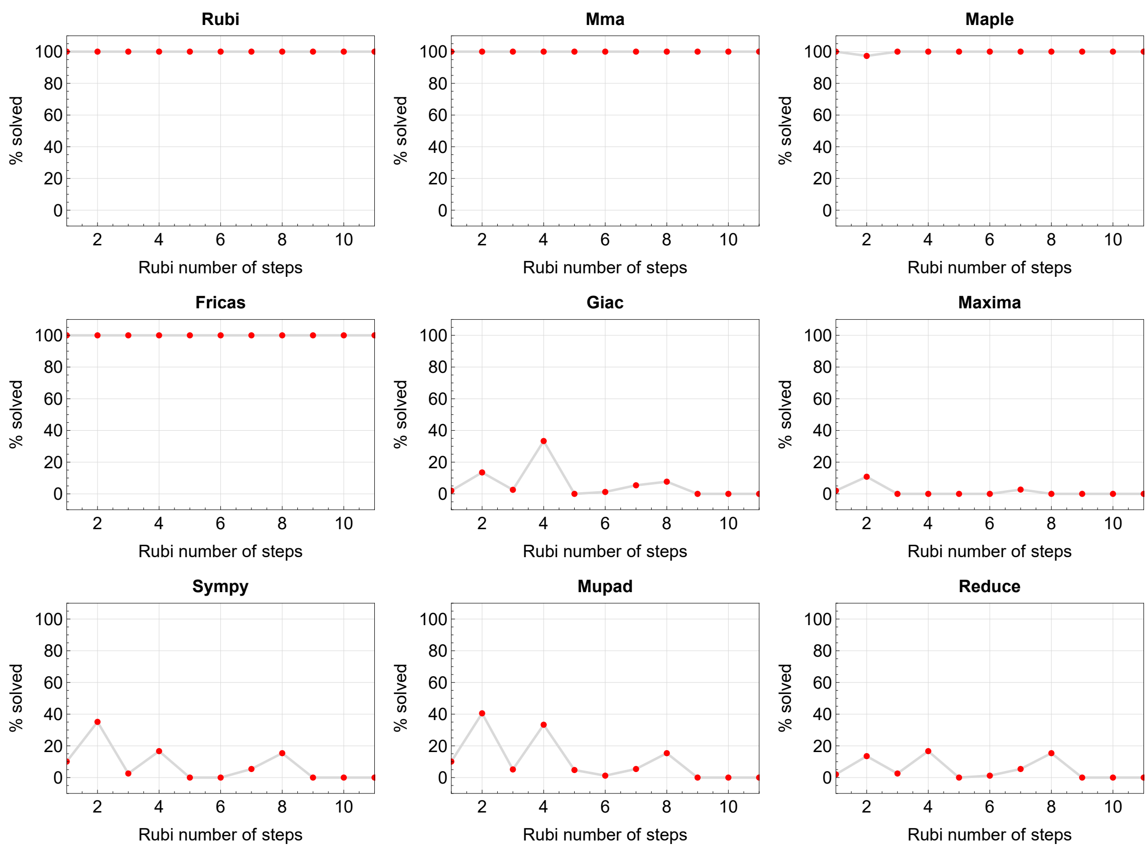


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

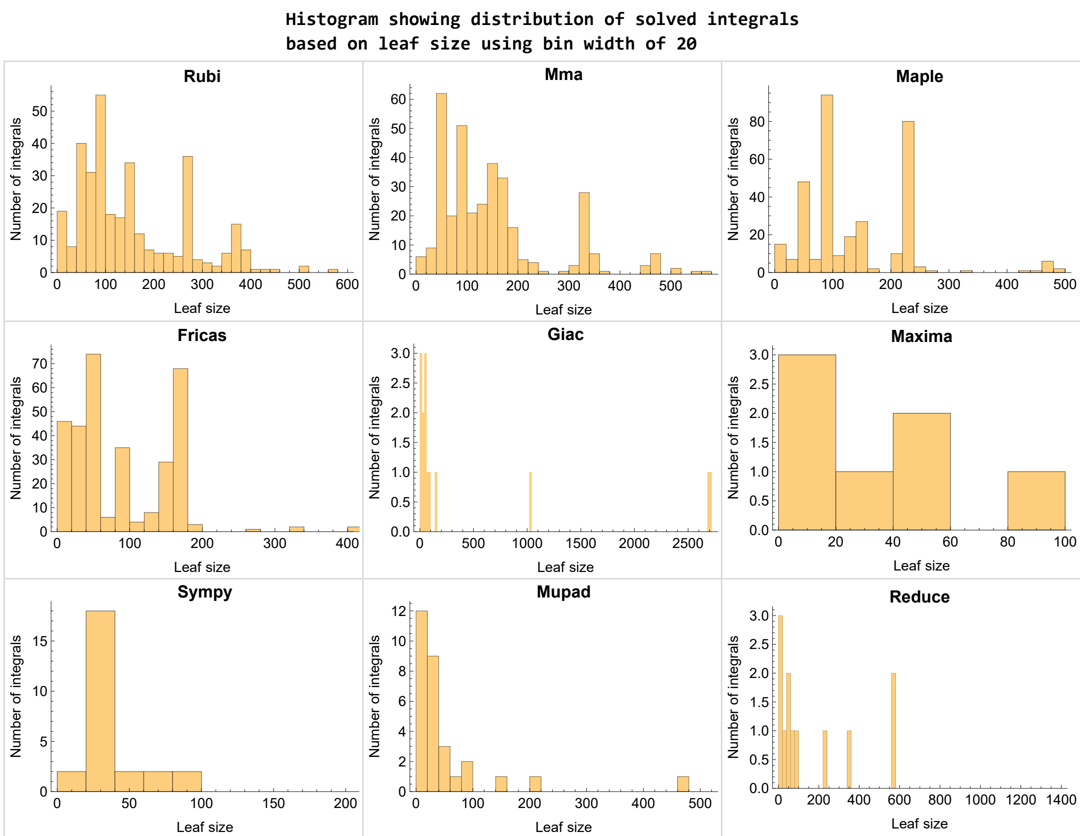


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

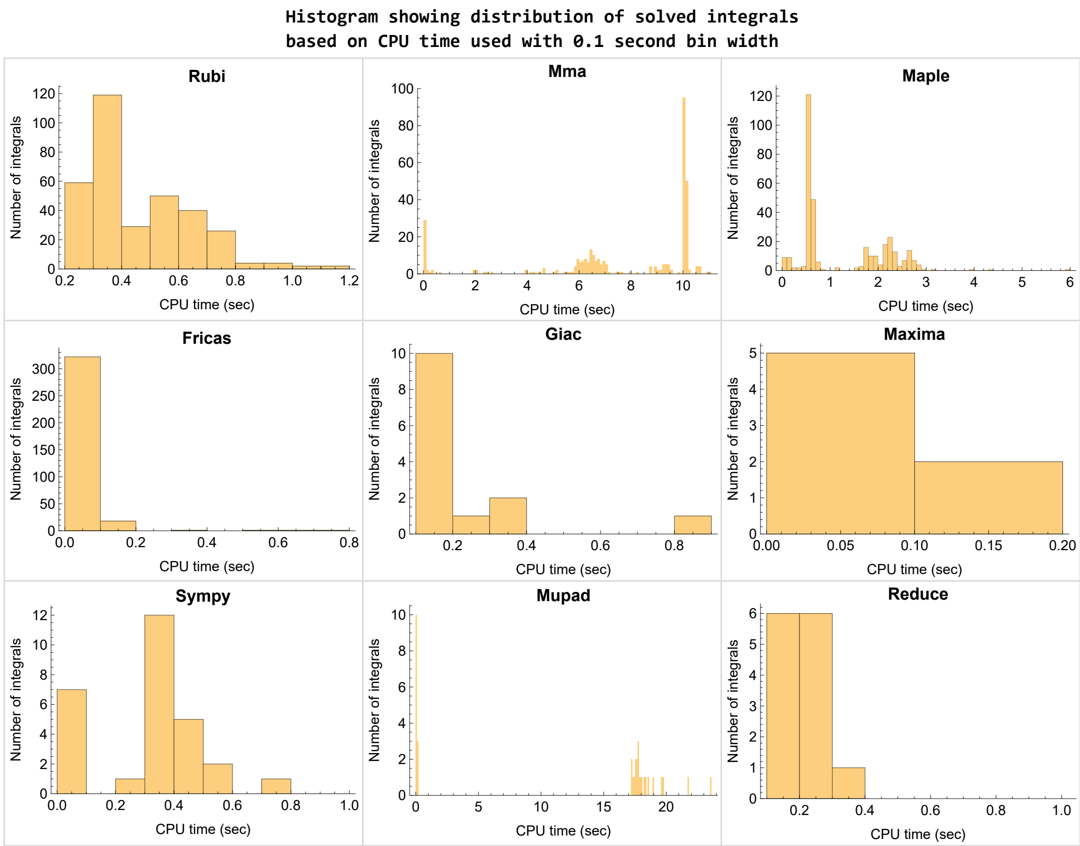


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

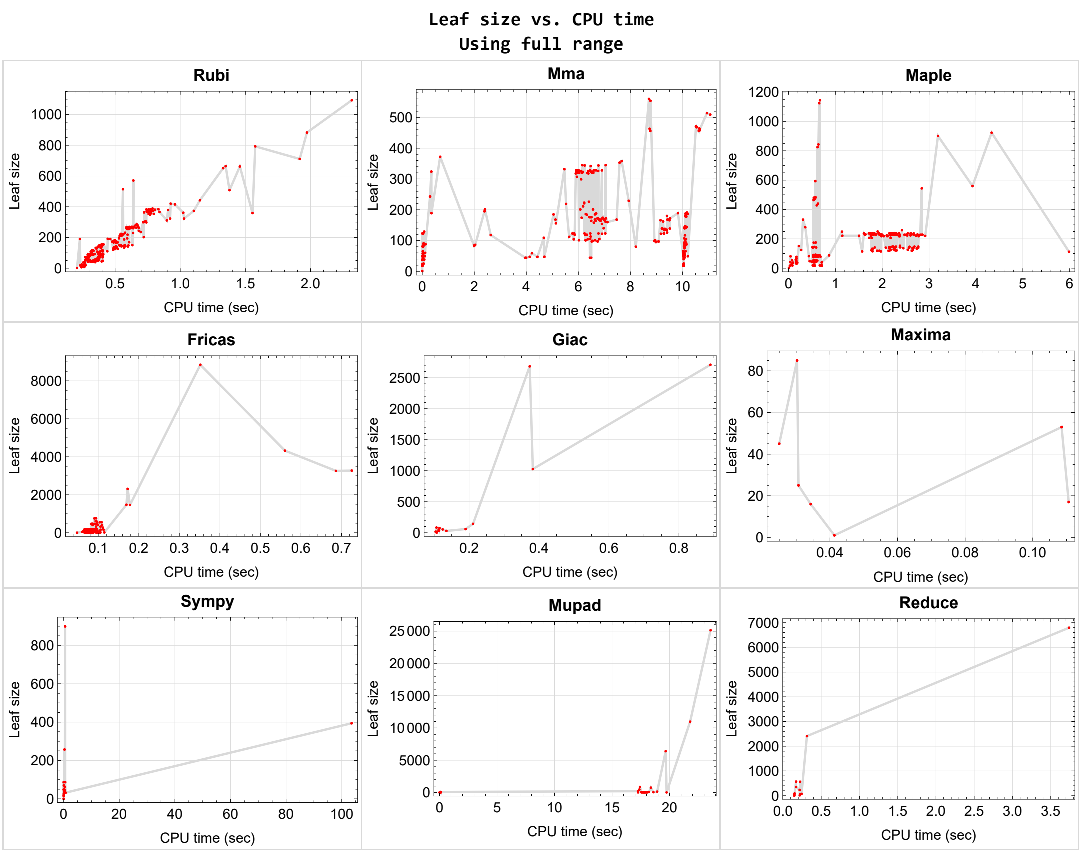


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {40, 41, 42, 43, 44, 59, 60, 61, 62, 63, 78, 93, 127, 133, 174, 182, 185, 197, 208, 210, 223, 225, 229, 243, 244, 259, 260, 261, 276, 291}

Mathematica {26, 27, 29, 45, 46, 48, 63, 64, 78, 93, 95, 98, 99, 103, 104, 107, 110, 111, 112, 116, 117, 118, 121, 125, 126, 130, 131, 134, 137, 138, 139, 143, 144, 145, 148, 153, 154, 155, 157, 159, 161, 162, 174, 175, 176, 180, 181, 182, 184, 185, 187, 188, 189, 193, 194, 195, 197, 198, 200, 201, 202, 203, 205, 206, 207, 210, 211, 212, 213, 215, 216, 217, 218, 220, 221, 222, 223, 226, 227, 228, 230, 231, 232, 244, 245, 246, 248, 260, 262, 264, 276, 279, 291, 293, 301, 302, 303, 305, 307, 309, 310}

Maple {128, 141, 209, 224}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

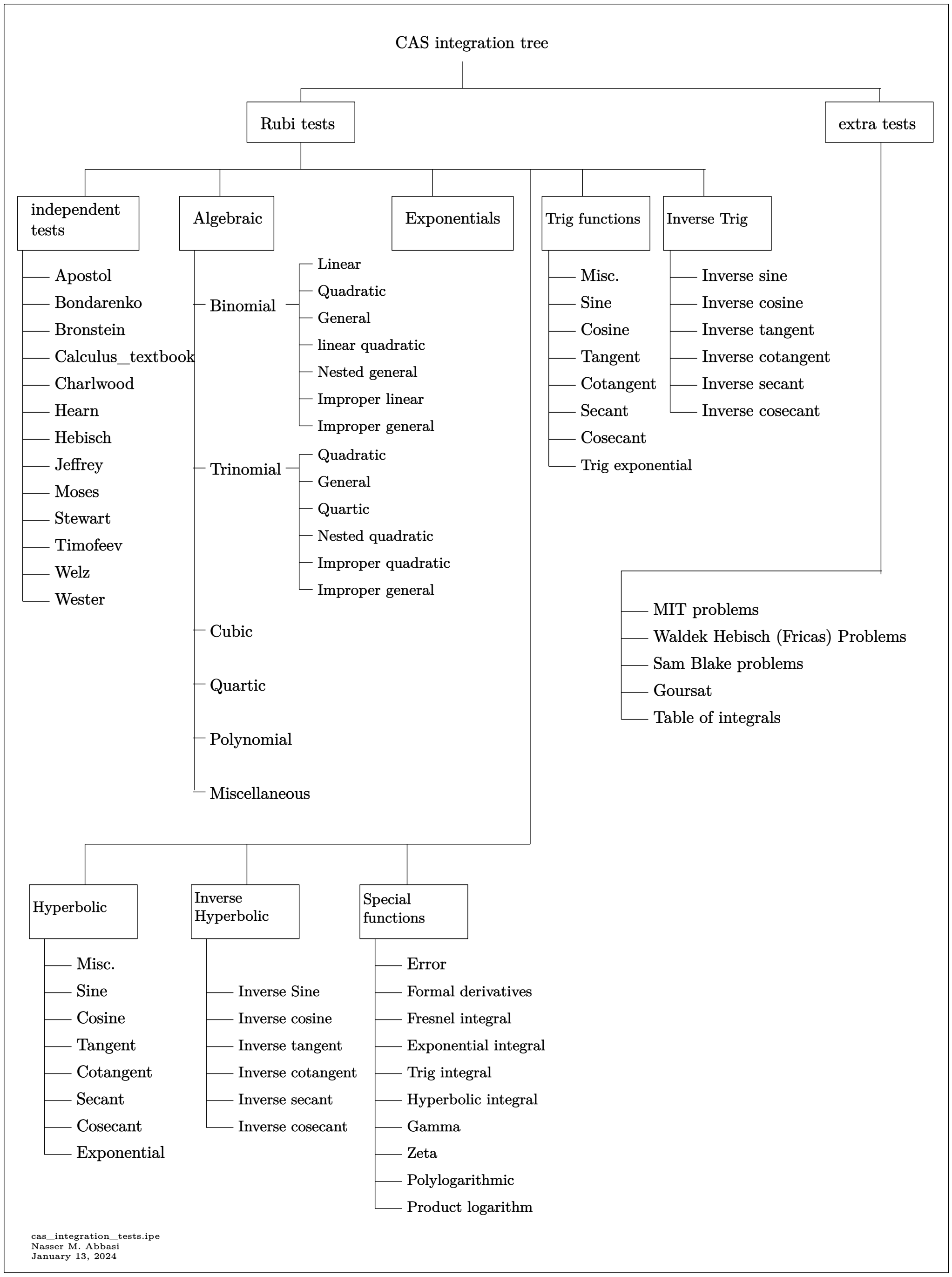
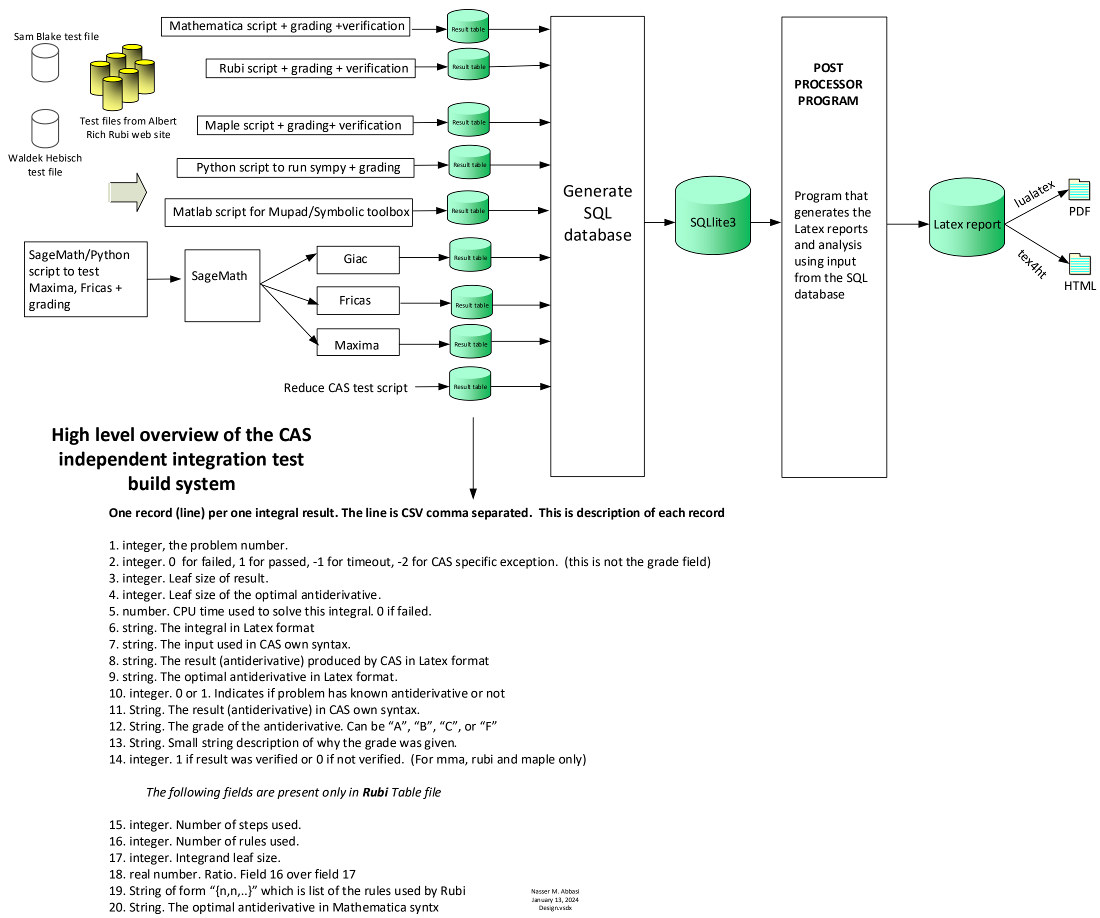


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344 }

B grade { 11, 16, 128, 141, 209, 224, 243, 259 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 22, 40, 41, 42, 43, 44, 59, 60, 61, 62, 63, 64, 66, 101, 102, 113, 114, 123, 127, 128, 136, 141, 142, 179, 183, 190, 196, 204, 208, 219, 225, 243, 244, 259, 260, 261, 262, 263, 264, 265, 278, 279, 293, 321, 322, 323, 324, 325, 326, 327, 329, 332, 335 }

B grade { 65, 67, 80, 81, 82, 95, 106, 119, 280, 331 }

C grade { 12, 13, 14, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 103, 104, 105, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 120, 121, 122, 124, 125, 126, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 195, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 328, 330, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 54, 59, 60, 61, 62, 63, 64, 77, 78, 79, 92, 93, 94, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 151, 152, 153, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 238, 243, 244, 245, 246, 247, 248, 249, 254, 259, 260, 261, 262, 275, 276, 277, 290, 291, 292, 294, 295, 296, 299, 300, 301, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 329, 332, 333, 336, 337, 341, 342 }

B grade { 17, 65, 66, 67, 80, 81, 82, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 154, 155, 156, 157, 158, 159, 160, 161, 162, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 263, 264, 265, 278, 279, 280, 293, 302, 303, 304, 305, 306, 307, 308, 309, 310, 331, 338, 339, 340, 343, 344 }

C grade { 31, 32, 33, 34, 36, 37, 38, 39, 50, 51, 52, 53, 55, 56, 57, 58, 68, 69, 70, 71, 72, 73, 74, 75, 76, 83, 84, 85, 86, 87, 88, 89, 90, 91, 101, 114, 128, 141, 149, 150, 178, 191, 201, 209, 224, 234, 235, 236, 237, 239, 240, 241, 242, 250, 251, 252, 253, 255, 256, 257, 258, 266, 267, 268, 269, 270, 271, 272, 273, 274, 281, 282, 283, 284, 285, 286, 287, 288, 289, 297, 298, 325, 326, 327, 328, 330, 334, 335 }

F normal fail { 3 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 317, 318, 319, 320, 321, 322, 323, 324, 329, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344 }

B grade { 17, 18, 24, 25, 303, 309, 315, 316, 325, 326, 327, 328, 330, 331, 335 }

C grade { 95, 293 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 321, 322, 323, 324, 332, 333 }

B grade { 331 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 325, 326, 327, 328, 330, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344 }

F(-1) timedout fail { }

F(-2) exception fail { 329 }

Giac

A grade { 1, 321, 322, 323, 324, 328, 329, 332, 333, 334 }

B grade { 325, 326, 327, 331 }

C grade { }

F normal fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 336, 337, 338, 339, 340, 341, 342, 343, 344 }

F(-1) timedout fail { 330 }

F(-2) exception fail { 335 }

Mupad

A grade { }

B grade { 4, 5, 6, 7, 35, 54, 72, 87, 101, 114, 128, 141, 178, 191, 209, 224, 238, 254, 270, 285, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 336, 337, 338, 339, 340, 341, 342, 343, 344 }

F(-2) exception fail { }

Sympy

A grade { 101, 114, 128, 141, 178, 191, 209, 224, 321, 322, 323, 324, 325, 326, 328, 330, 332, 333 }

B grade { 329, 331, 334 }

C grade { 35, 54, 72, 87, 238, 254, 270, 285 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 336, 337, 338, 339, 340, 341, 342, 343, 344 }

F(-1) timedout fail { 25, 201, 327, 335 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	108	88	77	0	177	0	59	59	0
N.S.	1	0.84	0.69	0.60	0.00	1.38	0.00	0.46	0.46	0.00
time (sec)	N/A	0.313	0.127	0.169	0.000	0.083	0.000	0.190	0.184	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	84	59	58	0	147	0	0	59	0
N.S.	1	0.92	0.65	0.64	0.00	1.62	0.00	0.00	0.65	0.00
time (sec)	N/A	0.301	0.077	0.155	0.000	0.074	0.000	0.000	0.187	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	49	0	0	90	0	0	22	0
N.S.	1	1.00	0.82	0.00	0.00	1.50	0.00	0.00	0.37	0.00
time (sec)	N/A	0.287	0.018	0.000	0.000	0.076	0.000	0.000	0.178	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	25	33	0	34	0	0	22	34
N.S.	1	1.00	0.74	0.97	0.00	1.00	0.00	0.00	0.65	1.00
time (sec)	N/A	0.267	0.053	0.158	0.000	0.066	0.000	0.000	0.183	17.603

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	79	38	44	0	58	0	0	83	45
N.S.	1	1.16	0.56	0.65	0.00	0.85	0.00	0.00	1.22	0.66
time (sec)	N/A	0.306	0.077	0.166	0.000	0.078	0.000	0.000	0.175	17.542

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	103	49	55	0	80	0	0	83	56
N.S.	1	0.98	0.47	0.52	0.00	0.76	0.00	0.00	0.79	0.53
time (sec)	N/A	0.321	0.097	0.167	0.000	0.075	0.000	0.000	0.180	18.599

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	127	60	66	0	102	0	0	145	141
N.S.	1	0.94	0.44	0.49	0.00	0.76	0.00	0.00	1.07	1.04
time (sec)	N/A	0.357	0.112	0.168	0.000	0.097	0.000	0.000	0.178	18.924

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	190	83	87	0	42	0	0	43	0
N.S.	1	1.96	0.86	0.90	0.00	0.43	0.00	0.00	0.44	0.00
time (sec)	N/A	0.463	1.992	0.594	0.000	0.097	0.000	0.000	0.190	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	86	90	0	40	0	0	46	0
N.S.	1	1.00	0.95	0.99	0.00	0.44	0.00	0.00	0.51	0.00
time (sec)	N/A	0.385	2.034	0.592	0.000	0.072	0.000	0.000	0.194	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	90	0	40	0	0	46	0
N.S.	1	1.00	0.91	0.98	0.00	0.43	0.00	0.00	0.50	0.00
time (sec)	N/A	0.393	1.999	0.566	0.000	0.079	0.000	0.000	0.189	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	191	85	92	0	39	0	0	47	0
N.S.	1	2.12	0.94	1.02	0.00	0.43	0.00	0.00	0.52	0.00
time (sec)	N/A	0.444	2.024	0.574	0.000	0.086	0.000	0.000	0.181	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	629	883	456	901	0	549	0	0	1045	0
N.S.	1	1.40	0.72	1.43	0.00	0.87	0.00	0.00	1.66	0.00
time (sec)	N/A	1.973	8.766	3.190	0.000	0.104	0.000	0.000	1.169	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	651	299	544	0	321	0	0	536	0
N.S.	1	1.60	0.73	1.33	0.00	0.79	0.00	0.00	1.31	0.00
time (sec)	N/A	1.329	6.106	2.844	0.000	0.106	0.000	0.000	0.631	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	514	195	250	0	158	0	0	167	0
N.S.	1	1.82	0.69	0.89	0.00	0.56	0.00	0.00	0.59	0.00
time (sec)	N/A	0.561	2.392	1.902	0.000	0.106	0.000	0.000	0.314	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	190	83	87	0	42	0	0	43	0
N.S.	1	1.96	0.86	0.90	0.00	0.43	0.00	0.00	0.44	0.00
time (sec)	N/A	0.230	0.043	0.526	0.000	0.101	0.000	0.000	0.181	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	571	221	428	0	412	0	0	108	0
N.S.	1	2.14	0.83	1.60	0.00	1.54	0.00	0.00	0.40	0.00
time (sec)	N/A	0.640	6.222	0.602	0.000	0.109	0.000	0.000	0.498	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	793	353	825	0	1461	0	0	207	0
N.S.	1	1.92	0.85	1.99	0.00	3.53	0.00	0.00	0.50	0.00
time (sec)	N/A	1.576	7.583	0.619	0.000	0.178	0.000	0.000	0.942	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	632	1093	554	1124	0	3257	0	0	340	0
N.S.	1	1.73	0.88	1.78	0.00	5.15	0.00	0.00	0.54	0.00
time (sec)	N/A	2.317	8.771	0.655	0.000	0.686	0.000	0.000	2.039	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	508	463	923	0	554	0	0	1093	0
N.S.	1	0.94	0.85	1.70	0.00	1.02	0.00	0.00	2.02	0.00
time (sec)	N/A	1.378	8.739	4.335	0.000	0.108	0.000	0.000	1.230	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	311	307	560	0	325	0	0	563	0
N.S.	1	0.88	0.86	1.58	0.00	0.92	0.00	0.00	1.59	0.00
time (sec)	N/A	0.897	5.945	3.927	0.000	0.095	0.000	0.000	0.662	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	187	201	259	0	163	0	0	177	0
N.S.	1	0.77	0.82	1.06	0.00	0.67	0.00	0.00	0.73	0.00
time (sec)	N/A	0.570	2.409	2.421	0.000	0.092	0.000	0.000	0.290	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	86	90	0	40	0	0	46	0
N.S.	1	1.00	0.95	0.99	0.00	0.44	0.00	0.00	0.51	0.00
time (sec)	N/A	0.336	0.051	0.535	0.000	0.081	0.000	0.000	0.180	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	243	226	440	0	421	0	0	109	0
N.S.	1	0.86	0.80	1.55	0.00	1.49	0.00	0.00	0.39	0.00
time (sec)	N/A	0.682	6.406	0.628	0.000	0.099	0.000	0.000	0.532	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	442	358	843	0	1472	0	0	210	0
N.S.	1	0.94	0.76	1.79	0.00	3.13	0.00	0.00	0.45	0.00
time (sec)	N/A	1.151	7.663	0.638	0.000	0.169	0.000	0.000	1.023	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	722	711	560	1144	0	3272	0	0	341	0
N.S.	1	0.98	0.78	1.58	0.00	4.53	0.00	0.00	0.47	0.00
time (sec)	N/A	1.917	8.713	0.671	0.000	0.725	0.000	0.000	2.463	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	110	105	78	0	41	0	0	30	0
N.S.	1	1.08	1.03	0.76	0.00	0.40	0.00	0.00	0.29	0.00
time (sec)	N/A	0.389	10.139	0.681	0.000	0.085	0.000	0.000	0.170	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	110	98	82	0	44	0	0	30	0
N.S.	1	1.05	0.93	0.78	0.00	0.42	0.00	0.00	0.29	0.00
time (sec)	N/A	0.440	10.110	0.610	0.000	0.083	0.000	0.000	0.166	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	60	61	50	0	16	0	0	30	0
N.S.	1	1.28	1.30	1.06	0.00	0.34	0.00	0.00	0.64	0.00
time (sec)	N/A	0.277	10.034	0.577	0.000	0.078	0.000	0.000	0.165	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	104	92	82	0	42	0	0	30	0
N.S.	1	1.03	0.91	0.81	0.00	0.42	0.00	0.00	0.30	0.00
time (sec)	N/A	0.368	10.064	0.615	0.000	0.085	0.000	0.000	0.164	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	52	58	44	0	11	0	0	30	0
N.S.	1	1.08	1.21	0.92	0.00	0.23	0.00	0.00	0.62	0.00
time (sec)	N/A	0.279	10.033	0.577	0.000	0.095	0.000	0.000	0.155	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	39	0	0	30	0
N.S.	1	1.00	1.60	0.97	0.00	0.43	0.00	0.00	0.33	0.00
time (sec)	N/A	0.329	10.103	0.600	0.000	0.078	0.000	0.000	0.187	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	36	0	0	30	0
N.S.	1	1.00	1.57	0.95	0.00	0.39	0.00	0.00	0.33	0.00
time (sec)	N/A	0.306	10.147	0.615	0.000	0.076	0.000	0.000	0.175	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	39	0	0	30	0
N.S.	1	1.00	1.57	0.95	0.00	0.42	0.00	0.00	0.33	0.00
time (sec)	N/A	0.311	10.113	0.605	0.000	0.080	0.000	0.000	0.171	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	142	85	0	36	0	0	26	0
N.S.	1	1.00	1.61	0.97	0.00	0.41	0.00	0.00	0.30	0.00
time (sec)	N/A	0.310	10.109	0.616	0.000	0.080	0.000	0.000	0.197	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	25	18	0	22	36	0	20	16
N.S.	1	1.00	0.35	0.25	0.00	0.31	0.50	0.00	0.28	0.22
time (sec)	N/A	0.276	10.049	0.563	0.000	0.079	0.335	0.000	0.170	0.105

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	36	0	0	30	0
N.S.	1	1.00	1.60	0.97	0.00	0.40	0.00	0.00	0.33	0.00
time (sec)	N/A	0.304	10.106	0.624	0.000	0.075	0.000	0.000	0.169	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	39	0	0	30	0
N.S.	1	1.00	1.60	0.97	0.00	0.43	0.00	0.00	0.33	0.00
time (sec)	N/A	0.306	10.111	0.618	0.000	0.078	0.000	0.000	0.161	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	36	0	0	30	0
N.S.	1	1.00	1.60	0.97	0.00	0.40	0.00	0.00	0.33	0.00
time (sec)	N/A	0.304	10.136	0.612	0.000	0.082	0.000	0.000	0.162	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	39	0	0	30	0
N.S.	1	1.00	1.64	0.99	0.00	0.44	0.00	0.00	0.34	0.00
time (sec)	N/A	0.304	10.116	0.600	0.000	0.078	0.000	0.000	0.171	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	92	53	42	0	9	0	0	30	0
N.S.	1	1.84	1.06	0.84	0.00	0.18	0.00	0.00	0.60	0.00
time (sec)	N/A	0.316	10.046	0.562	0.000	0.069	0.000	0.000	0.158	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	90	85	82	0	44	0	0	30	0
N.S.	1	0.87	0.83	0.80	0.00	0.43	0.00	0.00	0.29	0.00
time (sec)	N/A	0.315	10.112	0.579	0.000	0.071	0.000	0.000	0.166	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	92	58	49	0	16	0	0	30	0
N.S.	1	1.64	1.04	0.88	0.00	0.29	0.00	0.00	0.54	0.00
time (sec)	N/A	0.343	10.037	0.575	0.000	0.072	0.000	0.000	0.169	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	90	87	82	0	44	0	0	30	0
N.S.	1	0.90	0.87	0.82	0.00	0.44	0.00	0.00	0.30	0.00
time (sec)	N/A	0.332	10.151	0.581	0.000	0.072	0.000	0.000	0.163	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	92	91	78	0	41	0	0	30	0
N.S.	1	0.92	0.91	0.78	0.00	0.41	0.00	0.00	0.30	0.00
time (sec)	N/A	0.339	10.146	0.599	0.000	0.072	0.000	0.000	0.165	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	110	97	82	0	44	0	0	30	0
N.S.	1	1.09	0.96	0.81	0.00	0.44	0.00	0.00	0.30	0.00
time (sec)	N/A	0.407	10.102	0.636	0.000	0.077	0.000	0.000	0.166	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	110	98	82	0	44	0	0	30	0
N.S.	1	1.05	0.93	0.78	0.00	0.42	0.00	0.00	0.29	0.00
time (sec)	N/A	0.404	10.098	0.602	0.000	0.075	0.000	0.000	0.166	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	60	61	50	0	16	0	0	30	0
N.S.	1	1.28	1.30	1.06	0.00	0.34	0.00	0.00	0.64	0.00
time (sec)	N/A	0.282	10.031	0.596	0.000	0.074	0.000	0.000	0.156	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	104	90	82	0	34	0	0	30	0
N.S.	1	1.03	0.89	0.81	0.00	0.34	0.00	0.00	0.30	0.00
time (sec)	N/A	0.375	10.067	0.590	0.000	0.079	0.000	0.000	0.168	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	52	58	50	0	16	0	0	30	0
N.S.	1	0.93	1.04	0.89	0.00	0.29	0.00	0.00	0.54	0.00
time (sec)	N/A	0.271	10.034	0.583	0.000	0.080	0.000	0.000	0.162	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	39	0	0	30	0
N.S.	1	1.00	1.60	0.97	0.00	0.43	0.00	0.00	0.33	0.00
time (sec)	N/A	0.322	10.121	0.599	0.000	0.076	0.000	0.000	0.168	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	142	87	0	39	0	0	30	0
N.S.	1	1.00	1.54	0.95	0.00	0.42	0.00	0.00	0.33	0.00
time (sec)	N/A	0.329	10.131	0.603	0.000	0.077	0.000	0.000	0.166	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	39	0	0	30	0
N.S.	1	1.00	1.57	0.95	0.00	0.42	0.00	0.00	0.33	0.00
time (sec)	N/A	0.299	10.098	0.597	0.000	0.078	0.000	0.000	0.165	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	140	85	0	39	0	0	26	0
N.S.	1	1.00	1.59	0.97	0.00	0.44	0.00	0.00	0.30	0.00
time (sec)	N/A	0.302	10.084	0.604	0.000	0.081	0.000	0.000	0.172	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	25	18	0	16	36	0	20	16
N.S.	1	1.00	0.35	0.25	0.00	0.22	0.50	0.00	0.28	0.22
time (sec)	N/A	0.274	10.050	0.535	0.000	0.078	0.327	0.000	0.167	0.097

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	39	0	0	30	0
N.S.	1	1.00	1.58	0.97	0.00	0.43	0.00	0.00	0.33	0.00
time (sec)	N/A	0.306	10.079	0.635	0.000	0.084	0.000	0.000	0.163	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	39	0	0	30	0
N.S.	1	1.00	1.60	0.97	0.00	0.43	0.00	0.00	0.33	0.00
time (sec)	N/A	0.301	10.087	0.618	0.000	0.083	0.000	0.000	0.167	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	39	0	0	30	0
N.S.	1	1.00	1.58	0.97	0.00	0.43	0.00	0.00	0.33	0.00
time (sec)	N/A	0.307	10.112	0.628	0.000	0.081	0.000	0.000	0.162	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	39	0	0	30	0
N.S.	1	1.00	1.64	0.99	0.00	0.44	0.00	0.00	0.34	0.00
time (sec)	N/A	0.290	10.099	0.593	0.000	0.094	0.000	0.000	0.168	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	92	53	42	0	9	0	0	30	0
N.S.	1	1.84	1.06	0.84	0.00	0.18	0.00	0.00	0.60	0.00
time (sec)	N/A	0.303	10.034	0.559	0.000	0.079	0.000	0.000	0.163	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	90	81	82	0	36	0	0	30	0
N.S.	1	0.94	0.84	0.85	0.00	0.38	0.00	0.00	0.31	0.00
time (sec)	N/A	0.321	10.095	0.582	0.000	0.081	0.000	0.000	0.164	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	92	58	49	0	16	0	0	30	0
N.S.	1	1.64	1.04	0.88	0.00	0.29	0.00	0.00	0.54	0.00
time (sec)	N/A	0.331	10.032	0.570	0.000	0.081	0.000	0.000	0.166	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	90	89	82	0	44	0	0	30	0
N.S.	1	0.84	0.83	0.77	0.00	0.41	0.00	0.00	0.28	0.00
time (sec)	N/A	0.331	10.128	0.580	0.000	0.089	0.000	0.000	0.160	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	92	91	82	0	44	0	0	30	0
N.S.	1	0.92	0.91	0.82	0.00	0.44	0.00	0.00	0.30	0.00
time (sec)	N/A	0.326	10.115	0.589	0.000	0.083	0.000	0.000	0.163	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	71	97	87	0	36	0	0	94	0
N.S.	1	0.70	0.95	0.85	0.00	0.35	0.00	0.00	0.92	0.00
time (sec)	N/A	0.299	10.092	0.865	0.000	0.092	0.000	0.000	0.199	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	58	48	0	16	0	0	32	0
N.S.	1	1.00	3.05	2.53	0.00	0.84	0.00	0.00	1.68	0.00
time (sec)	N/A	0.262	10.037	0.601	0.000	0.099	0.000	0.000	0.162	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	81	82	0	44	0	0	32	0
N.S.	1	1.00	1.84	1.86	0.00	1.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.344	10.096	0.568	0.000	0.102	0.000	0.000	0.176	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	53	50	0	9	0	0	32	0
N.S.	1	1.00	3.79	3.57	0.00	0.64	0.00	0.00	2.29	0.00
time (sec)	N/A	0.256	10.033	0.556	0.000	0.097	0.000	0.000	0.167	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	44	0	0	32	0
N.S.	1	1.00	1.64	0.99	0.00	0.50	0.00	0.00	0.36	0.00
time (sec)	N/A	0.324	10.115	0.599	0.000	0.088	0.000	0.000	0.168	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	44	0	0	32	0
N.S.	1	1.00	1.58	0.97	0.00	0.49	0.00	0.00	0.36	0.00
time (sec)	N/A	0.299	10.126	0.614	0.000	0.092	0.000	0.000	0.180	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	39	0	0	32	0
N.S.	1	1.00	1.60	0.97	0.00	0.43	0.00	0.00	0.36	0.00
time (sec)	N/A	0.309	10.111	0.570	0.000	0.090	0.000	0.000	0.162	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	140	85	0	39	0	0	30	0
N.S.	1	1.00	1.59	0.97	0.00	0.44	0.00	0.00	0.34	0.00
time (sec)	N/A	0.308	10.089	0.583	0.000	0.092	0.000	0.000	0.178	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	47	19	0	16	39	0	22	31
N.S.	1	1.00	0.65	0.26	0.00	0.22	0.54	0.00	0.31	0.43
time (sec)	N/A	0.271	10.051	0.525	0.000	0.115	0.340	0.000	0.165	0.086

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	39	0	0	30	0
N.S.	1	1.00	1.58	0.97	0.00	0.43	0.00	0.00	0.33	0.00
time (sec)	N/A	0.300	10.094	0.596	0.000	0.082	0.000	0.000	0.168	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	39	0	0	32	0
N.S.	1	1.00	1.57	0.95	0.00	0.42	0.00	0.00	0.35	0.00
time (sec)	N/A	0.323	10.109	0.586	0.000	0.078	0.000	0.000	0.170	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	142	87	0	42	0	0	32	0
N.S.	1	1.00	1.54	0.95	0.00	0.46	0.00	0.00	0.35	0.00
time (sec)	N/A	0.307	10.141	0.582	0.000	0.081	0.000	0.000	0.174	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	44	0	0	32	0
N.S.	1	1.00	1.60	0.97	0.00	0.49	0.00	0.00	0.36	0.00
time (sec)	N/A	0.293	10.122	0.573	0.000	0.072	0.000	0.000	0.166	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	53	63	44	0	16	0	0	32	0
N.S.	1	1.32	1.58	1.10	0.00	0.40	0.00	0.00	0.80	0.00
time (sec)	N/A	0.289	10.043	0.541	0.000	0.082	0.000	0.000	0.165	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	118	92	82	0	42	0	0	32	0
N.S.	1	1.48	1.15	1.02	0.00	0.52	0.00	0.00	0.40	0.00
time (sec)	N/A	0.403	10.130	0.548	0.000	0.076	0.000	0.000	0.166	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	51	61	52	0	15	0	0	32	0
N.S.	1	1.55	1.85	1.58	0.00	0.45	0.00	0.00	0.97	0.00
time (sec)	N/A	0.306	10.035	0.547	0.000	0.074	0.000	0.000	0.155	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	58	48	0	16	0	0	32	0
N.S.	1	1.00	3.05	2.53	0.00	0.84	0.00	0.00	1.68	0.00
time (sec)	N/A	0.274	10.039	0.573	0.000	0.078	0.000	0.000	0.171	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	85	82	0	44	0	0	32	0
N.S.	1	1.00	2.02	1.95	0.00	1.05	0.00	0.00	0.76	0.00
time (sec)	N/A	0.345	10.106	0.570	0.000	0.087	0.000	0.000	0.177	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	53	42	0	9	0	0	32	0
N.S.	1	1.00	8.83	7.00	0.00	1.50	0.00	0.00	5.33	0.00
time (sec)	N/A	0.242	10.054	0.549	0.000	0.075	0.000	0.000	0.175	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	36	0	0	32	0
N.S.	1	1.00	1.64	0.99	0.00	0.41	0.00	0.00	0.36	0.00
time (sec)	N/A	0.312	10.120	0.579	0.000	0.073	0.000	0.000	0.162	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	36	0	0	32	0
N.S.	1	1.00	1.60	0.97	0.00	0.40	0.00	0.00	0.36	0.00
time (sec)	N/A	0.316	10.150	0.591	0.000	0.084	0.000	0.000	0.178	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	39	0	0	32	0
N.S.	1	1.00	1.60	0.97	0.00	0.43	0.00	0.00	0.36	0.00
time (sec)	N/A	0.312	10.113	0.573	0.000	0.074	0.000	0.000	0.175	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	142	85	0	36	0	0	30	0
N.S.	1	1.00	1.61	0.97	0.00	0.41	0.00	0.00	0.34	0.00
time (sec)	N/A	0.308	10.116	0.585	0.000	0.074	0.000	0.000	0.169	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	47	19	0	22	39	0	22	31
N.S.	1	1.00	0.65	0.26	0.00	0.31	0.54	0.00	0.31	0.43
time (sec)	N/A	0.285	10.051	0.517	0.000	0.081	0.338	0.000	0.172	18.086

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	36	0	0	30	0
N.S.	1	1.00	1.60	0.97	0.00	0.40	0.00	0.00	0.33	0.00
time (sec)	N/A	0.315	10.107	0.583	0.000	0.076	0.000	0.000	0.178	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	39	0	0	32	0
N.S.	1	1.00	1.57	0.95	0.00	0.42	0.00	0.00	0.35	0.00
time (sec)	N/A	0.322	10.109	0.578	0.000	0.080	0.000	0.000	0.171	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	36	0	0	32	0
N.S.	1	1.00	1.57	0.95	0.00	0.39	0.00	0.00	0.35	0.00
time (sec)	N/A	0.311	10.169	0.585	0.000	0.075	0.000	0.000	0.176	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	36	0	0	32	0
N.S.	1	1.00	1.60	0.97	0.00	0.40	0.00	0.00	0.36	0.00
time (sec)	N/A	0.321	10.111	0.563	0.000	0.076	0.000	0.000	0.171	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	52	63	50	0	11	0	0	32	0
N.S.	1	1.53	1.85	1.47	0.00	0.32	0.00	0.00	0.94	0.00
time (sec)	N/A	0.292	10.043	0.554	0.000	0.070	0.000	0.000	0.165	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	108	90	82	0	42	0	0	32	0
N.S.	1	1.42	1.18	1.08	0.00	0.55	0.00	0.00	0.42	0.00
time (sec)	N/A	0.383	10.127	0.556	0.000	0.084	0.000	0.000	0.164	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	51	61	52	0	15	0	0	32	0
N.S.	1	1.55	1.85	1.58	0.00	0.45	0.00	0.00	0.97	0.00
time (sec)	N/A	0.308	10.031	0.554	0.000	0.076	0.000	0.000	0.166	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	82	0	38	0	0	30	0
N.S.	1	1.00	2.23	2.10	0.00	0.97	0.00	0.00	0.77	0.00
time (sec)	N/A	0.347	10.137	0.434	0.000	0.075	0.000	0.000	0.176	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	9	0	0	32	0
N.S.	1	1.00	6.50	5.10	0.00	0.90	0.00	0.00	3.20	0.00
time (sec)	N/A	0.262	10.032	0.615	0.000	0.073	0.000	0.000	0.168	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	48	49	84	0	44	0	0	32	0
N.S.	1	1.09	1.11	1.91	0.00	1.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.406	10.065	0.609	0.000	0.087	0.000	0.000	0.177	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	80	0	41	0	0	32	0
N.S.	1	1.00	1.10	1.67	0.00	0.85	0.00	0.00	0.67	0.00
time (sec)	N/A	0.362	10.068	0.625	0.000	0.084	0.000	0.000	0.180	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	49	84	0	42	0	0	32	0
N.S.	1	1.00	1.11	1.91	0.00	0.95	0.00	0.00	0.73	0.00
time (sec)	N/A	0.347	10.055	0.621	0.000	0.078	0.000	0.000	0.170	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	63	41	0	9	0	0	30	0
N.S.	1	1.00	5.25	3.42	0.00	0.75	0.00	0.00	2.50	0.00
time (sec)	N/A	0.248	10.045	0.568	0.000	0.088	0.000	0.000	0.200	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	0	22	37	0	22	16
N.S.	1	1.00	1.00	1.00	0.00	1.22	2.06	0.00	1.22	0.89
time (sec)	N/A	0.234	10.043	0.554	0.000	0.071	0.339	0.000	0.249	17.888

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	49	0	16	0	0	30	0
N.S.	1	1.00	1.00	2.45	0.00	0.80	0.00	0.00	1.50	0.00
time (sec)	N/A	0.259	10.032	0.582	0.000	0.068	0.000	0.000	0.242	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	84	0	44	0	0	32	0
N.S.	1	1.00	1.21	2.00	0.00	1.05	0.00	0.00	0.76	0.00
time (sec)	N/A	0.321	10.071	0.621	0.000	0.069	0.000	0.000	0.267	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	80	0	41	0	0	32	0
N.S.	1	1.00	1.20	1.74	0.00	0.89	0.00	0.00	0.70	0.00
time (sec)	N/A	0.337	10.086	0.614	0.000	0.083	0.000	0.000	0.250	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	48	49	84	0	44	0	0	32	0
N.S.	1	1.09	1.11	1.91	0.00	1.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.361	10.208	0.602	0.000	0.071	0.000	0.000	0.241	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	54	50	0	16	0	0	32	0
N.S.	1	1.00	3.00	2.78	0.00	0.89	0.00	0.00	1.78	0.00
time (sec)	N/A	0.258	10.043	0.566	0.000	0.073	0.000	0.000	0.262	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	84	0	44	0	0	32	0
N.S.	1	1.00	1.16	1.87	0.00	0.98	0.00	0.00	0.71	0.00
time (sec)	N/A	0.323	10.052	0.673	0.000	0.076	0.000	0.000	0.238	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	43	84	0	35	0	0	32	0
N.S.	1	1.10	1.08	2.10	0.00	0.88	0.00	0.00	0.80	0.00
time (sec)	N/A	0.397	10.066	0.612	0.000	0.074	0.000	0.000	0.299	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	9	0	0	32	0
N.S.	1	1.00	6.50	5.10	0.00	0.90	0.00	0.00	3.20	0.00
time (sec)	N/A	0.265	10.121	0.559	0.000	0.075	0.000	0.000	0.187	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	84	0	44	0	0	32	0
N.S.	1	1.00	1.16	1.91	0.00	1.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.347	10.065	0.618	0.000	0.075	0.000	0.000	0.177	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	84	0	42	0	0	32	0
N.S.	1	1.00	1.11	1.87	0.00	0.93	0.00	0.00	0.71	0.00
time (sec)	N/A	0.322	10.064	0.627	0.000	0.090	0.000	0.000	0.180	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	49	84	0	42	0	0	32	0
N.S.	1	1.00	1.11	1.91	0.00	0.95	0.00	0.00	0.73	0.00
time (sec)	N/A	0.308	10.052	0.612	0.000	0.075	0.000	0.000	0.174	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	47	0	16	0	0	30	0
N.S.	1	1.00	1.00	2.35	0.00	0.80	0.00	0.00	1.50	0.00
time (sec)	N/A	0.251	10.045	0.569	0.000	0.077	0.000	0.000	0.168	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	0	16	37	0	22	16
N.S.	1	1.00	1.00	1.00	0.00	0.89	2.06	0.00	1.22	0.89
time (sec)	N/A	0.234	10.044	0.543	0.000	0.084	0.346	0.000	0.172	17.264

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	65	43	0	9	0	0	30	0
N.S.	1	1.00	5.42	3.58	0.00	0.75	0.00	0.00	2.50	0.00
time (sec)	N/A	0.245	10.030	0.567	0.000	0.102	0.000	0.000	0.169	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	84	0	44	0	0	32	0
N.S.	1	1.00	1.21	2.00	0.00	1.05	0.00	0.00	0.76	0.00
time (sec)	N/A	0.317	10.053	0.625	0.000	0.093	0.000	0.000	0.171	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	52	84	0	44	0	0	32	0
N.S.	1	1.00	1.21	1.95	0.00	1.02	0.00	0.00	0.74	0.00
time (sec)	N/A	0.292	10.076	0.628	0.000	0.078	0.000	0.000	0.176	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	84	0	44	0	0	32	0
N.S.	1	1.00	1.16	1.91	0.00	1.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.335	10.069	0.594	0.000	0.075	0.000	0.000	0.169	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	54	50	0	16	0	0	32	0
N.S.	1	1.00	3.00	2.78	0.00	0.89	0.00	0.00	1.78	0.00
time (sec)	N/A	0.253	10.033	0.553	0.000	0.080	0.000	0.000	0.169	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	45	84	0	35	0	0	32	0
N.S.	1	1.11	1.18	2.21	0.00	0.92	0.00	0.00	0.84	0.00
time (sec)	N/A	0.347	10.078	0.603	0.000	0.076	0.000	0.000	0.177	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	84	0	44	0	0	32	0
N.S.	1	1.00	1.16	1.87	0.00	0.98	0.00	0.00	0.71	0.00
time (sec)	N/A	0.309	10.055	0.620	0.000	0.085	0.000	0.000	0.167	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	43	0	9	0	0	30	0
N.S.	1	1.00	1.50	3.58	0.00	0.75	0.00	0.00	2.50	0.00
time (sec)	N/A	0.242	10.032	0.540	0.000	0.071	0.000	0.000	0.168	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	67	54	53	0	16	0	0	30	0
N.S.	1	1.24	1.00	0.98	0.00	0.30	0.00	0.00	0.56	0.00
time (sec)	N/A	0.278	10.041	0.597	0.000	0.077	0.000	0.000	0.161	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	141	81	84	0	44	0	0	30	0
N.S.	1	1.41	0.81	0.84	0.00	0.44	0.00	0.00	0.30	0.00
time (sec)	N/A	0.392	10.073	0.562	0.000	0.073	0.000	0.000	0.167	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	146	83	84	0	41	0	0	30	0
N.S.	1	1.49	0.85	0.86	0.00	0.42	0.00	0.00	0.31	0.00
time (sec)	N/A	0.392	10.083	0.577	0.000	0.072	0.000	0.000	0.174	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	141	83	84	0	44	0	0	30	0
N.S.	1	1.44	0.85	0.86	0.00	0.45	0.00	0.00	0.31	0.00
time (sec)	N/A	0.371	10.063	0.579	0.000	0.074	0.000	0.000	0.171	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	63	48	43	0	16	0	0	26	0
N.S.	1	1.17	0.89	0.80	0.00	0.30	0.00	0.00	0.48	0.00
time (sec)	N/A	0.288	10.039	0.546	0.000	0.066	0.000	0.000	0.161	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	115	40	40	0	22	34	0	20	31
N.S.	1	2.88	1.00	1.00	0.00	0.55	0.85	0.00	0.50	0.78
time (sec)	N/A	0.321	10.049	0.540	0.000	0.075	0.350	0.000	0.170	0.090

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	65	60	53	0	9	0	0	30	0
N.S.	1	1.30	1.20	1.06	0.00	0.18	0.00	0.00	0.60	0.00
time (sec)	N/A	0.286	10.033	0.552	0.000	0.072	0.000	0.000	0.163	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	148	81	84	0	42	0	0	30	0
N.S.	1	1.51	0.83	0.86	0.00	0.43	0.00	0.00	0.31	0.00
time (sec)	N/A	0.395	10.073	0.564	0.000	0.077	0.000	0.000	0.177	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	153	81	84	0	41	0	0	30	0
N.S.	1	1.56	0.83	0.86	0.00	0.42	0.00	0.00	0.31	0.00
time (sec)	N/A	0.402	10.079	0.568	0.000	0.080	0.000	0.000	0.165	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	148	81	84	0	44	0	0	30	0
N.S.	1	1.51	0.83	0.86	0.00	0.45	0.00	0.00	0.31	0.00
time (sec)	N/A	0.401	10.073	0.554	0.000	0.079	0.000	0.000	0.171	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	63	65	53	0	17	0	0	30	0
N.S.	1	1.29	1.33	1.08	0.00	0.35	0.00	0.00	0.61	0.00
time (sec)	N/A	0.284	10.033	0.544	0.000	0.070	0.000	0.000	0.169	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	148	80	84	0	44	0	0	30	0
N.S.	1	1.48	0.80	0.84	0.00	0.44	0.00	0.00	0.30	0.00
time (sec)	N/A	0.378	10.061	0.616	0.000	0.076	0.000	0.000	0.166	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	148	77	84	0	40	0	0	30	0
N.S.	1	1.57	0.82	0.89	0.00	0.43	0.00	0.00	0.32	0.00
time (sec)	N/A	0.402	10.088	0.564	0.000	0.091	0.000	0.000	0.162	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	67	54	53	0	16	0	0	30	0
N.S.	1	1.24	1.00	0.98	0.00	0.30	0.00	0.00	0.56	0.00
time (sec)	N/A	0.295	10.033	0.538	0.000	0.080	0.000	0.000	0.170	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	148	83	84	0	44	0	0	30	0
N.S.	1	1.41	0.79	0.80	0.00	0.42	0.00	0.00	0.29	0.00
time (sec)	N/A	0.402	10.090	0.575	0.000	0.067	0.000	0.000	0.165	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	146	80	84	0	44	0	0	30	0
N.S.	1	1.49	0.82	0.86	0.00	0.45	0.00	0.00	0.31	0.00
time (sec)	N/A	0.398	10.072	0.600	0.000	0.075	0.000	0.000	0.167	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	143	83	84	0	44	0	0	30	0
N.S.	1	1.46	0.85	0.86	0.00	0.45	0.00	0.00	0.31	0.00
time (sec)	N/A	0.383	10.062	0.562	0.000	0.077	0.000	0.000	0.164	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	63	63	51	0	9	0	0	26	0
N.S.	1	1.31	1.31	1.06	0.00	0.19	0.00	0.00	0.54	0.00
time (sec)	N/A	0.285	10.033	0.554	0.000	0.074	0.000	0.000	0.167	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	112	40	40	0	16	34	0	20	31
N.S.	1	2.80	1.00	1.00	0.00	0.40	0.85	0.00	0.50	0.78
time (sec)	N/A	0.333	10.049	0.546	0.000	0.080	0.351	0.000	0.173	0.085

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	65	51	45	0	16	0	0	30	0
N.S.	1	1.16	0.91	0.80	0.00	0.29	0.00	0.00	0.54	0.00
time (sec)	N/A	0.288	10.027	0.556	0.000	0.086	0.000	0.000	0.175	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	150	81	84	0	42	0	0	30	0
N.S.	1	1.53	0.83	0.86	0.00	0.43	0.00	0.00	0.31	0.00
time (sec)	N/A	0.396	10.064	0.581	0.000	0.077	0.000	0.000	0.168	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	153	78	84	0	42	0	0	30	0
N.S.	1	1.56	0.80	0.86	0.00	0.43	0.00	0.00	0.31	0.00
time (sec)	N/A	0.404	10.064	0.575	0.000	0.078	0.000	0.000	0.170	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	155	83	84	0	44	0	0	30	0
N.S.	1	1.50	0.81	0.82	0.00	0.43	0.00	0.00	0.29	0.00
time (sec)	N/A	0.398	10.079	0.566	0.000	0.068	0.000	0.000	0.172	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	63	65	53	0	17	0	0	30	0
N.S.	1	1.29	1.33	1.08	0.00	0.35	0.00	0.00	0.61	0.00
time (sec)	N/A	0.290	10.035	0.547	0.000	0.070	0.000	0.000	0.168	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	155	75	84	0	38	0	0	30	0
N.S.	1	1.65	0.80	0.89	0.00	0.40	0.00	0.00	0.32	0.00
time (sec)	N/A	0.411	10.072	0.557	0.000	0.076	0.000	0.000	0.172	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	153	80	84	0	44	0	0	30	0
N.S.	1	1.56	0.82	0.86	0.00	0.45	0.00	0.00	0.31	0.00
time (sec)	N/A	0.394	10.068	0.572	0.000	0.070	0.000	0.000	0.174	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	36	0	0	30	0
N.S.	1	1.00	1.57	0.95	0.00	0.39	0.00	0.00	0.33	0.00
time (sec)	N/A	0.370	10.146	0.665	0.000	0.072	0.000	0.000	0.176	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	147	87	0	36	0	0	30	0
N.S.	1	1.00	1.63	0.97	0.00	0.40	0.00	0.00	0.33	0.00
time (sec)	N/A	0.330	10.102	0.554	0.000	0.072	0.000	0.000	0.165	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	52	58	44	0	11	0	0	30	0
N.S.	1	1.08	1.21	0.92	0.00	0.23	0.00	0.00	0.62	0.00
time (sec)	N/A	0.272	0.018	0.507	0.000	0.078	0.000	0.000	0.167	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	58	58	48	0	16	0	0	30	0
N.S.	1	1.23	1.23	1.02	0.00	0.34	0.00	0.00	0.64	0.00
time (sec)	N/A	0.277	10.031	0.450	0.000	0.082	0.000	0.000	0.172	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	103	76	0	41	0	0	26	0
N.S.	1	1.08	1.03	0.76	0.00	0.41	0.00	0.00	0.26	0.00
time (sec)	N/A	0.362	10.103	0.536	0.000	0.074	0.000	0.000	0.177	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	55	80	0	41	0	0	30	0
N.S.	1	1.00	1.15	1.67	0.00	0.85	0.00	0.00	0.62	0.00
time (sec)	N/A	0.369	10.075	0.551	0.000	0.071	0.000	0.000	0.175	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	76	0	41	0	0	32	0
N.S.	1	1.00	1.16	1.69	0.00	0.91	0.00	0.00	0.71	0.00
time (sec)	N/A	0.319	10.054	0.528	0.000	0.071	0.000	0.000	0.178	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	9	0	0	32	0
N.S.	1	1.00	6.50	5.10	0.00	0.90	0.00	0.00	3.20	0.00
time (sec)	N/A	0.263	0.019	0.517	0.000	0.073	0.000	0.000	0.167	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	41	0	0	32	0
N.S.	1	1.00	1.14	1.63	0.00	0.84	0.00	0.00	0.65	0.00
time (sec)	N/A	0.371	10.071	0.568	0.000	0.071	0.000	0.000	0.177	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	52	80	0	41	0	0	32	0
N.S.	1	1.00	1.08	1.67	0.00	0.85	0.00	0.00	0.67	0.00
time (sec)	N/A	0.370	10.063	0.630	0.000	0.069	0.000	0.000	0.182	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	41	0	0	32	0
N.S.	1	1.00	1.14	1.63	0.00	0.84	0.00	0.00	0.65	0.00
time (sec)	N/A	0.345	10.065	0.631	0.000	0.070	0.000	0.000	0.181	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	65	43	0	9	0	0	32	0
N.S.	1	1.00	5.42	3.58	0.00	0.75	0.00	0.00	2.67	0.00
time (sec)	N/A	0.249	10.032	0.593	0.000	0.070	0.000	0.000	0.179	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	76	0	41	0	0	32	0
N.S.	1	1.00	1.16	1.69	0.00	0.91	0.00	0.00	0.71	0.00
time (sec)	N/A	0.325	10.058	0.542	0.000	0.069	0.000	0.000	0.177	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	41	0	0	32	0
N.S.	1	1.00	1.14	1.63	0.00	0.84	0.00	0.00	0.65	0.00
time (sec)	N/A	0.343	10.055	0.589	0.000	0.069	0.000	0.000	0.180	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	186	144	0	121	0	0	30	0
N.S.	1	1.00	1.63	1.26	0.00	1.06	0.00	0.00	0.26	0.00
time (sec)	N/A	0.334	10.147	0.533	0.000	0.077	0.000	0.000	0.174	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	180	143	0	121	0	0	32	0
N.S.	1	1.00	1.57	1.24	0.00	1.05	0.00	0.00	0.28	0.00
time (sec)	N/A	0.328	10.131	0.517	0.000	0.081	0.000	0.000	0.173	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	179	143	0	126	0	0	36	0
N.S.	1	1.00	1.53	1.22	0.00	1.08	0.00	0.00	0.31	0.00
time (sec)	N/A	0.329	10.113	0.518	0.000	0.085	0.000	0.000	0.178	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	181	142	0	126	0	0	33	0
N.S.	1	1.00	1.53	1.20	0.00	1.07	0.00	0.00	0.28	0.00
time (sec)	N/A	0.324	10.134	0.517	0.000	0.077	0.000	0.000	0.174	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	187	175	0	187	0	0	32	0
N.S.	1	1.00	1.41	1.32	0.00	1.41	0.00	0.00	0.24	0.00
time (sec)	N/A	0.389	10.204	0.581	0.000	0.083	0.000	0.000	0.181	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	191	174	0	187	0	0	34	0
N.S.	1	1.00	1.43	1.30	0.00	1.40	0.00	0.00	0.25	0.00
time (sec)	N/A	0.363	10.174	0.537	0.000	0.077	0.000	0.000	0.177	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	169	177	145	0	127	0	0	32	0
N.S.	1	1.30	1.36	1.12	0.00	0.98	0.00	0.00	0.25	0.00
time (sec)	N/A	0.608	10.143	0.550	0.000	0.078	0.000	0.000	0.173	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	194	179	144	0	126	0	0	34	0
N.S.	1	1.46	1.35	1.08	0.00	0.95	0.00	0.00	0.26	0.00
time (sec)	N/A	0.593	10.145	0.514	0.000	0.078	0.000	0.000	0.175	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	194	188	142	0	131	0	0	36	0
N.S.	1	1.46	1.41	1.07	0.00	0.98	0.00	0.00	0.27	0.00
time (sec)	N/A	0.530	10.122	0.516	0.000	0.081	0.000	0.000	0.179	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	171	182	141	0	132	0	0	36	0
N.S.	1	1.30	1.38	1.07	0.00	1.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.491	10.118	0.517	0.000	0.077	0.000	0.000	0.187	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	62	123	142	0	88	0	0	40	0
N.S.	1	1.09	2.16	2.49	0.00	1.54	0.00	0.00	0.70	0.00
time (sec)	N/A	0.364	9.291	2.385	0.000	0.079	0.000	0.000	0.178	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	150	135	230	0	155	0	0	40	0
N.S.	1	1.18	1.06	1.81	0.00	1.22	0.00	0.00	0.31	0.00
time (sec)	N/A	0.633	9.375	1.642	0.000	0.073	0.000	0.000	0.192	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	148	166	218	0	171	0	0	40	0
N.S.	1	1.13	1.27	1.66	0.00	1.31	0.00	0.00	0.31	0.00
time (sec)	N/A	0.576	9.498	2.371	0.000	0.080	0.000	0.000	0.197	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	148	135	230	0	176	0	0	40	0
N.S.	1	1.17	1.06	1.81	0.00	1.39	0.00	0.00	0.31	0.00
time (sec)	N/A	0.545	9.399	1.872	0.000	0.076	0.000	0.000	0.193	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	62	95	115	0	80	0	0	38	0
N.S.	1	1.09	1.67	2.02	0.00	1.40	0.00	0.00	0.67	0.00
time (sec)	N/A	0.361	8.983	1.793	0.000	0.069	0.000	0.000	0.182	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	44	18	0	52	37	0	25	16
N.S.	1	1.00	1.16	0.47	0.00	1.37	0.97	0.00	0.66	0.42
time (sec)	N/A	0.266	6.493	0.709	0.000	0.083	0.411	0.000	0.175	17.797

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	76	101	138	0	92	0	0	40	0
N.S.	1	1.01	1.35	1.84	0.00	1.23	0.00	0.00	0.53	0.00
time (sec)	N/A	0.382	8.953	2.270	0.000	0.072	0.000	0.000	0.177	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	142	139	230	0	177	0	0	40	0
N.S.	1	1.15	1.13	1.87	0.00	1.44	0.00	0.00	0.33	0.00
time (sec)	N/A	0.507	9.231	2.293	0.000	0.087	0.000	0.000	0.182	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	142	172	218	0	173	0	0	40	0
N.S.	1	1.12	1.35	1.72	0.00	1.36	0.00	0.00	0.31	0.00
time (sec)	N/A	0.548	9.523	2.778	0.000	0.080	0.000	0.000	0.191	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	148	135	230	0	159	0	0	40	0
N.S.	1	1.17	1.06	1.81	0.00	1.25	0.00	0.00	0.31	0.00
time (sec)	N/A	0.602	9.320	2.239	0.000	0.086	0.000	0.000	0.199	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	76	97	139	0	97	0	0	40	0
N.S.	1	1.01	1.29	1.85	0.00	1.29	0.00	0.00	0.53	0.00
time (sec)	N/A	0.387	9.078	2.754	0.000	0.081	0.000	0.000	0.177	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	136	168	228	0	178	0	0	40	0
N.S.	1	1.09	1.34	1.82	0.00	1.42	0.00	0.00	0.32	0.00
time (sec)	N/A	0.492	9.478	2.513	0.000	0.089	0.000	0.000	0.190	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	142	122	228	0	140	0	0	40	0
N.S.	1	1.19	1.03	1.92	0.00	1.18	0.00	0.00	0.34	0.00
time (sec)	N/A	0.598	9.171	1.628	0.000	0.080	0.000	0.000	0.192	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	62	123	142	0	88	0	0	40	0
N.S.	1	1.09	2.16	2.49	0.00	1.54	0.00	0.00	0.70	0.00
time (sec)	N/A	0.378	9.283	2.278	0.000	0.075	0.000	0.000	0.173	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	142	139	230	0	177	0	0	40	0
N.S.	1	1.12	1.09	1.81	0.00	1.39	0.00	0.00	0.31	0.00
time (sec)	N/A	0.599	9.514	2.296	0.000	0.080	0.000	0.000	0.188	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	136	162	228	0	176	0	0	40	0
N.S.	1	1.09	1.30	1.82	0.00	1.41	0.00	0.00	0.32	0.00
time (sec)	N/A	0.504	9.256	1.941	0.000	0.097	0.000	0.000	0.188	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	148	134	228	0	175	0	0	40	0
N.S.	1	1.17	1.06	1.80	0.00	1.38	0.00	0.00	0.31	0.00
time (sec)	N/A	0.546	9.301	1.858	0.000	0.080	0.000	0.000	0.195	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	98	132	0	96	0	0	38	0
N.S.	1	1.00	1.34	1.81	0.00	1.32	0.00	0.00	0.52	0.00
time (sec)	N/A	0.372	8.947	1.795	0.000	0.092	0.000	0.000	0.175	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	44	18	0	46	37	0	25	16
N.S.	1	1.00	1.16	0.47	0.00	1.21	0.97	0.00	0.66	0.42
time (sec)	N/A	0.257	6.445	0.707	0.000	0.097	0.403	0.000	0.169	17.782

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	64	99	121	0	76	0	0	40	0
N.S.	1	1.08	1.68	2.05	0.00	1.29	0.00	0.00	0.68	0.00
time (sec)	N/A	0.367	8.922	2.280	0.000	0.079	0.000	0.000	0.180	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	140	138	228	0	175	0	0	40	0
N.S.	1	1.16	1.14	1.88	0.00	1.45	0.00	0.00	0.33	0.00
time (sec)	N/A	0.503	9.328	2.311	0.000	0.074	0.000	0.000	0.188	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	130	166	228	0	177	0	0	40	0
N.S.	1	1.07	1.37	1.88	0.00	1.46	0.00	0.00	0.33	0.00
time (sec)	N/A	0.488	9.157	2.375	0.000	0.094	0.000	0.000	0.190	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	141	139	230	0	179	0	0	40	0
N.S.	1	1.11	1.09	1.81	0.00	1.41	0.00	0.00	0.31	0.00
time (sec)	N/A	0.565	9.211	2.676	0.000	0.077	0.000	0.000	0.193	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	76	97	139	0	97	0	0	40	0
N.S.	1	1.01	1.29	1.85	0.00	1.29	0.00	0.00	0.53	0.00
time (sec)	N/A	0.373	9.083	2.768	0.000	0.074	0.000	0.000	0.178	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	134	126	228	0	143	0	0	40	0
N.S.	1	1.19	1.12	2.02	0.00	1.27	0.00	0.00	0.35	0.00
time (sec)	N/A	0.570	9.410	2.227	0.000	0.077	0.000	0.000	0.190	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	136	168	228	0	179	0	0	40	0
N.S.	1	1.09	1.34	1.82	0.00	1.43	0.00	0.00	0.32	0.00
time (sec)	N/A	0.513	9.421	2.859	0.000	0.078	0.000	0.000	0.180	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	58	80	119	0	74	0	0	251	0
N.S.	1	1.02	1.40	2.09	0.00	1.30	0.00	0.00	4.40	0.00
time (sec)	N/A	0.358	8.210	1.937	0.000	0.079	0.000	0.000	0.193	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	148	189	220	0	171	0	0	44	0
N.S.	1	1.09	1.39	1.62	0.00	1.26	0.00	0.00	0.32	0.00
time (sec)	N/A	0.574	9.828	2.917	0.000	0.074	0.000	0.000	0.214	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	229	189	279	0	181	0	0	27	0
N.S.	1	1.07	0.88	1.30	0.00	0.84	0.00	0.00	0.13	0.00
time (sec)	N/A	0.638	0.354	0.357	0.000	0.079	0.000	0.000	200.015	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	372	180	228	0	173	0	0	40	0
N.S.	1	1.60	0.77	0.98	0.00	0.74	0.00	0.00	0.17	0.00
time (sec)	N/A	0.769	9.405	2.511	0.000	0.073	0.000	0.000	0.190	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	353	164	230	0	175	0	0	40	0
N.S.	1	1.57	0.73	1.02	0.00	0.78	0.00	0.00	0.18	0.00
time (sec)	N/A	0.797	6.851	2.316	0.000	0.081	0.000	0.000	0.181	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	223	97	147	0	97	0	0	40	0
N.S.	1	1.52	0.66	1.00	0.00	0.66	0.00	0.00	0.27	0.00
time (sec)	N/A	0.550	6.741	2.237	0.000	0.071	0.000	0.000	0.171	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	358	168	230	0	159	0	0	40	0
N.S.	1	1.50	0.70	0.96	0.00	0.67	0.00	0.00	0.17	0.00
time (sec)	N/A	0.775	6.866	1.758	0.000	0.076	0.000	0.000	0.188	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	368	172	228	0	173	0	0	40	0
N.S.	1	1.61	0.75	1.00	0.00	0.76	0.00	0.00	0.17	0.00
time (sec)	N/A	0.776	7.061	2.266	0.000	0.081	0.000	0.000	0.187	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	358	172	230	0	177	0	0	40	0
N.S.	1	1.52	0.73	0.98	0.00	0.75	0.00	0.00	0.17	0.00
time (sec)	N/A	0.776	6.733	1.780	0.000	0.073	0.000	0.000	0.183	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	211	98	122	0	90	0	0	38	0
N.S.	1	1.50	0.70	0.87	0.00	0.64	0.00	0.00	0.27	0.00
time (sec)	N/A	0.550	6.738	1.770	0.000	0.082	0.000	0.000	0.169	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	60	132	47	40	0	52	32	0	25	34
N.S.	1	2.20	0.78	0.67	0.00	0.87	0.53	0.00	0.42	0.57
time (sec)	N/A	0.351	4.687	0.708	0.000	0.072	0.712	0.000	0.164	17.597

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	218	105	147	0	82	0	0	40	0
N.S.	1	1.61	0.78	1.09	0.00	0.61	0.00	0.00	0.30	0.00
time (sec)	N/A	0.596	6.809	2.215	0.000	0.075	0.000	0.000	0.173	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	373	168	230	0	176	0	0	40	0
N.S.	1	1.59	0.71	0.98	0.00	0.75	0.00	0.00	0.17	0.00
time (sec)	N/A	0.778	6.637	2.219	0.000	0.089	0.000	0.000	0.178	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	381	168	228	0	171	0	0	40	0
N.S.	1	1.66	0.73	1.00	0.00	0.75	0.00	0.00	0.17	0.00
time (sec)	N/A	0.806	6.801	2.683	0.000	0.085	0.000	0.000	0.187	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	373	168	230	0	155	0	0	40	0
N.S.	1	1.59	0.71	0.98	0.00	0.66	0.00	0.00	0.17	0.00
time (sec)	N/A	0.790	6.645	1.973	0.000	0.076	0.000	0.000	0.179	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	218	123	147	0	100	0	0	40	0
N.S.	1	1.64	0.92	1.11	0.00	0.75	0.00	0.00	0.30	0.00
time (sec)	N/A	0.551	6.772	2.687	0.000	0.069	0.000	0.000	0.174	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	366	160	230	0	178	0	0	40	0
N.S.	1	1.63	0.71	1.02	0.00	0.79	0.00	0.00	0.18	0.00
time (sec)	N/A	0.841	7.118	2.752	0.000	0.074	0.000	0.000	0.189	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	383	174	228	0	173	0	0	40	0
N.S.	1	1.67	0.76	1.00	0.00	0.76	0.00	0.00	0.17	0.00
time (sec)	N/A	0.775	6.940	2.845	0.000	0.071	0.000	0.000	0.181	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	372	168	228	0	179	0	0	40	0
N.S.	1	1.56	0.70	0.95	0.00	0.75	0.00	0.00	0.17	0.00
time (sec)	N/A	0.754	7.472	2.474	0.000	0.075	0.000	0.000	0.183	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	373	158	228	0	148	0	0	40	0
N.S.	1	1.66	0.70	1.01	0.00	0.66	0.00	0.00	0.18	0.00
time (sec)	N/A	0.746	6.595	1.788	0.000	0.077	0.000	0.000	0.180	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	223	97	147	0	97	0	0	40	0
N.S.	1	1.52	0.66	1.00	0.00	0.66	0.00	0.00	0.27	0.00
time (sec)	N/A	0.543	6.514	2.293	0.000	0.084	0.000	0.000	0.172	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	371	172	230	0	179	0	0	40	0
N.S.	1	1.58	0.73	0.98	0.00	0.76	0.00	0.00	0.17	0.00
time (sec)	N/A	0.742	6.920	2.263	0.000	0.075	0.000	0.000	0.186	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	365	166	228	0	177	0	0	40	0
N.S.	1	1.55	0.71	0.97	0.00	0.75	0.00	0.00	0.17	0.00
time (sec)	N/A	0.762	7.006	1.953	0.000	0.077	0.000	0.000	0.180	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	368	170	228	0	175	0	0	40	0
N.S.	1	1.58	0.73	0.98	0.00	0.75	0.00	0.00	0.17	0.00
time (sec)	N/A	0.745	6.990	1.808	0.000	0.084	0.000	0.000	0.186	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	210	113	141	0	74	0	0	38	0
N.S.	1	1.63	0.88	1.09	0.00	0.57	0.00	0.00	0.29	0.00
time (sec)	N/A	0.535	6.471	1.840	0.000	0.070	0.000	0.000	0.170	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	60	132	47	40	0	46	32	0	25	34
N.S.	1	2.20	0.78	0.67	0.00	0.77	0.53	0.00	0.42	0.57
time (sec)	N/A	0.355	4.691	0.711	0.000	0.074	0.410	0.000	0.166	17.709

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	220	101	128	0	98	0	0	40	0
N.S.	1	1.50	0.69	0.87	0.00	0.67	0.00	0.00	0.27	0.00
time (sec)	N/A	0.563	6.643	2.244	0.000	0.084	0.000	0.000	0.177	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	383	166	228	0	175	0	0	40	0
N.S.	1	1.64	0.71	0.98	0.00	0.75	0.00	0.00	0.17	0.00
time (sec)	N/A	0.759	6.840	2.257	0.000	0.078	0.000	0.000	0.179	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	380	162	228	0	176	0	0	40	0
N.S.	1	1.62	0.69	0.97	0.00	0.75	0.00	0.00	0.17	0.00
time (sec)	N/A	0.771	7.035	2.304	0.000	0.089	0.000	0.000	0.194	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	384	172	230	0	177	0	0	40	0
N.S.	1	1.66	0.74	1.00	0.00	0.77	0.00	0.00	0.17	0.00
time (sec)	N/A	0.833	6.760	2.630	0.000	0.081	0.000	0.000	0.194	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	218	123	147	0	100	0	0	40	0
N.S.	1	1.64	0.92	1.11	0.00	0.75	0.00	0.00	0.30	0.00
time (sec)	N/A	0.542	7.035	2.769	0.000	0.082	0.000	0.000	0.176	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	388	154	228	0	145	0	0	40	0
N.S.	1	1.71	0.68	1.00	0.00	0.64	0.00	0.00	0.18	0.00
time (sec)	N/A	0.788	6.906	2.001	0.000	0.078	0.000	0.000	0.186	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	383	168	228	0	178	0	0	40	0
N.S.	1	1.63	0.71	0.97	0.00	0.76	0.00	0.00	0.17	0.00
time (sec)	N/A	0.741	6.793	2.812	0.000	0.090	0.000	0.000	0.175	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	286	186	224	0	174	0	0	40	0
N.S.	1	1.22	0.79	0.96	0.00	0.74	0.00	0.00	0.17	0.00
time (sec)	N/A	0.666	6.285	2.602	0.000	0.078	0.000	0.000	0.185	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	173	123	125	0	88	0	0	40	0
N.S.	1	1.44	1.02	1.04	0.00	0.73	0.00	0.00	0.33	0.00
time (sec)	N/A	0.510	5.771	2.652	0.000	0.092	0.000	0.000	0.170	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	263	328	237	0	148	0	0	40	0
N.S.	1	1.01	1.26	0.91	0.00	0.57	0.00	0.00	0.15	0.00
time (sec)	N/A	0.641	5.992	1.609	0.000	0.078	0.000	0.000	0.188	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	269	344	237	0	161	0	0	40	0
N.S.	1	1.03	1.32	0.91	0.00	0.62	0.00	0.00	0.15	0.00
time (sec)	N/A	0.631	6.244	2.265	0.000	0.078	0.000	0.000	0.192	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	267	328	237	0	166	0	0	40	0
N.S.	1	1.01	1.24	0.90	0.00	0.63	0.00	0.00	0.15	0.00
time (sec)	N/A	0.604	6.001	1.831	0.000	0.075	0.000	0.000	0.188	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	255	326	231	0	151	0	0	38	0
N.S.	1	1.00	1.28	0.91	0.00	0.59	0.00	0.00	0.15	0.00
time (sec)	N/A	0.604	6.102	1.847	0.000	0.079	0.000	0.000	0.190	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	44	18	0	52	36	0	25	16
N.S.	1	1.00	0.49	0.20	0.00	0.58	0.40	0.00	0.28	0.18
time (sec)	N/A	0.295	3.963	0.711	0.000	0.082	0.387	0.000	0.169	17.993

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	264	328	237	0	159	0	0	40	0
N.S.	1	1.01	1.25	0.90	0.00	0.61	0.00	0.00	0.15	0.00
time (sec)	N/A	0.591	5.944	2.292	0.000	0.093	0.000	0.000	0.187	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	264	328	237	0	165	0	0	40	0
N.S.	1	1.01	1.25	0.90	0.00	0.63	0.00	0.00	0.15	0.00
time (sec)	N/A	0.617	5.917	2.241	0.000	0.080	0.000	0.000	0.199	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	266	345	237	0	158	0	0	40	0
N.S.	1	1.04	1.34	0.92	0.00	0.61	0.00	0.00	0.16	0.00
time (sec)	N/A	0.619	6.153	2.672	0.000	0.079	0.000	0.000	0.185	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	260	328	237	0	153	0	0	40	0
N.S.	1	1.01	1.27	0.92	0.00	0.59	0.00	0.00	0.16	0.00
time (sec)	N/A	0.589	6.006	2.162	0.000	0.076	0.000	0.000	0.191	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	269	104	118	0	82	0	0	40	0
N.S.	1	2.05	0.79	0.90	0.00	0.63	0.00	0.00	0.31	0.00
time (sec)	N/A	0.608	5.827	2.135	0.000	0.069	0.000	0.000	0.175	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	267	184	224	0	179	0	0	40	0
N.S.	1	1.17	0.80	0.98	0.00	0.78	0.00	0.00	0.17	0.00
time (sec)	N/A	0.641	6.324	2.178	0.000	0.080	0.000	0.000	0.180	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	299	211	222	0	178	0	0	40	0
N.S.	1	1.27	0.89	0.94	0.00	0.75	0.00	0.00	0.17	0.00
time (sec)	N/A	0.715	6.181	2.721	0.000	0.074	0.000	0.000	0.185	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	298	223	224	0	178	0	0	40	0
N.S.	1	1.25	0.93	0.94	0.00	0.74	0.00	0.00	0.17	0.00
time (sec)	N/A	0.740	6.222	2.604	0.000	0.071	0.000	0.000	0.180	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	183	119	138	0	98	0	0	40	0
N.S.	1	1.50	0.98	1.13	0.00	0.80	0.00	0.00	0.33	0.00
time (sec)	N/A	0.497	5.858	2.647	0.000	0.074	0.000	0.000	0.177	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	279	201	222	0	141	0	0	40	0
N.S.	1	1.19	0.86	0.95	0.00	0.60	0.00	0.00	0.17	0.00
time (sec)	N/A	0.657	6.076	1.980	0.000	0.080	0.000	0.000	0.187	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	173	123	140	0	98	0	0	40	0
N.S.	1	1.28	0.91	1.04	0.00	0.73	0.00	0.00	0.30	0.00
time (sec)	N/A	0.478	5.837	2.570	0.000	0.072	0.000	0.000	0.170	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	265	322	237	0	165	0	0	40	0
N.S.	1	1.07	1.30	0.96	0.00	0.67	0.00	0.00	0.16	0.00
time (sec)	N/A	0.634	5.939	2.200	0.000	0.070	0.000	0.000	0.190	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	267	320	237	0	166	0	0	40	0
N.S.	1	1.02	1.23	0.91	0.00	0.64	0.00	0.00	0.15	0.00
time (sec)	N/A	0.593	6.037	1.832	0.000	0.075	0.000	0.000	0.187	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	267	326	235	0	164	0	0	40	0
N.S.	1	1.05	1.28	0.93	0.00	0.65	0.00	0.00	0.16	0.00
time (sec)	N/A	0.588	5.915	1.803	0.000	0.074	0.000	0.000	0.191	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	255	318	231	0	157	0	0	38	0
N.S.	1	1.02	1.28	0.93	0.00	0.63	0.00	0.00	0.15	0.00
time (sec)	N/A	0.595	5.901	1.842	0.000	0.072	0.000	0.000	0.194	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	44	18	0	46	36	0	25	16
N.S.	1	1.00	0.49	0.20	0.00	0.52	0.40	0.00	0.28	0.18
time (sec)	N/A	0.302	3.992	0.701	0.000	0.081	0.391	0.000	0.165	19.744

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	264	320	237	0	165	0	0	40	0
N.S.	1	1.03	1.25	0.92	0.00	0.64	0.00	0.00	0.16	0.00
time (sec)	N/A	0.614	5.988	2.255	0.000	0.074	0.000	0.000	0.191	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	262	326	235	0	164	0	0	40	0
N.S.	1	1.05	1.30	0.94	0.00	0.66	0.00	0.00	0.16	0.00
time (sec)	N/A	0.598	6.045	2.246	0.000	0.077	0.000	0.000	0.185	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	264	320	237	0	163	0	0	40	0
N.S.	1	1.03	1.25	0.92	0.00	0.63	0.00	0.00	0.16	0.00
time (sec)	N/A	0.683	6.146	2.302	0.000	0.070	0.000	0.000	0.196	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	262	323	237	0	166	0	0	40	0
N.S.	1	1.07	1.32	0.97	0.00	0.68	0.00	0.00	0.16	0.00
time (sec)	N/A	0.591	6.173	2.606	0.000	0.084	0.000	0.000	0.192	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	269	105	118	0	83	0	0	40	0
N.S.	1	2.07	0.81	0.91	0.00	0.64	0.00	0.00	0.31	0.00
time (sec)	N/A	0.627	5.825	2.152	0.000	0.074	0.000	0.000	0.170	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	265	174	222	0	141	0	0	40	0
N.S.	1	1.17	0.77	0.98	0.00	0.62	0.00	0.00	0.18	0.00
time (sec)	N/A	0.614	6.127	1.759	0.000	0.074	0.000	0.000	0.181	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	269	101	136	0	97	0	0	40	0
N.S.	1	1.78	0.67	0.90	0.00	0.64	0.00	0.00	0.26	0.00
time (sec)	N/A	0.642	6.023	2.155	0.000	0.071	0.000	0.000	0.169	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	231	229	249	0	171	0	0	598	0
N.S.	1	0.88	0.87	0.95	0.00	0.65	0.00	0.00	2.27	0.00
time (sec)	N/A	0.576	7.937	1.141	0.000	0.081	0.000	0.000	0.415	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	76	101	133	0	97	0	0	40	0
N.S.	1	1.07	1.42	1.87	0.00	1.37	0.00	0.00	0.56	0.00
time (sec)	N/A	0.381	6.280	2.737	0.000	0.074	0.000	0.000	0.175	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	130	176	221	0	157	0	0	40	0
N.S.	1	1.02	1.39	1.74	0.00	1.24	0.00	0.00	0.31	0.00
time (sec)	N/A	0.500	6.503	2.116	0.000	0.073	0.000	0.000	0.179	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	64	104	139	0	83	0	0	40	0
N.S.	1	1.05	1.70	2.28	0.00	1.36	0.00	0.00	0.66	0.00
time (sec)	N/A	0.369	6.359	2.668	0.000	0.082	0.000	0.000	0.169	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	263	322	237	0	179	0	0	40	0
N.S.	1	1.07	1.31	0.96	0.00	0.73	0.00	0.00	0.16	0.00
time (sec)	N/A	0.663	6.538	2.168	0.000	0.067	0.000	0.000	0.185	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	264	320	237	0	175	0	0	40	0
N.S.	1	1.03	1.25	0.92	0.00	0.68	0.00	0.00	0.16	0.00
time (sec)	N/A	0.613	6.574	1.784	0.000	0.072	0.000	0.000	0.183	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	263	326	235	0	164	0	0	40	0
N.S.	1	1.05	1.30	0.94	0.00	0.66	0.00	0.00	0.16	0.00
time (sec)	N/A	0.586	6.461	1.754	0.000	0.072	0.000	0.000	0.179	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	256	318	231	0	163	0	0	38	0
N.S.	1	1.03	1.28	0.93	0.00	0.65	0.00	0.00	0.15	0.00
time (sec)	N/A	0.607	6.384	1.783	0.000	0.074	0.000	0.000	0.189	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	47	19	0	46	37	0	25	34
N.S.	1	1.00	0.53	0.21	0.00	0.52	0.42	0.00	0.28	0.38
time (sec)	N/A	0.296	4.130	0.667	0.000	0.071	0.420	0.000	0.156	0.089

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	263	320	237	0	159	0	0	40	0
N.S.	1	1.02	1.25	0.92	0.00	0.62	0.00	0.00	0.16	0.00
time (sec)	N/A	0.592	6.346	2.190	0.000	0.067	0.000	0.000	0.191	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	266	326	235	0	164	0	0	40	0
N.S.	1	1.05	1.28	0.93	0.00	0.65	0.00	0.00	0.16	0.00
time (sec)	N/A	0.608	6.370	2.160	0.000	0.066	0.000	0.000	0.179	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	267	320	237	0	176	0	0	40	0
N.S.	1	1.02	1.23	0.91	0.00	0.67	0.00	0.00	0.15	0.00
time (sec)	N/A	0.605	6.521	2.208	0.000	0.066	0.000	0.000	0.179	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	264	323	237	0	175	0	0	40	0
N.S.	1	1.07	1.31	0.96	0.00	0.71	0.00	0.00	0.16	0.00
time (sec)	N/A	0.610	6.497	2.546	0.000	0.073	0.000	0.000	0.177	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	174	123	125	0	98	0	0	40	0
N.S.	1	1.67	1.18	1.20	0.00	0.94	0.00	0.00	0.38	0.00
time (sec)	N/A	0.486	6.236	2.180	0.000	0.073	0.000	0.000	0.161	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	304	201	221	0	149	0	0	40	0
N.S.	1	1.52	1.00	1.10	0.00	0.74	0.00	0.00	0.20	0.00
time (sec)	N/A	0.737	6.563	1.505	0.000	0.072	0.000	0.000	0.186	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	166	119	144	0	98	0	0	40	0
N.S.	1	1.77	1.27	1.53	0.00	1.04	0.00	0.00	0.43	0.00
time (sec)	N/A	0.511	6.326	2.186	0.000	0.073	0.000	0.000	0.165	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	76	101	133	0	97	0	0	40	0
N.S.	1	1.07	1.42	1.87	0.00	1.37	0.00	0.00	0.56	0.00
time (sec)	N/A	0.374	6.278	2.652	0.000	0.082	0.000	0.000	0.167	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	130	185	223	0	179	0	0	40	0
N.S.	1	1.03	1.47	1.77	0.00	1.42	0.00	0.00	0.32	0.00
time (sec)	N/A	0.514	6.639	2.533	0.000	0.091	0.000	0.000	0.181	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	50	105	118	0	82	0	0	40	0
N.S.	1	1.16	2.44	2.74	0.00	1.91	0.00	0.00	0.93	0.00
time (sec)	N/A	0.357	6.454	2.630	0.000	0.081	0.000	0.000	0.165	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	260	328	237	0	144	0	0	40	0
N.S.	1	1.01	1.27	0.92	0.00	0.56	0.00	0.00	0.16	0.00
time (sec)	N/A	0.638	6.591	1.716	0.000	0.085	0.000	0.000	0.185	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	267	344	237	0	158	0	0	40	0
N.S.	1	1.04	1.34	0.92	0.00	0.61	0.00	0.00	0.16	0.00
time (sec)	N/A	0.619	6.772	2.208	0.000	0.088	0.000	0.000	0.182	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	264	328	237	0	165	0	0	40	0
N.S.	1	1.01	1.25	0.90	0.00	0.63	0.00	0.00	0.15	0.00
time (sec)	N/A	0.615	6.445	1.742	0.000	0.076	0.000	0.000	0.186	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	256	326	231	0	157	0	0	38	0
N.S.	1	1.01	1.28	0.91	0.00	0.62	0.00	0.00	0.15	0.00
time (sec)	N/A	0.610	6.761	1.769	0.000	0.079	0.000	0.000	0.186	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	47	19	0	52	37	0	25	34
N.S.	1	1.00	0.53	0.21	0.00	0.58	0.42	0.00	0.28	0.38
time (sec)	N/A	0.298	4.426	0.678	0.000	0.077	0.402	0.000	0.158	17.673

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	263	328	237	0	153	0	0	40	0
N.S.	1	1.00	1.25	0.90	0.00	0.58	0.00	0.00	0.15	0.00
time (sec)	N/A	0.613	6.597	2.174	0.000	0.073	0.000	0.000	0.183	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	267	328	237	0	166	0	0	40	0
N.S.	1	1.01	1.24	0.90	0.00	0.63	0.00	0.00	0.15	0.00
time (sec)	N/A	0.610	6.679	2.151	0.000	0.075	0.000	0.000	0.184	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	268	345	237	0	161	0	0	40	0
N.S.	1	1.03	1.32	0.91	0.00	0.62	0.00	0.00	0.15	0.00
time (sec)	N/A	0.628	7.049	2.611	0.000	0.075	0.000	0.000	0.182	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	263	328	237	0	144	0	0	40	0
N.S.	1	1.01	1.26	0.91	0.00	0.55	0.00	0.00	0.15	0.00
time (sec)	N/A	0.636	6.905	1.923	0.000	0.071	0.000	0.000	0.191	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	168	123	140	0	87	0	0	40	0
N.S.	1	1.83	1.34	1.52	0.00	0.95	0.00	0.00	0.43	0.00
time (sec)	N/A	0.488	6.459	2.159	0.000	0.082	0.000	0.000	0.165	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	303	188	223	0	174	0	0	40	0
N.S.	1	1.53	0.95	1.13	0.00	0.88	0.00	0.00	0.20	0.00
time (sec)	N/A	0.727	6.683	2.095	0.000	0.068	0.000	0.000	0.172	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	166	119	144	0	98	0	0	40	0
N.S.	1	1.77	1.27	1.53	0.00	1.04	0.00	0.00	0.43	0.00
time (sec)	N/A	0.507	6.257	2.154	0.000	0.064	0.000	0.000	0.168	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	125	185	221	0	159	0	0	38	0
N.S.	1	1.06	1.57	1.87	0.00	1.35	0.00	0.00	0.32	0.00
time (sec)	N/A	0.522	5.046	1.148	0.000	0.079	0.000	0.000	0.176	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	168	123	140	0	87	0	0	40	0
N.S.	1	1.83	1.34	1.52	0.00	0.95	0.00	0.00	0.43	0.00
time (sec)	N/A	0.489	0.036	2.076	0.000	0.074	0.000	0.000	0.171	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	135	118	112	0	89	0	0	41	0
N.S.	1	1.41	1.23	1.17	0.00	0.93	0.00	0.00	0.43	0.00
time (sec)	N/A	0.394	2.633	5.987	0.000	0.071	0.000	0.000	0.236	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	168	123	141	0	87	0	0	40	0
N.S.	1	1.75	1.28	1.47	0.00	0.91	0.00	0.00	0.42	0.00
time (sec)	N/A	0.520	0.006	0.590	0.000	0.074	0.000	0.000	0.162	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	269	342	237	0	160	0	0	40	0
N.S.	1	1.03	1.31	0.91	0.00	0.61	0.00	0.00	0.15	0.00
time (sec)	N/A	0.675	6.473	2.359	0.000	0.075	0.000	0.000	0.184	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	262	332	237	0	162	0	0	40	0
N.S.	1	1.02	1.29	0.92	0.00	0.63	0.00	0.00	0.16	0.00
time (sec)	N/A	0.633	5.462	1.932	0.000	0.083	0.000	0.000	0.171	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	173	123	125	0	88	0	0	40	0
N.S.	1	1.44	1.02	1.04	0.00	0.73	0.00	0.00	0.33	0.00
time (sec)	N/A	0.489	0.014	2.545	0.000	0.079	0.000	0.000	0.172	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	172	109	132	0	98	0	0	40	0
N.S.	1	1.41	0.89	1.08	0.00	0.80	0.00	0.00	0.33	0.00
time (sec)	N/A	0.508	4.673	1.782	0.000	0.079	0.000	0.000	0.160	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	280	219	206	0	160	0	0	36	0
N.S.	1	1.20	0.94	0.88	0.00	0.68	0.00	0.00	0.15	0.00
time (sec)	N/A	0.672	5.527	2.178	0.000	0.081	0.000	0.000	0.172	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	143	176	218	0	162	0	0	38	0
N.S.	1	1.09	1.34	1.66	0.00	1.24	0.00	0.00	0.29	0.00
time (sec)	N/A	0.579	6.461	1.983	0.000	0.079	0.000	0.000	0.181	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	136	168	206	0	172	0	0	40	0
N.S.	1	1.09	1.34	1.65	0.00	1.38	0.00	0.00	0.32	0.00
time (sec)	N/A	0.495	5.120	1.727	0.000	0.076	0.000	0.000	0.172	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	62	123	142	0	88	0	0	40	0
N.S.	1	1.09	2.16	2.49	0.00	1.54	0.00	0.00	0.70	0.00
time (sec)	N/A	0.366	0.026	2.166	0.000	0.076	0.000	0.000	0.166	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	145	176	218	0	172	0	0	40	0
N.S.	1	1.09	1.32	1.64	0.00	1.29	0.00	0.00	0.30	0.00
time (sec)	N/A	0.563	6.423	2.034	0.000	0.075	0.000	0.000	0.185	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	148	140	218	0	172	0	0	40	0
N.S.	1	1.13	1.07	1.66	0.00	1.31	0.00	0.00	0.31	0.00
time (sec)	N/A	0.572	6.972	2.357	0.000	0.077	0.000	0.000	0.178	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	150	164	218	0	172	0	0	38	0
N.S.	1	1.13	1.23	1.64	0.00	1.29	0.00	0.00	0.29	0.00
time (sec)	N/A	0.535	6.660	2.376	0.000	0.090	0.000	0.000	0.180	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	64	99	121	0	83	0	0	40	0
N.S.	1	1.08	1.68	2.05	0.00	1.41	0.00	0.00	0.68	0.00
time (sec)	N/A	0.375	6.553	2.307	0.000	0.073	0.000	0.000	0.165	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	136	156	206	0	172	0	0	40	0
N.S.	1	1.09	1.25	1.65	0.00	1.38	0.00	0.00	0.32	0.00
time (sec)	N/A	0.504	5.138	1.782	0.000	0.074	0.000	0.000	0.169	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	145	164	218	0	172	0	0	40	0
N.S.	1	1.09	1.23	1.64	0.00	1.29	0.00	0.00	0.30	0.00
time (sec)	N/A	0.538	6.495	2.104	0.000	0.077	0.000	0.000	0.182	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	348	456	481	0	452	0	0	57	0
N.S.	1	0.99	1.29	1.36	0.00	1.28	0.00	0.00	0.16	0.00
time (sec)	N/A	0.769	10.628	0.549	0.000	0.089	0.000	0.000	0.211	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	353	469	482	0	456	0	0	58	0
N.S.	1	0.99	1.31	1.35	0.00	1.27	0.00	0.00	0.16	0.00
time (sec)	N/A	0.757	10.543	0.580	0.000	0.086	0.000	0.000	0.216	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	360	470	479	0	462	0	0	60	0
N.S.	1	0.98	1.28	1.31	0.00	1.26	0.00	0.00	0.16	0.00
time (sec)	N/A	0.760	10.519	0.554	0.000	0.082	0.000	0.000	0.215	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	363	464	479	0	460	0	0	59	0
N.S.	1	0.98	1.25	1.29	0.00	1.24	0.00	0.00	0.16	0.00
time (sec)	N/A	0.722	10.647	0.539	0.000	0.094	0.000	0.000	0.270	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	415	509	593	0	755	0	0	79	0
N.S.	1	1.02	1.26	1.46	0.00	1.86	0.00	0.00	0.20	0.00
time (sec)	N/A	0.960	11.068	0.567	0.000	0.091	0.000	0.000	0.289	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	420	514	594	0	759	0	0	80	0
N.S.	1	1.02	1.25	1.45	0.00	1.85	0.00	0.00	0.20	0.00
time (sec)	N/A	0.927	10.942	0.572	0.000	0.095	0.000	0.000	0.289	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	373	468	474	0	470	0	0	58	0
N.S.	1	0.93	1.17	1.18	0.00	1.18	0.00	0.00	0.14	0.00
time (sec)	N/A	1.104	10.542	0.557	0.000	0.084	0.000	0.000	0.216	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	662	462	473	0	466	0	0	59	0
N.S.	1	1.63	1.14	1.16	0.00	1.14	0.00	0.00	0.14	0.00
time (sec)	N/A	1.458	10.671	0.559	0.000	0.090	0.000	0.000	0.216	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	664	458	464	0	474	0	0	59	0
N.S.	1	1.63	1.13	1.14	0.00	1.16	0.00	0.00	0.14	0.00
time (sec)	N/A	1.349	10.663	0.536	0.000	0.086	0.000	0.000	0.211	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	379	471	466	0	480	0	0	60	0
N.S.	1	0.92	1.15	1.14	0.00	1.17	0.00	0.00	0.15	0.00
time (sec)	N/A	0.907	10.527	0.537	0.000	0.096	0.000	0.000	0.215	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	81	85	77	87	83	87	72
N.S.	1	1.00	1.00	1.00	1.05	0.95	1.07	1.02	1.07	0.89
time (sec)	N/A	0.407	0.010	0.042	0.030	0.079	0.023	0.107	0.152	18.213

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	45	41	48	43	46	42
N.S.	1	1.00	1.00	0.86	0.92	0.84	0.98	0.88	0.94	0.86
time (sec)	N/A	0.328	0.004	0.036	0.025	0.060	0.019	0.113	0.148	0.023

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	19	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.95	0.80
time (sec)	N/A	0.259	0.000	0.029	0.034	0.066	0.015	0.108	0.148	0.027

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.208	0.000	0.009	0.041	0.048	0.014	0.107	0.148	0.002

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	129	38	0	613	87	1026	351	763
N.S.	1	1.00	0.86	0.25	0.00	4.09	0.58	6.84	2.34	5.09
time (sec)	N/A	0.533	0.074	0.095	0.000	0.094	0.557	0.382	0.171	18.379

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	236	243	151	0	2309	394	2682	2409	6404
N.S.	1	0.94	0.97	0.60	0.00	9.24	1.58	10.73	9.64	25.62
time (sec)	N/A	0.697	0.300	0.220	0.000	0.172	103.570	0.373	0.313	19.663

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	361	372	331	0	4323	0	2707	6794	10979
N.S.	1	1.02	1.05	0.93	0.00	12.18	0.00	7.63	19.14	30.93
time (sec)	N/A	1.023	0.688	0.309	0.000	0.560	0.000	0.890	3.740	21.791

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	323	81	37	0	583	63	75	569	872
N.S.	1	1.42	0.36	0.16	0.00	2.57	0.28	0.33	2.51	3.84
time (sec)	N/A	1.029	0.033	0.174	0.000	0.097	0.385	0.115	0.171	17.429

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	32	0	269	257	31	70	85
N.S.	1	1.00	0.91	0.68	0.00	5.72	5.47	0.66	1.49	1.81
time (sec)	N/A	0.292	0.020	0.076	0.000	0.111	0.318	0.114	0.243	0.110

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	323	52	37	0	407	48	0	563	469
N.S.	1	1.42	0.23	0.16	0.00	1.79	0.21	0.00	2.48	2.07
time (sec)	N/A	0.922	0.025	0.187	0.000	0.086	0.298	0.000	0.223	17.380

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	26	25	25	26	29	25	11
N.S.	1	1.00	2.18	1.53	1.47	1.47	1.53	1.71	1.47	0.65
time (sec)	N/A	0.245	0.004	0.062	0.031	0.083	0.076	0.135	0.223	0.038

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	18	17	17	20	17	16	17
N.S.	1	1.00	1.00	0.75	0.71	0.71	0.83	0.71	0.67	0.71
time (sec)	N/A	0.245	0.010	0.082	0.111	0.080	0.061	0.104	0.219	0.052

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	71	91	54	53	53	70	53	51	83
N.S.	1	1.25	1.60	0.95	0.93	0.93	1.23	0.93	0.89	1.46
time (sec)	N/A	0.404	0.052	0.075	0.109	0.083	0.091	0.125	0.223	0.101

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	202	41	31	0	147	899	143	231	210
N.S.	1	1.47	0.30	0.23	0.00	1.07	6.56	1.04	1.69	1.53
time (sec)	N/A	0.720	0.028	0.173	0.000	0.075	0.531	0.211	0.218	17.255

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	359	324	124	0	8845	0	0	66	25137
N.S.	1	1.33	1.20	0.46	0.00	32.76	0.00	0.00	0.24	93.10
time (sec)	N/A	1.555	0.350	0.270	0.000	0.352	0.000	0.000	200.040	23.569

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	119	122	146	0	66	0	0	152	0
N.S.	1	1.10	1.13	1.35	0.00	0.61	0.00	0.00	1.41	0.00
time (sec)	N/A	0.484	6.910	2.582	0.000	0.099	0.000	0.000	0.259	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	112	136	0	57	0	0	116	0
N.S.	1	1.08	1.40	1.70	0.00	0.71	0.00	0.00	1.45	0.00
time (sec)	N/A	0.402	5.646	2.450	0.000	0.095	0.000	0.000	0.247	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	53	59	114	0	44	0	0	80	0
N.S.	1	1.10	1.23	2.38	0.00	0.92	0.00	0.00	1.67	0.00
time (sec)	N/A	0.341	4.216	1.972	0.000	0.088	0.000	0.000	0.229	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	43	0	9	0	0	30	0
N.S.	1	1.00	1.50	3.58	0.00	0.75	0.00	0.00	2.50	0.00
time (sec)	N/A	0.240	0.006	0.494	0.000	0.102	0.000	0.000	0.241	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	58	80	119	0	74	0	0	251	0
N.S.	1	1.02	1.40	2.09	0.00	1.30	0.00	0.00	4.40	0.00
time (sec)	N/A	0.349	0.028	1.889	0.000	0.104	0.000	0.000	0.274	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	91	107	143	0	117	0	0	50	0
N.S.	1	1.07	1.26	1.68	0.00	1.38	0.00	0.00	0.59	0.00
time (sec)	N/A	0.409	10.071	2.375	0.000	0.093	0.000	0.000	0.229	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	124	117	153	0	157	0	0	58	0
N.S.	1	1.10	1.04	1.35	0.00	1.39	0.00	0.00	0.51	0.00
time (sec)	N/A	0.498	10.097	2.379	0.000	0.113	0.000	0.000	0.179	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	53	59	114	0	44	0	0	80	0
N.S.	1	1.02	1.13	2.19	0.00	0.85	0.00	0.00	1.54	0.00
time (sec)	N/A	0.373	0.028	1.571	0.000	0.101	0.000	0.000	0.174	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	43	0	9	0	0	30	0
N.S.	1	1.00	1.50	3.58	0.00	0.75	0.00	0.00	2.50	0.00
time (sec)	N/A	0.262	0.006	0.500	0.000	0.105	0.000	0.000	0.167	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [324] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	0.84	22	0.182
2	A	3	3	0.92	22	0.136
3	A	2	2	1.00	22	0.091
4	A	2	2	1.00	22	0.091
5	A	3	3	1.16	22	0.136
6	A	4	4	0.98	22	0.182
7	A	5	5	0.94	22	0.227
8	A	2	2	1.96	19	0.105
9	A	3	3	1.00	20	0.150
10	A	3	3	1.00	20	0.150
11	B	2	2	2.12	21	0.095
12	A	7	7	1.40	25	0.280
13	A	6	6	1.60	25	0.240
14	A	5	5	1.82	25	0.200
15	A	1	1	1.96	25	0.040
16	B	6	6	2.14	25	0.240
17	A	8	8	1.92	25	0.320
18	A	10	10	1.73	25	0.400
19	A	9	9	0.94	27	0.333
20	A	7	7	0.88	27	0.259
21	A	5	5	0.77	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	27	0.074
23	A	7	7	0.86	27	0.259
24	A	9	9	0.94	27	0.333
25	A	11	11	0.98	27	0.407
26	A	1	1	1.08	16	0.062
27	A	1	1	1.05	16	0.062
28	A	1	1	1.28	16	0.062
29	A	1	1	1.03	16	0.062
30	A	1	1	1.08	16	0.062
31	A	1	1	1.00	16	0.062
32	A	1	1	1.00	16	0.062
33	A	1	1	1.00	16	0.062
34	A	1	1	1.00	14	0.071
35	A	1	1	1.00	11	0.091
36	A	1	1	1.00	16	0.062
37	A	1	1	1.00	16	0.062
38	A	1	1	1.00	16	0.062
39	A	1	1	1.00	16	0.062
40	A	1	1	1.84	16	0.062
41	A	1	1	0.87	16	0.062
42	A	1	1	1.64	16	0.062
43	A	1	1	0.90	16	0.062
44	A	1	1	0.92	16	0.062
45	A	1	1	1.09	16	0.062
46	A	1	1	1.05	16	0.062
47	A	1	1	1.28	16	0.062
48	A	1	1	1.03	16	0.062
49	A	1	1	0.93	16	0.062
50	A	1	1	1.00	16	0.062
51	A	1	1	1.00	16	0.062
52	A	1	1	1.00	16	0.062
53	A	1	1	1.00	14	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	1	1	1.00	11	0.091
55	A	1	1	1.00	16	0.062
56	A	1	1	1.00	16	0.062
57	A	1	1	1.00	16	0.062
58	A	1	1	1.00	16	0.062
59	A	1	1	1.84	16	0.062
60	A	1	1	0.94	16	0.062
61	A	1	1	1.64	16	0.062
62	A	1	1	0.84	16	0.062
63	A	1	1	0.92	16	0.062
64	A	1	1	0.70	19	0.053
65	A	3	3	1.00	16	0.188
66	A	3	3	1.00	16	0.188
67	A	3	3	1.00	16	0.188
68	A	1	1	1.00	16	0.062
69	A	1	1	1.00	16	0.062
70	A	1	1	1.00	16	0.062
71	A	1	1	1.00	14	0.071
72	A	1	1	1.00	11	0.091
73	A	1	1	1.00	16	0.062
74	A	1	1	1.00	16	0.062
75	A	1	1	1.00	16	0.062
76	A	1	1	1.00	16	0.062
77	A	3	3	1.32	16	0.188
78	A	3	3	1.48	16	0.188
79	A	3	3	1.55	16	0.188
80	A	3	3	1.00	16	0.188
81	A	3	3	1.00	16	0.188
82	A	3	3	1.00	16	0.188
83	A	1	1	1.00	16	0.062
84	A	1	1	1.00	16	0.062
85	A	1	1	1.00	16	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	1	1	1.00	14	0.071
87	A	1	1	1.00	11	0.091
88	A	1	1	1.00	16	0.062
89	A	1	1	1.00	16	0.062
90	A	1	1	1.00	16	0.062
91	A	1	1	1.00	16	0.062
92	A	3	3	1.53	16	0.188
93	A	3	3	1.42	16	0.188
94	A	3	3	1.55	16	0.188
95	A	2	2	1.00	16	0.125
96	A	3	3	1.00	16	0.188
97	A	3	3	1.09	16	0.188
98	A	2	2	1.00	16	0.125
99	A	3	3	1.00	16	0.188
100	A	3	3	1.00	14	0.214
101	A	1	1	1.00	11	0.091
102	A	3	3	1.00	16	0.188
103	A	3	3	1.00	16	0.188
104	A	2	2	1.00	16	0.125
105	A	3	3	1.09	16	0.188
106	A	3	3	1.00	16	0.188
107	A	2	2	1.00	16	0.125
108	A	3	3	1.10	16	0.188
109	A	3	3	1.00	16	0.188
110	A	3	3	1.00	16	0.188
111	A	2	2	1.00	16	0.125
112	A	3	3	1.00	16	0.188
113	A	3	3	1.00	14	0.214
114	A	1	1	1.00	11	0.091
115	A	3	3	1.00	16	0.188
116	A	3	3	1.00	16	0.188
117	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	3	3	1.00	16	0.188
119	A	3	3	1.00	16	0.188
120	A	3	3	1.11	16	0.188
121	A	2	2	1.00	16	0.125
122	A	3	3	1.00	16	0.188
123	A	1	1	1.24	16	0.062
124	A	1	1	1.41	16	0.062
125	A	1	1	1.49	16	0.062
126	A	1	1	1.44	16	0.062
127	A	1	1	1.17	14	0.071
128	B	1	1	2.88	11	0.091
129	A	1	1	1.30	16	0.062
130	A	1	1	1.51	16	0.062
131	A	1	1	1.56	16	0.062
132	A	1	1	1.51	16	0.062
133	A	1	1	1.29	16	0.062
134	A	1	1	1.48	16	0.062
135	A	1	1	1.57	16	0.062
136	A	1	1	1.24	16	0.062
137	A	1	1	1.41	16	0.062
138	A	1	1	1.49	16	0.062
139	A	1	1	1.46	16	0.062
140	A	1	1	1.31	14	0.071
141	B	1	1	2.80	11	0.091
142	A	1	1	1.16	16	0.062
143	A	1	1	1.53	16	0.062
144	A	1	1	1.56	16	0.062
145	A	1	1	1.50	16	0.062
146	A	1	1	1.29	16	0.062
147	A	1	1	1.65	16	0.062
148	A	1	1	1.56	16	0.062
149	A	1	1	1.00	16	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	1	1	1.00	16	0.062
151	A	1	1	1.08	16	0.062
152	A	1	1	1.23	16	0.062
153	A	1	1	1.08	14	0.071
154	A	2	2	1.00	16	0.125
155	A	2	2	1.00	16	0.125
156	A	3	3	1.00	16	0.188
157	A	2	2	1.00	16	0.125
158	A	2	2	1.00	16	0.125
159	A	2	2	1.00	16	0.125
160	A	3	3	1.00	16	0.188
161	A	2	2	1.00	16	0.125
162	A	2	2	1.00	16	0.125
163	A	1	1	1.00	16	0.062
164	A	1	1	1.00	17	0.059
165	A	1	1	1.00	19	0.053
166	A	1	1	1.00	20	0.050
167	A	1	1	1.00	17	0.059
168	A	1	1	1.00	18	0.056
169	A	2	2	1.30	17	0.118
170	A	2	2	1.46	18	0.111
171	A	2	2	1.46	18	0.111
172	A	2	2	1.30	19	0.105
173	A	7	7	1.09	16	0.438
174	A	7	7	1.18	16	0.438
175	A	6	6	1.13	16	0.375
176	A	7	7	1.17	16	0.438
177	A	7	7	1.09	14	0.500
178	A	2	2	1.00	11	0.182
179	A	7	7	1.01	16	0.438
180	A	7	7	1.15	16	0.438
181	A	6	6	1.12	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	7	7	1.17	16	0.438
183	A	7	7	1.01	16	0.438
184	A	6	6	1.09	16	0.375
185	A	7	7	1.19	16	0.438
186	A	7	7	1.09	16	0.438
187	A	7	7	1.12	16	0.438
188	A	6	6	1.09	16	0.375
189	A	7	7	1.17	16	0.438
190	A	7	7	1.00	14	0.500
191	A	2	2	1.00	11	0.182
192	A	7	7	1.08	16	0.438
193	A	7	7	1.16	16	0.438
194	A	6	6	1.07	16	0.375
195	A	7	7	1.11	16	0.438
196	A	7	7	1.01	16	0.438
197	A	7	7	1.19	16	0.438
198	A	6	6	1.09	16	0.375
199	A	7	7	1.02	16	0.438
200	A	6	6	1.09	16	0.375
201	A	7	7	1.07	33	0.212
202	A	5	5	1.60	16	0.312
203	A	6	6	1.57	16	0.375
204	A	6	6	1.52	16	0.375
205	A	6	6	1.50	16	0.375
206	A	5	5	1.61	16	0.312
207	A	6	6	1.52	16	0.375
208	A	6	6	1.50	14	0.429
209	B	2	2	2.20	11	0.182
210	A	6	6	1.61	16	0.375
211	A	6	6	1.59	16	0.375
212	A	6	6	1.66	16	0.375
213	A	6	6	1.59	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	6	6	1.64	16	0.375
215	A	6	6	1.63	16	0.375
216	A	6	6	1.67	16	0.375
217	A	5	5	1.56	16	0.312
218	A	6	6	1.66	16	0.375
219	A	6	6	1.52	16	0.375
220	A	6	6	1.58	16	0.375
221	A	5	5	1.55	16	0.312
222	A	6	6	1.58	16	0.375
223	A	6	6	1.63	14	0.429
224	B	2	2	2.20	11	0.182
225	A	6	6	1.50	16	0.375
226	A	6	6	1.64	16	0.375
227	A	6	6	1.62	16	0.375
228	A	6	6	1.66	16	0.375
229	A	6	6	1.64	16	0.375
230	A	6	6	1.71	16	0.375
231	A	6	6	1.63	16	0.375
232	A	5	5	1.22	16	0.312
233	A	5	5	1.44	16	0.312
234	A	6	6	1.01	16	0.375
235	A	6	6	1.03	16	0.375
236	A	6	6	1.01	16	0.375
237	A	6	6	1.00	14	0.429
238	A	2	2	1.00	11	0.182
239	A	6	6	1.01	16	0.375
240	A	6	6	1.01	16	0.375
241	A	6	6	1.04	16	0.375
242	A	6	6	1.01	16	0.375
243	B	6	6	2.05	16	0.375
244	A	6	6	1.17	16	0.375
245	A	5	5	1.27	16	0.312

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	5	5	1.25	16	0.312
247	A	5	5	1.50	16	0.312
248	A	5	5	1.19	16	0.312
249	A	5	5	1.28	16	0.312
250	A	6	6	1.07	16	0.375
251	A	6	6	1.02	16	0.375
252	A	6	6	1.05	16	0.375
253	A	6	6	1.02	14	0.429
254	A	2	2	1.00	11	0.182
255	A	6	6	1.03	16	0.375
256	A	6	6	1.05	16	0.375
257	A	6	6	1.03	16	0.375
258	A	6	6	1.07	16	0.375
259	B	6	6	2.07	16	0.375
260	A	6	6	1.17	16	0.375
261	A	6	6	1.78	16	0.375
262	A	4	4	0.88	19	0.211
263	A	7	7	1.07	16	0.438
264	A	7	7	1.02	16	0.438
265	A	7	7	1.05	16	0.438
266	A	6	6	1.07	16	0.375
267	A	6	6	1.03	16	0.375
268	A	6	6	1.05	16	0.375
269	A	6	6	1.03	14	0.429
270	A	2	2	1.00	11	0.182
271	A	6	6	1.02	16	0.375
272	A	6	6	1.05	16	0.375
273	A	6	6	1.02	16	0.375
274	A	6	6	1.07	16	0.375
275	A	8	8	1.67	16	0.500
276	A	8	8	1.52	16	0.500
277	A	8	8	1.77	16	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	7	7	1.07	16	0.438
279	A	7	7	1.03	16	0.438
280	A	7	7	1.16	16	0.438
281	A	6	6	1.01	16	0.375
282	A	6	6	1.04	16	0.375
283	A	6	6	1.01	16	0.375
284	A	6	6	1.01	14	0.429
285	A	2	2	1.00	11	0.182
286	A	6	6	1.00	16	0.375
287	A	6	6	1.01	16	0.375
288	A	6	6	1.03	16	0.375
289	A	6	6	1.01	16	0.375
290	A	8	8	1.83	16	0.500
291	A	8	8	1.53	16	0.500
292	A	8	8	1.77	16	0.500
293	A	5	5	1.06	16	0.312
294	A	8	8	1.83	16	0.500
295	A	5	5	1.41	21	0.238
296	A	9	9	1.75	19	0.474
297	A	6	6	1.03	16	0.375
298	A	5	5	1.02	16	0.312
299	A	5	5	1.44	16	0.312
300	A	5	5	1.41	16	0.312
301	A	4	4	1.20	14	0.286
302	A	6	6	1.09	16	0.375
303	A	6	6	1.09	16	0.375
304	A	7	7	1.09	16	0.438
305	A	6	6	1.09	16	0.375
306	A	6	6	1.13	16	0.375
307	A	6	6	1.13	16	0.375
308	A	7	7	1.08	16	0.438
309	A	6	6	1.09	16	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	6	6	1.09	16	0.375
311	A	6	6	0.99	16	0.375
312	A	6	6	0.99	17	0.353
313	A	6	6	0.98	19	0.316
314	A	6	6	0.98	20	0.300
315	A	5	5	1.02	17	0.294
316	A	5	5	1.02	18	0.278
317	A	7	7	0.93	17	0.412
318	A	8	8	1.63	18	0.444
319	A	8	8	1.63	18	0.444
320	A	7	7	0.92	19	0.368
321	A	2	2	1.00	14	0.143
322	A	2	2	1.00	14	0.143
323	A	1	1	1.00	12	0.083
324	A	1	1	1.00	1	1.000
325	A	2	2	1.00	14	0.143
326	A	4	4	0.94	14	0.286
327	A	6	6	1.02	14	0.429
328	A	8	7	1.42	16	0.438
329	A	3	3	1.00	16	0.188
330	A	8	7	1.42	16	0.438
331	A	2	2	1.00	12	0.167
332	A	2	2	1.00	12	0.167
333	A	7	6	1.25	12	0.500
334	A	7	6	1.47	12	0.500
335	A	2	2	1.33	64	0.031
336	A	11	11	1.10	16	0.688
337	A	9	9	1.08	16	0.562
338	A	7	7	1.10	16	0.438
339	A	3	3	1.00	16	0.188
340	A	7	7	1.02	16	0.438
341	A	9	9	1.07	16	0.562

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	11	11	1.10	16	0.688
343	A	8	8	1.02	17	0.471
344	A	4	4	1.00	17	0.235

CHAPTER 3

Listing of integrals

3.1 $\int (a^2+2abx^2+b^2x^4)^{3/4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/4}\,dx=\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{3ax(a^2+2abx^2+b^2x^4)^{3/4}}{8\left(a+bx^2\right)}+\frac{3\sqrt{a}(a^2+2abx^2+b^2x^4)^{3/4}\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

output
```
1/4*x*(b^2*x^4+2*a*b*x^2+a^2)^(3/4)+3*a*x*(b^2*x^4+2*a*b*x^2+a^2)^(3/4)/(8
*b*x^2+8*a)+3/8*a^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/4)*arcsinh(b^(1/2)*x/a^
(1/2))/b^(1/2)/(1+b*x^2/a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/4}\,dx=\frac{\left(\left(a+bx^2\right)^2\right)^{3/4}\left(\sqrt{b}x\sqrt{a+bx^2}(5a+2bx^2)-3a^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)\right)}{8\sqrt{b}\left(a+bx^2\right)^{3/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4),x]
```

output
```
(((a + b*x^2)^2)^(3/4)*(Sqrt[b]*x*Sqrt[a + b*x^2]*(5*a + 2*b*x^2) - 3*a^2*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]))/(8*Sqrt[b]*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1385, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/4}dx$$

$$\Big\downarrow \text{1385}$$

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/4}\int \left(\frac{bx^2}{a}+1\right)^{3/2}dx}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \text{211}$$

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/4}\left(\frac{3}{4}\int \sqrt{\frac{bx^2}{a}+1}dx+\frac{1}{4}x\left(\frac{bx^2}{a}+1\right)^{3/2}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \text{211}$$

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/4}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}}dx+\frac{1}{2}x\sqrt{\frac{bx^2}{a}+1}\right)+\frac{1}{4}x\left(\frac{bx^2}{a}+1\right)^{3/2}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{(a^2+2abx^2+b^2x^4)^{3/4}\left(\frac{3}{4}\left(\frac{\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{\frac{bx^2}{a}+1}\right)+\frac{1}{4}x\left(\frac{bx^2}{a}+1\right)^{3/2}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4),x]
```

output
```
((a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)*((x*(1 + (b*x^2)/a)^(3/2))/4 + (3*((x*S
qrt[1 + (b*x^2)/a])/2 + (Sqrt[a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[b])
))/4))/(1 + (b*x^2)/a)^(3/2)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.60

method	result	size
risch	$\frac{x(2b\,x^2+5a)(b\,x^2+a)}{8\left((b\,x^2+a)^2\right)^{\frac{1}{4}}}+\frac{3a^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)\sqrt{b\,x^2+a}}{8\sqrt{b}\left((b\,x^2+a)^2\right)^{\frac{1}{4}}}$	77

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*b*x^2+5*a)*(b*x^2+a)/((b*x^2+a)^2)^(1/4)+3/8*a^2*ln(b^(1/2)*x+(b*
x^2+a)^(1/2))/b^(1/2)/((b*x^2+a)^2)^(1/4)*(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.38

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/4}\,dx = \left[\frac{3\,a^2\sqrt{b}\log\left(-2\,bx^2-2\,(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}\sqrt{b}x-a\right)+2\,(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}(2\,b^2x^3+\cdots}{16\,b}\right., \left.-\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}\sqrt{-b}x}{bx^2+a}\right)-(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}(2\,b^2x^3+5\,abx)}{8\,b}\right]$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="fricas")
```

output
```
[1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sq
rt(b)*x - a) + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b^2*x^3 + 5*a*b*x))/
b, -1/8*(3*a^2*sqrt(-b)*arctan((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*
x/(b*x^2 + a)) - (b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b^2*x^3 + 5*a*b*x))/
b]
```

Sympy [F]

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/4}\,dx=\int \left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4), x)
```

Maxima [F]

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/4}\,dx=\int \left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/4), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/4}\,dx=-\frac{1}{8}\left(2\,bx^2+5\,a\right)\sqrt{-bx^2-a}x-\frac{3\,a^2\log\left(\left|-\sqrt{-b}x+\sqrt{-bx^2-a}\right|\right)}{8\,\sqrt{-b}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="giac")
```

output
```
-1/8*(2*b*x^2 + 5*a)*sqrt(-b*x^2 - a)*x - 3/8*a^2*log(abs(-sqrt(-b)*x + sq
rt(-b*x^2 - a)))/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/4}\,dx=\int \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/4}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4),x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/4}\,dx=\frac{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{3}{4}}x}{4}+\frac{3\left(\int \frac{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{3}{4}}}{b\,x^2+a}dx\right)a}{4}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x)
```

output
```
((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4)*x + 3*int((a**2 + 2*a*b*x**2 + b**
2*x**4)**(3/4)/(a + b*x**2),x)*a)/4
```

3.2 $\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4}+\frac{\sqrt{a}\sqrt[4]{a^2+2abx^2+b^2x^4}\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}\sqrt{1+\frac{bx^2}{a}}}$$

output
```
1/2*x*(b^2*x^4+2*a*b*x^2+a^2)^(1/4)+1/2*a^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^(1
/4)*arcsinh(b^(1/2)*x/a^(1/2))/b^(1/2)/(1+b*x^2/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.65

$$\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{2}\sqrt[4]{(a+bx^2)^2}\left(x-\frac{a\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}\sqrt{a+bx^2}}\right)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4),x]
```

output
```
(((a + b*x^2)^2)^(1/4)*(x - (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(Sqrt[
b]*Sqrt[a + b*x^2])))/2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1385, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx$$

$$\Big\downarrow \ 1385$$

$$\frac{\sqrt[4]{a^2+2abx^2+b^2x^4}\int \sqrt{\frac{bx^2}{a}+1}dx}{\sqrt{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \ 211$$

$$\frac{\sqrt[4]{a^2+2abx^2+b^2x^4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}}dx+\frac{1}{2}x\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \ 222$$

$$\frac{\sqrt[4]{a^2+2abx^2+b^2x^4}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4),x]
```

output
```
((a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)*((x*Sqrt[1 + (b*x^2)/a])/2 + (Sqrt[a]*A
rcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[b])))/Sqrt[1 + (b*x^2)/a]
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{x\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{1}{4}}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{1}{4}}}{2\sqrt{b}\,\sqrt{b\,x^{2}+a}}$	58

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*((b*x^2+a)^2)^(1/4)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*((b*
x^2+a)^2)^(1/4)/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.62

$$\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx = \left[\frac{a\sqrt{b}\log\left(-2\,bx^2-2\,(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}\sqrt{b}x-a\right)+2\,(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}bx}{4\,b}, -\frac{a\sqrt{-b}\arctan\left(\frac{(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}\sqrt{-b}x}{bx^2+a}\right)-(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}bx}{2\,b}\right]$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="fricas")
```

output
```
[1/4*(a*sqrt(b)*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(b)
*x - a) + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*b*x)/b, -1/2*(a*sqrt(-b)*arc
tan((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(b*x^2 + a)) - (b^2*x^4 +
 2*a*b*x^2 + a^2)^(1/4)*b*x)/b]
```

Sympy [F]

$$\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx = \int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4), x)
```

Maxima [F]

$$\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx=\int \left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx=\int \left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx=\int \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{1/4}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4),x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \sqrt[4]{a^2 + 2abx^2 + b^2x^4}\, dx = \frac{(b^2x^4 + 2ab\, x^2 + a^2)^{\frac{1}{4}}\, x}{2} + \frac{\left(\int \frac{(b^2x^4+2ab\, x^2+a^2)^{\frac{1}{4}}}{b\, x^2+a} dx\right) a}{2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

output
```
((a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4)*x + int((a**2 + 2*a*b*x**2 + b**2*
x**4)**(1/4)/(a + b*x**2),x)*a)/2
```

3.3 $\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 60

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx=\frac{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

output
```
a^(1/2)*(1+b*x^2/a)^(1/2)*arcsinh(b^(1/2)*x/a^(1/2))/b^(1/2)/(b^2*x^4+2*a*
b*x^2+a^2)^(1/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx=\frac{\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt[4]{(a+bx^2)^2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/4),x]
```

output
```
(Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*((a + b*x^
2)^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1385, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1385}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\text{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/4),x]
```

output
```
(Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(a^2 +
 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{1}{\left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

output
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx = \left[\frac{\log\left(-2\,bx^2-2\,(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}\sqrt{b}x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}\sqrt{-b}x}{bx^2+a}\right)}{b}\right]$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="fricas")
```

output
```
[1/2*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(b)*x - a)/sqr
t(b), -sqrt(-b)*arctan((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(b*x^2
 + a))/b]
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx$$

input
```
integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{1}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{1/4}}\,dx$$

input
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4),x)
```

output
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{1}{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

output
```
int(1/(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4),x)
```

3.4 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 34

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/4}}\,dx=\frac{x(a+bx^2)}{a\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

output
```
x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(3/4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/4}}\,dx=\frac{x(a+bx^2)}{a\left((a+bx^2)^2\right)^{3/4}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/4),x]
```

output
```
(x*(a + b*x^2))/(a*((a + b*x^2)^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/4}}dx$$

$$\Big\downarrow \text{1385}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/2}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/2}}dx}{(a^2+2abx^2+b^2x^4)^{3/4}}$$

$$\Big\downarrow \text{208}$$

$$\frac{x\left(\frac{bx^2}{a}+1\right)}{(a^2+2abx^2+b^2x^4)^{3/4}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/4),x]
```

output
```
(x*(1 + (b*x^2)/a))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
gosper	$\frac{x(b\,x^2+a)}{a(b^2x^4+2ab\,x^2+a^2)^{\frac{3}{4}}}$	33
orering	$\frac{x(b\,x^2+a)}{a(b^2x^4+2ab\,x^2+a^2)^{\frac{3}{4}}}$	33

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(3/4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/4}}\,dx = \frac{(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}x}{abx^2+a^2}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="fricas")
```

output
```
(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*x/(a*b*x^2 + a^2)
```

Sympy [F]

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/4}}\,dx = \int \frac{1}{(a^2+2abx^2+b^2x^4)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/4}}\,dx=\int \frac{1}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/4}}\,dx=\int \frac{1}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 17.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/4}}\,dx=\frac{x\,(a^2+2\,a\,b\,x^2+b^2\,x^4)^{1/4}}{a\,(b\,x^2+a)}$$

input
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4),x)
```

output
```
(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4))/(a*(a + b*x^2))
```

Reduce [F]

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/4}}\,dx = \int \frac{1}{(b^2x^4+2ab\,x^2+a^2)^{\frac{3}{4}}}dx$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x)
```

output
```
int(1/(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4),x)
```

3.5 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/4}}\,dx = \frac{x(a+bx^2)}{3a\,(a^2+2abx^2+b^2x^4)^{5/4}} + \frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

output
```
1/3*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(5/4)+2/3*x/a^2/(b^2*x^4+2*a*b*x
^2+a^2)^(1/4)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/4}}\,dx = \frac{(a+bx^2)\,(3ax+2bx^3)}{3a^2\left((a+bx^2)^2\right)^{5/4}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/4),x]
```

output
```
((a + b*x^2)*(3*a*x + 2*b*x^3))/(3*a^2*((a + b*x^2)^2)^(5/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1385, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/4}}dx$$

$$\Big\downarrow \text{1385}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/2}}dx}{a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\left(\frac{2}{3}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/2}}dx+\frac{x}{3\left(\frac{bx^2}{a}+1\right)^{3/2}}\right)}{a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\left(\frac{2x}{3\sqrt{\frac{bx^2}{a}+1}}+\frac{x}{3\left(\frac{bx^2}{a}+1\right)^{3/2}}\right)}{a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/4),x]
```

output
```
(Sqrt[1 + (b*x^2)/a]*(x/(3*(1 + (b*x^2)/a)^(3/2)) + (2*x)/(3*Sqrt[1 + (b*x
^2)/a])))/(a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)x\left(2 b\,x^{2}+3 a\right)}{3 a^{2} \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{\frac{5}{4}}}$	44
orering	$\frac{\left(b\,x^{2}+a\right)x\left(2 b\,x^{2}+3 a\right)}{3 a^{2} \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{\frac{5}{4}}}$	44

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^2+a)*x*(2*b*x^2+3*a)/a^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/4}}\,dx = \frac{(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}(2\,bx^3+3\,ax)}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="fricas")
```

output
```
1/3*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b*x^3 + 3*a*x)/(a^2*b^2*x^4 + 2*a
^3*b*x^2 + a^4)
```

Sympy [F]

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/4}}\,dx = \int \frac{1}{(a^2+2abx^2+b^2x^4)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/4),x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-5/4), x)
```

Maxima [F]

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/4}}\,dx = \int \frac{1}{(b^2x^4+2\,abx^2+a^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/4}}\,dx=\int \frac{1}{(b^2x^4+2\,abx^2+a^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 17.54 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/4}}\,dx=\frac{x\,(2\,b\,x^2+3\,a)\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/4}}{3\,a^2\,(b\,x^2+a)^3}$$

input
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/4),x)
```

output
```
(x*(3*a + 2*b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4))/(3*a^2*(a + b*x^2)^3
)
```

Reduce [F]

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/4}}\,dx=\int \frac{1}{(b^2x^4+2ab\,x^2+a^2)^{\frac{1}{4}}\,a^2+2\,(b^2x^4+2ab\,x^2+a^2)^{\frac{1}{4}}\,ab\,x^2+(b^2x^4+2ab\,x$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x)
```

output
```
int(1/((a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4)*a**2 + 2*(a**2 + 2*a*b*x**2
+ b**2*x**4)**(1/4)*a*b*x**2 + (a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4)*b**2
*x**4),x)
```

3.6 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}\,dx=\frac{x(a+bx^2)}{5a\left(a^2+2abx^2+b^2x^4\right)^{7/4}}+\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{8x(a+bx^2)}{15a^3\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

output
```
1/5*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(7/4)+4/15*x/a^2/(b^2*x^4+2*a*b*
x^2+a^2)^(3/4)+8/15*x*(b*x^2+a)/a^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/4)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.47

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}\,dx=\frac{(a+bx^2)\left(15a^2x+20abx^3+8b^2x^5\right)}{15a^3\left((a+bx^2)^2\right)^{7/4}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-7/4),x]
```

output
```
((a + b*x^2)*(15*a^2*x + 20*a*b*x^3 + 8*b^2*x^5))/(15*a^3*((a + b*x^2)^2)^
(7/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1385, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}dx$$

$$\Big\downarrow \text{1385}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/2}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{7/2}}dx}{a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/2}\left(\frac{4}{5}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/2}}dx+\frac{x}{5\left(\frac{bx^2}{a}+1\right)^{5/2}}\right)}{a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/2}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/2}}dx+\frac{x}{3\left(\frac{bx^2}{a}+1\right)^{3/2}}\right)+\frac{x}{5\left(\frac{bx^2}{a}+1\right)^{5/2}}\right)}{a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/2}\left(\frac{x}{5\left(\frac{bx^2}{a}+1\right)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{\frac{bx^2}{a}+1}}+\frac{x}{3\left(\frac{bx^2}{a}+1\right)^{3/2}}\right)\right)}{a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-7/4),x]
```

output
```
((1 + (b*x^2)/a)^(3/2)*(x/(5*(1 + (b*x^2)/a)^(5/2)) + (4*(x/(3*(1 + (b*x^2
)/a)^(3/2)) + (2*x)/(3*Sqrt[1 + (b*x^2)/a])))/5))/(a^2*(a^2 + 2*a*b*x^2 +
b^2*x^4)^(3/4))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)x\left(8b^{2}x^{4}+20ab\,x^{2}+15a^{2}\right)}{15a^{3}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{7}{4}}}$	55
orering	$\frac{\left(b\,x^{2}+a\right)x\left(8b^{2}x^{4}+20ab\,x^{2}+15a^{2}\right)}{15a^{3}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{7}{4}}}$	55

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
1/15*(b*x^2+a)*x*(8*b^2*x^4+20*a*b*x^2+15*a^2)/a^3/(b^2*x^4+2*a*b*x^2+a^2)
^(7/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}\,dx = \frac{(8\,b^2x^5+20\,abx^3+15\,a^2x)(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}}{15\,(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6)}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="fricas")
```

output
```
1/15*(8*b^2*x^5 + 20*a*b*x^3 + 15*a^2*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)
/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}\,dx = \int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(7/4),x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-7/4), x)
```

Maxima [F]

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}\,dx = \int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4), x)
```

Giac [F]

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{7/4}}\,dx = \int \frac{1}{(b^2x^4+2\,abx^2+a^2)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4), x)
```

Mupad [B] (verification not implemented)

Time = 18.60 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.53

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{7/4}}\,dx = \frac{x\,(a^2+2\,a\,b\,x^2+b^2\,x^4)^{1/4}\,(15\,a^2+20\,a\,b\,x^2+8\,b^2\,x^4)}{15\,a^3\,(b\,x^2+a)^3}$$

input
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(7/4),x)
```

output
```
(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4)*(15*a^2 + 8*b^2*x^4 + 20*a*b*x^2))/(1
5*a^3*(a + b*x^2)^3)
```

Reduce [F]

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{7/4}}\,dx = \int \frac{1}{(b^2x^4+2ab\,x^2+a^2)^{\frac{3}{4}}\,a^2+2\,(b^2x^4+2ab\,x^2+a^2)^{\frac{3}{4}}\,ab\,x^2+(b^2x^4+2ab\,x$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x)
```

output
```
int(1/((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4)*a**2 + 2*(a**2 + 2*a*b*x**2
+ b**2*x**4)**(3/4)*a*b*x**2 + (a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4)*b**2
*x**4),x)
```

3.7 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{9/4}}\,dx=\frac{x(a+bx^2)}{7a\left(a^2+2abx^2+b^2x^4\right)^{9/4}}+\frac{6x}{35a^2\left(a^2+2abx^2+b^2x^4\right)^{5/4}}+\frac{8x(a+bx^2)}{35a^3\left(a^2+2abx^2+b^2x^4\right)^{5/4}}+\frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

output
```
1/7*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(9/4)+6/35*x/a^2/(b^2*x^4+2*a*b*
x^2+a^2)^(5/4)+8/35*x*(b*x^2+a)/a^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/4)+16/35*x/
a^4/(b^2*x^4+2*a*b*x^2+a^2)^(1/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.44

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{9/4}}\,dx=\frac{(a+bx^2)\left(35a^3x+70a^2bx^3+56ab^2x^5+16b^3x^7\right)}{35a^4\left((a+bx^2)^2\right)^{9/4}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-9/4),x]
```

output
```
((a + b*x^2)*(35*a^3*x + 70*a^2*b*x^3 + 56*a*b^2*x^5 + 16*b^3*x^7))/(35*a^
4*((a + b*x^2)^2)^(9/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1385, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{9/4}}dx$$

$$\Big\downarrow \text{1385}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{9/2}}dx}{a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\left(\frac{6}{7}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{7/2}}dx+\frac{x}{7\left(\frac{bx^2}{a}+1\right)^{7/2}}\right)}{a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/2}}dx+\frac{x}{5\left(\frac{bx^2}{a}+1\right)^{5/2}}\right)+\frac{x}{7\left(\frac{bx^2}{a}+1\right)^{7/2}}\right)}{a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/2}}dx+\frac{x}{3\left(\frac{bx^2}{a}+1\right)^{3/2}}\right)+\frac{x}{5\left(\frac{bx^2}{a}+1\right)^{5/2}}\right)+\frac{x}{7\left(\frac{bx^2}{a}+1\right)^{7/2}}\right)}{a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\left(\frac{x}{7\left(\frac{bx^2}{a}+1\right)^{7/2}}+\frac{6}{7}\left(\frac{x}{5\left(\frac{bx^2}{a}+1\right)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{\frac{bx^2}{a}+1}}+\frac{x}{3\left(\frac{bx^2}{a}+1\right)^{3/2}}\right)\right)\right)}{a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-9/4),x]
```

output
```
(Sqrt[1 + (b*x^2)/a]*(x/(7*(1 + (b*x^2)/a)^(7/2)) + (6*(x/(5*(1 + (b*x^2)/
a)^(5/2)) + (4*(x/(3*(1 + (b*x^2)/a)^(3/2)) + (2*x)/(3*Sqrt[1 + (b*x^2)/a]
)))/5))/7))/(a^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{(b\,x^2+a)x(16b^3x^6+56b^2x^4a+70a^2b\,x^2+35a^3)}{35a^4(b^2x^4+2ab\,x^2+a^2)^{\frac{9}{4}}}$	66
orering	$\frac{(b\,x^2+a)x(16b^3x^6+56b^2x^4a+70a^2b\,x^2+35a^3)}{35a^4(b^2x^4+2ab\,x^2+a^2)^{\frac{9}{4}}}$	66

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
1/35*(b*x^2+a)*x*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/a^4/(b^2*x^
4+2*a*b*x^2+a^2)^(9/4)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{9/4}}\,dx = \frac{(16\,b^3x^7+56\,ab^2x^5+70\,a^2bx^3+35\,a^3x)(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}}{35\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="fricas")
```

output
```
1/35*(16*b^3*x^7 + 56*a*b^2*x^5 + 70*a^2*b*x^3 + 35*a^3*x)*(b^2*x^4 + 2*a*
b*x^2 + a^2)^(1/4)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*
x^2 + a^8)
```

Sympy [F]

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{9/4}}\,dx = \int \frac{1}{(a^2+2abx^2+b^2x^4)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(9/4),x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-9/4), x)
```

Maxima [F]

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{9/4}}\,dx = \int \frac{1}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4), x)
```

Giac [F]

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{9/4}}\,dx = \int \frac{1}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4), x)
```

Mupad [B] (verification not implemented)

Time = 18.92 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{9/4}}\,dx = \frac{x\,(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{3/4}}{7\,a\,(b\,x^2 + a)^5} + \frac{6\,x\,(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{3/4}}{35\,a^2\,(b\,x^2 + a)^4} + \frac{8\,x\,(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{3/4}}{35\,a^3\,(b\,x^2 + a)^3} + \frac{16\,x\,(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{3/4}}{35\,a^4\,(b\,x^2 + a)^2}$$

input
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(9/4),x)
```

output
```
(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4))/(7*a*(a + b*x^2)^5) + (6*x*(a^2 + b^
2*x^4 + 2*a*b*x^2)^(3/4))/(35*a^2*(a + b*x^2)^4) + (8*x*(a^2 + b^2*x^4 + 2
*a*b*x^2)^(3/4))/(35*a^3*(a + b*x^2)^3) + (16*x*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(3/4))/(35*a^4*(a + b*x^2)^2)
```

Reduce [F]

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{9/4}}\,dx = \int \frac{}{(b^2x^4+2ab\,x^2+a^2)^{\frac{1}{4}}\,a^4+4\,(b^2x^4+2ab\,x^2+a^2)^{\frac{1}{4}}\,a^3b\,x^2+6\,(b^2x^4+2a}$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x)
```

output
```
int(1/((a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4)*a**4 + 4*(a**2 + 2*a*b*x**2
+ b**2*x**4)**(1/4)*a**3*b*x**2 + 6*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4)
*a**2*b**2*x**4 + 4*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4)*a*b**3*x**6 + (
a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4)*b**4*x**8),x)
```

3.8 $\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \frac{\sqrt{a}(c+dx^2)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

output
```
a^(1/2)*(d*x^2+c)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1
/2))/b^(1/2)/c/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(a*c+(a*d+b*c)*x^2+b*d*x^4)
^(1/2)
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right),\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{(a+bx^2)(c+dx^2)}}$$

input
```
Integrate[1/Sqrt[(a + b*x^2)*(c + d*x^2)],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
 (a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[(a + b*x^2)*(c + d*x^2)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2048, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{1}{\sqrt{x^2(ad+bc)+ac+bdx^4}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x^2(ad+bc)+ac+bdx^4}}$$

input
```
Int[1/Sqrt[(a + b*x^2)*(c + d*x^2)],x]
```

output
```
((Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d
*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c
]*Sqrt[d]))/4])/(2*a^(1/4)*b^(1/4)*c^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*
x^2 + b*d*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}$	87
elliptic	$\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}$	87

input
```
int(1/((b*x^2+a)*(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx = -\frac{\sqrt{ac}\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc})}{bc}$$

input
```
integrate(1/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c))/(b*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$

input
```
integrate(1/((b*x**2+a)*(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/sqrt((a + b*x**2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(bx^2+a)(dx^2+c)}}\,dx$$

input
```
integrate(1/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((b*x^2 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(bx^2+a)(dx^2+c)}}\,dx$$

input
```
integrate(1/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((b*x^2 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}}\,dx$$

input
```
int(1/((a + b*x^2)*(c + d*x^2))^(1/2),x)
```

output
```
int(1/((a + b*x^2)*(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx$$

input
```
int(1/((b*x^2+a)*(d*x^2+c))^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x
**4),x)
```

3.9 $\int \frac{1}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{1}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx=\frac{\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{ac+(bc-ad)x^2-bdx^4}}$$

output
```
c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-
b*c/a/d)^(1/2))/d^(1/2)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)
```

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx=\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c-dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right),-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{(a+bx^2)(c-dx^2)}}$$

input
```
Integrate[1/Sqrt[(a + b*x^2)*(c - d*x^2)],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[(c - d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
 -((a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[(a + b*x^2)*(c - d*x^2)])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2048, 1417, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(a+bx^2)(c-dx^2)}}dx$$

$$\Big\downarrow \ 2048$$

$$\int \frac{1}{\sqrt{x^2(bc-ad)+ac-bdx^4}}dx$$

$$\Big\downarrow \ 1417$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\Big\downarrow \ 321$$

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

input
```
Int[1/Sqrt[(a + b*x^2)*(c - d*x^2)],x]
```

output
```
(Sqrt[c]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]
*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x
^4])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-x^{2}da+bc\,x^{2}+ac}}$	90
elliptic	$\frac{\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-x^{2}da+bc\,x^{2}+ac}}$	90

input
```
int(1/((b*x^2+a)*(-d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^
2+a*c)^(1/2)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx = \frac{\sqrt{ac}\sqrt{\frac{d}{c}}F\left(\arcsin\left(x\sqrt{\frac{d}{c}}\right)\mid -\frac{bc}{ad}\right)}{ad}$$

input
```
integrate(1/((b*x^2+a)*(-d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*c)*sqrt(d/c)*elliptic_f(arcsin(x*sqrt(d/c)), -b*c/(a*d))/(a*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx = \int \frac{1}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx$$

input
```
integrate(1/((b*x**2+a)*(-d*x**2+c))**(1/2),x)
```

output
```
Integral(1/sqrt((a + b*x**2)*(c - d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx = \int \frac{1}{\sqrt{-(bx^2+a)(dx^2-c)}}\,dx$$

input
```
integrate(1/((b*x^2+a)*(-d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(b*x^2 + a)*(d*x^2 - c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx = \int \frac{1}{\sqrt{-(bx^2+a)(dx^2-c)}}\,dx$$

input
```
integrate(1/((b*x^2+a)*(-d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-(b*x^2 + a)*(d*x^2 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx = \int \frac{1}{\sqrt{(b\,x^2+a)\,(c-d\,x^2)}}\,dx$$

input
```
int(1/((a + b*x^2)*(c - d*x^2))^(1/2),x)
```

output
```
int(1/((a + b*x^2)*(c - d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx = \int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}}{-bd\,x^4-ad\,x^2+bc\,x^2+ac}dx$$

input
```
int(1/((b*x^2+a)*(-d*x^2+c))^(1/2),x)
```

output
```
int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a*c - a*d*x**2 + b*c*x**2 - b*d*x
**4),x)
```

3.10 $\int \frac{1}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{1}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{ac-(bc-ad)x^2-bdx^4}}$$

output
```
a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-
a*d/b/c)^(1/2))/b^(1/2)/(a*c-(-a*d+b*c)*x^2-b*d*x^4)^(1/2)
```

Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx=\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right),-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{(a-bx^2)(c+dx^2)}}$$

input
```
Integrate[1/Sqrt[(a - b*x^2)*(c + d*x^2)],x]
```

output
```
(Sqrt[(a - b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[b/a]*x], -(
(a*d)/(b*c))])/(Sqrt[b/a]*Sqrt[(a - b*x^2)*(c + d*x^2)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2048, 1417, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(a-bx^2)(c+dx^2)}}dx$$

$$\Big\downarrow \text{ 2048}$$

$$\int \frac{1}{\sqrt{x^2(ad-bc)+ac-bdx^4}}dx$$

$$\Big\downarrow \text{ 1417}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{-x^2(bc-ad)+ac-bdx^4}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-x^2(bc-ad)+ac-bdx^4}}$$

input
```
Int[1/Sqrt[(a - b*x^2)*(c + d*x^2)],x]
```

output
```
(Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]
*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a*c - (b*c - a*d)*x^2 - b*d*x
^4])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+x^{2}da-bc\,x^{2}+ac}}$	90
elliptic	$\frac{\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+x^{2}da-bc\,x^{2}+ac}}$	90

input
```
int(1/((-b*x^2+a)*(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^
2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx = \frac{\sqrt{ac}\sqrt{\frac{b}{a}}F\left(\arcsin\left(x\sqrt{\frac{b}{a}}\right)\mid -\frac{ad}{bc}\right)}{bc}$$

input
```
integrate(1/((-b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*c)*sqrt(b/a)*elliptic_f(arcsin(x*sqrt(b/a)), -a*d/(b*c))/(b*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx$$

input
```
integrate(1/((-b*x**2+a)*(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/sqrt((a - b*x**2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{-(bx^2-a)(dx^2+c)}}\,dx$$

input
```
integrate(1/((-b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(b*x^2 - a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{-(bx^2-a)(dx^2+c)}}\,dx$$

input
```
integrate(1/((-b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-(b*x^2 - a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(a-b\,x^2)\,(d\,x^2+c)}}\,dx$$

input
```
int(1/((a - b*x^2)*(c + d*x^2))^(1/2),x)
```

output
```
int(1/((a - b*x^2)*(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx$$

input
```
int(1/((-b*x^2+a)*(d*x^2+c))^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c + a*d*x**2 - b*c*x**2 - b*d*x
**4),x)
```

3.11 $\int \frac{1}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{1}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx = \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{ac-(bc+ad)x^2+bdx^4}}$$

output
```
a^(1/2)*(1-b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(a
*d/b/c)^(1/2))/b^(1/2)/(a*c-(a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx = \frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{\frac{c-dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{(a-bx^2)(c-dx^2)}}$$

input
```
Integrate[1/Sqrt[(a - b*x^2)*(c - d*x^2)],x]
```

output
```
(Sqrt[(a - b*x^2)/a]*Sqrt[(c - d*x^2)/c]*EllipticF[ArcSin[Sqrt[b/a]*x], (a
*d)/(b*c)])/(Sqrt[b/a]*Sqrt[(a - b*x^2)*(c - d*x^2)])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(90) = 180.

Time = 0.44 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2048, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(a-bx^2)(c-dx^2)}}dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{1}{\sqrt{x^2(-ad-bc)+ac+bdx^4}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{-x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}+2\right)\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{c}\sqrt[4]{d}\sqrt{-x^2(ad+bc)+ac+bdx^4}}$$

input
```
Int[1/Sqrt[(a - b*x^2)*(c - d*x^2)],x]
```

output
```
((Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c - (b*c + a*d)*x^2 + b*d
*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 + (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c
]*Sqrt[d]))/4])/(2*a^(1/4)*b^(1/4)*c^(1/4)*d^(1/4)*Sqrt[a*c - (b*c + a*d)*
x^2 + b*d*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-x^{2}da-bc\,x^{2}+ac}}$	92
elliptic	$\frac{\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-x^{2}da-bc\,x^{2}+ac}}$	92

input
```
int(1/((-b*x^2+a)*(-d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1-b*x^2/a)^(1/2)/(b*d*x^4-a*d*x^2-b*c*x^2
+a*c)^(1/2)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d-b*c)/a/d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{(a-bx^2)\,(c-dx^2)}}\,dx = \frac{\sqrt{ac}\sqrt{\frac{d}{c}}F(\arcsin\left(x\sqrt{\frac{d}{c}}\right) \mid \frac{bc}{ad})}{ad}$$

input
```
integrate(1/((-b*x^2+a)*(-d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*c)*sqrt(d/c)*elliptic_f(arcsin(x*sqrt(d/c)), b*c/(a*d))/(a*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx = \int \frac{1}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx$$

input
```
integrate(1/((-b*x**2+a)*(-d*x**2+c))**(1/2),x)
```

output
```
Integral(1/sqrt((a - b*x**2)*(c - d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx = \int \frac{1}{\sqrt{(bx^2-a)(dx^2-c)}}\,dx$$

input
```
integrate(1/((-b*x^2+a)*(-d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((b*x^2 - a)*(d*x^2 - c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx = \int \frac{1}{\sqrt{(bx^2-a)(dx^2-c)}}\,dx$$

input
```
integrate(1/((-b*x^2+a)*(-d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((b*x^2 - a)*(d*x^2 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(a-bx^2)\,(c-dx^2)}}\,dx = \int \frac{1}{\sqrt{(a-b\,x^2)\,\left(c-d\,x^2\right)}}\,dx$$

input
```
int(1/((a - b*x^2)*(c - d*x^2))^(1/2),x)
```

output
```
int(1/((a - b*x^2)*(c - d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{(a-bx^2)\,(c-dx^2)}}\,dx = \int \frac{\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}}{bd\,x^4-ad\,x^2-bc\,x^2+ac}dx$$

input
```
int(1/((-b*x^2+a)*(-d*x^2+c))^(1/2),x)
```

output
```
int((sqrt(c - d*x**2)*sqrt(a - b*x**2))/(a*c - a*d*x**2 - b*c*x**2 + b*d*x
**4),x)
```

3.12 $\int (ac+(bc+ad)x^2+bdx^4)^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 629

$$\int (ac+(bc+ad)x^2 +bdx^4)^{5/2}\,dx=\frac{(bc+ad)\left(8b^4c^4-61ab^3c^3d+234a^2b^2c^2d^2-61a^3bcd^3+8a^4d^4\right)x(c+dx^2)}{693b^2d^3\sqrt{ac+(bc+ad)x^2+bdx^4}}$$
$$-\frac{x\left(4b^4c^4-11ab^3c^3d-210a^2b^2c^2d^2-11a^3bcd^3+4a^4d^4-12bd(bc+ad)\left(8abcd-(bc+ad)^2\right)x^2\right)\sqrt{ac+(}}{693b^2d^2}$$
$$+\frac{5x\left(3\left(6abcd+(bc+ad)^2\right)+7bd(bc+ad)x^2\right)\left(ac+(bc+ad)x^2+bdx^4\right)^{3/2}}{693bd}$$
$$+\frac{1}{11}x\left(ac+(bc+ad)x^2+bdx^4\right)^{5/2}$$
$$-\frac{\sqrt{a}(bc+ad)\left(8b^4c^4-61ab^3c^3d+234a^2b^2c^2d^2-61a^3bcd^3+8a^4d^4\right)(c+dx^2)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{693b^{5/2}d^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{ac+(bc+ad)x^2+bdx^4}}$$
$$+\frac{2a^{3/2}\left(2b^4c^4-13ab^3c^3d+150a^2b^2c^2d^2-13a^3bcd^3+2a^4d^4\right)(c+dx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{693b^{5/2}d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

output
```
1/693*(a*d+b*c)*(8*a^4*d^4-61*a^3*b*c*d^3+234*a^2*b^2*c^2*d^2-61*a*b^3*c^3
*d+8*b^4*c^4)*x*(d*x^2+c)/b^2/d^3/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)-1/693*
x*(4*b^4*c^4-11*a*b^3*c^3*d-210*a^2*b^2*c^2*d^2-11*a^3*b*c*d^3+4*a^4*d^4-1
2*b*d*(a*d+b*c)*(8*a*b*c*d-(a*d+b*c)^2)*x^2)*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(
1/2)/b^2/d^2+5/693*x*(18*a*b*c*d+3*(a*d+b*c)^2+7*b*d*(a*d+b*c)*x^2)*(a*c+(
a*d+b*c)*x^2+b*d*x^4)^(3/2)/b/d+1/11*x*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2)-1
/693*a^(1/2)*(a*d+b*c)*(8*a^4*d^4-61*a^3*b*c*d^3+234*a^2*b^2*c^2*d^2-61*a*
b^3*c^3*d+8*b^4*c^4)*(d*x^2+c)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/
2),(1-a*d/b/c)^(1/2))/b^(5/2)/d^3/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(a*c+(a*
d+b*c)*x^2+b*d*x^4)^(1/2)+2/693*a^(3/2)*(2*a^4*d^4-13*a^3*b*c*d^3+150*a^2*
b^2*c^2*d^2-13*a*b^3*c^3*d+2*b^4*c^4)*(d*x^2+c)*InverseJacobiAM(arctan(b^(
1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/d^2/(a*(d*x^2+c)/c/(b*x^2+a))^(
1/2)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.77 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.72

$$\int (ac+(bc+ad)x^2+bdx^4)^{5/2}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(-4a^4d^4+a^3bd^3(26c+3dx^2)+a^2b^2d^2(393c^2+356cdx^2+113d^2\cdots}{}$$

input
```
Integrate[(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(5/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*a^4*d^4 + a^3*b*d^3*(26*c + 3*d
*x^2) + a^2*b^2*d^2*(393*c^2 + 356*c*d*x^2 + 113*d^2*x^4) + a*b^3*d*(26*c^
3 + 356*c^2*d*x^2 + 442*c*d^2*x^4 + 161*d^3*x^6) + b^4*(-4*c^4 + 3*c^3*d*x
^2 + 113*c^2*d^2*x^4 + 161*c*d^3*x^6 + 63*d^4*x^8)) - I*c*(8*b^5*c^5 - 53*
a*b^4*c^4*d + 173*a^2*b^3*c^3*d^2 + 173*a^3*b^2*c^2*d^3 - 53*a^4*b*c*d^4 +
 8*a^5*d^5)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + I*c*(8*b^5*c^5 - 57*a*b^4*c^4*d + 199*a^2*b^3*c
^3*d^2 - 127*a^3*b^2*c^2*d^3 - 27*a^4*b*c*d^4 + 4*a^5*d^5)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(
693*a^2*(b/a)^(5/2)*d^3*Sqrt[(a + b*x^2)*(c + d*x^2)])
```

Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 883, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1404, 1490, 1490, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2(ad+bc)+ac+bdx^4\right)^{5/2} dx$$

$$\Big\downarrow \text{ 1404}$$

$$\frac{5}{11}\int \left((bc+ad)x^2+2ac\right)\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}dx+\frac{1}{11}x\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}$$

$$\Big\downarrow \text{ 1490}$$

$$\frac{5}{11}\left(\frac{\int \left(4(bc+ad)\left(8abcd-(bc+ad)^2\right)x^2+ac\left(36abcd-(bc+ad)^2\right)\right)\sqrt{bdx^4+(bc+ad)x^2+ac}dx}{21bd}+\frac{x\left(7bdx^2\right.}{}\right.$$
$$\frac{1}{11}x\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}$$

$$\Big\downarrow \text{ 1490}$$

$$\frac{5}{11}\left(\frac{\frac{\int \frac{(bc+ad)\left(8b^4c^4-61ab^3dc^3+234a^2b^2d^2c^2-61a^3bd^3c+8a^4d^4\right)x^2+2ac\left(2b^4c^4-13ab^3dc^3+150a^2b^2d^2c^2-13a^3bd^3c+2a^4d^4\right)}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{15bd}-\frac{x\left(4a^4d^4-11a^3bcd\right.}{}}{21bd}\right.$$
$$\frac{1}{11}x\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{5}{11}\left(\frac{\frac{\sqrt{a}\sqrt{c}\left(2\sqrt{a}\sqrt{c}\left(2a^4d^4-13a^3bcd^3+150a^2b^2c^2d^2-13ab^3c^3d+2b^4c^4\right)+\frac{(ad+bc)\left(8a^4d^4-61a^3bcd^3+234a^2b^2c^2d^2-61ab^3c^3d+8b^4c^4\right)}{\sqrt{b}\sqrt{d}}\right)\int \frac{}{\sqrt{bdx^4+(}}}{15bd}}{}\right.$$
$$\frac{1}{11}x\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{11}\left(\frac{\frac{\sqrt{a}\sqrt{c}\left(2\sqrt{a}\sqrt{c}(2a^4d^4-13a^3bcd^3+150a^2b^2c^2d^2-13ab^3c^3d+2b^4c^4)+\frac{(ad+bc)\left(8a^4d^4-61a^3bcd^3+234a^2b^2c^2d^2-61ab^3c^3d+8b^4c^4\right)}{\sqrt{b}\sqrt{d}}\right)\int\frac{}{\sqrt{bdx^4+(}}}{15bd}}{}\right.$$

$$\frac{1}{11}x\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}$$

$\downarrow$ 1416

$$\frac{5}{11}\left(\frac{\frac{\sqrt[4]{a}\sqrt[4]{c}\left(2\sqrt{a}\sqrt{c}\left(2a^4d^4-13a^3bcd^3+150a^2b^2c^2d^2-13ab^3c^3d+2b^4c^4\right)+\frac{(ad+bc)\left(8a^4d^4-61a^3bcd^3+234a^2b^2c^2d^2-61ab^3c^3d+8b^4c^4\right)}{\sqrt{b}\sqrt{d}}\right)\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x\right.}{2\sqrt[4]{b}\sqrt[4]{d}\sqrt{x^2(ad+bc)+ac+bdx^4}}}{}\right.$$

$$\frac{1}{11}x\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}$$

$\downarrow$ 1509

$$\frac{1}{11}x\left(bdx^4+(bc+ad)x^2+ac\right)^{5/2}+$$

$$\frac{5}{11}\left(\frac{x\left(7bd(bc+ad)x^2+3\left((bc+ad)^2+6abcd\right)\right)\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}}{63bd}+\frac{\frac{\sqrt[4]{a}\sqrt[4]{c}\left(2\sqrt{a}\sqrt{c}\left(2b^4c^4-13ab^3dc^3+150a\right.\right.}{}}{}\right.$$

input
```
Int[(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(5/2),x]
```

output
```
(x*(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(5/2))/11 + (5*((x*(3*(6*a*b*c*d + (b
*c + a*d)^2) + 7*b*d*(b*c + a*d)*x^2)*(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(3
/2))/(63*b*d) + (-1/15*(x*(4*b^4*c^4 - 11*a*b^3*c^3*d - 210*a^2*b^2*c^2*d^
2 - 11*a^3*b*c*d^3 + 4*a^4*d^4 - 12*b*d*(b*c + a*d)*(8*a*b*c*d - (b*c + a*
d)^2)*x^2)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(b*d) + (-(((b*c + a*d)*
(8*b^4*c^4 - 61*a*b^3*c^3*d + 234*a^2*b^2*c^2*d^2 - 61*a^3*b*c*d^3 + 8*a^4
*d^4)*(-((x*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c] + Sqrt
[b]*Sqrt[d]*x^2)) + (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^
2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[
d]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 -
 (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/4)*Sqrt[
a*c + (b*c + a*d)*x^2 + b*d*x^4])))/(Sqrt[b]*Sqrt[d])) + (a^(1/4)*c^(1/4)*
(2*Sqrt[a]*Sqrt[c]*(2*b^4*c^4 - 13*a*b^3*c^3*d + 150*a^2*b^2*c^2*d^2 - 13*
a^3*b*c*d^3 + 2*a^4*d^4) + ((b*c + a*d)*(8*b^4*c^4 - 61*a*b^3*c^3*d + 234*
a^2*b^2*c^2*d^2 - 61*a^3*b*c*d^3 + 8*a^4*d^4))/(Sqrt[b]*Sqrt[d]))*(Sqrt[a]
*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sq
rt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*d^(1/4
)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]
))/4])/(2*b^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4]))/(15*b*d)
)/(21*b*d)))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.43

method	result
risch	$-\frac{x\left(-63b^4d^4x^8-161a\,b^3d^4x^6-161b^4c\,d^3x^6-113a^2b^2d^4x^4-442a\,b^3c\,d^3x^4-113b^4c^2d^2x^4-3a^3b\,d^4x^2-356a^2b^2c\,d^3x^2-356a\,b^3c^2d^2\right.}{693b^2d^2\sqrt{(b\,x^2+a)(d\,x^2+c)}}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/693/b^2/d^2*x*(-63*b^4*d^4*x^8-161*a*b^3*d^4*x^6-161*b^4*c*d^3*x^6-113*
a^2*b^2*d^4*x^4-442*a*b^3*c*d^3*x^4-113*b^4*c^2*d^2*x^4-3*a^3*b*d^4*x^2-35
6*a^2*b^2*c*d^3*x^2-356*a*b^3*c^2*d^2*x^2-3*b^4*c^3*d*x^2+4*a^4*d^4-26*a^3
*b*c*d^3-393*a^2*b^2*c^2*d^2-26*a*b^3*c^3*d+4*b^4*c^4)*(b*x^2+a)*(d*x^2+c)
/((b*x^2+a)*(d*x^2+c))^(1/2)+1/693/b^2/d^2*(-(8*a^5*d^5-53*a^4*b*c*d^4+173
*a^3*b^2*c^2*d^3+173*a^2*b^3*c^3*d^2-53*a*b^4*c^4*d+8*b^5*c^5)*c/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+4*a*b^4*c^5/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+4*a^5*c*d^4/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-26*a^2*b^3*c^4*d/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+300*a^3*b^2*c^3*d^2/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-26*a^4*b*c^2*d^3/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.87

$$\int \left(ac+(bc+ad)x^2+bdx^4\right)^{5/2}\,dx =$$

$$-\frac{(8\,b^5c^6-53\,ab^4c^5d+173\,a^2b^3c^4d^2+173\,a^3b^2c^3d^3-53\,a^4bc^2d^4+8\,a^5cd^5)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid}{}$$

input
```
integrate((a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x, algorithm="fricas")
```

output
```
-1/693*((8*b^5*c^6 - 53*a*b^4*c^5*d + 173*a^2*b^3*c^4*d^2 + 173*a^3*b^2*c^
3*d^3 - 53*a^4*b*c^2*d^4 + 8*a^5*c*d^5)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(
arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (8*b^5*c^6 - 53*a*b^4*c^5*d + 4*a^5*d^6
 + (173*a^2*b^3 + 4*a*b^4)*c^4*d^2 + (173*a^3*b^2 - 26*a^2*b^3)*c^3*d^3 -
(53*a^4*b - 300*a^3*b^2)*c^2*d^4 + 2*(4*a^5 - 13*a^4*b)*c*d^5)*sqrt(b*d)*x
*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (63*b^5*d^6*x^10
 + 8*b^5*c^5*d - 53*a*b^4*c^4*d^2 + 173*a^2*b^3*c^3*d^3 + 173*a^3*b^2*c^2*
d^4 - 53*a^4*b*c*d^5 + 8*a^5*d^6 + 161*(b^5*c*d^5 + a*b^4*d^6)*x^8 + (113*
b^5*c^2*d^4 + 442*a*b^4*c*d^5 + 113*a^2*b^3*d^6)*x^6 + (3*b^5*c^3*d^3 + 35
6*a*b^4*c^2*d^4 + 356*a^2*b^3*c*d^5 + 3*a^3*b^2*d^6)*x^4 - (4*b^5*c^4*d^2
- 26*a*b^4*c^3*d^3 - 393*a^2*b^3*c^2*d^4 - 26*a^3*b^2*c*d^5 + 4*a^4*b*d^6)
*x^2)*sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c))/(b^3*d^4*x)
```

Sympy [F]

$$\int \left(ac+(bc+ad)x^2+bdx^4\right)^{5/2}\,dx=\int \left(ac+bdx^4+x^2(ad+bc)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x**2+b*d*x**4)**(5/2),x)
```

output
```
Integral((a*c + b*d*x**4 + x**2*(a*d + b*c))**(5/2), x)
```

Maxima [F]

$$\int \left(ac+(bc+ad)x^2+bdx^4\right)^{5/2}\,dx=\int \left(bdx^4+(bc+ad)x^2+ac\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(5/2), x)
```

Giac [F]

$$\int \left(ac+(bc+ad)x^2+bdx^4\right)^{5/2} dx = \int \left(bdx^4+(bc+ad)x^2+ac\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(ac+(bc+ad)x^2+bdx^4\right)^{5/2} dx = \int \left(b\,d\,x^4+(a\,d+b\,c)\,x^2+a\,c\right)^{5/2} dx$$

input
```
int((a*c + x^2*(a*d + b*c) + b*d*x^4)^(5/2),x)
```

output
```
int((a*c + x^2*(a*d + b*c) + b*d*x^4)^(5/2), x)
```

Reduce [F]

$$\int \left(ac+(bc+ad)x^2+bdx^4\right)^{5/2} dx = \text{Too large to display}$$

input
```
int((a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x)
```

output

```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**4*d**4*x + 26*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a**3*b*c*d**3*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**
3*b*d**4*x**3 + 393*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c**2*d**2*
x + 356*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c*d**3*x**3 + 113*sqrt
(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*d**4*x**5 + 26*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*b**3*c**3*d*x + 356*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b
**3*c**2*d**2*x**3 + 442*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c*d**3*x
**5 + 161*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*d**4*x**7 - 4*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*b**4*c**4*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*b**4*c**3*d*x**3 + 113*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c**2*d**2*x
**5 + 161*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c*d**3*x**7 + 63*sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*b**4*d**4*x**9 + 8*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**5*d**5 - 53*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b
*d*x**4),x)*a**4*b*c*d**4 + 173*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**
2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*b**2*c**2*d**3 + 173*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d
*x**4),x)*a**2*b**3*c**3*d**2 - 53*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*
x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**4*c**4*d + 8*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x...
```

3.13 $\int \left(ac+(bc+ad)x^2+bdx^4\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 408

$$\int \left(ac+(bc+ad)x^2+bdx^4\right)^{3/2} dx = \frac{2(bc+ad)\left(8abcd-(bc+ad)^2\right)x(c+dx^2)}{35bd^2\sqrt{ac+(bc+ad)x^2+bdx^4}}$$
$$+\frac{x\left(10abcd+(bc+ad)^2+3bd(bc+ad)x^2\right)\sqrt{ac+(bc+ad)x^2+bdx^4}}{35bd}$$
$$+\frac{1}{7}x\left(ac+(bc+ad)x^2+bdx^4\right)^{3/2}$$
$$-\frac{2\sqrt{a}(bc+ad)\left(8abcd-(bc+ad)^2\right)(c+dx^2)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{35b^{3/2}d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{ac+(bc+ad)x^2+bdx^4}}$$
$$+\frac{a^{3/2}\left(20abcd-(bc+ad)^2\right)(c+dx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{35b^{3/2}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

output
```
2/35*(a*d+b*c)*(8*a*b*c*d-(a*d+b*c)^2)*x*(d*x^2+c)/b/d^2/(a*c+(a*d+b*c)*x^
2+b*d*x^4)^(1/2)+1/35*x*(10*a*b*c*d+(a*d+b*c)^2+3*b*d*(a*d+b*c)*x^2)*(a*c+
(a*d+b*c)*x^2+b*d*x^4)^(1/2)/b/d+1/7*x*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2)-2
/35*a^(1/2)*(a*d+b*c)*(8*a*b*c*d-(a*d+b*c)^2)*(d*x^2+c)*EllipticE(b^(1/2)*
x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d^2/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)+1/35*a^(3/2)*(20*a*b*c*
d-(a*d+b*c)^2)*(d*x^2+c)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/
b/c)^(1/2))/b^(3/2)/d/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(a*c+(a*d+b*c)*x^2+b
*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.11 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.73

$$\int (ac+(bc+ad)x^2 +bdx^4)^{3/2} \, dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\,(c+dx^2)\,(a^2d^2+abd(17c+8dx^2)+b^2(c^2+8cdx^2+5d^2x^4))+2ic(b^3c^3-}{}$$

input
```
Integrate[(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(a^2*d^2 + a*b*d*(17*c + 8*d*x^2) +
 b^2*(c^2 + 8*c*d*x^2 + 5*d^2*x^4)) + (2*I)*c*(b^3*c^3 - 5*a*b^2*c^2*d - 5
*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(2*b^3*c^3 - 11*a*b^2*c^2*d + 8
*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(35*b*Sqrt[b/a]*d^2*Sqrt[(a + b*x^2)
*(c + d*x^2)])
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.60, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1404, 1490, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}dx$$

$$\Big\downarrow \text{1404}$$

$$\frac{3}{7}\int \left((bc+ad)x^2+2ac\right)\sqrt{bdx^4+(bc+ad)x^2+ac}dx+\frac{1}{7}x\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{3}{7}\left(\frac{\int \frac{2(bc+ad)\left(8abcd-(bc+ad)^2\right)x^2+ac\left(20abcd-(bc+ad)^2\right)}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{15bd}+\frac{x\sqrt{x^2(ad+bc)+ac+bdx^4}\left(3bdx^2(ad+bc)+(ad+bc)^2\right.}{15bd}\right.$$
$$\frac{1}{7}x\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}$$

$$\Big\downarrow \text{1511}$$

$$\frac{3}{7}\left(\frac{\sqrt{a}\sqrt{c}\left(\frac{2(ad+bc)\left(8abcd-(ad+bc)^2\right)}{\sqrt{b}\sqrt{d}}+\sqrt{a}\sqrt{c}\left(20abcd-(ad+bc)^2\right)\right)\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx-\frac{2\sqrt{a}\sqrt{c}(ad+bc)\left(8abcd-\right.}{}}{15bd}\right.$$
$$\frac{1}{7}x\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}\left(\frac{\sqrt{a}\sqrt{c}\left(\frac{2(ad+bc)\left(8abcd-(ad+bc)^2\right)}{\sqrt{b}\sqrt{d}}+\sqrt{a}\sqrt{c}\left(20abcd-(ad+bc)^2\right)\right)\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx-\frac{2(ad+bc)\left(8abcd-(ad+b\right.}{}}{15bd}\right.$$
$$\frac{1}{7}x\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}$$

$$\Big\downarrow \text{1416}$$

$$\frac{3}{7}\left(\frac{\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\frac{2(ad+bc)\left(8abcd-(ad+bc)^2\right)}{\sqrt{b}\sqrt{d}}+\sqrt{a}\sqrt{c}(20abcd-(ad+bc)^2)\right)\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right)\cdots\right.}{2\sqrt[4]{b}\sqrt[4]{d}\sqrt{x^2(ad+bc)+ac+bdx^4}}\cdots}{15bd}\cdots\right.$$

$$\frac{1}{7}x\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{3}{7}\left(\frac{\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\frac{2(ad+bc)\left(8abcd-(ad+bc)^2\right)}{\sqrt{b}\sqrt{d}}+\sqrt{a}\sqrt{c}(20abcd-(ad+bc)^2)\right)\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right)\cdots\right.}{2\sqrt[4]{b}\sqrt[4]{d}\sqrt{x^2(ad+bc)+ac+bdx^4}}\cdots}{\cdots}\cdots\right.$$

$$\frac{1}{7}x\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}$$

input
```
Int[(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(3/2),x]
```

output
```
(x*(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(3/2))/7 + (3*((x*(10*a*b*c*d + (b*c
+ a*d)^2 + 3*b*d*(b*c + a*d)*x^2)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(
15*b*d) + ((-2*(b*c + a*d)*(8*a*b*c*d - (b*c + a*d)^2)*(-((x*Sqrt[a*c + (b
*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)) + (a^(1
/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d
)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticE[2*Ar
cTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqr
t[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*x^2 + b
*d*x^4])))/(Sqrt[b]*Sqrt[d]) + (a^(1/4)*c^(1/4)*((2*(b*c + a*d)*(8*a*b*c*d
 - (b*c + a*d)^2))/(Sqrt[b]*Sqrt[d]) + Sqrt[a]*Sqrt[c]*(20*a*b*c*d - (b*c
+ a*d)^2))*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)
*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticF[2*Arc
Tan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt
[b]*Sqrt[c]*Sqrt[d]))/4])/(2*b^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*x^2 +
b*d*x^4]))/(15*b*d)))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.33

method	result
risch	$\frac{x\left(5b^{2}d^{2}x^{4}+8a\,d^{2}b\,x^{2}+8b^{2}cd\,x^{2}+a^{2}d^{2}+17abcd+b^{2}c^{2}\right)\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}{35bd\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}}-\frac{-\frac{\left(2a^{3}d^{3}-10a^{2}bc\,d^{2}-10a\,b^{2}c^{2}d+2b^{3}c^{3}\right)c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\ldots}}{\sqrt{-\frac{b}{a}}\ldots}}{\ldots}$
default	$\frac{bd\,x^{5}\sqrt{bd\,x^{4}+x^{2}da+bc\,x^{2}+ac}}{7}+\frac{\left(2ab\,d^{2}+2b^{2}cd-\frac{bd(6ad+6bc)}{7}\right)x^{3}\sqrt{bd\,x^{4}+x^{2}da+bc\,x^{2}+ac}}{5bd}+\frac{\left(a^{2}d^{2}+\frac{23abcd}{7}+b^{2}c^{2}-\frac{\left(2ab\,d^{2}+2\ldots\right.}{\ldots}\ldots\right.}{\ldots}$
elliptic	$\frac{bd\,x^{5}\sqrt{bd\,x^{4}+x^{2}da+bc\,x^{2}+ac}}{7}+\frac{\left(2ab\,d^{2}+2b^{2}cd-\frac{bd(6ad+6bc)}{7}\right)x^{3}\sqrt{bd\,x^{4}+x^{2}da+bc\,x^{2}+ac}}{5bd}+\frac{\left(a^{2}d^{2}+\frac{23abcd}{7}+b^{2}c^{2}-\frac{\left(2ab\,d^{2}+2\ldots\right.}{\ldots}\ldots\right.}{\ldots}$

input
```
int((a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35/b/d*x*(5*b^2*d^2*x^4+8*a*b*d^2*x^2+8*b^2*c*d*x^2+a^2*d^2+17*a*b*c*d+b
^2*c^2)*(b*x^2+a)*(d*x^2+c)/((b*x^2+a)*(d*x^2+c))^(1/2)-1/35/b/d*(-(2*a^3*
d^3-10*a^2*b*c*d^2-10*a*b^2*c^2*d+2*b^3*c^3)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2)))+a*b^2*c^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))+a^3*c*d^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))-18*a^2*c^2*b*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2
)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.79

$$\int (ac+(bc+ad)x^2+bdx^4)^{3/2}\,dx = \frac{2\,(b^3c^4-5\,ab^2c^3d-5\,a^2bc^2d^2+a^3cd^3)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(2\,b^3c^4-10\,ab^2\cdots}{\cdots}$$

input
```
integrate((a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x, algorithm="fricas")
```

output
```
1/35*(2*(b^3*c^4 - 5*a*b^2*c^3*d - 5*a^2*b*c^2*d^2 + a^3*c*d^3)*sqrt(b*d)*
x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (2*b^3*c^4 - 10
*a*b^2*c^3*d + a^3*d^4 - (10*a^2*b - a*b^2)*c^2*d^2 + 2*(a^3 - 9*a^2*b)*c*
d^3)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) +
(5*b^3*d^4*x^6 - 2*b^3*c^3*d + 10*a*b^2*c^2*d^2 + 10*a^2*b*c*d^3 - 2*a^3*d
^4 + 8*(b^3*c*d^3 + a*b^2*d^4)*x^4 + (b^3*c^2*d^2 + 17*a*b^2*c*d^3 + a^2*b
*d^4)*x^2)*sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c))/(b^2*d^3*x)
```

Sympy [F]

$$\int (ac+(bc+ad)x^2+bdx^4)^{3/2}\,dx = \int (ac+bdx^4+x^2(ad+bc))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x**2+b*d*x**4)**(3/2),x)
```

output
```
Integral((a*c + b*d*x**4 + x**2*(a*d + b*c))**(3/2), x)
```

Maxima [F]

$$\int \left(ac+(bc+ad)x^2+bdx^4\right)^{3/2}\,dx=\int \left(bdx^4+(bc+ad)x^2+ac\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(3/2), x)
```

Giac [F]

$$\int \left(ac+(bc+ad)x^2+bdx^4\right)^{3/2}\,dx=\int \left(bdx^4+(bc+ad)x^2+ac\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(ac+(bc+ad)x^2+bdx^4\right)^{3/2}\,dx=\int \left(b\,d\,x^4+(a\,d+b\,c)\,x^2+a\,c\right)^{3/2}\,dx$$

input
```
int((a*c + x^2*(a*d + b*c) + b*d*x^4)^(3/2),x)
```

output
```
int((a*c + x^2*(a*d + b*c) + b*d*x^4)^(3/2), x)
```

Reduce [F]

$$\int (ac + (bc + ad)x^2 + bdx^4)^{3/2} \, dx = \frac{\sqrt{d\,x^2 + c}\,\sqrt{b\,x^2 + a}\,a^2 d^2 x + 17\sqrt{d\,x^2 + c}\,\sqrt{b\,x^2 + a}\,abcdx + 8\sqrt{d\,x^2 + c}\,\sqrt{b\,x^2 + a}\,ab\,d^2 x^3}{}$$

input
```
int((a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*x + 17*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*a*b*c*d*x + 8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*x**3
+ sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*x + 8*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*b**2*c*d*x**3 + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*
x**5 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*
c*x**2 + b*d*x**4),x)*a**3*d**3 + 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*c*d**2 + 10*int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x
**4),x)*a*b**2*c**2*d - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*
c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**3*c**3 - int((sqrt(c + d*x**2)*s
qrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*c*d**2 + 1
8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d
*x**4),x)*a**2*b*c**2*d - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a
*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c**3)/(35*b*d)
```

3.14 $\int \sqrt{ac+(bc+ad)x^2+bdx^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 282

$$\begin{aligned}\int \sqrt{ac+(bc+ad)x^2+bdx^4}\,dx =& \frac{(bc+ad)x(c+dx^2)}{3d\sqrt{ac+(bc+ad)x^2+bdx^4}} \\ &+ \frac{1}{3}x\sqrt{ac+(bc+ad)x^2+bdx^4} \\ &- \frac{\sqrt{a}(bc+ad)\left(c+dx^2\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{b}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{ac+(bc+ad)x^2+bdx^4}} \\ &+ \frac{2a^{3/2}(c+dx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{ac+(bc+ad)x^2+bdx^4}}\end{aligned}$$

output

```
1/3*(a*d+b*c)*x*(d*x^2+c)/d/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)+1/3*x*(a*c+(
a*d+b*c)*x^2+b*d*x^4)^(1/2)-1/3*a^(1/2)*(a*d+b*c)*(d*x^2+c)*EllipticE(b^(1
/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/d/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)+2/3*a^(3/2)*(d*x^2+c)
*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/(a*(
d*x^2+c)/c/(b*x^2+a))^(1/2)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.69

$$\int \sqrt{ac+(bc+ad)x^2+bdx^4}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\,(c+dx^2)-ic(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic(-bc+ad)\sqrt{1-}}{3\sqrt{\frac{b}{a}}d\sqrt{(a+bx^2)\,(c+dx^2)}}$$

input
```
Integrate[Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - I*c*(b*c + a*d)*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*
c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*d*Sqrt[(a + b*x^2)*(c + d*x^2)
])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.82, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1404, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2(ad+bc)+ac+bdx^4}\,dx$$

$$\Big\downarrow \text{1404}$$

$$\frac{1}{3}\int \frac{(bc+ad)x^2+2ac}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx+\frac{1}{3}x\sqrt{x^2(ad+bc)+ac+bdx^4}$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{3}\left(\frac{\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\int\frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}(ad+bc)\int\frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{a}\sqrt{c}\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}\right)+$$
$$\frac{1}{3}x\sqrt{x^2(ad+bc)+ac+bdx^4}$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\int\frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{(ad+bc)\int\frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}\right)+$$
$$\frac{1}{3}x\sqrt{x^2(ad+bc)+ac+bdx^4}$$

$\downarrow$ 1416

$$\frac{1}{3}\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{}{\sqrt{}}\right.\right.}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}\right.$$
$$\frac{1}{3}x\sqrt{x^2(ad+bc)+ac+bdx^4}$$

$\downarrow$ 1509

$$\frac{1}{3}\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{}{\sqrt{}}\right.\right.}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}\right.$$
$$\frac{1}{3}x\sqrt{x^2(ad+bc)+ac+bdx^4}$$

input
```
Int[Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4],x]
```

output
```
(x*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/3 + (-(((b*c + a*d)*(-((x*Sqrt[a
*c + (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2))
+ (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*
c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt
[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*
x^2 + b*d*x^4])))/(Sqrt[b]*Sqrt[d])) + (a^(1/4)*c^(1/4)*(Sqrt[b]*Sqrt[c] +
 Sqrt[a]*Sqrt[d])^2*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b
*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqr
t[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(2*b^(3/4)*d^(3/4)*Sqrt[a*c + (b*c + a*
d)*x^2 + b*d*x^4]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{3}+\frac{2ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{3\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}-\frac{\left(\frac{ad}{3}+\frac{bc}{3}\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\text{Elli}\right.}{}$
elliptic	$\frac{x\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{3}+\frac{2ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{3\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}-\frac{\left(\frac{ad}{3}+\frac{bc}{3}\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\text{Elli}\right.}{}$
risch	$\frac{x(b\,x^2+a)(d\,x^2+c)}{3\sqrt{(b\,x^2+a)(d\,x^2+c)}}-\frac{(ad+bc)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{3\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}\,d}$

input
```
int((a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2/3*a*c/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(1/3*a*d+1/3*b*c)*c/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),
(-1+(a*d+b*c)/c/b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.56

$$\int \sqrt{ac+(bc+ad)x^2+bdx^4}\,dx =$$

$$-\frac{(bc^2+acd)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (bc^2+acd+2\,ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc})}{3\,bd^2x}$$

input
```
integrate((a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((b*c^2 + a*c*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/
x), a*d/(b*c)) - (b*c^2 + a*c*d + 2*a*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic
_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c
)*(b*d^2*x^2 + b*c*d + a*d^2))/(b*d^2*x)
```

Sympy [F]

$$\int \sqrt{ac+(bc+ad)x^2+bdx^4}\,dx = \int \sqrt{ac+bdx^4+x^2\,(ad+bc)}\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x**2+b*d*x**4)**(1/2),x)
```

output
```
Integral(sqrt(a*c + b*d*x**4 + x**2*(a*d + b*c)), x)
```

Maxima [F]

$$\int \sqrt{ac+(bc+ad)x^2+bdx^4}\,dx = \int \sqrt{bdx^4+(bc+ad)x^2+ac}\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c), x)
```

Giac [F]

$$\int \sqrt{ac+(bc+ad)x^2+bdx^4}\,dx=\int \sqrt{bdx^4+(bc+ad)x^2+ac}\,dx$$

input
```
integrate((a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ac+(bc+ad)x^2+bdx^4}\,dx=\int \sqrt{b\,d\,x^4+(a\,d+b\,c)\,x^2+a\,c}\,dx$$

input
```
int((a*c + x^2*(a*d + b*c) + b*d*x^4)^(1/2),x)
```

output
```
int((a*c + x^2*(a*d + b*c) + b*d*x^4)^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \sqrt{ac+(bc+ad)x^2+bdx^4}\,dx=&\ \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x}{3}+\frac{\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)ad}{3}\\&+\frac{\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)bc}{3}\\&+\frac{2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)ac}{3}\end{aligned}$$

input
```
int((a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x + int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*d + int((sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b*c
 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*a*c)/3
```

3.15 $\int \frac{1}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{1}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx=\frac{\sqrt{a}(c+dx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

output
```
a^(1/2)*(d*x^2+c)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1
/2))/b^(1/2)/c/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(a*c+(a*d+b*c)*x^2+b*d*x^4)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx=\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right),\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{(a+bx^2)\,(c+dx^2)}}$$

input
```
Integrate[1/Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
 (a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[(a + b*x^2)*(c + d*x^2)])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.96, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2(ad+bc)+ac+bdx^4}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x^2(ad+bc)+ac+bdx^4}}$$

input
```
Int[1/Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4],x]
```

output
```
((Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d
*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c
]*Sqrt[d]))/4])/(2*a^(1/4)*b^(1/4)*c^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*
x^2 + b*d*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}$	87
elliptic	$\frac{\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}$	87

input
```
int(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx = -\frac{\sqrt{ac}\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc})}{bc}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c))/(b*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{ac + (bc + ad)x^2 + bdx^4}}\, dx = \int \frac{1}{\sqrt{ac + bdx^4 + x^2\,(ad + bc)}}\, dx$$

input
```
integrate(1/(a*c+(a*d+b*c)*x**2+b*d*x**4)**(1/2),x)
```

output
```
Integral(1/sqrt(a*c + b*d*x**4 + x**2*(a*d + b*c)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ac + (bc + ad)x^2 + bdx^4}}\, dx = \int \frac{1}{\sqrt{bdx^4 + (bc + ad)x^2 + ac}}\, dx$$

input
```
integrate(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{ac + (bc + ad)x^2 + bdx^4}}\, dx = \int \frac{1}{\sqrt{bdx^4 + (bc + ad)x^2 + ac}}\, dx$$

input
```
integrate(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx=\int \frac{1}{\sqrt{b\,d\,x^4+(a\,d+b\,c)\,x^2+a\,c}}\,dx$$

input
```
int(1/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(1/2),x)
```

output
```
int(1/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx$$

input
```
int(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x
**4),x)
```

3.16 $\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 267

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{3/2}}\,dx = \frac{bx}{a(bc-ad)\sqrt{ac+(bc+ad)x^2+bdx^4}} + \frac{\sqrt{d}(bc+ad)\sqrt{ac+(bc+ad)x^2+bdx^4}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,(c+dx^2)} - \frac{2\sqrt{a}\sqrt{b}d(c+dx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{c(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

output
```
b*x/a/(-a*d+b*c)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)+d^(1/2)*(a*d+b*c)*(a*c+
(a*d+b*c)*x^2+b*d*x^4)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2)
,(1-b*c/a/d)^(1/2))/a/c^(1/2)/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
/(d*x^2+c)-2*a^(1/2)*b^(1/2)*d*(d*x^2+c)*InverseJacobiAM(arctan(b^(1/2)*x/
a^(1/2)),(1-a*d/b/c)^(1/2))/c/(-a*d+b*c)^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.22 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.83

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}x(a^2d^2+abd^2x^2+b^2c(c+dx^2))+ibc(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{\right.}$$

input
```
Integrate[(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(-3/2),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*x*(a^2*d^2 + a*b*d^2*x^2 + b^2*c*(c + d*x^2)) + I*b*
c*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c*(b*c
 - a*d)^2*Sqrt[(a + b*x^2)*(c + d*x^2)])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 571 vs. 2(267) = 534.

Time = 0.64 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2(ad+bc)+ac+bdx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 1405$$

$$\frac{x(a^2d^2+bdx^2(ad+bc)+b^2c^2)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{\int \frac{bd((bc+ad)x^2+2ac)}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{ac(bc-ad)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{bd\int\frac{(bc+ad)x^2+2ac}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{ac(bc-ad)^2}$$

$$\Big\downarrow \text{1511}$$

$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{bd\left(\frac{\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\int\frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}(ad+bc)\int\frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{a}\sqrt{c}\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}\right)}{ac(bc-ad)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{bd\left(\frac{\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\int\frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{(ad+bc)\int\frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}\right)}{ac(bc-ad)^2}$$

$$\Big\downarrow \text{1416}$$

$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{bd\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{(ad+bc)\int \cdots}{\cdots}\right.}{ac(bc-ad)^2}$$

$$\Big\downarrow \text{1509}$$

$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{bd\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{(ad+bc)\left(\cdots\right.}{\cdots}\right.}{ac(bc-ad)^2}$$

input
```
Int[(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(-3/2),x]
```

output
```
(x*(b^2*c^2 + a^2*d^2 + b*d*(b*c + a*d)*x^2))/(a*c*(b*c - a*d)^2*Sqrt[a*c
+ (b*c + a*d)*x^2 + b*d*x^4]) - (b*d*(-(((b*c + a*d)*(-((x*Sqrt[a*c + (b*c
 + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)) + (a^(1/4
)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*
x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticE[2*ArcT
an[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[
b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d
*x^4])))/(Sqrt[b]*Sqrt[d])) + (a^(1/4)*c^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[a]*
Sqrt[d])^2*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)
*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticF[2*Arc
Tan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt
[b]*Sqrt[c]*Sqrt[d]))/4])/(2*b^(3/4)*d^(3/4)*Sqrt[a*c + (b*c + a*d)*x^2 +
b*d*x^4])))/(a*c*(b*c - a*d)^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.60

method	result
default	$-\frac{2bd\left(-\frac{(ad+bc)x^3}{2ac\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{\left(a^2d^2+b^2c^2\right)x}{2ac\left(a^2d^2-2abcd+b^2c^2\right)bd}\right)}{\sqrt{\left(x^4+\frac{(ad+bc)x^2}{bd}+\frac{ac}{bd}\right)bd}}+\frac{\left(\frac{1}{ac}-\frac{a^2d^2+b^2c^2}{ac\left(a^2d^2-2abcd+b^2c^2\right)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\ \text{EllipticF}\left(x\sqrt{\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}$
elliptic	$-\frac{2bd\left(-\frac{(ad+bc)x^3}{2ac\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{\left(a^2d^2+b^2c^2\right)x}{2ac\left(a^2d^2-2abcd+b^2c^2\right)bd}\right)}{\sqrt{\left(x^4+\frac{(ad+bc)x^2}{bd}+\frac{ac}{bd}\right)bd}}+\frac{\left(\frac{1}{ac}-\frac{a^2d^2+b^2c^2}{ac\left(a^2d^2-2abcd+b^2c^2\right)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\ \text{EllipticF}\left(x\sqrt{\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}$

input
```
int(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*d*(-1/2/a/c*(a*d+b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x^3-1/2*(a^2*d^2+b^
2*c^2)/a/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d*x)/((x^4+(a*d+b*c)/b/d*x^2+a*c/
b/d)*b*d)^(1/2)+(1/a/c-(a^2*d^2+b^2*c^2)/a/c/(a^2*d^2-2*a*b*c*d+b^2*c^2))/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/a*(a*d+b*c
)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*b/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.54

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{3/2}}\,dx = -\frac{(ab^2c^2+a^2bcd+(b^3cd+ab^2d^2)x^4+(b^3c^2+2\,ab^2cd+a^2bd^2)x^2)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(ab}{a^3b^2c^4}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x, algorithm="fricas")
```

output
```
-((a*b^2*c^2 + a^2*b*c*d + (b^3*c*d + a*b^2*d^2)*x^4 + (b^3*c^2 + 2*a*b^2*
c*d + a^2*b*d^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a))
, a*d/(b*c)) - (a*b^2*c^2 + (b^3*c*d + (2*a^2*b + a*b^2)*d^2)*x^4 + (2*a^3
 + a^2*b)*c*d + (b^3*c^2 + 2*(a^2*b + a*b^2)*c*d + (2*a^3 + a^2*b)*d^2)*x^
2)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - sqrt
(b*d*x^4 + (b*c + a*d)*x^2 + a*c)*((a*b^2*c*d + a^2*b*d^2)*x^3 + (a*b^2*c^
2 + a^3*d^2)*x))/(a^3*b^2*c^4 - 2*a^4*b*c^3*d + a^5*c^2*d^2 + (a^2*b^3*c^3
*d - 2*a^3*b^2*c^2*d^2 + a^4*b*c*d^3)*x^4 + (a^2*b^3*c^4 - a^3*b^2*c^3*d -
 a^4*b*c^2*d^2 + a^5*c*d^3)*x^2)
```

Sympy [F]

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{3/2}}\,dx = \int \frac{1}{(ac+bdx^4+x^2(ad+bc))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*c+(a*d+b*c)*x**2+b*d*x**4)**(3/2),x)
```

output
```
Integral((a*c + b*d*x**4 + x**2*(a*d + b*c))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(ac + (bc + ad)x^2 + bdx^4)^{3/2}}\,dx = \int \frac{1}{(bdx^4 + (bc + ad)x^2 + ac)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(ac + (bc + ad)x^2 + bdx^4)^{3/2}}\,dx = \int \frac{1}{(bdx^4 + (bc + ad)x^2 + ac)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ac + (bc + ad)x^2 + bdx^4)^{3/2}}\,dx = \int \frac{1}{(b\,d\,x^4 + (a\,d + b\,c)\,x^2 + a\,c)^{3/2}}\,dx$$

input
```
int(1/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(3/2),x)
```

output
```
int(1/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b^2d^2x^8+2ab\,d^2x^6+2b^2cd\,x^6+a^2d^2x^4+4abcd\,x^4+b^2c^2x^4+2a^2c}$$

input
```
int(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c**2 + 2*a**2*c*d*x**2 + a**
2*d**2*x**4 + 2*a*b*c**2*x**2 + 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c*
*2*x**4 + 2*b**2*c*d*x**6 + b**2*d**2*x**8),x)
```

3.17 $\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 414

$$\begin{aligned}\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{5/2}}\,dx &= \frac{x(b^2c^2+a^2d^2+bd(bc+ad)x^2)}{3ac(bc-ad)^2\,(ac+(bc+ad)x^2+bdx^4)^{3/2}}\\ &+ \frac{d(b^2c^2+9abcd-2a^2d^2)\,x}{3ac^2(bc-ad)^3\sqrt{ac+(bc+ad)x^2+bdx^4}}\\ &+ \frac{2\sqrt{b}(bc+ad)\,(b^2c^2-6abcd+a^2d^2)\,(c+dx^2)\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}c^2(bc-ad)^4\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{ac+(bc+ad)x^2+bdx^4}}\\ &- \frac{\sqrt{b}d(b^2c^2-18abcd+a^2d^2)\,(c+dx^2)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{a}c^2(bc-ad)^4\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{ac+(bc+ad)x^2+bdx^4}}\end{aligned}$$

output
```
1/3*x*(b^2*c^2+a^2*d^2+b*d*(a*d+b*c)*x^2)/a/c/(-a*d+b*c)^2/(a*c+(a*d+b*c)*
x^2+b*d*x^4)^(3/2)+1/3*d*(-2*a^2*d^2+9*a*b*c*d+b^2*c^2)*x/a/c^2/(-a*d+b*c)
^3/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)+2/3*b^(1/2)*(a*d+b*c)*(a^2*d^2-6*a*b*
c*d+b^2*c^2)*(d*x^2+c)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*
d/b/c)^(1/2))/a^(3/2)/c^2/(-a*d+b*c)^4/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(a*
c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)-1/3*b^(1/2)*d*(a^2*d^2-18*a*b*c*d+b^2*c^2)*
(d*x^2+c)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(
1/2)/c^2/(-a*d+b*c)^4/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(a*c+(a*d+b*c)*x^2+b
*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.58 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.85

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{5/2}}\,dx = \frac{i\left(i\sqrt{\frac{b}{a}}x\left(a^2cd^3(bc-ad)\left(a+bx^2\right)^2-2a^2d^3(-5bc+ad)\left(a+bx^2\right)^2(\right.\right.}{}$$

input
```
Integrate[(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(-5/2),x]
```

output
```
((I/3)*(I*Sqrt[b/a]*x*(a^2*c*d^3*(b*c - a*d)*(a + b*x^2)^2 - 2*a^2*d^3*(-5
*b*c + a*d)*(a + b*x^2)^2*(c + d*x^2) + a*b^3*c^2*(-(b*c) + a*d)*(c + d*x^
2)^2 - 2*b^3*c^2*(b*c - 5*a*d)*(a + b*x^2)*(c + d*x^2)^2) + b*c*(a + b*x^2
)*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*(2*(b^3*c^3 - 5*a*b^
2*c^2*d - 5*a^2*b*c*d^2 + a^3*d^3)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)
/(b*c)] - (2*b^3*c^3 - 11*a*b^2*c^2*d + 8*a^2*b*c*d^2 + a^3*d^3)*EllipticF
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(a^2*Sqrt[b/a]*c^2*(b*c - a*d)^4*
((a + b*x^2)*(c + d*x^2))^(3/2))
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 793, normalized size of antiderivative = 1.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1405, 25, 1492, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2(ad+bc)+ac+bdx^4)^{5/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(a^2d^2+bdx^2(ad+bc)+b^2c^2)}{3ac(bc-ad)^2(x^2(ad+bc)+ac+bdx^4)^{3/2}}-\frac{\int -\frac{3bd(bc+ad)x^2+2(b^2c^2-3abdc+a^2d^2)}{(bdx^4+(bc+ad)x^2+ac)^{3/2}}dx}{3ac(bc-ad)^2}$$

$\downarrow$ 25

$$\frac{\int \frac{3bd(bc+ad)x^2+2\left(b^2c^2-3abdc+a^2d^2\right)}{\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}}dx}{3ac(bc-ad)^2}+\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}$$

$\downarrow$ 1492

$$\frac{\frac{x\left(2a^4d^4-9a^3bcd^3+2bdx^2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)-2a^2b^2c^2d^2-9ab^3c^3d+2b^4c^4\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{\int \frac{bd\left(2(bc+ad)\left(b^2c^2-6abdc+a^2d^2\right)x^2+ac\left(b^2c^2-18abdc+a\cdots\right.\right.}{\sqrt{bdx^4+(bc+ad)x^2+ac}}\cdots}{ac(bc-ad)^2}}{3ac(bc-ad)^2}$$
$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{x\left(2a^4d^4-9a^3bcd^3+2bdx^2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)-2a^2b^2c^2d^2-9ab^3c^3d+2b^4c^4\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{bd\int \frac{2(bc+ad)\left(b^2c^2-6abdc+a^2d^2\right)x^2+ac\left(b^2c^2-18abdc+a^2\cdots\right.}{\sqrt{bdx^4+(bc+ad)x^2+ac}}\cdots}{ac(bc-ad)^2}}{3ac(bc-ad)^2}$$
$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}$$

$\downarrow$ 1511

$$\frac{\frac{x\left(2a^4d^4-9a^3bcd^3+2bdx^2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)-2a^2b^2c^2d^2-9ab^3c^3d+2b^4c^4\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{bd\left(\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{c}\left(a^2d^2-18abcd+b^2c^2\right)+\frac{2(ad+bc)\left(a^2\cdots\right.}{\cdots}\cdots\right.\right.}{\cdots}}{3ac(bc-ad)^2}$$
$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{x\left(2a^4d^4-9a^3bcd^3+2bdx^2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)-2a^2b^2c^2d^2-9ab^3c^3d+2b^4c^4\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{bd\left(\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{c}\left(a^2d^2-18abcd+b^2c^2\right)+\frac{2(ad+bc)\left(a^2\cdots\right.}{\cdots}\cdots\right.\right.}{\cdots}}{3ac(bc-ad)^2}$$
$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}$$

$\downarrow$ 1416

$$\frac{\frac{x\left(2a^4d^4-9a^3bcd^3+2bdx^2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)-2a^2b^2c^2d^2-9ab^3c^3d+2b^4c^4\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{bd\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}\left(a^2d^2-18abcd+b^2c^2\right)+\frac{2(ad+bc)\left(a^2\cdots\right.}{\sqrt{\cdots}}\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}$$

$$\Big\downarrow \textbf{1509}$$

$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}+$$

$$\frac{\frac{x\left(2a^4d^4-9a^3bcd^3+2bdx^2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)-2a^2b^2c^2d^2-9ab^3c^3d+2b^4c^4\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{bd\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}\left(a^2d^2-18abcd+b^2c^2\right)+\frac{2(ad+bc)\left(a^2\cdots\right.}{\sqrt{\cdots}}\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Int[(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(-5/2),x]
```

output
```
(x*(b^2*c^2 + a^2*d^2 + b*d*(b*c + a*d)*x^2))/(3*a*c*(b*c - a*d)^2*(a*c +
(b*c + a*d)*x^2 + b*d*x^4)^(3/2)) + ((x*(2*b^4*c^4 - 9*a*b^3*c^3*d - 2*a^2
*b^2*c^2*d^2 - 9*a^3*b*c*d^3 + 2*a^4*d^4 + 2*b*d*(b*c + a*d)*(b^2*c^2 - 6*
a*b*c*d + a^2*d^2)*x^2))/(a*c*(b*c - a*d)^2*Sqrt[a*c + (b*c + a*d)*x^2 + b
*d*x^4]) - (b*d*((-2*(b*c + a*d)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*(-((x*Sqr
t[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2
)) + (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c +
(b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(S
qrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*
d)*x^2 + b*d*x^4])))/(Sqrt[b]*Sqrt[d]) + (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c]
*(b^2*c^2 - 18*a*b*c*d + a^2*d^2) + (2*(b*c + a*d)*(b^2*c^2 - 6*a*b*c*d +
a^2*d^2))/(Sqrt[b]*Sqrt[d]))*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[
(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c +
a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(2*b^(1/4)*d^(1/4)*Sqrt[a*c +
(b*c + a*d)*x^2 + b*d*x^4])))/(a*c*(b*c - a*d)^2))/(3*a*c*(b*c - a*d)^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. $2(393) = 786$.

Time = 0.62 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.99

method	result
default	$\frac{\left(\frac{(ad+bc)x^3}{3ac\left(a^2d^2-2abcd+b^2c^2\right)bd}+\frac{\left(a^2d^2+b^2c^2\right)x}{3ac\left(a^2d^2-2abcd+b^2c^2\right)b^2d^2}\right)\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{\left(x^4+\frac{(ad+bc)x^2}{bd}+\frac{ac}{bd}\right)^2}-\frac{2bd\left(-\frac{(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)x^3}{3a^2c^2\left(a^2d^2-2abcd+b^2c^2\right)^2}-\frac{(2a^4}{}\right.}{\sqrt{\left(x^4+\frac{(ad+}{b}\right.}}$
elliptic	$\frac{\left(\frac{(ad+bc)x^3}{3ac\left(a^2d^2-2abcd+b^2c^2\right)bd}+\frac{\left(a^2d^2+b^2c^2\right)x}{3ac\left(a^2d^2-2abcd+b^2c^2\right)b^2d^2}\right)\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{\left(x^4+\frac{(ad+bc)x^2}{bd}+\frac{ac}{bd}\right)^2}-\frac{2bd\left(-\frac{(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)x^3}{3a^2c^2\left(a^2d^2-2abcd+b^2c^2\right)^2}-\frac{(2a^4}{}\right.}{\sqrt{\left(x^4+\frac{(ad+}{b}\right.}}$

input
```
int(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/3/a/c*(a*d+b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d*x^3+1/3*(a^2*d^2+b^2*c^
2)/a/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/d^2*x)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/(x^4+(a*d+b*c)/b/d*x^2+a*c/b/d)^2-2*b*d*(-1/3*(a*d+b*c)*(a^2*d^2-6
*a*b*c*d+b^2*c^2)/a^2/c^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2*x^3-1/6*(2*a^4*d^4
-9*a^3*b*c*d^3-2*a^2*b^2*c^2*d^2-9*a*b^3*c^3*d+2*b^4*c^4)/a^2/c^2/(a^2*d^2
-2*a*b*c*d+b^2*c^2)^2/b/d*x)/((x^4+(a*d+b*c)/b/d*x^2+a*c/b/d)*b*d)^(1/2)+(
2/3*(a^2*d^2-3*a*b*c*d+b^2*c^2)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^2/c^2-1/3*(2
*a^4*d^4-9*a^3*b*c*d^3-2*a^2*b^2*c^2*d^2-9*a*b^3*c^3*d+2*b^4*c^4)/a^2/c^2/
(a^2*d^2-2*a*b*c*d+b^2*c^2)^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))-(4/3*b*d*(a^3*d^3-5*a^2*b*c*d^2-5*a*b^2*c^2*d+b^3*c^3)/
(a^2*d^2-2*a*b*c*d+b^2*c^2)^2/a^2/c^2-2*b*d*(a*d+b*c)*(a^2*d^2-6*a*b*c*d+b
^2*c^2)/a^2/c^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+
b*c)/c/b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1461 vs. $2(393) = 786$.

Time = 0.18 (sec) , antiderivative size = 1461, normalized size of antiderivative = 3.53

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(a^2*b^4*c^5 - 5*a^3*b^3*c^4*d - 5*a^4*b^2*c^3*d^2 + a^5*b*c^2*d^3
 + (b^6*c^3*d^2 - 5*a*b^5*c^2*d^3 - 5*a^2*b^4*c*d^4 + a^3*b^3*d^5)*x^8 + 2
*(b^6*c^4*d - 4*a*b^5*c^3*d^2 - 10*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^4
*b^2*d^5)*x^6 + (b^6*c^5 - a*b^5*c^4*d - 24*a^2*b^4*c^3*d^2 - 24*a^3*b^3*c
^2*d^3 - a^4*b^2*c*d^4 + a^5*b*d^5)*x^4 + 2*(a*b^5*c^5 - 4*a^2*b^4*c^4*d -
 10*a^3*b^3*c^3*d^2 - 4*a^4*b^2*c^2*d^3 + a^5*b*c*d^4)*x^2)*sqrt(a*c)*sqrt
(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*a^2*b^4*c^5 + (2*b
^6*c^3*d^2 + (a^2*b^4 - 10*a*b^5)*c^2*d^3 - 2*(9*a^3*b^3 + 5*a^2*b^4)*c*d^
4 + (a^4*b^2 + 2*a^3*b^3)*d^5)*x^8 + 2*(2*b^6*c^4*d + (a^2*b^4 - 8*a*b^5)*
c^3*d^2 - (17*a^3*b^3 + 20*a^2*b^4)*c^2*d^3 - (17*a^4*b^2 + 8*a^3*b^3)*c*d
^4 + (a^5*b + 2*a^4*b^2)*d^5)*x^6 + (a^4*b^2 - 10*a^3*b^3)*c^4*d - 2*(9*a^
5*b + 5*a^4*b^2)*c^3*d^2 + (a^6 + 2*a^5*b)*c^2*d^3 + (2*b^6*c^5 + (a^2*b^4
 - 2*a*b^5)*c^4*d - 2*(7*a^3*b^3 + 24*a^2*b^4)*c^3*d^2 - 2*(35*a^4*b^2 + 2
4*a^3*b^3)*c^2*d^3 - 2*(7*a^5*b + a^4*b^2)*c*d^4 + (a^6 + 2*a^5*b)*d^5)*x^
4 + 2*(2*a*b^5*c^5 + (a^3*b^3 - 8*a^2*b^4)*c^4*d - (17*a^4*b^2 + 20*a^3*b^
3)*c^3*d^2 - (17*a^5*b + 8*a^4*b^2)*c^2*d^3 + (a^6 + 2*a^5*b)*c*d^4)*x^2)*
sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*(a*b
^5*c^3*d^2 - 5*a^2*b^4*c^2*d^3 - 5*a^3*b^3*c*d^4 + a^4*b^2*d^5)*x^7 + (4*a
*b^5*c^4*d - 17*a^2*b^4*c^3*d^2 - 22*a^3*b^3*c^2*d^3 - 17*a^4*b^2*c*d^4 +
4*a^5*b*d^5)*x^5 + 2*(a*b^5*c^5 - 2*a^2*b^4*c^4*d - 11*a^3*b^3*c^3*d^2 ...
```

Sympy [F]

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{5/2}}\,dx=\int \frac{1}{(ac+bdx^4+x^2\,(ad+bc))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*c+(a*d+b*c)*x**2+b*d*x**4)**(5/2),x)
```

output
```
Integral((a*c + b*d*x**4 + x**2*(a*d + b*c))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{5/2}}\,dx = \int \frac{1}{(bdx^4+(bc+ad)x^2+ac)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{5/2}}\,dx = \int \frac{1}{(bdx^4+(bc+ad)x^2+ac)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{5/2}}\,dx = \int \frac{1}{(b\,d\,x^4+(a\,d+b\,c)\,x^2+a\,c)^{5/2}}\,dx$$

input
```
int(1/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(5/2),x)
```

output
```
int(1/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(ac+(bc+ad)x^2+bdx^4\right)^{5/2}}\,dx = \int \frac{}{b^3d^3x^{12}+3a\,b^2d^3x^{10}+3b^3c\,d^2x^{10}+3a^2b\,d^3x^8+9a\,b^2c\,d^2x^8+3b^3c}$$

input
```
int(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**3*c**3 + 3*a**3*c**2*d*x**2 +
3*a**3*c*d**2*x**4 + a**3*d**3*x**6 + 3*a**2*b*c**3*x**2 + 9*a**2*b*c**2*d
*x**4 + 9*a**2*b*c*d**2*x**6 + 3*a**2*b*d**3*x**8 + 3*a*b**2*c**3*x**4 + 9
*a*b**2*c**2*d*x**6 + 9*a*b**2*c*d**2*x**8 + 3*a*b**2*d**3*x**10 + b**3*c*
*3*x**6 + 3*b**3*c**2*d*x**8 + 3*b**3*c*d**2*x**10 + b**3*d**3*x**12),x)
```

3.18 $\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 632

$$\begin{aligned}\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{7/2}}\,dx &= \frac{x(b^2c^2+a^2d^2+bd(bc+ad)x^2)}{5ac(bc-ad)^2(ac+(bc+ad)x^2+bdx^4)^{5/2}}\\
&+ \frac{x(4b^4c^4-17ab^3c^3d-6a^2b^2c^2d^2-17a^3bcd^3+4a^4d^4-4bd(bc+ad)(8abcd-(bc+ad)^2)x^2)}{15a^2c^2(bc-ad)^4(ac+(bc+ad)x^2+bdx^4)^{3/2}}\\
&+ \frac{d(4b^4c^4-23ab^3c^3d-150a^2b^2c^2d^2+49a^3bcd^3-8a^4d^4)x}{15a^2c^3(bc-ad)^5\sqrt{ac+(bc+ad)x^2+bdx^4}}\\
&+ \frac{\sqrt{b}(bc+ad)(8b^4c^4-61ab^3c^3d+234a^2b^2c^2d^2-61a^3bcd^3+8a^4d^4)(c+dx^2)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}c^3(bc-ad)^6\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{ac+(bc+ad)x^2+bdx^4}}\\
&- \frac{2\sqrt{b}d(2b^4c^4-13ab^3c^3d+150a^2b^2c^2d^2-13a^3bcd^3+2a^4d^4)(c+dx^2)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15a^{3/2}c^3(bc-ad)^6\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{ac+(bc+ad)x^2+bdx^4}}\end{aligned}$$

output
```
1/5*x*(b^2*c^2+a^2*d^2+b*d*(a*d+b*c)*x^2)/a/c/(-a*d+b*c)^2/(a*c+(a*d+b*c)*
x^2+b*d*x^4)^(5/2)+1/15*x*(4*b^4*c^4-17*a*b^3*c^3*d-6*a^2*b^2*c^2*d^2-17*a
^3*b*c*d^3+4*a^4*d^4-4*b*d*(a*d+b*c)*(8*a*b*c*d-(a*d+b*c)^2)*x^2)/a^2/c^2/
(-a*d+b*c)^4/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2)+1/15*d*(-8*a^4*d^4+49*a^3*b
*c*d^3-150*a^2*b^2*c^2*d^2-23*a*b^3*c^3*d+4*b^4*c^4)*x/a^2/c^3/(-a*d+b*c)^
5/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)+1/15*b^(1/2)*(a*d+b*c)*(8*a^4*d^4-61*a
^3*b*c*d^3+234*a^2*b^2*c^2*d^2-61*a*b^3*c^3*d+8*b^4*c^4)*(d*x^2+c)*Ellipti
cE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(5/2)/c^3/(-a*
d+b*c)^6/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)
-2/15*b^(1/2)*d*(2*a^4*d^4-13*a^3*b*c*d^3+150*a^2*b^2*c^2*d^2-13*a*b^3*c^3
*d+2*b^4*c^4)*(d*x^2+c)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b
/c)^(1/2))/a^(3/2)/c^3/(-a*d+b*c)^6/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(a*c+(
a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.77 (sec) , antiderivative size = 554, normalized size of antiderivative = 0.88

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{7/2}}\,dx = \frac{\sqrt{(a+bx^2)(c+dx^2)}\left(\sqrt{\frac{b}{a}}x\left(3a^3c^2d^4(bc-ad)^2(a+bx^2)^3+a^3cd^4(-\right.\right.}{}$$

input
```
Integrate[(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(-7/2),x]
```

output
```
(Sqrt[(a + b*x^2)*(c + d*x^2)]*(Sqrt[b/a]*x*(3*a^3*c^2*d^4*(b*c - a*d)^2*(
a + b*x^2)^3 + a^3*c*d^4*(-(b*c) + a*d)*(-23*b*c + 4*a*d)*(a + b*x^2)^3*(c
 + d*x^2) + a^3*d^4*(173*b^2*c^2 - 53*a*b*c*d + 8*a^2*d^2)*(a + b*x^2)^3*(
c + d*x^2)^2 + 3*a^2*b^4*c^3*(b*c - a*d)^2*(c + d*x^2)^3 + a*b^4*c^3*(-(b*
c) + a*d)*(-4*b*c + 23*a*d)*(a + b*x^2)*(c + d*x^2)^3 + b^4*c^3*(8*b^2*c^2
 - 53*a*b*c*d + 173*a^2*d^2)*(a + b*x^2)^2*(c + d*x^2)^3) + I*b*c*(a + b*x
^2)^2*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*((8*b^5*c^5 -
53*a*b^4*c^4*d + 173*a^2*b^3*c^3*d^2 + 173*a^3*b^2*c^2*d^3 - 53*a^4*b*c*d^
4 + 8*a^5*d^5)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-8*b^5*c^
5 + 57*a*b^4*c^4*d - 199*a^2*b^3*c^3*d^2 + 127*a^3*b^2*c^2*d^3 + 27*a^4*b*
c*d^4 - 4*a^5*d^5)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(15*a
^3*Sqrt[b/a]*c^3*(b*c - a*d)^6*(a + b*x^2)^3*(c + d*x^2)^3)
```

Rubi [A] (verified)

Time = 2.32 (sec) , antiderivative size = 1093, normalized size of antiderivative = 1.73, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1405, 25, 1492, 25, 1492, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x^2(ad+bc)+ac+bdx^4\right)^{7/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{5ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}}-\frac{\int -\frac{7bd(bc+ad)x^2+2(bc-2ad)(2bc-ad)}{\left(bdx^4+(bc+ad)x^2+ac\right)^{5/2}}dx}{5ac(bc-ad)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7bd(bc+ad)x^2+2(bc-2ad)(2bc-ad)}{\left(bdx^4+(bc+ad)x^2+ac\right)^{5/2}}dx}{5ac(bc-ad)^2}+\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{5ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}}$$

$$\Big\downarrow \text{1492}$$

$$\frac{\frac{x\left(4a^4d^4-17a^3bcd^3+4bdx^2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)-6a^2b^2c^2d^2-17ab^3c^3d+4b^4c^4\right)}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}-\frac{\int -\frac{8b^4c^4-37ab^3dc^3+90a^2b^2d^2c^2-37a^3bd^3c+8a^4d^4+12\ldots}{\left(bdx^4+(bc+ad)x^2+ac\right)\ldots}\ldots}{3ac(bc-ad)^2}}{5ac(bc-ad)^2}+\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{5ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{8b^4c^4-37ab^3dc^3+90a^2b^2d^2c^2-37a^3bd^3c+8a^4d^4+12bd(bc+ad)\left(b^2c^2-6abdc+a^2d^2\right)x^2}{\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}}dx}{3ac(bc-ad)^2}+\frac{x\left(4a^4d^4-17a^3bcd^3+4bdx^2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\ldots\right.}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+b\ldots\right.}}{5ac(bc-ad)^2}+\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{5ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}}$$

$$\Big\downarrow \text{1492}$$

$$\frac{\frac{x\left(8a^6d^6-49a^5bcd^5+146a^4b^2c^2d^4+46a^3b^3c^3d^3+146a^2b^4c^4d^2+bdx^2(ad+bc)\left(8a^4d^4-61a^3bcd^3+234a^2b^2c^2d^2-61ab^3c^3d+8b^4c^4\right)-49ab^5c^5d+8b^6c^6\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{\int\frac{bd(\cdots}{\cdots}}{\cdots}}{3ac(bc-ad)^2}\cdots$$

$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{5ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}}$$

$\downarrow$ 27

$$\frac{\frac{x\left(8a^6d^6-49a^5bcd^5+146a^4b^2c^2d^4+46a^3b^3c^3d^3+146a^2b^4c^4d^2+bdx^2(ad+bc)\left(8a^4d^4-61a^3bcd^3+234a^2b^2c^2d^2-61ab^3c^3d+8b^4c^4\right)-49ab^5c^5d+8b^6c^6\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{bd\int\frac{(b\cdots}{\cdots}}{\cdots}}{3ac(bc-ad)^2}\cdots$$

$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{5ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}}$$

$\downarrow$ 1511

$$\frac{\frac{x\left(8a^6d^6-49a^5bcd^5+146a^4b^2c^2d^4+46a^3b^3c^3d^3+146a^2b^4c^4d^2+bdx^2(ad+bc)\left(8a^4d^4-61a^3bcd^3+234a^2b^2c^2d^2-61ab^3c^3d+8b^4c^4\right)-49ab^5c^5d+8b^6c^6\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{bd\left(\sqrt{a}\cdots\right)}{\cdots}}{\cdots}$$

$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{5ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}}$$

$\downarrow$ 27

$$\frac{\frac{x\left(8a^6d^6-49a^5bcd^5+146a^4b^2c^2d^4+46a^3b^3c^3d^3+146a^2b^4c^4d^2+bdx^2(ad+bc)\left(8a^4d^4-61a^3bcd^3+234a^2b^2c^2d^2-61ab^3c^3d+8b^4c^4\right)-49ab^5c^5d+8b^6c^6\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{bd\left(\sqrt{a}\cdots\right)}{\cdots}}{\cdots}$$

$$\frac{x\left(a^2d^2+bdx^2(ad+bc)+b^2c^2\right)}{5ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}}$$

$\downarrow$ 1416

$$\frac{x\left(b^2c^2+a^2d^2+bd(bc+ad)x^2\right)}{5ac(bc-ad)^2\left(bdx^4+(bc+ad)x^2+ac\right)^{5/2}}+$$

$$\frac{\frac{x\left(4b^4c^4-17ab^3dc^3-6a^2b^2d^2c^2-17a^3bd^3c+4a^4d^4+4bd(bc+ad)\left(b^2c^2-6abdc+a^2d^2\right)x^2\right)}{3ac(bc-ad)^2\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}}+\frac{x\left(8b^6c^6-49ab^5dc^5+146a^2b^4d^2c^4+46a^3b^3d^3c^3+146a^4b^2\right.}{}}{}$$

$\downarrow$ 1509

$$\frac{x\left(b^2c^2+a^2d^2+bd(bc+ad)x^2\right)}{5ac(bc-ad)^2\left(bdx^4+(bc+ad)x^2+ac\right)^{5/2}}+$$

$$\frac{\frac{x\left(4b^4c^4-17ab^3dc^3-6a^2b^2d^2c^2-17a^3bd^3c+4a^4d^4+4bd(bc+ad)\left(b^2c^2-6abdc+a^2d^2\right)x^2\right)}{3ac(bc-ad)^2\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}}+\frac{x\left(8b^6c^6-49ab^5dc^5+146a^2b^4d^2c^4+46a^3b^3d^3c^3+146a^4b^2\right.}{}}{}$$

input
```
Int[(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(-7/2),x]
```

output
```
(x*(b^2*c^2 + a^2*d^2 + b*d*(b*c + a*d)*x^2))/(5*a*c*(b*c - a*d)^2*(a*c +
(b*c + a*d)*x^2 + b*d*x^4)^(5/2)) + ((x*(4*b^4*c^4 - 17*a*b^3*c^3*d - 6*a^
2*b^2*c^2*d^2 - 17*a^3*b*c*d^3 + 4*a^4*d^4 + 4*b*d*(b*c + a*d)*(b^2*c^2 -
6*a*b*c*d + a^2*d^2)*x^2))/(3*a*c*(b*c - a*d)^2*(a*c + (b*c + a*d)*x^2 + b
*d*x^4)^(3/2)) + ((x*(8*b^6*c^6 - 49*a*b^5*c^5*d + 146*a^2*b^4*c^4*d^2 + 4
6*a^3*b^3*c^3*d^3 + 146*a^4*b^2*c^2*d^4 - 49*a^5*b*c*d^5 + 8*a^6*d^6 + b*d
*(b*c + a*d)*(8*b^4*c^4 - 61*a*b^3*c^3*d + 234*a^2*b^2*c^2*d^2 - 61*a^3*b*
c*d^3 + 8*a^4*d^4)*x^2))/(a*c*(b*c - a*d)^2*Sqrt[a*c + (b*c + a*d)*x^2 + b
*d*x^4]) - (b*d*(-(((b*c + a*d)*(8*b^4*c^4 - 61*a*b^3*c^3*d + 234*a^2*b^2*
c^2*d^2 - 61*a^3*b*c*d^3 + 8*a^4*d^4)*(-((x*Sqrt[a*c + (b*c + a*d)*x^2 + b
*d*x^4])/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)) + (a^(1/4)*c^(1/4)*(Sqrt
[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/
(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*d^(
1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt
[d]))/4])/(b^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])))/(Sqrt[
b]*Sqrt[d])) + (a^(1/4)*c^(1/4)*(2*Sqrt[a]*Sqrt[c]*(2*b^4*c^4 - 13*a*b^3*c
^3*d + 150*a^2*b^2*c^2*d^2 - 13*a^3*b*c*d^3 + 2*a^4*d^4) + ((b*c + a*d)*(8
*b^4*c^4 - 61*a*b^3*c^3*d + 234*a^2*b^2*c^2*d^2 - 61*a^3*b*c*d^3 + 8*a^4*d
^4))/(Sqrt[b]*Sqrt[d]))*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c
+ (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 1124, normalized size of antiderivative = 1.78

method	result	size
default	Expression too large to display	1124
elliptic	Expression too large to display	1124

input
```
int(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(1/5/a/c*(a*d+b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/d^2*x^3+1/5*(a^2*d^2+b^
2*c^2)/a/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/d^3*x)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)/(x^4+(a*d+b*c)/b/d*x^2+a*c/b/d)^3+(4/15*(a*d+b*c)/b/d*(a^2*d^2
-6*a*b*c*d+b^2*c^2)/a^2/c^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2*x^3+1/15*(4*a^4*
d^4-17*a^3*b*c*d^3-6*a^2*b^2*c^2*d^2-17*a*b^3*c^3*d+4*b^4*c^4)/a^2/c^2/(a^
2*d^2-2*a*b*c*d+b^2*c^2)^2/b^2/d^2*x)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/
(x^4+(a*d+b*c)/b/d*x^2+a*c/b/d)^2-2*b*d*(-1/30*(8*a^5*d^5-53*a^4*b*c*d^4+1
73*a^3*b^2*c^2*d^3+173*a^2*b^3*c^3*d^2-53*a*b^4*c^4*d+8*b^5*c^5)/a^3/c^3/(
a^2*d^2-2*a*b*c*d+b^2*c^2)^3*x^3-1/30*(8*a^6*d^6-49*a^5*b*c*d^5+146*a^4*b^
2*c^2*d^4+46*a^3*b^3*c^3*d^3+146*a^2*b^4*c^4*d^2-49*a*b^5*c^5*d+8*b^6*c^6)
/a^3/c^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)^3/b/d*x)/((x^4+(a*d+b*c)/b/d*x^2+a*c/
b/d)*b*d)^(1/2)+(1/15/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2*(8*a^4*d^4-37*a^3*b*c*
d^3+90*a^2*b^2*c^2*d^2-37*a*b^3*c^3*d+8*b^4*c^4)/a^3/c^3-1/15*(8*a^6*d^6-4
9*a^5*b*c*d^5+146*a^4*b^2*c^2*d^4+46*a^3*b^3*c^3*d^3+146*a^2*b^4*c^4*d^2-4
9*a*b^5*c^5*d+8*b^6*c^6)/a^3/c^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)^3)/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/15*b*(8*a^5*d^5-53*a
^4*b*c*d^4+173*a^3*b^2*c^2*d^3+173*a^2*b^3*c^3*d^2-53*a*b^4*c^4*d+8*b^5*c^
5)/(a^2*d^2-2*a*b*c*d+b^2*c^2)^3/a^3/c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3257 vs. $2(607) = 1214$.

Time = 0.69 (sec) , antiderivative size = 3257, normalized size of antiderivative = 5.15

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(7/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{7/2}}\,dx = \int \frac{1}{(ac+bdx^4+x^2\,(ad+bc))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*c+(a*d+b*c)*x**2+b*d*x**4)**(7/2),x)
```

output
```
Integral((a*c + b*d*x**4 + x**2*(a*d + b*c))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{7/2}}\,dx = \int \frac{1}{(bdx^4+(bc+ad)x^2+ac)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{7/2}}\,dx = \int \frac{1}{(bdx^4+(bc+ad)x^2+ac)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{7/2}}\,dx=\int \frac{1}{\left(b\,d\,x^4+\left(a\,d+b\,c\right)\,x^2+a\,c\right)^{7/2}}\,dx$$

input
```
int(1/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(7/2),x)
```

output
```
int(1/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(ac+(bc+ad)x^2+bdx^4)^{7/2}}\,dx=\int \frac{}{b^4d^4x^{16}+4a\,b^3d^4x^{14}+4b^4c\,d^3x^{14}+6a^2b^2d^4x^{12}+16a\,b^3c\,d^3x^{12}+}$$

input
```
int(1/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(7/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**4*c**4 + 4*a**4*c**3*d*x**2 +
6*a**4*c**2*d**2*x**4 + 4*a**4*c*d**3*x**6 + a**4*d**4*x**8 + 4*a**3*b*c**
4*x**2 + 16*a**3*b*c**3*d*x**4 + 24*a**3*b*c**2*d**2*x**6 + 16*a**3*b*c*d*
*3*x**8 + 4*a**3*b*d**4*x**10 + 6*a**2*b**2*c**4*x**4 + 24*a**2*b**2*c**3*
d*x**6 + 36*a**2*b**2*c**2*d**2*x**8 + 24*a**2*b**2*c*d**3*x**10 + 6*a**2*
b**2*d**4*x**12 + 4*a*b**3*c**4*x**6 + 16*a*b**3*c**3*d*x**8 + 24*a*b**3*c
**2*d**2*x**10 + 16*a*b**3*c*d**3*x**12 + 4*a*b**3*d**4*x**14 + b**4*c**4*
x**8 + 4*b**4*c**3*d*x**10 + 6*b**4*c**2*d**2*x**12 + 4*b**4*c*d**3*x**14
+ b**4*d**4*x**16),x)
```

3.19 $\int \left(ac+(bc-ad)x^2-bdx^4\right)^{5/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 542

$$\int \left(ac+(bc-ad)x^2-bdx^4\right)^{5/2}\,dx =$$

$$-\frac{x(4b^4c^4+11ab^3c^3d-210a^2b^2c^2d^2+11a^3bcd^3+4a^4d^4-12bd(bc-ad)\left(8abcd+(bc-ad)^2\right)x^2)\sqrt{ac+(}}{693b^2d^2}$$

$$+\frac{5x(3(6abcd-(bc-ad)^2)+7bd(bc-ad)x^2)\left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}}{693bd}$$

$$+\frac{1}{11}x\left(ac+(bc-ad)x^2-bdx^4\right)^{5/2}$$

$$+\frac{a\sqrt{c}(bc-ad)\left(8b^4c^4+61ab^3c^3d+234a^2b^2c^2d^2+61a^3bcd^3+8a^4d^4\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right|}{693b^3d^{5/2}\sqrt{ac+(bc-ad)x^2-bdx^4}}$$

$$-\frac{a\sqrt{c}(bc+ad)\left(4b^4c^4+23ab^3c^3d-150a^2b^2c^2d^2-49a^3bcd^3-8a^4d^4\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\mathrm{EllipticF}\left(\mathrm{arcsi}\right.}{693b^3d^{5/2}\sqrt{ac+(bc-ad)x^2-bdx^4}}$$

output
```
-1/693*x*(4*b^4*c^4+11*a*b^3*c^3*d-210*a^2*b^2*c^2*d^2+11*a^3*b*c*d^3+4*a^
4*d^4-12*b*d*(-a*d+b*c)*(8*a*b*c*d+(-a*d+b*c)^2)*x^2)*(a*c+(-a*d+b*c)*x^2-
b*d*x^4)^(1/2)/b^2/d^2+5/693*x*(18*a*b*c*d-3*(-a*d+b*c)^2+7*b*d*(-a*d+b*c)
*x^2)*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2)/b/d+1/11*x*(a*c+(-a*d+b*c)*x^2-b*
d*x^4)^(5/2)+1/693*a*c^(1/2)*(-a*d+b*c)*(8*a^4*d^4+61*a^3*b*c*d^3+234*a^2*
b^2*c^2*d^2+61*a*b^3*c^3*d+8*b^4*c^4)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*
EllipticE(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/b^3/d^(5/2)/(a*c+(-a*d+b*c)*
x^2-b*d*x^4)^(1/2)-1/693*a*c^(1/2)*(a*d+b*c)*(-8*a^4*d^4-49*a^3*b*c*d^3-15
0*a^2*b^2*c^2*d^2+23*a*b^3*c^3*d+4*b^4*c^4)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^
(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/b^3/d^(5/2)/(a*c+(-a*d
+b*c)*x^2-b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.74 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.85

$$\int (ac+(bc-ad)x^2 -bdx^4)^{5/2}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(-c+dx^2\right)\left(4a^4d^4+a^3bd^3(26c-3dx^2)+a^2b^2d^2(-393c^2+356cdx^2-113\ldots\right.}{}$$

input
```
Integrate[(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(5/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(-c + d*x^2)*(4*a^4*d^4 + a^3*b*d^3*(26*c - 3*d
*x^2) + a^2*b^2*d^2*(-393*c^2 + 356*c*d*x^2 - 113*d^2*x^4) + a*b^3*d*(26*c
^3 - 356*c^2*d*x^2 + 442*c*d^2*x^4 - 161*d^3*x^6) + b^4*(4*c^4 + 3*c^3*d*x
^2 - 113*c^2*d^2*x^4 + 161*c*d^3*x^6 - 63*d^4*x^8)) - I*c*(-8*b^5*c^5 - 53
*a*b^4*c^4*d - 173*a^2*b^3*c^3*d^2 + 173*a^3*b^2*c^2*d^3 + 53*a^4*b*c*d^4
+ 8*a^5*d^5)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[S
qrt[b/a]*x], -((a*d)/(b*c))] - I*c*(8*b^5*c^5 + 57*a*b^4*c^4*d + 199*a^2*b
^3*c^3*d^2 + 127*a^3*b^2*c^2*d^3 - 27*a^4*b*c*d^4 - 4*a^5*d^5)*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*
c))])/(693*a^2*(b/a)^(5/2)*d^3*Sqrt[(a + b*x^2)*(c - d*x^2)])
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 508, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1404, 1490, 25, 1490, 25, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2(bc-ad)+ac-bdx^4\right)^{5/2} dx$$

$$\Big\downarrow \text{ 1404}$$

$$\frac{5}{11}\int \left((bc-ad)x^2+2ac\right)\left(-bdx^4+(bc-ad)x^2+ac\right)^{3/2}dx+\frac{1}{11}x\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}$$

$$\Big\downarrow \text{ 1490}$$

$$\frac{5}{11}\left(\frac{x\left(7bdx^2(bc-ad)+3\left(6abcd-(bc-ad)^2\right)\right)\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}{63bd}-\frac{\int -\Big(\big(4(bc-ad)\left((bc-ad)^2+\right.}{}\right.$$
$$\frac{1}{11}x\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{5}{11}\left(\frac{\int \left(4(bc-ad)\left((bc-ad)^2+8abcd\right)x^2+ac\left((bc-ad)^2+36abcd\right)\right)\sqrt{-bdx^4+(bc-ad)x^2+ac}dx}{21bd}+\frac{x(7bdx}{}\right.$$
$$\frac{1}{11}x\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}$$

$$\Big\downarrow \text{ 1490}$$

$$\frac{5}{11}\left(\frac{-\frac{\int -\frac{(bc-ad)\left(8b^4c^4+61ab^3dc^3+234a^2b^2d^2c^2+61a^3bd^3c+8a^4d^4\right)x^2+2ac\left(2b^4c^4+13ab^3dc^3+150a^2b^2d^2c^2+13a^3bd^3c+2a^4d^4\right)}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{15bd}-\frac{x\sqrt{x^2(bc-ad)}}{}}{21bd}\right.$$
$$\frac{1}{11}x\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{5}{11}\left(\frac{\frac{\int \frac{(bc-ad)\left(8b^4c^4+61ab^3dc^3+234a^2b^2d^2c^2+61a^3bd^3c+8a^4d^4\right)x^2+2ac\left(2b^4c^4+13ab^3dc^3+150a^2b^2d^2c^2+13a^3bd^3c+2a^4d^4\right)}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{15bd}-\frac{x\left(4a^4d^4+11a^3bcd^3\right.}{}}{21bd}\right.$$

$$\frac{1}{11}x\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}$$

$\downarrow$ 1514

$$\frac{5}{11}\left(\frac{\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{(bc-ad)\left(8b^4c^4+61ab^3dc^3+234a^2b^2d^2c^2+61a^3bd^3c+8a^4d^4\right)x^2+2ac\left(2b^4c^4+13ab^3dc^3+150a^2b^2d^2c^2+13a^3bd^3c+2a^4d^4\right)}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{15bd\sqrt{x^2(bc-ad)+ac-bdx^4}}-}{21bd}\right.$$

$$\frac{1}{11}x\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}$$

$\downarrow$ 399

$$\frac{5}{11}\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a(bc-ad)\left(8a^4d^4+61a^3bcd^3+234a^2b^2c^2d^2+61ab^3c^3d+8b^4c^4\right)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}-\frac{a(ad+bc)\left(-8a^4d^4-49a^3bcd^3-150a^2b^2c^2d^2+23a\right.}{b}\right.}{15bd\sqrt{x^2(bc-ad)+ac-bdx^4}}\right.$$

$$\frac{1}{11}x\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}$$

$\downarrow$ 321

$$\frac{5}{11}\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a(bc-ad)\left(8a^4d^4+61a^3bcd^3+234a^2b^2c^2d^2+61ab^3c^3d+8b^4c^4\right)\int\frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}-\frac{a\sqrt{c}(ad+bc)\left(-8a^4d^4-49a^3bcd^3-150a^2b^2c^2d^2+2\right.}{b\sqrt{}}\right.}{15bd\sqrt{x^2(bc-ad)+ac-bdx^4}}\right.$$

$$\frac{1}{11}x\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}$$

$$\Big\downarrow \; 327$$

$$\frac{5}{11}\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a\sqrt{c}(bc-ad)\left(8a^4d^4+61a^3bcd^3+234a^2b^2c^2d^2+61ab^3c^3d+8b^4c^4\right)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}}-\frac{a\sqrt{c}(ad+bc)\left(-8a^4d^4-49a^3bcd^3-150\right.}{}\right.}{15bd\sqrt{x^2(bc-ad)+ac-bdx^4}}\right.$$

$$\frac{1}{11}x\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}$$

input
```
Int[(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(5/2),x]
```

output
```
(x*(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(5/2))/11 + (5*((x*(3*(6*a*b*c*d - (b
*c - a*d)^2) + 7*b*d*(b*c - a*d)*x^2)*(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(3
/2))/(63*b*d) + (-1/15*(x*(4*b^4*c^4 + 11*a*b^3*c^3*d - 210*a^2*b^2*c^2*d^
2 + 11*a^3*b*c*d^3 + 4*a^4*d^4 - 12*b*d*(b*c - a*d)*(8*a*b*c*d + (b*c - a*
d)^2)*x^2)*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4])/(b*d) + (Sqrt[1 + (b*x^2
)/a]*Sqrt[1 - (d*x^2)/c]*((a*Sqrt[c]*(b*c - a*d)*(8*b^4*c^4 + 61*a*b^3*c^3
*d + 234*a^2*b^2*c^2*d^2 + 61*a^3*b*c*d^3 + 8*a^4*d^4)*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]) - (a*Sqrt[c]*(b*c + a*d)*
(4*b^4*c^4 + 23*a*b^3*c^3*d - 150*a^2*b^2*c^2*d^2 - 49*a^3*b*c*d^3 - 8*a^4
*d^4)*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]))
)/(15*b*d*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4]))/(21*b*d)))/11
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 923, normalized size of antiderivative = 1.70

method	result
risch	$-\frac{x\left(-63b^4d^4x^8-161a\,b^3d^4x^6+161b^4c\,d^3x^6-113a^2b^2d^4x^4+442a\,b^3c\,d^3x^4-113b^4c^2d^2x^4-3a^3b\,d^4x^2+356a^2b^2c\,d^3x^2-356a\,b^3c^2d^2\right.}{693b^2d^2\sqrt{-(b\,x^2+a)(d\,x^2-c)}}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/693/b^2/d^2*x*(-63*b^4*d^4*x^8-161*a*b^3*d^4*x^6+161*b^4*c*d^3*x^6-113*
a^2*b^2*d^4*x^4+442*a*b^3*c*d^3*x^4-113*b^4*c^2*d^2*x^4-3*a^3*b*d^4*x^2+35
6*a^2*b^2*c*d^3*x^2-356*a*b^3*c^2*d^2*x^2+3*b^4*c^3*d*x^2+4*a^4*d^4+26*a^3
*b*c*d^3-393*a^2*b^2*c^2*d^2+26*a*b^3*c^3*d+4*b^4*c^4)*(b*x^2+a)*(-d*x^2+c
)/(-(b*x^2+a)*(d*x^2-c))^(1/2)+1/693/b^2/d^2*((8*a^5*d^5+53*a^4*b*c*d^4+17
3*a^3*b^2*c^2*d^3-173*a^2*b^3*c^3*d^2-53*a*b^4*c^4*d-8*b^5*c^5)*a/(d/c)^(1
/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/
2)/b*(EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))-EllipticE(x*(d/c)
^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2)))+4*a*b^4*c^5/(d/c)^(1/2)*(1-d*x^2/c)^(1/
2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(d/c
)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))+4*a^5*c*d^4/(d/c)^(1/2)*(1-d*x^2/c)^(1/
2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(d/c
)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))+26*a^2*b^3*c^4*d/(d/c)^(1/2)*(1-d*x^2/c
)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x
*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))+300*a^3*b^2*c^3*d^2/(d/c)^(1/2)*(1
-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*Ell
ipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))+26*a^4*b*c^2*d^3/(d/c)^(1/
2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2
)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.02

$$\int \left(ac+(bc-ad)x^2-bdx^4\right)^{5/2}\,dx =$$

$$-\frac{\left(8\,b^5c^6+53\,ab^4c^5d+173\,a^2b^3c^4d^2-173\,a^3b^2c^3d^3-53\,a^4bc^2d^4-8\,a^5cd^5\right)\sqrt{-bd}x\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\,|-}{}$$

input
```
integrate((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x, algorithm="fricas")
```

output
```
-1/693*((8*b^5*c^6 + 53*a*b^4*c^5*d + 173*a^2*b^3*c^4*d^2 - 173*a^3*b^2*c^
3*d^3 - 53*a^4*b*c^2*d^4 - 8*a^5*c*d^5)*sqrt(-b*d)*x*sqrt(c/d)*elliptic_e(
arcsin(sqrt(c/d)/x), -a*d/(b*c)) - (8*b^5*c^6 + 53*a*b^4*c^5*d + 4*a^5*d^6
 + (173*a^2*b^3 + 4*a*b^4)*c^4*d^2 - (173*a^3*b^2 - 26*a^2*b^3)*c^3*d^3 -
(53*a^4*b - 300*a^3*b^2)*c^2*d^4 - 2*(4*a^5 - 13*a^4*b)*c*d^5)*sqrt(-b*d)*
x*sqrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - (63*b^5*d^6*x^10
 - 8*b^5*c^5*d - 53*a*b^4*c^4*d^2 - 173*a^2*b^3*c^3*d^3 + 173*a^3*b^2*c^2*
d^4 + 53*a^4*b*c*d^5 + 8*a^5*d^6 - 161*(b^5*c*d^5 - a*b^4*d^6)*x^8 + (113*
b^5*c^2*d^4 - 442*a*b^4*c*d^5 + 113*a^2*b^3*d^6)*x^6 - (3*b^5*c^3*d^3 - 35
6*a*b^4*c^2*d^4 + 356*a^2*b^3*c*d^5 - 3*a^3*b^2*d^6)*x^4 - (4*b^5*c^4*d^2
+ 26*a*b^4*c^3*d^3 - 393*a^2*b^3*c^2*d^4 + 26*a^3*b^2*c*d^5 + 4*a^4*b*d^6)
*x^2)*sqrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c))/(b^3*d^4*x)
```

Sympy [F]

$$\int \left(ac+(bc-ad)x^2-bdx^4\right)^{5/2}\,dx=\int \left(ac-bdx^4+x^2(-ad+bc)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*c+(-a*d+b*c)*x**2-b*d*x**4)**(5/2),x)
```

output
```
Integral((a*c - b*d*x**4 + x**2*(-a*d + b*c))**(5/2), x)
```

Maxima [F]

$$\int \left(ac+(bc-ad)x^2-bdx^4\right)^{5/2}\,dx=\int \left(-bdx^4+(bc-ad)x^2+ac\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(5/2), x)
```

Giac [F]

$$\int \left(ac+(bc-ad)x^2-bdx^4\right)^{5/2}\,dx=\int \left(-bdx^4+(bc-ad)x^2+ac\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x, algorithm="giac")
```

output
```
integrate((-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(ac+(bc-ad)x^2-bdx^4\right)^{5/2}\,dx=\int \left(-b\,d\,x^4+(b\,c-a\,d)\,x^2+a\,c\right)^{5/2}\,dx$$

input
```
int((a*c - x^2*(a*d - b*c) - b*d*x^4)^(5/2),x)
```

output
```
int((a*c - x^2*(a*d - b*c) - b*d*x^4)^(5/2), x)
```

Reduce [F]

$$\int \left(ac+(bc-ad)x^2-bdx^4\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
int((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x)
```

output

```
( - 4*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a**4*d**4*x - 26*sqrt(c - d*x**2)*
sqrt(a + b*x**2)*a**3*b*c*d**3*x + 3*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a**
3*b*d**4*x**3 + 393*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c**2*d**2*
x - 356*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c*d**3*x**3 + 113*sqrt
(c - d*x**2)*sqrt(a + b*x**2)*a**2*b**2*d**4*x**5 - 26*sqrt(c - d*x**2)*sq
rt(a + b*x**2)*a*b**3*c**3*d*x + 356*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a*b
**3*c**2*d**2*x**3 - 442*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a*b**3*c*d**3*x
**5 + 161*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a*b**3*d**4*x**7 - 4*sqrt(c -
d*x**2)*sqrt(a + b*x**2)*b**4*c**4*x - 3*sqrt(c - d*x**2)*sqrt(a + b*x**2)
*b**4*c**3*d*x**3 + 113*sqrt(c - d*x**2)*sqrt(a + b*x**2)*b**4*c**2*d**2*x
**5 - 161*sqrt(c - d*x**2)*sqrt(a + b*x**2)*b**4*c*d**3*x**7 + 63*sqrt(c -
 d*x**2)*sqrt(a + b*x**2)*b**4*d**4*x**9 - 8*int((sqrt(c - d*x**2)*sqrt(a
+ b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a**5*d**5 - 53*i
nt((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b
*d*x**4),x)*a**4*b*c*d**4 - 173*int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**
2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a**3*b**2*c**2*d**3 + 173*int
((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d
*x**4),x)*a**2*b**3*c**3*d**2 + 53*int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*
x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a*b**4*c**4*d + 8*int((sqr
t(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x...
```

3.20 $\int (ac+(bc-ad)x^2-bdx^4)^{3/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 355

$$\int (ac+(bc-ad)x^2-bdx^4)^{3/2}\,dx = \frac{x(10abcd-(bc-ad)^2+3bd(bc-ad)x^2)\sqrt{ac+(bc-ad)x^2-bdx^4}}{35bd} + \frac{1}{7}x(ac+(bc-ad)x^2-bdx^4)^{3/2} + \frac{2a\sqrt{c}(bc-ad)\left(8abcd+(bc-ad)^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{35b^2d^{3/2}\sqrt{ac+(bc-ad)x^2-bdx^4}} - \frac{a\sqrt{c}(bc+ad)\left(b^2c^2-9abcd-2a^2d^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{35b^2d^{3/2}\sqrt{ac+(bc-ad)x^2-bdx^4}}$$

output
```
1/35*x*(10*a*b*c*d-(-a*d+b*c)^2+3*b*d*(-a*d+b*c)*x^2)*(a*c+(-a*d+b*c)*x^2-
b*d*x^4)^(1/2)/b/d+1/7*x*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2)+2/35*a*c^(1/2)
*(-a*d+b*c)*(8*a*b*c*d+(-a*d+b*c)^2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*E
llipticE(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/b^2/d^(3/2)/(a*c+(-a*d+b*c)*x
^2-b*d*x^4)^(1/2)-1/35*a*c^(1/2)*(a*d+b*c)*(-2*a^2*d^2-9*a*b*c*d+b^2*c^2)*
(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-b*c/a/d)
^(1/2))/b^2/d^(3/2)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.94 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.86

$$\int (ac+(bc-ad)x^2 -bdx^4)^{3/2} \, dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(-c+dx^2\right)\left(a^2d^2+abd(-17c+8dx^2)+b^2(c^2-8cdx^2+5d^2x^4)\right)-2ic(-}{}$$

input
```
Integrate[(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(-c + d*x^2)*(a^2*d^2 + a*b*d*(-17*c + 8*d*x^2)
 + b^2*(c^2 - 8*c*d*x^2 + 5*d^2*x^4)) - (2*I)*c*(-(b^3*c^3) - 5*a*b^2*c^2*
d + 5*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))] + I*c*(-2*b^3*c^3 - 11*a*b^2*
c^2*d - 8*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*E
llipticF[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))])/(35*b*Sqrt[b/a]*d^2*Sqrt
[(a + b*x^2)*(c - d*x^2)])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1404, 1490, 25, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2(bc-ad)+ac-bdx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1404}$$

$$\frac{3}{7}\int \left((bc-ad)x^2+2ac\right)\sqrt{-bdx^4+(bc-ad)x^2+ac}dx+\frac{1}{7}x\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}$$

$$\Big\downarrow \text{ 1490}$$

$$\frac{3}{7}\left(\frac{x\left(3bdx^2(bc-ad)-(bc-ad)^2+10abcd\right)\sqrt{x^2(bc-ad)+ac-bdx^4}}{15bd}-\frac{\int -\frac{2(bc-ad)\left((bc-ad)^2+8abcd\right)x^2+ac\left((bc-a\right.}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}}{15bd}\right.$$

$$\frac{1}{7}x\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}$$

$$\Big\downarrow 25$$

$$\frac{3}{7}\left(\frac{\int \frac{2(bc-ad)\left((bc-ad)^2+8abcd\right)x^2+ac\left((bc-ad)^2+20abcd\right)}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{15bd}+\frac{x\sqrt{x^2(bc-ad)+ac-bdx^4}\left(3bdx^2(bc-ad)-(bc-ad)^2\right.}{15bd}\right.$$

$$\frac{1}{7}x\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}$$

$$\Big\downarrow 1514$$

$$\frac{3}{7}\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{2(bc-ad)\left((bc-ad)^2+8abcd\right)x^2+ac\left((bc-ad)^2+20abcd\right)}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{15bd\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{x\sqrt{x^2(bc-ad)+ac-bdx^4}\left(3bdx^2(bc\right.}{15bd}\right.$$

$$\frac{1}{7}x\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}$$

$$\Big\downarrow 399$$

$$\frac{3}{7}\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{2a(bc-ad)\left((bc-ad)^2+8abcd\right)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}-\frac{a(ad+bc)\left(-2a^2d^2-9abcd+b^2c^2\right)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{b}\right)}{15bd\sqrt{x^2(bc-ad)+ac-bdx^4}}+\right.$$

$$\frac{1}{7}x\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}$$

$$\Big\downarrow 321$$

$$\frac{3}{7}\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{2a(bc-ad)\left((bc-ad)^2+8abcd\right)\int\frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}-\frac{a\sqrt{c}(ad+bc)\left(-2a^2d^2-9abcd+b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\right.}{b\sqrt{d}}\right.}{15bd\sqrt{x^2(bc-ad)+ac-bdx^4}}\right.$$

$$\frac{1}{7}x\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}$$

$$\Big\downarrow \text{327}$$

$$\frac{3}{7}\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{2a\sqrt{c}(bc-ad)\left((bc-ad)^2+8abcd\right)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}}-\frac{a\sqrt{c}(ad+bc)\left(-2a^2d^2-9abcd+b^2c^2\right)\text{EllipticF}\left(\text{arcsi}\right.}{b\sqrt{d}}\right.}{15bd\sqrt{x^2(bc-ad)+ac-bdx^4}}\right.$$

$$\frac{1}{7}x\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}$$

input
```
Int[(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(3/2),x]
```

output
```
(x*(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(3/2))/7 + (3*((x*(10*a*b*c*d - (b*c
- a*d)^2 + 3*b*d*(b*c - a*d)*x^2)*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4])/(
15*b*d) + (Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*((2*a*Sqrt[c]*(b*c - a*
d)*(8*a*b*c*d + (b*c - a*d)^2)*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b
*c)/(a*d))])/(b*Sqrt[d]) - (a*Sqrt[c]*(b*c + a*d)*(b^2*c^2 - 9*a*b*c*d - 2
*a^2*d^2)*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[
d])))/(15*b*d*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4])))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.58

method	result
risch	$-\frac{x(5b^2d^2x^4+8ad^2bx^2-8b^2cdx^2+a^2d^2-17abcd+b^2c^2)(bx^2+a)(-dx^2+c)}{35bd\sqrt{-(bx^2+a)(dx^2-c)}}+\frac{(2a^3d^3+10a^2bcd^2-10ab^2c^2d-2b^3c^3)a\sqrt{1-\frac{dx^2}{c}}\sqrt{1 \cdots}}{\sqrt{\frac{a}{c} \cdots}}$
default	$-\frac{bdx^5\sqrt{-bdx^4-x^2da+bcx^2+ac}}{7}-\frac{\left(2abd^2-2b^2cd+\frac{bd(-6ad+6bc)}{7}\right)x^3\sqrt{-bdx^4-x^2da+bcx^2+ac}}{5bd}-\frac{\left(a^2d^2-\frac{23abcd}{7}+b^2c^2+\cdots\right.}{\cdots}$
elliptic	$-\frac{bdx^5\sqrt{-bdx^4-x^2da+bcx^2+ac}}{7}-\frac{\left(2abd^2-2b^2cd+\frac{bd(-6ad+6bc)}{7}\right)x^3\sqrt{-bdx^4-x^2da+bcx^2+ac}}{5bd}-\frac{\left(a^2d^2-\frac{23abcd}{7}+b^2c^2+\cdots\right.}{\cdots}$

input
```
int((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35/b/d*x*(5*b^2*d^2*x^4+8*a*b*d^2*x^2-8*b^2*c*d*x^2+a^2*d^2-17*a*b*c*d+
b^2*c^2)*(b*x^2+a)*(-d*x^2+c)/(-(b*x^2+a)*(d*x^2-c))^(1/2)+1/35/b/d*((2*a^
3*d^3+10*a^2*b*c*d^2-10*a*b^2*c^2*d-2*b^3*c^3)*a/(d/c)^(1/2)*(1-d*x^2/c)^(
1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)/b*(EllipticF(x
*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))-EllipticE(x*(d/c)^(1/2),(-1-(-a*d+
b*c)/a/d)^(1/2)))+a*b^2*c^3/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2
)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b
*c)/a/d)^(1/2))+a^3*c*d^2/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/
(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c
)/a/d)^(1/2))+18*a^2*c^2*b*d/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/
2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d+
b*c)/a/d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.92

$$\int \left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}\,dx =$$

$$-\frac{2\,(b^3c^4+5\,ab^2c^3d-5\,a^2bc^2d^2-a^3cd^3)\sqrt{-bd}x\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid-\frac{ad}{bc})-(2\,b^3c^4+10\,ab^2c^3d+a^3d^4-}{}$$

input
```
integrate((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x, algorithm="fricas")
```

output
```
-1/35*(2*(b^3*c^4 + 5*a*b^2*c^3*d - 5*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(-b*d
)*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - (2*b^3*c^4 + 1
0*a*b^2*c^3*d + a^3*d^4 - (10*a^2*b - a*b^2)*c^2*d^2 - 2*(a^3 - 9*a^2*b)*c
*d^3)*sqrt(-b*d)*x*sqrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)) +
 (5*b^3*d^4*x^6 + 2*b^3*c^3*d + 10*a*b^2*c^2*d^2 - 10*a^2*b*c*d^3 - 2*a^3*
d^4 - 8*(b^3*c*d^3 - a*b^2*d^4)*x^4 + (b^3*c^2*d^2 - 17*a*b^2*c*d^3 + a^2*
b*d^4)*x^2)*sqrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c))/(b^2*d^3*x)
```

Sympy [F]

$$\int \left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}\,dx=\int \left(ac-bdx^4+x^2(-ad+bc)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*c+(-a*d+b*c)*x**2-b*d*x**4)**(3/2),x)
```

output
```
Integral((a*c - b*d*x**4 + x**2*(-a*d + b*c))**(3/2), x)
```

Maxima [F]

$$\int \left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}\,dx=\int \left(-bdx^4+(bc-ad)x^2+ac\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(3/2), x)
```

Giac [F]

$$\int \left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}\,dx=\int \left(-bdx^4+(bc-ad)x^2+ac\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}\,dx=\int {\left(-b\,d\,x^4+\left(b\,c-a\,d\right)\,x^2+a\,c\right)}^{3/2}\,dx$$

input
```
int((a*c - x^2*(a*d - b*c) - b*d*x^4)^(3/2),x)
```

output
```
int((a*c - x^2*(a*d - b*c) - b*d*x^4)^(3/2), x)
```

Reduce [F]

$$\int \left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}\,dx=\frac{-\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2d^2x+17\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,abcdx-8\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}}{}$$

input
```
int((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x)
```

output
```
( - sqrt(c - d*x**2)*sqrt(a + b*x**2)*a**2*d**2*x + 17*sqrt(c - d*x**2)*sq
rt(a + b*x**2)*a*b*c*d*x - 8*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a*b*d**2*x*
*3 - sqrt(c - d*x**2)*sqrt(a + b*x**2)*b**2*c**2*x + 8*sqrt(c - d*x**2)*sq
rt(a + b*x**2)*b**2*c*d*x**3 - 5*sqrt(c - d*x**2)*sqrt(a + b*x**2)*b**2*d*
*2*x**5 - 2*int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d*x**2 +
 b*c*x**2 - b*d*x**4),x)*a**3*d**3 - 10*int((sqrt(c - d*x**2)*sqrt(a + b*x
**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a**2*b*c*d**2 + 10*in
t((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*
d*x**4),x)*a*b**2*c**2*d + 2*int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/
(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*b**3*c**3 + int((sqrt(c - d*x**2
)*sqrt(a + b*x**2))/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a**3*c*d**2
+ 18*int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a*c - a*d*x**2 + b*c*x**2 -
b*d*x**4),x)*a**2*b*c**2*d + int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a*c
- a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a*b**2*c**3)/(35*b*d)
```

3.21 $\int \sqrt{ac+(bc-ad)x^2-bdx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 244

$$\int \sqrt{ac+(bc-ad)x^2-bdx^4}\,dx = \frac{1}{3}x\sqrt{ac+(bc-ad)x^2-bdx^4} + \frac{a\sqrt{c}(bc-ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{ac+(bc-ad)x^2-bdx^4}} + \frac{a\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{ac+(bc-ad)x^2-bdx^4}}$$

output
```
1/3*x*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)+1/3*a*c^(1/2)*(-a*d+b*c)*(1+b*x^2
/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/
b/d^(1/2)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)+1/3*a*c^(1/2)*(a*d+b*c)*(1+b*
x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2
))/b/d^(1/2)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.82

$$\int \sqrt{ac+(bc-ad)x^2-bdx^4}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c-dx^2)-ic(-bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)-ic(bc+ad)\sqrt{1}}{3\sqrt{\frac{b}{a}}d\sqrt{(a+bx^2)(c-dx^2)}}$$

input
```
Integrate[Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c - d*x^2) - I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))
] - I*c*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[I*Ar
cSinh[Sqrt[b/a]*x], -((a*d)/(b*c))])/(3*Sqrt[b/a]*d*Sqrt[(a + b*x^2)*(c -
d*x^2)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1404, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2(bc-ad)+ac-bdx^4}\,dx$$

$$\Big\downarrow \text{1404}$$

$$\frac{1}{3}\int \frac{(bc-ad)x^2+2ac}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx+\frac{1}{3}x\sqrt{x^2(bc-ad)+ac-bdx^4}$$

$$\Big\downarrow \text{1514}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{(bc-ad)x^2+2ac}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{3\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{1}{3}x\sqrt{x^2(bc-ad)+ac-bdx^4}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a(ad+bc)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{b}+\frac{a(bc-ad)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}\right)}{3\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{1}{3}x\sqrt{x^2(bc-ad)+ac-bdx^4}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a(bc-ad)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}+\frac{a\sqrt{c}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}}\right)}{3\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{1}{3}x\sqrt{x^2(bc-ad)+ac-bdx^4}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a\sqrt{c}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}}+\frac{a\sqrt{c}(bc-ad)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}}\right)}{3\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{1}{3}x\sqrt{x^2(bc-ad)+ac-bdx^4}$$

input
```
Int[Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4],x]
```

output
```
(x*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4])/3 + (Sqrt[1 + (b*x^2)/a]*Sqrt[1
- (d*x^2)/c]*((a*Sqrt[c]*(b*c - a*d)*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]]
, -((b*c)/(a*d))])/(b*Sqrt[d]) + (a*Sqrt[c]*(b*c + a*d)*EllipticF[ArcSin[(
Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d])))/(3*Sqrt[a*c + (b*c - a
*d)*x^2 - b*d*x^4])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.06

method	result
default	$\frac{x\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}}{3}+\frac{2ac\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{3\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}}-\frac{\left(-\frac{ad}{3}+\frac{bc}{3}\right)a\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\left(\right.}{}$
elliptic	$\frac{x\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}}{3}+\frac{2ac\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{3\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}}-\frac{\left(-\frac{ad}{3}+\frac{bc}{3}\right)a\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\left(\right.}{}$
risch	$\frac{x\left(b\,x^2+a\right)\left(-d\,x^2+c\right)}{3\sqrt{-\left(b\,x^2+a\right)\left(d\,x^2-c\right)}}+\frac{(ad-bc)a\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)\right)}{3\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}\,b}$

input
```
int((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)+2/3*a*c/(d/c)^(1/2)*(1-d*x^2/c)
^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))-(-1/3*a*d+1/3*b*c)*a/(d/c)^(1/2)*(1
-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)/b*(
EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))-EllipticE(x*(d/c)^(1/2)
,(-1-(-a*d+b*c)/a/d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.67

$$\int \sqrt{ac+(bc-ad)x^2-bdx^4}\,dx =$$

$$-\frac{(bc^2-acd)\sqrt{-bd}x\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid-\frac{ad}{bc})-(bc^2-acd+2\,ad^2)\sqrt{-bd}x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid-\frac{ad}{bc}}{3\,bd^2x}$$

input
```
integrate((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((b*c^2 - a*c*d)*sqrt(-b*d)*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x
), -a*d/(b*c)) - (b*c^2 - a*c*d + 2*a*d^2)*sqrt(-b*d)*x*sqrt(c/d)*elliptic
_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - sqrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*
c)*(b*d^2*x^2 - b*c*d + a*d^2))/(b*d^2*x)
```

Sympy [F]

$$\int \sqrt{ac+(bc-ad)x^2-bdx^4}\,dx = \int \sqrt{ac-bdx^4+x^2(-ad+bc)}\,dx$$

input
```
integrate((a*c+(-a*d+b*c)*x**2-b*d*x**4)**(1/2),x)
```

output
```
Integral(sqrt(a*c - b*d*x**4 + x**2*(-a*d + b*c)), x)
```

Maxima [F]

$$\int \sqrt{ac+(bc-ad)x^2-bdx^4}\,dx = \int \sqrt{-bdx^4+(bc-ad)x^2+ac}\,dx$$

input
```
integrate((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c), x)
```

Giac [F]

$$\int \sqrt{ac+(bc-ad)x^2-bdx^4}\,dx = \int \sqrt{-bdx^4+(bc-ad)x^2+ac}\,dx$$

input
```
integrate((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ac+(bc-ad)x^2-bdx^4}\,dx=\int \sqrt{-b\,d\,x^4+(b\,c-a\,d)\;x^2+a\,c}\,dx$$

input
```
int((a*c - x^2*(a*d - b*c) - b*d*x^4)^(1/2),x)
```

output
```
int((a*c - x^2*(a*d - b*c) - b*d*x^4)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{ac+(bc-ad)x^2-bdx^4}\,dx=\frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,x}{3}-\frac{\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{-bd\,x^4-ad\,x^2+bc\,x^2+ac}dx\right)ad}{3}+\frac{\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{-bd\,x^4-ad\,x^2+bc\,x^2+ac}dx\right)bc}{3}+\frac{2\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}}{-bd\,x^4-ad\,x^2+bc\,x^2+ac}dx\right)ac}{3}$$

input
```
int((a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x)
```

output
```
(sqrt(c - d*x**2)*sqrt(a + b*x**2)*x - int((sqrt(c - d*x**2)*sqrt(a + b*x*
*2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a*d + int((sqrt(c - d*
x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*b*c
 + 2*int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a*c - a*d*x**2 + b*c*x**2 -
b*d*x**4),x)*a*c)/3
```

3.22 $\int \frac{1}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 91

$$\int \frac{1}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx = \frac{\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{ac+(bc-ad)x^2-bdx^4}}$$

output
```
c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-
b*c/a/d)^(1/2))/d^(1/2)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c-dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right),-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{(a+bx^2)(c-dx^2)}}$$

input
```
Integrate[1/Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[(c - d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
 -((a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[(a + b*x^2)*(c - d*x^2)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1417, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2(bc-ad)+ac-bdx^4}}dx$$

$$\Big\downarrow \ 1417$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\Big\downarrow \ 321$$

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

input
```
Int[1/Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4],x]
```

output
```
(Sqrt[c]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]
*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x
^4])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-x^{2}da+bc\,x^{2}+ac}}$	90
elliptic	$\frac{\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-x^{2}da+bc\,x^{2}+ac}}$	90

input
```
int(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^
2+a*c)^(1/2)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx = \frac{\sqrt{ac}\sqrt{\frac{d}{c}}F\left(\arcsin\left(x\sqrt{\frac{d}{c}}\right) \mid -\frac{bc}{ad}\right)}{ad}$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*c)*sqrt(d/c)*elliptic_f(arcsin(x*sqrt(d/c)), -b*c/(a*d))/(a*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx = \int \frac{1}{\sqrt{ac-bdx^4+x^2(-ad+bc)}}\,dx$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x**2-b*d*x**4)**(1/2),x)
```

output
```
Integral(1/sqrt(a*c - b*d*x**4 + x**2*(-a*d + b*c)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx = \int \frac{1}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}\,dx$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx=\int \frac{1}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}\,dx$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx=\int \frac{1}{\sqrt{-b\,d\,x^4+(b\,c-a\,d)\,x^2+a\,c}}\,dx$$

input
```
int(1/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(1/2),x)
```

output
```
int(1/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx=\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}}{-bd\,x^4-ad\,x^2+bc\,x^2+ac}dx$$

input
```
int(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x)
```

output
```
int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a*c - a*d*x**2 + b*c*x**2 - b*d*x
**4),x)
```

3.23 $\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 283

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx = \frac{x(b^2c^2+a^2d^2-bd(bc-ad)x^2)}{ac(bc+ad)^2\sqrt{ac+(bc-ad)x^2-bdx^4}} + \frac{\sqrt{d}(bc-ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}(bc+ad)^2\sqrt{ac+(bc-ad)x^2-bdx^4}} + \frac{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{c}(bc+ad)\sqrt{ac+(bc-ad)x^2-bdx^4}}$$

output
```
x*(b^2*c^2+a^2*d^2-b*d*(-a*d+b*c)*x^2)/a/c/(a*d+b*c)^2/(a*c+(-a*d+b*c)*x^2
-b*d*x^4)^(1/2)+d^(1/2)*(-a*d+b*c)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*Ell
ipticE(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/c^(1/2)/(a*d+b*c)^2/(a*c+(-a*d+
b*c)*x^2-b*d*x^4)^(1/2)+d^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*Ellipt
icF(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/c^(1/2)/(a*d+b*c)/(a*c+(-a*d+b*c)*
x^2-b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.41 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.80

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}x(a^2d^2+abd^2x^2+b^2c(c-dx^2))-ibc(-bc+ad)\sqrt{1+\frac{bx^2}{a}}\right.}{}$$

input
```
Integrate[(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(-3/2),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*x*(a^2*d^2 + a*b*d^2*x^2 + b^2*c*(c - d*x^2)) - I*b*
c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSi
nh[Sqrt[b/a]*x], -((a*d)/(b*c))] - I*b*c*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*S
qrt[1 - (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))]))/(b*
c*(b*c + a*d)^2*Sqrt[(a + b*x^2)*(c - d*x^2)])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1405, 25, 27, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2(bc-ad)+ac-bdx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}-\frac{\int -\frac{bd((bc-ad)x^2+2ac)}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{ac(ad+bc)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bd((bc-ad)x^2+2ac)}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{ac(ad+bc)^2}+\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{bd\int \frac{(bc-ad)x^2+2ac}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{ac(ad+bc)^2}+\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\Big\downarrow \; 1514$$

$$\frac{bd\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{(bc-ad)x^2+2ac}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\Big\downarrow \; 399$$

$$\frac{bd\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a(ad+bc)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{b}+\frac{a(bc-ad)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\Big\downarrow \; 321$$

$$\frac{bd\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a(bc-ad)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}+\frac{a\sqrt{c}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}}\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{bd\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a\sqrt{c}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}}+\frac{a\sqrt{c}(bc-ad)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}}\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

input
```
Int[(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(-3/2),x]
```

output
```
(x*(b^2*c^2 + a^2*d^2 - b*d*(b*c - a*d)*x^2))/(a*c*(b*c + a*d)^2*Sqrt[a*c
+ (b*c - a*d)*x^2 - b*d*x^4]) + (b*d*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/
c]*((a*Sqrt[c]*(b*c - a*d)*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/
(a*d))])/(b*Sqrt[d]) + (a*Sqrt[c]*(b*c + a*d)*EllipticF[ArcSin[(Sqrt[d]*x)
/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d])))/(a*c*(b*c + a*d)^2*Sqrt[a*c + (b
*c - a*d)*x^2 - b*d*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.55

method	result
default	$\frac{2bd\left(\frac{(ad-bc)x^3}{2ac\left(a^2d^2+2abcd+b^2c^2\right)}+\frac{\left(a^2d^2+b^2c^2\right)x}{2ac\left(a^2d^2+2abcd+b^2c^2\right)bd}\right)}{\sqrt{-\left(x^4+\frac{(ad-bc)x^2}{bd}-\frac{ac}{bd}\right)bd}}+\frac{\left(\frac{1}{ac}-\frac{a^2d^2+b^2c^2}{ac\left(a^2d^2+2abcd+b^2c^2\right)}\right)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{bx^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\right.}{\sqrt{\frac{d}{c}}\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}}$
elliptic	$\frac{2bd\left(\frac{(ad-bc)x^3}{2ac\left(a^2d^2+2abcd+b^2c^2\right)}+\frac{\left(a^2d^2+b^2c^2\right)x}{2ac\left(a^2d^2+2abcd+b^2c^2\right)bd}\right)}{\sqrt{-\left(x^4+\frac{(ad-bc)x^2}{bd}-\frac{ac}{bd}\right)bd}}+\frac{\left(\frac{1}{ac}-\frac{a^2d^2+b^2c^2}{ac\left(a^2d^2+2abcd+b^2c^2\right)}\right)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{bx^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\right.}{\sqrt{\frac{d}{c}}\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}}$

input
```
int(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*b*d*(1/2/a/c*(a*d-b*c)/(a^2*d^2+2*a*b*c*d+b^2*c^2)*x^3+1/2*(a^2*d^2+b^2*
c^2)/a/c/(a^2*d^2+2*a*b*c*d+b^2*c^2)/b/d*x)/(-(x^4+(a*d-b*c)/b/d*x^2-a*c/b
/d)*b*d)^(1/2)+(1/a/c-(a^2*d^2+b^2*c^2)/a/c/(a^2*d^2+2*a*b*c*d+b^2*c^2))/(
d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a
*c)^(1/2)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))+1/c*(a*d-b*c)
/(a^2*d^2+2*a*b*c*d+b^2*c^2)*d/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(
1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(d/c)^(1/2),(-1-(-a
*d+b*c)/a/d)^(1/2))-EllipticE(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.49

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx = \frac{(abc^2d-a^2cd^2-(b^2cd^2-abd^3)x^4+(b^2c^2d-2\,abcd^2+a^2d^3)x^2)\sqrt{ac}}{}$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x, algorithm="fricas")
```

output
```
((a*b*c^2*d - a^2*c*d^2 - (b^2*c*d^2 - a*b*d^3)*x^4 + (b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*x^2)*sqrt(a*c)*sqrt(d/c)*elliptic_e(arcsin(x*sqrt(d/c)), -
b*c/(a*d)) + (2*a*b*c^3 - a*b*c^2*d + a^2*c*d^2 - (2*b^2*c^2*d - b^2*c*d^2
 + a*b*d^3)*x^4 + (2*b^2*c^3 + 2*a*b*c*d^2 - a^2*d^3 - (2*a*b + b^2)*c^2*d
)*x^2)*sqrt(a*c)*sqrt(d/c)*elliptic_f(arcsin(x*sqrt(d/c)), -b*c/(a*d)) - s
qrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c)*((b^2*c^2*d - a*b*c*d^2)*x^3 - (b^2*
c^3 + a^2*c*d^2)*x))/(a^2*b^2*c^5 + 2*a^3*b*c^4*d + a^4*c^3*d^2 - (a*b^3*c
^4*d + 2*a^2*b^2*c^3*d^2 + a^3*b*c^2*d^3)*x^4 + (a*b^3*c^5 + a^2*b^2*c^4*d
 - a^3*b*c^3*d^2 - a^4*c^2*d^3)*x^2)
```

Sympy [F]

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx = \int \frac{1}{(ac-bdx^4+x^2\,(-ad+bc))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x***2-b*d*x***4)**(3/2),x)
```

output
```
Integral((a*c - b*d*x***4 + x***2*(-a*d + b*c))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx = \int \frac{1}{(-bdx^4+(bc-ad)x^2+ac)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx = \int \frac{1}{(-bdx^4+(bc-ad)x^2+ac)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx = \int \frac{1}{(-b\,d\,x^4+(b\,c-a\,d)\,x^2+a\,c)^{3/2}}\,dx$$

input
```
int(1/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(3/2),x)
```

output
```
int(1/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx = \int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}}{b^2d^2x^8+2ab\,d^2x^6-2b^2cd\,x^6+a^2d^2x^4-4abcd\,x^4+b^2c^2x^4-2a^2c}$$

input
```
int(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x)
```

output
```
int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a**2*c**2 - 2*a**2*c*d*x**2 + a**
2*d**2*x**4 + 2*a*b*c**2*x**2 - 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c*
*2*x**4 - 2*b**2*c*d*x**6 + b**2*d**2*x**8),x)
```

3.24 $\int \frac{1}{\left(ac+(bc-ad)x^2-bdx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 471

$$\int \frac{1}{\left(ac+(bc-ad)x^2-bdx^4\right)^{5/2}}\,dx = \frac{x(b^2c^2+a^2d^2-bd(bc-ad)x^2)}{3ac(bc+ad)^2\left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}}$$
$$+\frac{x\left(2b^4c^4+9ab^3c^3d-2a^2b^2c^2d^2+9a^3bcd^3+2a^4d^4-2bd(bc-ad)\left(8abcd+(bc-ad)^2\right)x^2\right)}{3a^2c^2(bc+ad)^4\sqrt{ac+(bc-ad)x^2-bdx^4}}$$
$$+\frac{2\sqrt{d}(bc-ad)\left(b^2c^2+6abcd+a^2d^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{3ac^{3/2}(bc+ad)^4\sqrt{ac+(bc-ad)x^2-bdx^4}}$$
$$-\frac{\sqrt{d}(b^2c^2-9abcd-2a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{3ac^{3/2}(bc+ad)^3\sqrt{ac+(bc-ad)x^2-bdx^4}}$$

output

```
1/3*x*(b^2*c^2+a^2*d^2-b*d*(-a*d+b*c)*x^2)/a/c/(a*d+b*c)^2/(a*c+(-a*d+b*c)
*x^2-b*d*x^4)^(3/2)+1/3*x*(2*b^4*c^4+9*a*b^3*c^3*d-2*a^2*b^2*c^2*d^2+9*a^3
*b*c*d^3+2*a^4*d^4-2*b*d*(-a*d+b*c)*(8*a*b*c*d+(-a*d+b*c)^2)*x^2)/a^2/c^2/
(a*d+b*c)^4/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)+2/3*d^(1/2)*(-a*d+b*c)*(a^2
*d^2+6*a*b*c*d+b^2*c^2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticE(d^(1
/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/a/c^(3/2)/(a*d+b*c)^4/(a*c+(-a*d+b*c)*x^2-
b*d*x^4)^(1/2)-1/3*d^(1/2)*(-2*a^2*d^2-9*a*b*c*d+b^2*c^2)*(1+b*x^2/a)^(1/2
)*(1-d*x^2/c)^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/a/c^(3/2
)/(a*d+b*c)^3/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.66 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.76

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x\Big(a^2cd^3(bc+ad)\left(a+bx^2\right)^2+2a^2d^3(5bc+ad)\left(a+bx^2\right)^2(c-d}{}$$

input
```
Integrate[(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(-5/2),x]
```

output
```
(Sqrt[b/a]*x*(a^2*c*d^3*(b*c + a*d)*(a + b*x^2)^2 + 2*a^2*d^3*(5*b*c + a*d
)*(a + b*x^2)^2*(c - d*x^2) + a*b^3*c^2*(b*c + a*d)*(c - d*x^2)^2 + 2*b^3*
c^2*(b*c + 5*a*d)*(a + b*x^2)*(c - d*x^2)^2) + I*b*c*(a + b*x^2)*Sqrt[1 +
(b*x^2)/a]*(-c + d*x^2)*Sqrt[1 - (d*x^2)/c]*(-2*(b^3*c^3 + 5*a*b^2*c^2*d -
 5*a^2*b*c*d^2 - a^3*d^3)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))
] + (2*b^3*c^3 + 11*a*b^2*c^2*d + 8*a^2*b*c*d^2 - a^3*d^3)*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], -((a*d)/(b*c))]))/(3*a^2*Sqrt[b/a]*c^2*(b*c + a*d)^4*((
a + b*x^2)*(c - d*x^2))^(3/2))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1405, 25, 1492, 25, 27, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2(bc-ad)+ac-bdx^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}-\frac{\int -\frac{2(b^2c^2+3abdc+a^2d^2)-3bd(bc-ad)x^2}{(-bdx^4+(bc-ad)x^2+ac)^{3/2}}dx}{3ac(ad+bc)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2(b^2c^2+3abdc+a^2d^2)-3bd(bc-ad)x^2}{(-bdx^4+(bc-ad)x^2+ac)^{3/2}}dx}{3ac(ad+bc)^2}+\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}$$

$\downarrow$ 1492

$$\frac{\frac{x(2a^4d^4+9a^3bcd^3-2bdx^2(bc-ad)(a^2d^2+6abcd+b^2c^2)-2a^2b^2c^2d^2+9ab^3c^3d+2b^4c^4)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}-\frac{\int -\frac{bd\left(2(bc-ad)\left(b^2c^2+6abdc+a^2d^2\right)x^2+ac\left(b^2c^2+18abdc+\right.\right.}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}}{ac(ad+bc)^2}}{3ac(ad+bc)^2}$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{bd\left(2(bc-ad)\left(b^2c^2+6abdc+a^2d^2\right)x^2+ac\left(b^2c^2+18abdc+a^2d^2\right)\right)}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{ac(ad+bc)^2}+\frac{x(2a^4d^4+9a^3bcd^3-2bdx^2(bc-ad)(a^2d^2+6abcd+b^2c^2)-2a^2b^2c^2d^2+9ab^3c^3d}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}}{3ac(ad+bc)^2}$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{bd\int \frac{2(bc-ad)\left(b^2c^2+6abdc+a^2d^2\right)x^2+ac\left(b^2c^2+18abdc+a^2d^2\right)}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{ac(ad+bc)^2}+\frac{x(2a^4d^4+9a^3bcd^3-2bdx^2(bc-ad)(a^2d^2+6abcd+b^2c^2)-2a^2b^2c^2d^2+9ab^3c^3d+}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}}{3ac(ad+bc)^2}$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}$$

$\downarrow$ 1514

$$\frac{\frac{bd\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{2(bc-ad)\left(b^2c^2+6abdc+a^2d^2\right)x^2+ac\left(b^2c^2+18abdc+a^2d^2\right)}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{x(2a^4d^4+9a^3bcd^3-2bdx^2(bc-ad)(a^2d^2+6abcd+b^2c^2)-2a^2}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}}{3ac(ad+bc)^2}$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}$$

$\downarrow$ 399

$$\frac{\frac{bd\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{2a(bc-ad)\left(a^2d^2+6abcd+b^2c^2\right)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}-\frac{a(ad+bc)\left(-2a^2d^2-9abcd+b^2c^2\right)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{b}\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{x\left(2a^4d^4+9a^3bcd^3\right.}{}}{3ac(ad+bc)^2}$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}$$

$$\Big\downarrow \; 321$$

$$\frac{\frac{bd\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{2a(bc-ad)\left(a^2d^2+6abcd+b^2c^2\right)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}-\frac{a\sqrt{c}(ad+bc)\left(-2a^2d^2-9abcd+b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}}\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{x\left(2a^4d^4\right.}{}}{3ac(ad+bc)^2}$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}$$

$$\Big\downarrow \; 327$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}+$$

$$\frac{\frac{bd\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{2a\sqrt{c}(bc-ad)\left(a^2d^2+6abcd+b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}}-\frac{a\sqrt{c}(ad+bc)\left(-2a^2d^2-9abcd+b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}}\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}}{3ac(ad+bc)^2}$$

input
```
Int[(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(-5/2),x]
```

output
```
(x*(b^2*c^2 + a^2*d^2 - b*d*(b*c - a*d)*x^2))/(3*a*c*(b*c + a*d)^2*(a*c +
(b*c - a*d)*x^2 - b*d*x^4)^(3/2)) + ((x*(2*b^4*c^4 + 9*a*b^3*c^3*d - 2*a^2
*b^2*c^2*d^2 + 9*a^3*b*c*d^3 + 2*a^4*d^4 - 2*b*d*(b*c - a*d)*(b^2*c^2 + 6*
a*b*c*d + a^2*d^2)*x^2))/(a*c*(b*c + a*d)^2*Sqrt[a*c + (b*c - a*d)*x^2 - b
*d*x^4]) + (b*d*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*((2*a*Sqrt[c]*(b*c
 - a*d)*(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[
c]], -((b*c)/(a*d))])/(b*Sqrt[d]) - (a*Sqrt[c]*(b*c + a*d)*(b^2*c^2 - 9*a*
b*c*d - 2*a^2*d^2)*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])
/(b*Sqrt[d])))/(a*c*(b*c + a*d)^2*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4]))/
(3*a*c*(b*c + a*d)^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.79

method	result
default	$\frac{\left(\frac{(ad-bc)x^3}{3ac\left(a^2d^2+2abcd+b^2c^2\right)bd}+\frac{\left(a^2d^2+b^2c^2\right)x}{3ac\left(a^2d^2+2abcd+b^2c^2\right)b^2d^2}\right)\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}}{\left(x^4+\frac{(ad-bc)x^2}{bd}-\frac{ac}{bd}\right)^2}+\frac{2bd\left(\frac{(ad-bc)\left(a^2d^2+6abcd+b^2c^2\right)x^3}{3a^2c^2\left(a^2d^2+2abcd+b^2c^2\right)^2}+\frac{(2a^4}{}\right.}{\sqrt{-\left(x^4+\frac{(ad}{}\right.}}$
elliptic	$\frac{\left(\frac{(ad-bc)x^3}{3ac\left(a^2d^2+2abcd+b^2c^2\right)bd}+\frac{\left(a^2d^2+b^2c^2\right)x}{3ac\left(a^2d^2+2abcd+b^2c^2\right)b^2d^2}\right)\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}}{\left(x^4+\frac{(ad-bc)x^2}{bd}-\frac{ac}{bd}\right)^2}+\frac{2bd\left(\frac{(ad-bc)\left(a^2d^2+6abcd+b^2c^2\right)x^3}{3a^2c^2\left(a^2d^2+2abcd+b^2c^2\right)^2}+\frac{(2a^4}{}\right.}{\sqrt{-\left(x^4+\frac{(ad}{}\right.}}$

input
```
int(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/3/a/c*(a*d-b*c)/(a^2*d^2+2*a*b*c*d+b^2*c^2)/b/d*x^3+1/3*(a^2*d^2+b^2*c^
2)/a/c/(a^2*d^2+2*a*b*c*d+b^2*c^2)/b^2/d^2*x)*(-b*d*x^4-a*d*x^2+b*c*x^2+a*
c)^(1/2)/(x^4+(a*d-b*c)/b/d*x^2-a*c/b/d)^2+2*b*d*(1/3*(a*d-b*c)*(a^2*d^2+6
*a*b*c*d+b^2*c^2)/a^2/c^2/(a^2*d^2+2*a*b*c*d+b^2*c^2)^2*x^3+1/6*(2*a^4*d^4
+9*a^3*b*c*d^3-2*a^2*b^2*c^2*d^2+9*a*b^3*c^3*d+2*b^4*c^4)/a^2/c^2/(a^2*d^2
+2*a*b*c*d+b^2*c^2)^2/b/d*x)/(-(x^4+(a*d-b*c)/b/d*x^2-a*c/b/d)*b*d)^(1/2)+
(2/3*(a^2*d^2+3*a*b*c*d+b^2*c^2)/(a^2*d^2+2*a*b*c*d+b^2*c^2)/a^2/c^2-1/3*(
2*a^4*d^4+9*a^3*b*c*d^3-2*a^2*b^2*c^2*d^2+9*a*b^3*c^3*d+2*b^4*c^4)/a^2/c^2
/(a^2*d^2+2*a*b*c*d+b^2*c^2)^2)/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^
(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(d/c)^(1/2),(-1-(-a
*d+b*c)/a/d)^(1/2))-(4/3*b*d*(a^3*d^3+5*a^2*b*c*d^2-5*a*b^2*c^2*d-b^3*c^3)
/(a^2*d^2+2*a*b*c*d+b^2*c^2)^2/a^2/c^2-2*b*d*(a*d-b*c)*(a^2*d^2+6*a*b*c*d+
b^2*c^2)/a^2/c^2/(a^2*d^2+2*a*b*c*d+b^2*c^2)^2)*a/(d/c)^(1/2)*(1-d*x^2/c)^
(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)/b*(EllipticF(
x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))-EllipticE(x*(d/c)^(1/2),(-1-(-a*d
+b*c)/a/d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. $2(434) = 868$.

Time = 0.17 (sec) , antiderivative size = 1472, normalized size of antiderivative = 3.13

$$\int \frac{1}{(ac + (bc - ad)x^2 - bdx^4)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*(a^2*b^3*c^5*d + 5*a^3*b^2*c^4*d^2 - 5*a^4*b*c^3*d^3 - a^5*c^2*d^4
+ (b^5*c^3*d^3 + 5*a*b^4*c^2*d^4 - 5*a^2*b^3*c*d^5 - a^3*b^2*d^6)*x^8 - 2*
(b^5*c^4*d^2 + 4*a*b^4*c^3*d^3 - 10*a^2*b^3*c^2*d^4 + 4*a^3*b^2*c*d^5 + a^
4*b*d^6)*x^6 + (b^5*c^5*d + a*b^4*c^4*d^2 - 24*a^2*b^3*c^3*d^3 + 24*a^3*b^
2*c^2*d^4 - a^4*b*c*d^5 - a^5*d^6)*x^4 + 2*(a*b^4*c^5*d + 4*a^2*b^3*c^4*d^
2 - 10*a^3*b^2*c^3*d^3 + 4*a^4*b*c^2*d^4 + a^5*c*d^5)*x^2)*sqrt(a*c)*sqrt(
d/c)*elliptic_e(arcsin(x*sqrt(d/c)), -b*c/(a*d)) + (a^2*b^3*c^6 + 10*a^4*b
*c^3*d^3 + 2*a^5*c^2*d^4 + (b^5*c^4*d^2 + 10*a^2*b^3*c*d^5 + 2*a^3*b^2*d^6
 + 2*(9*a*b^4 - b^5)*c^3*d^3 + (a^2*b^3 - 10*a*b^4)*c^2*d^4)*x^8 + 2*(9*a^
3*b^2 - a^2*b^3)*c^5*d + (a^4*b - 10*a^3*b^2)*c^4*d^2 - 2*(b^5*c^5*d - 8*a
^3*b^2*c*d^5 - 2*a^4*b*d^6 + (17*a*b^4 - 2*b^5)*c^4*d^2 - (17*a^2*b^3 + 8*
a*b^4)*c^3*d^3 - (a^3*b^2 - 20*a^2*b^3)*c^2*d^4)*x^6 + (b^5*c^6 + 2*a^4*b*
c*d^5 + 2*a^5*d^6 + 2*(7*a*b^4 - b^5)*c^5*d - 2*(35*a^2*b^3 + a*b^4)*c^4*d
^2 + 2*(7*a^3*b^2 + 24*a^2*b^3)*c^3*d^3 + (a^4*b - 48*a^3*b^2)*c^2*d^4)*x^
4 + 2*(a*b^4*c^6 - 8*a^4*b*c^2*d^4 - 2*a^5*c*d^5 + (17*a^2*b^3 - 2*a*b^4)*
c^5*d - (17*a^3*b^2 + 8*a^2*b^3)*c^4*d^2 - (a^4*b - 20*a^3*b^2)*c^3*d^3)*x
^2)*sqrt(a*c)*sqrt(d/c)*elliptic_f(arcsin(x*sqrt(d/c)), -b*c/(a*d)) + (2*(
b^5*c^4*d^2 + 5*a*b^4*c^3*d^3 - 5*a^2*b^3*c^2*d^4 - a^3*b^2*c*d^5)*x^7 - (
4*b^5*c^5*d + 17*a*b^4*c^4*d^2 - 22*a^2*b^3*c^3*d^3 + 17*a^3*b^2*c^2*d^4 +
 4*a^4*b*c*d^5)*x^5 + 2*(b^5*c^6 + 2*a*b^4*c^5*d - 11*a^2*b^3*c^4*d^2 +...
```

Sympy [F]

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{5/2}}\,dx=\int \frac{1}{(ac-bdx^4+x^2\,(-ad+bc))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x**2-b*d*x**4)**(5/2),x)
```

output
```
Integral((a*c - b*d*x**4 + x**2*(-a*d + b*c))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{5/2}}\,dx = \int \frac{1}{(-bdx^4+(bc-ad)x^2+ac)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{5/2}}\,dx = \int \frac{1}{(-bdx^4+(bc-ad)x^2+ac)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x, algorithm="giac")
```

output
```
integrate((-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{5/2}}\,dx = \int \frac{1}{\left(-b\,d\,x^4+\left(b\,c-a\,d\right)\,x^2+a\,c\right)^{5/2}}\,dx$$

input
```
int(1/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(5/2),x)
```

output
```
int(1/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{5/2}}\,dx=\int \frac{}{-b^3d^3x^{12}-3a\,b^2d^3x^{10}+3b^3c\,d^2x^{10}-3a^2b\,d^3x^8+9a\,b^2c\,d^2x^8-3b$$

input
```
int(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x)
```

output
```
int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a**3*c**3 - 3*a**3*c**2*d*x**2 +
3*a**3*c*d**2*x**4 - a**3*d**3*x**6 + 3*a**2*b*c**3*x**2 - 9*a**2*b*c**2*d
*x**4 + 9*a**2*b*c*d**2*x**6 - 3*a**2*b*d**3*x**8 + 3*a*b**2*c**3*x**4 - 9
*a*b**2*c**2*d*x**6 + 9*a*b**2*c*d**2*x**8 - 3*a*b**2*d**3*x**10 + b**3*c*
*3*x**6 - 3*b**3*c**2*d*x**8 + 3*b**3*c*d**2*x**10 - b**3*d**3*x**12),x)
```

3.25 $\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 722

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{7/2}}\,dx = \frac{x(b^2c^2+a^2d^2-bd(bc-ad)x^2)}{5ac(bc+ad)^2\,(ac+(bc-ad)x^2-bdx^4)^{5/2}}$$
$$+\frac{x(4b^4c^4+17ab^3c^3d-6a^2b^2c^2d^2+17a^3bcd^3+4a^4d^4-4bd(bc-ad)\,(8abcd+(bc-ad)^2)\,x^2)}{15a^2c^2(bc+ad)^4\,(ac+(bc-ad)x^2-bdx^4)^{3/2}}$$
$$+\frac{x(8b^6c^6+49ab^5c^5d+146a^2b^4c^4d^2-46a^3b^3c^3d^3+146a^4b^2c^2d^4+49a^5bcd^5+8a^6d^6-bd(bc-ad)\,(8b^4c^4-}{15a^3c^3(bc+ad)^6\sqrt{ac+(bc-ad)x^2-bdx^4}}$$
$$+\frac{\sqrt{d}(bc-ad)\,(8b^4c^4+61ab^3c^3d+234a^2b^2c^2d^2+61a^3bcd^3+8a^4d^4)\,\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\right.}{15a^2c^{5/2}(bc+ad)^6\sqrt{ac+(bc-ad)x^2-bdx^4}}$$
$$-\frac{\sqrt{d}(4b^4c^4+23ab^3c^3d-150a^2b^2c^2d^2-49a^3bcd^3-8a^4d^4)\,\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\right.}{15a^2c^{5/2}(bc+ad)^5\sqrt{ac+(bc-ad)x^2-bdx^4}}$$

output
```
1/5*x*(b^2*c^2+a^2*d^2-b*d*(-a*d+b*c)*x^2)/a/c/(a*d+b*c)^2/(a*c+(-a*d+b*c)
*x^2-b*d*x^4)^(5/2)+1/15*x*(4*b^4*c^4+17*a*b^3*c^3*d-6*a^2*b^2*c^2*d^2+17*
a^3*b*c*d^3+4*a^4*d^4-4*b*d*(-a*d+b*c)*(8*a*b*c*d+(-a*d+b*c)^2)*x^2)/a^2/c
^2/(a*d+b*c)^4/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2)+1/15*x*(8*b^6*c^6+49*a*b
^5*c^5*d+146*a^2*b^4*c^4*d^2-46*a^3*b^3*c^3*d^3+146*a^4*b^2*c^2*d^4+49*a^5
*b*c*d^5+8*a^6*d^6-b*d*(-a*d+b*c)*(8*a^4*d^4+61*a^3*b*c*d^3+234*a^2*b^2*c^
2*d^2+61*a*b^3*c^3*d+8*b^4*c^4)*x^2)/a^3/c^3/(a*d+b*c)^6/(a*c+(-a*d+b*c)*x
^2-b*d*x^4)^(1/2)+1/15*d^(1/2)*(-a*d+b*c)*(8*a^4*d^4+61*a^3*b*c*d^3+234*a^
2*b^2*c^2*d^2+61*a*b^3*c^3*d+8*b^4*c^4)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2
)*EllipticE(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/a^2/c^(5/2)/(a*d+b*c)^6/(a
*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)-1/15*d^(1/2)*(-8*a^4*d^4-49*a^3*b*c*d^3-1
50*a^2*b^2*c^2*d^2+23*a*b^3*c^3*d+4*b^4*c^4)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)
^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/a^2/c^(5/2)/(a*d+b*c)
^5/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.71 (sec) , antiderivative size = 560, normalized size of antiderivative = 0.78

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{7/2}}\,dx = \frac{\sqrt{(a+bx^2)(c-dx^2)}\left(\sqrt{\frac{b}{a}}x\left(3a^3c^2d^4(bc+ad)^2(a+bx^2)^3+a^3cd^4(b\right.\right.}{}$$

input
```
Integrate[(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(-7/2),x]
```

output
```
(Sqrt[(a + b*x^2)*(c - d*x^2)]*(Sqrt[b/a]*x*(3*a^3*c^2*d^4*(b*c + a*d)^2*(
a + b*x^2)^3 + a^3*c*d^4*(b*c + a*d)*(23*b*c + 4*a*d)*(a + b*x^2)^3*(c - d
*x^2) + a^3*d^4*(173*b^2*c^2 + 53*a*b*c*d + 8*a^2*d^2)*(a + b*x^2)^3*(c -
d*x^2)^2 + 3*a^2*b^4*c^3*(b*c + a*d)^2*(c - d*x^2)^3 + a*b^4*c^3*(b*c + a*
d)*(4*b*c + 23*a*d)*(a + b*x^2)*(c - d*x^2)^3 + b^4*c^3*(8*b^2*c^2 + 53*a*
b*c*d + 173*a^2*d^2)*(a + b*x^2)^2*(c - d*x^2)^3) - I*b*c*(a + b*x^2)^2*Sq
rt[1 + (b*x^2)/a]*(c - d*x^2)^2*Sqrt[1 - (d*x^2)/c]*((-8*b^5*c^5 - 53*a*b^
4*c^4*d - 173*a^2*b^3*c^3*d^2 + 173*a^3*b^2*c^2*d^3 + 53*a^4*b*c*d^4 + 8*a
^5*d^5)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))] + (8*b^5*c^5 + 5
7*a*b^4*c^4*d + 199*a^2*b^3*c^3*d^2 + 127*a^3*b^2*c^2*d^3 - 27*a^4*b*c*d^4
 - 4*a^5*d^5)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))])))/(15*a^3
*Sqrt[b/a]*c^3*(b*c + a*d)^6*(a + b*x^2)^3*(c - d*x^2)^3)
```

Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 711, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {1405, 25, 1492, 25, 1492, 25, 27, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2(bc-ad)+ac-bdx^4)^{7/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(a^2d^2-bdx^2(bc-ad)+b^2c^2)}{5ac(ad+bc)^2(x^2(bc-ad)+ac-bdx^4)^{5/2}}-\frac{\int -\frac{2(2bc+ad)(bc+2ad)-7bd(bc-ad)x^2}{(-bdx^4+(bc-ad)x^2+ac)^{5/2}}dx}{5ac(ad+bc)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2(2bc+ad)(bc+2ad)-7bd(bc-ad)x^2}{(-bdx^4+(bc-ad)x^2+ac)^{5/2}}dx}{5ac(ad+bc)^2}+\frac{x(a^2d^2-bdx^2(bc-ad)+b^2c^2)}{5ac(ad+bc)^2(x^2(bc-ad)+ac-bdx^4)^{5/2}}$$

$$\Big\downarrow \text{1492}$$

$$\frac{\frac{x\left(-4bdx^2\left(a^2d^2+6abcd+b^2c^2\right)(bc-ad)+2(ad+2bc)(2ad+bc)\left(a^2d^2+b^2c^2\right)+7abcd(bc-ad)^2\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}-\frac{\int -\frac{8b^4c^4+37ab^3dc^3+90a^2b^2d^2c^2+37a^3bd^3c+8a^4d^4 \cdots}{\left(-bdx^4+(bc-ad)x^2 \cdots\right.}}{3ac(ad+bc) \cdots}}{5ac(ad+bc)^2}+\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{5ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{8b^4c^4+37ab^3dc^3+90a^2b^2d^2c^2+37a^3bd^3c+8a^4d^4-12bd(bc-ad)\left(b^2c^2+6abdc+a^2d^2\right)x^2}{\left(-bdx^4+(bc-ad)x^2+ac\right)^{3/2}}dx}{3ac(ad+bc)^2}+\frac{x\left(-4bdx^2\left(a^2d^2+6abcd+b^2c^2\right)(bc-ad)+2(ad+2bc)(2ad \cdots\right.}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac- \cdots\right.}}{5ac(ad+bc)^2}+\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{5ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}$$

$$\Big\downarrow \; 1492$$

$$\frac{\frac{\frac{x\left(8a^6d^6+49a^5bcd^5+146a^4b^2c^2d^4-46a^3b^3c^3d^3+146a^2b^4c^4d^2-bdx^2(bc-ad)\left(8a^4d^4+61a^3bcd^3+234a^2b^2c^2d^2+61ab^3c^3d+8b^4c^4\right)+49ab^5c^5d+8b^6c^6\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}-\frac{\int -\frac{bd \cdots}{\cdots}}{\cdots}}{3ac(ad+bc)^2} \cdots}{\cdots}+\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{5ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{bd\left((bc-ad)\left(8b^4c^4+61ab^3dc^3+234a^2b^2d^2c^2+61a^3bd^3c+8a^4d^4\right)x^2+2ac\left(2b^4c^4+13ab^3dc^3+150a^2b^2d^2c^2+13a^3bd^3c+2a^4d^4\right)\right)}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{ac(ad+bc)^2}+\frac{x\left(8a^6d^6+49a^5bcd^5+146 \cdots\right.}{\cdots}}{3ac(ad+bc)^2} \cdots}{\cdots}+\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{5ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{bd\int \frac{(bc-ad)\left(8b^4c^4+61ab^3dc^3+234a^2b^2d^2c^2+61a^3bd^3c+8a^4d^4\right)x^2+2ac\left(2b^4c^4+13ab^3dc^3+150a^2b^2d^2c^2+13a^3bd^3c+2a^4d^4\right)}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{ac(ad+bc)^2}+\frac{x\left(8a^6d^6+49a^5bcd^5+146a^4\right.}{}}{3ac(ad+bc)^2}$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{5ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}$$

$\Big\downarrow$ 1514

$$\frac{\frac{bd\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{(bc-ad)\left(8b^4c^4+61ab^3dc^3+234a^2b^2d^2c^2+61a^3bd^3c+8a^4d^4\right)x^2+2ac\left(2b^4c^4+13ab^3dc^3+150a^2b^2d^2c^2+13a^3bd^3c+2a^4d^4\right)}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{x\left(8a^6d^6\right.}{}}{3ac(ad+bc)^2}$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{5ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}$$

$\Big\downarrow$ 399

$$\frac{bd\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a(bc-ad)\left(8a^4d^4+61a^3bcd^3+234a^2b^2c^2d^2+61ab^3c^3d+8b^4c^4\right)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}-\frac{a(ad+bc)\left(-8a^4d^4-49a^3bcd^3-150a^2b^2c^2d^2+23ab^3c^3d+\right.}{b}\right.}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{5ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}$$

$\Big\downarrow$ 321

$$\frac{bd\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a(bc-ad)\left(8a^4d^4+61a^3bcd^3+234a^2b^2c^2d^2+61ab^3c^3d+8b^4c^4\right)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}-\frac{a\sqrt{c}(ad+bc)\left(-8a^4d^4-49a^3bcd^3-150a^2b^2c^2d^2+23ab^3c^3\right.}{b\sqrt{d}}\right.}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{5ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}$$

$\Big\downarrow$ 327

$$\frac{x\left(a^2d^2-bdx^2(bc-ad)+b^2c^2\right)}{5ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}+$$

$$\frac{\frac{x\left(-4bdx^2\left(a^2d^2+6abcd+b^2c^2\right)(bc-ad)+2(ad+2bc)(2ad+bc)\left(a^2d^2+b^2c^2\right)+7abcd(bc-ad)^2\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}+\frac{bd\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a\sqrt{c}(bc-ad)\left(8a^4d^4+61a^3bcd^3\right.}{}\right.}{}}{}$$

input
```
Int[(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(-7/2),x]
```

output
```
(x*(b^2*c^2 + a^2*d^2 - b*d*(b*c - a*d)*x^2))/(5*a*c*(b*c + a*d)^2*(a*c +
(b*c - a*d)*x^2 - b*d*x^4)^(5/2)) + ((x*(7*a*b*c*d*(b*c - a*d)^2 + 2*(2*b*
c + a*d)*(b*c + 2*a*d)*(b^2*c^2 + a^2*d^2) - 4*b*d*(b*c - a*d)*(b^2*c^2 +
6*a*b*c*d + a^2*d^2)*x^2))/(3*a*c*(b*c + a*d)^2*(a*c + (b*c - a*d)*x^2 - b
*d*x^4)^(3/2)) + ((x*(8*b^6*c^6 + 49*a*b^5*c^5*d + 146*a^2*b^4*c^4*d^2 - 4
6*a^3*b^3*c^3*d^3 + 146*a^4*b^2*c^2*d^4 + 49*a^5*b*c*d^5 + 8*a^6*d^6 - b*d
*(b*c - a*d)*(8*b^4*c^4 + 61*a*b^3*c^3*d + 234*a^2*b^2*c^2*d^2 + 61*a^3*b*
c*d^3 + 8*a^4*d^4)*x^2))/(a*c*(b*c + a*d)^2*Sqrt[a*c + (b*c - a*d)*x^2 - b
*d*x^4]) + (b*d*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*((a*Sqrt[c]*(b*c -
 a*d)*(8*b^4*c^4 + 61*a*b^3*c^3*d + 234*a^2*b^2*c^2*d^2 + 61*a^3*b*c*d^3 +
 8*a^4*d^4)*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqr
t[d]) - (a*Sqrt[c]*(b*c + a*d)*(4*b^4*c^4 + 23*a*b^3*c^3*d - 150*a^2*b^2*c
^2*d^2 - 49*a^3*b*c*d^3 - 8*a^4*d^4)*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]]
, -((b*c)/(a*d))])/(b*Sqrt[d])))/(a*c*(b*c + a*d)^2*Sqrt[a*c + (b*c - a*d)
*x^2 - b*d*x^4]))/(3*a*c*(b*c + a*d)^2))/(5*a*c*(b*c + a*d)^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 1144, normalized size of antiderivative = 1.58

method	result	size
default	Expression too large to display	1144
elliptic	Expression too large to display	1144

input

```
int(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
(-1/5/a/c*(a*d-b*c)/(a^2*d^2+2*a*b*c*d+b^2*c^2)/b^2/d^2*x^3-1/5*(a^2*d^2+b
^2*c^2)/a/c/(a^2*d^2+2*a*b*c*d+b^2*c^2)/b^3/d^3*x)*(-b*d*x^4-a*d*x^2+b*c*x
^2+a*c)^(1/2)/(x^4+(a*d-b*c)/b/d*x^2-a*c/b/d)^3+(4/15*(a*d-b*c)/b/d*(a^2*d
^2+6*a*b*c*d+b^2*c^2)/a^2/c^2/(a^2*d^2+2*a*b*c*d+b^2*c^2)^2*x^3+1/15*(4*a^
4*d^4+17*a^3*b*c*d^3-6*a^2*b^2*c^2*d^2+17*a*b^3*c^3*d+4*b^4*c^4)/a^2/c^2/(
a^2*d^2+2*a*b*c*d+b^2*c^2)^2/b^2/d^2*x)*(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/
2)/(x^4+(a*d-b*c)/b/d*x^2-a*c/b/d)^2+2*b*d*(1/30*(8*a^5*d^5+53*a^4*b*c*d^4
+173*a^3*b^2*c^2*d^3-173*a^2*b^3*c^3*d^2-53*a*b^4*c^4*d-8*b^5*c^5)/a^3/c^3
/(a^2*d^2+2*a*b*c*d+b^2*c^2)^3*x^3+1/30*(8*a^6*d^6+49*a^5*b*c*d^5+146*a^4*
b^2*c^2*d^4-46*a^3*b^3*c^3*d^3+146*a^2*b^4*c^4*d^2+49*a*b^5*c^5*d+8*b^6*c^
6)/a^3/c^3/(a^2*d^2+2*a*b*c*d+b^2*c^2)^3/b/d*x)/(-(x^4+(a*d-b*c)/b/d*x^2-a
*c/b/d)*b*d)^(1/2)+(1/15/(a^2*d^2+2*a*b*c*d+b^2*c^2)^2*(8*a^4*d^4+37*a^3*b
*c*d^3+90*a^2*b^2*c^2*d^2+37*a*b^3*c^3*d+8*b^4*c^4)/a^3/c^3-1/15*(8*a^6*d^
6+49*a^5*b*c*d^5+146*a^4*b^2*c^2*d^4-46*a^3*b^3*c^3*d^3+146*a^2*b^4*c^4*d^
2+49*a*b^5*c^5*d+8*b^6*c^6)/a^3/c^3/(a^2*d^2+2*a*b*c*d+b^2*c^2)^3)/(d/c)^(
1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1
/2)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))+1/15*d*(8*a^5*d^5+5
3*a^4*b*c*d^4+173*a^3*b^2*c^2*d^3-173*a^2*b^3*c^3*d^2-53*a*b^4*c^4*d-8*b^5
*c^5)/(a^2*d^2+2*a*b*c*d+b^2*c^2)^3/a^2/c^3/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*
(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(d/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3272 vs. 2(681) = 1362.

Time = 0.73 (sec) , antiderivative size = 3272, normalized size of antiderivative = 4.53

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(7/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x**2-b*d*x**4)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(ac+(bc-ad)x^2-bdx^4)^{7/2}}\,dx = \int \frac{1}{(-bdx^4+(bc-ad)x^2+ac)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{\left(ac+(bc-ad)x^2-bdx^4\right)^{7/2}}\,dx=\int \frac{1}{\left(-bdx^4+(bc-ad)x^2+ac\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(7/2),x, algorithm="giac")
```

output
```
integrate((-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(ac+(bc-ad)x^2-bdx^4\right)^{7/2}}\,dx=\int \frac{1}{{\left(-b\,d\,x^4+\left(b\,c-a\,d\right)\,x^2+a\,c\right)}^{7/2}}\,dx$$

input
```
int(1/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(7/2),x)
```

output
```
int(1/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(ac+(bc-ad)x^2-bdx^4\right)^{7/2}}\,dx=\int \frac{}{b^4d^4x^{16}+4a\,b^3d^4x^{14}-4b^4c\,d^3x^{14}+6a^2b^2d^4x^{12}-16a\,b^3c\,d^3x^{12}+}$$

input
```
int(1/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(7/2),x)
```

output

```
int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a**4*c**4 - 4*a**4*c**3*d*x**2 +
6*a**4*c**2*d**2*x**4 - 4*a**4*c*d**3*x**6 + a**4*d**4*x**8 + 4*a**3*b*c**
4*x**2 - 16*a**3*b*c**3*d*x**4 + 24*a**3*b*c**2*d**2*x**6 - 16*a**3*b*c*d*
*3*x**8 + 4*a**3*b*d**4*x**10 + 6*a**2*b**2*c**4*x**4 - 24*a**2*b**2*c**3*
d*x**6 + 36*a**2*b**2*c**2*d**2*x**8 - 24*a**2*b**2*c*d**3*x**10 + 6*a**2*
b**2*d**4*x**12 + 4*a*b**3*c**4*x**6 - 16*a*b**3*c**3*d*x**8 + 24*a*b**3*c
**2*d**2*x**10 - 16*a*b**3*c*d**3*x**12 + 4*a*b**3*d**4*x**14 + b**4*c**4*
x**8 - 4*b**4*c**3*d*x**10 + 6*b**4*c**2*d**2*x**12 - 4*b**4*c*d**3*x**14
+ b**4*d**4*x**16),x)
```

3.26 $\int \frac{1}{\sqrt{2+9x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 102

$$\int \frac{1}{\sqrt{2+9x^2+3x^4}}\,dx = \frac{\sqrt{4+\left(9-\sqrt{57}\right)x^2}\sqrt{4+\left(9+\sqrt{57}\right)x^2}\,\text{EllipticF}\left(\arctan\left(\frac{1}{2}\sqrt{9-\sqrt{57}}x\right),\frac{1}{4}\left(-19-3\sqrt{57}\right)\right)}{2\sqrt{9-\sqrt{57}}\sqrt{2+9x^2+3x^4}}$$

output
```
1/2*(4+(9-57^(1/2))*x^2)^(1/2)*(4+(9+57^(1/2))*x^2)^(1/2)*InverseJacobiAM(
arctan(1/2*(9-57^(1/2))^(1/2)*x),1/2*(-19-3*57^(1/2))^(1/2))/(9-57^(1/2))^
(1/2)/(3*x^4+9*x^2+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{2+9x^2+3x^4}}\,dx = -\frac{i\sqrt{9-\sqrt{57}+6x^2}\sqrt{9+\sqrt{57}+6x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{6}{9+\sqrt{57}}}x\right),\frac{23}{4}+\frac{3\sqrt{57}}{4}\right)}{\sqrt{6\left(9-\sqrt{57}\right)}\sqrt{2+9x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 + 9*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[9 - Sqrt[57] + 6*x^2]*Sqrt[9 + Sqrt[57] + 6*x^2]*EllipticF[I*Ar
cSinh[Sqrt[6/(9 + Sqrt[57])]*x], 23/4 + (3*Sqrt[57])/4])/(Sqrt[6*(9 - Sqrt
[57])]*Sqrt[2 + 9*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1412\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+9x^2+2}}\,dx$$

$$\Big\downarrow \text{ 1412}$$

$$\frac{\sqrt{\frac{\left(9-\sqrt{57}\right)x^2+4}{\left(9+\sqrt{57}\right)x^2+4}}\left(\left(9+\sqrt{57}\right)x^2+4\right)\text{EllipticF}\left(\arctan\left(\frac{1}{2}\sqrt{9+\sqrt{57}}x\right),\frac{1}{4}\left(-19+3\sqrt{57}\right)\right)}{2\sqrt{9+\sqrt{57}}\sqrt{3x^4+9x^2+2}}$$

input
```
Int[1/Sqrt[2 + 9*x^2 + 3*x^4],x]
```

output
```
(Sqrt[(4 + (9 - Sqrt[57])*x^2)/(4 + (9 + Sqrt[57])*x^2)]*(4 + (9 + Sqrt[57
])*x^2)*EllipticF[ArcTan[(Sqrt[9 + Sqrt[57]]*x)/2], (-19 + 3*Sqrt[57])/4])
/(2*Sqrt[9 + Sqrt[57]]*Sqrt[2 + 9*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{9}{4}+\frac{\sqrt{57}}{4}\right)x^{2}}\,\sqrt{1-\left(-\frac{9}{4}-\frac{\sqrt{57}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-9+\sqrt{57}}}{2},\frac{3\sqrt{6}}{4}+\frac{\sqrt{38}}{4}\right)}{\sqrt{-9+\sqrt{57}}\,\sqrt{3x^{4}+9x^{2}+2}}$	78
elliptic	$\frac{2\sqrt{1-\left(-\frac{9}{4}+\frac{\sqrt{57}}{4}\right)x^{2}}\,\sqrt{1-\left(-\frac{9}{4}-\frac{\sqrt{57}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-9+\sqrt{57}}}{2},\frac{3\sqrt{6}}{4}+\frac{\sqrt{38}}{4}\right)}{\sqrt{-9+\sqrt{57}}\,\sqrt{3x^{4}+9x^{2}+2}}$	78

input
```
int(1/(3*x^4+9*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-9+57^(1/2))^(1/2)*(1-(-9/4+1/4*57^(1/2))*x^2)^(1/2)*(1-(-9/4-1/4*57^(1
/2))*x^2)^(1/2)/(3*x^4+9*x^2+2)^(1/2)*EllipticF(1/2*x*(-9+57^(1/2))^(1/2),
3/4*6^(1/2)+1/4*38^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{2+9x^2+3x^4}}\,dx$$
$$= -\frac{1}{24}\left(\sqrt{57}\sqrt{2}+9\sqrt{2}\right)\sqrt{\sqrt{57}-9}F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{57}-9}\right) \mid \frac{3}{4}\sqrt{57}+\frac{23}{4})$$

input
```
integrate(1/(3*x^4+9*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(57)*sqrt(2) + 9*sqrt(2))*sqrt(sqrt(57) - 9)*elliptic_f(arcsin(
1/2*x*sqrt(sqrt(57) - 9)), 3/4*sqrt(57) + 23/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+9x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+9x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4+9*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 9*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+9x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+9\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+9*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 9*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+9x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+9\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+9*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 9*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+9x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+9\,x^2+2}}\,dx$$

input
```
int(1/(9*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(9*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+9x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4+9x^2+2}}{3x^4+9x^2+2}dx$$

input
```
int(1/(3*x^4+9*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 9*x**2 + 2)/(3*x**4 + 9*x**2 + 2),x)
```

3.27 $\int \frac{1}{\sqrt{2+8x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \frac{1}{\sqrt{2+8x^2+3x^4}}\,dx = \frac{\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}\sqrt{2+\left(4-\sqrt{10}\right)x^2}\sqrt{2+\left(4+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{2}\left(4-\sqrt{10}\right)}x\right),-\frac{2}{3}(5+2}{2\sqrt{2+8x^2+3x^4}}$$

output
```
1/6*(12+3*10^(1/2))^(1/2)*(2+(4-10^(1/2))*x^2)^(1/2)*(2+(4+10^(1/2))*x^2)^
(1/2)*InverseJacobiAM(arctan(1/2*(8-2*10^(1/2))^(1/2)*x),1/3*(-30-12*10^(1
/2))^(1/2))/(3*x^4+8*x^2+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{2+8x^2+3x^4}}\,dx = -\frac{i\sqrt{\frac{-4+\sqrt{10}-3x^2}{-4+\sqrt{10}}}\sqrt{4+\sqrt{10}+3x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2-\sqrt{\frac{5}{2}}}x\right),\frac{1}{3}\left(13+4\sqrt{10}\right)\right)}{\sqrt{6+24x^2+9x^4}}$$

input
```
Integrate[1/Sqrt[2 + 8*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[(-4 + Sqrt[10] - 3*x^2)/(-4 + Sqrt[10])]*Sqrt[4 + Sqrt[10] + 3*
x^2]*EllipticF[I*ArcSinh[Sqrt[2 - Sqrt[5/2]]*x], (13 + 4*Sqrt[10])/3])/Sqr
t[6 + 24*x^2 + 9*x^4]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+8x^2+2}}dx$$

$$\Big\downarrow \text{1412}$$

$$\frac{\sqrt{\frac{\left(4-\sqrt{10}\right)x^2+2}{\left(4+\sqrt{10}\right)x^2+2}}\left(\left(4+\sqrt{10}\right)x^2+2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{2}\left(4+\sqrt{10}\right)}x\right),-\frac{2}{3}\left(5-2\sqrt{10}\right)\right)}{\sqrt{2\left(4+\sqrt{10}\right)}\sqrt{3x^4+8x^2+2}}$$

input
```
Int[1/Sqrt[2 + 8*x^2 + 3*x^4],x]
```

output
```
(Sqrt[(2 + (4 - Sqrt[10])*x^2)/(2 + (4 + Sqrt[10])*x^2)]*(2 + (4 + Sqrt[10
])*x^2)*EllipticF[ArcTan[Sqrt[(4 + Sqrt[10])/2]*x], (-2*(5 - 2*Sqrt[10]))/
3])/(Sqrt[2*(4 + Sqrt[10])]*Sqrt[2 + 8*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{2\sqrt{1-\left(-2+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(-2-\frac{\sqrt{10}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-8+2\sqrt{10}}}{2},\frac{2\sqrt{6}}{3}+\frac{\sqrt{15}}{3}\right)}{\sqrt{-8+2\sqrt{10}}\,\sqrt{3x^4+8x^2+2}}$	82
elliptic	$\frac{2\sqrt{1-\left(-2+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(-2-\frac{\sqrt{10}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-8+2\sqrt{10}}}{2},\frac{2\sqrt{6}}{3}+\frac{\sqrt{15}}{3}\right)}{\sqrt{-8+2\sqrt{10}}\,\sqrt{3x^4+8x^2+2}}$	82

input
```
int(1/(3*x^4+8*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-8+2*10^(1/2))^(1/2)*(1-(-2+1/2*10^(1/2))*x^2)^(1/2)*(1-(-2-1/2*10^(1/2
))*x^2)^(1/2)/(3*x^4+8*x^2+2)^(1/2)*EllipticF(1/2*x*(-8+2*10^(1/2))^(1/2),
2/3*6^(1/2)+1/3*15^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{2+8x^2+3x^4}}\,dx$$
$$= -\frac{1}{6}\left(\sqrt{10}\sqrt{2}+4\sqrt{2}\right)\sqrt{\frac{1}{2}\sqrt{10}-2}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{10}-2}\right)\mid\frac{4}{3}\sqrt{10}+\frac{13}{3})$$

input
```
integrate(1/(3*x^4+8*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(10)*sqrt(2) + 4*sqrt(2))*sqrt(1/2*sqrt(10) - 2)*elliptic_f(arcs
in(x*sqrt(1/2*sqrt(10) - 2)), 4/3*sqrt(10) + 13/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+8x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+8x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4+8*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 8*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+8x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+8\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+8*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 8*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+8x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+8\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+8*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 8*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+8x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+8\,x^2+2}}\,dx$$

input
```
int(1/(8*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(8*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+8x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4+8x^2+2}}{3x^4+8x^2+2}dx$$

input
```
int(1/(3*x^4+8*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 8*x**2 + 2)/(3*x**4 + 8*x**2 + 2),x)
```

3.28 $\int \frac{1}{\sqrt{2+7x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 47

$$\int \frac{1}{\sqrt{2+7x^2+3x^4}}\,dx=\frac{\sqrt{2+x^2}\sqrt{1+3x^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{2}}\right),-5\right)}{\sqrt{2+7x^2+3x^4}}$$

output
```
(x^2+2)^(1/2)*(3*x^2+1)^(1/2)*InverseJacobiAM(arctan(1/2*x*2^(1/2)),I*5^(1
/2))/(3*x^4+7*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{2+7x^2+3x^4}}\,dx=-\frac{i\sqrt{2+x^2}\sqrt{1+3x^2}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{3}x\right),\frac{1}{6}\right)}{\sqrt{6}\sqrt{2+7x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 + 7*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[2 + x^2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], 1/6])/
(Sqrt[6]*Sqrt[2 + 7*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+7x^2+2}}\,dx$$

$$\Big\downarrow \text{1412}$$

$$\frac{\sqrt{\frac{x^2+2}{3x^2+1}}\,(3x^2+1)\,\text{EllipticF}\left(\arctan\left(\sqrt{3}x\right),\frac{5}{6}\right)}{\sqrt{6}\sqrt{3x^4+7x^2+2}}$$

input
```
Int[1/Sqrt[2 + 7*x^2 + 3*x^4],x]
```

output
```
(Sqrt[(2 + x^2)/(1 + 3*x^2)]*(1 + 3*x^2)*EllipticF[ArcTan[Sqrt[3]*x], 5/6]
)/(Sqrt[6]*Sqrt[2 + 7*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{3x^{2}+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{6}\right)}{2\sqrt{3x^{4}+7x^{2}+2}}$	50
elliptic	$-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{3x^{2}+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{6}\right)}{2\sqrt{3x^{4}+7x^{2}+2}}$	50

input
```
int(1/(3*x^4+7*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(3*x^2+1)^(1/2)/(3*x^4+7*x^2+2)^(1/2)*Ellip
ticF(1/2*I*x*2^(1/2),6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{2+7x^2+3x^4}}\,dx = -\sqrt{2}\sqrt{-\frac{1}{2}}F\left(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)\mid 6\right)$$

input
```
integrate(1/(3*x^4+7*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(2)*sqrt(-1/2)*elliptic_f(arcsin(sqrt(-1/2)*x), 6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+7x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+7x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4+7*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 7*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2 + 7x^2 + 3x^4}}\, dx = \int \frac{1}{\sqrt{3\,x^4 + 7\,x^2 + 2}}\, dx$$

input
```
integrate(1/(3*x^4+7*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 7*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2 + 7x^2 + 3x^4}}\, dx = \int \frac{1}{\sqrt{3\,x^4 + 7\,x^2 + 2}}\, dx$$

input
```
integrate(1/(3*x^4+7*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 7*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2 + 7x^2 + 3x^4}}\, dx = \int \frac{1}{\sqrt{3\,x^4 + 7\,x^2 + 2}}\, dx$$

input
```
int(1/(7*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(7*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+7x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4+7x^2+2}}{3x^4+7x^2+2}dx$$

input
```
int(1/(3*x^4+7*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 7*x**2 + 2)/(3*x**4 + 7*x**2 + 2),x)
```

3.29 $\int \frac{1}{\sqrt{2+6x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{1}{\sqrt{2+6x^2+3x^4}}\,dx = \frac{\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}\sqrt{2+\left(3-\sqrt{3}\right)x^2}\sqrt{2+\left(3+\sqrt{3}\right)x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{2}\left(3-\sqrt{3}\right)}x\right),-1-\sqrt{3}\right)}{2\sqrt{2+6x^2+3x^4}}$$

output
```
1/6*(9+3*3^(1/2))^(1/2)*(2+(3-3^(1/2))*x^2)^(1/2)*(2+(3+3^(1/2))*x^2)^(1/2
)*InverseJacobiAM(arctan(1/2*(6-2*3^(1/2))^(1/2)*x),(-1-3^(1/2))^(1/2))/(3
*x^4+6*x^2+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{2+6x^2+3x^4}}\,dx = -\frac{i\sqrt{\frac{-3+\sqrt{3}-3x^2}{-3+\sqrt{3}}}\sqrt{3+\sqrt{3}+3x^2}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right),2+\sqrt{3}\right)}{\sqrt{6+18x^2+9x^4}}$$

input
```
Integrate[1/Sqrt[2 + 6*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[(-3 + Sqrt[3] - 3*x^2)/(-3 + Sqrt[3])]*Sqrt[3 + Sqrt[3] + 3*x^2
]*EllipticF[I*ArcSinh[Sqrt[3/(3 + Sqrt[3])]*x], 2 + Sqrt[3]])/Sqrt[6 + 18*
x^2 + 9*x^4]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1412\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+6x^2+2}}dx$$
$$\Big\downarrow \text{1412}$$
$$\frac{\sqrt{\frac{\left(3-\sqrt{3}\right)x^2+2}{\left(3+\sqrt{3}\right)x^2+2}}\left(\left(3+\sqrt{3}\right)x^2+2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}x\right),-1+\sqrt{3}\right)}{\sqrt{2\left(3+\sqrt{3}\right)}\sqrt{3x^4+6x^2+2}}$$

input
```
Int[1/Sqrt[2 + 6*x^2 + 3*x^4],x]
```

output
```
(Sqrt[(2 + (3 - Sqrt[3])*x^2)/(2 + (3 + Sqrt[3])*x^2)]*(2 + (3 + Sqrt[3])*
x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[3])/2]*x], -1 + Sqrt[3]])/(Sqrt[2*(3
+ Sqrt[3])]*Sqrt[2 + 6*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^{2}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-6+2\sqrt{3}}}{2},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{-6+2\sqrt{3}}\,\sqrt{3x^{4}+6x^{2}+2}}$	82
elliptic	$\frac{2\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^{2}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-6+2\sqrt{3}}}{2},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{-6+2\sqrt{3}}\,\sqrt{3x^{4}+6x^{2}+2}}$	82

input
```
int(1/(3*x^4+6*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-6+2*3^(1/2))^(1/2)*(1-(-3/2+1/2*3^(1/2))*x^2)^(1/2)*(1-(-3/2-1/2*3^(1/
2))*x^2)^(1/2)/(3*x^4+6*x^2+2)^(1/2)*EllipticF(1/2*x*(-6+2*3^(1/2))^(1/2),
1/2*6^(1/2)+1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{2+6x^2+3x^4}}\,dx = -\frac{1}{6}\left(\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{\frac{1}{2}\sqrt{3}-\frac{3}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{3}-\frac{3}{2}}\right)\mid\sqrt{3}+2)$$

input
```
integrate(1/(3*x^4+6*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(1/2*sqrt(3) - 3/2)*elliptic_f(arcs
in(x*sqrt(1/2*sqrt(3) - 3/2)), sqrt(3) + 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+6x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+6x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4+6*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 6*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+6x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+6\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+6*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 6*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+6x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+6\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+6*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 6*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+6x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+6\,x^2+2}}\,dx$$

input
```
int(1/(6*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(6*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+6x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4+6x^2+2}}{3x^4+6x^2+2}dx$$

input
```
int(1/(3*x^4+6*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 6*x**2 + 2)/(3*x**4 + 6*x**2 + 2),x)
```

3.30 $\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx=\frac{\sqrt{1+x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
1/2*(x^2+1)^(1/2)*(3*x^2+2)^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))
*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx=-\frac{i\sqrt{1+x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)}{\sqrt{6+15x^2+9x^4}}$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3]
)/Sqrt[6 + 15*x^2 + 9*x^4]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1413}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx$$
$$\Big\downarrow \text{1413}$$
$$\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}$$

input
```
Int[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2
]*Sqrt[2 + 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}$	44
elliptic	$-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}$	44

input
```
int(1/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1
/2*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx = -\frac{1}{2}i\,\sqrt{2}F(\arcsin{(i\,x)}\mid\frac{3}{2})$$

input
```
integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*I*sqrt(2)*elliptic_f(arcsin(I*x), 3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx$$

input
```
int(1/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2),x)
```

3.31 $\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2+4x^2+3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4+4*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/6*(18-6*6^(1/2))^(1/2))*6^(3/4)/(3*
x^4+4*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx = -\frac{i\sqrt{1-\frac{3x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{-2+i\sqrt{2}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{3}{-2-i\sqrt{2}}}x\right),\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{2+4x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 + 4*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (3*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(-2 + I*Sqrt[2])
]*EllipticF[I*ArcSinh[Sqrt[-3/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 +
 I*Sqrt[2])])/(Sqrt[3]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[2 + 4*x^2 + 3*x^4
])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+4x^2+2}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3x^4+4x^2+2}}$$

input
```
Int[1/Sqrt[2 + 4*x^2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[2 + 4*x^2 + 3*
x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^{2}}\,\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-4+2i\sqrt{2}}}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{-4+2i\sqrt{2}}\,\sqrt{3x^{4}+4x^{2}+2}}$	87
elliptic	$\frac{2\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^{2}}\,\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-4+2i\sqrt{2}}}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{-4+2i\sqrt{2}}\,\sqrt{3x^{4}+4x^{2}+2}}$	87

input
```
int(1/(3*x^4+4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-4+2*I*2^(1/2))^(1/2)*(1-(-1+1/2*I*2^(1/2))*x^2)^(1/2)*(1-(-1-1/2*I*2^(
1/2))*x^2)^(1/2)/(3*x^4+4*x^2+2)^(1/2)*EllipticF(1/2*x*(-4+2*I*2^(1/2))^(1
/2),1/3*(3+6*I*2^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx$$
$$= -\frac{1}{6}\sqrt{2}(\sqrt{-2}+2)\sqrt{\frac{1}{2}\sqrt{-2}-1}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-2}-1}\right)\mid\frac{2}{3}\sqrt{-2}+\frac{1}{3})$$

input
```
integrate(1/(3*x^4+4*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2)*(sqrt(-2) + 2)*sqrt(1/2*sqrt(-2) - 1)*elliptic_f(arcsin(x*sqr
t(1/2*sqrt(-2) - 1)), 2/3*sqrt(-2) + 1/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+4x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4+4*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 4*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+4\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+4*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 4*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+4\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+4*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 4*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+4\,x^2+2}}\,dx$$

input
```
int(1/(4*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(4*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4+4x^2+2}}{3x^4+4x^2+2}dx$$

input
```
int(1/(3*x^4+4*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 4*x**2 + 2)/(3*x**4 + 4*x**2 + 2),x)
```

3.32 $\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2+3x^2+3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4+3*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/4*(8-2*6^(1/2))^(1/2))*6^(3/4)/(3*x
^4+3*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx = -\frac{i\sqrt{1-\frac{6x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{-3+i\sqrt{15}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{6}{-3-i\sqrt{15}}}x\right),\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{2+3x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 + 3*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (6*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(-3 + I*Sqrt[15
])]*EllipticF[I*ArcSinh[Sqrt[-6/(-3 - I*Sqrt[15])]*x], (-3 - I*Sqrt[15])/(
-3 + I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[2 + 3*x^2 +
 3*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+3x^2+2}}\,dx$$

$$\Big\downarrow \; 1416$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+3x^2+2}}$$

input
```
Int[1/Sqrt[2 + 3*x^2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[2 + 3*x^2 + 3*
x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-3+i\sqrt{15}}}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{-3+i\sqrt{15}}\,\sqrt{3x^{4}+3x^{2}+2}}$	87
elliptic	$\frac{2\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-3+i\sqrt{15}}}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{-3+i\sqrt{15}}\,\sqrt{3x^{4}+3x^{2}+2}}$	87

input
```
int(1/(3*x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-3+I*15^(1/2))^(1/2)*(1-(-3/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*I
*15^(1/2))*x^2)^(1/2)/(3*x^4+3*x^2+2)^(1/2)*EllipticF(1/2*x*(-3+I*15^(1/2)
)^(1/2),1/2*(-1+I*15^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx$$
$$= -\frac{1}{24}\,\sqrt{2}\left(\sqrt{-15}+3\right)\sqrt{\sqrt{-15}-3}F(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{-15}-3}\right) \mid \frac{1}{4}\,\sqrt{-15}-\frac{1}{4})$$

input
```
integrate(1/(3*x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*sqrt(2)*(sqrt(-15) + 3)*sqrt(sqrt(-15) - 3)*elliptic_f(arcsin(1/2*x*
sqrt(sqrt(-15) - 3)), 1/4*sqrt(-15) - 1/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+3x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 3*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+3\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+3\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4+3\,x^2+2}}\,dx$$

input
```
int(1/(3*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(3*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx=\int \frac{\sqrt{3x^4+3x^2+2}}{3x^4+3x^2+2}dx$$

input
```
int(1/(3*x^4+3*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 3*x**2 + 2)/(3*x**4 + 3*x**2 + 2),x)
```

3.33 $\int \frac{1}{\sqrt{2+2x^2+3x^4}} dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{1}{\sqrt{2+2x^2+3x^4}} dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2+2x^2+3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4+2*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/6*(18-3*6^(1/2))^(1/2))*6^(3/4)/(3*
x^4+2*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{2+2x^2+3x^4}} dx = -\frac{i\sqrt{1-\frac{3x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{-1+i\sqrt{5}}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{3}{-1-i\sqrt{5}}}x\right),\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{2+2x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 + 2*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (3*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(-1 + I*Sqrt[5])
]*EllipticF[I*ArcSinh[Sqrt[-3/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 +
 I*Sqrt[5])])/(Sqrt[3]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[2 + 2*x^2 + 3*x^4
])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+2x^2+2}}\,dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+2x^2+2}}$$

input
```
Int[1/Sqrt[2 + 2*x^2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[2 + 2*x^2 + 3
*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{i\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{5}}}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{-2+2i\sqrt{5}}\,\sqrt{3x^4+2x^2+2}}$	87
elliptic	$\frac{2\sqrt{1-\left(\frac{i\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{5}}}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{-2+2i\sqrt{5}}\,\sqrt{3x^4+2x^2+2}}$	87

input
```
int(1/(3*x^4+2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-2+2*I*5^(1/2))^(1/2)*(1-(1/2*I*5^(1/2)-1/2)*x^2)^(1/2)*(1-(-1/2-1/2*I*
5^(1/2))*x^2)^(1/2)/(3*x^4+2*x^2+2)^(1/2)*EllipticF(1/2*x*(-2+2*I*5^(1/2))
^(1/2),1/3*(-6+3*I*5^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{2+2x^2+3x^4}}\,dx$$
$$= -\frac{1}{6}\sqrt{2}(\sqrt{-5}+1)\sqrt{\frac{1}{2}\sqrt{-5}-\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-5}-\frac{1}{2}}\right)\mid\frac{1}{3}\sqrt{-5}-\frac{2}{3})$$

input
```
integrate(1/(3*x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2)*(sqrt(-5) + 1)*sqrt(1/2*sqrt(-5) - 1/2)*elliptic_f(arcsin(x*s
qrt(1/2*sqrt(-5) - 1/2)), 1/3*sqrt(-5) - 2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+2x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4+2*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 2*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+2\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 2*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+2\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+2*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 2*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+2\,x^2+2}}\,dx$$

input
```
int(1/(2*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(2*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+2x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4+2x^2+2}}{3x^4+2x^2+2}dx$$

input
```
int(1/(3*x^4+2*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 2*x**2 + 2)/(3*x**4 + 2*x**2 + 2),x)
```

3.34 $\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2+x^2+3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4+x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobi
AM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/12*(72-6*6^(1/2))^(1/2))*6^(3/4)/(3*x
^4+x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx = -\frac{i\sqrt{1-\frac{6x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{-1+i\sqrt{23}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{6}{-1-i\sqrt{23}}}x\right),\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{2+x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 + x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (6*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(-1 + I*Sqrt[23
])]*EllipticF[I*ArcSinh[Sqrt[-6/(-1 - I*Sqrt[23])]*x], (-1 - I*Sqrt[23])/(
-1 + I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[2 + x^2 + 3
*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+x^2+2}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+x^2+2}}$$

input
```
Int[1/Sqrt[2 + x^2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2
*ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 + x^2 + 3*x^
4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-1+i\sqrt{23}}}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{-1+i\sqrt{23}}\,\sqrt{3x^4+x^2+2}}$	85
elliptic	$\frac{2\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-1+i\sqrt{23}}}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{-1+i\sqrt{23}}\,\sqrt{3x^4+x^2+2}}$	85

input
```
int(1/(3*x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-1+I*23^(1/2))^(1/2)*(1-(-1/4+1/4*I*23^(1/2))*x^2)^(1/2)*(1-(-1/4-1/4*I
*23^(1/2))*x^2)^(1/2)/(3*x^4+x^2+2)^(1/2)*EllipticF(1/2*x*(-1+I*23^(1/2))^
(1/2),1/6*(-33+3*I*23^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx$$
$$= -\frac{1}{24}\sqrt{2}(\sqrt{-23}+1)\sqrt{\sqrt{-23}-1}F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{-23}-1}\right) \mid \frac{1}{12}\sqrt{-23}-\frac{11}{12})$$

input
```
integrate(1/(3*x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*sqrt(2)*(sqrt(-23) + 1)*sqrt(sqrt(-23) - 1)*elliptic_f(arcsin(1/2*x*
sqrt(sqrt(-23) - 1)), 1/12*sqrt(-23) - 11/12)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4+x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+x^2+2}}\,dx$$

input
```
int(1/(x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4+x^2+2}}{3x^4+x^2+2}dx$$

input
```
int(1/(3*x^4+x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + x**2 + 2)/(3*x**4 + x**2 + 2),x)
```

3.35 $\int \frac{1}{\sqrt{2+3x^4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 72

$$\int \frac{1}{\sqrt{2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{2+3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2
*arctan(1/2*3^(1/4)*2^(3/4)*x),1/2*2^(1/2))*6^(3/4)/(3*x^4+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.35

$$\int \frac{1}{\sqrt{2+3x^4}}\,dx = -\sqrt[4]{-\frac{1}{6}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-\frac{3}{2}}x\right),-1\right)$$

input
```
Integrate[1/Sqrt[2 + 3*x^4],x]
```

output
```
-((-1/6)^(1/4)*EllipticF[I*ArcSinh[(-3/2)^(1/4)*x], -1])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+2}}\,dx$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{3x^4+2}}$$

input
```
Int[1/Sqrt[2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTa
n[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[2 + 3*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.25

method	result	size
meijerg	$\frac{\sqrt{2}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-\frac{3x^4}{2}\right)}{2}$	18
default	$\frac{\sqrt{2}\,\sqrt{4-2i\sqrt{6}\,x^2}\,\sqrt{4+2i\sqrt{6}\,x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{i\sqrt{6}}}{2},i\right)}{4\sqrt{i\sqrt{6}}\,\sqrt{3x^4+2}}$	66
elliptic	$\frac{\sqrt{2}\,\sqrt{4-2i\sqrt{6}\,x^2}\,\sqrt{4+2i\sqrt{6}\,x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{i\sqrt{6}}}{2},i\right)}{4\sqrt{i\sqrt{6}}\,\sqrt{3x^4+2}}$	66

input
```
int(1/(3*x^4+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*x*hypergeom([1/4,1/2],[5/4],-3/2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt{2+3x^4}}\,dx = -\frac{1}{6}\,\sqrt{2}\sqrt{\frac{1}{2}}(-6)^{\frac{3}{4}}\,F(\arcsin\left(\sqrt{\frac{1}{2}}(-6)^{\frac{1}{4}}\,x\right)\,|-1)$$

input
```
integrate(1/(3*x^4+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2)*sqrt(1/2)*(-6)^(3/4)*elliptic_f(arcsin(sqrt(1/2)*(-6)^(1/4)*x
), -1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{2+3x^4}}\,dx = \frac{\sqrt{2}x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{3x^4e^{i\pi}}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(3*x**4+2)**(1/2),x)
```

output
```
sqrt(2)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(I*pi)/2)/(
8*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+2}}\,dx$$

input
```
integrate(1/(3*x^4+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+2}}\,dx$$

input
```
integrate(1/(3*x^4+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 2), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{2+3x^4}}\,dx = \frac{\sqrt{2}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{3x^4}{2}\right)}{2}$$

input
```
int(1/(3*x^4 + 2)^(1/2),x)
```

output
```
(2^(1/2)*x*hypergeom([1/4, 1/2], 5/4, -(3*x^4)/2))/2
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+3x^4}}\,dx = \int \frac{\sqrt{3x^4+2}}{3x^4+2}dx$$

input
```
int(1/(3*x^4+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 2)/(3*x**4 + 2),x)
```

3.36 $\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2-x^2+3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4-x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobi
AM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/12*(72+6*6^(1/2))^(1/2))*6^(3/4)/(3*x
^4-x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx = -\frac{i\sqrt{1-\frac{6x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{1+i\sqrt{23}}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{6}{1-i\sqrt{23}}}x\right),\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{2-x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 - x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (6*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(1 + I*Sqrt[23])
]*EllipticF[I*ArcSinh[Sqrt[-6/(1 - I*Sqrt[23])]*x], (1 - I*Sqrt[23])/(1 +
I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[2 - x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-x^2+2}}dx$$
$$\Big\downarrow \text{ 1416}$$
$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-x^2+2}}$$

input
```
Int[1/Sqrt[2 - x^2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2
*ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 - x^2 + 3*x^
4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^{2}}\,\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{1+i\sqrt{23}}}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{1+i\sqrt{23}}\,\sqrt{3x^{4}-x^{2}+2}}$	87
elliptic	$\frac{2\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^{2}}\,\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{1+i\sqrt{23}}}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{1+i\sqrt{23}}\,\sqrt{3x^{4}-x^{2}+2}}$	87

input
```
int(1/(3*x^4-x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(1+I*23^(1/2))^(1/2)*(1-(1/4+1/4*I*23^(1/2))*x^2)^(1/2)*(1-(1/4-1/4*I*23
^(1/2))*x^2)^(1/2)/(3*x^4-x^2+2)^(1/2)*EllipticF(1/2*x*(1+I*23^(1/2))^(1/2
),1/6*(-33-3*I*23^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx = -\frac{1}{24}\sqrt{2}\sqrt{\sqrt{-23}+1}(\sqrt{-23}-1)F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{-23}+1}\right) \mid -\frac{1}{12}\sqrt{-23}-\frac{11}{12})$$

input
```
integrate(1/(3*x^4-x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*sqrt(2)*sqrt(sqrt(-23) + 1)*(sqrt(-23) - 1)*elliptic_f(arcsin(1/2*x*
sqrt(sqrt(-23) + 1)), -1/12*sqrt(-23) - 11/12)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4-x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4-x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4-x^2+2}}\,dx$$

input
```
int(1/(3*x^4 - x^2 + 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx=\int \frac{\sqrt{3x^4-x^2+2}}{3x^4-x^2+2}dx$$

input
```
int(1/(3*x^4-x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - x**2 + 2)/(3*x**4 - x**2 + 2),x)
```

3.37 $\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2-2x^2+3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4-2*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/6*(18+3*6^(1/2))^(1/2))*6^(3/4)/(3*
x^4-2*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx = -\frac{i\sqrt{1-\frac{3x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{1+i\sqrt{5}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{3}{1-i\sqrt{5}}}x\right),\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{2-2x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 - 2*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (3*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(1 + I*Sqrt[5])]*
EllipticF[I*ArcSinh[Sqrt[-3/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sq
rt[5])])/(Sqrt[3]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[2 - 2*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-2x^2+2}}dx$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-2x^2+2}}$$

input
```
Int[1/Sqrt[2 - 2*x^2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[2 - 2*x^2 + 3
*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2+2i\sqrt{5}}}{2},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{2+2i\sqrt{5}}\,\sqrt{3x^4-2x^2+2}}$	87
elliptic	$\frac{2\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2+2i\sqrt{5}}}{2},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{2+2i\sqrt{5}}\,\sqrt{3x^4-2x^2+2}}$	87

input
```
int(1/(3*x^4-2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(2+2*I*5^(1/2))^(1/2)*(1-(1/2+1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2-1/2*I*5^
(1/2))*x^2)^(1/2)/(3*x^4-2*x^2+2)^(1/2)*EllipticF(1/2*x*(2+2*I*5^(1/2))^(1
/2),1/3*(-6-3*I*5^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx = -\frac{1}{6}\sqrt{2}(\sqrt{-5}-1)\sqrt{\frac{1}{2}\sqrt{-5}+\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-5}+\frac{1}{2}}\right) \mid -\frac{1}{3}\sqrt{-5}-\frac{2}{3})$$

input
```
integrate(1/(3*x^4-2*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2)*(sqrt(-5) - 1)*sqrt(1/2*sqrt(-5) + 1/2)*elliptic_f(arcsin(x*s
qrt(1/2*sqrt(-5) + 1/2)), -1/3*sqrt(-5) - 2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4-2x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4-2*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - 2*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-2\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-2*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - 2*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-2\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-2*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - 2*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-2\,x^2+2}}\,dx$$

input
```
int(1/(3*x^4 - 2*x^2 + 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - 2*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4-2x^2+2}}{3x^4-2x^2+2}dx$$

input
```
int(1/(3*x^4-2*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - 2*x**2 + 2)/(3*x**4 - 2*x**2 + 2),x)
```

3.38 $\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2-3x^2+3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4-3*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/4*(8+2*6^(1/2))^(1/2))*6^(3/4)/(3*x
^4-3*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx = -\frac{i\sqrt{1-\frac{6x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{3+i\sqrt{15}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{6}{3-i\sqrt{15}}}x\right),\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{2-3x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 - 3*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (6*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(3 + I*Sqrt[15])
]*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[15])]*x], (3 - I*Sqrt[15])/(3 +
I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[2 - 3*x^2 + 3*x^4
])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-3x^2+2}}dx$$
$$\Big\downarrow \ 1416$$
$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-3x^2+2}}$$

input
```
Int[1/Sqrt[2 - 3*x^2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[2 - 3*x^2 + 3*
x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{3+i\sqrt{15}}}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{3+i\sqrt{15}}\,\sqrt{3x^4-3x^2+2}}$	87
elliptic	$\frac{2\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{3+i\sqrt{15}}}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{3+i\sqrt{15}}\,\sqrt{3x^4-3x^2+2}}$	87

input
```
int(1/(3*x^4-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(3+I*15^(1/2))^(1/2)*(1-(3/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(3/4-1/4*I*15
^(1/2))*x^2)^(1/2)/(3*x^4-3*x^2+2)^(1/2)*EllipticF(1/2*x*(3+I*15^(1/2))^(1
/2),1/2*(-1-I*15^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx = -\frac{1}{24}\sqrt{2}\sqrt{\sqrt{-15}+3}\left(\sqrt{-15}-3\right)F\left(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{-15}+3}\right)\mid -\frac{1}{4}\sqrt{-15}-\frac{1}{4}\right)$$

input
```
integrate(1/(3*x^4-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*sqrt(2)*sqrt(sqrt(-15) + 3)*(sqrt(-15) - 3)*elliptic_f(arcsin(1/2*x*
sqrt(sqrt(-15) + 3)), -1/4*sqrt(-15) - 1/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4-3x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4-3*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - 3*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-3\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - 3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-3\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - 3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-3\,x^2+2}}\,dx$$

input
```
int(1/(3*x^4 - 3*x^2 + 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - 3*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4-3x^2+2}}{3x^4-3x^2+2}dx$$

input
```
int(1/(3*x^4-3*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - 3*x**2 + 2)/(3*x**4 - 3*x**2 + 2),x)
```

3.39 $\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2-4x^2+3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4-4*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/6*(18+6*6^(1/2))^(1/2))*6^(3/4)/(3*
x^4-4*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx = -\frac{i\sqrt{1-\frac{3x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{2+i\sqrt{2}}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{3}{2-i\sqrt{2}}}x\right),\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{2-4x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 - 4*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (3*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(2 + I*Sqrt[2])]*
EllipticF[I*ArcSinh[Sqrt[-3/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sq
rt[2])])/(Sqrt[3]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[2 - 4*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-4x^2+2}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3x^4-4x^2+2}}$$

input
```
Int[1/Sqrt[2 - 4*x^2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[2 - 4*x^2 + 3*
x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right)x^2}\,\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{4+2i\sqrt{2}}}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{4+2i\sqrt{2}}\,\sqrt{3x^4-4x^2+2}}$	87
elliptic	$\frac{2\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right)x^2}\,\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{4+2i\sqrt{2}}}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{4+2i\sqrt{2}}\,\sqrt{3x^4-4x^2+2}}$	87

input
```
int(1/(3*x^4-4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(4+2*I*2^(1/2))^(1/2)*(1-(1/2*I*2^(1/2)+1)*x^2)^(1/2)*(1-(1-1/2*I*2^(1/2
))*x^2)^(1/2)/(3*x^4-4*x^2+2)^(1/2)*EllipticF(1/2*x*(4+2*I*2^(1/2))^(1/2),
1/3*(3-6*I*2^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx = -\frac{1}{6}\sqrt{2}(\sqrt{-2}-2)\sqrt{\frac{1}{2}\sqrt{-2}+1}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-2}+1}\right) \mid -\frac{2}{3}\sqrt{-2}+\frac{1}{3})$$

input
```
integrate(1/(3*x^4-4*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2)*(sqrt(-2) - 2)*sqrt(1/2*sqrt(-2) + 1)*elliptic_f(arcsin(x*sqr
t(1/2*sqrt(-2) + 1)), -2/3*sqrt(-2) + 1/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4-4x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4-4*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - 4*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-4\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-4*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - 4*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-4\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-4*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - 4*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-4\,x^2+2}}\,dx$$

input
```
int(1/(3*x^4 - 4*x^2 + 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - 4*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4-4x^2+2}}{3x^4-4x^2+2}dx$$

input
```
int(1/(3*x^4-4*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - 4*x**2 + 2)/(3*x**4 - 4*x**2 + 2),x)
```

3.40 $\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 50

$$\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx=\frac{\sqrt{2-3x^2}\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),\frac{3}{2}\right)}{\sqrt{2}\sqrt{2-5x^2+3x^4}}$$

output
```
1/2*(-3*x^2+2)^(1/2)*(-x^2+1)^(1/2)*EllipticF(x,1/2*6^(1/2))*2^(1/2)/(3*x^
4-5*x^2+2)^(1/2)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx=\frac{\sqrt{2-3x^2}\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)}{\sqrt{6-15x^2+9x^4}}$$

input
```
Integrate[1/Sqrt[2 - 5*x^2 + 3*x^4],x]
```

output
```
(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3])/Sqrt[6
 - 15*x^2 + 9*x^4]
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.84, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1409}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-5x^2+2}}dx$$

$$\Big\downarrow \ 1409$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-5x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-5x^2+2}}$$

input
```
Int[1/Sqrt[2 - 5*x^2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 - 5*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 - 5*x^2
+ 3*x^4])
```

Defintions of rubi rules used

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\ \mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4-5x^2+2}}$	42
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\ \mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4-5x^2+2}}$	42

input
```
int(1/(3*x^4-5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(3*x^4-5*x^2+2)^(1/2)*EllipticF(x,1/2*
6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin{(x)} \mid \frac{3}{2})$$

input
```
integrate(1/(3*x^4-5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*elliptic_f(arcsin(x), 3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4-5x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4-5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-5\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-5\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-5\,x^2+2}}\,dx$$

input
```
int(1/(3*x^4 - 5*x^2 + 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - 5*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4-5x^2+2}}{3x^4-5x^2+2}dx$$

input
```
int(1/(3*x^4-5*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - 5*x**2 + 2)/(3*x**4 - 5*x**2 + 2),x)
```

3.41 $\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 103

$$\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx = \frac{\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}\sqrt{2-\left(3-\sqrt{3}\right)x^2}\sqrt{2-\left(3+\sqrt{3}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}x\right),2-\sqrt{3}\right)}{2\sqrt{2-6x^2+3x^4}}$$

output
```
1/6*(9-3*3^(1/2))^(1/2)*(2-(3-3^(1/2))*x^2)^(1/2)*(2-(3+3^(1/2))*x^2)^(1/2
)*EllipticF(1/2*(6+2*3^(1/2))^(1/2)*x,1/2*6^(1/2)-1/2*2^(1/2))/(3*x^4-6*x^
2+2)^(1/2)
```

Mathematica [A] (verified)

Time = 10.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx = \frac{\sqrt{3-\sqrt{3}-3x^2}\sqrt{2+\left(-3+\sqrt{3}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}x\right),2-\sqrt{3}\right)}{\sqrt{6}\sqrt{2-6x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 - 6*x^2 + 3*x^4],x]
```

output
```
(Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt[2 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[S
qrt[(3 + Sqrt[3])/2]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[2 - 6*x^2 + 3*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1409}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-6x^2+2}}dx$$

$$\Big\downarrow \text{1409}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-6x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-6x^2+2}}$$

input
```
Int[1/Sqrt[2 - 6*x^2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 - 6*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*Sqrt[2 - 6*x^2 + 3*
x^4])
```

Defintions of rubi rules used

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{6+2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{6+2\sqrt{3}}\,\sqrt{3x^{4}-6x^{2}+2}}$	82
elliptic	$\frac{2\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{6+2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{6+2\sqrt{3}}\,\sqrt{3x^{4}-6x^{2}+2}}$	82

input
```
int(1/(3*x^4-6*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(6+2*3^(1/2))^(1/2)*(1-(3/2+1/2*3^(1/2))*x^2)^(1/2)*(1-(3/2-1/2*3^(1/2))
*x^2)^(1/2)/(3*x^4-6*x^2+2)^(1/2)*EllipticF(1/2*(6+2*3^(1/2))^(1/2)*x,1/2*
6^(1/2)-1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx = -\frac{1}{6}\left(\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{\frac{1}{2}\sqrt{3}+\frac{3}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{3}+\frac{3}{2}}\right) \mid -\sqrt{3}+2)$$

input
```
integrate(1/(3*x^4-6*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(1/2*sqrt(3) + 3/2)*elliptic_f(arcs
in(x*sqrt(1/2*sqrt(3) + 3/2)), -sqrt(3) + 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4-6x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4-6*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - 6*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-6\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-6*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - 6*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-6\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-6*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - 6*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4-6\,x^2+2}}\,dx$$

input
```
int(1/(3*x^4 - 6*x^2 + 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - 6*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx=\int \frac{\sqrt{3x^4-6x^2+2}}{3x^4-6x^2+2}dx$$

input
```
int(1/(3*x^4-6*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - 6*x**2 + 2)/(3*x**4 - 6*x**2 + 2),x)
```

3.42 $\int \frac{1}{\sqrt{2-7x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 56

$$\int \frac{1}{\sqrt{2-7x^2+3x^4}}\,dx=\frac{\sqrt{1-3x^2}\sqrt{2-x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{3}x\right),\frac{1}{6}\right)}{\sqrt{6}\sqrt{2-7x^2+3x^4}}$$

output
```
1/6*(-3*x^2+1)^(1/2)*(-x^2+2)^(1/2)*EllipticF(x*3^(1/2),1/6*6^(1/2))*6^(1/
2)/(3*x^4-7*x^2+2)^(1/2)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{2-7x^2+3x^4}}\,dx=\frac{\sqrt{1-3x^2}\sqrt{1-\frac{x^2}{2}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{3}x\right),\frac{1}{6}\right)}{\sqrt{3}\sqrt{2-7x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 - 7*x^2 + 3*x^4],x]
```

output
```
(Sqrt[1 - 3*x^2]*Sqrt[1 - x^2/2]*EllipticF[ArcSin[Sqrt[3]*x], 1/6])/(Sqrt[
3]*Sqrt[2 - 7*x^2 + 3*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.64, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1409}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-7x^2+2}}\,dx$$

$$\Big\downarrow \text{ 1409}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-7x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-7x^2+2}}$$

input
```
Int[1/Sqrt[2 - 7*x^2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 - 7*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], (12 + 7*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 - 7*x^2
+ 3*x^4])
```

Defintions of rubi rules used

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\sqrt{3}\,\sqrt{-3x^2+1}\,\sqrt{-2x^2+4}\ \mathrm{EllipticF}\left(\sqrt{3}\,x,\frac{\sqrt{6}}{6}\right)}{6\sqrt{3x^4-7x^2+2}}$	49
elliptic	$\frac{\sqrt{3}\,\sqrt{-3x^2+1}\,\sqrt{-2x^2+4}\ \mathrm{EllipticF}\left(\sqrt{3}\,x,\frac{\sqrt{6}}{6}\right)}{6\sqrt{3x^4-7x^2+2}}$	49

input
```
int(1/(3*x^4-7*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*(-3*x^2+1)^(1/2)*(-2*x^2+4)^(1/2)/(3*x^4-7*x^2+2)^(1/2)*Ellipt
icF(3^(1/2)*x,1/6*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt{2-7x^2+3x^4}}\,dx = \frac{1}{6}\sqrt{3}\sqrt{2}F(\arcsin\left(\sqrt{3}x\right)\mid\frac{1}{6})$$

input
```
integrate(1/(3*x^4-7*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*sqrt(2)*elliptic_f(arcsin(sqrt(3)*x), 1/6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-7x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4-7x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4-7*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - 7*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-7x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-7\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-7*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - 7*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-7x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-7\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-7*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - 7*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-7x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-7\,x^2+2}}\,dx$$

input
```
int(1/(3*x^4 - 7*x^2 + 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - 7*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-7x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4-7x^2+2}}{3x^4-7x^2+2}dx$$

input
```
int(1/(3*x^4-7*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - 7*x**2 + 2)/(3*x**4 - 7*x**2 + 2),x)
```

3.43 $\int \frac{1}{\sqrt{2-8x^2+3x^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{1}{\sqrt{2-8x^2+3x^4}} \, dx = \frac{\sqrt{2-\left(4-\sqrt{10}\right)x^2}\sqrt{2-\left(4+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(4+\sqrt{10}\right)}x\right),\frac{1}{3}\left(13-4\sqrt{10}\right)\right)}{\sqrt{2\left(4+\sqrt{10}\right)}\sqrt{2-8x^2+3x^4}}$$

output
```
(2-(4-10^(1/2))*x^2)^(1/2)*(2-(4+10^(1/2))*x^2)^(1/2)*EllipticF(1/2*(8+2*1
0^(1/2))^(1/2)*x,2/3*6^(1/2)-1/3*15^(1/2))/(8+2*10^(1/2))^(1/2)/(3*x^4-8*x
^2+2)^(1/2)
```

Mathematica [A] (verified)

Time = 10.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{2-8x^2+3x^4}} \, dx = \frac{\sqrt{4-\sqrt{10}-3x^2}\sqrt{2+\left(-4+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2+\sqrt{\frac{5}{2}}}x\right),\frac{1}{3}\left(13-4\sqrt{10}\right)\right)}{\sqrt{6}\sqrt{2-8x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 - 8*x^2 + 3*x^4],x]
```

output
```
(Sqrt[4 - Sqrt[10] - 3*x^2]*Sqrt[2 + (-4 + Sqrt[10])*x^2]*EllipticF[ArcSin
[Sqrt[2 + Sqrt[5/2]]*x], (13 - 4*Sqrt[10])/3])/(Sqrt[6]*Sqrt[2 - 8*x^2 + 3
*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1409}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-8x^2+2}}dx$$

$$\Big\downarrow \text{ 1409}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-8x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}+\sqrt{\frac{2}{3}}\right)}{2\sqrt[4]{6}\sqrt{3x^4-8x^2+2}}$$

input
```
Int[1/Sqrt[2 - 8*x^2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 - 8*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], 1/2 + Sqrt[2/3]])/(2*6^(1/4)*Sqrt[2 - 8*x^2 + 3*
x^4])
```

Defintions of rubi rules used

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{2\sqrt{1-\left(2+\frac{\sqrt{10}}{2}\right)x^{2}}\,\sqrt{1-\left(2-\frac{\sqrt{10}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{8+2\sqrt{10}}\,x}{2},\frac{2\sqrt{6}}{3}-\frac{\sqrt{15}}{3}\right)}{\sqrt{8+2\sqrt{10}}\,\sqrt{3x^{4}-8x^{2}+2}}$	82
elliptic	$\frac{2\sqrt{1-\left(2+\frac{\sqrt{10}}{2}\right)x^{2}}\,\sqrt{1-\left(2-\frac{\sqrt{10}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{8+2\sqrt{10}}\,x}{2},\frac{2\sqrt{6}}{3}-\frac{\sqrt{15}}{3}\right)}{\sqrt{8+2\sqrt{10}}\,\sqrt{3x^{4}-8x^{2}+2}}$	82

input
```
int(1/(3*x^4-8*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(8+2*10^(1/2))^(1/2)*(1-(2+1/2*10^(1/2))*x^2)^(1/2)*(1-(2-1/2*10^(1/2))*
x^2)^(1/2)/(3*x^4-8*x^2+2)^(1/2)*EllipticF(1/2*(8+2*10^(1/2))^(1/2)*x,2/3*
6^(1/2)-1/3*15^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{2-8x^2+3x^4}}\,dx = -\frac{1}{6}\left(\sqrt{10}\sqrt{2}-4\sqrt{2}\right)\sqrt{\frac{1}{2}\sqrt{10}+2}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{10}+2}\right) \mid -\frac{4}{3}\sqrt{10}+\frac{13}{3})$$

input
```
integrate(1/(3*x^4-8*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(10)*sqrt(2) - 4*sqrt(2))*sqrt(1/2*sqrt(10) + 2)*elliptic_f(arcs
in(x*sqrt(1/2*sqrt(10) + 2)), -4/3*sqrt(10) + 13/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-8x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3x^4-8x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4-8*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - 8*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-8x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4-8\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-8*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - 8*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-8x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-8\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-8*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - 8*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-8x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-8\,x^2+2}}\,dx$$

input
```
int(1/(3*x^4 - 8*x^2 + 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - 8*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-8x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4-8x^2+2}}{3x^4-8x^2+2}dx$$

input
```
int(1/(3*x^4-8*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - 8*x**2 + 2)/(3*x**4 - 8*x**2 + 2),x)
```

3.44 $\int \frac{1}{\sqrt{2-9x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{1}{\sqrt{2-9x^2+3x^4}}\,dx = \frac{\sqrt{4-\left(9-\sqrt{57}\right)x^2}\sqrt{4-\left(9+\sqrt{57}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{9+\sqrt{57}}x\right),\frac{1}{4}\left(23-3\sqrt{57}\right)\right)}{2\sqrt{9+\sqrt{57}}\sqrt{2-9x^2+3x^4}}$$

output
```
1/2*(4-(9-57^(1/2))*x^2)^(1/2)*(4-(9+57^(1/2))*x^2)^(1/2)*EllipticF(1/2*(9
+57^(1/2))^(1/2)*x,3/4*6^(1/2)-1/4*38^(1/2))/(9+57^(1/2))^(1/2)/(3*x^4-9*x
^2+2)^(1/2)
```

Mathematica [A] (verified)

Time = 10.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{2-9x^2+3x^4}}\,dx = \frac{\sqrt{9-\sqrt{57}-6x^2}\sqrt{4+\left(-9+\sqrt{57}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{9+\sqrt{57}}x\right),\frac{23}{4}-\frac{3\sqrt{57}}{4}\right)}{2\sqrt{6}\sqrt{2-9x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[2 - 9*x^2 + 3*x^4],x]
```

output
```
(Sqrt[9 - Sqrt[57] - 6*x^2]*Sqrt[4 + (-9 + Sqrt[57])*x^2]*EllipticF[ArcSin
[(Sqrt[9 + Sqrt[57]]*x)/2], 23/4 - (3*Sqrt[57])/4])/(2*Sqrt[6]*Sqrt[2 - 9*
x^2 + 3*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1409\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-9x^2+2}}dx$$

$$\Big\downarrow \text{1409}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-9x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4+3\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-9x^2+2}}$$

input
```
Int[1/Sqrt[2 - 9*x^2 + 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 - 9*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], (4 + 3*Sqrt[6])/8])/(2*6^(1/4)*Sqrt[2 - 9*x^2 +
3*x^4])
```

Defintions of rubi rules used

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{9}{4}+\frac{\sqrt{57}}{4}\right)x^{2}}\,\sqrt{1-\left(\frac{9}{4}-\frac{\sqrt{57}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{9+\sqrt{57}}\,x}{2},\frac{3\sqrt{6}}{4}-\frac{\sqrt{38}}{4}\right)}{\sqrt{9+\sqrt{57}}\,\sqrt{3x^{4}-9x^{2}+2}}$	78
elliptic	$\frac{2\sqrt{1-\left(\frac{9}{4}+\frac{\sqrt{57}}{4}\right)x^{2}}\,\sqrt{1-\left(\frac{9}{4}-\frac{\sqrt{57}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{9+\sqrt{57}}\,x}{2},\frac{3\sqrt{6}}{4}-\frac{\sqrt{38}}{4}\right)}{\sqrt{9+\sqrt{57}}\,\sqrt{3x^{4}-9x^{2}+2}}$	78

input
```
int(1/(3*x^4-9*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(9+57^(1/2))^(1/2)*(1-(9/4+1/4*57^(1/2))*x^2)^(1/2)*(1-(9/4-1/4*57^(1/2)
)*x^2)^(1/2)/(3*x^4-9*x^2+2)^(1/2)*EllipticF(1/2*(9+57^(1/2))^(1/2)*x,3/4*
6^(1/2)-1/4*38^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{2-9x^2+3x^4}}\,dx = -\frac{1}{24}\left(\sqrt{57}\sqrt{2}-9\sqrt{2}\right)\sqrt{\sqrt{57}+9}F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{57}+9}\right) \mid -\frac{3}{4}\sqrt{57}+\frac{23}{4})$$

input
```
integrate(1/(3*x^4-9*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(57)*sqrt(2) - 9*sqrt(2))*sqrt(sqrt(57) + 9)*elliptic_f(arcsin(
1/2*x*sqrt(sqrt(57) + 9)), -3/4*sqrt(57) + 23/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-9x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4-9x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4-9*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - 9*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-9x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-9\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-9*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - 9*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-9x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-9\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4-9*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - 9*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-9x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-9\,x^2+2}}\,dx$$

input
```
int(1/(3*x^4 - 9*x^2 + 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - 9*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-9x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4-9x^2+2}}{3x^4-9x^2+2}dx$$

input
```
int(1/(3*x^4-9*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - 9*x**2 + 2)/(3*x**4 - 9*x**2 + 2),x)
```

3.45 $\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx = \frac{\sqrt{9+\sqrt{57}}\sqrt{6+\left(9-\sqrt{57}\right)x^2}\sqrt{6+\left(9+\sqrt{57}\right)x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(9-\sqrt{57}\right)}x\right),\frac{1}{4}\left(-19-3\sqrt{5\right.\right.}}{12\sqrt{3+9x^2+2x^4}}$$

output

```
1/12*(9+57^(1/2))^(1/2)*(6+(9-57^(1/2))*x^2)^(1/2)*(6+(9+57^(1/2))*x^2)^(1
/2)*InverseJacobiAM(arctan(1/6*(54-6*57^(1/2))^(1/2)*x),1/2*(-19-3*57^(1/2
))^(1/2))/(2*x^4+9*x^2+3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx = -\frac{i\sqrt{\frac{-9+\sqrt{57}-4x^2}{-9+\sqrt{57}}}\sqrt{9+\sqrt{57}+4x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{2x}{\sqrt{9+\sqrt{57}}}\right),\frac{23}{4}+\frac{3\sqrt{57}}{4}\right)}{2\sqrt{3+9x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 + 9*x^2 + 2*x^4],x]
```

output
```
((-1/2*I)*Sqrt[(-9 + Sqrt[57] - 4*x^2)/(-9 + Sqrt[57])]*Sqrt[9 + Sqrt[57]
+ 4*x^2]*EllipticF[I*ArcSinh[(2*x)/Sqrt[9 + Sqrt[57]]], 23/4 + (3*Sqrt[57]
)/4])/Sqrt[3 + 9*x^2 + 2*x^4]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1412\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+9x^2+3}}dx$$

$$\Big\downarrow \; 1412$$

$$\frac{\sqrt{\frac{\left(9-\sqrt{57}\right)x^2+6}{\left(9+\sqrt{57}\right)x^2+6}}\left(\left(9+\sqrt{57}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}x\right),\frac{1}{4}\left(-19+3\sqrt{57}\right)\right)}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{2x^4+9x^2+3}}$$

input
```
Int[1/Sqrt[3 + 9*x^2 + 2*x^4],x]
```

output
```
(Sqrt[(6 + (9 - Sqrt[57])*x^2)/(6 + (9 + Sqrt[57])*x^2)]*(6 + (9 + Sqrt[57
])*x^2)*EllipticF[ArcTan[Sqrt[(9 + Sqrt[57])/6]*x], (-19 + 3*Sqrt[57])/4])
/(Sqrt[6*(9 + Sqrt[57])]*Sqrt[3 + 9*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{6\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{57}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{57}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-54+6\sqrt{57}}}{6},\frac{3\sqrt{6}}{4}+\frac{\sqrt{38}}{4}\right)}{\sqrt{-54+6\sqrt{57}}\,\sqrt{2x^4+9x^2+3}}$	82
elliptic	$\frac{6\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{57}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{57}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-54+6\sqrt{57}}}{6},\frac{3\sqrt{6}}{4}+\frac{\sqrt{38}}{4}\right)}{\sqrt{-54+6\sqrt{57}}\,\sqrt{2x^4+9x^2+3}}$	82

input
```
int(1/(2*x^4+9*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(-54+6*57^(1/2))^(1/2)*(1-(-3/2+1/6*57^(1/2))*x^2)^(1/2)*(1-(-3/2-1/6*57
^(1/2))*x^2)^(1/2)/(2*x^4+9*x^2+3)^(1/2)*EllipticF(1/6*x*(-54+6*57^(1/2))^
(1/2),3/4*6^(1/2)+1/4*38^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx$$
$$= -\frac{1}{4}\left(\sqrt{\frac{19}{3}}\sqrt{3}+3\sqrt{3}\right)\sqrt{\frac{1}{2}\sqrt{\frac{19}{3}}-\frac{3}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{\frac{19}{3}}-\frac{3}{2}}\right)\mid\frac{9}{4}\sqrt{\frac{19}{3}}+\frac{23}{4})$$

input
```
integrate(1/(2*x^4+9*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(19/3)*sqrt(3) + 3*sqrt(3))*sqrt(1/2*sqrt(19/3) - 3/2)*elliptic_
f(arcsin(x*sqrt(1/2*sqrt(19/3) - 3/2)), 9/4*sqrt(19/3) + 23/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+9x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4+9*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 9*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+9\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+9*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 9*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+9\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+9*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 9*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+9\,x^2+3}}\,dx$$

input
```
int(1/(9*x^2 + 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(9*x^2 + 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+9x^2+3}}{2x^4+9x^2+3}dx$$

input
```
int(1/(2*x^4+9*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 9*x**2 + 3)/(2*x**4 + 9*x**2 + 3),x)
```

3.46 $\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx = \frac{\sqrt{\frac{1}{2}\left(4+\sqrt{10}\right)}\sqrt{3+\left(4-\sqrt{10}\right)x^2}\sqrt{3+\left(4+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(4-\sqrt{10}\right)}x\right),-\frac{2}{3}(5+2}{3\sqrt{3+8x^2+2x^4}}$$

output
```
1/6*(8+2*10^(1/2))^(1/2)*(3+(4-10^(1/2))*x^2)^(1/2)*(3+(4+10^(1/2))*x^2)^(
1/2)*InverseJacobiAM(arctan(1/3*(12-3*10^(1/2))^(1/2)*x),1/3*(-30-12*10^(1
/2))^(1/2))/(2*x^4+8*x^2+3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx = -\frac{i\sqrt{\frac{-4+\sqrt{10}-2x^2}{-4+\sqrt{10}}}\sqrt{4+\sqrt{10}+2x^2}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{\frac{2}{4+\sqrt{10}}}x\right),\frac{13}{3}+\frac{4\sqrt{10}}{3}\right)}{\sqrt{6+16x^2+4x^4}}$$

input
```
Integrate[1/Sqrt[3 + 8*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[(-4 + Sqrt[10] - 2*x^2)/(-4 + Sqrt[10])]*Sqrt[4 + Sqrt[10] + 2*
x^2]*EllipticF[I*ArcSinh[Sqrt[2/(4 + Sqrt[10])]*x], 13/3 + (4*Sqrt[10])/3]
)/Sqrt[6 + 16*x^2 + 4*x^4]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1412\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+8x^2+3}}\,dx$$

$$\Big\downarrow \; 1412$$

$$\frac{\sqrt{\frac{\left(4-\sqrt{10}\right)x^2+3}{\left(4+\sqrt{10}\right)x^2+3}}\left(\left(4+\sqrt{10}\right)x^2+3\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}x\right),-\frac{2}{3}\left(5-2\sqrt{10}\right)\right)}{\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{2x^4+8x^2+3}}$$

input
```
Int[1/Sqrt[3 + 8*x^2 + 2*x^4],x]
```

output
```
(Sqrt[(3 + (4 - Sqrt[10])*x^2)/(3 + (4 + Sqrt[10])*x^2)]*(3 + (4 + Sqrt[10
])*x^2)*EllipticF[ArcTan[Sqrt[(4 + Sqrt[10])/3]*x], (-2*(5 - 2*Sqrt[10]))/
3])/(Sqrt[3*(4 + Sqrt[10])]*Sqrt[3 + 8*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{3\sqrt{1-\left(-\frac{4}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{4}{3}-\frac{\sqrt{10}}{3}\right)x^2}\ \operatorname{EllipticF}\left(\frac{x\sqrt{-12+3\sqrt{10}}}{3},\frac{2\sqrt{6}}{3}+\frac{\sqrt{15}}{3}\right)}{\sqrt{-12+3\sqrt{10}}\,\sqrt{2x^4+8x^2+3}}$	82
elliptic	$\frac{3\sqrt{1-\left(-\frac{4}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{4}{3}-\frac{\sqrt{10}}{3}\right)x^2}\ \operatorname{EllipticF}\left(\frac{x\sqrt{-12+3\sqrt{10}}}{3},\frac{2\sqrt{6}}{3}+\frac{\sqrt{15}}{3}\right)}{\sqrt{-12+3\sqrt{10}}\,\sqrt{2x^4+8x^2+3}}$	82

input
```
int(1/(2*x^4+8*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(-12+3*10^(1/2))^(1/2)*(1-(-4/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-4/3-1/3*10
^(1/2))*x^2)^(1/2)/(2*x^4+8*x^2+3)^(1/2)*EllipticF(1/3*x*(-12+3*10^(1/2))^
(1/2),2/3*6^(1/2)+1/3*15^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx = -\frac{1}{6}\left(\sqrt{10}\sqrt{3}+4\sqrt{3}\right)\sqrt{\frac{1}{3}\sqrt{10}-\frac{4}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{10}-\frac{4}{3}}\right) \mid \frac{4}{3}\sqrt{10}+\frac{13}{3})$$

input
```
integrate(1/(2*x^4+8*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(10)*sqrt(3) + 4*sqrt(3))*sqrt(1/3*sqrt(10) - 4/3)*elliptic_f(ar
csin(x*sqrt(1/3*sqrt(10) - 4/3)), 4/3*sqrt(10) + 13/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+8x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4+8*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 8*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+8\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+8*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 8*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+8\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+8*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 8*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+8\,x^2+3}}\,dx$$

input
```
int(1/(8*x^2 + 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(8*x^2 + 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+8x^2+3}}{2x^4+8x^2+3}dx$$

input
```
int(1/(2*x^4+8*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 8*x**2 + 3)/(2*x**4 + 8*x**2 + 3),x)
```

3.47 $\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 47

$$\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx=\frac{\sqrt{3+x^2}\sqrt{1+2x^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}}\right),-5\right)}{\sqrt{3+7x^2+2x^4}}$$

output
```
(x^2+3)^(1/2)*(2*x^2+1)^(1/2)*InverseJacobiAM(arctan(1/3*x*3^(1/2)),I*5^(1
/2))/(2*x^4+7*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx=-\frac{i\sqrt{3+x^2}\sqrt{1+2x^2}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{2}x\right),\frac{1}{6}\right)}{\sqrt{6}\sqrt{3+7x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 + 7*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[3 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], 1/6])/
(Sqrt[6]*Sqrt[3 + 7*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+7x^2+3}}\,dx$$
$$\Big\downarrow \ 1412$$
$$\frac{\sqrt{\frac{x^2+3}{2x^2+1}}\left(2x^2+1\right)\text{EllipticF}\left(\arctan\left(\sqrt{2}x\right),\frac{5}{6}\right)}{\sqrt{6}\sqrt{2x^4+7x^2+3}}$$

input
```
Int[1/Sqrt[3 + 7*x^2 + 2*x^4],x]
```

output
```
(Sqrt[(3 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]*x], 5/6]
)/(Sqrt[6]*Sqrt[3 + 7*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{2x^2+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{3}}{3},\sqrt{6}\right)}{3\sqrt{2x^4+7x^2+3}}$	50
elliptic	$-\frac{i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{2x^2+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{3}}{3},\sqrt{6}\right)}{3\sqrt{2x^4+7x^2+3}}$	50

input
```
int(1/(2*x^4+7*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*3^(1/2)*(3*x^2+9)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+7*x^2+3)^(1/2)*Ellip
ticF(1/3*I*x*3^(1/2),6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx = -\sqrt{3}\sqrt{-\frac{1}{3}}F\left(\arcsin\left(\sqrt{-\frac{1}{3}}x\right)\mid 6\right)$$

input
```
integrate(1/(2*x^4+7*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(3)*sqrt(-1/3)*elliptic_f(arcsin(sqrt(-1/3)*x), 6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+7x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4+7*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 7*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+7\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+7*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 7*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+7\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+7*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 7*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+7\,x^2+3}}\,dx$$

input
```
int(1/(7*x^2 + 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(7*x^2 + 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+7x^2+3}}{2x^4+7x^2+3}dx$$

input
```
int(1/(2*x^4+7*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 7*x**2 + 3)/(2*x**4 + 7*x**2 + 3),x)
```

3.48 $\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx = \frac{\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}\sqrt{3+\left(3-\sqrt{3}\right)x^2}\sqrt{3+\left(3+\sqrt{3}\right)x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),-1-\sqrt{3}\right)}{3\sqrt{3+6x^2+2x^4}}$$

output
```
1/6*(6+2*3^(1/2))^(1/2)*(3+(3-3^(1/2))*x^2)^(1/2)*(3+(3+3^(1/2))*x^2)^(1/2
)*InverseJacobiAM(arctan(1/3*(9-3*3^(1/2))^(1/2)*x),(-1-3^(1/2))^(1/2))/(2
*x^4+6*x^2+3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx = -\frac{i\sqrt{\frac{-3+\sqrt{3}-2x^2}{-3+\sqrt{3}}}\sqrt{3+\sqrt{3}+2x^2}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{1-\frac{1}{\sqrt{3}}}x\right),2+\sqrt{3}\right)}{\sqrt{6+12x^2+4x^4}}$$

input
```
Integrate[1/Sqrt[3 + 6*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[(-3 + Sqrt[3] - 2*x^2)/(-3 + Sqrt[3])]*Sqrt[3 + Sqrt[3] + 2*x^2
]*EllipticF[I*ArcSinh[Sqrt[1 - 1/Sqrt[3]]*x], 2 + Sqrt[3]])/Sqrt[6 + 12*x^
2 + 4*x^4]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1412\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+6x^2+3}}dx$$

$$\Big\downarrow \text{1412}$$

$$\frac{\sqrt{\frac{\left(3-\sqrt{3}\right)x^2+3}{\left(3+\sqrt{3}\right)x^2+3}}\left(\left(3+\sqrt{3}\right)x^2+3\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right),-1+\sqrt{3}\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{2x^4+6x^2+3}}$$

input
```
Int[1/Sqrt[3 + 6*x^2 + 2*x^4],x]
```

output
```
(Sqrt[(3 + (3 - Sqrt[3])*x^2)/(3 + (3 + Sqrt[3])*x^2)]*(3 + (3 + Sqrt[3])*
x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[3])/3]*x], -1 + Sqrt[3]])/(Sqrt[3*(3
+ Sqrt[3])]*Sqrt[3 + 6*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{3\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-9+3\sqrt{3}}}{3},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{-9+3\sqrt{3}}\,\sqrt{2x^4+6x^2+3}}$	82
elliptic	$\frac{3\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-9+3\sqrt{3}}}{3},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{-9+3\sqrt{3}}\,\sqrt{2x^4+6x^2+3}}$	82

input
```
int(1/(2*x^4+6*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(-9+3*3^(1/2))^(1/2)*(1-(-1+1/3*3^(1/2))*x^2)^(1/2)*(1-(-1-1/3*3^(1/2))*
x^2)^(1/2)/(2*x^4+6*x^2+3)^(1/2)*EllipticF(1/3*x*(-9+3*3^(1/2))^(1/2),1/2*
6^(1/2)+1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx = -\frac{1}{2}\left(\sqrt{3}+1\right)\sqrt{\frac{1}{3}\sqrt{3}-1}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{3}-1}\right)\mid\sqrt{3}+2)$$

input
```
integrate(1/(2*x^4+6*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(3) + 1)*sqrt(1/3*sqrt(3) - 1)*elliptic_f(arcsin(x*sqrt(1/3*sqrt
(3) - 1)), sqrt(3) + 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+6x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4+6*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 6*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+6\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+6*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 6*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+6\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+6*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 6*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+6\,x^2+3}}\,dx$$

input
```
int(1/(6*x^2 + 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(6*x^2 + 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+6x^2+3}}{2x^4+6x^2+3}dx$$

input
```
int(1/(2*x^4+6*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 6*x**2 + 3)/(2*x**4 + 6*x**2 + 3),x)
```

3.49 $\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 56

$$\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx=\frac{\sqrt{1+x^2}\sqrt{3+2x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{2}{3}}x\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3+5x^2+2x^4}}$$

output
```
1/2*(x^2+1)^(1/2)*(2*x^2+3)^(1/2)*InverseJacobiAM(arctan(1/3*x*6^(1/2)),1/
2*I*2^(1/2))*2^(1/2)/(2*x^4+5*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx=-\frac{i\sqrt{1+x^2}\sqrt{3+2x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{3}}x\right),\frac{3}{2}\right)}{\sqrt{6+10x^2+4x^4}}$$

input
```
Integrate[1/Sqrt[3 + 5*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], 3/2]
)/Sqrt[6 + 10*x^2 + 4*x^4]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{1412}$$

$$\frac{(x^2+1)\sqrt{\frac{2x^2+3}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{3}\right)}{\sqrt{3}\sqrt{2x^4+5x^2+3}}$$

input
```
Int[1/Sqrt[3 + 5*x^2 + 2*x^4],x]
```

output
```
((1 + x^2)*Sqrt[(3 + 2*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/3])/(Sqrt[3]
*Sqrt[3 + 5*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{i\sqrt{6}\,\sqrt{6x^{2}+9}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{6\sqrt{2x^{4}+5x^{2}+3}}$	50
elliptic	$-\frac{i\sqrt{6}\,\sqrt{6x^{2}+9}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{6\sqrt{2x^{4}+5x^{2}+3}}$	50

input
```
int(1/(2*x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*6^(1/2)*(6*x^2+9)^(1/2)*(x^2+1)^(1/2)/(2*x^4+5*x^2+3)^(1/2)*Ellipti
cF(1/3*I*x*6^(1/2),1/2*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx = -\frac{1}{2}\sqrt{3}\sqrt{-\frac{2}{3}}F(\arcsin\left(\sqrt{-\frac{2}{3}}x\right)\mid\frac{3}{2})$$

input
```
integrate(1/(2*x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(3)*sqrt(-2/3)*elliptic_f(arcsin(sqrt(-2/3)*x), 3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+5x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 5*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+5\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 5*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+5\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 5*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+5\,x^2+3}}\,dx$$

input
```
int(1/(5*x^2 + 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(5*x^2 + 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+5x^2+3}}{2x^4+5x^2+3}dx$$

input
```
int(1/(2*x^4+5*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 5*x**2 + 3)/(2*x**4 + 5*x**2 + 3),x)
```

3.50 $\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx=\frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3+4x^2+2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4+4*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/6*(18-6*6^(1/2))^(1/2))*6^(3/4)/(2*
x^4+4*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx = -\frac{i\sqrt{1-\frac{2x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{-2+i\sqrt{2}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{2}{-2-i\sqrt{2}}}x\right),\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{3+4x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 + 4*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[1 - (2*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(-2 + I*Sqrt[2])
]*EllipticF[I*ArcSinh[Sqrt[-2/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 +
 I*Sqrt[2])])/(Sqrt[2]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[3 + 4*x^2 + 2*x^4
])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+4x^2+3}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4+4x^2+3}}$$

input
```
Int[1/Sqrt[3 + 4*x^2 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[3 + 4*x^2 + 2*
x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-6+3i\sqrt{2}}}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{-6+3i\sqrt{2}}\,\sqrt{2x^4+4x^2+3}}$	87
elliptic	$\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-6+3i\sqrt{2}}}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{-6+3i\sqrt{2}}\,\sqrt{2x^4+4x^2+3}}$	87

input
```
int(1/(2*x^4+4*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(-6+3*I*2^(1/2))^(1/2)*(1-(-2/3+1/3*I*2^(1/2))*x^2)^(1/2)*(1-(-2/3-1/3*I
*2^(1/2))*x^2)^(1/2)/(2*x^4+4*x^2+3)^(1/2)*EllipticF(1/3*x*(-6+3*I*2^(1/2)
)^(1/2),1/3*(3+6*I*2^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx$$
$$= -\frac{1}{6}\sqrt{3}(\sqrt{-2}+2)\sqrt{\frac{1}{3}\sqrt{-2}-\frac{2}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{-2}-\frac{2}{3}}\right) \mid \frac{2}{3}\sqrt{-2}+\frac{1}{3})$$

input
```
integrate(1/(2*x^4+4*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*(sqrt(-2) + 2)*sqrt(1/3*sqrt(-2) - 2/3)*elliptic_f(arcsin(x*s
qrt(1/3*sqrt(-2) - 2/3)), 2/3*sqrt(-2) + 1/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2x^4+4x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4+4*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 4*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4+4\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+4*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 4*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4+4\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+4*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 4*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+4\,x^2+3}}\,dx$$

input
```
int(1/(4*x^2 + 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(4*x^2 + 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+4x^2+3}}{2x^4+4x^2+3}dx$$

input
```
int(1/(2*x^4+4*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 4*x**2 + 3)/(2*x**4 + 4*x**2 + 3),x)
```

3.51 $\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3+3x^2+2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4+3*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/4*(8-2*6^(1/2))^(1/2))*6^(3/4)/(2*x
^4+3*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx = -\frac{i\sqrt{1-\frac{4x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{-3+i\sqrt{15}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(2\sqrt{-\frac{1}{-3-i\sqrt{15}}}x\right),\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{3+3x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 + 3*x^2 + 2*x^4],x]
```

output
```
((-1/2*I)*Sqrt[1 - (4*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(-3 + I*Sqr
t[15])]*EllipticF[I*ArcSinh[2*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*x], (-3 - I*Sq
rt[15])/(-3 + I*Sqrt[15])])/(Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[3 + 3*x^2
+ 2*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+3x^2+3}}dx$$
$$\Big\downarrow \text{1416}$$
$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+3x^2+3}}$$

input
```
Int[1/Sqrt[3 + 3*x^2 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[3 + 3*x^2 + 2*
x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-18+6i\sqrt{15}}}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{-18+6i\sqrt{15}}\,\sqrt{2x^4+3x^2+3}}$	87
elliptic	$\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-18+6i\sqrt{15}}}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{-18+6i\sqrt{15}}\,\sqrt{2x^4+3x^2+3}}$	87

input
```
int(1/(2*x^4+3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(-18+6*I*15^(1/2))^(1/2)*(1-(-1/2+1/6*I*15^(1/2))*x^2)^(1/2)*(1-(-1/2-1/
6*I*15^(1/2))*x^2)^(1/2)/(2*x^4+3*x^2+3)^(1/2)*EllipticF(1/6*x*(-18+6*I*15
^(1/2))^(1/2),1/2*(-1+I*15^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx = -\frac{1}{4}\sqrt{3}\left(\sqrt{-\frac{5}{3}}+1\right)\sqrt{\frac{1}{2}\sqrt{-\frac{5}{3}}-\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-\frac{5}{3}}-\frac{1}{2}}\right) \mid \frac{3}{4}\sqrt{-\frac{5}{3}}-\frac{1}{4})$$

input
```
integrate(1/(2*x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(3)*(sqrt(-5/3) + 1)*sqrt(1/2*sqrt(-5/3) - 1/2)*elliptic_f(arcsin
(x*sqrt(1/2*sqrt(-5/3) - 1/2)), 3/4*sqrt(-5/3) - 1/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+3x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4+3*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 3*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+3\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+3*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 3*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+3\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+3*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 3*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4+3\,x^2+3}}\,dx$$

input
```
int(1/(3*x^2 + 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(3*x^2 + 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx=\int \frac{\sqrt{2x^4+3x^2+3}}{2x^4+3x^2+3}dx$$

input
```
int(1/(2*x^4+3*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 3*x**2 + 3)/(2*x**4 + 3*x**2 + 3),x)
```

3.52 $\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3+2x^2+2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4+2*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/6*(18-3*6^(1/2))^(1/2))*6^(3/4)/(2*
x^4+2*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx = -\frac{i\sqrt{1-\frac{2x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{-1+i\sqrt{5}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{2}{-1-i\sqrt{5}}}x\right),\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{3+2x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 + 2*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[1 - (2*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(-1 + I*Sqrt[5])
]*EllipticF[I*ArcSinh[Sqrt[-2/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 +
 I*Sqrt[5])])/(Sqrt[2]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[3 + 2*x^2 + 2*x^4
])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+2x^2+3}}\,dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+2x^2+3}}$$

input
```
Int[1/Sqrt[3 + 2*x^2 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[3 + 2*x^2 + 2
*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-3+3i\sqrt{5}}}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{-3+3i\sqrt{5}}\,\sqrt{2x^4+2x^2+3}}$	87
elliptic	$\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-3+3i\sqrt{5}}}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{-3+3i\sqrt{5}}\,\sqrt{2x^4+2x^2+3}}$	87

input
```
int(1/(2*x^4+2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(-3+3*I*5^(1/2))^(1/2)*(1-(-1/3+1/3*I*5^(1/2))*x^2)^(1/2)*(1-(-1/3-1/3*I
*5^(1/2))*x^2)^(1/2)/(2*x^4+2*x^2+3)^(1/2)*EllipticF(1/3*x*(-3+3*I*5^(1/2)
)^(1/2),1/3*(-6+3*I*5^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx$$
$$= -\frac{1}{6}\sqrt{3}(\sqrt{-5}+1)\sqrt{\frac{1}{3}\sqrt{-5}-\frac{1}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{-5}-\frac{1}{3}}\right) \mid \frac{1}{3}\sqrt{-5}-\frac{2}{3})$$

input
```
integrate(1/(2*x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*(sqrt(-5) + 1)*sqrt(1/3*sqrt(-5) - 1/3)*elliptic_f(arcsin(x*s
qrt(1/3*sqrt(-5) - 1/3)), 1/3*sqrt(-5) - 2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+2x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4+2*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 2*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+2*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 2*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+2*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 2*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4+2\,x^2+3}}\,dx$$

input
```
int(1/(2*x^2 + 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(2*x^2 + 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx=\int \frac{\sqrt{2x^4+2x^2+3}}{2x^4+2x^2+3}dx$$

input
```
int(1/(2*x^4+2*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 2*x**2 + 3)/(2*x**4 + 2*x**2 + 3),x)
```

3.53 $\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3+x^2+2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4+x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobi
AM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/12*(72-6*6^(1/2))^(1/2))*6^(3/4)/(2*x
^4+x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx = -\frac{i\sqrt{1-\frac{4x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{-1+i\sqrt{23}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(2\sqrt{-\frac{1}{-1-i\sqrt{23}}}x\right),\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{3+x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 + x^2 + 2*x^4],x]
```

output
```
((-1/2*I)*Sqrt[1 - (4*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(-1 + I*Sqr
t[23])]*EllipticF[I*ArcSinh[2*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*x], (-1 - I*Sq
rt[23])/(-1 + I*Sqrt[23])])/(Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[3 + x^2 +
2*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+x^2+3}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+x^2+3}}$$

input
```
Int[1/Sqrt[3 + x^2 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2
*ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 + x^2 + 2*x^
4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{6\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\ \operatorname{EllipticF}\left(\frac{x\sqrt{-6+6i\sqrt{23}}}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{-6+6i\sqrt{23}}\,\sqrt{2x^4+x^2+3}}$	85
elliptic	$\frac{6\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\ \operatorname{EllipticF}\left(\frac{x\sqrt{-6+6i\sqrt{23}}}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{-6+6i\sqrt{23}}\,\sqrt{2x^4+x^2+3}}$	85

input
```
int(1/(2*x^4+x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(-6+6*I*23^(1/2))^(1/2)*(1-(-1/6+1/6*I*23^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6
*I*23^(1/2))*x^2)^(1/2)/(2*x^4+x^2+3)^(1/2)*EllipticF(1/6*x*(-6+6*I*23^(1/
2))^(1/2),1/6*(-33+3*I*23^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx$$
$$= -\frac{1}{12}\sqrt{3}(\sqrt{-23}+1)\sqrt{\frac{1}{6}\sqrt{-23}-\frac{1}{6}}F(\arcsin\left(x\sqrt{\frac{1}{6}\sqrt{-23}-\frac{1}{6}}\right)\mid\frac{1}{12}\sqrt{-23}-\frac{11}{12})$$

input
```
integrate(1/(2*x^4+x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*(sqrt(-23) + 1)*sqrt(1/6*sqrt(-23) - 1/6)*elliptic_f(arcsin(
x*sqrt(1/6*sqrt(-23) - 1/6)), 1/12*sqrt(-23) - 11/12)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4+x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4+x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+x^2+3}}\,dx$$

input
```
int(1/(x^2 + 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(x^2 + 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+x^2+3}}{2x^4+x^2+3}dx$$

input
```
int(1/(2*x^4+x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + x**2 + 3)/(2*x**4 + x**2 + 3),x)
```

3.54 $\int \frac{1}{\sqrt{3+2x^4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 72

$$\int \frac{1}{\sqrt{3+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{3+2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2
*arctan(1/3*2^(1/4)*3^(3/4)*x),1/2*2^(1/2))*6^(3/4)/(2*x^4+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.35

$$\int \frac{1}{\sqrt{3+2x^4}}\,dx = -\sqrt[4]{-\frac{1}{6}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt[4]{-\frac{2}{3}}x\right),-1\right)$$

input
```
Integrate[1/Sqrt[3 + 2*x^4],x]
```

output
```
-((-1/6)^(1/4)*EllipticF[I*ArcSinh[(-2/3)^(1/4)*x], -1])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+3}}\,dx$$

$$\Big\downarrow \ 761$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{2x^4+3}}$$

input
```
Int[1/Sqrt[3 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTa
n[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[3 + 2*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.25

method	result	size
meijerg	$\frac{\sqrt{3}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], -\frac{2 x^{4}}{3}\right)}{3}$	18
default	$\frac{\sqrt{3}\, \sqrt{9-3 i \sqrt{6}\, x^{2}}\, \sqrt{9+3 i \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{3}\, \sqrt{i \sqrt{6}}}{3}, i\right)}{9 \sqrt{i \sqrt{6}}\, \sqrt{2 x^{4}+3}}$	66
elliptic	$\frac{\sqrt{3}\, \sqrt{9-3 i \sqrt{6}\, x^{2}}\, \sqrt{9+3 i \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{3}\, \sqrt{i \sqrt{6}}}{3}, i\right)}{9 \sqrt{i \sqrt{6}}\, \sqrt{2 x^{4}+3}}$	66

input
```
int(1/(2*x^4+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*x*hypergeom([1/4,1/2],[5/4],-2/3*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{3+2x^4}}\, dx = -\frac{1}{2}\sqrt{3}\left(-\frac{2}{3}\right)^{\frac{3}{4}} F(\arcsin\left(\left(-\frac{2}{3}\right)^{\frac{1}{4}} x\right) \mid -1)$$

input
```
integrate(1/(2*x^4+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(3)*(-2/3)^(3/4)*elliptic_f(arcsin((-2/3)^(1/4)*x), -1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{3+2x^4}}\,dx = \frac{\sqrt{3}x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4e^{i\pi}}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(2*x**4+3)**(1/2),x)
```

output
```
sqrt(3)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(I*pi)/3)/(
12*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+3}}\,dx$$

input
```
integrate(1/(2*x^4+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+3}}\,dx$$

input
```
integrate(1/(2*x^4+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 3), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{3+2x^4}}\,dx = \frac{\sqrt{3}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{2x^4}{3}\right)}{3}$$

input
```
int(1/(2*x^4 + 3)^(1/2),x)
```

output
```
(3^(1/2)*x*hypergeom([1/4, 1/2], 5/4, -(2*x^4)/3))/3
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+2x^4}}\,dx = \int \frac{\sqrt{2x^4+3}}{2x^4+3}dx$$

input
```
int(1/(2*x^4+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 3)/(2*x**4 + 3),x)
```

3.55 $\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3-x^2+2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4-x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobi
AM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/12*(72+6*6^(1/2))^(1/2))*6^(3/4)/(2*x
^4-x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx = -\frac{i\sqrt{1-\frac{4x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{1+i\sqrt{23}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(2\sqrt{-\frac{1}{1-i\sqrt{23}}}x\right),\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{3-x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 - x^2 + 2*x^4],x]
```

output
```
((-1/2*I)*Sqrt[1 - (4*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(1 + I*Sqrt[
23])]*EllipticF[I*ArcSinh[2*Sqrt[-(1 - I*Sqrt[23])^(-1)]*x], (1 - I*Sqrt[2
3])/(1 + I*Sqrt[23])])/(Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[3 - x^2 + 2*x^4]
)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-x^2+3}}dx$$

$$\Big\downarrow \ 1416$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-x^2+3}}$$

input
```
Int[1/Sqrt[3 - x^2 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2
*ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - x^2 + 2*x^
4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{6\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{6+6i\sqrt{23}}}{6},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{6+6i\sqrt{23}}\,\sqrt{2x^4-x^2+3}}$	87
elliptic	$\frac{6\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{6+6i\sqrt{23}}}{6},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{6+6i\sqrt{23}}\,\sqrt{2x^4-x^2+3}}$	87

input
```
int(1/(2*x^4-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(6+6*I*23^(1/2))^(1/2)*(1-(1/6+1/6*I*23^(1/2))*x^2)^(1/2)*(1-(1/6-1/6*I*
23^(1/2))*x^2)^(1/2)/(2*x^4-x^2+3)^(1/2)*EllipticF(1/6*x*(6+6*I*23^(1/2))^
(1/2),1/6*(-33-3*I*23^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx$$
$$= -\frac{1}{12}\sqrt{3}(\sqrt{-23}-1)\sqrt{\frac{1}{6}\sqrt{-23}+\frac{1}{6}}F(\arcsin\left(x\sqrt{\frac{1}{6}\sqrt{-23}+\frac{1}{6}}\right)|-\frac{1}{12}\sqrt{-23}-\frac{11}{12})$$

input
```
integrate(1/(2*x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*(sqrt(-23) - 1)*sqrt(1/6*sqrt(-23) + 1/6)*elliptic_f(arcsin(
x*sqrt(1/6*sqrt(-23) + 1/6)), -1/12*sqrt(-23) - 11/12)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3 - x^2 + 2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4 - x^2 + 3}}\,dx$$

input
```
integrate(1/(2*x**4-x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3 - x^2 + 2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4 - x^2 + 3}}\,dx$$

input
```
integrate(1/(2*x^4-x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-x^2+3}}\,dx$$

input
```
int(1/(2*x^4 - x^2 + 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-x^2+3}}{2x^4-x^2+3}dx$$

input
```
int(1/(2*x^4-x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - x**2 + 3)/(2*x**4 - x**2 + 3),x)
```

3.56 $\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3-2x^2+2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4-2*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/6*(18+3*6^(1/2))^(1/2))*6^(3/4)/(2*
x^4-2*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx = -\frac{i\sqrt{1-\frac{2x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{1+i\sqrt{5}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{2}{1-i\sqrt{5}}}x\right),\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{3-2x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 - 2*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[1 - (2*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(1 + I*Sqrt[5])]*
EllipticF[I*ArcSinh[Sqrt[-2/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sq
rt[5])])/(Sqrt[2]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[3 - 2*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-2x^2+3}}dx$$
$$\Big\downarrow \text{ 1416}$$
$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-2x^2+3}}$$

input
```
Int[1/Sqrt[3 - 2*x^2 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[3 - 2*x^2 + 2
*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{3+3i\sqrt{5}}}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{3+3i\sqrt{5}}\,\sqrt{2x^4-2x^2+3}}$	87
elliptic	$\frac{3\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{3+3i\sqrt{5}}}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{3+3i\sqrt{5}}\,\sqrt{2x^4-2x^2+3}}$	87

input
```
int(1/(2*x^4-2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(3+3*I*5^(1/2))^(1/2)*(1-(1/3+1/3*I*5^(1/2))*x^2)^(1/2)*(1-(1/3-1/3*I*5^
(1/2))*x^2)^(1/2)/(2*x^4-2*x^2+3)^(1/2)*EllipticF(1/3*x*(3+3*I*5^(1/2))^(1
/2),1/3*(-6-3*I*5^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx = -\frac{1}{6}\sqrt{3}(\sqrt{-5}-1)\sqrt{\frac{1}{3}\sqrt{-5}+\frac{1}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{-5}+\frac{1}{3}}\right)\mid -\frac{1}{3}\sqrt{-5}-\frac{2}{3})$$

input
```
integrate(1/(2*x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*(sqrt(-5) - 1)*sqrt(1/3*sqrt(-5) + 1/3)*elliptic_f(arcsin(x*s
qrt(1/3*sqrt(-5) + 1/3)), -1/3*sqrt(-5) - 2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2x^4-2x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4-2*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 2*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4-2\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 2*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4-2\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 2*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-2\,x^2+3}}\,dx$$

input
```
int(1/(2*x^4 - 2*x^2 + 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 2*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-2x^2+3}}{2x^4-2x^2+3}dx$$

input
```
int(1/(2*x^4-2*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 2*x**2 + 3)/(2*x**4 - 2*x**2 + 3),x)
```

3.57 $\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3-3x^2+2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4-3*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/4*(8+2*6^(1/2))^(1/2))*6^(3/4)/(2*x
^4-3*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx = -\frac{i\sqrt{1-\frac{4x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{3+i\sqrt{15}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(2\sqrt{-\frac{1}{3-i\sqrt{15}}}x\right),\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{3-3x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 - 3*x^2 + 2*x^4],x]
```

output
```
((-1/2*I)*Sqrt[1 - (4*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(3 + I*Sqrt[
15])]*EllipticF[I*ArcSinh[2*Sqrt[-(3 - I*Sqrt[15])^(-1)]*x], (3 - I*Sqrt[1
5])/(3 + I*Sqrt[15])])/(Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[3 - 3*x^2 + 2*x^
4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-3x^2+3}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-3x^2+3}}$$

input
```
Int[1/Sqrt[3 - 3*x^2 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[3 - 3*x^2 + 2*
x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{6\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{18+6i\sqrt{15}}}{6},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{18+6i\sqrt{15}}\,\sqrt{2x^4-3x^2+3}}$	87
elliptic	$\frac{6\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{18+6i\sqrt{15}}}{6},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{18+6i\sqrt{15}}\,\sqrt{2x^4-3x^2+3}}$	87

input
```
int(1/(2*x^4-3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(18+6*I*15^(1/2))^(1/2)*(1-(1/2+1/6*I*15^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*I
*15^(1/2))*x^2)^(1/2)/(2*x^4-3*x^2+3)^(1/2)*EllipticF(1/6*x*(18+6*I*15^(1/
2))^(1/2),1/2*(-1-I*15^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx$$
$$= -\frac{1}{4}\sqrt{3}\left(\sqrt{-\frac{5}{3}}-1\right)\sqrt{\frac{1}{2}\sqrt{-\frac{5}{3}}+\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-\frac{5}{3}}+\frac{1}{2}}\right)\mid -\frac{3}{4}\sqrt{-\frac{5}{3}}-\frac{1}{4})$$

input
```
integrate(1/(2*x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(3)*(sqrt(-5/3) - 1)*sqrt(1/2*sqrt(-5/3) + 1/2)*elliptic_f(arcsin
(x*sqrt(1/2*sqrt(-5/3) + 1/2)), -3/4*sqrt(-5/3) - 1/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4-3x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4-3*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 3*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-3\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 3*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-3\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 3*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-3\,x^2+3}}\,dx$$

input
```
int(1/(2*x^4 - 3*x^2 + 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 3*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-3x^2+3}}{2x^4-3x^2+3}dx$$

input
```
int(1/(2*x^4-3*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 3*x**2 + 3)/(2*x**4 - 3*x**2 + 3),x)
```

3.58 $\int \frac{1}{\sqrt{3-4x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{1}{\sqrt{3-4x^2+2x^4}}\,dx=\frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3-4x^2+2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4-4*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/6*(18+6*6^(1/2))^(1/2))*6^(3/4)/(2*
x^4-4*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt{3-4x^2+2x^4}}\,dx = -\frac{i\sqrt{1-\frac{2x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{2+i\sqrt{2}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{2}{2-i\sqrt{2}}}x\right),\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{3-4x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 - 4*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[1 - (2*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(2 + I*Sqrt[2])]*
EllipticF[I*ArcSinh[Sqrt[-2/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sq
rt[2])])/(Sqrt[2]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[3 - 4*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-4x^2+3}}dx$$
$$\Big\downarrow \; 1416$$
$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4-4x^2+3}}$$

input
```
Int[1/Sqrt[3 - 4*x^2 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[3 - 4*x^2 + 2*
x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{6+3i\sqrt{2}}}{3},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{6+3i\sqrt{2}}\,\sqrt{2x^4-4x^2+3}}$	87
elliptic	$\frac{3\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{6+3i\sqrt{2}}}{3},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{6+3i\sqrt{2}}\,\sqrt{2x^4-4x^2+3}}$	87

input
```
int(1/(2*x^4-4*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(6+3*I*2^(1/2))^(1/2)*(1-(2/3+1/3*I*2^(1/2))*x^2)^(1/2)*(1-(2/3-1/3*I*2^
(1/2))*x^2)^(1/2)/(2*x^4-4*x^2+3)^(1/2)*EllipticF(1/3*x*(6+3*I*2^(1/2))^(1
/2),1/3*(3-6*I*2^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{3-4x^2+2x^4}}\,dx = -\frac{1}{6}\sqrt{3}(\sqrt{-2}-2)\sqrt{\frac{1}{3}\sqrt{-2}+\frac{2}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{-2}+\frac{2}{3}}\right) \mid -\frac{2}{3}\sqrt{-2}+\frac{1}{3})$$

input
```
integrate(1/(2*x^4-4*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*(sqrt(-2) - 2)*sqrt(1/3*sqrt(-2) + 2/3)*elliptic_f(arcsin(x*s
qrt(1/3*sqrt(-2) + 2/3)), -2/3*sqrt(-2) + 1/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-4x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2x^4-4x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4-4*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 4*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-4x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4-4\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-4*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 4*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-4x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4-4\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-4*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 4*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-4x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-4\,x^2+3}}\,dx$$

input
```
int(1/(2*x^4 - 4*x^2 + 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 4*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-4x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-4x^2+3}}{2x^4-4x^2+3}dx$$

input
```
int(1/(2*x^4-4*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 4*x**2 + 3)/(2*x**4 - 4*x**2 + 3),x)
```

3.59 $\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 50

$$\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx = \frac{\sqrt{3-2x^2}\sqrt{1-x^2}\operatorname{EllipticF}\left(\arcsin(x),\frac{2}{3}\right)}{\sqrt{3}\sqrt{3-5x^2+2x^4}}$$

output
```
1/3*(-2*x^2+3)^(1/2)*(-x^2+1)^(1/2)*EllipticF(x,1/3*6^(1/2))*3^(1/2)/(2*x^
4-5*x^2+3)^(1/2)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx = \frac{\sqrt{3-2x^2}\sqrt{1-x^2}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right),\frac{3}{2}\right)}{\sqrt{6-10x^2+4x^4}}$$

input
```
Integrate[1/Sqrt[3 - 5*x^2 + 2*x^4],x]
```

output
```
(Sqrt[3 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], 3/2])/Sqrt[6
 - 10*x^2 + 4*x^4]
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.84, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1409}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-5x^2+3}}\,dx$$

$$\Big\downarrow \; 1409$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-5x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-5x^2+3}}$$

input
```
Int[1/Sqrt[3 - 5*x^2 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 - 5*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - 5*x^2
+ 2*x^4])
```

Defintions of rubi rules used

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+9}\,\operatorname{EllipticF}\left(x,\frac{\sqrt{6}}{3}\right)}{3\sqrt{2x^4-5x^2+3}}$	42
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+9}\,\operatorname{EllipticF}\left(x,\frac{\sqrt{6}}{3}\right)}{3\sqrt{2x^4-5x^2+3}}$	42

input
```
int(1/(2*x^4-5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-x^2+1)^(1/2)*(-6*x^2+9)^(1/2)/(2*x^4-5*x^2+3)^(1/2)*EllipticF(x,1/3*
6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx = \frac{1}{3}\sqrt{3}F(\arcsin{(x)} \mid \frac{2}{3})$$

input
```
integrate(1/(2*x^4-5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*elliptic_f(arcsin(x), 2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4-5x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4-5*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 5*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-5\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 5*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-5\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 5*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-5\,x^2+3}}\,dx$$

input
```
int(1/(2*x^4 - 5*x^2 + 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 5*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-5x^2+3}}{2x^4-5x^2+3}dx$$

input
```
int(1/(2*x^4-5*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 5*x**2 + 3)/(2*x**4 - 5*x**2 + 3),x)
```

3.60 $\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx = \frac{\sqrt{3-\left(3-\sqrt{3}\right)x^2}\sqrt{3-\left(3+\sqrt{3}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right),2-\sqrt{3}\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{3-6x^2+2x^4}}$$

output
```
(3-(3-3^(1/2))*x^2)^(1/2)*(3-(3+3^(1/2))*x^2)^(1/2)*EllipticF(1/3*(9+3*3^(
1/2))^(1/2)*x,1/2*6^(1/2)-1/2*2^(1/2))/(9+3*3^(1/2))^(1/2)/(2*x^4-6*x^2+3)
^(1/2)
```

Mathematica [A] (verified)

Time = 10.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx = \frac{\sqrt{3-\sqrt{3}-2x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{1+\frac{1}{\sqrt{3}}}x\right),2-\sqrt{3}\right)}{\sqrt{6}\sqrt{3-6x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 - 6*x^2 + 2*x^4],x]
```

output
```
(Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[S
qrt[1 + 1/Sqrt[3]]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[3 - 6*x^2 + 2*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1409}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-6x^2+3}}dx$$

$$\Big\downarrow \text{1409}$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-6x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-6x^2+3}}$$

input
```
Int[1/Sqrt[3 - 6*x^2 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 - 6*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*Sqrt[3 - 6*x^2 + 2*
x^4])
```

Defintions of rubi rules used

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{3\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{9+3\sqrt{3}}\,x}{3}, \frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{9+3\sqrt{3}}\,\sqrt{2x^4-6x^2+3}}$	82
elliptic	$\frac{3\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{9+3\sqrt{3}}\,x}{3}, \frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{9+3\sqrt{3}}\,\sqrt{2x^4-6x^2+3}}$	82

input
```
int(1/(2*x^4-6*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(9+3*3^(1/2))^(1/2)*(1-(1+1/3*3^(1/2))*x^2)^(1/2)*(1-(1-1/3*3^(1/2))*x^2
)^(1/2)/(2*x^4-6*x^2+3)^(1/2)*EllipticF(1/3*(9+3*3^(1/2))^(1/2)*x,1/2*6^(1
/2)-1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.38

$$\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx = \frac{1}{2}\left(\sqrt{3}-1\right)\sqrt{\frac{1}{3}\sqrt{3}+1}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{3}+1}\right)|-\sqrt{3}+2)$$

input
```
integrate(1/(2*x^4-6*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(3) - 1)*sqrt(1/3*sqrt(3) + 1)*elliptic_f(arcsin(x*sqrt(1/3*sqrt(
3) + 1)), -sqrt(3) + 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4-6x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4-6*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 6*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-6\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-6*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 6*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-6\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-6*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 6*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-6\,x^2+3}}\,dx$$

input
```
int(1/(2*x^4 - 6*x^2 + 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 6*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-6x^2+3}}{2x^4-6x^2+3}dx$$

input
```
int(1/(2*x^4-6*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 6*x**2 + 3)/(2*x**4 - 6*x**2 + 3),x)
```

3.61 $\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 56

$$\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx=\frac{\sqrt{1-2x^2}\sqrt{3-x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),\frac{1}{6}\right)}{\sqrt{6}\sqrt{3-7x^2+2x^4}}$$

output
```
1/6*(-2*x^2+1)^(1/2)*(-x^2+3)^(1/2)*EllipticF(x*2^(1/2),1/6*6^(1/2))*6^(1/
2)/(2*x^4-7*x^2+3)^(1/2)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx=\frac{\sqrt{1-2x^2}\sqrt{1-\frac{x^2}{3}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),\frac{1}{6}\right)}{\sqrt{2}\sqrt{3-7x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 - 7*x^2 + 2*x^4],x]
```

output
```
(Sqrt[1 - 2*x^2]*Sqrt[1 - x^2/3]*EllipticF[ArcSin[Sqrt[2]*x], 1/6])/(Sqrt[
2]*Sqrt[3 - 7*x^2 + 2*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.64, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1409}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-7x^2+3}}\,dx$$
$$\Big\downarrow \ 1409$$
$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-7x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-7x^2+3}}$$

input
```
Int[1/Sqrt[3 - 7*x^2 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 - 7*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], (12 + 7*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - 7*x^2
+ 2*x^4])
```

Defintions of rubi rules used

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{-3x^2+9}\,\mathrm{EllipticF}\left(x\sqrt{2},\frac{\sqrt{6}}{6}\right)}{6\sqrt{2x^4-7x^2+3}}$	49
elliptic	$\frac{\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{-3x^2+9}\,\mathrm{EllipticF}\left(x\sqrt{2},\frac{\sqrt{6}}{6}\right)}{6\sqrt{2x^4-7x^2+3}}$	49

input
```
int(1/(2*x^4-7*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*(-2*x^2+1)^(1/2)*(-3*x^2+9)^(1/2)/(2*x^4-7*x^2+3)^(1/2)*Ellipt
icF(x*2^(1/2),1/6*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx = \frac{1}{6}\sqrt{3}\sqrt{2}F(\arcsin\left(\sqrt{2}x\right)\mid\frac{1}{6})$$

input
```
integrate(1/(2*x^4-7*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*sqrt(2)*elliptic_f(arcsin(sqrt(2)*x), 1/6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4-7x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4-7*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 7*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-7\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-7*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 7*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-7\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-7*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 7*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-7\,x^2+3}}\,dx$$

input
```
int(1/(2*x^4 - 7*x^2 + 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 7*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-7x^2+3}}{2x^4-7x^2+3}dx$$

input
```
int(1/(2*x^4-7*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 7*x**2 + 3)/(2*x**4 - 7*x**2 + 3),x)
```

3.62 $\int \frac{1}{\sqrt{3-8x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \frac{1}{\sqrt{3-8x^2+2x^4}}\,dx = \frac{\sqrt{\frac{1}{2}\left(4-\sqrt{10}\right)}\sqrt{3-\left(4-\sqrt{10}\right)x^2}\sqrt{3-\left(4+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}x\right),\frac{1}{3}\left(13-4\right.\right.}{3\sqrt{3-8x^2+2x^4}}$$

output
```
1/6*(8-2*10^(1/2))^(1/2)*(3-(4-10^(1/2))*x^2)^(1/2)*(3-(4+10^(1/2))*x^2)^(
1/2)*EllipticF(1/3*(12+3*10^(1/2))^(1/2)*x,2/3*6^(1/2)-1/3*15^(1/2))/(2*x^
4-8*x^2+3)^(1/2)
```

Mathematica [A] (verified)

Time = 10.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{3-8x^2+2x^4}}\,dx = \frac{\sqrt{4-\sqrt{10}-2x^2}\sqrt{3+\left(-4+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}x\right),\frac{13}{3}-\frac{4\sqrt{10}}{3}\right)}{\sqrt{6}\sqrt{3-8x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 - 8*x^2 + 2*x^4],x]
```

output
```
(Sqrt[4 - Sqrt[10] - 2*x^2]*Sqrt[3 + (-4 + Sqrt[10])*x^2]*EllipticF[ArcSin
[Sqrt[(4 + Sqrt[10])/3]*x], 13/3 - (4*Sqrt[10])/3])/(Sqrt[6]*Sqrt[3 - 8*x^
2 + 2*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.84, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1409}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-8x^2+3}}dx$$

$$\Big\downarrow \text{1409}$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-8x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}+\sqrt{\frac{2}{3}}\right)}{2\sqrt[4]{6}\sqrt{2x^4-8x^2+3}}$$

input
```
Int[1/Sqrt[3 - 8*x^2 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 - 8*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], 1/2 + Sqrt[2/3]])/(2*6^(1/4)*Sqrt[3 - 8*x^2 + 2*
x^4])
```

Defintions of rubi rules used

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{4}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(\frac{4}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{12+3\sqrt{10}}\,x}{3},\frac{2\sqrt{6}}{3}-\frac{\sqrt{15}}{3}\right)}{\sqrt{12+3\sqrt{10}}\,\sqrt{2x^4-8x^2+3}}$	82
elliptic	$\frac{3\sqrt{1-\left(\frac{4}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(\frac{4}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{12+3\sqrt{10}}\,x}{3},\frac{2\sqrt{6}}{3}-\frac{\sqrt{15}}{3}\right)}{\sqrt{12+3\sqrt{10}}\,\sqrt{2x^4-8x^2+3}}$	82

input
```
int(1/(2*x^4-8*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(12+3*10^(1/2))^(1/2)*(1-(4/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(4/3-1/3*10^(1
/2))*x^2)^(1/2)/(2*x^4-8*x^2+3)^(1/2)*EllipticF(1/3*(12+3*10^(1/2))^(1/2)*
x,2/3*6^(1/2)-1/3*15^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{3-8x^2+2x^4}}\,dx$$
$$= -\frac{1}{6}\left(\sqrt{10}\sqrt{3}-4\sqrt{3}\right)\sqrt{\frac{1}{3}\sqrt{10}+\frac{4}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{10}+\frac{4}{3}}\right)|-\frac{4}{3}\sqrt{10}+\frac{13}{3})$$

input
```
integrate(1/(2*x^4-8*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(10)*sqrt(3) - 4*sqrt(3))*sqrt(1/3*sqrt(10) + 4/3)*elliptic_f(ar
csin(x*sqrt(1/3*sqrt(10) + 4/3)), -4/3*sqrt(10) + 13/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-8x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4-8x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4-8*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 8*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-8x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-8\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-8*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 8*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-8x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-8\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-8*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 8*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-8x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4-8\,x^2+3}}\,dx$$

input
```
int(1/(2*x^4 - 8*x^2 + 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 8*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-8x^2+2x^4}}\,dx=\int \frac{\sqrt{2x^4-8x^2+3}}{2x^4-8x^2+3}dx$$

input
```
int(1/(2*x^4-8*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 8*x**2 + 3)/(2*x**4 - 8*x**2 + 3),x)
```

3.63 $\int \frac{1}{\sqrt{3-9x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{1}{\sqrt{3-9x^2+2x^4}}\,dx = \frac{\sqrt{6-\left(9-\sqrt{57}\right)x^2}\sqrt{6-\left(9+\sqrt{57}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}x\right),\frac{1}{4}\left(23-3\sqrt{57}\right)\right)}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{3-9x^2+2x^4}}$$

output
```
(6-(9-57^(1/2))*x^2)^(1/2)*(6-(9+57^(1/2))*x^2)^(1/2)*EllipticF(1/6*(54+6*
57^(1/2))^(1/2)*x,3/4*6^(1/2)-1/4*38^(1/2))/(54+6*57^(1/2))^(1/2)/(2*x^4-9
*x^2+3)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 10.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{3-9x^2+2x^4}}\,dx = \frac{\sqrt{9-\sqrt{57}-4x^2}\sqrt{6+\left(-9+\sqrt{57}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{9-\sqrt{57}}}\right),\frac{23}{4}-\frac{3\sqrt{57}}{4}\right)}{2\sqrt{6}\sqrt{3-9x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[3 - 9*x^2 + 2*x^4],x]
```

output
```
(Sqrt[9 - Sqrt[57] - 4*x^2]*Sqrt[6 + (-9 + Sqrt[57])*x^2]*EllipticF[ArcSin
[(2*x)/Sqrt[9 - Sqrt[57]]], 23/4 - (3*Sqrt[57])/4])/(2*Sqrt[6]*Sqrt[3 - 9*
x^2 + 2*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1409}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-9x^2+3}}dx$$

$$\Big\downarrow \text{ 1409}$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-9x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4+3\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-9x^2+3}}$$

input
```
Int[1/Sqrt[3 - 9*x^2 + 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 - 9*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], (4 + 3*Sqrt[6])/8])/(2*6^(1/4)*Sqrt[3 - 9*x^2 +
2*x^4])
```

Defintions of rubi rules used

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{6\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{57}}{6}\right)x^{2}}\,\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{57}}{6}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{54+6\sqrt{57}}\,x}{6},\frac{3\sqrt{6}}{4}-\frac{\sqrt{38}}{4}\right)}{\sqrt{54+6\sqrt{57}}\,\sqrt{2x^{4}-9x^{2}+3}}$	82
elliptic	$\frac{6\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{57}}{6}\right)x^{2}}\,\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{57}}{6}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{54+6\sqrt{57}}\,x}{6},\frac{3\sqrt{6}}{4}-\frac{\sqrt{38}}{4}\right)}{\sqrt{54+6\sqrt{57}}\,\sqrt{2x^{4}-9x^{2}+3}}$	82

input
```
int(1/(2*x^4-9*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(54+6*57^(1/2))^(1/2)*(1-(3/2+1/6*57^(1/2))*x^2)^(1/2)*(1-(3/2-1/6*57^(1
/2))*x^2)^(1/2)/(2*x^4-9*x^2+3)^(1/2)*EllipticF(1/6*(54+6*57^(1/2))^(1/2)*
x,3/4*6^(1/2)-1/4*38^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{3-9x^2+2x^4}}\,dx = -\frac{1}{4}\left(\sqrt{\frac{19}{3}}\sqrt{3}-3\sqrt{3}\right)\sqrt{\frac{1}{2}\sqrt{\frac{19}{3}}+\frac{3}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{\frac{19}{3}}+\frac{3}{2}}\right)|-\frac{9}{4}\sqrt{\frac{19}{3}}+\frac{23}{4})$$

input
```
integrate(1/(2*x^4-9*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(19/3)*sqrt(3) - 3*sqrt(3))*sqrt(1/2*sqrt(19/3) + 3/2)*elliptic_
f(arcsin(x*sqrt(1/2*sqrt(19/3) + 3/2)), -9/4*sqrt(19/3) + 23/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-9x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4-9x^2+3}}\,dx$$

input
```
integrate(1/(2*x**4-9*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 9*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-9x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-9\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-9*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 9*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-9x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-9\,x^2+3}}\,dx$$

input
```
integrate(1/(2*x^4-9*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 9*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-9x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-9\,x^2+3}}\,dx$$

input
```
int(1/(2*x^4 - 9*x^2 + 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 9*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-9x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-9x^2+3}}{2x^4-9x^2+3}dx$$

input
```
int(1/(2*x^4-9*x^2+3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 9*x**2 + 3)/(2*x**4 - 9*x**2 + 3),x)
```

3.64 $\int \frac{1}{\sqrt{1-\sqrt{5}x^2+x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int \frac{1}{\sqrt{1-\sqrt{5}x^2+x^4}}\,dx = \frac{\sqrt{2+\left(1-\sqrt{5}\right)x^2}\sqrt{2-\left(1+\sqrt{5}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x\right),\frac{1}{2}\left(3-\sqrt{5}\right)\right)}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{1-\sqrt{5}x^2+x^4}}$$

output
```
(2+(-5^(1/2)+1)*x^2)^(1/2)*(2-(5^(1/2)+1)*x^2)^(1/2)*EllipticF(1/2*(2+2*5^
(1/2))^(1/2)*x,1/2*5^(1/2)-1/2)/(2+2*5^(1/2))^(1/2)/(1-5^(1/2)*x^2+x^4)^(1
/2)
```

Mathematica [A] (warning: unable to verify)

Time = 10.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{1-\sqrt{5}x^2+x^4}}\,dx = \frac{\sqrt{-1+\sqrt{5}-2x^2}\sqrt{1+\sqrt{5}-2x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right),\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{1-\sqrt{5}x^2+x^4}}$$

input
```
Integrate[1/Sqrt[1 - Sqrt[5]*x^2 + x^4],x]
```

output
```
(Sqrt[-1 + Sqrt[5] - 2*x^2]*Sqrt[1 + Sqrt[5] - 2*x^2]*EllipticF[ArcSin[Sqr
t[2/(-1 + Sqrt[5])]*x], 3/2 - Sqrt[5]/2])/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[1 -
Sqrt[5]*x^2 + x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{1409\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^4-\sqrt{5}x^2+1}}dx$$

$$\Big\downarrow \text{1409}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4-\sqrt{5}x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\left(2+\sqrt{5}\right)\right)}{2\sqrt{x^4-\sqrt{5}x^2+1}}$$

input
```
Int[1/Sqrt[1 - Sqrt[5]*x^2 + x^4],x]
```

output
```
((1 + x^2)*Sqrt[(1 - Sqrt[5]*x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x]
, (2 + Sqrt[5])/4])/(2*Sqrt[1 - Sqrt[5]*x^2 + x^4])
```

Defintions of rubi rules used

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{2+2\sqrt{5}}\,x}{2},\sqrt{-1+\sqrt{5}\,\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}\right)}{\sqrt{2+2\sqrt{5}}\,\sqrt{1-\sqrt{5}\,x^{2}+x^{4}}}$	87
elliptic	$\frac{2\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{2+2\sqrt{5}}\,x}{2},\sqrt{-1+\sqrt{5}\,\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}\right)}{\sqrt{2+2\sqrt{5}}\,\sqrt{1-\sqrt{5}\,x^{2}+x^{4}}}$	87

input
```
int(1/(1-5^(1/2)*x^2+x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(2+2*5^(1/2))^(1/2)*(1-(1/2+1/2*5^(1/2))*x^2)^(1/2)*(1-(1/2*5^(1/2)-1/2)
*x^2)^(1/2)/(1-5^(1/2)*x^2+x^4)^(1/2)*EllipticF(1/2*(2+2*5^(1/2))^(1/2)*x,
(-1+5^(1/2)*(1/2*5^(1/2)-1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.35

$$\int \frac{1}{\sqrt{1-\sqrt{5}x^2+x^4}}\,dx = \frac{1}{2}\left(\sqrt{5}-1\right)\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right) \mid -\frac{1}{2}\sqrt{5} + \frac{3}{2})$$

input
```
integrate(1/(1-5^(1/2)*x^2+x^4)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(5) - 1)*sqrt(1/2*sqrt(5) + 1/2)*elliptic_f(arcsin(x*sqrt(1/2*sqr
t(5) + 1/2)), -1/2*sqrt(5) + 3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-\sqrt{5}x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4-\sqrt{5}x^2+1}}\,dx$$

input
```
integrate(1/(1-5**(1/2)*x**2+x**4)**(1/2),x)
```

output
```
Integral(1/sqrt(x**4 - sqrt(5)*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-\sqrt{5}x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4-\sqrt{5}x^2+1}}\,dx$$

input
```
integrate(1/(1-5^(1/2)*x^2+x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x^4 - sqrt(5)*x^2 + 1), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-\sqrt{5}x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4-\sqrt{5}x^2+1}}\,dx$$

input
```
integrate(1/(1-5^(1/2)*x^2+x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(x^4 - sqrt(5)*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-\sqrt{5}x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4-\sqrt{5}\,x^2+1}}\,dx$$

input
```
int(1/(x^4 - 5^(1/2)*x^2 + 1)^(1/2),x)
```

output
```
int(1/(x^4 - 5^(1/2)*x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-\sqrt{5}x^2+x^4}}\,dx=\sqrt{5}\left(\int \frac{\sqrt{-\sqrt{5}\,x^2+x^4+1}\,x^2}{x^8-3x^4+1}dx\right)+\int \frac{\sqrt{-\sqrt{5}\,x^2+x^4+1}}{x^8-3x^4+1}dx+\int \frac{\sqrt{-\sqrt{5}\,x^2+x^4+1}\,x^4}{x^8-3x^4+1}dx$$

input
```
int(1/(1-5^(1/2)*x^2+x^4)^(1/2),x)
```

output
```
sqrt(5)*int((sqrt( - sqrt(5)*x**2 + x**4 + 1)*x**2)/(x**8 - 3*x**4 + 1),x)
 + int(sqrt( - sqrt(5)*x**2 + x**4 + 1)/(x**8 - 3*x**4 + 1),x) + int((sqrt
( - sqrt(5)*x**2 + x**4 + 1)*x**4)/(x**8 - 3*x**4 + 1),x)
```

3.65 $\int \frac{1}{\sqrt{-3+7x^2-2x^4}} dx$

Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx = -\frac{\text{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{3}}\right),\frac{6}{5}\right)}{\sqrt{5}}$$

output
```
-1/5*InverseJacobiAM(arccos(1/3*x*3^(1/2)),1/5*30^(1/2))*5^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 58 vs. 2(19) = 38.

Time = 10.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.05

$$\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx = \frac{\sqrt{1-2x^2}\sqrt{1-\frac{x^2}{3}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),\frac{1}{6}\right)}{\sqrt{2}\sqrt{-3+7x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + 7*x^2 - 2*x^4],x]
```

output
```
(Sqrt[1 - 2*x^2]*Sqrt[1 - x^2/3]*EllipticF[ArcSin[Sqrt[2]*x], 1/6])/(Sqrt[
2]*Sqrt[-3 + 7*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 322}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+7x^2-3}}dx$$
$$\Big\downarrow \text{ 1408}$$
$$2\sqrt{2}\int \frac{1}{2\sqrt{2}\sqrt{3-x^2}\sqrt{2x^2-1}}dx$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{1}{\sqrt{3-x^2}\sqrt{2x^2-1}}dx$$
$$\Big\downarrow \text{ 322}$$
$$-\frac{\text{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{3}}\right),\frac{6}{5}\right)}{\sqrt{5}}$$

input
```
Int[1/Sqrt[-3 + 7*x^2 - 2*x^4],x]
```

output
```
-(EllipticF[ArcCos[x/Sqrt[3]], 6/5]/Sqrt[5])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(18) = 36$.

Time = 0.60 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

method	result	size
default	$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{-2x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3},\sqrt{6}\right)}{3\sqrt{-2x^4+7x^2-3}}$	48
elliptic	$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{-2x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3},\sqrt{6}\right)}{3\sqrt{-2x^4+7x^2-3}}$	48

input
```
int(1/(-2*x^4+7*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(-2*x^2+1)^(1/2)/(-2*x^4+7*x^2-3)^(1/2)*Ellip
ticF(1/3*3^(1/2)*x,6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx = -\frac{1}{6}\,\sqrt{2}\sqrt{-3}F(\arcsin\left(\sqrt{2}x\right)\mid\frac{1}{6})$$

input
```
integrate(1/(-2*x^4+7*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2)*sqrt(-3)*elliptic_f(arcsin(sqrt(2)*x), 1/6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+7x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4+7*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + 7*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+7\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+7*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 7*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+7\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+7*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 7*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+7\,x^2-3}}\,dx$$

input
```
int(1/(7*x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(7*x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+7x^2-3}}{2x^4-7x^2+3}dx\right)$$

input
```
int(1/(-2*x^4+7*x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 7*x**2 - 3)/(2*x**4 - 7*x**2 + 3),x)
```

3.66 $\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx=-\frac{\text{EllipticF}\left(\arccos\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

output
```
-1/6*InverseJacobiAM(arccos(1/3*(9-3*3^(1/2))^(1/2)*x),1/2*(2+2*3^(1/2))^(
1/2))*2^(1/2)*3^(3/4)
```

Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx =\frac{\sqrt{3-\sqrt{3}-2x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{1+\frac{1}{\sqrt{3}}}x\right),2-\sqrt{3}\right)}{\sqrt{6}\sqrt{-3+6x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + 6*x^2 - 2*x^4],x]
```

output
```
(Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[S
qrt[1 + 1/Sqrt[3]]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[-3 + 6*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1408, 27, 322}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+6x^2-3}}dx$$
$$\Big\downarrow \text{1408}$$
$$2\sqrt{2}\int \frac{1}{2\sqrt{-2x^2+\sqrt{3}+3}\sqrt{2x^2+\sqrt{3}-3}}dx$$
$$\Big\downarrow \text{27}$$
$$\sqrt{2}\int \frac{1}{\sqrt{-2x^2+\sqrt{3}+3}\sqrt{2x^2+\sqrt{3}-3}}dx$$
$$\Big\downarrow \text{322}$$
$$-\frac{\text{EllipticF}\left(\arccos\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

input
```
Int[1/Sqrt[-3 + 6*x^2 - 2*x^4],x]
```

output
```
-(EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sqrt[3])/2]/(Sqrt[2]*3^(
1/4)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(33) = 66$.

Time = 0.57 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.86

method	result	size
default	$\frac{3\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{9-3\sqrt{3}}\,x}{3},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{9-3\sqrt{3}}\,\sqrt{-2x^4+6x^2-3}}$	82
elliptic	$\frac{3\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{9-3\sqrt{3}}\,x}{3},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{9-3\sqrt{3}}\,\sqrt{-2x^4+6x^2-3}}$	82

input
```
int(1/(-2*x^4+6*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(9-3*3^(1/2))^(1/2)*(1-(1-1/3*3^(1/2))*x^2)^(1/2)*(1-(1+1/3*3^(1/2))*x^2
)^(1/2)/(-2*x^4+6*x^2-3)^(1/2)*EllipticF(1/3*(9-3*3^(1/2))^(1/2)*x,1/2*6^(
1/2)+1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx = \frac{1}{6}\left(\sqrt{3}\sqrt{-3}-3\sqrt{-3}\right)\sqrt{\frac{1}{3}\sqrt{3}+1}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{3}+1}\right)\mid -\sqrt{3}+2)$$

input
```
integrate(1/(-2*x^4+6*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(3)*sqrt(-3) - 3*sqrt(-3))*sqrt(1/3*sqrt(3) + 1)*elliptic_f(arcsi
n(x*sqrt(1/3*sqrt(3) + 1)), -sqrt(3) + 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+6x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4+6*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + 6*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+6\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+6*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 6*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+6\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+6*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 6*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+6\,x^2-3}}\,dx$$

input
```
int(1/(6*x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(6*x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+6x^2-3}}{2x^4-6x^2+3}dx\right)$$

input
```
int(1/(-2*x^4+6*x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 6*x**2 - 3)/(2*x**4 - 6*x**2 + 3),x)
```

3.67 $\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx=-\operatorname{EllipticF}\left(\arccos\left(\sqrt{\frac{2}{3}}x\right),3\right)$$

output
```
-InverseJacobiAM(arccos(1/3*x*6^(1/2)),3^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 53 vs. 2(14) = 28.

Time = 10.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx=\frac{\sqrt{3-2x^2}\sqrt{1-x^2}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right),\frac{3}{2}\right)}{\sqrt{-6+10x^2-4x^4}}$$

input
```
Integrate[1/Sqrt[-3 + 5*x^2 - 2*x^4],x]
```

output
```
(Sqrt[3 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], 3/2])/Sqrt[-
6 + 10*x^2 - 4*x^4]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 322}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+5x^2-3}}dx$$
$$\Big\downarrow \text{ 1408}$$
$$2\sqrt{2}\int \frac{1}{2\sqrt{2}\sqrt{3-2x^2}\sqrt{x^2-1}}dx$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{1}{\sqrt{3-2x^2}\sqrt{x^2-1}}dx$$
$$\Big\downarrow \text{ 322}$$
$$-\text{EllipticF}\left(\arccos\left(\sqrt{\frac{2}{3}}x\right),3\right)$$

input
```
Int[1/Sqrt[-3 + 5*x^2 - 2*x^4],x]
```

output
```
-EllipticF[ArcCos[Sqrt[2/3]*x], 3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(13) = 26$.

Time = 0.56 (sec) , antiderivative size = 50, normalized size of antiderivative = 3.57

method	result	size
default	$\frac{\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{6\sqrt{-2x^4+5x^2-3}}$	50
elliptic	$\frac{\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{6\sqrt{-2x^4+5x^2-3}}$	50

input
```
int(1/(-2*x^4+5*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*6^(1/2)*(-6*x^2+9)^(1/2)*(-x^2+1)^(1/2)/(-2*x^4+5*x^2-3)^(1/2)*Ellipti
cF(1/3*x*6^(1/2),1/2*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx = -\frac{1}{3}\,\sqrt{-3}F(\arcsin{(x)}\mid\frac{2}{3})$$

input
```
integrate(1/(-2*x^4+5*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(-3)*elliptic_f(arcsin(x), 2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+5x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4+5*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + 5*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+5\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+5*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 5*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+5\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+5*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 5*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+5\,x^2-3}}\,dx$$

input
```
int(1/(5*x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(5*x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+5x^2-3}}{2x^4-5x^2+3}dx\right)$$

input
```
int(1/(-2*x^4+5*x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 5*x**2 - 3)/(2*x**4 - 5*x**2 + 3),x)
```

3.68 $\int \frac{1}{\sqrt{-3+4x^2-2x^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{1}{\sqrt{-3+4x^2-2x^4}} \, dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3+4x^2-2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4-4*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/6*(18+6*6^(1/2))^(1/2))*6^(3/4)/(-2
*x^4+4*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt{-3+4x^2-2x^4}} \, dx = -\frac{i\sqrt{1-\frac{2x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{2+i\sqrt{2}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{2}{2-i\sqrt{2}}}x\right),\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{-3+4x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + 4*x^2 - 2*x^4],x]
```

output
```
((-I)*Sqrt[1 - (2*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(2 + I*Sqrt[2])]*
EllipticF[I*ArcSinh[Sqrt[-2/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sq
rt[2])])/(Sqrt[2]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[-3 + 4*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+4x^2-3}}dx$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2x^4+4x^2-3}}$$

input
```
Int[1/Sqrt[-3 + 4*x^2 - 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-3 + 4*x^2 - 2
*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{6-3i\sqrt{2}}\,\sqrt{-2x^4+4x^2-3}}$	87
elliptic	$\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{6-3i\sqrt{2}}\,\sqrt{-2x^4+4x^2-3}}$	87

input
```
int(1/(-2*x^4+4*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(6-3*I*2^(1/2))^(1/2)*(1-(2/3-1/3*I*2^(1/2))*x^2)^(1/2)*(1-(2/3+1/3*I*2^
(1/2))*x^2)^(1/2)/(-2*x^4+4*x^2-3)^(1/2)*EllipticF(1/3*(6-3*I*2^(1/2))^(1/
2)*x,1/3*(3+6*I*2^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx$$
$$=\frac{1}{6}\,(\sqrt{-2}\sqrt{-3}-2\,\sqrt{-3})\sqrt{\frac{1}{3}\,\sqrt{-2}+\frac{2}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\,\sqrt{-2}+\frac{2}{3}}\right)\,|-\frac{2}{3}\,\sqrt{-2}+\frac{1}{3})$$

input
```
integrate(1/(-2*x^4+4*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(-2)*sqrt(-3) - 2*sqrt(-3))*sqrt(1/3*sqrt(-2) + 2/3)*elliptic_f(a
rcsin(x*sqrt(1/3*sqrt(-2) + 2/3)), -2/3*sqrt(-2) + 1/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+4x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4+4*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + 4*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+4\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+4*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 4*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+4\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+4*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 4*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+4\,x^2-3}}\,dx$$

input
```
int(1/(4*x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(4*x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+4x^2-3}}{2x^4-4x^2+3}dx\right)$$

input
```
int(1/(-2*x^4+4*x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 4*x**2 - 3)/(2*x**4 - 4*x**2 + 3),x)
```

3.69 $\int \frac{1}{\sqrt{-3+3x^2-2x^4}} dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{-3+3x^2-2x^4}} dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3+3x^2-2x^4}}$$

output

```
1/12*(3+6^(1/2)*x^2)*((2*x^4-3*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/4*(8+2*6^(1/2))^(1/2))*6^(3/4)/(-2*
x^4+3*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{-3+3x^2-2x^4}} dx = -\frac{i\sqrt{1-\frac{4x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{3+i\sqrt{15}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(2\sqrt{-\frac{1}{3-i\sqrt{15}}}x\right),\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{-3+3x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + 3*x^2 - 2*x^4],x]
```

output
```
((-1/2*I)*Sqrt[1 - (4*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(3 + I*Sqrt[
15])]*EllipticF[I*ArcSinh[2*Sqrt[-(3 - I*Sqrt[15])^(-1)]*x], (3 - I*Sqrt[1
5])/(3 + I*Sqrt[15])])/(Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[-3 + 3*x^2 - 2*x
^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+3x^2-3}}dx$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+3x^2-3}}$$

input
```
Int[1/Sqrt[-3 + 3*x^2 - 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-3 + 3*x^2 - 2
*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{6\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{18-6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{18-6i\sqrt{15}}\,\sqrt{-2x^4+3x^2-3}}$	87
elliptic	$\frac{6\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{18-6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{18-6i\sqrt{15}}\,\sqrt{-2x^4+3x^2-3}}$	87

input
```
int(1/(-2*x^4+3*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(18-6*I*15^(1/2))^(1/2)*(1-(1/2-1/6*I*15^(1/2))*x^2)^(1/2)*(1-(1/2+1/6*I
*15^(1/2))*x^2)^(1/2)/(-2*x^4+3*x^2-3)^(1/2)*EllipticF(1/6*(18-6*I*15^(1/2
))^(1/2)*x,1/2*(-1+I*15^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx = \frac{1}{4}\left(\sqrt{-\frac{5}{3}}\sqrt{-3}-\sqrt{-3}\right)\sqrt{\frac{1}{2}\sqrt{-\frac{5}{3}}+\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-\frac{5}{3}}+\frac{1}{2}}\right)\mid-\frac{3}{4}\sqrt{-\frac{5}{3}}-\frac{1}{4})$$

input
```
integrate(1/(-2*x^4+3*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(-5/3)*sqrt(-3) - sqrt(-3))*sqrt(1/2*sqrt(-5/3) + 1/2)*elliptic_f
(arcsin(x*sqrt(1/2*sqrt(-5/3) + 1/2)), -3/4*sqrt(-5/3) - 1/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+3x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4+3*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + 3*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+3\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+3*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 3*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+3\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+3*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 3*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+3\,x^2-3}}\,dx$$

input
```
int(1/(3*x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(3*x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+3x^2-3}}{2x^4-3x^2+3}dx\right)$$

input
```
int(1/(-2*x^4+3*x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 3*x**2 - 3)/(2*x**4 - 3*x**2 + 3),x)
```

3.70 $\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3+2x^2-2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4-2*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/6*(18+3*6^(1/2))^(1/2))*6^(3/4)/(-2
*x^4+2*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx = -\frac{i\sqrt{1-\frac{2x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{1+i\sqrt{5}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{2}{1-i\sqrt{5}}}x\right),\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{-3+2x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + 2*x^2 - 2*x^4],x]
```

output
```
((-I)*Sqrt[1 - (2*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(1 + I*Sqrt[5])]*
EllipticF[I*ArcSinh[Sqrt[-2/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sq
rt[5])])/(Sqrt[2]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[-3 + 2*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+2x^2-3}}dx$$
$$\Big\downarrow \text{ 1416}$$
$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+2x^2-3}}$$

input
```
Int[1/Sqrt[-3 + 2*x^2 - 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-3 + 2*x^2 -
2*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^{2}}\,\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{3-3i\sqrt{5}}\,\sqrt{-2x^{4}+2x^{2}-3}}$	87
elliptic	$\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^{2}}\,\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{3-3i\sqrt{5}}\,\sqrt{-2x^{4}+2x^{2}-3}}$	87

input
```
int(1/(-2*x^4+2*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(3-3*I*5^(1/2))^(1/2)*(1-(1/3-1/3*I*5^(1/2))*x^2)^(1/2)*(1-(1/3+1/3*I*5^
(1/2))*x^2)^(1/2)/(-2*x^4+2*x^2-3)^(1/2)*EllipticF(1/3*(3-3*I*5^(1/2))^(1/
2)*x,1/3*(-6+3*I*5^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx = \frac{1}{6}\sqrt{-3}(\sqrt{-5}-1)\sqrt{\frac{1}{3}\sqrt{-5}+\frac{1}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{-5}+\frac{1}{3}}\right)\,|-\frac{1}{3}\sqrt{-5}-\frac{2}{3})$$

input
```
integrate(1/(-2*x^4+2*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(-3)*(sqrt(-5) - 1)*sqrt(1/3*sqrt(-5) + 1/3)*elliptic_f(arcsin(x*s
qrt(1/3*sqrt(-5) + 1/3)), -1/3*sqrt(-5) - 2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2x^4+2x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4+2*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + 2*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2\,x^4+2\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+2*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 2*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2\,x^4+2\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+2*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 2*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+2\,x^2-3}}\,dx$$

input
```
int(1/(2*x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(2*x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+2x^2-3}}{2x^4-2x^2+3}dx\right)$$

input
```
int(1/(-2*x^4+2*x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 2*x**2 - 3)/(2*x**4 - 2*x**2 + 3),x)
```

3.71 $\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3+x^2-2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4-x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobi
AM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/12*(72+6*6^(1/2))^(1/2))*6^(3/4)/(-2*
x^4+x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx = -\frac{i\sqrt{1-\frac{4x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{1+i\sqrt{23}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(2\sqrt{-\frac{1}{1-i\sqrt{23}}}x\right),\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{-3+x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + x^2 - 2*x^4],x]
```

output
```
((-1/2*I)*Sqrt[1 - (4*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(1 + I*Sqrt[
23])]*EllipticF[I*ArcSinh[2*Sqrt[-(1 - I*Sqrt[23])^(-1)]*x], (1 - I*Sqrt[2
3])/(1 + I*Sqrt[23])])/(Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[-3 + x^2 - 2*x^4
])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+x^2-3}}\,dx$$

$$\Big\downarrow \; 1416$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+x^2-3}}$$

input
```
Int[1/Sqrt[-3 + x^2 - 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2
*ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-3 + x^2 - 2*x
^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{6\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{6-6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{6-6i\sqrt{23}}\,\sqrt{-2x^4+x^2-3}}$	85
elliptic	$\frac{6\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{6-6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{6-6i\sqrt{23}}\,\sqrt{-2x^4+x^2-3}}$	85

input
```
int(1/(-2*x^4+x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(6-6*I*23^(1/2))^(1/2)*(1-(1/6-1/6*I*23^(1/2))*x^2)^(1/2)*(1-(1/6+1/6*I*
23^(1/2))*x^2)^(1/2)/(-2*x^4+x^2-3)^(1/2)*EllipticF(1/6*(6-6*I*23^(1/2))^(
1/2)*x,1/6*(-33+3*I*23^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx = \frac{1}{12}\sqrt{-3}(\sqrt{-23}-1)\sqrt{\frac{1}{6}\sqrt{-23}+\frac{1}{6}}F(\arcsin\left(x\sqrt{\frac{1}{6}\sqrt{-23}+\frac{1}{6}}\right)\mid -\frac{1}{12}\sqrt{-23} - \frac{11}{12})$$

input
```
integrate(1/(-2*x^4+x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(-3)*(sqrt(-23) - 1)*sqrt(1/6*sqrt(-23) + 1/6)*elliptic_f(arcsin(
x*sqrt(1/6*sqrt(-23) + 1/6)), -1/12*sqrt(-23) - 11/12)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4+x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4+x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+x^2-3}}\,dx$$

input
```
int(1/(x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+x^2-3}}{2x^4-x^2+3}dx\right)$$

input
```
int(1/(-2*x^4+x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + x**2 - 3)/(2*x**4 - x**2 + 3),x)
```

3.72 $\int \frac{1}{\sqrt{-3-2x^4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 72

$$\int \frac{1}{\sqrt{-3-2x^4}}\,dx=\frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-3-2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2
*arctan(1/3*2^(1/4)*3^(3/4)*x),1/2*2^(1/2))*6^(3/4)/(-2*x^4-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{-3-2x^4}}\,dx=-\frac{\sqrt[4]{-\frac{1}{6}}\sqrt{3+2x^4}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-\frac{2}{3}}x\right),-1\right)}{\sqrt{-3-2x^4}}$$

input
```
Integrate[1/Sqrt[-3 - 2*x^4],x]
```

output
```
-(((-1/6)^(1/4)*Sqrt[3 + 2*x^4]*EllipticF[I*ArcSinh[(-2/3)^(1/4)*x], -1])/
Sqrt[-3 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-3}}dx$$

$$\Big\downarrow \; 761$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-2x^4-3}}$$

input
```
Int[1/Sqrt[-3 - 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTa
n[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-3 - 2*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.26

method	result	size
meijerg	$-\frac{i\sqrt{3}\,x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-\frac{2x^{4}}{3}\right)}{3}$	19
default	$\frac{\sqrt{3}\,\sqrt{9+3i\sqrt{6}\,x^{2}}\,\sqrt{9-3i\sqrt{6}\,x^{2}}\,\text{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-i\sqrt{6}}}{3},i\right)}{9\sqrt{-i\sqrt{6}}\,\sqrt{-2x^{4}-3}}$	66
elliptic	$\frac{\sqrt{3}\,\sqrt{9+3i\sqrt{6}\,x^{2}}\,\sqrt{9-3i\sqrt{6}\,x^{2}}\,\text{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-i\sqrt{6}}}{3},i\right)}{9\sqrt{-i\sqrt{6}}\,\sqrt{-2x^{4}-3}}$	66

input
```
int(1/(-2*x^4-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*3^(1/2)*x*hypergeom([1/4,1/2],[5/4],-2/3*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{-3-2x^4}}\,dx = \frac{1}{2}\left(-\frac{2}{3}\right)^{\frac{3}{4}}\sqrt{-3}F\left(\arcsin\left(\left(-\frac{2}{3}\right)^{\frac{1}{4}}x\right)\,|\,-1\right)$$

input
```
integrate(1/(-2*x^4-3)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-2/3)^(3/4)*sqrt(-3)*elliptic_f(arcsin((-2/3)^(1/4)*x), -1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{-3-2x^4}}\,dx = -\frac{\sqrt{3}ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4e^{i\pi}}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-2*x**4-3)**(1/2),x)
```

output
```
-sqrt(3)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(I*pi)/3
)/(12*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-3}}\,dx$$

input
```
integrate(1/(-2*x^4-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-3}}\,dx$$

input
```
integrate(1/(-2*x^4-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - 3), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{-3-2x^4}}\,dx = \frac{x\,\sqrt{6\,x^4+9}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{2\,x^4}{3}\right)}{3\,\sqrt{-2\,x^4-3}}$$

input
```
int(1/(- 2*x^4 - 3)^(1/2),x)
```

output
```
(x*(6*x^4 + 9)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(2*x^4)/3))/(3*(- 2*x^4 -
 3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4-3}}{2x^4+3}dx\right)$$

input
```
int(1/(-2*x^4-3)^(1/2),x)
```

output
```
 - int(sqrt( - 2*x**4 - 3)/(2*x**4 + 3),x)
```

3.73 $\int \frac{1}{\sqrt{-3-x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{-3-x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3-x^2-2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4+x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobi
AM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/12*(72-6*6^(1/2))^(1/2))*6^(3/4)/(-2*
x^4-x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{-3-x^2-2x^4}}\,dx = -\frac{i\sqrt{1-\frac{4x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{-1+i\sqrt{23}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(2\sqrt{-\frac{1}{-1-i\sqrt{23}}}x\right),\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{-3-x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[-3 - x^2 - 2*x^4],x]
```

output
```
((-1/2*I)*Sqrt[1 - (4*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(-1 + I*Sqr
t[23])]*EllipticF[I*ArcSinh[2*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*x], (-1 - I*Sq
rt[23])/(-1 + I*Sqrt[23])])/(Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[-3 - x^2 -
 2*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-x^2-3}}\,dx$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-x^2-3}}$$

input
```
Int[1/Sqrt[-3 - x^2 - 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2
*ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-3 - x^2 - 2*x
^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{6\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\ \operatorname{EllipticF}\left(\frac{\sqrt{-6-6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{-6-6i\sqrt{23}}\,\sqrt{-2x^4-x^2-3}}$	87
elliptic	$\frac{6\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\ \operatorname{EllipticF}\left(\frac{\sqrt{-6-6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{-6-6i\sqrt{23}}\,\sqrt{-2x^4-x^2-3}}$	87

input
```
int(1/(-2*x^4-x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(-6-6*I*23^(1/2))^(1/2)*(1-(-1/6-1/6*I*23^(1/2))*x^2)^(1/2)*(1-(-1/6+1/6
*I*23^(1/2))*x^2)^(1/2)/(-2*x^4-x^2-3)^(1/2)*EllipticF(1/6*(-6-6*I*23^(1/2
))^(1/2)*x,1/6*(-33-3*I*23^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{-3-x^2-2x^4}}\,dx$$
$$=\frac{1}{12}\sqrt{-3}(\sqrt{-23}+1)\sqrt{\frac{1}{6}\sqrt{-23}-\frac{1}{6}}F(\arcsin\left(x\sqrt{\frac{1}{6}\sqrt{-23}-\frac{1}{6}}\right)\mid\frac{1}{12}\sqrt{-23}-\frac{11}{12})$$

input
```
integrate(1/(-2*x^4-x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(-3)*(sqrt(-23) + 1)*sqrt(1/6*sqrt(-23) - 1/6)*elliptic_f(arcsin(
x*sqrt(1/6*sqrt(-23) - 1/6)), 1/12*sqrt(-23) - 11/12)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4-x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4-x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2\,x^4-x^2-3}}\,dx$$

input
```
int(1/(- x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(- x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-x^2-2x^4}}\,dx=-\left(\int \frac{\sqrt{-2x^4-x^2-3}}{2x^4+x^2+3}dx\right)$$

input
```
int(1/(-2*x^4-x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - x**2 - 3)/(2*x**4 + x**2 + 3),x)
```

3.74 $\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3-2x^2-2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4+2*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/6*(18-3*6^(1/2))^(1/2))*6^(3/4)/(-2
*x^4-2*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx = -\frac{i\sqrt{1-\frac{2x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{-1+i\sqrt{5}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{2}{-1-i\sqrt{5}}}x\right),\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{-3-2x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[-3 - 2*x^2 - 2*x^4],x]
```

output
```
((-I)*Sqrt[1 - (2*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(-1 + I*Sqrt[5])
]*EllipticF[I*ArcSinh[Sqrt[-2/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 +
 I*Sqrt[5])])/(Sqrt[2]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[-3 - 2*x^2 - 2*x^
4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-2x^2-3}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-2x^2-3}}$$

input
```
Int[1/Sqrt[-3 - 2*x^2 - 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-3 - 2*x^2 -
2*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{-3-3i\sqrt{5}}\,\sqrt{-2x^4-2x^2-3}}$	87
elliptic	$\frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{-3-3i\sqrt{5}}\,\sqrt{-2x^4-2x^2-3}}$	87

input
```
int(1/(-2*x^4-2*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(-3-3*I*5^(1/2))^(1/2)*(1-(-1/3-1/3*I*5^(1/2))*x^2)^(1/2)*(1-(-1/3+1/3*I
*5^(1/2))*x^2)^(1/2)/(-2*x^4-2*x^2-3)^(1/2)*EllipticF(1/3*(-3-3*I*5^(1/2))
^(1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx = \frac{1}{6}\sqrt{-3}(\sqrt{-5}+1)\sqrt{\frac{1}{3}\sqrt{-5}-\frac{1}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{-5}-\frac{1}{3}}\right) \mid \frac{1}{3}\sqrt{-5}-\frac{2}{3})$$

input
```
integrate(1/(-2*x^4-2*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(-3)*(sqrt(-5) + 1)*sqrt(1/3*sqrt(-5) - 1/3)*elliptic_f(arcsin(x*s
qrt(1/3*sqrt(-5) - 1/3)), 1/3*sqrt(-5) - 2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2x^4-2x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4-2*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - 2*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2\,x^4-2\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-2*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - 2*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2\,x^4-2\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-2*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - 2*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-2\,x^2-3}}\,dx$$

input
```
int(1/(- 2*x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(- 2*x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4-2x^2-3}}{2x^4+2x^2+3}dx\right)$$

input
```
int(1/(-2*x^4-2*x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - 2*x**2 - 3)/(2*x**4 + 2*x**2 + 3),x)
```

3.75 $\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3-3x^2-2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4+3*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/4*(8-2*6^(1/2))^(1/2))*6^(3/4)/(-2*
x^4-3*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx = -\frac{i\sqrt{1-\frac{4x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{-3+i\sqrt{15}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(2\sqrt{-\frac{1}{-3-i\sqrt{15}}}x\right),\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{-3-3x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[-3 - 3*x^2 - 2*x^4],x]
```

output
```
((-1/2*I)*Sqrt[1 - (4*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(-3 + I*Sqr
t[15])]*EllipticF[I*ArcSinh[2*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*x], (-3 - I*Sq
rt[15])/(-3 + I*Sqrt[15])])/(Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[-3 - 3*x^2
 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-3x^2-3}}dx$$

$$\Big\downarrow \ 1416$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-3x^2-3}}$$

input
```
Int[1/Sqrt[-3 - 3*x^2 - 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-3 - 3*x^2 - 2
*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-18-6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{-18-6i\sqrt{15}}\,\sqrt{-2x^4-3x^2-3}}$	87
elliptic	$\frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-18-6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{-18-6i\sqrt{15}}\,\sqrt{-2x^4-3x^2-3}}$	87

input
```
int(1/(-2*x^4-3*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(-18-6*I*15^(1/2))^(1/2)*(1-(-1/2-1/6*I*15^(1/2))*x^2)^(1/2)*(1-(-1/2+1/
6*I*15^(1/2))*x^2)^(1/2)/(-2*x^4-3*x^2-3)^(1/2)*EllipticF(1/6*(-18-6*I*15^
(1/2))^(1/2)*x,1/2*(-1-I*15^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx$$
$$=\frac{1}{4}\left(\sqrt{-\frac{5}{3}}\sqrt{-3}+\sqrt{-3}\right)\sqrt{\frac{1}{2}\sqrt{-\frac{5}{3}}-\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-\frac{5}{3}}-\frac{1}{2}}\right)\mid\frac{3}{4}\sqrt{-\frac{5}{3}}-\frac{1}{4})$$

input
```
integrate(1/(-2*x^4-3*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(-5/3)*sqrt(-3) + sqrt(-3))*sqrt(1/2*sqrt(-5/3) - 1/2)*elliptic_f
(arcsin(x*sqrt(1/2*sqrt(-5/3) - 1/2)), 3/4*sqrt(-5/3) - 1/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2x^4-3x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4-3*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - 3*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2\,x^4-3\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-3*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - 3*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-3\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-3*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - 3*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-3\,x^2-3}}\,dx$$

input
```
int(1/(- 3*x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(- 3*x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4-3x^2-3}}{2x^4+3x^2+3}dx\right)$$

input
```
int(1/(-2*x^4-3*x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - 3*x**2 - 3)/(2*x**4 + 3*x**2 + 3),x)
```

3.76 $\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3-4x^2-2x^4}}$$

output
```
1/12*(3+6^(1/2)*x^2)*((2*x^4+4*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/6*(18-6*6^(1/2))^(1/2))*6^(3/4)/(-2
*x^4-4*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx = -\frac{i\sqrt{1-\frac{2x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{-2+i\sqrt{2}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{2}{-2-i\sqrt{2}}}x\right),\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{-3-4x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[-3 - 4*x^2 - 2*x^4],x]
```

output
```
((-I)*Sqrt[1 - (2*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(-2 + I*Sqrt[2])
]*EllipticF[I*ArcSinh[Sqrt[-2/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 +
 I*Sqrt[2])])/(Sqrt[2]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[-3 - 4*x^2 - 2*x^
4])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-4x^2-3}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2x^4-4x^2-3}}$$

input
```
Int[1/Sqrt[-3 - 4*x^2 - 2*x^4],x]
```

output
```
((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-3 - 4*x^2 - 2
*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^{2}}\,\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{-6-3i\sqrt{2}}\,\sqrt{-2x^{4}-4x^{2}-3}}$	87
elliptic	$\frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^{2}}\,\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{-6-3i\sqrt{2}}\,\sqrt{-2x^{4}-4x^{2}-3}}$	87

input
```
int(1/(-2*x^4-4*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(-6-3*I*2^(1/2))^(1/2)*(1-(-2/3-1/3*I*2^(1/2))*x^2)^(1/2)*(1-(-2/3+1/3*I
*2^(1/2))*x^2)^(1/2)/(-2*x^4-4*x^2-3)^(1/2)*EllipticF(1/3*(-6-3*I*2^(1/2))
^(1/2)*x,1/3*(3-6*I*2^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx = \frac{1}{6}\left(\sqrt{-2}\sqrt{-3}+2\sqrt{-3}\right)\sqrt{\frac{1}{3}\sqrt{-2}-\frac{2}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{-2}-\frac{2}{3}}\right) \mid \frac{2}{3}\sqrt{-2}+\frac{1}{3})$$

input
```
integrate(1/(-2*x^4-4*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(-2)*sqrt(-3) + 2*sqrt(-3))*sqrt(1/3*sqrt(-2) - 2/3)*elliptic_f(a
rcsin(x*sqrt(1/3*sqrt(-2) - 2/3)), 2/3*sqrt(-2) + 1/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4-4x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4-4*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - 4*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-4\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-4*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - 4*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-4\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-4*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - 4*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2\,x^4-4\,x^2-3}}\,dx$$

input
```
int(1/(- 4*x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(- 4*x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx=-\left(\int \frac{\sqrt{-2x^4-4x^2-3}}{2x^4+4x^2+3}dx\right)$$

input
```
int(1/(-2*x^4-4*x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - 4*x**2 - 3)/(2*x**4 + 4*x**2 + 3),x)
```

3.77 $\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx = \frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{2}{3}}x\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}}$$

output
```
1/2*(x^2+1)^(1/2)*InverseJacobiAM(arctan(1/3*x*6^(1/2)),1/2*I*2^(1/2))*2^(
1/2)/(-x^2-1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx = -\frac{i\sqrt{1+x^2}\sqrt{3+2x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{3}}x\right),\frac{3}{2}\right)}{\sqrt{2}\sqrt{-3-5x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[-3 - 5*x^2 - 2*x^4],x]
```

output
```
((-I)*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], 3/2]
)/(Sqrt[2]*Sqrt[-3 - 5*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-5x^2-3}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{2}\int \frac{1}{2\sqrt{2}\sqrt{-x^2-1}\sqrt{2x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{2x^2+3}}dx$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{2x^2+3}\,\text{EllipticF}\left(\arctan(x),\frac{1}{3}\right)}{\sqrt{3}\sqrt{-x^2-1}\sqrt{\frac{2x^2+3}{x^2+1}}}$$

input
```
Int[1/Sqrt[-3 - 5*x^2 - 2*x^4],x]
```

output
```
(Sqrt[3 + 2*x^2]*EllipticF[ArcTan[x], 1/3])/(Sqrt[3]*Sqrt[-1 - x^2]*Sqrt[(
3 + 2*x^2)/(1 + x^2)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+9}\;\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{3}\right)}{3\sqrt{-2x^{4}-5x^{2}-3}}$	44
elliptic	$-\frac{i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+9}\;\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{3}\right)}{3\sqrt{-2x^{4}-5x^{2}-3}}$	44

input
```
int(1/(-2*x^4-5*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*(x^2+1)^(1/2)*(6*x^2+9)^(1/2)/(-2*x^4-5*x^2-3)^(1/2)*EllipticF(I*x,
1/3*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx = \frac{1}{2}\sqrt{-\frac{2}{3}}\sqrt{-3}F(\arcsin\left(\sqrt{-\frac{2}{3}}x\right) | \frac{3}{2})$$

input
```
integrate(1/(-2*x^4-5*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-2/3)*sqrt(-3)*elliptic_f(arcsin(sqrt(-2/3)*x), 3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4-5x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4-5*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - 5*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-5\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-5*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - 5*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-5\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-5*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - 5*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-5\,x^2-3}}\,dx$$

input
```
int(1/(- 5*x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(- 5*x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4-5x^2-3}}{2x^4+5x^2+3}dx\right)$$

input
```
int(1/(-2*x^4-5*x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - 5*x**2 - 3)/(2*x**4 + 5*x**2 + 3),x)
```

3.78 $\int \frac{1}{\sqrt{-3-6x^2-2x^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 80

$$\int \frac{1}{\sqrt{-3-6x^2-2x^4}} \, dx = \frac{\sqrt{\frac{1}{6}\left(3+\sqrt{3}\right)}\sqrt{3-\sqrt{3}+2x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),-1-\sqrt{3}\right)}{\sqrt{-3+\sqrt{3}-2x^2}}$$

output
```
1/6*(18+6*3^(1/2))^(1/2)*(3-3^(1/2)+2*x^2)^(1/2)*InverseJacobiAM(arctan(1/
3*(9-3*3^(1/2))^(1/2)*x),(-1-3^(1/2))^(1/2))/(-3+3^(1/2)-2*x^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{-3-6x^2-2x^4}} \, dx = -\frac{i\sqrt{\frac{3}{2}-\frac{\sqrt{3}}{2}+x^2}\sqrt{3+\sqrt{3}+2x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1-\frac{1}{\sqrt{3}}}x\right),2+\sqrt{3}\right)}{\sqrt{\left(-3+\sqrt{3}\right)\left(3+6x^2+2x^4\right)}}$$

input
```
Integrate[1/Sqrt[-3 - 6*x^2 - 2*x^4],x]
```

output
```
((-I)*Sqrt[3/2 - Sqrt[3]/2 + x^2]*Sqrt[3 + Sqrt[3] + 2*x^2]*EllipticF[I*Ar
cSinh[Sqrt[1 - 1/Sqrt[3]]*x], 2 + Sqrt[3]])/Sqrt[(-3 + Sqrt[3])*(3 + 6*x^2
 + 2*x^4)]
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1408, 27, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-6x^2-3}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{2}\int \frac{1}{2\sqrt{-2x^2+\sqrt{3}-3}\sqrt{2x^2+\sqrt{3}+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\sqrt{2}\int \frac{1}{\sqrt{-2x^2+\sqrt{3}-3}\sqrt{2x^2+\sqrt{3}+3}}dx$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{\frac{6}{3-\sqrt{3}}}\sqrt{2x^2+\sqrt{3}+3}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right),-1+\sqrt{3}\right)}{\left(3+\sqrt{3}\right)\sqrt{-2x^2+\sqrt{3}-3}\sqrt{\frac{2x^2+\sqrt{3}+3}{2x^2-\sqrt{3}+3}}}$$

input
```
Int[1/Sqrt[-3 - 6*x^2 - 2*x^4],x]
```

output
```
(Sqrt[6/(3 - Sqrt[3])]*Sqrt[3 + Sqrt[3] + 2*x^2]*EllipticF[ArcTan[Sqrt[(3
+ Sqrt[3])/3]*x], -1 + Sqrt[3]])/((3 + Sqrt[3])*Sqrt[-3 + Sqrt[3] - 2*x^2]
*Sqrt[(3 + Sqrt[3] + 2*x^2)/(3 - Sqrt[3] + 2*x^2)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{3\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-9-3\sqrt{3}}\,x}{3},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{-9-3\sqrt{3}}\,\sqrt{-2x^4-6x^2-3}}$	82
elliptic	$\frac{3\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-9-3\sqrt{3}}\,x}{3},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{-9-3\sqrt{3}}\,\sqrt{-2x^4-6x^2-3}}$	82

input
```
int(1/(-2*x^4-6*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(-9-3*3^(1/2))^(1/2)*(1-(-1-1/3*3^(1/2))*x^2)^(1/2)*(1-(-1+1/3*3^(1/2))*
x^2)^(1/2)/(-2*x^4-6*x^2-3)^(1/2)*EllipticF(1/3*(-9-3*3^(1/2))^(1/2)*x,1/2
*6^(1/2)-1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{-3-6x^2-2x^4}}\,dx$$
$$= \frac{1}{6}\left(\sqrt{3}\sqrt{-3}+3\sqrt{-3}\right)\sqrt{\frac{1}{3}\sqrt{3}-1}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{3}-1}\right)\mid\sqrt{3}+2)$$

input
```
integrate(1/(-2*x^4-6*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(3)*sqrt(-3) + 3*sqrt(-3))*sqrt(1/3*sqrt(3) - 1)*elliptic_f(arcsi
n(x*sqrt(1/3*sqrt(3) - 1)), sqrt(3) + 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4-6x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4-6*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - 6*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-6\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-6*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - 6*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-6\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-6*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - 6*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-6\,x^2-3}}\,dx$$

input
```
int(1/(- 6*x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(- 6*x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-6x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4-6x^2-3}}{2x^4+6x^2+3}dx\right)$$

input
```
int(1/(-2*x^4-6*x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - 6*x**2 - 3)/(2*x**4 + 6*x**2 + 3),x)
```

3.79 $\int \frac{1}{\sqrt{-3-7x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{\sqrt{-3-7x^2-2x^4}}\,dx = \frac{\sqrt{1+2x^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}}\right),-5\right)}{\sqrt{-1-2x^2}}$$

output
```
(2*x^2+1)^(1/2)*InverseJacobiAM(arctan(1/3*x*3^(1/2)),I*5^(1/2))/(-2*x^2-1
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int \frac{1}{\sqrt{-3-7x^2-2x^4}}\,dx = -\frac{i\sqrt{3+x^2}\sqrt{1+2x^2}\,\mathrm{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{2}x\right),\frac{1}{6}\right)}{\sqrt{6}\sqrt{-3-7x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[-3 - 7*x^2 - 2*x^4],x]
```

output
```
((-I)*Sqrt[3 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], 1/6])/
(Sqrt[6]*Sqrt[-3 - 7*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1408, 27, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-7x^2-3}}dx$$
$$\Big\downarrow \text{ 1408}$$
$$2\sqrt{2}\int \frac{1}{2\sqrt{2}\sqrt{-2x^2-1}\sqrt{x^2+3}}dx$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{1}{\sqrt{-2x^2-1}\sqrt{x^2+3}}dx$$
$$\Big\downarrow \text{ 320}$$
$$-\frac{\sqrt{-2x^2-1}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}}\right),-5\right)}{\sqrt{x^2+3}\sqrt{\frac{2x^2+1}{x^2+3}}}$$

input
```
Int[1/Sqrt[-3 - 7*x^2 - 2*x^4],x]
```

output
```
-((Sqrt[-1 - 2*x^2]*EllipticF[ArcTan[x/Sqrt[3]], -5])/(Sqrt[3 + x^2]*Sqrt[
(1 + 2*x^2)/(3 + x^2)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{2x^{2}+1}\,\sqrt{3x^{2}+9}\,\operatorname{EllipticF}\left(ix\sqrt{2}, \frac{\sqrt{6}}{6}\right)}{6\sqrt{-2x^{4}-7x^{2}-3}}$	52
elliptic	$-\frac{i\sqrt{2}\,\sqrt{2x^{2}+1}\,\sqrt{3x^{2}+9}\,\operatorname{EllipticF}\left(ix\sqrt{2}, \frac{\sqrt{6}}{6}\right)}{6\sqrt{-2x^{4}-7x^{2}-3}}$	52

input
```
int(1/(-2*x^4-7*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*2^(1/2)*(2*x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-2*x^4-7*x^2-3)^(1/2)*Elli
pticF(I*2^(1/2)*x,1/6*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{-3-7x^2-2x^4}}\,dx = \sqrt{-\frac{1}{3}}\sqrt{-3}F\left(\arcsin\left(\sqrt{-\frac{1}{3}}x\right)\,|\,6\right)$$

input
```
integrate(1/(-2*x^4-7*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(-1/3)*sqrt(-3)*elliptic_f(arcsin(sqrt(-1/3)*x), 6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4-7x^2-3}}\,dx$$

input
```
integrate(1/(-2*x**4-7*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - 7*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-7\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-7*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - 7*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-7\,x^2-3}}\,dx$$

input
```
integrate(1/(-2*x^4-7*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - 7*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-7\,x^2-3}}\,dx$$

input
```
int(1/(- 7*x^2 - 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(- 7*x^2 - 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-7x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4-7x^2-3}}{2x^4+7x^2+3}dx\right)$$

input
```
int(1/(-2*x^4-7*x^2-3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - 7*x**2 - 3)/(2*x**4 + 7*x**2 + 3),x)
```

3.80 $\int \frac{1}{\sqrt{-2+7x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{1}{\sqrt{-2+7x^2-3x^4}}\,dx = -\frac{\operatorname{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{2}}\right),\frac{6}{5}\right)}{\sqrt{5}}$$

output
```
-1/5*InverseJacobiAM(arccos(1/2*x*2^(1/2)),1/5*30^(1/2))*5^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 58 vs. 2(19) = 38.

Time = 10.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.05

$$\int \frac{1}{\sqrt{-2+7x^2-3x^4}}\,dx = \frac{\sqrt{1-3x^2}\sqrt{1-\frac{x^2}{2}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{3}x\right),\frac{1}{6}\right)}{\sqrt{3}\sqrt{-2+7x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[-2 + 7*x^2 - 3*x^4],x]
```

output
```
(Sqrt[1 - 3*x^2]*Sqrt[1 - x^2/2]*EllipticF[ArcSin[Sqrt[3]*x], 1/6])/(Sqrt[
3]*Sqrt[-2 + 7*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 322}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4+7x^2-2}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{3}\int \frac{1}{2\sqrt{3}\sqrt{2-x^2}\sqrt{3x^2-1}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{3x^2-1}}dx$$

$$\Big\downarrow \text{322}$$

$$-\frac{\text{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{2}}\right),\frac{6}{5}\right)}{\sqrt{5}}$$

input
```
Int[1/Sqrt[-2 + 7*x^2 - 3*x^4],x]
```

output
```
-(EllipticF[ArcCos[x/Sqrt[2]], 6/5]/Sqrt[5])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(18) = 36$.

Time = 0.57 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{-3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},\sqrt{6}\right)}{2\sqrt{-3x^4+7x^2-2}}$	48
elliptic	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{-3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},\sqrt{6}\right)}{2\sqrt{-3x^4+7x^2-2}}$	48

input
```
int(1/(-3*x^4+7*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)/(-3*x^4+7*x^2-2)^(1/2)*Ellip
ticF(1/2*x*2^(1/2),6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{-2+7x^2-3x^4}}\,dx = -\frac{1}{6}\,\sqrt{3}\sqrt{-2}F(\arcsin\left(\sqrt{3}x\right)\mid\frac{1}{6})$$

input
```
integrate(1/(-3*x^4+7*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*sqrt(-2)*elliptic_f(arcsin(sqrt(3)*x), 1/6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2+7x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4+7x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4+7*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 + 7*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2+7x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+7\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+7*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + 7*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+7x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+7\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+7*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + 7*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2+7x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+7\,x^2-2}}\,dx$$

input
```
int(1/(7*x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(7*x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2+7x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4+7x^2-2}}{3x^4-7x^2+2}dx\right)$$

input
```
int(1/(-3*x^4+7*x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + 7*x**2 - 2)/(3*x**4 - 7*x**2 + 2),x)
```

3.81 $\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx = -\frac{\text{EllipticF}\left(\arccos\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right),\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

output
```
-1/6*InverseJacobiAM(arccos(3^(1/2)/(3+3^(1/2))^(1/2)*x),1/2*(2+2*3^(1/2))
^(1/2))*2^(1/2)*3^(3/4)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 85 vs. 2(42) = 84.

Time = 10.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx = \frac{\sqrt{3-\sqrt{3}-3x^2}\sqrt{2+\left(-3+\sqrt{3}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}x\right),2-\sqrt{3}\right)}{\sqrt{6}\sqrt{-2+6x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[-2 + 6*x^2 - 3*x^4],x]
```

output
```
(Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt[2 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[S
qrt[(3 + Sqrt[3])/2]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[-2 + 6*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 322}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4+6x^2-2}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{3}\int \frac{1}{2\sqrt{-3x^2+\sqrt{3}+3}\sqrt{3x^2+\sqrt{3}-3}}dx$$

$$\Big\downarrow \text{27}$$

$$\sqrt{3}\int \frac{1}{\sqrt{-3x^2+\sqrt{3}+3}\sqrt{3x^2+\sqrt{3}-3}}dx$$

$$\Big\downarrow \text{322}$$

$$-\frac{\text{EllipticF}\left(\arccos\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right),\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

input
```
Int[1/Sqrt[-2 + 6*x^2 - 3*x^4],x]
```

output
```
-(EllipticF[ArcCos[Sqrt[3/(3 + Sqrt[3])]*x], (1 + Sqrt[3])/2]/(Sqrt[2]*3^(
1/4)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(33) = 66$.

Time = 0.57 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{6-2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{6-2\sqrt{3}}\,\sqrt{-3x^{4}+6x^{2}-2}}$	82
elliptic	$\frac{2\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{6-2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{6-2\sqrt{3}}\,\sqrt{-3x^{4}+6x^{2}-2}}$	82

input
```
int(1/(-3*x^4+6*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(6-2*3^(1/2))^(1/2)*(1-(3/2-1/2*3^(1/2))*x^2)^(1/2)*(1-(3/2+1/2*3^(1/2))
*x^2)^(1/2)/(-3*x^4+6*x^2-2)^(1/2)*EllipticF(1/2*(6-2*3^(1/2))^(1/2)*x,1/2
*6^(1/2)+1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx = \frac{1}{6}\left(\sqrt{3}\sqrt{-2}-3\sqrt{-2}\right)\sqrt{\frac{1}{2}\sqrt{3}+\frac{3}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{3}+\frac{3}{2}}\right)|-\sqrt{3}+2)$$

input
```
integrate(1/(-3*x^4+6*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(3)*sqrt(-2) - 3*sqrt(-2))*sqrt(1/2*sqrt(3) + 3/2)*elliptic_f(arc
sin(x*sqrt(1/2*sqrt(3) + 3/2)), -sqrt(3) + 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4+6x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4+6*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 + 6*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+6\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+6*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + 6*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+6\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+6*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + 6*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+6\,x^2-2}}\,dx$$

input
```
int(1/(6*x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(6*x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4+6x^2-2}}{3x^4-6x^2+2}dx\right)$$

input
```
int(1/(-3*x^4+6*x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + 6*x**2 - 2)/(3*x**4 - 6*x**2 + 2),x)
```

3.82 $\int \frac{1}{\sqrt{-2+5x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 6

$$\int \frac{1}{\sqrt{-2+5x^2-3x^4}}\,dx = -\operatorname{EllipticF}(\arccos(x),3)$$

output
```
-InverseJacobiAM(arccos(x),3^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 53 vs. 2(6) = 12.

Time = 10.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 8.83

$$\int \frac{1}{\sqrt{-2+5x^2-3x^4}}\,dx = \frac{\sqrt{2-3x^2}\sqrt{1-x^2}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)}{\sqrt{-6+15x^2-9x^4}}$$

input
```
Integrate[1/Sqrt[-2 + 5*x^2 - 3*x^4],x]
```

output
```
(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3])/Sqrt[-
6 + 15*x^2 - 9*x^4]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1408, 27, 322}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4+5x^2-2}}dx$$
$$\Big\downarrow \text{ 1408}$$
$$2\sqrt{3}\int \frac{1}{2\sqrt{3}\sqrt{1-x^2}\sqrt{3x^2-2}}dx$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{1}{\sqrt{1-x^2}\sqrt{3x^2-2}}dx$$
$$\Big\downarrow \text{ 322}$$
$$-\text{EllipticF}(\arccos(x),3)$$

input
```
Int[1/Sqrt[-2 + 5*x^2 - 3*x^4],x]
```

output
```
-EllipticF[ArcCos[x], 3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(8) = 16$.

Time = 0.55 (sec) , antiderivative size = 42, normalized size of antiderivative = 7.00

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\ \text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{-3x^4+5x^2-2}}$	42
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\ \text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{-3x^4+5x^2-2}}$	42

input
```
int(1/(-3*x^4+5*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(-3*x^4+5*x^2-2)^(1/2)*EllipticF(x,1/2
*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{-2+5x^2-3x^4}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin{(x)}\mid\frac{3}{2})$$

input
```
integrate(1/(-3*x^4+5*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-2)*elliptic_f(arcsin(x), 3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2+5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4+5x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4+5*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 + 5*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2+5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+5\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+5*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + 5*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+5\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+5*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + 5*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2+5x^2-3x^4}}\,dx=\int \frac{1}{\sqrt{-3\,x^4+5\,x^2-2}}\,dx$$

input
```
int(1/(5*x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(5*x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2+5x^2-3x^4}}\,dx=-\left(\int \frac{\sqrt{-3x^4+5x^2-2}}{3x^4-5x^2+2}dx\right)$$

input
```
int(1/(-3*x^4+5*x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + 5*x**2 - 2)/(3*x**4 - 5*x**2 + 2),x)
```

3.83 $\int \frac{1}{\sqrt{-2+4x^2-3x^4}} dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2+4x^2-3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4-4*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/6*(18+6*6^(1/2))^(1/2))*6^(3/4)/(-3
*x^4+4*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx = -\frac{i\sqrt{1-\frac{3x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{2+i\sqrt{2}}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{3}{2-i\sqrt{2}}}x\right),\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{-2+4x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[-2 + 4*x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (3*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(2 + I*Sqrt[2])]*
EllipticF[I*ArcSinh[Sqrt[-3/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sq
rt[2])])/(Sqrt[3]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[-2 + 4*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4+4x^2-2}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3x^4+4x^2-2}}$$

input
```
Int[1/Sqrt[-2 + 4*x^2 - 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-2 + 4*x^2 - 3
*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{2\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right)x^2}\ \operatorname{EllipticF}\left(\frac{\sqrt{4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{4-2i\sqrt{2}}\,\sqrt{-3x^4+4x^2-2}}$	87
elliptic	$\frac{2\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right)x^2}\ \operatorname{EllipticF}\left(\frac{\sqrt{4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{4-2i\sqrt{2}}\,\sqrt{-3x^4+4x^2-2}}$	87

input
```
int(1/(-3*x^4+4*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(4-2*I*2^(1/2))^(1/2)*(1-(1-1/2*I*2^(1/2))*x^2)^(1/2)*(1-(1/2*I*2^(1/2)+
1)*x^2)^(1/2)/(-3*x^4+4*x^2-2)^(1/2)*EllipticF(1/2*(4-2*I*2^(1/2))^(1/2)*x
,1/3*(3+6*I*2^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx = -\frac{1}{3}\left(\sqrt{-2}+1\right)\sqrt{\frac{1}{2}\sqrt{-2}+1}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-2}+1}\right) \mid -\frac{2}{3}\sqrt{-2}+\frac{1}{3})$$

input
```
integrate(1/(-3*x^4+4*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(-2) + 1)*sqrt(1/2*sqrt(-2) + 1)*elliptic_f(arcsin(x*sqrt(1/2*sq
rt(-2) + 1)), -2/3*sqrt(-2) + 1/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4+4x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4+4*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 + 4*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+4\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+4*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + 4*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+4\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+4*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + 4*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx=\int \frac{1}{\sqrt{-3\,x^4+4\,x^2-2}}\,dx$$

input
```
int(1/(4*x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(4*x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx=-\left(\int \frac{\sqrt{-3x^4+4x^2-2}}{3x^4-4x^2+2}dx\right)$$

input
```
int(1/(-3*x^4+4*x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + 4*x**2 - 2)/(3*x**4 - 4*x**2 + 2),x)
```

3.84 $\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2+3x^2-3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4-3*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/4*(8+2*6^(1/2))^(1/2))*6^(3/4)/(-3*
x^4+3*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx = -\frac{i\sqrt{1-\frac{6x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{3+i\sqrt{15}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{6}{3-i\sqrt{15}}}x\right),\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{-2+3x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[-2 + 3*x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (6*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(3 + I*Sqrt[15])
]*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[15])]*x], (3 - I*Sqrt[15])/(3 +
I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[-2 + 3*x^2 - 3*x^
4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4+3x^2-2}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+3x^2-2}}$$

input
```
Int[1/Sqrt[-2 + 3*x^2 - 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-2 + 3*x^2 - 3
*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{3-i\sqrt{15}}\,\sqrt{-3x^4+3x^2-2}}$	87
elliptic	$\frac{2\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{3-i\sqrt{15}}\,\sqrt{-3x^4+3x^2-2}}$	87

input
```
int(1/(-3*x^4+3*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(3-I*15^(1/2))^(1/2)*(1-(3/4-1/4*I*15^(1/2))*x^2)^(1/2)*(1-(3/4+1/4*I*15
^(1/2))*x^2)^(1/2)/(-3*x^4+3*x^2-2)^(1/2)*EllipticF(1/2*(3-I*15^(1/2))^(1/
2)*x,1/2*(-1+I*15^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx = \frac{1}{24}\sqrt{-2}\sqrt{\sqrt{-15}+3}\left(\sqrt{-15}-3\right)F(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{-15}+3}\right)\mid -\frac{1}{4}\sqrt{-15}-\frac{1}{4})$$

input
```
integrate(1/(-3*x^4+3*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/24*sqrt(-2)*sqrt(sqrt(-15) + 3)*(sqrt(-15) - 3)*elliptic_f(arcsin(1/2*x*
sqrt(sqrt(-15) + 3)), -1/4*sqrt(-15) - 1/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4+3x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4+3*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 + 3*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+3\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+3*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + 3*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+3\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+3*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + 3*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx=\int \frac{1}{\sqrt{-3\,x^4+3\,x^2-2}}\,dx$$

input
```
int(1/(3*x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(3*x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx=-\left(\int \frac{\sqrt{-3x^4+3x^2-2}}{3x^4-3x^2+2}dx\right)$$

input
```
int(1/(-3*x^4+3*x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + 3*x**2 - 2)/(3*x**4 - 3*x**2 + 2),x)
```

3.85 $\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2+2x^2-3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4-2*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/6*(18+3*6^(1/2))^(1/2))*6^(3/4)/(-3
*x^4+2*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx = -\frac{i\sqrt{1-\frac{3x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{1+i\sqrt{5}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{3}{1-i\sqrt{5}}}x\right),\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{-2+2x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[-2 + 2*x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (3*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(1 + I*Sqrt[5])]*
EllipticF[I*ArcSinh[Sqrt[-3/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sq
rt[5])])/(Sqrt[3]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[-2 + 2*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4+2x^2-2}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+2x^2-2}}$$

input
```
Int[1/Sqrt[-2 + 2*x^2 - 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-2 + 2*x^2 -
3*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{2-2i\sqrt{5}}\,x}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{2-2i\sqrt{5}}\,\sqrt{-3x^4+2x^2-2}}$	87
elliptic	$\frac{2\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{2-2i\sqrt{5}}\,x}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{2-2i\sqrt{5}}\,\sqrt{-3x^4+2x^2-2}}$	87

input
```
int(1/(-3*x^4+2*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(2-2*I*5^(1/2))^(1/2)*(1-(1/2-1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2+1/2*I*5^
(1/2))*x^2)^(1/2)/(-3*x^4+2*x^2-2)^(1/2)*EllipticF(1/2*(2-2*I*5^(1/2))^(1/
2)*x,1/3*(-6+3*I*5^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx = \frac{1}{6}\sqrt{-2}(\sqrt{-5}-1)\sqrt{\frac{1}{2}\sqrt{-5}+\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-5}+\frac{1}{2}}\right)\mid -\frac{1}{3}\sqrt{-5}-\frac{2}{3})$$

input
```
integrate(1/(-3*x^4+2*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(-2)*(sqrt(-5) - 1)*sqrt(1/2*sqrt(-5) + 1/2)*elliptic_f(arcsin(x*s
qrt(1/2*sqrt(-5) + 1/2)), -1/3*sqrt(-5) - 2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4+2x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4+2*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 + 2*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+2\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+2*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + 2*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+2\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+2*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + 2*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+2\,x^2-2}}\,dx$$

input
```
int(1/(2*x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(2*x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4+2x^2-2}}{3x^4-2x^2+2}dx\right)$$

input
```
int(1/(-3*x^4+2*x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + 2*x**2 - 2)/(3*x**4 - 2*x**2 + 2),x)
```

3.86 $\int \frac{1}{\sqrt{-2+x^2-3x^4}} dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{1}{\sqrt{-2+x^2-3x^4}} dx = \frac{(2+\sqrt{6}x^2)\sqrt{\frac{2-x^2+3x^4}{(2+\sqrt{6}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}(12+\sqrt{6})\right)}{2\sqrt[4]{6}\sqrt{-2+x^2-3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4-x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobi
AM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/12*(72+6*6^(1/2))^(1/2))*6^(3/4)/(-3*
x^4+x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{-2+x^2-3x^4}} dx = -\frac{i\sqrt{1-\frac{6x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{1+i\sqrt{23}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{6}{1-i\sqrt{23}}}x\right),\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{-2+x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[-2 + x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (6*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(1 + I*Sqrt[23])
]*EllipticF[I*ArcSinh[Sqrt[-6/(1 - I*Sqrt[23])]*x], (1 - I*Sqrt[23])/(1 +
I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[-2 + x^2 - 3*x^4]
)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4+x^2-2}}dx$$

$$\Big\downarrow \ 1416$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+x^2-2}}$$

input
```
Int[1/Sqrt[-2 + x^2 - 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2
*ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-2 + x^2 - 3*x
^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{1-i\sqrt{23}}\,\sqrt{-3x^4+x^2-2}}$	85
elliptic	$\frac{2\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{1-i\sqrt{23}}\,\sqrt{-3x^4+x^2-2}}$	85

input
```
int(1/(-3*x^4+x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(1-I*23^(1/2))^(1/2)*(1-(1/4-1/4*I*23^(1/2))*x^2)^(1/2)*(1-(1/4+1/4*I*23
^(1/2))*x^2)^(1/2)/(-3*x^4+x^2-2)^(1/2)*EllipticF(1/2*(1-I*23^(1/2))^(1/2)
*x,1/6*(-33+3*I*23^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx$$
$$=\frac{1}{24}\sqrt{-2}\sqrt{\sqrt{-23}+1}\left(\sqrt{-23}-1\right)F\left(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{-23}+1}\right)\mid-\frac{1}{12}\sqrt{-23}-\frac{11}{12}\right)$$

input
```
integrate(1/(-3*x^4+x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/24*sqrt(-2)*sqrt(sqrt(-23) + 1)*(sqrt(-23) - 1)*elliptic_f(arcsin(1/2*x*
sqrt(sqrt(-23) + 1)), -1/12*sqrt(-23) - 11/12)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4+x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4+x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 + x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4+x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx=\int \frac{1}{\sqrt{-3\,x^4+x^2-2}}\,dx$$

input
```
int(1/(x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx=-\left(\int \frac{\sqrt{-3x^4+x^2-2}}{3x^4-x^2+2}dx\right)$$

input
```
int(1/(-3*x^4+x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + x**2 - 2)/(3*x**4 - x**2 + 2),x)
```

3.87 $\int \frac{1}{\sqrt{-2-3x^4}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 72

$$\int \frac{1}{\sqrt{-2-3x^4}} \, dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-2-3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2
*arctan(1/2*3^(1/4)*2^(3/4)*x),1/2*2^(1/2))*6^(3/4)/(-3*x^4-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{-2-3x^4}} \, dx = -\frac{\sqrt[4]{-\frac{1}{6}}\sqrt{2+3x^4}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-\frac{3}{2}}x\right),-1\right)}{\sqrt{-2-3x^4}}$$

input
```
Integrate[1/Sqrt[-2 - 3*x^4],x]
```

output
```
-(((-1/6)^(1/4)*Sqrt[2 + 3*x^4]*EllipticF[I*ArcSinh[(-3/2)^(1/4)*x], -1])/
Sqrt[-2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4-2}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-3x^4-2}}$$

input
```
Int[1/Sqrt[-2 - 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTa
n[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-2 - 3*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.26

method	result	size
meijerg	$-\frac{i\sqrt{2}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-\frac{3x^4}{2}\right)}{2}$	19
default	$\frac{\sqrt{2}\,\sqrt{4+2i\sqrt{6}\,x^2}\,\sqrt{4-2i\sqrt{6}\,x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{-i\sqrt{6}}\,x}{2},i\right)}{4\sqrt{-i\sqrt{6}}\,\sqrt{-3x^4-2}}$	66
elliptic	$\frac{\sqrt{2}\,\sqrt{4+2i\sqrt{6}\,x^2}\,\sqrt{4-2i\sqrt{6}\,x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{-i\sqrt{6}}\,x}{2},i\right)}{4\sqrt{-i\sqrt{6}}\,\sqrt{-3x^4-2}}$	66

input
```
int(1/(-3*x^4-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*2^(1/2)*x*hypergeom([1/4,1/2],[5/4],-3/2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt{-2-3x^4}}\,dx = \frac{1}{6}\sqrt{\frac{1}{2}}\sqrt{-2}(-6)^{\frac{3}{4}}\,F(\arcsin\left(\sqrt{\frac{1}{2}}(-6)^{\frac{1}{4}}\,x\right)|-1)$$

input
```
integrate(1/(-3*x^4-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(1/2)*sqrt(-2)*(-6)^(3/4)*elliptic_f(arcsin(sqrt(1/2)*(-6)^(1/4)*x
), -1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{-2-3x^4}}\,dx = -\frac{\sqrt{2}ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{3x^4e^{i\pi}}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-3*x**4-2)**(1/2),x)
```

output
```
-sqrt(2)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(I*pi)/2
)/(8*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-2}}\,dx$$

input
```
integrate(1/(-3*x^4-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-2}}\,dx$$

input
```
integrate(1/(-3*x^4-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 - 2), x)
```

Mupad [B] (verification not implemented)

Time = 18.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{-2-3x^4}}\,dx = \frac{x\,\sqrt{6\,x^4+4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{3\,x^4}{2}\right)}{2\,\sqrt{-3\,x^4-2}}$$

input
```
int(1/(- 3*x^4 - 2)^(1/2),x)
```

output
```
(x*(6*x^4 + 4)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(3*x^4)/2))/(2*(- 3*x^4 -
 2)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4-2}}{3x^4+2}dx\right)$$

input
```
int(1/(-3*x^4-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 - 2)/(3*x**4 + 2),x)
```

3.88 $\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2-x^2-3x^4}}$$

output

```
1/12*(2+6^(1/2)*x^2)*((3*x^4+x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobi
AM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/12*(72-6*6^(1/2))^(1/2))*6^(3/4)/(-3*
x^4-x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx = -\frac{i\sqrt{1-\frac{6x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{-1+i\sqrt{23}}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{6}{-1-i\sqrt{23}}}x\right),\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{-2-x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[-2 - x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (6*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(-1 + I*Sqrt[23
])]*EllipticF[I*ArcSinh[Sqrt[-6/(-1 - I*Sqrt[23])]*x], (-1 - I*Sqrt[23])/(
-1 + I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[-2 - x^2 -
3*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4-x^2-2}}dx$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-x^2-2}}$$

input
```
Int[1/Sqrt[-2 - x^2 - 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2
*ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-2 - x^2 - 3*x
^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{-1-i\sqrt{23}}\,\sqrt{-3x^4-x^2-2}}$	87
elliptic	$\frac{2\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{-1-i\sqrt{23}}\,\sqrt{-3x^4-x^2-2}}$	87

input
```
int(1/(-3*x^4-x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-1-I*23^(1/2))^(1/2)*(1-(-1/4-1/4*I*23^(1/2))*x^2)^(1/2)*(1-(-1/4+1/4*I
*23^(1/2))*x^2)^(1/2)/(-3*x^4-x^2-2)^(1/2)*EllipticF(1/2*(-1-I*23^(1/2))^(
1/2)*x,1/6*(-33-3*I*23^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx = \frac{1}{24}\sqrt{-2}\left(\sqrt{-23}+1\right)\sqrt{\sqrt{-23}-1}F\left(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{-23}-1}\right)\mid\frac{1}{12}\sqrt{-23}-\frac{11}{12}\right)$$

input
```
integrate(1/(-3*x^4-x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/24*sqrt(-2)*(sqrt(-23) + 1)*sqrt(sqrt(-23) - 1)*elliptic_f(arcsin(1/2*x*
sqrt(sqrt(-23) - 1)), 1/12*sqrt(-23) - 11/12)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4-x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4-x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 - x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 - x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 - x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-x^2-2}}\,dx$$

input
```
int(1/(- x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(- x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4-x^2-2}}{3x^4+x^2+2}dx\right)$$

input
```
int(1/(-3*x^4-x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 - x**2 - 2)/(3*x**4 + x**2 + 2),x)
```

3.89 $\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2-2x^2-3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4+2*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/6*(18-3*6^(1/2))^(1/2))*6^(3/4)/(-3
*x^4-2*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx = -\frac{i\sqrt{1-\frac{3x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{-1+i\sqrt{5}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{3}{-1-i\sqrt{5}}}x\right),\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{-2-2x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[-2 - 2*x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (3*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(-1 + I*Sqrt[5])
]*EllipticF[I*ArcSinh[Sqrt[-3/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 +
 I*Sqrt[5])])/(Sqrt[3]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[-2 - 2*x^2 - 3*x^
4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4-2x^2-2}}dx$$

$$\Big\downarrow \ 1416$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-2x^2-2}}$$

input
```
Int[1/Sqrt[-2 - 2*x^2 - 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-2 - 2*x^2 -
3*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\ \text{EllipticF}\left(\frac{\sqrt{-2-2i\sqrt{5}}\,x}{2}, \frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{-2-2i\sqrt{5}}\,\sqrt{-3x^4-2x^2-2}}$	87
elliptic	$\frac{2\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\ \text{EllipticF}\left(\frac{\sqrt{-2-2i\sqrt{5}}\,x}{2}, \frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{-2-2i\sqrt{5}}\,\sqrt{-3x^4-2x^2-2}}$	87

input
```
int(1/(-3*x^4-2*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-2-2*I*5^(1/2))^(1/2)*(1-(-1/2-1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2*I*5^(1
/2)-1/2)*x^2)^(1/2)/(-3*x^4-2*x^2-2)^(1/2)*EllipticF(1/2*(-2-2*I*5^(1/2))^
(1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx = \frac{1}{6}\sqrt{-2}(\sqrt{-5}+1)\sqrt{\frac{1}{2}\sqrt{-5}-\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-5}-\frac{1}{2}}\right) \mid \frac{1}{3}\sqrt{-5}-\frac{2}{3})$$

input
```
integrate(1/(-3*x^4-2*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(-2)*(sqrt(-5) + 1)*sqrt(1/2*sqrt(-5) - 1/2)*elliptic_f(arcsin(x*s
qrt(1/2*sqrt(-5) - 1/2)), 1/3*sqrt(-5) - 2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4-2x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4-2*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 - 2*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-2\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-2*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 - 2*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-2\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-2*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 - 2*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-2\,x^2-2}}\,dx$$

input
```
int(1/(- 2*x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(- 2*x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4-2x^2-2}}{3x^4+2x^2+2}dx\right)$$

input
```
int(1/(-3*x^4-2*x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 - 2*x**2 - 2)/(3*x**4 + 2*x**2 + 2),x)
```

3.90 $\int \frac{1}{\sqrt{-2-3x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{1}{\sqrt{-2-3x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2-3x^2-3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4+3*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/4*(8-2*6^(1/2))^(1/2))*6^(3/4)/(-3*
x^4-3*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{-2-3x^2-3x^4}}\,dx = -\frac{i\sqrt{1-\frac{6x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{-3+i\sqrt{15}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{6}{-3-i\sqrt{15}}}x\right),\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{-2-3x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[-2 - 3*x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (6*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(-3 + I*Sqrt[15
])]*EllipticF[I*ArcSinh[Sqrt[-6/(-3 - I*Sqrt[15])]*x], (-3 - I*Sqrt[15])/(
-3 + I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[-2 - 3*x^2
- 3*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4-3x^2-2}}\,dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-3x^2-2}}$$

input
```
Int[1/Sqrt[-2 - 3*x^2 - 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-2 - 3*x^2 - 3
*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{-3-i\sqrt{15}}\,\sqrt{-3x^4-3x^2-2}}$	87
elliptic	$\frac{2\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{-3-i\sqrt{15}}\,\sqrt{-3x^4-3x^2-2}}$	87

input
```
int(1/(-3*x^4-3*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-3-I*15^(1/2))^(1/2)*(1-(-3/4-1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-3/4+1/4*I
*15^(1/2))*x^2)^(1/2)/(-3*x^4-3*x^2-2)^(1/2)*EllipticF(1/2*(-3-I*15^(1/2))
^(1/2)*x,1/2*(-1-I*15^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{-2-3x^2-3x^4}}\,dx = \frac{1}{24}\sqrt{-2}(\sqrt{-15}+3)\sqrt{\sqrt{-15}-3}F(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{-15}-3}\right)\mid\frac{1}{4}\sqrt{-15}-\frac{1}{4})$$

input
```
integrate(1/(-3*x^4-3*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/24*sqrt(-2)*(sqrt(-15) + 3)*sqrt(sqrt(-15) - 3)*elliptic_f(arcsin(1/2*x*
sqrt(sqrt(-15) - 3)), 1/4*sqrt(-15) - 1/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2-3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4-3x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4-3*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 - 3*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-3\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-3*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 - 3*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-3\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-3*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 - 3*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-3\,x^2-2}}\,dx$$

input
```
int(1/(- 3*x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(- 3*x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-3x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4-3x^2-2}}{3x^4+3x^2+2}dx\right)$$

input
```
int(1/(-3*x^4-3*x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 - 3*x**2 - 2)/(3*x**4 + 3*x**2 + 2),x)
```

3.91 $\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2-4x^2-3x^4}}$$

output
```
1/12*(2+6^(1/2)*x^2)*((3*x^4+4*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/6*(18-6*6^(1/2))^(1/2))*6^(3/4)/(-3
*x^4-4*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx = -\frac{i\sqrt{1-\frac{3x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{-2+i\sqrt{2}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{3}{-2-i\sqrt{2}}}x\right),\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{-2-4x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[-2 - 4*x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - (3*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(-2 + I*Sqrt[2])
]*EllipticF[I*ArcSinh[Sqrt[-3/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 +
 I*Sqrt[2])])/(Sqrt[3]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[-2 - 4*x^2 - 3*x^
4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4-4x^2-2}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3x^4-4x^2-2}}$$

input
```
Int[1/Sqrt[-2 - 4*x^2 - 3*x^4],x]
```

output
```
((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF
[2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-2 - 4*x^2 - 3
*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{-4-2i\sqrt{2}}\,\sqrt{-3x^4-4x^2-2}}$	87
elliptic	$\frac{2\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{-4-2i\sqrt{2}}\,\sqrt{-3x^4-4x^2-2}}$	87

input
```
int(1/(-3*x^4-4*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-4-2*I*2^(1/2))^(1/2)*(1-(-1-1/2*I*2^(1/2))*x^2)^(1/2)*(1-(-1+1/2*I*2^(
1/2))*x^2)^(1/2)/(-3*x^4-4*x^2-2)^(1/2)*EllipticF(1/2*(-4-2*I*2^(1/2))^(1/
2)*x,1/3*(3-6*I*2^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx$$
$$= \frac{1}{3}\left(\sqrt{-2}-1\right)\sqrt{\frac{1}{2}\sqrt{-2}-1}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-2}-1}\right) \mid \frac{2}{3}\sqrt{-2}+\frac{1}{3})$$

input
```
integrate(1/(-3*x^4-4*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(-2) - 1)*sqrt(1/2*sqrt(-2) - 1)*elliptic_f(arcsin(x*sqrt(1/2*sqr
t(-2) - 1)), 2/3*sqrt(-2) + 1/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4-4x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4-4*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 - 4*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-4\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-4*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 - 4*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-4\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-4*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 - 4*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-4\,x^2-2}}\,dx$$

input
```
int(1/(- 4*x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(- 4*x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4-4x^2-2}}{3x^4+4x^2+2}dx\right)$$

input
```
int(1/(-3*x^4-4*x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 - 4*x**2 - 2)/(3*x**4 + 4*x**2 + 2),x)
```

3.92 $\int \frac{1}{\sqrt{-2-5x^2-3x^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{1}{\sqrt{-2-5x^2-3x^4}} \, dx = \frac{\sqrt{2+3x^2} \, \text{EllipticF}\left(\arctan(x), -\frac{1}{2}\right)}{\sqrt{2}\sqrt{-2-3x^2}}$$

output
```
1/2*(3*x^2+2)^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(-3*x
^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

$$\int \frac{1}{\sqrt{-2-5x^2-3x^4}} \, dx = -\frac{i\sqrt{1+x^2}\sqrt{2+3x^2} \, \text{EllipticF}\left(i \text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right), \frac{2}{3}\right)}{\sqrt{3}\sqrt{-2-5x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[-2 - 5*x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3]
)/(Sqrt[3]*Sqrt[-2 - 5*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1408, 27, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4-5x^2-2}}dx$$

$$\Big\downarrow \text{ 1408}$$

$$2\sqrt{3}\int \frac{1}{2\sqrt{3}\sqrt{-3x^2-2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 320}$$

$$-\frac{\sqrt{-3x^2-2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

input
```
Int[1/Sqrt[-2 - 5*x^2 - 3*x^4],x]
```

output
```
-((Sqrt[-2 - 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqr
t[(2 + 3*x^2)/(1 + x^2)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

method	result	size
default	$-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{6\sqrt{-3x^4-5x^2-2}}$	50
elliptic	$-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{6\sqrt{-3x^4-5x^2-2}}$	50

input
```
int(1/(-3*x^4-5*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*6^(1/2)*(6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-5*x^2-2)^(1/2)*Ellipt
icF(1/2*I*x*6^(1/2),1/3*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.32

$$\int \frac{1}{\sqrt{-2-5x^2-3x^4}}\,dx = \frac{1}{2}i\,\sqrt{-2}F(\arcsin{(i\,x)}\mid\frac{3}{2})$$

input
```
integrate(1/(-3*x^4-5*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*I*sqrt(-2)*elliptic_f(arcsin(I*x), 3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2-5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4-5x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4-5*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 - 5*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-5\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-5*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 - 5*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-5\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-5*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 - 5*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-5\,x^2-2}}\,dx$$

input
```
int(1/(- 5*x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(- 5*x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-5x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4-5x^2-2}}{3x^4+5x^2+2}dx\right)$$

input
```
int(1/(-3*x^4-5*x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 - 5*x**2 - 2)/(3*x**4 + 5*x**2 + 2),x)
```

3.93 $\int \frac{1}{\sqrt{-2-6x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{1}{\sqrt{-2-6x^2-3x^4}}\,dx = \frac{\sqrt{3-\sqrt{3}+3x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right),-1-\sqrt{3}\right)}{\sqrt{3-\sqrt{3}}\sqrt{-3+\sqrt{3}-3x^2}}$$

output
```
(3-3^(1/2)+3*x^2)^(1/2)*InverseJacobiAM(arctan(3^(1/2)/(3+3^(1/2))^(1/2)*x
),(-1-3^(1/2))^(1/2))/(3-3^(1/2))^(1/2)/(-3+3^(1/2)-3*x^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{-2-6x^2-3x^4}}\,dx = -\frac{i\sqrt{1-\frac{1}{\sqrt{3}}+x^2}\sqrt{3+\sqrt{3}+3x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right),2+\sqrt{3}\right)}{\sqrt{\left(-3+\sqrt{3}\right)\left(2+6x^2+3x^4\right)}}$$

input
```
Integrate[1/Sqrt[-2 - 6*x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - 1/Sqrt[3] + x^2]*Sqrt[3 + Sqrt[3] + 3*x^2]*EllipticF[I*ArcS
inh[Sqrt[3/(3 + Sqrt[3])]*x], 2 + Sqrt[3]])/Sqrt[(-3 + Sqrt[3])*(2 + 6*x^2
 + 3*x^4)]
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.42, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4-6x^2-2}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{3}\int \frac{1}{2\sqrt{-3x^2+\sqrt{3}-3}\sqrt{3x^2+\sqrt{3}+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\sqrt{3}\int \frac{1}{\sqrt{-3x^2+\sqrt{3}-3}\sqrt{3x^2+\sqrt{3}+3}}dx$$

$$\Big\downarrow \text{320}$$

$$-\frac{\sqrt{-3x^2+\sqrt{3}-3}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right),-1-\sqrt{3}\right)}{\sqrt{3-\sqrt{3}}\sqrt{\frac{3x^2-\sqrt{3}+3}{3x^2+\sqrt{3}+3}}\sqrt{3x^2+\sqrt{3}+3}}$$

input
```
Int[1/Sqrt[-2 - 6*x^2 - 3*x^4],x]
```

output
```
-((Sqrt[-3 + Sqrt[3] - 3*x^2]*EllipticF[ArcTan[Sqrt[3/(3 + Sqrt[3])]*x], -
1 - Sqrt[3]])/(Sqrt[3 - Sqrt[3]]*Sqrt[(3 - Sqrt[3] + 3*x^2)/(3 + Sqrt[3] +
 3*x^2)]*Sqrt[3 + Sqrt[3] + 3*x^2]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6-2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{-6-2\sqrt{3}}\,\sqrt{-3x^{4}-6x^{2}-2}}$	82
elliptic	$\frac{2\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6-2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{-6-2\sqrt{3}}\,\sqrt{-3x^{4}-6x^{2}-2}}$	82

input
```
int(1/(-3*x^4-6*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-6-2*3^(1/2))^(1/2)*(1-(-3/2-1/2*3^(1/2))*x^2)^(1/2)*(1-(-3/2+1/2*3^(1/
2))*x^2)^(1/2)/(-3*x^4-6*x^2-2)^(1/2)*EllipticF(1/2*(-6-2*3^(1/2))^(1/2)*x
,1/2*6^(1/2)-1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{-2-6x^2-3x^4}}\,dx$$
$$= \frac{1}{6}\left(\sqrt{3}\sqrt{-2}+3\sqrt{-2}\right)\sqrt{\frac{1}{2}\sqrt{3}-\frac{3}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{3}-\frac{3}{2}}\right)\mid\sqrt{3}+2)$$

input
```
integrate(1/(-3*x^4-6*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(3)*sqrt(-2) + 3*sqrt(-2))*sqrt(1/2*sqrt(3) - 3/2)*elliptic_f(arc
sin(x*sqrt(1/2*sqrt(3) - 3/2)), sqrt(3) + 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2-6x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4-6x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4-6*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 - 6*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-6x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-6\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-6*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 - 6*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-6x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-6\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-6*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 - 6*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-6x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-6\,x^2-2}}\,dx$$

input
```
int(1/(- 6*x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(- 6*x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-6x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4-6x^2-2}}{3x^4+6x^2+2}dx\right)$$

input
```
int(1/(-3*x^4-6*x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 - 6*x**2 - 2)/(3*x**4 + 6*x**2 + 2),x)
```

3.94 $\int \frac{1}{\sqrt{-2-7x^2-3x^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{\sqrt{-2-7x^2-3x^4}} \, dx = \frac{\sqrt{1+3x^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{2}}\right),-5\right)}{\sqrt{-1-3x^2}}$$

output
```
(3*x^2+1)^(1/2)*InverseJacobiAM(arctan(1/2*x*2^(1/2)),I*5^(1/2))/(-3*x^2-1
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int \frac{1}{\sqrt{-2-7x^2-3x^4}} \, dx = -\frac{i\sqrt{2+x^2}\sqrt{1+3x^2}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{3}x\right),\frac{1}{6}\right)}{\sqrt{6}\sqrt{-2-7x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[-2 - 7*x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[2 + x^2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], 1/6])/
(Sqrt[6]*Sqrt[-2 - 7*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1408, 27, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4-7x^2-2}}dx$$
$$\Big\downarrow \text{1408}$$
$$2\sqrt{3}\int \frac{1}{2\sqrt{3}\sqrt{-3x^2-1}\sqrt{x^2+2}}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{1}{\sqrt{-3x^2-1}\sqrt{x^2+2}}dx$$
$$\Big\downarrow \text{320}$$
$$-\frac{\sqrt{-3x^2-1}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{2}}\right),-5\right)}{\sqrt{x^2+2}\sqrt{\frac{3x^2+1}{x^2+2}}}$$

input
```
Int[1/Sqrt[-2 - 7*x^2 - 3*x^4],x]
```

output
```
-((Sqrt[-1 - 3*x^2]*EllipticF[ArcTan[x/Sqrt[2]], -5])/(Sqrt[2 + x^2]*Sqrt[
(1 + 3*x^2)/(2 + x^2)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

method	result	size
default	$-\frac{i\sqrt{3}\,\sqrt{3x^{2}+1}\,\sqrt{2x^{2}+4}\,\operatorname{EllipticF}\left(ix\sqrt{3},\frac{\sqrt{6}}{6}\right)}{6\sqrt{-3x^{4}-7x^{2}-2}}$	52
elliptic	$-\frac{i\sqrt{3}\,\sqrt{3x^{2}+1}\,\sqrt{2x^{2}+4}\,\operatorname{EllipticF}\left(ix\sqrt{3},\frac{\sqrt{6}}{6}\right)}{6\sqrt{-3x^{4}-7x^{2}-2}}$	52

input
```
int(1/(-3*x^4-7*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*3^(1/2)*(3*x^2+1)^(1/2)*(2*x^2+4)^(1/2)/(-3*x^4-7*x^2-2)^(1/2)*Elli
pticF(I*3^(1/2)*x,1/6*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{-2-7x^2-3x^4}}\,dx = \sqrt{-\frac{1}{2}}\sqrt{-2}F(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)\,|\,6)$$

input
```
integrate(1/(-3*x^4-7*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(-1/2)*sqrt(-2)*elliptic_f(arcsin(sqrt(-1/2)*x), 6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2-7x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4-7x^2-2}}\,dx$$

input
```
integrate(1/(-3*x**4-7*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 - 7*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-7x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-7\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-7*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 - 7*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-7x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-7\,x^2-2}}\,dx$$

input
```
integrate(1/(-3*x^4-7*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 - 7*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-7x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-7\,x^2-2}}\,dx$$

input
```
int(1/(- 7*x^2 - 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(- 7*x^2 - 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-7x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4-7x^2-2}}{3x^4+7x^2+2}dx\right)$$

input
```
int(1/(-3*x^4-7*x^2-2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 - 7*x**2 - 2)/(3*x**4 + 7*x**2 + 2),x)
```

3.95 $\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx = -\frac{\text{EllipticF}\left(\arccos\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right),\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{\sqrt[4]{21}}$$

output
```
-1/21*InverseJacobiAM(arccos(2^(1/2)/(5+21^(1/2))^(1/2)*x),1/42*(882+210*2
1^(1/2))^(1/2))*21^(3/4)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 10.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx = \frac{\sqrt{5-\sqrt{21}-2x^2}\sqrt{2+\left(-5+\sqrt{21}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right),\frac{23}{2}-\frac{5\sqrt{21}}{2}\right)}{2\sqrt{-1+5x^2-x^4}}$$

input
```
Integrate[1/Sqrt[-1 + 5*x^2 - x^4],x]
```

output
```
(Sqrt[5 - Sqrt[21] - 2*x^2]*Sqrt[2 + (-5 + Sqrt[21])*x^2]*EllipticF[ArcSin
[Sqrt[(5 + Sqrt[21])/2]*x], 23/2 - (5*Sqrt[21])/2])/(2*Sqrt[-1 + 5*x^2 - x
^4])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1408, 322}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^4+5x^2-1}}dx$$

$$\Big\downarrow \text{ 1408}$$

$$2\int \frac{1}{\sqrt{-2x^2+\sqrt{21}+5}\sqrt{2x^2+\sqrt{21}-5}}dx$$

$$\Big\downarrow \text{ 322}$$

$$-\frac{\text{EllipticF}\left(\arccos\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right),\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{\sqrt[4]{21}}$$

input
```
Int[1/Sqrt[-1 + 5*x^2 - x^4],x]
```

output
```
-(EllipticF[ArcCos[Sqrt[2/(5 + Sqrt[21])]*x], (21 + 5*Sqrt[21])/42]/21^(1/
4))
```

Defintions of rubi rules used

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(30) = 60$.

Time = 0.43 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

method	result	size
default	$\frac{\sqrt{1-\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)x^2}\,\sqrt{1-\left(\frac{5}{2}+\frac{\sqrt{21}}{2}\right)x^2}\ \mathrm{EllipticF}\left(x\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right),\frac{5}{2}+\frac{\sqrt{21}}{2}\right)}{\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right)\sqrt{-x^4+5x^2-1}}$	82
elliptic	$\frac{\sqrt{1-\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)x^2}\,\sqrt{1-\left(\frac{5}{2}+\frac{\sqrt{21}}{2}\right)x^2}\ \mathrm{EllipticF}\left(x\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right),\frac{5}{2}+\frac{\sqrt{21}}{2}\right)}{\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right)\sqrt{-x^4+5x^2-1}}$	82

input
```
int(1/(-x^4+5*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(1/2*7^(1/2)-1/2*3^(1/2))*(1-(5/2-1/2*21^(1/2))*x^2)^(1/2)*(1-(5/2+1/2*2
1^(1/2))*x^2)^(1/2)/(-x^4+5*x^2-1)^(1/2)*EllipticF(x*(1/2*7^(1/2)-1/2*3^(1
/2)),5/2+1/2*21^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}\sqrt{21}+\frac{5}{2}}\left(i\sqrt{21}-5i\right)F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{21}+\frac{5}{2}}\right)\mid -\frac{5}{2}\sqrt{21}+\frac{23}{2})$$

input
```
integrate(1/(-x^4+5*x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2*sqrt(21) + 5/2)*(I*sqrt(21) - 5*I)*elliptic_f(arcsin(x*sqrt(1
/2*sqrt(21) + 5/2)), -5/2*sqrt(21) + 23/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+5x^2-1}}\,dx$$

input
```
integrate(1/(-x**4+5*x**2-1)**(1/2),x)
```

output
```
Integral(1/sqrt(-x**4 + 5*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+5\,x^2-1}}\,dx$$

input
```
integrate(1/(-x^4+5*x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-x^4 + 5*x^2 - 1), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+5\,x^2-1}}\,dx$$

input
```
integrate(1/(-x^4+5*x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-x^4 + 5*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+5\,x^2-1}}\,dx$$

input
```
int(1/(5*x^2 - x^4 - 1)^(1/2),x)
```

output
```
int(1/(5*x^2 - x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4+5x^2-1}}{x^4-5x^2+1}dx\right)$$

input
```
int(1/(-x^4+5*x^2-1)^(1/2),x)
```

output
```
- int(sqrt( - x**4 + 5*x**2 - 1)/(x**4 - 5*x**2 + 1),x)
```

3.96 $\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = \text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-6\right)$$

output
```
EllipticF(1/2*x*2^(1/2),I*6^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 6.50

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = -\frac{i\sqrt{1-\frac{x^2}{2}}\sqrt{1+3x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{3}x\right),-\frac{1}{6}\right)}{\sqrt{3}\sqrt{2+5x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], -1/6
])/(Sqrt[3]*Sqrt[2 + 5*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{3}\int \frac{1}{2\sqrt{3}\sqrt{2-x^2}\sqrt{3x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{3x^2+1}}dx$$

$$\Big\downarrow \text{321}$$

$$\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-6\right)$$

input
```
Int[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

output
```
EllipticF[ArcSin[x/Sqrt[2]], -6]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(13) = 26$.

Time = 0.62 (sec) , antiderivative size = 51, normalized size of antiderivative = 5.10

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)}{2\sqrt{-3x^4+5x^2+2}}$	51
elliptic	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)}{2\sqrt{-3x^4+5x^2+2}}$	51

input
```
int(1/(-3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/2)*Ellipt
icF(1/2*x*2^(1/2),I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = F\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\mid -6\right)$$

input
```
integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
elliptic_f(arcsin(1/2*sqrt(2)*x), -6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(-3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 - 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 - 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4+5x^2+2}}{3x^4-5x^2-2}dx\right)$$

input
```
int(1/(-3*x^4+5*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + 5*x**2 + 2)/(3*x**4 - 5*x**2 - 2),x)
```

3.97 $\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{-2+\sqrt{10}}}$$

output
```
EllipticF(1/2*(-4+2*10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))/(-2+10
^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx=-\frac{i\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{1+\sqrt{\frac{5}{2}}}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

input
```
Integrate[1/Sqrt[2 + 4*x^2 - 3*x^4],x]
```

output
```
((-I)*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/2]]*x], (-7 + 2*Sqrt[10])/3])/Sq
rt[2 + Sqrt[10]]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4+4x^2+2}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{3}\int \frac{1}{2\sqrt{-3x^2+\sqrt{10}+2}\sqrt{3x^2+\sqrt{10}-2}}dx$$

$$\Big\downarrow \text{27}$$

$$\sqrt{3}\int \frac{1}{\sqrt{-3x^2+\sqrt{10}+2}\sqrt{3x^2+\sqrt{10}-2}}dx$$

$$\Big\downarrow \text{321}$$

$$\sqrt{\frac{1}{6}\left(2+\sqrt{10}\right)}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)$$

input
```
Int[1/Sqrt[2 + 4*x^2 - 3*x^4],x]
```

output
```
Sqrt[(2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(-2 + Sqrt[10])/2]*x], (-7 -
2*Sqrt[10])/3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(34) = 68$.

Time = 0.61 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.91

method	result	size
default	$\frac{2\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^{2}}\,\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-4+2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{-4+2\sqrt{10}}\,\sqrt{-3x^{4}+4x^{2}+2}}$	84
elliptic	$\frac{2\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^{2}}\,\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-4+2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{-4+2\sqrt{10}}\,\sqrt{-3x^{4}+4x^{2}+2}}$	84

input
```
int(1/(-3*x^4+4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-4+2*10^(1/2))^(1/2)*(1-(-1+1/2*10^(1/2))*x^2)^(1/2)*(1-(-1-1/2*10^(1/2
))*x^2)^(1/2)/(-3*x^4+4*x^2+2)^(1/2)*EllipticF(1/2*(-4+2*10^(1/2))^(1/2)*x
,1/3*I*6^(1/2)+1/3*I*15^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx = \frac{1}{6}\left(\sqrt{10}\sqrt{2}+2\sqrt{2}\right)\sqrt{\frac{1}{2}\sqrt{10}-1}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{10}-1}\right)\mid -\frac{2}{3}\sqrt{10}-\frac{7}{3})$$

input
```
integrate(1/(-3*x^4+4*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(10)*sqrt(2) + 2*sqrt(2))*sqrt(1/2*sqrt(10) - 1)*elliptic_f(arcsi
n(x*sqrt(1/2*sqrt(10) - 1)), -2/3*sqrt(10) - 7/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4+4x^2+2}}\,dx$$

input
```
integrate(1/(-3*x**4+4*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 + 4*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+4\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4+4*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + 4*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+4\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4+4*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + 4*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+4\,x^2+2}}\,dx$$

input
```
int(1/(4*x^2 - 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(4*x^2 - 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4+4x^2+2}}{3x^4-4x^2-2}dx\right)$$

input
```
int(1/(-3*x^4+4*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + 4*x**2 + 2)/(3*x**4 - 4*x**2 - 2),x)
```

3.98 $\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx = \sqrt{\frac{2}{-3+\sqrt{33}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

output
```
2^(1/2)/(-3+33^(1/2))^(1/2)*EllipticF(6^(1/2)/(3+33^(1/2))^(1/2)*x,1/4*I*6
^(1/2)+1/4*I*22^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx = -i\sqrt{\frac{2}{3+\sqrt{33}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

input
```
Integrate[1/Sqrt[2 + 3*x^2 - 3*x^4],x]
```

output
```
(-I)*Sqrt[2/(3 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*x]
, (-7 + Sqrt[33])/4]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1408, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{3}\int \frac{1}{\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}dx$$

$$\Big\downarrow \text{321}$$

$$\sqrt{\frac{2}{\sqrt{33}-3}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

input
```
Int[1/Sqrt[2 + 3*x^2 - 3*x^4],x]
```

output
```
Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(3 + Sqrt[33])]*x], (-7 -
Sqrt[33])/4]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(37) = 74$.

Time = 0.62 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3+\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{-3+\sqrt{33}}\,\sqrt{-3x^4+3x^2+2}}$	80
elliptic	$\frac{2\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3+\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{-3+\sqrt{33}}\,\sqrt{-3x^4+3x^2+2}}$	80

input
```
int(1/(-3*x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-3+33^(1/2))^(1/2)*(1-(-3/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*33^(1
/2))*x^2)^(1/2)/(-3*x^4+3*x^2+2)^(1/2)*EllipticF(1/2*(-3+33^(1/2))^(1/2)*x
,1/4*I*6^(1/2)+1/4*I*22^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx = \frac{1}{24}\left(\sqrt{33}\sqrt{2}+3\sqrt{2}\right)\sqrt{\sqrt{33}-3}F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{33}-3}\right) \mid -\frac{1}{4}\sqrt{33}-\frac{7}{4})$$

input
```
integrate(1/(-3*x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(sqrt(33)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(33) - 3)*elliptic_f(arcsin(1
/2*x*sqrt(sqrt(33) - 3)), -1/4*sqrt(33) - 7/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4+3x^2+2}}\,dx$$

input
```
integrate(1/(-3*x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 + 3*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+3\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + 3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+3\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + 3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+3\,x^2+2}}\,dx$$

input
```
int(1/(3*x^2 - 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(3*x^2 - 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4+3x^2+2}}{3x^4-3x^2-2}dx\right)$$

input
```
int(1/(-3*x^4+3*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + 3*x**2 + 2)/(3*x**4 - 3*x**2 - 2),x)
```

3.99 $\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{-1+\sqrt{7}}}$$

output
```
EllipticF(3^(1/2)/(1+7^(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))/(-1+7^
(1/2))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx=-\frac{i\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

input
```
Integrate[1/Sqrt[2 + 2*x^2 - 3*x^4],x]
```

output
```
((-I)*EllipticF[I*ArcSinh[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3])/Sq
rt[1 + Sqrt[7]]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4+2x^2+2}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{3}\int \frac{1}{2\sqrt{-3x^2+\sqrt{7}+1}\sqrt{3x^2+\sqrt{7}-1}}dx$$

$$\Big\downarrow \text{27}$$

$$\sqrt{3}\int \frac{1}{\sqrt{-3x^2+\sqrt{7}+1}\sqrt{3x^2+\sqrt{7}-1}}dx$$

$$\Big\downarrow \text{321}$$

$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

input
```
Int[1/Sqrt[2 + 2*x^2 - 3*x^4],x]
```

output
```
EllipticF[ArcSin[Sqrt[3/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt[-1 + Sqr
t[7]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(34) = 68$.

Time = 0.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.91

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^{2}}\;\mathrm{EllipticF}\left(\frac{\sqrt{-2+2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{-2+2\sqrt{7}}\,\sqrt{-3x^{4}+2x^{2}+2}}$	84
elliptic	$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^{2}}\;\mathrm{EllipticF}\left(\frac{\sqrt{-2+2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{-2+2\sqrt{7}}\,\sqrt{-3x^{4}+2x^{2}+2}}$	84

input
```
int(1/(-3*x^4+2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-2+2*7^(1/2))^(1/2)*(1-(-1/2+1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*7^(1/
2))*x^2)^(1/2)/(-3*x^4+2*x^2+2)^(1/2)*EllipticF(1/2*(-2+2*7^(1/2))^(1/2)*x
,1/6*I*6^(1/2)+1/6*I*42^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx = \frac{1}{6}\left(\sqrt{7}\sqrt{2}+\sqrt{2}\right)\sqrt{\frac{1}{2}\sqrt{7}-\frac{1}{2}}F\left(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{7}-\frac{1}{2}}\right)|-\frac{1}{3}\sqrt{7}-\frac{4}{3}\right)$$

input
```
integrate(1/(-3*x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(7)*sqrt(2) + sqrt(2))*sqrt(1/2*sqrt(7) - 1/2)*elliptic_f(arcsin(
x*sqrt(1/2*sqrt(7) - 1/2)), -1/3*sqrt(7) - 4/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4+2x^2+2}}\,dx$$

input
```
integrate(1/(-3*x**4+2*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 + 2*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+2\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + 2*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+2\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4+2*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + 2*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+2\,x^2+2}}\,dx$$

input
```
int(1/(2*x^2 - 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(2*x^2 - 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4+2x^2+2}}{3x^4-2x^2-2}dx\right)$$

input
```
int(1/(-3*x^4+2*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + 2*x**2 + 2)/(3*x**4 - 2*x**2 - 2),x)
```

3.100 $\int \frac{1}{\sqrt{2+x^2-3x^4}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{1}{\sqrt{2+x^2-3x^4}} \, dx = \frac{\text{EllipticF}\left(\arcsin(x), -\frac{3}{2}\right)}{\sqrt{2}}$$

output
```
1/2*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 5.25

$$\int \frac{1}{\sqrt{2+x^2-3x^4}} \, dx = -\frac{i\sqrt{1-x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right), -\frac{2}{3}\right)}{\sqrt{3}\sqrt{2+x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[2 + x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], -2/3
])/(Sqrt[3]*Sqrt[2 + x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4+x^2+2}}dx$$
$$\Big\downarrow \ 1408$$
$$2\sqrt{3}\int \frac{1}{2\sqrt{3}\sqrt{1-x^2}\sqrt{3x^2+2}}dx$$
$$\Big\downarrow \ 27$$
$$\int \frac{1}{\sqrt{1-x^2}\sqrt{3x^2+2}}dx$$
$$\Big\downarrow \ 321$$
$$\frac{\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[2 + x^2 - 3*x^4],x]
```

output
```
EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(13) = 26$.

Time = 0.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.42

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(x, \frac{i\sqrt{6}}{2}\right)}{2\sqrt{-3x^4+x^2+2}}$	41
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(x, \frac{i\sqrt{6}}{2}\right)}{2\sqrt{-3x^4+x^2+2}}$	41

input
```
int(1/(-3*x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(-3*x^4+x^2+2)^(1/2)*EllipticF(x,1/2*I*
6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{2+x^2-3x^4}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin{(x)}\mid -\frac{3}{2})$$

input
```
integrate(1/(-3*x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*elliptic_f(arcsin(x), -3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+x^2-3x^4}}\,dx=\int \frac{1}{\sqrt{-3x^4+x^2+2}}\,dx$$

input
```
integrate(1/(-3*x**4+x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 + x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+x^2-3x^4}}\,dx=\int \frac{1}{\sqrt{-3\,x^4+x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+x^2-3x^4}}\,dx=\int \frac{1}{\sqrt{-3\,x^4+x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+x^2+2}}\,dx$$

input
```
int(1/(x^2 - 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(x^2 - 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4+x^2+2}}{3x^4-x^2-2}dx\right)$$

input
```
int(1/(-3*x^4+x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + x**2 + 2)/(3*x**4 - x**2 - 2),x)
```

3.101 $\int \frac{1}{\sqrt{2-3x^4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{\sqrt{2-3x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{3}{2}}x\right),-1\right)}{\sqrt[4]{6}}$$

output
```
1/6*EllipticF(1/2*3^(1/4)*2^(3/4)*x,I)*6^(3/4)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-3x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{3}{2}}x\right),-1\right)}{\sqrt[4]{6}}$$

input
```
Integrate[1/Sqrt[2 - 3*x^4],x]
```

output
```
EllipticF[ArcSin[(3/2)^(1/4)*x], -1]/6^(1/4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x^4}}\,dx$$

$$\Big\downarrow \; 762$$

$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{3}{2}}x\right),-1\right)}{\sqrt[4]{6}}$$

input
```
Int[1/Sqrt[2 - 3*x^4],x]
```

output
```
EllipticF[ArcSin[(3/2)^(1/4)*x], -1]/6^(1/4)
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

method	result	size
meijerg	$\frac{\sqrt{2}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], \frac{3 x^{4}}{2}\right)}{2}$	18
default	$\frac{\sqrt{2}\, 6^{\frac{3}{4}} \sqrt{4-2 \sqrt{6}\, x^{2}}\, \sqrt{4+2 \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{2}\, 6^{\frac{1}{4}}}{2}, i\right)}{24 \sqrt{-3 x^{4}+2}}$	54
elliptic	$\frac{\sqrt{2}\, 6^{\frac{3}{4}} \sqrt{4-2 \sqrt{6}\, x^{2}}\, \sqrt{4+2 \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{2}\, 6^{\frac{1}{4}}}{2}, i\right)}{24 \sqrt{-3 x^{4}+2}}$	54

input
```
int(1/(-3*x^4+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*x*hypergeom([1/4,1/2],[5/4],3/2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{2-3 x^4}}\, dx = \frac{1}{6} \cdot 6^{\frac{3}{4}} \sqrt{2} \sqrt{\frac{1}{2}} F(\arcsin\left(6^{\frac{1}{4}} \sqrt{\frac{1}{2}} x\right) \mid -1)$$

input
```
integrate(1/(-3*x^4+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*6^(3/4)*sqrt(2)*sqrt(1/2)*elliptic_f(arcsin(6^(1/4)*sqrt(1/2)*x), -1)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{2-3 x^4}}\, dx = \frac{\sqrt{2} x \Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{3 x^{4} e^{2 i \pi}}{2}\right)}{8 \Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-3*x**4+2)**(1/2),x)
```

output
```
sqrt(2)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(2*I*pi)/2)
/(8*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+2}}\,dx$$

input
```
integrate(1/(-3*x^4+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+2}}\,dx$$

input
```
integrate(1/(-3*x^4+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + 2), x)
```

Mupad [B] (verification not implemented)

Time = 17.89 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{2-3x^4}}\,dx = \frac{\sqrt{2}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ \frac{3\,x^4}{2}\right)}{2}$$

input
```
int(1/(2 - 3*x^4)^(1/2),x)
```

output
```
(2^(1/2)*x*hypergeom([1/4, 1/2], 5/4, (3*x^4)/2))/2
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4+2}}{3x^4-2}dx\right)$$

input
```
int(1/(-3*x^4+2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + 2)/(3*x**4 - 2),x)
```

3.102 $\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx = \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx = \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[2 - x^2 - 3*x^4],x]
```

output
```
EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4-x^2+2}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{3}\int \frac{1}{2\sqrt{3}\sqrt{2-3x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{321}$$

$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[2 - x^2 - 3*x^4],x]
```

output
```
EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.58 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.45

method	result	size
default	$\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{x\sqrt{6}}{2}, \frac{i\sqrt{6}}{3}\right)}{6\sqrt{-3x^4-x^2+2}}$	49
elliptic	$\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{x\sqrt{6}}{2}, \frac{i\sqrt{6}}{3}\right)}{6\sqrt{-3x^4-x^2+2}}$	49

input
```
int(1/(-3*x^4-x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*6^(1/2)*(-6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-x^2+2)^(1/2)*EllipticF(
1/2*x*6^(1/2),1/3*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx = \frac{1}{3}\sqrt{2}\sqrt{\frac{3}{2}}F(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3})$$

input
```
integrate(1/(-3*x^4-x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2)*sqrt(3/2)*elliptic_f(arcsin(sqrt(3/2)*x), -2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4-x^2+2}}\,dx$$

input
```
integrate(1/(-3*x**4-x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 - x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4-x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 - x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4-x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 - x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-x^2+2}}\,dx$$

input
```
int(1/(2 - 3*x^4 - x^2)^(1/2),x)
```

output
```
int(1/(2 - 3*x^4 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4-x^2+2}}{3x^4+x^2-2}dx\right)$$

input
```
int(1/(-3*x^4-x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 - x**2 + 2)/(3*x**4 + x**2 - 2),x)
```

3.103 $\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx = \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

output
```
EllipticF(3^(1/2)/(-1+7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))/(1+7^
(1/2))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx = -\frac{i\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right),-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{-1+\sqrt{7}}}$$

input
```
Integrate[1/Sqrt[2 - 2*x^2 - 3*x^4],x]
```

output
```
((-I)*EllipticF[I*ArcSinh[Sqrt[3/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3])/Sqr
t[-1 + Sqrt[7]]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4-2x^2+2}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{3}\int \frac{1}{2\sqrt{-3x^2+\sqrt{7}-1}\sqrt{3x^2+\sqrt{7}+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\sqrt{3}\int \frac{1}{\sqrt{-3x^2+\sqrt{7}-1}\sqrt{3x^2+\sqrt{7}+1}}dx$$

$$\Big\downarrow \text{321}$$

$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

input
```
Int[1/Sqrt[2 - 2*x^2 - 3*x^4],x]
```

output
```
EllipticF[ArcSin[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqrt[1 + Sqr
t[7]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(34) = 68$.

Time = 0.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.00

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\ \text{EllipticF}\left(\frac{\sqrt{2+2\sqrt{7}}\,x}{2},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{2+2\sqrt{7}}\,\sqrt{-3x^4-2x^2+2}}$	84
elliptic	$\frac{2\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\ \text{EllipticF}\left(\frac{\sqrt{2+2\sqrt{7}}\,x}{2},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{2+2\sqrt{7}}\,\sqrt{-3x^4-2x^2+2}}$	84

input
```
int(1/(-3*x^4-2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(2+2*7^(1/2))^(1/2)*(1-(1/2+1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2*7^(1/2)+1/2
)*x^2)^(1/2)/(-3*x^4-2*x^2+2)^(1/2)*EllipticF(1/2*(2+2*7^(1/2))^(1/2)*x,1/
6*I*42^(1/2)-1/6*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx = \frac{1}{6}\left(\sqrt{7}\sqrt{2}-\sqrt{2}\right)\sqrt{\frac{1}{2}\sqrt{7}+\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{7}+\frac{1}{2}}\right) | \frac{1}{3}\sqrt{7} - \frac{4}{3})$$

input
```
integrate(1/(-3*x^4-2*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(7)*sqrt(2) - sqrt(2))*sqrt(1/2*sqrt(7) + 1/2)*elliptic_f(arcsin(
x*sqrt(1/2*sqrt(7) + 1/2)), 1/3*sqrt(7) - 4/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4-2x^2+2}}\,dx$$

input
```
integrate(1/(-3*x**4-2*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 - 2*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-2\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4-2*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 - 2*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-2\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4-2*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 - 2*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-2\,x^2+2}}\,dx$$

input
```
int(1/(2 - 3*x^4 - 2*x^2)^(1/2),x)
```

output
```
int(1/(2 - 3*x^4 - 2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4-2x^2+2}}{3x^4+2x^2-2}dx\right)$$

input
```
int(1/(-3*x^4-2*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 - 2*x**2 + 2)/(3*x**4 + 2*x**2 - 2),x)
```

3.104 $\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx = \sqrt{\frac{2}{3+\sqrt{33}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

output
```
2^(1/2)/(3+33^(1/2))^(1/2)*EllipticF(6^(1/2)/(-3+33^(1/2))^(1/2)*x,1/4*I*2
2^(1/2)-1/4*I*6^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx = -i\sqrt{\frac{2}{-3+\sqrt{33}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)$$

input
```
Integrate[1/Sqrt[2 - 3*x^2 - 3*x^4],x]
```

output
```
(-I)*Sqrt[2/(-3 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[6/(3 + Sqrt[33])]*x]
, -7/4 - Sqrt[33]/4]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1408, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4-3x^2+2}}dx$$

$$\Big\downarrow \text{ 1408}$$

$$2\sqrt{3}\int \frac{1}{\sqrt{-6x^2+\sqrt{33}-3}\sqrt{6x^2+\sqrt{33}+3}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\sqrt{\frac{2}{3+\sqrt{33}}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

input
```
Int[1/Sqrt[2 - 3*x^2 - 3*x^4],x]
```

output
```
Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(-3 + Sqrt[33])]*x], (-7 +
Sqrt[33])/4]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(37) = 74$.

Time = 0.61 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{3+\sqrt{33}}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{3+\sqrt{33}}\,\sqrt{-3x^4-3x^2+2}}$	80
elliptic	$\frac{2\sqrt{1-\left(\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{3+\sqrt{33}}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{3+\sqrt{33}}\,\sqrt{-3x^4-3x^2+2}}$	80

input
```
int(1/(-3*x^4-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(3+33^(1/2))^(1/2)*(1-(3/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(3/4-1/4*33^(1/2)
)*x^2)^(1/2)/(-3*x^4-3*x^2+2)^(1/2)*EllipticF(1/2*(3+33^(1/2))^(1/2)*x,1/4
*I*22^(1/2)-1/4*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx$$
$$= \frac{1}{24}\left(\sqrt{33}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{33}+3}F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{33}+3}\right) | \frac{1}{4}\sqrt{33}-\frac{7}{4})$$

input
```
integrate(1/(-3*x^4-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(sqrt(33)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(33) + 3)*elliptic_f(arcsin(1
/2*x*sqrt(sqrt(33) + 3)), 1/4*sqrt(33) - 7/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4-3x^2+2}}\,dx$$

input
```
integrate(1/(-3*x**4-3*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 - 3*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-3\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 - 3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-3\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 - 3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-3\,x^2+2}}\,dx$$

input
```
int(1/(2 - 3*x^4 - 3*x^2)^(1/2),x)
```

output
```
int(1/(2 - 3*x^4 - 3*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4-3x^2+2}}{3x^4+3x^2-2}dx\right)$$

input
```
int(1/(-3*x^4-3*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 - 3*x**2 + 2)/(3*x**4 + 3*x**2 - 2),x)
```

3.105 $\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

output
```
EllipticF(1/2*(4+2*10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))/(2+10^(
1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx=-\frac{i\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-1+\sqrt{\frac{5}{2}}}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{-2+\sqrt{10}}}$$

input
```
Integrate[1/Sqrt[2 - 4*x^2 - 3*x^4],x]
```

output
```
((-I)*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/2]]*x], (-7 - 2*Sqrt[10])/3])/S
qrt[-2 + Sqrt[10]]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4-4x^2+2}}dx$$
$$\Big\downarrow \text{1408}$$
$$2\sqrt{3}\int \frac{1}{2\sqrt{-3x^2+\sqrt{10}-2}\sqrt{3x^2+\sqrt{10}+2}}dx$$
$$\Big\downarrow \text{27}$$
$$\sqrt{3}\int \frac{1}{\sqrt{-3x^2+\sqrt{10}-2}\sqrt{3x^2+\sqrt{10}+2}}dx$$
$$\Big\downarrow \text{321}$$
$$\sqrt{\frac{1}{6}\left(\sqrt{10}-2\right)}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)$$

input
```
Int[1/Sqrt[2 - 4*x^2 - 3*x^4],x]
```

output
```
Sqrt[(-2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(2 + Sqrt[10])/2]*x], (-7 +
2*Sqrt[10])/3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(34) = 68$.

Time = 0.60 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.91

method	result	size
default	$\frac{2\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{4+2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{4+2\sqrt{10}}\,\sqrt{-3x^4-4x^2+2}}$	84
elliptic	$\frac{2\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{4+2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{4+2\sqrt{10}}\,\sqrt{-3x^4-4x^2+2}}$	84

input
```
int(1/(-3*x^4-4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(4+2*10^(1/2))^(1/2)*(1-(1+1/2*10^(1/2))*x^2)^(1/2)*(1-(1-1/2*10^(1/2))*
x^2)^(1/2)/(-3*x^4-4*x^2+2)^(1/2)*EllipticF(1/2*(4+2*10^(1/2))^(1/2)*x,1/3
*I*15^(1/2)-1/3*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx = \frac{1}{6}\left(\sqrt{10}\sqrt{2}-2\sqrt{2}\right)\sqrt{\frac{1}{2}\sqrt{10}+1}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{10}+1}\right) \mid \frac{2}{3}\sqrt{10}-\frac{7}{3})$$

input
```
integrate(1/(-3*x^4-4*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(10)*sqrt(2) - 2*sqrt(2))*sqrt(1/2*sqrt(10) + 1)*elliptic_f(arcsi
n(x*sqrt(1/2*sqrt(10) + 1)), 2/3*sqrt(10) - 7/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4-4x^2+2}}\,dx$$

input
```
integrate(1/(-3*x**4-4*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 - 4*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-4\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4-4*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 - 4*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-4\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4-4*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 - 4*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-4\,x^2+2}}\,dx$$

input
```
int(1/(2 - 3*x^4 - 4*x^2)^(1/2),x)
```

output
```
int(1/(2 - 3*x^4 - 4*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4-4x^2+2}}{3x^4+4x^2-2}dx\right)$$

input
```
int(1/(-3*x^4-4*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 - 4*x**2 + 2)/(3*x**4 + 4*x**2 - 2),x)
```

3.106 $\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx=\frac{\operatorname{EllipticF}\left(\arcsin\left(\sqrt{3}x\right),-\frac{1}{6}\right)}{\sqrt{6}}$$

output
```
1/6*EllipticF(x*3^(1/2),1/6*I*6^(1/2))*6^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 54 vs. 2(18) = 36.

Time = 10.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx=\frac{\sqrt{1-3x^2}\sqrt{2+x^2}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{3}x\right),-\frac{1}{6}\right)}{\sqrt{6}\sqrt{2-5x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[2 - 5*x^2 - 3*x^4],x]
```

output
```
(Sqrt[1 - 3*x^2]*Sqrt[2 + x^2]*EllipticF[ArcSin[Sqrt[3]*x], -1/6])/(Sqrt[6
]*Sqrt[2 - 5*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4-5x^2+2}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{3}\int \frac{1}{2\sqrt{3}\sqrt{1-3x^2}\sqrt{x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\sqrt{1-3x^2}\sqrt{x^2+2}}dx$$

$$\Big\downarrow \text{321}$$

$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{3}x\right),-\frac{1}{6}\right)}{\sqrt{6}}$$

input
```
Int[1/Sqrt[2 - 5*x^2 - 3*x^4],x]
```

output
```
EllipticF[ArcSin[Sqrt[3]*x], -1/6]/Sqrt[6]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(17) = 34$.

Time = 0.57 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

method	result	size
default	$\frac{\sqrt{3}\,\sqrt{-3x^2+1}\,\sqrt{2x^2+4}\ \mathrm{EllipticF}\left(\sqrt{3}\,x, \frac{i\sqrt{6}}{6}\right)}{6\sqrt{-3x^4-5x^2+2}}$	50
elliptic	$\frac{\sqrt{3}\,\sqrt{-3x^2+1}\,\sqrt{2x^2+4}\ \mathrm{EllipticF}\left(\sqrt{3}\,x, \frac{i\sqrt{6}}{6}\right)}{6\sqrt{-3x^4-5x^2+2}}$	50

input
```
int(1/(-3*x^4-5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*(-3*x^2+1)^(1/2)*(2*x^2+4)^(1/2)/(-3*x^4-5*x^2+2)^(1/2)*Ellipt
icF(3^(1/2)*x,1/6*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx = \frac{1}{6}\sqrt{3}\sqrt{2}F(\arcsin\left(\sqrt{3}x\right)\,|-\frac{1}{6})$$

input
```
integrate(1/(-3*x^4-5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*sqrt(2)*elliptic_f(arcsin(sqrt(3)*x), -1/6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4-5x^2+2}}\,dx$$

input
```
integrate(1/(-3*x**4-5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 - 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-5\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4-5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 - 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-5\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4-5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 - 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4-5\,x^2+2}}\,dx$$

input
```
int(1/(2 - 3*x^4 - 5*x^2)^(1/2),x)
```

output
```
int(1/(2 - 3*x^4 - 5*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx = -\left(\int \frac{\sqrt{-3x^4-5x^2+2}}{3x^4+5x^2-2}dx\right)$$

input
```
int(1/(-3*x^4-5*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 - 5*x**2 + 2)/(3*x**4 + 5*x**2 - 2),x)
```

3.107 $\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx=\sqrt{\frac{2}{-7+\sqrt{73}}}\,\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right),\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

output
```
2^(1/2)/(-7+73^(1/2))^(1/2)*EllipticF(2*x/(7+73^(1/2))^(1/2),7/12*I*6^(1/2
)+1/12*I*438^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx=-i\sqrt{\frac{2}{7+\sqrt{73}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right),\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

input
```
Integrate[1/Sqrt[3 + 7*x^2 - 2*x^4],x]
```

output
```
(-I)*Sqrt[2/(7 + Sqrt[73])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-7 + Sqrt[73]]]
, (-61 + 7*Sqrt[73])/12]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1408, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+7x^2+3}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{2}\int \frac{1}{\sqrt{-4x^2+\sqrt{73}+7}\sqrt{4x^2+\sqrt{73}-7}}dx$$

$$\Big\downarrow \text{321}$$

$$\sqrt{\frac{2}{\sqrt{73}-7}}\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right),\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

input
```
Int[1/Sqrt[3 + 7*x^2 - 2*x^4],x]
```

output
```
Sqrt[2/(-7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[7 + Sqrt[73]]], (-61 -
 7*Sqrt[73])/12]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(35) = 70$.

Time = 0.67 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.87

method	result	size
default	$\frac{6\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-42+6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{\sqrt{-42+6\sqrt{73}}\,\sqrt{-2x^4+7x^2+3}}$	84
elliptic	$\frac{6\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-42+6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{\sqrt{-42+6\sqrt{73}}\,\sqrt{-2x^4+7x^2+3}}$	84

input
```
int(1/(-2*x^4+7*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(-42+6*73^(1/2))^(1/2)*(1-(-7/6+1/6*73^(1/2))*x^2)^(1/2)*(1-(-7/6-1/6*73
^(1/2))*x^2)^(1/2)/(-2*x^4+7*x^2+3)^(1/2)*EllipticF(1/6*(-42+6*73^(1/2))^(
1/2)*x,7/12*I*6^(1/2)+1/12*I*438^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx = \frac{1}{12}\left(\sqrt{73}\sqrt{3}+7\sqrt{3}\right)\sqrt{\frac{1}{6}\sqrt{73}-\frac{7}{6}}F\left(\arcsin\left(x\sqrt{\frac{1}{6}\sqrt{73}-\frac{7}{6}}\right)\mid -\frac{7}{12}\sqrt{73}-\frac{61}{12}\right)$$

input
```
integrate(1/(-2*x^4+7*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(73)*sqrt(3) + 7*sqrt(3))*sqrt(1/6*sqrt(73) - 7/6)*elliptic_f(ar
csin(x*sqrt(1/6*sqrt(73) - 7/6)), -7/12*sqrt(73) - 61/12)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+7x^2+3}}\,dx$$

input
```
integrate(1/(-2*x**4+7*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + 7*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+7\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+7*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 7*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+7\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+7*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 7*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+7\,x^2+3}}\,dx$$

input
```
int(1/(7*x^2 - 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(7*x^2 - 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+7x^2+3}}{2x^4-7x^2-3}dx\right)$$

input
```
int(1/(-2*x^4+7*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 7*x**2 + 3)/(2*x**4 - 7*x**2 - 3),x)
```

3.108 $\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}x\right),-4-\sqrt{15}\right)}{\sqrt{-3+\sqrt{15}}}$$

output
```
EllipticF(1/3*(-9+3*15^(1/2))^(1/2)*x,1/2*I*6^(1/2)+1/2*I*10^(1/2))/(-3+15
^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx=-\frac{i\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{1+\sqrt{\frac{5}{3}}}x\right),-4+\sqrt{15}\right)}{\sqrt{3+\sqrt{15}}}$$

input
```
Integrate[1/Sqrt[3 + 6*x^2 - 2*x^4],x]
```

output
```
((-I)*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/3]]*x], -4 + Sqrt[15]])/Sqrt[3 +
 Sqrt[15]]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+6x^2+3}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{2}\int \frac{1}{2\sqrt{-2x^2+\sqrt{15}+3}\sqrt{2x^2+\sqrt{15}-3}}dx$$

$$\Big\downarrow \text{27}$$

$$\sqrt{2}\int \frac{1}{\sqrt{-2x^2+\sqrt{15}+3}\sqrt{2x^2+\sqrt{15}-3}}dx$$

$$\Big\downarrow \text{321}$$

$$\sqrt{\frac{1}{6}\left(3+\sqrt{15}\right)}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}x\right),-4-\sqrt{15}\right)$$

input
```
Int[1/Sqrt[3 + 6*x^2 - 2*x^4],x]
```

output
```
Sqrt[(3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(-3 + Sqrt[15])/3]*x], -4 - S
qrt[15]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(34) = 68$.

Time = 0.61 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.10

method	result	size
default	$\frac{3\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-9+3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{\sqrt{-9+3\sqrt{15}}\,\sqrt{-2x^4+6x^2+3}}$	84
elliptic	$\frac{3\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-9+3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{\sqrt{-9+3\sqrt{15}}\,\sqrt{-2x^4+6x^2+3}}$	84

input
```
int(1/(-2*x^4+6*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(-9+3*15^(1/2))^(1/2)*(1-(-1+1/3*15^(1/2))*x^2)^(1/2)*(1-(-1-1/3*15^(1/2
))*x^2)^(1/2)/(-2*x^4+6*x^2+3)^(1/2)*EllipticF(1/3*(-9+3*15^(1/2))^(1/2)*x
,1/2*I*6^(1/2)+1/2*I*10^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx = \frac{1}{2}\sqrt{3}\left(\sqrt{\frac{5}{3}}+1\right)\sqrt{\sqrt{\frac{5}{3}}-1}F\left(\arcsin\left(x\sqrt{\sqrt{\frac{5}{3}}-1}\right)\middle|-3\sqrt{\frac{5}{3}}-4\right)$$

input
```
integrate(1/(-2*x^4+6*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*(sqrt(5/3) + 1)*sqrt(sqrt(5/3) - 1)*elliptic_f(arcsin(x*sqrt(s
qrt(5/3) - 1)), -3*sqrt(5/3) - 4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+6x^2+3}}\,dx$$

input
```
integrate(1/(-2*x**4+6*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + 6*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+6\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+6*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 6*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+6\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+6*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 6*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+6\,x^2+3}}\,dx$$

input
```
int(1/(6*x^2 - 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(6*x^2 - 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+6x^2+3}}{2x^4-6x^2-3}dx\right)$$

input
```
int(1/(-2*x^4+6*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 6*x**2 + 3)/(2*x**4 - 6*x**2 - 3),x)
```

3.109 $\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx = \operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-6\right)$$

output
```
EllipticF(1/3*x*3^(1/2),I*6^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 6.50

$$\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx = -\frac{i\sqrt{1-\frac{x^2}{3}}\sqrt{1+2x^2}\,\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{2}x\right),-\frac{1}{6}\right)}{\sqrt{2}\sqrt{3+5x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[3 + 5*x^2 - 2*x^4],x]
```

output
```
((-I)*Sqrt[1 - x^2/3]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], -1/6
])/(Sqrt[2]*Sqrt[3 + 5*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+5x^2+3}}dx$$
$$\Big\downarrow \text{1408}$$
$$2\sqrt{2}\int \frac{1}{2\sqrt{2}\sqrt{3-x^2}\sqrt{2x^2+1}}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{1}{\sqrt{3-x^2}\sqrt{2x^2+1}}dx$$
$$\Big\downarrow \text{321}$$
$$\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-6\right)$$

input
```
Int[1/Sqrt[3 + 5*x^2 - 2*x^4],x]
```

output
```
EllipticF[ArcSin[x/Sqrt[3]], -6]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(13) = 26$.

Time = 0.56 (sec) , antiderivative size = 51, normalized size of antiderivative = 5.10

method	result	size
default	$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{2x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3}, i\sqrt{6}\right)}{3\sqrt{-2x^4+5x^2+3}}$	51
elliptic	$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{2x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3}, i\sqrt{6}\right)}{3\sqrt{-2x^4+5x^2+3}}$	51

input
```
int(1/(-2*x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(2*x^2+1)^(1/2)/(-2*x^4+5*x^2+3)^(1/2)*Ellipt
icF(1/3*3^(1/2)*x,I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx = F\left(\arcsin\left(\frac{1}{3}\sqrt{3}x\right)\middle|-6\right)$$

input
```
integrate(1/(-2*x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
elliptic_f(arcsin(1/3*sqrt(3)*x), -6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+5x^2+3}}\,dx$$

input
```
integrate(1/(-2*x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + 5*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+5\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 5*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+5\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 5*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2\,x^4+5\,x^2+3}}\,dx$$

input
```
int(1/(5*x^2 - 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(5*x^2 - 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx=-\left(\int \frac{\sqrt{-2x^4+5x^2+3}}{2x^4-5x^2-3}dx\right)$$

input
```
int(1/(-2*x^4+5*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 5*x**2 + 3)/(2*x**4 - 5*x**2 - 3),x)
```

3.110 $\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{-2+\sqrt{10}}}$$

output
```
EllipticF(2^(1/2)/(2+10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))/(-2+1
0^(1/2))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx=-\frac{i\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right),-\frac{7}{3}+\frac{2\sqrt{10}}{3}\right)}{\sqrt{2+\sqrt{10}}}$$

input
```
Integrate[1/Sqrt[3 + 4*x^2 - 2*x^4],x]
```

output
```
((-I)*EllipticF[I*ArcSinh[Sqrt[2/(-2 + Sqrt[10])]*x], -7/3 + (2*Sqrt[10])/
3])/Sqrt[2 + Sqrt[10]]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+4x^2+3}}dx$$
$$\Big\downarrow \text{ 1408}$$
$$2\sqrt{2}\int \frac{1}{2\sqrt{-2x^2+\sqrt{10}+2}\sqrt{2x^2+\sqrt{10}-2}}dx$$
$$\Big\downarrow \text{ 27}$$
$$\sqrt{2}\int \frac{1}{\sqrt{-2x^2+\sqrt{10}+2}\sqrt{2x^2+\sqrt{10}-2}}dx$$
$$\Big\downarrow \text{ 321}$$
$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}}$$

input
```
Int[1/Sqrt[3 + 4*x^2 - 2*x^4],x]
```

output
```
EllipticF[ArcSin[Sqrt[2/(2 + Sqrt[10])]*x], (-7 - 2*Sqrt[10])/3]/Sqrt[-2 +
 Sqrt[10]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(34) = 68$.

Time = 0.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.91

method	result	size
default	$\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6+3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{-6+3\sqrt{10}}\,\sqrt{-2x^4+4x^2+3}}$	84
elliptic	$\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6+3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{-6+3\sqrt{10}}\,\sqrt{-2x^4+4x^2+3}}$	84

input
```
int(1/(-2*x^4+4*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(-6+3*10^(1/2))^(1/2)*(1-(-2/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-2/3-1/3*10^
(1/2))*x^2)^(1/2)/(-2*x^4+4*x^2+3)^(1/2)*EllipticF(1/3*(-6+3*10^(1/2))^(1/
2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx = \frac{1}{6}\left(\sqrt{10}\sqrt{3}+2\sqrt{3}\right)\sqrt{\frac{1}{3}\sqrt{10}-\frac{2}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{10}-\frac{2}{3}}\right)\mid -\frac{2}{3}\sqrt{10}-\frac{7}{3})$$

input
```
integrate(1/(-2*x^4+4*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(10)*sqrt(3) + 2*sqrt(3))*sqrt(1/3*sqrt(10) - 2/3)*elliptic_f(arc
sin(x*sqrt(1/3*sqrt(10) - 2/3)), -2/3*sqrt(10) - 7/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+4x^2+3}}\,dx$$

input
```
integrate(1/(-2*x**4+4*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + 4*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+4\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+4*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 4*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+4\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+4*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 4*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+4\,x^2+3}}\,dx$$

input
```
int(1/(4*x^2 - 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(4*x^2 - 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+4x^2+3}}{2x^4-4x^2-3}dx\right)$$

input
```
int(1/(-2*x^4+4*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 4*x**2 + 3)/(2*x**4 - 4*x**2 - 3),x)
```

3.111 $\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx=\sqrt{\frac{2}{-3+\sqrt{33}}}\,\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

output
```
2^(1/2)/(-3+33^(1/2))^(1/2)*EllipticF(2*x/(3+33^(1/2))^(1/2),1/4*I*6^(1/2)
+1/4*I*22^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx=-i\sqrt{\frac{2}{3+\sqrt{33}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

input
```
Integrate[1/Sqrt[3 + 3*x^2 - 2*x^4],x]
```

output
```
(-I)*Sqrt[2/(3 + Sqrt[33])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-3 + Sqrt[33]]]
, (-7 + Sqrt[33])/4]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1408, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+3x^2+3}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{2}\int \frac{1}{\sqrt{-4x^2+\sqrt{33}+3}\sqrt{4x^2+\sqrt{33}-3}}dx$$

$$\Big\downarrow \text{321}$$

$$\sqrt{\frac{2}{\sqrt{33}-3}}\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

input
```
Int[1/Sqrt[3 + 3*x^2 - 2*x^4],x]
```

output
```
Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[3 + Sqrt[33]]], (-7 -
Sqrt[33])/4]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(35) = 70$.

Time = 0.63 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.87

method	result	size
default	$\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\;\mathrm{EllipticF}\left(\frac{\sqrt{-18+6\sqrt{33}}\,x}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{-18+6\sqrt{33}}\,\sqrt{-2x^4+3x^2+3}}$	84
elliptic	$\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\;\mathrm{EllipticF}\left(\frac{\sqrt{-18+6\sqrt{33}}\,x}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{-18+6\sqrt{33}}\,\sqrt{-2x^4+3x^2+3}}$	84

input
```
int(1/(-2*x^4+3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(-18+6*33^(1/2))^(1/2)*(1-(-1/2+1/6*33^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*33
^(1/2))*x^2)^(1/2)/(-2*x^4+3*x^2+3)^(1/2)*EllipticF(1/6*(-18+6*33^(1/2))^(
1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx = \frac{1}{4}\left(\sqrt{\frac{11}{3}}\sqrt{3}+\sqrt{3}\right)\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}-\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}-\frac{1}{2}}\right)|-\frac{3}{4}\sqrt{\frac{11}{3}}-\frac{7}{4})$$

input
```
integrate(1/(-2*x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(11/3)*sqrt(3) + sqrt(3))*sqrt(1/2*sqrt(11/3) - 1/2)*elliptic_f(a
rcsin(x*sqrt(1/2*sqrt(11/3) - 1/2)), -3/4*sqrt(11/3) - 7/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+3x^2+3}}\,dx$$

input
```
integrate(1/(-2*x**4+3*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + 3*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+3\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+3*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 3*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2\,x^4+3\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+3*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 3*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2\,x^4+3\,x^2+3}}\,dx$$

input
```
int(1/(3*x^2 - 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(3*x^2 - 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx=-\left(\int \frac{\sqrt{-2x^4+3x^2+3}}{2x^4-3x^2-3}dx\right)$$

input
```
int(1/(-2*x^4+3*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 3*x**2 + 3)/(2*x**4 - 3*x**2 - 3),x)
```

3.112 $\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx=\frac{\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{-1+\sqrt{7}}}$$

output
```
EllipticF(2^(1/2)/(1+7^(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))/(-1+7^
(1/2))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx=-\frac{i\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

input
```
Integrate[1/Sqrt[3 + 2*x^2 - 2*x^4],x]
```

output
```
((-I)*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3])/Sq
rt[1 + Sqrt[7]]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+2x^2+3}}dx$$
$$\Big\downarrow \text{ 1408}$$
$$2\sqrt{2}\int \frac{1}{2\sqrt{-2x^2+\sqrt{7}+1}\sqrt{2x^2+\sqrt{7}-1}}dx$$
$$\Big\downarrow \text{ 27}$$
$$\sqrt{2}\int \frac{1}{\sqrt{-2x^2+\sqrt{7}+1}\sqrt{2x^2+\sqrt{7}-1}}dx$$
$$\Big\downarrow \text{ 321}$$
$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

input
```
Int[1/Sqrt[3 + 2*x^2 - 2*x^4],x]
```

output
```
EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt[-1 + Sqr
t[7]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(34) = 68$.

Time = 0.61 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.91

method	result	size
default	$\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3+3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{-3+3\sqrt{7}}\,\sqrt{-2x^4+2x^2+3}}$	84
elliptic	$\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3+3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{-3+3\sqrt{7}}\,\sqrt{-2x^4+2x^2+3}}$	84

input
```
int(1/(-2*x^4+2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(-3+3*7^(1/2))^(1/2)*(1-(-1/3+1/3*7^(1/2))*x^2)^(1/2)*(1-(-1/3-1/3*7^(1/
2))*x^2)^(1/2)/(-2*x^4+2*x^2+3)^(1/2)*EllipticF(1/3*(-3+3*7^(1/2))^(1/2)*x
,1/6*I*6^(1/2)+1/6*I*42^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx = \frac{1}{6}\left(\sqrt{7}\sqrt{3}+\sqrt{3}\right)\sqrt{\frac{1}{3}\sqrt{7}-\frac{1}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{7}-\frac{1}{3}}\right) \mid -\frac{1}{3}\sqrt{7}-\frac{4}{3})$$

input
```
integrate(1/(-2*x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(7)*sqrt(3) + sqrt(3))*sqrt(1/3*sqrt(7) - 1/3)*elliptic_f(arcsin(
x*sqrt(1/3*sqrt(7) - 1/3)), -1/3*sqrt(7) - 4/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+2x^2+3}}\,dx$$

input
```
integrate(1/(-2*x**4+2*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + 2*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+2\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+2*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 2*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+2\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+2*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 2*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+2\,x^2+3}}\,dx$$

input
```
int(1/(2*x^2 - 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(2*x^2 - 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+2x^2+3}}{2x^4-2x^2-3}dx\right)$$

input
```
int(1/(-2*x^4+2*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 2*x**2 + 3)/(2*x**4 - 2*x**2 - 3),x)
```

3.113 $\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 20

$$\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{2}}$$

output
```
1/2*EllipticF(1/3*x*6^(1/2),1/2*I*6^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/Sqrt[3 + x^2 - 2*x^4],x]
```

output
```
EllipticF[ArcSin[Sqrt[2/3]*x], -3/2]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+x^2+3}}dx$$
$$\Big\downarrow \ 1408$$
$$2\sqrt{2}\int \frac{1}{2\sqrt{2}\sqrt{3-2x^2}\sqrt{x^2+1}}dx$$
$$\Big\downarrow \ 27$$
$$\int \frac{1}{\sqrt{3-2x^2}\sqrt{x^2+1}}dx$$
$$\Big\downarrow \ 321$$
$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[3 + x^2 - 2*x^4],x]
```

output
```
EllipticF[ArcSin[Sqrt[2/3]*x], -3/2]/Sqrt[2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(18) = 36$.

Time $= 0.57$ (sec) , antiderivative size $= 47$, normalized size of antiderivative $= 2.35$

method	result	size
default	$\frac{\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{x^2+1}\ \text{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{6\sqrt{-2x^4+x^2+3}}$	47
elliptic	$\frac{\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{x^2+1}\ \text{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{6\sqrt{-2x^4+x^2+3}}$	47

input
```
int(1/(-2*x^4+x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*6^(1/2)*(-6*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-2*x^4+x^2+3)^(1/2)*EllipticF(
1/3*x*6^(1/2),1/2*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx = \frac{1}{2}\sqrt{3}\sqrt{\frac{2}{3}}F(\arcsin\left(\sqrt{\frac{2}{3}}x\right)|-\frac{3}{2})$$

input
```
integrate(1/(-2*x^4+x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*sqrt(2/3)*elliptic_f(arcsin(sqrt(2/3)*x), -3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+x^2+3}}\,dx$$

input
```
integrate(1/(-2*x**4+x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4+x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+x^2+3}}\,dx$$

input
```
int(1/(x^2 - 2*x^4 + 3)^(1/2),x)
```

output
```
int(1/(x^2 - 2*x^4 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+x^2+3}}{2x^4-x^2-3}dx\right)$$

input
```
int(1/(-2*x^4+x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + x**2 + 3)/(2*x**4 - x**2 - 3),x)
```

3.114 $\int \frac{1}{\sqrt{3-2x^4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{\sqrt{3-2x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{\sqrt[4]{6}}$$

output
```
1/6*EllipticF(1/3*2^(1/4)*3^(3/4)*x,I)*6^(3/4)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{3-2x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{\sqrt[4]{6}}$$

input
```
Integrate[1/Sqrt[3 - 2*x^4],x]
```

output
```
EllipticF[ArcSin[(2/3)^(1/4)*x], -1]/6^(1/4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-2x^4}}\,dx$$
$$\Big\downarrow \text{762}$$
$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{\sqrt[4]{6}}$$

input
```
Int[1/Sqrt[3 - 2*x^4],x]
```

output
```
EllipticF[ArcSin[(2/3)^(1/4)*x], -1]/6^(1/4)
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

method	result	size
meijerg	$\frac{\sqrt{3}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], \frac{2 x^{4}}{3}\right)}{3}$	18
default	$\frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{9-3 \sqrt{6}\, x^{2}}\, \sqrt{9+3 \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{3}\, 6^{\frac{1}{4}}}{3}, i\right)}{54 \sqrt{-2 x^{4}+3}}$	54
elliptic	$\frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{9-3 \sqrt{6}\, x^{2}}\, \sqrt{9+3 \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{3}\, 6^{\frac{1}{4}}}{3}, i\right)}{54 \sqrt{-2 x^{4}+3}}$	54

input
```
int(1/(-2*x^4+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*x*hypergeom([1/4,1/2],[5/4],2/3*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{3-2x^4}}\, dx = \frac{1}{2}\sqrt{3}\left(\frac{2}{3}\right)^{\frac{3}{4}} F(\arcsin\left(\left(\frac{2}{3}\right)^{\frac{1}{4}} x\right) | -1)$$

input
```
integrate(1/(-2*x^4+3)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*(2/3)^(3/4)*elliptic_f(arcsin((2/3)^(1/4)*x), -1)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{3-2x^4}}\, dx = \frac{\sqrt{3} x \Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\right| \frac{2x^4 e^{2i\pi}}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-2*x**4+3)**(1/2),x)
```

output
```
sqrt(3)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(2*I*pi)/3)
/(12*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+3}}\,dx$$

input
```
integrate(1/(-2*x^4+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+3}}\,dx$$

input
```
integrate(1/(-2*x^4+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 3), x)
```

Mupad [B] (verification not implemented)

Time = 17.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{3-2x^4}}\,dx = \frac{\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ \frac{2\,x^4}{3}\right)}{3}$$

input
```
int(1/(3 - 2*x^4)^(1/2),x)
```

output
```
(3^(1/2)*x*hypergeom([1/4, 1/2], 5/4, (2*x^4)/3))/3
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+3}}{2x^4-3}dx\right)$$

input
```
int(1/(-2*x^4+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 3)/(2*x**4 - 3),x)
```

3.115 $\int \frac{1}{\sqrt{3-x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{1}{\sqrt{3-x^2-2x^4}}\,dx = \frac{\operatorname{EllipticF}\left(\arcsin(x),-\frac{2}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticF(x,1/3*I*6^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 5.42

$$\int \frac{1}{\sqrt{3-x^2-2x^4}}\,dx = -\frac{i\sqrt{1-x^2}\sqrt{3+2x^2}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{3-x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[3 - x^2 - 2*x^4],x]
```

output
```
((-I)*Sqrt[1 - x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], -3/2
])/(Sqrt[2]*Sqrt[3 - x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-x^2+3}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{2}\int \frac{1}{2\sqrt{2}\sqrt{1-x^2}\sqrt{2x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2x^2+3}}dx$$

$$\Big\downarrow \text{321}$$

$$\frac{\text{EllipticF}\left(\arcsin(x),-\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[3 - x^2 - 2*x^4],x]
```

output
```
EllipticF[ArcSin[x], -2/3]/Sqrt[3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(13) = 26$.

Time = 0.57 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+9}\ \mathrm{EllipticF}\left(x, \frac{i\sqrt{6}}{3}\right)}{3\sqrt{-2x^4-x^2+3}}$	43
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+9}\ \mathrm{EllipticF}\left(x, \frac{i\sqrt{6}}{3}\right)}{3\sqrt{-2x^4-x^2+3}}$	43

input
```
int(1/(-2*x^4-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-x^2+1)^(1/2)*(6*x^2+9)^(1/2)/(-2*x^4-x^2+3)^(1/2)*EllipticF(x,1/3*I*
6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{3-x^2-2x^4}}\,dx = \frac{1}{3}\sqrt{3}F(\arcsin{(x)}\mid -\frac{2}{3})$$

input
```
integrate(1/(-2*x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*elliptic_f(arcsin(x), -2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3 - x^2 - 2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4 - x^2 + 3}}\,dx$$

input
```
integrate(1/(-2*x**4-x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3 - x^2 - 2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4 - x^2 + 3}}\,dx$$

input
```
integrate(1/(-2*x^4-x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3 - x^2 - 2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4 - x^2 + 3}}\,dx$$

input
```
integrate(1/(-2*x^4-x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2\,x^4-x^2+3}}\,dx$$

input
```
int(1/(3 - 2*x^4 - x^2)^(1/2),x)
```

output
```
int(1/(3 - 2*x^4 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-x^2-2x^4}}\,dx=-\left(\int \frac{\sqrt{-2x^4-x^2+3}}{2x^4+x^2-3}dx\right)$$

input
```
int(1/(-2*x^4-x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - x**2 + 3)/(2*x**4 + x**2 - 3),x)
```

3.116 $\int \frac{1}{\sqrt{3-2x^2-2x^4}} dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{1}{\sqrt{3-2x^2-2x^4}} dx = \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right), \frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

output
```
EllipticF(2^(1/2)/(-1+7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))/(1+7^
(1/2))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{3-2x^2-2x^4}} dx = -\frac{i\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right), -\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{-1+\sqrt{7}}}$$

input
```
Integrate[1/Sqrt[3 - 2*x^2 - 2*x^4],x]
```

output
```
((-I)*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3])/Sqr
t[-1 + Sqrt[7]]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-2x^2+3}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{2}\int \frac{1}{2\sqrt{-2x^2+\sqrt{7}-1}\sqrt{2x^2+\sqrt{7}+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\sqrt{2}\int \frac{1}{\sqrt{-2x^2+\sqrt{7}-1}\sqrt{2x^2+\sqrt{7}+1}}dx$$

$$\Big\downarrow \text{321}$$

$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

input
```
Int[1/Sqrt[3 - 2*x^2 - 2*x^4],x]
```

output
```
EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqrt[1 + Sqr
t[7]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(34) = 68$.

Time = 0.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.00

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{3+3\sqrt{7}}\,x}{3},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{3+3\sqrt{7}}\,\sqrt{-2x^4-2x^2+3}}$	84
elliptic	$\frac{3\sqrt{1-\left(\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{3+3\sqrt{7}}\,x}{3},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{3+3\sqrt{7}}\,\sqrt{-2x^4-2x^2+3}}$	84

input
```
int(1/(-2*x^4-2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(3+3*7^(1/2))^(1/2)*(1-(1/3+1/3*7^(1/2))*x^2)^(1/2)*(1-(1/3-1/3*7^(1/2))
*x^2)^(1/2)/(-2*x^4-2*x^2+3)^(1/2)*EllipticF(1/3*(3+3*7^(1/2))^(1/2)*x,1/6
*I*42^(1/2)-1/6*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{3-2x^2-2x^4}}\,dx = \frac{1}{6}\left(\sqrt{7}\sqrt{3}-\sqrt{3}\right)\sqrt{\frac{1}{3}\sqrt{7}+\frac{1}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{7}+\frac{1}{3}}\right) | \frac{1}{3}\sqrt{7} - \frac{4}{3})$$

input
```
integrate(1/(-2*x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(7)*sqrt(3) - sqrt(3))*sqrt(1/3*sqrt(7) + 1/3)*elliptic_f(arcsin(
x*sqrt(1/3*sqrt(7) + 1/3)), 1/3*sqrt(7) - 4/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-2x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4-2x^2+3}}\,dx$$

input
```
integrate(1/(-2*x**4-2*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - 2*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-2x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-2\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - 2*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-2x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-2\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - 2*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-2x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-2\,x^2+3}}\,dx$$

input
```
int(1/(3 - 2*x^4 - 2*x^2)^(1/2),x)
```

output
```
int(1/(3 - 2*x^4 - 2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-2x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4-2x^2+3}}{2x^4+2x^2-3}dx\right)$$

input
```
int(1/(-2*x^4-2*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - 2*x**2 + 3)/(2*x**4 + 2*x**2 - 3),x)
```

3.117 $\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx=\sqrt{\frac{2}{3+\sqrt{33}}}\,\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

output
```
2^(1/2)/(3+33^(1/2))^(1/2)*EllipticF(2*x/(-3+33^(1/2))^(1/2),1/4*I*22^(1/2
)-1/4*I*6^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx=-i\sqrt{\frac{2}{-3+\sqrt{33}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right),-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)$$

input
```
Integrate[1/Sqrt[3 - 3*x^2 - 2*x^4],x]
```

output
```
(-I)*Sqrt[2/(-3 + Sqrt[33])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[3 + Sqrt[33]]]
, -7/4 - Sqrt[33]/4]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1408, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-3x^2+3}}dx$$

$$\Big\downarrow \text{ 1408}$$

$$2\sqrt{2}\int \frac{1}{\sqrt{-4x^2+\sqrt{33}-3}\sqrt{4x^2+\sqrt{33}+3}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\sqrt{\frac{2}{3+\sqrt{33}}}\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

input
```
Int[1/Sqrt[3 - 3*x^2 - 2*x^4],x]
```

output
```
Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[-3 + Sqrt[33]]], (-7 +
Sqrt[33])/4]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(35) = 70$.

Time = 0.63 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{6\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{18+6\sqrt{33}}\,x}{6},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{18+6\sqrt{33}}\,\sqrt{-2x^4-3x^2+3}}$	84
elliptic	$\frac{6\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{18+6\sqrt{33}}\,x}{6},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{18+6\sqrt{33}}\,\sqrt{-2x^4-3x^2+3}}$	84

input
```
int(1/(-2*x^4-3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(18+6*33^(1/2))^(1/2)*(1-(1/2+1/6*33^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*33^(1
/2))*x^2)^(1/2)/(-2*x^4-3*x^2+3)^(1/2)*EllipticF(1/6*(18+6*33^(1/2))^(1/2)
*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx = \frac{1}{4}\left(\sqrt{\frac{11}{3}}\sqrt{3}-\sqrt{3}\right)\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}+\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}+\frac{1}{2}}\right)\mid\frac{3}{4}\sqrt{\frac{11}{3}}-\frac{7}{4})$$

input
```
integrate(1/(-2*x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(11/3)*sqrt(3) - sqrt(3))*sqrt(1/2*sqrt(11/3) + 1/2)*elliptic_f(a
rcsin(x*sqrt(1/2*sqrt(11/3) + 1/2)), 3/4*sqrt(11/3) - 7/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4-3x^2+3}}\,dx$$

input
```
integrate(1/(-2*x**4-3*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - 3*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-3\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - 3*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-3\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - 3*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-3\,x^2+3}}\,dx$$

input
```
int(1/(3 - 2*x^4 - 3*x^2)^(1/2),x)
```

output
```
int(1/(3 - 2*x^4 - 3*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4-3x^2+3}}{2x^4+3x^2-3}dx\right)$$

input
```
int(1/(-2*x^4-3*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - 3*x**2 + 3)/(2*x**4 + 3*x**2 - 3),x)
```

3.118 $\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

output
```
EllipticF(2^(1/2)/(-2+10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))/(2+1
0^(1/2))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx=-\frac{i\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right),-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right)}{\sqrt{-2+\sqrt{10}}}$$

input
```
Integrate[1/Sqrt[3 - 4*x^2 - 2*x^4],x]
```

output
```
((-I)*EllipticF[I*ArcSinh[Sqrt[2/(2 + Sqrt[10])]*x], -7/3 - (2*Sqrt[10])/3
])/Sqrt[-2 + Sqrt[10]]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-4x^2+3}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{2}\int \frac{1}{2\sqrt{-2x^2+\sqrt{10}-2}\sqrt{2x^2+\sqrt{10}+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\sqrt{2}\int \frac{1}{\sqrt{-2x^2+\sqrt{10}-2}\sqrt{2x^2+\sqrt{10}+2}}dx$$

$$\Big\downarrow \text{321}$$

$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

input
```
Int[1/Sqrt[3 - 4*x^2 - 2*x^4],x]
```

output
```
EllipticF[ArcSin[Sqrt[2/(-2 + Sqrt[10])]*x], (-7 + 2*Sqrt[10])/3]/Sqrt[2 +
 Sqrt[10]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(34) = 68$.

Time = 0.59 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.91

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^{2}}\,\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{6+3\sqrt{10}}\,x}{3},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{6+3\sqrt{10}}\,\sqrt{-2x^{4}-4x^{2}+3}}$	84
elliptic	$\frac{3\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^{2}}\,\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{6+3\sqrt{10}}\,x}{3},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{6+3\sqrt{10}}\,\sqrt{-2x^{4}-4x^{2}+3}}$	84

input
```
int(1/(-2*x^4-4*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(6+3*10^(1/2))^(1/2)*(1-(2/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(2/3-1/3*10^(1/
2))*x^2)^(1/2)/(-2*x^4-4*x^2+3)^(1/2)*EllipticF(1/3*(6+3*10^(1/2))^(1/2)*x
,1/3*I*15^(1/2)-1/3*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx$$
$$=\frac{1}{6}\left(\sqrt{10}\sqrt{3}-2\sqrt{3}\right)\sqrt{\frac{1}{3}\sqrt{10}+\frac{2}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{10}+\frac{2}{3}}\right)\mid\frac{2}{3}\sqrt{10}-\frac{7}{3})$$

input
```
integrate(1/(-2*x^4-4*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(10)*sqrt(3) - 2*sqrt(3))*sqrt(1/3*sqrt(10) + 2/3)*elliptic_f(arc
sin(x*sqrt(1/3*sqrt(10) + 2/3)), 2/3*sqrt(10) - 7/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2x^4-4x^2+3}}\,dx$$

input
```
integrate(1/(-2*x**4-4*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - 4*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2\,x^4-4\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4-4*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - 4*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-4\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4-4*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - 4*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-4\,x^2+3}}\,dx$$

input
```
int(1/(3 - 2*x^4 - 4*x^2)^(1/2),x)
```

output
```
int(1/(3 - 2*x^4 - 4*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4-4x^2+3}}{2x^4+4x^2-3}dx\right)$$

input
```
int(1/(-2*x^4-4*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - 4*x**2 + 3)/(2*x**4 + 4*x**2 - 3),x)
```

3.119 $\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{6}\right)}{\sqrt{6}}$$

output
```
1/6*EllipticF(x*2^(1/2),1/6*I*6^(1/2))*6^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 54 vs. 2(18) = 36.

Time = 10.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx=\frac{\sqrt{1-2x^2}\sqrt{3+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{6}\right)}{\sqrt{6}\sqrt{3-5x^2-2x^4}}$$

input
```
Integrate[1/Sqrt[3 - 5*x^2 - 2*x^4],x]
```

output
```
(Sqrt[1 - 2*x^2]*Sqrt[3 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/6])/(Sqrt[6
]*Sqrt[3 - 5*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-5x^2+3}}dx$$
$$\Big\downarrow \text{1408}$$
$$2\sqrt{2}\int \frac{1}{2\sqrt{2}\sqrt{1-2x^2}\sqrt{x^2+3}}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{1}{\sqrt{1-2x^2}\sqrt{x^2+3}}dx$$
$$\Big\downarrow \text{321}$$
$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{6}\right)}{\sqrt{6}}$$

input
```
Int[1/Sqrt[3 - 5*x^2 - 2*x^4],x]
```

output
```
EllipticF[ArcSin[Sqrt[2]*x], -1/6]/Sqrt[6]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(17) = 34$.

Time = 0.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{3x^2+9}\ \mathrm{EllipticF}\left(x\sqrt{2}, \frac{i\sqrt{6}}{6}\right)}{6\sqrt{-2x^4-5x^2+3}}$	50
elliptic	$\frac{\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{3x^2+9}\ \mathrm{EllipticF}\left(x\sqrt{2}, \frac{i\sqrt{6}}{6}\right)}{6\sqrt{-2x^4-5x^2+3}}$	50

input
```
int(1/(-2*x^4-5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*(-2*x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-2*x^4-5*x^2+3)^(1/2)*Ellipt
icF(x*2^(1/2),1/6*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx = \frac{1}{6}\sqrt{3}\sqrt{2}F(\arcsin\left(\sqrt{2}x\right)\,|-\frac{1}{6})$$

input
```
integrate(1/(-2*x^4-5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*sqrt(2)*elliptic_f(arcsin(sqrt(2)*x), -1/6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4-5x^2+3}}\,dx$$

input
```
integrate(1/(-2*x**4-5*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - 5*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4-5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - 5*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4-5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - 5*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-5\,x^2+3}}\,dx$$

input
```
int(1/(3 - 2*x^4 - 5*x^2)^(1/2),x)
```

output
```
int(1/(3 - 2*x^4 - 5*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4-5x^2+3}}{2x^4+5x^2-3}dx\right)$$

input
```
int(1/(-2*x^4-5*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - 5*x**2 + 3)/(2*x**4 + 5*x**2 - 3),x)
```

3.120 $\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx=\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right),-4+\sqrt{15}\right)}{\sqrt{3+\sqrt{15}}}$$

output
```
EllipticF(1/3*(9+3*15^(1/2))^(1/2)*x,1/2*I*10^(1/2)-1/2*I*6^(1/2))/(3+15^(
1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx=-\frac{i\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-1+\sqrt{\frac{5}{3}}}x\right),-4-\sqrt{15}\right)}{\sqrt{-3+\sqrt{15}}}$$

input
```
Integrate[1/Sqrt[3 - 6*x^2 - 2*x^4],x]
```

output
```
((-I)*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/3]]*x], -4 - Sqrt[15]])/Sqrt[-3
 + Sqrt[15]]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-6x^2+3}}dx$$
$$\Big\downarrow \text{1408}$$
$$2\sqrt{2}\int \frac{1}{2\sqrt{-2x^2+\sqrt{15}-3}\sqrt{2x^2+\sqrt{15}+3}}dx$$
$$\Big\downarrow \text{27}$$
$$\sqrt{2}\int \frac{1}{\sqrt{-2x^2+\sqrt{15}-3}\sqrt{2x^2+\sqrt{15}+3}}dx$$
$$\Big\downarrow \text{321}$$
$$\sqrt{\frac{1}{6}\left(\sqrt{15}-3\right)}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right),-4+\sqrt{15}\right)$$

input
```
Int[1/Sqrt[3 - 6*x^2 - 2*x^4],x]
```

output
```
Sqrt[(-3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(3 + Sqrt[15])/3]*x], -4 + S
qrt[15]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(34) = 68$.

Time = 0.60 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.21

method	result	size
default	$\frac{3\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{9+3\sqrt{15}}\,x}{3},\frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{\sqrt{9+3\sqrt{15}}\,\sqrt{-2x^4-6x^2+3}}$	84
elliptic	$\frac{3\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{9+3\sqrt{15}}\,x}{3},\frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{\sqrt{9+3\sqrt{15}}\,\sqrt{-2x^4-6x^2+3}}$	84

input
```
int(1/(-2*x^4-6*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(9+3*15^(1/2))^(1/2)*(1-(1+1/3*15^(1/2))*x^2)^(1/2)*(1-(1-1/3*15^(1/2))*
x^2)^(1/2)/(-2*x^4-6*x^2+3)^(1/2)*EllipticF(1/3*(9+3*15^(1/2))^(1/2)*x,1/2
*I*10^(1/2)-1/2*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx = \frac{1}{2}\sqrt{3}\sqrt{\sqrt{\frac{5}{3}}+1}\left(\sqrt{\frac{5}{3}}-1\right)F\left(\arcsin\left(x\sqrt{\sqrt{\frac{5}{3}}+1}\right)|3\sqrt{\frac{5}{3}}-4\right)$$

input
```
integrate(1/(-2*x^4-6*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*sqrt(sqrt(5/3) + 1)*(sqrt(5/3) - 1)*elliptic_f(arcsin(x*sqrt(s
qrt(5/3) + 1)), 3*sqrt(5/3) - 4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4-6x^2+3}}\,dx$$

input
```
integrate(1/(-2*x**4-6*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - 6*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-6\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4-6*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - 6*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-6\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4-6*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - 6*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-6\,x^2+3}}\,dx$$

input
```
int(1/(3 - 2*x^4 - 6*x^2)^(1/2),x)
```

output
```
int(1/(3 - 2*x^4 - 6*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4-6x^2+3}}{2x^4+6x^2-3}dx\right)$$

input
```
int(1/(-2*x^4-6*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - 6*x**2 + 3)/(2*x**4 + 6*x**2 - 3),x)
```

3.121 $\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx=\sqrt{\frac{2}{7+\sqrt{73}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right),\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

output
```
2^(1/2)/(7+73^(1/2))^(1/2)*EllipticF(2*x/(-7+73^(1/2))^(1/2),1/12*I*438^(1
/2)-7/12*I*6^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx=-i\sqrt{\frac{2}{-7+\sqrt{73}}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right),\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

input
```
Integrate[1/Sqrt[3 - 7*x^2 - 2*x^4],x]
```

output
```
(-I)*Sqrt[2/(-7 + Sqrt[73])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[7 + Sqrt[73]]]
, (-61 - 7*Sqrt[73])/12]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1408, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4-7x^2+3}}dx$$
$$\Big\downarrow \text{1408}$$
$$2\sqrt{2}\int \frac{1}{\sqrt{-4x^2+\sqrt{73}-7}\sqrt{4x^2+\sqrt{73}+7}}dx$$
$$\Big\downarrow \text{321}$$
$$\sqrt{\frac{2}{7+\sqrt{73}}}\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right),\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

input
```
Int[1/Sqrt[3 - 7*x^2 - 2*x^4],x]
```

output
```
Sqrt[2/(7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[-7 + Sqrt[73]]], (-61 +
 7*Sqrt[73])/12]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(35) = 70$.

Time = 0.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.87

method	result	size
default	$\frac{6\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{\sqrt{73}}{6}+\frac{7}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{42+6\sqrt{73}}\,x}{6}, \frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{\sqrt{42+6\sqrt{73}}\,\sqrt{-2x^4-7x^2+3}}$	84
elliptic	$\frac{6\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{\sqrt{73}}{6}+\frac{7}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{42+6\sqrt{73}}\,x}{6}, \frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{\sqrt{42+6\sqrt{73}}\,\sqrt{-2x^4-7x^2+3}}$	84

input
```
int(1/(-2*x^4-7*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(42+6*73^(1/2))^(1/2)*(1-(7/6+1/6*73^(1/2))*x^2)^(1/2)*(1-(-1/6*73^(1/2)
+7/6)*x^2)^(1/2)/(-2*x^4-7*x^2+3)^(1/2)*EllipticF(1/6*(42+6*73^(1/2))^(1/2
)*x,1/12*I*438^(1/2)-7/12*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx$$
$$=\frac{1}{12}\left(\sqrt{73}\sqrt{3}-7\sqrt{3}\right)\sqrt{\frac{1}{6}\sqrt{73}+\frac{7}{6}}F(\arcsin\left(x\sqrt{\frac{1}{6}\sqrt{73}+\frac{7}{6}}\right)|\frac{7}{12}\sqrt{73}-\frac{61}{12})$$

input
```
integrate(1/(-2*x^4-7*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(73)*sqrt(3) - 7*sqrt(3))*sqrt(1/6*sqrt(73) + 7/6)*elliptic_f(ar
csin(x*sqrt(1/6*sqrt(73) + 7/6)), 7/12*sqrt(73) - 61/12)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2x^4-7x^2+3}}\,dx$$

input
```
integrate(1/(-2*x**4-7*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 - 7*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx=\int \frac{1}{\sqrt{-2\,x^4-7\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4-7*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 - 7*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-7\,x^2+3}}\,dx$$

input
```
integrate(1/(-2*x^4-7*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 - 7*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4-7\,x^2+3}}\,dx$$

input
```
int(1/(3 - 2*x^4 - 7*x^2)^(1/2),x)
```

output
```
int(1/(3 - 2*x^4 - 7*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4-7x^2+3}}{2x^4+7x^2-3}dx\right)$$

input
```
int(1/(-2*x^4-7*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 - 7*x**2 + 3)/(2*x**4 + 7*x**2 - 3),x)
```

3.122 $\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = -i\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right),-3\right)$$

input
```
Integrate[1/Sqrt[3 - 2*x^2 - x^4],x]
```

output
```
(-I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^4-2x^2+3}}dx$$
$$\Big\downarrow \text{1408}$$
$$2\int \frac{1}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx$$
$$\Big\downarrow \text{321}$$
$$\frac{\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[3 - 2*x^2 - x^4],x]
```

output
```
EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(13) = 26$.

Time = 0.54 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43

input
```
int(1/(-x^4-2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*
3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = \frac{1}{3}\sqrt{3}F(\arcsin{(x)}\mid -\frac{1}{3})$$

input
```
integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*elliptic_f(arcsin(x), -1/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4-2x^2+3}}\,dx$$

input
```
integrate(1/(-x**4-2*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-x**4 - 2*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

input
```
integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

input
```
integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

input
```
int(1/(3 - x^4 - 2*x^2)^(1/2),x)
```

output
```
int(1/(3 - x^4 - 2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4-2x^2+3}}{x^4+2x^2-3}dx\right)$$

input
```
int(1/(-x^4-2*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - x**4 - 2*x**2 + 3)/(x**4 + 2*x**2 - 3),x)
```

3.123 $\int \frac{1}{\sqrt{-2+5x^2+3x^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int \frac{1}{\sqrt{-2+5x^2+3x^4}} \, dx = \frac{\sqrt{1-3x^2}\sqrt{2+x^2}\, \text{EllipticF}\left(\arcsin\left(\sqrt{3}x\right), -\frac{1}{6}\right)}{\sqrt{6}\sqrt{-2+5x^2+3x^4}}$$

output
```
1/6*(-3*x^2+1)^(1/2)*(x^2+2)^(1/2)*EllipticF(x*3^(1/2),1/6*I*6^(1/2))*6^(1
/2)/(3*x^4+5*x^2-2)^(1/2)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-2+5x^2+3x^4}} \, dx = \frac{\sqrt{1-3x^2}\sqrt{2+x^2}\, \text{EllipticF}\left(\arcsin\left(\sqrt{3}x\right), -\frac{1}{6}\right)}{\sqrt{6}\sqrt{-2+5x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[-2 + 5*x^2 + 3*x^4],x]
```

output
```
(Sqrt[1 - 3*x^2]*Sqrt[2 + x^2]*EllipticF[ArcSin[Sqrt[3]*x], -1/6])/(Sqrt[6
]*Sqrt[-2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1410}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+5x^2-2}}\,dx$$
$$\Big\downarrow \; 1410$$
$$\frac{\sqrt{x^2+2}\sqrt{3x^2-1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{7}{2}}x}{\sqrt{3x^2-1}}\right),\frac{6}{7}\right)}{\sqrt{7}\sqrt{3x^4+5x^2-2}}$$

input
```
Int[1/Sqrt[-2 + 5*x^2 + 3*x^4],x]
```

output
```
(Sqrt[2 + x^2]*Sqrt[-1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7/2]*x)/Sqrt[-1 + 3
*x^2]], 6/7])/(Sqrt[7]*Sqrt[-2 + 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-3x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{2}}{2},i\sqrt{6}\right)}{2\sqrt{3x^4+5x^2-2}}$	53
elliptic	$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-3x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{2}}{2},i\sqrt{6}\right)}{2\sqrt{3x^4+5x^2-2}}$	53

input
```
int(1/(3*x^4+5*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)/(3*x^4+5*x^2-2)^(1/2)*Elli
pticF(1/2*I*x*2^(1/2),I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt{-2+5x^2+3x^4}}\,dx = -\frac{1}{6}\sqrt{3}\sqrt{-2}F(\arcsin\left(\sqrt{3}x\right)|-\frac{1}{6})$$

input
```
integrate(1/(3*x^4+5*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*sqrt(-2)*elliptic_f(arcsin(sqrt(3)*x), -1/6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2+5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+5x^2-2}}\,dx$$

input
```
integrate(1/(3*x**4+5*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 5*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2+5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+5\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4+5*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 5*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+5\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4+5*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 5*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2+5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+5\,x^2-2}}\,dx$$

input
```
int(1/(5*x^2 + 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(5*x^2 + 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2+5x^2+3x^4}}\,dx=\int \frac{\sqrt{3x^4+5x^2-2}}{3x^4+5x^2-2}dx$$

input
```
int(1/(3*x^4+5*x^2-2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 5*x**2 - 2)/(3*x**4 + 5*x**2 - 2),x)
```

3.124 $\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx = \frac{\sqrt{2-\left(2-\sqrt{10}\right)x^2}\sqrt{2-\left(2+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2\left(2+\sqrt{10}\right)}\sqrt{-2+4x^2+3x^4}}$$

output
```
(2-(2-10^(1/2))*x^2)^(1/2)*(2-(2+10^(1/2))*x^2)^(1/2)*EllipticF(1/2*(4+2*1
0^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))/(4+2*10^(1/2))^(1/2)/(3*x^4
+4*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx = -\frac{i\sqrt{2-4x^2-3x^4}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-1+\sqrt{\frac{5}{2}}}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{-2+\sqrt{10}}\sqrt{-2+4x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[-2 + 4*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[2 - 4*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/2]]*x],
 (-7 - 2*Sqrt[10])/3])/(Sqrt[-2 + Sqrt[10]]*Sqrt[-2 + 4*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.41, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1411}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+4x^2-2}}dx$$

$$\Big\downarrow \ 1411$$

$$\frac{\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-2}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{\left(2+\sqrt{10}\right)x^2-2}}\right),\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{3x^4+4x^2-2}}$$

input
```
Int[1/Sqrt[-2 + 4*x^2 + 3*x^4],x]
```

output
```
(Sqrt[(2 - (2 - Sqrt[10])*x^2)/(2 - (2 + Sqrt[10])*x^2)]*Sqrt[-2 + (2 + Sq
rt[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 + (2 + Sqrt[10])
*x^2]], (5 + Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2 - (2 + Sqrt[10])*x^2)^(-1)
]*Sqrt[-2 + 4*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{2\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\ \operatorname{EllipticF}\left(\frac{\sqrt{4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{4-2\sqrt{10}}\,\sqrt{3x^4+4x^2-2}}$	84
elliptic	$\frac{2\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\ \operatorname{EllipticF}\left(\frac{\sqrt{4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{4-2\sqrt{10}}\,\sqrt{3x^4+4x^2-2}}$	84

input
```
int(1/(3*x^4+4*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(4-2*10^(1/2))^(1/2)*(1-(1-1/2*10^(1/2))*x^2)^(1/2)*(1-(1+1/2*10^(1/2))*
x^2)^(1/2)/(3*x^4+4*x^2-2)^(1/2)*EllipticF(1/2*(4-2*10^(1/2))^(1/2)*x,1/3*
I*6^(1/2)+1/3*I*15^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx$$
$$= -\frac{1}{6}\left(\sqrt{10}\sqrt{-2}-2\sqrt{-2}\right)\sqrt{\frac{1}{2}\sqrt{10}+1}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{10}+1}\right)\mid\frac{2}{3}\sqrt{10}-\frac{7}{3})$$

input
```
integrate(1/(3*x^4+4*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(10)*sqrt(-2) - 2*sqrt(-2))*sqrt(1/2*sqrt(10) + 1)*elliptic_f(ar
csin(x*sqrt(1/2*sqrt(10) + 1)), 2/3*sqrt(10) - 7/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+4x^2-2}}\,dx$$

input
```
integrate(1/(3*x**4+4*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 4*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+4\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4+4*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 4*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+4\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4+4*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 4*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4+4\,x^2-2}}\,dx$$

input
```
int(1/(4*x^2 + 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(4*x^2 + 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx=\int \frac{\sqrt{3x^4+4x^2-2}}{3x^4+4x^2-2}dx$$

input
```
int(1/(3*x^4+4*x^2-2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 4*x**2 - 2)/(3*x**4 + 4*x**2 - 2),x)
```

3.125 $\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx = \frac{\sqrt{4-\left(3-\sqrt{33}\right)x^2}\sqrt{4-\left(3+\sqrt{33}\right)x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{3+\sqrt{33}}x\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{2\sqrt{3+\sqrt{33}}\sqrt{-2+3x^2+3x^4}}$$

output
```
1/2*(4-(3-33^(1/2))*x^2)^(1/2)*(4-(3+33^(1/2))*x^2)^(1/2)*EllipticF(1/2*(3
+33^(1/2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))/(3+33^(1/2))^(1/2)/(3*x^4
+3*x^2-2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx = -\frac{i\sqrt{4-6x^2-6x^4}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)}{\sqrt{-3+\sqrt{33}}\sqrt{-2+3x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[-2 + 3*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[4 - 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(3 + Sqrt[33])]*x
], -7/4 - Sqrt[33]/4])/(Sqrt[-3 + Sqrt[33]]*Sqrt[-2 + 3*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.49, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1411}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+3x^2-2}}\,dx$$

$$\Big\downarrow \text{1411}$$

$$\frac{\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-4}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{\left(3+\sqrt{33}\right)x^2-4}}\right),\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{3x^4+3x^2-2}}$$

input
```
Int[1/Sqrt[-2 + 3*x^2 + 3*x^4],x]
```

output
```
(Sqrt[(4 - (3 - Sqrt[33])*x^2)/(4 - (3 + Sqrt[33])*x^2)]*Sqrt[-4 + (3 + Sq
rt[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-4 + (3 + Sqrt[33]
)*x^2]], (11 + Sqrt[33])/22])/(2*Sqrt[2]*33^(1/4)*Sqrt[(4 - (3 + Sqrt[33])
*x^2)^(-1)]*Sqrt[-2 + 3*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^{2}}\,\sqrt{1-\left(\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3-\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{3-\sqrt{33}}\,\sqrt{3x^{4}+3x^{2}-2}}$	84
elliptic	$\frac{2\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^{2}}\,\sqrt{1-\left(\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3-\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{3-\sqrt{33}}\,\sqrt{3x^{4}+3x^{2}-2}}$	84

input
```
int(1/(3*x^4+3*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(3-33^(1/2))^(1/2)*(1-(3/4-1/4*33^(1/2))*x^2)^(1/2)*(1-(3/4+1/4*33^(1/2)
)*x^2)^(1/2)/(3*x^4+3*x^2-2)^(1/2)*EllipticF(1/2*(3-33^(1/2))^(1/2)*x,1/4*
I*6^(1/2)+1/4*I*22^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx = -\frac{1}{24}\left(\sqrt{33}\sqrt{-2}-3\sqrt{-2}\right)\sqrt{\sqrt{33}+3}F(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{33}+3}\right)\mid\frac{1}{4}\sqrt{33}-\frac{7}{4})$$

input
```
integrate(1/(3*x^4+3*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(33)*sqrt(-2) - 3*sqrt(-2))*sqrt(sqrt(33) + 3)*elliptic_f(arcsi
n(1/2*x*sqrt(sqrt(33) + 3)), 1/4*sqrt(33) - 7/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3x^4+3x^2-2}}\,dx$$

input
```
integrate(1/(3*x**4+3*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 3*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4+3\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4+3*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 3*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4+3\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4+3*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 3*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4+3\,x^2-2}}\,dx$$

input
```
int(1/(3*x^2 + 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(3*x^2 + 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx=\int \frac{\sqrt{3x^4+3x^2-2}}{3x^4+3x^2-2}dx$$

input
```
int(1/(3*x^4+3*x^2-2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 3*x**2 - 2)/(3*x**4 + 3*x**2 - 2),x)
```

3.126 $\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx = \frac{\sqrt{2-\left(1-\sqrt{7}\right)x^2}\sqrt{2-\left(1+\sqrt{7}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(1+\sqrt{7}\right)}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{2\left(1+\sqrt{7}\right)}\sqrt{-2+2x^2+3x^4}}$$

output
```
(2-(1-7^(1/2))*x^2)^(1/2)*(2-(1+7^(1/2))*x^2)^(1/2)*EllipticF(1/2*(2+2*7^(
1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))/(2+2*7^(1/2))^(1/2)/(3*x^4+2*x
^2-2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx = -\frac{i\sqrt{2-2x^2-3x^4}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right),-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{-1+\sqrt{7}}\sqrt{-2+2x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[-2 + 2*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[2 - 2*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(1 + Sqrt[7])]*x]
, -4/3 - Sqrt[7]/3])/(Sqrt[-1 + Sqrt[7]]*Sqrt[-2 + 2*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.44, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1411}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+2x^2-2}}\,dx$$

$$\Big\downarrow \; 1411$$

$$\frac{\sqrt{\frac{2-\left(1-\sqrt{7}\right)x^2}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-2}}\right),\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{3x^4+2x^2-2}}$$

input
```
Int[1/Sqrt[-2 + 2*x^2 + 3*x^4],x]
```

output
```
(Sqrt[(2 - (1 - Sqrt[7])*x^2)/(2 - (1 + Sqrt[7])*x^2)]*Sqrt[-2 + (1 + Sqrt
[7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 + (1 + Sqrt[7])*x^2
]], (7 + Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 - (1 + Sqrt[7])*x^2)^(-1)]*Sqrt[
-2 + 2*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{2-2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{2-2\sqrt{7}}\,\sqrt{3x^{4}+2x^{2}-2}}$	84
elliptic	$\frac{2\sqrt{1-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{2-2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{2-2\sqrt{7}}\,\sqrt{3x^{4}+2x^{2}-2}}$	84

input
```
int(1/(3*x^4+2*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(2-2*7^(1/2))^(1/2)*(1-(-1/2*7^(1/2)+1/2)*x^2)^(1/2)*(1-(1/2+1/2*7^(1/2)
)*x^2)^(1/2)/(3*x^4+2*x^2-2)^(1/2)*EllipticF(1/2*(2-2*7^(1/2))^(1/2)*x,1/6
*I*6^(1/2)+1/6*I*42^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx = -\frac{1}{6}\left(\sqrt{7}\sqrt{-2}-\sqrt{-2}\right)\sqrt{\frac{1}{2}\sqrt{7}+\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{7}+\frac{1}{2}}\right) \mid \frac{1}{3}\sqrt{7}-\frac{4}{3})$$

input
```
integrate(1/(3*x^4+2*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(7)*sqrt(-2) - sqrt(-2))*sqrt(1/2*sqrt(7) + 1/2)*elliptic_f(arcs
in(x*sqrt(1/2*sqrt(7) + 1/2)), 1/3*sqrt(7) - 4/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+2x^2-2}}\,dx$$

input
```
integrate(1/(3*x**4+2*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 2*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+2\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4+2*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 2*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+2\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4+2*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 2*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4+2\,x^2-2}}\,dx$$

input
```
int(1/(2*x^2 + 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(2*x^2 + 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx=\int \frac{\sqrt{3x^4+2x^2-2}}{3x^4+2x^2-2}dx$$

input
```
int(1/(3*x^4+2*x^2-2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 2*x**2 - 2)/(3*x**4 + 2*x**2 - 2),x)
```

3.127 $\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx=\frac{\sqrt{2-3x^2}\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-2+x^2+3x^4}}$$

output
```
1/3*(-3*x^2+2)^(1/2)*(x^2+1)^(1/2)*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*
3^(1/2)/(3*x^4+x^2-2)^(1/2)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx=\frac{\sqrt{\left(\frac{2}{3}-x^2\right)(1+x^2)}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{-2+x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[-2 + x^2 + 3*x^4],x]
```

output
```
(Sqrt[(2/3 - x^2)*(1 + x^2)]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/Sqrt[-2
 + x^2 + 3*x^4]
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1410}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+x^2-2}}\,dx$$

$$\Big\downarrow \text{1410}$$

$$\frac{\sqrt{x^2+1}\sqrt{3x^2-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{5}x}{\sqrt{3x^2-2}}\right),\frac{3}{5}\right)}{\sqrt{5}\sqrt{3x^4+x^2-2}}$$

input
```
Int[1/Sqrt[-2 + x^2 + 3*x^4],x]
```

output
```
(Sqrt[1 + x^2]*Sqrt[-2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt[-2 + 3*x
^2]], 3/5])/(Sqrt[5]*Sqrt[-2 + x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{i\sqrt{6}}{2}\right)}{2\sqrt{3x^4+x^2-2}}$	43
elliptic	$-\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{i\sqrt{6}}{2}\right)}{2\sqrt{3x^4+x^2-2}}$	43

input
```
int(1/(3*x^4+x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(3*x^4+x^2-2)^(1/2)*EllipticF(I*x,1/
2*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx = -\frac{1}{3}\sqrt{\frac{3}{2}}\sqrt{-2}F(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3})$$

input
```
integrate(1/(3*x^4+x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3/2)*sqrt(-2)*elliptic_f(arcsin(sqrt(3/2)*x), -2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+x^2-2}}\,dx$$

input
```
integrate(1/(3*x**4+x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4+x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4+x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4+x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4+x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4+x^2-2}}\,dx$$

input
```
int(1/(x^2 + 3*x^4 - 2)^(1/2),x)
```

output
```
int(1/(x^2 + 3*x^4 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx=\int \frac{\sqrt{3x^4+x^2-2}}{3x^4+x^2-2}dx$$

input
```
int(1/(3*x^4+x^2-2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + x**2 - 2)/(3*x**4 + x**2 - 2),x)
```

3.128 $\int \frac{1}{\sqrt{-2+3x^4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int \frac{1}{\sqrt{-2+3x^4}}\,dx=\frac{\sqrt{2-3x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{3}{2}}x\right),-1\right)}{\sqrt[4]{6}\sqrt{-2+3x^4}}$$

output
```
1/6*(-3*x^4+2)^(1/2)*EllipticF(1/2*3^(1/4)*2^(3/4)*x,I)*6^(3/4)/(3*x^4-2)^
(1/2)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-2+3x^4}}\,dx=\frac{\sqrt{2-3x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{3}{2}}x\right),-1\right)}{\sqrt[4]{6}\sqrt{-2+3x^4}}$$

input
```
Integrate[1/Sqrt[-2 + 3*x^4],x]
```

output
```
(Sqrt[2 - 3*x^4]*EllipticF[ArcSin[(3/2)^(1/4)*x], -1])/(6^(1/4)*Sqrt[-2 +
3*x^4])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. $2(40) = 80$.

Time = 0.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.88, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{764\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-2}}\,dx$$

$$\Big\downarrow \; 764$$

$$\frac{\sqrt{\sqrt{6}x^2-2}\sqrt{\frac{\sqrt{6}x^2+2}{2-\sqrt{6}x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{\sqrt{6}x^2-2}}\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{\frac{1}{2-\sqrt{6}x^2}}\sqrt{3x^4-2}}$$

input
```
Int[1/Sqrt[-2 + 3*x^4],x]
```

output
```
(Sqrt[-2 + Sqrt[6]*x^2]*Sqrt[(2 + Sqrt[6]*x^2)/(2 - Sqrt[6]*x^2)]*Elliptic
F[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-2 + Sqrt[6]*x^2]], 1/2])/(2*6^(1/4)*Sqr
t[(2 - Sqrt[6]*x^2)^(-1)]*Sqrt[-2 + 3*x^4])
```

Defintions of rubi rules used

rule 764
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[(a - q*x^2)/(a + q*x^2)]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[a + b*x^
4]*Sqrt[a/(a + q*x^2)]))*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2],
 x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.54 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

method	result	size
meijerg	$\frac{\sqrt{2}\,\sqrt{-\operatorname{signum}\left(-1+\frac{3x^4}{2}\right)}\,x\,\operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],\frac{3x^4}{2}\right)}{2\sqrt{\operatorname{signum}\left(-1+\frac{3x^4}{2}\right)}}$	40
default	$\frac{\sqrt{4+2\sqrt{6}\,x^2}\,\sqrt{4-2\sqrt{6}\,x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-2\sqrt{6}}}{2},i\right)}{2\sqrt{-2\sqrt{6}}\,\sqrt{3x^4-2}}$	56
elliptic	$\frac{\sqrt{4+2\sqrt{6}\,x^2}\,\sqrt{4-2\sqrt{6}\,x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-2\sqrt{6}}}{2},i\right)}{2\sqrt{-2\sqrt{6}}\,\sqrt{3x^4-2}}$	56

input
```
int(1/(3*x^4-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)/signum(-1+3/2*x^4)^(1/2)*(-signum(-1+3/2*x^4))^(1/2)*x*hyperge
om([1/4,1/2],[5/4],3/2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{-2+3x^4}}\,dx = -\frac{1}{6}\cdot 6^{\frac{3}{4}}\sqrt{\frac{1}{2}}\sqrt{-2}F(\arcsin\left(6^{\frac{1}{4}}\sqrt{\frac{1}{2}}x\right)\mid -1)$$

input
```
integrate(1/(3*x^4-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*6^(3/4)*sqrt(1/2)*sqrt(-2)*elliptic_f(arcsin(6^(1/4)*sqrt(1/2)*x), -1
)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-2+3x^4}}\,dx = -\frac{\sqrt{2}ix\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{3x^4}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(3*x**4-2)**(1/2),x)
```

output
```
-sqrt(2)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4/2)/(8*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-2}}\,dx$$

input
```
integrate(1/(3*x^4-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-2}}\,dx$$

input
```
integrate(1/(3*x^4-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - 2), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{-2+3x^4}}\,dx = \frac{x\,\sqrt{4-6\,x^4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ \frac{3\,x^4}{2}\right)}{2\,\sqrt{3\,x^4-2}}$$

input
```
int(1/(3*x^4 - 2)^(1/2),x)
```

output
```
(x*(4 - 6*x^4)^(1/2)*hypergeom([1/4, 1/2], 5/4, (3*x^4)/2))/(2*(3*x^4 - 2)
^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2+3x^4}}\,dx = \int \frac{\sqrt{3x^4-2}}{3x^4-2}dx$$

input
```
int(1/(3*x^4-2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - 2)/(3*x**4 - 2),x)
```

3.129 $\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 50

$$\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx = \frac{\sqrt{1-x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-2-x^2+3x^4}}$$

output
```
1/2*(-x^2+1)^(1/2)*(3*x^2+2)^(1/2)*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)/(3*x
^4-x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx = -\frac{i\sqrt{1-x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{-6-3x^2+9x^4}}$$

input
```
Integrate[1/Sqrt[-2 - x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], -2/3
])/Sqrt[-6 - 3*x^2 + 9*x^4]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1410}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-x^2-2}}\,dx$$

$$\Big\downarrow \; 1410$$

$$\frac{\sqrt{x^2-1}\sqrt{3x^2+2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{5}{2}}x}{\sqrt{x^2-1}}\right),\frac{2}{5}\right)}{\sqrt{5}\sqrt{3x^4-x^2-2}}$$

input
```
Int[1/Sqrt[-2 - x^2 + 3*x^4],x]
```

output
```
(Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5/2]*x)/Sqrt[-1 + x
^2]], 2/5])/(Sqrt[5]*Sqrt[-2 - x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{i\sqrt{6}\,\sqrt{6x^{2}+4}\,\sqrt{-x^{2}+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{3x^{4}-x^{2}-2}}$	53
elliptic	$-\frac{i\sqrt{6}\,\sqrt{6x^{2}+4}\,\sqrt{-x^{2}+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{3x^{4}-x^{2}-2}}$	53

input
```
int(1/(3*x^4-x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*6^(1/2)*(6*x^2+4)^(1/2)*(-x^2+1)^(1/2)/(3*x^4-x^2-2)^(1/2)*Elliptic
F(1/2*I*x*6^(1/2),1/3*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin(x)\,|-\frac{3}{2})$$

input
```
integrate(1/(3*x^4-x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-2)*elliptic_f(arcsin(x), -3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4-x^2-2}}\,dx$$

input
```
integrate(1/(3*x**4-x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4-x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4-x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-x^2-2}}\,dx$$

input
```
int(1/(3*x^4 - x^2 - 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - x^2 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx=\int \frac{\sqrt{3x^4-x^2-2}}{3x^4-x^2-2}dx$$

input
```
int(1/(3*x^4-x^2-2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - x**2 - 2)/(3*x**4 - x**2 - 2),x)
```

3.130 $\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx = \frac{\sqrt{2+\left(1-\sqrt{7}\right)x^2}\sqrt{2+\left(1+\sqrt{7}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{7}\right)}x\right),\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{2\left(-1+\sqrt{7}\right)}\sqrt{-2-2x^2+3x^4}}$$

output
```
(2+(1-7^(1/2))*x^2)^(1/2)*(2+(1+7^(1/2))*x^2)^(1/2)*EllipticF(1/2*(-2+2*7^
(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))/(-2+2*7^(1/2))^(1/2)/(3*x^4-2
*x^2-2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx = -\frac{i\sqrt{2+2x^2-3x^4}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}\sqrt{-2-2x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[-2 - 2*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[2 + 2*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(-1 + Sqrt[7])]*x
], (-4 + Sqrt[7])/3])/(Sqrt[1 + Sqrt[7]]*Sqrt[-2 - 2*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.51, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1411}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-2x^2-2}}dx$$

$$\Big\downarrow \text{1411}$$

$$\frac{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-2}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+2}{\left(1-\sqrt{7}\right)x^2+2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-2}}\right),\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+2}}\sqrt{3x^4-2x^2-2}}$$

input
```
Int[1/Sqrt[-2 - 2*x^2 + 3*x^4],x]
```

output
```
(Sqrt[-2 - (1 - Sqrt[7])*x^2]*Sqrt[(2 + (1 + Sqrt[7])*x^2)/(2 + (1 - Sqrt[
7])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[7])*x^2
]], (7 - Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 + (1 - Sqrt[7])*x^2)^(-1)]*Sqrt[
-2 - 2*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-2-2\sqrt{7}}\,x}{2},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{-2-2\sqrt{7}}\,\sqrt{3x^{4}-2x^{2}-2}}$	84
elliptic	$\frac{2\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-2-2\sqrt{7}}\,x}{2},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{-2-2\sqrt{7}}\,\sqrt{3x^{4}-2x^{2}-2}}$	84

input
```
int(1/(3*x^4-2*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-2-2*7^(1/2))^(1/2)*(1-(-1/2-1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2+1/2*7^(1/
2))*x^2)^(1/2)/(3*x^4-2*x^2-2)^(1/2)*EllipticF(1/2*(-2-2*7^(1/2))^(1/2)*x,
1/6*I*42^(1/2)-1/6*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx = -\frac{1}{6}\left(\sqrt{7}\sqrt{-2}+\sqrt{-2}\right)\sqrt{\frac{1}{2}\sqrt{7}-\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{7}-\frac{1}{2}}\right)|-\frac{1}{3}\sqrt{7}-\frac{4}{3})$$

input
```
integrate(1/(3*x^4-2*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(7)*sqrt(-2) + sqrt(-2))*sqrt(1/2*sqrt(7) - 1/2)*elliptic_f(arcs
in(x*sqrt(1/2*sqrt(7) - 1/2)), -1/3*sqrt(7) - 4/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4-2x^2-2}}\,dx$$

input
```
integrate(1/(3*x**4-2*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - 2*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-2\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4-2*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - 2*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-2\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4-2*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - 2*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-2\,x^2-2}}\,dx$$

input
```
int(1/(3*x^4 - 2*x^2 - 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - 2*x^2 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4-2x^2-2}}{3x^4-2x^2-2}dx$$

input
```
int(1/(3*x^4-2*x^2-2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - 2*x**2 - 2)/(3*x**4 - 2*x**2 - 2),x)
```

3.131 $\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx = \frac{\sqrt{4+\left(3-\sqrt{33}\right)x^2}\sqrt{4+\left(3+\sqrt{33}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-3+\sqrt{33}}x\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)}{2\sqrt{-3+\sqrt{33}}\sqrt{-2-3x^2+3x^4}}$$

output
```
1/2*(4+(3-33^(1/2))*x^2)^(1/2)*(4+(3+33^(1/2))*x^2)^(1/2)*EllipticF(1/2*(-
3+33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))/(-3+33^(1/2))^(1/2)/(3*x
^4-3*x^2-2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx = -\frac{i\sqrt{4+6x^2-6x^4}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{\sqrt{3+\sqrt{33}}\sqrt{-2-3x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[-2 - 3*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[4 + 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*
x], (-7 + Sqrt[33])/4])/(Sqrt[3 + Sqrt[33]]*Sqrt[-2 - 3*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.56, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1411}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-3x^2-2}}\,dx$$

$$\Big\downarrow \text{1411}$$

$$\frac{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-4}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+4}{\left(3-\sqrt{33}\right)x^2+4}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-4}}\right),\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+4}}\sqrt{3x^4-3x^2-2}}$$

input
```
Int[1/Sqrt[-2 - 3*x^2 + 3*x^4],x]
```

output
```
(Sqrt[-4 - (3 - Sqrt[33])*x^2]*Sqrt[(4 + (3 + Sqrt[33])*x^2)/(4 + (3 - Sqr
t[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-4 - (3 - Sqrt[33]
)*x^2]], (11 - Sqrt[33])/22])/(2*Sqrt[2]*33^(1/4)*Sqrt[(4 + (3 - Sqrt[33])
*x^2)^(-1)]*Sqrt[-2 - 3*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^{2}}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3-\sqrt{33}}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{-3-\sqrt{33}}\,\sqrt{3x^{4}-3x^{2}-2}}$	84
elliptic	$\frac{2\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^{2}}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3-\sqrt{33}}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{-3-\sqrt{33}}\,\sqrt{3x^{4}-3x^{2}-2}}$	84

input
```
int(1/(3*x^4-3*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-3-33^(1/2))^(1/2)*(1-(-3/4-1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4+1/4*33^(1
/2))*x^2)^(1/2)/(3*x^4-3*x^2-2)^(1/2)*EllipticF(1/2*(-3-33^(1/2))^(1/2)*x,
1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx$$
$$= -\frac{1}{24}\left(\sqrt{33}\sqrt{-2}+3\sqrt{-2}\right)\sqrt{\sqrt{33}-3}F(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{33}-3}\right)\mid -\frac{1}{4}\sqrt{33}-\frac{7}{4})$$

input
```
integrate(1/(3*x^4-3*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(33)*sqrt(-2) + 3*sqrt(-2))*sqrt(sqrt(33) - 3)*elliptic_f(arcsi
n(1/2*x*sqrt(sqrt(33) - 3)), -1/4*sqrt(33) - 7/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4-3x^2-2}}\,dx$$

input
```
integrate(1/(3*x**4-3*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - 3*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-3\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4-3*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - 3*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-3\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4-3*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - 3*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-3\,x^2-2}}\,dx$$

input
```
int(1/(3*x^4 - 3*x^2 - 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - 3*x^2 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4-3x^2-2}}{3x^4-3x^2-2}dx$$

input
```
int(1/(3*x^4-3*x^2-2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - 3*x**2 - 2)/(3*x**4 - 3*x**2 - 2),x)
```

3.132 $\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx = \frac{\sqrt{2+\left(2-\sqrt{10}\right)x^2}\sqrt{2+\left(2+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{2\left(-2+\sqrt{10}\right)}\sqrt{-2-4x^2+3x^4}}$$

output
```
(2+(2-10^(1/2))*x^2)^(1/2)*(2+(2+10^(1/2))*x^2)^(1/2)*EllipticF(1/2*(-4+2*
10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))/(-4+2*10^(1/2))^(1/2)/(3*x
^4-4*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx = -\frac{i\sqrt{2+4x^2-3x^4}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1+\sqrt{\frac{5}{2}}}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}\sqrt{-2-4x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[-2 - 4*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[2 + 4*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/2]]*x],
(-7 + 2*Sqrt[10])/3])/(Sqrt[2 + Sqrt[10]]*Sqrt[-2 - 4*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.51, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1411}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-4x^2-2}}dx$$

$$\Big\downarrow \ 1411$$

$$\frac{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-2}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+2}{\left(2-\sqrt{10}\right)x^2+2}}\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-2}}\right),\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+2}}\sqrt{3x^4-4x^2-2}}$$

input
```
Int[1/Sqrt[-2 - 4*x^2 + 3*x^4],x]
```

output
```
(Sqrt[-2 - (2 - Sqrt[10])*x^2]*Sqrt[(2 + (2 + Sqrt[10])*x^2)/(2 + (2 - Sqr
t[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 - (2 - Sqrt[10])
*x^2]], (5 - Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2 + (2 - Sqrt[10])*x^2)^(-1)
]*Sqrt[-2 - 4*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^{2}}\,\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{-4-2\sqrt{10}}\,\sqrt{3x^{4}-4x^{2}-2}}$	84
elliptic	$\frac{2\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^{2}}\,\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{-4-2\sqrt{10}}\,\sqrt{3x^{4}-4x^{2}-2}}$	84

input
```
int(1/(3*x^4-4*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-4-2*10^(1/2))^(1/2)*(1-(-1-1/2*10^(1/2))*x^2)^(1/2)*(1-(-1+1/2*10^(1/2
))*x^2)^(1/2)/(3*x^4-4*x^2-2)^(1/2)*EllipticF(1/2*(-4-2*10^(1/2))^(1/2)*x,
1/3*I*15^(1/2)-1/3*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx$$
$$= -\frac{1}{6}\left(\sqrt{10}\sqrt{-2}+2\sqrt{-2}\right)\sqrt{\frac{1}{2}\sqrt{10}-1}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{10}-1}\right) | -\frac{2}{3}\sqrt{10}-\frac{7}{3})$$

input
```
integrate(1/(3*x^4-4*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(10)*sqrt(-2) + 2*sqrt(-2))*sqrt(1/2*sqrt(10) - 1)*elliptic_f(ar
csin(x*sqrt(1/2*sqrt(10) - 1)), -2/3*sqrt(10) - 7/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3x^4-4x^2-2}}\,dx$$

input
```
integrate(1/(3*x**4-4*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - 4*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4-4\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4-4*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - 4*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4-4\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4-4*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - 4*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-4\,x^2-2}}\,dx$$

input
```
int(1/(3*x^4 - 4*x^2 - 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - 4*x^2 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4-4x^2-2}}{3x^4-4x^2-2}dx$$

input
```
int(1/(3*x^4-4*x^2-2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - 4*x**2 - 2)/(3*x**4 - 4*x**2 - 2),x)
```

3.133 $\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx = \frac{\sqrt{2-x^2}\sqrt{1+3x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-6\right)}{\sqrt{-2-5x^2+3x^4}}$$

output
```
(-x^2+2)^(1/2)*(3*x^2+1)^(1/2)*EllipticF(1/2*x*2^(1/2),I*6^(1/2))/(3*x^4-5
*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx = -\frac{i\sqrt{1-\frac{x^2}{2}}\sqrt{1+3x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{3}x\right),-\frac{1}{6}\right)}{\sqrt{3}\sqrt{-2-5x^2+3x^4}}$$

input
```
Integrate[1/Sqrt[-2 - 5*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], -1/6
])/(Sqrt[3]*Sqrt[-2 - 5*x^2 + 3*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1410\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4-5x^2-2}}\,dx$$

$$\Big\downarrow \text{ 1410}$$

$$\frac{\sqrt{x^2-2}\sqrt{3x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{7}x}{\sqrt{x^2-2}}\right),\frac{1}{7}\right)}{\sqrt{7}\sqrt{3x^4-5x^2-2}}$$

input
```
Int[1/Sqrt[-2 - 5*x^2 + 3*x^4],x]
```

output
```
(Sqrt[-2 + x^2]*Sqrt[1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt[-2 + x^2
]], 1/7])/(Sqrt[7]*Sqrt[-2 - 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{i\sqrt{3}\,\sqrt{3x^2+1}\,\sqrt{-2x^2+4}\ \mathrm{EllipticF}\left(ix\sqrt{3}, \frac{i\sqrt{6}}{6}\right)}{6\sqrt{3x^4-5x^2-2}}$	53
elliptic	$-\frac{i\sqrt{3}\,\sqrt{3x^2+1}\,\sqrt{-2x^2+4}\ \mathrm{EllipticF}\left(ix\sqrt{3}, \frac{i\sqrt{6}}{6}\right)}{6\sqrt{3x^4-5x^2-2}}$	53

input
```
int(1/(3*x^4-5*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*3^(1/2)*(3*x^2+1)^(1/2)*(-2*x^2+4)^(1/2)/(3*x^4-5*x^2-2)^(1/2)*Elli
pticF(I*3^(1/2)*x,1/6*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx = -\frac{1}{2}\sqrt{2}\sqrt{-2}F\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right) \mid -6\right)$$

input
```
integrate(1/(3*x^4-5*x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*sqrt(-2)*elliptic_f(arcsin(1/2*sqrt(2)*x), -6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4-5x^2-2}}\,dx$$

input
```
integrate(1/(3*x**4-5*x**2-2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 - 5*x**2 - 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-5\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4-5*x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 - 5*x^2 - 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-5\,x^2-2}}\,dx$$

input
```
integrate(1/(3*x^4-5*x^2-2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 - 5*x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4-5\,x^2-2}}\,dx$$

input
```
int(1/(3*x^4 - 5*x^2 - 2)^(1/2),x)
```

output
```
int(1/(3*x^4 - 5*x^2 - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4-5x^2-2}}{3x^4-5x^2-2}dx$$

input
```
int(1/(3*x^4-5*x^2-2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 - 5*x**2 - 2)/(3*x**4 - 5*x**2 - 2),x)
```

3.134 $\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx = \frac{\sqrt{6-\left(7-\sqrt{73}\right)x^2}\sqrt{6-\left(7+\sqrt{73}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{6}\left(7+\sqrt{73}\right)}x\right),\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)}{\sqrt{6\left(7+\sqrt{73}\right)}\sqrt{-3+7x^2+2x^4}}$$

output
```
(6-(-73^(1/2)+7)*x^2)^(1/2)*(6-(7+73^(1/2))*x^2)^(1/2)*EllipticF(1/6*(42+6
*73^(1/2))^(1/2)*x,1/12*I*438^(1/2)-7/12*I*6^(1/2))/(42+6*73^(1/2))^(1/2)/
(2*x^4+7*x^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx = -\frac{i\sqrt{6-14x^2-4x^4}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right),\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)}{\sqrt{-7+\sqrt{73}}\sqrt{-3+7x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + 7*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[6 - 14*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[7 + Sqrt[73]
]], (-61 - 7*Sqrt[73])/12])/(Sqrt[-7 + Sqrt[73]]*Sqrt[-3 + 7*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.48, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1411}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+7x^2-3}}dx$$

$$\Big\downarrow \; 1411$$

$$\frac{\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{\left(7+\sqrt{73}\right)x^2-6}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{\left(7+\sqrt{73}\right)x^2-6}}\right),\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{2\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{2x^4+7x^2-3}}$$

input
```
Int[1/Sqrt[-3 + 7*x^2 + 2*x^4],x]
```

output
```
(Sqrt[(6 - (7 - Sqrt[73])*x^2)/(6 - (7 + Sqrt[73])*x^2)]*Sqrt[-6 + (7 + Sq
rt[73])*x^2]*EllipticF[ArcSin[(Sqrt[2]*73^(1/4)*x)/Sqrt[-6 + (7 + Sqrt[73]
)*x^2]], (73 + 7*Sqrt[73])/146])/(2*Sqrt[3]*73^(1/4)*Sqrt[(6 - (7 + Sqrt[7
3])*x^2)^(-1)]*Sqrt[-3 + 7*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{6\sqrt{1-\left(-\frac{\sqrt{73}}{6}+\frac{7}{6}\right)x^2}\,\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{42-6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{\sqrt{42-6\sqrt{73}}\,\sqrt{2x^4+7x^2-3}}$	84
elliptic	$\frac{6\sqrt{1-\left(-\frac{\sqrt{73}}{6}+\frac{7}{6}\right)x^2}\,\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{42-6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{\sqrt{42-6\sqrt{73}}\,\sqrt{2x^4+7x^2-3}}$	84

input
```
int(1/(2*x^4+7*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(42-6*73^(1/2))^(1/2)*(1-(-1/6*73^(1/2)+7/6)*x^2)^(1/2)*(1-(7/6+1/6*73^(
1/2))*x^2)^(1/2)/(2*x^4+7*x^2-3)^(1/2)*EllipticF(1/6*(42-6*73^(1/2))^(1/2)
*x,7/12*I*6^(1/2)+1/12*I*438^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx$$
$$= -\frac{1}{12}\left(\sqrt{73}\sqrt{-3}-7\sqrt{-3}\right)\sqrt{\frac{1}{6}\sqrt{73}+\frac{7}{6}}F(\arcsin\left(x\sqrt{\frac{1}{6}\sqrt{73}+\frac{7}{6}}\right)\mid\frac{7}{12}\sqrt{73}-\frac{61}{12})$$

input
```
integrate(1/(2*x^4+7*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(73)*sqrt(-3) - 7*sqrt(-3))*sqrt(1/6*sqrt(73) + 7/6)*elliptic_f
(arcsin(x*sqrt(1/6*sqrt(73) + 7/6)), 7/12*sqrt(73) - 61/12)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+7x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4+7*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 7*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+7\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+7*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 7*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+7\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+7*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 7*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+7\,x^2-3}}\,dx$$

input
```
int(1/(7*x^2 + 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(7*x^2 + 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+7x^2-3}}{2x^4+7x^2-3}dx$$

input
```
int(1/(2*x^4+7*x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 7*x**2 - 3)/(2*x**4 + 7*x**2 - 3),x)
```

3.135 $\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 94

$$\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx = \frac{\sqrt{3-\left(3-\sqrt{15}\right)x^2}\sqrt{3-\left(3+\sqrt{15}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right),-4+\sqrt{15}\right)}{\sqrt{3\left(3+\sqrt{15}\right)}\sqrt{-3+6x^2+2x^4}}$$

output
```
(3-(3-15^(1/2))*x^2)^(1/2)*(3-(3+15^(1/2))*x^2)^(1/2)*EllipticF(1/3*(9+3*1
5^(1/2))^(1/2)*x,1/2*I*10^(1/2)-1/2*I*6^(1/2))/(9+3*15^(1/2))^(1/2)/(2*x^4
+6*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx = -\frac{i\sqrt{3-6x^2-2x^4}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-1+\sqrt{\frac{5}{3}}}x\right),-4-\sqrt{15}\right)}{\sqrt{-3+\sqrt{15}}\sqrt{-3+6x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + 6*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[3 - 6*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/3]]*x],
 -4 - Sqrt[15]])/(Sqrt[-3 + Sqrt[15]]*Sqrt[-3 + 6*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.57, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1411}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+6x^2-3}}\,dx$$

$$\Big\downarrow \text{ 1411}$$

$$\frac{\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{\left(3+\sqrt{15}\right)x^2-3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{15}x}{\sqrt{\left(3+\sqrt{15}\right)x^2-3}}\right),\frac{1}{10}\left(5+\sqrt{15}\right)\right)}{\sqrt{2}3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{2x^4+6x^2-3}}$$

input
```
Int[1/Sqrt[-3 + 6*x^2 + 2*x^4],x]
```

output
```
(Sqrt[(3 - (3 - Sqrt[15])*x^2)/(3 - (3 + Sqrt[15])*x^2)]*Sqrt[-3 + (3 + Sq
rt[15])*x^2]*EllipticF[ArcSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[-3 + (3 + Sqrt[15]
)*x^2]], (5 + Sqrt[15])/10])/(Sqrt[2]*3^(3/4)*5^(1/4)*Sqrt[(3 - (3 + Sqrt[
15])*x^2)^(-1)]*Sqrt[-3 + 6*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{3\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{9-3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{\sqrt{9-3\sqrt{15}}\,\sqrt{2x^4+6x^2-3}}$	84
elliptic	$\frac{3\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{9-3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{\sqrt{9-3\sqrt{15}}\,\sqrt{2x^4+6x^2-3}}$	84

input
```
int(1/(2*x^4+6*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(9-3*15^(1/2))^(1/2)*(1-(1-1/3*15^(1/2))*x^2)^(1/2)*(1-(1+1/3*15^(1/2))*
x^2)^(1/2)/(2*x^4+6*x^2-3)^(1/2)*EllipticF(1/3*(9-3*15^(1/2))^(1/2)*x,1/2*
I*6^(1/2)+1/2*I*10^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx$$
$$= -\frac{1}{2}\left(\sqrt{\frac{5}{3}}\sqrt{-3}-\sqrt{-3}\right)\sqrt{\sqrt{\frac{5}{3}}+1}F(\arcsin\left(x\sqrt{\sqrt{\frac{5}{3}}+1}\right)\,|3\sqrt{\frac{5}{3}}-4)$$

input
```
integrate(1/(2*x^4+6*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(5/3)*sqrt(-3) - sqrt(-3))*sqrt(sqrt(5/3) + 1)*elliptic_f(arcsin
(x*sqrt(sqrt(5/3) + 1)), 3*sqrt(5/3) - 4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+6x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4+6*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 6*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+6\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+6*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 6*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+6\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+6*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 6*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+6\,x^2-3}}\,dx$$

input
```
int(1/(6*x^2 + 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(6*x^2 + 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+6x^2-3}}{2x^4+6x^2-3}dx$$

input
```
int(1/(2*x^4+6*x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 6*x**2 - 3)/(2*x**4 + 6*x**2 - 3),x)
```

3.136 $\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx = \frac{\sqrt{1-2x^2}\sqrt{3+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{6}\right)}{\sqrt{6}\sqrt{-3+5x^2+2x^4}}$$

output
```
1/6*(-2*x^2+1)^(1/2)*(x^2+3)^(1/2)*EllipticF(x*2^(1/2),1/6*I*6^(1/2))*6^(1
/2)/(2*x^4+5*x^2-3)^(1/2)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx = \frac{\sqrt{1-2x^2}\sqrt{3+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{6}\right)}{\sqrt{6}\sqrt{-3+5x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + 5*x^2 + 2*x^4],x]
```

output
```
(Sqrt[1 - 2*x^2]*Sqrt[3 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/6])/(Sqrt[6
]*Sqrt[-3 + 5*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1410}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+5x^2-3}}\,dx$$

$$\Big\downarrow \text{ 1410}$$

$$\frac{\sqrt{x^2+3}\sqrt{2x^2-1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{7}{3}}x}{\sqrt{2x^2-1}}\right),\frac{6}{7}\right)}{\sqrt{7}\sqrt{2x^4+5x^2-3}}$$

input
```
Int[1/Sqrt[-3 + 5*x^2 + 2*x^4],x]
```

output
```
(Sqrt[3 + x^2]*Sqrt[-1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7/3]*x)/Sqrt[-1 + 2
*x^2]], 6/7])/(Sqrt[7]*Sqrt[-3 + 5*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{-2x^2+1}\,\text{EllipticF}\left(\frac{ix\sqrt{3}}{3},i\sqrt{6}\right)}{3\sqrt{2x^4+5x^2-3}}$	53
elliptic	$-\frac{i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{-2x^2+1}\,\text{EllipticF}\left(\frac{ix\sqrt{3}}{3},i\sqrt{6}\right)}{3\sqrt{2x^4+5x^2-3}}$	53

input
```
int(1/(2*x^4+5*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*3^(1/2)*(3*x^2+9)^(1/2)*(-2*x^2+1)^(1/2)/(2*x^4+5*x^2-3)^(1/2)*Elli
pticF(1/3*I*x*3^(1/2),I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx = -\frac{1}{6}\sqrt{2}\sqrt{-3}F\left(\arcsin\left(\sqrt{2}x\right)\mid-\frac{1}{6}\right)$$

input
```
integrate(1/(2*x^4+5*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2)*sqrt(-3)*elliptic_f(arcsin(sqrt(2)*x), -1/6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+5x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4+5*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 5*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+5\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+5*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 5*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+5\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+5*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 5*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+5\,x^2-3}}\,dx$$

input
```
int(1/(5*x^2 + 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(5*x^2 + 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+5x^2-3}}{2x^4+5x^2-3}dx$$

input
```
int(1/(2*x^4+5*x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 5*x**2 - 3)/(2*x**4 + 5*x**2 - 3),x)
```

3.137 $\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx = \frac{\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}\sqrt{3-\left(2-\sqrt{10}\right)x^2}\sqrt{3-\left(2+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(2+\sqrt{10}\right)}x\right),\frac{1}{3}(-7+$$

$$3\sqrt{-3+4x^2+2x^4}$$

output
```
1/6*(-4+2*10^(1/2))^(1/2)*(3-(2-10^(1/2))*x^2)^(1/2)*(3-(2+10^(1/2))*x^2)^
(1/2)*EllipticF(1/3*(6+3*10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))/(
2*x^4+4*x^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx = -\frac{i\sqrt{3-4x^2-2x^4}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right),-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right)}{\sqrt{-2+\sqrt{10}}\sqrt{-3+4x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + 4*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[3 - 4*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(2 + Sqrt[10])]*x
], -7/3 - (2*Sqrt[10])/3])/(Sqrt[-2 + Sqrt[10]]*Sqrt[-3 + 4*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.41, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1411\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+4x^2-3}}\,dx$$

$$\Big\downarrow \; 1411$$

$$\frac{\sqrt{\frac{3-\left(2-\sqrt{10}\right)x^2}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-3}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{\left(2+\sqrt{10}\right)x^2-3}}\right),\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{2x^4+4x^2-3}}$$

input
```
Int[1/Sqrt[-3 + 4*x^2 + 2*x^4],x]
```

output
```
(Sqrt[(3 - (2 - Sqrt[10])*x^2)/(3 - (2 + Sqrt[10])*x^2)]*Sqrt[-3 + (2 + Sq
rt[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 + (2 + Sqrt[10])
*x^2]], (5 + Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^(1/4)*Sqrt[(3 - (2 + Sqrt[1
0])*x^2)^(-1)]*Sqrt[-3 + 4*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{6-3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{6-3\sqrt{10}}\,\sqrt{2x^4+4x^2-3}}$	84
elliptic	$\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{6-3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{6-3\sqrt{10}}\,\sqrt{2x^4+4x^2-3}}$	84

input
```
int(1/(2*x^4+4*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(6-3*10^(1/2))^(1/2)*(1-(2/3-1/3*10^(1/2))*x^2)^(1/2)*(1-(2/3+1/3*10^(1/
2))*x^2)^(1/2)/(2*x^4+4*x^2-3)^(1/2)*EllipticF(1/3*(6-3*10^(1/2))^(1/2)*x,
1/3*I*6^(1/2)+1/3*I*15^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx$$
$$= -\frac{1}{6}\left(\sqrt{10}\sqrt{-3}-2\sqrt{-3}\right)\sqrt{\frac{1}{3}\sqrt{10}+\frac{2}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{10}+\frac{2}{3}}\right) \mid \frac{2}{3}\sqrt{10}-\frac{7}{3})$$

input
```
integrate(1/(2*x^4+4*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(10)*sqrt(-3) - 2*sqrt(-3))*sqrt(1/3*sqrt(10) + 2/3)*elliptic_f(
arcsin(x*sqrt(1/3*sqrt(10) + 2/3)), 2/3*sqrt(10) - 7/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+4x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4+4*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 4*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+4\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+4*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 4*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+4\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+4*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 4*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+4\,x^2-3}}\,dx$$

input
```
int(1/(4*x^2 + 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(4*x^2 + 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+4x^2-3}}{2x^4+4x^2-3}dx$$

input
```
int(1/(2*x^4+4*x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 4*x**2 - 3)/(2*x**4 + 4*x**2 - 3),x)
```

3.138 $\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx = \frac{\sqrt{6-\left(3-\sqrt{33}\right)x^2}\sqrt{6-\left(3+\sqrt{33}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{6}\left(3+\sqrt{33}\right)}x\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{\sqrt{6\left(3+\sqrt{33}\right)}\sqrt{-3+3x^2+2x^4}}$$

output
```
(6-(3-33^(1/2))*x^2)^(1/2)*(6-(3+33^(1/2))*x^2)^(1/2)*EllipticF(1/6*(18+6*
33^(1/2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))/(18+6*33^(1/2))^(1/2)/(2*x
^4+3*x^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx = -\frac{i\sqrt{6-6x^2-4x^4}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right),-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)}{\sqrt{-3+\sqrt{33}}\sqrt{-3+3x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + 3*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[6 - 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[3 + Sqrt[33]]
], -7/4 - Sqrt[33]/4])/(Sqrt[-3 + Sqrt[33]]*Sqrt[-3 + 3*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.49, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1411}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+3x^2-3}}dx$$

$$\Big\downarrow \; 1411$$

$$\frac{\sqrt{\frac{6-\left(3-\sqrt{33}\right)x^2}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-6}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{\left(3+\sqrt{33}\right)x^2-6}}\right),\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\ 3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{2x^4+3x^2-3}}$$

input
```
Int[1/Sqrt[-3 + 3*x^2 + 2*x^4],x]
```

output
```
(Sqrt[(6 - (3 - Sqrt[33])*x^2)/(6 - (3 + Sqrt[33])*x^2)]*Sqrt[-6 + (3 + Sq
rt[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6 + (3 + Sqrt[33]
)*x^2]], (11 + Sqrt[33])/22])/(2*3^(3/4)*11^(1/4)*Sqrt[(6 - (3 + Sqrt[33])
*x^2)^(-1)]*Sqrt[-3 + 3*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{6\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{18-6\sqrt{33}}\,x}{6}, \frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{18-6\sqrt{33}}\,\sqrt{2x^4+3x^2-3}}$	84
elliptic	$\frac{6\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{18-6\sqrt{33}}\,x}{6}, \frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{18-6\sqrt{33}}\,\sqrt{2x^4+3x^2-3}}$	84

input
```
int(1/(2*x^4+3*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(18-6*33^(1/2))^(1/2)*(1-(1/2-1/6*33^(1/2))*x^2)^(1/2)*(1-(1/2+1/6*33^(1
/2))*x^2)^(1/2)/(2*x^4+3*x^2-3)^(1/2)*EllipticF(1/6*(18-6*33^(1/2))^(1/2)*
x,1/4*I*6^(1/2)+1/4*I*22^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx$$
$$= -\frac{1}{4}\left(\sqrt{\frac{11}{3}}\sqrt{-3}-\sqrt{-3}\right)\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}+\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}+\frac{1}{2}}\right) | \frac{3}{4}\sqrt{\frac{11}{3}}-\frac{7}{4})$$

input
```
integrate(1/(2*x^4+3*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(11/3)*sqrt(-3) - sqrt(-3))*sqrt(1/2*sqrt(11/3) + 1/2)*elliptic_
f(arcsin(x*sqrt(1/2*sqrt(11/3) + 1/2)), 3/4*sqrt(11/3) - 7/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+3x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4+3*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 3*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+3\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+3*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 3*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+3\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+3*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 3*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+3\,x^2-3}}\,dx$$

input
```
int(1/(3*x^2 + 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(3*x^2 + 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+3x^2-3}}{2x^4+3x^2-3}dx$$

input
```
int(1/(2*x^4+3*x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 3*x**2 - 3)/(2*x**4 + 3*x**2 - 3),x)
```

3.139 $\int \frac{1}{\sqrt{-3+2x^2+2x^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{1}{\sqrt{-3+2x^2+2x^4}} \, dx = \frac{\sqrt{3-\left(1-\sqrt{7}\right)x^2}\sqrt{3-\left(1+\sqrt{7}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(1+\sqrt{7}\right)}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{3\left(1+\sqrt{7}\right)}\sqrt{-3+2x^2+2x^4}}$$

output
```
(3-(1-7^(1/2))*x^2)^(1/2)*(3-(1+7^(1/2))*x^2)^(1/2)*EllipticF(1/3*(3+3*7^(
1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))/(3+3*7^(1/2))^(1/2)/(2*x^4+2*x
^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-3+2x^2+2x^4}} \, dx = -\frac{i\sqrt{3-2x^2-2x^4}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right),-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{-1+\sqrt{7}}\sqrt{-3+2x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + 2*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[3 - 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[7])]*x]
, -4/3 - Sqrt[7]/3])/(Sqrt[-1 + Sqrt[7]]*Sqrt[-3 + 2*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.46, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1411\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+2x^2-3}}\,dx$$

$$\Big\downarrow \text{1411}$$

$$\frac{\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-3}}\right),\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{2x^4+2x^2-3}}$$

input
```
Int[1/Sqrt[-3 + 2*x^2 + 2*x^4],x]
```

output
```
(Sqrt[(3 - (1 - Sqrt[7])*x^2)/(3 - (1 + Sqrt[7])*x^2)]*Sqrt[-3 + (1 + Sqrt
[7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 + (1 + Sqrt[7])*x^2
]], (7 + Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3 - (1 + Sqrt[7])*x^2)^(-1)]
*Sqrt[-3 + 2*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^{2}}\,\sqrt{1-\left(\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3-3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{3-3\sqrt{7}}\,\sqrt{2x^{4}+2x^{2}-3}}$	84
elliptic	$\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^{2}}\,\sqrt{1-\left(\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3-3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{3-3\sqrt{7}}\,\sqrt{2x^{4}+2x^{2}-3}}$	84

input
```
int(1/(2*x^4+2*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(3-3*7^(1/2))^(1/2)*(1-(1/3-1/3*7^(1/2))*x^2)^(1/2)*(1-(1/3+1/3*7^(1/2))
*x^2)^(1/2)/(2*x^4+2*x^2-3)^(1/2)*EllipticF(1/3*(3-3*7^(1/2))^(1/2)*x,1/6*
I*6^(1/2)+1/6*I*42^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{-3+2x^2+2x^4}}\,dx$$
$$= -\frac{1}{6}\left(\sqrt{7}\sqrt{-3}-\sqrt{-3}\right)\sqrt{\frac{1}{3}\sqrt{7}+\frac{1}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{7}+\frac{1}{3}}\right)\mid\frac{1}{3}\sqrt{7}-\frac{4}{3})$$

input
```
integrate(1/(2*x^4+2*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(7)*sqrt(-3) - sqrt(-3))*sqrt(1/3*sqrt(7) + 1/3)*elliptic_f(arcs
in(x*sqrt(1/3*sqrt(7) + 1/3)), 1/3*sqrt(7) - 4/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+2x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4+2*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 2*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+2*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 2*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+2*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 2*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2-3}}\,dx$$

input
```
int(1/(2*x^2 + 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(2*x^2 + 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+2x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+2x^2-3}}{2x^4+2x^2-3}dx$$

input
```
int(1/(2*x^4+2*x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 2*x**2 - 3)/(2*x**4 + 2*x**2 - 3),x)
```

3.140 $\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx=\frac{\sqrt{1-x^2}\sqrt{3+2x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-3+x^2+2x^4}}$$

output
```
1/3*(-x^2+1)^(1/2)*(2*x^2+3)^(1/2)*EllipticF(x,1/3*I*6^(1/2))*3^(1/2)/(2*x
^4+x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx=-\frac{i\sqrt{1-x^2}\sqrt{3+2x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-3+x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[1 - x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], -3/2
])/(Sqrt[2]*Sqrt[-3 + x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.31, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1410}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+x^2-3}}\,dx$$

$$\Big\downarrow \text{1410}$$

$$\frac{\sqrt{x^2-1}\sqrt{2x^2+3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{x^2-1}}\right),\frac{3}{5}\right)}{\sqrt{5}\sqrt{2x^4+x^2-3}}$$

input
```
Int[1/Sqrt[-3 + x^2 + 2*x^4],x]
```

output
```
(Sqrt[-1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5/3]*x)/Sqrt[-1 + x
^2]], 3/5])/(Sqrt[5]*Sqrt[-3 + x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{6\sqrt{2x^4+x^2-3}}$	51
elliptic	$-\frac{i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{6\sqrt{2x^4+x^2-3}}$	51

input
```
int(1/(2*x^4+x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*6^(1/2)*(6*x^2+9)^(1/2)*(-x^2+1)^(1/2)/(2*x^4+x^2-3)^(1/2)*Elliptic
F(1/3*I*x*6^(1/2),1/2*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.19

$$\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx = -\frac{1}{3}\sqrt{-3}F(\arcsin{(x)}\,|-\frac{2}{3})$$

input
```
integrate(1/(2*x^4+x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(-3)*elliptic_f(arcsin(x), -2/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4+x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4+x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+x^2-3}}\,dx$$

input
```
int(1/(x^2 + 2*x^4 - 3)^(1/2),x)
```

output
```
int(1/(x^2 + 2*x^4 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+x^2-3}}{2x^4+x^2-3}dx$$

input
```
int(1/(2*x^4+x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + x**2 - 3)/(2*x**4 + x**2 - 3),x)
```

3.141 $\int \frac{1}{\sqrt{-3+2x^4}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int \frac{1}{\sqrt{-3+2x^4}} \, dx = \frac{\sqrt{3-2x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{\sqrt[4]{6}\sqrt{-3+2x^4}}$$

output
```
1/6*(-2*x^4+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*x,I)*6^(3/4)/(2*x^4-3)^
(1/2)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-3+2x^4}} \, dx = \frac{\sqrt{3-2x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{\sqrt[4]{6}\sqrt{-3+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 + 2*x^4],x]
```

output
```
(Sqrt[3 - 2*x^4]*EllipticF[ArcSin[(2/3)^(1/4)*x], -1])/(6^(1/4)*Sqrt[-3 +
2*x^4])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 112 vs. $2(40) = 80$.

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.80, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{764\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-3}}\,dx$$

$$\Big\downarrow \text{764}$$

$$\frac{\sqrt{\sqrt{6}x^2-3}\sqrt{\frac{\sqrt{6}x^2+3}{3-\sqrt{6}x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{\sqrt{6}x^2-3}}\right),\frac{1}{2}\right)}{6^{3/4}\sqrt{\frac{1}{3-\sqrt{6}x^2}}\sqrt{2x^4-3}}$$

input
```
Int[1/Sqrt[-3 + 2*x^4],x]
```

output
```
(Sqrt[-3 + Sqrt[6]*x^2]*Sqrt[(3 + Sqrt[6]*x^2)/(3 - Sqrt[6]*x^2)]*Elliptic
F[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-3 + Sqrt[6]*x^2]], 1/2])/(6^(3/4)*Sqrt[
(3 - Sqrt[6]*x^2)^(-1)]*Sqrt[-3 + 2*x^4])
```

Defintions of rubi rules used

rule 764
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[(a - q*x^2)/(a + q*x^2)]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[a + b*x^
4]*Sqrt[a/(a + q*x^2)]))*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2],
 x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

method	result	size
meijerg	$\frac{\sqrt{3}\,\sqrt{-\operatorname{signum}\left(-1+\frac{2x^4}{3}\right)}\,x\,\operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],\frac{2x^4}{3}\right)}{3\sqrt{\operatorname{signum}\left(-1+\frac{2x^4}{3}\right)}}$	40
default	$\frac{\sqrt{9+3\sqrt{6}\,x^2}\,\sqrt{9-3\sqrt{6}\,x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3\sqrt{6}}\,x}{3},i\right)}{3\sqrt{-3\sqrt{6}}\,\sqrt{2x^4-3}}$	56
elliptic	$\frac{\sqrt{9+3\sqrt{6}\,x^2}\,\sqrt{9-3\sqrt{6}\,x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3\sqrt{6}}\,x}{3},i\right)}{3\sqrt{-3\sqrt{6}}\,\sqrt{2x^4-3}}$	56

input
```
int(1/(2*x^4-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)/signum(-1+2/3*x^4)^(1/2)*(-signum(-1+2/3*x^4))^(1/2)*x*hyperge
om([1/4,1/2],[5/4],2/3*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{-3+2x^4}}\,dx = -\frac{1}{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\sqrt{-3}F(\arcsin\left(\left(\frac{2}{3}\right)^{\frac{1}{4}}x\right)|-1)$$

input
```
integrate(1/(2*x^4-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2/3)^(3/4)*sqrt(-3)*elliptic_f(arcsin((2/3)^(1/4)*x), -1)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-3+2x^4}}\,dx = -\frac{\sqrt{3}ix\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{2x^4}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(2*x**4-3)**(1/2),x)
```

output
```
-sqrt(3)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4/3)/(12*gamma(5/4)
)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-3}}\,dx$$

input
```
integrate(1/(2*x^4-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-3}}\,dx$$

input
```
integrate(1/(2*x^4-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 3), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{-3+2x^4}}\,dx = \frac{x\,\sqrt{9-6\,x^4}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ \frac{2\,x^4}{3}\right)}{3\,\sqrt{2\,x^4-3}}$$

input
```
int(1/(2*x^4 - 3)^(1/2),x)
```

output
```
(x*(9 - 6*x^4)^(1/2)*hypergeom([1/4, 1/2], 5/4, (2*x^4)/3))/(3*(2*x^4 - 3)
^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3+2x^4}}\,dx = \int \frac{\sqrt{2x^4-3}}{2x^4-3}dx$$

input
```
int(1/(2*x^4-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 3)/(2*x**4 - 3),x)
```

3.142 $\int \frac{1}{\sqrt{-3-x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 56

$$\int \frac{1}{\sqrt{-3-x^2+2x^4}}\,dx=\frac{\sqrt{3-2x^2}\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-3-x^2+2x^4}}$$

output
```
1/2*(-2*x^2+3)^(1/2)*(x^2+1)^(1/2)*EllipticF(1/3*x*6^(1/2),1/2*I*6^(1/2))*
2^(1/2)/(2*x^4-x^2-3)^(1/2)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{-3-x^2+2x^4}}\,dx=\frac{\sqrt{3-2x^2}\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{-6-2x^2+4x^4}}$$

input
```
Integrate[1/Sqrt[-3 - x^2 + 2*x^4],x]
```

output
```
(Sqrt[3 - 2*x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], -3/2])/Sqrt[
-6 - 2*x^2 + 4*x^4]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1410}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-x^2-3}}\,dx$$

$$\Big\downarrow \text{1410}$$

$$\frac{\sqrt{x^2+1}\sqrt{2x^2-3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{5}x}{\sqrt{2x^2-3}}\right),\frac{2}{5}\right)}{\sqrt{5}\sqrt{2x^4-x^2-3}}$$

input
```
Int[1/Sqrt[-3 - x^2 + 2*x^4],x]
```

output
```
(Sqrt[1 + x^2]*Sqrt[-3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt[-3 + 2*x
^2]], 2/5])/(Sqrt[5]*Sqrt[-3 - x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+9}\ \mathrm{EllipticF}\left(ix,\frac{i\sqrt{6}}{3}\right)}{3\sqrt{2x^4-x^2-3}}$	45
elliptic	$-\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+9}\ \mathrm{EllipticF}\left(ix,\frac{i\sqrt{6}}{3}\right)}{3\sqrt{2x^4-x^2-3}}$	45

input
```
int(1/(2*x^4-x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*(x^2+1)^(1/2)*(-6*x^2+9)^(1/2)/(2*x^4-x^2-3)^(1/2)*EllipticF(I*x,1/
3*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt{-3-x^2+2x^4}}\,dx = -\frac{1}{2}\sqrt{\frac{2}{3}}\sqrt{-3}F(\arcsin\left(\sqrt{\frac{2}{3}}x\right)\,|-\frac{3}{2})$$

input
```
integrate(1/(2*x^4-x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2/3)*sqrt(-3)*elliptic_f(arcsin(sqrt(2/3)*x), -3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4-x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4-x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3 - x^2 + 2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4 - x^2 - 3}}\,dx$$

input
```
integrate(1/(2*x^4-x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3 - x^2 + 2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4 - x^2 - 3}}\,dx$$

input
```
integrate(1/(2*x^4-x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3 - x^2 + 2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4 - x^2 - 3}}\,dx$$

input
```
int(1/(2*x^4 - x^2 - 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - x^2 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-x^2-3}}{2x^4-x^2-3}dx$$

input
```
int(1/(2*x^4-x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - x**2 - 3)/(2*x**4 - x**2 - 3),x)
```

3.143 $\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx = \frac{\sqrt{3+\left(1-\sqrt{7}\right)x^2}\sqrt{3+\left(1+\sqrt{7}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-1+\sqrt{7}\right)}x\right),\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{3\left(-1+\sqrt{7}\right)}\sqrt{-3-2x^2+2x^4}}$$

output
```
(3+(1-7^(1/2))*x^2)^(1/2)*(3+(1+7^(1/2))*x^2)^(1/2)*EllipticF(1/3*(-3+3*7^
(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))/(-3+3*7^(1/2))^(1/2)/(2*x^4-2
*x^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx = -\frac{i\sqrt{3+2x^2-2x^4}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}\sqrt{-3-2x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 - 2*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[3 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[7])]*x
], (-4 + Sqrt[7])/3])/(Sqrt[1 + Sqrt[7]]*Sqrt[-3 - 2*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.53, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1411}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-2x^2-3}}dx$$

$$\Big\downarrow \text{1411}$$

$$\frac{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-3}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+3}{\left(1-\sqrt{7}\right)x^2+3}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-3}}\right),\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+3}}\sqrt{2x^4-2x^2-3}}$$

input
```
Int[1/Sqrt[-3 - 2*x^2 + 2*x^4],x]
```

output
```
(Sqrt[-3 - (1 - Sqrt[7])*x^2]*Sqrt[(3 + (1 + Sqrt[7])*x^2)/(3 + (1 - Sqrt[
7])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 - (1 - Sqrt[7])*x^2
]], (7 - Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3 + (1 - Sqrt[7])*x^2)^(-1)]
*Sqrt[-3 - 2*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3-3\sqrt{7}}\,x}{3},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{-3-3\sqrt{7}}\,\sqrt{2x^{4}-2x^{2}-3}}$	84
elliptic	$\frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3-3\sqrt{7}}\,x}{3},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{-3-3\sqrt{7}}\,\sqrt{2x^{4}-2x^{2}-3}}$	84

input
```
int(1/(2*x^4-2*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(-3-3*7^(1/2))^(1/2)*(1-(-1/3-1/3*7^(1/2))*x^2)^(1/2)*(1-(-1/3+1/3*7^(1/
2))*x^2)^(1/2)/(2*x^4-2*x^2-3)^(1/2)*EllipticF(1/3*(-3-3*7^(1/2))^(1/2)*x,
1/6*I*42^(1/2)-1/6*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx = -\frac{1}{6}\left(\sqrt{7}\sqrt{-3}+\sqrt{-3}\right)\sqrt{\frac{1}{3}\sqrt{7}-\frac{1}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{7}-\frac{1}{3}}\right) | -\frac{1}{3}\sqrt{7}-\frac{4}{3})$$

input
```
integrate(1/(2*x^4-2*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(7)*sqrt(-3) + sqrt(-3))*sqrt(1/3*sqrt(7) - 1/3)*elliptic_f(arcs
in(x*sqrt(1/3*sqrt(7) - 1/3)), -1/3*sqrt(7) - 4/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4-2x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4-2*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 2*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-2\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4-2*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 2*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-2\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4-2*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 2*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-2\,x^2-3}}\,dx$$

input
```
int(1/(2*x^4 - 2*x^2 - 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 2*x^2 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-2x^2-3}}{2x^4-2x^2-3}dx$$

input
```
int(1/(2*x^4-2*x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 2*x**2 - 3)/(2*x**4 - 2*x**2 - 3),x)
```

3.144 $\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx = \frac{\sqrt{6+\left(3-\sqrt{33}\right)x^2}\sqrt{6+\left(3+\sqrt{33}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{6}\left(-3+\sqrt{33}\right)}x\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)}{\sqrt{6\left(-3+\sqrt{33}\right)}\sqrt{-3-3x^2+2x^4}}$$

output
```
(6+(3-33^(1/2))*x^2)^(1/2)*(6+(3+33^(1/2))*x^2)^(1/2)*EllipticF(1/6*(-18+6
*33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))/(-18+6*33^(1/2))^(1/2)/(2
*x^4-3*x^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx = -\frac{i\sqrt{6+6x^2-4x^4}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{\sqrt{3+\sqrt{33}}\sqrt{-3-3x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 - 3*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[6 + 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-3 + Sqrt[33]
]], (-7 + Sqrt[33])/4])/(Sqrt[3 + Sqrt[33]]*Sqrt[-3 - 3*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.56, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1411}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-3x^2-3}}\,dx$$

$$\Big\downarrow \ 1411$$

$$\frac{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-6}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+6}{\left(3-\sqrt{33}\right)x^2+6}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-6}}\right),\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\ 3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+6}}\sqrt{2x^4-3x^2-3}}$$

input
```
Int[1/Sqrt[-3 - 3*x^2 + 2*x^4],x]
```

output
```
(Sqrt[-6 - (3 - Sqrt[33])*x^2]*Sqrt[(6 + (3 + Sqrt[33])*x^2)/(6 + (3 - Sqr
t[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6 - (3 - Sqrt[33]
)*x^2]], (11 - Sqrt[33])/22])/(2*3^(3/4)*11^(1/4)*Sqrt[(6 + (3 - Sqrt[33])
*x^2)^(-1)]*Sqrt[-3 - 3*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-18-6\sqrt{33}}}{6},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{-18-6\sqrt{33}}\,\sqrt{2x^4-3x^2-3}}$	84
elliptic	$\frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-18-6\sqrt{33}}}{6},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{-18-6\sqrt{33}}\,\sqrt{2x^4-3x^2-3}}$	84

input
```
int(1/(2*x^4-3*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(-18-6*33^(1/2))^(1/2)*(1-(-1/2-1/6*33^(1/2))*x^2)^(1/2)*(1-(-1/2+1/6*33
^(1/2))*x^2)^(1/2)/(2*x^4-3*x^2-3)^(1/2)*EllipticF(1/6*x*(-18-6*33^(1/2))^
(1/2),1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx$$
$$= -\frac{1}{4}\left(\sqrt{\frac{11}{3}}\sqrt{-3}+\sqrt{-3}\right)\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}-\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}-\frac{1}{2}}\right)|-\frac{3}{4}\sqrt{\frac{11}{3}}-\frac{7}{4})$$

input
```
integrate(1/(2*x^4-3*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(11/3)*sqrt(-3) + sqrt(-3))*sqrt(1/2*sqrt(11/3) - 1/2)*elliptic_
f(arcsin(x*sqrt(1/2*sqrt(11/3) - 1/2)), -3/4*sqrt(11/3) - 7/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4-3x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4-3*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 3*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-3\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4-3*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 3*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-3\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4-3*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 3*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-3\,x^2-3}}\,dx$$

input
```
int(1/(2*x^4 - 3*x^2 - 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 3*x^2 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-3x^2-3}}{2x^4-3x^2-3}dx$$

input
```
int(1/(2*x^4-3*x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 3*x**2 - 3)/(2*x**4 - 3*x**2 - 3),x)
```

3.145 $\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 103

$$\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx = \frac{\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}\sqrt{3+\left(2-\sqrt{10}\right)x^2}\sqrt{3+\left(2+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-2+\sqrt{10}\right)}x\right),\frac{1}{3}(-7-\right.}{3\sqrt{-3-4x^2+2x^4}}$$

output
```
1/6*(4+2*10^(1/2))^(1/2)*(3+(2-10^(1/2))*x^2)^(1/2)*(3+(2+10^(1/2))*x^2)^(
1/2)*EllipticF(1/3*(-6+3*10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))/(
2*x^4-4*x^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx = -\frac{i\sqrt{3+4x^2-2x^4}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right),-\frac{7}{3}+\frac{2\sqrt{10}}{3}\right)}{\sqrt{2+\sqrt{10}}\sqrt{-3-4x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 - 4*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[3 + 4*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-2 + Sqrt[10])]*
x], -7/3 + (2*Sqrt[10])/3])/(Sqrt[2 + Sqrt[10]]*Sqrt[-3 - 4*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.50, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1411\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-4x^2-3}}dx$$

$$\Big\downarrow \text{1411}$$

$$\frac{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-3}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+3}{\left(2-\sqrt{10}\right)x^2+3}}\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-3}}\right),\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+3}}\sqrt{2x^4-4x^2-3}}$$

input
```
Int[1/Sqrt[-3 - 4*x^2 + 2*x^4],x]
```

output
```
(Sqrt[-3 - (2 - Sqrt[10])*x^2]*Sqrt[(3 + (2 + Sqrt[10])*x^2)/(3 + (2 - Sqr
t[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 - (2 - Sqrt[10])
*x^2]], (5 - Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^(1/4)*Sqrt[(3 + (2 - Sqrt[1
0])*x^2)^(-1)]*Sqrt[-3 - 4*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^{2}}\,\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6-3\sqrt{10}}\,x}{3},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{-6-3\sqrt{10}}\,\sqrt{2x^{4}-4x^{2}-3}}$	84
elliptic	$\frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^{2}}\,\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6-3\sqrt{10}}\,x}{3},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{-6-3\sqrt{10}}\,\sqrt{2x^{4}-4x^{2}-3}}$	84

input
```
int(1/(2*x^4-4*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(-6-3*10^(1/2))^(1/2)*(1-(-2/3-1/3*10^(1/2))*x^2)^(1/2)*(1-(-2/3+1/3*10^
(1/2))*x^2)^(1/2)/(2*x^4-4*x^2-3)^(1/2)*EllipticF(1/3*(-6-3*10^(1/2))^(1/2
)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx = -\frac{1}{6}\left(\sqrt{10}\sqrt{-3}+2\sqrt{-3}\right)\sqrt{\frac{1}{3}\sqrt{10}-\frac{2}{3}}F(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{10}-\frac{2}{3}}\right)\mid -\frac{2}{3}\sqrt{10}-\frac{7}{3})$$

input
```
integrate(1/(2*x^4-4*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(10)*sqrt(-3) + 2*sqrt(-3))*sqrt(1/3*sqrt(10) - 2/3)*elliptic_f(
arcsin(x*sqrt(1/3*sqrt(10) - 2/3)), -2/3*sqrt(10) - 7/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4-4x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4-4*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 4*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-4\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4-4*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 4*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-4\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4-4*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 4*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-4\,x^2-3}}\,dx$$

input
```
int(1/(2*x^4 - 4*x^2 - 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 4*x^2 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-4x^2-3}}{2x^4-4x^2-3}dx$$

input
```
int(1/(2*x^4-4*x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 4*x**2 - 3)/(2*x**4 - 4*x**2 - 3),x)
```

3.146 $\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx = \frac{\sqrt{3-x^2}\sqrt{1+2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-6\right)}{\sqrt{-3-5x^2+2x^4}}$$

output
```
(-x^2+3)^(1/2)*(2*x^2+1)^(1/2)*EllipticF(1/3*x*3^(1/2),I*6^(1/2))/(2*x^4-5
*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx = -\frac{i\sqrt{1-\frac{x^2}{3}}\sqrt{1+2x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}x\right),-\frac{1}{6}\right)}{\sqrt{2}\sqrt{-3-5x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 - 5*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[1 - x^2/3]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], -1/6
])/(Sqrt[2]*Sqrt[-3 - 5*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1410\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-5x^2-3}}\,dx$$
$$\Big\downarrow \text{1410}$$
$$\frac{\sqrt{x^2-3}\sqrt{2x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{7}x}{\sqrt{x^2-3}}\right),\frac{1}{7}\right)}{\sqrt{7}\sqrt{2x^4-5x^2-3}}$$

input
```
Int[1/Sqrt[-3 - 5*x^2 + 2*x^4],x]
```

output
```
(Sqrt[-3 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt[-3 + x^2
]], 1/7])/(Sqrt[7]*Sqrt[-3 - 5*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{2x^2+1}\,\sqrt{-3x^2+9}\ \mathrm{EllipticF}\left(ix\sqrt{2}, \frac{i\sqrt{6}}{6}\right)}{6\sqrt{2x^4-5x^2-3}}$	53
elliptic	$-\frac{i\sqrt{2}\,\sqrt{2x^2+1}\,\sqrt{-3x^2+9}\ \mathrm{EllipticF}\left(ix\sqrt{2}, \frac{i\sqrt{6}}{6}\right)}{6\sqrt{2x^4-5x^2-3}}$	53

input
```
int(1/(2*x^4-5*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*2^(1/2)*(2*x^2+1)^(1/2)*(-3*x^2+9)^(1/2)/(2*x^4-5*x^2-3)^(1/2)*Elli
pticF(I*2^(1/2)*x,1/6*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx = -\frac{1}{3}\,\sqrt{3}\sqrt{-3}F\left(\arcsin\left(\frac{1}{3}\,\sqrt{3}x\right)\,|-6\right)$$

input
```
integrate(1/(2*x^4-5*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*sqrt(-3)*elliptic_f(arcsin(1/3*sqrt(3)*x), -6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4-5x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4-5*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 5*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-5\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4-5*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 5*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-5\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4-5*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 5*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-5\,x^2-3}}\,dx$$

input
```
int(1/(2*x^4 - 5*x^2 - 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 5*x^2 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-5x^2-3}}{2x^4-5x^2-3}dx$$

input
```
int(1/(2*x^4-5*x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 5*x**2 - 3)/(2*x**4 - 5*x**2 - 3),x)
```

3.147 $\int \frac{1}{\sqrt{-3-6x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 94

$$\int \frac{1}{\sqrt{-3-6x^2+2x^4}}\,dx = \frac{\sqrt{3+\left(3-\sqrt{15}\right)x^2}\sqrt{3+\left(3+\sqrt{15}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}x\right),-4-\sqrt{15}\right)}{\sqrt{3\left(-3+\sqrt{15}\right)}\sqrt{-3-6x^2+2x^4}}$$

output
```
(3+(3-15^(1/2))*x^2)^(1/2)*(3+(3+15^(1/2))*x^2)^(1/2)*EllipticF(1/3*(-9+3*
15^(1/2))^(1/2)*x,1/2*I*6^(1/2)+1/2*I*10^(1/2))/(-9+3*15^(1/2))^(1/2)/(2*x
^4-6*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-3-6x^2+2x^4}}\,dx = -\frac{i\sqrt{3+6x^2-2x^4}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{1+\sqrt{\frac{5}{3}}}x\right),-4+\sqrt{15}\right)}{\sqrt{3+\sqrt{15}}\sqrt{-3-6x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 - 6*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[3 + 6*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/3]]*x],
-4 + Sqrt[15]])/(Sqrt[3 + Sqrt[15]]*Sqrt[-3 - 6*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.65, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1411\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-6x^2-3}}dx$$
$$\Big\downarrow \text{1411}$$
$$\frac{\sqrt{-\left(\left(3-\sqrt{15}\right)x^2\right)-3}\sqrt{\frac{\left(3+\sqrt{15}\right)x^2+3}{\left(3-\sqrt{15}\right)x^2+3}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{15}x}{\sqrt{-\left(\left(3-\sqrt{15}\right)x^2\right)-3}}\right),\frac{1}{10}\left(5-\sqrt{15}\right)\right)}{\sqrt{2}3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{\left(3-\sqrt{15}\right)x^2+3}}\sqrt{2x^4-6x^2-3}}$$

input
```
Int[1/Sqrt[-3 - 6*x^2 + 2*x^4],x]
```

output
```
(Sqrt[-3 - (3 - Sqrt[15])*x^2]*Sqrt[(3 + (3 + Sqrt[15])*x^2)/(3 + (3 - Sqr
t[15])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[-3 - (3 - Sqrt[15]
)*x^2]], (5 - Sqrt[15])/10])/(Sqrt[2]*3^(3/4)*5^(1/4)*Sqrt[(3 + (3 - Sqrt[
15])*x^2)^(-1)]*Sqrt[-3 - 6*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{3\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^{2}}\,\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-9-3\sqrt{15}}\,x}{3},\frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{\sqrt{-9-3\sqrt{15}}\,\sqrt{2x^{4}-6x^{2}-3}}$	84
elliptic	$\frac{3\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^{2}}\,\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-9-3\sqrt{15}}\,x}{3},\frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{\sqrt{-9-3\sqrt{15}}\,\sqrt{2x^{4}-6x^{2}-3}}$	84

input
```
int(1/(2*x^4-6*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/(-9-3*15^(1/2))^(1/2)*(1-(-1-1/3*15^(1/2))*x^2)^(1/2)*(1-(-1+1/3*15^(1/2
))*x^2)^(1/2)/(2*x^4-6*x^2-3)^(1/2)*EllipticF(1/3*(-9-3*15^(1/2))^(1/2)*x,
1/2*I*10^(1/2)-1/2*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{-3-6x^2+2x^4}}\,dx =$$

$$-\frac{1}{2}\left(\sqrt{\frac{5}{3}}\sqrt{-3}+\sqrt{-3}\right)\sqrt{\sqrt{\frac{5}{3}}-1}F(\arcsin\left(x\sqrt{\sqrt{\frac{5}{3}}-1}\right)\mid -3\sqrt{\frac{5}{3}}-4)$$

input
```
integrate(1/(2*x^4-6*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(5/3)*sqrt(-3) + sqrt(-3))*sqrt(sqrt(5/3) - 1)*elliptic_f(arcsin
(x*sqrt(sqrt(5/3) - 1)), -3*sqrt(5/3) - 4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4-6x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4-6*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 6*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-6\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4-6*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 6*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-6\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4-6*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 6*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-6x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-6\,x^2-3}}\,dx$$

input
```
int(1/(2*x^4 - 6*x^2 - 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 6*x^2 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-6x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4-6x^2-3}}{2x^4-6x^2-3}dx$$

input
```
int(1/(2*x^4-6*x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 6*x**2 - 3)/(2*x**4 - 6*x**2 - 3),x)
```

3.148 $\int \frac{1}{\sqrt{-3-7x^2+2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{1}{\sqrt{-3-7x^2+2x^4}}\,dx = \frac{\sqrt{6+\left(7-\sqrt{73}\right)x^2}\sqrt{6+\left(7+\sqrt{73}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{6}\left(-7+\sqrt{73}\right)}x\right),\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)}{\sqrt{6\left(-7+\sqrt{73}\right)}\sqrt{-3-7x^2+2x^4}}$$

output
```
(6+(-73^(1/2)+7)*x^2)^(1/2)*(6+(7+73^(1/2))*x^2)^(1/2)*EllipticF(1/6*(-42+
6*73^(1/2))^(1/2)*x,7/12*I*6^(1/2)+1/12*I*438^(1/2))/(-42+6*73^(1/2))^(1/2
)/(2*x^4-7*x^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{-3-7x^2+2x^4}}\,dx = -\frac{i\sqrt{6+14x^2-4x^4}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right),\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)}{\sqrt{7+\sqrt{73}}\sqrt{-3-7x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[-3 - 7*x^2 + 2*x^4],x]
```

output
```
((-I)*Sqrt[6 + 14*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-7 + Sqrt[73
]]], (-61 + 7*Sqrt[73])/12])/(Sqrt[7 + Sqrt[73]]*Sqrt[-3 - 7*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.56, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1411}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4-7x^2-3}}\,dx$$

$$\Big\downarrow \text{ 1411}$$

$$\frac{\sqrt{-\left(\left(7-\sqrt{73}\right)x^2\right)-6}\sqrt{\frac{\left(7+\sqrt{73}\right)x^2+6}{\left(7-\sqrt{73}\right)x^2+6}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{-\left(\left(7-\sqrt{73}\right)x^2\right)-6}}\right),\frac{1}{146}\left(73-7\sqrt{73}\right)\right)}{2\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{\left(7-\sqrt{73}\right)x^2+6}}\sqrt{2x^4-7x^2-3}}$$

input
```
Int[1/Sqrt[-3 - 7*x^2 + 2*x^4],x]
```

output
```
(Sqrt[-6 - (7 - Sqrt[73])*x^2]*Sqrt[(6 + (7 + Sqrt[73])*x^2)/(6 + (7 - Sqr
t[73])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*73^(1/4)*x)/Sqrt[-6 - (7 - Sqrt[73]
)*x^2]], (73 - 7*Sqrt[73])/146])/(2*Sqrt[3]*73^(1/4)*Sqrt[(6 + (7 - Sqrt[7
3])*x^2)^(-1)]*Sqrt[-3 - 7*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{6\sqrt{1-\left(-\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\ \operatorname{EllipticF}\left(\frac{\sqrt{-42-6\sqrt{73}}\,x}{6},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{\sqrt{-42-6\sqrt{73}}\,\sqrt{2x^4-7x^2-3}}$	84
elliptic	$\frac{6\sqrt{1-\left(-\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\ \operatorname{EllipticF}\left(\frac{\sqrt{-42-6\sqrt{73}}\,x}{6},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{\sqrt{-42-6\sqrt{73}}\,\sqrt{2x^4-7x^2-3}}$	84

input
```
int(1/(2*x^4-7*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(-42-6*73^(1/2))^(1/2)*(1-(-7/6-1/6*73^(1/2))*x^2)^(1/2)*(1-(-7/6+1/6*73
^(1/2))*x^2)^(1/2)/(2*x^4-7*x^2-3)^(1/2)*EllipticF(1/6*(-42-6*73^(1/2))^(1
/2)*x,1/12*I*438^(1/2)-7/12*I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{-3-7x^2+2x^4}}\,dx = -\frac{1}{12}\left(\sqrt{73}\sqrt{-3}+7\sqrt{-3}\right)\sqrt{\frac{1}{6}\sqrt{73}-\frac{7}{6}}F(\arcsin\left(x\sqrt{\frac{1}{6}\sqrt{73}-\frac{7}{6}}\right)|-\frac{7}{12}\sqrt{73}-\frac{61}{12})$$

input
```
integrate(1/(2*x^4-7*x^2-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(73)*sqrt(-3) + 7*sqrt(-3))*sqrt(1/6*sqrt(73) - 7/6)*elliptic_f
(arcsin(x*sqrt(1/6*sqrt(73) - 7/6)), -7/12*sqrt(73) - 61/12)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4-7x^2-3}}\,dx$$

input
```
integrate(1/(2*x**4-7*x**2-3)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 - 7*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-7x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4-7\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4-7*x^2-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 - 7*x^2 - 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3-7x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4-7\,x^2-3}}\,dx$$

input
```
integrate(1/(2*x^4-7*x^2-3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 - 7*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-7x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4-7\,x^2-3}}\,dx$$

input
```
int(1/(2*x^4 - 7*x^2 - 3)^(1/2),x)
```

output
```
int(1/(2*x^4 - 7*x^2 - 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-3-7x^2+2x^4}}\,dx=\int \frac{\sqrt{2x^4-7x^2-3}}{2x^4-7x^2-3}dx$$

input
```
int(1/(2*x^4-7*x^2-3)^(1/2),x)
```

output
```
int(sqrt(2*x**4 - 7*x**2 - 3)/(2*x**4 - 7*x**2 - 3),x)
```

3.149 $\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx = \frac{\left(2+\sqrt{10}x^2\right)\sqrt{\frac{2+5x^2+5x^4}{\left(2+\sqrt{10}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{5}{2}}x\right),\frac{1}{8}\left(4-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{2+5x^2+5x^4}}$$

output
```
1/20*(2+10^(1/2)*x^2)*((5*x^4+5*x^2+2)/(2+10^(1/2)*x^2)^2)^(1/2)*InverseJa
cobiAM(2*arctan(1/2*5^(1/4)*2^(3/4)*x),1/4*(8-2*10^(1/2))^(1/2))*10^(3/4)/
(5*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx = -\frac{i\sqrt{1-\frac{10x^2}{-5-i\sqrt{15}}}\sqrt{1-\frac{10x^2}{-5+i\sqrt{15}}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{10}{-5-i\sqrt{15}}}x\right),\frac{-5-i\sqrt{15}}{-5+i\sqrt{15}}\right)}{\sqrt{10}\sqrt{-\frac{1}{-5-i\sqrt{15}}}\sqrt{2+5x^2+5x^4}}$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 + 5*x^4],x]
```

output
```
((-I)*Sqrt[1 - (10*x^2)/(-5 - I*Sqrt[15])]*Sqrt[1 - (10*x^2)/(-5 + I*Sqrt[
15])]*EllipticF[I*ArcSinh[Sqrt[-10/(-5 - I*Sqrt[15])]*x], (-5 - I*Sqrt[15]
)/(-5 + I*Sqrt[15])])/(Sqrt[10]*Sqrt[-(-5 - I*Sqrt[15])^(-1)]*Sqrt[2 + 5*x
^2 + 5*x^4])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{5x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\left(\sqrt{10}x^2+2\right)\sqrt{\frac{5x^4+5x^2+2}{\left(\sqrt{10}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{5}{2}}x\right),\frac{1}{8}\left(4-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{5x^4+5x^2+2}}$$

input
```
Int[1/Sqrt[2 + 5*x^2 + 5*x^4],x]
```

output
```
((2 + Sqrt[10]*x^2)*Sqrt[(2 + 5*x^2 + 5*x^4)/(2 + Sqrt[10]*x^2)^2]*Ellipti
cF[2*ArcTan[(5/2)^(1/4)*x], (4 - Sqrt[10])/8])/(2*10^(1/4)*Sqrt[2 + 5*x^2
+ 5*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{15}}}{2},\frac{\sqrt{1+i\sqrt{15}}}{2}\right)}{\sqrt{-5+i\sqrt{15}}\,\sqrt{5x^4+5x^2+2}}$	87
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{15}}}{2},\frac{\sqrt{1+i\sqrt{15}}}{2}\right)}{\sqrt{-5+i\sqrt{15}}\,\sqrt{5x^4+5x^2+2}}$	87

input
```
int(1/(5*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-5+I*15^(1/2))^(1/2)*(1-(-5/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*I
*15^(1/2))*x^2)^(1/2)/(5*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+I*15^(1/2)
)^(1/2),1/2*(1+I*15^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx$$
$$= -\frac{1}{40}\sqrt{2}(\sqrt{-15}+5)\sqrt{\sqrt{-15}-5}F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{-15}-5}\right) \mid \frac{1}{4}\sqrt{-15}+\frac{1}{4})$$

input
```
integrate(1/(5*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/40*sqrt(2)*(sqrt(-15) + 5)*sqrt(sqrt(-15) - 5)*elliptic_f(arcsin(1/2*x*
sqrt(sqrt(-15) - 5)), 1/4*sqrt(-15) + 1/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx = \int \frac{1}{\sqrt{5x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(5*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(5*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx = \int \frac{1}{\sqrt{5\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(5*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(5*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx = \int \frac{1}{\sqrt{5\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(5*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(5*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx = \int \frac{1}{\sqrt{5\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 + 5*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 + 5*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx = \int \frac{\sqrt{5x^4+5x^2+2}}{5x^4+5x^2+2}dx$$

input
```
int(1/(5*x^4+5*x^2+2)^(1/2),x)
```

output
```
int(sqrt(5*x**4 + 5*x**2 + 2)/(5*x**4 + 5*x**2 + 2),x)
```

3.150 $\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx=\frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{2+5x^2+4x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{2+5x^2+4x^4}}$$

output
```
1/4*(1+x^2*2^(1/2))*((4*x^4+5*x^2+2)/(1+x^2*2^(1/2))^2)^(1/2)*InverseJacob
iAM(2*arctan(2^(1/4)*x),1/4*(8-5*2^(1/2))^(1/2))*2^(1/4)/(4*x^4+5*x^2+2)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.63

$$\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx =-\frac{i\sqrt{1-\frac{8x^2}{-5-i\sqrt{7}}}\sqrt{1-\frac{8x^2}{-5+i\sqrt{7}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(2\sqrt{-\frac{2}{-5-i\sqrt{7}}}x\right),\frac{-5-i\sqrt{7}}{-5+i\sqrt{7}}\right)}{2\sqrt{2}\sqrt{-\frac{1}{-5-i\sqrt{7}}}\sqrt{2+5x^2+4x^4}}$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 + 4*x^4],x]
```

output
```
((-1/2*I)*Sqrt[1 - (8*x^2)/(-5 - I*Sqrt[7])]*Sqrt[1 - (8*x^2)/(-5 + I*Sqrt
[7])]*EllipticF[I*ArcSinh[2*Sqrt[-2/(-5 - I*Sqrt[7])]*x], (-5 - I*Sqrt[7])
/(-5 + I*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-5 - I*Sqrt[7])^(-1)]*Sqrt[2 + 5*x^2 +
 4*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4x^4+5x^2+2}}\,dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{4x^4+5x^2+2}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{2\ 2^{3/4}\sqrt{4x^4+5x^2+2}}$$

input
```
Int[1/Sqrt[2 + 5*x^2 + 4*x^4],x]
```

output
```
((1 + Sqrt[2]*x^2)*Sqrt[(2 + 5*x^2 + 4*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF
[2*ArcTan[2^(1/4)*x], (8 - 5*Sqrt[2])/16])/(2*2^(3/4)*Sqrt[2 + 5*x^2 + 4*x
^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{7}}{4}\right)x^{2}}\,\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{7}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{7}}}{2},\frac{\sqrt{9+5i\sqrt{7}}}{4}\right)}{\sqrt{-5+i\sqrt{7}}\,\sqrt{4x^{4}+5x^{2}+2}}$	87
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{7}}{4}\right)x^{2}}\,\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{7}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{7}}}{2},\frac{\sqrt{9+5i\sqrt{7}}}{4}\right)}{\sqrt{-5+i\sqrt{7}}\,\sqrt{4x^{4}+5x^{2}+2}}$	87

input
```
int(1/(4*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-5+I*7^(1/2))^(1/2)*(1-(-5/4+1/4*I*7^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*I*7
^(1/2))*x^2)^(1/2)/(4*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+I*7^(1/2))^(1
/2),1/4*(9+5*I*7^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx = -\frac{1}{32}\,\sqrt{2}(\sqrt{-7}+5)\sqrt{\sqrt{-7}-5}F(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{-7}-5}\right) \mid \frac{5}{16}\,\sqrt{-7}+\frac{9}{16})$$

input
```
integrate(1/(4*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/32*sqrt(2)*(sqrt(-7) + 5)*sqrt(sqrt(-7) - 5)*elliptic_f(arcsin(1/2*x*sq
rt(sqrt(-7) - 5)), 5/16*sqrt(-7) + 9/16)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(4*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(4*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(4*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(4*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(4*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(4*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 + 4*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 + 4*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx = \int \frac{\sqrt{4x^4+5x^2+2}}{4x^4+5x^2+2}dx$$

input
```
int(1/(4*x^4+5*x^2+2)^(1/2),x)
```

output
```
int(sqrt(4*x**4 + 5*x**2 + 2)/(4*x**4 + 5*x**2 + 2),x)
```

3.151 $\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx=\frac{\sqrt{1+x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
1/2*(x^2+1)^(1/2)*(3*x^2+2)^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))
*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx=-\frac{i\sqrt{1+x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)}{\sqrt{6+15x^2+9x^4}}$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3]
)/Sqrt[6 + 15*x^2 + 9*x^4]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1413}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+5x^2+2}}\,dx$$
$$\Big\downarrow \text{ 1413}$$
$$\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}$$

input
```
Int[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2
]*Sqrt[2 + 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}$	44
elliptic	$-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}$	44

input
```
int(1/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1
/2*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx = -\frac{1}{2}i\,\sqrt{2}F(\arcsin{(i\,x)}\mid\frac{3}{2})$$

input
```
integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*I*sqrt(2)*elliptic_f(arcsin(I*x), 3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx$$

input
```
int(1/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2),x)
```

3.152 $\int \frac{1}{\sqrt{2+5x^2+2x^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 47

$$\int \frac{1}{\sqrt{2+5x^2+2x^4}} \, dx = \frac{\sqrt{2+x^2}\sqrt{1+2x^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{2}}\right),-3\right)}{\sqrt{2+5x^2+2x^4}}$$

output
```
(x^2+2)^(1/2)*(2*x^2+1)^(1/2)*InverseJacobiAM(arctan(1/2*x*2^(1/2)),I*3^(1
/2))/(2*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{2+5x^2+2x^4}} \, dx = -\frac{i\sqrt{2+x^2}\sqrt{1+2x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}x\right),\frac{1}{4}\right)}{2\sqrt{2+5x^2+2x^4}}$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 + 2*x^4],x]
```

output
```
((-1/2*I)*Sqrt[2 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], 1/
4])/Sqrt[2 + 5*x^2 + 2*x^4]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^4+5x^2+2}}\,dx$$

$$\Big\downarrow \text{ 1412}$$

$$\frac{\sqrt{\frac{x^2+2}{2x^2+1}}\,(2x^2+1)\,\text{EllipticF}\left(\arctan\left(\sqrt{2}x\right),\frac{3}{4}\right)}{2\sqrt{2x^4+5x^2+2}}$$

input
```
Int[1/Sqrt[2 + 5*x^2 + 2*x^4],x]
```

output
```
(Sqrt[(2 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]*x], 3/4]
)/(2*Sqrt[2 + 5*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{2x^{2}+1}\ \operatorname{EllipticF}\left(\frac{ix\sqrt{2}}{2},2\right)}{2\sqrt{2x^{4}+5x^{2}+2}}$	48
elliptic	$-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{2x^{2}+1}\ \operatorname{EllipticF}\left(\frac{ix\sqrt{2}}{2},2\right)}{2\sqrt{2x^{4}+5x^{2}+2}}$	48

input
```
int(1/(2*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+5*x^2+2)^(1/2)*Ellip
ticF(1/2*I*x*2^(1/2),2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{2+5x^2+2x^4}}\,dx = -\sqrt{2}\sqrt{-\frac{1}{2}}F\left(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)\mid 4\right)$$

input
```
integrate(1/(2*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(2)*sqrt(-1/2)*elliptic_f(arcsin(sqrt(-1/2)*x), 4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(2*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(2*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(2*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(2*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 + 2*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 + 2*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2+2x^4}}\,dx = \int \frac{\sqrt{2x^4+5x^2+2}}{2x^4+5x^2+2}dx$$

input
```
int(1/(2*x^4+5*x^2+2)^(1/2),x)
```

output
```
int(sqrt(2*x**4 + 5*x**2 + 2)/(2*x**4 + 5*x**2 + 2),x)
```

3.153 $\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 100

$$\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx = \frac{\sqrt{4+\left(5-\sqrt{17}\right)x^2}\sqrt{4+\left(5+\sqrt{17}\right)x^2}\,\text{EllipticF}\left(\arctan\left(\frac{1}{2}\sqrt{5-\sqrt{17}}x\right),\frac{1}{4}\left(-17-5\sqrt{17}\right)\right)}{2\sqrt{5-\sqrt{17}}\sqrt{2+5x^2+x^4}}$$

output
```
1/2*(4+(5-17^(1/2))*x^2)^(1/2)*(4+(5+17^(1/2))*x^2)^(1/2)*InverseJacobiAM(
arctan(1/2*(5-17^(1/2))^(1/2)*x),1/2*(-17-5*17^(1/2))^(1/2))/(5-17^(1/2))^
(1/2)/(x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx = -\frac{i\sqrt{5-\sqrt{17}+2x^2}\sqrt{5+\sqrt{17}+2x^2}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{17}}}x\right),\frac{21}{4}+\frac{5\sqrt{17}}{4}\right)}{\sqrt{2\left(5-\sqrt{17}\right)}\sqrt{2+5x^2+x^4}}$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 + x^4],x]
```

output
```
((-I)*Sqrt[5 - Sqrt[17] + 2*x^2]*Sqrt[5 + Sqrt[17] + 2*x^2]*EllipticF[I*Ar
cSinh[Sqrt[2/(5 + Sqrt[17])]*x], 21/4 + (5*Sqrt[17])/4])/(Sqrt[2*(5 - Sqrt
[17])]*Sqrt[2 + 5*x^2 + x^4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^4+5x^2+2}}\,dx$$

$$\Big\downarrow \text{1412}$$

$$\frac{\sqrt{\frac{\left(5-\sqrt{17}\right)x^2+4}{\left(5+\sqrt{17}\right)x^2+4}}\left(\left(5+\sqrt{17}\right)x^2+4\right)\text{EllipticF}\left(\arctan\left(\frac{1}{2}\sqrt{5+\sqrt{17}}x\right),\frac{1}{4}\left(-17+5\sqrt{17}\right)\right)}{2\sqrt{5+\sqrt{17}}\sqrt{x^4+5x^2+2}}$$

input
```
Int[1/Sqrt[2 + 5*x^2 + x^4],x]
```

output
```
(Sqrt[(4 + (5 - Sqrt[17])*x^2)/(4 + (5 + Sqrt[17])*x^2)]*(4 + (5 + Sqrt[17
])*x^2)*EllipticF[ArcTan[(Sqrt[5 + Sqrt[17]]*x)/2], (-17 + 5*Sqrt[17])/4])
/(2*Sqrt[5 + Sqrt[17]]*Sqrt[2 + 5*x^2 + x^4])
```

Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{17}}{4}\right)x^{2}}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{17}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{17}}}{2},\frac{5\sqrt{2}}{4}+\frac{\sqrt{34}}{4}\right)}{\sqrt{-5+\sqrt{17}}\,\sqrt{x^{4}+5x^{2}+2}}$	76
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{17}}{4}\right)x^{2}}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{17}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{17}}}{2},\frac{5\sqrt{2}}{4}+\frac{\sqrt{34}}{4}\right)}{\sqrt{-5+\sqrt{17}}\,\sqrt{x^{4}+5x^{2}+2}}$	76

input
```
int(1/(x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-5+17^(1/2))^(1/2)*(1-(-5/4+1/4*17^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*17^(1
/2))*x^2)^(1/2)/(x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+17^(1/2))^(1/2),5/
4*2^(1/2)+1/4*34^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx = -\frac{1}{8}\left(\sqrt{17}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{17}-5}F(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{17}-5}\right)\mid\frac{5}{4}\sqrt{17}+\frac{21}{4})$$

input
```
integrate(1/(x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(17)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(17) - 5)*elliptic_f(arcsin(1
/2*x*sqrt(sqrt(17) - 5)), 5/4*sqrt(17) + 21/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 + x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 + x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx=\int \frac{\sqrt{x^4+5x^2+2}}{x^4+5x^2+2}dx$$

input
```
int(1/(x^4+5*x^2+2)^(1/2),x)
```

output
```
int(sqrt(x**4 + 5*x**2 + 2)/(x**4 + 5*x**2 + 2),x)
```

3.154 $\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx = \sqrt{\frac{2}{-5+\sqrt{33}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{5+\sqrt{33}}}x\right),\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)$$

output
```
2^(1/2)/(-5+33^(1/2))^(1/2)*EllipticF(2^(1/2)/(5+33^(1/2))^(1/2)*x,5/4*I*2
^(1/2)+1/4*I*66^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx = -i\sqrt{\frac{2}{5+\sqrt{33}}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{-5+\sqrt{33}}}x\right),-\frac{29}{4}+\frac{5\sqrt{33}}{4}\right)$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 - x^4],x]
```

output
```
(-I)*Sqrt[2/(5 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[2/(-5 + Sqrt[33])]*x]
, -29/4 + (5*Sqrt[33])/4]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1408, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{ 1408}$$

$$2\int \frac{1}{\sqrt{-2x^2+\sqrt{33}+5}\sqrt{2x^2+\sqrt{33}-5}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\sqrt{\frac{2}{\sqrt{33}-5}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{5+\sqrt{33}}}x\right),\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)$$

input
```
Int[1/Sqrt[2 + 5*x^2 - x^4],x]
```

output
```
Sqrt[2/(-5 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[2/(5 + Sqrt[33])]*x], (-29 -
 5*Sqrt[33])/4]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(37) = 74$.

Time = 0.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{33}}{4}\right)x^{2}}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{33}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{33}}}{2},\frac{5i\sqrt{2}}{4}+\frac{i\sqrt{66}}{4}\right)}{\sqrt{-5+\sqrt{33}}\,\sqrt{-x^{4}+5x^{2}+2}}$	80
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{33}}{4}\right)x^{2}}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{33}}{4}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{33}}}{2},\frac{5i\sqrt{2}}{4}+\frac{i\sqrt{66}}{4}\right)}{\sqrt{-5+\sqrt{33}}\,\sqrt{-x^{4}+5x^{2}+2}}$	80

input
```
int(1/(-x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-5+33^(1/2))^(1/2)*(1-(-5/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*33^(1
/2))*x^2)^(1/2)/(-x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+33^(1/2))^(1/2),5
/4*I*2^(1/2)+1/4*I*66^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx = \frac{1}{8}\left(\sqrt{33}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{33}-5}F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{33}-5}\right)\mid -\frac{5}{4}\sqrt{33}-\frac{29}{4})$$

input
```
integrate(1/(-x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(33)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(33) - 5)*elliptic_f(arcsin(1/
2*x*sqrt(sqrt(33) - 5)), -5/4*sqrt(33) - 29/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(-x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 - x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 - x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4+5x^2+2}}{x^4-5x^2-2}dx\right)$$

input
```
int(1/(-x^4+5*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - x**4 + 5*x**2 + 2)/(x**4 - 5*x**2 - 2),x)
```

3.155 $\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx = \sqrt{\frac{2}{-5+\sqrt{41}}}\,\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right),\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

output
```
2^(1/2)/(-5+41^(1/2))^(1/2)*EllipticF(2*x/(5+41^(1/2))^(1/2),5/4*I+1/4*I*4
1^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx = -i\sqrt{\frac{2}{5+\sqrt{41}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{2x}{\sqrt{-5+\sqrt{41}}}\right),-\frac{33}{8}+\frac{5\sqrt{41}}{8}\right)$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 - 2*x^4],x]
```

output
```
(-I)*Sqrt[2/(5 + Sqrt[41])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-5 + Sqrt[41]]]
, -33/8 + (5*Sqrt[41])/8]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1408, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{2}\int \frac{1}{\sqrt{-4x^2+\sqrt{41}+5}\sqrt{4x^2+\sqrt{41}-5}}dx$$

$$\Big\downarrow \text{321}$$

$$\sqrt{\frac{2}{\sqrt{41}-5}}\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right),\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

input
```
Int[1/Sqrt[2 + 5*x^2 - 2*x^4],x]
```

output
```
Sqrt[2/(-5 + Sqrt[41])]*EllipticF[ArcSin[(2*x)/Sqrt[5 + Sqrt[41]]], (-33 -
 5*Sqrt[41])/8]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.

Time = 0.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{41}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{41}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{41}}}{2},\frac{5i}{4}+\frac{i\sqrt{41}}{4}\right)}{\sqrt{-5+\sqrt{41}}\,\sqrt{-2x^4+5x^2+2}}$	76
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{41}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{41}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{41}}}{2},\frac{5i}{4}+\frac{i\sqrt{41}}{4}\right)}{\sqrt{-5+\sqrt{41}}\,\sqrt{-2x^4+5x^2+2}}$	76

input
```
int(1/(-2*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-5+41^(1/2))^(1/2)*(1-(-5/4+1/4*41^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*41^(1
/2))*x^2)^(1/2)/(-2*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+41^(1/2))^(1/2)
,5/4*I+1/4*I*41^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx = \frac{1}{16}\left(\sqrt{41}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{41}-5}F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{41}-5}\right)| -\frac{5}{8}\sqrt{41}-\frac{33}{8})$$

input
```
integrate(1/(-2*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/16*(sqrt(41)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(41) - 5)*elliptic_f(arcsin(1
/2*x*sqrt(sqrt(41) - 5)), -5/8*sqrt(41) - 33/8)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(-2*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-2*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-2*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-2*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-2*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-2*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx = \int \frac{1}{\sqrt{-2\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 - 2*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 - 2*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx = -\left(\int \frac{\sqrt{-2x^4+5x^2+2}}{2x^4-5x^2-2}dx\right)$$

input
```
int(1/(-2*x^4+5*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 2*x**4 + 5*x**2 + 2)/(2*x**4 - 5*x**2 - 2),x)
```

3.156 $\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = \text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-6\right)$$

output
```
EllipticF(1/2*x*2^(1/2),I*6^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 6.50

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = -\frac{i\sqrt{1-\frac{x^2}{2}}\sqrt{1+3x^2}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{3}x\right),-\frac{1}{6}\right)}{\sqrt{3}\sqrt{2+5x^2-3x^4}}$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

output
```
((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], -1/6
])/(Sqrt[3]*Sqrt[2 + 5*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{3}\int \frac{1}{2\sqrt{3}\sqrt{2-x^2}\sqrt{3x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{3x^2+1}}dx$$

$$\Big\downarrow \text{321}$$

$$\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-6\right)$$

input
```
Int[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

output
```
EllipticF[ArcSin[x/Sqrt[2]], -6]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(13) = 26$.

Time = 0.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 5.10

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{6}\right)}{2\sqrt{-3x^4+5x^2+2}}$	51
elliptic	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{6}\right)}{2\sqrt{-3x^4+5x^2+2}}$	51

input
```
int(1/(-3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/2)*Ellipt
icF(1/2*x*2^(1/2),I*6^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = F\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\,|-6\right)$$

input
```
integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
elliptic_f(arcsin(1/2*sqrt(2)*x), -6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(-3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-3*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = \int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx=\int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 - 3*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 - 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx=-\left(\int \frac{\sqrt{-3x^4+5x^2+2}}{3x^4-5x^2-2}dx\right)$$

input
```
int(1/(-3*x^4+5*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 3*x**4 + 5*x**2 + 2)/(3*x**4 - 5*x**2 - 2),x)
```

3.157 $\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx=\sqrt{\frac{2}{-5+\sqrt{57}}}\,\text{EllipticF}\left(\arcsin\left(2\sqrt{\frac{2}{5+\sqrt{57}}}x\right),\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)$$

output
```
2^(1/2)/(-5+57^(1/2))^(1/2)*EllipticF(2*2^(1/2)/(5+57^(1/2))^(1/2)*x,5/8*I
*2^(1/2)+1/8*I*114^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx=-i\sqrt{\frac{2}{5+\sqrt{57}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(2\sqrt{\frac{2}{-5+\sqrt{57}}}x\right),\frac{1}{16}\left(-41+5\sqrt{57}\right)\right)$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 - 4*x^4],x]
```

output
```
(-I)*Sqrt[2/(5 + Sqrt[57])]*EllipticF[I*ArcSinh[2*Sqrt[2/(-5 + Sqrt[57])]*
x], (-41 + 5*Sqrt[57])/16]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1408, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-4x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{1408}$$

$$4\int \frac{1}{\sqrt{-8x^2+\sqrt{57}+5}\sqrt{8x^2+\sqrt{57}-5}}dx$$

$$\Big\downarrow \text{321}$$

$$\sqrt{\frac{2}{\sqrt{57}-5}}\text{EllipticF}\left(\arcsin\left(2\sqrt{\frac{2}{5+\sqrt{57}}}x\right),\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)$$

input
```
Int[1/Sqrt[2 + 5*x^2 - 4*x^4],x]
```

output
```
Sqrt[2/(-5 + Sqrt[57])]*EllipticF[ArcSin[2*Sqrt[2/(5 + Sqrt[57])]*x], (-41
 - 5*Sqrt[57])/16]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(38) = 76$.

Time = 0.57 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{57}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{57}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{57}}}{2},\frac{5i\sqrt{2}}{8}+\frac{i\sqrt{114}}{8}\right)}{\sqrt{-5+\sqrt{57}}\,\sqrt{-4x^4+5x^2+2}}$	80
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{57}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{57}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{57}}}{2},\frac{5i\sqrt{2}}{8}+\frac{i\sqrt{114}}{8}\right)}{\sqrt{-5+\sqrt{57}}\,\sqrt{-4x^4+5x^2+2}}$	80

input
```
int(1/(-4*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-5+57^(1/2))^(1/2)*(1-(-5/4+1/4*57^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*57^(1
/2))*x^2)^(1/2)/(-4*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+57^(1/2))^(1/2)
,5/8*I*2^(1/2)+1/8*I*114^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx = \frac{1}{32}\left(\sqrt{57}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{57}-5}F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{57}-5}\right) \mid -\frac{5}{16}\sqrt{57}-\frac{41}{16})$$

input
```
integrate(1/(-4*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/32*(sqrt(57)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(57) - 5)*elliptic_f(arcsin(1
/2*x*sqrt(sqrt(57) - 5)), -5/16*sqrt(57) - 41/16)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx = \int \frac{1}{\sqrt{-4x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(-4*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-4*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx = \int \frac{1}{\sqrt{-4\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-4*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-4*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx = \int \frac{1}{\sqrt{-4\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-4*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-4*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx = \int \frac{1}{\sqrt{-4\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 - 4*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 - 4*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx = -\left(\int \frac{\sqrt{-4x^4+5x^2+2}}{4x^4-5x^2-2}dx\right)$$

input
```
int(1/(-4*x^4+5*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 4*x**4 + 5*x**2 + 2)/(4*x**4 - 5*x**2 - 2),x)
```

3.158 $\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx = \sqrt{\frac{2}{-5+\sqrt{65}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{10}{5+\sqrt{65}}}x\right),\frac{1}{4}\left(-9-\sqrt{65}\right)\right)$$

output
```
2^(1/2)/(-5+65^(1/2))^(1/2)*EllipticF(10^(1/2)/(5+65^(1/2))^(1/2)*x,1/4*I*
10^(1/2)+1/4*I*26^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx = -i\sqrt{\frac{2}{5+\sqrt{65}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{1}{2}\sqrt{5+\sqrt{65}}x\right),\frac{1}{4}\left(-9+\sqrt{65}\right)\right)$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 - 5*x^4],x]
```

output
```
(-I)*Sqrt[2/(5 + Sqrt[65])]*EllipticF[I*ArcSinh[(Sqrt[5 + Sqrt[65]]*x)/2],
 (-9 + Sqrt[65])/4]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1408, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-5x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{ 1408}$$

$$2\sqrt{5}\int \frac{1}{\sqrt{-10x^2+\sqrt{65}+5}\sqrt{10x^2+\sqrt{65}-5}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\sqrt{\frac{2}{\sqrt{65}-5}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{10}{5+\sqrt{65}}}x\right),\frac{1}{4}\left(-9-\sqrt{65}\right)\right)$$

input
```
Int[1/Sqrt[2 + 5*x^2 - 5*x^4],x]
```

output
```
Sqrt[2/(-5 + Sqrt[65])]*EllipticF[ArcSin[Sqrt[10/(5 + Sqrt[65])]*x], (-9 -
 Sqrt[65])/4]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(37) = 74$.

Time = 0.63 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{65}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{65}}{4}\right)x^2}\ \operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{65}}}{2},\frac{i\sqrt{10}}{4}+\frac{i\sqrt{26}}{4}\right)}{\sqrt{-5+\sqrt{65}}\,\sqrt{-5x^4+5x^2+2}}$	80
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{65}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{65}}{4}\right)x^2}\ \operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{65}}}{2},\frac{i\sqrt{10}}{4}+\frac{i\sqrt{26}}{4}\right)}{\sqrt{-5+\sqrt{65}}\,\sqrt{-5x^4+5x^2+2}}$	80

input
```
int(1/(-5*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-5+65^(1/2))^(1/2)*(1-(-5/4+1/4*65^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*65^(1
/2))*x^2)^(1/2)/(-5*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+65^(1/2))^(1/2)
,1/4*I*10^(1/2)+1/4*I*26^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx = \frac{1}{40}\left(\sqrt{65}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{65}-5}F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{65}-5}\right)| -\frac{1}{4}\sqrt{65}-\frac{9}{4})$$

input
```
integrate(1/(-5*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/40*(sqrt(65)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(65) - 5)*elliptic_f(arcsin(1
/2*x*sqrt(sqrt(65) - 5)), -1/4*sqrt(65) - 9/4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx = \int \frac{1}{\sqrt{-5x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(-5*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-5*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx = \int \frac{1}{\sqrt{-5\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-5*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-5*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx = \int \frac{1}{\sqrt{-5\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-5*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-5*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx = \int \frac{1}{\sqrt{-5\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 - 5*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 - 5*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx = -\left(\int \frac{\sqrt{-5x^4+5x^2+2}}{5x^4-5x^2-2}dx\right)$$

input
```
int(1/(-5*x^4+5*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 5*x**4 + 5*x**2 + 2)/(5*x**4 - 5*x**2 - 2),x)
```

3.159 $\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx=\sqrt{\frac{2}{-5+\sqrt{73}}}\,\text{EllipticF}\left(\arcsin\left(2\sqrt{\frac{3}{5+\sqrt{73}}}x\right),\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)$$

output
```
2^(1/2)/(-5+73^(1/2))^(1/2)*EllipticF(2*3^(1/2)/(5+73^(1/2))^(1/2)*x,5/12*
I*3^(1/2)+1/12*I*219^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx=-i\sqrt{\frac{2}{5+\sqrt{73}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(2\sqrt{\frac{3}{-5+\sqrt{73}}}x\right),\frac{1}{24}\left(-49+5\sqrt{73}\right)\right)$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 - 6*x^4],x]
```

output
```
(-I)*Sqrt[2/(5 + Sqrt[73])]*EllipticF[I*ArcSinh[2*Sqrt[3/(-5 + Sqrt[73])]*
x], (-49 + 5*Sqrt[73])/24]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1408, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-6x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{1408}$$

$$2\sqrt{6}\int \frac{1}{\sqrt{-12x^2+\sqrt{73}+5}\sqrt{12x^2+\sqrt{73}-5}}dx$$

$$\Big\downarrow \text{321}$$

$$\sqrt{\frac{2}{\sqrt{73}-5}}\text{EllipticF}\left(\arcsin\left(2\sqrt{\frac{3}{5+\sqrt{73}}}x\right),\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)$$

input
```
Int[1/Sqrt[2 + 5*x^2 - 6*x^4],x]
```

output
```
Sqrt[2/(-5 + Sqrt[73])]*EllipticF[ArcSin[2*Sqrt[3/(5 + Sqrt[73])]*x], (-49
 - 5*Sqrt[73])/24]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(38) = 76.

Time = 0.63 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{73}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{73}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{73}}}{2},\frac{5i\sqrt{3}}{12}+\frac{i\sqrt{219}}{12}\right)}{\sqrt{-5+\sqrt{73}}\,\sqrt{-6x^4+5x^2+2}}$	80
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{73}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{73}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{73}}}{2},\frac{5i\sqrt{3}}{12}+\frac{i\sqrt{219}}{12}\right)}{\sqrt{-5+\sqrt{73}}\,\sqrt{-6x^4+5x^2+2}}$	80

input
```
int(1/(-6*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-5+73^(1/2))^(1/2)*(1-(-5/4+1/4*73^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*73^(1
/2))*x^2)^(1/2)/(-6*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+73^(1/2))^(1/2)
,5/12*I*3^(1/2)+1/12*I*219^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx = \frac{1}{48}\left(\sqrt{73}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{73}-5}F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{73}-5}\right)\mid -\frac{5}{24}\sqrt{73}-\frac{49}{24})$$

input
```
integrate(1/(-6*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/48*(sqrt(73)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(73) - 5)*elliptic_f(arcsin(1
/2*x*sqrt(sqrt(73) - 5)), -5/24*sqrt(73) - 49/24)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx = \int \frac{1}{\sqrt{-6x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(-6*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-6*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx = \int \frac{1}{\sqrt{-6\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-6*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-6*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx = \int \frac{1}{\sqrt{-6\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-6*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-6*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx = \int \frac{1}{\sqrt{-6\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 - 6*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 - 6*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx = -\left(\int \frac{\sqrt{-6x^4+5x^2+2}}{6x^4-5x^2-2}dx\right)$$

input
```
int(1/(-6*x^4+5*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 6*x**4 + 5*x**2 + 2)/(6*x**4 - 5*x**2 - 2),x)
```

3.160 $\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),-\frac{7}{2}\right)}{\sqrt{2}}$$

output
```
1/2*EllipticF(x,1/2*I*14^(1/2))*2^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 5.42

$$\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx = -\frac{i\sqrt{1-x^2}\sqrt{2+7x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{7}{2}}x\right),-\frac{2}{7}\right)}{\sqrt{7}\sqrt{2+5x^2-7x^4}}$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 - 7*x^4],x]
```

output
```
((-I)*Sqrt[1 - x^2]*Sqrt[2 + 7*x^2]*EllipticF[I*ArcSinh[Sqrt[7/2]*x], -2/7
])/(Sqrt[7]*Sqrt[2 + 5*x^2 - 7*x^4])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-7x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{ 1408}$$

$$2\sqrt{7}\int \frac{1}{2\sqrt{7}\sqrt{1-x^2}\sqrt{7x^2+2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{7x^2+2}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\text{EllipticF}\left(\arcsin(x),-\frac{7}{2}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[2 + 5*x^2 - 7*x^4],x]
```

output
```
EllipticF[ArcSin[x], -7/2]/Sqrt[2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(13) = 26$.

Time = 0.59 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{14x^2+4}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{14}}{2}\right)}{2\sqrt{-7x^4+5x^2+2}}$	43
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{14x^2+4}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{14}}{2}\right)}{2\sqrt{-7x^4+5x^2+2}}$	43

input
```
int(1/(-7*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-x^2+1)^(1/2)*(14*x^2+4)^(1/2)/(-7*x^4+5*x^2+2)^(1/2)*EllipticF(x,1/2
*I*14^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin{(x)}\mid -\frac{7}{2})$$

input
```
integrate(1/(-7*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*elliptic_f(arcsin(x), -7/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx = \int \frac{1}{\sqrt{-7x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(-7*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-7*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx = \int \frac{1}{\sqrt{-7\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-7*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-7*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx = \int \frac{1}{\sqrt{-7\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-7*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-7*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx = \int \frac{1}{\sqrt{-7\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 - 7*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 - 7*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx = -\left(\int \frac{\sqrt{-7x^4+5x^2+2}}{7x^4-5x^2-2}dx\right)$$

input
```
int(1/(-7*x^4+5*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 7*x**4 + 5*x**2 + 2)/(7*x**4 - 5*x**2 - 2),x)
```

3.161 $\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx=\sqrt{\frac{2}{-5+\sqrt{89}}}\,\text{EllipticF}\left(\arcsin\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right),\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

output
```
2^(1/2)/(-5+89^(1/2))^(1/2)*EllipticF(4*x/(5+89^(1/2))^(1/2),5/8*I+1/8*I*8
9^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx=-i\sqrt{\frac{2}{5+\sqrt{89}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{4x}{\sqrt{-5+\sqrt{89}}}\right),\frac{1}{32}\left(-57+5\sqrt{89}\right)\right)$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 - 8*x^4],x]
```

output
```
(-I)*Sqrt[2/(5 + Sqrt[89])]*EllipticF[I*ArcSinh[(4*x)/Sqrt[-5 + Sqrt[89]]]
, (-57 + 5*Sqrt[89])/32]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1408, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-8x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{1408}$$

$$4\sqrt{2}\int \frac{1}{\sqrt{-16x^2+\sqrt{89}+5}\sqrt{16x^2+\sqrt{89}-5}}dx$$

$$\Big\downarrow \text{321}$$

$$\sqrt{\frac{2}{\sqrt{89}-5}}\,\text{EllipticF}\left(\arcsin\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right),\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

input
```
Int[1/Sqrt[2 + 5*x^2 - 8*x^4],x]
```

output
```
Sqrt[2/(-5 + Sqrt[89])]*EllipticF[ArcSin[(4*x)/Sqrt[5 + Sqrt[89]]], (-57 -
 5*Sqrt[89])/32]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.

Time = 0.54 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{89}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{89}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{89}}}{2},\frac{5i}{8}+\frac{i\sqrt{89}}{8}\right)}{\sqrt{-5+\sqrt{89}}\,\sqrt{-8x^4+5x^2+2}}$	76
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{89}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{89}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{89}}}{2},\frac{5i}{8}+\frac{i\sqrt{89}}{8}\right)}{\sqrt{-5+\sqrt{89}}\,\sqrt{-8x^4+5x^2+2}}$	76

input
```
int(1/(-8*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-5+89^(1/2))^(1/2)*(1-(-5/4+1/4*89^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*89^(1
/2))*x^2)^(1/2)/(-8*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+89^(1/2))^(1/2)
,5/8*I+1/8*I*89^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx = \frac{1}{64}\left(\sqrt{89}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{89}-5}F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{89}-5}\right)\mid -\frac{5}{32}\sqrt{89}-\frac{57}{32})$$

input
```
integrate(1/(-8*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/64*(sqrt(89)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(89) - 5)*elliptic_f(arcsin(1
/2*x*sqrt(sqrt(89) - 5)), -5/32*sqrt(89) - 57/32)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx = \int \frac{1}{\sqrt{-8x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(-8*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-8*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx = \int \frac{1}{\sqrt{-8\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-8*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-8*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx = \int \frac{1}{\sqrt{-8\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-8*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-8*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx = \int \frac{1}{\sqrt{-8\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 - 8*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 - 8*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx = -\left(\int \frac{\sqrt{-8x^4+5x^2+2}}{8x^4-5x^2-2}dx\right)$$

input
```
int(1/(-8*x^4+5*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 8*x**4 + 5*x**2 + 2)/(8*x**4 - 5*x**2 - 2),x)
```

3.162 $\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx=\sqrt{\frac{2}{-5+\sqrt{97}}}\,\text{EllipticF}\left(\arcsin\left(3\sqrt{\frac{2}{5+\sqrt{97}}}x\right),\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)$$

output
```
2^(1/2)/(-5+97^(1/2))^(1/2)*EllipticF(3*2^(1/2)/(5+97^(1/2))^(1/2)*x,5/12*
I*2^(1/2)+1/12*I*194^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx=-i\sqrt{\frac{2}{5+\sqrt{97}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(3\sqrt{\frac{2}{-5+\sqrt{97}}}x\right),\frac{1}{36}\left(-61+5\sqrt{97}\right)\right)$$

input
```
Integrate[1/Sqrt[2 + 5*x^2 - 9*x^4],x]
```

output
```
(-I)*Sqrt[2/(5 + Sqrt[97])]*EllipticF[I*ArcSinh[3*Sqrt[2/(-5 + Sqrt[97])]*
x], (-61 + 5*Sqrt[97])/36]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1408, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-9x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{ 1408}$$

$$6\int \frac{1}{\sqrt{-18x^2+\sqrt{97}+5}\sqrt{18x^2+\sqrt{97}-5}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\sqrt{\frac{2}{\sqrt{97}-5}}\text{EllipticF}\left(\arcsin\left(3\sqrt{\frac{2}{5+\sqrt{97}}}x\right),\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)$$

input
```
Int[1/Sqrt[2 + 5*x^2 - 9*x^4],x]
```

output
```
Sqrt[2/(-5 + Sqrt[97])]*EllipticF[ArcSin[3*Sqrt[2/(5 + Sqrt[97])]*x], (-61
 - 5*Sqrt[97])/36]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(38) = 76.

Time = 0.59 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{97}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{97}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{97}}}{2},\frac{5i\sqrt{2}}{12}+\frac{i\sqrt{194}}{12}\right)}{\sqrt{-5+\sqrt{97}}\,\sqrt{-9x^4+5x^2+2}}$	80
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{97}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{97}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{97}}}{2},\frac{5i\sqrt{2}}{12}+\frac{i\sqrt{194}}{12}\right)}{\sqrt{-5+\sqrt{97}}\,\sqrt{-9x^4+5x^2+2}}$	80

input
```
int(1/(-9*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-5+97^(1/2))^(1/2)*(1-(-5/4+1/4*97^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*97^(1
/2))*x^2)^(1/2)/(-9*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+97^(1/2))^(1/2)
,5/12*I*2^(1/2)+1/12*I*194^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx = \frac{1}{72}\left(\sqrt{97}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{97}-5}F(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{97}-5}\right)\mid -\frac{5}{36}\sqrt{97}-\frac{61}{36})$$

input
```
integrate(1/(-9*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/72*(sqrt(97)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(97) - 5)*elliptic_f(arcsin(1
/2*x*sqrt(sqrt(97) - 5)), -5/36*sqrt(97) - 61/36)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx = \int \frac{1}{\sqrt{-9x^4+5x^2+2}}\,dx$$

input
```
integrate(1/(-9*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-9*x**4 + 5*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx = \int \frac{1}{\sqrt{-9\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-9*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-9*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx = \int \frac{1}{\sqrt{-9\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(1/(-9*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-9*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx = \int \frac{1}{\sqrt{-9\,x^4+5\,x^2+2}}\,dx$$

input
```
int(1/(5*x^2 - 9*x^4 + 2)^(1/2),x)
```

output
```
int(1/(5*x^2 - 9*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx = -\left(\int \frac{\sqrt{-9x^4+5x^2+2}}{9x^4-5x^2-2}dx\right)$$

input
```
int(1/(-9*x^4+5*x^2+2)^(1/2),x)
```

output
```
- int(sqrt( - 9*x**4 + 5*x**2 + 2)/(9*x**4 - 5*x**2 - 2),x)
```

3.163 $\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 114

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output
```
1/2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*
InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2
))/a^(1/4)/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.63

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[1/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c
/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

input
```
Int[1/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$	144
elliptic	$\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$	144

input
```
int(1/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{\frac{1}{2}}\left(a\sqrt{\frac{b^2-4\,ac}{a^2}}+b\right)\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}F(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}\right)\mid\frac{ab\sqrt{\frac{b^2-4\,ac}{a^2}}+b^2-2\,ac}{2\,ac})}{2\,\sqrt{ac}}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*(a*sqrt((b^2 - 4*a*c)/a^2) + b)*sqrt((a*sqrt((b^2 - 4*a*c)/
a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2)
 - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c))/(sqrt(a)
*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(1/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int(1/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)
```

3.164 $\int \frac{1}{\sqrt{a-bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 115

$$\int \frac{1}{\sqrt{a-bx^2+cx^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a-bx^2+cx^4}}$$

output
```
1/2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4-b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*
InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2
))/a^(1/4)/c^(1/4)/(c*x^4-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{a-bx^2+cx^4}}\,dx = -\frac{i\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b-\sqrt{b^2-4ac}}}x\right),\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{-\frac{c}{b-\sqrt{b^2-4ac}}}\sqrt{a-bx^2+cx^4}}$$

input
```
Integrate[1/Sqrt[a - b*x^2 + c*x^4],x]
```

output
```
((-I)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b - Sqrt[b^2 - 4
*a*c]))]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sq
rt[-(c/(b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a - b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 1416$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a-bx^2+cx^4}}$$

input
```
Int[1/Sqrt[a - b*x^2 + c*x^4],x]
```

output
```
((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/
(2*a^(1/4)*c^(1/4)*Sqrt[a - b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{\sqrt{2}\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(-b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}-b\,x^{2}+a}}$	143
elliptic	$\frac{\sqrt{2}\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(-b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}-b\,x^{2}+a}}$	143

input
```
int(1/(c*x^4-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4-b*x^2+a)^(1/2
)*EllipticF(1/2*x*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(-b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{a-bx^2+cx^4}}\,dx = -\frac{\sqrt{\frac{1}{2}}\left(a\sqrt{\frac{b^2-4\,ac}{a^2}}-b\right)\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}+b}{a}}F(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}+b}{a}}\right) \mid -\frac{ab\sqrt{\frac{b^2-4\,ac}{a^2}}-b^2+2\,ac}{2\,ac})}{2\,\sqrt{ac}}$$

input
```
integrate(1/(c*x^4-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*(a*sqrt((b^2 - 4*a*c)/a^2) - b)*sqrt((a*sqrt((b^2 - 4*a*c)/
a^2) + b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2)
 + b)/a)), -1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) - b^2 + 2*a*c)/(a*c))/(sqrt(a
)*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{a-bx^2+cx^4}}\,dx$$

input
```
integrate(1/(c*x**4-b*x**2+a)**(1/2),x)
```

output
```
Integral(1/sqrt(a - b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4-bx^2+a}}\,dx$$

input
```
integrate(1/(c*x^4-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 - b*x^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4-bx^2+a}}\,dx$$

input
```
integrate(1/(c*x^4-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^4 - b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{c\,x^4-b\,x^2+a}}\,dx$$

input
```
int(1/(a - b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(1/(a - b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a-bx^2+cx^4}}\,dx=\int \frac{\sqrt{c\,x^4-b\,x^2+a}}{c\,x^4-b\,x^2+a}dx$$

input
```
int(1/(c*x^4-b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a - b*x**2 + c*x**4)/(a - b*x**2 + c*x**4),x)
```

3.165 $\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 117

$$\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{-a+bx^2-cx^4}}$$

output
```
1/2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4-b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*
InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2
))/a^(1/4)/c^(1/4)/(-c*x^4+b*x^2-a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx = -\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}x\right),-\frac{b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{-a+bx^2-cx^4}}$$

input
```
Integrate[1/Sqrt[-a + b*x^2 - c*x^4],x]
```

output
```
((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 -
4*a*c]))]*x], -((b + Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c]))])/(Sqrt[
2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[-a + b*x^2 - c*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx$$

$$\Big\downarrow \; 1416$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{-a+bx^2-cx^4}}$$

input
```
Int[1/Sqrt[-a + b*x^2 - c*x^4],x]
```

output
```
((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/
(2*a^(1/4)*c^(1/4)*Sqrt[-a + b*x^2 - c*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}\,\sqrt{-c\,x^4+b\,x^2-a}}$	143
elliptic	$\frac{\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}\,\sqrt{-c\,x^4+b\,x^2-a}}$	143

input
```
int(1/(-c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2-a)^(1/2)*E
llipticF(1/2*x*(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(a\sqrt{\frac{b^2-4\,ac}{a^2}}-b\right)\sqrt{-a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}+b}{a}}F(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}+b}{a}}\right)\mid -\frac{ab\sqrt{\frac{b^2-4\,ac}{a^2}}-b^2+2\,ac}{2\,ac})}{2\,ac}$$

input
```
integrate(1/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*(a*sqrt((b^2 - 4*a*c)/a^2) - b)*sqrt(-a)*sqrt((a*sqrt((b^2 -
 4*a*c)/a^2) + b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a
*c)/a^2) + b)/a)), -1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) - b^2 + 2*a*c)/(a*c))
/(a*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx$$

input
```
integrate(1/(-c*x**4+b*x**2-a)**(1/2),x)
```

output
```
Integral(1/sqrt(-a + b*x**2 - c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2-a}}\,dx$$

input
```
integrate(1/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-c*x^4 + b*x^2 - a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx=\int \frac{1}{\sqrt{-cx^4+bx^2-a}}\,dx$$

input
```
integrate(1/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-c*x^4 + b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx=\int \frac{1}{\sqrt{-c\,x^4+b\,x^2-a}}\,dx$$

input
```
int(1/(b*x^2 - a - c*x^4)^(1/2),x)
```

output
```
int(1/(b*x^2 - a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx=-\left(\int \frac{\sqrt{-c\,x^4+b\,x^2-a}}{c\,x^4-b\,x^2+a}dx\right)$$

input
```
int(1/(-c*x^4+b*x^2-a)^(1/2),x)
```

output
```
- int(sqrt( - a + b*x**2 - c*x**4)/(a - b*x**2 + c*x**4),x)
```

3.166 $\int \frac{1}{\sqrt{-a-bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 118

$$\int \frac{1}{\sqrt{-a-bx^2-cx^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{-a-bx^2-cx^4}}$$

output

```
1/2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*
InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2
))/a^(1/4)/c^(1/4)/(-c*x^4-b*x^2-a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{-a-bx^2-cx^4}}\,dx = -\frac{i\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b-\sqrt{b^2-4ac}}}x\right),\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b-\sqrt{b^2-4ac}}}\sqrt{-a-x^2\left(b+cx^2\right)}}$$

input
```
Integrate[1/Sqrt[-a - b*x^2 - c*x^4],x]
```

output
```
((-I)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b - Sqrt[b^2 - 4*a
*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[
c/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[-a - x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-a-bx^2-cx^4}}\,dx$$

$$\Big\downarrow \; 1416$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{-a-bx^2-cx^4}}$$

input
```
Int[1/Sqrt[-a - b*x^2 - c*x^4],x]
```

output
```
((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(2*a^(1/4)*c^(1/4)*Sqrt[-a - b*x^2 - c*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(-b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)}{a}}\,\sqrt{-c\,x^{4}-b\,x^{2}-a}}$	142
elliptic	$\frac{\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(-b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)}{a}}\,\sqrt{-c\,x^{4}-b\,x^{2}-a}}$	142

input
```
int(1/(-c*x^4-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4-b*x^2-a)^(1/2)*El
lipticF(1/2*x*(-2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(-b+(-4*a*c+
b^2)^(1/2))/a/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{-a-bx^2-cx^4}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(a\sqrt{\frac{b^2-4\,ac}{a^2}}+b\right)\sqrt{-a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}F(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}\right)\mid\frac{ab\sqrt{\frac{b^2-4\,ac}{a^2}}+b^2-2\,ac}{2\,ac})}{2\,ac}$$

input
```
integrate(1/(-c*x^4-b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*(a*sqrt((b^2 - 4*a*c)/a^2) + b)*sqrt(-a)*sqrt((a*sqrt((b^2 -
 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a
*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c))/
(a*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-a-bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-a-bx^2-cx^4}}\,dx$$

input
```
integrate(1/(-c*x**4-b*x**2-a)**(1/2),x)
```

output
```
Integral(1/sqrt(-a - b*x**2 - c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-a-bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4-bx^2-a}}\,dx$$

input
```
integrate(1/(-c*x^4-b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-c*x^4 - b*x^2 - a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-a-bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4-bx^2-a}}\,dx$$

input
```
integrate(1/(-c*x^4-b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-c*x^4 - b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-a-bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-c\,x^4-b\,x^2-a}}\,dx$$

input
```
int(1/(- a - b*x^2 - c*x^4)^(1/2),x)
```

output
```
int(1/(- a - b*x^2 - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-a-bx^2-cx^4}}\,dx = -\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right) i$$

input
```
int(1/(-c*x^4-b*x^2-a)^(1/2),x)
```

output
```
- int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*i
```

3.167 $\int \frac{1}{\sqrt{\mathrm{a1}+\mathrm{a2}+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 133

$$\int \frac{1}{\sqrt{\mathrm{a1}+\mathrm{a2}+bx^2+cx^4}}\,dx = \frac{\sqrt[4]{\mathrm{a1}+\mathrm{a2}}\left(1+\frac{\sqrt{c}x^2}{\sqrt{\mathrm{a1}+\mathrm{a2}}}\right)\sqrt{\frac{\mathrm{a1}+\mathrm{a2}+bx^2+cx^4}{(\mathrm{a1}+\mathrm{a2})\left(1+\frac{\sqrt{c}x^2}{\sqrt{\mathrm{a1}+\mathrm{a2}}}\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{\mathrm{a1}+\mathrm{a2}}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{\mathrm{a1}+\mathrm{a2}}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{\mathrm{a1}+\mathrm{a2}+bx^2+cx^4}}$$

output
```
1/2*(a1+a2)^(1/4)*(1+c^(1/2)*x^2/(a1+a2)^(1/2))*((c*x^4+b*x^2+a1+a2)/(a1+a
2)/(1+c^(1/2)*x^2/(a1+a2)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)
*x/(a1+a2)^(1/4)),1/2*(2-b/(a1+a2)^(1/2)/c^(1/2))^(1/2))/c^(1/4)/(c*x^4+b*
x^2+a1+a2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{\text{a1} + \text{a2} + bx^2 + cx^4}}\,dx = -\frac{i\sqrt{1-\frac{2cx^2}{-b+\sqrt{b^2-4(\text{a1}+\text{a2})c}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4(\text{a1}+\text{a2})c}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4(\text{a1}+\text{a2})c}}}x\right),-\frac{b+\sqrt{b^2-4(\text{a1}+\text{a2})c}}{-b+\sqrt{b^2-4(\text{a1}+\text{a2})c}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4(\text{a1}+\text{a2})c}}}\sqrt{\text{a1}+\text{a2}+bx^2+cx^4}}$$

input
```
Integrate[1/Sqrt[a1 + a2 + b*x^2 + c*x^4],x]
```

output
```
((-I)*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*(a1 + a2)*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*(a1 + a2)*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
 + Sqrt[b^2 - 4*(a1 + a2)*c])]*x], -((b + Sqrt[b^2 - 4*(a1 + a2)*c])/(-b +
 Sqrt[b^2 - 4*(a1 + a2)*c]))])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*(a1 + a2)
*c])]*Sqrt[a1 + a2 + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\text{a1} + \text{a2} + bx^2 + cx^4}}\,dx$$

$$\Big\downarrow \; 1416$$

$$\frac{\sqrt[4]{\mathrm{a1}+\mathrm{a2}}\left(\frac{\sqrt{c}x^2}{\sqrt{\mathrm{a1}+\mathrm{a2}}}+1\right)\sqrt{\frac{\mathrm{a1}+\mathrm{a2}+bx^2+cx^4}{(\mathrm{a1}+\mathrm{a2})\left(\frac{\sqrt{c}x^2}{\sqrt{\mathrm{a1}+\mathrm{a2}}}+1\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{\mathrm{a1}+\mathrm{a2}}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{\mathrm{a1}+\mathrm{a2}}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{\mathrm{a1}+\mathrm{a2}+bx^2+cx^4}}$$

input
```
Int[1/Sqrt[a1 + a2 + b*x^2 + c*x^4],x]
```

output
```
((a1 + a2)^(1/4)*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2])*Sqrt[(a1 + a2 + b*x^2 +
 c*x^4)/((a1 + a2)*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2])^2)]*EllipticF[2*ArcTa
n[(c^(1/4)*x)/(a1 + a2)^(1/4)], (2 - b/(Sqrt[a1 + a2]*Sqrt[c]))/4])/(2*c^(
1/4)*Sqrt[a1 + a2 + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.32

method	result
default	$\frac{\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}\right)x^2}{\mathrm{a1}+\mathrm{a2}}}\sqrt{4+\frac{2\left(b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}\right)x^2}{\mathrm{a1}+\mathrm{a2}}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}}{\mathrm{a1}+\mathrm{a2}}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4\,\mathrm{a}\right.}}{(\mathrm{a1}}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}}{\mathrm{a1}+\mathrm{a2}}}\sqrt{c\,x^4+b\,x^2+\mathrm{a1}+\mathrm{a2}}}$
elliptic	$\frac{\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}\right)x^2}{\mathrm{a1}+\mathrm{a2}}}\sqrt{4+\frac{2\left(b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}\right)x^2}{\mathrm{a1}+\mathrm{a2}}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}}{\mathrm{a1}+\mathrm{a2}}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4\,\mathrm{a}\right.}}{(\mathrm{a1}}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}}{\mathrm{a1}+\mathrm{a2}}}\sqrt{c\,x^4+b\,x^2+\mathrm{a1}+\mathrm{a2}}}$

input
```
int(1/(c*x^4+b*x^2+a1+a2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)/((-b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2))^(1/2)*(4-2*(-b+(-4*a
1*c-4*a2*c+b^2)^(1/2))/(a1+a2)*x^2)^(1/2)*(4+2*(b+(-4*a1*c-4*a2*c+b^2)^(1/
2))/(a1+a2)*x^2)^(1/2)/(c*x^4+b*x^2+a1+a2)^(1/2)*EllipticF(1/2*x*2^(1/2)*(
(-b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2))^(1/2),1/2*(-4+2*b*(b+(-4*a1*c-4*a
2*c+b^2)^(1/2))/(a1+a2)/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{\mathrm{a1}+\mathrm{a2}+bx^2+cx^4}}\,dx = -\frac{\sqrt{\frac{1}{2}}\left((a_1+a_2)\sqrt{\frac{b^2-4\,(a_1+a_2)c}{a_1^2+2\,a_1a_2+a_2^2}}+b\right)\sqrt{\frac{(a_1+a_2)\sqrt{\frac{b^2-4\,(a_1+a_2)c}{a_1^2+2\,a_1a_2+a_2^2}}-b}{a_1+a_2}}F\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{(a_1+a_2)\sqrt{\frac{b^2-4\,(a_1+a_2)c}{a_1^2+2\,a_1a_2+a_2^2}}-b}{a_1+a_2}}\right)\mid\cdots\right.}{2\sqrt{a_1+a_2}c}$$

input
```
integrate(1/(c*x^4+b*x^2+a1+a2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*((a1 + a2)*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^
2)) + b)*sqrt(((a1 + a2)*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2
)) - b)/(a1 + a2))*elliptic_f(arcsin(sqrt(1/2)*x*sqrt(((a1 + a2)*sqrt((b^2
 - 4*(a1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2)) - b)/(a1 + a2))), 1/2*((a1 + a2
)*b*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2)) + b^2 - 2*(a1 + a2
)*c)/((a1 + a2)*c))/(sqrt(a1 + a2)*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\mathrm{a1} + \mathrm{a2} + bx^2 + cx^4}}\,dx = \int \frac{1}{\sqrt{a_1 + a_2 + bx^2 + cx^4}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**2+a1+a2)**(1/2),x)
```

output
```
Integral(1/sqrt(a1 + a2 + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\mathrm{a1} + \mathrm{a2} + bx^2 + cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4 + bx^2 + a_1 + a_2}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a1+a2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2 + a1 + a2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\mathrm{a1} + \mathrm{a2} + bx^2 + cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4 + bx^2 + a_1 + a_2}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a1+a2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2 + a1 + a2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\text{a1} + \text{a2} + bx^2 + cx^4}}\, dx = \int \frac{1}{\sqrt{c\,x^4 + b\,x^2 + a_1 + a_2}}\, dx$$

input
```
int(1/(a1 + a2 + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(1/(a1 + a2 + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\text{a1} + \text{a2} + bx^2 + cx^4}}\, dx = \int \frac{\sqrt{c\,x^4 + b\,x^2 + a1 + a2}}{c\,x^4 + b\,x^2 + a1 + a2} dx$$

input
```
int(1/(c*x^4+b*x^2+a1+a2)^(1/2),x)
```

output
```
int(sqrt(a1 + a2 + b*x**2 + c*x**4)/(a1 + a2 + b*x**2 + c*x**4),x)
```

3.168 $\int \frac{1}{\sqrt{\mathbf{a1}+\mathbf{a2}-bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 134

$$\int \frac{1}{\sqrt{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}}\,dx = \frac{\sqrt[4]{\mathrm{a1}+\mathrm{a2}}\left(1+\frac{\sqrt{c}x^2}{\sqrt{\mathrm{a1}+\mathrm{a2}}}\right)\sqrt{\frac{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}{(\mathrm{a1}+\mathrm{a2})\left(1+\frac{\sqrt{c}x^2}{\sqrt{\mathrm{a1}+\mathrm{a2}}}\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{\mathrm{a1}+\mathrm{a2}}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{\mathrm{a1}+\mathrm{a2}}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}}$$

output
```
1/2*(a1+a2)^(1/4)*(1+c^(1/2)*x^2/(a1+a2)^(1/2))*((c*x^4-b*x^2+a1+a2)/(a1+a
2)/(1+c^(1/2)*x^2/(a1+a2)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)
*x/(a1+a2)^(1/4)),1/2*(2+b/(a1+a2)^(1/2)/c^(1/2))^(1/2))/c^(1/4)/(c*x^4-b*
x^2+a1+a2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}}\,dx =$$

$$-\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4(\mathrm{a1}+\mathrm{a2})c}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4(\mathrm{a1}+\mathrm{a2})c}}}\,\mathrm{EllipticF}\left(i\,\mathrm{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{-b+\sqrt{b^2-4(\mathrm{a1}+\mathrm{a2})c}}}x\right),\frac{b-\sqrt{}}{b+\sqrt{}}\right.}{\sqrt{2}\sqrt{\frac{c}{-b+\sqrt{b^2-4(\mathrm{a1}+\mathrm{a2})c}}}\sqrt{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}}$$

input
```
Integrate[1/Sqrt[a1 + a2 - b*x^2 + c*x^4],x]
```

output
```
((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*(a1 + a2)*c])]*Sqrt[1 - (2*c*x
^2)/(b + Sqrt[b^2 - 4*(a1 + a2)*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-
b + Sqrt[b^2 - 4*(a1 + a2)*c])]*x], (b - Sqrt[b^2 - 4*(a1 + a2)*c])/(b + S
qrt[b^2 - 4*(a1 + a2)*c])])/(Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*(a1 + a2)*c
])]*Sqrt[a1 + a2 - b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 1416$$

$$\frac{\sqrt[4]{\mathrm{a1}+\mathrm{a2}}\left(\frac{\sqrt{c}x^2}{\sqrt{\mathrm{a1}+\mathrm{a2}}}+1\right)\sqrt{\frac{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}{(\mathrm{a1}+\mathrm{a2})\left(\frac{\sqrt{c}x^2}{\sqrt{\mathrm{a1}+\mathrm{a2}}}+1\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{\mathrm{a1}+\mathrm{a2}}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{\mathrm{a1}+\mathrm{a2}}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{c}\sqrt{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}}$$

input
```
Int[1/Sqrt[a1 + a2 - b*x^2 + c*x^4],x]
```

output
```
((a1 + a2)^(1/4)*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2])*Sqrt[(a1 + a2 - b*x^2 +
 c*x^4)/((a1 + a2)*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2])^2)]*EllipticF[2*ArcTa
n[(c^(1/4)*x)/(a1 + a2)^(1/4)], (2 + b/(Sqrt[a1 + a2]*Sqrt[c]))/4])/(2*c^(
1/4)*Sqrt[a1 + a2 - b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.30

method	result
default	$\frac{\sqrt{2}\sqrt{4-\frac{2\left(b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}\right)x^2}{\mathrm{a1}+\mathrm{a2}}}\sqrt{4+\frac{2\left(-b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}\right)x^2}{\mathrm{a1}+\mathrm{a2}}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}}{\mathrm{a1}+\mathrm{a2}}}}{2},\frac{\sqrt{-4-\frac{2b\left(-b+\sqrt{-4\,\mathrm{a}}\right.}{(\mathrm{a1}+}}}{2}\right.}{4\sqrt{\frac{b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}}{\mathrm{a1}+\mathrm{a2}}}\sqrt{c\,x^4-b\,x^2+\mathrm{a1}+\mathrm{a2}}}$
elliptic	$\frac{\sqrt{2}\sqrt{4-\frac{2\left(b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}\right)x^2}{\mathrm{a1}+\mathrm{a2}}}\sqrt{4+\frac{2\left(-b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}\right)x^2}{\mathrm{a1}+\mathrm{a2}}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}}{\mathrm{a1}+\mathrm{a2}}}}{2},\frac{\sqrt{-4-\frac{2b\left(-b+\sqrt{-4\,\mathrm{a}}\right.}{(\mathrm{a1}+}}}{2}\right.}{4\sqrt{\frac{b+\sqrt{-4\,\mathrm{a1}\,c-4c\,\mathrm{a2}+b^2}}{\mathrm{a1}+\mathrm{a2}}}\sqrt{c\,x^4-b\,x^2+\mathrm{a1}+\mathrm{a2}}}$

input
```
int(1/(c*x^4-b*x^2+a1+a2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)/((b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2))^(1/2)*(4-2*(b+(-4*a1*
c-4*a2*c+b^2)^(1/2))/(a1+a2)*x^2)^(1/2)*(4+2*(-b+(-4*a1*c-4*a2*c+b^2)^(1/2
))/(a1+a2)*x^2)^(1/2)/(c*x^4-b*x^2+a1+a2)^(1/2)*EllipticF(1/2*x*2^(1/2)*((
b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2))^(1/2),1/2*(-4-2*b*(-b+(-4*a1*c-4*a2
*c+b^2)^(1/2))/(a1+a2)/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}}\,dx = -\frac{\sqrt{\frac{1}{2}}\left((a_1+a_2)\sqrt{\frac{b^2-4\,(a_1+a_2)c}{a_1^2+2\,a_1a_2+a_2^2}}-b\right)\sqrt{\frac{(a_1+a_2)\sqrt{\frac{b^2-4\,(a_1+a_2)c}{a_1^2+2\,a_1a_2+a_2^2}}+b}{a_1+a_2}}F(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{(a_1+a_2)\sqrt{\frac{b^2-4\,(a_1+a_2)c}{a_1^2+2\,a_1a_2+a_2^2}}+b}{a_1+a_2}}\right)\mid -}{2\sqrt{a_1+a_2}c}$$

input
```
integrate(1/(c*x^4-b*x^2+a1+a2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*((a1 + a2)*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^
2)) - b)*sqrt(((a1 + a2)*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2
)) + b)/(a1 + a2))*elliptic_f(arcsin(sqrt(1/2)*x*sqrt(((a1 + a2)*sqrt((b^2
 - 4*(a1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2)) + b)/(a1 + a2))), -1/2*((a1 + a
2)*b*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2)) - b^2 + 2*(a1 + a
2)*c)/((a1 + a2)*c))/(sqrt(a1 + a2)*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\text{a1} + \text{a2} - bx^2 + cx^4}}\,dx = \int \frac{1}{\sqrt{a_1 + a_2 - bx^2 + cx^4}}\,dx$$

input
```
integrate(1/(c*x**4-b*x**2+a1+a2)**(1/2),x)
```

output
```
Integral(1/sqrt(a1 + a2 - b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\text{a1} + \text{a2} - bx^2 + cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4 - bx^2 + a_1 + a_2}}\,dx$$

input
```
integrate(1/(c*x^4-b*x^2+a1+a2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 - b*x^2 + a1 + a2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\text{a1} + \text{a2} - bx^2 + cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4 - bx^2 + a_1 + a_2}}\,dx$$

input
```
integrate(1/(c*x^4-b*x^2+a1+a2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^4 - b*x^2 + a1 + a2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\mathrm{a1} + \mathrm{a2} - bx^2 + cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4 - b\,x^2 + a_1 + a_2}}\,dx$$

input
```
int(1/(a1 + a2 - b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(1/(a1 + a2 - b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\mathrm{a1} + \mathrm{a2} - bx^2 + cx^4}}\,dx = \int \frac{\sqrt{c\,x^4 - b\,x^2 + a1 + a2}}{c\,x^4 - b\,x^2 + a1 + a2}\,dx$$

input
```
int(1/(c*x^4-b*x^2+a1+a2)^(1/2),x)
```

output
```
int(sqrt(a1 + a2 - b*x**2 + c*x**4)/(a1 + a2 - b*x**2 + c*x**4),x)
```

3.169 $\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 130

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\sqrt[4]{-a}\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)\sqrt{\frac{a+bx^2-cx^4}{a\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{-a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{-a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2-cx^4}}$$

output
```
1/2*(-a)^(1/4)*(1+c^(1/2)*x^2/(-a)^(1/2))*((-c*x^4+b*x^2+a)/a/(1+c^(1/2)*x
^2/(-a)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/(-a)^(1/4)),1/2
*(2+b/(-a)^(1/2)/c^(1/2))^(1/2))/c^(1/4)/(-c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right),-\frac{b+\sqrt{b^2+4ac}}{-b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{a+bx^2-cx^4}}$$

input
```
Integrate[1/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
 Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 +
4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))])/(Sqrt[
2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1417, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}dx$$

$$\Big\downarrow \ 1417$$

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{\sqrt{a+bx^2-cx^4}}$$

$$\Big\downarrow \ 321$$

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

input
```
Int[1/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
(Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*S
qrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2
+ 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}}$	145
elliptic	$\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}}$	145

input
```
int(1/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(
4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(a^{\frac{3}{2}}\sqrt{\frac{b^2+4\,ac}{a^2}}+\sqrt{a}b\right)\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}-b}{a}}F(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}-b}{a}}\right)\,|\,-\frac{ab\sqrt{\frac{b^2+4\,ac}{a^2}}+b^2+2\,ac}{2\,ac})}{2\,ac}$$

input
```
integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*(a^(3/2)*sqrt((b^2 + 4*a*c)/a^2) + sqrt(a)*b)*sqrt((a*sqrt((
b^2 + 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2
+ 4*a*c)/a^2) - b)/a)), -1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) + b^2 + 2*a*c)/(
a*c))/(a*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate(1/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*x**2 - c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{1}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{1}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{1}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/(a + b*x^2 - c*x^4)^(1/2),x)
```

output
```
int(1/(a + b*x^2 - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{\sqrt{-c\,x^4+b\,x^2+a}}{-c\,x^4+b\,x^2+a}dx$$

input
```
int(1/(-c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**2 - c*x**4)/(a + b*x**2 - c*x**4),x)
```

3.170 $\int \frac{1}{\sqrt{a-bx^2-cx^4}} dx$

Optimal result

Integrand size = 18, antiderivative size = 133

$$\int \frac{1}{\sqrt{a-bx^2-cx^4}}\,dx = \frac{\sqrt[4]{-a}\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)\sqrt{\frac{a-bx^2-cx^4}{a\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{-a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{-a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a-bx^2-cx^4}}$$

output
```
1/2*(-a)^(1/4)*(1+c^(1/2)*x^2/(-a)^(1/2))*((-c*x^4-b*x^2+a)/a/(1+c^(1/2)*x
^2/(-a)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/(-a)^(1/4)),1/2
*(2-b/(-a)^(1/2)/c^(1/2))^(1/2))/c^(1/4)/(-c*x^4-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt{a-bx^2-cx^4}}\,dx = -\frac{i\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b-\sqrt{b^2+4ac}}}x\right),\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b-\sqrt{b^2+4ac}}}\sqrt{a-x^2(b+cx^2)}}$$

input
```
Integrate[1/Sqrt[a - b*x^2 - c*x^4],x]
```

output
```
((-I)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b - Sqrt[b^2 + 4*a
*c])]*x], (b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[
c/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[a - x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1417, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^2-cx^4}}dx$$

$$\Big\downarrow \text{ 1417}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\int \frac{1}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx}{\sqrt{a-bx^2-cx^4}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{a-bx^2-cx^4}}$$

input
```
Int[1/Sqrt[a - b*x^2 - c*x^4],x]
```

output
```
(Sqrt[b + Sqrt[b^2 + 4*a*c]]*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*Ellip
ticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2
 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/
(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a -
 b*x^2 - c*x^4])
```

Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{b+\sqrt{4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(-b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4-b\,x^2+a}}$	144
elliptic	$\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{b+\sqrt{4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(-b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4-b\,x^2+a}}$	144

input
```
int(1/(-c*x^4-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)/((b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4-b*x^2+a)^(1/2)*
EllipticF(1/2*x*2^(1/2)*((b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(-b+(4
*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{a-bx^2-cx^4}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(a^{\frac{3}{2}}\sqrt{\frac{b^2+4\,ac}{a^2}}-\sqrt{a}b\right)\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}+b}{a}}F(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}+b}{a}}\right) \mid \frac{ab\sqrt{\frac{b^2+4\,ac}{a^2}}-b^2-2\,ac}{2\,ac})}{2\,ac}$$

input
```
integrate(1/(-c*x^4-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*(a^(3/2)*sqrt((b^2 + 4*a*c)/a^2) - sqrt(a)*b)*sqrt((a*sqrt((
b^2 + 4*a*c)/a^2) + b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2
+ 4*a*c)/a^2) + b)/a)), 1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) - b^2 - 2*a*c)/(a
*c))/(a*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{a-bx^2-cx^4}}\,dx$$

input
```
integrate(1/(-c*x**4-b*x**2+a)**(1/2),x)
```

output
```
Integral(1/sqrt(a - b*x**2 - c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4-bx^2+a}}\,dx$$

input
```
integrate(1/(-c*x^4-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-c*x^4 - b*x^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4-bx^2+a}}\,dx$$

input
```
integrate(1/(-c*x^4-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-c*x^4 - b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-c\,x^4-b\,x^2+a}}\,dx$$

input
```
int(1/(a - b*x^2 - c*x^4)^(1/2),x)
```

output
```
int(1/(a - b*x^2 - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a-bx^2-cx^4}}\,dx=\int \frac{\sqrt{-c\,x^4-b\,x^2+a}}{-c\,x^4-b\,x^2+a}dx$$

input
```
int(1/(-c*x^4-b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a - b*x**2 - c*x**4)/(a - b*x**2 - c*x**4),x)
```

3.171 $\int \frac{1}{\sqrt{-a+bx^2+cx^4}} dx$

Optimal result

Integrand size = 18, antiderivative size = 133

$$\int \frac{1}{\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\sqrt[4]{-a}\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)\sqrt{\frac{a-bx^2-cx^4}{a\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{-a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{-a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{-a+bx^2+cx^4}}$$

output
```
1/2*(-a)^(1/4)*(1+c^(1/2)*x^2/(-a)^(1/2))*((-c*x^4-b*x^2+a)/a/(1+c^(1/2)*x
^2/(-a)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/(-a)^(1/4)),1/2
*(2-b/(-a)^(1/2)/c^(1/2))^(1/2))/c^(1/4)/(c*x^4+b*x^2-a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{-a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2+4ac}+2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}x\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{-a+bx^2+cx^4}}$$

input
```
Integrate[1/Sqrt[-a + b*x^2 + c*x^4],x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c
/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*
c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[-a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1417, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1417}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\int \frac{1}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx}{\sqrt{-a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}$$

input
```
Int[1/Sqrt[-a + b*x^2 + c*x^4],x]
```

output
```
(Sqrt[b + Sqrt[b^2 + 4*a*c]]*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*Ellip
ticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2
 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/
(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[-a
+ b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^{2}}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}$	142
elliptic	$\frac{\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^{2}}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}$	142

input
```
int(1/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*Ellip
ticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^
(1/2))/a/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{-a+bx^2+cx^4}}\,dx = -\frac{\sqrt{\frac{1}{2}}\left(\sqrt{-a}a\sqrt{\frac{b^2+4\,ac}{a^2}}-\sqrt{-a}b\right)\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}+b}{a}}F(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}+b}{a}}\right)\mid\frac{ab\sqrt{\frac{b^2+4\,ac}{a^2}}-b^2-2\,ac}{2\,ac})}{2\,ac}$$

input
```
integrate(1/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*(sqrt(-a)*a*sqrt((b^2 + 4*a*c)/a^2) - sqrt(-a)*b)*sqrt((a*s
qrt((b^2 + 4*a*c)/a^2) + b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt(
(b^2 + 4*a*c)/a^2) + b)/a)), 1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) - b^2 - 2*a*
c)/(a*c))/(a*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{-a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**2-a)**(1/2),x)
```

output
```
Integral(1/sqrt(-a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2-a}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2 - a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2-a}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

input
```
int(1/(b*x^2 - a + c*x^4)^(1/2),x)
```

output
```
int(1/(b*x^2 - a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-a+bx^2+cx^4}}\,dx = -\left(\int \frac{\sqrt{c\,x^4+b\,x^2-a}}{-c\,x^4-b\,x^2+a}dx\right)$$

input
```
int(1/(c*x^4+b*x^2-a)^(1/2),x)
```

output
```
 - int(sqrt( - a + b*x**2 + c*x**4)/(a - b*x**2 - c*x**4),x)
```

3.172 $\int \frac{1}{\sqrt{-a-bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 132

$$\int \frac{1}{\sqrt{-a-bx^2+cx^4}}\,dx = \frac{\sqrt[4]{-a}\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)\sqrt{\frac{a+bx^2-cx^4}{a\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{-a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{-a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{-a-bx^2+cx^4}}$$

output
```
1/2*(-a)^(1/4)*(1+c^(1/2)*x^2/(-a)^(1/2))*((-c*x^4+b*x^2+a)/a/(1+c^(1/2)*x
^2/(-a)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/(-a)^(1/4)),1/2
*(2+b/(-a)^(1/2)/c^(1/2))^(1/2))/c^(1/4)/(c*x^4-b*x^2-a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{-a-bx^2+cx^4}}\,dx = -\frac{i\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b-\sqrt{b^2+4ac}}}x\right),\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{-\frac{c}{b-\sqrt{b^2+4ac}}}\sqrt{-a-bx^2+cx^4}}$$

input
```
Integrate[1/Sqrt[-a - b*x^2 + c*x^4],x]
```

output
```
((-I)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b - Sqrt[b^2 + 4
*a*c]))]*x], (b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sq
rt[-(c/(b - Sqrt[b^2 + 4*a*c]))]*Sqrt[-a - b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1417, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-a-bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1417}$$

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{\sqrt{-a-bx^2+cx^4}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-a-bx^2+cx^4}}$$

input
```
Int[1/Sqrt[-a - b*x^2 + c*x^4],x]
```

output
```
(Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*S
qrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2
+ 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[-a - b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(b+\sqrt{4ac+b^{2}}\right)}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(-b+\sqrt{4ac+b^{2}}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(b+\sqrt{4ac+b^{2}}\right)}{a}}\,\sqrt{c\,x^{4}-b\,x^{2}-a}}$	141
elliptic	$\frac{\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(b+\sqrt{4ac+b^{2}}\right)}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(-b+\sqrt{4ac+b^{2}}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(b+\sqrt{4ac+b^{2}}\right)}{a}}\,\sqrt{c\,x^{4}-b\,x^{2}-a}}$	141

input
```
int(1/(c*x^4-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-2*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4-b*x^2-a)^(1/2)*Ellipt
icF(1/2*x*(-2*(b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(-b+(4*a*c+b^2)^(
1/2))/a/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-a-bx^2+cx^4}}\,dx = -\frac{\sqrt{\frac{1}{2}}\left(\sqrt{-a}a\sqrt{\frac{b^2+4\,ac}{a^2}}+\sqrt{-a}b\right)\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}-b}{a}}F\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}-b}{a}}\right)\mid-\frac{ab\sqrt{\frac{b^2+4\,ac}{a^2}}+b^2+2\,ac}{2\,ac}\right)}{2\,ac}$$

input
```
integrate(1/(c*x^4-b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*(sqrt(-a)*a*sqrt((b^2 + 4*a*c)/a^2) + sqrt(-a)*b)*sqrt((a*s
qrt((b^2 + 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt(
(b^2 + 4*a*c)/a^2) - b)/a)), -1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) + b^2 + 2*a
*c)/(a*c))/(a*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-a-bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{-a-bx^2+cx^4}}\,dx$$

input
```
integrate(1/(c*x**4-b*x**2-a)**(1/2),x)
```

output
```
Integral(1/sqrt(-a - b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-a - bx^2 + cx^4}} \, dx = \int \frac{1}{\sqrt{cx^4 - bx^2 - a}} \, dx$$

input
```
integrate(1/(c*x^4-b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 - b*x^2 - a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-a - bx^2 + cx^4}} \, dx = \int \frac{1}{\sqrt{cx^4 - bx^2 - a}} \, dx$$

input
```
integrate(1/(c*x^4-b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^4 - b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-a - bx^2 + cx^4}} \, dx = \int \frac{1}{\sqrt{c\,x^4 - b\,x^2 - a}} \, dx$$

input
```
int(1/(c*x^4 - b*x^2 - a)^(1/2),x)
```

output
```
int(1/(c*x^4 - b*x^2 - a)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-a-bx^2+cx^4}}\,dx = -\left(\int \frac{\sqrt{c\,x^4-b\,x^2-a}}{-c\,x^4+b\,x^2+a}dx\right)$$

input
```
int(1/(c*x^4-b*x^2-a)^(1/2),x)
```

output
```
- int(sqrt( - a - b*x**2 + c*x**4)/(a + b*x**2 - c*x**4),x)
```

3.173 $\int \frac{1}{(2+5x^2-3x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{(2+5x^2-3x^4)^{3/2}} \, dx = \frac{x(37-15x^2)}{98\sqrt{2+5x^2-3x^4}} + \frac{5}{98}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right) + \frac{1}{14}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-6\right)$$

output
```
1/98*x*(-15*x^2+37)/(-3*x^4+5*x^2+2)^(1/2)+5/98*EllipticE(1/2*x*2^(1/2),I*
6^(1/2))+1/14*EllipticF(1/2*x*2^(1/2),I*6^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.16

$$\int \frac{1}{(2+5x^2-3x^4)^{3/2}} \, dx = \frac{37x-15x^3+5i\sqrt{6}\sqrt{2-x^2}\sqrt{1+3x^2}E\left(i\text{arcsinh}\left(\sqrt{3}x\right)\middle|-\frac{1}{6}\right)-7i\sqrt{6}\sqrt{2-x^2}\sqrt{}}{98\sqrt{2+5x^2-3x^4}}$$

input
```
Integrate[(2 + 5*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(37*x - 15*x^3 + (5*I)*Sqrt[6]*Sqrt[2 - x^2]*Sqrt[1 + 3*x^2]*EllipticE[I*A
rcSinh[Sqrt[3]*x], -1/6] - (7*I)*Sqrt[6]*Sqrt[2 - x^2]*Sqrt[1 + 3*x^2]*Ell
ipticF[I*ArcSinh[Sqrt[3]*x], -1/6])/(98*Sqrt[2 + 5*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \ 1405$$

$$\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}-\frac{1}{98}\int -\frac{3(5x^2+4)}{\sqrt{-3x^4+5x^2+2}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{3}{98}\int \frac{5x^2+4}{\sqrt{-3x^4+5x^2+2}}dx+\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}$$

$$\Big\downarrow \ 1494$$

$$\frac{3}{49}\sqrt{3}\int \frac{5x^2+4}{2\sqrt{3}\sqrt{2-x^2}\sqrt{3x^2+1}}dx+\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{98}\int \frac{5x^2+4}{\sqrt{2-x^2}\sqrt{3x^2+1}}dx+\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}$$

$$\Big\downarrow \ 399$$

$$\frac{3}{98}\left(\frac{7}{3}\int \frac{1}{\sqrt{2-x^2}\sqrt{3x^2+1}}dx+\frac{5}{3}\int \frac{\sqrt{3x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}$$

$$\Big\downarrow \ 321$$

$$\frac{3}{98}\left(\frac{5}{3}\int \frac{\sqrt{3x^2+1}}{\sqrt{2-x^2}}dx+\frac{7}{3}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-6\right)\right)+\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{3}{98}\left(\frac{7}{3}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-6\right)+\frac{5}{3}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)\right)+\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(x*(37 - 15*x^2))/(98*Sqrt[2 + 5*x^2 - 3*x^4]) + (3*((5*EllipticE[ArcSin[x
/Sqrt[2]], -6])/3 + (7*EllipticF[ArcSin[x/Sqrt[2]], -6])/3))/98
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(55) = 110.

Time = 2.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.49

method	result
risch	$-\frac{x(15x^2-37)}{98\sqrt{-3x^4+5x^2+2}}+\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)}{49\sqrt{-3x^4+5x^2+2}}-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\ (\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)-\mathrm{Ellip}}{196\sqrt{-3x^4+5x^2+2}}$
default	$\frac{\frac{37}{98}x-\frac{15}{98}x^3}{\sqrt{-3x^4+5x^2+2}}+\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)}{49\sqrt{-3x^4+5x^2+2}}-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\ (\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)-\mathrm{EllipticE}(}{196\sqrt{-3x^4+5x^2+2}}$
elliptic	$\frac{\frac{37}{98}x-\frac{15}{98}x^3}{\sqrt{-3x^4+5x^2+2}}+\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)}{49\sqrt{-3x^4+5x^2+2}}-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\ (\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)-\mathrm{EllipticE}(}{196\sqrt{-3x^4+5x^2+2}}$

input
```
int(1/(-3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/98*x*(15*x^2-37)/(-3*x^4+5*x^2+2)^(1/2)+3/49*2^(1/2)*(-2*x^2+4)^(1/2)*(
3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*6^(1/2))-5
/196*2^(1/2)*(-2*x^2+4)^(1/2)*(3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/2)*(Elli
pticF(1/2*x*2^(1/2),I*6^(1/2))-EllipticE(1/2*x*2^(1/2),I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.54

$$\int \frac{1}{(2+5x^2-3x^4)^{3/2}}\,dx = \frac{5\,(3\,x^4-5\,x^2-2)E(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\mid -6)+19\,(3\,x^4-5\,x^2-2)F(\arcsin\left(\frac{1}{2}\sqrt{\right.}}{196\,(3\,x^4-5\,x^2-2)}$$

input
```
integrate(1/(-3*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/196*(5*(3*x^4 - 5*x^2 - 2)*elliptic_e(arcsin(1/2*sqrt(2)*x), -6) + 19*(3
*x^4 - 5*x^2 - 2)*elliptic_f(arcsin(1/2*sqrt(2)*x), -6) + 2*sqrt(-3*x^4 +
5*x^2 + 2)*(15*x^3 - 37*x))/(3*x^4 - 5*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((-3*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(2+5x^2-3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-3\,x^4+5\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2+5x^2-3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-3\,x^4+5\,x^2+2\right)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - 3*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 - 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(2+5x^2-3x^4\right)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4+5x^2+2}}{9x^8-30x^6+13x^4+20x^2+4}dx$$

input
```
int(1/(-3*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + 5*x**2 + 2)/(9*x**8 - 30*x**6 + 13*x**4 + 20*x**2 + 4
),x)
```

3.174 $\int \frac{1}{(2+4x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 127

$$\int \frac{1}{(2+4x^2-3x^4)^{3/2}}\,dx = \frac{x(7-3x^2)}{20\sqrt{2+4x^2-3x^4}} + \frac{1}{20}\sqrt{-2+\sqrt{10}}E\left(\arcsin\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right) + \frac{1}{4}\sqrt{\frac{1}{10}\left(-2+\sqrt{10}\right)}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)$$

output
```
1/20*x*(-3*x^2+7)/(-3*x^4+4*x^2+2)^(1/2)+1/20*(-2+10^(1/2))^(1/2)*Elliptic
E(1/2*(-4+2*10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))+1/40*(-20+10*1
0^(1/2))^(1/2)*EllipticF(1/2*(-4+2*10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*1
5^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\int \frac{1}{(2+4x^2-3x^4)^{3/2}}\,dx = \frac{1}{20}\left(\frac{x(7-3x^2)}{\sqrt{2+4x^2-3x^4}}\right.$$

$$+ i\sqrt{2+\sqrt{10}}E\left(i\operatorname{arcsinh}\left(\sqrt{1+\sqrt{\frac{5}{2}}}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right) - \frac{i(5+\sqrt{10})\,\text{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{1+\sqrt{\frac{5}{2}}\right.\right.}{\sqrt{2+\sqrt{10}}}$$

input

```
Integrate[(2 + 4*x^2 - 3*x^4)^(-3/2),x]
```

output

```
((x*(7 - 3*x^2))/Sqrt[2 + 4*x^2 - 3*x^4] + I*Sqrt[2 + Sqrt[10]]*EllipticE[
I*ArcSinh[Sqrt[1 + Sqrt[5/2]]*x], (-7 + 2*Sqrt[10])/3] - (I*(5 + Sqrt[10])
*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/2]]*x], (-7 + 2*Sqrt[10])/3])/Sqrt[2
+ Sqrt[10]])/20
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4+4x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1405$$

$$\frac{x(7-3x^2)}{20\sqrt{-3x^4+4x^2+2}}-\frac{1}{80}\int -\frac{12(x^2+1)}{\sqrt{-3x^4+4x^2+2}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{3}{20}\int \frac{x^2+1}{\sqrt{-3x^4+4x^2+2}}dx+\frac{x(7-3x^2)}{20\sqrt{-3x^4+4x^2+2}}$$

$$\Big\downarrow \; 1494$$

$$\frac{3}{10}\sqrt{3}\int \frac{x^2+1}{2\sqrt{-3x^2+\sqrt{10}+2}\sqrt{3x^2+\sqrt{10}-2}}dx+\frac{x(7-3x^2)}{20\sqrt{-3x^4+4x^2+2}}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{20}\sqrt{3}\int \frac{x^2+1}{\sqrt{-3x^2+\sqrt{10}+2}\sqrt{3x^2+\sqrt{10}-2}}dx+\frac{x(7-3x^2)}{20\sqrt{-3x^4+4x^2+2}}$$

$$\Big\downarrow \; 399$$

$$\frac{3}{20}\sqrt{3}\left(\frac{1}{3}\left(5-\sqrt{10}\right)\int \frac{1}{\sqrt{-3x^2+\sqrt{10}+2}\sqrt{3x^2+\sqrt{10}-2}}dx+\frac{1}{3}\int \frac{\sqrt{3x^2+\sqrt{10}-2}}{\sqrt{-3x^2+\sqrt{10}+2}}dx\right)+\frac{x(7-3x^2)}{20\sqrt{-3x^4+4x^2+2}}$$

$$\Big\downarrow \; 321$$

$$\frac{3}{20}\sqrt{3}\left(\frac{1}{3}\int \frac{\sqrt{3x^2+\sqrt{10}-2}}{\sqrt{-3x^2+\sqrt{10}+2}}dx+\frac{1}{9}\left(5-\sqrt{10}\right)\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(\right.\right.\right.$$
$$\frac{x(7-3x^2)}{20\sqrt{-3x^4+4x^2+2}}$$

$$\Big\downarrow \; 327$$

$$\frac{3}{20}\sqrt{3}\left(\frac{1}{9}\left(5-\sqrt{10}\right)\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)+\frac{1}{3}\sqrt{\frac{2}{2+\sqrt{10}}}\right.$$
$$\frac{x(7-3x^2)}{20\sqrt{-3x^4+4x^2+2}}$$

input
```
Int[(2 + 4*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(x*(7 - 3*x^2))/(20*Sqrt[2 + 4*x^2 - 3*x^4]) + (3*Sqrt[3]*((Sqrt[2/(2 + Sq
rt[10])]*EllipticE[ArcSin[Sqrt[(-2 + Sqrt[10])/2]*x], (-7 - 2*Sqrt[10])/3]
)/3 + ((5 - Sqrt[10])*Sqrt[(2 + Sqrt[10])/2]*EllipticF[ArcSin[Sqrt[(-2 + S
qrt[10])/2]*x], (-7 - 2*Sqrt[10])/3])/9))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(97) = 194$.

Time = 1.64 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.81

method	result
risch	$-\frac{x(3x^2-7)}{20\sqrt{-3x^4+4x^2+2}}+\frac{3\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-4+2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{10\sqrt{-4+2\sqrt{10}}\,\sqrt{-3x^4+4x^2+2}}-\frac{6\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}}{}$
default	$\frac{\frac{7}{20}x-\frac{3}{20}x^3}{\sqrt{-3x^4+4x^2+2}}+\frac{3\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-4+2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{10\sqrt{-4+2\sqrt{10}}\,\sqrt{-3x^4+4x^2+2}}-\frac{6\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1}}{}$
elliptic	$\frac{\frac{7}{20}x-\frac{3}{20}x^3}{\sqrt{-3x^4+4x^2+2}}+\frac{3\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-4+2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{10\sqrt{-4+2\sqrt{10}}\,\sqrt{-3x^4+4x^2+2}}-\frac{6\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1}}{}$

input
```
int(1/(-3*x^4+4*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*x*(3*x^2-7)/(-3*x^4+4*x^2+2)^(1/2)+3/10/(-4+2*10^(1/2))^(1/2)*(1-(-1
+1/2*10^(1/2))*x^2)^(1/2)*(1-(-1-1/2*10^(1/2))*x^2)^(1/2)/(-3*x^4+4*x^2+2)
^(1/2)*EllipticF(1/2*(-4+2*10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))
-6/5/(-4+2*10^(1/2))^(1/2)*(1-(-1+1/2*10^(1/2))*x^2)^(1/2)*(1-(-1-1/2*10^(
1/2))*x^2)^(1/2)/(-3*x^4+4*x^2+2)^(1/2)/(4+2*10^(1/2))*(EllipticF(1/2*(-4+
2*10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))-EllipticE(1/2*(-4+2*10^(
1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.22

$$\int \frac{1}{(2+4x^2-3x^4)^{3/2}}\,dx = \frac{4\sqrt{2}(3\,x^4-4\,x^2-2)\sqrt{\frac{1}{2}\sqrt{10}-1}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{10}-1}\right)|-\frac{2}{3}\sqrt{10}-\frac{7}{3})+}{}$$

input
```
integrate(1/(-3*x^4+4*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/40*(4*sqrt(2)*(3*x^4 - 4*x^2 - 2)*sqrt(1/2*sqrt(10) - 1)*elliptic_f(arcs
in(x*sqrt(1/2*sqrt(10) - 1)), -2/3*sqrt(10) - 7/3) + (sqrt(10)*sqrt(2)*(3*
x^4 - 4*x^2 - 2) - 2*sqrt(2)*(3*x^4 - 4*x^2 - 2))*sqrt(1/2*sqrt(10) - 1)*e
lliptic_e(arcsin(x*sqrt(1/2*sqrt(10) - 1)), -2/3*sqrt(10) - 7/3) + 2*sqrt(
-3*x^4 + 4*x^2 + 2)*(3*x^3 - 7*x))/(3*x^4 - 4*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4+4x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4+4*x**2+2)**(3/2),x)
```

output
```
Integral((-3*x**4 + 4*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+4\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+4*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + 4*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+4\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+4*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + 4*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+4\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(4*x^2 - 3*x^4 + 2)^(3/2),x)
```

output
```
int(1/(4*x^2 - 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+4x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4+4x^2+2}}{9x^8-24x^6+4x^4+16x^2+4}dx$$

input
```
int(1/(-3*x^4+4*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + 4*x**2 + 2)/(9*x**8 - 24*x**6 + 4*x**4 + 16*x**2 + 4)
,x)
```

3.175 $\int \frac{1}{(2+3x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 131

$$\int \frac{1}{(2+3x^2-3x^4)^{3/2}}\,dx = \frac{x(7-3x^2)}{22\sqrt{2+3x^2-3x^4}} + \frac{1}{22}\sqrt{\frac{1}{2}\left(-3+\sqrt{33}\right)}\,E\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right) + \frac{1}{2}\sqrt{\frac{1}{66}\left(-3+\sqrt{33}\right)}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

output
```
1/22*x*(-3*x^2+7)/(-3*x^4+3*x^2+2)^(1/2)+1/44*(-6+2*33^(1/2))^(1/2)*Ellipt
icE(6^(1/2)/(3+33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))+1/132*(-198
+66*33^(1/2))^(1/2)*EllipticF(6^(1/2)/(3+33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1
/4*I*22^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.50 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.27

$$\int \frac{1}{(2+3x^2-3x^4)^{3/2}}\,dx = \frac{84x-36x^3+6i\sqrt{3+\sqrt{33}}\sqrt{4+6x^2-6x^4}E\left(i\text{arcsinh}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right)|\frac{1}{4}(-7+\cdots}{264\sqrt{2+3x^2-\cdots}}$$

input
```
Integrate[(2 + 3*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(84*x - 36*x^3 + (6*I)*Sqrt[3 + Sqrt[33]]*Sqrt[4 + 6*x^2 - 6*x^4]*Elliptic
E[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*x], (-7 + Sqrt[33])/4] - ((6*I)*(11 +
Sqrt[33])*Sqrt[4 + 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33
])]*x], (-7 + Sqrt[33])/4])/Sqrt[3 + Sqrt[33]])/(264*Sqrt[2 + 3*x^2 - 3*x^
4])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4+3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(7-3x^2)}{22\sqrt{-3x^4+3x^2+2}}-\frac{1}{66}\int -\frac{3(3x^2+4)}{\sqrt{-3x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{22}\int \frac{3x^2+4}{\sqrt{-3x^4+3x^2+2}}dx+\frac{x(7-3x^2)}{22\sqrt{-3x^4+3x^2+2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{11}\sqrt{3}\int \frac{3x^2+4}{\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}dx+\frac{x(7-3x^2)}{22\sqrt{-3x^4+3x^2+2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{11}\sqrt{3}\left(\frac{1}{2}\left(11-\sqrt{33}\right)\int \frac{1}{\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}dx+\frac{1}{2}\int \frac{\sqrt{6x^2+\sqrt{33}-3}}{\sqrt{-6x^2+\sqrt{33}+3}}dx\right)+\frac{x(7-3x^2)}{22\sqrt{-3x^4+3x^2+2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{11}\sqrt{3}\left(\frac{1}{2}\int \frac{\sqrt{6x^2+\sqrt{33}-3}}{\sqrt{-6x^2+\sqrt{33}+3}}dx+\frac{\left(11-\sqrt{33}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)}{2\sqrt{6\left(\sqrt{33}-3\right)}}\right)+\frac{x(7-3x^2)}{22\sqrt{-3x^4+3x^2+2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{11}\sqrt{3}\left(\frac{\left(11-\sqrt{33}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)}{2\sqrt{6\left(\sqrt{33}-3\right)}}+\frac{1}{2}\sqrt{\frac{1}{6}\left(\sqrt{33}-3\right)}E\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right.\right.\right.$$
$$\frac{x(7-3x^2)}{22\sqrt{-3x^4+3x^2+2}}$$

input
```
Int[(2 + 3*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(x*(7 - 3*x^2))/(22*Sqrt[2 + 3*x^2 - 3*x^4]) + (Sqrt[3]*((Sqrt[(-3 + Sqrt[
33])/6]*EllipticE[ArcSin[Sqrt[6/(3 + Sqrt[33])]*x], (-7 - Sqrt[33])/4])/2
+ ((11 - Sqrt[33])*EllipticF[ArcSin[Sqrt[6/(3 + Sqrt[33])]*x], (-7 - Sqrt[
33])/4])/(2*Sqrt[6*(-3 + Sqrt[33])])))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(99) = 198$.

Time = 2.37 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.66

method	result
risch	$-\frac{x(3x^2-7)}{22\sqrt{-3x^4+3x^2+2}} + \frac{4\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3+\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{11\sqrt{-3+\sqrt{33}}\sqrt{-3x^4+3x^2+2}} - \frac{12\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)\cdots}}{}$
default	$\frac{\frac{7}{22}x-\frac{3}{22}x^3}{\sqrt{-3x^4+3x^2+2}} + \frac{4\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3+\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{11\sqrt{-3+\sqrt{33}}\sqrt{-3x^4+3x^2+2}} - \frac{12\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{\cdots}}{}$
elliptic	$\frac{\frac{7}{22}x-\frac{3}{22}x^3}{\sqrt{-3x^4+3x^2+2}} + \frac{4\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3+\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{11\sqrt{-3+\sqrt{33}}\sqrt{-3x^4+3x^2+2}} - \frac{12\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{\cdots}}{}$

input
```
int(1/(-3*x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/22*x*(3*x^2-7)/(-3*x^4+3*x^2+2)^(1/2)+4/11/(-3+33^(1/2))^(1/2)*(1-(-3/4
+1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*33^(1/2))*x^2)^(1/2)/(-3*x^4+3*x^2+
2)^(1/2)*EllipticF(1/2*(-3+33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))
-12/11/(-3+33^(1/2))^(1/2)*(1-(-3/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*
33^(1/2))*x^2)^(1/2)/(-3*x^4+3*x^2+2)^(1/2)/(3+33^(1/2))*(EllipticF(1/2*(-
3+33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))-EllipticE(1/2*(-3+33^(1/
2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.31

$$\int \frac{1}{(2+3x^2-3x^4)^{3/2}}\,dx = \frac{3\left(\sqrt{33}\sqrt{2}(3x^4-3x^2-2)-3\sqrt{2}(3x^4-3x^2-2)\right)\sqrt{\sqrt{33}-3}\,E\left(\arcsin\left(\frac{1}{2}x\sqrt{\cdots}\right.\right.}{\cdots}$$

input
```
integrate(1/(-3*x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/528*(3*(sqrt(33)*sqrt(2)*(3*x^4 - 3*x^2 - 2) - 3*sqrt(2)*(3*x^4 - 3*x^2
- 2))*sqrt(sqrt(33) - 3)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(33) - 3)), -1/4
*sqrt(33) - 7/4) + (sqrt(33)*sqrt(2)*(3*x^4 - 3*x^2 - 2) + 21*sqrt(2)*(3*x
^4 - 3*x^2 - 2))*sqrt(sqrt(33) - 3)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(33)
- 3)), -1/4*sqrt(33) - 7/4) + 24*sqrt(-3*x^4 + 3*x^2 + 2)*(3*x^3 - 7*x))/(
3*x^4 - 3*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4+3x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral((-3*x**4 + 3*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + 3*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + 3*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+3\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(3*x^2 - 3*x^4 + 2)^(3/2),x)
```

output
```
int(1/(3*x^2 - 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+3x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4+3x^2+2}}{9x^8-18x^6-3x^4+12x^2+4}dx$$

input
```
int(1/(-3*x^4+3*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + 3*x**2 + 2)/(9*x**8 - 18*x**6 - 3*x**4 + 12*x**2 + 4)
,x)
```

3.176 $\int \frac{1}{(2+2x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 127

$$\int \frac{1}{(2+2x^2-3x^4)^{3/2}}\,dx = \frac{x(8-3x^2)}{28\sqrt{2+2x^2-3x^4}} + \frac{1}{28}\sqrt{-1+\sqrt{7}}E\left(\arcsin\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right) + \frac{1}{4}\sqrt{\frac{1}{7}\left(-1+\sqrt{7}\right)}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4-\sqrt{7}\right)\right)$$

output
```
1/28*x*(-3*x^2+8)/(-3*x^4+2*x^2+2)^(1/2)+1/28*(-1+7^(1/2))^(1/2)*EllipticE
(3^(1/2)/(1+7^(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))+1/28*(-7+7*7^(1
/2))^(1/2)*EllipticF(3^(1/2)/(1+7^(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1
/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\int \frac{1}{(2+2x^2-3x^4)^{3/2}}\,dx = \frac{1}{28}\left(\frac{x(8-3x^2)}{\sqrt{2+2x^2-3x^4}}\right.$$
$$+i\sqrt{1+\sqrt{7}}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)-\frac{i(7+\sqrt{7})\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right),\frac{1}{3}(\right.}{\sqrt{1+\sqrt{7}}}$$

input
```
Integrate[(2 + 2*x^2 - 3*x^4)^(-3/2),x]
```

output
```
((x*(8 - 3*x^2))/Sqrt[2 + 2*x^2 - 3*x^4] + I*Sqrt[1 + Sqrt[7]]*EllipticE[I
*ArcSinh[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3] - (I*(7 + Sqrt[7])*E
llipticF[I*ArcSinh[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3])/Sqrt[1 +
Sqrt[7]])/28
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4+2x^2+2)^{3/2}}dx$$

$$\Big\downarrow \; 1405$$

$$\frac{x(8-3x^2)}{28\sqrt{-3x^4+2x^2+2}}-\frac{1}{56}\int -\frac{6(x^2+2)}{\sqrt{-3x^4+2x^2+2}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{3}{28}\int \frac{x^2+2}{\sqrt{-3x^4+2x^2+2}}dx+\frac{x(8-3x^2)}{28\sqrt{-3x^4+2x^2+2}}$$

$$\Big\downarrow \; 1494$$

$$\frac{3}{14}\sqrt{3}\int \frac{x^2+2}{2\sqrt{-3x^2+\sqrt{7}+1}\sqrt{3x^2+\sqrt{7}-1}}dx+\frac{x(8-3x^2)}{28\sqrt{-3x^4+2x^2+2}}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{28}\sqrt{3}\int \frac{x^2+2}{\sqrt{-3x^2+\sqrt{7}+1}\sqrt{3x^2+\sqrt{7}-1}}dx+\frac{x(8-3x^2)}{28\sqrt{-3x^4+2x^2+2}}$$

$$\Big\downarrow \; 399$$

$$\frac{3}{28}\sqrt{3}\left(\frac{1}{3}\left(7-\sqrt{7}\right)\int \frac{1}{\sqrt{-3x^2+\sqrt{7}+1}\sqrt{3x^2+\sqrt{7}-1}}dx+\frac{1}{3}\int \frac{\sqrt{3x^2+\sqrt{7}-1}}{\sqrt{-3x^2+\sqrt{7}+1}}dx\right)+\frac{x(8-3x^2)}{28\sqrt{-3x^4+2x^2+2}}$$

$$\Big\downarrow \; 321$$

$$\frac{3}{28}\sqrt{3}\left(\frac{1}{3}\int \frac{\sqrt{3x^2+\sqrt{7}-1}}{\sqrt{-3x^2+\sqrt{7}+1}}dx+\frac{(7-\sqrt{7})\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right),\frac{1}{3}(-4-\sqrt{7})\right)}{3\sqrt{3\left(\sqrt{7}-1\right)}}\right)+\frac{x(8-3x^2)}{28\sqrt{-3x^4+2x^2+2}}$$

$$\Big\downarrow \; 327$$

$$\frac{3}{28}\sqrt{3}\left(\frac{(7-\sqrt{7})\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right),\frac{1}{3}(-4-\sqrt{7})\right)}{3\sqrt{3\left(\sqrt{7}-1\right)}}+\frac{1}{3}\sqrt{\frac{1}{3}\left(\sqrt{7}-1\right)}E\left(\arcsin\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-\right.\right.\right.$$
$$\frac{x(8-3x^2)}{28\sqrt{-3x^4+2x^2+2}}$$

input

```
Int[(2 + 2*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(x*(8 - 3*x^2))/(28*Sqrt[2 + 2*x^2 - 3*x^4]) + (3*Sqrt[3]*((Sqrt[(-1 + Sqr
t[7])/3]*EllipticE[ArcSin[Sqrt[3/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3])/3 +
 ((7 - Sqrt[7])*EllipticF[ArcSin[Sqrt[3/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/
3])/(3*Sqrt[3*(-1 + Sqrt[7])])))/28
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(97) = 194$.

Time = 1.87 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.81

method	result
risch	$-\frac{x(3x^2-8)}{28\sqrt{-3x^4+2x^2+2}}+\frac{3\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-2+2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{7\sqrt{-2+2\sqrt{7}}\sqrt{-3x^4+2x^2+2}}-\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\sqrt{}}{}$
default	$\frac{\frac{2}{7}x-\frac{3}{28}x^3}{\sqrt{-3x^4+2x^2+2}}+\frac{3\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-2+2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{7\sqrt{-2+2\sqrt{7}}\sqrt{-3x^4+2x^2+2}}-\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\sqrt{1-(}}{}$
elliptic	$\frac{\frac{2}{7}x-\frac{3}{28}x^3}{\sqrt{-3x^4+2x^2+2}}+\frac{3\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-2+2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{7\sqrt{-2+2\sqrt{7}}\sqrt{-3x^4+2x^2+2}}-\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\sqrt{1-(}}{}$

input
```
int(1/(-3*x^4+2*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/28*x*(3*x^2-8)/(-3*x^4+2*x^2+2)^(1/2)+3/7/(-2+2*7^(1/2))^(1/2)*(1-(-1/2
+1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*7^(1/2))*x^2)^(1/2)/(-3*x^4+2*x^2+2)
^(1/2)*EllipticF(1/2*(-2+2*7^(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))-
6/7/(-2+2*7^(1/2))^(1/2)*(1-(-1/2+1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*7^(
1/2))*x^2)^(1/2)/(-3*x^4+2*x^2+2)^(1/2)/(2+2*7^(1/2))*(EllipticF(1/2*(-2+2
*7^(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))-EllipticE(1/2*(-2+2*7^(1/2
))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.39

$$\int \frac{1}{(2+2x^2-3x^4)^{3/2}}\,dx = \frac{\left(\sqrt{7}\sqrt{2}(3\,x^4-2\,x^2-2)-\sqrt{2}(3\,x^4-2\,x^2-2)\right)\sqrt{\frac{1}{2}\sqrt{7}-\frac{1}{2}}E\left(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{\cdots}}\right.\right.}{\cdots}$$

input
```
integrate(1/(-3*x^4+2*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/56*((sqrt(7)*sqrt(2)*(3*x^4 - 2*x^2 - 2) - sqrt(2)*(3*x^4 - 2*x^2 - 2))*
sqrt(1/2*sqrt(7) - 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(7) - 1/2)), -1/3
*sqrt(7) - 4/3) + (sqrt(7)*sqrt(2)*(3*x^4 - 2*x^2 - 2) + 3*sqrt(2)*(3*x^4
- 2*x^2 - 2))*sqrt(1/2*sqrt(7) - 1/2)*elliptic_f(arcsin(x*sqrt(1/2*sqrt(7)
 - 1/2)), -1/3*sqrt(7) - 4/3) + 2*sqrt(-3*x^4 + 2*x^2 + 2)*(3*x^3 - 8*x))/
(3*x^4 - 2*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+2x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4+2x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4+2*x**2+2)**(3/2),x)
```

output
```
Integral((-3*x**4 + 2*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+2x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+2\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+2*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + 2*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(2+2x^2-3x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-3\,x^4+2\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+2*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + 2*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2+2x^2-3x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-3\,x^4+2\,x^2+2\right)^{3/2}}\,dx$$

input
```
int(1/(2*x^2 - 3*x^4 + 2)^(3/2),x)
```

output
```
int(1/(2*x^2 - 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(2+2x^2-3x^4\right)^{3/2}}\,dx=\int \frac{\sqrt{-3x^4+2x^2+2}}{9x^8-12x^6-8x^4+8x^2+4}dx$$

input
```
int(1/(-3*x^4+2*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + 2*x**2 + 2)/(9*x**8 - 12*x**6 - 8*x**4 + 8*x**2 + 4),
x)
```

3.177 $\int \frac{1}{(2+x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{1}{(2+x^2-3x^4)^{3/2}}\,dx=\frac{x(13-3x^2)}{50\sqrt{2+x^2-3x^4}}+\frac{E\left(\arcsin(x)\left|-\frac{3}{2}\right.\right)}{25\sqrt{2}}+\frac{\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{5\sqrt{2}}$$

output
```
1/50*x*(-3*x^2+13)/(-3*x^4+x^2+2)^(1/2)+1/50*EllipticE(x,1/2*I*6^(1/2))*2^
(1/2)+1/10*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.98 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.67

$$\int \frac{1}{(2+x^2-3x^4)^{3/2}}\,dx=\frac{1}{50}\left(\frac{13x}{\sqrt{2+x^2-3x^4}}-\frac{3x^3}{\sqrt{2+x^2-3x^4}}+i\sqrt{3}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\left|-\frac{2}{3}\right.\right)-5i\sqrt{3}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)\right)$$

input
```
Integrate[(2 + x^2 - 3*x^4)^(-3/2),x]
```

output
```
((13*x)/Sqrt[2 + x^2 - 3*x^4] - (3*x^3)/Sqrt[2 + x^2 - 3*x^4] + I*Sqrt[3]*
EllipticE[I*ArcSinh[Sqrt[3/2]*x], -2/3] - (5*I)*Sqrt[3]*EllipticF[I*ArcSin
h[Sqrt[3/2]*x], -2/3])/50
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4+x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(13-3x^2)}{50\sqrt{-3x^4+x^2+2}}-\frac{1}{50}\int -\frac{3(x^2+4)}{\sqrt{-3x^4+x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{50}\int \frac{x^2+4}{\sqrt{-3x^4+x^2+2}}dx+\frac{x(13-3x^2)}{50\sqrt{-3x^4+x^2+2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{3}{25}\sqrt{3}\int \frac{x^2+4}{2\sqrt{3}\sqrt{1-x^2}\sqrt{3x^2+2}}dx+\frac{x(13-3x^2)}{50\sqrt{-3x^4+x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{50}\int \frac{x^2+4}{\sqrt{1-x^2}\sqrt{3x^2+2}}dx+\frac{x(13-3x^2)}{50\sqrt{-3x^4+x^2+2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{3}{50}\left(\frac{10}{3}\int \frac{1}{\sqrt{1-x^2}\sqrt{3x^2+2}}dx+\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{1-x^2}}dx\right)+\frac{x(13-3x^2)}{50\sqrt{-3x^4+x^2+2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{3}{50}\left(\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{1-x^2}}dx+\frac{5}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)\right)+\frac{x(13-3x^2)}{50\sqrt{-3x^4+x^2+2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{3}{50}\left(\frac{5}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)+\frac{1}{3}\sqrt{2}E\left(\arcsin(x)\left|-\frac{3}{2}\right.\right)\right)+\frac{x(13-3x^2)}{50\sqrt{-3x^4+x^2+2}}$$

input
```
Int[(2 + x^2 - 3*x^4)^(-3/2),x]
```

output
```
(x*(13 - 3*x^2))/(50*Sqrt[2 + x^2 - 3*x^4]) + (3*((Sqrt[2]*EllipticE[ArcSi
n[x], -3/2])/3 + (5*Sqrt[2]*EllipticF[ArcSin[x], -3/2])/3))/50
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(49) = 98$.

Time = 1.79 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.02

method	result
risch	$-\frac{x(3x^2-13)}{50\sqrt{-3x^4+x^2+2}}+\frac{3\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(x,\frac{i\sqrt{6}}{2}\right)}{25\sqrt{-3x^4+x^2+2}}-\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,\left(\operatorname{EllipticF}\left(x,\frac{i\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(x,\frac{i\sqrt{6}}{2}\right)\right)}{50\sqrt{-3x^4+x^2+2}}$
default	$\frac{\frac{13}{50}x-\frac{3}{50}x^3}{\sqrt{-3x^4+x^2+2}}+\frac{3\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(x,\frac{i\sqrt{6}}{2}\right)}{25\sqrt{-3x^4+x^2+2}}-\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,\left(\operatorname{EllipticF}\left(x,\frac{i\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(x,\frac{i\sqrt{6}}{2}\right)\right)}{50\sqrt{-3x^4+x^2+2}}$
elliptic	$\frac{\frac{13}{50}x-\frac{3}{50}x^3}{\sqrt{-3x^4+x^2+2}}+\frac{3\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(x,\frac{i\sqrt{6}}{2}\right)}{25\sqrt{-3x^4+x^2+2}}-\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,\left(\operatorname{EllipticF}\left(x,\frac{i\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(x,\frac{i\sqrt{6}}{2}\right)\right)}{50\sqrt{-3x^4+x^2+2}}$

input
```
int(1/(-3*x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/50*x*(3*x^2-13)/(-3*x^4+x^2+2)^(1/2)+3/25*(-x^2+1)^(1/2)*(6*x^2+4)^(1/2
)/(-3*x^4+x^2+2)^(1/2)*EllipticF(x,1/2*I*6^(1/2))-1/50*(-x^2+1)^(1/2)*(6*x
^2+4)^(1/2)/(-3*x^4+x^2+2)^(1/2)*(EllipticF(x,1/2*I*6^(1/2))-EllipticE(x,1
/2*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \frac{1}{(2+x^2-3x^4)^{3/2}}\,dx = \frac{\sqrt{2}(3\,x^4-x^2-2)E(\arcsin{(x)}\mid-\frac{3}{2})+5\,\sqrt{2}(3\,x^4-x^2-2)F(\arcsin{(x)}\mid-\frac{3}{2})+}{50\,(3\,x^4-x^2-2)}$$

input
```
integrate(1/(-3*x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/50*(sqrt(2)*(3*x^4 - x^2 - 2)*elliptic_e(arcsin(x), -3/2) + 5*sqrt(2)*(3
*x^4 - x^2 - 2)*elliptic_f(arcsin(x), -3/2) + sqrt(-3*x^4 + x^2 + 2)*(3*x^
3 - 13*x))/(3*x^4 - x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4+x**2+2)**(3/2),x)
```

output
```
Integral((-3*x**4 + x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+x^2+2)^{3/2}}\,dx$$

input
```
int(1/(x^2 - 3*x^4 + 2)^(3/2),x)
```

output
```
int(1/(x^2 - 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4+x^2+2}}{9x^8-6x^6-11x^4+4x^2+4}dx$$

input
```
int(1/(-3*x^4+x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + x**2 + 2)/(9*x**8 - 6*x**6 - 11*x**4 + 4*x**2 + 4),x)
```

3.178 $\int \frac{1}{(2-3x^4)^{3/2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{1}{(2-3x^4)^{3/2}} dx = \frac{x}{4\sqrt{2-3x^4}} + \frac{\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{3}{2}}x\right), -1\right)}{4\sqrt[4]{6}}$$

output
```
1/4*x/(-3*x^4+2)^(1/2)+1/24*EllipticF(1/2*3^(1/4)*2^(3/4)*x,I)*6^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.49 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{1}{(2-3x^4)^{3/2}} dx = \frac{x}{4\sqrt{2-3x^4}} + \frac{x \,\text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{3x^4}{2}\right)}{4\sqrt{2}}$$

input
```
Integrate[(2 - 3*x^4)^(-3/2),x]
```

output
```
x/(4*Sqrt[2 - 3*x^4]) + (x*Hypergeometric2F1[1/4, 1/2, 5/4, (3*x^4)/2])/(4
*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{749, 762\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-3x^4)^{3/2}} \, dx$$

$$\Big\downarrow \; 749$$

$$\frac{1}{4}\int \frac{1}{\sqrt{2-3x^4}}dx+\frac{x}{4\sqrt{2-3x^4}}$$

$$\Big\downarrow \; 762$$

$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{3}{2}}x\right),-1\right)}{4\sqrt[4]{6}}+\frac{x}{4\sqrt{2-3x^4}}$$

input
```
Int[(2 - 3*x^4)^(-3/2),x]
```

output
```
x/(4*Sqrt[2 - 3*x^4]) + EllipticF[ArcSin[(3/2)^(1/4)*x], -1]/(4*6^(1/4))
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.71 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.47

method	result	size
meijerg	$\frac{\sqrt{2}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right], \left[\frac{5}{4}\right], \frac{3 x^{4}}{2}\right)}{4}$	18
default	$\frac{x}{4 \sqrt{-3 x^{4}+2}}+\frac{\sqrt{2}\, 6^{\frac{3}{4}} \sqrt{4-2 \sqrt{6}\, x^{2}}\, \sqrt{4+2 \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{2}\, 6^{\frac{1}{4}}}{2}, i\right)}{96 \sqrt{-3 x^{4}+2}}$	67
risch	$\frac{x}{4 \sqrt{-3 x^{4}+2}}+\frac{\sqrt{2}\, 6^{\frac{3}{4}} \sqrt{4-2 \sqrt{6}\, x^{2}}\, \sqrt{4+2 \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{2}\, 6^{\frac{1}{4}}}{2}, i\right)}{96 \sqrt{-3 x^{4}+2}}$	67
elliptic	$\frac{x}{4 \sqrt{-3 x^{4}+2}}+\frac{\sqrt{2}\, 6^{\frac{3}{4}} \sqrt{4-2 \sqrt{6}\, x^{2}}\, \sqrt{4+2 \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{2}\, 6^{\frac{1}{4}}}{2}, i\right)}{96 \sqrt{-3 x^{4}+2}}$	67

input
```
int(1/(-3*x^4+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*x*hypergeom([1/4,3/2],[5/4],3/2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{1}{(2-3x^4)^{3/2}}\,dx = \frac{6^{\frac{3}{4}}\sqrt{2}\sqrt{\frac{1}{2}}(3\,x^4-2)F\left(\arcsin\left(6^{\frac{1}{4}}\sqrt{\frac{1}{2}}x\right)\,|-1\right)-6\,\sqrt{-3\,x^4+2}x}{24\,(3\,x^4-2)}$$

input
```
integrate(1/(-3*x^4+2)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(6^(3/4)*sqrt(2)*sqrt(1/2)*(3*x^4 - 2)*elliptic_f(arcsin(6^(1/4)*sqrt
(1/2)*x), -1) - 6*sqrt(-3*x^4 + 2)*x)/(3*x^4 - 2)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{(2-3x^4)^{3/2}}\,dx = \frac{\sqrt{2}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{3x^4e^{2i\pi}}{2}\right)}{16\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-3*x**4+2)**(3/2),x)
```

output
```
sqrt(2)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), 3*x**4*exp_polar(2*I*pi)/2)
/(16*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + 2)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 17.80 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.42

$$\int \frac{1}{(2-3x^4)^{3/2}}\,dx = \frac{\sqrt{2}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ \frac{3\,x^4}{2}\right)}{4}$$

input
```
int(1/(2 - 3*x^4)^(3/2),x)
```

output
```
(2^(1/2)*x*hypergeom([1/4, 3/2], 5/4, (3*x^4)/2))/4
```

Reduce [F]

$$\int \frac{1}{(2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4+2}}{9x^8-12x^4+4}dx$$

input
```
int(1/(-3*x^4+2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + 2)/(9*x**8 - 12*x**4 + 4),x)
```

3.179 $\int \frac{1}{(2-x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \frac{1}{(2-x^2-3x^4)^{3/2}}\,dx = \frac{x(13+3x^2)}{50\sqrt{2-x^2-3x^4}} - \frac{1}{50}\sqrt{3}E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right) + \frac{1}{10}\sqrt{3}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)$$

output
```
1/50*x*(3*x^2+13)/(-3*x^4-x^2+2)^(1/2)-1/50*3^(1/2)*EllipticE(1/2*x*6^(1/2
),1/3*I*6^(1/2))+1/10*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 8.95 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.35

$$\int \frac{1}{(2-x^2-3x^4)^{3/2}}\,dx = \frac{13x+3x^3-\sqrt{6-9x^2}\sqrt{1+x^2}E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)+5\sqrt{6-9x^2}\sqrt{1+x^2}\,\text{El}}{50\sqrt{2-x^2-3x^4}}$$

input
```
Integrate[(2 - x^2 - 3*x^4)^(-3/2),x]
```

output
```
(13*x + 3*x^3 - Sqrt[6 - 9*x^2]*Sqrt[1 + x^2]*EllipticE[ArcSin[Sqrt[3/2]*x
], -2/3] + 5*Sqrt[6 - 9*x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x],
-2/3])/(50*Sqrt[2 - x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-3x^4-x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x\left(3x^2+13\right)}{50\sqrt{-3x^4-x^2+2}}-\frac{1}{50}\int -\frac{3\left(4-x^2\right)}{\sqrt{-3x^4-x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{50}\int \frac{4-x^2}{\sqrt{-3x^4-x^2+2}}dx+\frac{x\left(3x^2+13\right)}{50\sqrt{-3x^4-x^2+2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{3}{25}\sqrt{3}\int \frac{4-x^2}{2\sqrt{3}\sqrt{2-3x^2}\sqrt{x^2+1}}dx+\frac{x\left(3x^2+13\right)}{50\sqrt{-3x^4-x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{50}\int \frac{4-x^2}{\sqrt{2-3x^2}\sqrt{x^2+1}}dx+\frac{x\left(3x^2+13\right)}{50\sqrt{-3x^4-x^2+2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{3}{50}\left(5\int \frac{1}{\sqrt{2-3x^2}\sqrt{x^2+1}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{2-3x^2}}dx\right)+\frac{x\left(3x^2+13\right)}{50\sqrt{-3x^4-x^2+2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{3}{50}\left(\frac{5\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}-\int\frac{\sqrt{x^2+1}}{\sqrt{2-3x^2}}dx\right)+\frac{x(3x^2+13)}{50\sqrt{-3x^4-x^2+2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{3}{50}\left(\frac{5\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}-\frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}\right)+\frac{x(3x^2+13)}{50\sqrt{-3x^4-x^2+2}}$$

input
```
Int[(2 - x^2 - 3*x^4)^(-3/2),x]
```

output
```
(x*(13 + 3*x^2))/(50*Sqrt[2 - x^2 - 3*x^4]) + (3*(-(EllipticE[ArcSin[Sqrt[
3/2]*x], -2/3]/Sqrt[3]) + (5*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/Sqrt[3]
))/50
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(61) = 122.

Time = 2.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.84

method	result
risch	$\frac{x(3x^2+13)}{50\sqrt{-3x^4-x^2+2}}+\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{25\sqrt{-3x^4-x^2+2}}+\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2}\right.\right.}{100\sqrt{-3x^4-x^2+2}}$
default	$\frac{\frac{13}{50}x+\frac{3}{50}x^3}{\sqrt{-3x^4-x^2+2}}+\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{25\sqrt{-3x^4-x^2+2}}+\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\right.\right.}{100\sqrt{-3x^4-x^2+2}}$
elliptic	$\frac{\frac{13}{50}x+\frac{3}{50}x^3}{\sqrt{-3x^4-x^2+2}}+\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{25\sqrt{-3x^4-x^2+2}}+\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\right.\right.}{100\sqrt{-3x^4-x^2+2}}$

input
```
int(1/(-3*x^4-x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/50*x*(3*x^2+13)/(-3*x^4-x^2+2)^(1/2)+1/25*6^(1/2)*(-6*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-3*x^4-x^2+2)^(1/2)*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))+1/100
*6^(1/2)*(-6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-x^2+2)^(1/2)*(EllipticF(1/
2*x*6^(1/2),1/3*I*6^(1/2))-EllipticE(1/2*x*6^(1/2),1/3*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{1}{(2-x^2-3x^4)^{3/2}}\,dx = -\frac{3\sqrt{2}\sqrt{\frac{3}{2}}(3x^4+x^2-2)E(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}) - 11\sqrt{2}\sqrt{\frac{3}{2}}(3x^4+x^2-2)F(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}) + 2\sqrt{}}{100\,(3x^4+x^2-2)}$$

input
```
integrate(1/(-3*x^4-x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/100*(3*sqrt(2)*sqrt(3/2)*(3*x^4 + x^2 - 2)*elliptic_e(arcsin(sqrt(3/2)*
x), -2/3) - 11*sqrt(2)*sqrt(3/2)*(3*x^4 + x^2 - 2)*elliptic_f(arcsin(sqrt(
3/2)*x), -2/3) + 2*sqrt(-3*x^4 - x^2 + 2)*(3*x^3 + 13*x))/(3*x^4 + x^2 - 2
)
```

Sympy [F]

$$\int \frac{1}{(2-x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4-x**2+2)**(3/2),x)
```

output
```
Integral((-3*x**4 - x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2-x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2-x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-x^2+2)^{3/2}}\,dx$$

input
```
int(1/(2 - 3*x^4 - x^2)^(3/2),x)
```

output
```
int(1/(2 - 3*x^4 - x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2-x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-x^2+2}}{9x^8+6x^6-11x^4-4x^2+4}dx$$

input
```
int(1/(-3*x^4-x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - x**2 + 2)/(9*x**8 + 6*x**6 - 11*x**4 - 4*x**2 + 4),x)
```

3.180 $\int \frac{1}{(2-2x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 123

$$\int \frac{1}{(2-2x^2-3x^4)^{3/2}}\,dx = \frac{x(8+3x^2)}{28\sqrt{2-2x^2-3x^4}} - \frac{1}{28}\sqrt{1+\sqrt{7}}E\left(\arcsin\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right) + \frac{1}{4}\sqrt{\frac{1}{7}\left(1+\sqrt{7}\right)}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)$$

output
```
1/28*x*(3*x^2+8)/(-3*x^4-2*x^2+2)^(1/2)-1/28*(1+7^(1/2))^(1/2)*EllipticE(3
^(1/2)/(-1+7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))+1/28*(7+7*7^(1/2
))^(1/2)*EllipticF(3^(1/2)/(-1+7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/
2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.13

$$\int \frac{1}{(2-2x^2-3x^4)^{3/2}}\,dx = \frac{1}{28}\left(\frac{x(8+3x^2)}{\sqrt{2-2x^2-3x^4}} - i\sqrt{-1+\sqrt{7}}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right)\middle|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right) + \frac{i(-7+\sqrt{7})\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right),-\frac{4}{3}-\right.}{\sqrt{-1+\sqrt{7}}}\right.$$

input
```
Integrate[(2 - 2*x^2 - 3*x^4)^(-3/2),x]
```

output
```
((x*(8 + 3*x^2))/Sqrt[2 - 2*x^2 - 3*x^4] - I*Sqrt[-1 + Sqrt[7]]*EllipticE[
I*ArcSinh[Sqrt[3/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3] + (I*(-7 + Sqrt[7])*
EllipticF[I*ArcSinh[Sqrt[3/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3])/Sqrt[-1 +
 Sqrt[7]])/28
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4-2x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(3x^2+8)}{28\sqrt{-3x^4-2x^2+2}} - \frac{1}{56}\int -\frac{6(2-x^2)}{\sqrt{-3x^4-2x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{28}\int \frac{2-x^2}{\sqrt{-3x^4-2x^2+2}}dx+\frac{x\left(3x^2+8\right)}{28\sqrt{-3x^4-2x^2+2}}$$

$\Big\downarrow$ 1494

$$\frac{3}{14}\sqrt{3}\int \frac{2-x^2}{2\sqrt{-3x^2+\sqrt{7}-1}\sqrt{3x^2+\sqrt{7}+1}}dx+\frac{x\left(3x^2+8\right)}{28\sqrt{-3x^4-2x^2+2}}$$

$\Big\downarrow$ 27

$$\frac{3}{28}\sqrt{3}\int \frac{2-x^2}{\sqrt{-3x^2+\sqrt{7}-1}\sqrt{3x^2+\sqrt{7}+1}}dx+\frac{x\left(3x^2+8\right)}{28\sqrt{-3x^4-2x^2+2}}$$

$\Big\downarrow$ 399

$$\frac{3}{28}\sqrt{3}\left(\frac{1}{3}\left(7+\sqrt{7}\right)\int \frac{1}{\sqrt{-3x^2+\sqrt{7}-1}\sqrt{3x^2+\sqrt{7}+1}}dx-\frac{1}{3}\int \frac{\sqrt{3x^2+\sqrt{7}+1}}{\sqrt{-3x^2+\sqrt{7}-1}}dx\right)+\frac{x\left(3x^2+8\right)}{28\sqrt{-3x^4-2x^2+2}}$$

$\Big\downarrow$ 321

$$\frac{3}{28}\sqrt{3}\left(\frac{\left(7+\sqrt{7}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{3\sqrt{3\left(1+\sqrt{7}\right)}}-\frac{1}{3}\int \frac{\sqrt{3x^2+\sqrt{7}+1}}{\sqrt{-3x^2+\sqrt{7}-1}}dx\right)+\frac{x\left(3x^2+8\right)}{28\sqrt{-3x^4-2x^2+2}}$$

$\Big\downarrow$ 327

$$\frac{3}{28}\sqrt{3}\left(\frac{\left(7+\sqrt{7}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{3\sqrt{3\left(1+\sqrt{7}\right)}}-\frac{1}{3}\sqrt{\frac{1}{3}\left(1+\sqrt{7}\right)}E\left(\arcsin\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right)|\right.\right.$$
$$\frac{x\left(3x^2+8\right)}{28\sqrt{-3x^4-2x^2+2}}$$

input
```
Int[(2 - 2*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(x*(8 + 3*x^2))/(28*Sqrt[2 - 2*x^2 - 3*x^4]) + (3*Sqrt[3]*(-1/3*(Sqrt[(1 +
 Sqrt[7])/3]*EllipticE[ArcSin[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]
) + ((7 + Sqrt[7])*EllipticF[ArcSin[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[
7])/3])/(3*Sqrt[3*(1 + Sqrt[7])])))/28
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(97) = 194.

Time = 2.29 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.87

method	result
risch	$\frac{x(3x^2+8)}{28\sqrt{-3x^4-2x^2+2}}+\frac{3\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\sqrt{1-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{2+2\sqrt{7}}\,x}{2},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{7\sqrt{2+2\sqrt{7}}\sqrt{-3x^4-2x^2+2}}+\frac{6\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\sqrt{1-\left(-\right.}}{}$
default	$\frac{\frac{2}{7}x+\frac{3}{28}x^3}{\sqrt{-3x^4-2x^2+2}}+\frac{3\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\sqrt{1-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{2+2\sqrt{7}}\,x}{2},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{7\sqrt{2+2\sqrt{7}}\sqrt{-3x^4-2x^2+2}}+\frac{6\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\sqrt{1-\left(-\frac{\sqrt{7}}{2}\right.}}{}$
elliptic	$\frac{\frac{2}{7}x+\frac{3}{28}x^3}{\sqrt{-3x^4-2x^2+2}}+\frac{3\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\sqrt{1-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{2+2\sqrt{7}}\,x}{2},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{7\sqrt{2+2\sqrt{7}}\sqrt{-3x^4-2x^2+2}}+\frac{6\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\sqrt{1-\left(-\frac{\sqrt{7}}{2}\right.}}{}$

input
```
int(1/(-3*x^4-2*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/28*x*(3*x^2+8)/(-3*x^4-2*x^2+2)^(1/2)+3/7/(2+2*7^(1/2))^(1/2)*(1-(1/2+1/
2*7^(1/2))*x^2)^(1/2)*(1-(-1/2*7^(1/2)+1/2)*x^2)^(1/2)/(-3*x^4-2*x^2+2)^(1
/2)*EllipticF(1/2*(2+2*7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))+6/7/
(2+2*7^(1/2))^(1/2)*(1-(1/2+1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2*7^(1/2)+1/2)*
x^2)^(1/2)/(-3*x^4-2*x^2+2)^(1/2)/(-2+2*7^(1/2))*(EllipticF(1/2*(2+2*7^(1/
2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))-EllipticE(1/2*(2+2*7^(1/2))^(1/2
)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.44

$$\int \frac{1}{(2-2x^2-3x^4)^{3/2}}\,dx =$$
$$-\frac{\left(\sqrt{7}\sqrt{2}(3\,x^4+2\,x^2-2)+\sqrt{2}(3\,x^4+2\,x^2-2)\right)\sqrt{\frac{1}{2}\sqrt{7}+\frac{1}{2}}E(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{7}+\frac{1}{2}}\right)\mid\frac{1}{3}\sqrt{7}-\frac{4}{3})-(3\sqrt{}}{}$$

input
```
integrate(1/(-3*x^4-2*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/56*((sqrt(7)*sqrt(2)*(3*x^4 + 2*x^2 - 2) + sqrt(2)*(3*x^4 + 2*x^2 - 2))
*sqrt(1/2*sqrt(7) + 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(7) + 1/2)), 1/3
*sqrt(7) - 4/3) - (3*sqrt(7)*sqrt(2)*(3*x^4 + 2*x^2 - 2) - sqrt(2)*(3*x^4
+ 2*x^2 - 2))*sqrt(1/2*sqrt(7) + 1/2)*elliptic_f(arcsin(x*sqrt(1/2*sqrt(7)
 + 1/2)), 1/3*sqrt(7) - 4/3) + 2*sqrt(-3*x^4 - 2*x^2 + 2)*(3*x^3 + 8*x))/(
3*x^4 + 2*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2-2x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-2x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4-2*x**2+2)**(3/2),x)
```

output
```
Integral((-3*x**4 - 2*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2-2x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-2\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-2*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - 2*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2-2x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-2\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-2*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - 2*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-2x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-2\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(2 - 3*x^4 - 2*x^2)^(3/2),x)
```

output
```
int(1/(2 - 3*x^4 - 2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2-2x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-2x^2+2}}{9x^8+12x^6-8x^4-8x^2+4}dx$$

input
```
int(1/(-3*x^4-2*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - 2*x**2 + 2)/(9*x**8 + 12*x**6 - 8*x**4 - 8*x**2 + 4),
x)
```

3.181 $\int \frac{1}{(2-3x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 127

$$\int \frac{1}{(2-3x^2-3x^4)^{3/2}}\,dx = \frac{x(7+3x^2)}{22\sqrt{2-3x^2-3x^4}} - \frac{1}{22}\sqrt{\frac{1}{2}\left(3+\sqrt{33}\right)}E\left(\arcsin\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right) + \frac{1}{2}\sqrt{\frac{1}{66}\left(3+\sqrt{33}\right)}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

output

```
1/22*x*(3*x^2+7)/(-3*x^4-3*x^2+2)^(1/2)-1/44*(6+2*33^(1/2))^(1/2)*Elliptic
E(6^(1/2)/(-3+33^(1/2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))+1/132*(198+6
6*33^(1/2))^(1/2)*EllipticF(6^(1/2)/(-3+33^(1/2))^(1/2)*x,1/4*I*22^(1/2)-1
/4*I*6^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.52 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.35

$$\int \frac{1}{(2-3x^2-3x^4)^{3/2}}\,dx = \frac{12x(7+3x^2)-6i\sqrt{-3+\sqrt{33}}\sqrt{4-6x^2-6x^4}E\left(i\text{arcsinh}\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right)|-\frac{7}{4}-\right.}{264\sqrt{2-3x^2-3}}$$

input
```
Integrate[(2 - 3*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(12*x*(7 + 3*x^2) - (6*I)*Sqrt[-3 + Sqrt[33]]*Sqrt[4 - 6*x^2 - 6*x^4]*Elli
pticE[I*ArcSinh[Sqrt[6/(3 + Sqrt[33])]*x], -7/4 - Sqrt[33]/4] + ((6*I)*(-1
1 + Sqrt[33])*Sqrt[4 - 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(3 + Sqrt
[33])]*x], -7/4 - Sqrt[33]/4])/Sqrt[-3 + Sqrt[33]])/(264*Sqrt[2 - 3*x^2 -
3*x^4])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4-3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x(3x^2+7)}{22\sqrt{-3x^4-3x^2+2}}-\frac{1}{66}\int -\frac{3(4-3x^2)}{\sqrt{-3x^4-3x^2+2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{22}\int \frac{4-3x^2}{\sqrt{-3x^4-3x^2+2}}dx+\frac{x(3x^2+7)}{22\sqrt{-3x^4-3x^2+2}}$$

$$\Big\downarrow \; 1494$$

$$\frac{1}{11}\sqrt{3}\int \frac{4-3x^2}{\sqrt{-6x^2+\sqrt{33}-3}\sqrt{6x^2+\sqrt{33}+3}}dx+\frac{x(3x^2+7)}{22\sqrt{-3x^4-3x^2+2}}$$

$$\Big\downarrow \; 399$$

$$\frac{1}{11}\sqrt{3}\left(\frac{1}{2}\left(11+\sqrt{33}\right)\int \frac{1}{\sqrt{-6x^2+\sqrt{33}-3}\sqrt{6x^2+\sqrt{33}+3}}dx-\frac{1}{2}\int \frac{\sqrt{6x^2+\sqrt{33}+3}}{\sqrt{-6x^2+\sqrt{33}-3}}dx\right)+\frac{x(3x^2+7)}{22\sqrt{-3x^4-3x^2+2}}$$

$$\Big\downarrow \; 321$$

$$\frac{1}{11}\sqrt{3}\left(\frac{\left(11+\sqrt{33}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{2\sqrt{6\left(3+\sqrt{33}\right)}}-\frac{1}{2}\int \frac{\sqrt{6x^2+\sqrt{33}+3}}{\sqrt{-6x^2+\sqrt{33}-3}}dx\right)+\frac{x(3x^2+7)}{22\sqrt{-3x^4-3x^2+2}}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{11}\sqrt{3}\left(\frac{\left(11+\sqrt{33}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{2\sqrt{6\left(3+\sqrt{33}\right)}}-\frac{1}{2}\sqrt{\frac{1}{6}\left(3+\sqrt{33}\right)}E\left(\arcsin\left(\sqrt{\frac{6}{-3+\sqrt{3}}}\right.\right.\right.$$
$$\frac{x(3x^2+7)}{22\sqrt{-3x^4-3x^2+2}}$$

input
```
Int[(2 - 3*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(x*(7 + 3*x^2))/(22*Sqrt[2 - 3*x^2 - 3*x^4]) + (Sqrt[3]*(-1/2*(Sqrt[(3 + S
qrt[33])/6]*EllipticE[ArcSin[Sqrt[6/(-3 + Sqrt[33])]*x], (-7 + Sqrt[33])/4
]) + ((11 + Sqrt[33])*EllipticF[ArcSin[Sqrt[6/(-3 + Sqrt[33])]*x], (-7 + S
qrt[33])/4])/(2*Sqrt[6*(3 + Sqrt[33])])))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(99) = 198$.

Time = 2.78 (sec), antiderivative size = 218, normalized size of antiderivative = 1.72

method	result
risch	$\frac{x(3x^2+7)}{22\sqrt{-3x^4-3x^2+2}}+\frac{4\sqrt{1-\left(\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3+\sqrt{33}}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{11\sqrt{3+\sqrt{33}}\sqrt{-3x^4-3x^2+2}}+\frac{12\sqrt{1-\left(\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\sqrt{1-(}{}$
default	$\frac{\frac{7}{22}x+\frac{3}{22}x^3}{\sqrt{-3x^4-3x^2+2}}+\frac{4\sqrt{1-\left(\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3+\sqrt{33}}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{11\sqrt{3+\sqrt{33}}\sqrt{-3x^4-3x^2+2}}+\frac{12\sqrt{1-\left(\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\sqrt{1-\left(\frac{3}{4}-\right.}{}$
elliptic	$\frac{\frac{7}{22}x+\frac{3}{22}x^3}{\sqrt{-3x^4-3x^2+2}}+\frac{4\sqrt{1-\left(\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3+\sqrt{33}}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{11\sqrt{3+\sqrt{33}}\sqrt{-3x^4-3x^2+2}}+\frac{12\sqrt{1-\left(\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\sqrt{1-\left(\frac{3}{4}-\right.}{}$

input
```
int(1/(-3*x^4-3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/22*x*(3*x^2+7)/(-3*x^4-3*x^2+2)^(1/2)+4/11/(3+33^(1/2))^(1/2)*(1-(3/4+1/
4*33^(1/2))*x^2)^(1/2)*(1-(3/4-1/4*33^(1/2))*x^2)^(1/2)/(-3*x^4-3*x^2+2)^(
1/2)*EllipticF(1/2*(3+33^(1/2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))+12/1
1/(3+33^(1/2))^(1/2)*(1-(3/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(3/4-1/4*33^(1/2)
)*x^2)^(1/2)/(-3*x^4-3*x^2+2)^(1/2)/(-3+33^(1/2))*(EllipticF(1/2*(3+33^(1/
2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))-EllipticE(1/2*(3+33^(1/2))^(1/2)
*x,1/4*I*22^(1/2)-1/4*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec), antiderivative size = 173, normalized size of antiderivative = 1.36

$$\int \frac{1}{(2-3x^2-3x^4)^{3/2}}\,dx =$$
$$-\frac{3\left(\sqrt{33}\sqrt{2}(3\,x^4+3\,x^2-2)+3\,\sqrt{2}(3\,x^4+3\,x^2-2)\right)\sqrt{\sqrt{33}+3}E(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{33}+3}\right)\mid\frac{1}{4}\,\sqrt{33}-\frac{7}{4})-}{}$$

input
```
integrate(1/(-3*x^4-3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/528*(3*(sqrt(33)*sqrt(2)*(3*x^4 + 3*x^2 - 2) + 3*sqrt(2)*(3*x^4 + 3*x^2
 - 2))*sqrt(sqrt(33) + 3)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(33) + 3)), 1/4
*sqrt(33) - 7/4) - (7*sqrt(33)*sqrt(2)*(3*x^4 + 3*x^2 - 2) - 3*sqrt(2)*(3*
x^4 + 3*x^2 - 2))*sqrt(sqrt(33) + 3)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(33)
 + 3)), 1/4*sqrt(33) - 7/4) + 24*sqrt(-3*x^4 - 3*x^2 + 2)*(3*x^3 + 7*x))/(
3*x^4 + 3*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2-3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-3x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4-3*x**2+2)**(3/2),x)
```

output
```
Integral((-3*x**4 - 3*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2-3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - 3*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2-3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - 3*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-3\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(2 - 3*x^4 - 3*x^2)^(3/2),x)
```

output
```
int(1/(2 - 3*x^4 - 3*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2-3x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-3x^2+2}}{9x^8+18x^6-3x^4-12x^2+4}dx$$

input
```
int(1/(-3*x^4-3*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - 3*x**2 + 2)/(9*x**8 + 18*x**6 - 3*x**4 - 12*x**2 + 4)
,x)
```

3.182 $\int \frac{1}{(2-4x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 127

$$\int \frac{1}{(2-4x^2-3x^4)^{3/2}}\,dx = \frac{x(7+3x^2)}{20\sqrt{2-4x^2-3x^4}} - \frac{1}{20}\sqrt{2+\sqrt{10}}E\left(\arcsin\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right) + \frac{1}{4}\sqrt{\frac{1}{10}\left(2+\sqrt{10}\right)}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)$$

output
```
1/20*x*(3*x^2+7)/(-3*x^4-4*x^2+2)^(1/2)-1/20*(2+10^(1/2))^(1/2)*EllipticE(
1/2*(4+2*10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))+1/40*(20+10*10^(1
/2))^(1/2)*EllipticF(1/2*(4+2*10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/
2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\int \frac{1}{(2-4x^2-3x^4)^{3/2}}\,dx = \frac{1}{20}\left(\frac{x(7+3x^2)}{\sqrt{2-4x^2-3x^4}}\right.$$

$$-i\sqrt{-2+\sqrt{10}}E\left(i\text{arcsinh}\left(\sqrt{-1+\sqrt{\frac{5}{2}}}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)+\frac{i\left(-5+\sqrt{10}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\right.\right.}{\sqrt{-2+\sqrt{1}}}$$

input

```
Integrate[(2 - 4*x^2 - 3*x^4)^(-3/2),x]
```

output

```
((x*(7 + 3*x^2))/Sqrt[2 - 4*x^2 - 3*x^4] - I*Sqrt[-2 + Sqrt[10]]*EllipticE
[I*ArcSinh[Sqrt[-1 + Sqrt[5/2]]*x], (-7 - 2*Sqrt[10])/3] + (I*(-5 + Sqrt[1
0])*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/2]]*x], (-7 - 2*Sqrt[10])/3])/Sqr
t[-2 + Sqrt[10]])/20
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4-4x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x(3x^2+7)}{20\sqrt{-3x^4-4x^2+2}}-\frac{1}{80}\int -\frac{12(1-x^2)}{\sqrt{-3x^4-4x^2+2}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{3}{20}\int \frac{1-x^2}{\sqrt{-3x^4-4x^2+2}}dx+\frac{x(3x^2+7)}{20\sqrt{-3x^4-4x^2+2}}$$

$$\Big\downarrow \; 1494$$

$$\frac{3}{10}\sqrt{3}\int \frac{1-x^2}{2\sqrt{-3x^2+\sqrt{10}-2}\sqrt{3x^2+\sqrt{10}+2}}dx+\frac{x(3x^2+7)}{20\sqrt{-3x^4-4x^2+2}}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{20}\sqrt{3}\int \frac{1-x^2}{\sqrt{-3x^2+\sqrt{10}-2}\sqrt{3x^2+\sqrt{10}+2}}dx+\frac{x(3x^2+7)}{20\sqrt{-3x^4-4x^2+2}}$$

$$\Big\downarrow \; 399$$

$$\frac{3}{20}\sqrt{3}\left(\frac{1}{3}\left(5+\sqrt{10}\right)\int \frac{1}{\sqrt{-3x^2+\sqrt{10}-2}\sqrt{3x^2+\sqrt{10}+2}}dx-\frac{1}{3}\int \frac{\sqrt{3x^2+\sqrt{10}+2}}{\sqrt{-3x^2+\sqrt{10}-2}}dx\right)+\frac{x(3x^2+7)}{20\sqrt{-3x^4-4x^2+2}}$$

$$\Big\downarrow \; 321$$

$$\frac{3}{20}\sqrt{3}\left(\frac{1}{9}\sqrt{\frac{1}{2}\left(\sqrt{10}-2\right)}\left(5+\sqrt{10}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)-\frac{1}{3}\int \frac{\sqrt{3x^2+}}{\sqrt{-3x^2+}}\right.$$
$$\frac{x(3x^2+7)}{20\sqrt{-3x^4-4x^2+2}}$$

$$\Big\downarrow \; 327$$

$$\frac{3}{20}\sqrt{3}\left(\frac{1}{9}\sqrt{\frac{1}{2}\left(\sqrt{10}-2\right)}\left(5+\sqrt{10}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)-\frac{1}{3}\sqrt{\frac{2}{\sqrt{10}-2}}E\right.$$
$$\frac{x(3x^2+7)}{20\sqrt{-3x^4-4x^2+2}}$$

input
```
Int[(2 - 4*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(x*(7 + 3*x^2))/(20*Sqrt[2 - 4*x^2 - 3*x^4]) + (3*Sqrt[3]*(-1/3*(Sqrt[2/(-
2 + Sqrt[10])]*EllipticE[ArcSin[Sqrt[(2 + Sqrt[10])/2]*x], (-7 + 2*Sqrt[10
])/3]) + (Sqrt[(-2 + Sqrt[10])/2]*(5 + Sqrt[10])*EllipticF[ArcSin[Sqrt[(2
+ Sqrt[10])/2]*x], (-7 + 2*Sqrt[10])/3])/9))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(97) = 194.

Time = 2.24 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.81

method	result
risch	$\frac{x(3x^2+7)}{20\sqrt{-3x^4-4x^2+2}}+\frac{3\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{4+2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{10\sqrt{4+2\sqrt{10}}\,\sqrt{-3x^4-4x^2+2}}+\frac{6\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(1-\right.}}{}$
default	$\frac{\frac{7}{20}x+\frac{3}{20}x^3}{\sqrt{-3x^4-4x^2+2}}+\frac{3\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{4+2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{10\sqrt{4+2\sqrt{10}}\,\sqrt{-3x^4-4x^2+2}}+\frac{6\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(1-\right.}}{}$
elliptic	$\frac{\frac{7}{20}x+\frac{3}{20}x^3}{\sqrt{-3x^4-4x^2+2}}+\frac{3\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{4+2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{10\sqrt{4+2\sqrt{10}}\,\sqrt{-3x^4-4x^2+2}}+\frac{6\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(1-\right.}}{}$

input
```
int(1/(-3*x^4-4*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*x*(3*x^2+7)/(-3*x^4-4*x^2+2)^(1/2)+3/10/(4+2*10^(1/2))^(1/2)*(1-(1+1/
2*10^(1/2))*x^2)^(1/2)*(1-(1-1/2*10^(1/2))*x^2)^(1/2)/(-3*x^4-4*x^2+2)^(1/
2)*EllipticF(1/2*(4+2*10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))+6/5/
(4+2*10^(1/2))^(1/2)*(1-(1+1/2*10^(1/2))*x^2)^(1/2)*(1-(1-1/2*10^(1/2))*x^
2)^(1/2)/(-3*x^4-4*x^2+2)^(1/2)/(-4+2*10^(1/2))*(EllipticF(1/2*(4+2*10^(1/
2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))-EllipticE(1/2*(4+2*10^(1/2))^(1/
2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.25

$$\int \frac{1}{(2-4x^2-3x^4)^{3/2}}\,dx = \frac{2\sqrt{10}\sqrt{2}(3\,x^4+4\,x^2-2)\sqrt{\frac{1}{2}\sqrt{10}+1}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{10}+1}\right)\mid\frac{2}{3}\sqrt{10}-\frac{7}{3})}{}$$

input
```
integrate(1/(-3*x^4-4*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/40*(2*sqrt(10)*sqrt(2)*(3*x^4 + 4*x^2 - 2)*sqrt(1/2*sqrt(10) + 1)*ellipt
ic_f(arcsin(x*sqrt(1/2*sqrt(10) + 1)), 2/3*sqrt(10) - 7/3) - (sqrt(10)*sqr
t(2)*(3*x^4 + 4*x^2 - 2) + 2*sqrt(2)*(3*x^4 + 4*x^2 - 2))*sqrt(1/2*sqrt(10
) + 1)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(10) + 1)), 2/3*sqrt(10) - 7/3) -
2*sqrt(-3*x^4 - 4*x^2 + 2)*(3*x^3 + 7*x))/(3*x^4 + 4*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2-4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-4x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4-4*x**2+2)**(3/2),x)
```

output
```
Integral((-3*x**4 - 4*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2-4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-4\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-4*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - 4*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2-4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-4\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-4*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - 4*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-4\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(2 - 3*x^4 - 4*x^2)^(3/2),x)
```

output
```
int(1/(2 - 3*x^4 - 4*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2-4x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-4x^2+2}}{9x^8+24x^6+4x^4-16x^2+4}dx$$

input
```
int(1/(-3*x^4-4*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - 4*x**2 + 2)/(9*x**8 + 24*x**6 + 4*x**4 - 16*x**2 + 4)
,x)
```

3.183 $\int \frac{1}{(2-5x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \frac{1}{(2-5x^2-3x^4)^{3/2}}\,dx = \frac{x(37+15x^2)}{98\sqrt{2-5x^2-3x^4}} - \frac{5}{49}\sqrt{\frac{3}{2}}E\left(\arcsin\left(\sqrt{3}x\right)|-\frac{1}{6}\right) + \frac{1}{7}\sqrt{\frac{3}{2}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{3}x\right),-\frac{1}{6}\right)$$

output
```
1/98*x*(15*x^2+37)/(-3*x^4-5*x^2+2)^(1/2)-5/98*6^(1/2)*EllipticE(x*3^(1/2)
,1/6*I*6^(1/2))+1/14*EllipticF(x*3^(1/2),1/6*I*6^(1/2))*6^(1/2)
```

Mathematica [A] (verified)

Time = 9.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \frac{1}{(2-5x^2-3x^4)^{3/2}}\,dx = \frac{37x+15x^3-5\sqrt{6-18x^2}\sqrt{2+x^2}E\left(\arcsin\left(\sqrt{3}x\right)|-\frac{1}{6}\right)+7\sqrt{6-18x^2}\sqrt{2+x}}{98\sqrt{2-5x^2-3x^4}}$$

input
```
Integrate[(2 - 5*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(37*x + 15*x^3 - 5*Sqrt[6 - 18*x^2]*Sqrt[2 + x^2]*EllipticE[ArcSin[Sqrt[3]
*x], -1/6] + 7*Sqrt[6 - 18*x^2]*Sqrt[2 + x^2]*EllipticF[ArcSin[Sqrt[3]*x],
 -1/6])/(98*Sqrt[2 - 5*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4-5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(15x^2+37)}{98\sqrt{-3x^4-5x^2+2}}-\frac{1}{98}\int -\frac{3(4-5x^2)}{\sqrt{-3x^4-5x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{98}\int \frac{4-5x^2}{\sqrt{-3x^4-5x^2+2}}dx+\frac{x(15x^2+37)}{98\sqrt{-3x^4-5x^2+2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{3}{49}\sqrt{3}\int \frac{4-5x^2}{2\sqrt{3}\sqrt{1-3x^2}\sqrt{x^2+2}}dx+\frac{x(15x^2+37)}{98\sqrt{-3x^4-5x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{98}\int \frac{4-5x^2}{\sqrt{1-3x^2}\sqrt{x^2+2}}dx+\frac{x(15x^2+37)}{98\sqrt{-3x^4-5x^2+2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{3}{98}\left(14\int \frac{1}{\sqrt{1-3x^2}\sqrt{x^2+2}}dx-5\int \frac{\sqrt{x^2+2}}{\sqrt{1-3x^2}}dx\right)+\frac{x(15x^2+37)}{98\sqrt{-3x^4-5x^2+2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{3}{98}\left(7\sqrt{\frac{2}{3}}\text{EllipticF}\left(\arcsin\left(\sqrt{3}x\right),-\frac{1}{6}\right)-5\int \frac{\sqrt{x^2+2}}{\sqrt{1-3x^2}}dx\right)+\frac{x(15x^2+37)}{98\sqrt{-3x^4-5x^2+2}}$$

$$\Big\downarrow 327$$

$$\frac{3}{98}\left(7\sqrt{\frac{2}{3}}\text{EllipticF}\left(\arcsin\left(\sqrt{3}x\right),-\frac{1}{6}\right)-5\sqrt{\frac{2}{3}}E\left(\arcsin\left(\sqrt{3}x\right)|-\frac{1}{6}\right)\right)+\frac{x(15x^2+37)}{98\sqrt{-3x^4-5x^2+2}}$$

input
```
Int[(2 - 5*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(x*(37 + 15*x^2))/(98*Sqrt[2 - 5*x^2 - 3*x^4]) + (3*(-5*Sqrt[2/3]*Elliptic
E[ArcSin[Sqrt[3]*x], -1/6] + 7*Sqrt[2/3]*EllipticF[ArcSin[Sqrt[3]*x], -1/6
]))/98
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(59) = 118$.

Time = 2.75 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.85

method	result
risch	$\frac{x(15x^2+37)}{98\sqrt{-3x^4-5x^2+2}}+\frac{\sqrt{3}\,\sqrt{-3x^2+1}\,\sqrt{2x^2+4}\,\text{EllipticF}\left(\sqrt{3}\,x,\frac{i\sqrt{6}}{6}\right)}{49\sqrt{-3x^4-5x^2+2}}+\frac{5\sqrt{3}\,\sqrt{-3x^2+1}\,\sqrt{2x^2+4}\,\left(\text{EllipticF}\left(\sqrt{3}\,x,\frac{i\sqrt{6}}{6}\right)-\text{Elliptic}$
default	$\frac{\frac{37}{98}x+\frac{15}{98}x^3}{\sqrt{-3x^4-5x^2+2}}+\frac{\sqrt{3}\,\sqrt{-3x^2+1}\,\sqrt{2x^2+4}\,\text{EllipticF}\left(\sqrt{3}\,x,\frac{i\sqrt{6}}{6}\right)}{49\sqrt{-3x^4-5x^2+2}}+\frac{5\sqrt{3}\,\sqrt{-3x^2+1}\,\sqrt{2x^2+4}\,\left(\text{EllipticF}\left(\sqrt{3}\,x,\frac{i\sqrt{6}}{6}\right)-\text{EllipticE}\right.}{98\sqrt{-3x^4-5x^2+2}}$
elliptic	$\frac{\frac{37}{98}x+\frac{15}{98}x^3}{\sqrt{-3x^4-5x^2+2}}+\frac{\sqrt{3}\,\sqrt{-3x^2+1}\,\sqrt{2x^2+4}\,\text{EllipticF}\left(\sqrt{3}\,x,\frac{i\sqrt{6}}{6}\right)}{49\sqrt{-3x^4-5x^2+2}}+\frac{5\sqrt{3}\,\sqrt{-3x^2+1}\,\sqrt{2x^2+4}\,\left(\text{EllipticF}\left(\sqrt{3}\,x,\frac{i\sqrt{6}}{6}\right)-\text{EllipticE}\right.}{98\sqrt{-3x^4-5x^2+2}}$

input
```
int(1/(-3*x^4-5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/98*x*(15*x^2+37)/(-3*x^4-5*x^2+2)^(1/2)+1/49*3^(1/2)*(-3*x^2+1)^(1/2)*(2
*x^2+4)^(1/2)/(-3*x^4-5*x^2+2)^(1/2)*EllipticF(3^(1/2)*x,1/6*I*6^(1/2))+5/
98*3^(1/2)*(-3*x^2+1)^(1/2)*(2*x^2+4)^(1/2)/(-3*x^4-5*x^2+2)^(1/2)*(Ellipt
icF(3^(1/2)*x,1/6*I*6^(1/2))-EllipticE(3^(1/2)*x,1/6*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \frac{1}{(2-5x^2-3x^4)^{3/2}}\,dx = -\frac{15\sqrt{3}\sqrt{2}(3x^4+5x^2-2)E\left(\arcsin\left(\sqrt{3}x\right)\mid -\frac{1}{6}\right) - 17\sqrt{3}\sqrt{2}(3x^4+5x^2-2)F\left(\arcsin\left(\sqrt{3}x\right)\mid -\frac{1}{6}\right) + \sqrt{}}{98\,(3x^4+5x^2-2)}$$

input
```
integrate(1/(-3*x^4-5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/98*(15*sqrt(3)*sqrt(2)*(3*x^4 + 5*x^2 - 2)*elliptic_e(arcsin(sqrt(3)*x)
, -1/6) - 17*sqrt(3)*sqrt(2)*(3*x^4 + 5*x^2 - 2)*elliptic_f(arcsin(sqrt(3)
*x), -1/6) + sqrt(-3*x^4 - 5*x^2 + 2)*(15*x^3 + 37*x))/(3*x^4 + 5*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4-5*x**2+2)**(3/2),x)
```

output
```
Integral((-3*x**4 - 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(2 - 3*x^4 - 5*x^2)^(3/2),x)
```

output
```
int(1/(2 - 3*x^4 - 5*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-5x^2+2}}{9x^8+30x^6+13x^4-20x^2+4}dx$$

input
```
int(1/(-3*x^4-5*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - 5*x**2 + 2)/(9*x**8 + 30*x**6 + 13*x**4 - 20*x**2 + 4
),x)
```

3.184 $\int \frac{1}{(3+7x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{(3+7x^2-2x^4)^{3/2}}\,dx = \frac{x(61-14x^2)}{219\sqrt{3+7x^2-2x^4}} + \frac{7}{219}\sqrt{\frac{1}{2}\left(-7+\sqrt{73}\right)}E\left(\arcsin\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right) + \frac{1}{3}\sqrt{\frac{1}{146}\left(-7+\sqrt{73}\right)}\,\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right),\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

output
```
1/219*x*(-14*x^2+61)/(-2*x^4+7*x^2+3)^(1/2)+7/438*(-14+2*73^(1/2))^(1/2)*E
llipticE(2*x/(7+73^(1/2))^(1/2),7/12*I*6^(1/2)+1/12*I*438^(1/2))+1/438*(-1
022+146*73^(1/2))^(1/2)*EllipticF(2*x/(7+73^(1/2))^(1/2),7/12*I*6^(1/2)+1/
12*I*438^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.48 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.34

$$\int \frac{1}{(3+7x^2-2x^4)^{3/2}}\,dx = \frac{244x-56x^3+14i\sqrt{2\left(7+\sqrt{73}\right)}\sqrt{3+7x^2-2x^4}E\left(i\text{arcsinh}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)|\frac{1}{12}(-\right.}{876\sqrt{3+}}$$

input
```
Integrate[(3 + 7*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(244*x - 56*x^3 + (14*I)*Sqrt[2*(7 + Sqrt[73])]*Sqrt[3 + 7*x^2 - 2*x^4]*El
lipticE[I*ArcSinh[(2*x)/Sqrt[-7 + Sqrt[73]]], (-61 + 7*Sqrt[73])/12] - ((2
*I)*(73 + 7*Sqrt[73])*Sqrt[6 + 14*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/S
qrt[-7 + Sqrt[73]]], (-61 + 7*Sqrt[73])/12])/Sqrt[7 + Sqrt[73]])/(876*Sqrt
[3 + 7*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+7x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(61-14x^2)}{219\sqrt{-2x^4+7x^2+3}}-\frac{1}{219}\int -\frac{2(7x^2+6)}{\sqrt{-2x^4+7x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{219}\int \frac{7x^2+6}{\sqrt{-2x^4+7x^2+3}}dx+\frac{x(61-14x^2)}{219\sqrt{-2x^4+7x^2+3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{4}{219}\sqrt{2}\int \frac{7x^2+6}{\sqrt{-4x^2+\sqrt{73}+7}\sqrt{4x^2+\sqrt{73}-7}}dx+\frac{x(61-14x^2)}{219\sqrt{-2x^4+7x^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{4}{219}\sqrt{2}\left(\frac{1}{4}\left(73-7\sqrt{73}\right)\int \frac{1}{\sqrt{-4x^2+\sqrt{73}+7}\sqrt{4x^2+\sqrt{73}-7}}dx+\frac{7}{4}\int \frac{\sqrt{4x^2+\sqrt{73}-7}}{\sqrt{-4x^2+\sqrt{73}+7}}dx\right)+\frac{x(61-14x^2)}{219\sqrt{-2x^4+7x^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{4}{219}\sqrt{2}\left(\frac{7}{4}\int \frac{\sqrt{4x^2+\sqrt{73}-7}}{\sqrt{-4x^2+\sqrt{73}+7}}dx+\frac{\left(73-7\sqrt{73}\right)\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right),\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)}{8\sqrt{\sqrt{73}-7}}\right)+\frac{x(61-14x^2)}{219\sqrt{-2x^4+7x^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{4}{219}\sqrt{2}\left(\frac{\left(73-7\sqrt{73}\right)\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right),\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)}{8\sqrt{\sqrt{73}-7}}+\frac{7}{8}\sqrt{\sqrt{73}-7}E\left(\arcsin\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)\right.\right.$$
$$\frac{x(61-14x^2)}{219\sqrt{-2x^4+7x^2+3}}$$

input
```
Int[(3 + 7*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(61 - 14*x^2))/(219*Sqrt[3 + 7*x^2 - 2*x^4]) + (4*Sqrt[2]*((7*Sqrt[-7 +
 Sqrt[73]]*EllipticE[ArcSin[(2*x)/Sqrt[7 + Sqrt[73]]], (-61 - 7*Sqrt[73])/
12])/8 + ((73 - 7*Sqrt[73])*EllipticF[ArcSin[(2*x)/Sqrt[7 + Sqrt[73]]], (-
61 - 7*Sqrt[73])/12])/(8*Sqrt[-7 + Sqrt[73]])))/219
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(95) = 190$.

Time = 2.51 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{x(14x^2-61)}{219\sqrt{-2x^4+7x^2+3}}+\frac{24\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-42+6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{73\sqrt{-42+6\sqrt{73}}\,\sqrt{-2x^4+7x^2+3}}-\frac{168\sqrt{1-\left(-\frac{7}{6}\right.}}{}$
default	$\frac{\frac{61}{219}x-\frac{14}{219}x^3}{\sqrt{-2x^4+7x^2+3}}+\frac{24\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-42+6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{73\sqrt{-42+6\sqrt{73}}\,\sqrt{-2x^4+7x^2+3}}-\frac{168\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right.}}{}$
elliptic	$\frac{\frac{61}{219}x-\frac{14}{219}x^3}{\sqrt{-2x^4+7x^2+3}}+\frac{24\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-42+6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{73\sqrt{-42+6\sqrt{73}}\,\sqrt{-2x^4+7x^2+3}}-\frac{168\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right.}}{}$

input
```
int(1/(-2*x^4+7*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/219*x*(14*x^2-61)/(-2*x^4+7*x^2+3)^(1/2)+24/73/(-42+6*73^(1/2))^(1/2)*(
1-(-7/6+1/6*73^(1/2))*x^2)^(1/2)*(1-(-7/6-1/6*73^(1/2))*x^2)^(1/2)/(-2*x^4
+7*x^2+3)^(1/2)*EllipticF(1/6*(-42+6*73^(1/2))^(1/2)*x,7/12*I*6^(1/2)+1/12
*I*438^(1/2))-168/73/(-42+6*73^(1/2))^(1/2)*(1-(-7/6+1/6*73^(1/2))*x^2)^(1
/2)*(1-(-7/6-1/6*73^(1/2))*x^2)^(1/2)/(-2*x^4+7*x^2+3)^(1/2)/(7+73^(1/2))*
(EllipticF(1/6*(-42+6*73^(1/2))^(1/2)*x,7/12*I*6^(1/2)+1/12*I*438^(1/2))-E
llipticE(1/6*(-42+6*73^(1/2))^(1/2)*x,7/12*I*6^(1/2)+1/12*I*438^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(3+7x^2-2x^4\right)^{3/2}}\,dx = \frac{7\left(\sqrt{73}\sqrt{3}(2\,x^4-7\,x^2-3)-7\,\sqrt{3}(2\,x^4-7\,x^2-3)\right)\sqrt{\frac{1}{6}\sqrt{73}-\frac{7}{6}}E(\arcsin\left(x\right.}{}$$

input
```
integrate(1/(-2*x^4+7*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/1314*(7*(sqrt(73)*sqrt(3)*(2*x^4 - 7*x^2 - 3) - 7*sqrt(3)*(2*x^4 - 7*x^2
 - 3))*sqrt(1/6*sqrt(73) - 7/6)*elliptic_e(arcsin(x*sqrt(1/6*sqrt(73) - 7/
6)), -7/12*sqrt(73) - 61/12) - (sqrt(73)*sqrt(3)*(2*x^4 - 7*x^2 - 3) - 91*
sqrt(3)*(2*x^4 - 7*x^2 - 3))*sqrt(1/6*sqrt(73) - 7/6)*elliptic_f(arcsin(x*
sqrt(1/6*sqrt(73) - 7/6)), -7/12*sqrt(73) - 61/12) + 6*sqrt(-2*x^4 + 7*x^2
 + 3)*(14*x^3 - 61*x))/(2*x^4 - 7*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3+7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+7x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+7*x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 + 7*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+7\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+7*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 7*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+7\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+7*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 7*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+7\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(7*x^2 - 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(7*x^2 - 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+7x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+7x^2+3}}{4x^8-28x^6+37x^4+42x^2+9}dx$$

input
```
int(1/(-2*x^4+7*x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + 7*x**2 + 3)/(4*x**8 - 28*x**6 + 37*x**4 + 42*x**2 + 9
),x)
```

3.185 $\int \frac{1}{(3+6x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 119

$$\int \frac{1}{(3+6x^2-2x^4)^{3/2}}\,dx = \frac{x(4-x^2)}{15\sqrt{3+6x^2-2x^4}} + \frac{1}{30}\sqrt{-3+\sqrt{15}}E\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}x\right)|-4-\sqrt{15}\right) + \frac{1}{6}\sqrt{\frac{1}{15}\left(-3+\sqrt{15}\right)}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}x\right),-4-\sqrt{15}\right)$$

output
```
1/15*x*(-x^2+4)/(-2*x^4+6*x^2+3)^(1/2)+1/30*(-3+15^(1/2))^(1/2)*EllipticE(
1/3*(-9+3*15^(1/2))^(1/2)*x,1/2*I*6^(1/2)+1/2*I*10^(1/2))+1/90*(-45+15*15^
(1/2))^(1/2)*EllipticF(1/3*(-9+3*15^(1/2))^(1/2)*x,1/2*I*6^(1/2)+1/2*I*10^
(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{1}{(3+6x^2-2x^4)^{3/2}}\,dx = \frac{1}{30}\left(-\frac{2x(-4+x^2)}{\sqrt{3+6x^2-2x^4}}\right.$$
$$+i\sqrt{3+\sqrt{15}}E\left(i\text{arcsinh}\left(\sqrt{1+\sqrt{\frac{5}{3}}}x\right)|-4+\sqrt{15}\right)-\frac{i(5+\sqrt{15})\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1+\sqrt{\frac{5}{3}}}x\right),-\right.}{\sqrt{3+\sqrt{15}}}$$

input
```
Integrate[(3 + 6*x^2 - 2*x^4)^(-3/2),x]
```

output
```
((-2*x*(-4 + x^2))/Sqrt[3 + 6*x^2 - 2*x^4] + I*Sqrt[3 + Sqrt[15]]*Elliptic
E[I*ArcSinh[Sqrt[1 + Sqrt[5/3]]*x], -4 + Sqrt[15]] - (I*(5 + Sqrt[15])*Ell
ipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/3]]*x], -4 + Sqrt[15]])/Sqrt[3 + Sqrt[15]
])/30
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+6x^2+3)^{3/2}}\,dx$$
$$\Big\downarrow \text{1405}$$

$$\frac{x(4-x^2)}{15\sqrt{-2x^4+6x^2+3}}-\frac{1}{180}\int -\frac{12(x^2+1)}{\sqrt{-2x^4+6x^2+3}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{15}\int \frac{x^2+1}{\sqrt{-2x^4+6x^2+3}}dx+\frac{x(4-x^2)}{15\sqrt{-2x^4+6x^2+3}}$$

$$\Big\downarrow \; 1494$$

$$\frac{2}{15}\sqrt{2}\int \frac{x^2+1}{2\sqrt{-2x^2+\sqrt{15}+3}\sqrt{2x^2+\sqrt{15}-3}}dx+\frac{x(4-x^2)}{15\sqrt{-2x^4+6x^2+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{15}\sqrt{2}\int \frac{x^2+1}{\sqrt{-2x^2+\sqrt{15}+3}\sqrt{2x^2+\sqrt{15}-3}}dx+\frac{x(4-x^2)}{15\sqrt{-2x^4+6x^2+3}}$$

$$\Big\downarrow \; 399$$

$$\frac{1}{15}\sqrt{2}\left(\frac{1}{2}\left(5-\sqrt{15}\right)\int \frac{1}{\sqrt{-2x^2+\sqrt{15}+3}\sqrt{2x^2+\sqrt{15}-3}}dx+\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{15}-3}}{\sqrt{-2x^2+\sqrt{15}+3}}dx\right)+\frac{x(4-x^2)}{15\sqrt{-2x^4+6x^2+3}}$$

$$\Big\downarrow \; 321$$

$$\frac{1}{15}\sqrt{2}\left(\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{15}-3}}{\sqrt{-2x^2+\sqrt{15}+3}}dx+\frac{1}{4}\left(5-\sqrt{15}\right)\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}x\right),-4\right.\right.$$
$$\frac{x(4-x^2)}{15\sqrt{-2x^4+6x^2+3}}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{15}\sqrt{2}\left(\frac{1}{4}\left(5-\sqrt{15}\right)\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}x\right),-4-\sqrt{15}\right)+\frac{1}{2}\sqrt{\frac{3}{3+\sqrt{15}}}E\left(\text{ar}\right.\right.$$
$$\frac{x(4-x^2)}{15\sqrt{-2x^4+6x^2+3}}$$

input
```
Int[(3 + 6*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(4 - x^2))/(15*Sqrt[3 + 6*x^2 - 2*x^4]) + (Sqrt[2]*((Sqrt[3/(3 + Sqrt[1
5])]*EllipticE[ArcSin[Sqrt[(-3 + Sqrt[15])/3]*x], -4 - Sqrt[15]])/2 + ((5
- Sqrt[15])*Sqrt[(3 + Sqrt[15])/3]*EllipticF[ArcSin[Sqrt[(-3 + Sqrt[15])/3
]*x], -4 - Sqrt[15]])/4))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(97) = 194$.

Time = 1.63 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.92

method	result
risch	$-\frac{x(x^2-4)}{15\sqrt{-2x^4+6x^2+3}}+\frac{\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-9+3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{5\sqrt{-9+3\sqrt{15}}\,\sqrt{-2x^4+6x^2+3}}-\frac{6\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^2}}{}$
default	$\frac{\frac{4}{15}x-\frac{1}{15}x^3}{\sqrt{-2x^4+6x^2+3}}+\frac{\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-9+3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{5\sqrt{-9+3\sqrt{15}}\,\sqrt{-2x^4+6x^2+3}}-\frac{6\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-}}{}$
elliptic	$\frac{\frac{4}{15}x-\frac{1}{15}x^3}{\sqrt{-2x^4+6x^2+3}}+\frac{\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-9+3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{5\sqrt{-9+3\sqrt{15}}\,\sqrt{-2x^4+6x^2+3}}-\frac{6\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-}}{}$

input
```
int(1/(-2*x^4+6*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*x*(x^2-4)/(-2*x^4+6*x^2+3)^(1/2)+1/5/(-9+3*15^(1/2))^(1/2)*(1-(-1+1/
3*15^(1/2))*x^2)^(1/2)*(1-(-1-1/3*15^(1/2))*x^2)^(1/2)/(-2*x^4+6*x^2+3)^(1
/2)*EllipticF(1/3*(-9+3*15^(1/2))^(1/2)*x,1/2*I*6^(1/2)+1/2*I*10^(1/2))-6/
5/(-9+3*15^(1/2))^(1/2)*(1-(-1+1/3*15^(1/2))*x^2)^(1/2)*(1-(-1-1/3*15^(1/2
))*x^2)^(1/2)/(-2*x^4+6*x^2+3)^(1/2)/(6+2*15^(1/2))*(EllipticF(1/3*(-9+3*1
5^(1/2))^(1/2)*x,1/2*I*6^(1/2)+1/2*I*10^(1/2))-EllipticE(1/3*(-9+3*15^(1/2
))^(1/2)*x,1/2*I*6^(1/2)+1/2*I*10^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18

$$\int \frac{1}{(3+6x^2-2x^4)^{3/2}}\,dx =$$

$$-\frac{\sqrt{3}\left(2\,x^4-6\,x^2-\sqrt{\frac{5}{3}}(2\,x^4-6\,x^2-3)-3\right)\sqrt{\sqrt{\frac{5}{3}}-1}E(\arcsin\left(x\sqrt{\sqrt{\frac{5}{3}}-1}\right)|-3\sqrt{\frac{5}{3}}-4)-2\sqrt{3}(2\,}{30\,(2\,x^4-6\,x}$$

input
```
integrate(1/(-2*x^4+6*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/30*(sqrt(3)*(2*x^4 - 6*x^2 - sqrt(5/3)*(2*x^4 - 6*x^2 - 3) - 3)*sqrt(sq
rt(5/3) - 1)*elliptic_e(arcsin(x*sqrt(sqrt(5/3) - 1)), -3*sqrt(5/3) - 4) -
 2*sqrt(3)*(2*x^4 - 6*x^2 - 3)*sqrt(sqrt(5/3) - 1)*elliptic_f(arcsin(x*sqr
t(sqrt(5/3) - 1)), -3*sqrt(5/3) - 4) - 2*sqrt(-2*x^4 + 6*x^2 + 3)*(x^3 - 4
*x))/(2*x^4 - 6*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3+6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+6x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+6*x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 + 6*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+6\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+6*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 6*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+6\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+6*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 6*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+6\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(6*x^2 - 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(6*x^2 - 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+6x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+6x^2+3}}{4x^8-24x^6+24x^4+36x^2+9}dx$$

input
```
int(1/(-2*x^4+6*x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + 6*x**2 + 3)/(4*x**8 - 24*x**6 + 24*x**4 + 36*x**2 + 9
),x)
```

3.186 $\int \frac{1}{(3+5x^2-2x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{(3+5x^2-2x^4)^{3/2}} \, dx = \frac{x(37-10x^2)}{147\sqrt{3+5x^2-2x^4}} + \frac{5}{147} E\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle| -6\right) + \frac{1}{21} \operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right), -6\right)$$

output
```
1/147*x*(-10*x^2+37)/(-2*x^4+5*x^2+3)^(1/2)+5/147*EllipticE(1/3*x*3^(1/2),
I*6^(1/2))+1/21*EllipticF(1/3*x*3^(1/2),I*6^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.16

$$\int \frac{1}{(3+5x^2-2x^4)^{3/2}} \, dx = \frac{37x-10x^3+5i\sqrt{6}\sqrt{3-x^2}\sqrt{1+2x^2}E\left(i\text{arcsinh}\left(\sqrt{2}x\right)\middle|-\frac{1}{6}\right)-7i\sqrt{6}\sqrt{3-x^2}\sqrt{}}{147\sqrt{3+5x^2-2x^4}}$$

input
```
Integrate[(3 + 5*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(37*x - 10*x^3 + (5*I)*Sqrt[6]*Sqrt[3 - x^2]*Sqrt[1 + 2*x^2]*EllipticE[I*A
rcSinh[Sqrt[2]*x], -1/6] - (7*I)*Sqrt[6]*Sqrt[3 - x^2]*Sqrt[1 + 2*x^2]*Ell
ipticF[I*ArcSinh[Sqrt[2]*x], -1/6])/(147*Sqrt[3 + 5*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+5x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(37-10x^2)}{147\sqrt{-2x^4+5x^2+3}}-\frac{1}{147}\int -\frac{2(5x^2+6)}{\sqrt{-2x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{147}\int \frac{5x^2+6}{\sqrt{-2x^4+5x^2+3}}dx+\frac{x(37-10x^2)}{147\sqrt{-2x^4+5x^2+3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{4}{147}\sqrt{2}\int \frac{5x^2+6}{2\sqrt{2}\sqrt{3-x^2}\sqrt{2x^2+1}}dx+\frac{x(37-10x^2)}{147\sqrt{-2x^4+5x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{147}\int \frac{5x^2+6}{\sqrt{3-x^2}\sqrt{2x^2+1}}dx+\frac{x(37-10x^2)}{147\sqrt{-2x^4+5x^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{2}{147}\left(\frac{7}{2}\int \frac{1}{\sqrt{3-x^2}\sqrt{2x^2+1}}dx+\frac{5}{2}\int \frac{\sqrt{2x^2+1}}{\sqrt{3-x^2}}dx\right)+\frac{x(37-10x^2)}{147\sqrt{-2x^4+5x^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2}{147}\left(\frac{5}{2}\int \frac{\sqrt{2x^2+1}}{\sqrt{3-x^2}}dx+\frac{7}{2}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-6\right)\right)+\frac{x(37-10x^2)}{147\sqrt{-2x^4+5x^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{2}{147}\left(\frac{7}{2}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-6\right)+\frac{5}{2}E\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\bigg|-6\right)\right)+\frac{x(37-10x^2)}{147\sqrt{-2x^4+5x^2+3}}$$

input
```
Int[(3 + 5*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(37 - 10*x^2))/(147*Sqrt[3 + 5*x^2 - 2*x^4]) + (2*((5*EllipticE[ArcSin[
x/Sqrt[3]], -6])/2 + (7*EllipticF[ArcSin[x/Sqrt[3]], -6])/2))/147
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(55) = 110$.

Time = 2.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.49

method	result
risch	$-\frac{x(10x^2-37)}{147\sqrt{-2x^4+5x^2+3}}+\frac{4\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{2x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\sqrt{6}\right)}{147\sqrt{-2x^4+5x^2+3}}-\frac{5\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{2x^2+1}\left(\mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\sqrt{6}\right)-\mathrm{Ell}\right.}{441\sqrt{-2x^4+5x^2+3}}$
default	$\frac{\frac{37}{147}x-\frac{10}{147}x^3}{\sqrt{-2x^4+5x^2+3}}+\frac{4\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{2x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\sqrt{6}\right)}{147\sqrt{-2x^4+5x^2+3}}-\frac{5\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{2x^2+1}\left(\mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\sqrt{6}\right)-\mathrm{EllipticE}\right.}{441\sqrt{-2x^4+5x^2+3}}$
elliptic	$\frac{\frac{37}{147}x-\frac{10}{147}x^3}{\sqrt{-2x^4+5x^2+3}}+\frac{4\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{2x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\sqrt{6}\right)}{147\sqrt{-2x^4+5x^2+3}}-\frac{5\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{2x^2+1}\left(\mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\sqrt{6}\right)-\mathrm{EllipticE}\right.}{441\sqrt{-2x^4+5x^2+3}}$

input
```
int(1/(-2*x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/147*x*(10*x^2-37)/(-2*x^4+5*x^2+3)^(1/2)+4/147*3^(1/2)*(-3*x^2+9)^(1/2)
*(2*x^2+1)^(1/2)/(-2*x^4+5*x^2+3)^(1/2)*EllipticF(1/3*3^(1/2)*x,I*6^(1/2))
-5/441*3^(1/2)*(-3*x^2+9)^(1/2)*(2*x^2+1)^(1/2)/(-2*x^4+5*x^2+3)^(1/2)*(El
lipticF(1/3*3^(1/2)*x,I*6^(1/2))-EllipticE(1/3*3^(1/2)*x,I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.54

$$\int \frac{1}{(3+5x^2-2x^4)^{3/2}}\,dx = \frac{5\,(2\,x^4-5\,x^2-3)E\left(\arcsin\left(\frac{1}{3}\,\sqrt{3}x\right)\,|-6\right)+31\,(2\,x^4-5\,x^2-3)F\left(\arcsin\left(\frac{1}{3}\,\sqrt{\right.\right.}}{441\,(2\,x^4-5\,x^2-3)}$$

input
```
integrate(1/(-2*x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/441*(5*(2*x^4 - 5*x^2 - 3)*elliptic_e(arcsin(1/3*sqrt(3)*x), -6) + 31*(2
*x^4 - 5*x^2 - 3)*elliptic_f(arcsin(1/3*sqrt(3)*x), -6) + 3*sqrt(-2*x^4 +
5*x^2 + 3)*(10*x^3 - 37*x))/(2*x^4 - 5*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3+5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 + 5*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 5*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 5*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+5\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(5*x^2 - 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+5x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+5x^2+3}}{4x^8-20x^6+13x^4+30x^2+9}dx$$

input
```
int(1/(-2*x^4+5*x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + 5*x**2 + 3)/(4*x**8 - 20*x**6 + 13*x**4 + 30*x**2 + 9
),x)
```

3.187 $\int \frac{1}{(3+4x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 127

$$\begin{aligned}\int \frac{1}{(3+4x^2-2x^4)^{3/2}}\,dx &= \frac{x(7-2x^2)}{30\sqrt{3+4x^2-2x^4}} \\ &+ \frac{1}{30}\sqrt{-2+\sqrt{10}}E\left(\arcsin\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right) \\ &+ \frac{1}{6}\sqrt{\frac{1}{10}\left(-2+\sqrt{10}\right)}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)\end{aligned}$$

output

```
1/30*x*(-2*x^2+7)/(-2*x^4+4*x^2+3)^(1/2)+1/30*(-2+10^(1/2))^(1/2)*Elliptic
E(2^(1/2)/(2+10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))+1/60*(-20+10*
10^(1/2))^(1/2)*EllipticF(2^(1/2)/(2+10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I
*15^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.51 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09

$$\int \frac{1}{(3+4x^2-2x^4)^{3/2}}\,dx = \frac{1}{30}\left(\frac{x(7-2x^2)}{\sqrt{3+4x^2-2x^4}} + i\sqrt{2+\sqrt{10}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right)\middle|-\frac{7}{3}+\frac{2\sqrt{10}}{3}\right) - \frac{i(5+\sqrt{10})\,\text{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right),\right.}{\sqrt{2+\sqrt{10}}}\right.$$

input
```
Integrate[(3 + 4*x^2 - 2*x^4)^(-3/2),x]
```

output
```
((x*(7 - 2*x^2))/Sqrt[3 + 4*x^2 - 2*x^4] + I*Sqrt[2 + Sqrt[10]]*EllipticE[
I*ArcSinh[Sqrt[2/(-2 + Sqrt[10])]*x], -7/3 + (2*Sqrt[10])/3] - (I*(5 + Sqr
t[10])*EllipticF[I*ArcSinh[Sqrt[2/(-2 + Sqrt[10])]*x], -7/3 + (2*Sqrt[10])
/3])/Sqrt[2 + Sqrt[10]])/30
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+4x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(7-2x^2)}{30\sqrt{-2x^4+4x^2+3}} - \frac{1}{120}\int -\frac{4(2x^2+3)}{\sqrt{-2x^4+4x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{30}\int \frac{2x^2+3}{\sqrt{-2x^4+4x^2+3}}dx+\frac{x(7-2x^2)}{30\sqrt{-2x^4+4x^2+3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{15}\sqrt{2}\int \frac{2x^2+3}{2\sqrt{-2x^2+\sqrt{10}+2}\sqrt{2x^2+\sqrt{10}-2}}dx+\frac{x(7-2x^2)}{30\sqrt{-2x^4+4x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2x^2+3}{\sqrt{-2x^2+\sqrt{10}+2}\sqrt{2x^2+\sqrt{10}-2}}dx}{15\sqrt{2}}+\frac{x(7-2x^2)}{30\sqrt{-2x^4+4x^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{(5-\sqrt{10})\int \frac{1}{\sqrt{-2x^2+\sqrt{10}+2}\sqrt{2x^2+\sqrt{10}-2}}dx+\int \frac{\sqrt{2x^2+\sqrt{10}-2}}{\sqrt{-2x^2+\sqrt{10}+2}}dx}{15\sqrt{2}}+\frac{x(7-2x^2)}{30\sqrt{-2x^4+4x^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\int \frac{\sqrt{2x^2+\sqrt{10}-2}}{\sqrt{-2x^2+\sqrt{10}+2}}dx+\frac{\left(5-\sqrt{10}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{2\left(\sqrt{10}-2\right)}}}{15\sqrt{2}}+\frac{x(7-2x^2)}{30\sqrt{-2x^4+4x^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\left(5-\sqrt{10}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{2\left(\sqrt{10}-2\right)}}+\sqrt{\frac{1}{2}\left(\sqrt{10}-2\right)}E\left(\arcsin\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{15\sqrt{2}}+\frac{x(7-2x^2)}{30\sqrt{-2x^4+4x^2+3}}$$

input
```
Int[(3 + 4*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(7 - 2*x^2))/(30*Sqrt[3 + 4*x^2 - 2*x^4]) + (Sqrt[(-2 + Sqrt[10])/2]*El
lipticE[ArcSin[Sqrt[2/(2 + Sqrt[10])]*x], (-7 - 2*Sqrt[10])/3] + ((5 - Sqr
t[10])*EllipticF[ArcSin[Sqrt[2/(2 + Sqrt[10])]*x], (-7 - 2*Sqrt[10])/3])/S
qrt[2*(-2 + Sqrt[10])])/(15*Sqrt[2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(97) = 194$.

Time = 2.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.81

method	result
risch	$-\frac{x(2x^2-7)}{30\sqrt{-2x^4+4x^2+3}}+\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6+3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{10\sqrt{-6+3\sqrt{10}}\,\sqrt{-2x^4+4x^2+3}}-\frac{6\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\cdots}{\cdots}$
default	$\frac{\frac{7}{30}x-\frac{1}{15}x^3}{\sqrt{-2x^4+4x^2+3}}+\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6+3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{10\sqrt{-6+3\sqrt{10}}\,\sqrt{-2x^4+4x^2+3}}-\frac{6\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{\cdots}}{\cdots}$
elliptic	$\frac{\frac{7}{30}x-\frac{1}{15}x^3}{\sqrt{-2x^4+4x^2+3}}+\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6+3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{10\sqrt{-6+3\sqrt{10}}\,\sqrt{-2x^4+4x^2+3}}-\frac{6\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{\cdots}}{\cdots}$

input
```
int(1/(-2*x^4+4*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*x*(2*x^2-7)/(-2*x^4+4*x^2+3)^(1/2)+3/10/(-6+3*10^(1/2))^(1/2)*(1-(-2
/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-2/3-1/3*10^(1/2))*x^2)^(1/2)/(-2*x^4+4*x^
2+3)^(1/2)*EllipticF(1/3*(-6+3*10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1
/2))-6/5/(-6+3*10^(1/2))^(1/2)*(1-(-2/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-2/3-
1/3*10^(1/2))*x^2)^(1/2)/(-2*x^4+4*x^2+3)^(1/2)/(4+2*10^(1/2))*(EllipticF(
1/3*(-6+3*10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))-EllipticE(1/3*(-
6+3*10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.39

$$\int \frac{1}{(3+4x^2-2x^4)^{3/2}}\,dx = \frac{2\left(\sqrt{10}\sqrt{3}(2\,x^4-4\,x^2-3)-2\,\sqrt{3}(2\,x^4-4\,x^2-3)\right)\sqrt{\frac{1}{3}\,\sqrt{10}-\frac{2}{3}}E\left(\arcsin\left(x\cdots\right.\right.}{\cdots}$$

input
```
integrate(1/(-2*x^4+4*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/180*(2*(sqrt(10)*sqrt(3)*(2*x^4 - 4*x^2 - 3) - 2*sqrt(3)*(2*x^4 - 4*x^2
- 3))*sqrt(1/3*sqrt(10) - 2/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(10) - 2/3
)), -2/3*sqrt(10) - 7/3) + (sqrt(10)*sqrt(3)*(2*x^4 - 4*x^2 - 3) + 10*sqrt
(3)*(2*x^4 - 4*x^2 - 3))*sqrt(1/3*sqrt(10) - 2/3)*elliptic_f(arcsin(x*sqrt
(1/3*sqrt(10) - 2/3)), -2/3*sqrt(10) - 7/3) + 6*sqrt(-2*x^4 + 4*x^2 + 3)*(
2*x^3 - 7*x))/(2*x^4 - 4*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3+4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+4x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+4*x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 + 4*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+4\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+4*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 4*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+4\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+4*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 4*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+4\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(4*x^2 - 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(4*x^2 - 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+4x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+4x^2+3}}{4x^8-16x^6+4x^4+24x^2+9}dx$$

input
```
int(1/(-2*x^4+4*x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + 4*x**2 + 3)/(4*x**8 - 16*x**6 + 4*x**4 + 24*x**2 + 9)
,x)
```

3.188 $\int \frac{1}{(3+3x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{(3+3x^2-2x^4)^{3/2}}\,dx = \frac{x(7-2x^2)}{33\sqrt{3+3x^2-2x^4}} + \frac{1}{33}\sqrt{\frac{1}{2}\left(-3+\sqrt{33}\right)}E\left(\arcsin\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right) + \frac{1}{3}\sqrt{\frac{1}{66}\left(-3+\sqrt{33}\right)}\,\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

output
```
1/33*x*(-2*x^2+7)/(-2*x^4+3*x^2+3)^(1/2)+1/66*(-6+2*33^(1/2))^(1/2)*Ellipt
icE(2*x/(3+33^(1/2))^(1/2),1/4*I*6^(1/2)+1/4*I*22^(1/2))+1/198*(-198+66*33
^(1/2))^(1/2)*EllipticF(2*x/(3+33^(1/2))^(1/2),1/4*I*6^(1/2)+1/4*I*22^(1/2
))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.30

$$\int \frac{1}{(3+3x^2-2x^4)^{3/2}}\,dx = \frac{4x(7-2x^2)+2i\sqrt{3+\sqrt{33}}\sqrt{6+6x^2-4x^4}E\left(i\text{arcsinh}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}(-7+\sqrt{\cdots}\right.}{132\sqrt{3+3x^2-\cdots}}$$

input
```
Integrate[(3 + 3*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(4*x*(7 - 2*x^2) + (2*I)*Sqrt[3 + Sqrt[33]]*Sqrt[6 + 6*x^2 - 4*x^4]*Ellipt
icE[I*ArcSinh[(2*x)/Sqrt[-3 + Sqrt[33]]], (-7 + Sqrt[33])/4] - ((2*I)*(11
+ Sqrt[33])*Sqrt[6 + 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-3 + Sq
rt[33]]], (-7 + Sqrt[33])/4])/Sqrt[3 + Sqrt[33]])/(132*Sqrt[3 + 3*x^2 - 2*
x^4])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+3x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(7-2x^2)}{33\sqrt{-2x^4+3x^2+3}}-\frac{1}{99}\int -\frac{6(x^2+2)}{\sqrt{-2x^4+3x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{33}\int \frac{x^2+2}{\sqrt{-2x^4+3x^2+3}}dx+\frac{x(7-2x^2)}{33\sqrt{-2x^4+3x^2+3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{4}{33}\sqrt{2}\int \frac{x^2+2}{\sqrt{-4x^2+\sqrt{33}+3}\sqrt{4x^2+\sqrt{33}-3}}dx+\frac{x(7-2x^2)}{33\sqrt{-2x^4+3x^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{4}{33}\sqrt{2}\left(\frac{1}{4}\left(11-\sqrt{33}\right)\int \frac{1}{\sqrt{-4x^2+\sqrt{33}+3}\sqrt{4x^2+\sqrt{33}-3}}dx+\frac{1}{4}\int \frac{\sqrt{4x^2+\sqrt{33}-3}}{\sqrt{-4x^2+\sqrt{33}+3}}dx\right)+\frac{x(7-2x^2)}{33\sqrt{-2x^4+3x^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{4}{33}\sqrt{2}\left(\frac{1}{4}\int \frac{\sqrt{4x^2+\sqrt{33}-3}}{\sqrt{-4x^2+\sqrt{33}+3}}dx+\frac{\left(11-\sqrt{33}\right)\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)}{8\sqrt{\sqrt{33}-3}}\right)+\frac{x(7-2x^2)}{33\sqrt{-2x^4+3x^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{4}{33}\sqrt{2}\left(\frac{\left(11-\sqrt{33}\right)\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)}{8\sqrt{\sqrt{33}-3}}+\frac{1}{8}\sqrt{\sqrt{33}-3}E\left(\arcsin\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|\frac{1}{4}\left(-\right.\right.\right.$$
$$\frac{x(7-2x^2)}{33\sqrt{-2x^4+3x^2+3}}$$

input
```
Int[(3 + 3*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(7 - 2*x^2))/(33*Sqrt[3 + 3*x^2 - 2*x^4]) + (4*Sqrt[2]*((Sqrt[-3 + Sqrt
[33]]*EllipticE[ArcSin[(2*x)/Sqrt[3 + Sqrt[33]]], (-7 - Sqrt[33])/4])/8 +
((11 - Sqrt[33])*EllipticF[ArcSin[(2*x)/Sqrt[3 + Sqrt[33]]], (-7 - Sqrt[33
])/4])/(8*Sqrt[-3 + Sqrt[33]])))/33
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(95) = 190$.

Time = 1.94 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{x(2x^2-7)}{33\sqrt{-2x^4+3x^2+3}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-18+6\sqrt{33}}\,x}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{11\sqrt{-18+6\sqrt{33}}\,\sqrt{-2x^4+3x^2+3}}-\frac{24\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right.}}{}$
default	$\frac{\frac{7}{33}x-\frac{2}{33}x^3}{\sqrt{-2x^4+3x^2+3}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-18+6\sqrt{33}}\,x}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{11\sqrt{-18+6\sqrt{33}}\,\sqrt{-2x^4+3x^2+3}}-\frac{24\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}}{}$
elliptic	$\frac{\frac{7}{33}x-\frac{2}{33}x^3}{\sqrt{-2x^4+3x^2+3}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-18+6\sqrt{33}}\,x}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{11\sqrt{-18+6\sqrt{33}}\,\sqrt{-2x^4+3x^2+3}}-\frac{24\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}}{}$

input
```
int(1/(-2*x^4+3*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/33*x*(2*x^2-7)/(-2*x^4+3*x^2+3)^(1/2)+8/11/(-18+6*33^(1/2))^(1/2)*(1-(-
1/2+1/6*33^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*33^(1/2))*x^2)^(1/2)/(-2*x^4+3*x
^2+3)^(1/2)*EllipticF(1/6*(-18+6*33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^
(1/2))-24/11/(-18+6*33^(1/2))^(1/2)*(1-(-1/2+1/6*33^(1/2))*x^2)^(1/2)*(1-(
-1/2-1/6*33^(1/2))*x^2)^(1/2)/(-2*x^4+3*x^2+3)^(1/2)/(3+33^(1/2))*(Ellipti
cF(1/6*(-18+6*33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))-EllipticE(1/
6*(-18+6*33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.41

$$\int \frac{1}{(3+3x^2-2x^4)^{3/2}}\,dx = \frac{\left(\sqrt{\frac{11}{3}}\sqrt{3}(2\,x^4-3\,x^2-3)-\sqrt{3}(2\,x^4-3\,x^2-3)\right)\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}-\frac{1}{2}}E(\arcsin\left(x\sqrt{\right.}}{}$$

input
```
integrate(1/(-2*x^4+3*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/66*((sqrt(11/3)*sqrt(3)*(2*x^4 - 3*x^2 - 3) - sqrt(3)*(2*x^4 - 3*x^2 - 3
))*sqrt(1/2*sqrt(11/3) - 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(11/3) - 1/
2)), -3/4*sqrt(11/3) - 7/4) + (sqrt(11/3)*sqrt(3)*(2*x^4 - 3*x^2 - 3) + 3*
sqrt(3)*(2*x^4 - 3*x^2 - 3))*sqrt(1/2*sqrt(11/3) - 1/2)*elliptic_f(arcsin(
x*sqrt(1/2*sqrt(11/3) - 1/2)), -3/4*sqrt(11/3) - 7/4) + 2*sqrt(-2*x^4 + 3*
x^2 + 3)*(2*x^3 - 7*x))/(2*x^4 - 3*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3+3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+3x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+3*x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 + 3*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+3\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+3*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 3*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+3\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+3*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 3*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+3\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(3*x^2 - 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(3*x^2 - 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+3x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+3x^2+3}}{4x^8-12x^6-3x^4+18x^2+9}dx$$

input
```
int(1/(-2*x^4+3*x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + 3*x**2 + 3)/(4*x**8 - 12*x**6 - 3*x**4 + 18*x**2 + 9)
,x)
```

3.189 $\int \frac{1}{\left(3+2x^2-2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 127

$$\int \frac{1}{(3+2x^2-2x^4)^{3/2}}\,dx = \frac{x(4-x^2)}{21\sqrt{3+2x^2-2x^4}} + \frac{1}{42}\sqrt{-1+\sqrt{7}}E\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right) + \frac{1}{6}\sqrt{\frac{1}{7}\left(-1+\sqrt{7}\right)}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4-\sqrt{7}\right)\right)$$

output
```
1/21*x*(-x^2+4)/(-2*x^4+2*x^2+3)^(1/2)+1/42*(-1+7^(1/2))^(1/2)*EllipticE(2
^(1/2)/(1+7^(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))+1/42*(-7+7*7^(1/2
))^(1/2)*EllipticF(2^(1/2)/(1+7^(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2
))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\int \frac{1}{(3+2x^2-2x^4)^{3/2}}\,dx = \frac{1}{42}\left(-\frac{2x(-4+x^2)}{\sqrt{3+2x^2-2x^4}} + i\sqrt{1+\sqrt{7}}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right) - \frac{i(7+\sqrt{7})\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right),\frac{1}{3}(\right.}{\sqrt{1+\sqrt{7}}}\right.$$

input
```
Integrate[(3 + 2*x^2 - 2*x^4)^(-3/2),x]
```

output
```
((-2*x*(-4 + x^2))/Sqrt[3 + 2*x^2 - 2*x^4] + I*Sqrt[1 + Sqrt[7]]*EllipticE
[I*ArcSinh[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3] - (I*(7 + Sqrt[7])
*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3])/Sqrt[1
+ Sqrt[7]])/42
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+2x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x(4-x^2)}{21\sqrt{-2x^4+2x^2+3}} - \frac{1}{84}\int -\frac{4(x^2+3)}{\sqrt{-2x^4+2x^2+3}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{21}\int \frac{x^2+3}{\sqrt{-2x^4+2x^2+3}}dx+\frac{x\left(4-x^2\right)}{21\sqrt{-2x^4+2x^2+3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{2}{21}\sqrt{2}\int \frac{x^2+3}{2\sqrt{-2x^2+\sqrt{7}+1}\sqrt{2x^2+\sqrt{7}-1}}dx+\frac{x\left(4-x^2\right)}{21\sqrt{-2x^4+2x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\sqrt{2}\int \frac{x^2+3}{\sqrt{-2x^2+\sqrt{7}+1}\sqrt{2x^2+\sqrt{7}-1}}dx+\frac{x\left(4-x^2\right)}{21\sqrt{-2x^4+2x^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{21}\sqrt{2}\left(\frac{1}{2}\left(7-\sqrt{7}\right)\int \frac{1}{\sqrt{-2x^2+\sqrt{7}+1}\sqrt{2x^2+\sqrt{7}-1}}dx+\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{7}-1}}{\sqrt{-2x^2+\sqrt{7}+1}}dx\right)+\frac{x\left(4-x^2\right)}{21\sqrt{-2x^4+2x^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{21}\sqrt{2}\left(\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{7}-1}}{\sqrt{-2x^2+\sqrt{7}+1}}dx+\frac{\left(7-\sqrt{7}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{2\sqrt{2\left(\sqrt{7}-1\right)}}\right)+\frac{x\left(4-x^2\right)}{21\sqrt{-2x^4+2x^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{21}\sqrt{2}\left(\frac{\left(7-\sqrt{7}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{2\sqrt{2\left(\sqrt{7}-1\right)}}+\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{7}-1\right)}E\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-\right.\right.\right.$$
$$\frac{x\left(4-x^2\right)}{21\sqrt{-2x^4+2x^2+3}}$$

input
```
Int[(3 + 2*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(4 - x^2))/(21*Sqrt[3 + 2*x^2 - 2*x^4]) + (Sqrt[2]*((Sqrt[(-1 + Sqrt[7]
)/2]*EllipticE[ArcSin[Sqrt[2/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3])/2 + ((7
 - Sqrt[7])*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3])/
(2*Sqrt[2*(-1 + Sqrt[7])])))/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(97) = 194.

Time = 1.86 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.80

method	result
risch	$-\frac{x(x^2-4)}{21\sqrt{-2x^4+2x^2+3}}+\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3+3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{7\sqrt{-3+3\sqrt{7}}\,\sqrt{-2x^4+2x^2+3}}-6\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{\cdots}$
default	$\frac{\frac{4}{21}x-\frac{1}{21}x^3}{\sqrt{-2x^4+2x^2+3}}+\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3+3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{7\sqrt{-3+3\sqrt{7}}\,\sqrt{-2x^4+2x^2+3}}-6\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-(\cdots}$
elliptic	$\frac{\frac{4}{21}x-\frac{1}{21}x^3}{\sqrt{-2x^4+2x^2+3}}+\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3+3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{7\sqrt{-3+3\sqrt{7}}\,\sqrt{-2x^4+2x^2+3}}-6\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-(\cdots}$

input
```
int(1/(-2*x^4+2*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*x*(x^2-4)/(-2*x^4+2*x^2+3)^(1/2)+3/7/(-3+3*7^(1/2))^(1/2)*(1-(-1/3+1
/3*7^(1/2))*x^2)^(1/2)*(1-(-1/3-1/3*7^(1/2))*x^2)^(1/2)/(-2*x^4+2*x^2+3)^(
1/2)*EllipticF(1/3*(-3+3*7^(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))-6/
7/(-3+3*7^(1/2))^(1/2)*(1-(-1/3+1/3*7^(1/2))*x^2)^(1/2)*(1-(-1/3-1/3*7^(1/
2))*x^2)^(1/2)/(-2*x^4+2*x^2+3)^(1/2)/(2+2*7^(1/2))*(EllipticF(1/3*(-3+3*7
^(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))-EllipticE(1/3*(-3+3*7^(1/2))
^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.38

$$\int \frac{1}{(3+2x^2-2x^4)^{3/2}}\,dx = \frac{\left(\sqrt{7}\sqrt{3}(2\,x^4-2\,x^2-3)-\sqrt{3}(2\,x^4-2\,x^2-3)\right)\sqrt{\frac{1}{3}\sqrt{7}-\frac{1}{3}}E\left(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{\right.\right.}}}{}$$

input
```
integrate(1/(-2*x^4+2*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/126*((sqrt(7)*sqrt(3)*(2*x^4 - 2*x^2 - 3) - sqrt(3)*(2*x^4 - 2*x^2 - 3))
*sqrt(1/3*sqrt(7) - 1/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(7) - 1/3)), -1/
3*sqrt(7) - 4/3) + 2*(sqrt(7)*sqrt(3)*(2*x^4 - 2*x^2 - 3) + 2*sqrt(3)*(2*x
^4 - 2*x^2 - 3))*sqrt(1/3*sqrt(7) - 1/3)*elliptic_f(arcsin(x*sqrt(1/3*sqrt
(7) - 1/3)), -1/3*sqrt(7) - 4/3) + 6*sqrt(-2*x^4 + 2*x^2 + 3)*(x^3 - 4*x))
/(2*x^4 - 2*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3+2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+2x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+2*x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 + 2*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+2\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+2*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 2*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+2\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+2*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 2*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+2\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(2*x^2 - 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(2*x^2 - 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+2x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+2x^2+3}}{4x^8-8x^6-8x^4+12x^2+9}dx$$

input
```
int(1/(-2*x^4+2*x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + 2*x**2 + 3)/(4*x**8 - 8*x**6 - 8*x**4 + 12*x**2 + 9),
x)
```

3.190 $\int \frac{1}{\left(3+x^2-2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int \frac{1}{(3+x^2-2x^4)^{3/2}}\,dx = \frac{x(13-2x^2)}{75\sqrt{3+x^2-2x^4}} + \frac{1}{75}\sqrt{2}E\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right)|-\frac{3}{2}\right) + \frac{1}{15}\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)$$

output
```
1/75*x*(-2*x^2+13)/(-2*x^4+x^2+3)^(1/2)+1/75*2^(1/2)*EllipticE(1/3*x*6^(1/
2),1/2*I*6^(1/2))+1/15*EllipticF(1/3*x*6^(1/2),1/2*I*6^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 8.95 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.34

$$\int \frac{1}{(3+x^2-2x^4)^{3/2}}\,dx = \frac{13x-2x^3+\sqrt{6-4x^2}\sqrt{1+x^2}E\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right)|-\frac{3}{2}\right)+5\sqrt{6-4x^2}\sqrt{1+x^2}\,\text{Ell}}{75\sqrt{3+x^2-2x^4}}$$

input
```
Integrate[(3 + x^2 - 2*x^4)^(-3/2),x]
```

output
```
(13*x - 2*x^3 + Sqrt[6 - 4*x^2]*Sqrt[1 + x^2]*EllipticE[ArcSin[Sqrt[2/3]*x
], -3/2] + 5*Sqrt[6 - 4*x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2/3]*x],
-3/2])/(75*Sqrt[3 + x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-2x^4+x^2+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x\left(13-2x^2\right)}{75\sqrt{-2x^4+x^2+3}}-\frac{1}{75}\int -\frac{2\left(x^2+6\right)}{\sqrt{-2x^4+x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{75}\int \frac{x^2+6}{\sqrt{-2x^4+x^2+3}}dx+\frac{x\left(13-2x^2\right)}{75\sqrt{-2x^4+x^2+3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{4}{75}\sqrt{2}\int \frac{x^2+6}{2\sqrt{2}\sqrt{3-2x^2}\sqrt{x^2+1}}dx+\frac{x\left(13-2x^2\right)}{75\sqrt{-2x^4+x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{75}\int \frac{x^2+6}{\sqrt{3-2x^2}\sqrt{x^2+1}}dx+\frac{x\left(13-2x^2\right)}{75\sqrt{-2x^4+x^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{2}{75}\left(5\int \frac{1}{\sqrt{3-2x^2}\sqrt{x^2+1}}dx+\int \frac{\sqrt{x^2+1}}{\sqrt{3-2x^2}}dx\right)+\frac{x\left(13-2x^2\right)}{75\sqrt{-2x^4+x^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2}{75}\left(\int \frac{\sqrt{x^2+1}}{\sqrt{3-2x^2}}dx+\frac{5\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{2}}\right)+\frac{x(13-2x^2)}{75\sqrt{-2x^4+x^2+3}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{2}{75}\left(\frac{5\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{2}}+\frac{E\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right)|-\frac{3}{2}\right)}{\sqrt{2}}\right)+\frac{x(13-2x^2)}{75\sqrt{-2x^4+x^2+3}}$$

input
```
Int[(3 + x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(13 - 2*x^2))/(75*Sqrt[3 + x^2 - 2*x^4]) + (2*(EllipticE[ArcSin[Sqrt[2/
3]*x], -3/2]/Sqrt[2] + (5*EllipticF[ArcSin[Sqrt[2/3]*x], -3/2])/Sqrt[2]))/
75
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(59) = 118$.

Time = 1.80 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.81

method	result
risch	$-\frac{x(2x^2-13)}{75\sqrt{-2x^4+x^2+3}}+\frac{2\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{x^2+1}\ \text{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{75\sqrt{-2x^4+x^2+3}}-\frac{\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)-\text{EllipticE}\right.}{225\sqrt{-2x^4+x^2+3}}$
default	$\frac{\frac{13}{75}x-\frac{2}{75}x^3}{\sqrt{-2x^4+x^2+3}}+\frac{2\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{x^2+1}\ \text{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{75\sqrt{-2x^4+x^2+3}}-\frac{\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)-\text{EllipticE}\left(\frac{x\sqrt{6}}{3}\right.\right.}{225\sqrt{-2x^4+x^2+3}}$
elliptic	$\frac{\frac{13}{75}x-\frac{2}{75}x^3}{\sqrt{-2x^4+x^2+3}}+\frac{2\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{x^2+1}\ \text{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{75\sqrt{-2x^4+x^2+3}}-\frac{\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)-\text{EllipticE}\left(\frac{x\sqrt{6}}{3}\right.\right.}{225\sqrt{-2x^4+x^2+3}}$

input
```
int(1/(-2*x^4+x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/75*x*(2*x^2-13)/(-2*x^4+x^2+3)^(1/2)+2/75*6^(1/2)*(-6*x^2+9)^(1/2)*(x^2
+1)^(1/2)/(-2*x^4+x^2+3)^(1/2)*EllipticF(1/3*x*6^(1/2),1/2*I*6^(1/2))-1/22
5*6^(1/2)*(-6*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-2*x^4+x^2+3)^(1/2)*(EllipticF(1
/3*x*6^(1/2),1/2*I*6^(1/2))-EllipticE(1/3*x*6^(1/2),1/2*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

$$\int \frac{1}{(3+x^2-2x^4)^{3/2}}\,dx = \frac{2\sqrt{3}\sqrt{\frac{2}{3}}(2\,x^4-x^2-3)E(\arcsin\left(\sqrt{\frac{2}{3}}x\right)\mid -\frac{3}{2}) + 16\sqrt{3}\sqrt{\frac{2}{3}}(2\,x^4-x^2-3)F(\text{ar}}{225\,(2\,x^4-x^2-3)}$$

input
```
integrate(1/(-2*x^4+x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/225*(2*sqrt(3)*sqrt(2/3)*(2*x^4 - x^2 - 3)*elliptic_e(arcsin(sqrt(2/3)*x
), -3/2) + 16*sqrt(3)*sqrt(2/3)*(2*x^4 - x^2 - 3)*elliptic_f(arcsin(sqrt(2
/3)*x), -3/2) + 3*sqrt(-2*x^4 + x^2 + 3)*(2*x^3 - 13*x))/(2*x^4 - x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3+x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 + x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+x^2+3)^{3/2}}\,dx$$

input
```
int(1/(x^2 - 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(x^2 - 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+x^2+3}}{4x^8-4x^6-11x^4+6x^2+9}dx$$

input
```
int(1/(-2*x^4+x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + x**2 + 3)/(4*x**8 - 4*x**6 - 11*x**4 + 6*x**2 + 9),x)
```

3.191 $\int \frac{1}{(3-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{1}{(3-2x^4)^{3/2}}\,dx=\frac{x}{6\sqrt{3-2x^4}}+\frac{\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{6\sqrt[4]{6}}$$

output
```
1/6*x/(-2*x^4+3)^(1/2)+1/36*EllipticF(1/3*2^(1/4)*3^(3/4)*x,I)*6^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{1}{(3-2x^4)^{3/2}}\,dx=\frac{x}{6\sqrt{3-2x^4}}+\frac{x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{2x^4}{3}\right)}{6\sqrt{3}}$$

input
```
Integrate[(3 - 2*x^4)^(-3/2),x]
```

output
```
x/(6*Sqrt[3 - 2*x^4]) + (x*Hypergeometric2F1[1/4, 1/2, 5/4, (2*x^4)/3])/(6
*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {749, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{749}$$

$$\frac{1}{6}\int \frac{1}{\sqrt{3-2x^4}}dx+\frac{x}{6\sqrt{3-2x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{6\sqrt[4]{6}}+\frac{x}{6\sqrt{3-2x^4}}$$

input
```
Int[(3 - 2*x^4)^(-3/2),x]
```

output
```
x/(6*Sqrt[3 - 2*x^4]) + EllipticF[ArcSin[(2/3)^(1/4)*x], -1]/(6*6^(1/4))
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.71 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.47

method	result	size
meijerg	$\frac{\sqrt{3}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],\frac{2 x^{4}}{3}\right)}{9}$	18
default	$\frac{x}{6 \sqrt{-2 x^{4}+3}}+\frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{9-3 \sqrt{6}\, x^{2}}\, \sqrt{9+3 \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{3}\, 6^{\frac{1}{4}}}{3}, i\right)}{324 \sqrt{-2 x^{4}+3}}$	67
risch	$\frac{x}{6 \sqrt{-2 x^{4}+3}}+\frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{9-3 \sqrt{6}\, x^{2}}\, \sqrt{9+3 \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{3}\, 6^{\frac{1}{4}}}{3}, i\right)}{324 \sqrt{-2 x^{4}+3}}$	67
elliptic	$\frac{x}{6 \sqrt{-2 x^{4}+3}}+\frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{9-3 \sqrt{6}\, x^{2}}\, \sqrt{9+3 \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{3}\, 6^{\frac{1}{4}}}{3}, i\right)}{324 \sqrt{-2 x^{4}+3}}$	67

input
```
int(1/(-2*x^4+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*3^(1/2)*x*hypergeom([1/4,3/2],[5/4],2/3*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{1}{(3-2x^4)^{3/2}}\,dx = \frac{\sqrt{3}\left(\frac{2}{3}\right)^{\frac{3}{4}}(2\,x^4-3)F(\arcsin\left(\left(\frac{2}{3}\right)^{\frac{1}{4}}x\right)|-1)-2\sqrt{-2\,x^4+3}x}{12\,(2\,x^4-3)}$$

input
```
integrate(1/(-2*x^4+3)^(3/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(3)*(2/3)^(3/4)*(2*x^4 - 3)*elliptic_f(arcsin((2/3)^(1/4)*x), -1
) - 2*sqrt(-2*x^4 + 3)*x)/(2*x^4 - 3)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{(3-2x^4)^{3/2}}\,dx = \frac{\sqrt{3}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4e^{2i\pi}}{3}\right)}{36\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-2*x**4+3)**(3/2),x)
```

output
```
sqrt(3)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), 2*x**4*exp_polar(2*I*pi)/3)
/(36*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(3-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 3)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 17.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.42

$$\int \frac{1}{(3-2x^4)^{3/2}}\,dx = \frac{\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ \frac{2\,x^4}{3}\right)}{9}$$

input
```
int(1/(3 - 2*x^4)^(3/2),x)
```

output
```
(3^(1/2)*x*hypergeom([1/4, 3/2], 5/4, (2*x^4)/3))/9
```

Reduce [F]

$$\int \frac{1}{(3-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+3}}{4x^8-12x^4+9}dx$$

input
```
int(1/(-2*x^4+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x***4 + 3)/(4*x***8 - 12*x***4 + 9),x)
```

3.192 $\int \frac{1}{(3-x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{1}{(3-x^2-2x^4)^{3/2}}\,dx=\frac{x(13+2x^2)}{75\sqrt{3-x^2-2x^4}}-\frac{E\left(\arcsin(x)\left|-\frac{2}{3}\right.\right)}{25\sqrt{3}}+\frac{\operatorname{EllipticF}\left(\arcsin(x),-\frac{2}{3}\right)}{5\sqrt{3}}$$

output
```
1/75*x*(2*x^2+13)/(-2*x^4-x^2+3)^(1/2)-1/75*EllipticE(x,1/3*I*6^(1/2))*3^(
1/2)+1/15*EllipticF(x,1/3*I*6^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.92 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

$$\int \frac{1}{(3-x^2-2x^4)^{3/2}}\,dx=\frac{1}{75}\left(\frac{13x}{\sqrt{3-x^2-2x^4}}+\frac{2x^3}{\sqrt{3-x^2-2x^4}}-i\sqrt{2}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{3}}x\right)\left|-\frac{3}{2}\right.\right)-5i\sqrt{2}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)\right)$$

input
```
Integrate[(3 - x^2 - 2*x^4)^(-3/2),x]
```

output
```
((13*x)/Sqrt[3 - x^2 - 2*x^4] + (2*x^3)/Sqrt[3 - x^2 - 2*x^4] - I*Sqrt[2]*
EllipticE[I*ArcSinh[Sqrt[2/3]*x], -3/2] - (5*I)*Sqrt[2]*EllipticF[I*ArcSin
h[Sqrt[2/3]*x], -3/2])/75
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(2x^2+13)}{75\sqrt{-2x^4-x^2+3}}-\frac{1}{75}\int -\frac{2(6-x^2)}{\sqrt{-2x^4-x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{75}\int \frac{6-x^2}{\sqrt{-2x^4-x^2+3}}dx+\frac{x(2x^2+13)}{75\sqrt{-2x^4-x^2+3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{4}{75}\sqrt{2}\int \frac{6-x^2}{2\sqrt{2}\sqrt{1-x^2}\sqrt{2x^2+3}}dx+\frac{x(2x^2+13)}{75\sqrt{-2x^4-x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{75}\int \frac{6-x^2}{\sqrt{1-x^2}\sqrt{2x^2+3}}dx+\frac{x(2x^2+13)}{75\sqrt{-2x^4-x^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{2}{75}\left(\frac{15}{2}\int \frac{1}{\sqrt{1-x^2}\sqrt{2x^2+3}}dx-\frac{1}{2}\int \frac{\sqrt{2x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{x(2x^2+13)}{75\sqrt{-2x^4-x^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2}{75}\left(\frac{5}{2}\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{2}{3}\right)-\frac{1}{2}\int\frac{\sqrt{2x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{x(2x^2+13)}{75\sqrt{-2x^4-x^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{2}{75}\left(\frac{5}{2}\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{2}{3}\right)-\frac{1}{2}\sqrt{3}E\left(\arcsin(x)\left|-\frac{2}{3}\right.\right)\right)+\frac{x(2x^2+13)}{75\sqrt{-2x^4-x^2+3}}$$

input
```
Int[(3 - x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(13 + 2*x^2))/(75*Sqrt[3 - x^2 - 2*x^4]) + (2*(-1/2*(Sqrt[3]*EllipticE[
ArcSin[x], -2/3]) + (5*Sqrt[3]*EllipticF[ArcSin[x], -2/3])/2))/75
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(51) = 102$.

Time = 2.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.05

method	result	si
risch	$\frac{x(2x^2+13)}{75\sqrt{-2x^4-x^2+3}}+\frac{4\sqrt{-x^2+1}\,\sqrt{6x^2+9}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{6}}{3}\right)}{75\sqrt{-2x^4-x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+9}\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{6}}{3}\right)\right)}{75\sqrt{-2x^4-x^2+3}}$	12
default	$\frac{\frac{13}{75}x+\frac{2}{75}x^3}{\sqrt{-2x^4-x^2+3}}+\frac{4\sqrt{-x^2+1}\,\sqrt{6x^2+9}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{6}}{3}\right)}{75\sqrt{-2x^4-x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+9}\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{6}}{3}\right)\right)}{75\sqrt{-2x^4-x^2+3}}$	12
elliptic	$\frac{\frac{13}{75}x+\frac{2}{75}x^3}{\sqrt{-2x^4-x^2+3}}+\frac{4\sqrt{-x^2+1}\,\sqrt{6x^2+9}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{6}}{3}\right)}{75\sqrt{-2x^4-x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+9}\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{6}}{3}\right)\right)}{75\sqrt{-2x^4-x^2+3}}$	12

input
```
int(1/(-2*x^4-x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/75*x*(2*x^2+13)/(-2*x^4-x^2+3)^(1/2)+4/75*(-x^2+1)^(1/2)*(6*x^2+9)^(1/2)
/(-2*x^4-x^2+3)^(1/2)*EllipticF(x,1/3*I*6^(1/2))+1/75*(-x^2+1)^(1/2)*(6*x^
2+9)^(1/2)/(-2*x^4-x^2+3)^(1/2)*(EllipticF(x,1/3*I*6^(1/2))-EllipticE(x,1/
3*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{1}{(3-x^2-2x^4)^{3/2}}\,dx = -\frac{\sqrt{3}(2\,x^4+x^2-3)E(\arcsin{(x)}\,|\,-\frac{2}{3})-5\,\sqrt{3}(2\,x^4+x^2-3)F(\arcsin{(x)}\,|\,-\frac{2}{3})+\sqrt{-2\,x^4-x^2+3}(2\,x^3+13\,x)}{75\,(2\,x^4+x^2-3)}$$

input
```
integrate(1/(-2*x^4-x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/75*(sqrt(3)*(2*x^4 + x^2 - 3)*elliptic_e(arcsin(x), -2/3) - 5*sqrt(3)*(
2*x^4 + x^2 - 3)*elliptic_f(arcsin(x), -2/3) + sqrt(-2*x^4 - x^2 + 3)*(2*x
^3 + 13*x))/(2*x^4 + x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3-x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 - x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-x^2+3)^{3/2}}\,dx$$

input
```
int(1/(3 - 2*x^4 - x^2)^(3/2),x)
```

output
```
int(1/(3 - 2*x^4 - x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-x^2+3}}{4x^8+4x^6-11x^4-6x^2+9}dx$$

input
```
int(1/(-2*x^4-x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - x**2 + 3)/(4*x**8 + 4*x**6 - 11*x**4 - 6*x**2 + 9),x)
```

3.193 $\int \frac{1}{(3-2x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\int \frac{1}{(3-2x^2-2x^4)^{3/2}}\,dx = \frac{x(4+x^2)}{21\sqrt{3-2x^2-2x^4}} - \frac{1}{42}\sqrt{1+\sqrt{7}}E\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right) + \frac{1}{6}\sqrt{\frac{1}{7}\left(1+\sqrt{7}\right)}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)$$

output
```
1/21*x*(x^2+4)/(-2*x^4-2*x^2+3)^(1/2)-1/42*(1+7^(1/2))^(1/2)*EllipticE(2^(
1/2)/(-1+7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))+1/42*(7+7*7^(1/2))
^(1/2)*EllipticF(2^(1/2)/(-1+7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2)
)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

$$\int \frac{1}{(3-2x^2-2x^4)^{3/2}}\,dx = \frac{1}{42}\left(\frac{2x(4+x^2)}{\sqrt{3-2x^2-2x^4}}\right.$$
$$-i\sqrt{-1+\sqrt{7}}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right)|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)+\frac{i(-7+\sqrt{7})\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right),-\frac{4}{3}-\right.}{\sqrt{-1+\sqrt{7}}}$$

input
```
Integrate[(3 - 2*x^2 - 2*x^4)^(-3/2),x]
```

output
```
((2*x*(4 + x^2))/Sqrt[3 - 2*x^2 - 2*x^4] - I*Sqrt[-1 + Sqrt[7]]*EllipticE[
I*ArcSinh[Sqrt[2/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3] + (I*(-7 + Sqrt[7])*
EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3])/Sqrt[-1 +
 Sqrt[7]])/42
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-2x^2+3)^{3/2}}dx$$
$$\Big\downarrow \text{1405}$$
$$\frac{x(x^2+4)}{21\sqrt{-2x^4-2x^2+3}}-\frac{1}{84}\int -\frac{4(3-x^2)}{\sqrt{-2x^4-2x^2+3}}dx$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\int \frac{3-x^2}{\sqrt{-2x^4-2x^2+3}}dx+\frac{x\left(x^2+4\right)}{21\sqrt{-2x^4-2x^2+3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{2}{21}\sqrt{2}\int \frac{3-x^2}{2\sqrt{-2x^2+\sqrt{7}-1}\sqrt{2x^2+\sqrt{7}+1}}dx+\frac{x\left(x^2+4\right)}{21\sqrt{-2x^4-2x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\sqrt{2}\int \frac{3-x^2}{\sqrt{-2x^2+\sqrt{7}-1}\sqrt{2x^2+\sqrt{7}+1}}dx+\frac{x\left(x^2+4\right)}{21\sqrt{-2x^4-2x^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{21}\sqrt{2}\left(\frac{1}{2}\left(7+\sqrt{7}\right)\int \frac{1}{\sqrt{-2x^2+\sqrt{7}-1}\sqrt{2x^2+\sqrt{7}+1}}dx-\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{7}+1}}{\sqrt{-2x^2+\sqrt{7}-1}}dx\right)+\frac{x\left(x^2+4\right)}{21\sqrt{-2x^4-2x^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{21}\sqrt{2}\left(\frac{\left(7+\sqrt{7}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{2\sqrt{2\left(1+\sqrt{7}\right)}}-\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{7}+1}}{\sqrt{-2x^2+\sqrt{7}-1}}dx\right)+\frac{x\left(x^2+4\right)}{21\sqrt{-2x^4-2x^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{21}\sqrt{2}\left(\frac{\left(7+\sqrt{7}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{2\sqrt{2\left(1+\sqrt{7}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{7}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right)|\right.\right.$$

$$\frac{x\left(x^2+4\right)}{21\sqrt{-2x^4-2x^2+3}}$$

input

```
Int[(3 - 2*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(4 + x^2))/(21*Sqrt[3 - 2*x^2 - 2*x^4]) + (Sqrt[2]*(-1/2*(Sqrt[(1 + Sqr
t[7])/2]*EllipticE[ArcSin[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]) +
((7 + Sqrt[7])*EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/
3])/(2*Sqrt[2*(1 + Sqrt[7])])))/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(95) = 190$.

Time = 2.31 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.88

method	result
risch	$\frac{x(x^2+4)}{21\sqrt{-2x^4-2x^2+3}} + \frac{3\sqrt{1-\left(\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\ \text{EllipticF}\left(\frac{\sqrt{3+3\sqrt{7}}\,x}{3},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{7\sqrt{3+3\sqrt{7}}\,\sqrt{-2x^4-2x^2+3}} + 6\sqrt{1-\left(\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}-\right.}$
default	$\frac{\frac{4}{21}x+\frac{1}{21}x^3}{\sqrt{-2x^4-2x^2+3}} + \frac{3\sqrt{1-\left(\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\ \text{EllipticF}\left(\frac{\sqrt{3+3\sqrt{7}}\,x}{3},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{7\sqrt{3+3\sqrt{7}}\,\sqrt{-2x^4-2x^2+3}} + 6\sqrt{1-\left(\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right.}$
elliptic	$\frac{\frac{4}{21}x+\frac{1}{21}x^3}{\sqrt{-2x^4-2x^2+3}} + \frac{3\sqrt{1-\left(\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\ \text{EllipticF}\left(\frac{\sqrt{3+3\sqrt{7}}\,x}{3},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{7\sqrt{3+3\sqrt{7}}\,\sqrt{-2x^4-2x^2+3}} + 6\sqrt{1-\left(\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right.}$

input
```
int(1/(-2*x^4-2*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*x*(x^2+4)/(-2*x^4-2*x^2+3)^(1/2)+3/7/(3+3*7^(1/2))^(1/2)*(1-(1/3+1/3*
7^(1/2))*x^2)^(1/2)*(1-(1/3-1/3*7^(1/2))*x^2)^(1/2)/(-2*x^4-2*x^2+3)^(1/2)
*EllipticF(1/3*(3+3*7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))+6/7/(3+
3*7^(1/2))^(1/2)*(1-(1/3+1/3*7^(1/2))*x^2)^(1/2)*(1-(1/3-1/3*7^(1/2))*x^2)
^(1/2)/(-2*x^4-2*x^2+3)^(1/2)/(-2+2*7^(1/2))*(EllipticF(1/3*(3+3*7^(1/2))^
(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))-EllipticE(1/3*(3+3*7^(1/2))^(1/2)*x,
1/6*I*42^(1/2)-1/6*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.45

$$\int \frac{1}{(3-2x^2-2x^4)^{3/2}}\,dx =$$
$$-\frac{\left(\sqrt{7}\sqrt{3}(2\,x^4+2\,x^2-3)+\sqrt{3}(2\,x^4+2\,x^2-3)\right)\sqrt{\frac{1}{3}\,\sqrt{7}+\frac{1}{3}}E(\arcsin\left(x\sqrt{\frac{1}{3}\,\sqrt{7}+\frac{1}{3}}\right)\mid\frac{1}{3}\,\sqrt{7}-\frac{4}{3})-2\,(2}{}$$

input
```
integrate(1/(-2*x^4-2*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/126*((sqrt(7)*sqrt(3)*(2*x^4 + 2*x^2 - 3) + sqrt(3)*(2*x^4 + 2*x^2 - 3)
)*sqrt(1/3*sqrt(7) + 1/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(7) + 1/3)), 1/
3*sqrt(7) - 4/3) - 2*(2*sqrt(7)*sqrt(3)*(2*x^4 + 2*x^2 - 3) - sqrt(3)*(2*x
^4 + 2*x^2 - 3))*sqrt(1/3*sqrt(7) + 1/3)*elliptic_f(arcsin(x*sqrt(1/3*sqrt
(7) + 1/3)), 1/3*sqrt(7) - 4/3) + 6*sqrt(-2*x^4 - 2*x^2 + 3)*(x^3 + 4*x))/
(2*x^4 + 2*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3-2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-2x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-2*x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 - 2*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-2\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-2*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - 2*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-2\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-2*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - 2*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-2\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(3 - 2*x^4 - 2*x^2)^(3/2),x)
```

output
```
int(1/(3 - 2*x^4 - 2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-2x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-2x^2+3}}{4x^8+8x^6-8x^4-12x^2+9}dx$$

input
```
int(1/(-2*x^4-2*x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - 2*x**2 + 3)/(4*x**8 + 8*x**6 - 8*x**4 - 12*x**2 + 9),
x)
```

3.194 $\int \frac{1}{(3-3x^2-2x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\begin{aligned}\int \frac{1}{(3-3x^2-2x^4)^{3/2}} \, dx &= \frac{x(7+2x^2)}{33\sqrt{3-3x^2-2x^4}} \\ &- \frac{1}{33}\sqrt{\frac{1}{2}\left(3+\sqrt{33}\right)}E\left(\arcsin\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right) \\ &+ \frac{1}{3}\sqrt{\frac{1}{66}\left(3+\sqrt{33}\right)}\,\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)\end{aligned}$$

output
```
1/33*x*(2*x^2+7)/(-2*x^4-3*x^2+3)^(1/2)-1/66*(6+2*33^(1/2))^(1/2)*Elliptic
E(2*x/(-3+33^(1/2))^(1/2),1/4*I*22^(1/2)-1/4*I*6^(1/2))+1/198*(198+66*33^(
1/2))^(1/2)*EllipticF(2*x/(-3+33^(1/2))^(1/2),1/4*I*22^(1/2)-1/4*I*6^(1/2)
)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.37

$$\int \frac{1}{(3-3x^2-2x^4)^{3/2}}\,dx = \frac{4x(7+2x^2)-2i\sqrt{-3+\sqrt{33}}\sqrt{6-6x^2-4x^4}E\left(i\text{arcsinh}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)\middle|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right.}{132\sqrt{3-3x^2-2x^4}}$$

input
```
Integrate[(3 - 3*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(4*x*(7 + 2*x^2) - (2*I)*Sqrt[-3 + Sqrt[33]]*Sqrt[6 - 6*x^2 - 4*x^4]*Ellip
ticE[I*ArcSinh[(2*x)/Sqrt[3 + Sqrt[33]]], -7/4 - Sqrt[33]/4] + ((2*I)*(-11
 + Sqrt[33])*Sqrt[6 - 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[3 + Sq
rt[33]]], -7/4 - Sqrt[33]/4])/Sqrt[-3 + Sqrt[33]])/(132*Sqrt[3 - 3*x^2 - 2
*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-3x^2+3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x(2x^2+7)}{33\sqrt{-2x^4-3x^2+3}}-\frac{1}{99}\int -\frac{6(2-x^2)}{\sqrt{-2x^4-3x^2+3}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{33}\int \frac{2-x^2}{\sqrt{-2x^4-3x^2+3}}dx+\frac{x(2x^2+7)}{33\sqrt{-2x^4-3x^2+3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{4}{33}\sqrt{2}\int \frac{2-x^2}{\sqrt{-4x^2+\sqrt{33}-3}\sqrt{4x^2+\sqrt{33}+3}}dx+\frac{x(2x^2+7)}{33\sqrt{-2x^4-3x^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{4}{33}\sqrt{2}\left(\frac{1}{4}\left(11+\sqrt{33}\right)\int \frac{1}{\sqrt{-4x^2+\sqrt{33}-3}\sqrt{4x^2+\sqrt{33}+3}}dx-\frac{1}{4}\int \frac{\sqrt{4x^2+\sqrt{33}+3}}{\sqrt{-4x^2+\sqrt{33}-3}}dx\right)+\frac{x(2x^2+7)}{33\sqrt{-2x^4-3x^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{4}{33}\sqrt{2}\left(\frac{\left(11+\sqrt{33}\right)\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{8\sqrt{3+\sqrt{33}}}-\frac{1}{4}\int \frac{\sqrt{4x^2+\sqrt{33}+3}}{\sqrt{-4x^2+\sqrt{33}-3}}dx\right)+\frac{x(2x^2+7)}{33\sqrt{-2x^4-3x^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{4}{33}\sqrt{2}\left(\frac{\left(11+\sqrt{33}\right)\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{8\sqrt{3+\sqrt{33}}}-\frac{1}{8}\sqrt{3+\sqrt{33}}E\left(\arcsin\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}\right.\right.$$
$$\frac{x(2x^2+7)}{33\sqrt{-2x^4-3x^2+3}}$$

input
```
Int[(3 - 3*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(7 + 2*x^2))/(33*Sqrt[3 - 3*x^2 - 2*x^4]) + (4*Sqrt[2]*(-1/8*(Sqrt[3 +
Sqrt[33]]*EllipticE[ArcSin[(2*x)/Sqrt[-3 + Sqrt[33]]], (-7 + Sqrt[33])/4])
 + ((11 + Sqrt[33])*EllipticF[ArcSin[(2*x)/Sqrt[-3 + Sqrt[33]]], (-7 + Sqr
t[33])/4])/(8*Sqrt[3 + Sqrt[33]])))/33
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(95) = 190$.

Time = 2.38 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.88

method	result
risch	$\frac{x(2x^2+7)}{33\sqrt{-2x^4-3x^2+3}} + \frac{8\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{18+6\sqrt{33}}\,x}{6}, \frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{11\sqrt{18+6\sqrt{33}}\sqrt{-2x^4-3x^2+3}} + \frac{24\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-}$
default	$\frac{\frac{7}{33}x+\frac{2}{33}x^3}{\sqrt{-2x^4-3x^2+3}} + \frac{8\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{18+6\sqrt{33}}\,x}{6}, \frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{11\sqrt{18+6\sqrt{33}}\sqrt{-2x^4-3x^2+3}} + \frac{24\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-(}$
elliptic	$\frac{\frac{7}{33}x+\frac{2}{33}x^3}{\sqrt{-2x^4-3x^2+3}} + \frac{8\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{18+6\sqrt{33}}\,x}{6}, \frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{11\sqrt{18+6\sqrt{33}}\sqrt{-2x^4-3x^2+3}} + \frac{24\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-(}$

input
```
int(1/(-2*x^4-3*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/33*x*(2*x^2+7)/(-2*x^4-3*x^2+3)^(1/2)+8/11/(18+6*33^(1/2))^(1/2)*(1-(1/2
+1/6*33^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*33^(1/2))*x^2)^(1/2)/(-2*x^4-3*x^2+3
)^(1/2)*EllipticF(1/6*(18+6*33^(1/2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2)
)+24/11/(18+6*33^(1/2))^(1/2)*(1-(1/2+1/6*33^(1/2))*x^2)^(1/2)*(1-(1/2-1/6
*33^(1/2))*x^2)^(1/2)/(-2*x^4-3*x^2+3)^(1/2)/(-3+33^(1/2))*(EllipticF(1/6*
(18+6*33^(1/2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))-EllipticE(1/6*(18+6*
33^(1/2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.46

$$\int \frac{1}{(3-3x^2-2x^4)^{3/2}}\,dx =$$

$$-\frac{\left(\sqrt{\frac{11}{3}}\sqrt{3}(2\,x^4+3\,x^2-3)+\sqrt{3}(2\,x^4+3\,x^2-3)\right)\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}+\frac{1}{2}}E(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}+\frac{1}{2}}\right)\mid\frac{3}{4}\sqrt{\frac{11}{3}}-\frac{7}{4})}{}$$

input
```
integrate(1/(-2*x^4-3*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/66*((sqrt(11/3)*sqrt(3)*(2*x^4 + 3*x^2 - 3) + sqrt(3)*(2*x^4 + 3*x^2 -
3))*sqrt(1/2*sqrt(11/3) + 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(11/3) + 1
/2)), 3/4*sqrt(11/3) - 7/4) - (3*sqrt(11/3)*sqrt(3)*(2*x^4 + 3*x^2 - 3) -
sqrt(3)*(2*x^4 + 3*x^2 - 3))*sqrt(1/2*sqrt(11/3) + 1/2)*elliptic_f(arcsin(
x*sqrt(1/2*sqrt(11/3) + 1/2)), 3/4*sqrt(11/3) - 7/4) + 2*sqrt(-2*x^4 - 3*x
^2 + 3)*(2*x^3 + 7*x))/(2*x^4 + 3*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3-3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-3x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-3*x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 - 3*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-3\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-3*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - 3*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-3\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-3*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - 3*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-3\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(3 - 2*x^4 - 3*x^2)^(3/2),x)
```

output
```
int(1/(3 - 2*x^4 - 3*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-3x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-3x^2+3}}{4x^8+12x^6-3x^4-18x^2+9}dx$$

input
```
int(1/(-2*x^4-3*x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - 3*x**2 + 3)/(4*x**8 + 12*x**6 - 3*x**4 - 18*x**2 + 9)
,x)
```

3.195 $\int \frac{1}{(3-4x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 127

$$\int \frac{1}{(3-4x^2-2x^4)^{3/2}}\,dx = \frac{x(7+2x^2)}{30\sqrt{3-4x^2-2x^4}} - \frac{1}{30}\sqrt{2+\sqrt{10}}E\left(\arcsin\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right) + \frac{1}{6}\sqrt{\frac{1}{10}\left(2+\sqrt{10}\right)}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)$$

output
```
1/30*x*(2*x^2+7)/(-2*x^4-4*x^2+3)^(1/2)-1/30*(2+10^(1/2))^(1/2)*EllipticE(
2^(1/2)/(-2+10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))+1/60*(20+10*10
^(1/2))^(1/2)*EllipticF(2^(1/2)/(-2+10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I
*6^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09

$$\int \frac{1}{(3-4x^2-2x^4)^{3/2}}\,dx = \frac{1}{30}\left(\frac{x(7+2x^2)}{\sqrt{3-4x^2-2x^4}} - i\sqrt{-2+\sqrt{10}}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right)|-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right)+\frac{i(-5+\sqrt{10})\text{ EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right)\right.}{\sqrt{-2+\sqrt{10}}}\right.$$

input
```
Integrate[(3 - 4*x^2 - 2*x^4)^(-3/2),x]
```

output
```
((x*(7 + 2*x^2))/Sqrt[3 - 4*x^2 - 2*x^4] - I*Sqrt[-2 + Sqrt[10]]*EllipticE
[I*ArcSinh[Sqrt[2/(2 + Sqrt[10])]*x], -7/3 - (2*Sqrt[10])/3] + (I*(-5 + Sq
rt[10])*EllipticF[I*ArcSinh[Sqrt[2/(2 + Sqrt[10])]*x], -7/3 - (2*Sqrt[10])
/3])/Sqrt[-2 + Sqrt[10]])/30
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-4x^2+3)^{3/2}}\,dx$$

$$\Big\downarrow \; 1405$$

$$\frac{x(2x^2+7)}{30\sqrt{-2x^4-4x^2+3}} - \frac{1}{120}\int -\frac{4(3-2x^2)}{\sqrt{-2x^4-4x^2+3}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{30}\int \frac{3-2x^2}{\sqrt{-2x^4-4x^2+3}}dx+\frac{x(2x^2+7)}{30\sqrt{-2x^4-4x^2+3}}$$

$\Big\downarrow$ 1494

$$\frac{1}{15}\sqrt{2}\int \frac{3-2x^2}{2\sqrt{-2x^2+\sqrt{10}-2}\sqrt{2x^2+\sqrt{10}+2}}dx+\frac{x(2x^2+7)}{30\sqrt{-2x^4-4x^2+3}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{3-2x^2}{\sqrt{-2x^2+\sqrt{10}-2}\sqrt{2x^2+\sqrt{10}+2}}dx}{15\sqrt{2}}+\frac{x(2x^2+7)}{30\sqrt{-2x^4-4x^2+3}}$$

$\Big\downarrow$ 399

$$\frac{(5+\sqrt{10})\int \frac{1}{\sqrt{-2x^2+\sqrt{10}-2}\sqrt{2x^2+\sqrt{10}+2}}dx-\int \frac{\sqrt{2x^2+\sqrt{10}+2}}{\sqrt{-2x^2+\sqrt{10}-2}}dx}{15\sqrt{2}}+\frac{x(2x^2+7)}{30\sqrt{-2x^4-4x^2+3}}$$

$\Big\downarrow$ 321

$$\frac{\frac{(5+\sqrt{10})\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2\left(2+\sqrt{10}\right)}}-\int \frac{\sqrt{2x^2+\sqrt{10}+2}}{\sqrt{-2x^2+\sqrt{10}-2}}dx}{15\sqrt{2}}+\frac{x(2x^2+7)}{30\sqrt{-2x^4-4x^2+3}}$$

$\Big\downarrow$ 327

$$\frac{\frac{(5+\sqrt{10})\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2\left(2+\sqrt{10}\right)}}-\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{15\sqrt{2}}+\frac{x(2x^2+7)}{30\sqrt{-2x^4-4x^2+3}}$$

input
```
Int[(3 - 4*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(7 + 2*x^2))/(30*Sqrt[3 - 4*x^2 - 2*x^4]) + (-(Sqrt[(2 + Sqrt[10])/2]*E
llipticE[ArcSin[Sqrt[2/(-2 + Sqrt[10])]*x], (-7 + 2*Sqrt[10])/3]) + ((5 +
Sqrt[10])*EllipticF[ArcSin[Sqrt[2/(-2 + Sqrt[10])]*x], (-7 + 2*Sqrt[10])/3
])/Sqrt[2*(2 + Sqrt[10])])/(15*Sqrt[2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(97) = 194.

Time = 2.68 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.81

method	result
risch	$\frac{x(2x^2+7)}{30\sqrt{-2x^4-4x^2+3}}+\frac{3\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{6+3\sqrt{10}}\,x}{3},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{10\sqrt{6+3\sqrt{10}}\,\sqrt{-2x^4-4x^2+3}}+\frac{6\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(\right.}}{}$
default	$\frac{\frac{7}{30}x+\frac{1}{15}x^3}{\sqrt{-2x^4-4x^2+3}}+\frac{3\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{6+3\sqrt{10}}\,x}{3},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{10\sqrt{6+3\sqrt{10}}\,\sqrt{-2x^4-4x^2+3}}+\frac{6\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}-\right.}}{}$
elliptic	$\frac{\frac{7}{30}x+\frac{1}{15}x^3}{\sqrt{-2x^4-4x^2+3}}+\frac{3\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{6+3\sqrt{10}}\,x}{3},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{10\sqrt{6+3\sqrt{10}}\,\sqrt{-2x^4-4x^2+3}}+\frac{6\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}-\right.}}{}$

input
```
int(1/(-2*x^4-4*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*x*(2*x^2+7)/(-2*x^4-4*x^2+3)^(1/2)+3/10/(6+3*10^(1/2))^(1/2)*(1-(2/3+
1/3*10^(1/2))*x^2)^(1/2)*(1-(2/3-1/3*10^(1/2))*x^2)^(1/2)/(-2*x^4-4*x^2+3)
^(1/2)*EllipticF(1/3*(6+3*10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))+
6/5/(6+3*10^(1/2))^(1/2)*(1-(2/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(2/3-1/3*10^(
1/2))*x^2)^(1/2)/(-2*x^4-4*x^2+3)^(1/2)/(-4+2*10^(1/2))*(EllipticF(1/3*(6+
3*10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))-EllipticE(1/3*(6+3*10^(1
/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.41

$$\int \frac{1}{(3-4x^2-2x^4)^{3/2}}\,dx =$$

$$-\frac{2\left(\sqrt{10}\sqrt{3}(2x^4+4x^2-3)+2\sqrt{3}(2x^4+4x^2-3)\right)\sqrt{\frac{1}{3}\sqrt{10}+\frac{2}{3}}E(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{10}+\frac{2}{3}}\right)\mid\frac{2}{3}\sqrt{10}-\frac{7}{3}}{}$$

input
```
integrate(1/(-2*x^4-4*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/180*(2*(sqrt(10)*sqrt(3)*(2*x^4 + 4*x^2 - 3) + 2*sqrt(3)*(2*x^4 + 4*x^2
 - 3))*sqrt(1/3*sqrt(10) + 2/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(10) + 2/
3)), 2/3*sqrt(10) - 7/3) - (5*sqrt(10)*sqrt(3)*(2*x^4 + 4*x^2 - 3) - 2*sqr
t(3)*(2*x^4 + 4*x^2 - 3))*sqrt(1/3*sqrt(10) + 2/3)*elliptic_f(arcsin(x*sqr
t(1/3*sqrt(10) + 2/3)), 2/3*sqrt(10) - 7/3) + 6*sqrt(-2*x^4 - 4*x^2 + 3)*(
2*x^3 + 7*x))/(2*x^4 + 4*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3-4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-4x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-4*x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 - 4*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-4\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-4*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - 4*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-4\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-4*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - 4*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-4\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(3 - 2*x^4 - 4*x^2)^(3/2),x)
```

output
```
int(1/(3 - 2*x^4 - 4*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-4x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-4x^2+3}}{4x^8+16x^6+4x^4-24x^2+9}dx$$

input
```
int(1/(-2*x^4-4*x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - 4*x**2 + 3)/(4*x**8 + 16*x**6 + 4*x**4 - 24*x**2 + 9)
,x)
```

3.196 $\int \frac{1}{(3-5x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \frac{1}{(3-5x^2-2x^4)^{3/2}}\,dx=\frac{x(37+10x^2)}{147\sqrt{3-5x^2-2x^4}}-\frac{5}{49}\sqrt{\frac{2}{3}}E\left(\arcsin\left(\sqrt{2}x\right)|-\frac{1}{6}\right)+\frac{1}{7}\sqrt{\frac{2}{3}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{6}\right)$$

output
```
1/147*x*(10*x^2+37)/(-2*x^4-5*x^2+3)^(1/2)-5/147*6^(1/2)*EllipticE(x*2^(1/
2),1/6*I*6^(1/2))+1/21*EllipticF(x*2^(1/2),1/6*I*6^(1/2))*6^(1/2)
```

Mathematica [A] (verified)

Time = 9.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \frac{1}{(3-5x^2-2x^4)^{3/2}}\,dx=\frac{37x+10x^3-5\sqrt{6-12x^2}\sqrt{3+x^2}E\left(\arcsin\left(\sqrt{2}x\right)|-\frac{1}{6}\right)+7\sqrt{6-12x^2}\sqrt{3+x}}{147\sqrt{3-5x^2-2x^4}}$$

input
```
Integrate[(3 - 5*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(37*x + 10*x^3 - 5*Sqrt[6 - 12*x^2]*Sqrt[3 + x^2]*EllipticE[ArcSin[Sqrt[2]
*x], -1/6] + 7*Sqrt[6 - 12*x^2]*Sqrt[3 + x^2]*EllipticF[ArcSin[Sqrt[2]*x],
 -1/6])/(147*Sqrt[3 - 5*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-5x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(10x^2+37)}{147\sqrt{-2x^4-5x^2+3}}-\frac{1}{147}\int -\frac{2(6-5x^2)}{\sqrt{-2x^4-5x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{147}\int \frac{6-5x^2}{\sqrt{-2x^4-5x^2+3}}dx+\frac{x(10x^2+37)}{147\sqrt{-2x^4-5x^2+3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{4}{147}\sqrt{2}\int \frac{6-5x^2}{2\sqrt{2}\sqrt{1-2x^2}\sqrt{x^2+3}}dx+\frac{x(10x^2+37)}{147\sqrt{-2x^4-5x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{147}\int \frac{6-5x^2}{\sqrt{1-2x^2}\sqrt{x^2+3}}dx+\frac{x(10x^2+37)}{147\sqrt{-2x^4-5x^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{2}{147}\left(21\int \frac{1}{\sqrt{1-2x^2}\sqrt{x^2+3}}dx-5\int \frac{\sqrt{x^2+3}}{\sqrt{1-2x^2}}dx\right)+\frac{x(10x^2+37)}{147\sqrt{-2x^4-5x^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2}{147}\left(7\sqrt{\frac{3}{2}}\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{6}\right)-5\int \frac{\sqrt{x^2+3}}{\sqrt{1-2x^2}}dx\right)+\frac{x(10x^2+37)}{147\sqrt{-2x^4-5x^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{2}{147}\left(7\sqrt{\frac{3}{2}}\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{6}\right)-5\sqrt{\frac{3}{2}}E\left(\arcsin\left(\sqrt{2}x\right)|-\frac{1}{6}\right)\right)+\frac{x(10x^2+37)}{147\sqrt{-2x^4-5x^2+3}}$$

input
```
Int[(3 - 5*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(37 + 10*x^2))/(147*Sqrt[3 - 5*x^2 - 2*x^4]) + (2*(-5*Sqrt[3/2]*Ellipti
cE[ArcSin[Sqrt[2]*x], -1/6] + 7*Sqrt[3/2]*EllipticF[ArcSin[Sqrt[2]*x], -1/
6]))/147
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(59) = 118$.

Time = 2.77 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.85

method	result
risch	$\frac{x(10x^2+37)}{147\sqrt{-2x^4-5x^2+3}} + \frac{2\sqrt{2}\sqrt{-2x^2+1}\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x\sqrt{2},\frac{i\sqrt{6}}{6}\right)}{147\sqrt{-2x^4-5x^2+3}} + \frac{5\sqrt{2}\sqrt{-2x^2+1}\sqrt{3x^2+9}\left(\mathrm{EllipticF}\left(x\sqrt{2},\frac{i\sqrt{6}}{6}\right)-\mathrm{Ellipti}\right.}{147\sqrt{-2x^4-5x^2+3}}$
default	$\frac{\frac{37}{147}x+\frac{10}{147}x^3}{\sqrt{-2x^4-5x^2+3}} + \frac{2\sqrt{2}\sqrt{-2x^2+1}\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x\sqrt{2},\frac{i\sqrt{6}}{6}\right)}{147\sqrt{-2x^4-5x^2+3}} + \frac{5\sqrt{2}\sqrt{-2x^2+1}\sqrt{3x^2+9}\left(\mathrm{EllipticF}\left(x\sqrt{2},\frac{i\sqrt{6}}{6}\right)-\mathrm{EllipticE}(\right.}{147\sqrt{-2x^4-5x^2+3}}$
elliptic	$\frac{\frac{37}{147}x+\frac{10}{147}x^3}{\sqrt{-2x^4-5x^2+3}} + \frac{2\sqrt{2}\sqrt{-2x^2+1}\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x\sqrt{2},\frac{i\sqrt{6}}{6}\right)}{147\sqrt{-2x^4-5x^2+3}} + \frac{5\sqrt{2}\sqrt{-2x^2+1}\sqrt{3x^2+9}\left(\mathrm{EllipticF}\left(x\sqrt{2},\frac{i\sqrt{6}}{6}\right)-\mathrm{EllipticE}(\right.}{147\sqrt{-2x^4-5x^2+3}}$

input
```
int(1/(-2*x^4-5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/147*x*(10*x^2+37)/(-2*x^4-5*x^2+3)^(1/2)+2/147*2^(1/2)*(-2*x^2+1)^(1/2)*
(3*x^2+9)^(1/2)/(-2*x^4-5*x^2+3)^(1/2)*EllipticF(x*2^(1/2),1/6*I*6^(1/2))+
5/147*2^(1/2)*(-2*x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-2*x^4-5*x^2+3)^(1/2)*(Ell
ipticF(x*2^(1/2),1/6*I*6^(1/2))-EllipticE(x*2^(1/2),1/6*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \frac{1}{(3-5x^2-2x^4)^{3/2}}\,dx =$$
$$-\frac{10\sqrt{3}\sqrt{2}(2x^4+5x^2-3)E\left(\arcsin\left(\sqrt{2}x\right) | -\frac{1}{6}\right) - 12\sqrt{3}\sqrt{2}(2x^4+5x^2-3)F\left(\arcsin\left(\sqrt{2}x\right) | -\frac{1}{6}\right) + \sqrt{-}}{147\,(2x^4+5x^2-3)}$$

input
```
integrate(1/(-2*x^4-5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/147*(10*sqrt(3)*sqrt(2)*(2*x^4 + 5*x^2 - 3)*elliptic_e(arcsin(sqrt(2)*x
), -1/6) - 12*sqrt(3)*sqrt(2)*(2*x^4 + 5*x^2 - 3)*elliptic_f(arcsin(sqrt(2
)*x), -1/6) + sqrt(-2*x^4 - 5*x^2 + 3)*(10*x^3 + 37*x))/(2*x^4 + 5*x^2 - 3
)
```

Sympy [F]

$$\int \frac{1}{(3-5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-5x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-5*x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 - 5*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - 5*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - 5*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-5\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(3 - 2*x^4 - 5*x^2)^(3/2),x)
```

output
```
int(1/(3 - 2*x^4 - 5*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-5x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-5x^2+3}}{4x^8+20x^6+13x^4-30x^2+9}dx$$

input
```
int(1/(-2*x^4-5*x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - 5*x**2 + 3)/(4*x**8 + 20*x**6 + 13*x**4 - 30*x**2 + 9
),x)
```

3.197 $\int \frac{1}{(3-6x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{1}{(3-6x^2-2x^4)^{3/2}}\,dx = \frac{x(4+x^2)}{15\sqrt{3-6x^2-2x^4}} - \frac{1}{30}\sqrt{3+\sqrt{15}}E\left(\arcsin\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right)|-4+\sqrt{15}\right) + \frac{1}{6}\sqrt{\frac{1}{15}\left(3+\sqrt{15}\right)}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right),-4+\sqrt{15}\right)$$

output
```
1/15*x*(x^2+4)/(-2*x^4-6*x^2+3)^(1/2)-1/30*(3+15^(1/2))^(1/2)*EllipticE(1/
3*(9+3*15^(1/2))^(1/2)*x,1/2*I*10^(1/2)-1/2*I*6^(1/2))+1/90*(45+15*15^(1/2
))^(1/2)*EllipticF(1/3*(9+3*15^(1/2))^(1/2)*x,1/2*I*10^(1/2)-1/2*I*6^(1/2)
)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.41 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{1}{(3-6x^2-2x^4)^{3/2}}\,dx = \frac{1}{30}\left(\frac{2x(4+x^2)}{\sqrt{3-6x^2-2x^4}}\right.$$

$$-i\sqrt{-3+\sqrt{15}}E\left(i\operatorname{arcsinh}\left(\sqrt{-1+\sqrt{\frac{5}{3}}}x\right)|-4-\sqrt{15}\right)+\frac{i(-5+\sqrt{15})\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-1+\sqrt{}}\right.\right.}{\sqrt{-3+\sqrt{15}}}$$

input
```
Integrate[(3 - 6*x^2 - 2*x^4)^(-3/2),x]
```

output
```
((2*x*(4 + x^2))/Sqrt[3 - 6*x^2 - 2*x^4] - I*Sqrt[-3 + Sqrt[15]]*EllipticE
[I*ArcSinh[Sqrt[-1 + Sqrt[5/3]]*x], -4 - Sqrt[15]] + (I*(-5 + Sqrt[15])*El
lipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/3]]*x], -4 - Sqrt[15]])/Sqrt[-3 + Sqrt[
15]])/30
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-6x^2+3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(x^2+4)}{15\sqrt{-2x^4-6x^2+3}}-\frac{1}{180}\int -\frac{12(1-x^2)}{\sqrt{-2x^4-6x^2+3}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{15}\int \frac{1-x^2}{\sqrt{-2x^4-6x^2+3}}dx+\frac{x(x^2+4)}{15\sqrt{-2x^4-6x^2+3}}$$

$$\Big\downarrow \; 1494$$

$$\frac{2}{15}\sqrt{2}\int \frac{1-x^2}{2\sqrt{-2x^2+\sqrt{15}-3}\sqrt{2x^2+\sqrt{15}+3}}dx+\frac{x(x^2+4)}{15\sqrt{-2x^4-6x^2+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{15}\sqrt{2}\int \frac{1-x^2}{\sqrt{-2x^2+\sqrt{15}-3}\sqrt{2x^2+\sqrt{15}+3}}dx+\frac{x(x^2+4)}{15\sqrt{-2x^4-6x^2+3}}$$

$$\Big\downarrow \; 399$$

$$\frac{1}{15}\sqrt{2}\left(\frac{1}{2}\left(5+\sqrt{15}\right)\int \frac{1}{\sqrt{-2x^2+\sqrt{15}-3}\sqrt{2x^2+\sqrt{15}+3}}dx-\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{15}+3}}{\sqrt{-2x^2+\sqrt{15}-3}}dx\right)+\frac{x(x^2+4)}{15\sqrt{-2x^4-6x^2+3}}$$

$$\Big\downarrow \; 321$$

$$\frac{1}{15}\sqrt{2}\left(\frac{1}{4}\sqrt{\frac{1}{3}\left(\sqrt{15}-3\right)}\left(5+\sqrt{15}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right),-4+\sqrt{15}\right)-\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{15}+}}{\sqrt{-2x^2+\sqrt{15}}}\right.$$
$$\frac{x(x^2+4)}{15\sqrt{-2x^4-6x^2+3}}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{15}\sqrt{2}\left(\frac{1}{4}\sqrt{\frac{1}{3}\left(\sqrt{15}-3\right)}\left(5+\sqrt{15}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right),-4+\sqrt{15}\right)-\frac{1}{2}\sqrt{\frac{3}{\sqrt{15}-3}}E\left(\text{arcs}\right.\right.$$
$$\frac{x(x^2+4)}{15\sqrt{-2x^4-6x^2+3}}$$

input
```
Int[(3 - 6*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(4 + x^2))/(15*Sqrt[3 - 6*x^2 - 2*x^4]) + (Sqrt[2]*(-1/2*(Sqrt[3/(-3 +
Sqrt[15])]*EllipticE[ArcSin[Sqrt[(3 + Sqrt[15])/3]*x], -4 + Sqrt[15]]) + (
Sqrt[(-3 + Sqrt[15])/3]*(5 + Sqrt[15])*EllipticF[ArcSin[Sqrt[(3 + Sqrt[15]
)/3]*x], -4 + Sqrt[15]])/4))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(95) = 190$.

Time = 2.23 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.02

method	result
risch	$\frac{x(x^2+4)}{15\sqrt{-2x^4-6x^2+3}} + \frac{\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{9+3\sqrt{15}}\,x}{3}, \frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{5\sqrt{9+3\sqrt{15}}\,\sqrt{-2x^4-6x^2+3}} + \frac{6\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1-\right.}}{}$
default	$\frac{\frac{4}{15}x+\frac{1}{15}x^3}{\sqrt{-2x^4-6x^2+3}} + \frac{\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{9+3\sqrt{15}}\,x}{3}, \frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{5\sqrt{9+3\sqrt{15}}\,\sqrt{-2x^4-6x^2+3}} + \frac{6\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1-\right.}}{}$
elliptic	$\frac{\frac{4}{15}x+\frac{1}{15}x^3}{\sqrt{-2x^4-6x^2+3}} + \frac{\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{9+3\sqrt{15}}\,x}{3}, \frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{5\sqrt{9+3\sqrt{15}}\,\sqrt{-2x^4-6x^2+3}} + \frac{6\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1-\right.}}{}$

input
```
int(1/(-2*x^4-6*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(x^2+4)/(-2*x^4-6*x^2+3)^(1/2)+1/5/(9+3*15^(1/2))^(1/2)*(1-(1+1/3*1
5^(1/2))*x^2)^(1/2)*(1-(1-1/3*15^(1/2))*x^2)^(1/2)/(-2*x^4-6*x^2+3)^(1/2)*
EllipticF(1/3*(9+3*15^(1/2))^(1/2)*x,1/2*I*10^(1/2)-1/2*I*6^(1/2))+6/5/(9+
3*15^(1/2))^(1/2)*(1-(1+1/3*15^(1/2))*x^2)^(1/2)*(1-(1-1/3*15^(1/2))*x^2)^
(1/2)/(-2*x^4-6*x^2+3)^(1/2)/(-6+2*15^(1/2))*(EllipticF(1/3*(9+3*15^(1/2))
^(1/2)*x,1/2*I*10^(1/2)-1/2*I*6^(1/2))-EllipticE(1/3*(9+3*15^(1/2))^(1/2)*
x,1/2*I*10^(1/2)-1/2*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.27

$$\int \frac{1}{(3-6x^2-2x^4)^{3/2}}\,dx = \frac{2\sqrt{3}\sqrt{\frac{5}{3}}(2x^4+6x^2-3)\sqrt{\sqrt{\frac{5}{3}}+1}F(\arcsin\left(x\sqrt{\sqrt{\frac{5}{3}}+1}\right)|3\sqrt{\frac{5}{3}}-4)-\sqrt{3}$$

input
```
integrate(1/(-2*x^4-6*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/30*(2*sqrt(3)*sqrt(5/3)*(2*x^4 + 6*x^2 - 3)*sqrt(sqrt(5/3) + 1)*elliptic
_f(arcsin(x*sqrt(sqrt(5/3) + 1)), 3*sqrt(5/3) - 4) - sqrt(3)*(2*x^4 + 6*x^
2 + sqrt(5/3)*(2*x^4 + 6*x^2 - 3) - 3)*sqrt(sqrt(5/3) + 1)*elliptic_e(arcs
in(x*sqrt(sqrt(5/3) + 1)), 3*sqrt(5/3) - 4) - 2*sqrt(-2*x^4 - 6*x^2 + 3)*(
x^3 + 4*x))/(2*x^4 + 6*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3-6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-6x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-6*x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 - 6*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-6\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-6*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - 6*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-6\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-6*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - 6*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-6\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(3 - 2*x^4 - 6*x^2)^(3/2),x)
```

output
```
int(1/(3 - 2*x^4 - 6*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-6x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-6x^2+3}}{4x^8+24x^6+24x^4-36x^2+9}dx$$

input
```
int(1/(-2*x^4-6*x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - 6*x**2 + 3)/(4*x**8 + 24*x**6 + 24*x**4 - 36*x**2 + 9
),x)
```

3.198 $\int \frac{1}{(3-7x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{(3-7x^2-2x^4)^{3/2}}\,dx = \frac{x(61+14x^2)}{219\sqrt{3-7x^2-2x^4}} - \frac{7}{219}\sqrt{\frac{1}{2}\left(7+\sqrt{73}\right)}E\left(\arcsin\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right) + \frac{1}{3}\sqrt{\frac{1}{146}\left(7+\sqrt{73}\right)}\,\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right),\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

output
```
1/219*x*(14*x^2+61)/(-2*x^4-7*x^2+3)^(1/2)-7/438*(14+2*73^(1/2))^(1/2)*Ell
ipticE(2*x/(-7+73^(1/2))^(1/2),1/12*I*438^(1/2)-7/12*I*6^(1/2))+1/438*(102
2+146*73^(1/2))^(1/2)*EllipticF(2*x/(-7+73^(1/2))^(1/2),1/12*I*438^(1/2)-7
/12*I*6^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.34

$$\int \frac{1}{(3-7x^2-2x^4)^{3/2}}\,dx = \frac{4x(61+14x^2)-14i\sqrt{-7+\sqrt{73}}\sqrt{6-14x^2-4x^4}E\left(i\text{arcsinh}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}(-\right.}{876\sqrt{3-}}$$

input
```
Integrate[(3 - 7*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(4*x*(61 + 14*x^2) - (14*I)*Sqrt[-7 + Sqrt[73]]*Sqrt[6 - 14*x^2 - 4*x^4]*E
llipticE[I*ArcSinh[(2*x)/Sqrt[7 + Sqrt[73]]], (-61 - 7*Sqrt[73])/12] + ((2
*I)*(-73 + 7*Sqrt[73])*Sqrt[6 - 14*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/
Sqrt[7 + Sqrt[73]]], (-61 - 7*Sqrt[73])/12])/Sqrt[-7 + Sqrt[73]])/(876*Sqr
t[3 - 7*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-7x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(14x^2+61)}{219\sqrt{-2x^4-7x^2+3}}-\frac{1}{219}\int -\frac{2(6-7x^2)}{\sqrt{-2x^4-7x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{219}\int \frac{6-7x^2}{\sqrt{-2x^4-7x^2+3}}dx+\frac{x(14x^2+61)}{219\sqrt{-2x^4-7x^2+3}}$$

$$\Big\downarrow \ 1494$$

$$\frac{4}{219}\sqrt{2}\int \frac{6-7x^2}{\sqrt{-4x^2+\sqrt{73}-7}\sqrt{4x^2+\sqrt{73}+7}}dx+\frac{x(14x^2+61)}{219\sqrt{-2x^4-7x^2+3}}$$

$$\Big\downarrow \ 399$$

$$\frac{4}{219}\sqrt{2}\left(\frac{1}{4}\left(73+7\sqrt{73}\right)\int \frac{1}{\sqrt{-4x^2+\sqrt{73}-7}\sqrt{4x^2+\sqrt{73}+7}}dx-\frac{7}{4}\int \frac{\sqrt{4x^2+\sqrt{73}+7}}{\sqrt{-4x^2+\sqrt{73}-7}}dx\right)+\frac{x(14x^2+61)}{219\sqrt{-2x^4-7x^2+3}}$$

$$\Big\downarrow \ 321$$

$$\frac{4}{219}\sqrt{2}\left(\frac{\left(73+7\sqrt{73}\right)\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right),\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)}{8\sqrt{7+\sqrt{73}}}-\frac{7}{4}\int \frac{\sqrt{4x^2+\sqrt{73}+7}}{\sqrt{-4x^2+\sqrt{73}-7}}dx\right)+\frac{x(14x^2+61)}{219\sqrt{-2x^4-7x^2+3}}$$

$$\Big\downarrow \ 327$$

$$\frac{4}{219}\sqrt{2}\left(\frac{\left(73+7\sqrt{73}\right)\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right),\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)}{8\sqrt{7+\sqrt{73}}}-\frac{7}{8}\sqrt{7+\sqrt{73}}E\left(\arcsin\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right.\right.\right.$$
$$\frac{x(14x^2+61)}{219\sqrt{-2x^4-7x^2+3}}$$

input
```
Int[(3 - 7*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(61 + 14*x^2))/(219*Sqrt[3 - 7*x^2 - 2*x^4]) + (4*Sqrt[2]*((-7*Sqrt[7 +
 Sqrt[73]]*EllipticE[ArcSin[(2*x)/Sqrt[-7 + Sqrt[73]]], (-61 + 7*Sqrt[73])
/12])/8 + ((73 + 7*Sqrt[73])*EllipticF[ArcSin[(2*x)/Sqrt[-7 + Sqrt[73]]],
(-61 + 7*Sqrt[73])/12])/(8*Sqrt[7 + Sqrt[73]])))/219
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(95) = 190.

Time = 2.86 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.82

method	result
risch	$\frac{x(14x^2+61)}{219\sqrt{-2x^4-7x^2+3}}+\frac{24\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\sqrt{1-\left(-\frac{\sqrt{73}}{6}+\frac{7}{6}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{42+6\sqrt{73}}\,x}{6},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{73\sqrt{42+6\sqrt{73}}\sqrt{-2x^4-7x^2+3}}+\frac{168\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}}{}$
default	$\frac{\frac{61}{219}x+\frac{14}{219}x^3}{\sqrt{-2x^4-7x^2+3}}+\frac{24\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\sqrt{1-\left(-\frac{\sqrt{73}}{6}+\frac{7}{6}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{42+6\sqrt{73}}\,x}{6},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{73\sqrt{42+6\sqrt{73}}\sqrt{-2x^4-7x^2+3}}+\frac{168\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}}{}$
elliptic	$\frac{\frac{61}{219}x+\frac{14}{219}x^3}{\sqrt{-2x^4-7x^2+3}}+\frac{24\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\sqrt{1-\left(-\frac{\sqrt{73}}{6}+\frac{7}{6}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{42+6\sqrt{73}}\,x}{6},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{73\sqrt{42+6\sqrt{73}}\sqrt{-2x^4-7x^2+3}}+\frac{168\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}}{}$

input
```
int(1/(-2*x^4-7*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/219*x*(14*x^2+61)/(-2*x^4-7*x^2+3)^(1/2)+24/73/(42+6*73^(1/2))^(1/2)*(1-
(7/6+1/6*73^(1/2))*x^2)^(1/2)*(1-(-1/6*73^(1/2)+7/6)*x^2)^(1/2)/(-2*x^4-7*
x^2+3)^(1/2)*EllipticF(1/6*(42+6*73^(1/2))^(1/2)*x,1/12*I*438^(1/2)-7/12*I
*6^(1/2))+168/73/(42+6*73^(1/2))^(1/2)*(1-(7/6+1/6*73^(1/2))*x^2)^(1/2)*(1
-(-1/6*73^(1/2)+7/6)*x^2)^(1/2)/(-2*x^4-7*x^2+3)^(1/2)/(-7+73^(1/2))*(Elli
pticF(1/6*(42+6*73^(1/2))^(1/2)*x,1/12*I*438^(1/2)-7/12*I*6^(1/2))-Ellipti
cE(1/6*(42+6*73^(1/2))^(1/2)*x,1/12*I*438^(1/2)-7/12*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.43

$$\int \frac{1}{(3-7x^2-2x^4)^{3/2}}\,dx =$$

$$-\frac{7\left(\sqrt{73}\sqrt{3}(2\,x^4+7\,x^2-3)+7\,\sqrt{3}(2\,x^4+7\,x^2-3)\right)\sqrt{\frac{1}{6}\,\sqrt{73}+\frac{7}{6}}E(\arcsin\left(x\sqrt{\frac{1}{6}\,\sqrt{73}+\frac{7}{6}}\right)\mid\frac{7}{12}\,\sqrt{73}-}{}$$

input
```
integrate(1/(-2*x^4-7*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/1314*(7*(sqrt(73)*sqrt(3)*(2*x^4 + 7*x^2 - 3) + 7*sqrt(3)*(2*x^4 + 7*x^
2 - 3))*sqrt(1/6*sqrt(73) + 7/6)*elliptic_e(arcsin(x*sqrt(1/6*sqrt(73) + 7
/6)), 7/12*sqrt(73) - 61/12) - (13*sqrt(73)*sqrt(3)*(2*x^4 + 7*x^2 - 3) +
7*sqrt(3)*(2*x^4 + 7*x^2 - 3))*sqrt(1/6*sqrt(73) + 7/6)*elliptic_f(arcsin(
x*sqrt(1/6*sqrt(73) + 7/6)), 7/12*sqrt(73) - 61/12) + 6*sqrt(-2*x^4 - 7*x^
2 + 3)*(14*x^3 + 61*x))/(2*x^4 + 7*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3-7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-7x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-7*x**2+3)**(3/2),x)
```

output
```
Integral((-2*x**4 - 7*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-7\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-7*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - 7*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-7\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-7*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - 7*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-7\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(3 - 2*x^4 - 7*x^2)^(3/2),x)
```

output
```
int(1/(3 - 2*x^4 - 7*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-7x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-7x^2+3}}{4x^8+28x^6+37x^4-42x^2+9}dx$$

input
```
int(1/(-2*x^4-7*x^2+3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - 7*x**2 + 3)/(4*x**8 + 28*x**6 + 37*x**4 - 42*x**2 + 9
),x)
```

3.199 $\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx=\frac{x(5+x^2)}{24\sqrt{3-2x^2-x^4}}-\frac{E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)}{8\sqrt{3}}+\frac{\operatorname{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{4\sqrt{3}}$$

output
```
1/24*x*(x^2+5)/(-x^4-2*x^2+3)^(1/2)-1/24*EllipticE(x,1/3*I*3^(1/2))*3^(1/2
)+1/12*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx=\frac{1}{24}\left(\frac{5x}{\sqrt{3-2x^2-x^4}}+\frac{x^3}{\sqrt{3-2x^2-x^4}}-iE\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-2i\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{3}}\right),-3\right)\right)$$

input
```
Integrate[(3 - 2*x^2 - x^4)^(-3/2),x]
```

output
```
((5*x)/Sqrt[3 - 2*x^2 - x^4] + x^3/Sqrt[3 - 2*x^2 - x^4] - I*EllipticE[I*A
rcSinh[x/Sqrt[3]], -3] - (2*I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/24
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^4-2x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}-\frac{1}{48}\int -\frac{2(3-x^2)}{\sqrt{-x^4-2x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\int \frac{3-x^2}{\sqrt{-x^4-2x^2+3}}dx+\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{12}\int \frac{3-x^2}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\int \frac{3-x^2}{\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{24}\left(6\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx-\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{24}\left(2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-\int\frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{24}\left(2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-\sqrt{3}E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)\right)+\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}$$

input
```
Int[(3 - 2*x^2 - x^4)^(-3/2),x]
```

output
```
(x*(5 + x^2))/(24*Sqrt[3 - 2*x^2 - x^4]) + (-(Sqrt[3]*EllipticE[ArcSin[x],
 -1/3]) + 2*Sqrt[3]*EllipticF[ArcSin[x], -1/3])/24
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(49) = 98$.

Time = 1.94 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.09

method	result	siz
risch	$\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{24\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\operatorname{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{24\sqrt{-x^4-2x^2+3}}$	11
default	$\frac{\frac{5}{24}x+\frac{1}{24}x^3}{\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{24\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\operatorname{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{24\sqrt{-x^4-2x^2+3}}$	12
elliptic	$\frac{\frac{5}{24}x+\frac{1}{24}x^3}{\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{24\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\operatorname{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{24\sqrt{-x^4-2x^2+3}}$	12

input
```
int(1/(-x^4-2*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*x*(x^2+5)/(-x^4-2*x^2+3)^(1/2)+1/24*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-
x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+1/24*(-x^2+1)^(1/2)*(3*x^2+9
)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I
*3^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx =$$
$$-\frac{\sqrt{3}(x^4+2\,x^2-3)E(\arcsin{(x)}\,|-\frac{1}{3})-2\,\sqrt{3}(x^4+2\,x^2-3)F(\arcsin{(x)}\,|-\frac{1}{3})+\sqrt{-x^4-2\,x^2+3}(x^3+}{24\,(x^4+2\,x^2-3)}$$

input
```
integrate(1/(-x^4-2*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(3)*(x^4 + 2*x^2 - 3)*elliptic_e(arcsin(x), -1/3) - 2*sqrt(3)*(
x^4 + 2*x^2 - 3)*elliptic_f(arcsin(x), -1/3) + sqrt(-x^4 - 2*x^2 + 3)*(x^3
 + 5*x))/(x^4 + 2*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4-2x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x**4-2*x**2+3)**(3/2),x)
```

output
```
Integral((-x**4 - 2*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4-2\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4-2*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 - 2*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4-2\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4-2*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 - 2*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4-2\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(3 - x^4 - 2*x^2)^(3/2),x)
```

output
```
int(1/(3 - x^4 - 2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx = \frac{-\sqrt{-x^4-2x^2+3}\,x+3\left(\int \frac{\sqrt{-x^4-2x^2+3}}{x^6+x^4-5x^2+3}dx\right)x^4+6\left(\int \frac{\sqrt{-x^4-2x^2+3}}{x^6+x^4-5x^2+3}dx\right)x^2-9\left(\int\right.}{1}$$

input
```
int(1/(-x^4-2*x^2+3)^(3/2),x)
```

output
```
( - sqrt( - x**4 - 2*x**2 + 3)*x + 3*int(sqrt( - x**4 - 2*x**2 + 3)/(x**6
+ x**4 - 5*x**2 + 3),x)*x**4 + 6*int(sqrt( - x**4 - 2*x**2 + 3)/(x**6 + x*
*4 - 5*x**2 + 3),x)*x**2 - 9*int(sqrt( - x**4 - 2*x**2 + 3)/(x**6 + x**4 -
 5*x**2 + 3),x) - int((sqrt( - x**4 - 2*x**2 + 3)*x**2)/(x**6 + x**4 - 5*x
**2 + 3),x)*x**4 - 2*int((sqrt( - x**4 - 2*x**2 + 3)*x**2)/(x**6 + x**4 -
5*x**2 + 3),x)*x**2 + 3*int((sqrt( - x**4 - 2*x**2 + 3)*x**2)/(x**6 + x**4
 - 5*x**2 + 3),x))/(12*(x**4 + 2*x**2 - 3))
```

3.200 $\int \frac{1}{(24+36x^2-36x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 136

$$\int \frac{1}{(24+36x^2-36x^4)^{3/2}}\,dx = \frac{x(7-3x^2)}{528\sqrt{3}\sqrt{2+3x^2-3x^4}} + \frac{1}{528}\sqrt{\frac{1}{6}\left(-3+\sqrt{33}\right)}E\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right) + \frac{1}{144}\sqrt{\frac{1}{22}\left(-3+\sqrt{33}\right)}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

output
```
1/1584*x*(-3*x^2+7)*3^(1/2)/(-3*x^4+3*x^2+2)^(1/2)+1/3168*(-18+6*33^(1/2))
^(1/2)*EllipticE(6^(1/2)/(3+33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2)
)+1/3168*(-66+22*33^(1/2))^(1/2)*EllipticF(6^(1/2)/(3+33^(1/2))^(1/2)*x,1/
4*I*6^(1/2)+1/4*I*22^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.83 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.39

$$\int \frac{1}{(24+36x^2-36x^4)^{3/2}}\,dx = \frac{2\sqrt{3\left(3+\sqrt{33}\right)}x(7-3x^2)+3i\left(\sqrt{3}+\sqrt{11}\right)\sqrt{4+6x^2-6x^4}E\left(i\text{arcsinh}\left(\sqrt{\right.\right.}$$

input
```
Integrate[(24 + 36*x^2 - 36*x^4)^(-3/2),x]
```

output
```
(2*Sqrt[3*(3 + Sqrt[33])]*x*(7 - 3*x^2) + (3*I)*(Sqrt[3] + Sqrt[11])*Sqrt[
4 + 6*x^2 - 6*x^4]*EllipticE[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*x], (-7 + S
qrt[33])/4] - I*(11*Sqrt[3] + 3*Sqrt[11])*Sqrt[4 + 6*x^2 - 6*x^4]*Elliptic
F[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*x], (-7 + Sqrt[33])/4])/(3168*Sqrt[3 +
 Sqrt[33]]*Sqrt[2 + 3*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1405, 27, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-36x^4+36x^2+24)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(7-3x^2)}{528\sqrt{3}\sqrt{-3x^4+3x^2+2}}-\frac{\int -\frac{72\sqrt{3}(3x^2+4)}{\sqrt{-3x^4+3x^2+2}}dx}{114048}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3x^2+4}{\sqrt{-3x^4+3x^2+2}}dx}{528\sqrt{3}}+\frac{x(7-3x^2)}{528\sqrt{3}\sqrt{-3x^4+3x^2+2}}$$

$$\downarrow \text{1494}$$

$$\frac{1}{264}\int \frac{3x^2+4}{\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}dx+\frac{x(7-3x^2)}{528\sqrt{3}\sqrt{-3x^4+3x^2+2}}$$

$$\downarrow \text{399}$$

$$\frac{1}{264}\left(\frac{1}{2}\left(11-\sqrt{33}\right)\int \frac{1}{\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}dx+\frac{1}{2}\int \frac{\sqrt{6x^2+\sqrt{33}-3}}{\sqrt{-6x^2+\sqrt{33}+3}}dx\right)+\frac{x(7-3x^2)}{528\sqrt{3}\sqrt{-3x^4+3x^2+2}}$$

$$\downarrow \text{321}$$

$$\frac{1}{264}\left(\frac{1}{2}\int \frac{\sqrt{6x^2+\sqrt{33}-3}}{\sqrt{-6x^2+\sqrt{33}+3}}dx+\frac{(11-\sqrt{33})\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),\frac{1}{4}(-7-\sqrt{33})\right)}{2\sqrt{6\left(\sqrt{33}-3\right)}}\right)+\frac{x(7-3x^2)}{528\sqrt{3}\sqrt{-3x^4+3x^2+2}}$$

$$\downarrow \text{327}$$

$$\frac{1}{264}\left(\frac{(11-\sqrt{33})\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),\frac{1}{4}(-7-\sqrt{33})\right)}{2\sqrt{6\left(\sqrt{33}-3\right)}}+\frac{1}{2}\sqrt{\frac{1}{6}\left(\sqrt{33}-3\right)}E\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right)\right.\right.$$
$$\frac{x(7-3x^2)}{528\sqrt{3}\sqrt{-3x^4+3x^2+2}}$$

input
```
Int[(24 + 36*x^2 - 36*x^4)^(-3/2),x]
```

output
```
(x*(7 - 3*x^2))/(528*Sqrt[3]*Sqrt[2 + 3*x^2 - 3*x^4]) + ((Sqrt[(-3 + Sqrt[
33])/6]*EllipticE[ArcSin[Sqrt[6/(3 + Sqrt[33])]*x], (-7 - Sqrt[33])/4])/2
+ ((11 - Sqrt[33])*EllipticF[ArcSin[Sqrt[6/(3 + Sqrt[33])]*x], (-7 - Sqrt[
33])/4])/(2*Sqrt[6*(-3 + Sqrt[33])]))/264
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(102) = 204$.

Time = 2.92 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.62

method	result
risch	$-\frac{x(3x^2-7)}{528\sqrt{-9x^4+9x^2+6}}+\frac{\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-3+\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{66\sqrt{-3+\sqrt{33}}\,\sqrt{-9x^4+9x^2+6}}-\frac{3\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x}}{}$
default	$\frac{\frac{7}{528}x-\frac{1}{176}x^3}{\sqrt{-9x^4+9x^2+6}}+\frac{\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-3+\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{66\sqrt{-3+\sqrt{33}}\,\sqrt{-9x^4+9x^2+6}}-\frac{3\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-}}{}$
elliptic	$\frac{\frac{7}{528}x-\frac{1}{176}x^3}{\sqrt{-9x^4+9x^2+6}}+\frac{\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-3+\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{66\sqrt{-3+\sqrt{33}}\,\sqrt{-9x^4+9x^2+6}}-\frac{6\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-}}{}$

input
```
int(1/(-36*x^4+36*x^2+24)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/528*x*(3*x^2-7)/(-9*x^4+9*x^2+6)^(1/2)+1/66/(-3+33^(1/2))^(1/2)*(1-(-3/
4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*33^(1/2))*x^2)^(1/2)/(-9*x^4+9*x^2
+6)^(1/2)*EllipticF(1/2*(-3+33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2)
)-3/22/(-3+33^(1/2))^(1/2)*(1-(-3/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*
33^(1/2))*x^2)^(1/2)/(-9*x^4+9*x^2+6)^(1/2)/(9+3*33^(1/2))*(EllipticF(1/2*
(-3+33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))-EllipticE(1/2*(-3+33^(
1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.26

$$\int \frac{1}{(24+36x^2-36x^4)^{3/2}}\,dx = \frac{3\left(\sqrt{33}\sqrt{6}(3\,x^4-3\,x^2-2)-3\,\sqrt{6}(3\,x^4-3\,x^2-2)\right)\sqrt{\sqrt{33}-3}E(\arcsin\left(\right.}{}$$

input
```
integrate(1/(-36*x^4+36*x^2+24)^(3/2),x, algorithm="fricas")
```

output
```
1/38016*(3*(sqrt(33)*sqrt(6)*(3*x^4 - 3*x^2 - 2) - 3*sqrt(6)*(3*x^4 - 3*x^
2 - 2))*sqrt(sqrt(33) - 3)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(33) - 3)), -1
/4*sqrt(33) - 7/4) + (sqrt(33)*sqrt(6)*(3*x^4 - 3*x^2 - 2) + 21*sqrt(6)*(3
*x^4 - 3*x^2 - 2))*sqrt(sqrt(33) - 3)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(33
) - 3)), -1/4*sqrt(33) - 7/4) + 12*sqrt(-36*x^4 + 36*x^2 + 24)*(3*x^3 - 7*
x))/(3*x^4 - 3*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(24+36x^2-36x^4)^{3/2}}\,dx = \frac{\sqrt{3}\int \frac{1}{-3x^4\sqrt{-3x^4+3x^2+2}+3x^2\sqrt{-3x^4+3x^2+2}+2\sqrt{-3x^4+3x^2+2}}\,dx}{72}$$

input
```
integrate(1/(-36*x**4+36*x**2+24)**(3/2),x)
```

output
```
sqrt(3)*Integral(1/(-3*x**4*sqrt(-3*x**4 + 3*x**2 + 2) + 3*x**2*sqrt(-3*x*
*4 + 3*x**2 + 2) + 2*sqrt(-3*x**4 + 3*x**2 + 2)), x)/72
```

Maxima [F]

$$\int \frac{1}{(24+36x^2-36x^4)^{3/2}}\,dx = \int \frac{1}{(-36\,x^4+36\,x^2+24)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-36*x^4+36*x^2+24)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-36*x^4 + 36*x^2 + 24)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(24+36x^2-36x^4)^{3/2}}\,dx = \int \frac{1}{(-36\,x^4+36\,x^2+24)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-36*x^4+36*x^2+24)^(3/2),x, algorithm="giac")
```

output
```
integrate((-36*x^4 + 36*x^2 + 24)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(24+36x^2-36x^4)^{3/2}}\,dx = \int \frac{1}{(-36\,x^4+36\,x^2+24)^{3/2}}\,dx$$

input
```
int(1/(36*x^2 - 36*x^4 + 24)^(3/2),x)
```

output
```
int(1/(36*x^2 - 36*x^4 + 24)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(24+36x^2-36x^4)^{3/2}}\,dx = \frac{\sqrt{3}\left(\int \frac{\sqrt{-3x^4+3x^2+2}}{9x^8-18x^6-3x^4+12x^2+4}dx\right)}{72}$$

input
```
int(1/(-36*x^4+36*x^2+24)^(3/2),x)
```

output
```
(sqrt(3)*int(sqrt( - 3*x**4 + 3*x**2 + 2)/(9*x**8 - 18*x**6 - 3*x**4 + 12*
x**2 + 4),x))/72
```

3.201 $\int \frac{1}{\left(3+\sqrt{33}-6x^2\right)^{3/2}\left(-3+\sqrt{33}+6x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 215

$$\int \frac{1}{\left(3+\sqrt{33}-6x^2\right)^{3/2}\left(-3+\sqrt{33}+6x^2\right)^{3/2}}\,dx = \frac{x}{6\left(11+\sqrt{33}\right)\sqrt{3+\sqrt{33}-6x^2}\sqrt{-3+\sqrt{33}+6x^2}} + \frac{1}{528}\sqrt{\frac{1}{6}\left(3+\sqrt{33}\right)}\sqrt{\frac{1}{-3+\sqrt{33}+6x^2}}\sqrt{-3+\sqrt{33}+6x^2}\,E\left(\arctan\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right)|\frac{1}{4}\left(11-\sqrt{33}\right)\right)$$

output
```
1/6*x/(11+33^(1/2))/(3+33^(1/2)-6*x^2)^(1/2)/(-3+33^(1/2)+6*x^2)^(1/2)+1/3
168*(18+6*33^(1/2))^(1/2)*(1/(-3+33^(1/2)+6*x^2))^(1/2)*(-3+33^(1/2)+6*x^2
)^(1/2)*EllipticE(6^(1/2)/(-3+33^(1/2))^(1/2)*x/(1+6/(-3+33^(1/2))*x^2)^(1
/2),1/2*(11-33^(1/2))^(1/2))+1/66*(1/(-3+33^(1/2)+6*x^2))^(1/2)*(-3+33^(1/
2)+6*x^2)^(1/2)*InverseJacobiAM(arctan(6^(1/2)/(-3+33^(1/2))^(1/2)*x),1/2*
(11-33^(1/2))^(1/2))/(18+6*33^(1/2))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(3+\sqrt{33}-6x^2\right)^{3/2}\left(-3+\sqrt{33}+6x^2\right)^{3/2}}\,dx = \frac{2\sqrt{3\left(3+\sqrt{33}\right)}x(7-3x^2)+3i\left(\sqrt{3}+\sqrt{11}\right)\sqrt{4+6x^2}}{}$$

input
```
Integrate[1/((3 + Sqrt[33] - 6*x^2)^(3/2)*(-3 + Sqrt[33] + 6*x^2)^(3/2)),x
]
```

output
```
(2*Sqrt[3*(3 + Sqrt[33])]*x*(7 - 3*x^2) + (3*I)*(Sqrt[3] + Sqrt[11])*Sqrt[
4 + 6*x^2 - 6*x^4]*EllipticE[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*x], (-7 + S
qrt[33])/4] - I*(11*Sqrt[3] + 3*Sqrt[11])*Sqrt[4 + 6*x^2 - 6*x^4]*Elliptic
F[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*x], (-7 + Sqrt[33])/4])/(3168*Sqrt[3 +
 Sqrt[33]]*Sqrt[2 + 3*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {316, 27, 402, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-6x^2+\sqrt{33}+3\right)^{3/2}\left(6x^2+\sqrt{33}-3\right)^{3/2}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{6\left(6x^2+\sqrt{33}+3\right)}{\sqrt{-6x^2+\sqrt{33}+3}\left(6x^2+\sqrt{33}-3\right)^{3/2}}dx}{36\left(11+\sqrt{33}\right)}+\frac{x}{6\left(11+\sqrt{33}\right)\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{6x^2+\sqrt{33}+3}{\sqrt{-6x^2+\sqrt{33}+3}\left(6x^2+\sqrt{33}-3\right)^{3/2}}dx}{6\left(11+\sqrt{33}\right)}+\frac{x}{6\left(11+\sqrt{33}\right)\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}$$

$\downarrow$ 402

$$\frac{\frac{x\sqrt{-6x^2+\sqrt{33}+3}}{\left(11-\sqrt{33}\right)\sqrt{6x^2+\sqrt{33}-3}}-\frac{\int -\frac{72\left(3x^2+4\right)}{\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}dx}{36\left(11-\sqrt{33}\right)}}{6\left(11+\sqrt{33}\right)}+\frac{x}{6\left(11+\sqrt{33}\right)\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}$$

$\downarrow$ 27

$$\frac{\frac{2\int \frac{3x^2+4}{\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}dx}{11-\sqrt{33}}+\frac{\sqrt{-6x^2+\sqrt{33}+3}x}{\left(11-\sqrt{33}\right)\sqrt{6x^2+\sqrt{33}-3}}}{6\left(11+\sqrt{33}\right)}+\frac{x}{6\left(11+\sqrt{33}\right)\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}$$

$\downarrow$ 399

$$\frac{\frac{2\left(\frac{1}{2}\left(11-\sqrt{33}\right)\int \frac{1}{\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}dx+\frac{1}{2}\int \frac{\sqrt{6x^2+\sqrt{33}-3}}{\sqrt{-6x^2+\sqrt{33}+3}}dx\right)}{11-\sqrt{33}}+\frac{\sqrt{-6x^2+\sqrt{33}+3}x}{\left(11-\sqrt{33}\right)\sqrt{6x^2+\sqrt{33}-3}}}{6\left(11+\sqrt{33}\right)}+\frac{x}{6\left(11+\sqrt{33}\right)\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}$$

$\downarrow$ 321

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{\sqrt{6x^2+\sqrt{33}-3}}{\sqrt{-6x^2+\sqrt{33}+3}}dx+\frac{\left(11-\sqrt{33}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)}{2\sqrt{6\left(\sqrt{33}-3\right)}}\right)}{11-\sqrt{33}}+\frac{\sqrt{-6x^2+\sqrt{33}+3}x}{\left(11-\sqrt{33}\right)\sqrt{6x^2+\sqrt{33}-3}}}{6\left(11+\sqrt{33}\right)}+\frac{x}{6\left(11+\sqrt{33}\right)\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}$$

$\downarrow$ 327

$$\frac{\frac{2\left(\frac{\left(11-\sqrt{33}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)}{2\sqrt{6\left(\sqrt{33}-3\right)}}+\frac{1}{2}\sqrt{\frac{1}{6}\left(\sqrt{33}-3\right)}E\left(\arcsin\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)\right)}{11-\sqrt{33}}+\frac{\sqrt{-6x^2+\sqrt{33}+3}x}{\left(11-\sqrt{33}\right)\sqrt{6x^2+\sqrt{33}-3}}}{6\left(11+\sqrt{33}\right)}$$

$$\frac{x}{6\left(11+\sqrt{33}\right)\sqrt{-6x^2+\sqrt{33}+3}\sqrt{6x^2+\sqrt{33}-3}}$$

input
```
Int[1/((3 + Sqrt[33] - 6*x^2)^(3/2)*(-3 + Sqrt[33] + 6*x^2)^(3/2)),x]
```

output
```
x/(6*(11 + Sqrt[33])*Sqrt[3 + Sqrt[33] - 6*x^2]*Sqrt[-3 + Sqrt[33] + 6*x^2
]) + ((x*Sqrt[3 + Sqrt[33] - 6*x^2])/((11 - Sqrt[33])*Sqrt[-3 + Sqrt[33] +
 6*x^2]) + (2*((Sqrt[(-3 + Sqrt[33])/6]*EllipticE[ArcSin[Sqrt[6/(3 + Sqrt[
33])]*x], (-7 - Sqrt[33])/4])/2 + ((11 - Sqrt[33])*EllipticF[ArcSin[Sqrt[6
/(3 + Sqrt[33])]*x], (-7 - Sqrt[33])/4])/(2*Sqrt[6*(-3 + Sqrt[33])])))/(11
 - Sqrt[33]))/(6*(11 + Sqrt[33]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.30

method	result
risch	$-\frac{x\left(3x^2-7\right)}{264\sqrt{3+\sqrt{33}-6x^2}\,\sqrt{-3+\sqrt{33}+6x^2}}+\frac{\left(\frac{\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3+\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{66\sqrt{-3+\sqrt{33}}\,\sqrt{-9x^4+9x^2+6}}-\frac{6\sqrt{1-\left(-\right.}}{}\right.}{}$
elliptic	$-\frac{12\left(3x^4-3x^2-2\right)\sqrt{\left(3+\sqrt{33}-6x^2\right)\left(-3+\sqrt{33}+6x^2\right)}\left(\frac{\frac{7}{528}x-\frac{1}{176}x^3}{\sqrt{-9x^4+9x^2+6}}+\frac{\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3+\sqrt{}}}{2}\right.}{66\sqrt{-3+\sqrt{33}}\,\sqrt{-9x^4+9x^2+6}}\right.}{\left(3+\sqrt{33}-6x^2\right)}$

input
```
int(1/(3+33^(1/2)-6*x^2)^(3/2)/(-3+33^(1/2)+6*x^2)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/264*x*(3*x^2-7)/(3+33^(1/2)-6*x^2)^(1/2)/(-3+33^(1/2)+6*x^2)^(1/2)+(1/6
6/(-3+33^(1/2))^(1/2)*(1-(-3/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*33^(1
/2))*x^2)^(1/2)/(-9*x^4+9*x^2+6)^(1/2)*EllipticF(1/2*(-3+33^(1/2))^(1/2)*x
,1/4*I*6^(1/2)+1/4*I*22^(1/2))-6/11/(-3+33^(1/2))^(1/2)*(1-(-3/4+1/4*33^(1
/2))*x^2)^(1/2)*(1-(-3/4-1/4*33^(1/2))*x^2)^(1/2)/(-9*x^4+9*x^2+6)^(1/2)/(
36+12*33^(1/2))*(EllipticF(1/2*(-3+33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*2
2^(1/2))-EllipticE(1/2*(-3+33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))
))*((3+33^(1/2)-6*x^2)*(-3+33^(1/2)+6*x^2))^(1/2)/(3+33^(1/2)-6*x^2)^(1/2)
/(-3+33^(1/2)+6*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(3+\sqrt{33}-6x^2\right)^{3/2}\left(-3+\sqrt{33}+6x^2\right)^{3/2}}\,dx = \frac{3\left(\sqrt{33}\sqrt{6}(3\,x^4-3\,x^2-2)-3\,\sqrt{6}(3\,x^4-3\,x^2-2)\right)\sqrt{\cdots}}{\cdots}$$

input
```
integrate(1/(3+33^(1/2)-6*x^2)^(3/2)/(-3+33^(1/2)+6*x^2)^(3/2),x, algorith
m="fricas")
```

output
```
1/38016*(3*(sqrt(33)*sqrt(6)*(3*x^4 - 3*x^2 - 2) - 3*sqrt(6)*(3*x^4 - 3*x^
2 - 2))*sqrt(sqrt(33) - 3)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(33) - 3)), -1
/4*sqrt(33) - 7/4) + (sqrt(33)*sqrt(6)*(3*x^4 - 3*x^2 - 2) + 21*sqrt(6)*(3
*x^4 - 3*x^2 - 2))*sqrt(sqrt(33) - 3)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(33
) - 3)), -1/4*sqrt(33) - 7/4) + 12*(3*x^3 - 7*x)*sqrt(6*x^2 + sqrt(33) - 3
)*sqrt(-6*x^2 + sqrt(33) + 3))/(3*x^4 - 3*x^2 - 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(3+\sqrt{33}-6x^2\right)^{3/2}\left(-3+\sqrt{33}+6x^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(3+33**(1/2)-6*x**2)**(3/2)/(-3+33**(1/2)+6*x**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(3+\sqrt{33}-6x^2\right)^{3/2}\left(-3+\sqrt{33}+6x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(6\,x^2+\sqrt{33}-3\right)^{\frac{3}{2}}\left(-6\,x^2+\sqrt{33}+3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3+33^(1/2)-6*x^2)^(3/2)/(-3+33^(1/2)+6*x^2)^(3/2),x, algorith
m="maxima")
```

output
```
integrate(1/((6*x^2 + sqrt(33) - 3)^(3/2)*(-6*x^2 + sqrt(33) + 3)^(3/2)),
x)
```

Giac [F]

$$\int \frac{1}{\left(3+\sqrt{33}-6x^2\right)^{3/2}\left(-3+\sqrt{33}+6x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(6\,x^2+\sqrt{33}-3\right)^{\frac{3}{2}}\left(-6\,x^2+\sqrt{33}+3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3+33^(1/2)-6*x^2)^(3/2)/(-3+33^(1/2)+6*x^2)^(3/2),x, algorith
m="giac")
```

output
```
integrate(1/((6*x^2 + sqrt(33) - 3)^(3/2)*(-6*x^2 + sqrt(33) + 3)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(3+\sqrt{33}-6x^2\right)^{3/2}\left(-3+\sqrt{33}+6x^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-6\,x^2+\sqrt{33}+3\right)^{3/2}\left(6\,x^2+\sqrt{33}-3\right)^{3/2}}\,dx$$

input
```
int(1/((33^(1/2) - 6*x^2 + 3)^(3/2)*(33^(1/2) + 6*x^2 - 3)^(3/2)),x)
```

output
```
int(1/((33^(1/2) - 6*x^2 + 3)^(3/2)*(33^(1/2) + 6*x^2 - 3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(3+\sqrt{33}-6x^2\right)^{3/2}\left(-3+\sqrt{33}+6x^2\right)^{3/2}}\,dx=\int \frac{1}{\left(3+\sqrt{33}-6x^2\right)^{\frac{3}{2}}\left(-3+\sqrt{33}+6x^2\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(3+33^(1/2)-6*x^2)^(3/2)/(-3+33^(1/2)+6*x^2)^(3/2),x)
```

output
```
int(1/(3+33^(1/2)-6*x^2)^(3/2)/(-3+33^(1/2)+6*x^2)^(3/2),x)
```

3.202 $\int \frac{1}{(-2+7x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 233

$$\int \frac{1}{(-2+7x^2+3x^4)^{3/2}}\,dx = -\frac{x(61+21x^2)}{146\sqrt{-2+7x^2+3x^4}}$$
$$+\frac{7\sqrt{7+\sqrt{73}}\sqrt{4-\left(7-\sqrt{73}\right)x^2}\sqrt{4-\left(7+\sqrt{73}\right)x^2}E\left(\arcsin\left(\frac{1}{2}\sqrt{7+\sqrt{73}}x\right)|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)}{584\sqrt{-2+7x^2+3x^4}}$$
$$-\frac{\sqrt{\frac{1}{73}\left(7+\sqrt{73}\right)}\sqrt{4-\left(7-\sqrt{73}\right)x^2}\sqrt{4-\left(7+\sqrt{73}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{7+\sqrt{73}}x\right),\frac{1}{12}\left(-61+7\sqrt{\right.\right.}}{8\sqrt{-2+7x^2+3x^4}}$$

output
```
-1/146*x*(21*x^2+61)/(3*x^4+7*x^2-2)^(1/2)+7/584*(7+73^(1/2))^(1/2)*(4-(-7
3^(1/2)+7)*x^2)^(1/2)*(4-(7+73^(1/2))*x^2)^(1/2)*EllipticE(1/2*(7+73^(1/2)
)^(1/2)*x,1/12*I*438^(1/2)-7/12*I*6^(1/2))/(3*x^4+7*x^2-2)^(1/2)-1/584*(51
1+73*73^(1/2))^(1/2)*(4-(-73^(1/2)+7)*x^2)^(1/2)*(4-(7+73^(1/2))*x^2)^(1/2
)*EllipticF(1/2*(7+73^(1/2))^(1/2)*x,1/12*I*438^(1/2)-7/12*I*6^(1/2))/(3*x
^4+7*x^2-2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.41 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.77

$$\int \frac{1}{(-2+7x^2+3x^4)^{3/2}}\,dx =$$

$$-\frac{2x(61+21x^2)-7i\sqrt{2\left(-7+\sqrt{73}\right)}\sqrt{2-7x^2-3x^4}E\left(i\text{arcsinh}\left(\sqrt{\frac{6}{7+\sqrt{73}}}x\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)+i\sqrt{\frac{}{-7}}}{292\sqrt{-2+7x^2+3}}$$

input
```
Integrate[(-2 + 7*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/292*(2*x*(61 + 21*x^2) - (7*I)*Sqrt[2*(-7 + Sqrt[73])]*Sqrt[2 - 7*x^2 -
 3*x^4]*EllipticE[I*ArcSinh[Sqrt[6/(7 + Sqrt[73])]*x], (-61 - 7*Sqrt[73])/
12] + I*Sqrt[2/(-7 + Sqrt[73])]*(-73 + 7*Sqrt[73])*Sqrt[2 - 7*x^2 - 3*x^4]
*EllipticF[I*ArcSinh[Sqrt[6/(7 + Sqrt[73])]*x], (-61 - 7*Sqrt[73])/12])/Sq
rt[-2 + 7*x^2 + 3*x^4]
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.60, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1405, 27, 1501, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+7x^2-2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{146}\int -\frac{3(4-7x^2)}{\sqrt{3x^4+7x^2-2}}dx-\frac{x(21x^2+61)}{146\sqrt{3x^4+7x^2-2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{146}\int \frac{4-7x^2}{\sqrt{3x^4+7x^2-2}}dx-\frac{x\left(21x^2+61\right)}{146\sqrt{3x^4+7x^2-2}}$$

$\Big\downarrow$ 1501

$$-\frac{3}{146}\left(\frac{1}{6}\left(73-7\sqrt{73}\right)\int \frac{1}{\sqrt{3x^4+7x^2-2}}dx-\frac{7}{6}\int \frac{6x^2-\sqrt{73}+7}{\sqrt{3x^4+7x^2-2}}dx\right)-\frac{x\left(21x^2+61\right)}{146\sqrt{3x^4+7x^2-2}}$$

$\Big\downarrow$ 1411

$$-\frac{3}{146}\left(\frac{\left(73-7\sqrt{73}\right)\sqrt{\frac{4-\left(7-\sqrt{73}\right)x^2}{4-\left(7+\sqrt{73}\right)x^2}}\sqrt{\left(7+\sqrt{73}\right)x^2-4}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{\left(7+\sqrt{73}\right)x^2-4}}\right),\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{12\sqrt{2}\sqrt[4]{73}\sqrt{\frac{1}{4-\left(7+\sqrt{73}\right)x^2}}\sqrt{3x^4+7x^2-2}}\right.$$

$$\frac{x\left(21x^2+61\right)}{146\sqrt{3x^4+7x^2-2}}$$

$\Big\downarrow$ 1498

$$-\frac{3}{146}\left(\frac{\left(73-7\sqrt{73}\right)\sqrt{\frac{4-\left(7-\sqrt{73}\right)x^2}{4-\left(7+\sqrt{73}\right)x^2}}\sqrt{\left(7+\sqrt{73}\right)x^2-4}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{\left(7+\sqrt{73}\right)x^2-4}}\right),\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{12\sqrt{2}\sqrt[4]{73}\sqrt{\frac{1}{4-\left(7+\sqrt{73}\right)x^2}}\sqrt{3x^4+7x^2-2}}\right.$$

$$\frac{x\left(21x^2+61\right)}{146\sqrt{3x^4+7x^2-2}}$$

input
```
Int[(-2 + 7*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/146*(x*(61 + 21*x^2))/Sqrt[-2 + 7*x^2 + 3*x^4] - (3*((-7*((x*(7 + Sqrt[
73] + 6*x^2))/Sqrt[-2 + 7*x^2 + 3*x^4] - (73^(1/4)*Sqrt[(4 - (7 - Sqrt[73]
)*x^2)/(4 - (7 + Sqrt[73])*x^2)]*Sqrt[-4 + (7 + Sqrt[73])*x^2]*EllipticE[A
rcSin[(Sqrt[2]*73^(1/4)*x)/Sqrt[-4 + (7 + Sqrt[73])*x^2]], (73 + 7*Sqrt[73
])/146])/(Sqrt[2]*Sqrt[(4 - (7 + Sqrt[73])*x^2)^(-1)]*Sqrt[-2 + 7*x^2 + 3*
x^4])))/6 + ((73 - 7*Sqrt[73])*Sqrt[(4 - (7 - Sqrt[73])*x^2)/(4 - (7 + Sqr
t[73])*x^2)]*Sqrt[-4 + (7 + Sqrt[73])*x^2]*EllipticF[ArcSin[(Sqrt[2]*73^(1
/4)*x)/Sqrt[-4 + (7 + Sqrt[73])*x^2]], (73 + 7*Sqrt[73])/146])/(12*Sqrt[2]
*73^(1/4)*Sqrt[(4 - (7 + Sqrt[73])*x^2)^(-1)]*Sqrt[-2 + 7*x^2 + 3*x^4])))/
146
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{x(21x^2+61)}{146\sqrt{3x^4+7x^2-2}}-\frac{12\sqrt{1-\left(\frac{7}{4}-\frac{\sqrt{73}}{4}\right)x^2}\,\sqrt{1-\left(\frac{7}{4}+\frac{\sqrt{73}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-\sqrt{73}+7}}{2},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{73\sqrt{-\sqrt{73}+7}\,\sqrt{3x^4+7x^2-2}}+\frac{84\sqrt{1-\left(\frac{7}{4}-\frac{\sqrt{73}}{4}\right)x^2}\ \cdots}{\cdots}$
default	$-\frac{6\left(\frac{61}{876}x+\frac{7}{292}x^3\right)}{\sqrt{3x^4+7x^2-2}}-\frac{12\sqrt{1-\left(\frac{7}{4}-\frac{\sqrt{73}}{4}\right)x^2}\,\sqrt{1-\left(\frac{7}{4}+\frac{\sqrt{73}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-\sqrt{73}+7}}{2},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{73\sqrt{-\sqrt{73}+7}\,\sqrt{3x^4+7x^2-2}}+\frac{84\sqrt{1-\left(\frac{7}{4}-\frac{\sqrt{73}}{4}\right)x^2}\ \cdots}{\cdots}$
elliptic	$-\frac{6\left(\frac{61}{876}x+\frac{7}{292}x^3\right)}{\sqrt{3x^4+7x^2-2}}-\frac{12\sqrt{1-\left(\frac{7}{4}-\frac{\sqrt{73}}{4}\right)x^2}\,\sqrt{1-\left(\frac{7}{4}+\frac{\sqrt{73}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-\sqrt{73}+7}}{2},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{73\sqrt{-\sqrt{73}+7}\,\sqrt{3x^4+7x^2-2}}+\frac{84\sqrt{1-\left(\frac{7}{4}-\frac{\sqrt{73}}{4}\right)x^2}\ \cdots}{\cdots}$

input
```
int(1/(3*x^4+7*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/146*x*(21*x^2+61)/(3*x^4+7*x^2-2)^(1/2)-12/73/(-73^(1/2)+7)^(1/2)*(1-(7
/4-1/4*73^(1/2))*x^2)^(1/2)*(1-(7/4+1/4*73^(1/2))*x^2)^(1/2)/(3*x^4+7*x^2-
2)^(1/2)*EllipticF(1/2*x*(-73^(1/2)+7)^(1/2),7/12*I*6^(1/2)+1/12*I*438^(1/
2))+84/73/(-73^(1/2)+7)^(1/2)*(1-(7/4-1/4*73^(1/2))*x^2)^(1/2)*(1-(7/4+1/4
*73^(1/2))*x^2)^(1/2)/(3*x^4+7*x^2-2)^(1/2)/(7+73^(1/2))*(EllipticF(1/2*x*
(-73^(1/2)+7)^(1/2),7/12*I*6^(1/2)+1/12*I*438^(1/2))-EllipticE(1/2*x*(-73^
(1/2)+7)^(1/2),7/12*I*6^(1/2)+1/12*I*438^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-2+7x^2+3x^4)^{3/2}}\,dx =$$
$$-\frac{7\left(\sqrt{73}\sqrt{-2}(3\,x^4+7\,x^2-2)+7\,\sqrt{-2}(3\,x^4+7\,x^2-2)\right)\sqrt{\sqrt{73}+7}E(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{73}+7}\right)\mid\frac{7}{12}\,\sqrt{73}-$$

input
```
integrate(1/(3*x^4+7*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
-1/1168*(7*(sqrt(73)*sqrt(-2)*(3*x^4 + 7*x^2 - 2) + 7*sqrt(-2)*(3*x^4 + 7*
x^2 - 2))*sqrt(sqrt(73) + 7)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(73) + 7)),
7/12*sqrt(73) - 61/12) - (11*sqrt(73)*sqrt(-2)*(3*x^4 + 7*x^2 - 2) + 21*sq
rt(-2)*(3*x^4 + 7*x^2 - 2))*sqrt(sqrt(73) + 7)*elliptic_f(arcsin(1/2*x*sqr
t(sqrt(73) + 7)), 7/12*sqrt(73) - 61/12) + 8*sqrt(3*x^4 + 7*x^2 - 2)*(21*x
^3 + 61*x))/(3*x^4 + 7*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(-2+7x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+7x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+7*x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 + 7*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2+7x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+7\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+7*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 7*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+7x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+7\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+7*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 7*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+7x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+7\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(7*x^2 + 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(7*x^2 + 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+7x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+7x^2-2}}{9x^8+42x^6+37x^4-28x^2+4}dx$$

input
```
int(1/(3*x^4+7*x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 7*x**2 - 2)/(9*x**8 + 42*x**6 + 37*x**4 - 28*x**2 + 4),x
)
```

3.203 $\int \frac{1}{\left(-2+6x^2+3x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 225

$$\int \frac{1}{\left(-2+6x^2+3x^4\right)^{3/2}}\,dx = -\frac{x(8+3x^2)}{20\sqrt{-2+6x^2+3x^4}} + \frac{\sqrt{\frac{3}{-3+\sqrt{15}}}\sqrt{2-\left(3-\sqrt{15}\right)x^2}\sqrt{2-\left(3+\sqrt{15}\right)x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{2}\left(3+\sqrt{15}\right)}x\right)\middle|-4+\sqrt{15}\right)}{20\sqrt{-2+6x^2+3x^4}} - \frac{\sqrt{2-\left(3-\sqrt{15}\right)x^2}\sqrt{2-\left(3+\sqrt{15}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(3+\sqrt{15}\right)}x\right),-4+\sqrt{15}\right)}{4\sqrt{5\left(-3+\sqrt{15}\right)}\sqrt{-2+6x^2+3x^4}}$$

output
```
-1/20*x*(3*x^2+8)/(3*x^4+6*x^2-2)^(1/2)+1/20*3^(1/2)/(-3+15^(1/2))^(1/2)*(
2-(3-15^(1/2))*x^2)^(1/2)*(2-(3+15^(1/2))*x^2)^(1/2)*EllipticE(1/2*(6+2*15
^(1/2))^(1/2)*x,1/2*I*10^(1/2)-1/2*I*6^(1/2))/(3*x^4+6*x^2-2)^(1/2)-1/4*(2
-(3-15^(1/2))*x^2)^(1/2)*(2-(3+15^(1/2))*x^2)^(1/2)*EllipticF(1/2*(6+2*15^
(1/2))^(1/2)*x,1/2*I*10^(1/2)-1/2*I*6^(1/2))/(-15+5*15^(1/2))^(1/2)/(3*x^4
+6*x^2-2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.85 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-2+6x^2+3x^4)^{3/2}}\,dx = \frac{-3x(8+3x^2)+3i\sqrt{-3+\sqrt{15}}\sqrt{2-6x^2-3x^4}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{3+\sqrt{15}}}x\right)|-4 \cdots\right.}{60\sqrt{-2+6x^2+\cdots}}$$

input
```
Integrate[(-2 + 6*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(-3*x*(8 + 3*x^2) + (3*I)*Sqrt[-3 + Sqrt[15]]*Sqrt[2 - 6*x^2 - 3*x^4]*Elli
pticE[I*ArcSinh[Sqrt[3/(3 + Sqrt[15])]*x], -4 - Sqrt[15]] - ((3*I)*(-5 + S
qrt[15])*Sqrt[2 - 6*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(3 + Sqrt[15])
]*x], -4 - Sqrt[15]])/Sqrt[-3 + Sqrt[15]])/(60*Sqrt[-2 + 6*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.57, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1405, 27, 1501, 27, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+6x^2-2)^{3/2}}dx$$

$$\Big\downarrow \ 1405$$

$$\frac{1}{120}\int -\frac{6(2-3x^2)}{\sqrt{3x^4+6x^2-2}}dx-\frac{x(3x^2+8)}{20\sqrt{3x^4+6x^2-2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{20}\int \frac{2-3x^2}{\sqrt{3x^4+6x^2-2}}dx-\frac{x(3x^2+8)}{20\sqrt{3x^4+6x^2-2}}$$

$$\Big\downarrow \ 1501$$

$$\frac{1}{20}\left(\frac{1}{2}\int \frac{2\left(3x^2-\sqrt{15}+3\right)}{\sqrt{3x^4+6x^2-2}}dx-\left(5-\sqrt{15}\right)\int \frac{1}{\sqrt{3x^4+6x^2-2}}dx\right)-\frac{x\left(3x^2+8\right)}{20\sqrt{3x^4+6x^2-2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{20}\left(\int \frac{3x^2-\sqrt{15}+3}{\sqrt{3x^4+6x^2-2}}dx-\left(5-\sqrt{15}\right)\int \frac{1}{\sqrt{3x^4+6x^2-2}}dx\right)-\frac{x\left(3x^2+8\right)}{20\sqrt{3x^4+6x^2-2}}$$

$$\Big\downarrow \; 1411$$

$$\frac{1}{20}\left(\int \frac{3x^2-\sqrt{15}+3}{\sqrt{3x^4+6x^2-2}}dx-\frac{\left(5-\sqrt{15}\right)\sqrt{\frac{2-\left(3-\sqrt{15}\right)x^2}{2-\left(3+\sqrt{15}\right)x^2}}\sqrt{\left(3+\sqrt{15}\right)x^2-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{15}x}{\sqrt{\left(3+\sqrt{15}\right)x^2-2}}\right)\cdots\right.}{2\sqrt[4]{15}\sqrt{\frac{1}{2-\left(3+\sqrt{15}\right)x^2}}\sqrt{3x^4+6x^2-2}}\cdots\right.$$

$$\frac{x\left(3x^2+8\right)}{20\sqrt{3x^4+6x^2-2}}$$

$$\Big\downarrow \; 1498$$

$$\frac{1}{20}\left(-\frac{\left(5-\sqrt{15}\right)\sqrt{\frac{2-\left(3-\sqrt{15}\right)x^2}{2-\left(3+\sqrt{15}\right)x^2}}\sqrt{\left(3+\sqrt{15}\right)x^2-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{15}x}{\sqrt{\left(3+\sqrt{15}\right)x^2-2}}\right),\frac{1}{10}\left(5+\sqrt{15}\right)\right)}{2\sqrt[4]{15}\sqrt{\frac{1}{2-\left(3+\sqrt{15}\right)x^2}}\sqrt{3x^4+6x^2-2}}-\frac{\sqrt[4]{1}\cdots}{\cdots}\right.$$

$$\frac{x\left(3x^2+8\right)}{20\sqrt{3x^4+6x^2-2}}$$

input

```
Int[(-2 + 6*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/20*(x*(8 + 3*x^2))/Sqrt[-2 + 6*x^2 + 3*x^4] + ((x*(3 + Sqrt[15] + 3*x^2
))/Sqrt[-2 + 6*x^2 + 3*x^4] - (15^(1/4)*Sqrt[(2 - (3 - Sqrt[15])*x^2)/(2 -
 (3 + Sqrt[15])*x^2)]*Sqrt[-2 + (3 + Sqrt[15])*x^2]*EllipticE[ArcSin[(Sqrt
[2]*15^(1/4)*x)/Sqrt[-2 + (3 + Sqrt[15])*x^2]], (5 + Sqrt[15])/10])/(Sqrt[
(2 - (3 + Sqrt[15])*x^2)^(-1)]*Sqrt[-2 + 6*x^2 + 3*x^4]) - ((5 - Sqrt[15])
*Sqrt[(2 - (3 - Sqrt[15])*x^2)/(2 - (3 + Sqrt[15])*x^2)]*Sqrt[-2 + (3 + Sq
rt[15])*x^2]*EllipticF[ArcSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[-2 + (3 + Sqrt[15]
)*x^2]], (5 + Sqrt[15])/10])/(2*15^(1/4)*Sqrt[(2 - (3 + Sqrt[15])*x^2)^(-1
)]*Sqrt[-2 + 6*x^2 + 3*x^4]))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{x(3x^2+8)}{20\sqrt{3x^4+6x^2-2}} - \frac{\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x^2}\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{15}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6-2\sqrt{15}}}{2}, \frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{5\sqrt{6-2\sqrt{15}}\sqrt{3x^4+6x^2-2}} + \frac{6\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x^2}\sqrt{1-\left(\frac{3}{2}\right.}}{}$
default	$-\frac{6\left(\frac{1}{15}x+\frac{1}{40}x^3\right)}{\sqrt{3x^4+6x^2-2}} - \frac{\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x^2}\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{15}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6-2\sqrt{15}}}{2}, \frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{5\sqrt{6-2\sqrt{15}}\sqrt{3x^4+6x^2-2}} + \frac{6\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x^2}\sqrt{1-\left(\frac{3}{2}+\right.}}{}$
elliptic	$-\frac{6\left(\frac{1}{15}x+\frac{1}{40}x^3\right)}{\sqrt{3x^4+6x^2-2}} - \frac{\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x^2}\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{15}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6-2\sqrt{15}}}{2}, \frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{5\sqrt{6-2\sqrt{15}}\sqrt{3x^4+6x^2-2}} + \frac{6\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x^2}\sqrt{1-\left(\frac{3}{2}+\right.}}{}$

input
```
int(1/(3*x^4+6*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*x*(3*x^2+8)/(3*x^4+6*x^2-2)^(1/2)-1/5/(6-2*15^(1/2))^(1/2)*(1-(3/2-1
/2*15^(1/2))*x^2)^(1/2)*(1-(3/2+1/2*15^(1/2))*x^2)^(1/2)/(3*x^4+6*x^2-2)^(
1/2)*EllipticF(1/2*x*(6-2*15^(1/2))^(1/2),1/2*I*6^(1/2)+1/2*I*10^(1/2))+6/
5/(6-2*15^(1/2))^(1/2)*(1-(3/2-1/2*15^(1/2))*x^2)^(1/2)*(1-(3/2+1/2*15^(1/
2))*x^2)^(1/2)/(3*x^4+6*x^2-2)^(1/2)/(6+2*15^(1/2))*(EllipticF(1/2*x*(6-2*
15^(1/2))^(1/2),1/2*I*6^(1/2)+1/2*I*10^(1/2))-EllipticE(1/2*x*(6-2*15^(1/2
))^(1/2),1/2*I*6^(1/2)+1/2*I*10^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.78

$$\int \frac{1}{(-2+6x^2+3x^4)^{3/2}}\,dx =$$
$$-\frac{3\left(\sqrt{15}\sqrt{-2}(3\,x^4+6\,x^2-2)+3\,\sqrt{-2}(3\,x^4+6\,x^2-2)\right)\sqrt{\frac{1}{2}\,\sqrt{15}+\frac{3}{2}}E(\arcsin\left(x\sqrt{\frac{1}{2}\,\sqrt{15}+\frac{3}{2}}\right)\mid\sqrt{15}-$$

input
```
integrate(1/(3*x^4+6*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
-1/120*(3*(sqrt(15)*sqrt(-2)*(3*x^4 + 6*x^2 - 2) + 3*sqrt(-2)*(3*x^4 + 6*x
^2 - 2))*sqrt(1/2*sqrt(15) + 3/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(15) +
3/2)), sqrt(15) - 4) - (5*sqrt(15)*sqrt(-2)*(3*x^4 + 6*x^2 - 2) + 3*sqrt(-
2)*(3*x^4 + 6*x^2 - 2))*sqrt(1/2*sqrt(15) + 3/2)*elliptic_f(arcsin(x*sqrt(
1/2*sqrt(15) + 3/2)), sqrt(15) - 4) + 6*sqrt(3*x^4 + 6*x^2 - 2)*(3*x^3 + 8
*x))/(3*x^4 + 6*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(-2+6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+6x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+6*x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 + 6*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2+6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+6\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+6*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 6*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+6\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+6*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 6*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+6\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(6*x^2 + 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(6*x^2 + 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+6x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+6x^2-2}}{9x^8+36x^6+24x^4-24x^2+4}dx$$

input
```
int(1/(3*x^4+6*x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 6*x**2 - 2)/(9*x**8 + 36*x**6 + 24*x**4 - 24*x**2 + 4),x
)
```

3.204 $\int \frac{1}{(-2+5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 147

$$\int \frac{1}{(-2+5x^2+3x^4)^{3/2}}\,dx = -\frac{x(37+15x^2)}{98\sqrt{-2+5x^2+3x^4}} + \frac{5\sqrt{\frac{3}{2}}\sqrt{1-3x^2}\sqrt{2+x^2}E\left(\arcsin\left(\sqrt{3}x\right)\middle|-\frac{1}{6}\right)}{49\sqrt{-2+5x^2+3x^4}} - \frac{\sqrt{\frac{3}{2}}\sqrt{1-3x^2}\sqrt{2+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{3}x\right),-\frac{1}{6}\right)}{7\sqrt{-2+5x^2+3x^4}}$$

output

```
-1/98*x*(15*x^2+37)/(3*x^4+5*x^2-2)^(1/2)+5/98*6^(1/2)*(-3*x^2+1)^(1/2)*(x
^2+2)^(1/2)*EllipticE(x*3^(1/2),1/6*I*6^(1/2))/(3*x^4+5*x^2-2)^(1/2)-1/14*
(-3*x^2+1)^(1/2)*(x^2+2)^(1/2)*EllipticF(x*3^(1/2),1/6*I*6^(1/2))*6^(1/2)/
(3*x^4+5*x^2-2)^(1/2)
```

Mathematica [A] (verified)

Time = 6.74 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

$$\int \frac{1}{(-2+5x^2+3x^4)^{3/2}}\,dx = \frac{-37x-15x^3+5\sqrt{6-18x^2}\sqrt{2+x^2}E\left(\arcsin\left(\sqrt{3}x\right)|-\frac{1}{6}\right)-7\sqrt{6-18x^2}\sqrt{2}}{98\sqrt{-2+5x^2+3x^4}}$$

input
```
Integrate[(-2 + 5*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(-37*x - 15*x^3 + 5*Sqrt[6 - 18*x^2]*Sqrt[2 + x^2]*EllipticE[ArcSin[Sqrt[3
]*x], -1/6] - 7*Sqrt[6 - 18*x^2]*Sqrt[2 + x^2]*EllipticF[ArcSin[Sqrt[3]*x]
, -1/6])/(98*Sqrt[-2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.52, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1410, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+5x^2-2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{98}\int -\frac{3(4-5x^2)}{\sqrt{3x^4+5x^2-2}}dx-\frac{x(15x^2+37)}{98\sqrt{3x^4+5x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{98}\int \frac{4-5x^2}{\sqrt{3x^4+5x^2-2}}dx-\frac{x(15x^2+37)}{98\sqrt{3x^4+5x^2-2}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{3}{98}\left(\frac{7}{3}\int \frac{1}{\sqrt{3x^4+5x^2-2}}dx-\frac{5}{6}\int -\frac{2(1-3x^2)}{\sqrt{3x^4+5x^2-2}}dx\right)-\frac{x(15x^2+37)}{98\sqrt{3x^4+5x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{98}\left(\frac{7}{3}\int \frac{1}{\sqrt{3x^4+5x^2-2}}dx+\frac{5}{3}\int \frac{1-3x^2}{\sqrt{3x^4+5x^2-2}}dx\right)-\frac{x(15x^2+37)}{98\sqrt{3x^4+5x^2-2}}$$

$\Big\downarrow$ 1410

$$-\frac{3}{98}\left(\frac{5}{3}\int \frac{1-3x^2}{\sqrt{3x^4+5x^2-2}}dx+\frac{\sqrt{7}\sqrt{x^2+2}\sqrt{3x^2-1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{7}{2}}x}{\sqrt{3x^2-1}}\right),\frac{6}{7}\right)}{3\sqrt{3x^4+5x^2-2}}\right)-\frac{x(15x^2+37)}{98\sqrt{3x^4+5x^2-2}}$$

$\Big\downarrow$ 1498

$$-\frac{3}{98}\left(\frac{\sqrt{7}\sqrt{x^2+2}\sqrt{3x^2-1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{7}{2}}x}{\sqrt{3x^2-1}}\right),\frac{6}{7}\right)}{3\sqrt{3x^4+5x^2-2}}+\frac{5}{3}\left(\frac{\sqrt{7}\sqrt{\frac{x^2+2}{1-3x^2}}\sqrt{3x^2-1}E\left(\arcsin\left(\frac{\sqrt{\frac{7}{2}}x}{\sqrt{3x^2-1}}\right)|\right.}{\sqrt{\frac{1}{1-3x^2}}\sqrt{3x^4+5x^2-2}}\right.\right.$$

$$\frac{x(15x^2+37)}{98\sqrt{3x^4+5x^2-2}}$$

input
```
Int[(-2 + 5*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/98*(x*(37 + 15*x^2))/Sqrt[-2 + 5*x^2 + 3*x^4] - (3*((5*((-3*x*(2 + x^2)
)/Sqrt[-2 + 5*x^2 + 3*x^4] + (Sqrt[7]*Sqrt[(2 + x^2)/(1 - 3*x^2)]*Sqrt[-1
+ 3*x^2]*EllipticE[ArcSin[(Sqrt[7/2]*x)/Sqrt[-1 + 3*x^2]], 6/7])/(Sqrt[(1
- 3*x^2)^(-1)]*Sqrt[-2 + 5*x^2 + 3*x^4])))/3 + (Sqrt[7]*Sqrt[2 + x^2]*Sqrt
[-1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7/2]*x)/Sqrt[-1 + 3*x^2]], 6/7])/(3*Sq
rt[-2 + 5*x^2 + 3*x^4])))/98
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{x(15x^2+37)}{98\sqrt{3x^4+5x^2-2}}+\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-3x^2+1}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},i\sqrt{6}\right)}{49\sqrt{3x^4+5x^2-2}}-\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-3x^2+1}\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},i\sqrt{6}\right)-\mathrm{Elli}\right.}{196\sqrt{3x^4+5x^2-2}}$
default	$-\frac{6\left(\frac{37}{588}x+\frac{5}{196}x^3\right)}{\sqrt{3x^4+5x^2-2}}+\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-3x^2+1}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},i\sqrt{6}\right)}{49\sqrt{3x^4+5x^2-2}}-\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-3x^2+1}\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},i\sqrt{6}\right)-\mathrm{Elli}\right.}{196\sqrt{3x^4+5x^2-2}}$
elliptic	$-\frac{6\left(\frac{37}{588}x+\frac{5}{196}x^3\right)}{\sqrt{3x^4+5x^2-2}}+\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-3x^2+1}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},i\sqrt{6}\right)}{49\sqrt{3x^4+5x^2-2}}-\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-3x^2+1}\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},i\sqrt{6}\right)-\mathrm{Elli}\right.}{196\sqrt{3x^4+5x^2-2}}$

input
```
int(1/(3*x^4+5*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/98*x*(15*x^2+37)/(3*x^4+5*x^2-2)^(1/2)+3/49*I*2^(1/2)*(2*x^2+4)^(1/2)*(
-3*x^2+1)^(1/2)/(3*x^4+5*x^2-2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),I*6^(1/2))
-5/196*I*2^(1/2)*(2*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)/(3*x^4+5*x^2-2)^(1/2)*(E
llipticF(1/2*I*x*2^(1/2),I*6^(1/2))-EllipticE(1/2*I*x*2^(1/2),I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

$$\int \frac{1}{(-2+5x^2+3x^4)^{3/2}}\,dx =$$
$$-\frac{15\sqrt{3}\sqrt{-2}(3x^4+5x^2-2)E\left(\arcsin\left(\sqrt{3}x\right)\,|-\frac{1}{6}\right)-17\sqrt{3}\sqrt{-2}(3x^4+5x^2-2)F\left(\arcsin\left(\sqrt{3}x\right)\,|-\frac{1}{6}\right)+}{98\,(3x^4+5x^2-2)}$$

input
```
integrate(1/(3*x^4+5*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
-1/98*(15*sqrt(3)*sqrt(-2)*(3*x^4 + 5*x^2 - 2)*elliptic_e(arcsin(sqrt(3)*x
), -1/6) - 17*sqrt(3)*sqrt(-2)*(3*x^4 + 5*x^2 - 2)*elliptic_f(arcsin(sqrt(
3)*x), -1/6) + sqrt(3*x^4 + 5*x^2 - 2)*(15*x^3 + 37*x))/(3*x^4 + 5*x^2 - 2
)
```

Sympy [F]

$$\int \frac{1}{(-2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+5x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+5*x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 + 5*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+5\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+5*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 5*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+5\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+5*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 5*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{1}{{\left(3\,x^4+5\,x^2-2\right)}^{3/2}}\,dx$$

input
```
int(1/(5*x^2 + 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(5*x^2 + 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{\sqrt{3x^4+5x^2-2}}{9x^8+30x^6+13x^4-20x^2+4}dx$$

input
```
int(1/(3*x^4+5*x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 5*x**2 - 2)/(9*x**8 + 30*x**6 + 13*x**4 - 20*x**2 + 4),x
)
```

3.205 $\int \frac{1}{(-2+4x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 239

$$\int \frac{1}{(-2+4x^2+3x^4)^{3/2}}\,dx = -\frac{x(7+3x^2)}{20\sqrt{-2+4x^2+3x^4}} + \frac{\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}\sqrt{2-\left(2-\sqrt{10}\right)x^2}\sqrt{2-\left(2+\sqrt{10}\right)x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{20\sqrt{-2+4x^2+3x^4}} - \frac{\sqrt{\frac{1}{5}\left(2+\sqrt{10}\right)}\sqrt{2-\left(2-\sqrt{10}\right)x^2}\sqrt{2-\left(2+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right),\frac{1}{3}(-7+2}{8\sqrt{-2+4x^2+3x^4}}$$

output
```
-1/20*x*(3*x^2+7)/(3*x^4+4*x^2-2)^(1/2)+1/40*(4+2*10^(1/2))^(1/2)*(2-(2-10
^(1/2))*x^2)^(1/2)*(2-(2+10^(1/2))*x^2)^(1/2)*EllipticE(1/2*(4+2*10^(1/2))
^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))/(3*x^4+4*x^2-2)^(1/2)-1/40*(10+5*10
^(1/2))^(1/2)*(2-(2-10^(1/2))*x^2)^(1/2)*(2-(2+10^(1/2))*x^2)^(1/2)*Ellipt
icF(1/2*(4+2*10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))/(3*x^4+4*x^2-
2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.87 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.70

$$\int \frac{1}{(-2+4x^2+3x^4)^{3/2}}\,dx = \frac{-3x(7+3x^2)+3i\sqrt{-2+\sqrt{10}}\sqrt{2-4x^2-3x^4}E\left(i\text{arcsinh}\left(\sqrt{-1+\sqrt{\frac{5}{2}}}x\right)\right.}{60\sqrt{-2}}$$

input
```
Integrate[(-2 + 4*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(-3*x*(7 + 3*x^2) + (3*I)*Sqrt[-2 + Sqrt[10]]*Sqrt[2 - 4*x^2 - 3*x^4]*Elli
pticE[I*ArcSinh[Sqrt[-1 + Sqrt[5/2]]*x], (-7 - 2*Sqrt[10])/3] - ((3*I)*(-5
 + Sqrt[10])*Sqrt[2 - 4*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/
2]]*x], (-7 - 2*Sqrt[10])/3])/Sqrt[-2 + Sqrt[10]])/(60*Sqrt[-2 + 4*x^2 + 3
*x^4])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.50, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+4x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{80}\int -\frac{12(1-x^2)}{\sqrt{3x^4+4x^2-2}}dx - \frac{x(3x^2+7)}{20\sqrt{3x^4+4x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{20}\int \frac{1-x^2}{\sqrt{3x^4+4x^2-2}}dx - \frac{x(3x^2+7)}{20\sqrt{3x^4+4x^2-2}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{3}{20}\left(\frac{1}{3}\left(5-\sqrt{10}\right)\int\frac{1}{\sqrt{3x^4+4x^2-2}}dx-\frac{1}{6}\int\frac{2\left(3x^2-\sqrt{10}+2\right)}{\sqrt{3x^4+4x^2-2}}dx\right)-\frac{x\left(3x^2+7\right)}{20\sqrt{3x^4+4x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{20}\left(\frac{1}{3}\left(5-\sqrt{10}\right)\int\frac{1}{\sqrt{3x^4+4x^2-2}}dx-\frac{1}{3}\int\frac{3x^2-\sqrt{10}+2}{\sqrt{3x^4+4x^2-2}}dx\right)-\frac{x\left(3x^2+7\right)}{20\sqrt{3x^4+4x^2-2}}$$

$$\Big\downarrow \text{1411}$$

$$-\frac{3}{20}\left(\frac{\left(5-\sqrt{10}\right)\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-2}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{\left(2+\sqrt{10}\right)x^2-2}}\right),\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{6\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{3x^4+4x^2-2}}-\frac{1}{3}\int\right.$$

$$\frac{x\left(3x^2+7\right)}{20\sqrt{3x^4+4x^2-2}}$$

$$\Big\downarrow \text{1498}$$

$$-\frac{3}{20}\left(\frac{\left(5-\sqrt{10}\right)\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-2}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{\left(2+\sqrt{10}\right)x^2-2}}\right),\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{6\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{3x^4+4x^2-2}}+\frac{1}{3}\left(\right.\right.$$

$$\frac{x\left(3x^2+7\right)}{20\sqrt{3x^4+4x^2-2}}$$

input
```
Int[(-2 + 4*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/20*(x*(7 + 3*x^2))/Sqrt[-2 + 4*x^2 + 3*x^4] - (3*((-((x*(2 + Sqrt[10] +
 3*x^2))/Sqrt[-2 + 4*x^2 + 3*x^4]) + (10^(1/4)*Sqrt[(2 - (2 - Sqrt[10])*x^
2)/(2 - (2 + Sqrt[10])*x^2)]*Sqrt[-2 + (2 + Sqrt[10])*x^2]*EllipticE[ArcSi
n[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 + (2 + Sqrt[10])*x^2]], (5 + Sqrt[10])/10])/
(Sqrt[(2 - (2 + Sqrt[10])*x^2)^(-1)]*Sqrt[-2 + 4*x^2 + 3*x^4]))/3 + ((5 -
Sqrt[10])*Sqrt[(2 - (2 - Sqrt[10])*x^2)/(2 - (2 + Sqrt[10])*x^2)]*Sqrt[-2
+ (2 + Sqrt[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 + (2 +
Sqrt[10])*x^2]], (5 + Sqrt[10])/10])/(6*10^(1/4)*Sqrt[(2 - (2 + Sqrt[10])*
x^2)^(-1)]*Sqrt[-2 + 4*x^2 + 3*x^4])))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{x(3x^2+7)}{20\sqrt{3x^4+4x^2-2}}-\frac{3\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{10\sqrt{4-2\sqrt{10}}\sqrt{3x^4+4x^2-2}}+\frac{6\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1-(1}}{}$
default	$-\frac{6\left(\frac{7}{120}x+\frac{1}{40}x^3\right)}{\sqrt{3x^4+4x^2-2}}-\frac{3\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{10\sqrt{4-2\sqrt{10}}\sqrt{3x^4+4x^2-2}}+\frac{6\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1-(1+}}{}$
elliptic	$-\frac{6\left(\frac{7}{120}x+\frac{1}{40}x^3\right)}{\sqrt{3x^4+4x^2-2}}-\frac{3\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{10\sqrt{4-2\sqrt{10}}\sqrt{3x^4+4x^2-2}}+\frac{6\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1-(1+}}{}$

input

```
int(1/(3*x^4+4*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/20*x*(3*x^2+7)/(3*x^4+4*x^2-2)^(1/2)-3/10/(4-2*10^(1/2))^(1/2)*(1-(1-1/
2*10^(1/2))*x^2)^(1/2)*(1-(1+1/2*10^(1/2))*x^2)^(1/2)/(3*x^4+4*x^2-2)^(1/2
)*EllipticF(1/2*(4-2*10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))+6/5/(
4-2*10^(1/2))^(1/2)*(1-(1-1/2*10^(1/2))*x^2)^(1/2)*(1-(1+1/2*10^(1/2))*x^2
)^(1/2)/(3*x^4+4*x^2-2)^(1/2)/(4+2*10^(1/2))*(EllipticF(1/2*(4-2*10^(1/2))
^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))-EllipticE(1/2*(4-2*10^(1/2))^(1/2)*
x,1/3*I*6^(1/2)+1/3*I*15^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.67

$$\int \frac{1}{(-2+4x^2+3x^4)^{3/2}}\,dx = \frac{2\sqrt{10}\sqrt{-2}(3\,x^4+4\,x^2-2)\sqrt{\frac{1}{2}\sqrt{10}+1}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{10}+1}\right)\mid\frac{2}{3}\sqrt{10}-}{}$$

input
```
integrate(1/(3*x^4+4*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/40*(2*sqrt(10)*sqrt(-2)*(3*x^4 + 4*x^2 - 2)*sqrt(1/2*sqrt(10) + 1)*ellip
tic_f(arcsin(x*sqrt(1/2*sqrt(10) + 1)), 2/3*sqrt(10) - 7/3) - (sqrt(10)*sq
rt(-2)*(3*x^4 + 4*x^2 - 2) + 2*sqrt(-2)*(3*x^4 + 4*x^2 - 2))*sqrt(1/2*sqrt
(10) + 1)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(10) + 1)), 2/3*sqrt(10) - 7/3)
 - 2*sqrt(3*x^4 + 4*x^2 - 2)*(3*x^3 + 7*x))/(3*x^4 + 4*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(-2+4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+4x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+4*x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 + 4*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2+4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+4\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+4*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 4*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+4\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+4*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 4*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+4\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(4*x^2 + 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(4*x^2 + 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+4x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+4x^2-2}}{9x^8+24x^6+4x^4-16x^2+4}dx$$

input
```
int(1/(3*x^4+4*x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 4*x**2 - 2)/(9*x**8 + 24*x**6 + 4*x**4 - 16*x**2 + 4),x)
```

3.206 $\int \frac{1}{(-2+3x^2+3x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 229

$$\int \frac{1}{(-2+3x^2+3x^4)^{3/2}} \, dx = -\frac{x(7+3x^2)}{22\sqrt{-2+3x^2+3x^4}} + \frac{\sqrt{3+\sqrt{33}}\sqrt{4-\left(3-\sqrt{33}\right)x^2}\sqrt{4-\left(3+\sqrt{33}\right)x^2}E\left(\arcsin\left(\frac{1}{2}\sqrt{3+\sqrt{33}}x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{88\sqrt{-2+3x^2+3x^4}} - \frac{\sqrt{\frac{1}{33}\left(3+\sqrt{33}\right)}\sqrt{4-\left(3-\sqrt{33}\right)x^2}\sqrt{4-\left(3+\sqrt{33}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{3+\sqrt{33}}x\right),\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{8\sqrt{-2+3x^2+3x^4}}$$

output
```
-1/22*x*(3*x^2+7)/(3*x^4+3*x^2-2)^(1/2)+1/88*(3+33^(1/2))^(1/2)*(4-(3-33^(
1/2))*x^2)^(1/2)*(4-(3+33^(1/2))*x^2)^(1/2)*EllipticE(1/2*(3+33^(1/2))^(1/
2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))/(3*x^4+3*x^2-2)^(1/2)-1/264*(99+33*33^(
1/2))^(1/2)*(4-(3-33^(1/2))*x^2)^(1/2)*(4-(3+33^(1/2))*x^2)^(1/2)*Elliptic
F(1/2*(3+33^(1/2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))/(3*x^4+3*x^2-2)^(
1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-2+3x^2+3x^4)^{3/2}}\,dx = \frac{-12x(7+3x^2)+6i\sqrt{-3+\sqrt{33}}\sqrt{4-6x^2-6x^4}E\left(i\text{arcsinh}\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right)|-\frac{7}{4}\right.}{264\sqrt{-2+3x^2}}$$

input
```
Integrate[(-2 + 3*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(-12*x*(7 + 3*x^2) + (6*I)*Sqrt[-3 + Sqrt[33]]*Sqrt[4 - 6*x^2 - 6*x^4]*Ell
ipticE[I*ArcSinh[Sqrt[6/(3 + Sqrt[33])]*x], -7/4 - Sqrt[33]/4] - ((6*I)*(-
11 + Sqrt[33])*Sqrt[4 - 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(3 + Sqr
t[33])]*x], -7/4 - Sqrt[33]/4])/Sqrt[-3 + Sqrt[33]])/(264*Sqrt[-2 + 3*x^2
+ 3*x^4])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.61, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1405, 27, 1501, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+3x^2-2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{66}\int -\frac{3(4-3x^2)}{\sqrt{3x^4+3x^2-2}}dx-\frac{x(3x^2+7)}{22\sqrt{3x^4+3x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{22}\int \frac{4-3x^2}{\sqrt{3x^4+3x^2-2}}dx-\frac{x(3x^2+7)}{22\sqrt{3x^4+3x^2-2}}$$

$$\Big\downarrow \text{1501}$$

$$\frac{1}{22}\left(\frac{1}{2}\int \frac{6x^2-\sqrt{33}+3}{\sqrt{3x^4+3x^2-2}}dx-\frac{1}{2}\left(11-\sqrt{33}\right)\int \frac{1}{\sqrt{3x^4+3x^2-2}}dx\right)-\frac{x\left(3x^2+7\right)}{22\sqrt{3x^4+3x^2-2}}$$

$$\Big\downarrow \text{1411}$$

$$\frac{1}{22}\left(\frac{1}{2}\int \frac{6x^2-\sqrt{33}+3}{\sqrt{3x^4+3x^2-2}}dx-\frac{\left(11-\sqrt{33}\right)\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-4}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{\left(3+\sqrt{33}\right)x^2-}}\right.\right.}{4\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{3x^4+3x^2-2}}\right.$$

$$\frac{x\left(3x^2+7\right)}{22\sqrt{3x^4+3x^2-2}}$$

$$\Big\downarrow \text{1498}$$

$$\frac{1}{22}\left(\frac{1}{2}\left(\frac{x\left(6x^2+\sqrt{33}+3\right)}{\sqrt{3x^4+3x^2-2}}-\frac{\sqrt[4]{33}\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-4}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{\left(3+\sqrt{33}\right)x^2-4}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right.\right.}{\sqrt{2}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{3x^4+3x^2-2}}\right.\right.$$

$$\frac{x\left(3x^2+7\right)}{22\sqrt{3x^4+3x^2-2}}$$

input
```
Int[(-2 + 3*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/22*(x*(7 + 3*x^2))/Sqrt[-2 + 3*x^2 + 3*x^4] + (((x*(3 + Sqrt[33] + 6*x^
2))/Sqrt[-2 + 3*x^2 + 3*x^4] - (33^(1/4)*Sqrt[(4 - (3 - Sqrt[33])*x^2)/(4
- (3 + Sqrt[33])*x^2)]*Sqrt[-4 + (3 + Sqrt[33])*x^2]*EllipticE[ArcSin[(Sqr
t[2]*33^(1/4)*x)/Sqrt[-4 + (3 + Sqrt[33])*x^2]], (11 + Sqrt[33])/22])/(Sqr
t[2]*Sqrt[(4 - (3 + Sqrt[33])*x^2)^(-1)]*Sqrt[-2 + 3*x^2 + 3*x^4]))/2 - ((
11 - Sqrt[33])*Sqrt[(4 - (3 - Sqrt[33])*x^2)/(4 - (3 + Sqrt[33])*x^2)]*Sqr
t[-4 + (3 + Sqrt[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-4 +
 (3 + Sqrt[33])*x^2]], (11 + Sqrt[33])/22])/(4*Sqrt[2]*33^(1/4)*Sqrt[(4 -
(3 + Sqrt[33])*x^2)^(-1)]*Sqrt[-2 + 3*x^2 + 3*x^4]))/22
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{x(3x^2+7)}{22\sqrt{3x^4+3x^2-2}}-\frac{4\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{3-\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{11\sqrt{3-\sqrt{33}}\,\sqrt{3x^4+3x^2-2}}+\frac{12\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(\right.}}{}$
default	$-\frac{6\left(\frac{7}{132}x+\frac{1}{44}x^3\right)}{\sqrt{3x^4+3x^2-2}}-\frac{4\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{3-\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{11\sqrt{3-\sqrt{33}}\,\sqrt{3x^4+3x^2-2}}+\frac{12\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(\right.}}{}$
elliptic	$-\frac{6\left(\frac{7}{132}x+\frac{1}{44}x^3\right)}{\sqrt{3x^4+3x^2-2}}-\frac{4\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{3-\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{11\sqrt{3-\sqrt{33}}\,\sqrt{3x^4+3x^2-2}}+\frac{12\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(\right.}}{}$

input
```
int(1/(3*x^4+3*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/22*x*(3*x^2+7)/(3*x^4+3*x^2-2)^(1/2)-4/11/(3-33^(1/2))^(1/2)*(1-(3/4-1/
4*33^(1/2))*x^2)^(1/2)*(1-(3/4+1/4*33^(1/2))*x^2)^(1/2)/(3*x^4+3*x^2-2)^(1
/2)*EllipticF(1/2*(3-33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))+12/11
/(3-33^(1/2))^(1/2)*(1-(3/4-1/4*33^(1/2))*x^2)^(1/2)*(1-(3/4+1/4*33^(1/2))
*x^2)^(1/2)/(3*x^4+3*x^2-2)^(1/2)/(3+33^(1/2))*(EllipticF(1/2*(3-33^(1/2))
^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))-EllipticE(1/2*(3-33^(1/2))^(1/2)*x,
1/4*I*6^(1/2)+1/4*I*22^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(-2+3x^2+3x^4\right)^{3/2}}\,dx =$$

$$-\frac{3\left(\sqrt{33}\sqrt{-2}(3\,x^4+3\,x^2-2)+3\,\sqrt{-2}(3\,x^4+3\,x^2-2)\right)\sqrt{\sqrt{33}+3}\,E\left(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{33}+3}\right)\mid\frac{1}{4}\,\sqrt{33}-\right.}{}$$

input
```
integrate(1/(3*x^4+3*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
-1/528*(3*(sqrt(33)*sqrt(-2)*(3*x^4 + 3*x^2 - 2) + 3*sqrt(-2)*(3*x^4 + 3*x
^2 - 2))*sqrt(sqrt(33) + 3)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(33) + 3)), 1
/4*sqrt(33) - 7/4) - (7*sqrt(33)*sqrt(-2)*(3*x^4 + 3*x^2 - 2) - 3*sqrt(-2)
*(3*x^4 + 3*x^2 - 2))*sqrt(sqrt(33) + 3)*elliptic_f(arcsin(1/2*x*sqrt(sqrt
(33) + 3)), 1/4*sqrt(33) - 7/4) + 24*sqrt(3*x^4 + 3*x^2 - 2)*(3*x^3 + 7*x)
)/(3*x^4 + 3*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(-2+3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+3x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+3*x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 + 3*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2+3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+3\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+3*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 3*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+3\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+3*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 3*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+3\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(3*x^2 + 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(3*x^2 + 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+3x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+3x^2-2}}{9x^8+18x^6-3x^4-12x^2+4}dx$$

input
```
int(1/(3*x^4+3*x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 3*x**2 - 2)/(9*x**8 + 18*x**6 - 3*x**4 - 12*x**2 + 4),x)
```

3.207 $\int \frac{1}{(-2+2x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 235

$$\int \frac{1}{(-2+2x^2+3x^4)^{3/2}}\,dx = -\frac{x(8+3x^2)}{28\sqrt{-2+2x^2+3x^4}} + \frac{\sqrt{\frac{1}{2}\left(1+\sqrt{7}\right)}\sqrt{2-\left(1-\sqrt{7}\right)x^2}\sqrt{2-\left(1+\sqrt{7}\right)x^2}E\left(\arcsin\left(\sqrt{\frac{1}{2}\left(1+\sqrt{7}\right)}x\right)|\frac{1}{3}(-4+\sqrt{7})\right)}{28\sqrt{-2+2x^2+3x^4}} - \frac{\sqrt{\frac{1}{14}\left(1+\sqrt{7}\right)}\sqrt{2-\left(1-\sqrt{7}\right)x^2}\sqrt{2-\left(1+\sqrt{7}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(1+\sqrt{7}\right)}x\right),\frac{1}{3}(-4+\sqrt{7})\right)}{4\sqrt{-2+2x^2+3x^4}}$$

output
```
-1/28*x*(3*x^2+8)/(3*x^4+2*x^2-2)^(1/2)+1/56*(2+2*7^(1/2))^(1/2)*(2-(1-7^(
1/2))*x^2)^(1/2)*(2-(1+7^(1/2))*x^2)^(1/2)*EllipticE(1/2*(2+2*7^(1/2))^(1/
2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))/(3*x^4+2*x^2-2)^(1/2)-1/56*(14+14*7^(1/
2))^(1/2)*(2-(1-7^(1/2))*x^2)^(1/2)*(2-(1+7^(1/2))*x^2)^(1/2)*EllipticF(1/
2*(2+2*7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))/(3*x^4+2*x^2-2)^(1/2
)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.73 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-2+2x^2+3x^4)^{3/2}}\,dx = \frac{-3x(8+3x^2)+3i\sqrt{-1+\sqrt{7}}\sqrt{2-2x^2-3x^4}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right)|-\frac{4}{3}-\right.}{84\sqrt{-2+2x^2+3x}}$$

input
```
Integrate[(-2 + 2*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(-3*x*(8 + 3*x^2) + (3*I)*Sqrt[-1 + Sqrt[7]]*Sqrt[2 - 2*x^2 - 3*x^4]*Ellip
ticE[I*ArcSinh[Sqrt[3/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3] - ((3*I)*(-7 +
Sqrt[7])*Sqrt[2 - 2*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(1 + Sqrt[7])]
*x], -4/3 - Sqrt[7]/3])/Sqrt[-1 + Sqrt[7]])/(84*Sqrt[-2 + 2*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.52, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+2x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{56}\int -\frac{6(2-x^2)}{\sqrt{3x^4+2x^2-2}}dx-\frac{x(3x^2+8)}{28\sqrt{3x^4+2x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{28}\int \frac{2-x^2}{\sqrt{3x^4+2x^2-2}}dx-\frac{x(3x^2+8)}{28\sqrt{3x^4+2x^2-2}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{3}{28}\left(\frac{1}{3}\left(7-\sqrt{7}\right)\int \frac{1}{\sqrt{3x^4+2x^2-2}}dx-\frac{1}{6}\int \frac{2\left(3x^2-\sqrt{7}+1\right)}{\sqrt{3x^4+2x^2-2}}dx\right)-\frac{x\left(3x^2+8\right)}{28\sqrt{3x^4+2x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{28}\left(\frac{1}{3}\left(7-\sqrt{7}\right)\int \frac{1}{\sqrt{3x^4+2x^2-2}}dx-\frac{1}{3}\int \frac{3x^2-\sqrt{7}+1}{\sqrt{3x^4+2x^2-2}}dx\right)-\frac{x\left(3x^2+8\right)}{28\sqrt{3x^4+2x^2-2}}$$

$$\Big\downarrow \text{1411}$$

$$-\frac{3}{28}\left(\frac{\left(7-\sqrt{7}\right)\sqrt{\frac{2-\left(1-\sqrt{7}\right)x^2}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-2}}\right),\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{6\sqrt[4]{7}\sqrt{\frac{1}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{3x^4+2x^2-2}}-\frac{1}{3}\int \frac{3x}{\sqrt{3x}}\right.$$

$$\frac{x\left(3x^2+8\right)}{28\sqrt{3x^4+2x^2-2}}$$

$$\Big\downarrow \text{1498}$$

$$-\frac{3}{28}\left(\frac{\left(7-\sqrt{7}\right)\sqrt{\frac{2-\left(1-\sqrt{7}\right)x^2}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-2}}\right),\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{6\sqrt[4]{7}\sqrt{\frac{1}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{3x^4+2x^2-2}}+\frac{1}{3}\left(\frac{\sqrt[4]{7}}{}\right.\right.$$

$$\frac{x\left(3x^2+8\right)}{28\sqrt{3x^4+2x^2-2}}$$

input
```
Int[(-2 + 2*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/28*(x*(8 + 3*x^2))/Sqrt[-2 + 2*x^2 + 3*x^4] - (3*((-((x*(1 + Sqrt[7] +
3*x^2))/Sqrt[-2 + 2*x^2 + 3*x^4]) + (7^(1/4)*Sqrt[(2 - (1 - Sqrt[7])*x^2)/
(2 - (1 + Sqrt[7])*x^2)]*Sqrt[-2 + (1 + Sqrt[7])*x^2]*EllipticE[ArcSin[(Sq
rt[2]*7^(1/4)*x)/Sqrt[-2 + (1 + Sqrt[7])*x^2]], (7 + Sqrt[7])/14])/(Sqrt[(
2 - (1 + Sqrt[7])*x^2)^(-1)]*Sqrt[-2 + 2*x^2 + 3*x^4]))/3 + ((7 - Sqrt[7])
*Sqrt[(2 - (1 - Sqrt[7])*x^2)/(2 - (1 + Sqrt[7])*x^2)]*Sqrt[-2 + (1 + Sqrt
[7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 + (1 + Sqrt[7])*x^2
]], (7 + Sqrt[7])/14])/(6*7^(1/4)*Sqrt[(2 - (1 + Sqrt[7])*x^2)^(-1)]*Sqrt[
-2 + 2*x^2 + 3*x^4])))/28
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{x(3x^2+8)}{28\sqrt{3x^4+2x^2-2}}-\frac{3\sqrt{1-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{2-2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{7\sqrt{2-2\sqrt{7}}\,\sqrt{3x^4+2x^2-2}}+\frac{6\sqrt{1-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\sqrt{1-(}}{}$
default	$-\frac{6\left(\frac{1}{21}x+\frac{1}{56}x^3\right)}{\sqrt{3x^4+2x^2-2}}-\frac{3\sqrt{1-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{2-2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{7\sqrt{2-2\sqrt{7}}\,\sqrt{3x^4+2x^2-2}}+\frac{6\sqrt{1-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}\right.}}{}$
elliptic	$-\frac{6\left(\frac{1}{21}x+\frac{1}{56}x^3\right)}{\sqrt{3x^4+2x^2-2}}-\frac{3\sqrt{1-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{2-2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{7\sqrt{2-2\sqrt{7}}\,\sqrt{3x^4+2x^2-2}}+\frac{6\sqrt{1-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}\right.}}{}$

input
```
int(1/(3*x^4+2*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/28*x*(3*x^2+8)/(3*x^4+2*x^2-2)^(1/2)-3/7/(2-2*7^(1/2))^(1/2)*(1-(-1/2*7
^(1/2)+1/2)*x^2)^(1/2)*(1-(1/2+1/2*7^(1/2))*x^2)^(1/2)/(3*x^4+2*x^2-2)^(1/
2)*EllipticF(1/2*(2-2*7^(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))+6/7/(
2-2*7^(1/2))^(1/2)*(1-(-1/2*7^(1/2)+1/2)*x^2)^(1/2)*(1-(1/2+1/2*7^(1/2))*x
^2)^(1/2)/(3*x^4+2*x^2-2)^(1/2)/(2+2*7^(1/2))*(EllipticF(1/2*(2-2*7^(1/2))
^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))-EllipticE(1/2*(2-2*7^(1/2))^(1/2)*x
,1/6*I*6^(1/2)+1/6*I*42^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-2+2x^2+3x^4)^{3/2}}\,dx =$$

$$-\frac{\left(\sqrt{7}\sqrt{-2}(3\,x^4+2\,x^2-2)+\sqrt{-2}(3\,x^4+2\,x^2-2)\right)\sqrt{\frac{1}{2}\,\sqrt{7}+\frac{1}{2}}E(\arcsin\left(x\sqrt{\frac{1}{2}\,\sqrt{7}+\frac{1}{2}}\right)\mid\frac{1}{3}\,\sqrt{7}-\frac{4}{3})-}{}$$

input
```
integrate(1/(3*x^4+2*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
-1/56*((sqrt(7)*sqrt(-2)*(3*x^4 + 2*x^2 - 2) + sqrt(-2)*(3*x^4 + 2*x^2 - 2
))*sqrt(1/2*sqrt(7) + 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(7) + 1/2)), 1
/3*sqrt(7) - 4/3) - (3*sqrt(7)*sqrt(-2)*(3*x^4 + 2*x^2 - 2) - sqrt(-2)*(3*
x^4 + 2*x^2 - 2))*sqrt(1/2*sqrt(7) + 1/2)*elliptic_f(arcsin(x*sqrt(1/2*sqr
t(7) + 1/2)), 1/3*sqrt(7) - 4/3) + 2*sqrt(3*x^4 + 2*x^2 - 2)*(3*x^3 + 8*x)
)/(3*x^4 + 2*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(-2+2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+2x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+2*x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 + 2*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2+2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+2\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+2*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 2*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+2\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+2*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 2*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+2\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(2*x^2 + 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(2*x^2 + 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+2x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+2x^2-2}}{9x^8+12x^6-8x^4-8x^2+4}dx$$

input
```
int(1/(3*x^4+2*x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 2*x**2 - 2)/(9*x**8 + 12*x**6 - 8*x**4 - 8*x**2 + 4),x)
```

3.208 $\int \frac{1}{(-2+x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 141

$$\int \frac{1}{(-2+x^2+3x^4)^{3/2}}\,dx = -\frac{x(13+3x^2)}{50\sqrt{-2+x^2+3x^4}} + \frac{\sqrt{3}\sqrt{2-3x^2}\sqrt{1+x^2}E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{50\sqrt{-2+x^2+3x^4}} - \frac{\sqrt{3}\sqrt{2-3x^2}\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{10\sqrt{-2+x^2+3x^4}}$$

output
```
-1/50*x*(3*x^2+13)/(3*x^4+x^2-2)^(1/2)+1/50*3^(1/2)*(-3*x^2+2)^(1/2)*(x^2+
1)^(1/2)*EllipticE(1/2*x*6^(1/2),1/3*I*6^(1/2))/(3*x^4+x^2-2)^(1/2)-1/10*(
-3*x^2+2)^(1/2)*(x^2+1)^(1/2)*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/
2)/(3*x^4+x^2-2)^(1/2)
```

Mathematica [A] (verified)

Time = 6.74 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.70

$$\int \frac{1}{(-2+x^2+3x^4)^{3/2}}\,dx = \frac{-13x-3x^3+\sqrt{6-9x^2}\sqrt{1+x^2}E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)-5\sqrt{6-9x^2}\sqrt{1+x^2}}{50\sqrt{-2+x^2+3x^4}}$$

input
```
Integrate[(-2 + x^2 + 3*x^4)^(-3/2),x]
```

output
```
(-13*x - 3*x^3 + Sqrt[6 - 9*x^2]*Sqrt[1 + x^2]*EllipticE[ArcSin[Sqrt[3/2]*
x], -2/3] - 5*Sqrt[6 - 9*x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x],
 -2/3])/(50*Sqrt[-2 + x^2 + 3*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.50, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1405, 27, 1501, 27, 1410, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{50}\int -\frac{3(4-x^2)}{\sqrt{3x^4+x^2-2}}dx-\frac{x(3x^2+13)}{50\sqrt{3x^4+x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{50}\int \frac{4-x^2}{\sqrt{3x^4+x^2-2}}dx-\frac{x(3x^2+13)}{50\sqrt{3x^4+x^2-2}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{3}{50}\left(\frac{10}{3}\int \frac{1}{\sqrt{3x^4+x^2-2}}dx-\frac{1}{6}\int -\frac{2(2-3x^2)}{\sqrt{3x^4+x^2-2}}dx\right)-\frac{x(3x^2+13)}{50\sqrt{3x^4+x^2-2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{3}{50}\left(\frac{10}{3}\int \frac{1}{\sqrt{3x^4+x^2-2}}dx+\frac{1}{3}\int \frac{2-3x^2}{\sqrt{3x^4+x^2-2}}dx\right)-\frac{x(3x^2+13)}{50\sqrt{3x^4+x^2-2}}$$

$$\Big\downarrow \; 1410$$

$$-\frac{3}{50}\left(\frac{1}{3}\int \frac{2-3x^2}{\sqrt{3x^4+x^2-2}}dx+\frac{2\sqrt{5}\sqrt{x^2+1}\sqrt{3x^2-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{5}x}{\sqrt{3x^2-2}}\right),\frac{3}{5}\right)}{3\sqrt{3x^4+x^2-2}}\right)-\frac{x(3x^2+13)}{50\sqrt{3x^4+x^2-2}}$$

$$\Big\downarrow \; 1498$$

$$-\frac{3}{50}\left(\frac{2\sqrt{5}\sqrt{x^2+1}\sqrt{3x^2-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{5}x}{\sqrt{3x^2-2}}\right),\frac{3}{5}\right)}{3\sqrt{3x^4+x^2-2}}+\frac{1}{3}\left(\frac{\sqrt{5}\sqrt{\frac{x^2+1}{2-3x^2}}\sqrt{3x^2-2}E\left(\arcsin\left(\frac{\sqrt{5}x}{\sqrt{3x^2-2}}\right)|\frac{3}{5}\right)}{\sqrt{\frac{1}{2-3x^2}}\sqrt{3x^4+x^2-2}}\right.\right.$$

$$\frac{x(3x^2+13)}{50\sqrt{3x^4+x^2-2}}$$

input
```
Int[(-2 + x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/50*(x*(13 + 3*x^2))/Sqrt[-2 + x^2 + 3*x^4] - (3*(((-3*x*(1 + x^2))/Sqrt
[-2 + x^2 + 3*x^4] + (Sqrt[5]*Sqrt[(1 + x^2)/(2 - 3*x^2)]*Sqrt[-2 + 3*x^2]
*EllipticE[ArcSin[(Sqrt[5]*x)/Sqrt[-2 + 3*x^2]], 3/5])/(Sqrt[(2 - 3*x^2)^(
-1)]*Sqrt[-2 + x^2 + 3*x^4]))/3 + (2*Sqrt[5]*Sqrt[1 + x^2]*Sqrt[-2 + 3*x^2
]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt[-2 + 3*x^2]], 3/5])/(3*Sqrt[-2 + x^2 +
 3*x^4])))/50
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{x(3x^2+13)}{50\sqrt{3x^4+x^2-2}}+\frac{3i\sqrt{x^2+1}\,\sqrt{-6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{i\sqrt{6}}{2}\right)}{25\sqrt{3x^4+x^2-2}}-\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{i\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{i\sqrt{6}}{2}\right)\right)}{50\sqrt{3x^4+x^2-2}}$
default	$-\frac{6\left(\frac{13}{300}x+\frac{1}{100}x^3\right)}{\sqrt{3x^4+x^2-2}}+\frac{3i\sqrt{x^2+1}\,\sqrt{-6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{i\sqrt{6}}{2}\right)}{25\sqrt{3x^4+x^2-2}}-\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{i\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{i\sqrt{6}}{2}\right)\right)}{50\sqrt{3x^4+x^2-2}}$
elliptic	$-\frac{6\left(\frac{13}{300}x+\frac{1}{100}x^3\right)}{\sqrt{3x^4+x^2-2}}+\frac{3i\sqrt{x^2+1}\,\sqrt{-6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{i\sqrt{6}}{2}\right)}{25\sqrt{3x^4+x^2-2}}-\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{i\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{i\sqrt{6}}{2}\right)\right)}{50\sqrt{3x^4+x^2-2}}$

input
```
int(1/(3*x^4+x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/50*x*(3*x^2+13)/(3*x^4+x^2-2)^(1/2)+3/25*I*(x^2+1)^(1/2)*(-6*x^2+4)^(1/
2)/(3*x^4+x^2-2)^(1/2)*EllipticF(I*x,1/2*I*6^(1/2))-1/50*I*(x^2+1)^(1/2)*(
-6*x^2+4)^(1/2)/(3*x^4+x^2-2)^(1/2)*(EllipticF(I*x,1/2*I*6^(1/2))-Elliptic
E(I*x,1/2*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.64

$$\int \frac{1}{(-2+x^2+3x^4)^{3/2}}\,dx =$$
$$-\frac{3\sqrt{\frac{3}{2}}\sqrt{-2}(3\,x^4+x^2-2)E(\arcsin\left(\sqrt{\frac{3}{2}}x\right)\,|-\frac{2}{3})-11\sqrt{\frac{3}{2}}\sqrt{-2}(3\,x^4+x^2-2)F(\arcsin\left(\sqrt{\frac{3}{2}}x\right)\,|-\frac{2}{3})+}{100\,(3\,x^4+x^2-2)}$$

input
```
integrate(1/(3*x^4+x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
-1/100*(3*sqrt(3/2)*sqrt(-2)*(3*x^4 + x^2 - 2)*elliptic_e(arcsin(sqrt(3/2)
*x), -2/3) - 11*sqrt(3/2)*sqrt(-2)*(3*x^4 + x^2 - 2)*elliptic_f(arcsin(sqr
t(3/2)*x), -2/3) + 2*sqrt(3*x^4 + x^2 - 2)*(3*x^3 + 13*x))/(3*x^4 + x^2 -
2)
```

Sympy [F]

$$\int \frac{1}{(-2+x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 + x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2+x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+x^2-2)^{3/2}}\,dx$$

input
```
int(1/(x^2 + 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(x^2 + 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+x^2-2}}{9x^8+6x^6-11x^4-4x^2+4}dx$$

input
```
int(1/(3*x^4+x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + x**2 - 2)/(9*x**8 + 6*x**6 - 11*x**4 - 4*x**2 + 4),x)
```

3.209 $\int \frac{1}{(-2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{1}{(-2+3x^4)^{3/2}}\,dx=-\frac{x}{4\sqrt{-2+3x^4}}-\frac{\sqrt{2-3x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{3}{2}}x\right),-1\right)}{4\sqrt[4]{6}\sqrt{-2+3x^4}}$$

output
```
-1/4*x/(3*x^4-2)^(1/2)-1/24*(-3*x^4+2)^(1/2)*EllipticF(1/2*3^(1/4)*2^(3/4)
*x,I)*6^(3/4)/(3*x^4-2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.69 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{1}{(-2+3x^4)^{3/2}}\,dx=-\frac{x\left(2+\sqrt{4-6x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{3x^4}{2}\right)\right)}{8\sqrt{-2+3x^4}}$$

input
```
Integrate[(-2 + 3*x^4)^(-3/2),x]
```

output
```
-1/8*(x*(2 + Sqrt[4 - 6*x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, (3*x^4)/2]))
/Sqrt[-2 + 3*x^4]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 132 vs. $2(60) = 120$.

Time = 0.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{749, 764\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-2)^{3/2}}dx$$

$$\Big\downarrow \text{749}$$

$$-\frac{1}{4}\int \frac{1}{\sqrt{3x^4-2}}dx-\frac{x}{4\sqrt{3x^4-2}}$$

$$\Big\downarrow \text{764}$$

$$-\frac{\sqrt{\sqrt{6}x^2-2}\sqrt{\frac{\sqrt{6}x^2+2}{2-\sqrt{6}x^2}}\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{\sqrt{6}x^2-2}}\right),\frac{1}{2}\right)}{8\sqrt[4]{6}\sqrt{\frac{1}{2-\sqrt{6}x^2}}\sqrt{3x^4-2}}-\frac{x}{4\sqrt{3x^4-2}}$$

input
```
Int[(-2 + 3*x^4)^(-3/2),x]
```

output
```
-1/4*x/Sqrt[-2 + 3*x^4] - (Sqrt[-2 + Sqrt[6]*x^2]*Sqrt[(2 + Sqrt[6]*x^2)/(
2 - Sqrt[6]*x^2)]*EllipticF[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-2 + Sqrt[6]*x
^2]], 1/2])/(8*6^(1/4)*Sqrt[(2 - Sqrt[6]*x^2)^(-1)]*Sqrt[-2 + 3*x^4])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 764
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[(a - q*x^2)/(a + q*x^2)]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[a + b*x^
4]*Sqrt[a/(a + q*x^2)]))*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2],
 x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.71 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

method	result	size
meijerg	$\frac{\sqrt{2}\left(-\operatorname{signum}\left(-1+\frac{3x^4}{2}\right)\right)^{\frac{3}{2}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],\frac{3x^4}{2}\right)}{4\operatorname{signum}\left(-1+\frac{3x^4}{2}\right)^{\frac{3}{2}}}$	40
default	$-\frac{x}{4\sqrt{3x^4-2}}-\frac{\sqrt{4+2\sqrt{6}\,x^2}\,\sqrt{4-2\sqrt{6}\,x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-2\sqrt{6}}}{2},i\right)}{8\sqrt{-2\sqrt{6}}\,\sqrt{3x^4-2}}$	69
risch	$-\frac{x}{4\sqrt{3x^4-2}}-\frac{\sqrt{4+2\sqrt{6}\,x^2}\,\sqrt{4-2\sqrt{6}\,x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-2\sqrt{6}}}{2},i\right)}{8\sqrt{-2\sqrt{6}}\,\sqrt{3x^4-2}}$	69
elliptic	$-\frac{x}{4\sqrt{3x^4-2}}-\frac{\sqrt{4+2\sqrt{6}\,x^2}\,\sqrt{4-2\sqrt{6}\,x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-2\sqrt{6}}}{2},i\right)}{8\sqrt{-2\sqrt{6}}\,\sqrt{3x^4-2}}$	69

input
```
int(1/(3*x^4-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)/signum(-1+3/2*x^4)^(3/2)*(-signum(-1+3/2*x^4))^(3/2)*x*hyperge
om([1/4,3/2],[5/4],3/2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{1}{(-2+3x^4)^{3/2}}\,dx = \frac{6^{\frac{3}{4}}\sqrt{\frac{1}{2}}\sqrt{-2}(3\,x^4-2)F\left(\arcsin\left(6^{\frac{1}{4}}\sqrt{\frac{1}{2}}x\right)\,|-1\right)-6\,\sqrt{3\,x^4-2}x}{24\,(3\,x^4-2)}$$

input
```
integrate(1/(3*x^4-2)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(6^(3/4)*sqrt(1/2)*sqrt(-2)*(3*x^4 - 2)*elliptic_f(arcsin(6^(1/4)*sqr
t(1/2)*x), -1) - 6*sqrt(3*x^4 - 2)*x)/(3*x^4 - 2)
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.53

$$\int \frac{1}{(-2+3x^4)^{3/2}}\,dx = \frac{\sqrt{2}ix\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\right|\frac{3x^4}{2}\right)}{16\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(3*x**4-2)**(3/2),x)
```

output
```
sqrt(2)*I*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), 3*x**4/2)/(16*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(-2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - 2)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 17.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{1}{(-2+3x^4)^{3/2}}\,dx = \frac{\sqrt{2}\,x\,(2-3\,x^4)^{3/2}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\;\frac{5}{4};\;\frac{3\,x^4}{2}\right)}{4\,(3\,x^4-2)^{3/2}}$$

input
```
int(1/(3*x^4 - 2)^(3/2),x)
```

output
```
(2^(1/2)*x*(2 - 3*x^4)^(3/2)*hypergeom([1/4, 3/2], 5/4, (3*x^4)/2))/(4*(3*
x^4 - 2)^(3/2))
```

Reduce [F]

$$\int \frac{1}{(-2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-2}}{9x^8-12x^4+4}dx$$

input
```
int(1/(3*x^4-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - 2)/(9*x**8 - 12*x**4 + 4),x)
```

3.210 $\int \frac{1}{(-2-x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{1}{(-2-x^2+3x^4)^{3/2}}\,dx = -\frac{x(13-3x^2)}{50\sqrt{-2-x^2+3x^4}} - \frac{\sqrt{1-x^2}\sqrt{2+3x^2}E\left(\arcsin(x)\left|-\frac{3}{2}\right.\right)}{25\sqrt{2}\sqrt{-2-x^2+3x^4}} - \frac{\sqrt{1-x^2}\sqrt{2+3x^2}\,\mathrm{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{5\sqrt{2}\sqrt{-2-x^2+3x^4}}$$

output
```
-1/50*x*(-3*x^2+13)/(3*x^4-x^2-2)^(1/2)-1/50*(-x^2+1)^(1/2)*(3*x^2+2)^(1/2
)*EllipticE(x,1/2*I*6^(1/2))*2^(1/2)/(3*x^4-x^2-2)^(1/2)-1/10*(-x^2+1)^(1/
2)*(3*x^2+2)^(1/2)*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)/(3*x^4-x^2-2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.81 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int \frac{1}{(-2-x^2+3x^4)^{3/2}}\,dx = \frac{-13x+3x^3-i\sqrt{6+3x^2-9x^4}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)+5i\sqrt{6+3x^2-9x^4}}{50\sqrt{-2-x^2+3x^4}}$$

input
```
Integrate[(-2 - x^2 + 3*x^4)^(-3/2),x]
```

output
```
(-13*x + 3*x^3 - I*Sqrt[6 + 3*x^2 - 9*x^4]*EllipticE[I*ArcSinh[Sqrt[3/2]*x
], -2/3] + (5*I)*Sqrt[6 + 3*x^2 - 9*x^4]*EllipticF[I*ArcSinh[Sqrt[3/2]*x],
 -2/3])/(50*Sqrt[-2 - x^2 + 3*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.61, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1410, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-x^2-2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{50}\int -\frac{3(x^2+4)}{\sqrt{3x^4-x^2-2}}dx-\frac{x(13-3x^2)}{50\sqrt{3x^4-x^2-2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{50}\int \frac{x^2+4}{\sqrt{3x^4-x^2-2}}dx-\frac{x(13-3x^2)}{50\sqrt{3x^4-x^2-2}}$$

$$\Big\downarrow \text{ 1501}$$

$$-\frac{3}{50}\left(5\int \frac{1}{\sqrt{3x^4-x^2-2}}dx+\frac{1}{6}\int -\frac{6(1-x^2)}{\sqrt{3x^4-x^2-2}}dx\right)-\frac{x(13-3x^2)}{50\sqrt{3x^4-x^2-2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{3}{50}\left(5\int \frac{1}{\sqrt{3x^4-x^2-2}}dx-\int \frac{1-x^2}{\sqrt{3x^4-x^2-2}}dx\right)-\frac{x(13-3x^2)}{50\sqrt{3x^4-x^2-2}}$$

$$\Big\downarrow \; 1410$$

$$-\frac{3}{50}\left(\frac{\sqrt{5}\sqrt{x^2-1}\sqrt{3x^2+2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{5}{2}}x}{\sqrt{x^2-1}}\right),\frac{2}{5}\right)}{\sqrt{3x^4-x^2-2}}-\int \frac{1-x^2}{\sqrt{3x^4-x^2-2}}dx\right)-\frac{x(13-3x^2)}{50\sqrt{3x^4-x^2-2}}$$

$$\Big\downarrow \; 1498$$

$$-\frac{3}{50}\left(\frac{\sqrt{5}\sqrt{x^2-1}\sqrt{3x^2+2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{5}{2}}x}{\sqrt{x^2-1}}\right),\frac{2}{5}\right)}{\sqrt{3x^4-x^2-2}}-\frac{\sqrt{5}\sqrt{x^2-1}\sqrt{\frac{3x^2+2}{1-x^2}}E\left(\arcsin\left(\frac{\sqrt{\frac{5}{2}}x}{\sqrt{x^2-1}}\right)|\frac{2}{5}\right)}{3\sqrt{\frac{1}{1-x^2}}\sqrt{3x^4-x^2-2}}+ \right.$$

$$\frac{x(13-3x^2)}{50\sqrt{3x^4-x^2-2}}$$

input
```
Int[(-2 - x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/50*(x*(13 - 3*x^2))/Sqrt[-2 - x^2 + 3*x^4] - (3*((x*(2 + 3*x^2))/(3*Sqr
t[-2 - x^2 + 3*x^4]) - (Sqrt[5]*Sqrt[-1 + x^2]*Sqrt[(2 + 3*x^2)/(1 - x^2)]
*EllipticE[ArcSin[(Sqrt[5/2]*x)/Sqrt[-1 + x^2]], 2/5])/(3*Sqrt[(1 - x^2)^(
-1)]*Sqrt[-2 - x^2 + 3*x^4]) + (Sqrt[5]*Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]*Ell
ipticF[ArcSin[(Sqrt[5/2]*x)/Sqrt[-1 + x^2]], 2/5])/Sqrt[-2 - x^2 + 3*x^4])
)/50
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

method	result
risch	$\frac{x(3x^2-13)}{50\sqrt{3x^4-x^2-2}}+\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{-x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{25\sqrt{3x^4-x^2-2}}+\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{-x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)-\text{EllipticE}\left(\frac{i}{}\right.\right.}{100\sqrt{3x^4-x^2-2}}$
default	$-\frac{6\left(\frac{13}{300}x-\frac{1}{100}x^3\right)}{\sqrt{3x^4-x^2-2}}+\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{-x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{25\sqrt{3x^4-x^2-2}}+\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{-x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)-\text{Elliptic}\right.}{100\sqrt{3x^4-x^2-2}}$
elliptic	$-\frac{6\left(\frac{13}{300}x-\frac{1}{100}x^3\right)}{\sqrt{3x^4-x^2-2}}+\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{-x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{25\sqrt{3x^4-x^2-2}}+\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{-x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)-\text{Elliptic}\right.}{100\sqrt{3x^4-x^2-2}}$

input
```
int(1/(3*x^4-x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/50*x*(3*x^2-13)/(3*x^4-x^2-2)^(1/2)+1/25*I*6^(1/2)*(6*x^2+4)^(1/2)*(-x^2
+1)^(1/2)/(3*x^4-x^2-2)^(1/2)*EllipticF(1/2*I*x*6^(1/2),1/3*I*6^(1/2))+1/1
00*I*6^(1/2)*(6*x^2+4)^(1/2)*(-x^2+1)^(1/2)/(3*x^4-x^2-2)^(1/2)*(EllipticF
(1/2*I*x*6^(1/2),1/3*I*6^(1/2))-EllipticE(1/2*I*x*6^(1/2),1/3*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.61

$$\int \frac{1}{(-2-x^2+3x^4)^{3/2}}\,dx = \frac{\sqrt{-2}(3\,x^4-x^2-2)E(\arcsin{(x)}\mid-\frac{3}{2})+5\,\sqrt{-2}(3\,x^4-x^2-2)F(\arcsin{(x)}\mid}{50\,(3\,x^4-x^2-2)}$$

input
```
integrate(1/(3*x^4-x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/50*(sqrt(-2)*(3*x^4 - x^2 - 2)*elliptic_e(arcsin(x), -3/2) + 5*sqrt(-2)*
(3*x^4 - x^2 - 2)*elliptic_f(arcsin(x), -3/2) + sqrt(3*x^4 - x^2 - 2)*(3*x
^3 - 13*x))/(3*x^4 - x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(-2-x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4-x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4-x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 - x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2-x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 - x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-x^2-2)^{3/2}}\,dx$$

input
```
int(1/(3*x^4 - x^2 - 2)^(3/2),x)
```

output
```
int(1/(3*x^4 - x^2 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-x^2-2}}{9x^8-6x^6-11x^4+4x^2+4}dx$$

input
```
int(1/(3*x^4-x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - x**2 - 2)/(9*x**8 - 6*x**6 - 11*x**4 + 4*x**2 + 4),x)
```

3.211 $\int \frac{1}{(-2-2x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 235

$$\int \frac{1}{(-2-2x^2+3x^4)^{3/2}}\,dx = -\frac{x(8-3x^2)}{28\sqrt{-2-2x^2+3x^4}} - \frac{\sqrt{\frac{1}{2}(-1+\sqrt{7})}\sqrt{2+(1-\sqrt{7})x^2}\sqrt{2+(1+\sqrt{7})x^2}E\left(\arcsin\left(\sqrt{\frac{1}{2}(-1+\sqrt{7})}x\right)|\frac{1}{3}(-4-\sqrt{7})\right)}{28\sqrt{-2-2x^2+3x^4}} - \frac{\sqrt{\frac{1}{14}(-1+\sqrt{7})}\sqrt{2+(1-\sqrt{7})x^2}\sqrt{2+(1+\sqrt{7})x^2}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}(-1+\sqrt{7})}x\right),\frac{1}{3}(-4-\sqrt{7})\right)}{4\sqrt{-2-2x^2+3x^4}}$$

output
```
-1/28*x*(-3*x^2+8)/(3*x^4-2*x^2-2)^(1/2)-1/56*(-2+2*7^(1/2))^(1/2)*(2+(1-7
^(1/2))*x^2)^(1/2)*(2+(1+7^(1/2))*x^2)^(1/2)*EllipticE(1/2*(-2+2*7^(1/2))^
(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))/(3*x^4-2*x^2-2)^(1/2)-1/56*(-14+14*7
^(1/2))^(1/2)*(2+(1-7^(1/2))*x^2)^(1/2)*(2+(1+7^(1/2))*x^2)^(1/2)*Elliptic
F(1/2*(-2+2*7^(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))/(3*x^4-2*x^2-2)
^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-2-2x^2+3x^4)^{3/2}}\,dx = \frac{3x(-8+3x^2)-3i\sqrt{1+\sqrt{7}}\sqrt{2+2x^2-3x^4}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right)|\frac{1}{3}(-4 \ldots\right.}{84\sqrt{-2-2x^2+ \ldots}}$$

input
```
Integrate[(-2 - 2*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(3*x*(-8 + 3*x^2) - (3*I)*Sqrt[1 + Sqrt[7]]*Sqrt[2 + 2*x^2 - 3*x^4]*Ellipt
icE[I*ArcSinh[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3] + ((3*I)*(7 + S
qrt[7])*Sqrt[2 + 2*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(-1 + Sqrt[7])]
*x], (-4 + Sqrt[7])/3])/Sqrt[1 + Sqrt[7]])/(84*Sqrt[-2 - 2*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.59, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-2x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{56}\int -\frac{6(x^2+2)}{\sqrt{3x^4-2x^2-2}}dx-\frac{x(8-3x^2)}{28\sqrt{3x^4-2x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{28}\int \frac{x^2+2}{\sqrt{3x^4-2x^2-2}}dx-\frac{x(8-3x^2)}{28\sqrt{3x^4-2x^2-2}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{3}{28}\left(\frac{1}{3}\left(7+\sqrt{7}\right)\int\frac{1}{\sqrt{3x^4-2x^2-2}}dx+\frac{1}{6}\int-\frac{2\left(-3x^2+\sqrt{7}+1\right)}{\sqrt{3x^4-2x^2-2}}dx\right)-\frac{x\left(8-3x^2\right)}{28\sqrt{3x^4-2x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{28}\left(\frac{1}{3}\left(7+\sqrt{7}\right)\int\frac{1}{\sqrt{3x^4-2x^2-2}}dx-\frac{1}{3}\int\frac{-3x^2+\sqrt{7}+1}{\sqrt{3x^4-2x^2-2}}dx\right)-\frac{x\left(8-3x^2\right)}{28\sqrt{3x^4-2x^2-2}}$$

$$\Big\downarrow \text{1411}$$

$$-\frac{3}{28}\left(\frac{\left(7+\sqrt{7}\right)\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-2}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+2}{\left(1-\sqrt{7}\right)x^2+2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-2}}\right),\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{6\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+2}}\sqrt{3x^4-2x^2-2}}-\right.$$

$$\frac{x\left(8-3x^2\right)}{28\sqrt{3x^4-2x^2-2}}$$

$$\Big\downarrow \text{1498}$$

$$-\frac{3}{28}\left(\frac{\left(7+\sqrt{7}\right)\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-2}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+2}{\left(1-\sqrt{7}\right)x^2+2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-2}}\right),\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{6\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+2}}\sqrt{3x^4-2x^2-2}}+\right.$$

$$\frac{x\left(8-3x^2\right)}{28\sqrt{3x^4-2x^2-2}}$$

input
```
Int[(-2 - 2*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/28*(x*(8 - 3*x^2))/Sqrt[-2 - 2*x^2 + 3*x^4] - (3*((-((x*(1 - Sqrt[7] -
3*x^2))/Sqrt[-2 - 2*x^2 + 3*x^4]) - (7^(1/4)*Sqrt[-2 - (1 - Sqrt[7])*x^2]*
Sqrt[(2 + (1 + Sqrt[7])*x^2)/(2 + (1 - Sqrt[7])*x^2)]*EllipticE[ArcSin[(Sq
rt[2]*7^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[7])*x^2]], (7 - Sqrt[7])/14])/(Sqrt[(
2 + (1 - Sqrt[7])*x^2)^(-1)]*Sqrt[-2 - 2*x^2 + 3*x^4]))/3 + ((7 + Sqrt[7])
*Sqrt[-2 - (1 - Sqrt[7])*x^2]*Sqrt[(2 + (1 + Sqrt[7])*x^2)/(2 + (1 - Sqrt[
7])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[7])*x^2
]], (7 - Sqrt[7])/14])/(6*7^(1/4)*Sqrt[(2 + (1 - Sqrt[7])*x^2)^(-1)]*Sqrt[
-2 - 2*x^2 + 3*x^4])))/28
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.98

method	result
risch	$\frac{x(3x^2-8)}{28\sqrt{3x^4-2x^2-2}} - \frac{3\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-2-2\sqrt{7}}\,x}{2}, \frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{7\sqrt{-2-2\sqrt{7}}\,\sqrt{3x^4-2x^2-2}} - \frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\sqrt{1-(}{}$
default	$-\frac{6\left(\frac{1}{21}x-\frac{1}{56}x^3\right)}{\sqrt{3x^4-2x^2-2}} - \frac{3\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-2-2\sqrt{7}}\,x}{2}, \frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{7\sqrt{-2-2\sqrt{7}}\,\sqrt{3x^4-2x^2-2}} - \frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\sqrt{1-}{}$
elliptic	$-\frac{6\left(\frac{1}{21}x-\frac{1}{56}x^3\right)}{\sqrt{3x^4-2x^2-2}} - \frac{3\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-2-2\sqrt{7}}\,x}{2}, \frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{7\sqrt{-2-2\sqrt{7}}\,\sqrt{3x^4-2x^2-2}} - \frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\sqrt{1-}{}$

input
```
int(1/(3*x^4-2*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/28*x*(3*x^2-8)/(3*x^4-2*x^2-2)^(1/2)-3/7/(-2-2*7^(1/2))^(1/2)*(1-(-1/2-1
/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2+1/2*7^(1/2))*x^2)^(1/2)/(3*x^4-2*x^2-2)^(1
/2)*EllipticF(1/2*(-2-2*7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))-6/7
/(-2-2*7^(1/2))^(1/2)*(1-(-1/2-1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2+1/2*7^(1/2
))*x^2)^(1/2)/(3*x^4-2*x^2-2)^(1/2)/(-2+2*7^(1/2))*(EllipticF(1/2*(-2-2*7^
(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))-EllipticE(1/2*(-2-2*7^(1/2))^
(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(-2-2x^2+3x^4\right)^{3/2}}\,dx = \frac{\left(\sqrt{7}\sqrt{-2}(3\,x^4-2\,x^2-2)-\sqrt{-2}(3\,x^4-2\,x^2-2)\right)\sqrt{\frac{1}{2}\sqrt{7}-\frac{1}{2}}E\left(\arcsin\left(x\right.\right.}{}$$

input
```
integrate(1/(3*x^4-2*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/56*((sqrt(7)*sqrt(-2)*(3*x^4 - 2*x^2 - 2) - sqrt(-2)*(3*x^4 - 2*x^2 - 2)
)*sqrt(1/2*sqrt(7) - 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(7) - 1/2)), -1
/3*sqrt(7) - 4/3) + (sqrt(7)*sqrt(-2)*(3*x^4 - 2*x^2 - 2) + 3*sqrt(-2)*(3*
x^4 - 2*x^2 - 2))*sqrt(1/2*sqrt(7) - 1/2)*elliptic_f(arcsin(x*sqrt(1/2*sqr
t(7) - 1/2)), -1/3*sqrt(7) - 4/3) + 2*sqrt(3*x^4 - 2*x^2 - 2)*(3*x^3 - 8*x
))/(3*x^4 - 2*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{\left(-2-2x^2+3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(3x^4-2x^2-2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4-2*x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 - 2*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(-2-2x^2+3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(3\,x^4-2\,x^2-2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-2*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 - 2*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-2\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-2*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - 2*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-2\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(3*x^4 - 2*x^2 - 2)^(3/2),x)
```

output
```
int(1/(3*x^4 - 2*x^2 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-2x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-2x^2-2}}{9x^8-12x^6-8x^4+8x^2+4}dx$$

input
```
int(1/(3*x^4-2*x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - 2*x**2 - 2)/(9*x**8 - 12*x**6 - 8*x**4 + 8*x**2 + 4),x)
```

3.212 $\int \frac{1}{(-2-3x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 229

$$\int \frac{1}{(-2-3x^2+3x^4)^{3/2}}\,dx = -\frac{x(7-3x^2)}{22\sqrt{-2-3x^2+3x^4}} - \frac{\sqrt{-3+\sqrt{33}}\sqrt{4+\left(3-\sqrt{33}\right)x^2}\sqrt{4+\left(3+\sqrt{33}\right)x^2}E\left(\arcsin\left(\frac{1}{2}\sqrt{-3+\sqrt{33}}x\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)}{88\sqrt{-2-3x^2+3x^4}} - \frac{\sqrt{\frac{1}{33}\left(-3+\sqrt{33}\right)}\sqrt{4+\left(3-\sqrt{33}\right)x^2}\sqrt{4+\left(3+\sqrt{33}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-3+\sqrt{33}}x\right),\frac{1}{4}(-7-\right.}{8\sqrt{-2-3x^2+3x^4}}$$

output
```
-1/22*x*(-3*x^2+7)/(3*x^4-3*x^2-2)^(1/2)-1/88*(-3+33^(1/2))^(1/2)*(4+(3-33
^(1/2))*x^2)^(1/2)*(4+(3+33^(1/2))*x^2)^(1/2)*EllipticE(1/2*(-3+33^(1/2))^
(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))/(3*x^4-3*x^2-2)^(1/2)-1/264*(-99+33*
33^(1/2))^(1/2)*(4+(3-33^(1/2))*x^2)^(1/2)*(4+(3+33^(1/2))*x^2)^(1/2)*Elli
pticF(1/2*(-3+33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))/(3*x^4-3*x^2
-2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.80 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-2-3x^2+3x^4)^{3/2}}\,dx = \frac{12x(-7+3x^2)-6i\sqrt{3+\sqrt{33}}\sqrt{4+6x^2-6x^4}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right)|\frac{1}{4}(-\right.}{264\sqrt{-2-3}}$$

input
```
Integrate[(-2 - 3*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(12*x*(-7 + 3*x^2) - (6*I)*Sqrt[3 + Sqrt[33]]*Sqrt[4 + 6*x^2 - 6*x^4]*Elli
pticE[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*x], (-7 + Sqrt[33])/4] + ((6*I)*(1
1 + Sqrt[33])*Sqrt[4 + 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(-3 + Sqr
t[33])]*x], (-7 + Sqrt[33])/4])/Sqrt[3 + Sqrt[33]])/(264*Sqrt[-2 - 3*x^2 +
 3*x^4])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.66, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 25, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-3x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{66}\int -\frac{3(3x^2+4)}{\sqrt{3x^4-3x^2-2}}dx - \frac{x(7-3x^2)}{22\sqrt{3x^4-3x^2-2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{22}\int \frac{3x^2+4}{\sqrt{3x^4-3x^2-2}}dx - \frac{x(7-3x^2)}{22\sqrt{3x^4-3x^2-2}}$$

$$\Big\downarrow \; 1501$$

$$\frac{1}{22}\left(-\frac{1}{2}\left(11+\sqrt{33}\right)\int \frac{1}{\sqrt{3x^4-3x^2-2}}dx-\frac{1}{2}\int -\frac{-6x^2+\sqrt{33}+3}{\sqrt{3x^4-3x^2-2}}dx\right)-\frac{x(7-3x^2)}{22\sqrt{3x^4-3x^2-2}}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{22}\left(\frac{1}{2}\int \frac{-6x^2+\sqrt{33}+3}{\sqrt{3x^4-3x^2-2}}dx-\frac{1}{2}\left(11+\sqrt{33}\right)\int \frac{1}{\sqrt{3x^4-3x^2-2}}dx\right)-\frac{x(7-3x^2)}{22\sqrt{3x^4-3x^2-2}}$$

$$\Big\downarrow \; 1411$$

$$\frac{1}{22}\left(\frac{1}{2}\int \frac{-6x^2+\sqrt{33}+3}{\sqrt{3x^4-3x^2-2}}dx-\frac{\left(11+\sqrt{33}\right)\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-4}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+4}{\left(3-\sqrt{33}\right)x^2+4}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\cdots}{\sqrt{-\left(\left(3-\cdots\right.\right.}}\right.\right.}{4\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+4}}\sqrt{3x^4-3x^2-2}}\right.$$

$$\frac{x(7-3x^2)}{22\sqrt{3x^4-3x^2-2}}$$

$$\Big\downarrow \; 1498$$

$$\frac{1}{22}\left(\frac{1}{2}\left(\frac{\sqrt[4]{33}\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-4}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+4}{\left(3-\sqrt{33}\right)x^2+4}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-4}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{\sqrt{2}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+4}}\sqrt{3x^4-3x^2-2}}+\frac{x(-6x\cdots}{\sqrt{3x\cdots}}\right.\right.$$

$$\frac{x(7-3x^2)}{22\sqrt{3x^4-3x^2-2}}$$

input
```
Int[(-2 - 3*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/22*(x*(7 - 3*x^2))/Sqrt[-2 - 3*x^2 + 3*x^4] + (((x*(3 - Sqrt[33] - 6*x^
2))/Sqrt[-2 - 3*x^2 + 3*x^4] + (33^(1/4)*Sqrt[-4 - (3 - Sqrt[33])*x^2]*Sqr
t[(4 + (3 + Sqrt[33])*x^2)/(4 + (3 - Sqrt[33])*x^2)]*EllipticE[ArcSin[(Sqr
t[2]*33^(1/4)*x)/Sqrt[-4 - (3 - Sqrt[33])*x^2]], (11 - Sqrt[33])/22])/(Sqr
t[2]*Sqrt[(4 + (3 - Sqrt[33])*x^2)^(-1)]*Sqrt[-2 - 3*x^2 + 3*x^4]))/2 - ((
11 + Sqrt[33])*Sqrt[-4 - (3 - Sqrt[33])*x^2]*Sqrt[(4 + (3 + Sqrt[33])*x^2)
/(4 + (3 - Sqrt[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-4 -
 (3 - Sqrt[33])*x^2]], (11 - Sqrt[33])/22])/(4*Sqrt[2]*33^(1/4)*Sqrt[(4 +
(3 - Sqrt[33])*x^2)^(-1)]*Sqrt[-2 - 3*x^2 + 3*x^4]))/22
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

method	result
risch	$\frac{x(3x^2-7)}{22\sqrt{3x^4-3x^2-2}}-\frac{4\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-3-\sqrt{33}}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{11\sqrt{-3-\sqrt{33}}\,\sqrt{3x^4-3x^2-2}}-\frac{12\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{}}{}$
default	$-\frac{6\left(\frac{7}{132}x-\frac{1}{44}x^3\right)}{\sqrt{3x^4-3x^2-2}}-\frac{4\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-3-\sqrt{33}}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{11\sqrt{-3-\sqrt{33}}\,\sqrt{3x^4-3x^2-2}}-\frac{12\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}}{}$
elliptic	$-\frac{6\left(\frac{7}{132}x-\frac{1}{44}x^3\right)}{\sqrt{3x^4-3x^2-2}}-\frac{4\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-3-\sqrt{33}}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{11\sqrt{-3-\sqrt{33}}\,\sqrt{3x^4-3x^2-2}}-\frac{12\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}}{}$

input
```
int(1/(3*x^4-3*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/22*x*(3*x^2-7)/(3*x^4-3*x^2-2)^(1/2)-4/11/(-3-33^(1/2))^(1/2)*(1-(-3/4-1
/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4+1/4*33^(1/2))*x^2)^(1/2)/(3*x^4-3*x^2-2)^
(1/2)*EllipticF(1/2*(-3-33^(1/2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))-12
/11/(-3-33^(1/2))^(1/2)*(1-(-3/4-1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4+1/4*33^
(1/2))*x^2)^(1/2)/(3*x^4-3*x^2-2)^(1/2)/(-3+33^(1/2))*(EllipticF(1/2*(-3-3
3^(1/2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))-EllipticE(1/2*(-3-33^(1/2))
^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-2-3x^2+3x^4)^{3/2}}\,dx = \frac{3\left(\sqrt{33}\sqrt{-2}(3\,x^4-3\,x^2-2)-3\,\sqrt{-2}(3\,x^4-3\,x^2-2)\right)\sqrt{\sqrt{33}-3}E(\arcsin}{}$$

input
```
integrate(1/(3*x^4-3*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/528*(3*(sqrt(33)*sqrt(-2)*(3*x^4 - 3*x^2 - 2) - 3*sqrt(-2)*(3*x^4 - 3*x^
2 - 2))*sqrt(sqrt(33) - 3)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(33) - 3)), -1
/4*sqrt(33) - 7/4) + (sqrt(33)*sqrt(-2)*(3*x^4 - 3*x^2 - 2) + 21*sqrt(-2)*
(3*x^4 - 3*x^2 - 2))*sqrt(sqrt(33) - 3)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(
33) - 3)), -1/4*sqrt(33) - 7/4) + 24*sqrt(3*x^4 - 3*x^2 - 2)*(3*x^3 - 7*x)
)/(3*x^4 - 3*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(-2-3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4-3x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4-3*x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 - 3*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2-3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-3\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-3*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 - 3*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-3\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-3*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - 3*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-3\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(3*x^4 - 3*x^2 - 2)^(3/2),x)
```

output
```
int(1/(3*x^4 - 3*x^2 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-3x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-3x^2-2}}{9x^8-18x^6-3x^4+12x^2+4}dx$$

input
```
int(1/(3*x^4-3*x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - 3*x**2 - 2)/(9*x**8 - 18*x**6 - 3*x**4 + 12*x**2 + 4),x)
```

3.213 $\int \frac{1}{(-2-4x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 235

$$\int \frac{1}{(-2-4x^2+3x^4)^{3/2}}\,dx = -\frac{x(7-3x^2)}{20\sqrt{-2-4x^2+3x^4}} - \frac{\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}\sqrt{2+\left(2-\sqrt{10}\right)x^2}\sqrt{2+\left(2+\sqrt{10}\right)x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right)\middle|\frac{1}{3}(-7-2\sqrt{10}\right.}{20\sqrt{-2-4x^2+3x^4}} - \frac{\sqrt{\frac{1}{5}\left(-2+\sqrt{10}\right)}\sqrt{2+\left(2-\sqrt{10}\right)x^2}\sqrt{2+\left(2+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right),\frac{1}{3}(-7\right.}{8\sqrt{-2-4x^2+3x^4}}$$

output
```
-1/20*x*(-3*x^2+7)/(3*x^4-4*x^2-2)^(1/2)-1/40*(-4+2*10^(1/2))^(1/2)*(2+(2-
10^(1/2))*x^2)^(1/2)*(2+(2+10^(1/2))*x^2)^(1/2)*EllipticE(1/2*(-4+2*10^(1/
2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))/(3*x^4-4*x^2-2)^(1/2)-1/40*(-10+
5*10^(1/2))^(1/2)*(2+(2-10^(1/2))*x^2)^(1/2)*(2+(2+10^(1/2))*x^2)^(1/2)*El
lipticF(1/2*(-4+2*10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))/(3*x^4-4
*x^2-2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.65 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-2-4x^2+3x^4)^{3/2}}\,dx = \frac{3x(-7+3x^2)-3i\sqrt{2+\sqrt{10}}\sqrt{2+4x^2-3x^4}E\left(i\text{arcsinh}\left(\sqrt{1+\sqrt{\frac{5}{2}}}x\right)|\frac{1}{3}(\right.}{60\sqrt{-2-}}$$

input
```
Integrate[(-2 - 4*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(3*x*(-7 + 3*x^2) - (3*I)*Sqrt[2 + Sqrt[10]]*Sqrt[2 + 4*x^2 - 3*x^4]*Ellip
ticE[I*ArcSinh[Sqrt[1 + Sqrt[5/2]]*x], (-7 + 2*Sqrt[10])/3] + ((3*I)*(5 +
Sqrt[10])*Sqrt[2 + 4*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/2]]*
x], (-7 + 2*Sqrt[10])/3])/Sqrt[2 + Sqrt[10]])/(60*Sqrt[-2 - 4*x^2 + 3*x^4]
)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.59, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-4x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{80}\int -\frac{12(x^2+1)}{\sqrt{3x^4-4x^2-2}}dx-\frac{x(7-3x^2)}{20\sqrt{3x^4-4x^2-2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{20}\int \frac{x^2+1}{\sqrt{3x^4-4x^2-2}}dx-\frac{x(7-3x^2)}{20\sqrt{3x^4-4x^2-2}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{3}{20}\left(\frac{1}{3}\left(5+\sqrt{10}\right)\int \frac{1}{\sqrt{3x^4-4x^2-2}}dx+\frac{1}{6}\int -\frac{2\left(-3x^2+\sqrt{10}+2\right)}{\sqrt{3x^4-4x^2-2}}dx\right)-\frac{x\left(7-3x^2\right)}{20\sqrt{3x^4-4x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{20}\left(\frac{1}{3}\left(5+\sqrt{10}\right)\int \frac{1}{\sqrt{3x^4-4x^2-2}}dx-\frac{1}{3}\int \frac{-3x^2+\sqrt{10}+2}{\sqrt{3x^4-4x^2-2}}dx\right)-\frac{x\left(7-3x^2\right)}{20\sqrt{3x^4-4x^2-2}}$$

$$\Big\downarrow \text{1411}$$

$$-\frac{3}{20}\left(\frac{\left(5+\sqrt{10}\right)\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-2}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+2}{\left(2-\sqrt{10}\right)x^2+2}}\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-2}}\right),\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{6\sqrt[4]{10}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+2}}\sqrt{3x^4-4x^2-2}}\right.$$

$$\frac{x\left(7-3x^2\right)}{20\sqrt{3x^4-4x^2-2}}$$

$$\Big\downarrow \text{1498}$$

$$-\frac{3}{20}\left(\frac{\left(5+\sqrt{10}\right)\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-2}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+2}{\left(2-\sqrt{10}\right)x^2+2}}\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-2}}\right),\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{6\sqrt[4]{10}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+2}}\sqrt{3x^4-4x^2-2}}\right.$$

$$\frac{x\left(7-3x^2\right)}{20\sqrt{3x^4-4x^2-2}}$$

input
```
Int[(-2 - 4*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/20*(x*(7 - 3*x^2))/Sqrt[-2 - 4*x^2 + 3*x^4] - (3*((-((x*(2 - Sqrt[10] -
 3*x^2))/Sqrt[-2 - 4*x^2 + 3*x^4]) - (10^(1/4)*Sqrt[-2 - (2 - Sqrt[10])*x^
2]*Sqrt[(2 + (2 + Sqrt[10])*x^2)/(2 + (2 - Sqrt[10])*x^2)]*EllipticE[ArcSi
n[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 - (2 - Sqrt[10])*x^2]], (5 - Sqrt[10])/10])/
(Sqrt[(2 + (2 - Sqrt[10])*x^2)^(-1)]*Sqrt[-2 - 4*x^2 + 3*x^4]))/3 + ((5 +
Sqrt[10])*Sqrt[-2 - (2 - Sqrt[10])*x^2]*Sqrt[(2 + (2 + Sqrt[10])*x^2)/(2 +
 (2 - Sqrt[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 - (2 -
Sqrt[10])*x^2]], (5 - Sqrt[10])/10])/(6*10^(1/4)*Sqrt[(2 + (2 - Sqrt[10])*
x^2)^(-1)]*Sqrt[-2 - 4*x^2 + 3*x^4])))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.98

method	result
risch	$\frac{x(3x^2-7)}{20\sqrt{3x^4-4x^2-2}}-\frac{3\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{10\sqrt{-4-2\sqrt{10}}\sqrt{3x^4-4x^2-2}}-\frac{6\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1}}{}$
default	$-\frac{6\left(\frac{7}{120}x-\frac{1}{40}x^3\right)}{\sqrt{3x^4-4x^2-2}}-\frac{3\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{10\sqrt{-4-2\sqrt{10}}\sqrt{3x^4-4x^2-2}}-\frac{6\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{}}{}$
elliptic	$-\frac{6\left(\frac{7}{120}x-\frac{1}{40}x^3\right)}{\sqrt{3x^4-4x^2-2}}-\frac{3\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{10\sqrt{-4-2\sqrt{10}}\sqrt{3x^4-4x^2-2}}-\frac{6\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{}}{}$

input
```
int(1/(3*x^4-4*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*x*(3*x^2-7)/(3*x^4-4*x^2-2)^(1/2)-3/10/(-4-2*10^(1/2))^(1/2)*(1-(-1-1
/2*10^(1/2))*x^2)^(1/2)*(1-(-1+1/2*10^(1/2))*x^2)^(1/2)/(3*x^4-4*x^2-2)^(1
/2)*EllipticF(1/2*(-4-2*10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))-6/
5/(-4-2*10^(1/2))^(1/2)*(1-(-1-1/2*10^(1/2))*x^2)^(1/2)*(1-(-1+1/2*10^(1/2
))*x^2)^(1/2)/(3*x^4-4*x^2-2)^(1/2)/(-4+2*10^(1/2))*(EllipticF(1/2*(-4-2*1
0^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))-EllipticE(1/2*(-4-2*10^(1/2
))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.66

$$\int \frac{1}{(-2-4x^2+3x^4)^{3/2}}\,dx = \frac{4\sqrt{-2}(3\,x^4-4\,x^2-2)\sqrt{\frac{1}{2}\sqrt{10}-1}F\left(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{10}-1}\right)\mid -\frac{2}{3}\sqrt{10}-\frac{7}{3}\right.}{}$$

input
```
integrate(1/(3*x^4-4*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/40*(4*sqrt(-2)*(3*x^4 - 4*x^2 - 2)*sqrt(1/2*sqrt(10) - 1)*elliptic_f(arc
sin(x*sqrt(1/2*sqrt(10) - 1)), -2/3*sqrt(10) - 7/3) + (sqrt(10)*sqrt(-2)*(
3*x^4 - 4*x^2 - 2) - 2*sqrt(-2)*(3*x^4 - 4*x^2 - 2))*sqrt(1/2*sqrt(10) - 1
)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(10) - 1)), -2/3*sqrt(10) - 7/3) + 2*sq
rt(3*x^4 - 4*x^2 - 2)*(3*x^3 - 7*x))/(3*x^4 - 4*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(-2-4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4-4x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4-4*x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 - 4*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2-4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-4\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-4*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 - 4*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-4\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-4*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - 4*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-4\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(3*x^4 - 4*x^2 - 2)^(3/2),x)
```

output
```
int(1/(3*x^4 - 4*x^2 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-4x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-4x^2-2}}{9x^8-24x^6+4x^4+16x^2+4}dx$$

input
```
int(1/(3*x^4-4*x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - 4*x**2 - 2)/(9*x**8 - 24*x**6 + 4*x**4 + 16*x**2 + 4),x)
```

3.214 $\int \frac{1}{(-2-5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{1}{(-2-5x^2+3x^4)^{3/2}}\,dx = -\frac{x(37-15x^2)}{98\sqrt{-2-5x^2+3x^4}} - \frac{5\sqrt{2-x^2}\sqrt{1+3x^2}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)}{98\sqrt{-2-5x^2+3x^4}} - \frac{\sqrt{2-x^2}\sqrt{1+3x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-6\right)}{14\sqrt{-2-5x^2+3x^4}}$$

output
```
-1/98*x*(-15*x^2+37)/(3*x^4-5*x^2-2)^(1/2)-5/98*(-x^2+2)^(1/2)*(3*x^2+1)^(
1/2)*EllipticE(1/2*x*2^(1/2),I*6^(1/2))/(3*x^4-5*x^2-2)^(1/2)-1/14*(-x^2+2
)^(1/2)*(3*x^2+1)^(1/2)*EllipticF(1/2*x*2^(1/2),I*6^(1/2))/(3*x^4-5*x^2-2)
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-2-5x^2+3x^4)^{3/2}}\,dx = \frac{-37x+15x^3-5i\sqrt{6}\sqrt{2-x^2}\sqrt{1+3x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{3}x\right)\mid-\frac{1}{6}\right)+7i\sqrt{6}\sqrt{2-}}{98\sqrt{-2-5x^2+3x^4}}$$

input
```
Integrate[(-2 - 5*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(-37*x + 15*x^3 - (5*I)*Sqrt[6]*Sqrt[2 - x^2]*Sqrt[1 + 3*x^2]*EllipticE[I*
ArcSinh[Sqrt[3]*x], -1/6] + (7*I)*Sqrt[6]*Sqrt[2 - x^2]*Sqrt[1 + 3*x^2]*El
lipticF[I*ArcSinh[Sqrt[3]*x], -1/6])/(98*Sqrt[-2 - 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.64, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1410, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-5x^2-2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{98}\int -\frac{3(5x^2+4)}{\sqrt{3x^4-5x^2-2}}dx-\frac{x(37-15x^2)}{98\sqrt{3x^4-5x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{98}\int \frac{5x^2+4}{\sqrt{3x^4-5x^2-2}}dx-\frac{x(37-15x^2)}{98\sqrt{3x^4-5x^2-2}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{3}{98}\left(14\int \frac{1}{\sqrt{3x^4-5x^2-2}}dx+\frac{5}{6}\int -\frac{6(2-x^2)}{\sqrt{3x^4-5x^2-2}}dx\right)-\frac{x(37-15x^2)}{98\sqrt{3x^4-5x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{98}\left(14\int \frac{1}{\sqrt{3x^4-5x^2-2}}dx-5\int \frac{2-x^2}{\sqrt{3x^4-5x^2-2}}dx\right)-\frac{x(37-15x^2)}{98\sqrt{3x^4-5x^2-2}}$$

$$\Big\downarrow \text{1410}$$

$$-\frac{3}{98}\left(\frac{2\sqrt{7}\sqrt{x^2-2}\sqrt{3x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{7}x}{\sqrt{x^2-2}}\right),\frac{1}{7}\right)}{\sqrt{3x^4-5x^2-2}}-5\int \frac{2-x^2}{\sqrt{3x^4-5x^2-2}}dx\right)-\frac{x(37-15x^2)}{98\sqrt{3x^4-5x^2-2}}$$

$$\Big\downarrow \text{1498}$$

$$-\frac{3}{98}\left(\frac{2\sqrt{7}\sqrt{x^2-2}\sqrt{3x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{7}x}{\sqrt{x^2-2}}\right),\frac{1}{7}\right)}{\sqrt{3x^4-5x^2-2}}-5\left(\frac{\sqrt{7}\sqrt{x^2-2}\sqrt{\frac{3x^2+1}{2-x^2}}E\left(\arcsin\left(\frac{\sqrt{7}x}{\sqrt{x^2-2}}\right)|\frac{1}{7}\right)}{3\sqrt{\frac{1}{2-x^2}}\sqrt{3x^4-5x^2-2}}-\right.\right.$$
$$\frac{x(37-15x^2)}{98\sqrt{3x^4-5x^2-2}}$$

input
```
Int[(-2 - 5*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/98*(x*(37 - 15*x^2))/Sqrt[-2 - 5*x^2 + 3*x^4] - (3*(-5*(-1/3*(x*(1 + 3*
x^2))/Sqrt[-2 - 5*x^2 + 3*x^4] + (Sqrt[7]*Sqrt[-2 + x^2]*Sqrt[(1 + 3*x^2)/
(2 - x^2)]*EllipticE[ArcSin[(Sqrt[7]*x)/Sqrt[-2 + x^2]], 1/7])/(3*Sqrt[(2
- x^2)^(-1)]*Sqrt[-2 - 5*x^2 + 3*x^4])) + (2*Sqrt[7]*Sqrt[-2 + x^2]*Sqrt[1
 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt[-2 + x^2]], 1/7])/Sqrt[-2 - 5*
x^2 + 3*x^4]))/98
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11

method	result
risch	$\frac{x(15x^2-37)}{98\sqrt{3x^4-5x^2-2}}+\frac{i\sqrt{3}\,\sqrt{3x^2+1}\,\sqrt{-2x^2+4}\ \mathrm{EllipticF}\left(ix\sqrt{3},\frac{i\sqrt{6}}{6}\right)}{49\sqrt{3x^4-5x^2-2}}+\frac{5i\sqrt{3}\,\sqrt{3x^2+1}\,\sqrt{-2x^2+4}\left(\mathrm{EllipticF}\left(ix\sqrt{3},\frac{i\sqrt{6}}{6}\right)-\mathrm{Elliptic}\right.}{98\sqrt{3x^4-5x^2-2}}$
default	$-\frac{6\left(\frac{37}{588}x-\frac{5}{196}x^3\right)}{\sqrt{3x^4-5x^2-2}}+\frac{i\sqrt{3}\,\sqrt{3x^2+1}\,\sqrt{-2x^2+4}\ \mathrm{EllipticF}\left(ix\sqrt{3},\frac{i\sqrt{6}}{6}\right)}{49\sqrt{3x^4-5x^2-2}}+\frac{5i\sqrt{3}\,\sqrt{3x^2+1}\,\sqrt{-2x^2+4}\left(\mathrm{EllipticF}\left(ix\sqrt{3},\frac{i\sqrt{6}}{6}\right)-\mathrm{Ellip}\right.}{98\sqrt{3x^4-5x^2-2}}$
elliptic	$-\frac{6\left(\frac{37}{588}x-\frac{5}{196}x^3\right)}{\sqrt{3x^4-5x^2-2}}+\frac{i\sqrt{3}\,\sqrt{3x^2+1}\,\sqrt{-2x^2+4}\ \mathrm{EllipticF}\left(ix\sqrt{3},\frac{i\sqrt{6}}{6}\right)}{49\sqrt{3x^4-5x^2-2}}+\frac{5i\sqrt{3}\,\sqrt{3x^2+1}\,\sqrt{-2x^2+4}\left(\mathrm{EllipticF}\left(ix\sqrt{3},\frac{i\sqrt{6}}{6}\right)-\mathrm{Ellip}\right.}{98\sqrt{3x^4-5x^2-2}}$

input
```
int(1/(3*x^4-5*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/98*x*(15*x^2-37)/(3*x^4-5*x^2-2)^(1/2)+1/49*I*3^(1/2)*(3*x^2+1)^(1/2)*(-
2*x^2+4)^(1/2)/(3*x^4-5*x^2-2)^(1/2)*EllipticF(I*3^(1/2)*x,1/6*I*6^(1/2))+
5/98*I*3^(1/2)*(3*x^2+1)^(1/2)*(-2*x^2+4)^(1/2)/(3*x^4-5*x^2-2)^(1/2)*(Ell
ipticF(I*3^(1/2)*x,1/6*I*6^(1/2))-EllipticE(I*3^(1/2)*x,1/6*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-2-5x^2+3x^4)^{3/2}}\,dx=\frac{5\,\sqrt{2}\sqrt{-2}(3\,x^4-5\,x^2-2)E(\arcsin\left(\frac{1}{2}\,\sqrt{2}x\right)\,|-6)+19\,\sqrt{2}\sqrt{-2}(3\,x^4-5\,x^2}{392\,(3\,x^4-5\,x^2}$$

input
```
integrate(1/(3*x^4-5*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/392*(5*sqrt(2)*sqrt(-2)*(3*x^4 - 5*x^2 - 2)*elliptic_e(arcsin(1/2*sqrt(2
)*x), -6) + 19*sqrt(2)*sqrt(-2)*(3*x^4 - 5*x^2 - 2)*elliptic_f(arcsin(1/2*
sqrt(2)*x), -6) + 4*sqrt(3*x^4 - 5*x^2 - 2)*(15*x^3 - 37*x))/(3*x^4 - 5*x^
2 - 2)
```

Sympy [F]

$$\int \frac{1}{(-2-5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4-5x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4-5*x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 - 5*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2-5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-5\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-5*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 - 5*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-5\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-5*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - 5*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-5\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(3*x^4 - 5*x^2 - 2)^(3/2),x)
```

output
```
int(1/(3*x^4 - 5*x^2 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-5x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-5x^2-2}}{9x^8-30x^6+13x^4+20x^2+4}dx$$

input
```
int(1/(3*x^4-5*x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - 5*x**2 - 2)/(9*x**8 - 30*x**6 + 13*x**4 + 20*x**2 + 4),x
)
```

3.215 $\int \frac{1}{(-2-6x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 225

$$\int \frac{1}{(-2-6x^2+3x^4)^{3/2}}\,dx = -\frac{x(8-3x^2)}{20\sqrt{-2-6x^2+3x^4}} - \frac{\sqrt{\frac{3}{3+\sqrt{15}}}\sqrt{2+\left(3-\sqrt{15}\right)x^2}\sqrt{2+\left(3+\sqrt{15}\right)x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{2}\left(-3+\sqrt{15}\right)}x\right)\middle|-4-\sqrt{15}\right)}{20\sqrt{-2-6x^2+3x^4}} - \frac{\sqrt{2+\left(3-\sqrt{15}\right)x^2}\sqrt{2+\left(3+\sqrt{15}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(-3+\sqrt{15}\right)}x\right),-4-\sqrt{15}\right)}{4\sqrt{5\left(3+\sqrt{15}\right)}\sqrt{-2-6x^2+3x^4}}$$

output
```
-1/20*x*(-3*x^2+8)/(3*x^4-6*x^2-2)^(1/2)-1/20*3^(1/2)/(3+15^(1/2))^(1/2)*(
2+(3-15^(1/2))*x^2)^(1/2)*(2+(3+15^(1/2))*x^2)^(1/2)*EllipticE(1/2*(-6+2*1
5^(1/2))^(1/2)*x,1/2*I*6^(1/2)+1/2*I*10^(1/2))/(3*x^4-6*x^2-2)^(1/2)-1/4*(
2+(3-15^(1/2))*x^2)^(1/2)*(2+(3+15^(1/2))*x^2)^(1/2)*EllipticF(1/2*(-6+2*1
5^(1/2))^(1/2)*x,1/2*I*6^(1/2)+1/2*I*10^(1/2))/(15+5*15^(1/2))^(1/2)/(3*x^
4-6*x^2-2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-2-6x^2+3x^4)^{3/2}}\,dx = \frac{3x(-8+3x^2)-3i\sqrt{3+\sqrt{15}}\sqrt{2+6x^2-3x^4}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{-3+\sqrt{15}}}x\right)|-4+\ldots}{60\sqrt{-2-6x^2+\ldots}}$$

input
```
Integrate[(-2 - 6*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(3*x*(-8 + 3*x^2) - (3*I)*Sqrt[3 + Sqrt[15]]*Sqrt[2 + 6*x^2 - 3*x^4]*Ellip
ticE[I*ArcSinh[Sqrt[3/(-3 + Sqrt[15])]*x], -4 + Sqrt[15]] + ((3*I)*(5 + Sq
rt[15])*Sqrt[2 + 6*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(-3 + Sqrt[15])
]*x], -4 + Sqrt[15]])/Sqrt[3 + Sqrt[15]])/(60*Sqrt[-2 - 6*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.63, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-6x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{120}\int -\frac{6(3x^2+2)}{\sqrt{3x^4-6x^2-2}}dx - \frac{x(8-3x^2)}{20\sqrt{3x^4-6x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{20}\int \frac{3x^2+2}{\sqrt{3x^4-6x^2-2}}dx - \frac{x(8-3x^2)}{20\sqrt{3x^4-6x^2-2}}$$

$$\Big\downarrow \text{1501}$$

$$\frac{1}{20}\left(-\left(\left(5+\sqrt{15}\right)\int \frac{1}{\sqrt{3x^4-6x^2-2}}dx\right)-\frac{1}{2}\int -\frac{2\left(-3x^2+\sqrt{15}+3\right)}{\sqrt{3x^4-6x^2-2}}dx\right)-\frac{x\left(8-3x^2\right)}{20\sqrt{3x^4-6x^2-2}}$$

$\Big\downarrow$ 27

$$\frac{1}{20}\left(\int \frac{-3x^2+\sqrt{15}+3}{\sqrt{3x^4-6x^2-2}}dx-\left(5+\sqrt{15}\right)\int \frac{1}{\sqrt{3x^4-6x^2-2}}dx\right)-\frac{x\left(8-3x^2\right)}{20\sqrt{3x^4-6x^2-2}}$$

$\Big\downarrow$ 1411

$$\frac{1}{20}\left(\int \frac{-3x^2+\sqrt{15}+3}{\sqrt{3x^4-6x^2-2}}dx-\frac{\left(5+\sqrt{15}\right)\sqrt{-\left(\left(3-\sqrt{15}\right)x^2\right)-2}\sqrt{\frac{\left(3+\sqrt{15}\right)x^2+2}{\left(3-\sqrt{15}\right)x^2+2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{15}}{\sqrt{-\left(\left(3-\sqrt{15}\right.\right.}}\right.\right.}{2\sqrt[4]{15}\sqrt{\frac{1}{\left(3-\sqrt{15}\right)x^2+2}}\sqrt{3x^4-6x^2-2}}\right.$$
$$\frac{x\left(8-3x^2\right)}{20\sqrt{3x^4-6x^2-2}}$$

$\Big\downarrow$ 1498

$$\frac{1}{20}\left(-\frac{\left(5+\sqrt{15}\right)\sqrt{-\left(\left(3-\sqrt{15}\right)x^2\right)-2}\sqrt{\frac{\left(3+\sqrt{15}\right)x^2+2}{\left(3-\sqrt{15}\right)x^2+2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{15}x}{\sqrt{-\left(\left(3-\sqrt{15}\right)x^2\right)-2}}\right),\frac{1}{10}\left(5-\sqrt{15}\right)\right.}{2\sqrt[4]{15}\sqrt{\frac{1}{\left(3-\sqrt{15}\right)x^2+2}}\sqrt{3x^4-6x^2-2}}\right.$$
$$\frac{x\left(8-3x^2\right)}{20\sqrt{3x^4-6x^2-2}}$$

input
```
Int[(-2 - 6*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/20*(x*(8 - 3*x^2))/Sqrt[-2 - 6*x^2 + 3*x^4] + ((x*(3 - Sqrt[15] - 3*x^2
))/Sqrt[-2 - 6*x^2 + 3*x^4] + (15^(1/4)*Sqrt[-2 - (3 - Sqrt[15])*x^2]*Sqrt
[(2 + (3 + Sqrt[15])*x^2)/(2 + (3 - Sqrt[15])*x^2)]*EllipticE[ArcSin[(Sqrt
[2]*15^(1/4)*x)/Sqrt[-2 - (3 - Sqrt[15])*x^2]], (5 - Sqrt[15])/10])/(Sqrt[
(2 + (3 - Sqrt[15])*x^2)^(-1)]*Sqrt[-2 - 6*x^2 + 3*x^4]) - ((5 + Sqrt[15])
*Sqrt[-2 - (3 - Sqrt[15])*x^2]*Sqrt[(2 + (3 + Sqrt[15])*x^2)/(2 + (3 - Sqr
t[15])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[-2 - (3 - Sqrt[15]
)*x^2]], (5 - Sqrt[15])/10])/(2*15^(1/4)*Sqrt[(2 + (3 - Sqrt[15])*x^2)^(-1
)]*Sqrt[-2 - 6*x^2 + 3*x^4]))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.02

method	result
risch	$\frac{x(3x^2-8)}{20\sqrt{3x^4-6x^2-2}}-\frac{\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x^2}\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{15}}{2}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{-6-2\sqrt{15}}}{2},\frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{5\sqrt{-6-2\sqrt{15}}\sqrt{3x^4-6x^2-2}}-\frac{6\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x^2}\sqrt{1}}{}$
default	$-\frac{6\left(\frac{1}{15}x-\frac{1}{40}x^3\right)}{\sqrt{3x^4-6x^2-2}}-\frac{\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x^2}\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{15}}{2}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{-6-2\sqrt{15}}}{2},\frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{5\sqrt{-6-2\sqrt{15}}\sqrt{3x^4-6x^2-2}}-\frac{6\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x^2}\sqrt{1}}{}$
elliptic	$-\frac{6\left(\frac{1}{15}x-\frac{1}{40}x^3\right)}{\sqrt{3x^4-6x^2-2}}-\frac{\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x^2}\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{15}}{2}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{-6-2\sqrt{15}}}{2},\frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{5\sqrt{-6-2\sqrt{15}}\sqrt{3x^4-6x^2-2}}-\frac{6\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x^2}\sqrt{1}}{}$

input
```
int(1/(3*x^4-6*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*x*(3*x^2-8)/(3*x^4-6*x^2-2)^(1/2)-1/5/(-6-2*15^(1/2))^(1/2)*(1-(-3/2-
1/2*15^(1/2))*x^2)^(1/2)*(1-(-3/2+1/2*15^(1/2))*x^2)^(1/2)/(3*x^4-6*x^2-2)
^(1/2)*EllipticF(1/2*x*(-6-2*15^(1/2))^(1/2),1/2*I*10^(1/2)-1/2*I*6^(1/2))
-6/5/(-6-2*15^(1/2))^(1/2)*(1-(-3/2-1/2*15^(1/2))*x^2)^(1/2)*(1-(-3/2+1/2*
15^(1/2))*x^2)^(1/2)/(3*x^4-6*x^2-2)^(1/2)/(-6+2*15^(1/2))*(EllipticF(1/2*
x*(-6-2*15^(1/2))^(1/2),1/2*I*10^(1/2)-1/2*I*6^(1/2))-EllipticE(1/2*x*(-6-
2*15^(1/2))^(1/2),1/2*I*10^(1/2)-1/2*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(-2-6x^2+3x^4\right)^{3/2}}\,dx = \frac{3\left(\sqrt{15}\sqrt{-2}(3\,x^4-6\,x^2-2)-3\,\sqrt{-2}(3\,x^4-6\,x^2-2)\right)\sqrt{\frac{1}{2}\,\sqrt{15}-\frac{3}{2}}E(\text{arcsi}}{}$$

input
```
integrate(1/(3*x^4-6*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/120*(3*(sqrt(15)*sqrt(-2)*(3*x^4 - 6*x^2 - 2) - 3*sqrt(-2)*(3*x^4 - 6*x^
2 - 2))*sqrt(1/2*sqrt(15) - 3/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(15) - 3
/2)), -sqrt(15) - 4) - (sqrt(15)*sqrt(-2)*(3*x^4 - 6*x^2 - 2) - 15*sqrt(-2
)*(3*x^4 - 6*x^2 - 2))*sqrt(1/2*sqrt(15) - 3/2)*elliptic_f(arcsin(x*sqrt(1
/2*sqrt(15) - 3/2)), -sqrt(15) - 4) + 6*sqrt(3*x^4 - 6*x^2 - 2)*(3*x^3 - 8
*x))/(3*x^4 - 6*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{\left(-2-6x^2+3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(3x^4-6x^2-2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4-6*x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 - 6*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(-2-6x^2+3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(3\,x^4-6\,x^2-2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-6*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 - 6*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-6\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-6*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - 6*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-6\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(3*x^4 - 6*x^2 - 2)^(3/2),x)
```

output
```
int(1/(3*x^4 - 6*x^2 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-6x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-6x^2-2}}{9x^8-36x^6+24x^4+24x^2+4}dx$$

input
```
int(1/(3*x^4-6*x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - 6*x**2 - 2)/(9*x**8 - 36*x**6 + 24*x**4 + 24*x**2 + 4),x
)
```

3.216 $\int \frac{1}{(-2-7x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 229

$$\int \frac{1}{(-2-7x^2+3x^4)^{3/2}}\,dx = -\frac{x(61-21x^2)}{146\sqrt{-2-7x^2+3x^4}}$$
$$-\frac{7\sqrt{-7+\sqrt{73}}\sqrt{4+\left(7-\sqrt{73}\right)x^2}\sqrt{4+\left(7+\sqrt{73}\right)x^2}\,E\left(\arcsin\left(\frac{1}{2}\sqrt{-7+\sqrt{73}}x\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)}{584\sqrt{-2-7x^2+3x^4}}$$
$$-\frac{\sqrt{\frac{1}{73}\left(-7+\sqrt{73}\right)}\sqrt{4+\left(7-\sqrt{73}\right)x^2}\sqrt{4+\left(7+\sqrt{73}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-7+\sqrt{73}}x\right),\frac{1}{12}(-61-\right.}{8\sqrt{-2-7x^2+3x^4}}$$

output
```
-1/146*x*(-21*x^2+61)/(3*x^4-7*x^2-2)^(1/2)-7/584*(-7+73^(1/2))^(1/2)*(4+(
-73^(1/2)+7)*x^2)^(1/2)*(4+(7+73^(1/2))*x^2)^(1/2)*EllipticE(1/2*(-7+73^(1
/2))^(1/2)*x,7/12*I*6^(1/2)+1/12*I*438^(1/2))/(3*x^4-7*x^2-2)^(1/2)-1/584*
(-511+73*73^(1/2))^(1/2)*(4+(-73^(1/2)+7)*x^2)^(1/2)*(4+(7+73^(1/2))*x^2)^
(1/2)*EllipticF(1/2*(-7+73^(1/2))^(1/2)*x,7/12*I*6^(1/2)+1/12*I*438^(1/2))
/(3*x^4-7*x^2-2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.94 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.76

$$\int \frac{1}{(-2-7x^2+3x^4)^{3/2}}\,dx = \frac{12x(-61+21x^2)-42i\sqrt{7+\sqrt{73}}\sqrt{4+14x^2-6x^4}\,E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{6}{-7+\sqrt{73}}}x\right)\right.}{1752\sqrt{}}$$

input
```
Integrate[(-2 - 7*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(12*x*(-61 + 21*x^2) - (42*I)*Sqrt[7 + Sqrt[73]]*Sqrt[4 + 14*x^2 - 6*x^4]*
EllipticE[I*ArcSinh[Sqrt[6/(-7 + Sqrt[73])]*x], (-61 + 7*Sqrt[73])/12] + (
(6*I)*(73 + 7*Sqrt[73])*Sqrt[4 + 14*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[
6/(-7 + Sqrt[73])]*x], (-61 + 7*Sqrt[73])/12])/Sqrt[7 + Sqrt[73]])/(1752*S
qrt[-2 - 7*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.67, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 25, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-7x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{146}\int -\frac{3(7x^2+4)}{\sqrt{3x^4-7x^2-2}}dx-\frac{x(61-21x^2)}{146\sqrt{3x^4-7x^2-2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{146}\int \frac{7x^2+4}{\sqrt{3x^4-7x^2-2}}dx-\frac{x(61-21x^2)}{146\sqrt{3x^4-7x^2-2}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{3}{146}\left(\frac{1}{6}\left(73+7\sqrt{73}\right)\int \frac{1}{\sqrt{3x^4-7x^2-2}}dx+\frac{7}{6}\int -\frac{-6x^2+\sqrt{73}+7}{\sqrt{3x^4-7x^2-2}}dx\right)-\frac{x\left(61-21x^2\right)}{146\sqrt{3x^4-7x^2-2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3}{146}\left(\frac{1}{6}\left(73+7\sqrt{73}\right)\int \frac{1}{\sqrt{3x^4-7x^2-2}}dx-\frac{7}{6}\int \frac{-6x^2+\sqrt{73}+7}{\sqrt{3x^4-7x^2-2}}dx\right)-\frac{x\left(61-21x^2\right)}{146\sqrt{3x^4-7x^2-2}}$$

$$\Big\downarrow \text{1411}$$

$$-\frac{3}{146}\left(\frac{\left(73+7\sqrt{73}\right)\sqrt{-\left(\left(7-\sqrt{73}\right)x^2\right)-4}\sqrt{\frac{\left(7+\sqrt{73}\right)x^2+4}{\left(7-\sqrt{73}\right)x^2+4}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{-\left(\left(7-\sqrt{73}\right)x^2\right)-4}}\right),\frac{1}{146}\left(73-\right.\right.}{12\sqrt{2}\sqrt[4]{73}\sqrt{\frac{1}{\left(7-\sqrt{73}\right)x^2+4}}\sqrt{3x^4-7x^2-2}}\right.$$

$$\frac{x\left(61-21x^2\right)}{146\sqrt{3x^4-7x^2-2}}$$

$$\Big\downarrow \text{1498}$$

$$-\frac{3}{146}\left(\frac{\left(73+7\sqrt{73}\right)\sqrt{-\left(\left(7-\sqrt{73}\right)x^2\right)-4}\sqrt{\frac{\left(7+\sqrt{73}\right)x^2+4}{\left(7-\sqrt{73}\right)x^2+4}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{-\left(\left(7-\sqrt{73}\right)x^2\right)-4}}\right),\frac{1}{146}\left(73-\right.\right.}{12\sqrt{2}\sqrt[4]{73}\sqrt{\frac{1}{\left(7-\sqrt{73}\right)x^2+4}}\sqrt{3x^4-7x^2-2}}\right.$$

$$\frac{x\left(61-21x^2\right)}{146\sqrt{3x^4-7x^2-2}}$$

input
```
Int[(-2 - 7*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/146*(x*(61 - 21*x^2))/Sqrt[-2 - 7*x^2 + 3*x^4] - (3*((-7*((x*(7 - Sqrt[
73] - 6*x^2))/Sqrt[-2 - 7*x^2 + 3*x^4] + (73^(1/4)*Sqrt[-4 - (7 - Sqrt[73]
)*x^2]*Sqrt[(4 + (7 + Sqrt[73])*x^2)/(4 + (7 - Sqrt[73])*x^2)]*EllipticE[A
rcSin[(Sqrt[2]*73^(1/4)*x)/Sqrt[-4 - (7 - Sqrt[73])*x^2]], (73 - 7*Sqrt[73
])/146])/(Sqrt[2]*Sqrt[(4 + (7 - Sqrt[73])*x^2)^(-1)]*Sqrt[-2 - 7*x^2 + 3*
x^4])))/6 + ((73 + 7*Sqrt[73])*Sqrt[-4 - (7 - Sqrt[73])*x^2]*Sqrt[(4 + (7
+ Sqrt[73])*x^2)/(4 + (7 - Sqrt[73])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*73^(1
/4)*x)/Sqrt[-4 - (7 - Sqrt[73])*x^2]], (73 - 7*Sqrt[73])/146])/(12*Sqrt[2]
*73^(1/4)*Sqrt[(4 + (7 - Sqrt[73])*x^2)^(-1)]*Sqrt[-2 - 7*x^2 + 3*x^4])))/
146
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

method	result
risch	$\frac{x(21x^2-61)}{146\sqrt{3x^4-7x^2-2}}-\frac{12\sqrt{1-\left(-\frac{7}{4}-\frac{\sqrt{73}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{7}{4}+\frac{\sqrt{73}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-7-\sqrt{73}}}{2},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{73\sqrt{-7-\sqrt{73}}\,\sqrt{3x^4-7x^2-2}}-\frac{84\sqrt{1-\left(-\frac{7}{4}-\frac{\sqrt{73}}{4}\right)x}}{}$
default	$-\frac{6\left(\frac{61}{876}x-\frac{7}{292}x^3\right)}{\sqrt{3x^4-7x^2-2}}-\frac{12\sqrt{1-\left(-\frac{7}{4}-\frac{\sqrt{73}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{7}{4}+\frac{\sqrt{73}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-7-\sqrt{73}}}{2},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{73\sqrt{-7-\sqrt{73}}\,\sqrt{3x^4-7x^2-2}}-\frac{84\sqrt{1-\left(-\frac{7}{4}-\frac{\sqrt{73}}{4}\right.}}{}$
elliptic	$-\frac{6\left(\frac{61}{876}x-\frac{7}{292}x^3\right)}{\sqrt{3x^4-7x^2-2}}-\frac{12\sqrt{1-\left(-\frac{7}{4}-\frac{\sqrt{73}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{7}{4}+\frac{\sqrt{73}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-7-\sqrt{73}}}{2},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{73\sqrt{-7-\sqrt{73}}\,\sqrt{3x^4-7x^2-2}}-\frac{84\sqrt{1-\left(-\frac{7}{4}-\frac{\sqrt{73}}{4}\right.}}{}$

input
```
int(1/(3*x^4-7*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/146*x*(21*x^2-61)/(3*x^4-7*x^2-2)^(1/2)-12/73/(-7-73^(1/2))^(1/2)*(1-(-7
/4-1/4*73^(1/2))*x^2)^(1/2)*(1-(-7/4+1/4*73^(1/2))*x^2)^(1/2)/(3*x^4-7*x^2
-2)^(1/2)*EllipticF(1/2*x*(-7-73^(1/2))^(1/2),1/12*I*438^(1/2)-7/12*I*6^(1
/2))-84/73/(-7-73^(1/2))^(1/2)*(1-(-7/4-1/4*73^(1/2))*x^2)^(1/2)*(1-(-7/4+
1/4*73^(1/2))*x^2)^(1/2)/(3*x^4-7*x^2-2)^(1/2)/(-7+73^(1/2))*(EllipticF(1/
2*x*(-7-73^(1/2))^(1/2),1/12*I*438^(1/2)-7/12*I*6^(1/2))-EllipticE(1/2*x*(
-7-73^(1/2))^(1/2),1/12*I*438^(1/2)-7/12*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.76

$$\int \frac{1}{(-2-7x^2+3x^4)^{3/2}}\,dx = \frac{7\left(\sqrt{73}\sqrt{-2}(3\,x^4-7\,x^2-2)-7\,\sqrt{-2}(3\,x^4-7\,x^2-2)\right)\sqrt{\sqrt{73}-7}E(\arcsin}{}$$

input
```
integrate(1/(3*x^4-7*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/1168*(7*(sqrt(73)*sqrt(-2)*(3*x^4 - 7*x^2 - 2) - 7*sqrt(-2)*(3*x^4 - 7*x
^2 - 2))*sqrt(sqrt(73) - 7)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(73) - 7)), -
7/12*sqrt(73) - 61/12) - (3*sqrt(73)*sqrt(-2)*(3*x^4 - 7*x^2 - 2) - 77*sqr
t(-2)*(3*x^4 - 7*x^2 - 2))*sqrt(sqrt(73) - 7)*elliptic_f(arcsin(1/2*x*sqrt
(sqrt(73) - 7)), -7/12*sqrt(73) - 61/12) + 8*sqrt(3*x^4 - 7*x^2 - 2)*(21*x
^3 - 61*x))/(3*x^4 - 7*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(-2-7x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4-7x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4-7*x**2-2)**(3/2),x)
```

output
```
Integral((3*x**4 - 7*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2-7x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-7\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-7*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 - 7*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-7x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-7\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-7*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - 7*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-7x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-7\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(3*x^4 - 7*x^2 - 2)^(3/2),x)
```

output
```
int(1/(3*x^4 - 7*x^2 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-7x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-7x^2-2}}{9x^8-42x^6+37x^4+28x^2+4}dx$$

input
```
int(1/(3*x^4-7*x^2-2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - 7*x**2 - 2)/(9*x**8 - 42*x**6 + 37*x**4 + 28*x**2 + 4),x
)
```

3.217 $\int \frac{1}{(-3+7x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 239

$$\int \frac{1}{(-3+7x^2+2x^4)^{3/2}}\,dx = -\frac{x(61+14x^2)}{219\sqrt{-3+7x^2+2x^4}} + \frac{7\sqrt{\frac{1}{6}(7+\sqrt{73})}\sqrt{6-(7-\sqrt{73})x^2}\sqrt{6-(7+\sqrt{73})x^2}E\left(\arcsin\left(\sqrt{\frac{1}{6}(7+\sqrt{73})}x\right)|\frac{1}{12}(-61+7\sqrt{73})\right)}{438\sqrt{-3+7x^2+2x^4}} - \frac{\sqrt{\frac{1}{438}(7+\sqrt{73})}\sqrt{6-(7-\sqrt{73})x^2}\sqrt{6-(7+\sqrt{73})x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{6}(7+\sqrt{73})}x\right),\frac{1}{12}(-61\right.}{6\sqrt{-3+7x^2+2x^4}}$$

output
```
-1/219*x*(14*x^2+61)/(2*x^4+7*x^2-3)^(1/2)+7/2628*(42+6*73^(1/2))^(1/2)*(6
-(-73^(1/2)+7)*x^2)^(1/2)*(6-(7+73^(1/2))*x^2)^(1/2)*EllipticE(1/6*(42+6*7
3^(1/2))^(1/2)*x,1/12*I*438^(1/2)-7/12*I*6^(1/2))/(2*x^4+7*x^2-3)^(1/2)-1/
2628*(3066+438*73^(1/2))^(1/2)*(6-(-73^(1/2)+7)*x^2)^(1/2)*(6-(7+73^(1/2))
*x^2)^(1/2)*EllipticF(1/6*(42+6*73^(1/2))^(1/2)*x,1/12*I*438^(1/2)-7/12*I*
6^(1/2))/(2*x^4+7*x^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.47 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.70

$$\int \frac{1}{(-3+7x^2+2x^4)^{3/2}}\,dx = \frac{-4x(61+14x^2)+14i\sqrt{-7+\sqrt{73}}\sqrt{6-14x^2-4x^4}E\left(i\text{arcsinh}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\right.}{876\sqrt{-3}}$$

input
```
Integrate[(-3 + 7*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(-4*x*(61 + 14*x^2) + (14*I)*Sqrt[-7 + Sqrt[73]]*Sqrt[6 - 14*x^2 - 4*x^4]*
EllipticE[I*ArcSinh[(2*x)/Sqrt[7 + Sqrt[73]]], (-61 - 7*Sqrt[73])/12] - ((
2*I)*(-73 + 7*Sqrt[73])*Sqrt[6 - 14*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)
/Sqrt[7 + Sqrt[73]]], (-61 - 7*Sqrt[73])/12])/Sqrt[-7 + Sqrt[73]])/(876*Sq
rt[-3 + 7*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.56, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1405, 27, 1501, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+7x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{219}\int -\frac{2(6-7x^2)}{\sqrt{2x^4+7x^2-3}}dx - \frac{x(14x^2+61)}{219\sqrt{2x^4+7x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{219}\int \frac{6-7x^2}{\sqrt{2x^4+7x^2-3}}dx - \frac{x(14x^2+61)}{219\sqrt{2x^4+7x^2-3}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{2}{219}\left(\frac{1}{4}\left(73-7\sqrt{73}\right)\int \frac{1}{\sqrt{2x^4+7x^2-3}}dx-\frac{7}{4}\int \frac{4x^2-\sqrt{73}+7}{\sqrt{2x^4+7x^2-3}}dx\right)-\frac{x\left(14x^2+61\right)}{219\sqrt{2x^4+7x^2-3}}$$

$$\Big\downarrow \text{1411}$$

$$-\frac{2}{219}\left(\frac{\left(73-7\sqrt{73}\right)\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{\left(7+\sqrt{73}\right)x^2-6}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{\left(7+\sqrt{73}\right)x^2-6}}\right),\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{8\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{2x^4+7x^2-3}}\right.$$

$$\frac{x\left(14x^2+61\right)}{219\sqrt{2x^4+7x^2-3}}$$

$$\Big\downarrow \text{1498}$$

$$-\frac{2}{219}\left(\frac{\left(73-7\sqrt{73}\right)\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{\left(7+\sqrt{73}\right)x^2-6}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{\left(7+\sqrt{73}\right)x^2-6}}\right),\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{8\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{2x^4+7x^2-3}}\right.$$

$$\frac{x\left(14x^2+61\right)}{219\sqrt{2x^4+7x^2-3}}$$

input
```
Int[(-3 + 7*x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/219*(x*(61 + 14*x^2))/Sqrt[-3 + 7*x^2 + 2*x^4] - (2*((-7*((x*(7 + Sqrt[
73] + 4*x^2))/Sqrt[-3 + 7*x^2 + 2*x^4] - (73^(1/4)*Sqrt[(6 - (7 - Sqrt[73]
)*x^2)/(6 - (7 + Sqrt[73])*x^2)]*Sqrt[-6 + (7 + Sqrt[73])*x^2]*EllipticE[A
rcSin[(Sqrt[2]*73^(1/4)*x)/Sqrt[-6 + (7 + Sqrt[73])*x^2]], (73 + 7*Sqrt[73
])/146])/(Sqrt[3]*Sqrt[(6 - (7 + Sqrt[73])*x^2)^(-1)]*Sqrt[-3 + 7*x^2 + 2*
x^4])))/4 + ((73 - 7*Sqrt[73])*Sqrt[(6 - (7 - Sqrt[73])*x^2)/(6 - (7 + Sqr
t[73])*x^2)]*Sqrt[-6 + (7 + Sqrt[73])*x^2]*EllipticF[ArcSin[(Sqrt[2]*73^(1
/4)*x)/Sqrt[-6 + (7 + Sqrt[73])*x^2]], (73 + 7*Sqrt[73])/146])/(8*Sqrt[3]*
73^(1/4)*Sqrt[(6 - (7 + Sqrt[73])*x^2)^(-1)]*Sqrt[-3 + 7*x^2 + 2*x^4])))/2
19
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{x(14x^2+61)}{219\sqrt{2x^4+7x^2-3}}-\frac{24\sqrt{1-\left(-\frac{\sqrt{73}}{6}+\frac{7}{6}\right)x^2}\,\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{42-6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{73\sqrt{42-6\sqrt{73}}\,\sqrt{2x^4+7x^2-3}}+\frac{168\sqrt{1-\left(-\frac{\sqrt{73}}{6}+\frac{7}{6}\right.}}{}$
default	$-\frac{4\left(\frac{61}{876}x+\frac{7}{438}x^3\right)}{\sqrt{2x^4+7x^2-3}}-\frac{24\sqrt{1-\left(-\frac{\sqrt{73}}{6}+\frac{7}{6}\right)x^2}\,\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{42-6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{73\sqrt{42-6\sqrt{73}}\,\sqrt{2x^4+7x^2-3}}+\frac{168\sqrt{1-\left(-\frac{\sqrt{73}}{6}+\frac{7}{6}\right)}}{}$
elliptic	$-\frac{4\left(\frac{61}{876}x+\frac{7}{438}x^3\right)}{\sqrt{2x^4+7x^2-3}}-\frac{24\sqrt{1-\left(-\frac{\sqrt{73}}{6}+\frac{7}{6}\right)x^2}\,\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{42-6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{73\sqrt{42-6\sqrt{73}}\,\sqrt{2x^4+7x^2-3}}+\frac{168\sqrt{1-\left(-\frac{\sqrt{73}}{6}+\frac{7}{6}\right)}}{}$

input

```
int(1/(2*x^4+7*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/219*x*(14*x^2+61)/(2*x^4+7*x^2-3)^(1/2)-24/73/(42-6*73^(1/2))^(1/2)*(1-
(-1/6*73^(1/2)+7/6)*x^2)^(1/2)*(1-(7/6+1/6*73^(1/2))*x^2)^(1/2)/(2*x^4+7*x
^2-3)^(1/2)*EllipticF(1/6*(42-6*73^(1/2))^(1/2)*x,7/12*I*6^(1/2)+1/12*I*43
8^(1/2))+168/73/(42-6*73^(1/2))^(1/2)*(1-(-1/6*73^(1/2)+7/6)*x^2)^(1/2)*(1
-(7/6+1/6*73^(1/2))*x^2)^(1/2)/(2*x^4+7*x^2-3)^(1/2)/(7+73^(1/2))*(Ellipti
cF(1/6*(42-6*73^(1/2))^(1/2)*x,7/12*I*6^(1/2)+1/12*I*438^(1/2))-EllipticE(
1/6*(42-6*73^(1/2))^(1/2)*x,7/12*I*6^(1/2)+1/12*I*438^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-3+7x^2+2x^4)^{3/2}}\,dx =$$

$$-\frac{7\left(\sqrt{73}\sqrt{-3}(2\,x^4+7\,x^2-3)+7\,\sqrt{-3}(2\,x^4+7\,x^2-3)\right)\sqrt{\frac{1}{6}\,\sqrt{73}+\frac{7}{6}}E(\arcsin\left(x\sqrt{\frac{1}{6}\,\sqrt{73}+\frac{7}{6}}\right)\mid\frac{7}{12}\,\sqrt{73}}{}$$

input
```
integrate(1/(2*x^4+7*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
-1/1314*(7*(sqrt(73)*sqrt(-3)*(2*x^4 + 7*x^2 - 3) + 7*sqrt(-3)*(2*x^4 + 7*
x^2 - 3))*sqrt(1/6*sqrt(73) + 7/6)*elliptic_e(arcsin(x*sqrt(1/6*sqrt(73) +
 7/6)), 7/12*sqrt(73) - 61/12) - (13*sqrt(73)*sqrt(-3)*(2*x^4 + 7*x^2 - 3)
 + 7*sqrt(-3)*(2*x^4 + 7*x^2 - 3))*sqrt(1/6*sqrt(73) + 7/6)*elliptic_f(arc
sin(x*sqrt(1/6*sqrt(73) + 7/6)), 7/12*sqrt(73) - 61/12) + 6*sqrt(2*x^4 + 7
*x^2 - 3)*(14*x^3 + 61*x))/(2*x^4 + 7*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(-3+7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+7x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+7*x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 + 7*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3+7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+7\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+7*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 7*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3+7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+7\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+7*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 7*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3+7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+7\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(7*x^2 + 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(7*x^2 + 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3+7x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+7x^2-3}}{4x^8+28x^6+37x^4-42x^2+9}dx$$

input
```
int(1/(2*x^4+7*x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 7*x**2 - 3)/(4*x**8 + 28*x**6 + 37*x**4 - 42*x**2 + 9),x
)
```

3.218 $\int \frac{1}{(-3+6x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 225

$$\int \frac{1}{(-3+6x^2+2x^4)^{3/2}}\,dx = -\frac{x(4+x^2)}{15\sqrt{-3+6x^2+2x^4}} + \frac{\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}\sqrt{3-\left(3-\sqrt{15}\right)x^2}\sqrt{3-\left(3+\sqrt{15}\right)x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right)\middle|-4+\sqrt{15}\right)}{30\sqrt{-3+6x^2+2x^4}} - \frac{\sqrt{\frac{1}{5}\left(3+\sqrt{15}\right)}\sqrt{3-\left(3-\sqrt{15}\right)x^2}\sqrt{3-\left(3+\sqrt{15}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right),-4+\sqrt{15}\right)}{18\sqrt{-3+6x^2+2x^4}}$$

output
```
-1/15*x*(x^2+4)/(2*x^4+6*x^2-3)^(1/2)+1/90*(9+3*15^(1/2))^(1/2)*(3-(3-15^(
1/2))*x^2)^(1/2)*(3-(3+15^(1/2))*x^2)^(1/2)*EllipticE(1/3*(9+3*15^(1/2))^(
1/2)*x,1/2*I*10^(1/2)-1/2*I*6^(1/2))/(2*x^4+6*x^2-3)^(1/2)-1/90*(15+5*15^(
1/2))^(1/2)*(3-(3-15^(1/2))*x^2)^(1/2)*(3-(3+15^(1/2))*x^2)^(1/2)*Elliptic
F(1/3*(9+3*15^(1/2))^(1/2)*x,1/2*I*10^(1/2)-1/2*I*6^(1/2))/(2*x^4+6*x^2-3)
^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.59 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.70

$$\int \frac{1}{(-3+6x^2+2x^4)^{3/2}}\,dx = \frac{-4x(4+x^2)+2i\sqrt{-3+\sqrt{15}}\sqrt{3-6x^2-2x^4}E\left(i\text{arcsinh}\left(\sqrt{-1+\sqrt{\frac{5}{3}}}x\right)\right|}{60\sqrt{-3+6x}}$$

input
```
Integrate[(-3 + 6*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(-4*x*(4 + x^2) + (2*I)*Sqrt[-3 + Sqrt[15]]*Sqrt[3 - 6*x^2 - 2*x^4]*Ellipt
icE[I*ArcSinh[Sqrt[-1 + Sqrt[5/3]]*x], -4 - Sqrt[15]] - ((2*I)*(-5 + Sqrt[
15])*Sqrt[3 - 6*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/3]]*x],
-4 - Sqrt[15]])/Sqrt[-3 + Sqrt[15]])/(60*Sqrt[-3 + 6*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.66, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+6x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{180}\int -\frac{12(1-x^2)}{\sqrt{2x^4+6x^2-3}}dx - \frac{x(x^2+4)}{15\sqrt{2x^4+6x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}\int \frac{1-x^2}{\sqrt{2x^4+6x^2-3}}dx - \frac{x(x^2+4)}{15\sqrt{2x^4+6x^2-3}}$$

$$\Big\downarrow \text{1501}$$

$$\frac{1}{15}\left(\frac{1}{4}\int \frac{2\left(2x^2-\sqrt{15}+3\right)}{\sqrt{2x^4+6x^2-3}}dx-\frac{1}{2}\left(5-\sqrt{15}\right)\int \frac{1}{\sqrt{2x^4+6x^2-3}}dx\right)-\frac{x\left(x^2+4\right)}{15\sqrt{2x^4+6x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(\frac{1}{2}\int \frac{2x^2-\sqrt{15}+3}{\sqrt{2x^4+6x^2-3}}dx-\frac{1}{2}\left(5-\sqrt{15}\right)\int \frac{1}{\sqrt{2x^4+6x^2-3}}dx\right)-\frac{x\left(x^2+4\right)}{15\sqrt{2x^4+6x^2-3}}$$

$$\Big\downarrow \text{1411}$$

$$\frac{1}{15}\left(\frac{1}{2}\int \frac{2x^2-\sqrt{15}+3}{\sqrt{2x^4+6x^2-3}}dx-\frac{\left(5-\sqrt{15}\right)\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{\left(3+\sqrt{15}\right)x^2-3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{15}x}{\sqrt{\left(3+\sqrt{15}\right)x^2-3}}\right.\right.}{2\sqrt{2}3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{2x^4+6x^2-3}}\right.$$

$$\frac{x\left(x^2+4\right)}{15\sqrt{2x^4+6x^2-3}}$$

$$\Big\downarrow \text{1498}$$

$$\frac{1}{15}\left(\frac{1}{2}\left(\frac{x\left(2x^2+\sqrt{15}+3\right)}{\sqrt{2x^4+6x^2-3}}-\frac{\sqrt[4]{\frac{5}{3}}\sqrt{2}\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{\left(3+\sqrt{15}\right)x^2-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{15}x}{\sqrt{\left(3+\sqrt{15}\right)x^2-3}}\right)|\frac{1}{10}\left(5+\sqrt{\right.\right.}{\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{2x^4+6x^2-3}}\right.\right.$$

$$\frac{x\left(x^2+4\right)}{15\sqrt{2x^4+6x^2-3}}$$

input
```
Int[(-3 + 6*x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/15*(x*(4 + x^2))/Sqrt[-3 + 6*x^2 + 2*x^4] + (((x*(3 + Sqrt[15] + 2*x^2)
)/Sqrt[-3 + 6*x^2 + 2*x^4] - ((5/3)^(1/4)*Sqrt[2]*Sqrt[(3 - (3 - Sqrt[15])
*x^2)/(3 - (3 + Sqrt[15])*x^2)]*Sqrt[-3 + (3 + Sqrt[15])*x^2]*EllipticE[Ar
cSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[-3 + (3 + Sqrt[15])*x^2]], (5 + Sqrt[15])/1
0])/(Sqrt[(3 - (3 + Sqrt[15])*x^2)^(-1)]*Sqrt[-3 + 6*x^2 + 2*x^4]))/2 - ((
5 - Sqrt[15])*Sqrt[(3 - (3 - Sqrt[15])*x^2)/(3 - (3 + Sqrt[15])*x^2)]*Sqrt
[-3 + (3 + Sqrt[15])*x^2]*EllipticF[ArcSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[-3 +
(3 + Sqrt[15])*x^2]], (5 + Sqrt[15])/10])/(2*Sqrt[2]*3^(3/4)*5^(1/4)*Sqrt[
(3 - (3 + Sqrt[15])*x^2)^(-1)]*Sqrt[-3 + 6*x^2 + 2*x^4]))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{x(x^2+4)}{15\sqrt{2x^4+6x^2-3}}-\frac{\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\;\mathrm{EllipticF}\left(\frac{\sqrt{9-3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{5\sqrt{9-3\sqrt{15}}\,\sqrt{2x^4+6x^2-3}}+\frac{6\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-(1+}{}$
default	$-\frac{4\left(\frac{1}{15}x+\frac{1}{60}x^3\right)}{\sqrt{2x^4+6x^2-3}}-\frac{\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\;\mathrm{EllipticF}\left(\frac{\sqrt{9-3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{5\sqrt{9-3\sqrt{15}}\,\sqrt{2x^4+6x^2-3}}+\frac{6\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-(1+}{}$
elliptic	$-\frac{4\left(\frac{1}{15}x+\frac{1}{60}x^3\right)}{\sqrt{2x^4+6x^2-3}}-\frac{\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\;\mathrm{EllipticF}\left(\frac{\sqrt{9-3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{5\sqrt{9-3\sqrt{15}}\,\sqrt{2x^4+6x^2-3}}+\frac{6\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-(1+}{}$

input
```
int(1/(2*x^4+6*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*x*(x^2+4)/(2*x^4+6*x^2-3)^(1/2)-1/5/(9-3*15^(1/2))^(1/2)*(1-(1-1/3*1
5^(1/2))*x^2)^(1/2)*(1-(1+1/3*15^(1/2))*x^2)^(1/2)/(2*x^4+6*x^2-3)^(1/2)*E
llipticF(1/3*(9-3*15^(1/2))^(1/2)*x,1/2*I*6^(1/2)+1/2*I*10^(1/2))+6/5/(9-3
*15^(1/2))^(1/2)*(1-(1-1/3*15^(1/2))*x^2)^(1/2)*(1-(1+1/3*15^(1/2))*x^2)^(
1/2)/(2*x^4+6*x^2-3)^(1/2)/(6+2*15^(1/2))*(EllipticF(1/3*(9-3*15^(1/2))^(1
/2)*x,1/2*I*6^(1/2)+1/2*I*10^(1/2))-EllipticE(1/3*(9-3*15^(1/2))^(1/2)*x,1
/2*I*6^(1/2)+1/2*I*10^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.66

$$\int \frac{1}{(-3+6x^2+2x^4)^{3/2}}\,dx = \frac{2\sqrt{\frac{5}{3}}\sqrt{-3}(2\,x^4+6\,x^2-3)\sqrt{\sqrt{\frac{5}{3}}+1}F(\arcsin\left(x\sqrt{\sqrt{\frac{5}{3}}+1}\right)|3\sqrt{\frac{5}{3}}-4)-}{}$$

input
```
integrate(1/(2*x^4+6*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/30*(2*sqrt(5/3)*sqrt(-3)*(2*x^4 + 6*x^2 - 3)*sqrt(sqrt(5/3) + 1)*ellipti
c_f(arcsin(x*sqrt(sqrt(5/3) + 1)), 3*sqrt(5/3) - 4) - (sqrt(5/3)*sqrt(-3)*
(2*x^4 + 6*x^2 - 3) + sqrt(-3)*(2*x^4 + 6*x^2 - 3))*sqrt(sqrt(5/3) + 1)*el
liptic_e(arcsin(x*sqrt(sqrt(5/3) + 1)), 3*sqrt(5/3) - 4) - 2*sqrt(2*x^4 +
6*x^2 - 3)*(x^3 + 4*x))/(2*x^4 + 6*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(-3+6x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+6x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+6*x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 + 6*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3+6x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+6\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+6*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 6*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3+6x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+6\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+6*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 6*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3+6x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+6\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(6*x^2 + 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(6*x^2 + 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3+6x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+6x^2-3}}{4x^8+24x^6+24x^4-36x^2+9}dx$$

input
```
int(1/(2*x^4+6*x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 6*x**2 - 3)/(4*x**8 + 24*x**6 + 24*x**4 - 36*x**2 + 9),x
)
```

3.219 $\int \frac{1}{(-3+5x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 147

$$\int \frac{1}{(-3+5x^2+2x^4)^{3/2}}\,dx = -\frac{x(37+10x^2)}{147\sqrt{-3+5x^2+2x^4}} + \frac{5\sqrt{\frac{2}{3}}\sqrt{1-2x^2}\sqrt{3+x^2}E\left(\arcsin\left(\sqrt{2}x\right)|-\frac{1}{6}\right)}{49\sqrt{-3+5x^2+2x^4}} - \frac{\sqrt{\frac{2}{3}}\sqrt{1-2x^2}\sqrt{3+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{6}\right)}{7\sqrt{-3+5x^2+2x^4}}$$

output
```
-1/147*x*(10*x^2+37)/(2*x^4+5*x^2-3)^(1/2)+5/147*6^(1/2)*(-2*x^2+1)^(1/2)*
(x^2+3)^(1/2)*EllipticE(x*2^(1/2),1/6*I*6^(1/2))/(2*x^4+5*x^2-3)^(1/2)-1/2
1*(-2*x^2+1)^(1/2)*(x^2+3)^(1/2)*EllipticF(x*2^(1/2),1/6*I*6^(1/2))*6^(1/2
)/(2*x^4+5*x^2-3)^(1/2)
```

Mathematica [A] (verified)

Time = 6.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

$$\int \frac{1}{\left(-3+5x^2+2x^4\right)^{3/2}}\,dx = \frac{-37x-10x^3+5\sqrt{6-12x^2}\sqrt{3+x^2}E\left(\arcsin\left(\sqrt{2}x\right)|-\frac{1}{6}\right)-7\sqrt{6-12x^2}\sqrt{3}}{147\sqrt{-3+5x^2+2x^4}}$$

input
```
Integrate[(-3 + 5*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(-37*x - 10*x^3 + 5*Sqrt[6 - 12*x^2]*Sqrt[3 + x^2]*EllipticE[ArcSin[Sqrt[2
]*x], -1/6] - 7*Sqrt[6 - 12*x^2]*Sqrt[3 + x^2]*EllipticF[ArcSin[Sqrt[2]*x]
, -1/6])/(147*Sqrt[-3 + 5*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.52, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1405, 27, 1501, 27, 1410, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(2x^4+5x^2-3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{147}\int -\frac{2\left(6-5x^2\right)}{\sqrt{2x^4+5x^2-3}}dx-\frac{x\left(10x^2+37\right)}{147\sqrt{2x^4+5x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{147}\int \frac{6-5x^2}{\sqrt{2x^4+5x^2-3}}dx-\frac{x\left(10x^2+37\right)}{147\sqrt{2x^4+5x^2-3}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{2}{147}\left(\frac{7}{2}\int \frac{1}{\sqrt{2x^4+5x^2-3}}dx-\frac{5}{4}\int -\frac{2\left(1-2x^2\right)}{\sqrt{2x^4+5x^2-3}}dx\right)-\frac{x\left(10x^2+37\right)}{147\sqrt{2x^4+5x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{147}\left(\frac{7}{2}\int \frac{1}{\sqrt{2x^4+5x^2-3}}dx+\frac{5}{2}\int \frac{1-2x^2}{\sqrt{2x^4+5x^2-3}}dx\right)-\frac{x(10x^2+37)}{147\sqrt{2x^4+5x^2-3}}$$

$\Big\downarrow$ 1410

$$-\frac{2}{147}\left(\frac{5}{2}\int \frac{1-2x^2}{\sqrt{2x^4+5x^2-3}}dx+\frac{\sqrt{7}\sqrt{x^2+3}\sqrt{2x^2-1}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{7}{3}}x}{\sqrt{2x^2-1}}\right),\frac{6}{7}\right)}{2\sqrt{2x^4+5x^2-3}}\right)-\frac{x(10x^2+37)}{147\sqrt{2x^4+5x^2-3}}$$

$\Big\downarrow$ 1498

$$-\frac{2}{147}\left(\frac{\sqrt{7}\sqrt{x^2+3}\sqrt{2x^2-1}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{7}{3}}x}{\sqrt{2x^2-1}}\right),\frac{6}{7}\right)}{2\sqrt{2x^4+5x^2-3}}+\frac{5}{2}\left(\frac{\sqrt{7}\sqrt{\frac{x^2+3}{1-2x^2}}\sqrt{2x^2-1}E\left(\arcsin\left(\frac{\sqrt{\frac{7}{3}}x}{\sqrt{2x^2-1}}\right)\right.}{\sqrt{\frac{1}{1-2x^2}}\sqrt{2x^4+5x^2-3}}\right.\right.$$

$$\frac{x(10x^2+37)}{147\sqrt{2x^4+5x^2-3}}$$

input
```
Int[(-3 + 5*x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/147*(x*(37 + 10*x^2))/Sqrt[-3 + 5*x^2 + 2*x^4] - (2*((5*((-2*x*(3 + x^2
))/Sqrt[-3 + 5*x^2 + 2*x^4] + (Sqrt[7]*Sqrt[(3 + x^2)/(1 - 2*x^2)]*Sqrt[-1
 + 2*x^2]*EllipticE[ArcSin[(Sqrt[7/3]*x)/Sqrt[-1 + 2*x^2]], 6/7])/(Sqrt[(1
 - 2*x^2)^(-1)]*Sqrt[-3 + 5*x^2 + 2*x^4])))/2 + (Sqrt[7]*Sqrt[3 + x^2]*Sqr
t[-1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7/3]*x)/Sqrt[-1 + 2*x^2]], 6/7])/(2*S
qrt[-3 + 5*x^2 + 2*x^4])))/147
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{x(10x^2+37)}{147\sqrt{2x^4+5x^2-3}}+\frac{4i\sqrt{3}\sqrt{3x^2+9}\sqrt{-2x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{3}}{3},i\sqrt{6}\right)}{147\sqrt{2x^4+5x^2-3}}-\frac{5i\sqrt{3}\sqrt{3x^2+9}\sqrt{-2x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{3}}{3},i\sqrt{6}\right)-\text{El}\right.}{441\sqrt{2x^4+5x^2-3}}$
default	$-\frac{4\left(\frac{37}{588}x+\frac{5}{294}x^3\right)}{\sqrt{2x^4+5x^2-3}}+\frac{4i\sqrt{3}\sqrt{3x^2+9}\sqrt{-2x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{3}}{3},i\sqrt{6}\right)}{147\sqrt{2x^4+5x^2-3}}-\frac{5i\sqrt{3}\sqrt{3x^2+9}\sqrt{-2x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{3}}{3},i\sqrt{6}\right)-\text{Elli}\right.}{441\sqrt{2x^4+5x^2-3}}$
elliptic	$-\frac{4\left(\frac{37}{588}x+\frac{5}{294}x^3\right)}{\sqrt{2x^4+5x^2-3}}+\frac{4i\sqrt{3}\sqrt{3x^2+9}\sqrt{-2x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{3}}{3},i\sqrt{6}\right)}{147\sqrt{2x^4+5x^2-3}}-\frac{5i\sqrt{3}\sqrt{3x^2+9}\sqrt{-2x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{3}}{3},i\sqrt{6}\right)-\text{Elli}\right.}{441\sqrt{2x^4+5x^2-3}}$

input
```
int(1/(2*x^4+5*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/147*x*(10*x^2+37)/(2*x^4+5*x^2-3)^(1/2)+4/147*I*3^(1/2)*(3*x^2+9)^(1/2)
*(-2*x^2+1)^(1/2)/(2*x^4+5*x^2-3)^(1/2)*EllipticF(1/3*I*x*3^(1/2),I*6^(1/2
))-5/441*I*3^(1/2)*(3*x^2+9)^(1/2)*(-2*x^2+1)^(1/2)/(2*x^4+5*x^2-3)^(1/2)*
(EllipticF(1/3*I*x*3^(1/2),I*6^(1/2))-EllipticE(1/3*I*x*3^(1/2),I*6^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

$$\int \frac{1}{(-3+5x^2+2x^4)^{3/2}}\,dx =$$
$$-\frac{10\sqrt{2}\sqrt{-3}(2x^4+5x^2-3)E(\arcsin(\sqrt{2}x)\mid-\frac{1}{6})-12\sqrt{2}\sqrt{-3}(2x^4+5x^2-3)F(\arcsin(\sqrt{2}x)\mid-\frac{1}{6})+}{147(2x^4+5x^2-3)}$$

input
```
integrate(1/(2*x^4+5*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
-1/147*(10*sqrt(2)*sqrt(-3)*(2*x^4 + 5*x^2 - 3)*elliptic_e(arcsin(sqrt(2)*
x), -1/6) - 12*sqrt(2)*sqrt(-3)*(2*x^4 + 5*x^2 - 3)*elliptic_f(arcsin(sqrt
(2)*x), -1/6) + sqrt(2*x^4 + 5*x^2 - 3)*(10*x^3 + 37*x))/(2*x^4 + 5*x^2 -
3)
```

Sympy [F]

$$\int \frac{1}{(-3+5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+5x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+5*x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 + 5*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3+5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+5\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+5*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 5*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3+5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+5\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+5*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 5*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3+5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+5\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 + 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(5*x^2 + 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3+5x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+5x^2-3}}{4x^8+20x^6+13x^4-30x^2+9}dx$$

input
```
int(1/(2*x^4+5*x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 5*x**2 - 3)/(4*x**8 + 20*x**6 + 13*x**4 - 30*x**2 + 9),x
)
```

3.220 $\int \frac{1}{\left(-3+4x^2+2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 235

$$\int \frac{1}{\left(-3+4x^2+2x^4\right)^{3/2}}\,dx=-\frac{x(7+2x^2)}{30\sqrt{-3+4x^2+2x^4}}+\frac{\sqrt{3-\left(2-\sqrt{10}\right)x^2}\sqrt{3-\left(2+\sqrt{10}\right)x^2}E\left(\arcsin\left(\sqrt{\frac{1}{3}\left(2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{15\sqrt{2\left(-2+\sqrt{10}\right)}\sqrt{-3+4x^2+2x^4}}-\frac{\sqrt{3-\left(2-\sqrt{10}\right)x^2}\sqrt{3-\left(2+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{6\sqrt{5\left(-2+\sqrt{10}\right)}\sqrt{-3+4x^2+2x^4}}$$

output
```
-1/30*x*(2*x^2+7)/(2*x^4+4*x^2-3)^(1/2)+1/15*(3-(2-10^(1/2))*x^2)^(1/2)*(3
-(2+10^(1/2))*x^2)^(1/2)*EllipticE(1/3*(6+3*10^(1/2))^(1/2)*x,1/3*I*15^(1/
2)-1/3*I*6^(1/2))/(-4+2*10^(1/2))^(1/2)/(2*x^4+4*x^2-3)^(1/2)-1/6*(3-(2-10
^(1/2))*x^2)^(1/2)*(3-(2+10^(1/2))*x^2)^(1/2)*EllipticF(1/3*(6+3*10^(1/2))
^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))/(-10+5*10^(1/2))^(1/2)/(2*x^4+4*x^2
-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.92 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-3+4x^2+2x^4)^{3/2}}\,dx = \frac{-2x(7+2x^2)+2i\sqrt{-2+\sqrt{10}}\sqrt{3-4x^2-2x^4}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right)|-\frac{7}{3}\right.}{60\sqrt{-3+4x^2}}$$

input
```
Integrate[(-3 + 4*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(-2*x*(7 + 2*x^2) + (2*I)*Sqrt[-2 + Sqrt[10]]*Sqrt[3 - 4*x^2 - 2*x^4]*Elli
pticE[I*ArcSinh[Sqrt[2/(2 + Sqrt[10])]*x], -7/3 - (2*Sqrt[10])/3] - ((2*I)
*(-5 + Sqrt[10])*Sqrt[3 - 4*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(2 + S
qrt[10])]*x], -7/3 - (2*Sqrt[10])/3])/Sqrt[-2 + Sqrt[10]])/(60*Sqrt[-3 + 4
*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.58, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+4x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{120}\int -\frac{4(3-2x^2)}{\sqrt{2x^4+4x^2-3}}dx - \frac{x(2x^2+7)}{30\sqrt{2x^4+4x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{30}\int \frac{3-2x^2}{\sqrt{2x^4+4x^2-3}}dx - \frac{x(2x^2+7)}{30\sqrt{2x^4+4x^2-3}}$$

$$\Big\downarrow \; 1501$$

$$\frac{1}{30}\left(\frac{1}{2}\int \frac{2\left(2x^2-\sqrt{10}+2\right)}{\sqrt{2x^4+4x^2-3}}dx-\left(5-\sqrt{10}\right)\int \frac{1}{\sqrt{2x^4+4x^2-3}}dx\right)-\frac{x\left(2x^2+7\right)}{30\sqrt{2x^4+4x^2-3}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{30}\left(\int \frac{2x^2-\sqrt{10}+2}{\sqrt{2x^4+4x^2-3}}dx-\left(5-\sqrt{10}\right)\int \frac{1}{\sqrt{2x^4+4x^2-3}}dx\right)-\frac{x\left(2x^2+7\right)}{30\sqrt{2x^4+4x^2-3}}$$

$$\Big\downarrow \; 1411$$

$$\frac{1}{30}\left(\int \frac{2x^2-\sqrt{10}+2}{\sqrt{2x^4+4x^2-3}}dx-\frac{(5-\sqrt{10})\sqrt{\frac{3-\left(2-\sqrt{10}\right)x^2}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{(2+\sqrt{10})\,x^2-3}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{\left(2+\sqrt{10}\right)x^2-3}}\right)\right.}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{2x^4+4x^2-3}}\right.$$

$$\frac{x\left(2x^2+7\right)}{30\sqrt{2x^4+4x^2-3}}$$

$$\Big\downarrow \; 1498$$

$$\frac{1}{30}\left(-\frac{(5-\sqrt{10})\sqrt{\frac{3-\left(2-\sqrt{10}\right)x^2}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{(2+\sqrt{10})\,x^2-3}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{\left(2+\sqrt{10}\right)x^2-3}}\right),\frac{1}{10}(5+\sqrt{10})\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{2x^4+4x^2-3}}-\frac{2^{3/}}{}\right.$$

$$\frac{x\left(2x^2+7\right)}{30\sqrt{2x^4+4x^2-3}}$$

input
```
Int[(-3 + 4*x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/30*(x*(7 + 2*x^2))/Sqrt[-3 + 4*x^2 + 2*x^4] + ((x*(2 + Sqrt[10] + 2*x^2
))/Sqrt[-3 + 4*x^2 + 2*x^4] - (2^(3/4)*5^(1/4)*Sqrt[(3 - (2 - Sqrt[10])*x^
2)/(3 - (2 + Sqrt[10])*x^2)]*Sqrt[-3 + (2 + Sqrt[10])*x^2]*EllipticE[ArcSi
n[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 + (2 + Sqrt[10])*x^2]], (5 + Sqrt[10])/10])/
(Sqrt[3]*Sqrt[(3 - (2 + Sqrt[10])*x^2)^(-1)]*Sqrt[-3 + 4*x^2 + 2*x^4]) - (
(5 - Sqrt[10])*Sqrt[(3 - (2 - Sqrt[10])*x^2)/(3 - (2 + Sqrt[10])*x^2)]*Sqr
t[-3 + (2 + Sqrt[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 +
(2 + Sqrt[10])*x^2]], (5 + Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^(1/4)*Sqrt[(3
 - (2 + Sqrt[10])*x^2)^(-1)]*Sqrt[-3 + 4*x^2 + 2*x^4]))/30
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{x(2x^2+7)}{30\sqrt{2x^4+4x^2-3}}-\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{10\sqrt{6-3\sqrt{10}}\sqrt{2x^4+4x^2-3}}+\frac{6\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\sqrt{1-(\cdots}{\cdots}$
default	$-\frac{4\left(\frac{7}{120}x+\frac{1}{60}x^3\right)}{\sqrt{2x^4+4x^2-3}}-\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{10\sqrt{6-3\sqrt{10}}\sqrt{2x^4+4x^2-3}}+\frac{6\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\sqrt{1-(\cdots}{\cdots}$
elliptic	$-\frac{4\left(\frac{7}{120}x+\frac{1}{60}x^3\right)}{\sqrt{2x^4+4x^2-3}}-\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{10\sqrt{6-3\sqrt{10}}\sqrt{2x^4+4x^2-3}}+\frac{6\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\sqrt{1-(\cdots}{\cdots}$

input
```
int(1/(2*x^4+4*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*x*(2*x^2+7)/(2*x^4+4*x^2-3)^(1/2)-3/10/(6-3*10^(1/2))^(1/2)*(1-(2/3-
1/3*10^(1/2))*x^2)^(1/2)*(1-(2/3+1/3*10^(1/2))*x^2)^(1/2)/(2*x^4+4*x^2-3)^
(1/2)*EllipticF(1/3*(6-3*10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))+6
/5/(6-3*10^(1/2))^(1/2)*(1-(2/3-1/3*10^(1/2))*x^2)^(1/2)*(1-(2/3+1/3*10^(1
/2))*x^2)^(1/2)/(2*x^4+4*x^2-3)^(1/2)/(4+2*10^(1/2))*(EllipticF(1/3*(6-3*1
0^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))-EllipticE(1/3*(6-3*10^(1/2)
)^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.76

$$\int \frac{1}{(-3+4x^2+2x^4)^{3/2}}\,dx =$$

$$-\frac{2\left(\sqrt{10}\sqrt{-3}(2\,x^4+4\,x^2-3)+2\,\sqrt{-3}(2\,x^4+4\,x^2-3)\right)\sqrt{\frac{1}{3}\,\sqrt{10}+\frac{2}{3}}E(\arcsin\left(x\sqrt{\frac{1}{3}\,\sqrt{10}+\frac{2}{3}}\right)\mid\frac{2}{3}\,\sqrt{10}$$

input
```
integrate(1/(2*x^4+4*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
-1/180*(2*(sqrt(10)*sqrt(-3)*(2*x^4 + 4*x^2 - 3) + 2*sqrt(-3)*(2*x^4 + 4*x
^2 - 3))*sqrt(1/3*sqrt(10) + 2/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(10) +
2/3)), 2/3*sqrt(10) - 7/3) - (5*sqrt(10)*sqrt(-3)*(2*x^4 + 4*x^2 - 3) - 2*
sqrt(-3)*(2*x^4 + 4*x^2 - 3))*sqrt(1/3*sqrt(10) + 2/3)*elliptic_f(arcsin(x
*sqrt(1/3*sqrt(10) + 2/3)), 2/3*sqrt(10) - 7/3) + 6*sqrt(2*x^4 + 4*x^2 - 3
)*(2*x^3 + 7*x))/(2*x^4 + 4*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(-3+4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+4x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+4*x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 + 4*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3+4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+4\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+4*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 4*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3+4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+4\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+4*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 4*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3+4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+4\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(4*x^2 + 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(4*x^2 + 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3+4x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+4x^2-3}}{4x^8+16x^6+4x^4-24x^2+9}dx$$

input
```
int(1/(2*x^4+4*x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 4*x**2 - 3)/(4*x**8 + 16*x**6 + 4*x**4 - 24*x**2 + 9),x)
```

3.221 $\int \frac{1}{(-3+3x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 235

$$\int \frac{1}{(-3+3x^2+2x^4)^{3/2}}\,dx = -\frac{x(7+2x^2)}{33\sqrt{-3+3x^2+2x^4}}$$
$$+\frac{\sqrt{\frac{1}{6}\left(3+\sqrt{33}\right)}\sqrt{6-\left(3-\sqrt{33}\right)x^2}\sqrt{6-\left(3+\sqrt{33}\right)x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{6}\left(3+\sqrt{33}\right)}x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{66\sqrt{-3+3x^2+2x^4}}$$
$$-\frac{\sqrt{\frac{1}{22}\left(3+\sqrt{33}\right)}\sqrt{6-\left(3-\sqrt{33}\right)x^2}\sqrt{6-\left(3+\sqrt{33}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{6}\left(3+\sqrt{33}\right)}x\right),\frac{1}{4}(-7+}{18\sqrt{-3+3x^2+2x^4}}$$

output
```
-1/33*x*(2*x^2+7)/(2*x^4+3*x^2-3)^(1/2)+1/396*(18+6*33^(1/2))^(1/2)*(6-(3-
33^(1/2))*x^2)^(1/2)*(6-(3+33^(1/2))*x^2)^(1/2)*EllipticE(1/6*(18+6*33^(1/
2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))/(2*x^4+3*x^2-3)^(1/2)-1/396*(66+
22*33^(1/2))^(1/2)*(6-(3-33^(1/2))*x^2)^(1/2)*(6-(3+33^(1/2))*x^2)^(1/2)*E
llipticF(1/6*(18+6*33^(1/2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))/(2*x^4+
3*x^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.01 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-3+3x^2+2x^4)^{3/2}}\,dx = \frac{-4x(7+2x^2)+2i\sqrt{-3+\sqrt{33}}\sqrt{6-6x^2-4x^4}E\left(i\text{arcsinh}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|-\frac{7}{4}-\right.}{132\sqrt{-3+3x^2+}}$$

input
```
Integrate[(-3 + 3*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(-4*x*(7 + 2*x^2) + (2*I)*Sqrt[-3 + Sqrt[33]]*Sqrt[6 - 6*x^2 - 4*x^4]*Elli
pticE[I*ArcSinh[(2*x)/Sqrt[3 + Sqrt[33]]], -7/4 - Sqrt[33]/4] - ((2*I)*(-1
1 + Sqrt[33])*Sqrt[6 - 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[3 + S
qrt[33]]], -7/4 - Sqrt[33]/4])/Sqrt[-3 + Sqrt[33]])/(132*Sqrt[-3 + 3*x^2 +
 2*x^4])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.55, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1405, 27, 1501, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+3x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{99}\int -\frac{6(2-x^2)}{\sqrt{2x^4+3x^2-3}}dx - \frac{x(2x^2+7)}{33\sqrt{2x^4+3x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{33}\int \frac{2-x^2}{\sqrt{2x^4+3x^2-3}}dx - \frac{x(2x^2+7)}{33\sqrt{2x^4+3x^2-3}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{2}{33}\left(\frac{1}{4}\left(11-\sqrt{33}\right)\int \frac{1}{\sqrt{2x^4+3x^2-3}}dx-\frac{1}{4}\int \frac{4x^2-\sqrt{33}+3}{\sqrt{2x^4+3x^2-3}}dx\right)-\frac{x\left(2x^2+7\right)}{33\sqrt{2x^4+3x^2-3}}$$

$$\Big\downarrow \text{1411}$$

$$-\frac{2}{33}\left(\frac{\left(11-\sqrt{33}\right)\sqrt{\frac{6-\left(3-\sqrt{33}\right)x^2}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-6}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{\left(3+\sqrt{33}\right)x^2-6}}\right),\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{8\ 3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{2x^4+3x^2-3}}-\frac{1}{4}\cdots\right)-$$

$$\frac{x\left(2x^2+7\right)}{33\sqrt{2x^4+3x^2-3}}$$

$$\Big\downarrow \text{1498}$$

$$-\frac{2}{33}\left(\frac{\left(11-\sqrt{33}\right)\sqrt{\frac{6-\left(3-\sqrt{33}\right)x^2}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-6}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{\left(3+\sqrt{33}\right)x^2-6}}\right),\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{8\ 3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{2x^4+3x^2-3}}+\frac{1}{4}\cdots\right)-$$

$$\frac{x\left(2x^2+7\right)}{33\sqrt{2x^4+3x^2-3}}$$

input
```
Int[(-3 + 3*x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/33*(x*(7 + 2*x^2))/Sqrt[-3 + 3*x^2 + 2*x^4] - (2*((-((x*(3 + Sqrt[33] +
 4*x^2))/Sqrt[-3 + 3*x^2 + 2*x^4]) + ((11/3)^(1/4)*Sqrt[(6 - (3 - Sqrt[33]
)*x^2)/(6 - (3 + Sqrt[33])*x^2)]*Sqrt[-6 + (3 + Sqrt[33])*x^2]*EllipticE[A
rcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6 + (3 + Sqrt[33])*x^2]], (11 + Sqrt[33])
/22])/(Sqrt[(6 - (3 + Sqrt[33])*x^2)^(-1)]*Sqrt[-3 + 3*x^2 + 2*x^4]))/4 +
((11 - Sqrt[33])*Sqrt[(6 - (3 - Sqrt[33])*x^2)/(6 - (3 + Sqrt[33])*x^2)]*S
qrt[-6 + (3 + Sqrt[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6
 + (3 + Sqrt[33])*x^2]], (11 + Sqrt[33])/22])/(8*3^(3/4)*11^(1/4)*Sqrt[(6
- (3 + Sqrt[33])*x^2)^(-1)]*Sqrt[-3 + 3*x^2 + 2*x^4])))/33
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{x(2x^2+7)}{33\sqrt{2x^4+3x^2-3}}-\frac{8\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{18-6\sqrt{33}}\,x}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{11\sqrt{18-6\sqrt{33}}\sqrt{2x^4+3x^2-3}}+\frac{24\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-}$
default	$-\frac{4\left(\frac{7}{132}x+\frac{1}{66}x^3\right)}{\sqrt{2x^4+3x^2-3}}-\frac{8\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{18-6\sqrt{33}}\,x}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{11\sqrt{18-6\sqrt{33}}\sqrt{2x^4+3x^2-3}}+\frac{24\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-}$
elliptic	$-\frac{4\left(\frac{7}{132}x+\frac{1}{66}x^3\right)}{\sqrt{2x^4+3x^2-3}}-\frac{8\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{18-6\sqrt{33}}\,x}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{11\sqrt{18-6\sqrt{33}}\sqrt{2x^4+3x^2-3}}+\frac{24\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-}$

input
```
int(1/(2*x^4+3*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/33*x*(2*x^2+7)/(2*x^4+3*x^2-3)^(1/2)-8/11/(18-6*33^(1/2))^(1/2)*(1-(1/2
-1/6*33^(1/2))*x^2)^(1/2)*(1-(1/2+1/6*33^(1/2))*x^2)^(1/2)/(2*x^4+3*x^2-3)
^(1/2)*EllipticF(1/6*(18-6*33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))
+24/11/(18-6*33^(1/2))^(1/2)*(1-(1/2-1/6*33^(1/2))*x^2)^(1/2)*(1-(1/2+1/6*
33^(1/2))*x^2)^(1/2)/(2*x^4+3*x^2-3)^(1/2)/(3+33^(1/2))*(EllipticF(1/6*(18
-6*33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))-EllipticE(1/6*(18-6*33^
(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-3+3x^2+2x^4)^{3/2}}\,dx =$$

$$-\frac{\left(\sqrt{\frac{11}{3}}\sqrt{-3}(2\,x^4+3\,x^2-3)+\sqrt{-3}(2\,x^4+3\,x^2-3)\right)\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}+\frac{1}{2}}E(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}+\frac{1}{2}}\right)\mid\frac{3}{4}\sqrt{\frac{11}{3}}-}{}$$

input
```
integrate(1/(2*x^4+3*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
-1/66*((sqrt(11/3)*sqrt(-3)*(2*x^4 + 3*x^2 - 3) + sqrt(-3)*(2*x^4 + 3*x^2
- 3))*sqrt(1/2*sqrt(11/3) + 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(11/3) +
 1/2)), 3/4*sqrt(11/3) - 7/4) - (3*sqrt(11/3)*sqrt(-3)*(2*x^4 + 3*x^2 - 3)
 - sqrt(-3)*(2*x^4 + 3*x^2 - 3))*sqrt(1/2*sqrt(11/3) + 1/2)*elliptic_f(arc
sin(x*sqrt(1/2*sqrt(11/3) + 1/2)), 3/4*sqrt(11/3) - 7/4) + 2*sqrt(2*x^4 +
3*x^2 - 3)*(2*x^3 + 7*x))/(2*x^4 + 3*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{\left(-3+3x^2+2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(2x^4+3x^2-3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+3*x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 + 3*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(-3+3x^2+2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(2\,x^4+3\,x^2-3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+3*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 3*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3+3x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+3\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+3*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 3*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3+3x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+3\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(3*x^2 + 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(3*x^2 + 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3+3x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+3x^2-3}}{4x^8+12x^6-3x^4-18x^2+9}dx$$

input
```
int(1/(2*x^4+3*x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 3*x**2 - 3)/(4*x**8 + 12*x**6 - 3*x**4 - 18*x**2 + 9),x)
```

3.222 $\int \frac{1}{\left(-3+2x^2+2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 233

$$\int \frac{1}{\left(-3+2x^2+2x^4\right)^{3/2}}\,dx = -\frac{x(4+x^2)}{21\sqrt{-3+2x^2+2x^4}} + \frac{\sqrt{\frac{1}{3}\left(1+\sqrt{7}\right)}\sqrt{3-\left(1-\sqrt{7}\right)x^2}\sqrt{3-\left(1+\sqrt{7}\right)x^2}E\left(\arcsin\left(\sqrt{\frac{1}{3}\left(1+\sqrt{7}\right)}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{42\sqrt{-3+2x^2+2x^4}} - \frac{\sqrt{\frac{1}{21}\left(1+\sqrt{7}\right)}\sqrt{3-\left(1-\sqrt{7}\right)x^2}\sqrt{3-\left(1+\sqrt{7}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(1+\sqrt{7}\right)}x\right),\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{6\sqrt{-3+2x^2+2x^4}}$$

output
```
-1/21*x*(x^2+4)/(2*x^4+2*x^2-3)^(1/2)+1/126*(3+3*7^(1/2))^(1/2)*(3-(1-7^(1
/2))*x^2)^(1/2)*(3-(1+7^(1/2))*x^2)^(1/2)*EllipticE(1/3*(3+3*7^(1/2))^(1/2
)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))/(2*x^4+2*x^2-3)^(1/2)-1/126*(21+21*7^(1/
2))^(1/2)*(3-(1-7^(1/2))*x^2)^(1/2)*(3-(1+7^(1/2))*x^2)^(1/2)*EllipticF(1/
3*(3+3*7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))/(2*x^4+2*x^2-3)^(1/2
)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.99 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-3+2x^2+2x^4)^{3/2}}\,dx = \frac{-4x(4+x^2)+2i\sqrt{-1+\sqrt{7}}\sqrt{3-2x^2-2x^4}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right)|-\frac{4}{3}-\frac{\sqrt{}}{3}\right.}{84\sqrt{-3+2x^2+2x}}$$

input
```
Integrate[(-3 + 2*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(-4*x*(4 + x^2) + (2*I)*Sqrt[-1 + Sqrt[7]]*Sqrt[3 - 2*x^2 - 2*x^4]*Ellipti
cE[I*ArcSinh[Sqrt[2/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3] - ((2*I)*(-7 + Sq
rt[7])*Sqrt[3 - 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[7])]*x
], -4/3 - Sqrt[7]/3])/Sqrt[-1 + Sqrt[7]])/(84*Sqrt[-3 + 2*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.58, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+2x^2-3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{84}\int -\frac{4(3-x^2)}{\sqrt{2x^4+2x^2-3}}dx - \frac{x(x^2+4)}{21\sqrt{2x^4+2x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{21}\int \frac{3-x^2}{\sqrt{2x^4+2x^2-3}}dx - \frac{x(x^2+4)}{21\sqrt{2x^4+2x^2-3}}$$

$$\Big\downarrow \text{1501}$$

$$\frac{1}{21}\left(\frac{1}{4}\int \frac{2\left(2x^2-\sqrt{7}+1\right)}{\sqrt{2x^4+2x^2-3}}dx-\frac{1}{2}\left(7-\sqrt{7}\right)\int \frac{1}{\sqrt{2x^4+2x^2-3}}dx\right)-\frac{x\left(x^2+4\right)}{21\sqrt{2x^4+2x^2-3}}$$

$\Big\downarrow$ 27

$$\frac{1}{21}\left(\frac{1}{2}\int \frac{2x^2-\sqrt{7}+1}{\sqrt{2x^4+2x^2-3}}dx-\frac{1}{2}\left(7-\sqrt{7}\right)\int \frac{1}{\sqrt{2x^4+2x^2-3}}dx\right)-\frac{x\left(x^2+4\right)}{21\sqrt{2x^4+2x^2-3}}$$

$\Big\downarrow$ 1411

$$\frac{1}{21}\left(\frac{1}{2}\int \frac{2x^2-\sqrt{7}+1}{\sqrt{2x^4+2x^2-3}}dx-\frac{\left(7-\sqrt{7}\right)\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-3}}\right),\right.}{2\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{2x^4+2x^2-3}}\right.$$

$$\frac{x\left(x^2+4\right)}{21\sqrt{2x^4+2x^2-3}}$$

$\Big\downarrow$ 1498

$$\frac{1}{21}\left(\frac{1}{2}\left(\frac{x\left(2x^2+\sqrt{7}+1\right)}{\sqrt{2x^4+2x^2-3}}-\frac{\sqrt{\frac{2}{3}}\sqrt[4]{7}\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-3}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{2x^4+2x^2-3}}\right.\right.$$

$$\frac{x\left(x^2+4\right)}{21\sqrt{2x^4+2x^2-3}}$$

input
```
Int[(-3 + 2*x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/21*(x*(4 + x^2))/Sqrt[-3 + 2*x^2 + 2*x^4] + (((x*(1 + Sqrt[7] + 2*x^2))
/Sqrt[-3 + 2*x^2 + 2*x^4] - (Sqrt[2/3]*7^(1/4)*Sqrt[(3 - (1 - Sqrt[7])*x^2
)/(3 - (1 + Sqrt[7])*x^2)]*Sqrt[-3 + (1 + Sqrt[7])*x^2]*EllipticE[ArcSin[(
Sqrt[2]*7^(1/4)*x)/Sqrt[-3 + (1 + Sqrt[7])*x^2]], (7 + Sqrt[7])/14])/(Sqrt
[(3 - (1 + Sqrt[7])*x^2)^(-1)]*Sqrt[-3 + 2*x^2 + 2*x^4]))/2 - ((7 - Sqrt[7
])*Sqrt[(3 - (1 - Sqrt[7])*x^2)/(3 - (1 + Sqrt[7])*x^2)]*Sqrt[-3 + (1 + Sq
rt[7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 + (1 + Sqrt[7])*x
^2]], (7 + Sqrt[7])/14])/(2*Sqrt[6]*7^(1/4)*Sqrt[(3 - (1 + Sqrt[7])*x^2)^(
-1)]*Sqrt[-3 + 2*x^2 + 2*x^4]))/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{x(x^2+4)}{21\sqrt{2x^4+2x^2-3}}-\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{3-3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{7\sqrt{3-3\sqrt{7}}\,\sqrt{2x^4+2x^2-3}}+\frac{6\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}+\right.}}{}$
default	$-\frac{4\left(\frac{1}{21}x+\frac{1}{84}x^3\right)}{\sqrt{2x^4+2x^2-3}}-\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{3-3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{7\sqrt{3-3\sqrt{7}}\,\sqrt{2x^4+2x^2-3}}+\frac{6\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}+\right.}}{}$
elliptic	$-\frac{4\left(\frac{1}{21}x+\frac{1}{84}x^3\right)}{\sqrt{2x^4+2x^2-3}}-\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{3-3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{7\sqrt{3-3\sqrt{7}}\,\sqrt{2x^4+2x^2-3}}+\frac{6\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}+\right.}}{}$

input
```
int(1/(2*x^4+2*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*x*(x^2+4)/(2*x^4+2*x^2-3)^(1/2)-3/7/(3-3*7^(1/2))^(1/2)*(1-(1/3-1/3*
7^(1/2))*x^2)^(1/2)*(1-(1/3+1/3*7^(1/2))*x^2)^(1/2)/(2*x^4+2*x^2-3)^(1/2)*
EllipticF(1/3*(3-3*7^(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))+6/7/(3-3
*7^(1/2))^(1/2)*(1-(1/3-1/3*7^(1/2))*x^2)^(1/2)*(1-(1/3+1/3*7^(1/2))*x^2)^
(1/2)/(2*x^4+2*x^2-3)^(1/2)/(2+2*7^(1/2))*(EllipticF(1/3*(3-3*7^(1/2))^(1/
2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))-EllipticE(1/3*(3-3*7^(1/2))^(1/2)*x,1/6
*I*6^(1/2)+1/6*I*42^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-3+2x^2+2x^4)^{3/2}}\,dx = -\frac{\left(\sqrt{7}\sqrt{-3}(2\,x^4+2\,x^2-3)+\sqrt{-3}(2\,x^4+2\,x^2-3)\right)\sqrt{\frac{1}{3}\,\sqrt{7}+\frac{1}{3}}E(\arcsin\left(x\sqrt{\frac{1}{3}\,\sqrt{7}+\frac{1}{3}}\right)\mid\frac{1}{3}\,\sqrt{7}-\frac{4}{3})-}{}$$

input
```
integrate(1/(2*x^4+2*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
-1/126*((sqrt(7)*sqrt(-3)*(2*x^4 + 2*x^2 - 3) + sqrt(-3)*(2*x^4 + 2*x^2 -
3))*sqrt(1/3*sqrt(7) + 1/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(7) + 1/3)),
1/3*sqrt(7) - 4/3) - 2*(2*sqrt(7)*sqrt(-3)*(2*x^4 + 2*x^2 - 3) - sqrt(-3)*
(2*x^4 + 2*x^2 - 3))*sqrt(1/3*sqrt(7) + 1/3)*elliptic_f(arcsin(x*sqrt(1/3*
sqrt(7) + 1/3)), 1/3*sqrt(7) - 4/3) + 6*sqrt(2*x^4 + 2*x^2 - 3)*(x^3 + 4*x
))/(2*x^4 + 2*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(-3+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+2x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+2*x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 + 2*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+2\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+2*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 2*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+2\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+2*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 2*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+2\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(2*x^2 + 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(2*x^2 + 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(-3+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{\sqrt{2x^4+2x^2-3}}{4x^8+8x^6-8x^4-12x^2+9}dx$$

input
```
int(1/(2*x^4+2*x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 2*x**2 - 3)/(4*x**8 + 8*x**6 - 8*x**4 - 12*x**2 + 9),x)
```

3.223 $\int \frac{1}{\left(-3+x^2+2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 129

$$\int \frac{1}{\left(-3+x^2+2x^4\right)^{3/2}}\,dx = -\frac{x(13+2x^2)}{75\sqrt{-3+x^2+2x^4}} + \frac{\sqrt{1-x^2}\sqrt{3+2x^2}\,E\left(\arcsin(x)\,\middle|-\frac{2}{3}\right)}{25\sqrt{3}\sqrt{-3+x^2+2x^4}} - \frac{\sqrt{1-x^2}\sqrt{3+2x^2}\,\operatorname{EllipticF}\left(\arcsin(x),-\frac{2}{3}\right)}{5\sqrt{3}\sqrt{-3+x^2+2x^4}}$$

output
```
-1/75*x*(2*x^2+13)/(2*x^4+x^2-3)^(1/2)+1/75*(-x^2+1)^(1/2)*(2*x^2+3)^(1/2)
*EllipticE(x,1/3*I*6^(1/2))*3^(1/2)/(2*x^4+x^2-3)^(1/2)-1/15*(-x^2+1)^(1/2
)*(2*x^2+3)^(1/2)*EllipticF(x,1/3*I*6^(1/2))*3^(1/2)/(2*x^4+x^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.47 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88

$$\int \frac{1}{(-3+x^2+2x^4)^{3/2}}\,dx = \frac{-13x-2x^3+i\sqrt{2}\sqrt{3-x^2-2x^4}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{3}}x\right)|-\frac{3}{2}\right)+5i\sqrt{2}\sqrt{3-x^2-}}{75\sqrt{-3+x^2+2x^4}}$$

input
```
Integrate[(-3 + x^2 + 2*x^4)^(-3/2),x]
```

output
```
(-13*x - 2*x^3 + I*Sqrt[2]*Sqrt[3 - x^2 - 2*x^4]*EllipticE[I*ArcSinh[Sqrt[
2/3]*x], -3/2] + (5*I)*Sqrt[2]*Sqrt[3 - x^2 - 2*x^4]*EllipticF[I*ArcSinh[S
qrt[2/3]*x], -3/2])/(75*Sqrt[-3 + x^2 + 2*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.63, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1405, 27, 1501, 27, 1410, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{75}\int -\frac{2(6-x^2)}{\sqrt{2x^4+x^2-3}}dx-\frac{x(2x^2+13)}{75\sqrt{2x^4+x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{75}\int \frac{6-x^2}{\sqrt{2x^4+x^2-3}}dx-\frac{x(2x^2+13)}{75\sqrt{2x^4+x^2-3}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{2}{75}\left(5\int \frac{1}{\sqrt{2x^4+x^2-3}}dx-\frac{1}{4}\int -\frac{4(1-x^2)}{\sqrt{2x^4+x^2-3}}dx\right)-\frac{x(2x^2+13)}{75\sqrt{2x^4+x^2-3}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2}{75}\left(5\int \frac{1}{\sqrt{2x^4+x^2-3}}dx+\int \frac{1-x^2}{\sqrt{2x^4+x^2-3}}dx\right)-\frac{x(2x^2+13)}{75\sqrt{2x^4+x^2-3}}$$

$$\Big\downarrow \; 1410$$

$$-\frac{2}{75}\left(\int \frac{1-x^2}{\sqrt{2x^4+x^2-3}}dx+\frac{\sqrt{5}\sqrt{x^2-1}\sqrt{2x^2+3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{x^2-1}}\right),\frac{3}{5}\right)}{\sqrt{2x^4+x^2-3}}\right)-\frac{x(2x^2+13)}{75\sqrt{2x^4+x^2-3}}$$

$$\Big\downarrow \; 1498$$

$$-\frac{2}{75}\left(\frac{\sqrt{5}\sqrt{x^2-1}\sqrt{2x^2+3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{x^2-1}}\right),\frac{3}{5}\right)}{\sqrt{2x^4+x^2-3}}+\frac{\sqrt{5}\sqrt{x^2-1}\sqrt{\frac{2x^2+3}{1-x^2}}E\left(\arcsin\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{x^2-1}}\right)|\frac{3}{5}\right)}{2\sqrt{\frac{1}{1-x^2}}\sqrt{2x^4+x^2-3}}-\right.$$

$$\frac{x(2x^2+13)}{75\sqrt{2x^4+x^2-3}}$$

input
```
Int[(-3 + x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/75*(x*(13 + 2*x^2))/Sqrt[-3 + x^2 + 2*x^4] - (2*(-1/2*(x*(3 + 2*x^2))/S
qrt[-3 + x^2 + 2*x^4] + (Sqrt[5]*Sqrt[-1 + x^2]*Sqrt[(3 + 2*x^2)/(1 - x^2)
]*EllipticE[ArcSin[(Sqrt[5/3]*x)/Sqrt[-1 + x^2]], 3/5])/(2*Sqrt[(1 - x^2)^
(-1)]*Sqrt[-3 + x^2 + 2*x^4]) + (Sqrt[5]*Sqrt[-1 + x^2]*Sqrt[3 + 2*x^2]*El
lipticF[ArcSin[(Sqrt[5/3]*x)/Sqrt[-1 + x^2]], 3/5])/Sqrt[-3 + x^2 + 2*x^4]
))/75
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{x(2x^2+13)}{75\sqrt{2x^4+x^2-3}}+\frac{2i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{75\sqrt{2x^4+x^2-3}}-\frac{i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{-x^2+1}\,\left(\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)-\operatorname{Elliptic}\right.}{225\sqrt{2x^4+x^2-3}}$
default	$-\frac{4\left(\frac{13}{300}x+\frac{1}{150}x^3\right)}{\sqrt{2x^4+x^2-3}}+\frac{2i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{75\sqrt{2x^4+x^2-3}}-\frac{i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{-x^2+1}\,\left(\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)-\operatorname{Elliptic}\right.}{225\sqrt{2x^4+x^2-3}}$
elliptic	$-\frac{4\left(\frac{13}{300}x+\frac{1}{150}x^3\right)}{\sqrt{2x^4+x^2-3}}+\frac{2i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{75\sqrt{2x^4+x^2-3}}-\frac{i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{-x^2+1}\,\left(\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)-\operatorname{Elliptic}\right.}{225\sqrt{2x^4+x^2-3}}$

input
```
int(1/(2*x^4+x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/75*x*(2*x^2+13)/(2*x^4+x^2-3)^(1/2)+2/75*I*6^(1/2)*(6*x^2+9)^(1/2)*(-x^
2+1)^(1/2)/(2*x^4+x^2-3)^(1/2)*EllipticF(1/3*I*x*6^(1/2),1/2*I*6^(1/2))-1/
225*I*6^(1/2)*(6*x^2+9)^(1/2)*(-x^2+1)^(1/2)/(2*x^4+x^2-3)^(1/2)*(Elliptic
F(1/3*I*x*6^(1/2),1/2*I*6^(1/2))-EllipticE(1/3*I*x*6^(1/2),1/2*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.57

$$\int \frac{1}{(-3+x^2+2x^4)^{3/2}}\,dx =$$
$$-\frac{\sqrt{-3}(2\,x^4+x^2-3)E(\arcsin{(x)}\mid-\frac{2}{3})-5\,\sqrt{-3}(2\,x^4+x^2-3)F(\arcsin{(x)}\mid-\frac{2}{3})+\sqrt{2\,x^4+x^2-3}(2\,x^3}{75\,(2\,x^4+x^2-3)}$$

input
```
integrate(1/(2*x^4+x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
-1/75*(sqrt(-3)*(2*x^4 + x^2 - 3)*elliptic_e(arcsin(x), -2/3) - 5*sqrt(-3)
*(2*x^4 + x^2 - 3)*elliptic_f(arcsin(x), -2/3) + sqrt(2*x^4 + x^2 - 3)*(2*
x^3 + 13*x))/(2*x^4 + x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(-3+x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 + x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3+x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3+x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3+x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+x^2-3)^{3/2}}\,dx$$

input
```
int(1/(x^2 + 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(x^2 + 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3+x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+x^2-3}}{4x^8+4x^6-11x^4-6x^2+9}dx$$

input
```
int(1/(2*x^4+x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + x**2 - 3)/(4*x**8 + 4*x**6 - 11*x**4 - 6*x**2 + 9),x)
```

3.224 $\int \frac{1}{\left(-3+2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{1}{\left(-3+2x^4\right)^{3/2}}\,dx=-\frac{x}{6\sqrt{-3+2x^4}}-\frac{\sqrt{3-2x^4}\operatorname{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{6\sqrt[4]{6}\sqrt{-3+2x^4}}$$

output
```
-1/6*x/(2*x^4-3)^(1/2)-1/36*(-2*x^4+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)
*x,I)*6^(3/4)/(2*x^4-3)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.69 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(-3+2x^4\right)^{3/2}}\,dx=-\frac{x\left(3+\sqrt{9-6x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{2x^4}{3}\right)\right)}{18\sqrt{-3+2x^4}}$$

input
```
Integrate[(-3 + 2*x^4)^(-3/2),x]
```

output
```
-1/18*(x*(3 + Sqrt[9 - 6*x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, (2*x^4)/3])
)/Sqrt[-3 + 2*x^4]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 132 vs. $2(60) = 120$.

Time = 0.36 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {749, 764}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-3)^{3/2}}\,dx$$
$$\Big\downarrow \text{749}$$
$$-\frac{1}{6}\int \frac{1}{\sqrt{2x^4-3}}dx-\frac{x}{6\sqrt{2x^4-3}}$$
$$\Big\downarrow \text{764}$$
$$-\frac{\sqrt{\sqrt{6}x^2-3}\sqrt{\frac{\sqrt{6}x^2+3}{3-\sqrt{6}x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{\sqrt{6}x^2-3}}\right),\frac{1}{2}\right)}{6\ 6^{3/4}\sqrt{\frac{1}{3-\sqrt{6}x^2}}\sqrt{2x^4-3}}-\frac{x}{6\sqrt{2x^4-3}}$$

input
```
Int[(-3 + 2*x^4)^(-3/2),x]
```

output
```
-1/6*x/Sqrt[-3 + 2*x^4] - (Sqrt[-3 + Sqrt[6]*x^2]*Sqrt[(3 + Sqrt[6]*x^2)/(
3 - Sqrt[6]*x^2)]*EllipticF[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-3 + Sqrt[6]*x
^2]], 1/2])/(6*6^(3/4)*Sqrt[(3 - Sqrt[6]*x^2)^(-1)]*Sqrt[-3 + 2*x^4])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 764
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[(a - q*x^2)/(a + q*x^2)]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[a + b*x^
4]*Sqrt[a/(a + q*x^2)]))*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2],
 x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.71 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

method	result	size
meijerg	$\frac{\sqrt{3}\left(-\operatorname{signum}\left(-1+\frac{2 x^{4}}{3}\right)\right)^{\frac{3}{2}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right],\left[\frac{5}{4}\right], \frac{2 x^{4}}{3}\right)}{9 \operatorname{signum}\left(-1+\frac{2 x^{4}}{3}\right)^{\frac{3}{2}}}$	40
default	$-\frac{x}{6 \sqrt{2 x^{4}-3}}-\frac{\sqrt{9+3 \sqrt{6}\, x^{2}}\, \sqrt{9-3 \sqrt{6}\, x^{2}}\, \operatorname{EllipticF}\left(\frac{\sqrt{-3 \sqrt{6}}\, x}{3}, i\right)}{18 \sqrt{-3 \sqrt{6}}\, \sqrt{2 x^{4}-3}}$	69
risch	$-\frac{x}{6 \sqrt{2 x^{4}-3}}-\frac{\sqrt{9+3 \sqrt{6}\, x^{2}}\, \sqrt{9-3 \sqrt{6}\, x^{2}}\, \operatorname{EllipticF}\left(\frac{\sqrt{-3 \sqrt{6}}\, x}{3}, i\right)}{18 \sqrt{-3 \sqrt{6}}\, \sqrt{2 x^{4}-3}}$	69
elliptic	$-\frac{x}{6 \sqrt{2 x^{4}-3}}-\frac{\sqrt{9+3 \sqrt{6}\, x^{2}}\, \sqrt{9-3 \sqrt{6}\, x^{2}}\, \operatorname{EllipticF}\left(\frac{\sqrt{-3 \sqrt{6}}\, x}{3}, i\right)}{18 \sqrt{-3 \sqrt{6}}\, \sqrt{2 x^{4}-3}}$	69

input
```
int(1/(2*x^4-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*3^(1/2)/signum(-1+2/3*x^4)^(3/2)*(-signum(-1+2/3*x^4))^(3/2)*x*hyperge
om([1/4,3/2],[5/4],2/3*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{1}{(-3+2x^4)^{3/2}}\,dx = \frac{\left(\frac{2}{3}\right)^{\frac{3}{4}}\sqrt{-3}(2\,x^4-3)F(\arcsin\left(\left(\frac{2}{3}\right)^{\frac{1}{4}}x\right)|-1)-2\,\sqrt{2\,x^4-3}x}{12\,(2\,x^4-3)}$$

input
```
integrate(1/(2*x^4-3)^(3/2),x, algorithm="fricas")
```

output
```
1/12*((2/3)^(3/4)*sqrt(-3)*(2*x^4 - 3)*elliptic_f(arcsin((2/3)^(1/4)*x), -
1) - 2*sqrt(2*x^4 - 3)*x)/(2*x^4 - 3)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.53

$$\int \frac{1}{(-3+2x^4)^{3/2}}\,dx = \frac{\sqrt{3}ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4}{3}\right)}{36\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(2*x**4-3)**(3/2),x)
```

output
```
sqrt(3)*I*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), 2*x**4/3)/(36*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(-3+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - 3)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 17.71 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{1}{(-3+2x^4)^{3/2}}\,dx = \frac{\sqrt{3}\,x\,(3-2\,x^4)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ \frac{2\,x^4}{3}\right)}{9\,(2\,x^4-3)^{3/2}}$$

input
```
int(1/(2*x^4 - 3)^(3/2),x)
```

output
```
(3^(1/2)*x*(3 - 2*x^4)^(3/2)*hypergeom([1/4, 3/2], 5/4, (2*x^4)/3))/(9*(2*
x^4 - 3)^(3/2))
```

Reduce [F]

$$\int \frac{1}{(-3+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4-3}}{4x^8-12x^4+9}dx$$

input
```
int(1/(2*x^4-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - 3)/(4*x**8 - 12*x**4 + 9),x)
```

3.225 $\int \frac{1}{\left(-3-x^2+2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 147

$$\int \frac{1}{\left(-3-x^2+2x^4\right)^{3/2}}\,dx = -\frac{x(13-2x^2)}{75\sqrt{-3-x^2+2x^4}} - \frac{\sqrt{2}\sqrt{3-2x^2}\sqrt{1+x^2}E\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right)\middle|-\frac{3}{2}\right)}{75\sqrt{-3-x^2+2x^4}} - \frac{\sqrt{2}\sqrt{3-2x^2}\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{15\sqrt{-3-x^2+2x^4}}$$

output
```
-1/75*x*(-2*x^2+13)/(2*x^4-x^2-3)^(1/2)-1/75*2^(1/2)*(-2*x^2+3)^(1/2)*(x^2
+1)^(1/2)*EllipticE(1/3*x*6^(1/2),1/2*I*6^(1/2))/(2*x^4-x^2-3)^(1/2)-1/15*
(-2*x^2+3)^(1/2)*(x^2+1)^(1/2)*EllipticF(1/3*x*6^(1/2),1/2*I*6^(1/2))*2^(1
/2)/(2*x^4-x^2-3)^(1/2)
```

Mathematica [A] (verified)

Time = 6.64 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-3-x^2+2x^4)^{3/2}}\,dx = \frac{-13x+2x^3-\sqrt{6-4x^2}\sqrt{1+x^2}E\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right)|-\frac{3}{2}\right)-5\sqrt{6-4x^2}\sqrt{1+x}}{75\sqrt{-3-x^2+2x^4}}$$

input
```
Integrate[(-3 - x^2 + 2*x^4)^(-3/2),x]
```

output
```
(-13*x + 2*x^3 - Sqrt[6 - 4*x^2]*Sqrt[1 + x^2]*EllipticE[ArcSin[Sqrt[2/3]*
x], -3/2] - 5*Sqrt[6 - 4*x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2/3]*x],
 -3/2])/(75*Sqrt[-3 - x^2 + 2*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.50, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1410, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{75}\int -\frac{2(x^2+6)}{\sqrt{2x^4-x^2-3}}dx-\frac{x(13-2x^2)}{75\sqrt{2x^4-x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{75}\int \frac{x^2+6}{\sqrt{2x^4-x^2-3}}dx-\frac{x(13-2x^2)}{75\sqrt{2x^4-x^2-3}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{2}{75}\left(\frac{15}{2}\int \frac{1}{\sqrt{2x^4-x^2-3}}dx+\frac{1}{4}\int -\frac{2(3-2x^2)}{\sqrt{2x^4-x^2-3}}dx\right)-\frac{x(13-2x^2)}{75\sqrt{2x^4-x^2-3}}$$

$$\big\downarrow \; 27$$

$$-\frac{2}{75}\left(\frac{15}{2}\int \frac{1}{\sqrt{2x^4-x^2-3}}dx-\frac{1}{2}\int \frac{3-2x^2}{\sqrt{2x^4-x^2-3}}dx\right)-\frac{x\left(13-2x^2\right)}{75\sqrt{2x^4-x^2-3}}$$

$$\big\downarrow \; 1410$$

$$-\frac{2}{75}\left(\frac{3\sqrt{5}\sqrt{x^2+1}\sqrt{2x^2-3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{5}x}{\sqrt{2x^2-3}}\right),\frac{2}{5}\right)}{2\sqrt{2x^4-x^2-3}}-\frac{1}{2}\int \frac{3-2x^2}{\sqrt{2x^4-x^2-3}}dx\right)-\frac{x\left(13-2x^2\right)}{75\sqrt{2x^4-x^2-3}}$$

$$\big\downarrow \; 1498$$

$$-\frac{2}{75}\left(\frac{3\sqrt{5}\sqrt{x^2+1}\sqrt{2x^2-3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{5}x}{\sqrt{2x^2-3}}\right),\frac{2}{5}\right)}{2\sqrt{2x^4-x^2-3}}+\frac{1}{2}\left(\frac{2x\left(x^2+1\right)}{\sqrt{2x^4-x^2-3}}-\frac{\sqrt{5}\sqrt{\frac{x^2+1}{3-2x^2}}\sqrt{2x^2-3}E\left(\text{ar}\right.}{\sqrt{\frac{1}{3-2x^2}}\sqrt{2x^4-}}\right.\right.$$

$$\frac{x\left(13-2x^2\right)}{75\sqrt{2x^4-x^2-3}}$$

input
```
Int[(-3 - x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/75*(x*(13 - 2*x^2))/Sqrt[-3 - x^2 + 2*x^4] - (2*(((2*x*(1 + x^2))/Sqrt[
-3 - x^2 + 2*x^4] - (Sqrt[5]*Sqrt[(1 + x^2)/(3 - 2*x^2)]*Sqrt[-3 + 2*x^2]*
EllipticE[ArcSin[(Sqrt[5]*x)/Sqrt[-3 + 2*x^2]], 2/5])/(Sqrt[(3 - 2*x^2)^(-
1)]*Sqrt[-3 - x^2 + 2*x^4]))/2 + (3*Sqrt[5]*Sqrt[1 + x^2]*Sqrt[-3 + 2*x^2]
*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt[-3 + 2*x^2]], 2/5])/(2*Sqrt[-3 - x^2 +
2*x^4])))/75
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.87

method	result
risch	$\frac{x(2x^2-13)}{75\sqrt{2x^4-x^2-3}}+\frac{4i\sqrt{x^2+1}\,\sqrt{-6x^2+9}\ \text{EllipticF}\left(ix,\frac{i\sqrt{6}}{3}\right)}{75\sqrt{2x^4-x^2-3}}+\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+9}\left(\text{EllipticF}\left(ix,\frac{i\sqrt{6}}{3}\right)-\text{EllipticE}\left(ix,\frac{i\sqrt{6}}{3}\right)\right)}{75\sqrt{2x^4-x^2-3}}$
default	$-\frac{4\left(\frac{13}{300}x-\frac{1}{150}x^3\right)}{\sqrt{2x^4-x^2-3}}+\frac{4i\sqrt{x^2+1}\,\sqrt{-6x^2+9}\ \text{EllipticF}\left(ix,\frac{i\sqrt{6}}{3}\right)}{75\sqrt{2x^4-x^2-3}}+\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+9}\left(\text{EllipticF}\left(ix,\frac{i\sqrt{6}}{3}\right)-\text{EllipticE}\left(ix,\frac{i\sqrt{6}}{3}\right)\right)}{75\sqrt{2x^4-x^2-3}}$
elliptic	$-\frac{4\left(\frac{13}{300}x-\frac{1}{150}x^3\right)}{\sqrt{2x^4-x^2-3}}+\frac{4i\sqrt{x^2+1}\,\sqrt{-6x^2+9}\ \text{EllipticF}\left(ix,\frac{i\sqrt{6}}{3}\right)}{75\sqrt{2x^4-x^2-3}}+\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+9}\left(\text{EllipticF}\left(ix,\frac{i\sqrt{6}}{3}\right)-\text{EllipticE}\left(ix,\frac{i\sqrt{6}}{3}\right)\right)}{75\sqrt{2x^4-x^2-3}}$

input
```
int(1/(2*x^4-x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/75*x*(2*x^2-13)/(2*x^4-x^2-3)^(1/2)+4/75*I*(x^2+1)^(1/2)*(-6*x^2+9)^(1/2
)/(2*x^4-x^2-3)^(1/2)*EllipticF(I*x,1/3*I*6^(1/2))+1/75*I*(x^2+1)^(1/2)*(-
6*x^2+9)^(1/2)/(2*x^4-x^2-3)^(1/2)*(EllipticF(I*x,1/3*I*6^(1/2))-EllipticE
(I*x,1/3*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.67

$$\int \frac{1}{(-3-x^2+2x^4)^{3/2}}\,dx = \frac{2\sqrt{\frac{2}{3}}\sqrt{-3}(2\,x^4-x^2-3)E(\arcsin\left(\sqrt{\frac{2}{3}}x\right)\mid -\frac{3}{2})+16\sqrt{\frac{2}{3}}\sqrt{-3}(2\,x^4-x^2-3)}{225\,(2\,x^4-x^2-3)}$$

input
```
integrate(1/(2*x^4-x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/225*(2*sqrt(2/3)*sqrt(-3)*(2*x^4 - x^2 - 3)*elliptic_e(arcsin(sqrt(2/3)*
x), -3/2) + 16*sqrt(2/3)*sqrt(-3)*(2*x^4 - x^2 - 3)*elliptic_f(arcsin(sqrt
(2/3)*x), -3/2) + 3*sqrt(2*x^4 - x^2 - 3)*(2*x^3 - 13*x))/(2*x^4 - x^2 - 3
)
```

Sympy [F]

$$\int \frac{1}{(-3-x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4-x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 - x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-3-x^2+2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(2\,x^4-x^2-3\right)^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - x^2 - 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - x^2 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(-3-x^2+2x^4\right)^{3/2}}\,dx=\int \frac{\sqrt{2x^4-x^2-3}}{4x^8-4x^6-11x^4+6x^2+9}dx$$

input
```
int(1/(2*x^4-x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - x**2 - 3)/(4*x**8 - 4*x**6 - 11*x**4 + 6*x**2 + 9),x)
```

3.226 $\int \frac{1}{(-3-2x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 233

$$\int \frac{1}{(-3-2x^2+2x^4)^{3/2}}\,dx = -\frac{x(4-x^2)}{21\sqrt{-3-2x^2+2x^4}} - \frac{\sqrt{\frac{1}{3}\left(-1+\sqrt{7}\right)}\sqrt{3+\left(1-\sqrt{7}\right)x^2}\sqrt{3+\left(1+\sqrt{7}\right)x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-1+\sqrt{7}\right)}x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{42\sqrt{-3-2x^2+2x^4}} - \frac{\sqrt{3+\left(1-\sqrt{7}\right)x^2}\sqrt{3+\left(1+\sqrt{7}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-1+\sqrt{7}\right)}x\right),\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{3\sqrt{14\left(1+\sqrt{7}\right)}\sqrt{-3-2x^2+2x^4}}$$

output
```
-1/21*x*(-x^2+4)/(2*x^4-2*x^2-3)^(1/2)-1/126*(-3+3*7^(1/2))^(1/2)*(3+(1-7^
(1/2))*x^2)^(1/2)*(3+(1+7^(1/2))*x^2)^(1/2)*EllipticE(1/3*(-3+3*7^(1/2))^(
1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))/(2*x^4-2*x^2-3)^(1/2)-1/3*(3+(1-7^(1/
2))*x^2)^(1/2)*(3+(1+7^(1/2))*x^2)^(1/2)*EllipticF(1/3*(-3+3*7^(1/2))^(1/2
)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))/(14+14*7^(1/2))^(1/2)/(2*x^4-2*x^2-3)^(1
/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.84 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-3-2x^2+2x^4)^{3/2}}\,dx = \frac{4x(-4+x^2)-2i\sqrt{1+\sqrt{7}}\sqrt{3+2x^2-2x^4}\,E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right)|\frac{1}{3}(-4+\cdots\right.}{84\sqrt{-3-2x^2+\cdots}}$$

input
```
Integrate[(-3 - 2*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(4*x*(-4 + x^2) - (2*I)*Sqrt[1 + Sqrt[7]]*Sqrt[3 + 2*x^2 - 2*x^4]*Elliptic
E[I*ArcSinh[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3] + ((2*I)*(7 + Sqr
t[7])*Sqrt[3 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[7])]*x
], (-4 + Sqrt[7])/3])/Sqrt[1 + Sqrt[7]])/(84*Sqrt[-3 - 2*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.64, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-2x^2-3)^{3/2}}dx$$

$$\Big\downarrow \; 1405$$

$$\frac{1}{84}\int -\frac{4(x^2+3)}{\sqrt{2x^4-2x^2-3}}dx-\frac{x(4-x^2)}{21\sqrt{2x^4-2x^2-3}}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{21}\int \frac{x^2+3}{\sqrt{2x^4-2x^2-3}}dx-\frac{x(4-x^2)}{21\sqrt{2x^4-2x^2-3}}$$

$$\Big\downarrow \; 1501$$

$$\frac{1}{21}\left(-\frac{1}{2}\left(7+\sqrt{7}\right)\int \frac{1}{\sqrt{2x^4-2x^2-3}}dx-\frac{1}{4}\int -\frac{2\left(-2x^2+\sqrt{7}+1\right)}{\sqrt{2x^4-2x^2-3}}dx\right)-\frac{x\left(4-x^2\right)}{21\sqrt{2x^4-2x^2-3}}$$

$\big\downarrow$ 27

$$\frac{1}{21}\left(\frac{1}{2}\int \frac{-2x^2+\sqrt{7}+1}{\sqrt{2x^4-2x^2-3}}dx-\frac{1}{2}\left(7+\sqrt{7}\right)\int \frac{1}{\sqrt{2x^4-2x^2-3}}dx\right)-\frac{x\left(4-x^2\right)}{21\sqrt{2x^4-2x^2-3}}$$

$\big\downarrow$ 1411

$$\frac{1}{21}\left(\frac{1}{2}\int \frac{-2x^2+\sqrt{7}+1}{\sqrt{2x^4-2x^2-3}}dx-\frac{\left(7+\sqrt{7}\right)\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-3}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+3}{\left(1-\sqrt{7}\right)x^2+3}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right.}}\right.\right.}{2\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+3}}\sqrt{2x^4-2x^2-3}}\right.$$

$$\frac{x\left(4-x^2\right)}{21\sqrt{2x^4-2x^2-3}}$$

$\big\downarrow$ 1498

$$\frac{1}{21}\left(\frac{1}{2}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{7}\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-3}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+3}{\left(1-\sqrt{7}\right)x^2+3}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-3}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+3}}\sqrt{2x^4-2x^2-3}}+\frac{x\left(-2x^2\right.}{\sqrt{2x^4-}}\right.\right.$$

$$\frac{x\left(4-x^2\right)}{21\sqrt{2x^4-2x^2-3}}$$

input
```
Int[(-3 - 2*x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/21*(x*(4 - x^2))/Sqrt[-3 - 2*x^2 + 2*x^4] + (((x*(1 - Sqrt[7] - 2*x^2))
/Sqrt[-3 - 2*x^2 + 2*x^4] + (Sqrt[2/3]*7^(1/4)*Sqrt[-3 - (1 - Sqrt[7])*x^2
]*Sqrt[(3 + (1 + Sqrt[7])*x^2)/(3 + (1 - Sqrt[7])*x^2)]*EllipticE[ArcSin[(
Sqrt[2]*7^(1/4)*x)/Sqrt[-3 - (1 - Sqrt[7])*x^2]], (7 - Sqrt[7])/14])/(Sqrt
[(3 + (1 - Sqrt[7])*x^2)^(-1)]*Sqrt[-3 - 2*x^2 + 2*x^4]))/2 - ((7 + Sqrt[7
])*Sqrt[-3 - (1 - Sqrt[7])*x^2]*Sqrt[(3 + (1 + Sqrt[7])*x^2)/(3 + (1 - Sqr
t[7])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 - (1 - Sqrt[7])*x
^2]], (7 - Sqrt[7])/14])/(2*Sqrt[6]*7^(1/4)*Sqrt[(3 + (1 - Sqrt[7])*x^2)^(
-1)]*Sqrt[-3 - 2*x^2 + 2*x^4]))/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.98

method	result
risch	$\frac{x(x^2-4)}{21\sqrt{2x^4-2x^2-3}} - \frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-3-3\sqrt{7}}\,x}{3}, \frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{7\sqrt{-3-3\sqrt{7}}\sqrt{2x^4-2x^2-3}} - \frac{6\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\sqrt{1-(}}{}$
default	$-\frac{4\left(\frac{1}{21}x-\frac{1}{84}x^3\right)}{\sqrt{2x^4-2x^2-3}} - \frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-3-3\sqrt{7}}\,x}{3}, \frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{7\sqrt{-3-3\sqrt{7}}\sqrt{2x^4-2x^2-3}} - \frac{6\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\sqrt{1-}}{}$
elliptic	$-\frac{4\left(\frac{1}{21}x-\frac{1}{84}x^3\right)}{\sqrt{2x^4-2x^2-3}} - \frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-3-3\sqrt{7}}\,x}{3}, \frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{7\sqrt{-3-3\sqrt{7}}\sqrt{2x^4-2x^2-3}} - \frac{6\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\sqrt{1-}}{}$

input
```
int(1/(2*x^4-2*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*x*(x^2-4)/(2*x^4-2*x^2-3)^(1/2)-3/7/(-3-3*7^(1/2))^(1/2)*(1-(-1/3-1/3
*7^(1/2))*x^2)^(1/2)*(1-(-1/3+1/3*7^(1/2))*x^2)^(1/2)/(2*x^4-2*x^2-3)^(1/2
)*EllipticF(1/3*(-3-3*7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))-6/7/(
-3-3*7^(1/2))^(1/2)*(1-(-1/3-1/3*7^(1/2))*x^2)^(1/2)*(1-(-1/3+1/3*7^(1/2))
*x^2)^(1/2)/(2*x^4-2*x^2-3)^(1/2)/(-2+2*7^(1/2))*(EllipticF(1/3*(-3-3*7^(1
/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))-EllipticE(1/3*(-3-3*7^(1/2))^(1
/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-3-2x^2+2x^4)^{3/2}}\,dx = \frac{\left(\sqrt{7}\sqrt{-3}(2\,x^4-2\,x^2-3)-\sqrt{-3}(2\,x^4-2\,x^2-3)\right)\sqrt{\frac{1}{3}\sqrt{7}-\frac{1}{3}}E\left(\arcsin\left(x\right.\right.}{}$$

input
```
integrate(1/(2*x^4-2*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/126*((sqrt(7)*sqrt(-3)*(2*x^4 - 2*x^2 - 3) - sqrt(-3)*(2*x^4 - 2*x^2 - 3
))*sqrt(1/3*sqrt(7) - 1/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(7) - 1/3)), -
1/3*sqrt(7) - 4/3) + 2*(sqrt(7)*sqrt(-3)*(2*x^4 - 2*x^2 - 3) + 2*sqrt(-3)*
(2*x^4 - 2*x^2 - 3))*sqrt(1/3*sqrt(7) - 1/3)*elliptic_f(arcsin(x*sqrt(1/3*
sqrt(7) - 1/3)), -1/3*sqrt(7) - 4/3) + 6*sqrt(2*x^4 - 2*x^2 - 3)*(x^3 - 4*
x))/(2*x^4 - 2*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(-3-2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4-2x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-2*x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 - 2*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-2\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-2*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - 2*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-2x^2+2x^4)^{3/2}}\,dx=\int \frac{1}{(2\,x^4-2\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-2*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - 2*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3-2x^2+2x^4)^{3/2}}\,dx=\int \frac{1}{(2\,x^4-2\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - 2*x^2 - 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - 2*x^2 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3-2x^2+2x^4)^{3/2}}\,dx=\int \frac{\sqrt{2x^4-2x^2-3}}{4x^8-8x^6-8x^4+12x^2+9}dx$$

input
```
int(1/(2*x^4-2*x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - 2*x**2 - 3)/(4*x**8 - 8*x**6 - 8*x**4 + 12*x**2 + 9),x)
```

3.227 $\int \frac{1}{(-3-3x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 235

$$\int \frac{1}{(-3-3x^2+2x^4)^{3/2}}\,dx = -\frac{x(7-2x^2)}{33\sqrt{-3-3x^2+2x^4}}$$
$$-\frac{\sqrt{\frac{1}{6}(-3+\sqrt{33})}\sqrt{6+(3-\sqrt{33})\,x^2}\sqrt{6+(3+\sqrt{33})\,x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{6}(-3+\sqrt{33})}x\right)|\frac{1}{4}(-7-\sqrt{33})\right.}{66\sqrt{-3-3x^2+2x^4}}$$
$$-\frac{\sqrt{\frac{1}{22}(-3+\sqrt{33})}\sqrt{6+(3-\sqrt{33})\,x^2}\sqrt{6+(3+\sqrt{33})\,x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{6}(-3+\sqrt{33})}x\right),\frac{1}{4}(-7\right.}{18\sqrt{-3-3x^2+2x^4}}$$

output
```
-1/33*x*(-2*x^2+7)/(2*x^4-3*x^2-3)^(1/2)-1/396*(-18+6*33^(1/2))^(1/2)*(6+(
3-33^(1/2))*x^2)^(1/2)*(6+(3+33^(1/2))*x^2)^(1/2)*EllipticE(1/6*(-18+6*33^
(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))/(2*x^4-3*x^2-3)^(1/2)-1/396*(
-66+22*33^(1/2))^(1/2)*(6+(3-33^(1/2))*x^2)^(1/2)*(6+(3+33^(1/2))*x^2)^(1/
2)*EllipticF(1/6*(-18+6*33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))/(2
*x^4-3*x^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-3-3x^2+2x^4)^{3/2}}\,dx = \frac{4x(-7+2x^2)-2i\sqrt{3+\sqrt{33}}\sqrt{6+6x^2-4x^4}\,E\left(i\text{arcsinh}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}(-7}{132\sqrt{-3-3x}}$$

input
```
Integrate[(-3 - 3*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(4*x*(-7 + 2*x^2) - (2*I)*Sqrt[3 + Sqrt[33]]*Sqrt[6 + 6*x^2 - 4*x^4]*Ellip
ticE[I*ArcSinh[(2*x)/Sqrt[-3 + Sqrt[33]]], (-7 + Sqrt[33])/4] + ((2*I)*(11
 + Sqrt[33])*Sqrt[6 + 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-3 + S
qrt[33]]], (-7 + Sqrt[33])/4])/Sqrt[3 + Sqrt[33]])/(132*Sqrt[-3 - 3*x^2 +
2*x^4])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.62, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 25, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-3x^2-3)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{99}\int -\frac{6(x^2+2)}{\sqrt{2x^4-3x^2-3}}dx-\frac{x(7-2x^2)}{33\sqrt{2x^4-3x^2-3}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2}{33}\int \frac{x^2+2}{\sqrt{2x^4-3x^2-3}}dx-\frac{x(7-2x^2)}{33\sqrt{2x^4-3x^2-3}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{2}{33}\left(\frac{1}{4}\left(11+\sqrt{33}\right)\int\frac{1}{\sqrt{2x^4-3x^2-3}}dx+\frac{1}{4}\int-\frac{-4x^2+\sqrt{33}+3}{\sqrt{2x^4-3x^2-3}}dx\right)-\frac{x(7-2x^2)}{33\sqrt{2x^4-3x^2-3}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2}{33}\left(\frac{1}{4}\left(11+\sqrt{33}\right)\int\frac{1}{\sqrt{2x^4-3x^2-3}}dx-\frac{1}{4}\int\frac{-4x^2+\sqrt{33}+3}{\sqrt{2x^4-3x^2-3}}dx\right)-\frac{x(7-2x^2)}{33\sqrt{2x^4-3x^2-3}}$$

$$\Big\downarrow \text{1411}$$

$$-\frac{2}{33}\left(\frac{\left(11+\sqrt{33}\right)\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-6}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+6}{\left(3-\sqrt{33}\right)x^2+6}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-6}}\right),\frac{1}{22}\left(11-\sqrt{33}\right.\right.}{8\ 3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+6}}\sqrt{2x^4-3x^2-3}}\right.$$

$$\frac{x(7-2x^2)}{33\sqrt{2x^4-3x^2-3}}$$

$$\Big\downarrow \text{1498}$$

$$-\frac{2}{33}\left(\frac{\left(11+\sqrt{33}\right)\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-6}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+6}{\left(3-\sqrt{33}\right)x^2+6}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-6}}\right),\frac{1}{22}\left(11-\sqrt{33}\right.\right.}{8\ 3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+6}}\sqrt{2x^4-3x^2-3}}\right.$$

$$\frac{x(7-2x^2)}{33\sqrt{2x^4-3x^2-3}}$$

input
```
Int[(-3 - 3*x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/33*(x*(7 - 2*x^2))/Sqrt[-3 - 3*x^2 + 2*x^4] - (2*((-((x*(3 - Sqrt[33] -
 4*x^2))/Sqrt[-3 - 3*x^2 + 2*x^4]) - ((11/3)^(1/4)*Sqrt[-6 - (3 - Sqrt[33]
)*x^2]*Sqrt[(6 + (3 + Sqrt[33])*x^2)/(6 + (3 - Sqrt[33])*x^2)]*EllipticE[A
rcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6 - (3 - Sqrt[33])*x^2]], (11 - Sqrt[33])
/22])/(Sqrt[(6 + (3 - Sqrt[33])*x^2)^(-1)]*Sqrt[-3 - 3*x^2 + 2*x^4]))/4 +
((11 + Sqrt[33])*Sqrt[-6 - (3 - Sqrt[33])*x^2]*Sqrt[(6 + (3 + Sqrt[33])*x^
2)/(6 + (3 - Sqrt[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6
 - (3 - Sqrt[33])*x^2]], (11 - Sqrt[33])/22])/(8*3^(3/4)*11^(1/4)*Sqrt[(6
+ (3 - Sqrt[33])*x^2)^(-1)]*Sqrt[-3 - 3*x^2 + 2*x^4])))/33
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.97

method	result
risch	$\frac{x(2x^2-7)}{33\sqrt{2x^4-3x^2-3}}-\frac{8\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-18-6\sqrt{33}}}{6},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{11\sqrt{-18-6\sqrt{33}}\,\sqrt{2x^4-3x^2-3}}-\frac{24\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}}{}$
default	$-\frac{4\left(\frac{7}{132}x-\frac{1}{66}x^3\right)}{\sqrt{2x^4-3x^2-3}}-\frac{8\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-18-6\sqrt{33}}}{6},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{11\sqrt{-18-6\sqrt{33}}\,\sqrt{2x^4-3x^2-3}}-\frac{24\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x}}{}$
elliptic	$-\frac{4\left(\frac{7}{132}x-\frac{1}{66}x^3\right)}{\sqrt{2x^4-3x^2-3}}-\frac{8\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-18-6\sqrt{33}}}{6},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{11\sqrt{-18-6\sqrt{33}}\,\sqrt{2x^4-3x^2-3}}-\frac{24\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x}}{}$

input
```
int(1/(2*x^4-3*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/33*x*(2*x^2-7)/(2*x^4-3*x^2-3)^(1/2)-8/11/(-18-6*33^(1/2))^(1/2)*(1-(-1/
2-1/6*33^(1/2))*x^2)^(1/2)*(1-(-1/2+1/6*33^(1/2))*x^2)^(1/2)/(2*x^4-3*x^2-
3)^(1/2)*EllipticF(1/6*x*(-18-6*33^(1/2))^(1/2),1/4*I*22^(1/2)-1/4*I*6^(1/
2))-24/11/(-18-6*33^(1/2))^(1/2)*(1-(-1/2-1/6*33^(1/2))*x^2)^(1/2)*(1-(-1/
2+1/6*33^(1/2))*x^2)^(1/2)/(2*x^4-3*x^2-3)^(1/2)/(-3+33^(1/2))*(EllipticF(
1/6*x*(-18-6*33^(1/2))^(1/2),1/4*I*22^(1/2)-1/4*I*6^(1/2))-EllipticE(1/6*x
*(-18-6*33^(1/2))^(1/2),1/4*I*22^(1/2)-1/4*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(-3-3x^2+2x^4\right)^{3/2}}\,dx = \frac{\left(\sqrt{\frac{11}{3}}\sqrt{-3}(2\,x^4-3\,x^2-3)-\sqrt{-3}(2\,x^4-3\,x^2-3)\right)\sqrt{\frac{1}{2}\sqrt{\frac{11}{3}}-\frac{1}{2}}E(\arcsin$$

input
```
integrate(1/(2*x^4-3*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/66*((sqrt(11/3)*sqrt(-3)*(2*x^4 - 3*x^2 - 3) - sqrt(-3)*(2*x^4 - 3*x^2 -
 3))*sqrt(1/2*sqrt(11/3) - 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(11/3) -
1/2)), -3/4*sqrt(11/3) - 7/4) + (sqrt(11/3)*sqrt(-3)*(2*x^4 - 3*x^2 - 3) +
 3*sqrt(-3)*(2*x^4 - 3*x^2 - 3))*sqrt(1/2*sqrt(11/3) - 1/2)*elliptic_f(arc
sin(x*sqrt(1/2*sqrt(11/3) - 1/2)), -3/4*sqrt(11/3) - 7/4) + 2*sqrt(2*x^4 -
 3*x^2 - 3)*(2*x^3 - 7*x))/(2*x^4 - 3*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{\left(-3-3x^2+2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(2x^4-3x^2-3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-3*x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 - 3*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-3x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-3\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-3*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - 3*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-3x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-3\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-3*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - 3*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3-3x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-3\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - 3*x^2 - 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - 3*x^2 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3-3x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4-3x^2-3}}{4x^8-12x^6-3x^4+18x^2+9}dx$$

input
```
int(1/(2*x^4-3*x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - 3*x**2 - 3)/(4*x**8 - 12*x**6 - 3*x**4 + 18*x**2 + 9),x)
```

3.228 $\int \frac{1}{(-3-4x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 231

$$\int \frac{1}{(-3-4x^2+2x^4)^{3/2}}\,dx = -\frac{x(7-2x^2)}{30\sqrt{-3-4x^2+2x^4}} - \frac{\sqrt{3+(2-\sqrt{10})\,x^2}\sqrt{3+(2+\sqrt{10})\,x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{15\sqrt{2\left(2+\sqrt{10}\right)}\sqrt{-3-4x^2+2x^4}} - \frac{\sqrt{3+(2-\sqrt{10})\,x^2}\sqrt{3+(2+\sqrt{10})\,x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{6\sqrt{5\left(2+\sqrt{10}\right)}\sqrt{-3-4x^2+2x^4}}$$

output
```
-1/30*x*(-2*x^2+7)/(2*x^4-4*x^2-3)^(1/2)-1/15*(3+(2-10^(1/2))*x^2)^(1/2)*(
3+(2+10^(1/2))*x^2)^(1/2)*EllipticE(1/3*(-6+3*10^(1/2))^(1/2)*x,1/3*I*6^(1
/2)+1/3*I*15^(1/2))/(4+2*10^(1/2))^(1/2)/(2*x^4-4*x^2-3)^(1/2)-1/6*(3+(2-1
0^(1/2))*x^2)^(1/2)*(3+(2+10^(1/2))*x^2)^(1/2)*EllipticF(1/3*(-6+3*10^(1/2
))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))/(10+5*10^(1/2))^(1/2)/(2*x^4-4*x^
2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.76 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-3-4x^2+2x^4)^{3/2}}\,dx = \frac{2x(-7+2x^2)-2i\sqrt{2+\sqrt{10}}\sqrt{3+4x^2-2x^4}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right)|-\frac{7}{3}-\right.}{60\sqrt{-3-4x^2+}}$$

input
```
Integrate[(-3 - 4*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(2*x*(-7 + 2*x^2) - (2*I)*Sqrt[2 + Sqrt[10]]*Sqrt[3 + 4*x^2 - 2*x^4]*Ellip
ticE[I*ArcSinh[Sqrt[2/(-2 + Sqrt[10])]*x], -7/3 + (2*Sqrt[10])/3] + ((2*I)
*(5 + Sqrt[10])*Sqrt[3 + 4*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-2 + S
qrt[10])]*x], -7/3 + (2*Sqrt[10])/3])/Sqrt[2 + Sqrt[10]])/(60*Sqrt[-3 - 4*
x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.66, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-4x^2-3)^{3/2}}dx$$

$$\Big\downarrow \; 1405$$

$$\frac{1}{120}\int -\frac{4(2x^2+3)}{\sqrt{2x^4-4x^2-3}}dx-\frac{x(7-2x^2)}{30\sqrt{2x^4-4x^2-3}}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{30}\int \frac{2x^2+3}{\sqrt{2x^4-4x^2-3}}dx-\frac{x(7-2x^2)}{30\sqrt{2x^4-4x^2-3}}$$

$\downarrow$ 1501

$$\frac{1}{30}\left(-\left(\left(5+\sqrt{10}\right)\int\frac{1}{\sqrt{2x^4-4x^2-3}}dx\right)-\frac{1}{2}\int-\frac{2\left(-2x^2+\sqrt{10}+2\right)}{\sqrt{2x^4-4x^2-3}}dx\right)-\frac{x\left(7-2x^2\right)}{30\sqrt{2x^4-4x^2-3}}$$

$\downarrow$ 27

$$\frac{1}{30}\left(\int\frac{-2x^2+\sqrt{10}+2}{\sqrt{2x^4-4x^2-3}}dx-\left(5+\sqrt{10}\right)\int\frac{1}{\sqrt{2x^4-4x^2-3}}dx\right)-\frac{x\left(7-2x^2\right)}{30\sqrt{2x^4-4x^2-3}}$$

$\downarrow$ 1411

$$\frac{1}{30}\left(\int\frac{-2x^2+\sqrt{10}+2}{\sqrt{2x^4-4x^2-3}}dx-\frac{\left(5+\sqrt{10}\right)\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-3}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+3}{\left(2-\sqrt{10}\right)x^2+3}}\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{5}}{\sqrt{-\left(\left(2-\sqrt{10}\right.\right.}}\right.\right.}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+3}}\sqrt{2x^4-4x^2-3}}\right.$$

$$\frac{x\left(7-2x^2\right)}{30\sqrt{2x^4-4x^2-3}}$$

$\downarrow$ 1498

$$\frac{1}{30}\left(-\frac{\left(5+\sqrt{10}\right)\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-3}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+3}{\left(2-\sqrt{10}\right)x^2+3}}\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-3}}\right),\frac{1}{10}\left(5-\sqrt{10}\right)\right.}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+3}}\sqrt{2x^4-4x^2-3}}\right.$$

$$\frac{x\left(7-2x^2\right)}{30\sqrt{2x^4-4x^2-3}}$$

input
```
Int[(-3 - 4*x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/30*(x*(7 - 2*x^2))/Sqrt[-3 - 4*x^2 + 2*x^4] + ((x*(2 - Sqrt[10] - 2*x^2
))/Sqrt[-3 - 4*x^2 + 2*x^4] + (2^(3/4)*5^(1/4)*Sqrt[-3 - (2 - Sqrt[10])*x^
2]*Sqrt[(3 + (2 + Sqrt[10])*x^2)/(3 + (2 - Sqrt[10])*x^2)]*EllipticE[ArcSi
n[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 - (2 - Sqrt[10])*x^2]], (5 - Sqrt[10])/10])/
(Sqrt[3]*Sqrt[(3 + (2 - Sqrt[10])*x^2)^(-1)]*Sqrt[-3 - 4*x^2 + 2*x^4]) - (
(5 + Sqrt[10])*Sqrt[-3 - (2 - Sqrt[10])*x^2]*Sqrt[(3 + (2 + Sqrt[10])*x^2)
/(3 + (2 - Sqrt[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 -
(2 - Sqrt[10])*x^2]], (5 - Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^(1/4)*Sqrt[(3
 + (2 - Sqrt[10])*x^2)^(-1)]*Sqrt[-3 - 4*x^2 + 2*x^4]))/30
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00

method	result
risch	$\frac{x(2x^2-7)}{30\sqrt{2x^4-4x^2-3}} - \frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6-3\sqrt{10}}\,x}{3}, \frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{10\sqrt{-6-3\sqrt{10}}\,\sqrt{2x^4-4x^2-3}} - \frac{6\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{}}{}$
default	$-\frac{4\left(\frac{7}{120}x-\frac{1}{60}x^3\right)}{\sqrt{2x^4-4x^2-3}} - \frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6-3\sqrt{10}}\,x}{3}, \frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{10\sqrt{-6-3\sqrt{10}}\,\sqrt{2x^4-4x^2-3}} - \frac{6\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}}{}$
elliptic	$-\frac{4\left(\frac{7}{120}x-\frac{1}{60}x^3\right)}{\sqrt{2x^4-4x^2-3}} - \frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6-3\sqrt{10}}\,x}{3}, \frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{10\sqrt{-6-3\sqrt{10}}\,\sqrt{2x^4-4x^2-3}} - \frac{6\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}}{}$

input
```
int(1/(2*x^4-4*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*x*(2*x^2-7)/(2*x^4-4*x^2-3)^(1/2)-3/10/(-6-3*10^(1/2))^(1/2)*(1-(-2/3
-1/3*10^(1/2))*x^2)^(1/2)*(1-(-2/3+1/3*10^(1/2))*x^2)^(1/2)/(2*x^4-4*x^2-3
)^(1/2)*EllipticF(1/3*(-6-3*10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2)
)-6/5/(-6-3*10^(1/2))^(1/2)*(1-(-2/3-1/3*10^(1/2))*x^2)^(1/2)*(1-(-2/3+1/3
*10^(1/2))*x^2)^(1/2)/(2*x^4-4*x^2-3)^(1/2)/(-4+2*10^(1/2))*(EllipticF(1/3
*(-6-3*10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))-EllipticE(1/3*(-6-3
*10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.77

$$\int \frac{1}{(-3-4x^2+2x^4)^{3/2}}\,dx = \frac{2\left(\sqrt{10}\sqrt{-3}(2\,x^4-4\,x^2-3)-2\,\sqrt{-3}(2\,x^4-4\,x^2-3)\right)\sqrt{\frac{1}{3}\,\sqrt{10}-\frac{2}{3}}E(\mathrm{arcsi}}{}$$

input
```
integrate(1/(2*x^4-4*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/180*(2*(sqrt(10)*sqrt(-3)*(2*x^4 - 4*x^2 - 3) - 2*sqrt(-3)*(2*x^4 - 4*x^
2 - 3))*sqrt(1/3*sqrt(10) - 2/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(10) - 2
/3)), -2/3*sqrt(10) - 7/3) + (sqrt(10)*sqrt(-3)*(2*x^4 - 4*x^2 - 3) + 10*s
qrt(-3)*(2*x^4 - 4*x^2 - 3))*sqrt(1/3*sqrt(10) - 2/3)*elliptic_f(arcsin(x*
sqrt(1/3*sqrt(10) - 2/3)), -2/3*sqrt(10) - 7/3) + 6*sqrt(2*x^4 - 4*x^2 - 3
)*(2*x^3 - 7*x))/(2*x^4 - 4*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(-3-4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4-4x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-4*x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 - 4*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-4\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-4*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - 4*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-4\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-4*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - 4*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3-4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-4\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - 4*x^2 - 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - 4*x^2 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3-4x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4-4x^2-3}}{4x^8-16x^6+4x^4+24x^2+9}dx$$

input
```
int(1/(2*x^4-4*x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - 4*x**2 - 3)/(4*x**8 - 16*x**6 + 4*x**4 + 24*x**2 + 9),x)
```

3.229 $\int \frac{1}{(-3-5x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{1}{(-3-5x^2+2x^4)^{3/2}}\,dx = -\frac{x(37-10x^2)}{147\sqrt{-3-5x^2+2x^4}} - \frac{5\sqrt{3-x^2}\sqrt{1+2x^2}E\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-6\right)}{147\sqrt{-3-5x^2+2x^4}} - \frac{\sqrt{3-x^2}\sqrt{1+2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-6\right)}{21\sqrt{-3-5x^2+2x^4}}$$

output

```
-1/147*x*(-10*x^2+37)/(2*x^4-5*x^2-3)^(1/2)-5/147*(-x^2+3)^(1/2)*(2*x^2+1)
^(1/2)*EllipticE(1/3*x*3^(1/2),I*6^(1/2))/(2*x^4-5*x^2-3)^(1/2)-1/21*(-x^2
+3)^(1/2)*(2*x^2+1)^(1/2)*EllipticF(1/3*x*3^(1/2),I*6^(1/2))/(2*x^4-5*x^2-
3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-3-5x^2+2x^4)^{3/2}}\,dx = \frac{-37x+10x^3-5i\sqrt{6}\sqrt{3-x^2}\sqrt{1+2x^2}E\left(i\text{arcsinh}\left(\sqrt{2}x\right)\left|-\frac{1}{6}\right.\right)+7i\sqrt{6}\sqrt{3-}}{147\sqrt{-3-5x^2+2x^4}}$$

input
```
Integrate[(-3 - 5*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(-37*x + 10*x^3 - (5*I)*Sqrt[6]*Sqrt[3 - x^2]*Sqrt[1 + 2*x^2]*EllipticE[I*
ArcSinh[Sqrt[2]*x], -1/6] + (7*I)*Sqrt[6]*Sqrt[3 - x^2]*Sqrt[1 + 2*x^2]*El
lipticF[I*ArcSinh[Sqrt[2]*x], -1/6])/(147*Sqrt[-3 - 5*x^2 + 2*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.64, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1410, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-5x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{147}\int -\frac{2(5x^2+6)}{\sqrt{2x^4-5x^2-3}}dx-\frac{x(37-10x^2)}{147\sqrt{2x^4-5x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{147}\int \frac{5x^2+6}{\sqrt{2x^4-5x^2-3}}dx-\frac{x(37-10x^2)}{147\sqrt{2x^4-5x^2-3}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{2}{147}\left(21\int \frac{1}{\sqrt{2x^4-5x^2-3}}dx+\frac{5}{4}\int -\frac{4(3-x^2)}{\sqrt{2x^4-5x^2-3}}dx\right)-\frac{x(37-10x^2)}{147\sqrt{2x^4-5x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{147}\left(21\int \frac{1}{\sqrt{2x^4-5x^2-3}}dx-5\int \frac{3-x^2}{\sqrt{2x^4-5x^2-3}}dx\right)-\frac{x(37-10x^2)}{147\sqrt{2x^4-5x^2-3}}$$

$$\Big\downarrow \text{1410}$$

$$-\frac{2}{147}\left(\frac{3\sqrt{7}\sqrt{x^2-3}\sqrt{2x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{7}x}{\sqrt{x^2-3}}\right),\frac{1}{7}\right)}{\sqrt{2x^4-5x^2-3}}-5\int \frac{3-x^2}{\sqrt{2x^4-5x^2-3}}dx\right)-\frac{x(37-10x^2)}{147\sqrt{2x^4-5x^2-3}}$$

$$\Big\downarrow \text{1498}$$

$$-\frac{2}{147}\left(\frac{3\sqrt{7}\sqrt{x^2-3}\sqrt{2x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{7}x}{\sqrt{x^2-3}}\right),\frac{1}{7}\right)}{\sqrt{2x^4-5x^2-3}}-5\left(\frac{\sqrt{7}\sqrt{x^2-3}\sqrt{\frac{2x^2+1}{3-x^2}}E\left(\arcsin\left(\frac{\sqrt{7}x}{\sqrt{x^2-3}}\right)|\frac{1}{7}\right)}{2\sqrt{\frac{1}{3-x^2}}\sqrt{2x^4-5x^2-3}} -\right.\right.$$

$$\frac{x(37-10x^2)}{147\sqrt{2x^4-5x^2-3}}$$

input

```
Int[(-3 - 5*x^2 + 2*x^4)^(-3/2),x]
```

output

```
-1/147*(x*(37 - 10*x^2))/Sqrt[-3 - 5*x^2 + 2*x^4] - (2*(-5*(-1/2*(x*(1 + 2
*x^2))/Sqrt[-3 - 5*x^2 + 2*x^4] + (Sqrt[7]*Sqrt[-3 + x^2]*Sqrt[(1 + 2*x^2)
/(3 - x^2)]*EllipticE[ArcSin[(Sqrt[7]*x)/Sqrt[-3 + x^2]], 1/7])/(2*Sqrt[(3
 - x^2)^(-1)]*Sqrt[-3 - 5*x^2 + 2*x^4])) + (3*Sqrt[7]*Sqrt[-3 + x^2]*Sqrt[
1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt[-3 + x^2]], 1/7])/Sqrt[-3 - 5
*x^2 + 2*x^4]))/147
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1410
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11

method	result
risch	$\frac{x(10x^2-37)}{147\sqrt{2x^4-5x^2-3}}+\frac{2i\sqrt{2}\sqrt{2x^2+1}\sqrt{-3x^2+9}\ \mathrm{EllipticF}\left(ix\sqrt{2},\frac{i\sqrt{6}}{6}\right)}{147\sqrt{2x^4-5x^2-3}}+\frac{5i\sqrt{2}\sqrt{2x^2+1}\sqrt{-3x^2+9}\left(\mathrm{EllipticF}\left(ix\sqrt{2},\frac{i\sqrt{6}}{6}\right)-\mathrm{Ellip}\right.}{147\sqrt{2x^4-5x^2-3}}$
default	$-\frac{4\left(\frac{37}{588}x-\frac{5}{294}x^3\right)}{\sqrt{2x^4-5x^2-3}}+\frac{2i\sqrt{2}\sqrt{2x^2+1}\sqrt{-3x^2+9}\ \mathrm{EllipticF}\left(ix\sqrt{2},\frac{i\sqrt{6}}{6}\right)}{147\sqrt{2x^4-5x^2-3}}+\frac{5i\sqrt{2}\sqrt{2x^2+1}\sqrt{-3x^2+9}\left(\mathrm{EllipticF}\left(ix\sqrt{2},\frac{i\sqrt{6}}{6}\right)-\mathrm{Elli}\right.}{147\sqrt{2x^4-5x^2-3}}$
elliptic	$-\frac{4\left(\frac{37}{588}x-\frac{5}{294}x^3\right)}{\sqrt{2x^4-5x^2-3}}+\frac{2i\sqrt{2}\sqrt{2x^2+1}\sqrt{-3x^2+9}\ \mathrm{EllipticF}\left(ix\sqrt{2},\frac{i\sqrt{6}}{6}\right)}{147\sqrt{2x^4-5x^2-3}}+\frac{5i\sqrt{2}\sqrt{2x^2+1}\sqrt{-3x^2+9}\left(\mathrm{EllipticF}\left(ix\sqrt{2},\frac{i\sqrt{6}}{6}\right)-\mathrm{Elli}\right.}{147\sqrt{2x^4-5x^2-3}}$

input
```
int(1/(2*x^4-5*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/147*x*(10*x^2-37)/(2*x^4-5*x^2-3)^(1/2)+2/147*I*2^(1/2)*(2*x^2+1)^(1/2)*
(-3*x^2+9)^(1/2)/(2*x^4-5*x^2-3)^(1/2)*EllipticF(I*2^(1/2)*x,1/6*I*6^(1/2)
)+5/147*I*2^(1/2)*(2*x^2+1)^(1/2)*(-3*x^2+9)^(1/2)/(2*x^4-5*x^2-3)^(1/2)*(
EllipticF(I*2^(1/2)*x,1/6*I*6^(1/2))-EllipticE(I*2^(1/2)*x,1/6*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-3-5x^2+2x^4)^{3/2}}\,dx = \frac{5\sqrt{3}\sqrt{-3}(2x^4-5x^2-3)E\left(\arcsin\left(\frac{1}{3}\sqrt{3}x\right)\mid -6\right)+31\sqrt{3}\sqrt{-3}(2x^4-5x^2}{1323\,(2x^4-5x^2}$$

input
```
integrate(1/(2*x^4-5*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/1323*(5*sqrt(3)*sqrt(-3)*(2*x^4 - 5*x^2 - 3)*elliptic_e(arcsin(1/3*sqrt(
3)*x), -6) + 31*sqrt(3)*sqrt(-3)*(2*x^4 - 5*x^2 - 3)*elliptic_f(arcsin(1/3
*sqrt(3)*x), -6) + 9*sqrt(2*x^4 - 5*x^2 - 3)*(10*x^3 - 37*x))/(2*x^4 - 5*x
^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(-3-5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4-5x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-5*x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 - 5*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-5\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-5*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - 5*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-5\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-5*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - 5*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-3-5x^2+2x^4\right)^{3/2}}\,dx=\int \frac{1}{{\left(2\,x^4-5\,x^2-3\right)}^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - 5*x^2 - 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - 5*x^2 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(-3-5x^2+2x^4\right)^{3/2}}\,dx=\int \frac{\sqrt{2x^4-5x^2-3}}{4x^8-20x^6+13x^4+30x^2+9}dx$$

input
```
int(1/(2*x^4-5*x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - 5*x**2 - 3)/(4*x**8 - 20*x**6 + 13*x**4 + 30*x**2 + 9),x
)
```

3.230 $\int \frac{1}{\left(-3-6x^2+2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 227

$$\int \frac{1}{\left(-3-6x^2+2x^4\right)^{3/2}}\,dx = -\frac{x(4-x^2)}{15\sqrt{-3-6x^2+2x^4}} - \frac{\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}\sqrt{3+\left(3-\sqrt{15}\right)x^2}\sqrt{3+\left(3+\sqrt{15}\right)x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}x\right)|-4-\sqrt{15}\right)}{30\sqrt{-3-6x^2+2x^4}} - \frac{\sqrt{\frac{1}{5}\left(-3+\sqrt{15}\right)}\sqrt{3+\left(3-\sqrt{15}\right)x^2}\sqrt{3+\left(3+\sqrt{15}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}x\right),-4-\right.}{18\sqrt{-3-6x^2+2x^4}}$$

output
```
-1/15*x*(-x^2+4)/(2*x^4-6*x^2-3)^(1/2)-1/90*(-9+3*15^(1/2))^(1/2)*(3+(3-15
^(1/2))*x^2)^(1/2)*(3+(3+15^(1/2))*x^2)^(1/2)*EllipticE(1/3*(-9+3*15^(1/2)
)^(1/2)*x,1/2*I*6^(1/2)+1/2*I*10^(1/2))/(2*x^4-6*x^2-3)^(1/2)-1/90*(-15+5*
15^(1/2))^(1/2)*(3+(3-15^(1/2))*x^2)^(1/2)*(3+(3+15^(1/2))*x^2)^(1/2)*Elli
pticF(1/3*(-9+3*15^(1/2))^(1/2)*x,1/2*I*6^(1/2)+1/2*I*10^(1/2))/(2*x^4-6*x
^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.91 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.68

$$\int \frac{1}{(-3-6x^2+2x^4)^{3/2}}\,dx = \frac{4x(-4+x^2)-2i\sqrt{3+\sqrt{15}}\sqrt{3+6x^2-2x^4}E\left(i\text{arcsinh}\left(\sqrt{1+\sqrt{\frac{5}{3}}}x\right)|-4\right.}{60\sqrt{-3-6x^2+}}$$

input
```
Integrate[(-3 - 6*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(4*x*(-4 + x^2) - (2*I)*Sqrt[3 + Sqrt[15]]*Sqrt[3 + 6*x^2 - 2*x^4]*Ellipti
cE[I*ArcSinh[Sqrt[1 + Sqrt[5/3]]*x], -4 + Sqrt[15]] + ((2*I)*(5 + Sqrt[15]
)*Sqrt[3 + 6*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/3]]*x], -4 +
 Sqrt[15]])/Sqrt[3 + Sqrt[15]])/(60*Sqrt[-3 - 6*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.71, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 27, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-6x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{180}\int -\frac{12(x^2+1)}{\sqrt{2x^4-6x^2-3}}dx - \frac{x(4-x^2)}{15\sqrt{2x^4-6x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}\int \frac{x^2+1}{\sqrt{2x^4-6x^2-3}}dx - \frac{x(4-x^2)}{15\sqrt{2x^4-6x^2-3}}$$

$$\Big\downarrow \text{1501}$$

$$\frac{1}{15}\left(-\frac{1}{2}\left(5+\sqrt{15}\right)\int \frac{1}{\sqrt{2x^4-6x^2-3}}dx-\frac{1}{4}\int -\frac{2\left(-2x^2+\sqrt{15}+3\right)}{\sqrt{2x^4-6x^2-3}}dx\right)-\frac{x\left(4-x^2\right)}{15\sqrt{2x^4-6x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(\frac{1}{2}\int \frac{-2x^2+\sqrt{15}+3}{\sqrt{2x^4-6x^2-3}}dx-\frac{1}{2}\left(5+\sqrt{15}\right)\int \frac{1}{\sqrt{2x^4-6x^2-3}}dx\right)-\frac{x\left(4-x^2\right)}{15\sqrt{2x^4-6x^2-3}}$$

$$\Big\downarrow \text{1411}$$

$$\frac{1}{15}\left(\frac{1}{2}\int \frac{-2x^2+\sqrt{15}+3}{\sqrt{2x^4-6x^2-3}}dx-\frac{\left(5+\sqrt{15}\right)\sqrt{-\left(\left(3-\sqrt{15}\right)x^2\right)-3}\sqrt{\frac{\left(3+\sqrt{15}\right)x^2+3}{\left(3-\sqrt{15}\right)x^2+3}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt{-\left(\left(3-\cdots\right)\right)}}\cdots\right)\right)}{2\sqrt{2}3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{\left(3-\sqrt{15}\right)x^2+3}}\sqrt{2x^4-6x^2-3}}\cdots\right)$$

$$\frac{x\left(4-x^2\right)}{15\sqrt{2x^4-6x^2-3}}$$

$$\Big\downarrow \text{1498}$$

$$\frac{1}{15}\left(\frac{1}{2}\left(\frac{\sqrt[4]{\frac{5}{3}}\sqrt{2}\sqrt{-\left(\left(3-\sqrt{15}\right)x^2\right)-3}\sqrt{\frac{\left(3+\sqrt{15}\right)x^2+3}{\left(3-\sqrt{15}\right)x^2+3}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{15}x}{\sqrt{-\left(\left(3-\sqrt{15}\right)x^2\right)-3}}\right)|\frac{1}{10}\left(5-\sqrt{15}\right)\right)}{\sqrt{\frac{1}{\left(3-\sqrt{15}\right)x^2+3}}\sqrt{2x^4-6x^2-3}}+\frac{x\left(-2\cdots\right)}{\sqrt{2\cdots}}\cdots\right)\right)$$

$$\frac{x\left(4-x^2\right)}{15\sqrt{2x^4-6x^2-3}}$$

input
```
Int[(-3 - 6*x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/15*(x*(4 - x^2))/Sqrt[-3 - 6*x^2 + 2*x^4] + (((x*(3 - Sqrt[15] - 2*x^2)
)/Sqrt[-3 - 6*x^2 + 2*x^4] + ((5/3)^(1/4)*Sqrt[2]*Sqrt[-3 - (3 - Sqrt[15])
*x^2]*Sqrt[(3 + (3 + Sqrt[15])*x^2)/(3 + (3 - Sqrt[15])*x^2)]*EllipticE[Ar
cSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[-3 - (3 - Sqrt[15])*x^2]], (5 - Sqrt[15])/1
0])/(Sqrt[(3 + (3 - Sqrt[15])*x^2)^(-1)]*Sqrt[-3 - 6*x^2 + 2*x^4]))/2 - ((
5 + Sqrt[15])*Sqrt[-3 - (3 - Sqrt[15])*x^2]*Sqrt[(3 + (3 + Sqrt[15])*x^2)/
(3 + (3 - Sqrt[15])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[-3 -
(3 - Sqrt[15])*x^2]], (5 - Sqrt[15])/10])/(2*Sqrt[2]*3^(3/4)*5^(1/4)*Sqrt[
(3 + (3 - Sqrt[15])*x^2)^(-1)]*Sqrt[-3 - 6*x^2 + 2*x^4]))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

method	result
risch	$\frac{x(x^2-4)}{15\sqrt{2x^4-6x^2-3}}-\frac{\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-9-3\sqrt{15}}\,x}{3},\frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{5\sqrt{-9-3\sqrt{15}}\,\sqrt{2x^4-6x^2-3}}-\frac{6\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-}$
default	$-\frac{4\left(\frac{1}{15}x-\frac{1}{60}x^3\right)}{\sqrt{2x^4-6x^2-3}}-\frac{\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-9-3\sqrt{15}}\,x}{3},\frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{5\sqrt{-9-3\sqrt{15}}\,\sqrt{2x^4-6x^2-3}}-\frac{6\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-}$
elliptic	$-\frac{4\left(\frac{1}{15}x-\frac{1}{60}x^3\right)}{\sqrt{2x^4-6x^2-3}}-\frac{\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-9-3\sqrt{15}}\,x}{3},\frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{5\sqrt{-9-3\sqrt{15}}\,\sqrt{2x^4-6x^2-3}}-\frac{6\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-}$

input
```
int(1/(2*x^4-6*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(x^2-4)/(2*x^4-6*x^2-3)^(1/2)-1/5/(-9-3*15^(1/2))^(1/2)*(1-(-1-1/3*
15^(1/2))*x^2)^(1/2)*(1-(-1+1/3*15^(1/2))*x^2)^(1/2)/(2*x^4-6*x^2-3)^(1/2)
*EllipticF(1/3*(-9-3*15^(1/2))^(1/2)*x,1/2*I*10^(1/2)-1/2*I*6^(1/2))-6/5/(
-9-3*15^(1/2))^(1/2)*(1-(-1-1/3*15^(1/2))*x^2)^(1/2)*(1-(-1+1/3*15^(1/2))*
x^2)^(1/2)/(2*x^4-6*x^2-3)^(1/2)/(-6+2*15^(1/2))*(EllipticF(1/3*(-9-3*15^(
1/2))^(1/2)*x,1/2*I*10^(1/2)-1/2*I*6^(1/2))-EllipticE(1/3*(-9-3*15^(1/2))^
(1/2)*x,1/2*I*10^(1/2)-1/2*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.64

$$\int \frac{1}{(-3-6x^2+2x^4)^{3/2}}\,dx = \frac{2\sqrt{-3}(2\,x^4-6\,x^2-3)\sqrt{\sqrt{\frac{5}{3}}-1}F(\arcsin\left(x\sqrt{\sqrt{\frac{5}{3}}-1}\right)|-3\sqrt{\frac{5}{3}}-4)+\left(\right.}{}$$

input
```
integrate(1/(2*x^4-6*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/30*(2*sqrt(-3)*(2*x^4 - 6*x^2 - 3)*sqrt(sqrt(5/3) - 1)*elliptic_f(arcsin
(x*sqrt(sqrt(5/3) - 1)), -3*sqrt(5/3) - 4) + (sqrt(5/3)*sqrt(-3)*(2*x^4 -
6*x^2 - 3) - sqrt(-3)*(2*x^4 - 6*x^2 - 3))*sqrt(sqrt(5/3) - 1)*elliptic_e(
arcsin(x*sqrt(sqrt(5/3) - 1)), -3*sqrt(5/3) - 4) + 2*sqrt(2*x^4 - 6*x^2 -
3)*(x^3 - 4*x))/(2*x^4 - 6*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(-3-6x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4-6x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-6*x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 - 6*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-6x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-6\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-6*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - 6*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-6x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-6\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-6*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - 6*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3-6x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-6\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - 6*x^2 - 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - 6*x^2 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3-6x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4-6x^2-3}}{4x^8-24x^6+24x^4+36x^2+9}dx$$

input
```
int(1/(2*x^4-6*x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - 6*x**2 - 3)/(4*x**8 - 24*x**6 + 24*x**4 + 36*x**2 + 9),x
)
```

3.231 $\int \frac{1}{(-3-7x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 235

$$\int \frac{1}{(-3-7x^2+2x^4)^{3/2}}\,dx = -\frac{x(61-14x^2)}{219\sqrt{-3-7x^2+2x^4}} - \frac{7\sqrt{\frac{1}{6}\left(-7+\sqrt{73}\right)}\sqrt{6+\left(7-\sqrt{73}\right)x^2}\sqrt{6+\left(7+\sqrt{73}\right)x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{6}\left(-7+\sqrt{73}\right)}x\right)|\frac{1}{12}\left(-61-7\sqrt{}\right.\right.}{438\sqrt{-3-7x^2+2x^4}} - \frac{\sqrt{\frac{1}{438}\left(-7+\sqrt{73}\right)}\sqrt{6+\left(7-\sqrt{73}\right)x^2}\sqrt{6+\left(7+\sqrt{73}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{6}\left(-7+\sqrt{73}\right)}x\right),\frac{1}{12}\left(-\right.\right.}{6\sqrt{-3-7x^2+2x^4}}$$

output
```
-1/219*x*(-14*x^2+61)/(2*x^4-7*x^2-3)^(1/2)-7/2628*(-42+6*73^(1/2))^(1/2)*
(6+(-73^(1/2)+7)*x^2)^(1/2)*(6+(7+73^(1/2))*x^2)^(1/2)*EllipticE(1/6*(-42+
6*73^(1/2))^(1/2)*x,7/12*I*6^(1/2)+1/12*I*438^(1/2))/(2*x^4-7*x^2-3)^(1/2)
-1/2628*(-3066+438*73^(1/2))^(1/2)*(6+(-73^(1/2)+7)*x^2)^(1/2)*(6+(7+73^(1
/2))*x^2)^(1/2)*EllipticF(1/6*(-42+6*73^(1/2))^(1/2)*x,7/12*I*6^(1/2)+1/12
*I*438^(1/2))/(2*x^4-7*x^2-3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.79 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-3-7x^2+2x^4)^{3/2}}\,dx = \frac{4x(-61+14x^2)-14i\sqrt{7+\sqrt{73}}\sqrt{6+14x^2-4x^4}E\left(i\text{arcsinh}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)|\frac{1}{12}\right.}{876\sqrt{-3}}$$

input
```
Integrate[(-3 - 7*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(4*x*(-61 + 14*x^2) - (14*I)*Sqrt[7 + Sqrt[73]]*Sqrt[6 + 14*x^2 - 4*x^4]*E
llipticE[I*ArcSinh[(2*x)/Sqrt[-7 + Sqrt[73]]], (-61 + 7*Sqrt[73])/12] + ((
2*I)*(73 + 7*Sqrt[73])*Sqrt[6 + 14*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/
Sqrt[-7 + Sqrt[73]]], (-61 + 7*Sqrt[73])/12])/Sqrt[7 + Sqrt[73]])/(876*Sqr
t[-3 - 7*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.63, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1501, 25, 1411, 1498}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-7x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{219}\int -\frac{2(7x^2+6)}{\sqrt{2x^4-7x^2-3}}dx - \frac{x(61-14x^2)}{219\sqrt{2x^4-7x^2-3}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2}{219}\int \frac{7x^2+6}{\sqrt{2x^4-7x^2-3}}dx - \frac{x(61-14x^2)}{219\sqrt{2x^4-7x^2-3}}$$

$$\Big\downarrow \text{1501}$$

$$-\frac{2}{219}\left(\frac{1}{4}\left(73+7\sqrt{73}\right)\int \frac{1}{\sqrt{2x^4-7x^2-3}}dx+\frac{7}{4}\int -\frac{-4x^2+\sqrt{73}+7}{\sqrt{2x^4-7x^2-3}}dx\right)-\frac{x(61-14x^2)}{219\sqrt{2x^4-7x^2-3}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2}{219}\left(\frac{1}{4}\left(73+7\sqrt{73}\right)\int \frac{1}{\sqrt{2x^4-7x^2-3}}dx-\frac{7}{4}\int \frac{-4x^2+\sqrt{73}+7}{\sqrt{2x^4-7x^2-3}}dx\right)-\frac{x(61-14x^2)}{219\sqrt{2x^4-7x^2-3}}$$

$$\Big\downarrow \text{1411}$$

$$-\frac{2}{219}\left(\frac{\left(73+7\sqrt{73}\right)\sqrt{-\left(\left(7-\sqrt{73}\right)x^2\right)-6}\sqrt{\frac{\left(7+\sqrt{73}\right)x^2+6}{\left(7-\sqrt{73}\right)x^2+6}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{-\left(\left(7-\sqrt{73}\right)x^2\right)-6}}\right),\frac{1}{146}(73-\right.}{8\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{\left(7-\sqrt{73}\right)x^2+6}}\sqrt{2x^4-7x^2-3}}\right.$$

$$\frac{x(61-14x^2)}{219\sqrt{2x^4-7x^2-3}}$$

$$\Big\downarrow \text{1498}$$

$$-\frac{2}{219}\left(\frac{\left(73+7\sqrt{73}\right)\sqrt{-\left(\left(7-\sqrt{73}\right)x^2\right)-6}\sqrt{\frac{\left(7+\sqrt{73}\right)x^2+6}{\left(7-\sqrt{73}\right)x^2+6}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{-\left(\left(7-\sqrt{73}\right)x^2\right)-6}}\right),\frac{1}{146}(73-\right.}{8\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{\left(7-\sqrt{73}\right)x^2+6}}\sqrt{2x^4-7x^2-3}}\right.$$

$$\frac{x(61-14x^2)}{219\sqrt{2x^4-7x^2-3}}$$

input
```
Int[(-3 - 7*x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/219*(x*(61 - 14*x^2))/Sqrt[-3 - 7*x^2 + 2*x^4] - (2*((-7*((x*(7 - Sqrt[
73] - 4*x^2))/Sqrt[-3 - 7*x^2 + 2*x^4] + (73^(1/4)*Sqrt[-6 - (7 - Sqrt[73]
)*x^2]*Sqrt[(6 + (7 + Sqrt[73])*x^2)/(6 + (7 - Sqrt[73])*x^2)]*EllipticE[A
rcSin[(Sqrt[2]*73^(1/4)*x)/Sqrt[-6 - (7 - Sqrt[73])*x^2]], (73 - 7*Sqrt[73
])/146])/(Sqrt[3]*Sqrt[(6 + (7 - Sqrt[73])*x^2)^(-1)]*Sqrt[-3 - 7*x^2 + 2*
x^4])))/4 + ((73 + 7*Sqrt[73])*Sqrt[-6 - (7 - Sqrt[73])*x^2]*Sqrt[(6 + (7
+ Sqrt[73])*x^2)/(6 + (7 - Sqrt[73])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*73^(1
/4)*x)/Sqrt[-6 - (7 - Sqrt[73])*x^2]], (73 - 7*Sqrt[73])/146])/(8*Sqrt[3]*
73^(1/4)*Sqrt[(6 + (7 - Sqrt[73])*x^2)^(-1)]*Sqrt[-3 - 7*x^2 + 2*x^4])))/2
19
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1411
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1498
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[e*x*((b + q + 2*c*x^2)/(2*c*Sqrt[
a + b*x^2 + c*x^4])), x] - Simp[e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)
*x^2)]*(Sqrt[(2*a + (b + q)*x^2)/q]/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*
a + (b + q)*x^2)]))*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b
 + q)/(2*q)], x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1501
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*d - e*(b - q))/(2*c)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[e/(2*c)   Int[(b - q + 2*c*x^2)/Sqr
t[a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b,
c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.97

method	result
risch	$\frac{x(14x^2-61)}{219\sqrt{2x^4-7x^2-3}}-\frac{24\sqrt{1-\left(-\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-42-6\sqrt{73}}\,x}{6},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{73\sqrt{-42-6\sqrt{73}}\,\sqrt{2x^4-7x^2-3}}-\frac{168\sqrt{1-\left(-\frac{7}{6}-\frac{\sqrt{7}}{6}\right.}}{}$
default	$-\frac{4\left(\frac{61}{876}x-\frac{7}{438}x^3\right)}{\sqrt{2x^4-7x^2-3}}-\frac{24\sqrt{1-\left(-\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-42-6\sqrt{73}}\,x}{6},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{73\sqrt{-42-6\sqrt{73}}\,\sqrt{2x^4-7x^2-3}}-\frac{168\sqrt{1-\left(-\frac{7}{6}-\right.}}{}$
elliptic	$-\frac{4\left(\frac{61}{876}x-\frac{7}{438}x^3\right)}{\sqrt{2x^4-7x^2-3}}-\frac{24\sqrt{1-\left(-\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-42-6\sqrt{73}}\,x}{6},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{73\sqrt{-42-6\sqrt{73}}\,\sqrt{2x^4-7x^2-3}}-\frac{168\sqrt{1-\left(-\frac{7}{6}-\right.}}{}$

input
```
int(1/(2*x^4-7*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/219*x*(14*x^2-61)/(2*x^4-7*x^2-3)^(1/2)-24/73/(-42-6*73^(1/2))^(1/2)*(1-
(-7/6-1/6*73^(1/2))*x^2)^(1/2)*(1-(-7/6+1/6*73^(1/2))*x^2)^(1/2)/(2*x^4-7*
x^2-3)^(1/2)*EllipticF(1/6*(-42-6*73^(1/2))^(1/2)*x,1/12*I*438^(1/2)-7/12*
I*6^(1/2))-168/73/(-42-6*73^(1/2))^(1/2)*(1-(-7/6-1/6*73^(1/2))*x^2)^(1/2)
*(1-(-7/6+1/6*73^(1/2))*x^2)^(1/2)/(2*x^4-7*x^2-3)^(1/2)/(-7+73^(1/2))*(El
lipticF(1/6*(-42-6*73^(1/2))^(1/2)*x,1/12*I*438^(1/2)-7/12*I*6^(1/2))-Elli
pticE(1/6*(-42-6*73^(1/2))^(1/2)*x,1/12*I*438^(1/2)-7/12*I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.76

$$\int \frac{1}{(-3-7x^2+2x^4)^{3/2}}\,dx = \frac{7\left(\sqrt{73}\sqrt{-3}(2\,x^4-7\,x^2-3)-7\,\sqrt{-3}(2\,x^4-7\,x^2-3)\right)\sqrt{\frac{1}{6}\,\sqrt{73}-\frac{7}{6}}E(\mathrm{arcsi}}{}$$

input
```
integrate(1/(2*x^4-7*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/1314*(7*(sqrt(73)*sqrt(-3)*(2*x^4 - 7*x^2 - 3) - 7*sqrt(-3)*(2*x^4 - 7*x
^2 - 3))*sqrt(1/6*sqrt(73) - 7/6)*elliptic_e(arcsin(x*sqrt(1/6*sqrt(73) -
7/6)), -7/12*sqrt(73) - 61/12) - (sqrt(73)*sqrt(-3)*(2*x^4 - 7*x^2 - 3) -
91*sqrt(-3)*(2*x^4 - 7*x^2 - 3))*sqrt(1/6*sqrt(73) - 7/6)*elliptic_f(arcsi
n(x*sqrt(1/6*sqrt(73) - 7/6)), -7/12*sqrt(73) - 61/12) + 6*sqrt(2*x^4 - 7*
x^2 - 3)*(14*x^3 - 61*x))/(2*x^4 - 7*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(-3-7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4-7x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-7*x**2-3)**(3/2),x)
```

output
```
Integral((2*x**4 - 7*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-7\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-7*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - 7*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-7\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-7*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - 7*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3-7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-7\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - 7*x^2 - 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - 7*x^2 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3-7x^2+2x^4)^{3/2}}\,dx=\int \frac{\sqrt{2x^4-7x^2-3}}{4x^8-28x^6+37x^4+42x^2+9}dx$$

input
```
int(1/(2*x^4-7*x^2-3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - 7*x**2 - 3)/(4*x**8 - 28*x**6 + 37*x**4 + 42*x**2 + 9),x
)
```

3.232 $\int \frac{1}{(2+6x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 234

$$\int \frac{1}{(2+6x^2+3x^4)^{3/2}}\,dx = \frac{\sqrt{3}x}{2\left(3-\sqrt{3}\right)\sqrt{2+6x^2+3x^4}} + \frac{3\sqrt{2+6x^2+3x^4}E\left(\arctan\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right)|-1-\sqrt{3}\right)}{2\sqrt{2\left(3+\sqrt{3}\right)}\sqrt{3-\sqrt{3}+3x^2}\sqrt{3+\sqrt{3}+3x^2}} - \frac{\sqrt{2+\left(3-\sqrt{3}\right)x^2}\sqrt{2+\left(3+\sqrt{3}\right)x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{2}\left(3-\sqrt{3}\right)}x\right),-1-\sqrt{3}\right)}{2\sqrt{2\left(3-\sqrt{3}\right)}\sqrt{2+6x^2+3x^4}}$$

output
```
1/2*3^(1/2)*x/(3-3^(1/2))/(3*x^4+6*x^2+2)^(1/2)+3/2*(3*x^4+6*x^2+2)^(1/2)*
EllipticE(3^(1/2)/(3+3^(1/2))^(1/2)*x/(1+3/(3+3^(1/2))*x^2)^(1/2),(-1-3^(1
/2))^(1/2))/(6+2*3^(1/2))^(1/2)/(3-3^(1/2)+3*x^2)^(1/2)/(3+3^(1/2)+3*x^2)^
(1/2)-1/2*(2+(3-3^(1/2))*x^2)^(1/2)*(2+(3+3^(1/2))*x^2)^(1/2)*InverseJacob
iAM(arctan(1/2*(6-2*3^(1/2))^(1/2)*x),(-1-3^(1/2))^(1/2))/(6-2*3^(1/2))^(1
/2)/(3*x^4+6*x^2+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.79

$$\int \frac{1}{(2+6x^2+3x^4)^{3/2}}\,dx = \frac{12x+9x^3+3i(-1+\sqrt{3})\sqrt{\frac{-3+\sqrt{3}-3x^2}{-3+\sqrt{3}}}\sqrt{3+\sqrt{3}+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right)\right.}{}$$

input
```
Integrate[(2 + 6*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(12*x + 9*x^3 + (3*I)*(-1 + Sqrt[3])*Sqrt[(-3 + Sqrt[3] - 3*x^2)/(-3 + Sqr
t[3])]*Sqrt[3 + Sqrt[3] + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/(3 + Sqrt[3])]
*x], 2 + Sqrt[3]] + I*Sqrt[(-3 + Sqrt[3])*(-3 + Sqrt[3] - 3*x^2)]*Sqrt[3 +
 Sqrt[3] + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/(3 + Sqrt[3])]*x], 2 + Sqrt[3
]])/(12*Sqrt[2 + 6*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1405, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+6x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(3x^2+4)}{4\sqrt{3x^4+6x^2+2}}-\frac{1}{24}\int \frac{6(3x^2+2)}{\sqrt{3x^4+6x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(3x^2+4)}{4\sqrt{3x^4+6x^2+2}}-\frac{1}{4}\int \frac{3x^2+2}{\sqrt{3x^4+6x^2+2}}dx$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{4}\left(-2\int \frac{1}{\sqrt{3x^4+6x^2+2}}dx-3\int \frac{x^2}{\sqrt{3x^4+6x^2+2}}dx\right)+\frac{x(3x^2+4)}{4\sqrt{3x^4+6x^2+2}}$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{4}\left(-3\int \frac{x^2}{\sqrt{3x^4+6x^2+2}}dx-\frac{\sqrt{\frac{2}{3+\sqrt{3}}}\sqrt{\frac{(3-\sqrt{3})x^2+2}{(3+\sqrt{3})x^2+2}}\left(\left(3+\sqrt{3}\right)x^2+2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}x\right),-\right.}{\sqrt{3x^4+6x^2+2}}\right.$$

$$\frac{x(3x^2+4)}{4\sqrt{3x^4+6x^2+2}}$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{4}\left(-\frac{\sqrt{\frac{2}{3+\sqrt{3}}}\sqrt{\frac{(3-\sqrt{3})x^2+2}{(3+\sqrt{3})x^2+2}}\left(\left(3+\sqrt{3}\right)x^2+2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}x\right),-1+\sqrt{3}\right)}{\sqrt{3x^4+6x^2+2}}-3\left(\frac{x(3x^2+\sqrt{}}{3\sqrt{3x^4+6}}\right.\right.$$

$$\frac{x(3x^2+4)}{4\sqrt{3x^4+6x^2+2}}$$

input
```
Int[(2 + 6*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(x*(4 + 3*x^2))/(4*Sqrt[2 + 6*x^2 + 3*x^4]) + (-3*((x*(3 + Sqrt[3] + 3*x^2
))/(3*Sqrt[2 + 6*x^2 + 3*x^4]) - (Sqrt[(3 + Sqrt[3])/2]*Sqrt[(2 + (3 - Sqr
t[3])*x^2)/(2 + (3 + Sqrt[3])*x^2)]*(2 + (3 + Sqrt[3])*x^2)*EllipticE[ArcT
an[Sqrt[(3 + Sqrt[3])/2]*x], -1 + Sqrt[3]])/(3*Sqrt[2 + 6*x^2 + 3*x^4])) -
 (Sqrt[2/(3 + Sqrt[3])]*Sqrt[(2 + (3 - Sqrt[3])*x^2)/(2 + (3 + Sqrt[3])*x^
2)]*(2 + (3 + Sqrt[3])*x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[3])/2]*x], -1
+ Sqrt[3]])/Sqrt[2 + 6*x^2 + 3*x^4])/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.96

method	result
risch	$\frac{x(3x^2+4)}{4\sqrt{3x^4+6x^2+2}} - \frac{\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2\sqrt{3}}}{2},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{-6+2\sqrt{3}}\,\sqrt{3x^4+6x^2+2}} + 6\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{3}{2}-\right.}$
default	$-\frac{6\left(-\frac{1}{8}x^3-\frac{1}{6}x\right)}{\sqrt{3x^4+6x^2+2}} - \frac{\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2\sqrt{3}}}{2},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{-6+2\sqrt{3}}\,\sqrt{3x^4+6x^2+2}} + 6\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\right.}$
elliptic	$-\frac{6\left(-\frac{1}{8}x^3-\frac{1}{6}x\right)}{\sqrt{3x^4+6x^2+2}} - \frac{\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2\sqrt{3}}}{2},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{-6+2\sqrt{3}}\,\sqrt{3x^4+6x^2+2}} + 6\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\right.}$

input
```
int(1/(3*x^4+6*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(3*x^2+4)/(3*x^4+6*x^2+2)^(1/2)-1/(-6+2*3^(1/2))^(1/2)*(1-(-3/2+1/2*
3^(1/2))*x^2)^(1/2)*(1-(-3/2-1/2*3^(1/2))*x^2)^(1/2)/(3*x^4+6*x^2+2)^(1/2)
*EllipticF(1/2*x*(-6+2*3^(1/2))^(1/2),1/2*6^(1/2)+1/2*2^(1/2))+6/(-6+2*3^(
1/2))^(1/2)*(1-(-3/2+1/2*3^(1/2))*x^2)^(1/2)*(1-(-3/2-1/2*3^(1/2))*x^2)^(1
/2)/(3*x^4+6*x^2+2)^(1/2)/(6+2*3^(1/2))*(EllipticF(1/2*x*(-6+2*3^(1/2))^(1
/2),1/2*6^(1/2)+1/2*2^(1/2))-EllipticE(1/2*x*(-6+2*3^(1/2))^(1/2),1/2*6^(1
/2)+1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(2+6x^2+3x^4\right)^{3/2}}\,dx = \frac{3\left(\sqrt{3}\sqrt{2}(3\,x^4+6\,x^2+2)-3\,\sqrt{2}(3\,x^4+6\,x^2+2)\right)\sqrt{\frac{1}{2}\,\sqrt{3}-\frac{3}{2}}\,E\left(\arcsin\left(x\sqrt{\tfrac{1}{2}\right.\right.}}{}$$

input
```
integrate(1/(3*x^4+6*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(3*(sqrt(3)*sqrt(2)*(3*x^4 + 6*x^2 + 2) - 3*sqrt(2)*(3*x^4 + 6*x^2 +
2))*sqrt(1/2*sqrt(3) - 3/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(3) - 3/2)),
sqrt(3) + 2) - (sqrt(3)*sqrt(2)*(3*x^4 + 6*x^2 + 2) - 15*sqrt(2)*(3*x^4 +
6*x^2 + 2))*sqrt(1/2*sqrt(3) - 3/2)*elliptic_f(arcsin(x*sqrt(1/2*sqrt(3) -
 3/2)), sqrt(3) + 2) + 6*sqrt(3*x^4 + 6*x^2 + 2)*(3*x^3 + 4*x))/(3*x^4 + 6
*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2+6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+6x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+6*x**2+2)**(3/2),x)
```

output
```
Integral((3*x**4 + 6*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+6\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+6*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 6*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+6\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+6*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 6*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+6\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(6*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int(1/(6*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+6x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+6x^2+2}}{9x^8+36x^6+48x^4+24x^2+4}dx$$

input
```
int(1/(3*x^4+6*x^2+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 6*x**2 + 2)/(9*x**8 + 36*x**6 + 48*x**4 + 24*x**2 + 4),x
)
```

3.233 $\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 120

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{3x}{2\sqrt{2+5x^2+3x^4}} + \frac{5\sqrt{2+5x^2+3x^4}\,E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{2+3x^2}} - \frac{3\sqrt{2}\sqrt{1+x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2+5x^2+3x^4}}$$

output
```
3/2*x/(3*x^4+5*x^2+2)^(1/2)+5/2*(3*x^4+5*x^2+2)^(1/2)*EllipticE(x/(x^2+1)^
(1/2),1/2*I*2^(1/2))*2^(1/2)/(x^2+1)^(1/2)/(3*x^2+2)^(1/2)-3*(x^2+1)^(1/2)
*(3*x^2+2)^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5
*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{13x+15x^3+5i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}\,E\left(i\,\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\left|\frac{2}{3}\right.\right)-i\sqrt{3}\sqrt{1+x^2}\sqrt{}}{2\sqrt{2+5x^2+3x^4}}$$

input
```
Integrate[(2 + 5*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(13*x + 15*x^3 + (5*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*A
rcSinh[Sqrt[3/2]*x], 2/3] - I*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Ellipt
icF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(2*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1405, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(15x^2+13)}{2\sqrt{3x^4+5x^2+2}}-\frac{1}{2}\int \frac{3(5x^2+4)}{\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(15x^2+13)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\int \frac{5x^2+4}{\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{1503}$$

$$\frac{x(15x^2+13)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\left(4\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+5\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)$$

$$\Big\downarrow \text{1413}$$

$$\frac{x(15x^2+13)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\left(5\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{2\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right)$$

$$\Big\downarrow \text{1456}$$

$$\frac{x(15x^2+13)}{2\sqrt{3x^4+5x^2+2}}-$$
$$\frac{3}{2}\left(\frac{2\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+5\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\right|}{3\sqrt{3x^4+5x^2+2}}\right.\right.$$

input
```
Int[(2 + 5*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(x*(13 + 15*x^2))/(2*Sqrt[2 + 5*x^2 + 3*x^4]) - (3*(5*((x*(2 + 3*x^2))/(3*
Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*
EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (2*Sqrt[2]*(1 +
 x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x
^2 + 3*x^4]))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04

method	result	si
risch	$\frac{x\left(15x^{2}+13\right)}{2\sqrt{3x^{4}+5x^{2}+2}}+\frac{3i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^{4}+5x^{2}+2}}-\frac{5i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{3x^{4}+5x^{2}+2}}$	12
default	$-\frac{6\left(-\frac{5}{4}x^{3}-\frac{13}{12}x\right)}{\sqrt{3x^{4}+5x^{2}+2}}+\frac{3i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^{4}+5x^{2}+2}}-\frac{5i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{3x^{4}+5x^{2}+2}}$	12
elliptic	$-\frac{6\left(-\frac{5}{4}x^{3}-\frac{13}{12}x\right)}{\sqrt{3x^{4}+5x^{2}+2}}+\frac{3i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^{4}+5x^{2}+2}}-\frac{5i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{3x^{4}+5x^{2}+2}}$	12

input
```
int(1/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(15*x^2+13)/(3*x^4+5*x^2+2)^(1/2)+3*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/
(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))-5/2*I*(x^2+1)^(1/2)*(6*x^
2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x
,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx =$$
$$-\frac{5\sqrt{2}(3i\,x^4+5i\,x^2+2i)E(\arcsin{(i\,x)}\mid\frac{3}{2})+11\sqrt{2}(-3i\,x^4-5i\,x^2-2i)F(\arcsin{(i\,x)}\mid\frac{3}{2})-\sqrt{3\,x^4+5\,x}}{2\,(3\,x^4+5\,x^2+2)}$$

input
```
integrate(1/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(5*sqrt(2)*(3*I*x^4 + 5*I*x^2 + 2*I)*elliptic_e(arcsin(I*x), 3/2) + 1
1*sqrt(2)*(-3*I*x^4 - 5*I*x^2 - 2*I)*elliptic_f(arcsin(I*x), 3/2) - sqrt(3
*x^4 + 5*x^2 + 2)*(15*x^3 + 13*x))/(3*x^4 + 5*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((3*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+5x^2+2}}{9x^8+30x^6+37x^4+20x^2+4}dx$$

input
```
int(1/(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x
)
```

3.234 $\int \frac{1}{(2+4x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 260

$$\int \frac{1}{(2+4x^2+3x^4)^{3/2}}\,dx = -\frac{x(1+3x^2)}{4\sqrt{2+4x^2+3x^4}}+\frac{3x\sqrt{2+4x^2+3x^4}}{4\left(\sqrt{6}+3x^2\right)} - \frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\,2^{3/4}\sqrt{2+4x^2+3x^4}} + \frac{\sqrt[4]{3}(2+\sqrt{6})\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{8\,2^{3/4}\sqrt{2+4x^2+3x^4}}$$

output
```
-1/4*x*(3*x^2+1)/(3*x^4+4*x^2+2)^(1/2)+3*x*(3*x^4+4*x^2+2)^(1/2)/(4*6^(1/2
)+12*x^2)-1/4*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4+4*x^2+2)/(2+6^(1/2)*x^2)^2)^
(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18-6*6^(1/2))^(1
/2))*2^(1/4)/(3*x^4+4*x^2+2)^(1/2)+1/16*3^(1/4)*(2+6^(1/2))*(2+6^(1/2)*x^2
)*((3*x^4+4*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*3
^(1/4)*2^(3/4)*x),1/6*(18-6*6^(1/2))^(1/2))*2^(1/4)/(3*x^4+4*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.99 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.26

$$\int \frac{1}{(2+4x^2+3x^4)^{3/2}}\,dx = \frac{-3\sqrt{-\frac{i}{-2i+\sqrt{2}}}x(1+3x^2)-\sqrt{3}(2i+\sqrt{2})\sqrt{\frac{-2i+\sqrt{2}-3ix^2}{-2i+\sqrt{2}}}\sqrt{\frac{2i+\sqrt{2}+3ix^2}{2i+\sqrt{2}}}E\Big(i\text{arcsinh}}{}$$

input
```
Integrate[(2 + 4*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(-3*Sqrt[(-I)/(-2*I + Sqrt[2])]*x*(1 + 3*x^2) - Sqrt[3]*(2*I + Sqrt[2])*Sq
rt[(-2*I + Sqrt[2] - (3*I)*x^2)/(-2*I + Sqrt[2])]*Sqrt[(2*I + Sqrt[2] + (3
*I)*x^2)/(2*I + Sqrt[2])]*EllipticE[I*ArcSinh[Sqrt[(-3*I)/(-2*I + Sqrt[2])
]*x], (2*I - Sqrt[2])/(2*I + Sqrt[2])] + Sqrt[3]*(-I + Sqrt[2])*Sqrt[(-2*I
 + Sqrt[2] - (3*I)*x^2)/(-2*I + Sqrt[2])]*Sqrt[(2*I + Sqrt[2] + (3*I)*x^2)
/(2*I + Sqrt[2])]*EllipticF[I*ArcSinh[Sqrt[(-3*I)/(-2*I + Sqrt[2])]*x], (2
*I - Sqrt[2])/(2*I + Sqrt[2])])/(12*Sqrt[(-I)/(-2*I + Sqrt[2])]*Sqrt[2 + 4
*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+4x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{16}\int \frac{12(x^2+1)}{\sqrt{3x^4+4x^2+2}}dx-\frac{x(3x^2+1)}{4\sqrt{3x^4+4x^2+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{4}\int \frac{x^2+1}{\sqrt{3x^4+4x^2+2}}dx-\frac{x(3x^2+1)}{4\sqrt{3x^4+4x^2+2}}$$

$\Big\downarrow$ 1511

$$\frac{3}{4}\left(\frac{1}{3}\left(3+\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4+4x^2+2}}dx-\sqrt{\frac{2}{3}}\int \frac{2-\sqrt{6}x^2}{2\sqrt{3x^4+4x^2+2}}dx\right)-\frac{x(3x^2+1)}{4\sqrt{3x^4+4x^2+2}}$$

$\Big\downarrow$ 27

$$\frac{3}{4}\left(\frac{1}{3}\left(3+\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4+4x^2+2}}dx-\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4+4x^2+2}}dx}{\sqrt{6}}\right)-\frac{x(3x^2+1)}{4\sqrt{3x^4+4x^2+2}}$$

$\Big\downarrow$ 1416

$$\frac{3}{4}\left(\frac{\left(3+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{6\sqrt[4]{6}\sqrt{3x^4+4x^2+2}}-\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4+4x^2+2}}dx}{\sqrt{6}}\right)-\frac{x(3x^2+1)}{4\sqrt{3x^4+4x^2+2}}$$

$\Big\downarrow$ 1509

$$\frac{3}{4}\left(\frac{\left(3+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{6\sqrt[4]{6}\sqrt{3x^4+4x^2+2}}-\frac{\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}E\left(2\text{arc}\right.}{\sqrt[4]{3}\sqrt{3x^4+4x^2+}}\right.$$

$$\frac{x(3x^2+1)}{4\sqrt{3x^4+4x^2+2}}$$

input
```
Int[(2 + 4*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/4*(x*(1 + 3*x^2))/Sqrt[2 + 4*x^2 + 3*x^4] + (3*(-(((-2*x*Sqrt[2 + 4*x^2
 + 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2
+ 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/S
qrt[6]])/(3^(1/4)*Sqrt[2 + 4*x^2 + 3*x^4]))/Sqrt[6]) + ((3 + Sqrt[6])*(2 +
 Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*Ar
cTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(6*6^(1/4)*Sqrt[2 + 4*x^2 + 3*x^4])
))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{x(3x^2+1)}{4\sqrt{3x^4+4x^2+2}}+\frac{3\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-4+2i\sqrt{2}}}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{2\sqrt{-4+2i\sqrt{2}}\sqrt{3x^4+4x^2+2}}-\frac{6\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\sqrt{1-}$
default	$-\frac{6\left(\frac{1}{24}x+\frac{1}{8}x^3\right)}{\sqrt{3x^4+4x^2+2}}+\frac{3\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-4+2i\sqrt{2}}}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{2\sqrt{-4+2i\sqrt{2}}\sqrt{3x^4+4x^2+2}}-\frac{6\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\sqrt{1-(}$
elliptic	$-\frac{6\left(\frac{1}{24}x+\frac{1}{8}x^3\right)}{\sqrt{3x^4+4x^2+2}}+\frac{3\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-4+2i\sqrt{2}}}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{2\sqrt{-4+2i\sqrt{2}}\sqrt{3x^4+4x^2+2}}-\frac{6\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\sqrt{1-(}$

input
```
int(1/(3*x^4+4*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x*(3*x^2+1)/(3*x^4+4*x^2+2)^(1/2)+3/2/(-4+2*I*2^(1/2))^(1/2)*(1-(-1+1
/2*I*2^(1/2))*x^2)^(1/2)*(1-(-1-1/2*I*2^(1/2))*x^2)^(1/2)/(3*x^4+4*x^2+2)^
(1/2)*EllipticF(1/2*x*(-4+2*I*2^(1/2))^(1/2),1/3*(3+6*I*2^(1/2))^(1/2))-6/
(-4+2*I*2^(1/2))^(1/2)*(1-(-1+1/2*I*2^(1/2))*x^2)^(1/2)*(1-(-1-1/2*I*2^(1/
2))*x^2)^(1/2)/(3*x^4+4*x^2+2)^(1/2)/(4+2*I*2^(1/2))*(EllipticF(1/2*x*(-4+
2*I*2^(1/2))^(1/2),1/3*(3+6*I*2^(1/2))^(1/2))-EllipticE(1/2*x*(-4+2*I*2^(1
/2))^(1/2),1/3*(3+6*I*2^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.57

$$\int \frac{1}{(2+4x^2+3x^4)^{3/2}}\,dx = \frac{\sqrt{2}\left(6\,x^4+8\,x^2-\sqrt{-2}(3\,x^4+4\,x^2+2)+4\right)\sqrt{\frac{1}{2}\sqrt{-2}-1}E(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-2}}\right.}{}$$

input
```
integrate(1/(3*x^4+4*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*(6*x^4 + 8*x^2 - sqrt(-2)*(3*x^4 + 4*x^2 + 2) + 4)*sqrt(1/2*s
qrt(-2) - 1)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(-2) - 1)), 2/3*sqrt(-2) + 1
/3) - 4*sqrt(2)*(3*x^4 + 4*x^2 + 2)*sqrt(1/2*sqrt(-2) - 1)*elliptic_f(arcs
in(x*sqrt(1/2*sqrt(-2) - 1)), 2/3*sqrt(-2) + 1/3) - 2*sqrt(3*x^4 + 4*x^2 +
 2)*(3*x^3 + x))/(3*x^4 + 4*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2+4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+4x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+4*x**2+2)**(3/2),x)
```

output
```
Integral((3*x**4 + 4*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+4\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+4*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 4*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+4\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+4*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 4*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+4\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(4*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int(1/(4*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+4x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+4x^2+2}}{9x^8+24x^6+28x^4+16x^2+4}dx$$

input
```
int(1/(3*x^4+4*x^2+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 4*x**2 + 2)/(9*x**8 + 24*x**6 + 28*x**4 + 16*x**2 + 4),x
)
```

3.235 $\int \frac{1}{(2+3x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 261

$$\int \frac{1}{(2+3x^2+3x^4)^{3/2}}\,dx = \frac{x(1-3x^2)}{10\sqrt{2+3x^2+3x^4}} + \frac{3x\sqrt{2+3x^2+3x^4}}{10\left(\sqrt{6}+3x^2\right)} - \frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)\middle|\frac{1}{8}(4-\sqrt{6})\right)}{5\ 2^{3/4}\sqrt{2+3x^2+3x^4}} + \frac{\left(3+2\sqrt{6}\right)\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}(4-\sqrt{6})\right)}{10\ 6^{3/4}\sqrt{2+3x^2+3x^4}}$$

output

```
1/10*x*(-3*x^2+1)/(3*x^4+3*x^2+2)^(1/2)+3*x*(3*x^4+3*x^2+2)^(1/2)/(10*6^(1
/2)+30*x^2)-1/10*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4+3*x^2+2)/(2+6^(1/2)*x^2)^
2)^(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/4*(8-2*6^(1/2))^
(1/2))*2^(1/4)/(3*x^4+3*x^2+2)^(1/2)+1/60*(3+2*6^(1/2))*(2+6^(1/2)*x^2)*((
3*x^4+3*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*3^(1/
4)*2^(3/4)*x),1/4*(8-2*6^(1/2))^(1/2))*6^(1/4)/(3*x^4+3*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.24 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.32

$$\int \frac{1}{(2+3x^2+3x^4)^{3/2}}\,dx = \frac{12\sqrt{-\frac{i}{-3i+\sqrt{15}}}x(1-3x^2)-3i\sqrt{2}(\sqrt{3}-i\sqrt{5})\sqrt{\frac{-3i+\sqrt{15}-6ix^2}{-3i+\sqrt{15}}}\sqrt{\frac{3i+\sqrt{15}+6ix^2}{3i+\sqrt{15}}}E\Big(i}{}$$

input
```
Integrate[(2 + 3*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(12*Sqrt[(-I)/(-3*I + Sqrt[15])]*x*(1 - 3*x^2) - (3*I)*Sqrt[2]*(Sqrt[3] -
I*Sqrt[5])*Sqrt[(-3*I + Sqrt[15] - (6*I)*x^2)/(-3*I + Sqrt[15])]*Sqrt[(3*I
 + Sqrt[15] + (6*I)*x^2)/(3*I + Sqrt[15])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)
/(-3*I + Sqrt[15])]*x], (3*I - Sqrt[15])/(3*I + Sqrt[15])] + Sqrt[2]*((-5*
I)*Sqrt[3] + 3*Sqrt[5])*Sqrt[(-3*I + Sqrt[15] - (6*I)*x^2)/(-3*I + Sqrt[15
])]*Sqrt[(3*I + Sqrt[15] + (6*I)*x^2)/(3*I + Sqrt[15])]*EllipticF[I*ArcSin
h[Sqrt[(-6*I)/(-3*I + Sqrt[15])]*x], (3*I - Sqrt[15])/(3*I + Sqrt[15])])/(
120*Sqrt[(-I)/(-3*I + Sqrt[15])]*Sqrt[2 + 3*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{30}\int \frac{3(3x^2+4)}{\sqrt{3x^4+3x^2+2}}dx+\frac{x(1-3x^2)}{10\sqrt{3x^4+3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\int \frac{3x^2+4}{\sqrt{3x^4+3x^2+2}}dx+\frac{x(1-3x^2)}{10\sqrt{3x^4+3x^2+2}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{10}\left(\left(4+\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4+3x^2+2}}dx-\sqrt{6}\int \frac{2-\sqrt{6}x^2}{2\sqrt{3x^4+3x^2+2}}dx\right)+\frac{x(1-3x^2)}{10\sqrt{3x^4+3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\left(4+\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4+3x^2+2}}dx-\sqrt{\frac{3}{2}}\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4+3x^2+2}}dx\right)+\frac{x(1-3x^2)}{10\sqrt{3x^4+3x^2+2}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{10}\left(\frac{\left(4+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+3x^2+2}}-\sqrt{\frac{3}{2}}\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4+3x^2+2}}dx\right.$$

$$\frac{x(1-3x^2)}{10\sqrt{3x^4+3x^2+2}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{10}\left(\frac{\left(4+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+3x^2+2}}-\sqrt{\frac{3}{2}}\left(\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\ldots}}{\ldots}\right.\right.$$

$$\frac{x(1-3x^2)}{10\sqrt{3x^4+3x^2+2}}$$

input
```
Int[(2 + 3*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(x*(1 - 3*x^2))/(10*Sqrt[2 + 3*x^2 + 3*x^4]) + (-(Sqrt[3/2]*((-2*x*Sqrt[2
+ 3*x^2 + 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 +
 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], (4
 - Sqrt[6])/8])/(3^(1/4)*Sqrt[2 + 3*x^2 + 3*x^4]))) + ((4 + Sqrt[6])*(2 +
Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*Arc
Tan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[2 + 3*x^2 + 3*x^4]))
/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{x(3x^2-1)}{10\sqrt{3x^4+3x^2+2}}+\frac{4\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-3+i\sqrt{15}}}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{5\sqrt{-3+i\sqrt{15}}\,\sqrt{3x^4+3x^2+2}}-\frac{12\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right.}}{}$
default	$-\frac{6\left(-\frac{1}{60}x+\frac{1}{20}x^3\right)}{\sqrt{3x^4+3x^2+2}}+\frac{4\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-3+i\sqrt{15}}}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{5\sqrt{-3+i\sqrt{15}}\,\sqrt{3x^4+3x^2+2}}-\frac{12\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right.}}{}$
elliptic	$-\frac{6\left(-\frac{1}{60}x+\frac{1}{20}x^3\right)}{\sqrt{3x^4+3x^2+2}}+\frac{4\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-3+i\sqrt{15}}}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{5\sqrt{-3+i\sqrt{15}}\,\sqrt{3x^4+3x^2+2}}-\frac{12\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right.}}{}$

input
```
int(1/(3*x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*x*(3*x^2-1)/(3*x^4+3*x^2+2)^(1/2)+4/5/(-3+I*15^(1/2))^(1/2)*(1-(-3/4
+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*I*15^(1/2))*x^2)^(1/2)/(3*x^4+3*x
^2+2)^(1/2)*EllipticF(1/2*x*(-3+I*15^(1/2))^(1/2),1/2*(-1+I*15^(1/2))^(1/2
))-12/5/(-3+I*15^(1/2))^(1/2)*(1-(-3/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-3/4
-1/4*I*15^(1/2))*x^2)^(1/2)/(3*x^4+3*x^2+2)^(1/2)/(3+I*15^(1/2))*(Elliptic
F(1/2*x*(-3+I*15^(1/2))^(1/2),1/2*(-1+I*15^(1/2))^(1/2))-EllipticE(1/2*x*(
-3+I*15^(1/2))^(1/2),1/2*(-1+I*15^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.62

$$\int \frac{1}{(2+3x^2+3x^4)^{3/2}}\,dx = \frac{3\sqrt{2}\left(9\,x^4+9\,x^2-\sqrt{-15}(3\,x^4+3\,x^2+2)+6\right)\sqrt{\sqrt{-15}-3}E(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{}\right.}{}$$

input
```
integrate(1/(3*x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/240*(3*sqrt(2)*(9*x^4 + 9*x^2 - sqrt(-15)*(3*x^4 + 3*x^2 + 2) + 6)*sqrt(
sqrt(-15) - 3)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(-15) - 3)), 1/4*sqrt(-15)
 - 1/4) - sqrt(2)*(63*x^4 + 63*x^2 + sqrt(-15)*(3*x^4 + 3*x^2 + 2) + 42)*s
qrt(sqrt(-15) - 3)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(-15) - 3)), 1/4*sqrt(
-15) - 1/4) - 24*sqrt(3*x^4 + 3*x^2 + 2)*(3*x^3 - x))/(3*x^4 + 3*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2+3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+3x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral((3*x**4 + 3*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 3*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 3*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+3\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(3*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int(1/(3*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+3x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+3x^2+2}}{9x^8+18x^6+21x^4+12x^2+4}dx$$

input
```
int(1/(3*x^4+3*x^2+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 3*x**2 + 2)/(9*x**8 + 18*x**6 + 21*x**4 + 12*x**2 + 4),x
)
```

3.236 $\int \frac{1}{(2+2x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 264

$$\begin{aligned}\int \frac{1}{(2+2x^2+3x^4)^{3/2}}\,dx &= \frac{x(4-3x^2)}{20\sqrt{2+2x^2+3x^4}}+\frac{3x\sqrt{2+2x^2+3x^4}}{20\left(\sqrt{6}+3x^2\right)} \\ &\quad -\frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)\mid\frac{1}{12}(6-\sqrt{6})\right)}{10\ 2^{3/4}\sqrt{2+2x^2+3x^4}} \\ &\quad +\frac{\sqrt[4]{3}(1+\sqrt{6})\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}(6-\sqrt{6})\right)}{20\ 2^{3/4}\sqrt{2+2x^2+3x^4}}\end{aligned}$$

output
```
1/20*x*(-3*x^2+4)/(3*x^4+2*x^2+2)^(1/2)+3*x*(3*x^4+2*x^2+2)^(1/2)/(20*6^(1
/2)+60*x^2)-1/20*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4+2*x^2+2)/(2+6^(1/2)*x^2)^
2)^(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18-3*6^(1/2))
^(1/2))*2^(1/4)/(3*x^4+2*x^2+2)^(1/2)+1/40*3^(1/4)*(1+6^(1/2))*(2+6^(1/2)*
x^2)*((3*x^4+2*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/
2*3^(1/4)*2^(3/4)*x),1/6*(18-3*6^(1/2))^(1/2))*2^(1/4)/(3*x^4+2*x^2+2)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.00 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.24

$$\int \frac{1}{(2+2x^2+3x^4)^{3/2}}\,dx = \frac{3\sqrt{-\frac{i}{-i+\sqrt{5}}}x(4-3x^2)-\sqrt{3}(i+\sqrt{5})\sqrt{\frac{-i+\sqrt{5}-3ix^2}{-i+\sqrt{5}}}\sqrt{\frac{i+\sqrt{5}+3ix^2}{i+\sqrt{5}}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\right.\right.}{60}$$

input
```
Integrate[(2 + 2*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(3*Sqrt[(-I)/(-I + Sqrt[5])]*x*(4 - 3*x^2) - Sqrt[3]*(I + Sqrt[5])*Sqrt[(-
I + Sqrt[5] - (3*I)*x^2)/(-I + Sqrt[5])]*Sqrt[(I + Sqrt[5] + (3*I)*x^2)/(I
 + Sqrt[5])]*EllipticE[I*ArcSinh[Sqrt[(-3*I)/(-I + Sqrt[5])]*x], (I - Sqrt
[5])/(I + Sqrt[5])] + Sqrt[3]*(-5*I + Sqrt[5])*Sqrt[(-I + Sqrt[5] - (3*I)*
x^2)/(-I + Sqrt[5])]*Sqrt[(I + Sqrt[5] + (3*I)*x^2)/(I + Sqrt[5])]*Ellipti
cF[I*ArcSinh[Sqrt[(-3*I)/(-I + Sqrt[5])]*x], (I - Sqrt[5])/(I + Sqrt[5])])
/(60*Sqrt[(-I)/(-I + Sqrt[5])]*Sqrt[2 + 2*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+2x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{40}\int \frac{6(x^2+2)}{\sqrt{3x^4+2x^2+2}}dx+\frac{x(4-3x^2)}{20\sqrt{3x^4+2x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{20}\int \frac{x^2+2}{\sqrt{3x^4+2x^2+2}}dx+\frac{x(4-3x^2)}{20\sqrt{3x^4+2x^2+2}}$$

$$\Big\downarrow \; 1511$$

$$\frac{3}{20}\left(\frac{1}{3}\left(6+\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4+2x^2+2}}dx-\sqrt{\frac{2}{3}}\int \frac{2-\sqrt{6}x^2}{2\sqrt{3x^4+2x^2+2}}dx\right)+\frac{x(4-3x^2)}{20\sqrt{3x^4+2x^2+2}}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{20}\left(\frac{1}{3}\left(6+\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4+2x^2+2}}dx-\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4+2x^2+2}}dx}{\sqrt{6}}\right)+\frac{x(4-3x^2)}{20\sqrt{3x^4+2x^2+2}}$$

$$\Big\downarrow \; 1416$$

$$\frac{3}{20}\left(\frac{\left(6+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{3x^4+2x^2+2}}-\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4+2x^2+2}}dx}{\sqrt{6}}\right)+\frac{x(4-3x^2)}{20\sqrt{3x^4+2x^2+2}}$$

$$\Big\downarrow \; 1509$$

$$\frac{3}{20}\left(\frac{\left(6+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{3x^4+2x^2+2}}-\frac{\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}E}{\sqrt[4]{3}\sqrt{3}}}{}\right.$$
$$\frac{x(4-3x^2)}{20\sqrt{3x^4+2x^2+2}}$$

input
```
Int[(2 + 2*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(x*(4 - 3*x^2))/(20*Sqrt[2 + 2*x^2 + 3*x^4]) + (3*(-(((-2*x*Sqrt[2 + 2*x^2
 + 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2
+ 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt
[6])/12])/(3^(1/4)*Sqrt[2 + 2*x^2 + 3*x^4]))/Sqrt[6]) + ((6 + Sqrt[6])*(2
+ Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*A
rcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(6*6^(1/4)*Sqrt[2 + 2*x^2 + 3*x^4
])))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{x(3x^2-4)}{20\sqrt{3x^4+2x^2+2}}+\frac{3\sqrt{1-\left(\frac{i\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{5}}}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{5\sqrt{-2+2i\sqrt{5}}\,\sqrt{3x^4+2x^2+2}}-\frac{6\sqrt{1-\left(\frac{i\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\sqrt{1-}}{}$
default	$-\frac{6\left(-\frac{1}{30}x+\frac{1}{40}x^3\right)}{\sqrt{3x^4+2x^2+2}}+\frac{3\sqrt{1-\left(\frac{i\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{5}}}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{5\sqrt{-2+2i\sqrt{5}}\,\sqrt{3x^4+2x^2+2}}-\frac{6\sqrt{1-\left(\frac{i\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\sqrt{1-}}{}$
elliptic	$-\frac{6\left(-\frac{1}{30}x+\frac{1}{40}x^3\right)}{\sqrt{3x^4+2x^2+2}}+\frac{3\sqrt{1-\left(\frac{i\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{5}}}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{5\sqrt{-2+2i\sqrt{5}}\,\sqrt{3x^4+2x^2+2}}-\frac{6\sqrt{1-\left(\frac{i\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\sqrt{1-}}{}$

input
```
int(1/(3*x^4+2*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*x*(3*x^2-4)/(3*x^4+2*x^2+2)^(1/2)+3/5/(-2+2*I*5^(1/2))^(1/2)*(1-(1/2
*I*5^(1/2)-1/2)*x^2)^(1/2)*(1-(-1/2-1/2*I*5^(1/2))*x^2)^(1/2)/(3*x^4+2*x^2
+2)^(1/2)*EllipticF(1/2*x*(-2+2*I*5^(1/2))^(1/2),1/3*(-6+3*I*5^(1/2))^(1/2
))-6/5/(-2+2*I*5^(1/2))^(1/2)*(1-(1/2*I*5^(1/2)-1/2)*x^2)^(1/2)*(1-(-1/2-1
/2*I*5^(1/2))*x^2)^(1/2)/(3*x^4+2*x^2+2)^(1/2)/(2+2*I*5^(1/2))*(EllipticF(
1/2*x*(-2+2*I*5^(1/2))^(1/2),1/3*(-6+3*I*5^(1/2))^(1/2))-EllipticE(1/2*x*(
-2+2*I*5^(1/2))^(1/2),1/3*(-6+3*I*5^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.63

$$\int \frac{1}{(2+2x^2+3x^4)^{3/2}}\,dx = \frac{\sqrt{2}\left(3\,x^4+2\,x^2-\sqrt{-5}(3\,x^4+2\,x^2+2)+2\right)\sqrt{\frac{1}{2}\,\sqrt{-5}-\frac{1}{2}}E(\arcsin\left(x\sqrt{\frac{1}{2}\,\sqrt{-}\right.}{}$$

input
```
integrate(1/(3*x^4+2*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/40*(sqrt(2)*(3*x^4 + 2*x^2 - sqrt(-5)*(3*x^4 + 2*x^2 + 2) + 2)*sqrt(1/2*
sqrt(-5) - 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(-5) - 1/2)), 1/3*sqrt(-5
) - 2/3) - sqrt(2)*(9*x^4 + 6*x^2 + sqrt(-5)*(3*x^4 + 2*x^2 + 2) + 6)*sqrt
(1/2*sqrt(-5) - 1/2)*elliptic_f(arcsin(x*sqrt(1/2*sqrt(-5) - 1/2)), 1/3*sq
rt(-5) - 2/3) - 2*sqrt(3*x^4 + 2*x^2 + 2)*(3*x^3 - 4*x))/(3*x^4 + 2*x^2 +
2)
```

Sympy [F]

$$\int \frac{1}{(2+2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+2x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+2*x**2+2)**(3/2),x)
```

output
```
Integral((3*x**4 + 2*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+2\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+2*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 2*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+2\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+2*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 2*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+2\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(2*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int(1/(2*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+2x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+2x^2+2}}{9x^8+12x^6+16x^4+8x^2+4}dx$$

input
```
int(1/(3*x^4+2*x^2+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 2*x**2 + 2)/(9*x**8 + 12*x**6 + 16*x**4 + 8*x**2 + 4),x)
```

3.237 $\int \frac{1}{(2+x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 254

$$\int \frac{1}{(2+x^2+3x^4)^{3/2}}\,dx = \frac{x(11-3x^2)}{46\sqrt{2+x^2+3x^4}} + \frac{3x\sqrt{2+x^2+3x^4}}{46\left(\sqrt{6}+3x^2\right)} - \frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}(12-\sqrt{6})\right)}{23\ 2^{3/4}\sqrt{2+x^2+3x^4}} + \frac{\sqrt[4]{3}(1+2\sqrt{6})\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}(12-\sqrt{6})\right)}{46\ 2^{3/4}\sqrt{2+x^2+3x^4}}$$

output
```
1/46*x*(-3*x^2+11)/(3*x^4+x^2+2)^(1/2)+3*x*(3*x^4+x^2+2)^(1/2)/(46*6^(1/2)
+138*x^2)-1/46*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4+x^2+2)/(2+6^(1/2)*x^2)^2)^(
1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72-6*6^(1/2))^(1
/2))*2^(1/4)/(3*x^4+x^2+2)^(1/2)+1/92*3^(1/4)*(1+2*6^(1/2))*(2+6^(1/2)*x^2
)*((3*x^4+x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*3^(
1/4)*2^(3/4)*x),1/12*(72-6*6^(1/2))^(1/2))*2^(1/4)/(3*x^4+x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.10 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.28

$$\int \frac{1}{(2+x^2+3x^4)^{3/2}}\,dx = \frac{12\sqrt{-\frac{i}{-i+\sqrt{23}}}x(11-3x^2)-\sqrt{6}(i+\sqrt{23})\sqrt{\frac{-i+\sqrt{23}-6ix^2}{-i+\sqrt{23}}}\sqrt{\frac{i+\sqrt{23}+6ix^2}{i+\sqrt{23}}}E\left(i\text{arcsinh}\right.}{}$$

input
```
Integrate[(2 + x^2 + 3*x^4)^(-3/2),x]
```

output
```
(12*Sqrt[(-I)/(-I + Sqrt[23])]*x*(11 - 3*x^2) - Sqrt[6]*(I + Sqrt[23])*Sqr
t[(-I + Sqrt[23] - (6*I)*x^2)/(-I + Sqrt[23])]*Sqrt[(I + Sqrt[23] + (6*I)*
x^2)/(I + Sqrt[23])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(-I + Sqrt[23])]*x],
(I - Sqrt[23])/(I + Sqrt[23])] + Sqrt[6]*(-23*I + Sqrt[23])*Sqrt[(-I + Sqr
t[23] - (6*I)*x^2)/(-I + Sqrt[23])]*Sqrt[(I + Sqrt[23] + (6*I)*x^2)/(I + S
qrt[23])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(-I + Sqrt[23])]*x], (I - Sqrt[2
3])/(I + Sqrt[23])])/(552*Sqrt[(-I)/(-I + Sqrt[23])]*Sqrt[2 + x^2 + 3*x^4]
)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{46}\int \frac{3(x^2+4)}{\sqrt{3x^4+x^2+2}}dx+\frac{x(11-3x^2)}{46\sqrt{3x^4+x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{46}\int \frac{x^2+4}{\sqrt{3x^4+x^2+2}}dx+\frac{x(11-3x^2)}{46\sqrt{3x^4+x^2+2}}$$

$\downarrow$ 1511

$$\frac{3}{46}\left(\frac{1}{3}\left(12+\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4+x^2+2}}dx-\sqrt{\frac{2}{3}}\int \frac{2-\sqrt{6}x^2}{2\sqrt{3x^4+x^2+2}}dx\right)+\frac{x(11-3x^2)}{46\sqrt{3x^4+x^2+2}}$$

$\downarrow$ 27

$$\frac{3}{46}\left(\frac{1}{3}\left(12+\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4+x^2+2}}dx-\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4+x^2+2}}dx}{\sqrt{6}}\right)+\frac{x(11-3x^2)}{46\sqrt{3x^4+x^2+2}}$$

$\downarrow$ 1416

$$\frac{3}{46}\left(\frac{\left(12+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{3x^4+x^2+2}}-\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4+x^2+2}}dx}{\sqrt{6}}\right)+\frac{x(11-3x^2)}{46\sqrt{3x^4+x^2+2}}$$

$\downarrow$ 1509

$$\frac{3}{46}\left(\frac{\left(12+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{3x^4+x^2+2}}-\frac{\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)}}}{\sqrt[4]{}}}{}\right.$$

$$\frac{x(11-3x^2)}{46\sqrt{3x^4+x^2+2}}$$

input
```
Int[(2 + x^2 + 3*x^4)^(-3/2),x]
```

output
```
(x*(11 - 3*x^2))/(46*Sqrt[2 + x^2 + 3*x^4]) + (3*(-(((-2*x*Sqrt[2 + x^2 +
3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x
^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])
/24])/(3^(1/4)*Sqrt[2 + x^2 + 3*x^4]))/Sqrt[6]) + ((12 + Sqrt[6])*(2 + Sqr
t[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan[(
3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(6*6^(1/4)*Sqrt[2 + x^2 + 3*x^4])))/46
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{x(3x^2-11)}{46\sqrt{3x^4+x^2+2}}+\frac{12\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-1+i\sqrt{23}}}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{23\sqrt{-1+i\sqrt{23}}\sqrt{3x^4+x^2+2}}-\frac{12\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{2}}{4}\right.}}{}$
default	$-\frac{6\left(-\frac{11}{276}x+\frac{1}{92}x^3\right)}{\sqrt{3x^4+x^2+2}}+\frac{12\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-1+i\sqrt{23}}}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{23\sqrt{-1+i\sqrt{23}}\sqrt{3x^4+x^2+2}}-\frac{12\sqrt{1-\left(-\frac{1}{4}+\right.}}{}$
elliptic	$-\frac{6\left(-\frac{11}{276}x+\frac{1}{92}x^3\right)}{\sqrt{3x^4+x^2+2}}+\frac{12\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-1+i\sqrt{23}}}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{23\sqrt{-1+i\sqrt{23}}\sqrt{3x^4+x^2+2}}-\frac{12\sqrt{1-\left(-\frac{1}{4}+\right.}}{}$

input
```
int(1/(3*x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/46*x*(3*x^2-11)/(3*x^4+x^2+2)^(1/2)+12/23/(-1+I*23^(1/2))^(1/2)*(1-(-1/
4+1/4*I*23^(1/2))*x^2)^(1/2)*(1-(-1/4-1/4*I*23^(1/2))*x^2)^(1/2)/(3*x^4+x^
2+2)^(1/2)*EllipticF(1/2*x*(-1+I*23^(1/2))^(1/2),1/6*(-33+3*I*23^(1/2))^(1
/2))-12/23/(-1+I*23^(1/2))^(1/2)*(1-(-1/4+1/4*I*23^(1/2))*x^2)^(1/2)*(1-(-
1/4-1/4*I*23^(1/2))*x^2)^(1/2)/(3*x^4+x^2+2)^(1/2)/(1+I*23^(1/2))*(Ellipti
cF(1/2*x*(-1+I*23^(1/2))^(1/2),1/6*(-33+3*I*23^(1/2))^(1/2))-EllipticE(1/2
*x*(-1+I*23^(1/2))^(1/2),1/6*(-33+3*I*23^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.59

$$\int \frac{1}{(2+x^2+3x^4)^{3/2}}\,dx = \frac{\sqrt{2}(3\,x^4+x^2-\sqrt{-23}(3\,x^4+x^2+2)+2)\sqrt{\sqrt{-23}-1}E(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{-23}-\right.}$$

input
```
integrate(1/(3*x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/368*(sqrt(2)*(3*x^4 + x^2 - sqrt(-23)*(3*x^4 + x^2 + 2) + 2)*sqrt(sqrt(-
23) - 1)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(-23) - 1)), 1/12*sqrt(-23) - 11
/12) - sqrt(2)*(15*x^4 + 5*x^2 + 3*sqrt(-23)*(3*x^4 + x^2 + 2) + 10)*sqrt(
sqrt(-23) - 1)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(-23) - 1)), 1/12*sqrt(-23
) - 11/12) - 8*sqrt(3*x^4 + x^2 + 2)*(3*x^3 - 11*x))/(3*x^4 + x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2+x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+x**2+2)**(3/2),x)
```

output
```
Integral((3*x**4 + x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+x^2+3x^4)^{3/2}}\,dx=\int \frac{1}{(3\,x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+x^2+3x^4)^{3/2}}\,dx=\int \frac{1}{(3\,x^4+x^2+2)^{3/2}}\,dx$$

input
```
int(1/(x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int(1/(x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+x^2+3x^4)^{3/2}}\,dx=\int \frac{\sqrt{3x^4+x^2+2}}{9x^8+6x^6+13x^4+4x^2+4}dx$$

input
```
int(1/(3*x^4+x^2+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + x**2 + 2)/(9*x**8 + 6*x**6 + 13*x**4 + 4*x**2 + 4),x)
```

3.238 $\int \frac{1}{(2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 89

$$\int \frac{1}{(2+3x^4)^{3/2}}\,dx = \frac{x}{4\sqrt{2+3x^4}} + \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}\right)}{8\sqrt[4]{6}\sqrt{2+3x^4}}$$

output
```
1/4*x/(3*x^4+2)^(1/2)+1/48*(2+6^(1/2)*x^2)*((3*x^4+2)/(2+6^(1/2)*x^2)^2)^(
1/2)*InverseJacobiAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/2*2^(1/2))*6^(3/4)/
(3*x^4+2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.96 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.49

$$\int \frac{1}{(2+3x^4)^{3/2}}\,dx = \frac{x}{4\sqrt{2+3x^4}} + \frac{x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{3x^4}{2}\right)}{4\sqrt{2}}$$

input
```
Integrate[(2 + 3*x^4)^(-3/2),x]
```

output
```
x/(4*Sqrt[2 + 3*x^4]) + (x*Hypergeometric2F1[1/4, 1/2, 5/4, (-3*x^4)/2])/(
4*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{749}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{3x^4+2}}dx+\frac{x}{4\sqrt{3x^4+2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}\right)}{8\sqrt[4]{6}\sqrt{3x^4+2}}+\frac{x}{4\sqrt{3x^4+2}}$$

input
```
Int[(2 + 3*x^4)^(-3/2),x]
```

output
```
x/(4*Sqrt[2 + 3*x^4]) + ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x
^2)^2]*EllipticF[2*ArcTan[(3/2)^(1/4)*x], 1/2])/(8*6^(1/4)*Sqrt[2 + 3*x^4]
)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.20

method	result	size
meijerg	$\frac{\sqrt{2}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right], \left[\frac{5}{4}\right], -\frac{3 x^{4}}{2}\right)}{4}$	18
default	$\frac{x}{4 \sqrt{3 x^{4}+2}}+\frac{\sqrt{2}\, \sqrt{4-2 i \sqrt{6}\, x^{2}}\, \sqrt{4+2 i \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{2}\, \sqrt{i \sqrt{6}}}{2}, i\right)}{16 \sqrt{i \sqrt{6}}\, \sqrt{3 x^{4}+2}}$	79
risch	$\frac{x}{4 \sqrt{3 x^{4}+2}}+\frac{\sqrt{2}\, \sqrt{4-2 i \sqrt{6}\, x^{2}}\, \sqrt{4+2 i \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{2}\, \sqrt{i \sqrt{6}}}{2}, i\right)}{16 \sqrt{i \sqrt{6}}\, \sqrt{3 x^{4}+2}}$	79
elliptic	$\frac{x}{4 \sqrt{3 x^{4}+2}}+\frac{\sqrt{2}\, \sqrt{4-2 i \sqrt{6}\, x^{2}}\, \sqrt{4+2 i \sqrt{6}\, x^{2}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{2}\, \sqrt{i \sqrt{6}}}{2}, i\right)}{16 \sqrt{i \sqrt{6}}\, \sqrt{3 x^{4}+2}}$	79

input
```
int(1/(3*x^4+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*x*hypergeom([1/4,3/2],[5/4],-3/2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{1}{(2+3x^4)^{3/2}}\,dx = -\frac{\sqrt{2}\sqrt{\frac{1}{2}}(-6)^{\frac{3}{4}}\,(3\,x^4+2)F(\arcsin\left(\sqrt{\frac{1}{2}}(-6)^{\frac{1}{4}}\,x\right)\,|-1) - 6\,\sqrt{3\,x^4+2}x}{24\,(3\,x^4+2)}$$

input
```
integrate(1/(3*x^4+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(2)*sqrt(1/2)*(-6)^(3/4)*(3*x^4 + 2)*elliptic_f(arcsin(sqrt(1/2
)*(-6)^(1/4)*x), -1) - 6*sqrt(3*x^4 + 2)*x)/(3*x^4 + 2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

$$\int \frac{1}{(2+3x^4)^{3/2}}\,dx = \frac{\sqrt{2}x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{3x^4e^{i\pi}}{2}\right)}{16\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(3*x**4+2)**(3/2),x)
```

output
```
sqrt(2)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), 3*x**4*exp_polar(I*pi)/2)/(
16*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 2)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 17.99 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.18

$$\int \frac{1}{(2+3x^4)^{3/2}}\,dx = \frac{\sqrt{2}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{3\,x^4}{2}\right)}{4}$$

input
```
int(1/(3*x^4 + 2)^(3/2),x)
```

output
```
(2^(1/2)*x*hypergeom([1/4, 3/2], 5/4, -(3*x^4)/2))/4
```

Reduce [F]

$$\int \frac{1}{(2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+2}}{9x^8+12x^4+4}dx$$

input
```
int(1/(3*x^4+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 2)/(9*x**8 + 12*x**4 + 4),x)
```

3.239 $\int \frac{1}{(2-x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 262

$$\int \frac{1}{(2-x^2+3x^4)^{3/2}}\,dx = \frac{x(11+3x^2)}{46\sqrt{2-x^2+3x^4}} - \frac{3x\sqrt{2-x^2+3x^4}}{46\left(\sqrt{6}+3x^2\right)} + \frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}(12+\sqrt{6})\right)}{23\ 2^{3/4}\sqrt{2-x^2+3x^4}} - \frac{\sqrt[4]{3}(1-2\sqrt{6})\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}(12+\sqrt{6})\right)}{46\ 2^{3/4}\sqrt{2-x^2+3x^4}}$$

output
```
1/46*x*(3*x^2+11)/(3*x^4-x^2+2)^(1/2)-3*x*(3*x^4-x^2+2)^(1/2)/(46*6^(1/2)+
138*x^2)+1/46*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4-x^2+2)/(2+6^(1/2)*x^2)^2)^(1
/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72+6*6^(1/2))^(1/
2))*2^(1/4)/(3*x^4-x^2+2)^(1/2)-1/92*3^(1/4)*(1-2*6^(1/2))*(2+6^(1/2)*x^2)
*((3*x^4-x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*3^(1
/4)*2^(3/4)*x),1/12*(72+6*6^(1/2))^(1/2))*2^(1/4)/(3*x^4-x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.94 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.25

$$\int \frac{1}{(2-x^2+3x^4)^{3/2}}\,dx = \frac{12\sqrt{-\frac{i}{i+\sqrt{23}}}x(11+3x^2)+\sqrt{6}(-i+\sqrt{23})\sqrt{\frac{i+\sqrt{23}-6ix^2}{i+\sqrt{23}}}\sqrt{\frac{-i+\sqrt{23}+6ix^2}{-i+\sqrt{23}}}E\left(i\text{arcsinh}\right.}{}$$

input
```
Integrate[(2 - x^2 + 3*x^4)^(-3/2),x]
```

output
```
(12*Sqrt[(-I)/(I + Sqrt[23])]*x*(11 + 3*x^2) + Sqrt[6]*(-I + Sqrt[23])*Sqr
t[(I + Sqrt[23] - (6*I)*x^2)/(I + Sqrt[23])]*Sqrt[(-I + Sqrt[23] + (6*I)*x
^2)/(-I + Sqrt[23])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(I + Sqrt[23])]*x], (
I + Sqrt[23])/(I - Sqrt[23])] - Sqrt[6]*(23*I + Sqrt[23])*Sqrt[(I + Sqrt[2
3] - (6*I)*x^2)/(I + Sqrt[23])]*Sqrt[(-I + Sqrt[23] + (6*I)*x^2)/(-I + Sqr
t[23])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(I + Sqrt[23])]*x], (I + Sqrt[23])
/(I - Sqrt[23])])/(552*Sqrt[(-I)/(I + Sqrt[23])]*Sqrt[2 - x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-x^2+2)^{3/2}}dx$$

$$\Big\downarrow \; 1405$$

$$\frac{1}{46}\int \frac{3(4-x^2)}{\sqrt{3x^4-x^2+2}}dx+\frac{x(3x^2+11)}{46\sqrt{3x^4-x^2+2}}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{46}\int \frac{4-x^2}{\sqrt{3x^4-x^2+2}}dx+\frac{x(3x^2+11)}{46\sqrt{3x^4-x^2+2}}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{3}{46}\left(\frac{1}{3}\left(12-\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4-x^2+2}}dx+\sqrt{\frac{2}{3}}\int \frac{2-\sqrt{6}x^2}{2\sqrt{3x^4-x^2+2}}dx\right)+\frac{x(3x^2+11)}{46\sqrt{3x^4-x^2+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{46}\left(\frac{1}{3}\left(12-\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4-x^2+2}}dx+\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4-x^2+2}}dx}{\sqrt{6}}\right)+\frac{x(3x^2+11)}{46\sqrt{3x^4-x^2+2}}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{3}{46}\left(\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4-x^2+2}}dx}{\sqrt{6}}+\frac{\left(12-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{3x^4-x^2+2}}\right)+\frac{x(3x^2+11)}{46\sqrt{3x^4-x^2+2}}$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{3}{46}\left(\frac{\left(12-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{3x^4-x^2+2}}+\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)}}}{\sqrt[4]{}}\right.$$

$$\frac{x(3x^2+11)}{46\sqrt{3x^4-x^2+2}}$$

input
```
Int[(2 - x^2 + 3*x^4)^(-3/2),x]
```

output
```
(x*(11 + 3*x^2))/(46*Sqrt[2 - x^2 + 3*x^4]) + (3*(((-2*x*Sqrt[2 - x^2 + 3*
x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4
)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/2
4])/(3^(1/4)*Sqrt[2 - x^2 + 3*x^4]))/Sqrt[6] + ((12 - Sqrt[6])*(2 + Sqrt[6
]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan[(3/2
)^(1/4)*x], (12 + Sqrt[6])/24])/(6*6^(1/4)*Sqrt[2 - x^2 + 3*x^4])))/46
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.29 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.90

method	result
risch	$\frac{x(3x^2+11)}{46\sqrt{3x^4-x^2+2}}+\frac{12\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{1+i\sqrt{23}}}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{23\sqrt{1+i\sqrt{23}}\,\sqrt{3x^4-x^2+2}}+\frac{12\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-}}{}$
default	$-\frac{6\left(-\frac{11}{276}x-\frac{1}{92}x^3\right)}{\sqrt{3x^4-x^2+2}}+\frac{12\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{1+i\sqrt{23}}}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{23\sqrt{1+i\sqrt{23}}\,\sqrt{3x^4-x^2+2}}+\frac{12\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}}{}$
elliptic	$-\frac{6\left(-\frac{11}{276}x-\frac{1}{92}x^3\right)}{\sqrt{3x^4-x^2+2}}+\frac{12\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{1+i\sqrt{23}}}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{23\sqrt{1+i\sqrt{23}}\,\sqrt{3x^4-x^2+2}}+\frac{12\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}}{}$

input
```
int(1/(3*x^4-x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/46*x*(3*x^2+11)/(3*x^4-x^2+2)^(1/2)+12/23/(1+I*23^(1/2))^(1/2)*(1-(1/4+1
/4*I*23^(1/2))*x^2)^(1/2)*(1-(1/4-1/4*I*23^(1/2))*x^2)^(1/2)/(3*x^4-x^2+2)
^(1/2)*EllipticF(1/2*x*(1+I*23^(1/2))^(1/2),1/6*(-33-3*I*23^(1/2))^(1/2))+
12/23/(1+I*23^(1/2))^(1/2)*(1-(1/4+1/4*I*23^(1/2))*x^2)^(1/2)*(1-(1/4-1/4*
I*23^(1/2))*x^2)^(1/2)/(3*x^4-x^2+2)^(1/2)/(-1+I*23^(1/2))*(EllipticF(1/2*
x*(1+I*23^(1/2))^(1/2),1/6*(-33-3*I*23^(1/2))^(1/2))-EllipticE(1/2*x*(1+I*
23^(1/2))^(1/2),1/6*(-33-3*I*23^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.61

$$\int \frac{1}{(2-x^2+3x^4)^{3/2}}\,dx = \frac{\sqrt{2}\left(3\,x^4-x^2+\sqrt{-23}(3\,x^4-x^2+2)+2\right)\sqrt{\sqrt{-23}+1}E(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{-23}+\right.}{}$$

input
```
integrate(1/(3*x^4-x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/368*(sqrt(2)*(3*x^4 - x^2 + sqrt(-23)*(3*x^4 - x^2 + 2) + 2)*sqrt(sqrt(-
23) + 1)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(-23) + 1)), -1/12*sqrt(-23) - 1
1/12) + sqrt(2)*(9*x^4 - 3*x^2 - 5*sqrt(-23)*(3*x^4 - x^2 + 2) + 6)*sqrt(s
qrt(-23) + 1)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(-23) + 1)), -1/12*sqrt(-23
) - 11/12) + 8*sqrt(3*x^4 - x^2 + 2)*(3*x^3 + 11*x))/(3*x^4 - x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2-x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4-x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4-x**2+2)**(3/2),x)
```

output
```
Integral((3*x**4 - x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2-x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 - x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2-x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-x^2+2)^{3/2}}\,dx$$

input
```
int(1/(3*x^4 - x^2 + 2)^(3/2),x)
```

output
```
int(1/(3*x^4 - x^2 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2-x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-x^2+2}}{9x^8-6x^6+13x^4-4x^2+4}dx$$

input
```
int(1/(3*x^4-x^2+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - x**2 + 2)/(9*x**8 - 6*x**6 + 13*x**4 - 4*x**2 + 4),x)
```

3.240 $\int \frac{1}{(2-2x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 262

$$\int \frac{1}{(2-2x^2+3x^4)^{3/2}}\,dx = \frac{x(4+3x^2)}{20\sqrt{2-2x^2+3x^4}} - \frac{3x\sqrt{2-2x^2+3x^4}}{20\left(\sqrt{6}+3x^2\right)} + \frac{\sqrt[4]{3}\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)\middle|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{10\ 2^{3/4}\sqrt{2-2x^2+3x^4}} - \frac{\sqrt[4]{3}\left(1-\sqrt{6}\right)\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{20\ 2^{3/4}\sqrt{2-2x^2+3x^4}}$$

output
```
1/20*x*(3*x^2+4)/(3*x^4-2*x^2+2)^(1/2)-3*x*(3*x^4-2*x^2+2)^(1/2)/(20*6^(1/
2)+60*x^2)+1/20*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4-2*x^2+2)/(2+6^(1/2)*x^2)^2
)^(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18+3*6^(1/2))^
(1/2))*2^(1/4)/(3*x^4-2*x^2+2)^(1/2)-1/40*3^(1/4)*(1-6^(1/2))*(2+6^(1/2)*x
^2)*((3*x^4-2*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2
*3^(1/4)*2^(3/4)*x),1/6*(18+3*6^(1/2))^(1/2))*2^(1/4)/(3*x^4-2*x^2+2)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.92 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.25

$$\int \frac{1}{(2-2x^2+3x^4)^{3/2}}\,dx = \frac{3\sqrt{-\frac{i}{i+\sqrt{5}}}x(4+3x^2)+\sqrt{3}(-i+\sqrt{5})\sqrt{\frac{i+\sqrt{5}-3ix^2}{i+\sqrt{5}}}\sqrt{\frac{-i+\sqrt{5}+3ix^2}{-i+\sqrt{5}}}E\left(i\operatorname{arcsinh}\left(\sqrt{\right.\right.}{60\sqrt{}}$$

input
```
Integrate[(2 - 2*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(3*Sqrt[(-I)/(I + Sqrt[5])]*x*(4 + 3*x^2) + Sqrt[3]*(-I + Sqrt[5])*Sqrt[(I
 + Sqrt[5] - (3*I)*x^2)/(I + Sqrt[5])]*Sqrt[(-I + Sqrt[5] + (3*I)*x^2)/(-I
 + Sqrt[5])]*EllipticE[I*ArcSinh[Sqrt[(-3*I)/(I + Sqrt[5])]*x], (I + Sqrt[
5])/(I - Sqrt[5])] - Sqrt[3]*(5*I + Sqrt[5])*Sqrt[(I + Sqrt[5] - (3*I)*x^2
)/(I + Sqrt[5])]*Sqrt[(-I + Sqrt[5] + (3*I)*x^2)/(-I + Sqrt[5])]*EllipticF
[I*ArcSinh[Sqrt[(-3*I)/(I + Sqrt[5])]*x], (I + Sqrt[5])/(I - Sqrt[5])])/(6
0*Sqrt[(-I)/(I + Sqrt[5])]*Sqrt[2 - 2*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-2x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{40}\int \frac{6(2-x^2)}{\sqrt{3x^4-2x^2+2}}dx+\frac{x(3x^2+4)}{20\sqrt{3x^4-2x^2+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{20}\int \frac{2-x^2}{\sqrt{3x^4-2x^2+2}}dx+\frac{x(3x^2+4)}{20\sqrt{3x^4-2x^2+2}}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{3}{20}\left(\frac{1}{3}\left(6-\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4-2x^2+2}}dx+\sqrt{\frac{2}{3}}\int \frac{2-\sqrt{6}x^2}{2\sqrt{3x^4-2x^2+2}}dx\right)+\frac{x(3x^2+4)}{20\sqrt{3x^4-2x^2+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{20}\left(\frac{1}{3}\left(6-\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4-2x^2+2}}dx+\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4-2x^2+2}}dx}{\sqrt{6}}\right)+\frac{x(3x^2+4)}{20\sqrt{3x^4-2x^2+2}}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{3}{20}\left(\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4-2x^2+2}}dx}{\sqrt{6}}+\frac{\left(6-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{3x^4-2x^2+2}}\right)+$$
$$\frac{x(3x^2+4)}{20\sqrt{3x^4-2x^2+2}}$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{3}{20}\left(\frac{\left(6-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{3x^4-2x^2+2}}+\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}E}{\sqrt[4]{3}\sqrt{3}}\right.$$
$$\frac{x(3x^2+4)}{20\sqrt{3x^4-2x^2+2}}$$

input
```
Int[(2 - 2*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(x*(4 + 3*x^2))/(20*Sqrt[2 - 2*x^2 + 3*x^4]) + (3*(((-2*x*Sqrt[2 - 2*x^2 +
 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 +
3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6
])/12])/(3^(1/4)*Sqrt[2 - 2*x^2 + 3*x^4]))/Sqrt[6] + ((6 - Sqrt[6])*(2 + S
qrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcT
an[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(6*6^(1/4)*Sqrt[2 - 2*x^2 + 3*x^4]))
)/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.90

method	result
risch	$\frac{x(3x^2+4)}{20\sqrt{3x^4-2x^2+2}} + \frac{3\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{2+2i\sqrt{5}}}{2}, \frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{5\sqrt{2+2i\sqrt{5}}\,\sqrt{3x^4-2x^2+2}} + \frac{6\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\right.}}{}$
default	$-\frac{6\left(-\frac{1}{30}x-\frac{1}{40}x^3\right)}{\sqrt{3x^4-2x^2+2}} + \frac{3\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{2+2i\sqrt{5}}}{2}, \frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{5\sqrt{2+2i\sqrt{5}}\,\sqrt{3x^4-2x^2+2}} + \frac{6\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}\right.}}{}$
elliptic	$-\frac{6\left(-\frac{1}{30}x-\frac{1}{40}x^3\right)}{\sqrt{3x^4-2x^2+2}} + \frac{3\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{2+2i\sqrt{5}}}{2}, \frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{5\sqrt{2+2i\sqrt{5}}\,\sqrt{3x^4-2x^2+2}} + \frac{6\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}\right.}}{}$

input
```
int(1/(3*x^4-2*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*x*(3*x^2+4)/(3*x^4-2*x^2+2)^(1/2)+3/5/(2+2*I*5^(1/2))^(1/2)*(1-(1/2+1
/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2-1/2*I*5^(1/2))*x^2)^(1/2)/(3*x^4-2*x^2+2)
^(1/2)*EllipticF(1/2*x*(2+2*I*5^(1/2))^(1/2),1/3*(-6-3*I*5^(1/2))^(1/2))+6
/5/(2+2*I*5^(1/2))^(1/2)*(1-(1/2+1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2-1/2*I*5
^(1/2))*x^2)^(1/2)/(3*x^4-2*x^2+2)^(1/2)/(-2+2*I*5^(1/2))*(EllipticF(1/2*x
*(2+2*I*5^(1/2))^(1/2),1/3*(-6-3*I*5^(1/2))^(1/2))-EllipticE(1/2*x*(2+2*I*
5^(1/2))^(1/2),1/3*(-6-3*I*5^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.63

$$\int \frac{1}{(2-2x^2+3x^4)^{3/2}}\,dx = \frac{\sqrt{2}\left(3\,x^4-2\,x^2+\sqrt{-5}(3\,x^4-2\,x^2+2)+2\right)\sqrt{\frac{1}{2}\,\sqrt{-5}+\frac{1}{2}}E\left(\arcsin\left(x\sqrt{\frac{1}{2}\,\sqrt{-}\right.\right.}{}$$

input
```
integrate(1/(3*x^4-2*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/40*(sqrt(2)*(3*x^4 - 2*x^2 + sqrt(-5)*(3*x^4 - 2*x^2 + 2) + 2)*sqrt(1/2*
sqrt(-5) + 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(-5) + 1/2)), -1/3*sqrt(-
5) - 2/3) + sqrt(2)*(3*x^4 - 2*x^2 - 3*sqrt(-5)*(3*x^4 - 2*x^2 + 2) + 2)*s
qrt(1/2*sqrt(-5) + 1/2)*elliptic_f(arcsin(x*sqrt(1/2*sqrt(-5) + 1/2)), -1/
3*sqrt(-5) - 2/3) + 2*sqrt(3*x^4 - 2*x^2 + 2)*(3*x^3 + 4*x))/(3*x^4 - 2*x^
2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2-2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4-2x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4-2*x**2+2)**(3/2),x)
```

output
```
Integral((3*x**4 - 2*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2-2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-2\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-2*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 - 2*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2-2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-2\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-2*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - 2*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-2x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-2\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(3*x^4 - 2*x^2 + 2)^(3/2),x)
```

output
```
int(1/(3*x^4 - 2*x^2 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2-2x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-2x^2+2}}{9x^8-12x^6+16x^4-8x^2+4}dx$$

input
```
int(1/(3*x^4-2*x^2+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - 2*x**2 + 2)/(9*x**8 - 12*x**6 + 16*x**4 - 8*x**2 + 4),x)
```

3.241 $\int \frac{1}{(2-3x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 257

$$\int \frac{1}{(2-3x^2+3x^4)^{3/2}}\,dx = \frac{x(1+3x^2)}{10\sqrt{2-3x^2+3x^4}} - \frac{3x\sqrt{2-3x^2+3x^4}}{10\left(\sqrt{6}+3x^2\right)} + \frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}(4+\sqrt{6})\right)}{5\ 2^{3/4}\sqrt{2-3x^2+3x^4}} - \frac{\left(3-2\sqrt{6}\right)\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}(4+\sqrt{6})\right)}{10\ 6^{3/4}\sqrt{2-3x^2+3x^4}}$$

output
```
1/10*x*(3*x^2+1)/(3*x^4-3*x^2+2)^(1/2)-3*x*(3*x^4-3*x^2+2)^(1/2)/(10*6^(1/
2)+30*x^2)+1/10*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4-3*x^2+2)/(2+6^(1/2)*x^2)^2
)^(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/4*(8+2*6^(1/2))^(
1/2))*2^(1/4)/(3*x^4-3*x^2+2)^(1/2)-1/60*(3-2*6^(1/2))*(2+6^(1/2)*x^2)*((3
*x^4-3*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*3^(1/4
)*2^(3/4)*x),1/4*(8+2*6^(1/2))^(1/2))*6^(1/4)/(3*x^4-3*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.15 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.34

$$\int \frac{1}{(2-3x^2+3x^4)^{3/2}}\,dx = \frac{12\sqrt{-\frac{i}{3i+\sqrt{15}}}x(1+3x^2)+3\sqrt{2}\left(-i\sqrt{3}+\sqrt{5}\right)\sqrt{\frac{3i+\sqrt{15}-6ix^2}{3i+\sqrt{15}}}\sqrt{\frac{-3i+\sqrt{15}+6ix^2}{-3i+\sqrt{15}}}E\Big(ia$$

input
```
Integrate[(2 - 3*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(12*Sqrt[(-I)/(3*I + Sqrt[15])]*x*(1 + 3*x^2) + 3*Sqrt[2]*((-I)*Sqrt[3] +
Sqrt[5])*Sqrt[(3*I + Sqrt[15] - (6*I)*x^2)/(3*I + Sqrt[15])]*Sqrt[(-3*I +
Sqrt[15] + (6*I)*x^2)/(-3*I + Sqrt[15])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(
3*I + Sqrt[15])]*x], (3*I + Sqrt[15])/(3*I - Sqrt[15])] - I*Sqrt[2]*(5*Sqr
t[3] - (3*I)*Sqrt[5])*Sqrt[(3*I + Sqrt[15] - (6*I)*x^2)/(3*I + Sqrt[15])]*
Sqrt[(-3*I + Sqrt[15] + (6*I)*x^2)/(-3*I + Sqrt[15])]*EllipticF[I*ArcSinh[
Sqrt[(-6*I)/(3*I + Sqrt[15])]*x], (3*I + Sqrt[15])/(3*I - Sqrt[15])])/(120
*Sqrt[(-I)/(3*I + Sqrt[15])]*Sqrt[2 - 3*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{30}\int \frac{3(4-3x^2)}{\sqrt{3x^4-3x^2+2}}dx+\frac{x(3x^2+1)}{10\sqrt{3x^4-3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\int \frac{4-3x^2}{\sqrt{3x^4-3x^2+2}}dx+\frac{x(3x^2+1)}{10\sqrt{3x^4-3x^2+2}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{10}\left(\left(4-\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4-3x^2+2}}dx+\sqrt{6}\int \frac{2-\sqrt{6}x^2}{2\sqrt{3x^4-3x^2+2}}dx\right)+\frac{x(3x^2+1)}{10\sqrt{3x^4-3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\left(4-\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4-3x^2+2}}dx+\sqrt{\frac{3}{2}}\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4-3x^2+2}}dx\right)+\frac{x(3x^2+1)}{10\sqrt{3x^4-3x^2+2}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{10}\left(\sqrt{\frac{3}{2}}\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4-3x^2+2}}dx+\frac{(4-\sqrt{6})(\sqrt{6}x^2+2)\sqrt{\frac{3x^4-3x^2+2}{(\sqrt{6}x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}(4+\sqrt{6})\right)}{2\sqrt[4]{6}\sqrt{3x^4-3x^2+2}}\right.$$

$$\frac{x(3x^2+1)}{10\sqrt{3x^4-3x^2+2}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{10}\left(\frac{(4-\sqrt{6})(\sqrt{6}x^2+2)\sqrt{\frac{3x^4-3x^2+2}{(\sqrt{6}x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}(4+\sqrt{6})\right)}{2\sqrt[4]{6}\sqrt{3x^4-3x^2+2}}+\sqrt{\frac{3}{2}}\left(\frac{2^{3/4}(\sqrt{6}x^2+2)\sqrt{\cdots}}{\cdots}\right.\right.$$

$$\frac{x(3x^2+1)}{10\sqrt{3x^4-3x^2+2}}$$

input
```
Int[(2 - 3*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(x*(1 + 3*x^2))/(10*Sqrt[2 - 3*x^2 + 3*x^4]) + (Sqrt[3/2]*((-2*x*Sqrt[2 -
3*x^2 + 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 - 3
*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], (4 +
 Sqrt[6])/8])/(3^(1/4)*Sqrt[2 - 3*x^2 + 3*x^4])) + ((4 - Sqrt[6])*(2 + Sqr
t[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[2 - 3*x^2 + 3*x^4]))/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.92

method	result
risch	$\frac{x(3x^2+1)}{10\sqrt{3x^4-3x^2+2}}+\frac{4\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+i\sqrt{15}}}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{5\sqrt{3+i\sqrt{15}}\,\sqrt{3x^4-3x^2+2}}+\frac{12\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-(}{}$
default	$-\frac{6\left(-\frac{1}{60}x-\frac{1}{20}x^3\right)}{\sqrt{3x^4-3x^2+2}}+\frac{4\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+i\sqrt{15}}}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{5\sqrt{3+i\sqrt{15}}\,\sqrt{3x^4-3x^2+2}}+\frac{12\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1}}{}$
elliptic	$-\frac{6\left(-\frac{1}{60}x-\frac{1}{20}x^3\right)}{\sqrt{3x^4-3x^2+2}}+\frac{4\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+i\sqrt{15}}}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{5\sqrt{3+i\sqrt{15}}\,\sqrt{3x^4-3x^2+2}}+\frac{12\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1}}{}$

input
```
int(1/(3*x^4-3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*x*(3*x^2+1)/(3*x^4-3*x^2+2)^(1/2)+4/5/(3+I*15^(1/2))^(1/2)*(1-(3/4+1/
4*I*15^(1/2))*x^2)^(1/2)*(1-(3/4-1/4*I*15^(1/2))*x^2)^(1/2)/(3*x^4-3*x^2+2
)^(1/2)*EllipticF(1/2*x*(3+I*15^(1/2))^(1/2),1/2*(-1-I*15^(1/2))^(1/2))+12
/5/(3+I*15^(1/2))^(1/2)*(1-(3/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(3/4-1/4*I*1
5^(1/2))*x^2)^(1/2)/(3*x^4-3*x^2+2)^(1/2)/(-3+I*15^(1/2))*(EllipticF(1/2*x
*(3+I*15^(1/2))^(1/2),1/2*(-1-I*15^(1/2))^(1/2))-EllipticE(1/2*x*(3+I*15^(
1/2))^(1/2),1/2*(-1-I*15^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.61

$$\int \frac{1}{(2-3x^2+3x^4)^{3/2}}\,dx=\frac{3\sqrt{2}\left(9x^4-9x^2+\sqrt{-15}(3x^4-3x^2+2)+6\right)\sqrt{\sqrt{-15}+3}\,E\left(\arcsin\left(\frac{1}{2}x\sqrt{\sqrt{}}\right.\right.}{}$$

input
```
integrate(1/(3*x^4-3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/240*(3*sqrt(2)*(9*x^4 - 9*x^2 + sqrt(-15)*(3*x^4 - 3*x^2 + 2) + 6)*sqrt(
sqrt(-15) + 3)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(-15) + 3)), -1/4*sqrt(-15
) - 1/4) + sqrt(2)*(9*x^4 - 9*x^2 - 7*sqrt(-15)*(3*x^4 - 3*x^2 + 2) + 6)*s
qrt(sqrt(-15) + 3)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(-15) + 3)), -1/4*sqrt
(-15) - 1/4) + 24*sqrt(3*x^4 - 3*x^2 + 2)*(3*x^3 + x))/(3*x^4 - 3*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2-3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4-3x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4-3*x**2+2)**(3/2),x)
```

output
```
Integral((3*x**4 - 3*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2-3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 - 3*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2-3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - 3*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-3x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-3\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(3*x^4 - 3*x^2 + 2)^(3/2),x)
```

output
```
int(1/(3*x^4 - 3*x^2 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2-3x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-3x^2+2}}{9x^8-18x^6+21x^4-12x^2+4}dx$$

input
```
int(1/(3*x^4-3*x^2+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - 3*x**2 + 2)/(9*x**8 - 18*x**6 + 21*x**4 - 12*x**2 + 4),x
)
```

3.242 $\int \frac{1}{(2-4x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 258

$$\int \frac{1}{(2-4x^2+3x^4)^{3/2}}\,dx = -\frac{x(1-3x^2)}{4\sqrt{2-4x^2+3x^4}} - \frac{3x\sqrt{2-4x^2+3x^4}}{4\left(\sqrt{6}+3x^2\right)} + \frac{\sqrt[4]{3}\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\,2^{3/4}\sqrt{2-4x^2+3x^4}} - \frac{\sqrt[4]{3}\left(2-\sqrt{6}\right)\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{8\,2^{3/4}\sqrt{2-4x^2+3x^4}}$$

output

```
-1/4*x*(-3*x^2+1)/(3*x^4-4*x^2+2)^(1/2)-3*x*(3*x^4-4*x^2+2)^(1/2)/(4*6^(1/
2)+12*x^2)+1/4*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4-4*x^2+2)/(2+6^(1/2)*x^2)^2)
^(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18+6*6^(1/2))^(
1/2))*2^(1/4)/(3*x^4-4*x^2+2)^(1/2)-1/16*3^(1/4)*(2-6^(1/2))*(2+6^(1/2)*x^
2)*((3*x^4-4*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*
3^(1/4)*2^(3/4)*x),1/6*(18+6*6^(1/2))^(1/2))*2^(1/4)/(3*x^4-4*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.01 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.27

$$\int \frac{1}{(2-4x^2+3x^4)^{3/2}}\,dx = \frac{3\sqrt{-\frac{i}{2i+\sqrt{2}}}x(-1+3x^2)+\sqrt{3}(-2i+\sqrt{2})\sqrt{\frac{2i+\sqrt{2}-3ix^2}{2i+\sqrt{2}}}\sqrt{\frac{-2i+\sqrt{2}+3ix^2}{-2i+\sqrt{2}}}E\Big(i\text{arcsin}$$

input
```
Integrate[(2 - 4*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(3*Sqrt[(-I)/(2*I + Sqrt[2])]*x*(-1 + 3*x^2) + Sqrt[3]*(-2*I + Sqrt[2])*Sq
rt[(2*I + Sqrt[2] - (3*I)*x^2)/(2*I + Sqrt[2])]*Sqrt[(-2*I + Sqrt[2] + (3*
I)*x^2)/(-2*I + Sqrt[2])]*EllipticE[I*ArcSinh[Sqrt[(-3*I)/(2*I + Sqrt[2])]
*x], (2*I + Sqrt[2])/(2*I - Sqrt[2])] - Sqrt[3]*(I + Sqrt[2])*Sqrt[(2*I +
Sqrt[2] - (3*I)*x^2)/(2*I + Sqrt[2])]*Sqrt[(-2*I + Sqrt[2] + (3*I)*x^2)/(-
2*I + Sqrt[2])]*EllipticF[I*ArcSinh[Sqrt[(-3*I)/(2*I + Sqrt[2])]*x], (2*I
+ Sqrt[2])/(2*I - Sqrt[2])])/(12*Sqrt[(-I)/(2*I + Sqrt[2])]*Sqrt[2 - 4*x^2
 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-4x^2+2)^{3/2}}dx$$

$$\Big\downarrow \; 1405$$

$$\frac{1}{16}\int \frac{12(1-x^2)}{\sqrt{3x^4-4x^2+2}}dx - \frac{x(1-3x^2)}{4\sqrt{3x^4-4x^2+2}}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{4}\int \frac{1-x^2}{\sqrt{3x^4-4x^2+2}}dx-\frac{x(1-3x^2)}{4\sqrt{3x^4-4x^2+2}}$$

$\big\downarrow$ 1511

$$\frac{3}{4}\left(\frac{1}{3}\left(3-\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4-4x^2+2}}dx+\sqrt{\frac{2}{3}}\int \frac{2-\sqrt{6}x^2}{2\sqrt{3x^4-4x^2+2}}dx\right)-\frac{x(1-3x^2)}{4\sqrt{3x^4-4x^2+2}}$$

$\big\downarrow$ 27

$$\frac{3}{4}\left(\frac{1}{3}\left(3-\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4-4x^2+2}}dx+\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4-4x^2+2}}dx}{\sqrt{6}}\right)-\frac{x(1-3x^2)}{4\sqrt{3x^4-4x^2+2}}$$

$\big\downarrow$ 1416

$$\frac{3}{4}\left(\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4-4x^2+2}}dx}{\sqrt{6}}+\frac{\left(3-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{6\sqrt[4]{6}\sqrt{3x^4-4x^2+2}}\right)-\frac{x(1-3x^2)}{4\sqrt{3x^4-4x^2+2}}$$

$\big\downarrow$ 1509

$$\frac{3}{4}\left(\frac{\left(3-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{6\sqrt[4]{6}\sqrt{3x^4-4x^2+2}}+\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}E\left(2\text{arc}\right.}{\sqrt[4]{3}\sqrt{3x^4-4x^2+}}\right.$$

$$\frac{x(1-3x^2)}{4\sqrt{3x^4-4x^2+2}}$$

input
```
Int[(2 - 4*x^2 + 3*x^4)^(-3/2),x]
```

output
```
-1/4*(x*(1 - 3*x^2))/Sqrt[2 - 4*x^2 + 3*x^4] + (3*(((-2*x*Sqrt[2 - 4*x^2 +
 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 +
3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqr
t[6]])/(3^(1/4)*Sqrt[2 - 4*x^2 + 3*x^4]))/Sqrt[6] + ((3 - Sqrt[6])*(2 + Sq
rt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTa
n[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(6*6^(1/4)*Sqrt[2 - 4*x^2 + 3*x^4])))/
4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.92

method	result
risch	$\frac{x(3x^2-1)}{4\sqrt{3x^4-4x^2+2}}+\frac{3\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right)x^2}\,\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{4+2i\sqrt{2}}}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{2\sqrt{4+2i\sqrt{2}}\,\sqrt{3x^4-4x^2+2}}+\frac{6\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right)x^2}\,\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)\ldots}}{\ldots}$
default	$-\frac{6\left(\frac{1}{24}x-\frac{1}{8}x^3\right)}{\sqrt{3x^4-4x^2+2}}+\frac{3\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right)x^2}\,\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{4+2i\sqrt{2}}}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{2\sqrt{4+2i\sqrt{2}}\,\sqrt{3x^4-4x^2+2}}+\frac{6\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right)x^2}\,\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\ldots\right.}}{\ldots}$
elliptic	$-\frac{6\left(\frac{1}{24}x-\frac{1}{8}x^3\right)}{\sqrt{3x^4-4x^2+2}}+\frac{3\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right)x^2}\,\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{4+2i\sqrt{2}}}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{2\sqrt{4+2i\sqrt{2}}\,\sqrt{3x^4-4x^2+2}}+\frac{6\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right)x^2}\,\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\ldots\right.}}{\ldots}$

input
```
int(1/(3*x^4-4*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(3*x^2-1)/(3*x^4-4*x^2+2)^(1/2)+3/2/(4+2*I*2^(1/2))^(1/2)*(1-(1/2*I*
2^(1/2)+1)*x^2)^(1/2)*(1-(1-1/2*I*2^(1/2))*x^2)^(1/2)/(3*x^4-4*x^2+2)^(1/2
)*EllipticF(1/2*x*(4+2*I*2^(1/2))^(1/2),1/3*(3-6*I*2^(1/2))^(1/2))+6/(4+2*
I*2^(1/2))^(1/2)*(1-(1/2*I*2^(1/2)+1)*x^2)^(1/2)*(1-(1-1/2*I*2^(1/2))*x^2)
^(1/2)/(3*x^4-4*x^2+2)^(1/2)/(-4+2*I*2^(1/2))*(EllipticF(1/2*x*(4+2*I*2^(1
/2))^(1/2),1/3*(3-6*I*2^(1/2))^(1/2))-EllipticE(1/2*x*(4+2*I*2^(1/2))^(1/2
),1/3*(3-6*I*2^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.59

$$\int \frac{1}{(2-4x^2+3x^4)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{2}\sqrt{-2}(3\,x^4-4\,x^2+2)\sqrt{\frac{1}{2}\sqrt{-2}+1}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-2}+1}\right)\,|-\frac{2}{3}\sqrt{-2}+\frac{1}{3})-\sqrt{2}(6\,x^4-8\,x^2+\ldots}{8\,(3\ldots}$$

input
```
integrate(1/(3*x^4-4*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(2)*sqrt(-2)*(3*x^4 - 4*x^2 + 2)*sqrt(1/2*sqrt(-2) + 1)*ellipt
ic_f(arcsin(x*sqrt(1/2*sqrt(-2) + 1)), -2/3*sqrt(-2) + 1/3) - sqrt(2)*(6*x
^4 - 8*x^2 + sqrt(-2)*(3*x^4 - 4*x^2 + 2) + 4)*sqrt(1/2*sqrt(-2) + 1)*elli
ptic_e(arcsin(x*sqrt(1/2*sqrt(-2) + 1)), -2/3*sqrt(-2) + 1/3) - 2*sqrt(3*x
^4 - 4*x^2 + 2)*(3*x^3 - x))/(3*x^4 - 4*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2-4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4-4x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4-4*x**2+2)**(3/2),x)
```

output
```
Integral((3*x**4 - 4*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2-4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-4\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-4*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 - 4*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2-4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-4\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-4*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - 4*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-4x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-4\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(3*x^4 - 4*x^2 + 2)^(3/2),x)
```

output
```
int(1/(3*x^4 - 4*x^2 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2-4x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-4x^2+2}}{9x^8-24x^6+28x^4-16x^2+4}dx$$

input
```
int(1/(3*x^4-4*x^2+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - 4*x**2 + 2)/(9*x**8 - 24*x**6 + 28*x**4 - 16*x**2 + 4),x
)
```

3.243 $\int \frac{1}{(2-5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 131

$$\int \frac{1}{(2-5x^2+3x^4)^{3/2}}\,dx = \frac{x(13-15x^2)}{2\sqrt{2-5x^2+3x^4}} - \frac{5\sqrt{2-3x^2}\sqrt{1-x^2}\,E\left(\arcsin(x)\,\middle|\,\frac{3}{2}\right)}{\sqrt{2}\sqrt{2-5x^2+3x^4}} - \frac{\sqrt{2-3x^2}\sqrt{1-x^2}\,\operatorname{EllipticF}\left(\arcsin(x),\frac{3}{2}\right)}{\sqrt{2}\sqrt{2-5x^2+3x^4}}$$

output
```
1/2*x*(-15*x^2+13)/(3*x^4-5*x^2+2)^(1/2)-5/2*(-3*x^2+2)^(1/2)*(-x^2+1)^(1/
2)*EllipticE(x,1/2*6^(1/2))*2^(1/2)/(3*x^4-5*x^2+2)^(1/2)-1/2*(-3*x^2+2)^(
1/2)*(-x^2+1)^(1/2)*EllipticF(x,1/2*6^(1/2))*2^(1/2)/(3*x^4-5*x^2+2)^(1/2)
```

Mathematica [A] (verified)

Time = 5.83 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79

$$\int \frac{1}{(2-5x^2+3x^4)^{3/2}}\,dx = \frac{13x-15x^3-5\sqrt{6-9x^2}\sqrt{1-x^2}\,E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|\frac{2}{3}\right)+\sqrt{6-9x^2}\sqrt{1-x^2}\,\mathrm{El}}{2\sqrt{2-5x^2+3x^4}}$$

input
```
Integrate[(2 - 5*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(13*x - 15*x^3 - 5*Sqrt[6 - 9*x^2]*Sqrt[1 - x^2]*EllipticE[ArcSin[Sqrt[3/2
]*x], 2/3] + Sqrt[6 - 9*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x],
2/3])/(2*Sqrt[2 - 5*x^2 + 3*x^4])
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 269 vs. $2(131) = 262$.

Time = 0.61 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1497, 27, 1409, 1496}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-5x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(13-15x^2)}{2\sqrt{3x^4-5x^2+2}}-\frac{1}{2}\int \frac{3(4-5x^2)}{\sqrt{3x^4-5x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(13-15x^2)}{2\sqrt{3x^4-5x^2+2}}-\frac{3}{2}\int \frac{4-5x^2}{\sqrt{3x^4-5x^2+2}}dx$$

$$\Big\downarrow \text{1497}$$

$$\frac{x(13-15x^2)}{2\sqrt{3x^4-5x^2+2}}-\frac{3}{2}\left(\frac{1}{3}\left(12-5\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4-5x^2+2}}dx+5\sqrt{\frac{2}{3}}\int \frac{2-\sqrt{6}x^2}{2\sqrt{3x^4-5x^2+2}}dx\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{x(13-15x^2)}{2\sqrt{3x^4-5x^2+2}}-\frac{3}{2}\left(\frac{1}{3}\left(12-5\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4-5x^2+2}}dx+\frac{5\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4-5x^2+2}}dx}{\sqrt{6}}\right)$$

$$\Big\downarrow \text{1409}$$

$$\frac{x(13-15x^2)}{2\sqrt{3x^4-5x^2+2}}-\frac{3}{2}\left(\frac{5\int\frac{2-\sqrt{6}x^2}{\sqrt{3x^4-5x^2+2}}dx}{\sqrt{6}}+\frac{\left(12-5\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-5x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{3x^4-5x^2+2}}\right)$$

$$\Big\downarrow \text{1496}$$

$$\frac{x(13-15x^2)}{2\sqrt{3x^4-5x^2+2}}-\frac{3}{2}\left(\frac{\left(12-5\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-5x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{3x^4-5x^2+2}}+\frac{5\left(\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-\ldots}{(\sqrt{6}\ldots}}}{\ldots}\right.}{\ldots}\right.$$

input
```
Int[(2 - 5*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(x*(13 - 15*x^2))/(2*Sqrt[2 - 5*x^2 + 3*x^4]) - (3*((5*((-2*x*Sqrt[2 - 5*x
^2 + 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 - 5*x^
2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], (12 + 5
*Sqrt[6])/24])/(3^(1/4)*Sqrt[2 - 5*x^2 + 3*x^4])))/Sqrt[6] + ((12 - 5*Sqrt
[6])*(2 + Sqrt[6]*x^2)*Sqrt[(2 - 5*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*Ellip
ticF[2*ArcTan[(3/2)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(6*6^(1/4)*Sqrt[2 - 5*
x^2 + 3*x^4])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

rule 1496
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2
- 4*a*c, 0] && GtQ[c/a, 0] && LtQ[b/a, 0]
```

rule 1497
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ
[c/a, 0] && LtQ[b/a, 0]
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{x(15x^2-13)}{2\sqrt{3x^4-5x^2+2}}-\frac{3\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^4-5x^2+2}}+\frac{5\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\left(\mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(x,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{3x^4-5x^2+2}}$
default	$-\frac{6\left(\frac{5}{4}x^3-\frac{13}{12}x\right)}{\sqrt{3x^4-5x^2+2}}-\frac{3\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^4-5x^2+2}}+\frac{5\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\left(\mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(x,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{3x^4-5x^2+2}}$
elliptic	$-\frac{6\left(\frac{5}{4}x^3-\frac{13}{12}x\right)}{\sqrt{3x^4-5x^2+2}}-\frac{3\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^4-5x^2+2}}+\frac{5\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\left(\mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(x,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{3x^4-5x^2+2}}$

input
```
int(1/(3*x^4-5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(15*x^2-13)/(3*x^4-5*x^2+2)^(1/2)-3*(-x^2+1)^(1/2)*(-6*x^2+4)^(1/2)
/(3*x^4-5*x^2+2)^(1/2)*EllipticF(x,1/2*6^(1/2))+5/2*(-x^2+1)^(1/2)*(-6*x^2
+4)^(1/2)/(3*x^4-5*x^2+2)^(1/2)*(EllipticF(x,1/2*6^(1/2))-EllipticE(x,1/2*
6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int \frac{1}{(2-5x^2+3x^4)^{3/2}}\,dx =$$
$$-\frac{5\sqrt{2}(3x^4-5x^2+2)E(\arcsin(x)\mid\frac{3}{2})+\sqrt{2}(3x^4-5x^2+2)F(\arcsin(x)\mid\frac{3}{2})+\sqrt{3x^4-5x^2+2}(15x^3}{2(3x^4-5x^2+2)}$$

input
```
integrate(1/(3*x^4-5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(5*sqrt(2)*(3*x^4 - 5*x^2 + 2)*elliptic_e(arcsin(x), 3/2) + sqrt(2)*(
3*x^4 - 5*x^2 + 2)*elliptic_f(arcsin(x), 3/2) + sqrt(3*x^4 - 5*x^2 + 2)*(1
5*x^3 - 13*x))/(3*x^4 - 5*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2-5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4-5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4-5*x**2+2)**(3/2),x)
```

output
```
Integral((3*x**4 - 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2-5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 - 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2-5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{\left(3\,x^4-5\,x^2+2\right)^{3/2}}\,dx$$

input
```
int(1/(3*x^4 - 5*x^2 + 2)^(3/2),x)
```

output
```
int(1/(3*x^4 - 5*x^2 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2-5x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-5x^2+2}}{9x^8-30x^6+37x^4-20x^2+4}dx$$

input
```
int(1/(3*x^4-5*x^2+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - 5*x**2 + 2)/(9*x**8 - 30*x**6 + 37*x**4 - 20*x**2 + 4),x
)
```

3.244 $\int \frac{1}{(2-6x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 229

$$\int \frac{1}{(2-6x^2+3x^4)^{3/2}}\,dx = \frac{x(4-3x^2)}{4\sqrt{2-6x^2+3x^4}} - \frac{\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}\sqrt{2-\left(3-\sqrt{3}\right)x^2}\sqrt{2-\left(3+\sqrt{3}\right)x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}x\right)|2-\sqrt{3}\right)}{4\sqrt{2-6x^2+3x^4}} + \frac{\sqrt{2-\left(3-\sqrt{3}\right)x^2}\sqrt{2-\left(3+\sqrt{3}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}x\right),2-\sqrt{3}\right)}{4\sqrt{3-\sqrt{3}}\sqrt{2-6x^2+3x^4}}$$

output

```
1/4*x*(-3*x^2+4)/(3*x^4-6*x^2+2)^(1/2)-1/8*(6+2*3^(1/2))^(1/2)*(2-(3-3^(1/
2))*x^2)^(1/2)*(2-(3+3^(1/2))*x^2)^(1/2)*EllipticE(1/2*(6+2*3^(1/2))^(1/2)
*x,1/2*6^(1/2)-1/2*2^(1/2))/(3*x^4-6*x^2+2)^(1/2)+1/4*(2-(3-3^(1/2))*x^2)^
(1/2)*(2-(3+3^(1/2))*x^2)^(1/2)*EllipticF(1/2*(6+2*3^(1/2))^(1/2)*x,1/2*6^
(1/2)-1/2*2^(1/2))/(3-3^(1/2))^(1/2)/(3*x^4-6*x^2+2)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 6.32 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.80

$$\int \frac{1}{(2-6x^2+3x^4)^{3/2}}\,dx = \frac{6x(4-3x^2)-3\sqrt{2}(1+\sqrt{3})\sqrt{3-\sqrt{3}-3x^2}\sqrt{2+(-3+\sqrt{3})\,x^2}E\Big(\arcsin\Big(}{}$$

input
```
Integrate[(2 - 6*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(6*x*(4 - 3*x^2) - 3*Sqrt[2]*(1 + Sqrt[3])*Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt[
2 + (-3 + Sqrt[3])*x^2]*EllipticE[ArcSin[Sqrt[(3 + Sqrt[3])/2]*x], 2 - Sqr
t[3]] + Sqrt[2]*(3 + Sqrt[3])*Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt[2 + (-3 + Sqr
t[3])*x^2]*EllipticF[ArcSin[Sqrt[(3 + Sqrt[3])/2]*x], 2 - Sqrt[3]])/(24*Sq
rt[2 - 6*x^2 + 3*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1497, 27, 1409, 1496}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4-6x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(4-3x^2)}{4\sqrt{3x^4-6x^2+2}}-\frac{1}{24}\int \frac{6(2-3x^2)}{\sqrt{3x^4-6x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(4-3x^2)}{4\sqrt{3x^4-6x^2+2}}-\frac{1}{4}\int \frac{2-3x^2}{\sqrt{3x^4-6x^2+2}}dx$$

$$\Big\downarrow \text{1497}$$

$$\frac{1}{4}\left(-\left(\left(2-\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4-6x^2+2}}dx\right)-\sqrt{6}\int \frac{2-\sqrt{6}x^2}{2\sqrt{3x^4-6x^2+2}}dx\right)+\frac{x(4-3x^2)}{4\sqrt{3x^4-6x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\left(\left(2-\sqrt{6}\right)\int \frac{1}{\sqrt{3x^4-6x^2+2}}dx\right)-\sqrt{\frac{3}{2}}\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4-6x^2+2}}dx\right)+\frac{x(4-3x^2)}{4\sqrt{3x^4-6x^2+2}}$$

$$\Big\downarrow \text{1409}$$

$$\frac{1}{4}\left(-\sqrt{\frac{3}{2}}\int \frac{2-\sqrt{6}x^2}{\sqrt{3x^4-6x^2+2}}dx-\frac{(2-\sqrt{6})(\sqrt{6}x^2+2)\sqrt{\frac{3x^4-6x^2+2}{(\sqrt{6}x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{4}(2+\sqrt{6})\right)}{2\sqrt[4]{6}\sqrt{3x^4-6x^2+2}}\right.$$

$$\frac{x(4-3x^2)}{4\sqrt{3x^4-6x^2+2}}$$

$$\Big\downarrow \text{1496}$$

$$\frac{1}{4}\left(-\frac{(2-\sqrt{6})(\sqrt{6}x^2+2)\sqrt{\frac{3x^4-6x^2+2}{(\sqrt{6}x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{4}(2+\sqrt{6})\right)}{2\sqrt[4]{6}\sqrt{3x^4-6x^2+2}}-\sqrt{\frac{3}{2}}\left(\frac{2^{3/4}(\sqrt{6}x^2+2)\sqrt{}}{}\right.\right.$$

$$\frac{x(4-3x^2)}{4\sqrt{3x^4-6x^2+2}}$$

input
```
Int[(2 - 6*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(x*(4 - 3*x^2))/(4*Sqrt[2 - 6*x^2 + 3*x^4]) + (-(Sqrt[3/2]*((-2*x*Sqrt[2 -
 6*x^2 + 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 -
6*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], (2
+ Sqrt[6])/4])/(3^(1/4)*Sqrt[2 - 6*x^2 + 3*x^4]))) - ((2 - Sqrt[6])*(2 + S
qrt[6]*x^2)*Sqrt[(2 - 6*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcT
an[(3/2)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*Sqrt[2 - 6*x^2 + 3*x^4]))/
4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

rule 1496
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2
- 4*a*c, 0] && GtQ[c/a, 0] && LtQ[b/a, 0]
```

rule 1497
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ
[c/a, 0] && LtQ[b/a, 0]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{x(3x^2-4)}{4\sqrt{3x^4-6x^2+2}}-\frac{\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{6+2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{6+2\sqrt{3}}\,\sqrt{3x^4-6x^2+2}}-\frac{6\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)}}{}$
default	$-\frac{6\left(\frac{1}{8}x^3-\frac{1}{6}x\right)}{\sqrt{3x^4-6x^2+2}}-\frac{\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{6+2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{6+2\sqrt{3}}\,\sqrt{3x^4-6x^2+2}}-\frac{6\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x}}{}$
elliptic	$-\frac{6\left(\frac{1}{8}x^3-\frac{1}{6}x\right)}{\sqrt{3x^4-6x^2+2}}-\frac{\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{6+2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{6+2\sqrt{3}}\,\sqrt{3x^4-6x^2+2}}-\frac{6\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x}}{}$

input
```
int(1/(3*x^4-6*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x*(3*x^2-4)/(3*x^4-6*x^2+2)^(1/2)-1/(6+2*3^(1/2))^(1/2)*(1-(3/2+1/2*3
^(1/2))*x^2)^(1/2)*(1-(3/2-1/2*3^(1/2))*x^2)^(1/2)/(3*x^4-6*x^2+2)^(1/2)*E
llipticF(1/2*(6+2*3^(1/2))^(1/2)*x,1/2*6^(1/2)-1/2*2^(1/2))-6/(6+2*3^(1/2)
)^(1/2)*(1-(3/2+1/2*3^(1/2))*x^2)^(1/2)*(1-(3/2-1/2*3^(1/2))*x^2)^(1/2)/(3
*x^4-6*x^2+2)^(1/2)/(-6+2*3^(1/2))*(EllipticF(1/2*(6+2*3^(1/2))^(1/2)*x,1/
2*6^(1/2)-1/2*2^(1/2))-EllipticE(1/2*(6+2*3^(1/2))^(1/2)*x,1/2*6^(1/2)-1/2
*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.78

$$\int \frac{1}{(2-6x^2+3x^4)^{3/2}}\,dx =$$

$$-\frac{3\left(\sqrt{3}\sqrt{2}(3\,x^4-6\,x^2+2)+3\,\sqrt{2}(3\,x^4-6\,x^2+2)\right)\sqrt{\frac{1}{2}\,\sqrt{3}+\frac{3}{2}}E(\arcsin\left(x\sqrt{\frac{1}{2}\,\sqrt{3}+\frac{3}{2}}\right)\mid-\sqrt{3}+2)-(}{}$$

input
```
integrate(1/(3*x^4-6*x^2+2)^(3/2),x, algorithm="fricas")
```

output

```
-1/24*(3*(sqrt(3)*sqrt(2)*(3*x^4 - 6*x^2 + 2) + 3*sqrt(2)*(3*x^4 - 6*x^2 +
 2))*sqrt(1/2*sqrt(3) + 3/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(3) + 3/2)),
 -sqrt(3) + 2) - (5*sqrt(3)*sqrt(2)*(3*x^4 - 6*x^2 + 2) + 3*sqrt(2)*(3*x^4
 - 6*x^2 + 2))*sqrt(1/2*sqrt(3) + 3/2)*elliptic_f(arcsin(x*sqrt(1/2*sqrt(3
) + 3/2)), -sqrt(3) + 2) + 6*sqrt(3*x^4 - 6*x^2 + 2)*(3*x^3 - 4*x))/(3*x^4
 - 6*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2-6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4-6x^2+2)^{\frac{3}{2}}}\,dx$$

input

```
integrate(1/(3*x**4-6*x**2+2)**(3/2),x)
```

output

```
Integral((3*x**4 - 6*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2-6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-6\,x^2+2)^{\frac{3}{2}}}\,dx$$

input

```
integrate(1/(3*x^4-6*x^2+2)^(3/2),x, algorithm="maxima")
```

output

```
integrate((3*x^4 - 6*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2-6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-6\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4-6*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 - 6*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-6x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4-6\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(3*x^4 - 6*x^2 + 2)^(3/2),x)
```

output
```
int(1/(3*x^4 - 6*x^2 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2-6x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4-6x^2+2}}{9x^8-36x^6+48x^4-24x^2+4}dx$$

input
```
int(1/(3*x^4-6*x^2+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 - 6*x**2 + 2)/(9*x**8 - 36*x**6 + 48*x**4 - 24*x**2 + 4),x
)
```

3.245 $\int \frac{1}{(3+9x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 236

$$\int \frac{1}{(3+9x^2+2x^4)^{3/2}}\,dx = \frac{4x}{\sqrt{57}\left(9-\sqrt{57}\right)\sqrt{3+9x^2+2x^4}} + \frac{2\sqrt{\frac{6}{9+\sqrt{57}}}\sqrt{3+9x^2+2x^4}\,E\left(\arctan\left(\frac{2x}{\sqrt{9+\sqrt{57}}}\right)|\frac{1}{4}\left(-19-3\sqrt{57}\right)\right)}{19\sqrt{\frac{6}{9+\sqrt{57}}+x^2}\sqrt{9+\sqrt{57}+4x^2}} - \frac{\sqrt{9+\sqrt{57}}\sqrt{6+\left(9-\sqrt{57}\right)x^2}\sqrt{6+\left(9+\sqrt{57}\right)x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(9-\sqrt{57}\right)}x\right),\frac{1}{4}\left(-19-3\sqrt{5\right.}\right.}{171\sqrt{3+9x^2+2x^4}}$$

output
```
4/57*x*57^(1/2)/(9-57^(1/2))/(2*x^4+9*x^2+3)^(1/2)+2/19*6^(1/2)/(9+57^(1/2
))^(1/2)*(2*x^4+9*x^2+3)^(1/2)*EllipticE(2*x/(9+57^(1/2))^(1/2)/(1+4*x^2/(
9+57^(1/2)))^(1/2),1/2*(-19-3*57^(1/2))^(1/2))/(6/(9+57^(1/2))+x^2)^(1/2)/
(9+57^(1/2)+4*x^2)^(1/2)-1/171*(9+57^(1/2))^(1/2)*(6+(9-57^(1/2))*x^2)^(1/
2)*(6+(9+57^(1/2))*x^2)^(1/2)*InverseJacobiAM(arctan(1/6*(54-6*57^(1/2))^(
1/2)*x),1/2*(-19-3*57^(1/2))^(1/2))/(2*x^4+9*x^2+3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.89

$$\int \frac{1}{(3+9x^2+2x^4)^{3/2}}\,dx = \frac{4\sqrt{9-\sqrt{57}}x(23+6x^2)-3i\left(-9+\sqrt{57}\right)\sqrt{9-\sqrt{57}+4x^2}\sqrt{9+\sqrt{57}+4x^2}E}{}$$

input
```
Integrate[(3 + 9*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(4*Sqrt[9 - Sqrt[57]]*x*(23 + 6*x^2) - (3*I)*(-9 + Sqrt[57])*Sqrt[9 - Sqrt
[57] + 4*x^2]*Sqrt[9 + Sqrt[57] + 4*x^2]*EllipticE[I*ArcSinh[(2*x)/Sqrt[9
+ Sqrt[57]]], 23/4 + (3*Sqrt[57])/4] + I*(-19 + 3*Sqrt[57])*Sqrt[9 - Sqrt[
57] + 4*x^2]*Sqrt[9 + Sqrt[57] + 4*x^2]*EllipticF[I*ArcSinh[(2*x)/Sqrt[9 +
 Sqrt[57]]], 23/4 + (3*Sqrt[57])/4])/(228*Sqrt[-((-9 + Sqrt[57])*(3 + 9*x^
2 + 2*x^4))])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1405, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+9x^2+3)^{3/2}}dx$$

$$\Big\downarrow \; 1405$$

$$\frac{x(6x^2+23)}{57\sqrt{2x^4+9x^2+3}}-\frac{1}{171}\int \frac{6(3x^2+2)}{\sqrt{2x^4+9x^2+3}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{x(6x^2+23)}{57\sqrt{2x^4+9x^2+3}}-\frac{2}{57}\int \frac{3x^2+2}{\sqrt{2x^4+9x^2+3}}dx$$

$$\Big\downarrow \text{1503}$$

$$\frac{x(6x^2+23)}{57\sqrt{2x^4+9x^2+3}}-\frac{2}{57}\left(2\int\frac{1}{\sqrt{2x^4+9x^2+3}}dx+3\int\frac{x^2}{\sqrt{2x^4+9x^2+3}}dx\right)$$

$$\Big\downarrow \text{1412}$$

$$\frac{x(6x^2+23)}{57\sqrt{2x^4+9x^2+3}}-\frac{2}{57}\left(3\int\frac{x^2}{\sqrt{2x^4+9x^2+3}}dx+\frac{\sqrt{\frac{2}{3(9+\sqrt{57})}}\sqrt{\frac{(9-\sqrt{57})x^2+6}{(9+\sqrt{57})x^2+6}}\left((9+\sqrt{57})x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(9+\sqrt{57})}\right.\right.}{\sqrt{2x^4+9x^2+3}}\right.$$

$$\Big\downarrow \text{1455}$$

$$\frac{x(6x^2+23)}{57\sqrt{2x^4+9x^2+3}}-\frac{2}{57}\left(\frac{\sqrt{\frac{2}{3(9+\sqrt{57})}}\sqrt{\frac{(9-\sqrt{57})x^2+6}{(9+\sqrt{57})x^2+6}}\left((9+\sqrt{57})x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(9+\sqrt{57})}x\right),\frac{1}{4}(-19+3\sqrt{57})\right)}{\sqrt{2x^4+9x^2+3}}+3\right.$$

input
```
Int[(3 + 9*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(23 + 6*x^2))/(57*Sqrt[3 + 9*x^2 + 2*x^4]) - (2*(3*((x*(9 + Sqrt[57] +
4*x^2))/(4*Sqrt[3 + 9*x^2 + 2*x^4]) - (Sqrt[(9 + Sqrt[57])/6]*Sqrt[(6 + (9
 - Sqrt[57])*x^2)/(6 + (9 + Sqrt[57])*x^2)]*(6 + (9 + Sqrt[57])*x^2)*Ellip
ticE[ArcTan[Sqrt[(9 + Sqrt[57])/6]*x], (-19 + 3*Sqrt[57])/4])/(4*Sqrt[3 +
9*x^2 + 2*x^4])) + (Sqrt[2/(3*(9 + Sqrt[57]))]*Sqrt[(6 + (9 - Sqrt[57])*x^
2)/(6 + (9 + Sqrt[57])*x^2)]*(6 + (9 + Sqrt[57])*x^2)*EllipticF[ArcTan[Sqr
t[(9 + Sqrt[57])/6]*x], (-19 + 3*Sqrt[57])/4])/Sqrt[3 + 9*x^2 + 2*x^4]))/5
7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.94

method	result
risch	$\frac{x(6x^2+23)}{57\sqrt{2x^4+9x^2+3}}-\frac{8\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{57}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{57}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-54+6\sqrt{57}}}{6},\frac{3\sqrt{6}}{4}+\frac{\sqrt{38}}{4}\right)}{19\sqrt{-54+6\sqrt{57}}\,\sqrt{2x^4+9x^2+3}}+\frac{72\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{57}}{6}\right)x^2}}{}$
default	$-\frac{4\left(-\frac{23}{228}x-\frac{1}{38}x^3\right)}{\sqrt{2x^4+9x^2+3}}-\frac{8\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{57}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{57}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-54+6\sqrt{57}}}{6},\frac{3\sqrt{6}}{4}+\frac{\sqrt{38}}{4}\right)}{19\sqrt{-54+6\sqrt{57}}\,\sqrt{2x^4+9x^2+3}}+\frac{72\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{57}}{6}\right)}}{}$
elliptic	$-\frac{4\left(-\frac{23}{228}x-\frac{1}{38}x^3\right)}{\sqrt{2x^4+9x^2+3}}-\frac{8\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{57}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{57}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-54+6\sqrt{57}}}{6},\frac{3\sqrt{6}}{4}+\frac{\sqrt{38}}{4}\right)}{19\sqrt{-54+6\sqrt{57}}\,\sqrt{2x^4+9x^2+3}}+\frac{72\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{57}}{6}\right)}}{}$

input
```
int(1/(2*x^4+9*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/57*x*(6*x^2+23)/(2*x^4+9*x^2+3)^(1/2)-8/19/(-54+6*57^(1/2))^(1/2)*(1-(-3
/2+1/6*57^(1/2))*x^2)^(1/2)*(1-(-3/2-1/6*57^(1/2))*x^2)^(1/2)/(2*x^4+9*x^2
+3)^(1/2)*EllipticF(1/6*x*(-54+6*57^(1/2))^(1/2),3/4*6^(1/2)+1/4*38^(1/2))
+72/19/(-54+6*57^(1/2))^(1/2)*(1-(-3/2+1/6*57^(1/2))*x^2)^(1/2)*(1-(-3/2-1
/6*57^(1/2))*x^2)^(1/2)/(2*x^4+9*x^2+3)^(1/2)/(9+57^(1/2))*(EllipticF(1/6*
x*(-54+6*57^(1/2))^(1/2),3/4*6^(1/2)+1/4*38^(1/2))-EllipticE(1/6*x*(-54+6*
57^(1/2))^(1/2),3/4*6^(1/2)+1/4*38^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.75

$$\int \frac{1}{(3+9x^2+2x^4)^{3/2}}\,dx=\frac{3\left(\sqrt{\frac{19}{3}}\sqrt{3}(2\,x^4+9\,x^2+3)-3\,\sqrt{3}(2\,x^4+9\,x^2+3)\right)\sqrt{\frac{1}{2}\,\sqrt{\frac{19}{3}}-\frac{3}{2}}E(\arcsin\left(x\right.}{}$$

input
```
integrate(1/(2*x^4+9*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/114*(3*(sqrt(19/3)*sqrt(3)*(2*x^4 + 9*x^2 + 3) - 3*sqrt(3)*(2*x^4 + 9*x^
2 + 3))*sqrt(1/2*sqrt(19/3) - 3/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(19/3)
 - 3/2)), 9/4*sqrt(19/3) + 23/4) - (sqrt(19/3)*sqrt(3)*(2*x^4 + 9*x^2 + 3)
 - 15*sqrt(3)*(2*x^4 + 9*x^2 + 3))*sqrt(1/2*sqrt(19/3) - 3/2)*elliptic_f(a
rcsin(x*sqrt(1/2*sqrt(19/3) - 3/2)), 9/4*sqrt(19/3) + 23/4) + 2*sqrt(2*x^4
 + 9*x^2 + 3)*(6*x^3 + 23*x))/(2*x^4 + 9*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(3+9x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+9x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+9*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 + 9*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+9x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+9\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+9*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 9*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+9x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+9\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+9*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 9*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+9x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+9\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(9*x^2 + 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(9*x^2 + 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+9x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+9x^2+3}}{4x^8+36x^6+93x^4+54x^2+9}dx$$

input
```
int(1/(2*x^4+9*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 9*x**2 + 3)/(4*x**8 + 36*x**6 + 93*x**4 + 54*x**2 + 9),x
)
```

3.246 $\int \frac{1}{(3+8x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 239

$$\int \frac{1}{(3+8x^2+2x^4)^{3/2}}\,dx = \frac{x}{\sqrt{10}\left(4-\sqrt{10}\right)\sqrt{3+8x^2+2x^4}} + \frac{2\sqrt{3+8x^2+2x^4}\,E\left(\arctan\left(\sqrt{\frac{2}{4+\sqrt{10}}}x\right)\middle|-\frac{2}{3}\left(5+2\sqrt{10}\right)\right)}{5\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{4-\sqrt{10}+2x^2}\sqrt{4+\sqrt{10}+2x^2}} - \frac{\sqrt{3+\left(4-\sqrt{10}\right)x^2}\sqrt{3+\left(4+\sqrt{10}\right)x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(4-\sqrt{10}\right)}x\right),-\frac{2}{3}\left(5+2\sqrt{10}\right)\right)}{10\sqrt{3\left(4-\sqrt{10}\right)}\sqrt{3+8x^2+2x^4}}$$

output
```
1/10*x*10^(1/2)/(4-10^(1/2))/(2*x^4+8*x^2+3)^(1/2)+2/5*(2*x^4+8*x^2+3)^(1/
2)*EllipticE(2^(1/2)/(4+10^(1/2))^(1/2)*x/(1+2/(4+10^(1/2))*x^2)^(1/2),1/3
*(-30-12*10^(1/2))^(1/2))/(12+3*10^(1/2))^(1/2)/(4-10^(1/2)+2*x^2)^(1/2)/(
4+10^(1/2)+2*x^2)^(1/2)-1/10*(3+(4-10^(1/2))*x^2)^(1/2)*(3+(4+10^(1/2))*x^
2)^(1/2)*InverseJacobiAM(arctan(1/3*(12-3*10^(1/2))^(1/2)*x),1/3*(-30-12*1
0^(1/2))^(1/2))/(12-3*10^(1/2))^(1/2)/(2*x^4+8*x^2+3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.22 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int \frac{1}{(3+8x^2+2x^4)^{3/2}}\,dx = \frac{26x+8x^3+4i(2\sqrt{2}-\sqrt{5})\sqrt{\frac{-4+\sqrt{10}-2x^2}{-4+\sqrt{10}}}\sqrt{4+\sqrt{10}+2x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{4+\sqrt{1}}}\right.\right.}{}$$

input
```
Integrate[(3 + 8*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(26*x + 8*x^3 + (4*I)*(2*Sqrt[2] - Sqrt[5])*Sqrt[(-4 + Sqrt[10] - 2*x^2)/(
-4 + Sqrt[10])]*Sqrt[4 + Sqrt[10] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(4 +
 Sqrt[10])]*x], 13/3 + (4*Sqrt[10])/3] - I*(5*Sqrt[2] - 4*Sqrt[5])*Sqrt[(-
4 + Sqrt[10] - 2*x^2)/(-4 + Sqrt[10])]*Sqrt[4 + Sqrt[10] + 2*x^2]*Elliptic
F[I*ArcSinh[Sqrt[2/(4 + Sqrt[10])]*x], 13/3 + (4*Sqrt[10])/3])/(60*Sqrt[3
+ 8*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1405, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+8x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(4x^2+13)}{30\sqrt{2x^4+8x^2+3}} - \frac{1}{120}\int \frac{4(4x^2+3)}{\sqrt{2x^4+8x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(4x^2+13)}{30\sqrt{2x^4+8x^2+3}} - \frac{1}{30}\int \frac{4x^2+3}{\sqrt{2x^4+8x^2+3}}dx$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{30}\left(-3\int \frac{1}{\sqrt{2x^4+8x^2+3}}dx-4\int \frac{x^2}{\sqrt{2x^4+8x^2+3}}dx\right)+\frac{x\left(4x^2+13\right)}{30\sqrt{2x^4+8x^2+3}}$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{30}\left(-4\int \frac{x^2}{\sqrt{2x^4+8x^2+3}}dx-\frac{\sqrt{\frac{3}{4+\sqrt{10}}}\sqrt{\frac{\left(4-\sqrt{10}\right)x^2+3}{\left(4+\sqrt{10}\right)x^2+3}}\left(\left(4+\sqrt{10}\right)x^2+3\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}x\right.\right.}{\sqrt{2x^4+8x^2+3}}\right.$$

$$\frac{x\left(4x^2+13\right)}{30\sqrt{2x^4+8x^2+3}}$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{30}\left(-\frac{\sqrt{\frac{3}{4+\sqrt{10}}}\sqrt{\frac{\left(4-\sqrt{10}\right)x^2+3}{\left(4+\sqrt{10}\right)x^2+3}}\left(\left(4+\sqrt{10}\right)x^2+3\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}x\right),-\frac{2}{3}\left(5-2\sqrt{10}\right)\right)}{\sqrt{2x^4+8x^2+3}}-4\left(\right.\right.$$

$$\frac{x\left(4x^2+13\right)}{30\sqrt{2x^4+8x^2+3}}$$

input
```
Int[(3 + 8*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(13 + 4*x^2))/(30*Sqrt[3 + 8*x^2 + 2*x^4]) + (-4*((x*(4 + Sqrt[10] + 2*
x^2))/(2*Sqrt[3 + 8*x^2 + 2*x^4]) - (Sqrt[(4 + Sqrt[10])/3]*Sqrt[(3 + (4 -
 Sqrt[10])*x^2)/(3 + (4 + Sqrt[10])*x^2)]*(3 + (4 + Sqrt[10])*x^2)*Ellipti
cE[ArcTan[Sqrt[(4 + Sqrt[10])/3]*x], (-2*(5 - 2*Sqrt[10]))/3])/(2*Sqrt[3 +
 8*x^2 + 2*x^4])) - (Sqrt[3/(4 + Sqrt[10])]*Sqrt[(3 + (4 - Sqrt[10])*x^2)/
(3 + (4 + Sqrt[10])*x^2)]*(3 + (4 + Sqrt[10])*x^2)*EllipticF[ArcTan[Sqrt[(
4 + Sqrt[10])/3]*x], (-2*(5 - 2*Sqrt[10]))/3])/Sqrt[3 + 8*x^2 + 2*x^4])/30
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.94

method	result
risch	$\frac{x(4x^2+13)}{30\sqrt{2x^4+8x^2+3}} - \frac{3\sqrt{1-\left(-\frac{4}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{4}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-12+3\sqrt{10}}}{3},\frac{2\sqrt{6}}{3}+\frac{\sqrt{15}}{3}\right)}{10\sqrt{-12+3\sqrt{10}}\,\sqrt{2x^4+8x^2+3}} + \frac{12\sqrt{1-\left(-\frac{4}{3}+\frac{\sqrt{10}}{3}\right)x^2}}{}$
default	$-\frac{4\left(-\frac{13}{120}x-\frac{1}{30}x^3\right)}{\sqrt{2x^4+8x^2+3}} - \frac{3\sqrt{1-\left(-\frac{4}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{4}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-12+3\sqrt{10}}}{3},\frac{2\sqrt{6}}{3}+\frac{\sqrt{15}}{3}\right)}{10\sqrt{-12+3\sqrt{10}}\,\sqrt{2x^4+8x^2+3}} + \frac{12\sqrt{1-\left(-\frac{4}{3}+\frac{\sqrt{10}}{3}\right)}}{}$
elliptic	$-\frac{4\left(-\frac{13}{120}x-\frac{1}{30}x^3\right)}{\sqrt{2x^4+8x^2+3}} - \frac{3\sqrt{1-\left(-\frac{4}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{4}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-12+3\sqrt{10}}}{3},\frac{2\sqrt{6}}{3}+\frac{\sqrt{15}}{3}\right)}{10\sqrt{-12+3\sqrt{10}}\,\sqrt{2x^4+8x^2+3}} + \frac{12\sqrt{1-\left(-\frac{4}{3}+\frac{\sqrt{10}}{3}\right)}}{}$

input
```
int(1/(2*x^4+8*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*x*(4*x^2+13)/(2*x^4+8*x^2+3)^(1/2)-3/10/(-12+3*10^(1/2))^(1/2)*(1-(-4
/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-4/3-1/3*10^(1/2))*x^2)^(1/2)/(2*x^4+8*x^2
+3)^(1/2)*EllipticF(1/3*x*(-12+3*10^(1/2))^(1/2),2/3*6^(1/2)+1/3*15^(1/2))
+12/5/(-12+3*10^(1/2))^(1/2)*(1-(-4/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-4/3-1/
3*10^(1/2))*x^2)^(1/2)/(2*x^4+8*x^2+3)^(1/2)/(8+2*10^(1/2))*(EllipticF(1/3
*x*(-12+3*10^(1/2))^(1/2),2/3*6^(1/2)+1/3*15^(1/2))-EllipticE(1/3*x*(-12+3
*10^(1/2))^(1/2),2/3*6^(1/2)+1/3*15^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(3+8x^2+2x^4\right)^{3/2}}\,dx = \frac{4\left(\sqrt{10}\sqrt{3}(2\,x^4+8\,x^2+3)-4\,\sqrt{3}(2\,x^4+8\,x^2+3)\right)\sqrt{\frac{1}{3}\,\sqrt{10}-\frac{4}{3}}\,E(\arcsin\left(x\sqrt{\right.}}{}$$

input
```
integrate(1/(2*x^4+8*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/180*(4*(sqrt(10)*sqrt(3)*(2*x^4 + 8*x^2 + 3) - 4*sqrt(3)*(2*x^4 + 8*x^2
+ 3))*sqrt(1/3*sqrt(10) - 4/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(10) - 4/3
)), 4/3*sqrt(10) + 13/3) - (sqrt(10)*sqrt(3)*(2*x^4 + 8*x^2 + 3) - 28*sqrt
(3)*(2*x^4 + 8*x^2 + 3))*sqrt(1/3*sqrt(10) - 4/3)*elliptic_f(arcsin(x*sqrt
(1/3*sqrt(10) - 4/3)), 4/3*sqrt(10) + 13/3) + 6*sqrt(2*x^4 + 8*x^2 + 3)*(4
*x^3 + 13*x))/(2*x^4 + 8*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(3+8x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+8x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+8*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 + 8*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+8x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+8\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+8*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 8*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+8x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+8\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+8*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 8*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+8x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+8\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(8*x^2 + 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(8*x^2 + 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+8x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+8x^2+3}}{4x^8+32x^6+76x^4+48x^2+9}dx$$

input
```
int(1/(2*x^4+8*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 8*x**2 + 3)/(4*x**8 + 32*x**6 + 76*x**4 + 48*x**2 + 9),x
)
```

3.247 $\int \frac{1}{(3+7x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 122

$$\int \frac{1}{(3+7x^2+2x^4)^{3/2}}\,dx = \frac{2x}{5\sqrt{3+7x^2+2x^4}} + \frac{7\sqrt{3+7x^2+2x^4}\,E\left(\arctan\left(\frac{x}{\sqrt{3}}\right)\middle|-5\right)}{75\sqrt{3+x^2}\sqrt{1+2x^2}} - \frac{4\sqrt{3+x^2}\sqrt{1+2x^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}}\right),-5\right)}{25\sqrt{3+7x^2+2x^4}}$$

output
```
2/5*x/(2*x^4+7*x^2+3)^(1/2)+7/75*(2*x^4+7*x^2+3)^(1/2)*EllipticE(x*3^(1/2)
/(3*x^2+9)^(1/2),I*5^(1/2))/(x^2+3)^(1/2)/(2*x^2+1)^(1/2)-4/25*(x^2+3)^(1/
2)*(2*x^2+1)^(1/2)*InverseJacobiAM(arctan(1/3*x*3^(1/2)),I*5^(1/2))/(2*x^4
+7*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.86 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{1}{(3+7x^2+2x^4)^{3/2}}\,dx = \frac{37x+14x^3+7i\sqrt{6}\sqrt{3+x^2}\sqrt{1+2x^2}E\left(i\text{arcsinh}\left(\sqrt{2}x\right)\,|\frac{1}{6}\right)-5i\sqrt{6}\sqrt{3+x^2}\sqrt{1}}{75\sqrt{3+7x^2+2x^4}}$$

input
```
Integrate[(3 + 7*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(37*x + 14*x^3 + (7*I)*Sqrt[6]*Sqrt[3 + x^2]*Sqrt[1 + 2*x^2]*EllipticE[I*A
rcSinh[Sqrt[2]*x], 1/6] - (5*I)*Sqrt[6]*Sqrt[3 + x^2]*Sqrt[1 + 2*x^2]*Elli
pticF[I*ArcSinh[Sqrt[2]*x], 1/6])/(75*Sqrt[3 + 7*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.50, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1405, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+7x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x(14x^2+37)}{75\sqrt{2x^4+7x^2+3}}-\frac{1}{75}\int \frac{2(7x^2+6)}{\sqrt{2x^4+7x^2+3}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x(14x^2+37)}{75\sqrt{2x^4+7x^2+3}}-\frac{2}{75}\int \frac{7x^2+6}{\sqrt{2x^4+7x^2+3}}dx$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{x(14x^2+37)}{75\sqrt{2x^4+7x^2+3}}-\frac{2}{75}\left(6\int \frac{1}{\sqrt{2x^4+7x^2+3}}dx+7\int \frac{x^2}{\sqrt{2x^4+7x^2+3}}dx\right)$$

$$\Big\downarrow \ 1412$$

$$\frac{x(14x^2+37)}{75\sqrt{2x^4+7x^2+3}}-\frac{2}{75}\left(7\int \frac{x^2}{\sqrt{2x^4+7x^2+3}}dx+\frac{\sqrt{6}\sqrt{\frac{x^2+3}{2x^2+1}}(2x^2+1)\operatorname{EllipticF}\left(\arctan\left(\sqrt{2}x\right),\frac{5}{6}\right)}{\sqrt{2x^4+7x^2+3}}\right)$$

$$\Big\downarrow \ 1455$$

$$\frac{x(14x^2+37)}{75\sqrt{2x^4+7x^2+3}}-\frac{2}{75}\left(\frac{\sqrt{6}\sqrt{\frac{x^2+3}{2x^2+1}}(2x^2+1)\operatorname{EllipticF}\left(\arctan\left(\sqrt{2}x\right),\frac{5}{6}\right)}{\sqrt{2x^4+7x^2+3}}+7\left(\frac{x(x^2+3)}{\sqrt{2x^4+7x^2+3}}-\frac{\sqrt{\frac{3}{2}}\sqrt{\frac{x^2+3}{2x^2+1}}(2x^2+1)E(\arctan}{\sqrt{2x^4+7x^2+3}}\right.\right.$$

input
```
Int[(3 + 7*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(37 + 14*x^2))/(75*Sqrt[3 + 7*x^2 + 2*x^4]) - (2*(7*((x*(3 + x^2))/Sqrt
[3 + 7*x^2 + 2*x^4] - (Sqrt[3/2]*Sqrt[(3 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*E
llipticE[ArcTan[Sqrt[2]*x], 5/6])/Sqrt[3 + 7*x^2 + 2*x^4]) + (Sqrt[6]*Sqrt
[(3 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]*x], 5/6])/Sqr
t[3 + 7*x^2 + 2*x^4]))/75
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.13

method	result
risch	$\frac{x(14x^2+37)}{75\sqrt{2x^4+7x^2+3}}+\frac{4i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{2x^2+1}\,\text{EllipticF}\left(\frac{ix\sqrt{3}}{3},\sqrt{6}\right)}{75\sqrt{2x^4+7x^2+3}}-\frac{7i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{2x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{3}}{3},\sqrt{6}\right)-\text{EllipticE}\left(\frac{i}{}\right.\right.}{225\sqrt{2x^4+7x^2+3}}$
default	$-\frac{4\left(-\frac{37}{300}x-\frac{7}{150}x^3\right)}{\sqrt{2x^4+7x^2+3}}+\frac{4i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{2x^2+1}\,\text{EllipticF}\left(\frac{ix\sqrt{3}}{3},\sqrt{6}\right)}{75\sqrt{2x^4+7x^2+3}}-\frac{7i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{2x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{3}}{3},\sqrt{6}\right)-\text{Elliptic}\right.}{225\sqrt{2x^4+7x^2+3}}$
elliptic	$-\frac{4\left(-\frac{37}{300}x-\frac{7}{150}x^3\right)}{\sqrt{2x^4+7x^2+3}}+\frac{4i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{2x^2+1}\,\text{EllipticF}\left(\frac{ix\sqrt{3}}{3},\sqrt{6}\right)}{75\sqrt{2x^4+7x^2+3}}-\frac{7i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{2x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{3}}{3},\sqrt{6}\right)-\text{Elliptic}\right.}{225\sqrt{2x^4+7x^2+3}}$

input
```
int(1/(2*x^4+7*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/75*x*(14*x^2+37)/(2*x^4+7*x^2+3)^(1/2)+4/75*I*3^(1/2)*(3*x^2+9)^(1/2)*(2
*x^2+1)^(1/2)/(2*x^4+7*x^2+3)^(1/2)*EllipticF(1/3*I*x*3^(1/2),6^(1/2))-7/2
25*I*3^(1/2)*(3*x^2+9)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+7*x^2+3)^(1/2)*(Ellipt
icF(1/3*I*x*3^(1/2),6^(1/2))-EllipticE(1/3*I*x*3^(1/2),6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \frac{1}{(3+7x^2+2x^4)^{3/2}}\,dx = -\frac{7\sqrt{3}\sqrt{-\frac{1}{3}}(2\,x^4+7\,x^2+3)E(\arcsin\left(\sqrt{-\frac{1}{3}}x\right)\,|\,6)-43\sqrt{3}\sqrt{-\frac{1}{3}}(2\,x^4+7\,x^2+3)F(\arcsin\left(\sqrt{-\frac{1}{3}}x\right)\,|\,6)}{225\,(2\,x^4+7\,x^2+3)}$$

input
```
integrate(1/(2*x^4+7*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/225*(7*sqrt(3)*sqrt(-1/3)*(2*x^4 + 7*x^2 + 3)*elliptic_e(arcsin(sqrt(-1
/3)*x), 6) - 43*sqrt(3)*sqrt(-1/3)*(2*x^4 + 7*x^2 + 3)*elliptic_f(arcsin(s
qrt(-1/3)*x), 6) - 3*sqrt(2*x^4 + 7*x^2 + 3)*(14*x^3 + 37*x))/(2*x^4 + 7*x
^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(3+7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+7x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+7*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 + 7*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+7\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+7*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 7*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+7\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+7*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 7*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+7\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(7*x^2 + 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(7*x^2 + 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+7x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+7x^2+3}}{4x^8+28x^6+61x^4+42x^2+9}dx$$

input
```
int(1/(2*x^4+7*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 7*x**2 + 3)/(4*x**8 + 28*x**6 + 61*x**4 + 42*x**2 + 9),x
)
```

3.248 $\int \frac{1}{(3+6x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 234

$$\int \frac{1}{(3+6x^2+2x^4)^{3/2}}\,dx = \frac{x}{\sqrt{3}\left(3-\sqrt{3}\right)\sqrt{3+6x^2+2x^4}} + \frac{\sqrt{3-\sqrt{3}}\sqrt{3+6x^2+2x^4}\,E\left(\arctan\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|-1-\sqrt{3}\right)}{6\sqrt{\frac{3}{3+\sqrt{3}}+x^2}\sqrt{3+\sqrt{3}+2x^2}} - \frac{\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}\sqrt{3+\left(3-\sqrt{3}\right)x^2}\sqrt{3+\left(3+\sqrt{3}\right)x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),-1-\sqrt{3}\right)}{9\sqrt{3+6x^2+2x^4}}$$

output
```
1/3*x*3^(1/2)/(3-3^(1/2))/(2*x^4+6*x^2+3)^(1/2)+1/6*(3-3^(1/2))^(1/2)*(2*x
^4+6*x^2+3)^(1/2)*EllipticE((9-3*3^(1/2))^(1/2)*x/(9+(9-3*3^(1/2))*x^2)^(1
/2),(-1-3^(1/2))^(1/2))/(3/(3+3^(1/2))+x^2)^(1/2)/(3+3^(1/2)+2*x^2)^(1/2)-
1/18*(6+2*3^(1/2))^(1/2)*(3+(3-3^(1/2))*x^2)^(1/2)*(3+(3+3^(1/2))*x^2)^(1/
2)*InverseJacobiAM(arctan(1/3*(9-3*3^(1/2))^(1/2)*x),(-1-3^(1/2))^(1/2))/(
2*x^4+6*x^2+3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.86

$$\int \frac{1}{(3+6x^2+2x^4)^{3/2}}\,dx = \frac{4x(2+x^2)-i\sqrt{2}(-3+\sqrt{3})\sqrt{\frac{-3+\sqrt{3}-2x^2}{-3+\sqrt{3}}}\sqrt{3+\sqrt{3}+2x^2}E\left(i\text{arcsinh}\left(\sqrt{1-\right.\right.}$$

input
```
Integrate[(3 + 6*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(4*x*(2 + x^2) - I*Sqrt[2]*(-3 + Sqrt[3])*Sqrt[(-3 + Sqrt[3] - 2*x^2)/(-3
+ Sqrt[3])]*Sqrt[3 + Sqrt[3] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - 1/Sqrt[
3]]*x], 2 + Sqrt[3]] + I*Sqrt[2]*(-1 + Sqrt[3])*Sqrt[(-3 + Sqrt[3] - 2*x^2
)/(-3 + Sqrt[3])]*Sqrt[3 + Sqrt[3] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - 1
/Sqrt[3]]*x], 2 + Sqrt[3]])/(12*Sqrt[3 + 6*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1405, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+6x^2+3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(x^2+2)}{3\sqrt{2x^4+6x^2+3}}-\frac{1}{36}\int \frac{12(x^2+1)}{\sqrt{2x^4+6x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(x^2+2)}{3\sqrt{2x^4+6x^2+3}}-\frac{1}{3}\int \frac{x^2+1}{\sqrt{2x^4+6x^2+3}}dx$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{3}\left(-\int \frac{1}{\sqrt{2x^4+6x^2+3}}dx-\int \frac{x^2}{\sqrt{2x^4+6x^2+3}}dx\right)+\frac{x(x^2+2)}{3\sqrt{2x^4+6x^2+3}}$$

$$\Big\downarrow \; 1412$$

$$\frac{1}{3}\left(-\int \frac{x^2}{\sqrt{2x^4+6x^2+3}}dx-\frac{\sqrt{\frac{(3-\sqrt{3})x^2+3}{(3+\sqrt{3})x^2+3}}\left(\left(3+\sqrt{3}\right)x^2+3\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right),-1+\sqrt{3}\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{2x^4+6x^2+3}}\right.$$

$$\frac{x(x^2+2)}{3\sqrt{2x^4+6x^2+3}}$$

$$\Big\downarrow \; 1455$$

$$\frac{1}{3}\left(-\frac{\sqrt{\frac{(3-\sqrt{3})x^2+3}{(3+\sqrt{3})x^2+3}}\left(\left(3+\sqrt{3}\right)x^2+3\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right),-1+\sqrt{3}\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{2x^4+6x^2+3}}+\frac{\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}\sqrt{\frac{(3-\sqrt{3}}{(3+\sqrt{3}}}}{}\right.$$

$$\frac{x(x^2+2)}{3\sqrt{2x^4+6x^2+3}}$$

input

```
Int[(3 + 6*x^2 + 2*x^4)^(-3/2),x]
```

output

```
(x*(2 + x^2))/(3*Sqrt[3 + 6*x^2 + 2*x^4]) + (-1/2*(x*(3 + Sqrt[3] + 2*x^2)
)/Sqrt[3 + 6*x^2 + 2*x^4] + (Sqrt[(3 + Sqrt[3])/3]*Sqrt[(3 + (3 - Sqrt[3])
*x^2)/(3 + (3 + Sqrt[3])*x^2)]*(3 + (3 + Sqrt[3])*x^2)*EllipticE[ArcTan[Sq
rt[(3 + Sqrt[3])/3]*x], -1 + Sqrt[3]])/(2*Sqrt[3 + 6*x^2 + 2*x^4]) - (Sqrt
[(3 + (3 - Sqrt[3])*x^2)/(3 + (3 + Sqrt[3])*x^2)]*(3 + (3 + Sqrt[3])*x^2)*
EllipticF[ArcTan[Sqrt[(3 + Sqrt[3])/3]*x], -1 + Sqrt[3]])/(Sqrt[3*(3 + Sqr
t[3])]*Sqrt[3 + 6*x^2 + 2*x^4]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.95

method	result
risch	$\frac{(x^2+2)x}{3\sqrt{2x^4+6x^2+3}} - \frac{\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-9+3\sqrt{3}}}{3}, \frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{-9+3\sqrt{3}}\,\sqrt{2x^4+6x^2+3}} + \frac{6\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(-1-\right.}}{}$
default	$-\frac{4\left(-\frac{1}{6}x-\frac{1}{12}x^3\right)}{\sqrt{2x^4+6x^2+3}} - \frac{\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-9+3\sqrt{3}}}{3}, \frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{-9+3\sqrt{3}}\,\sqrt{2x^4+6x^2+3}} + \frac{6\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(-1\right.}}{}$
elliptic	$-\frac{4\left(-\frac{1}{6}x-\frac{1}{12}x^3\right)}{\sqrt{2x^4+6x^2+3}} - \frac{\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-9+3\sqrt{3}}}{3}, \frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{-9+3\sqrt{3}}\,\sqrt{2x^4+6x^2+3}} + \frac{6\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(-1\right.}}{}$

input
```
int(1/(2*x^4+6*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2+2)*x/(2*x^4+6*x^2+3)^(1/2)-1/(-9+3*3^(1/2))^(1/2)*(1-(-1+1/3*3^(1
/2))*x^2)^(1/2)*(1-(-1-1/3*3^(1/2))*x^2)^(1/2)/(2*x^4+6*x^2+3)^(1/2)*Ellip
ticF(1/3*x*(-9+3*3^(1/2))^(1/2),1/2*6^(1/2)+1/2*2^(1/2))+6/(-9+3*3^(1/2))^
(1/2)*(1-(-1+1/3*3^(1/2))*x^2)^(1/2)*(1-(-1-1/3*3^(1/2))*x^2)^(1/2)/(2*x^4
+6*x^2+3)^(1/2)/(6+2*3^(1/2))*(EllipticF(1/3*x*(-9+3*3^(1/2))^(1/2),1/2*6^
(1/2)+1/2*2^(1/2))-EllipticE(1/3*x*(-9+3*3^(1/2))^(1/2),1/2*6^(1/2)+1/2*2^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.60

$$\int \frac{1}{(3+6x^2+2x^4)^{3/2}}\,dx = \frac{2\sqrt{3}(2x^4+6x^2+3)\sqrt{\frac{1}{3}\sqrt{3}-1}\,F\left(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{3}-1}\right) \mid \sqrt{3}+2\right) + (2x^4 +}{}$$

input
```
integrate(1/(2*x^4+6*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*(2*x^4 + 6*x^2 + 3)*sqrt(1/3*sqrt(3) - 1)*elliptic_f(arcsin
(x*sqrt(1/3*sqrt(3) - 1)), sqrt(3) + 2) + (2*x^4 + 6*x^2 - sqrt(3)*(2*x^4
+ 6*x^2 + 3) + 3)*sqrt(1/3*sqrt(3) - 1)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(
3) - 1)), sqrt(3) + 2) + 2*sqrt(2*x^4 + 6*x^2 + 3)*(x^3 + 2*x))/(2*x^4 + 6
*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(3+6x^2+2x^4)^{3/2}}\,dx=\int \frac{1}{(2x^4+6x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+6*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 + 6*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+6x^2+2x^4)^{3/2}}\,dx=\int \frac{1}{(2\,x^4+6\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+6*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 6*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+6x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+6\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+6*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 6*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+6x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+6\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(6*x^2 + 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(6*x^2 + 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+6x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+6x^2+3}}{4x^8+24x^6+48x^4+36x^2+9}dx$$

input
```
int(1/(2*x^4+6*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 6*x**2 + 3)/(4*x**8 + 24*x**6 + 48*x**4 + 36*x**2 + 9),x
)
```

3.249 $\int \frac{1}{(3+5x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{1}{(3+5x^2+2x^4)^{3/2}}\,dx = \frac{x}{\sqrt{3+5x^2+2x^4}} + \frac{5\sqrt{2}\sqrt{3+5x^2+2x^4}E\left(\arctan\left(\sqrt{\frac{2}{3}}x\right)|-\frac{1}{2}\right)}{3\sqrt{1+x^2}\sqrt{3+2x^2}} - \frac{2\sqrt{2}\sqrt{1+x^2}\sqrt{3+2x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{2}{3}}x\right),-\frac{1}{2}\right)}{\sqrt{3+5x^2+2x^4}}$$

output
```
x/(2*x^4+5*x^2+3)^(1/2)+5/3*2^(1/2)*(2*x^4+5*x^2+3)^(1/2)*EllipticE(x*6^(1
/2)/(6*x^2+9)^(1/2),1/2*I*2^(1/2))/(x^2+1)^(1/2)/(2*x^2+3)^(1/2)-2*(x^2+1)
^(1/2)*(2*x^2+3)^(1/2)*InverseJacobiAM(arctan(1/3*x*6^(1/2)),1/2*I*2^(1/2)
)*2^(1/2)/(2*x^4+5*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.84 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.91

$$\int \frac{1}{(3+5x^2+2x^4)^{3/2}}\,dx = \frac{13x+10x^3+5i\sqrt{2}\sqrt{1+x^2}\sqrt{3+2x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{3}}x\right)|\frac{3}{2}\right)+i\sqrt{2}\sqrt{1+x^2}\sqrt{3}}{3\sqrt{3+5x^2+2x^4}}$$

input
```
Integrate[(3 + 5*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(13*x + 10*x^3 + (5*I)*Sqrt[2]*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*EllipticE[I*A
rcSinh[Sqrt[2/3]*x], 3/2] + I*Sqrt[2]*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*Ellipt
icF[I*ArcSinh[Sqrt[2/3]*x], 3/2])/(3*Sqrt[3 + 5*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1405, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+5x^2+3)^{3/2}}dx$$

$$\Big\downarrow \ 1405$$

$$\frac{x(10x^2+13)}{3\sqrt{2x^4+5x^2+3}}-\frac{1}{3}\int \frac{2(5x^2+6)}{\sqrt{2x^4+5x^2+3}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{x(10x^2+13)}{3\sqrt{2x^4+5x^2+3}}-\frac{2}{3}\int \frac{5x^2+6}{\sqrt{2x^4+5x^2+3}}dx$$

$$\Big\downarrow \ 1503$$

$$\frac{x(10x^2+13)}{3\sqrt{2x^4+5x^2+3}}-\frac{2}{3}\left(6\int \frac{1}{\sqrt{2x^4+5x^2+3}}dx+5\int \frac{x^2}{\sqrt{2x^4+5x^2+3}}dx\right)$$

$$\Big\downarrow \ 1412$$

$$\frac{x(10x^2+13)}{3\sqrt{2x^4+5x^2+3}}-\frac{2}{3}\left(5\int \frac{x^2}{\sqrt{2x^4+5x^2+3}}dx+\frac{2\sqrt{3}(x^2+1)\sqrt{\frac{2x^2+3}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{3}\right)}{\sqrt{2x^4+5x^2+3}}\right)$$

$$\Big\downarrow \ 1455$$

$$\frac{x(10x^2+13)}{3\sqrt{2x^4+5x^2+3}}-\frac{2}{3}\left(\frac{2\sqrt{3}(x^2+1)\sqrt{\frac{2x^2+3}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{3}\right)}{\sqrt{2x^4+5x^2+3}}+5\left(\frac{x(2x^2+3)}{2\sqrt{2x^4+5x^2+3}}-\frac{\sqrt{3}(x^2+1)\sqrt{\frac{2x^2+3}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{3}\right.\right)}{2\sqrt{2x^4+5x^2+3}}\right.\right.$$

input
```
Int[(3 + 5*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(13 + 10*x^2))/(3*Sqrt[3 + 5*x^2 + 2*x^4]) - (2*(5*((x*(3 + 2*x^2))/(2*
Sqrt[3 + 5*x^2 + 2*x^4]) - (Sqrt[3]*(1 + x^2)*Sqrt[(3 + 2*x^2)/(1 + x^2)]*
EllipticE[ArcTan[x], 1/3])/(2*Sqrt[3 + 5*x^2 + 2*x^4])) + (2*Sqrt[3]*(1 +
x^2)*Sqrt[(3 + 2*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/3])/Sqrt[3 + 5*x^2
 + 2*x^4]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04

method	result
risch	$\frac{x(10x^2+13)}{3\sqrt{2x^4+5x^2+3}}+\frac{2i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{3\sqrt{2x^4+5x^2+3}}-\frac{5i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(\frac{ix\sqrt{\cdots}}{3}\cdots\right)\right)}{9\sqrt{2x^4+5x^2+3}}$
default	$-\frac{4\left(-\frac{5}{6}x^3-\frac{13}{12}x\right)}{\sqrt{2x^4+5x^2+3}}+\frac{2i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{3\sqrt{2x^4+5x^2+3}}-\frac{5i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(\cdots\right)\right)}{9\sqrt{2x^4+5x^2+3}}$
elliptic	$-\frac{4\left(-\frac{5}{6}x^3-\frac{13}{12}x\right)}{\sqrt{2x^4+5x^2+3}}+\frac{2i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{3\sqrt{2x^4+5x^2+3}}-\frac{5i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(\cdots\right)\right)}{9\sqrt{2x^4+5x^2+3}}$

input
```
int(1/(2*x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(10*x^2+13)/(2*x^4+5*x^2+3)^(1/2)+2/3*I*6^(1/2)*(6*x^2+9)^(1/2)*(x^2
+1)^(1/2)/(2*x^4+5*x^2+3)^(1/2)*EllipticF(1/3*I*x*6^(1/2),1/2*6^(1/2))-5/9
*I*6^(1/2)*(6*x^2+9)^(1/2)*(x^2+1)^(1/2)/(2*x^4+5*x^2+3)^(1/2)*(EllipticF(
1/3*I*x*6^(1/2),1/2*6^(1/2))-EllipticE(1/3*I*x*6^(1/2),1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int \frac{1}{(3+5x^2+2x^4)^{3/2}}\,dx =$$
$$-\frac{10\sqrt{3}\sqrt{-\frac{2}{3}}(2\,x^4+5\,x^2+3)E(\arcsin\left(\sqrt{-\frac{2}{3}}x\right) \mid \frac{3}{2}) - 28\sqrt{3}\sqrt{-\frac{2}{3}}(2\,x^4+5\,x^2+3)F(\arcsin\left(\sqrt{-\frac{2}{3}}x\right) \mid}{9\,(2\,x^4+5\,x^2+3)}$$

input
```
integrate(1/(2*x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/9*(10*sqrt(3)*sqrt(-2/3)*(2*x^4 + 5*x^2 + 3)*elliptic_e(arcsin(sqrt(-2/
3)*x), 3/2) - 28*sqrt(3)*sqrt(-2/3)*(2*x^4 + 5*x^2 + 3)*elliptic_f(arcsin(
sqrt(-2/3)*x), 3/2) - 3*sqrt(2*x^4 + 5*x^2 + 3)*(10*x^3 + 13*x))/(2*x^4 +
5*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(3+5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 + 5*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 5*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 5*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+5\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 + 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(5*x^2 + 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+5x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+5x^2+3}}{4x^8+20x^6+37x^4+30x^2+9}dx$$

input
```
int(1/(2*x^4+5*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 5*x**2 + 3)/(4*x**8 + 20*x**6 + 37*x**4 + 30*x**2 + 9),x
)
```

3.250 $\int \frac{1}{(3+4x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 248

$$\int \frac{1}{(3+4x^2+2x^4)^{3/2}}\,dx = -\frac{x(1+2x^2)}{6\sqrt{3+4x^2+2x^4}}+\frac{x\sqrt{3+4x^2+2x^4}}{3\left(\sqrt{6}+2x^2\right)} - \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{6^{3/4}\sqrt{3+4x^2+2x^4}} + \frac{\left(2+\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{4\ 6^{3/4}\sqrt{3+4x^2+2x^4}}$$

output

```
-1/6*x*(2*x^2+1)/(2*x^4+4*x^2+3)^(1/2)+x*(2*x^4+4*x^2+3)^(1/2)/(3*6^(1/2)+
6*x^2)-1/6*(3+6^(1/2)*x^2)*((2*x^4+4*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*Ellip
ticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18-6*6^(1/2))^(1/2))*6^(1/4
)/(2*x^4+4*x^2+3)^(1/2)+1/24*(2+6^(1/2))*(3+6^(1/2)*x^2)*((2*x^4+4*x^2+3)/
(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1
/6*(18-6*6^(1/2))^(1/2))*6^(1/4)/(2*x^4+4*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.94 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.30

$$\int \frac{1}{(3+4x^2+2x^4)^{3/2}}\,dx = \frac{-2\sqrt{-\frac{i}{-2i+\sqrt{2}}}x(1+2x^2)-2i\left(-i+\sqrt{2}\right)\sqrt{\frac{-2i+\sqrt{2}-2ix^2}{-2i+\sqrt{2}}}\sqrt{\frac{2i+\sqrt{2}+2ix^2}{2i+\sqrt{2}}}E\Big(i\text{arcsinh}$$

input
```
Integrate[(3 + 4*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(-2*Sqrt[(-I)/(-2*I + Sqrt[2])]*x*(1 + 2*x^2) - (2*I)*(-I + Sqrt[2])*Sqrt[
(-2*I + Sqrt[2] - (2*I)*x^2)/(-2*I + Sqrt[2])]*Sqrt[(2*I + Sqrt[2] + (2*I)
*x^2)/(2*I + Sqrt[2])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-2*I + Sqrt[2])]*x
], (2*I - Sqrt[2])/(2*I + Sqrt[2])] + (2 - I*Sqrt[2])*Sqrt[(-2*I + Sqrt[2]
 - (2*I)*x^2)/(-2*I + Sqrt[2])]*Sqrt[(2*I + Sqrt[2] + (2*I)*x^2)/(2*I + Sq
rt[2])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-2*I + Sqrt[2])]*x], (2*I - Sqrt[
2])/(2*I + Sqrt[2])])/(12*Sqrt[(-I)/(-2*I + Sqrt[2])]*Sqrt[3 + 4*x^2 + 2*x
^4])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+4x^2+3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{24}\int \frac{4(2x^2+3)}{\sqrt{2x^4+4x^2+3}}dx - \frac{x(2x^2+1)}{6\sqrt{2x^4+4x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \frac{2x^2+3}{\sqrt{2x^4+4x^2+3}}dx-\frac{x(2x^2+1)}{6\sqrt{2x^4+4x^2+3}}$$

$\Big\downarrow$ 1511

$$\frac{1}{6}\left(\left(3+\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4+4x^2+3}}dx-\sqrt{6}\int \frac{3-\sqrt{6}x^2}{3\sqrt{2x^4+4x^2+3}}dx\right)-\frac{x(2x^2+1)}{6\sqrt{2x^4+4x^2+3}}$$

$\Big\downarrow$ 27

$$\frac{1}{6}\left(\left(3+\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4+4x^2+3}}dx-\sqrt{\frac{2}{3}}\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4+4x^2+3}}dx\right)-\frac{x(2x^2+1)}{6\sqrt{2x^4+4x^2+3}}$$

$\Big\downarrow$ 1416

$$\frac{1}{6}\left(\frac{\left(3+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4+4x^2+3}}-\sqrt{\frac{2}{3}}\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4+4x^2+3}}dx\right)-$$

$$\frac{x(2x^2+1)}{6\sqrt{2x^4+4x^2+3}}$$

$\Big\downarrow$ 1509

$$\frac{1}{6}\left(\frac{\left(3+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4+4x^2+3}}-\sqrt{\frac{2}{3}}\left(\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4}{\left(\sqrt{6}x^2\right.}}}{\sqrt[4]{2}}\right.\right.$$

$$\frac{x(2x^2+1)}{6\sqrt{2x^4+4x^2+3}}$$

input
```
Int[(3 + 4*x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/6*(x*(1 + 2*x^2))/Sqrt[3 + 4*x^2 + 2*x^4] + (-(Sqrt[2/3]*((-3*x*Sqrt[3
+ 4*x^2 + 2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 +
 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], 1/
2 - 1/Sqrt[6]])/(2^(1/4)*Sqrt[3 + 4*x^2 + 2*x^4]))) + ((3 + Sqrt[6])*(3 +
Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*Arc
Tan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[3 + 4*x^2 + 2*x^4]))
/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.20 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{x(2x^2+1)}{6\sqrt{2x^4+4x^2+3}}+\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-6+3i\sqrt{2}}}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{2\sqrt{-6+3i\sqrt{2}}\,\sqrt{2x^4+4x^2+3}}-\frac{6\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1 \cdots}}{\cdots}$
default	$-\frac{4\left(\frac{1}{12}x^3+\frac{1}{24}x\right)}{\sqrt{2x^4+4x^2+3}}+\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-6+3i\sqrt{2}}}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{2\sqrt{-6+3i\sqrt{2}}\,\sqrt{2x^4+4x^2+3}}-\frac{6\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1 \cdots}}{\cdots}$
elliptic	$-\frac{4\left(\frac{1}{12}x^3+\frac{1}{24}x\right)}{\sqrt{2x^4+4x^2+3}}+\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-6+3i\sqrt{2}}}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{2\sqrt{-6+3i\sqrt{2}}\,\sqrt{2x^4+4x^2+3}}-\frac{6\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1 \cdots}}{\cdots}$

input
```
int(1/(2*x^4+4*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x*(2*x^2+1)/(2*x^4+4*x^2+3)^(1/2)+3/2/(-6+3*I*2^(1/2))^(1/2)*(1-(-2/3
+1/3*I*2^(1/2))*x^2)^(1/2)*(1-(-2/3-1/3*I*2^(1/2))*x^2)^(1/2)/(2*x^4+4*x^2
+3)^(1/2)*EllipticF(1/3*x*(-6+3*I*2^(1/2))^(1/2),1/3*(3+6*I*2^(1/2))^(1/2)
)-6/(-6+3*I*2^(1/2))^(1/2)*(1-(-2/3+1/3*I*2^(1/2))*x^2)^(1/2)*(1-(-2/3-1/3
*I*2^(1/2))*x^2)^(1/2)/(2*x^4+4*x^2+3)^(1/2)/(4+2*I*2^(1/2))*(EllipticF(1/
3*x*(-6+3*I*2^(1/2))^(1/2),1/3*(3+6*I*2^(1/2))^(1/2))-EllipticE(1/3*x*(-6+
3*I*2^(1/2))^(1/2),1/3*(3+6*I*2^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.67

$$\int \frac{1}{(3+4x^2+2x^4)^{3/2}}\,dx = \frac{2\sqrt{3}\left(4\,x^4+8\,x^2-\sqrt{-2}(2\,x^4+4\,x^2+3)+6\right)\sqrt{\frac{1}{3}\sqrt{-2}-\frac{2}{3}}E\left(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{-}\right.\right.}{}$$

input
```
integrate(1/(2*x^4+4*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/36*(2*sqrt(3)*(4*x^4 + 8*x^2 - sqrt(-2)*(2*x^4 + 4*x^2 + 3) + 6)*sqrt(1/
3*sqrt(-2) - 2/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(-2) - 2/3)), 2/3*sqrt(
-2) + 1/3) - sqrt(3)*(20*x^4 + 40*x^2 + sqrt(-2)*(2*x^4 + 4*x^2 + 3) + 30)
*sqrt(1/3*sqrt(-2) - 2/3)*elliptic_f(arcsin(x*sqrt(1/3*sqrt(-2) - 2/3)), 2
/3*sqrt(-2) + 1/3) - 6*sqrt(2*x^4 + 4*x^2 + 3)*(2*x^3 + x))/(2*x^4 + 4*x^2
 + 3)
```

Sympy [F]

$$\int \frac{1}{(3+4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+4x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+4*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 + 4*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+4\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+4*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 4*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(3+4x^2+2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(2\,x^4+4\,x^2+3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+4*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 4*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(3+4x^2+2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(2\,x^4+4\,x^2+3\right)^{3/2}}\,dx$$

input
```
int(1/(4*x^2 + 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(4*x^2 + 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(3+4x^2+2x^4\right)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+4x^2+3}}{4x^8+16x^6+28x^4+24x^2+9}dx$$

input
```
int(1/(2*x^4+4*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 4*x**2 + 3)/(4*x**8 + 16*x**6 + 28*x**4 + 24*x**2 + 9),x
)
```

3.251 $\int \frac{1}{(3+3x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 261

$$\int \frac{1}{(3+3x^2+2x^4)^{3/2}}\,dx = \frac{x(1-2x^2)}{15\sqrt{3+3x^2+2x^4}}+\frac{2x\sqrt{3+3x^2+2x^4}}{15\left(\sqrt{6}+2x^2\right)} - \frac{\sqrt[4]{2}(3+\sqrt{6}x^2)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}(4-\sqrt{6})\right)}{5\ 3^{3/4}\sqrt{3+3x^2+2x^4}} + \frac{\left(3+2\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}(4-\sqrt{6})\right)}{15\ 6^{3/4}\sqrt{3+3x^2+2x^4}}$$

output
```
1/15*x*(-2*x^2+1)/(2*x^4+3*x^2+3)^(1/2)+2*x*(2*x^4+3*x^2+3)^(1/2)/(15*6^(1
/2)+30*x^2)-1/15*2^(1/4)*(3+6^(1/2)*x^2)*((2*x^4+3*x^2+3)/(3+6^(1/2)*x^2)^
2)^(1/2)*EllipticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/4*(8-2*6^(1/2))^
(1/2))*3^(1/4)/(2*x^4+3*x^2+3)^(1/2)+1/90*(3+2*6^(1/2))*(3+6^(1/2)*x^2)*((
2*x^4+3*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/
4)*3^(3/4)*x),1/4*(8-2*6^(1/2))^(1/2))*6^(1/4)/(2*x^4+3*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.04 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.23

$$\int \frac{1}{(3+3x^2+2x^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{-3i+\sqrt{15}}}x(1-2x^2)-(3i+\sqrt{15})\sqrt{\frac{-3i+\sqrt{15}-4ix^2}{-3i+\sqrt{15}}}\sqrt{\frac{3i+\sqrt{15}+4ix^2}{3i+\sqrt{15}}}E\Big(i\text{arcsinh}$$

input
```
Integrate[(3 + 3*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(4*Sqrt[(-I)/(-3*I + Sqrt[15])]*x*(1 - 2*x^2) - (3*I + Sqrt[15])*Sqrt[(-3*
I + Sqrt[15] - (4*I)*x^2)/(-3*I + Sqrt[15])]*Sqrt[(3*I + Sqrt[15] + (4*I)*
x^2)/(3*I + Sqrt[15])]*EllipticE[I*ArcSinh[2*Sqrt[(-I)/(-3*I + Sqrt[15])]*
x], (3*I - Sqrt[15])/(3*I + Sqrt[15])] + (-5*I + Sqrt[15])*Sqrt[(-3*I + Sq
rt[15] - (4*I)*x^2)/(-3*I + Sqrt[15])]*Sqrt[(3*I + Sqrt[15] + (4*I)*x^2)/(
3*I + Sqrt[15])]*EllipticF[I*ArcSinh[2*Sqrt[(-I)/(-3*I + Sqrt[15])]*x], (3
*I - Sqrt[15])/(3*I + Sqrt[15])])/(60*Sqrt[(-I)/(-3*I + Sqrt[15])]*Sqrt[3
+ 3*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+3x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{45}\int \frac{6(x^2+2)}{\sqrt{2x^4+3x^2+3}}dx+\frac{x(1-2x^2)}{15\sqrt{2x^4+3x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{15}\int \frac{x^2+2}{\sqrt{2x^4+3x^2+3}}dx+\frac{x(1-2x^2)}{15\sqrt{2x^4+3x^2+3}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{2}{15}\left(\frac{1}{2}\left(4+\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4+3x^2+3}}dx-\sqrt{\frac{3}{2}}\int \frac{3-\sqrt{6}x^2}{3\sqrt{2x^4+3x^2+3}}dx\right)+\frac{x(1-2x^2)}{15\sqrt{2x^4+3x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{15}\left(\frac{1}{2}\left(4+\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4+3x^2+3}}dx-\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4+3x^2+3}}dx}{\sqrt{6}}\right)+\frac{x(1-2x^2)}{15\sqrt{2x^4+3x^2+3}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{2}{15}\left(\frac{\left(4+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4+3x^2+3}}-\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4+3x^2+3}}dx}{\sqrt{6}}\right)+$$
$$\frac{x(1-2x^2)}{15\sqrt{2x^4+3x^2+3}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{2}{15}\left(\frac{\left(4+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4+3x^2+3}}-\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}E\left(\right.}{\sqrt[4]{2}\sqrt{2x}}\right.$$
$$\frac{x(1-2x^2)}{15\sqrt{2x^4+3x^2+3}}$$

input
```
Int[(3 + 3*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(1 - 2*x^2))/(15*Sqrt[3 + 3*x^2 + 2*x^4]) + (2*(-(((-3*x*Sqrt[3 + 3*x^2
 + 2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2
+ 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt
[6])/8])/(2^(1/4)*Sqrt[3 + 3*x^2 + 2*x^4]))/Sqrt[6]) + ((4 + Sqrt[6])*(3 +
 Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*Ar
cTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(4*6^(1/4)*Sqrt[3 + 3*x^2 + 2*x^4])
))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{x(2x^2-1)}{15\sqrt{2x^4+3x^2+3}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{-18+6i\sqrt{15}}}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{5\sqrt{-18+6i\sqrt{15}}\sqrt{2x^4+3x^2+3}}-\frac{24\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{1}}{6}\right.}}{}$
default	$-\frac{4\left(-\frac{1}{60}x+\frac{1}{30}x^3\right)}{\sqrt{2x^4+3x^2+3}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{-18+6i\sqrt{15}}}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{5\sqrt{-18+6i\sqrt{15}}\sqrt{2x^4+3x^2+3}}-\frac{24\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right.}}{}$
elliptic	$-\frac{4\left(-\frac{1}{60}x+\frac{1}{30}x^3\right)}{\sqrt{2x^4+3x^2+3}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{-18+6i\sqrt{15}}}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{5\sqrt{-18+6i\sqrt{15}}\sqrt{2x^4+3x^2+3}}-\frac{24\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right.}}{}$

input
```
int(1/(2*x^4+3*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*x*(2*x^2-1)/(2*x^4+3*x^2+3)^(1/2)+8/5/(-18+6*I*15^(1/2))^(1/2)*(1-(-
1/2+1/6*I*15^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*I*15^(1/2))*x^2)^(1/2)/(2*x^4+
3*x^2+3)^(1/2)*EllipticF(1/6*x*(-18+6*I*15^(1/2))^(1/2),1/2*(-1+I*15^(1/2)
)^(1/2))-24/5/(-18+6*I*15^(1/2))^(1/2)*(1-(-1/2+1/6*I*15^(1/2))*x^2)^(1/2)
*(1-(-1/2-1/6*I*15^(1/2))*x^2)^(1/2)/(2*x^4+3*x^2+3)^(1/2)/(3+I*15^(1/2))*
(EllipticF(1/6*x*(-18+6*I*15^(1/2))^(1/2),1/2*(-1+I*15^(1/2))^(1/2))-Ellip
ticE(1/6*x*(-18+6*I*15^(1/2))^(1/2),1/2*(-1+I*15^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.64

$$\int \frac{1}{(3+3x^2+2x^4)^{3/2}}\,dx = \frac{\sqrt{3}\left(2\,x^4+3\,x^2-\sqrt{-\frac{5}{3}}(2\,x^4+3\,x^2+3)+3\right)\sqrt{\frac{1}{2}\,\sqrt{-\frac{5}{3}}-\frac{1}{2}}E\left(\arcsin\left(x\sqrt{\frac{1}{2}\,\sqrt{\right.\right.}}}{}$$

input
```
integrate(1/(2*x^4+3*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/30*(sqrt(3)*(2*x^4 + 3*x^2 - sqrt(-5/3)*(2*x^4 + 3*x^2 + 3) + 3)*sqrt(1/
2*sqrt(-5/3) - 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(-5/3) - 1/2)), 3/4*s
qrt(-5/3) - 1/4) - sqrt(3)*(6*x^4 + 9*x^2 + sqrt(-5/3)*(2*x^4 + 3*x^2 + 3)
 + 9)*sqrt(1/2*sqrt(-5/3) - 1/2)*elliptic_f(arcsin(x*sqrt(1/2*sqrt(-5/3) -
 1/2)), 3/4*sqrt(-5/3) - 1/4) - 2*sqrt(2*x^4 + 3*x^2 + 3)*(2*x^3 - x))/(2*
x^4 + 3*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(3+3x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+3x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+3*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 + 3*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+3x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+3\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+3*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 3*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+3x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+3\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+3*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 3*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+3x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+3\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(3*x^2 + 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(3*x^2 + 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+3x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+3x^2+3}}{4x^8+12x^6+21x^4+18x^2+9}dx$$

input
```
int(1/(2*x^4+3*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 3*x**2 + 3)/(4*x**8 + 12*x**6 + 21*x**4 + 18*x**2 + 9),x
)
```

3.252 $\int \frac{1}{(3+2x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 254

$$\begin{aligned}\int \frac{1}{(3+2x^2+2x^4)^{3/2}}\,dx &= \frac{x(2-x^2)}{15\sqrt{3+2x^2+2x^4}}+\frac{x\sqrt{3+2x^2+2x^4}}{15\left(\sqrt{6}+2x^2\right)}\\ &\quad-\frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{5\,6^{3/4}\sqrt{3+2x^2+2x^4}}\\ &\quad+\frac{\left(1+\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{10\,6^{3/4}\sqrt{3+2x^2+2x^4}}\end{aligned}$$

output
```
1/15*x*(-x^2+2)/(2*x^4+2*x^2+3)^(1/2)+x*(2*x^4+2*x^2+3)^(1/2)/(15*6^(1/2)+
30*x^2)-1/30*(3+6^(1/2)*x^2)*((2*x^4+2*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*Ell
ipticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18-3*6^(1/2))^(1/2))*6^(1
/4)/(2*x^4+2*x^2+3)^(1/2)+1/60*(1+6^(1/2))*(3+6^(1/2)*x^2)*((2*x^4+2*x^2+3
)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x)
,1/6*(18-3*6^(1/2))^(1/2))*6^(1/4)/(2*x^4+2*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.91 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(3+2x^2+2x^4\right)^{3/2}}\,dx = \frac{-4\sqrt{-\frac{i}{-i+\sqrt{5}}}x\left(-2+x^2\right)-\sqrt{2}\left(i+\sqrt{5}\right)\sqrt{\frac{-i+\sqrt{5}-2ix^2}{-i+\sqrt{5}}}\sqrt{\frac{i+\sqrt{5}+2ix^2}{i+\sqrt{5}}}E\left(i\text{arcsinh}\left(\right.\right.}{6}$$

input
```
Integrate[(3 + 2*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(-4*Sqrt[(-I)/(-I + Sqrt[5])]*x*(-2 + x^2) - Sqrt[2]*(I + Sqrt[5])*Sqrt[(-
I + Sqrt[5] - (2*I)*x^2)/(-I + Sqrt[5])]*Sqrt[(I + Sqrt[5] + (2*I)*x^2)/(I
 + Sqrt[5])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[5])]*x], (I - Sqrt
[5])/(I + Sqrt[5])] + Sqrt[2]*(-5*I + Sqrt[5])*Sqrt[(-I + Sqrt[5] - (2*I)*
x^2)/(-I + Sqrt[5])]*Sqrt[(I + Sqrt[5] + (2*I)*x^2)/(I + Sqrt[5])]*Ellipti
cF[I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[5])]*x], (I - Sqrt[5])/(I + Sqrt[5])])
/(60*Sqrt[(-I)/(-I + Sqrt[5])]*Sqrt[3 + 2*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(2x^4+2x^2+3\right)^{3/2}}dx$$

$$\Big\downarrow \; 1405$$

$$\frac{1}{60}\int \frac{4\left(x^2+3\right)}{\sqrt{2x^4+2x^2+3}}dx+\frac{x\left(2-x^2\right)}{15\sqrt{2x^4+2x^2+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{15}\int \frac{x^2+3}{\sqrt{2x^4+2x^2+3}}dx+\frac{x(2-x^2)}{15\sqrt{2x^4+2x^2+3}}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{1}{15}\left(\frac{1}{2}\left(6+\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4+2x^2+3}}dx-\sqrt{\frac{3}{2}}\int \frac{3-\sqrt{6}x^2}{3\sqrt{2x^4+2x^2+3}}dx\right)+\frac{x(2-x^2)}{15\sqrt{2x^4+2x^2+3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{15}\left(\frac{1}{2}\left(6+\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4+2x^2+3}}dx-\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4+2x^2+3}}dx}{\sqrt{6}}\right)+\frac{x(2-x^2)}{15\sqrt{2x^4+2x^2+3}}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{1}{15}\left(\frac{\left(6+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4+2x^2+3}}-\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4+2x^2+3}}dx}{\sqrt{6}}\right)+\frac{x(2-x^2)}{15\sqrt{2x^4+2x^2+3}}$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{1}{15}\left(\frac{\left(6+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4+2x^2+3}}-\frac{\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}E}{\sqrt[4]{2}\sqrt{2}}}{}\right.$$

$$\frac{x(2-x^2)}{15\sqrt{2x^4+2x^2+3}}$$

input

```
Int[(3 + 2*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(2 - x^2))/(15*Sqrt[3 + 2*x^2 + 2*x^4]) + (-(((-3*x*Sqrt[3 + 2*x^2 + 2*
x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x
^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/
12])/(2^(1/4)*Sqrt[3 + 2*x^2 + 2*x^4]))/Sqrt[6]) + ((6 + Sqrt[6])*(3 + Sqr
t[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(4*6^(1/4)*Sqrt[3 + 2*x^2 + 2*x^4]))/1
5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{x\left(x^2-2\right)}{15\sqrt{2x^4+2x^2+3}}+\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3+3i\sqrt{5}}}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{5\sqrt{-3+3i\sqrt{5}}\,\sqrt{2x^4+2x^2+3}}-\frac{6\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}}{}$
default	$-\frac{4\left(-\frac{1}{30}x+\frac{1}{60}x^3\right)}{\sqrt{2x^4+2x^2+3}}+\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3+3i\sqrt{5}}}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{5\sqrt{-3+3i\sqrt{5}}\,\sqrt{2x^4+2x^2+3}}-\frac{6\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}}{}$
elliptic	$-\frac{4\left(-\frac{1}{30}x+\frac{1}{60}x^3\right)}{\sqrt{2x^4+2x^2+3}}+\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3+3i\sqrt{5}}}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{5\sqrt{-3+3i\sqrt{5}}\,\sqrt{2x^4+2x^2+3}}-\frac{6\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}}{}$

input
```
int(1/(2*x^4+2*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*x*(x^2-2)/(2*x^4+2*x^2+3)^(1/2)+3/5/(-3+3*I*5^(1/2))^(1/2)*(1-(-1/3+
1/3*I*5^(1/2))*x^2)^(1/2)*(1-(-1/3-1/3*I*5^(1/2))*x^2)^(1/2)/(2*x^4+2*x^2+
3)^(1/2)*EllipticF(1/3*x*(-3+3*I*5^(1/2))^(1/2),1/3*(-6+3*I*5^(1/2))^(1/2)
)-6/5/(-3+3*I*5^(1/2))^(1/2)*(1-(-1/3+1/3*I*5^(1/2))*x^2)^(1/2)*(1-(-1/3-1
/3*I*5^(1/2))*x^2)^(1/2)/(2*x^4+2*x^2+3)^(1/2)/(2+2*I*5^(1/2))*(EllipticF(
1/3*x*(-3+3*I*5^(1/2))^(1/2),1/3*(-6+3*I*5^(1/2))^(1/2))-EllipticE(1/3*x*(
-3+3*I*5^(1/2))^(1/2),1/3*(-6+3*I*5^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.65

$$\int \frac{1}{(3+2x^2+2x^4)^{3/2}}\,dx = \frac{\sqrt{3}\left(2\,x^4+2\,x^2-\sqrt{-5}(2\,x^4+2\,x^2+3)+3\right)\sqrt{\frac{1}{3}\,\sqrt{-5}-\frac{1}{3}}E\left(\arcsin\left(x\sqrt{\frac{1}{3}\,\sqrt{-}\right.\right.}{}$$

input
```
integrate(1/(2*x^4+2*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/90*(sqrt(3)*(2*x^4 + 2*x^2 - sqrt(-5)*(2*x^4 + 2*x^2 + 3) + 3)*sqrt(1/3*
sqrt(-5) - 1/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(-5) - 1/3)), 1/3*sqrt(-5
) - 2/3) - 2*sqrt(3)*(4*x^4 + 4*x^2 + sqrt(-5)*(2*x^4 + 2*x^2 + 3) + 6)*sq
rt(1/3*sqrt(-5) - 1/3)*elliptic_f(arcsin(x*sqrt(1/3*sqrt(-5) - 1/3)), 1/3*
sqrt(-5) - 2/3) - 6*sqrt(2*x^4 + 2*x^2 + 3)*(x^3 - 2*x))/(2*x^4 + 2*x^2 +
3)
```

Sympy [F]

$$\int \frac{1}{(3+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+2x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+2*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 + 2*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+2\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+2*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 2*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+2\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+2*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 2*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+2\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(2*x^2 + 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(2*x^2 + 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+2x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+2x^2+3}}{4x^8+8x^6+16x^4+12x^2+9}dx$$

input
```
int(1/(2*x^4+2*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 2*x**2 + 3)/(4*x**8 + 8*x**6 + 16*x**4 + 12*x**2 + 9),x)
```

3.253 $\int \frac{1}{(3+x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 249

$$\int \frac{1}{(3+x^2+2x^4)^{3/2}}\,dx = \frac{x(11-2x^2)}{69\sqrt{3+x^2+2x^4}}+\frac{2x\sqrt{3+x^2+2x^4}}{69\left(\sqrt{6}+2x^2\right)} - \frac{\sqrt[4]{2}(3+\sqrt{6}x^2)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|\frac{1}{24}(12-\sqrt{6})\right)}{23\,3^{3/4}\sqrt{3+x^2+2x^4}} + \frac{\left(1+2\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}(12-\sqrt{6})\right)}{23\,6^{3/4}\sqrt{3+x^2+2x^4}}$$

output
```
1/69*x*(-2*x^2+11)/(2*x^4+x^2+3)^(1/2)+2*x*(2*x^4+x^2+3)^(1/2)/(69*6^(1/2)
+138*x^2)-1/69*2^(1/4)*(3+6^(1/2)*x^2)*((2*x^4+x^2+3)/(3+6^(1/2)*x^2)^2)^(
1/2)*EllipticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72-6*6^(1/2))^(1
/2))*3^(1/4)/(2*x^4+x^2+3)^(1/2)+1/138*(1+2*6^(1/2))*(3+6^(1/2)*x^2)*((2*x
^4+x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4)*3^
(3/4)*x),1/12*(72-6*6^(1/2))^(1/2))*6^(1/4)/(2*x^4+x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.90 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.28

$$\int \frac{1}{(3+x^2+2x^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{-i+\sqrt{23}}}x(11-2x^2)-\left(i+\sqrt{23}\right)\sqrt{\frac{-i+\sqrt{23}-4ix^2}{-i+\sqrt{23}}}\sqrt{\frac{i+\sqrt{23}+4ix^2}{i+\sqrt{23}}}E\left(i\text{arcsinh}\left(2\sqrt{\right.\right.}{}$$

input
```
Integrate[(3 + x^2 + 2*x^4)^(-3/2),x]
```

output
```
(4*Sqrt[(-I)/(-I + Sqrt[23])]*x*(11 - 2*x^2) - (I + Sqrt[23])*Sqrt[(-I + S
qrt[23] - (4*I)*x^2)/(-I + Sqrt[23])]*Sqrt[(I + Sqrt[23] + (4*I)*x^2)/(I +
 Sqrt[23])]*EllipticE[I*ArcSinh[2*Sqrt[(-I)/(-I + Sqrt[23])]*x], (I - Sqrt
[23])/(I + Sqrt[23])] + (-23*I + Sqrt[23])*Sqrt[(-I + Sqrt[23] - (4*I)*x^2
)/(-I + Sqrt[23])]*Sqrt[(I + Sqrt[23] + (4*I)*x^2)/(I + Sqrt[23])]*Ellipti
cF[I*ArcSinh[2*Sqrt[(-I)/(-I + Sqrt[23])]*x], (I - Sqrt[23])/(I + Sqrt[23]
)])/(276*Sqrt[(-I)/(-I + Sqrt[23])]*Sqrt[3 + x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+x^2+3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{69}\int \frac{2(x^2+6)}{\sqrt{2x^4+x^2+3}}dx+\frac{x(11-2x^2)}{69\sqrt{2x^4+x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{69}\int \frac{x^2+6}{\sqrt{2x^4+x^2+3}}dx+\frac{x(11-2x^2)}{69\sqrt{2x^4+x^2+3}}$$

$\Big\downarrow$ 1511

$$\frac{2}{69}\left(\frac{1}{2}\left(12+\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4+x^2+3}}dx-\sqrt{\frac{3}{2}}\int \frac{3-\sqrt{6}x^2}{3\sqrt{2x^4+x^2+3}}dx\right)+\frac{x(11-2x^2)}{69\sqrt{2x^4+x^2+3}}$$

$\Big\downarrow$ 27

$$\frac{2}{69}\left(\frac{1}{2}\left(12+\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4+x^2+3}}dx-\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4+x^2+3}}dx}{\sqrt{6}}\right)+\frac{x(11-2x^2)}{69\sqrt{2x^4+x^2+3}}$$

$\Big\downarrow$ 1416

$$\frac{2}{69}\left(\frac{\left(12+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4+x^2+3}}-\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4+x^2+3}}dx}{\sqrt{6}}\right)+$$

$$\frac{x(11-2x^2)}{69\sqrt{2x^4+x^2+3}}$$

$\Big\downarrow$ 1509

$$\frac{2}{69}\left(\frac{\left(12+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4+x^2+3}}-\frac{\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)}}}{\sqrt[4]{}}}{}\right.$$

$$\frac{x(11-2x^2)}{69\sqrt{2x^4+x^2+3}}$$

input
```
Int[(3 + x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(11 - 2*x^2))/(69*Sqrt[3 + x^2 + 2*x^4]) + (2*(-(((-3*x*Sqrt[3 + x^2 +
2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x
^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])
/24])/(2^(1/4)*Sqrt[3 + x^2 + 2*x^4]))/Sqrt[6]) + ((12 + Sqrt[6])*(3 + Sqr
t[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan[(
2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(4*6^(1/4)*Sqrt[3 + x^2 + 2*x^4])))/69
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{x(2x^2-11)}{69\sqrt{2x^4+x^2+3}}+\frac{24\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+6i\sqrt{23}}}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{23\sqrt{-6+6i\sqrt{23}}\,\sqrt{2x^4+x^2+3}}-\frac{24\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{}}{6}\right.}}{}$
default	$-\frac{4\left(-\frac{11}{276}x+\frac{1}{138}x^3\right)}{\sqrt{2x^4+x^2+3}}+\frac{24\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+6i\sqrt{23}}}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{23\sqrt{-6+6i\sqrt{23}}\,\sqrt{2x^4+x^2+3}}-\frac{24\sqrt{1-\left(-\frac{1}{6}+\right.}}{}$
elliptic	$-\frac{4\left(-\frac{11}{276}x+\frac{1}{138}x^3\right)}{\sqrt{2x^4+x^2+3}}+\frac{24\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+6i\sqrt{23}}}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{23\sqrt{-6+6i\sqrt{23}}\,\sqrt{2x^4+x^2+3}}-\frac{24\sqrt{1-\left(-\frac{1}{6}+\right.}}{}$

input
```
int(1/(2*x^4+x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/69*x*(2*x^2-11)/(2*x^4+x^2+3)^(1/2)+24/23/(-6+6*I*23^(1/2))^(1/2)*(1-(-
1/6+1/6*I*23^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*I*23^(1/2))*x^2)^(1/2)/(2*x^4+
x^2+3)^(1/2)*EllipticF(1/6*x*(-6+6*I*23^(1/2))^(1/2),1/6*(-33+3*I*23^(1/2)
)^(1/2))-24/23/(-6+6*I*23^(1/2))^(1/2)*(1-(-1/6+1/6*I*23^(1/2))*x^2)^(1/2)
*(1-(-1/6-1/6*I*23^(1/2))*x^2)^(1/2)/(2*x^4+x^2+3)^(1/2)/(1+I*23^(1/2))*(E
llipticF(1/6*x*(-6+6*I*23^(1/2))^(1/2),1/6*(-33+3*I*23^(1/2))^(1/2))-Ellip
ticE(1/6*x*(-6+6*I*23^(1/2))^(1/2),1/6*(-33+3*I*23^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.63

$$\int \frac{1}{(3+x^2+2x^4)^{3/2}}\,dx = \frac{\sqrt{3}\left(2\,x^4+x^2-\sqrt{-23}(2\,x^4+x^2+3)+3\right)\sqrt{\frac{1}{6}\,\sqrt{-23}-\frac{1}{6}}E\left(\arcsin\left(x\sqrt{\frac{1}{6}\,\sqrt{-23}\right.\right.}{}$$

input
```
integrate(1/(2*x^4+x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/414*(sqrt(3)*(2*x^4 + x^2 - sqrt(-23)*(2*x^4 + x^2 + 3) + 3)*sqrt(1/6*sq
rt(-23) - 1/6)*elliptic_e(arcsin(x*sqrt(1/6*sqrt(-23) - 1/6)), 1/12*sqrt(-
23) - 11/12) - sqrt(3)*(14*x^4 + 7*x^2 + 5*sqrt(-23)*(2*x^4 + x^2 + 3) + 2
1)*sqrt(1/6*sqrt(-23) - 1/6)*elliptic_f(arcsin(x*sqrt(1/6*sqrt(-23) - 1/6)
), 1/12*sqrt(-23) - 11/12) - 6*sqrt(2*x^4 + x^2 + 3)*(2*x^3 - 11*x))/(2*x^
4 + x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(3+x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 + x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3+x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+x^2+3)^{3/2}}\,dx$$

input
```
int(1/(x^2 + 2*x^4 + 3)^(3/2),x)
```

output
```
int(1/(x^2 + 2*x^4 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3+x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+x^2+3}}{4x^8+4x^6+13x^4+6x^2+9}dx$$

input
```
int(1/(2*x^4+x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + x**2 + 3)/(4*x**8 + 4*x**6 + 13*x**4 + 6*x**2 + 9),x)
```

3.254 $\int \frac{1}{(3+2x^4)^{3/2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 89

$$\int \frac{1}{(3+2x^4)^{3/2}} dx = \frac{x}{6\sqrt{3+2x^4}} + \frac{(3+\sqrt{6}x^2)\sqrt{\frac{3+2x^4}{(3+\sqrt{6}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}\right)}{12\sqrt[4]{6}\sqrt{3+2x^4}}$$

output
```
1/6*x/(2*x^4+3)^(1/2)+1/72*(3+6^(1/2)*x^2)*((2*x^4+3)/(3+6^(1/2)*x^2)^2)^(
1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/2*2^(1/2))*6^(3/4)/
(2*x^4+3)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.99 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.49

$$\int \frac{1}{(3+2x^4)^{3/2}} dx = \frac{x}{6\sqrt{3+2x^4}} + \frac{x \operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{2x^4}{3}\right)}{6\sqrt{3}}$$

input
```
Integrate[(3 + 2*x^4)^(-3/2),x]
```

output
```
x/(6*Sqrt[3 + 2*x^4]) + (x*Hypergeometric2F1[1/4, 1/2, 5/4, (-2*x^4)/3])/(
6*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+3)^{3/2}}\,dx$$

$$\Big\downarrow \ 749$$

$$\frac{1}{6}\int \frac{1}{\sqrt{2x^4+3}}dx+\frac{x}{6\sqrt{2x^4+3}}$$

$$\Big\downarrow \ 761$$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}\right)}{12\sqrt[4]{6}\sqrt{2x^4+3}}+\frac{x}{6\sqrt{2x^4+3}}$$

input
```
Int[(3 + 2*x^4)^(-3/2),x]
```

output
```
x/(6*Sqrt[3 + 2*x^4]) + ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x
^2)^2]*EllipticF[2*ArcTan[(2/3)^(1/4)*x], 1/2])/(12*6^(1/4)*Sqrt[3 + 2*x^4
])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.20

method	result	size
meijerg	$\frac{\sqrt{3}\, x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right], \left[\frac{5}{4}\right], -\frac{2 x^{4}}{3}\right)}{9}$	18
default	$\frac{x}{6 \sqrt{2 x^{4}+3}}+\frac{\sqrt{3}\, \sqrt{9-3 i \sqrt{6}\, x^{2}}\, \sqrt{9+3 i \sqrt{6}\, x^{2}}\, \operatorname{EllipticF}\left(\frac{x \sqrt{3}\, \sqrt{i \sqrt{6}}}{3}, i\right)}{54 \sqrt{i \sqrt{6}}\, \sqrt{2 x^{4}+3}}$	79
risch	$\frac{x}{6 \sqrt{2 x^{4}+3}}+\frac{\sqrt{3}\, \sqrt{9-3 i \sqrt{6}\, x^{2}}\, \sqrt{9+3 i \sqrt{6}\, x^{2}}\, \operatorname{EllipticF}\left(\frac{x \sqrt{3}\, \sqrt{i \sqrt{6}}}{3}, i\right)}{54 \sqrt{i \sqrt{6}}\, \sqrt{2 x^{4}+3}}$	79
elliptic	$\frac{x}{6 \sqrt{2 x^{4}+3}}+\frac{\sqrt{3}\, \sqrt{9-3 i \sqrt{6}\, x^{2}}\, \sqrt{9+3 i \sqrt{6}\, x^{2}}\, \operatorname{EllipticF}\left(\frac{x \sqrt{3}\, \sqrt{i \sqrt{6}}}{3}, i\right)}{54 \sqrt{i \sqrt{6}}\, \sqrt{2 x^{4}+3}}$	79

input
```
int(1/(2*x^4+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*3^(1/2)*x*hypergeom([1/4,3/2],[5/4],-2/3*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{1}{(3+2x^4)^{3/2}}\,dx = -\frac{\sqrt{3}\left(-\frac{2}{3}\right)^{\frac{3}{4}}(2\,x^4+3)F(\arcsin\left(\left(-\frac{2}{3}\right)^{\frac{1}{4}}x\right)\,|-1)-2\,\sqrt{2\,x^4+3}x}{12\,(2\,x^4+3)}$$

input
```
integrate(1/(2*x^4+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(3)*(-2/3)^(3/4)*(2*x^4 + 3)*elliptic_f(arcsin((-2/3)^(1/4)*x),
 -1) - 2*sqrt(2*x^4 + 3)*x)/(2*x^4 + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

$$\int \frac{1}{(3+2x^4)^{3/2}}\,dx = \frac{\sqrt{3}x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{2x^4e^{i\pi}}{3}\right)}{36\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(2*x**4+3)**(3/2),x)
```

output
```
sqrt(3)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), 2*x**4*exp_polar(I*pi)/3)/(
36*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(3+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 3)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 19.74 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.18

$$\int \frac{1}{(3+2x^4)^{3/2}}\,dx = \frac{\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{2\,x^4}{3}\right)}{9}$$

input
```
int(1/(2*x^4 + 3)^(3/2),x)
```

output
```
(3^(1/2)*x*hypergeom([1/4, 3/2], 5/4, -(2*x^4)/3))/9
```

Reduce [F]

$$\int \frac{1}{(3+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+3}}{4x^8+12x^4+9}dx$$

input
```
int(1/(2*x^4+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 3)/(4*x**8 + 12*x**4 + 9),x)
```

3.255 $\int \frac{1}{(3-x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 257

$$\int \frac{1}{(3-x^2+2x^4)^{3/2}}\,dx=\frac{x(11+2x^2)}{69\sqrt{3-x^2+2x^4}}-\frac{2x\sqrt{3-x^2+2x^4}}{69\left(\sqrt{6}+2x^2\right)}+\frac{\sqrt[4]{2}(3+\sqrt{6}x^2)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|\frac{1}{24}(12+\sqrt{6})\right)}{23\ 3^{3/4}\sqrt{3-x^2+2x^4}}-\frac{(1-2\sqrt{6})\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}(12+\sqrt{6})\right)}{23\ 6^{3/4}\sqrt{3-x^2+2x^4}}$$

output
```
1/69*x*(2*x^2+11)/(2*x^4-x^2+3)^(1/2)-2*x*(2*x^4-x^2+3)^(1/2)/(69*6^(1/2)+
138*x^2)+1/69*2^(1/4)*(3+6^(1/2)*x^2)*((2*x^4-x^2+3)/(3+6^(1/2)*x^2)^2)^(1
/2)*EllipticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72+6*6^(1/2))^(1/
2))*3^(1/4)/(2*x^4-x^2+3)^(1/2)-1/138*(1-2*6^(1/2))*(3+6^(1/2)*x^2)*((2*x^
4-x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4)*3^(
3/4)*x),1/12*(72+6*6^(1/2))^(1/2))*6^(1/4)/(2*x^4-x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.99 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.25

$$\int \frac{1}{(3-x^2+2x^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{i+\sqrt{23}}}x(11+2x^2)+\left(-i+\sqrt{23}\right)\sqrt{\frac{i+\sqrt{23}-4ix^2}{i+\sqrt{23}}}\sqrt{\frac{-i+\sqrt{23}+4ix^2}{-i+\sqrt{23}}}E\left(i\operatorname{arcsinh}\left(2\right.\right.}{2}$$

input
```
Integrate[(3 - x^2 + 2*x^4)^(-3/2),x]
```

output
```
(4*Sqrt[(-I)/(I + Sqrt[23])]*x*(11 + 2*x^2) + (-I + Sqrt[23])*Sqrt[(I + Sq
rt[23] - (4*I)*x^2)/(I + Sqrt[23])]*Sqrt[(-I + Sqrt[23] + (4*I)*x^2)/(-I +
 Sqrt[23])]*EllipticE[I*ArcSinh[2*Sqrt[(-I)/(I + Sqrt[23])]*x], (I + Sqrt[
23])/(I - Sqrt[23])] - (23*I + Sqrt[23])*Sqrt[(I + Sqrt[23] - (4*I)*x^2)/(
I + Sqrt[23])]*Sqrt[(-I + Sqrt[23] + (4*I)*x^2)/(-I + Sqrt[23])]*EllipticF
[I*ArcSinh[2*Sqrt[(-I)/(I + Sqrt[23])]*x], (I + Sqrt[23])/(I - Sqrt[23])])
/(276*Sqrt[(-I)/(I + Sqrt[23])]*Sqrt[3 - x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{69}\int \frac{2(6-x^2)}{\sqrt{2x^4-x^2+3}}dx+\frac{x(2x^2+11)}{69\sqrt{2x^4-x^2+3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{69}\int \frac{6-x^2}{\sqrt{2x^4-x^2+3}}dx+\frac{x(2x^2+11)}{69\sqrt{2x^4-x^2+3}}$$

$$\Big\downarrow \; 1511$$

$$\frac{2}{69}\left(\frac{1}{2}\left(12-\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4-x^2+3}}dx+\sqrt{\frac{3}{2}}\int \frac{3-\sqrt{6}x^2}{3\sqrt{2x^4-x^2+3}}dx\right)+\frac{x(2x^2+11)}{69\sqrt{2x^4-x^2+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{69}\left(\frac{1}{2}\left(12-\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4-x^2+3}}dx+\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4-x^2+3}}dx}{\sqrt{6}}\right)+\frac{x(2x^2+11)}{69\sqrt{2x^4-x^2+3}}$$

$$\Big\downarrow \; 1416$$

$$\frac{2}{69}\left(\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4-x^2+3}}dx}{\sqrt{6}}+\frac{\left(12-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4-x^2+3}}\right)+$$
$$\frac{x(2x^2+11)}{69\sqrt{2x^4-x^2+3}}$$

$$\Big\downarrow \; 1509$$

$$\frac{2}{69}\left(\frac{\left(12-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4-x^2+3}}+\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)}}}{\sqrt[4]{}}\right.$$
$$\frac{x(2x^2+11)}{69\sqrt{2x^4-x^2+3}}$$

input
```
Int[(3 - x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(11 + 2*x^2))/(69*Sqrt[3 - x^2 + 2*x^4]) + (2*(((-3*x*Sqrt[3 - x^2 + 2*
x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4
)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/2
4])/(2^(1/4)*Sqrt[3 - x^2 + 2*x^4]))/Sqrt[6] + ((12 - Sqrt[6])*(3 + Sqrt[6
]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan[(2/3
)^(1/4)*x], (12 + Sqrt[6])/24])/(4*6^(1/4)*Sqrt[3 - x^2 + 2*x^4])))/69
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.92

method	result
risch	$\frac{x(2x^2+11)}{69\sqrt{2x^4-x^2+3}}+\frac{24\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6+6i\sqrt{23}}}{6},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{23\sqrt{6+6i\sqrt{23}}\,\sqrt{2x^4-x^2+3}}+\frac{24\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1}}{}$
default	$-\frac{4\left(-\frac{11}{276}x-\frac{1}{138}x^3\right)}{\sqrt{2x^4-x^2+3}}+\frac{24\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6+6i\sqrt{23}}}{6},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{23\sqrt{6+6i\sqrt{23}}\,\sqrt{2x^4-x^2+3}}+\frac{24\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}}{}$
elliptic	$-\frac{4\left(-\frac{11}{276}x-\frac{1}{138}x^3\right)}{\sqrt{2x^4-x^2+3}}+\frac{24\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6+6i\sqrt{23}}}{6},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{23\sqrt{6+6i\sqrt{23}}\,\sqrt{2x^4-x^2+3}}+\frac{24\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}}{}$

input
```
int(1/(2*x^4-x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/69*x*(2*x^2+11)/(2*x^4-x^2+3)^(1/2)+24/23/(6+6*I*23^(1/2))^(1/2)*(1-(1/6
+1/6*I*23^(1/2))*x^2)^(1/2)*(1-(1/6-1/6*I*23^(1/2))*x^2)^(1/2)/(2*x^4-x^2+
3)^(1/2)*EllipticF(1/6*x*(6+6*I*23^(1/2))^(1/2),1/6*(-33-3*I*23^(1/2))^(1/
2))+24/23/(6+6*I*23^(1/2))^(1/2)*(1-(1/6+1/6*I*23^(1/2))*x^2)^(1/2)*(1-(1/
6-1/6*I*23^(1/2))*x^2)^(1/2)/(2*x^4-x^2+3)^(1/2)/(-1+I*23^(1/2))*(Elliptic
F(1/6*x*(6+6*I*23^(1/2))^(1/2),1/6*(-33-3*I*23^(1/2))^(1/2))-EllipticE(1/6
*x*(6+6*I*23^(1/2))^(1/2),1/6*(-33-3*I*23^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.64

$$\int \frac{1}{(3-x^2+2x^4)^{3/2}}\,dx = \frac{\sqrt{3}(2\,x^4-x^2+\sqrt{-23}(2\,x^4-x^2+3)+3)\sqrt{\frac{1}{6}\,\sqrt{-23}+\frac{1}{6}}E(\arcsin\left(x\sqrt{\frac{1}{6}\,\sqrt{-23}\right.}$$

input
```
integrate(1/(2*x^4-x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/414*(sqrt(3)*(2*x^4 - x^2 + sqrt(-23)*(2*x^4 - x^2 + 3) + 3)*sqrt(1/6*sq
rt(-23) + 1/6)*elliptic_e(arcsin(x*sqrt(1/6*sqrt(-23) + 1/6)), -1/12*sqrt(
-23) - 11/12) + sqrt(3)*(10*x^4 - 5*x^2 - 7*sqrt(-23)*(2*x^4 - x^2 + 3) +
15)*sqrt(1/6*sqrt(-23) + 1/6)*elliptic_f(arcsin(x*sqrt(1/6*sqrt(-23) + 1/6
)), -1/12*sqrt(-23) - 11/12) + 6*sqrt(2*x^4 - x^2 + 3)*(2*x^3 + 11*x))/(2*
x^4 - x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(3-x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4-x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 - x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-x^2+2x^4)^{3/2}}\,dx=\int \frac{1}{(2\,x^4-x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x^2+2x^4)^{3/2}}\,dx=\int \frac{1}{(2\,x^4-x^2+3)^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - x^2 + 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - x^2 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-x^2+2x^4)^{3/2}}\,dx=\int \frac{\sqrt{2x^4-x^2+3}}{4x^8-4x^6+13x^4-6x^2+9}dx$$

input
```
int(1/(2*x^4-x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - x**2 + 3)/(4*x**8 - 4*x**6 + 13*x**4 - 6*x**2 + 9),x)
```

3.256 $\int \frac{1}{(3-2x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 250

$$\int \frac{1}{(3-2x^2+2x^4)^{3/2}}\,dx = \frac{x(2+x^2)}{15\sqrt{3-2x^2+2x^4}} - \frac{x\sqrt{3-2x^2+2x^4}}{15\left(\sqrt{6}+2x^2\right)} + \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{5\ 6^{3/4}\sqrt{3-2x^2+2x^4}} - \frac{\left(1-\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{10\ 6^{3/4}\sqrt{3-2x^2+2x^4}}$$

output
```
1/15*x*(x^2+2)/(2*x^4-2*x^2+3)^(1/2)-x*(2*x^4-2*x^2+3)^(1/2)/(15*6^(1/2)+3
0*x^2)+1/30*(3+6^(1/2)*x^2)*((2*x^4-2*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*Elli
pticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18+3*6^(1/2))^(1/2))*6^(1/
4)/(2*x^4-2*x^2+3)^(1/2)-1/60*(1-6^(1/2))*(3+6^(1/2)*x^2)*((2*x^4-2*x^2+3)
/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),
1/6*(18+3*6^(1/2))^(1/2))*6^(1/4)/(2*x^4-2*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.05 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.30

$$\int \frac{1}{(3-2x^2+2x^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{i+\sqrt{5}}}x(2+x^2)+\sqrt{2}(-i+\sqrt{5})\sqrt{\frac{i+\sqrt{5}-2ix^2}{i+\sqrt{5}}}\sqrt{\frac{-i+\sqrt{5}+2ix^2}{-i+\sqrt{5}}}E\left(i\text{arcsinh}\left(\sqrt{-\right.\right.}{60\sqrt{}}$$

input
```
Integrate[(3 - 2*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(4*Sqrt[(-I)/(I + Sqrt[5])]*x*(2 + x^2) + Sqrt[2]*(-I + Sqrt[5])*Sqrt[(I +
 Sqrt[5] - (2*I)*x^2)/(I + Sqrt[5])]*Sqrt[(-I + Sqrt[5] + (2*I)*x^2)/(-I +
 Sqrt[5])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(I + Sqrt[5])]*x], (I + Sqrt[5]
)/(I - Sqrt[5])] - Sqrt[2]*(5*I + Sqrt[5])*Sqrt[(I + Sqrt[5] - (2*I)*x^2)/
(I + Sqrt[5])]*Sqrt[(-I + Sqrt[5] + (2*I)*x^2)/(-I + Sqrt[5])]*EllipticF[I
*ArcSinh[Sqrt[(-2*I)/(I + Sqrt[5])]*x], (I + Sqrt[5])/(I - Sqrt[5])])/(60*
Sqrt[(-I)/(I + Sqrt[5])]*Sqrt[3 - 2*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-2x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{60}\int \frac{4(3-x^2)}{\sqrt{2x^4-2x^2+3}}dx+\frac{x(x^2+2)}{15\sqrt{2x^4-2x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\int \frac{3-x^2}{\sqrt{2x^4-2x^2+3}}dx+\frac{x(x^2+2)}{15\sqrt{2x^4-2x^2+3}}$$

$\Big\downarrow$ 1511

$$\frac{1}{15}\left(\frac{1}{2}\left(6-\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4-2x^2+3}}dx+\sqrt{\frac{3}{2}}\int \frac{3-\sqrt{6}x^2}{3\sqrt{2x^4-2x^2+3}}dx\right)+\frac{x(x^2+2)}{15\sqrt{2x^4-2x^2+3}}$$

$\Big\downarrow$ 27

$$\frac{1}{15}\left(\frac{1}{2}\left(6-\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4-2x^2+3}}dx+\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4-2x^2+3}}dx}{\sqrt{6}}\right)+\frac{x(x^2+2)}{15\sqrt{2x^4-2x^2+3}}$$

$\Big\downarrow$ 1416

$$\frac{1}{15}\left(\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4-2x^2+3}}dx}{\sqrt{6}}+\frac{\left(6-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4-2x^2+3}}\right)+$$

$$\frac{x(x^2+2)}{15\sqrt{2x^4-2x^2+3}}$$

$\Big\downarrow$ 1509

$$\frac{1}{15}\left(\frac{\left(6-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4-2x^2+3}}+\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}E}{\sqrt[4]{2}\sqrt{2}}\right.$$

$$\frac{x(x^2+2)}{15\sqrt{2x^4-2x^2+3}}$$

input
```
Int[(3 - 2*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(2 + x^2))/(15*Sqrt[3 - 2*x^2 + 2*x^4]) + (((-3*x*x*Sqrt[3 - 2*x^2 + 2*x^
4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4
)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12
])/(2^(1/4)*Sqrt[3 - 2*x^2 + 2*x^4]))/Sqrt[6] + ((6 - Sqrt[6])*(3 + Sqrt[6
]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan[(2
/3)^(1/4)*x], (6 + Sqrt[6])/12])/(4*6^(1/4)*Sqrt[3 - 2*x^2 + 2*x^4]))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.94

method	result
risch	$\frac{x(x^2+2)}{15\sqrt{2x^4-2x^2+3}}+\frac{3\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+3i\sqrt{5}}}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{5\sqrt{3+3i\sqrt{5}}\sqrt{2x^4-2x^2+3}}+\frac{6\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(\frac{1}{3}-\right.}}{}$
default	$-\frac{4\left(-\frac{1}{30}x-\frac{1}{60}x^3\right)}{\sqrt{2x^4-2x^2+3}}+\frac{3\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+3i\sqrt{5}}}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{5\sqrt{3+3i\sqrt{5}}\sqrt{2x^4-2x^2+3}}+\frac{6\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(\frac{1}{3}\right.}}{}$
elliptic	$-\frac{4\left(-\frac{1}{30}x-\frac{1}{60}x^3\right)}{\sqrt{2x^4-2x^2+3}}+\frac{3\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+3i\sqrt{5}}}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{5\sqrt{3+3i\sqrt{5}}\sqrt{2x^4-2x^2+3}}+\frac{6\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(\frac{1}{3}\right.}}{}$

input
```
int(1/(2*x^4-2*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(x^2+2)/(2*x^4-2*x^2+3)^(1/2)+3/5/(3+3*I*5^(1/2))^(1/2)*(1-(1/3+1/3
*I*5^(1/2))*x^2)^(1/2)*(1-(1/3-1/3*I*5^(1/2))*x^2)^(1/2)/(2*x^4-2*x^2+3)^(
1/2)*EllipticF(1/3*x*(3+3*I*5^(1/2))^(1/2),1/3*(-6-3*I*5^(1/2))^(1/2))+6/5
/(3+3*I*5^(1/2))^(1/2)*(1-(1/3+1/3*I*5^(1/2))*x^2)^(1/2)*(1-(1/3-1/3*I*5^(
1/2))*x^2)^(1/2)/(2*x^4-2*x^2+3)^(1/2)/(-2+2*I*5^(1/2))*(EllipticF(1/3*x*(
3+3*I*5^(1/2))^(1/2),1/3*(-6-3*I*5^(1/2))^(1/2))-EllipticE(1/3*x*(3+3*I*5^
(1/2))^(1/2),1/3*(-6-3*I*5^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.66

$$\int \frac{1}{(3-2x^2+2x^4)^{3/2}}\,dx = \frac{\sqrt{3}\left(2\,x^4-2\,x^2+\sqrt{-5}(2\,x^4-2\,x^2+3)+3\right)\sqrt{\frac{1}{3}\sqrt{-5}+\frac{1}{3}}E\left(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{-}\right.\right.}{}$$

input
```
integrate(1/(2*x^4-2*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/90*(sqrt(3)*(2*x^4 - 2*x^2 + sqrt(-5)*(2*x^4 - 2*x^2 + 3) + 3)*sqrt(1/3*
sqrt(-5) + 1/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(-5) + 1/3)), -1/3*sqrt(-
5) - 2/3) + 2*sqrt(3)*(2*x^4 - 2*x^2 - 2*sqrt(-5)*(2*x^4 - 2*x^2 + 3) + 3)
*sqrt(1/3*sqrt(-5) + 1/3)*elliptic_f(arcsin(x*sqrt(1/3*sqrt(-5) + 1/3)), -
1/3*sqrt(-5) - 2/3) + 6*sqrt(2*x^4 - 2*x^2 + 3)*(x^3 + 2*x))/(2*x^4 - 2*x^
2 + 3)
```

Sympy [F]

$$\int \frac{1}{(3-2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4-2x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-2*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 - 2*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-2\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-2*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - 2*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-2\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-2*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - 2*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-2\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - 2*x^2 + 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - 2*x^2 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-2x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4-2x^2+3}}{4x^8-8x^6+16x^4-12x^2+9}dx$$

input
```
int(1/(2*x^4-2*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - 2*x**2 + 3)/(4*x**8 - 8*x**6 + 16*x**4 - 12*x**2 + 9),x)
```

3.257 $\int \frac{1}{(3-3x^2+2x^4)^{3/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 257

$$\int \frac{1}{(3-3x^2+2x^4)^{3/2}}\,dx=\frac{x(1+2x^2)}{15\sqrt{3-3x^2+2x^4}}-\frac{2x\sqrt{3-3x^2+2x^4}}{15\left(\sqrt{6}+2x^2\right)}+\frac{\sqrt[4]{2}(3+\sqrt{6}x^2)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}(4+\sqrt{6})\right)}{5\ 3^{3/4}\sqrt{3-3x^2+2x^4}}-\frac{\left(3-2\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}(4+\sqrt{6})\right)}{15\ 6^{3/4}\sqrt{3-3x^2+2x^4}}$$

output
```
1/15*x*(2*x^2+1)/(2*x^4-3*x^2+3)^(1/2)-2*x*(2*x^4-3*x^2+3)^(1/2)/(15*6^(1/
2)+30*x^2)+1/15*2^(1/4)*(3+6^(1/2)*x^2)*((2*x^4-3*x^2+3)/(3+6^(1/2)*x^2)^2
)^(1/2)*EllipticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/4*(8+2*6^(1/2))^(
1/2))*3^(1/4)/(2*x^4-3*x^2+3)^(1/2)-1/90*(3-2*6^(1/2))*(3+6^(1/2)*x^2)*((2
*x^4-3*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4
)*3^(3/4)*x),1/4*(8+2*6^(1/2))^(1/2))*6^(1/4)/(2*x^4-3*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.15 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.25

$$\int \frac{1}{(3-3x^2+2x^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{3i+\sqrt{15}}}x(1+2x^2)+\left(-3i+\sqrt{15}\right)\sqrt{\frac{3i+\sqrt{15}-4ix^2}{3i+\sqrt{15}}}\sqrt{\frac{-3i+\sqrt{15}+4ix^2}{-3i+\sqrt{15}}}E\left(i\text{arcsinh}\right.}{}$$

input
```
Integrate[(3 - 3*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(4*Sqrt[(-I)/(3*I + Sqrt[15])]*x*(1 + 2*x^2) + (-3*I + Sqrt[15])*Sqrt[(3*I
 + Sqrt[15] - (4*I)*x^2)/(3*I + Sqrt[15])]*Sqrt[(-3*I + Sqrt[15] + (4*I)*x
^2)/(-3*I + Sqrt[15])]*EllipticE[I*ArcSinh[2*Sqrt[(-I)/(3*I + Sqrt[15])]*x
], (3*I + Sqrt[15])/(3*I - Sqrt[15])] - (5*I + Sqrt[15])*Sqrt[(3*I + Sqrt[
15] - (4*I)*x^2)/(3*I + Sqrt[15])]*Sqrt[(-3*I + Sqrt[15] + (4*I)*x^2)/(-3*
I + Sqrt[15])]*EllipticF[I*ArcSinh[2*Sqrt[(-I)/(3*I + Sqrt[15])]*x], (3*I
+ Sqrt[15])/(3*I - Sqrt[15])])/(60*Sqrt[(-I)/(3*I + Sqrt[15])]*Sqrt[3 - 3*
x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-3x^2+3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{45}\int \frac{6(2-x^2)}{\sqrt{2x^4-3x^2+3}}dx+\frac{x(2x^2+1)}{15\sqrt{2x^4-3x^2+3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{15}\int \frac{2-x^2}{\sqrt{2x^4-3x^2+3}}dx+\frac{x(2x^2+1)}{15\sqrt{2x^4-3x^2+3}}$$

$\Big\downarrow$ 1511

$$\frac{2}{15}\left(\frac{1}{2}\left(4-\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4-3x^2+3}}dx+\sqrt{\frac{3}{2}}\int \frac{3-\sqrt{6}x^2}{3\sqrt{2x^4-3x^2+3}}dx\right)+\frac{x(2x^2+1)}{15\sqrt{2x^4-3x^2+3}}$$

$\Big\downarrow$ 27

$$\frac{2}{15}\left(\frac{1}{2}\left(4-\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4-3x^2+3}}dx+\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4-3x^2+3}}dx}{\sqrt{6}}\right)+\frac{x(2x^2+1)}{15\sqrt{2x^4-3x^2+3}}$$

$\Big\downarrow$ 1416

$$\frac{2}{15}\left(\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4-3x^2+3}}dx}{\sqrt{6}}+\frac{\left(4-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4-3x^2+3}}\right)+$$

$$\frac{x(2x^2+1)}{15\sqrt{2x^4-3x^2+3}}$$

$\Big\downarrow$ 1509

$$\frac{2}{15}\left(\frac{\left(4-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4-3x^2+3}}+\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}E\left(\right.}{\sqrt[4]{2}\sqrt{2x}}\right.$$

$$\frac{x(2x^2+1)}{15\sqrt{2x^4-3x^2+3}}$$

input
```
Int[(3 - 3*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(1 + 2*x^2))/(15*Sqrt[3 - 3*x^2 + 2*x^4]) + (2*(((-3*x*Sqrt[3 - 3*x^2 +
 2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 +
2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6
])/8])/(2^(1/4)*Sqrt[3 - 3*x^2 + 2*x^4]))/Sqrt[6] + ((4 - Sqrt[6])*(3 + Sq
rt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTa
n[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(4*6^(1/4)*Sqrt[3 - 3*x^2 + 2*x^4])))/
15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.92

method	result
risch	$\frac{x(2x^2+1)}{15\sqrt{2x^4-3x^2+3}}+\frac{8\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{18+6i\sqrt{15}}}{6},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{5\sqrt{18+6i\sqrt{15}}\,\sqrt{2x^4-3x^2+3}}+\frac{24\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-}$
default	$-\frac{4\left(-\frac{1}{60}x-\frac{1}{30}x^3\right)}{\sqrt{2x^4-3x^2+3}}+\frac{8\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{18+6i\sqrt{15}}}{6},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{5\sqrt{18+6i\sqrt{15}}\,\sqrt{2x^4-3x^2+3}}+\frac{24\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{}$
elliptic	$-\frac{4\left(-\frac{1}{60}x-\frac{1}{30}x^3\right)}{\sqrt{2x^4-3x^2+3}}+\frac{8\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{18+6i\sqrt{15}}}{6},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{5\sqrt{18+6i\sqrt{15}}\,\sqrt{2x^4-3x^2+3}}+\frac{24\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{}$

input
```
int(1/(2*x^4-3*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(2*x^2+1)/(2*x^4-3*x^2+3)^(1/2)+8/5/(18+6*I*15^(1/2))^(1/2)*(1-(1/2
+1/6*I*15^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*I*15^(1/2))*x^2)^(1/2)/(2*x^4-3*x^
2+3)^(1/2)*EllipticF(1/6*x*(18+6*I*15^(1/2))^(1/2),1/2*(-1-I*15^(1/2))^(1/
2))+24/5/(18+6*I*15^(1/2))^(1/2)*(1-(1/2+1/6*I*15^(1/2))*x^2)^(1/2)*(1-(1/
2-1/6*I*15^(1/2))*x^2)^(1/2)/(2*x^4-3*x^2+3)^(1/2)/(-3+I*15^(1/2))*(Ellipt
icF(1/6*x*(18+6*I*15^(1/2))^(1/2),1/2*(-1-I*15^(1/2))^(1/2))-EllipticE(1/6
*x*(18+6*I*15^(1/2))^(1/2),1/2*(-1-I*15^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.63

$$\int \frac{1}{(3-3x^2+2x^4)^{3/2}}\,dx = \frac{\sqrt{3}\left(2\,x^4-3\,x^2+\sqrt{-\frac{5}{3}}(2\,x^4-3\,x^2+3)+3\right)\sqrt{\frac{1}{2}\,\sqrt{-\frac{5}{3}}+\frac{1}{2}}E(\arcsin\left(x\sqrt{\frac{1}{2}\,\sqrt{\right.$$

input
```
integrate(1/(2*x^4-3*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/30*(sqrt(3)*(2*x^4 - 3*x^2 + sqrt(-5/3)*(2*x^4 - 3*x^2 + 3) + 3)*sqrt(1/
2*sqrt(-5/3) + 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(-5/3) + 1/2)), -3/4*
sqrt(-5/3) - 1/4) + sqrt(3)*(2*x^4 - 3*x^2 - 3*sqrt(-5/3)*(2*x^4 - 3*x^2 +
 3) + 3)*sqrt(1/2*sqrt(-5/3) + 1/2)*elliptic_f(arcsin(x*sqrt(1/2*sqrt(-5/3
) + 1/2)), -3/4*sqrt(-5/3) - 1/4) + 2*sqrt(2*x^4 - 3*x^2 + 3)*(2*x^3 + x))
/(2*x^4 - 3*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(3-3x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4-3x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-3*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 - 3*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-3x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-3\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-3*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - 3*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-3x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-3\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-3*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - 3*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-3x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-3\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - 3*x^2 + 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - 3*x^2 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(3-3x^2+2x^4\right)^{3/2}}\,dx = \int \frac{\sqrt{2x^4-3x^2+3}}{4x^8-12x^6+21x^4-18x^2+9}dx$$

input
```
int(1/(2*x^4-3*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - 3*x**2 + 3)/(4*x**8 - 12*x**6 + 21*x**4 - 18*x**2 + 9),x
)
```

3.258 $\int \frac{1}{(3-4x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 245

$$\int \frac{1}{(3-4x^2+2x^4)^{3/2}}\,dx = -\frac{x(1-2x^2)}{6\sqrt{3-4x^2+2x^4}} - \frac{x\sqrt{3-4x^2+2x^4}}{3\left(\sqrt{6}+2x^2\right)} + \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{6^{3/4}\sqrt{3-4x^2+2x^4}} - \frac{\left(2-\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{4\ 6^{3/4}\sqrt{3-4x^2+2x^4}}$$

output
```
-1/6*x*(-2*x^2+1)/(2*x^4-4*x^2+3)^(1/2)-x*(2*x^4-4*x^2+3)^(1/2)/(3*6^(1/2)
+6*x^2)+1/6*(3+6^(1/2)*x^2)*((2*x^4-4*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*Elli
pticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18+6*6^(1/2))^(1/2))*6^(1/
4)/(2*x^4-4*x^2+3)^(1/2)-1/24*(2-6^(1/2))*(3+6^(1/2)*x^2)*((2*x^4-4*x^2+3)
/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),
1/6*(18+6*6^(1/2))^(1/2))*6^(1/4)/(2*x^4-4*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.17 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.32

$$\int \frac{1}{(3-4x^2+2x^4)^{3/2}}\,dx = \frac{2\sqrt{-\frac{i}{2i+\sqrt{2}}}x(-1+2x^2)+2(1-i\sqrt{2})\sqrt{\frac{2i+\sqrt{2}-2ix^2}{2i+\sqrt{2}}}\sqrt{\frac{-2i+\sqrt{2}+2ix^2}{-2i+\sqrt{2}}}E\Big(i\text{arcsinh}\Big(}{1}$$

input
```
Integrate[(3 - 4*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(2*Sqrt[(-I)/(2*I + Sqrt[2])]*x*(-1 + 2*x^2) + 2*(1 - I*Sqrt[2])*Sqrt[(2*I
 + Sqrt[2] - (2*I)*x^2)/(2*I + Sqrt[2])]*Sqrt[(-2*I + Sqrt[2] + (2*I)*x^2)
/(-2*I + Sqrt[2])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(2*I + Sqrt[2])]*x], (2
*I + Sqrt[2])/(2*I - Sqrt[2])] - I*(-2*I + Sqrt[2])*Sqrt[(2*I + Sqrt[2] -
(2*I)*x^2)/(2*I + Sqrt[2])]*Sqrt[(-2*I + Sqrt[2] + (2*I)*x^2)/(-2*I + Sqrt
[2])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(2*I + Sqrt[2])]*x], (2*I + Sqrt[2])
/(2*I - Sqrt[2])])/(12*Sqrt[(-I)/(2*I + Sqrt[2])]*Sqrt[3 - 4*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-4x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{24}\int \frac{4(3-2x^2)}{\sqrt{2x^4-4x^2+3}}dx - \frac{x(1-2x^2)}{6\sqrt{2x^4-4x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \frac{3-2x^2}{\sqrt{2x^4-4x^2+3}}dx-\frac{x(1-2x^2)}{6\sqrt{2x^4-4x^2+3}}$$

$$\Big\downarrow \; 1511$$

$$\frac{1}{6}\left(\left(3-\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4-4x^2+3}}dx+\sqrt{6}\int \frac{3-\sqrt{6}x^2}{3\sqrt{2x^4-4x^2+3}}dx\right)-\frac{x(1-2x^2)}{6\sqrt{2x^4-4x^2+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\left(\left(3-\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4-4x^2+3}}dx+\sqrt{\frac{2}{3}}\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4-4x^2+3}}dx\right)-\frac{x(1-2x^2)}{6\sqrt{2x^4-4x^2+3}}$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{6}\left(\sqrt{\frac{2}{3}}\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4-4x^2+3}}dx+\frac{\left(3-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4-4x^2+3}}\right)-\frac{x(1-2x^2)}{6\sqrt{2x^4-4x^2+3}}$$

$$\Big\downarrow \; 1509$$

$$\frac{1}{6}\left(\frac{\left(3-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4-4x^2+3}}+\sqrt{\frac{2}{3}}\left(\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4}{\left(\sqrt{6}x^2\right.}}}{\sqrt[4]{2}}\right.\right.$$

$$\frac{x(1-2x^2)}{6\sqrt{2x^4-4x^2+3}}$$

input
```
Int[(3 - 4*x^2 + 2*x^4)^(-3/2),x]
```

output
```
-1/6*(x*(1 - 2*x^2))/Sqrt[3 - 4*x^2 + 2*x^4] + (Sqrt[2/3]*((-3*x*Sqrt[3 -
4*x^2 + 2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 - 4
*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], 1/2
+ 1/Sqrt[6]])/(2^(1/4)*Sqrt[3 - 4*x^2 + 2*x^4])) + ((3 - Sqrt[6])*(3 + Sqr
t[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[3 - 4*x^2 + 2*x^4]))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.97

method	result
risch	$\frac{x(2x^2-1)}{6\sqrt{2x^4-4x^2+3}} + \frac{3\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{6+3i\sqrt{2}}}{3}, \frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{2\sqrt{6+3i\sqrt{2}}\sqrt{2x^4-4x^2+3}} + \frac{6\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{}}{3}\right.}}{}$
default	$-\frac{4\left(-\frac{1}{12}x^3+\frac{1}{24}x\right)}{\sqrt{2x^4-4x^2+3}} + \frac{3\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{6+3i\sqrt{2}}}{3}, \frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{2\sqrt{6+3i\sqrt{2}}\sqrt{2x^4-4x^2+3}} + \frac{6\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}-\right.}}{}$
elliptic	$-\frac{4\left(-\frac{1}{12}x^3+\frac{1}{24}x\right)}{\sqrt{2x^4-4x^2+3}} + \frac{3\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{6+3i\sqrt{2}}}{3}, \frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{2\sqrt{6+3i\sqrt{2}}\sqrt{2x^4-4x^2+3}} + \frac{6\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}-\right.}}{}$

input
```
int(1/(2*x^4-4*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*x*(2*x^2-1)/(2*x^4-4*x^2+3)^(1/2)+3/2/(6+3*I*2^(1/2))^(1/2)*(1-(2/3+1/
3*I*2^(1/2))*x^2)^(1/2)*(1-(2/3-1/3*I*2^(1/2))*x^2)^(1/2)/(2*x^4-4*x^2+3)^
(1/2)*EllipticF(1/3*x*(6+3*I*2^(1/2))^(1/2),1/3*(3-6*I*2^(1/2))^(1/2))+6/(
6+3*I*2^(1/2))^(1/2)*(1-(2/3+1/3*I*2^(1/2))*x^2)^(1/2)*(1-(2/3-1/3*I*2^(1/
2))*x^2)^(1/2)/(2*x^4-4*x^2+3)^(1/2)/(-4+2*I*2^(1/2))*(EllipticF(1/3*x*(6+
3*I*2^(1/2))^(1/2),1/3*(3-6*I*2^(1/2))^(1/2))-EllipticE(1/3*x*(6+3*I*2^(1/
2))^(1/2),1/3*(3-6*I*2^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.68

$$\int \frac{1}{(3-4x^2+2x^4)^{3/2}}\,dx = \frac{2\sqrt{3}\left(4x^4-8x^2+\sqrt{-2}(2x^4-4x^2+3)+6\right)\sqrt{\frac{1}{3}\sqrt{-2}+\frac{2}{3}}E\left(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{\right.\right.}}\right.}{}$$

input
```
integrate(1/(2*x^4-4*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/36*(2*sqrt(3)*(4*x^4 - 8*x^2 + sqrt(-2)*(2*x^4 - 4*x^2 + 3) + 6)*sqrt(1/
3*sqrt(-2) + 2/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(-2) + 2/3)), -2/3*sqrt
(-2) + 1/3) + sqrt(3)*(4*x^4 - 8*x^2 - 5*sqrt(-2)*(2*x^4 - 4*x^2 + 3) + 6)
*sqrt(1/3*sqrt(-2) + 2/3)*elliptic_f(arcsin(x*sqrt(1/3*sqrt(-2) + 2/3)), -
2/3*sqrt(-2) + 1/3) + 6*sqrt(2*x^4 - 4*x^2 + 3)*(2*x^3 - x))/(2*x^4 - 4*x^
2 + 3)
```

Sympy [F]

$$\int \frac{1}{(3-4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4-4x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-4*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 - 4*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-4\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-4*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - 4*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-4\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-4*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - 4*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-4x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-4\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - 4*x^2 + 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - 4*x^2 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-4x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4-4x^2+3}}{4x^8-16x^6+28x^4-24x^2+9}dx$$

input
```
int(1/(2*x^4-4*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - 4*x**2 + 3)/(4*x**8 - 16*x**6 + 28*x**4 - 24*x**2 + 9),x
)
```

3.259 $\int \frac{1}{(3-5x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 130

$$\int \frac{1}{(3-5x^2+2x^4)^{3/2}}\,dx = \frac{x(13-10x^2)}{3\sqrt{3-5x^2+2x^4}} - \frac{5\sqrt{3-2x^2}\sqrt{1-x^2}\,E\left(\arcsin(x)\,\middle|\,\frac{2}{3}\right)}{\sqrt{3}\sqrt{3-5x^2+2x^4}} + \frac{\sqrt{3-2x^2}\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),\frac{2}{3}\right)}{\sqrt{3}\sqrt{3-5x^2+2x^4}}$$

output
```
1/3*x*(-10*x^2+13)/(2*x^4-5*x^2+3)^(1/2)-5/3*(-2*x^2+3)^(1/2)*(-x^2+1)^(1/
2)*EllipticE(x,1/3*6^(1/2))*3^(1/2)/(2*x^4-5*x^2+3)^(1/2)+1/3*(-2*x^2+3)^(
1/2)*(-x^2+1)^(1/2)*EllipticF(x,1/3*6^(1/2))*3^(1/2)/(2*x^4-5*x^2+3)^(1/2)
```

Mathematica [A] (verified)

Time = 5.82 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

$$\int \frac{1}{(3-5x^2+2x^4)^{3/2}}\,dx = \frac{13x-10x^3-5\sqrt{6-4x^2}\sqrt{1-x^2}\,E\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right)\,\middle|\,\frac{3}{2}\right)-\sqrt{6-4x^2}\sqrt{1-x^2}\,\text{El}}{3\sqrt{3-5x^2+2x^4}}$$

input
```
Integrate[(3 - 5*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(13*x - 10*x^3 - 5*Sqrt[6 - 4*x^2]*Sqrt[1 - x^2]*EllipticE[ArcSin[Sqrt[2/3
]*x], 3/2] - Sqrt[6 - 4*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2/3]*x],
3/2])/(3*Sqrt[3 - 5*x^2 + 2*x^4])
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 269 vs. $2(130) = 260$.

Time = 0.63 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1497, 27, 1409, 1496}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-5x^2+3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(13-10x^2)}{3\sqrt{2x^4-5x^2+3}}-\frac{1}{3}\int \frac{2(6-5x^2)}{\sqrt{2x^4-5x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(13-10x^2)}{3\sqrt{2x^4-5x^2+3}}-\frac{2}{3}\int \frac{6-5x^2}{\sqrt{2x^4-5x^2+3}}dx$$

$$\Big\downarrow \text{1497}$$

$$\frac{x(13-10x^2)}{3\sqrt{2x^4-5x^2+3}}-\frac{2}{3}\left(\frac{1}{2}\left(12-5\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4-5x^2+3}}dx+5\sqrt{\frac{3}{2}}\int \frac{3-\sqrt{6}x^2}{3\sqrt{2x^4-5x^2+3}}dx\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{x(13-10x^2)}{3\sqrt{2x^4-5x^2+3}}-\frac{2}{3}\left(\frac{1}{2}\left(12-5\sqrt{6}\right)\int \frac{1}{\sqrt{2x^4-5x^2+3}}dx+\frac{5\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4-5x^2+3}}dx}{\sqrt{6}}\right)$$

$$\Big\downarrow \text{1409}$$

$$\frac{x(13-10x^2)}{3\sqrt{2x^4-5x^2+3}}-$$
$$\frac{2}{3}\left(\frac{5\int \frac{3-\sqrt{6}x^2}{\sqrt{2x^4-5x^2+3}}dx}{\sqrt{6}}+\frac{(12-5\sqrt{6})\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-5x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}(12+5\sqrt{6})\right)}{4\sqrt[4]{6}\sqrt{2x^4-5x^2+3}}\right)$$

$$\Big\downarrow \text{1496}$$

$$\frac{x(13-10x^2)}{3\sqrt{2x^4-5x^2+3}}-$$
$$\frac{2}{3}\left(\frac{(12-5\sqrt{6})\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-5x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}(12+5\sqrt{6})\right)}{4\sqrt[4]{6}\sqrt{2x^4-5x^2+3}}+\frac{5\left(\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-\cdots}{(\sqrt{6}\cdots}}}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[(3 - 5*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(13 - 10*x^2))/(3*Sqrt[3 - 5*x^2 + 2*x^4]) - (2*((5*((-3*x*Sqrt[3 - 5*x
^2 + 2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 - 5*x^
2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (12 + 5
*Sqrt[6])/24])/(2^(1/4)*Sqrt[3 - 5*x^2 + 2*x^4])))/Sqrt[6] + ((12 - 5*Sqrt
[6])*(3 + Sqrt[6]*x^2)*Sqrt[(3 - 5*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*Ellip
ticF[2*ArcTan[(2/3)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(4*6^(1/4)*Sqrt[3 - 5*
x^2 + 2*x^4])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

rule 1496
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2
- 4*a*c, 0] && GtQ[c/a, 0] && LtQ[b/a, 0]
```

rule 1497
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ
[c/a, 0] && LtQ[b/a, 0]
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{x(10x^2-13)}{3\sqrt{2x^4-5x^2+3}}-\frac{4\sqrt{-x^2+1}\,\sqrt{-6x^2+9}\ \text{EllipticF}\left(x,\frac{\sqrt{6}}{3}\right)}{3\sqrt{2x^4-5x^2+3}}+\frac{5\sqrt{-x^2+1}\,\sqrt{-6x^2+9}\left(\text{EllipticF}\left(x,\frac{\sqrt{6}}{3}\right)-\text{EllipticE}\left(x,\frac{\sqrt{6}}{3}\right)\right)}{3\sqrt{2x^4-5x^2+3}}$
default	$-\frac{4\left(\frac{5}{6}x^3-\frac{13}{12}x\right)}{\sqrt{2x^4-5x^2+3}}-\frac{4\sqrt{-x^2+1}\,\sqrt{-6x^2+9}\ \text{EllipticF}\left(x,\frac{\sqrt{6}}{3}\right)}{3\sqrt{2x^4-5x^2+3}}+\frac{5\sqrt{-x^2+1}\,\sqrt{-6x^2+9}\left(\text{EllipticF}\left(x,\frac{\sqrt{6}}{3}\right)-\text{EllipticE}\left(x,\frac{\sqrt{6}}{3}\right)\right)}{3\sqrt{2x^4-5x^2+3}}$
elliptic	$-\frac{4\left(\frac{5}{6}x^3-\frac{13}{12}x\right)}{\sqrt{2x^4-5x^2+3}}-\frac{4\sqrt{-x^2+1}\,\sqrt{-6x^2+9}\ \text{EllipticF}\left(x,\frac{\sqrt{6}}{3}\right)}{3\sqrt{2x^4-5x^2+3}}+\frac{5\sqrt{-x^2+1}\,\sqrt{-6x^2+9}\left(\text{EllipticF}\left(x,\frac{\sqrt{6}}{3}\right)-\text{EllipticE}\left(x,\frac{\sqrt{6}}{3}\right)\right)}{3\sqrt{2x^4-5x^2+3}}$

input
```
int(1/(2*x^4-5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(10*x^2-13)/(2*x^4-5*x^2+3)^(1/2)-4/3*(-x^2+1)^(1/2)*(-6*x^2+9)^(1/
2)/(2*x^4-5*x^2+3)^(1/2)*EllipticF(x,1/3*6^(1/2))+5/3*(-x^2+1)^(1/2)*(-6*x
^2+9)^(1/2)/(2*x^4-5*x^2+3)^(1/2)*(EllipticF(x,1/3*6^(1/2))-EllipticE(x,1/
3*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.64

$$\int \frac{1}{(3-5x^2+2x^4)^{3/2}}\,dx =$$
$$-\frac{5\sqrt{3}(2\,x^4-5\,x^2+3)E(\arcsin{(x)}\mid\frac{2}{3})-\sqrt{3}(2\,x^4-5\,x^2+3)F(\arcsin{(x)}\mid\frac{2}{3})+\sqrt{2\,x^4-5\,x^2+3}(10\,x^3}{3\,(2\,x^4-5\,x^2+3)}$$

input
```
integrate(1/(2*x^4-5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(5*sqrt(3)*(2*x^4 - 5*x^2 + 3)*elliptic_e(arcsin(x), 2/3) - sqrt(3)*(
2*x^4 - 5*x^2 + 3)*elliptic_f(arcsin(x), 2/3) + sqrt(2*x^4 - 5*x^2 + 3)*(1
0*x^3 - 13*x))/(2*x^4 - 5*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(3-5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4-5x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-5*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 - 5*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - 5*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - 5*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-5x^2+2x^4)^{3/2}}\,dx=\int \frac{1}{(2\,x^4-5\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - 5*x^2 + 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - 5*x^2 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-5x^2+2x^4)^{3/2}}\,dx=\int \frac{\sqrt{2x^4-5x^2+3}}{4x^8-20x^6+37x^4-30x^2+9}dx$$

input
```
int(1/(2*x^4-5*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - 5*x**2 + 3)/(4*x**8 - 20*x**6 + 37*x**4 - 30*x**2 + 9),x
)
```

3.260 $\int \frac{1}{(3-6x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 227

$$\int \frac{1}{(3-6x^2+2x^4)^{3/2}}\,dx = \frac{x(2-x^2)}{3\sqrt{3-6x^2+2x^4}} - \frac{\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}\sqrt{3-\left(3-\sqrt{3}\right)x^2}\sqrt{3-\left(3+\sqrt{3}\right)x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right)|2-\sqrt{3}\right)}{6\sqrt{3-6x^2+2x^4}} + \frac{\sqrt{3+\sqrt{3}}\sqrt{3-\left(3-\sqrt{3}\right)x^2}\sqrt{3-\left(3+\sqrt{3}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right),2-\sqrt{3}\right)}{18\sqrt{3-6x^2+2x^4}}$$

output
```
1/3*x*(-x^2+2)/(2*x^4-6*x^2+3)^(1/2)-1/18*(9+3*3^(1/2))^(1/2)*(3-(3-3^(1/2
))*x^2)^(1/2)*(3-(3+3^(1/2))*x^2)^(1/2)*EllipticE(1/3*(9+3*3^(1/2))^(1/2)*
x,1/2*6^(1/2)-1/2*2^(1/2))/(2*x^4-6*x^2+3)^(1/2)+1/18*(3+3^(1/2))^(1/2)*(3
-(3-3^(1/2))*x^2)^(1/2)*(3-(3+3^(1/2))*x^2)^(1/2)*EllipticF(1/3*(9+3*3^(1/
2))^(1/2)*x,1/2*6^(1/2)-1/2*2^(1/2))/(2*x^4-6*x^2+3)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 6.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.77

$$\int \frac{1}{(3-6x^2+2x^4)^{3/2}}\,dx = \frac{-12x(-2+x^2)-3\sqrt{2}(1+\sqrt{3})\sqrt{3-\sqrt{3}-2x^2}\sqrt{3+(-3+\sqrt{3})x^2}E\Big(\text{arcsin}$$

input
```
Integrate[(3 - 6*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(-12*x*(-2 + x^2) - 3*Sqrt[2]*(1 + Sqrt[3])*Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt
[3 + (-3 + Sqrt[3])*x^2]*EllipticE[ArcSin[Sqrt[1 + 1/Sqrt[3]]*x], 2 - Sqrt
[3]] + Sqrt[2]*(3 + Sqrt[3])*Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt[3 + (-3 + Sqrt
[3])*x^2]*EllipticF[ArcSin[Sqrt[1 + 1/Sqrt[3]]*x], 2 - Sqrt[3]])/(36*Sqrt[
3 - 6*x^2 + 2*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1405, 27, 1497, 27, 1409, 1496}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-6x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(2-x^2)}{3\sqrt{2x^4-6x^2+3}}-\frac{1}{36}\int \frac{12(1-x^2)}{\sqrt{2x^4-6x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(2-x^2)}{3\sqrt{2x^4-6x^2+3}}-\frac{1}{3}\int \frac{1-x^2}{\sqrt{2x^4-6x^2+3}}dx$$

$$\Big\downarrow \text{1497}$$

$$\frac{1}{3}\left(-\frac{1}{2}\left(2-\sqrt{6}\right)\int\frac{1}{\sqrt{2x^4-6x^2+3}}dx-\sqrt{\frac{3}{2}}\int\frac{3-\sqrt{6}x^2}{3\sqrt{2x^4-6x^2+3}}dx\right)+\frac{x(2-x^2)}{3\sqrt{2x^4-6x^2+3}}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(-\frac{1}{2}\left(2-\sqrt{6}\right)\int\frac{1}{\sqrt{2x^4-6x^2+3}}dx-\frac{\int\frac{3-\sqrt{6}x^2}{\sqrt{2x^4-6x^2+3}}dx}{\sqrt{6}}\right)+\frac{x(2-x^2)}{3\sqrt{2x^4-6x^2+3}}$$

$\Big\downarrow$ 1409

$$\frac{1}{3}\left(-\frac{\int\frac{3-\sqrt{6}x^2}{\sqrt{2x^4-6x^2+3}}dx}{\sqrt{6}}-\frac{\left(2-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-6x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4-6x^2+3}}\right)+\frac{x(2-x^2)}{3\sqrt{2x^4-6x^2+3}}$$

$\Big\downarrow$ 1496

$$\frac{1}{3}\left(-\frac{\left(2-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-6x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4-6x^2+3}}-\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-6x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}E}{\sqrt[4]{2}\sqrt{2x}}\right.$$

$$\frac{x(2-x^2)}{3\sqrt{2x^4-6x^2+3}}$$

input
```
Int[(3 - 6*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(2 - x^2))/(3*Sqrt[3 - 6*x^2 + 2*x^4]) + (-(((-3*x*x*Sqrt[3 - 6*x^2 + 2*x
^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 - 6*x^2 + 2*x^
4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (2 + Sqrt[6])/4
])/(2^(1/4)*Sqrt[3 - 6*x^2 + 2*x^4]))/Sqrt[6]) - ((2 - Sqrt[6])*(3 + Sqrt[
6]*x^2)*Sqrt[(3 - 6*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan[(
2/3)^(1/4)*x], (2 + Sqrt[6])/4])/(4*6^(1/4)*Sqrt[3 - 6*x^2 + 2*x^4]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

rule 1496
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2
- 4*a*c, 0] && GtQ[c/a, 0] && LtQ[b/a, 0]
```

rule 1497
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ
[c/a, 0] && LtQ[b/a, 0]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{x(x^2-2)}{3\sqrt{2x^4-6x^2+3}}-\frac{\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{9+3\sqrt{3}}\,x}{3},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{9+3\sqrt{3}}\,\sqrt{2x^4-6x^2+3}}-\frac{6\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}}{}$
default	$-\frac{4\left(-\frac{1}{6}x+\frac{1}{12}x^3\right)}{\sqrt{2x^4-6x^2+3}}-\frac{\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{9+3\sqrt{3}}\,x}{3},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{9+3\sqrt{3}}\,\sqrt{2x^4-6x^2+3}}-\frac{6\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}}{}$
elliptic	$-\frac{4\left(-\frac{1}{6}x+\frac{1}{12}x^3\right)}{\sqrt{2x^4-6x^2+3}}-\frac{\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{9+3\sqrt{3}}\,x}{3},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{9+3\sqrt{3}}\,\sqrt{2x^4-6x^2+3}}-\frac{6\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}}{}$

input
```
int(1/(2*x^4-6*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(x^2-2)/(2*x^4-6*x^2+3)^(1/2)-1/(9+3*3^(1/2))^(1/2)*(1-(1+1/3*3^(1/
2))*x^2)^(1/2)*(1-(1-1/3*3^(1/2))*x^2)^(1/2)/(2*x^4-6*x^2+3)^(1/2)*Ellipti
cF(1/3*(9+3*3^(1/2))^(1/2)*x,1/2*6^(1/2)-1/2*2^(1/2))-6/(9+3*3^(1/2))^(1/2
)*(1-(1+1/3*3^(1/2))*x^2)^(1/2)*(1-(1-1/3*3^(1/2))*x^2)^(1/2)/(2*x^4-6*x^2
+3)^(1/2)/(-6+2*3^(1/2))*(EllipticF(1/3*(9+3*3^(1/2))^(1/2)*x,1/2*6^(1/2)-
1/2*2^(1/2))-EllipticE(1/3*(9+3*3^(1/2))^(1/2)*x,1/2*6^(1/2)-1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.62

$$\int \frac{1}{(3-6x^2+2x^4)^{3/2}}\,dx =$$

$$-\frac{\left(2\,x^4-6\,x^2+\sqrt{3}(2\,x^4-6\,x^2+3)+3\right)\sqrt{\frac{1}{3}\,\sqrt{3}+1}E(\arcsin\left(x\sqrt{\frac{1}{3}\,\sqrt{3}+1}\right)\,|-\sqrt{3}+2)-2\,(2\,x^4-6\,x}{6\,(2\,x^4-6\,x^2+3)}$$

input
```
integrate(1/(2*x^4-6*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*((2*x^4 - 6*x^2 + sqrt(3)*(2*x^4 - 6*x^2 + 3) + 3)*sqrt(1/3*sqrt(3) +
 1)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(3) + 1)), -sqrt(3) + 2) - 2*(2*x^4 -
 6*x^2 + 3)*sqrt(1/3*sqrt(3) + 1)*elliptic_f(arcsin(x*sqrt(1/3*sqrt(3) + 1
)), -sqrt(3) + 2) + 2*sqrt(2*x^4 - 6*x^2 + 3)*(x^3 - 2*x))/(2*x^4 - 6*x^2
+ 3)
```

Sympy [F]

$$\int \frac{1}{(3-6x^2+2x^4)^{3/2}}\,dx=\int \frac{1}{(2x^4-6x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-6*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 - 6*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-6x^2+2x^4)^{3/2}}\,dx=\int \frac{1}{(2\,x^4-6\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-6*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - 6*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-6x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-6\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-6*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - 6*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-6x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-6\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - 6*x^2 + 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - 6*x^2 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-6x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4-6x^2+3}}{4x^8-24x^6+48x^4-36x^2+9}dx$$

input
```
int(1/(2*x^4-6*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - 6*x**2 + 3)/(4*x**8 - 24*x**6 + 48*x**4 - 36*x**2 + 9),x
)
```

3.261 $\int \frac{1}{(3-7x^2+2x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 151

$$\int \frac{1}{(3-7x^2+2x^4)^{3/2}} \, dx = \frac{x(37-14x^2)}{75\sqrt{3-7x^2+2x^4}} - \frac{7\sqrt{\frac{2}{3}}\sqrt{1-2x^2}\sqrt{3-x^2}E\left(\arcsin\left(\sqrt{2}x\right)|\frac{1}{6}\right)}{25\sqrt{3-7x^2+2x^4}} + \frac{\sqrt{\frac{2}{3}}\sqrt{1-2x^2}\sqrt{3-x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),\frac{1}{6}\right)}{5\sqrt{3-7x^2+2x^4}}$$

output
```
1/75*x*(-14*x^2+37)/(2*x^4-7*x^2+3)^(1/2)-7/75*6^(1/2)*(-2*x^2+1)^(1/2)*(-
x^2+3)^(1/2)*EllipticE(x*2^(1/2),1/6*6^(1/2))/(2*x^4-7*x^2+3)^(1/2)+1/15*(
-2*x^2+1)^(1/2)*(-x^2+3)^(1/2)*EllipticF(x*2^(1/2),1/6*6^(1/2))*6^(1/2)/(2
*x^4-7*x^2+3)^(1/2)
```

Mathematica [A] (verified)

Time = 6.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.67

$$\int \frac{1}{(3-7x^2+2x^4)^{3/2}}\,dx = \frac{37x-14x^3-7\sqrt{6-12x^2}\sqrt{3-x^2}E\left(\arcsin\left(\sqrt{2}x\right)|\frac{1}{6}\right)+5\sqrt{6-12x^2}\sqrt{3-x^2}}{75\sqrt{3-7x^2+2x^4}}$$

input
```
Integrate[(3 - 7*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(37*x - 14*x^3 - 7*Sqrt[6 - 12*x^2]*Sqrt[3 - x^2]*EllipticE[ArcSin[Sqrt[2]
*x], 1/6] + 5*Sqrt[6 - 12*x^2]*Sqrt[3 - x^2]*EllipticF[ArcSin[Sqrt[2]*x],
1/6])/(75*Sqrt[3 - 7*x^2 + 2*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.78, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1497, 27, 1409, 1496}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4-7x^2+3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x(37-14x^2)}{75\sqrt{2x^4-7x^2+3}}-\frac{1}{75}\int \frac{2(6-7x^2)}{\sqrt{2x^4-7x^2+3}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x(37-14x^2)}{75\sqrt{2x^4-7x^2+3}}-\frac{2}{75}\int \frac{6-7x^2}{\sqrt{2x^4-7x^2+3}}dx$$

$$\Big\downarrow \text{ 1497}$$

$$\frac{x(37-14x^2)}{75\sqrt{2x^4-7x^2+3}}-\frac{2}{75}\left(\frac{1}{2}\left(12-7\sqrt{6}\right)\int\frac{1}{\sqrt{2x^4-7x^2+3}}dx+7\sqrt{\frac{3}{2}}\int\frac{3-\sqrt{6}x^2}{3\sqrt{2x^4-7x^2+3}}dx\right)$$

$\downarrow$ 27

$$\frac{x(37-14x^2)}{75\sqrt{2x^4-7x^2+3}}-\frac{2}{75}\left(\frac{1}{2}\left(12-7\sqrt{6}\right)\int\frac{1}{\sqrt{2x^4-7x^2+3}}dx+\frac{7\int\frac{3-\sqrt{6}x^2}{\sqrt{2x^4-7x^2+3}}dx}{\sqrt{6}}\right)$$

$\downarrow$ 1409

$$\frac{x(37-14x^2)}{75\sqrt{2x^4-7x^2+3}}-\frac{2}{75}\left(\frac{7\int\frac{3-\sqrt{6}x^2}{\sqrt{2x^4-7x^2+3}}dx}{\sqrt{6}}+\frac{\left(12-7\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-7x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4-7x^2+3}}\right)$$

$\downarrow$ 1496

$$\frac{x(37-14x^2)}{75\sqrt{2x^4-7x^2+3}}-\frac{2}{75}\left(\frac{\left(12-7\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-7x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{2x^4-7x^2+3}}+\frac{7\left(\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4}{(\sqrt{}}}}{}\right.}{}\right.$$

input
```
Int[(3 - 7*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(37 - 14*x^2))/(75*Sqrt[3 - 7*x^2 + 2*x^4]) - (2*((7*((-3*x*Sqrt[3 - 7*
x^2 + 2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 - 7*x
^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (12 +
7*Sqrt[6])/24])/(2^(1/4)*Sqrt[3 - 7*x^2 + 2*x^4])))/Sqrt[6] + ((12 - 7*Sqr
t[6])*(3 + Sqrt[6]*x^2)*Sqrt[(3 - 7*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*Elli
pticF[2*ArcTan[(2/3)^(1/4)*x], (12 + 7*Sqrt[6])/24])/(4*6^(1/4)*Sqrt[3 - 7
*x^2 + 2*x^4])))/75
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

rule 1496
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2
- 4*a*c, 0] && GtQ[c/a, 0] && LtQ[b/a, 0]
```

rule 1497
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ
[c/a, 0] && LtQ[b/a, 0]
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{x(14x^2-37)}{75\sqrt{2x^4-7x^2+3}}-\frac{2\sqrt{2}\sqrt{-2x^2+1}\sqrt{-3x^2+9}\,\text{EllipticF}\left(x\sqrt{2},\frac{\sqrt{6}}{6}\right)}{75\sqrt{2x^4-7x^2+3}}+\frac{7\sqrt{2}\sqrt{-2x^2+1}\sqrt{-3x^2+9}\left(\text{EllipticF}\left(x\sqrt{2},\frac{\sqrt{6}}{6}\right)-\text{Ellip}\right.}{75\sqrt{2x^4-7x^2+3}}$
default	$-\frac{4\left(-\frac{37}{300}x+\frac{7}{150}x^3\right)}{\sqrt{2x^4-7x^2+3}}-\frac{2\sqrt{2}\sqrt{-2x^2+1}\sqrt{-3x^2+9}\,\text{EllipticF}\left(x\sqrt{2},\frac{\sqrt{6}}{6}\right)}{75\sqrt{2x^4-7x^2+3}}+\frac{7\sqrt{2}\sqrt{-2x^2+1}\sqrt{-3x^2+9}\left(\text{EllipticF}\left(x\sqrt{2},\frac{\sqrt{6}}{6}\right)-\text{Ell}\right.}{75\sqrt{2x^4-7x^2+3}}$
elliptic	$-\frac{4\left(-\frac{37}{300}x+\frac{7}{150}x^3\right)}{\sqrt{2x^4-7x^2+3}}-\frac{2\sqrt{2}\sqrt{-2x^2+1}\sqrt{-3x^2+9}\,\text{EllipticF}\left(x\sqrt{2},\frac{\sqrt{6}}{6}\right)}{75\sqrt{2x^4-7x^2+3}}+\frac{7\sqrt{2}\sqrt{-2x^2+1}\sqrt{-3x^2+9}\left(\text{EllipticF}\left(x\sqrt{2},\frac{\sqrt{6}}{6}\right)-\text{Ell}\right.}{75\sqrt{2x^4-7x^2+3}}$

input
```
int(1/(2*x^4-7*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/75*x*(14*x^2-37)/(2*x^4-7*x^2+3)^(1/2)-2/75*2^(1/2)*(-2*x^2+1)^(1/2)*(-
3*x^2+9)^(1/2)/(2*x^4-7*x^2+3)^(1/2)*EllipticF(x*2^(1/2),1/6*6^(1/2))+7/75
*2^(1/2)*(-2*x^2+1)^(1/2)*(-3*x^2+9)^(1/2)/(2*x^4-7*x^2+3)^(1/2)*(Elliptic
F(x*2^(1/2),1/6*6^(1/2))-EllipticE(x*2^(1/2),1/6*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.64

$$\int \frac{1}{(3-7x^2+2x^4)^{3/2}}\,dx =$$
$$-\frac{14\sqrt{3}\sqrt{2}(2\,x^4-7\,x^2+3)E(\arcsin\left(\sqrt{2}x\right)\mid\frac{1}{6})-12\sqrt{3}\sqrt{2}(2\,x^4-7\,x^2+3)F(\arcsin\left(\sqrt{2}x\right)\mid\frac{1}{6})+\sqrt{2\,x^4}}{75\left(2\,x^4-7\,x^2+3\right)}$$

input
```
integrate(1/(2*x^4-7*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/75*(14*sqrt(3)*sqrt(2)*(2*x^4 - 7*x^2 + 3)*elliptic_e(arcsin(sqrt(2)*x)
, 1/6) - 12*sqrt(3)*sqrt(2)*(2*x^4 - 7*x^2 + 3)*elliptic_f(arcsin(sqrt(2)*
x), 1/6) + sqrt(2*x^4 - 7*x^2 + 3)*(14*x^3 - 37*x))/(2*x^4 - 7*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(3-7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4-7x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4-7*x**2+3)**(3/2),x)
```

output
```
Integral((2*x**4 - 7*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-7\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-7*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 - 7*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-7\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4-7*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 - 7*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-7x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4-7\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(2*x^4 - 7*x^2 + 3)^(3/2),x)
```

output
```
int(1/(2*x^4 - 7*x^2 + 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-7x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4-7x^2+3}}{4x^8-28x^6+61x^4-42x^2+9}dx$$

input
```
int(1/(2*x^4-7*x^2+3)^(3/2),x)
```

output
```
int(sqrt(2*x**4 - 7*x**2 + 3)/(4*x**8 - 28*x**6 + 61*x**4 - 42*x**2 + 9),x
)
```

3.262 $\int \frac{1}{\left(1-5\sqrt{5}x^2+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 263

$$\int \frac{1}{\left(1-5\sqrt{5}x^2+x^4\right)^{3/2}}\,dx = \frac{x\left(123-5\sqrt{5}x^2\right)}{121\sqrt{1-5\sqrt{5}x^2+x^4}}$$
$$-\frac{5\sqrt{\frac{5}{2}\left(11+5\sqrt{5}\right)}\sqrt{2+\left(11-5\sqrt{5}\right)x^2}\sqrt{2-\left(11+5\sqrt{5}\right)x^2}\,E\left(\arcsin\left(\sqrt{\frac{1}{2}\left(11+5\sqrt{5}\right)}x\right)|\frac{1}{2}(123-55\sqrt{}\right.}{242\sqrt{1-5\sqrt{5}x^2+x^4}}$$
$$+\frac{\sqrt{\frac{1}{2}\left(11+5\sqrt{5}\right)}\sqrt{2+\left(11-5\sqrt{5}\right)x^2}\sqrt{2-\left(11+5\sqrt{5}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1}{2}\left(11+5\sqrt{5}\right)}x\right),\frac{1}{2}(123\right.}{22\sqrt{1-5\sqrt{5}x^2+x^4}}$$

output
```
1/121*x*(123-5*5^(1/2)*x^2)/(1-5*5^(1/2)*x^2+x^4)^(1/2)-5/484*(110+50*5^(1
/2))^(1/2)*(2+(11-5*5^(1/2))*x^2)^(1/2)*(2-(11+5*5^(1/2))*x^2)^(1/2)*Ellip
ticE(1/2*(22+10*5^(1/2))^(1/2)*x,5/2*5^(1/2)-11/2)/(1-5*5^(1/2)*x^2+x^4)^(
1/2)+1/44*(22+10*5^(1/2))^(1/2)*(2+(11-5*5^(1/2))*x^2)^(1/2)*(2-(11+5*5^(1
/2))*x^2)^(1/2)*EllipticF(1/2*(22+10*5^(1/2))^(1/2)*x,5/2*5^(1/2)-11/2)/(1
-5*5^(1/2)*x^2+x^4)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 7.94 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(1-5\sqrt{5}x^2+x^4\right)^{3/2}}\,dx = \frac{246x-10\sqrt{5}x^3-5\sqrt{5}\sqrt{11+5\sqrt{5}-2x^2}\sqrt{2-\left(11+5\sqrt{5}\right)x^2}E\left(\arcsin\left(\sqrt{\right.\right.}}{}$$

input
```
Integrate[(1 - 5*Sqrt[5]*x^2 + x^4)^(-3/2),x]
```

output
```
(246*x - 10*Sqrt[5]*x^3 - 5*Sqrt[5]*Sqrt[11 + 5*Sqrt[5] - 2*x^2]*Sqrt[2 -
(11 + 5*Sqrt[5])*x^2]*EllipticE[ArcSin[Sqrt[(11 + 5*Sqrt[5])/2]*x], 123/2
- (55*Sqrt[5])/2] + Sqrt[11 + 5*Sqrt[5] - 2*x^2]*(-2*Sqrt[2/(11 + 5*Sqrt[5
])]*Sqrt[-11 + 5*Sqrt[5] - 2*x^2] + 5*Sqrt[10 - 5*(11 + 5*Sqrt[5])*x^2])*E
llipticF[ArcSin[Sqrt[(11 + 5*Sqrt[5])/2]*x], 123/2 - (55*Sqrt[5])/2])/(242
*Sqrt[1 - 5*Sqrt[5]*x^2 + x^4])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1405, 1497, 1409, 1496}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x^4-5\sqrt{5}x^2+1\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x\left(123-5\sqrt{5}x^2\right)}{121\sqrt{x^4-5\sqrt{5}x^2+1}}-\frac{1}{121}\int \frac{2-5\sqrt{5}x^2}{\sqrt{x^4-5\sqrt{5}x^2+1}}dx$$

$$\Big\downarrow \text{1497}$$

$$\frac{1}{121}\left(-\left(\left(2-5\sqrt{5}\right)\int \frac{1}{\sqrt{x^4-5\sqrt{5}x^2+1}}dx\right)-5\sqrt{5}\int \frac{1-x^2}{\sqrt{x^4-5\sqrt{5}x^2+1}}dx\right)+\frac{x\left(123-5\sqrt{5}x^2\right)}{121\sqrt{x^4-5\sqrt{5}x^2+1}}$$

$\Big\downarrow$ 1409

$$\frac{1}{121}\left(-5\sqrt{5}\int \frac{1-x^2}{\sqrt{x^4-5\sqrt{5}x^2+1}}dx-\frac{\left(2-5\sqrt{5}\right)\left(x^2+1\right)\sqrt{\frac{x^4-5\sqrt{5}x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\left(2+5\sqrt{5}\right)\right)}{2\sqrt{x^4-5\sqrt{5}x^2+1}}\right)$$
$$\frac{x\left(123-5\sqrt{5}x^2\right)}{121\sqrt{x^4-5\sqrt{5}x^2+1}}$$

$\Big\downarrow$ 1496

$$\frac{1}{121}\left(-\frac{\left(2-5\sqrt{5}\right)\left(x^2+1\right)\sqrt{\frac{x^4-5\sqrt{5}x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\left(2+5\sqrt{5}\right)\right)}{2\sqrt{x^4-5\sqrt{5}x^2+1}}-5\sqrt{5}\left(\frac{\left(x^2+1\right)\sqrt{\frac{x^4-5\sqrt{5}x^2+1}{(x^2+1)^2}}E}{\sqrt{x^4-}}\right.\right.$$
$$\frac{x\left(123-5\sqrt{5}x^2\right)}{121\sqrt{x^4-5\sqrt{5}x^2+1}}$$

input
```
Int[(1 - 5*Sqrt[5]*x^2 + x^4)^(-3/2),x]
```

output
```
(x*(123 - 5*Sqrt[5]*x^2))/(121*Sqrt[1 - 5*Sqrt[5]*x^2 + x^4]) + (-5*Sqrt[5
]*(-((x*Sqrt[1 - 5*Sqrt[5]*x^2 + x^4])/(1 + x^2)) + ((1 + x^2)*Sqrt[(1 - 5
*Sqrt[5]*x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], (2 + 5*Sqrt[5])/4]
)/Sqrt[1 - 5*Sqrt[5]*x^2 + x^4]) - ((2 - 5*Sqrt[5])*(1 + x^2)*Sqrt[(1 - 5*
Sqrt[5]*x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], (2 + 5*Sqrt[5])/4])
/(2*Sqrt[1 - 5*Sqrt[5]*x^2 + x^4]))/121
```

Defintions of rubi rules used

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

rule 1496
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2
- 4*a*c, 0] && GtQ[c/a, 0] && LtQ[b/a, 0]
```

rule 1497
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ
[c/a, 0] && LtQ[b/a, 0]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.95

method	result
default	$-\frac{2\left(\frac{5\sqrt{5}\,x^{3}}{242}-\frac{123x}{242}\right)}{\sqrt{1-5\sqrt{5}\,x^{2}+x^{4}}}-\frac{4\sqrt{1-\left(\frac{11}{2}+\frac{5\sqrt{5}}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{5\sqrt{5}}{2}-\frac{11}{2}\right)x^{2}}\,\text{EllipticF}\left(\frac{\sqrt{22+10\sqrt{5}}\,x}{2},\sqrt{-1+5\sqrt{5}\left(\frac{5\sqrt{5}}{2}-\frac{11}{2}\right)}\right)}{121\sqrt{22+10\sqrt{5}}\,\sqrt{1-5\sqrt{5}\,x^{2}+x^{4}}}-\frac{20\sqrt{5}\,\sqrt{1}}{}$
elliptic	$-\frac{2\left(\frac{5\sqrt{5}\,x^{3}}{242}-\frac{123x}{242}\right)}{\sqrt{1-5\sqrt{5}\,x^{2}+x^{4}}}-\frac{4\sqrt{1-\left(\frac{11}{2}+\frac{5\sqrt{5}}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{5\sqrt{5}}{2}-\frac{11}{2}\right)x^{2}}\,\text{EllipticF}\left(\frac{\sqrt{22+10\sqrt{5}}\,x}{2},\sqrt{-1+5\sqrt{5}\left(\frac{5\sqrt{5}}{2}-\frac{11}{2}\right)}\right)}{121\sqrt{22+10\sqrt{5}}\,\sqrt{1-5\sqrt{5}\,x^{2}+x^{4}}}-\frac{20\sqrt{5}\,\sqrt{1}}{}$
risch	$-\frac{\sqrt{5}\,x\left(25x^{2}-123\sqrt{5}\right)}{605\sqrt{1-5\sqrt{5}\,x^{2}+x^{4}}}+\frac{\sqrt{5}\left(-\frac{4\sqrt{5}\,\sqrt{1-\left(\frac{11}{2}+\frac{5\sqrt{5}}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{5\sqrt{5}}{2}-\frac{11}{2}\right)x^{2}}\,\text{EllipticF}\left(\frac{\sqrt{22+10\sqrt{5}}\,x}{2},\sqrt{-1+5\sqrt{5}\left(\frac{5\sqrt{5}}{2}-\frac{11}{2}\right)}\right)}{\sqrt{22+10\sqrt{5}}\,\sqrt{1-5\sqrt{5}\,x^{2}+x^{4}}}-\frac{100\sqrt{}}{}\right.}{}$

input
```
int(1/(1-5*5^(1/2)*x^2+x^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(5/242*5^(1/2)*x^3-123/242*x)/(1-5*5^(1/2)*x^2+x^4)^(1/2)-4/121/(22+10*
5^(1/2))^(1/2)*(1-(11/2+5/2*5^(1/2))*x^2)^(1/2)*(1-(5/2*5^(1/2)-11/2)*x^2)
^(1/2)/(1-5*5^(1/2)*x^2+x^4)^(1/2)*EllipticF(1/2*(22+10*5^(1/2))^(1/2)*x,(
-1+5*5^(1/2)*(5/2*5^(1/2)-11/2))^(1/2))-20/121*5^(1/2)/(22+10*5^(1/2))^(1/
2)*(1-(11/2+5/2*5^(1/2))*x^2)^(1/2)*(1-(5/2*5^(1/2)-11/2)*x^2)^(1/2)/(1-5*
5^(1/2)*x^2+x^4)^(1/2)/(11-5*5^(1/2))*(EllipticF(1/2*(22+10*5^(1/2))^(1/2)
*x,(-1+5*5^(1/2)*(5/2*5^(1/2)-11/2))^(1/2))-EllipticE(1/2*(22+10*5^(1/2))^
(1/2)*x,(-1+5*5^(1/2)*(5/2*5^(1/2)-11/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(1-5\sqrt{5}x^2+x^4\right)^{3/2}}\,dx =$$

$$-\frac{5\left(25\,x^{8}-3075\,x^{4}+11\,\sqrt{5}(x^{8}-123\,x^{4}+1)+25\right)\sqrt{\frac{5}{2}\,\sqrt{5}+\frac{11}{2}}\,E(\arcsin\left(x\sqrt{\frac{5}{2}\,\sqrt{5}+\frac{11}{2}}\right)\,|-\frac{55}{2}\,\sqrt{5}+\frac{12}{2}}{}$$

input
```
integrate(1/(1-5*5^(1/2)*x^2+x^4)^(3/2),x, algorithm="fricas")
```

output
```
-1/242*(5*(25*x^8 - 3075*x^4 + 11*sqrt(5)*(x^8 - 123*x^4 + 1) + 25)*sqrt(5
/2*sqrt(5) + 11/2)*elliptic_e(arcsin(x*sqrt(5/2*sqrt(5) + 11/2)), -55/2*sq
rt(5) + 123/2) - 3*(49*x^8 - 6027*x^4 + 15*sqrt(5)*(x^8 - 123*x^4 + 1) + 4
9)*sqrt(5/2*sqrt(5) + 11/2)*elliptic_f(arcsin(x*sqrt(5/2*sqrt(5) + 11/2)),
 -55/2*sqrt(5) + 123/2) + 2*(2*x^5 + 5*sqrt(5)*(x^7 - 122*x^3) - 123*x)*sq
rt(x^4 - 5*sqrt(5)*x^2 + 1))/(x^8 - 123*x^4 + 1)
```

Sympy [F]

$$\int \frac{1}{\left(1-5\sqrt{5}x^2+x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(x^4-5\sqrt{5}x^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1-5*5**(1/2)*x**2+x**4)**(3/2),x)
```

output
```
Integral((x**4 - 5*sqrt(5)*x**2 + 1)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(1-5\sqrt{5}x^2+x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(x^4-5\,\sqrt{5}x^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1-5*5^(1/2)*x^2+x^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 - 5*sqrt(5)*x^2 + 1)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(1-5\sqrt{5}x^2+x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(x^4-5\,\sqrt{5}x^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1-5*5^(1/2)*x^2+x^4)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 - 5*sqrt(5)*x^2 + 1)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1-5\sqrt{5}x^2+x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(x^4-5\,\sqrt{5}\,x^2+1\right)^{3/2}}\,dx$$

input
```
int(1/(x^4 - 5*5^(1/2)*x^2 + 1)^(3/2),x)
```

output
```
int(1/(x^4 - 5*5^(1/2)*x^2 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(1-5\sqrt{5}x^2+x^4\right)^{3/2}}\,dx=\frac{10\sqrt{-5\sqrt{5}\,x^2+x^4+1}\,\sqrt{5}\,x+11\sqrt{-5\sqrt{5}\,x^2+x^4+1}\,x^3-5\sqrt{5}\left(\int \frac{\sqrt{-}}{x^{16}-246x}\right.}{}$$

input
```
int(1/(1-5*5^(1/2)*x^2+x^4)^(3/2),x)
```

output
```
(10*sqrt( - 5*sqrt(5)*x**2 + x**4 + 1)*sqrt(5)*x + 11*sqrt( - 5*sqrt(5)*x*
*2 + x**4 + 1)*x**3 - 5*sqrt(5)*int(sqrt( - 5*sqrt(5)*x**2 + x**4 + 1)/(x*
*16 - 246*x**12 + 15131*x**8 - 246*x**4 + 1),x)*x**8 + 615*sqrt(5)*int(sqr
t( - 5*sqrt(5)*x**2 + x**4 + 1)/(x**16 - 246*x**12 + 15131*x**8 - 246*x**4
 + 1),x)*x**4 - 5*sqrt(5)*int(sqrt( - 5*sqrt(5)*x**2 + x**4 + 1)/(x**16 -
246*x**12 + 15131*x**8 - 246*x**4 + 1),x) - 1770*sqrt(5)*int((sqrt( - 5*sq
rt(5)*x**2 + x**4 + 1)*x**4)/(x**16 - 246*x**12 + 15131*x**8 - 246*x**4 +
1),x)*x**8 + 217710*sqrt(5)*int((sqrt( - 5*sqrt(5)*x**2 + x**4 + 1)*x**4)/
(x**16 - 246*x**12 + 15131*x**8 - 246*x**4 + 1),x)*x**4 - 1770*sqrt(5)*int
((sqrt( - 5*sqrt(5)*x**2 + x**4 + 1)*x**4)/(x**16 - 246*x**12 + 15131*x**8
 - 246*x**4 + 1),x) + 33*int((sqrt( - 5*sqrt(5)*x**2 + x**4 + 1)*x**10)/(x
**16 - 246*x**12 + 15131*x**8 - 246*x**4 + 1),x)*x**8 - 4059*int((sqrt( -
5*sqrt(5)*x**2 + x**4 + 1)*x**10)/(x**16 - 246*x**12 + 15131*x**8 - 246*x*
*4 + 1),x)*x**4 + 33*int((sqrt( - 5*sqrt(5)*x**2 + x**4 + 1)*x**10)/(x**16
 - 246*x**12 + 15131*x**8 - 246*x**4 + 1),x) + 467*int((sqrt( - 5*sqrt(5)*
x**2 + x**4 + 1)*x**2)/(x**16 - 246*x**12 + 15131*x**8 - 246*x**4 + 1),x)*
x**8 - 57441*int((sqrt( - 5*sqrt(5)*x**2 + x**4 + 1)*x**2)/(x**16 - 246*x*
*12 + 15131*x**8 - 246*x**4 + 1),x)*x**4 + 467*int((sqrt( - 5*sqrt(5)*x**2
 + x**4 + 1)*x**2)/(x**16 - 246*x**12 + 15131*x**8 - 246*x**4 + 1),x))/(5*
sqrt(5)*(x**8 - 123*x**4 + 1))
```

3.263 $\int \frac{1}{(-3+7x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{1}{(-3+7x^2-2x^4)^{3/2}}\,dx = -\frac{x(37-14x^2)}{75\sqrt{-3+7x^2-2x^4}} + \frac{7E\left(\arccos\left(\frac{x}{\sqrt{3}}\right)|\frac{6}{5}\right)}{15\sqrt{5}} - \frac{\operatorname{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{3}}\right),\frac{6}{5}\right)}{15\sqrt{5}}$$

output
```
-1/75*x*(-14*x^2+37)/(-2*x^4+7*x^2-3)^(1/2)+7/75*EllipticE(1/3*(-3*x^2+9)^
(1/2),1/5*30^(1/2))*5^(1/2)-1/75*InverseJacobiAM(arccos(1/3*x*3^(1/2)),1/5
*30^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 6.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42

$$\int \frac{1}{(-3+7x^2-2x^4)^{3/2}}\,dx = \frac{-37x+14x^3+7\sqrt{6-12x^2}\sqrt{3-x^2}E\left(\arcsin\left(\sqrt{2}x\right)|\frac{1}{6}\right)-5\sqrt{6-12x^2}\sqrt{3-}}{75\sqrt{-3+7x^2-2x^4}}$$

input
```
Integrate[(-3 + 7*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(-37*x + 14*x^3 + 7*Sqrt[6 - 12*x^2]*Sqrt[3 - x^2]*EllipticE[ArcSin[Sqrt[2
]*x], 1/6] - 5*Sqrt[6 - 12*x^2]*Sqrt[3 - x^2]*EllipticF[ArcSin[Sqrt[2]*x],
 1/6])/(75*Sqrt[-3 + 7*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 322, 328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+7x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{75}\int \frac{2(6-7x^2)}{\sqrt{-2x^4+7x^2-3}}dx-\frac{x(37-14x^2)}{75\sqrt{-2x^4+7x^2-3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{75}\int \frac{6-7x^2}{\sqrt{-2x^4+7x^2-3}}dx-\frac{x(37-14x^2)}{75\sqrt{-2x^4+7x^2-3}}$$

$$\Big\downarrow \text{ 1494}$$

$$\frac{4}{75}\sqrt{2}\int \frac{6-7x^2}{2\sqrt{2}\sqrt{3-x^2}\sqrt{2x^2-1}}dx-\frac{x(37-14x^2)}{75\sqrt{-2x^4+7x^2-3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{75}\int \frac{6-7x^2}{\sqrt{3-x^2}\sqrt{2x^2-1}}dx-\frac{x(37-14x^2)}{75\sqrt{-2x^4+7x^2-3}}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{2}{75}\left(\frac{5}{2}\int \frac{1}{\sqrt{3-x^2}\sqrt{2x^2-1}}dx-\frac{7}{2}\int \frac{\sqrt{2x^2-1}}{\sqrt{3-x^2}}dx\right)-\frac{x(37-14x^2)}{75\sqrt{-2x^4+7x^2-3}}$$

$$\Big\downarrow \text{ 322}$$

$$\frac{2}{75}\left(-\frac{7}{2}\int \frac{\sqrt{2x^2-1}}{\sqrt{3-x^2}}dx-\frac{1}{2}\sqrt{5}\,\text{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{3}}\right),\frac{6}{5}\right)\right)-\frac{x(37-14x^2)}{75\sqrt{-2x^4+7x^2-3}}$$

$$\Big\downarrow \text{328}$$

$$\frac{2}{75}\left(\frac{7}{2}\sqrt{5}E\left(\arccos\left(\frac{x}{\sqrt{3}}\right)|\frac{6}{5}\right)-\frac{1}{2}\sqrt{5}\,\text{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{3}}\right),\frac{6}{5}\right)\right)-\frac{x(37-14x^2)}{75\sqrt{-2x^4+7x^2-3}}$$

input
```
Int[(-3 + 7*x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/75*(x*(37 - 14*x^2))/Sqrt[-3 + 7*x^2 - 2*x^4] + (2*((7*Sqrt[5]*Elliptic
E[ArcCos[x/Sqrt[3]], 6/5])/2 - (Sqrt[5]*EllipticF[ArcCos[x/Sqrt[3]], 6/5])
/2))/75
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(65) = 130$.

Time = 2.74 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.87

method	result
risch	$\frac{x(14x^2-37)}{75\sqrt{-2x^4+7x^2-3}}+\frac{4\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{-2x^2+1}\;\text{EllipticF}\left(\frac{\sqrt{3}\,x}{3},\sqrt{6}\right)}{75\sqrt{-2x^4+7x^2-3}}-\frac{7\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{-2x^2+1}\,\left(\text{EllipticF}\left(\frac{\sqrt{3}\,x}{3},\sqrt{6}\right)-\text{Ellip}\right.}{225\sqrt{-2x^4+7x^2-3}}$
default	$\frac{-\frac{37}{75}x+\frac{14}{75}x^3}{\sqrt{-2x^4+7x^2-3}}+\frac{4\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{-2x^2+1}\;\text{EllipticF}\left(\frac{\sqrt{3}\,x}{3},\sqrt{6}\right)}{75\sqrt{-2x^4+7x^2-3}}-\frac{7\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{-2x^2+1}\,\left(\text{EllipticF}\left(\frac{\sqrt{3}\,x}{3},\sqrt{6}\right)-\text{Elliptic}\right.}{225\sqrt{-2x^4+7x^2-3}}$
elliptic	$\frac{-\frac{37}{75}x+\frac{14}{75}x^3}{\sqrt{-2x^4+7x^2-3}}+\frac{4\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{-2x^2+1}\;\text{EllipticF}\left(\frac{\sqrt{3}\,x}{3},\sqrt{6}\right)}{75\sqrt{-2x^4+7x^2-3}}-\frac{7\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{-2x^2+1}\,\left(\text{EllipticF}\left(\frac{\sqrt{3}\,x}{3},\sqrt{6}\right)-\text{Elliptic}\right.}{225\sqrt{-2x^4+7x^2-3}}$

input
```
int(1/(-2*x^4+7*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/75*x*(14*x^2-37)/(-2*x^4+7*x^2-3)^(1/2)+4/75*3^(1/2)*(-3*x^2+9)^(1/2)*(-
2*x^2+1)^(1/2)/(-2*x^4+7*x^2-3)^(1/2)*EllipticF(1/3*3^(1/2)*x,6^(1/2))-7/2
25*3^(1/2)*(-3*x^2+9)^(1/2)*(-2*x^2+1)^(1/2)/(-2*x^4+7*x^2-3)^(1/2)*(Ellip
ticF(1/3*3^(1/2)*x,6^(1/2))-EllipticE(1/3*3^(1/2)*x,6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int \frac{1}{(-3+7x^2-2x^4)^{3/2}}\,dx = -\frac{14\sqrt{2}\sqrt{-3}(2x^4-7x^2+3)E(\arcsin(\sqrt{2}x)\mid\frac{1}{6}) - 12\sqrt{2}\sqrt{-3}(2x^4-7x^2+3)F(\arcsin(\sqrt{2}x)\mid\frac{1}{6}) + \sqrt{-}}{75(2x^4-7x^2+3)}$$

input
```
integrate(1/(-2*x^4+7*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
-1/75*(14*sqrt(2)*sqrt(-3)*(2*x^4 - 7*x^2 + 3)*elliptic_e(arcsin(sqrt(2)*x
), 1/6) - 12*sqrt(2)*sqrt(-3)*(2*x^4 - 7*x^2 + 3)*elliptic_f(arcsin(sqrt(2
)*x), 1/6) + sqrt(-2*x^4 + 7*x^2 - 3)*(14*x^3 - 37*x))/(2*x^4 - 7*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(-3+7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+7x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+7*x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 + 7*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3+7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+7\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+7*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 7*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3+7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+7\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+7*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 7*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3+7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+7\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(7*x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(7*x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3+7x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+7x^2-3}}{4x^8-28x^6+61x^4-42x^2+9}dx$$

input
```
int(1/(-2*x^4+7*x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + 7*x**2 - 3)/(4*x**8 - 28*x**6 + 61*x**4 - 42*x**2 + 9
),x)
```

3.264 $\int \frac{1}{(-3+6x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 127

$$\int \frac{1}{(-3+6x^2-2x^4)^{3/2}}\,dx = -\frac{x(2-x^2)}{3\sqrt{-3+6x^2-2x^4}} + \frac{E\left(\arccos\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}3^{3/4}} + \frac{\left(1-\sqrt{3}\right)\operatorname{EllipticF}\left(\arccos\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{6\sqrt{2}\sqrt[4]{3}}$$

output
```
-1/3*x*(-x^2+2)/(-2*x^4+6*x^2-3)^(1/2)+1/6*EllipticE(1/3*(9-(9-3*3^(1/2))*
x^2)^(1/2),1/2*(2+2*3^(1/2))^(1/2))*2^(1/2)*3^(1/4)+1/36*(1-3^(1/2))*Inver
seJacobiAM(arccos(1/3*(9-3*3^(1/2))^(1/2)*x),1/2*(2+2*3^(1/2))^(1/2))*2^(1
/2)*3^(3/4)
```

Mathematica [A] (warning: unable to verify)

Time = 6.50 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.39

$$\int \frac{1}{(-3+6x^2-2x^4)^{3/2}}\,dx = \frac{4x(-2+x^2)+\sqrt{2}(1+\sqrt{3})\,\sqrt{3-\sqrt{3}-2x^2}\sqrt{3+(-3+\sqrt{3})\,x^2}E\Big(\arcsin\Big(}{}$$

input
```
Integrate[(-3 + 6*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(4*x*(-2 + x^2) + Sqrt[2]*(1 + Sqrt[3])*Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt[3 +
 (-3 + Sqrt[3])*x^2]*EllipticE[ArcSin[Sqrt[1 + 1/Sqrt[3]]*x], 2 - Sqrt[3]]
 - (Sqrt[2]*(3 + Sqrt[3])*Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt[3 + (-3 + Sqrt[3]
)*x^2]*EllipticF[ArcSin[Sqrt[1 + 1/Sqrt[3]]*x], 2 - Sqrt[3]])/3)/(12*Sqrt[
-3 + 6*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 322, 328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+6x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \ 1405$$

$$\frac{1}{36}\int \frac{12(1-x^2)}{\sqrt{-2x^4+6x^2-3}}dx - \frac{x(2-x^2)}{3\sqrt{-2x^4+6x^2-3}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\int \frac{1-x^2}{\sqrt{-2x^4+6x^2-3}}dx - \frac{x(2-x^2)}{3\sqrt{-2x^4+6x^2-3}}$$

$$\Big\downarrow \ 1494$$

$$\frac{2}{3}\sqrt{2}\int \frac{1-x^2}{2\sqrt{-2x^2+\sqrt{3}+3}\sqrt{2x^2+\sqrt{3}-3}}dx - \frac{x(2-x^2)}{3\sqrt{-2x^4+6x^2-3}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\sqrt{2}\int \frac{1-x^2}{\sqrt{-2x^2+\sqrt{3}+3}\sqrt{2x^2+\sqrt{3}-3}}dx - \frac{x(2-x^2)}{3\sqrt{-2x^4+6x^2-3}}$$

$$\Big\downarrow \; 399$$

$$\frac{1}{3}\sqrt{2}\left(-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{-2x^2+\sqrt{3}+3}\sqrt{2x^2+\sqrt{3}-3}}dx - \frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{3}-3}}{\sqrt{-2x^2+\sqrt{3}+3}}dx\right) - \frac{x(2-x^2)}{3\sqrt{-2x^4+6x^2-3}}$$

$$\Big\downarrow \; 322$$

$$\frac{1}{3}\sqrt{2}\left(\frac{(1-\sqrt{3})\,\text{EllipticF}\left(\arccos\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),\frac{1}{2}(1+\sqrt{3})\right)}{4\sqrt[4]{3}} - \frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{3}-3}}{\sqrt{-2x^2+\sqrt{3}+3}}dx\right) - \frac{x(2-x^2)}{3\sqrt{-2x^4+6x^2-3}}$$

$$\Big\downarrow \; 328$$

$$\frac{1}{3}\sqrt{2}\left(\frac{(1-\sqrt{3})\,\text{EllipticF}\left(\arccos\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),\frac{1}{2}(1+\sqrt{3})\right)}{4\sqrt[4]{3}} + \frac{1}{2}\sqrt[4]{3}E\left(\arccos\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|\frac{1}{2}(1+\right.\right. - \frac{x(2-x^2)}{3\sqrt{-2x^4+6x^2-3}}$$

input
```
Int[(-3 + 6*x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/3*(x*(2 - x^2))/Sqrt[-3 + 6*x^2 - 2*x^4] + (Sqrt[2]*((3^(1/4)*EllipticE
[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sqrt[3])/2])/2 + ((1 - Sqrt[3])*Ell
ipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sqrt[3])/2])/(4*3^(1/4))))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(103) = 206$.

Time = 2.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.74

method	result
risch	$\frac{x(x^2-2)}{3\sqrt{-2x^4+6x^2-3}} + \frac{\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{9-3\sqrt{3}}\,x}{3}, \frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{9-3\sqrt{3}}\,\sqrt{-2x^4+6x^2-3}} - \frac{6\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}}{\sqrt{}}$
default	$\frac{-\frac{2}{3}x+\frac{1}{3}x^3}{\sqrt{-2x^4+6x^2-3}} + \frac{\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{9-3\sqrt{3}}\,x}{3}, \frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{9-3\sqrt{3}}\,\sqrt{-2x^4+6x^2-3}} - \frac{6\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\ (}{\sqrt{9}}$
elliptic	$\frac{-\frac{2}{3}x+\frac{1}{3}x^3}{\sqrt{-2x^4+6x^2-3}} + \frac{\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{9-3\sqrt{3}}\,x}{3}, \frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{9-3\sqrt{3}}\,\sqrt{-2x^4+6x^2-3}} - \frac{6\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\ (}{\sqrt{9}}$

input
```
int(1/(-2*x^4+6*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(x^2-2)/(-2*x^4+6*x^2-3)^(1/2)+1/(9-3*3^(1/2))^(1/2)*(1-(1-1/3*3^(1/
2))*x^2)^(1/2)*(1-(1+1/3*3^(1/2))*x^2)^(1/2)/(-2*x^4+6*x^2-3)^(1/2)*Ellipt
icF(1/3*(9-3*3^(1/2))^(1/2)*x,1/2*6^(1/2)+1/2*2^(1/2))-6/(9-3*3^(1/2))^(1/
2)*(1-(1-1/3*3^(1/2))*x^2)^(1/2)*(1-(1+1/3*3^(1/2))*x^2)^(1/2)/(-2*x^4+6*x
^2-3)^(1/2)/(6+2*3^(1/2))*(EllipticF(1/3*(9-3*3^(1/2))^(1/2)*x,1/2*6^(1/2)
+1/2*2^(1/2))-EllipticE(1/3*(9-3*3^(1/2))^(1/2)*x,1/2*6^(1/2)+1/2*2^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.24

$$\int \frac{1}{(-3+6x^2-2x^4)^{3/2}}\,dx = \frac{2\sqrt{3}\sqrt{-3}(2x^4-6x^2+3)\sqrt{\frac{1}{3}\sqrt{3}+1}\,F\left(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{3}+1}\right) \mid -\sqrt{3}+2\right) -}{}$$

input
```
integrate(1/(-2*x^4+6*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/18*(2*sqrt(3)*sqrt(-3)*(2*x^4 - 6*x^2 + 3)*sqrt(1/3*sqrt(3) + 1)*ellipti
c_f(arcsin(x*sqrt(1/3*sqrt(3) + 1)), -sqrt(3) + 2) - (sqrt(3)*sqrt(-3)*(2*
x^4 - 6*x^2 + 3) + 3*sqrt(-3)*(2*x^4 - 6*x^2 + 3))*sqrt(1/3*sqrt(3) + 1)*e
lliptic_e(arcsin(x*sqrt(1/3*sqrt(3) + 1)), -sqrt(3) + 2) - 6*sqrt(-2*x^4 +
 6*x^2 - 3)*(x^3 - 2*x))/(2*x^4 - 6*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(-3+6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+6x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+6*x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 + 6*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3+6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+6\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+6*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 6*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3+6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+6\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+6*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 6*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3+6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+6\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(6*x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(6*x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3+6x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+6x^2-3}}{4x^8-24x^6+48x^4-36x^2+9}dx$$

input
```
int(1/(-2*x^4+6*x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + 6*x**2 - 3)/(4*x**8 - 24*x**6 + 48*x**4 - 36*x**2 + 9
),x)
```

3.265 $\int \frac{1}{(-3+5x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int \frac{1}{(-3+5x^2-2x^4)^{3/2}}\,dx = -\frac{x(13-10x^2)}{3\sqrt{-3+5x^2-2x^4}} + \frac{5}{3}E\left(\arccos\left(\sqrt{\frac{2}{3}}x\right)\middle|3\right) - \frac{2}{3}\operatorname{EllipticF}\left(\arccos\left(\sqrt{\frac{2}{3}}x\right),3\right)$$

output
```
-1/3*x*(-10*x^2+13)/(-2*x^4+5*x^2-3)^(1/2)+5/3*EllipticE(1/3*(-6*x^2+9)^(1
/2),3^(1/2))-2/3*InverseJacobiAM(arccos(1/3*x*6^(1/2)),3^(1/2))
```

Mathematica [A] (verified)

Time = 6.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.70

$$\int \frac{1}{(-3+5x^2-2x^4)^{3/2}}\,dx = \frac{-13x+10x^3+5\sqrt{6-4x^2}\sqrt{1-x^2}E\left(\arcsin\left(\sqrt{\frac{2}{3}}x\right)|\frac{3}{2}\right)+\sqrt{6-4x^2}\sqrt{1-x}}{3\sqrt{-3+5x^2-2x^4}}$$

input
```
Integrate[(-3 + 5*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(-13*x + 10*x^3 + 5*Sqrt[6 - 4*x^2]*Sqrt[1 - x^2]*EllipticE[ArcSin[Sqrt[2/
3]*x], 3/2] + Sqrt[6 - 4*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2/3]*x],
 3/2])/(3*Sqrt[-3 + 5*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 322, 328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+5x^2-3)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{3}\int \frac{2(6-5x^2)}{\sqrt{-2x^4+5x^2-3}}dx-\frac{x(13-10x^2)}{3\sqrt{-2x^4+5x^2-3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}\int \frac{6-5x^2}{\sqrt{-2x^4+5x^2-3}}dx-\frac{x(13-10x^2)}{3\sqrt{-2x^4+5x^2-3}}$$

$$\Big\downarrow \text{ 1494}$$

$$\frac{4}{3}\sqrt{2}\int \frac{6-5x^2}{2\sqrt{2}\sqrt{3-2x^2}\sqrt{x^2-1}}dx-\frac{x(13-10x^2)}{3\sqrt{-2x^4+5x^2-3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}\int \frac{6-5x^2}{\sqrt{3-2x^2}\sqrt{x^2-1}}dx-\frac{x(13-10x^2)}{3\sqrt{-2x^4+5x^2-3}}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{2}{3}\left(\int \frac{1}{\sqrt{3-2x^2}\sqrt{x^2-1}}dx-5\int \frac{\sqrt{x^2-1}}{\sqrt{3-2x^2}}dx\right)-\frac{x(13-10x^2)}{3\sqrt{-2x^4+5x^2-3}}$$

$$\Big\downarrow \text{ 322}$$

$$\frac{2}{3}\left(-5\int \frac{\sqrt{x^2-1}}{\sqrt{3-2x^2}}dx-\text{EllipticF}\left(\arccos\left(\sqrt{\frac{2}{3}}x\right),3\right)\right)-\frac{x(13-10x^2)}{3\sqrt{-2x^4+5x^2-3}}$$

$$\Big\downarrow \text{328}$$

$$\frac{2}{3}\left(\frac{5}{2}E\left(\arccos\left(\sqrt{\frac{2}{3}}x\right)\middle|3\right)-\text{EllipticF}\left(\arccos\left(\sqrt{\frac{2}{3}}x\right),3\right)\right)-\frac{x(13-10x^2)}{3\sqrt{-2x^4+5x^2-3}}$$

input
```
Int[(-3 + 5*x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/3*(x*(13 - 10*x^2))/Sqrt[-3 + 5*x^2 - 2*x^4] + (2*((5*EllipticE[ArcCos[
Sqrt[2/3]*x], 3])/2 - EllipticF[ArcCos[Sqrt[2/3]*x], 3]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(55) = 110.

Time = 2.67 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.28

method	result
risch	$\frac{x(10x^2-13)}{3\sqrt{-2x^4+5x^2-3}}+\frac{2\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{3\sqrt{-2x^4+5x^2-3}}-\frac{5\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{-x^2+1}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\right.}{9\sqrt{-2x^4+5x^2-3}}$
default	$\frac{\frac{10}{3}x^3-\frac{13}{3}x}{\sqrt{-2x^4+5x^2-3}}+\frac{2\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{3\sqrt{-2x^4+5x^2-3}}-\frac{5\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{-x^2+1}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}(\right.}{9\sqrt{-2x^4+5x^2-3}}$
elliptic	$\frac{\frac{10}{3}x^3-\frac{13}{3}x}{\sqrt{-2x^4+5x^2-3}}+\frac{2\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{3\sqrt{-2x^4+5x^2-3}}-\frac{5\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{-x^2+1}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}(\right.}{9\sqrt{-2x^4+5x^2-3}}$

input
```
int(1/(-2*x^4+5*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(10*x^2-13)/(-2*x^4+5*x^2-3)^(1/2)+2/3*6^(1/2)*(-6*x^2+9)^(1/2)*(-x^
2+1)^(1/2)/(-2*x^4+5*x^2-3)^(1/2)*EllipticF(1/3*x*6^(1/2),1/2*6^(1/2))-5/9
*6^(1/2)*(-6*x^2+9)^(1/2)*(-x^2+1)^(1/2)/(-2*x^4+5*x^2-3)^(1/2)*(EllipticF
(1/3*x*6^(1/2),1/2*6^(1/2))-EllipticE(1/3*x*6^(1/2),1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.36

$$\int \frac{1}{(-3+5x^2-2x^4)^{3/2}}\,dx = -\frac{5\sqrt{-3}(2x^4-5x^2+3)E(\arcsin(x)\mid\frac{2}{3}) - \sqrt{-3}(2x^4-5x^2+3)F(\arcsin(x)\mid\frac{2}{3}) + \sqrt{-2x^4+5x^2-3}(}{3(2x^4-5x^2+3)}$$

input
```
integrate(1/(-2*x^4+5*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(5*sqrt(-3)*(2*x^4 - 5*x^2 + 3)*elliptic_e(arcsin(x), 2/3) - sqrt(-3)
*(2*x^4 - 5*x^2 + 3)*elliptic_f(arcsin(x), 2/3) + sqrt(-2*x^4 + 5*x^2 - 3)
*(10*x^3 - 13*x))/(2*x^4 - 5*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(-3+5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+5x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+5*x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 + 5*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3+5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+5\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+5*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 5*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3+5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+5\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+5*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 5*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3+5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+5\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(5*x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3+5x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+5x^2-3}}{4x^8-20x^6+37x^4-30x^2+9}dx$$

input
```
int(1/(-2*x^4+5*x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + 5*x**2 - 3)/(4*x**8 - 20*x**6 + 37*x**4 - 30*x**2 + 9
),x)
```

3.266 $\int \frac{1}{(-3+4x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 246

$$\int \frac{1}{(-3+4x^2-2x^4)^{3/2}}\,dx = \frac{x(1-2x^2)}{6\sqrt{-3+4x^2-2x^4}} - \frac{x\sqrt{-3+4x^2-2x^4}}{3\left(\sqrt{6}+2x^2\right)} - \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{6^{3/4}\sqrt{-3+4x^2-2x^4}} + \frac{\left(2-\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{4\,6^{3/4}\sqrt{-3+4x^2-2x^4}}$$

output
```
1/6*x*(-2*x^2+1)/(-2*x^4+4*x^2-3)^(1/2)-x*(-2*x^4+4*x^2-3)^(1/2)/(3*6^(1/2
)+6*x^2)-1/6*(3+6^(1/2)*x^2)*((2*x^4-4*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*Ell
ipticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18+6*6^(1/2))^(1/2))*6^(1
/4)/(-2*x^4+4*x^2-3)^(1/2)+1/24*(2-6^(1/2))*(3+6^(1/2)*x^2)*((2*x^4-4*x^2+
3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x
),1/6*(18+6*6^(1/2))^(1/2))*6^(1/4)/(-2*x^4+4*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.54 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.31

$$\int \frac{1}{(-3+4x^2-2x^4)^{3/2}}\,dx = \frac{2\sqrt{-\frac{i}{2i+\sqrt{2}}}x(1-2x^2)+2i(i+\sqrt{2})\sqrt{\frac{2i+\sqrt{2}-2ix^2}{2i+\sqrt{2}}}\sqrt{\frac{-2i+\sqrt{2}+2ix^2}{-2i+\sqrt{2}}}E\Big(i\text{arcsinh}\Big(\sqrt{}}{12}$$

input
```
Integrate[(-3 + 4*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(2*Sqrt[(-I)/(2*I + Sqrt[2])]*x*(1 - 2*x^2) + (2*I)*(I + Sqrt[2])*Sqrt[(2*
I + Sqrt[2] - (2*I)*x^2)/(2*I + Sqrt[2])]*Sqrt[(-2*I + Sqrt[2] + (2*I)*x^2
)/(-2*I + Sqrt[2])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(2*I + Sqrt[2])]*x], (
2*I + Sqrt[2])/(2*I - Sqrt[2])] + (2 + I*Sqrt[2])*Sqrt[(2*I + Sqrt[2] - (2
*I)*x^2)/(2*I + Sqrt[2])]*Sqrt[(-2*I + Sqrt[2] + (2*I)*x^2)/(-2*I + Sqrt[2
])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(2*I + Sqrt[2])]*x], (2*I + Sqrt[2])/(
2*I - Sqrt[2])])/(12*Sqrt[(-I)/(2*I + Sqrt[2])]*Sqrt[-3 + 4*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+4x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x(1-2x^2)}{6\sqrt{-2x^4+4x^2-3}}-\frac{1}{24}\int \frac{4(3-2x^2)}{\sqrt{-2x^4+4x^2-3}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x(1-2x^2)}{6\sqrt{-2x^4+4x^2-3}}-\frac{1}{6}\int \frac{3-2x^2}{\sqrt{-2x^4+4x^2-3}}dx$$

$$\Big\downarrow \ 1511$$

$$\frac{1}{6}\left(-\left(\left(3-\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4+4x^2-3}}dx\right)-\sqrt{6}\int \frac{3-\sqrt{6}x^2}{3\sqrt{-2x^4+4x^2-3}}dx\right)+\frac{x(1-2x^2)}{6\sqrt{-2x^4+4x^2-3}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{6}\left(-\left(\left(3-\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4+4x^2-3}}dx\right)-\sqrt{\frac{2}{3}}\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4+4x^2-3}}dx\right)+\frac{x(1-2x^2)}{6\sqrt{-2x^4+4x^2-3}}$$

$$\Big\downarrow \ 1416$$

$$\frac{1}{6}\left(-\sqrt{\frac{2}{3}}\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4+4x^2-3}}dx-\frac{\left(3-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2x^4+4x^2-3}}\right)+\frac{x(1-2x^2)}{6\sqrt{-2x^4+4x^2-3}}$$

$$\Big\downarrow \ 1509$$

$$\frac{1}{6}\left(-\frac{\left(3-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2x^4+4x^2-3}}-\sqrt{\frac{2}{3}}\left(\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-\cdots}{(\sqrt{6}\cdots}}}{\sqrt[4]{2}\cdots}\right.\right.$$

$$\frac{x(1-2x^2)}{6\sqrt{-2x^4+4x^2-3}}$$

input
```
Int[(-3 + 4*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(1 - 2*x^2))/(6*Sqrt[-3 + 4*x^2 - 2*x^4]) + (-(Sqrt[2/3]*((3*x*Sqrt[-3
+ 4*x^2 - 2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 -
 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], 1/
2 + 1/Sqrt[6]])/(2^(1/4)*Sqrt[-3 + 4*x^2 - 2*x^4]))) - ((3 - Sqrt[6])*(3 +
 Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*Ar
cTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-3 + 4*x^2 - 2*x^4]
))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{x(2x^2-1)}{6\sqrt{-2x^4+4x^2-3}}-\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{2\sqrt{6-3i\sqrt{2}}\,\sqrt{-2x^4+4x^2-3}}+\frac{6\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}\cdots\right.}}{\cdots}$
default	$\frac{-\frac{1}{3}x^3+\frac{1}{6}x}{\sqrt{-2x^4+4x^2-3}}-\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{2\sqrt{6-3i\sqrt{2}}\,\sqrt{-2x^4+4x^2-3}}+\frac{6\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}+\cdots\right.}}{\cdots}$
elliptic	$\frac{-\frac{1}{3}x^3+\frac{1}{6}x}{\sqrt{-2x^4+4x^2-3}}-\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{2\sqrt{6-3i\sqrt{2}}\,\sqrt{-2x^4+4x^2-3}}+\frac{6\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}+\cdots\right.}}{\cdots}$

input
```
int(1/(-2*x^4+4*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x*(2*x^2-1)/(-2*x^4+4*x^2-3)^(1/2)-3/2/(6-3*I*2^(1/2))^(1/2)*(1-(2/3-
1/3*I*2^(1/2))*x^2)^(1/2)*(1-(2/3+1/3*I*2^(1/2))*x^2)^(1/2)/(-2*x^4+4*x^2-
3)^(1/2)*EllipticF(1/3*(6-3*I*2^(1/2))^(1/2)*x,1/3*(3+6*I*2^(1/2))^(1/2))+
6/(6-3*I*2^(1/2))^(1/2)*(1-(2/3-1/3*I*2^(1/2))*x^2)^(1/2)*(1-(2/3+1/3*I*2^
(1/2))*x^2)^(1/2)/(-2*x^4+4*x^2-3)^(1/2)/(4+2*I*2^(1/2))*(EllipticF(1/3*(6
-3*I*2^(1/2))^(1/2)*x,1/3*(3+6*I*2^(1/2))^(1/2))-EllipticE(1/3*(6-3*I*2^(1
/2))^(1/2)*x,1/3*(3+6*I*2^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-3+4x^2-2x^4)^{3/2}}\,dx = \frac{2\left(\sqrt{-2}\sqrt{-3}(2\,x^4-4\,x^2+3)+2\,\sqrt{-3}(2\,x^4-4\,x^2+3)\right)\sqrt{\frac{1}{3}\,\sqrt{-2}+\frac{2}{3}}E(\mathrm{arcs}$$

input
```
integrate(1/(-2*x^4+4*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/36*(2*(sqrt(-2)*sqrt(-3)*(2*x^4 - 4*x^2 + 3) + 2*sqrt(-3)*(2*x^4 - 4*x^2
 + 3))*sqrt(1/3*sqrt(-2) + 2/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(-2) + 2/
3)), -2/3*sqrt(-2) + 1/3) - (5*sqrt(-2)*sqrt(-3)*(2*x^4 - 4*x^2 + 3) - 2*s
qrt(-3)*(2*x^4 - 4*x^2 + 3))*sqrt(1/3*sqrt(-2) + 2/3)*elliptic_f(arcsin(x*
sqrt(1/3*sqrt(-2) + 2/3)), -2/3*sqrt(-2) + 1/3) + 6*sqrt(-2*x^4 + 4*x^2 -
3)*(2*x^3 - x))/(2*x^4 - 4*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(-3+4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+4x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+4*x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 + 4*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3+4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+4\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+4*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 4*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(-3+4x^2-2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-2\,x^4+4\,x^2-3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+4*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 4*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-3+4x^2-2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-2\,x^4+4\,x^2-3\right)^{3/2}}\,dx$$

input
```
int(1/(4*x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(4*x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(-3+4x^2-2x^4\right)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+4x^2-3}}{4x^8-16x^6+28x^4-24x^2+9}dx$$

input
```
int(1/(-2*x^4+4*x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + 4*x**2 - 3)/(4*x**8 - 16*x**6 + 28*x**4 - 24*x**2 + 9
),x)
```

3.267 $\int \frac{1}{(-3+3x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 257

$$\int \frac{1}{(-3+3x^2-2x^4)^{3/2}}\,dx = -\frac{x(1+2x^2)}{15\sqrt{-3+3x^2-2x^4}}-\frac{2x\sqrt{-3+3x^2-2x^4}}{15\left(\sqrt{6}+2x^2\right)} - \frac{\sqrt[4]{2}(3+\sqrt{6}x^2)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}(4+\sqrt{6})\right)}{5\ 3^{3/4}\sqrt{-3+3x^2-2x^4}} + \frac{\left(3-2\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}(4+\sqrt{6})\right)}{15\ 6^{3/4}\sqrt{-3+3x^2-2x^4}}$$

output

```
-1/15*x*(2*x^2+1)/(-2*x^4+3*x^2-3)^(1/2)-2*x*(-2*x^4+3*x^2-3)^(1/2)/(15*6^
(1/2)+30*x^2)-1/15*2^(1/4)*(3+6^(1/2)*x^2)*((2*x^4-3*x^2+3)/(3+6^(1/2)*x^2
)^2)^(1/2)*EllipticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/4*(8+2*6^(1/2)
)^(1/2))*3^(1/4)/(-2*x^4+3*x^2-3)^(1/2)+1/90*(3-2*6^(1/2))*(3+6^(1/2)*x^2)
*((2*x^4-3*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^
(1/4)*3^(3/4)*x),1/4*(8+2*6^(1/2))^(1/2))*6^(1/4)/(-2*x^4+3*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.57 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.25

$$\int \frac{1}{(-3+3x^2-2x^4)^{3/2}}\,dx = \frac{-4\sqrt{-\frac{i}{3i+\sqrt{15}}}x(1+2x^2)-\left(-3i+\sqrt{15}\right)\sqrt{\frac{3i+\sqrt{15}-4ix^2}{3i+\sqrt{15}}}\sqrt{\frac{-3i+\sqrt{15}+4ix^2}{-3i+\sqrt{15}}}E\Big(i\text{arc}$$

input
```
Integrate[(-3 + 3*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(-4*Sqrt[(-I)/(3*I + Sqrt[15])]*x*(1 + 2*x^2) - (-3*I + Sqrt[15])*Sqrt[(3*
I + Sqrt[15] - (4*I)*x^2)/(3*I + Sqrt[15])]*Sqrt[(-3*I + Sqrt[15] + (4*I)*
x^2)/(-3*I + Sqrt[15])]*EllipticE[I*ArcSinh[2*Sqrt[(-I)/(3*I + Sqrt[15])]*
x], (3*I + Sqrt[15])/(3*I - Sqrt[15])] + (5*I + Sqrt[15])*Sqrt[(3*I + Sqrt
[15] - (4*I)*x^2)/(3*I + Sqrt[15])]*Sqrt[(-3*I + Sqrt[15] + (4*I)*x^2)/(-3
*I + Sqrt[15])]*EllipticF[I*ArcSinh[2*Sqrt[(-I)/(3*I + Sqrt[15])]*x], (3*I
 + Sqrt[15])/(3*I - Sqrt[15])])/(60*Sqrt[(-I)/(3*I + Sqrt[15])]*Sqrt[-3 +
3*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+3x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$-\frac{1}{45}\int \frac{6(2-x^2)}{\sqrt{-2x^4+3x^2-3}}dx-\frac{x(2x^2+1)}{15\sqrt{-2x^4+3x^2-3}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2}{15}\int \frac{2-x^2}{\sqrt{-2x^4+3x^2-3}}dx-\frac{x(2x^2+1)}{15\sqrt{-2x^4+3x^2-3}}$$

$$\Big\downarrow \text{1511}$$

$$-\frac{2}{15}\left(\frac{1}{2}\left(4-\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4+3x^2-3}}dx+\sqrt{\frac{3}{2}}\int \frac{3-\sqrt{6}x^2}{3\sqrt{-2x^4+3x^2-3}}dx\right)-\frac{x(2x^2+1)}{15\sqrt{-2x^4+3x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{15}\left(\frac{1}{2}\left(4-\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4+3x^2-3}}dx+\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4+3x^2-3}}dx}{\sqrt{6}}\right)-\frac{x(2x^2+1)}{15\sqrt{-2x^4+3x^2-3}}$$

$$\Big\downarrow \text{1416}$$

$$-\frac{2}{15}\left(\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4+3x^2-3}}dx}{\sqrt{6}}+\frac{\left(4-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{-2x^4+3x^2-3}}\right)-\frac{x(2x^2+1)}{15\sqrt{-2x^4+3x^2-3}}$$

$$\Big\downarrow \text{1509}$$

$$-\frac{2}{15}\left(\frac{\left(4-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{-2x^4+3x^2-3}}+\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\cdots}{\sqrt[4]{2}\sqrt{-\cdots}}\right.$$

$$\frac{x(2x^2+1)}{15\sqrt{-2x^4+3x^2-3}}$$

input
```
Int[(-3 + 3*x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/15*(x*(1 + 2*x^2))/Sqrt[-3 + 3*x^2 - 2*x^4] - (2*(((3*x*Sqrt[-3 + 3*x^2
 - 2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2
+ 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt
[6])/8])/(2^(1/4)*Sqrt[-3 + 3*x^2 - 2*x^4]))/Sqrt[6] + ((4 - Sqrt[6])*(3 +
 Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*Ar
cTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(4*6^(1/4)*Sqrt[-3 + 3*x^2 - 2*x^4]
)))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{x(2x^2+1)}{15\sqrt{-2x^4+3x^2-3}}-\frac{8\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{18-6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{5\sqrt{18-6i\sqrt{15}}\,\sqrt{-2x^4+3x^2-3}}+\frac{24\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}}{}$
default	$\frac{-\frac{1}{15}x-\frac{2}{15}x^3}{\sqrt{-2x^4+3x^2-3}}-\frac{8\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{18-6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{5\sqrt{18-6i\sqrt{15}}\,\sqrt{-2x^4+3x^2-3}}+\frac{24\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-}}{}$
elliptic	$\frac{-\frac{1}{15}x-\frac{2}{15}x^3}{\sqrt{-2x^4+3x^2-3}}-\frac{8\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{18-6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{5\sqrt{18-6i\sqrt{15}}\,\sqrt{-2x^4+3x^2-3}}+\frac{24\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\sqrt{1-}}{}$

input
```
int(1/(-2*x^4+3*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*x*(2*x^2+1)/(-2*x^4+3*x^2-3)^(1/2)-8/5/(18-6*I*15^(1/2))^(1/2)*(1-(1
/2-1/6*I*15^(1/2))*x^2)^(1/2)*(1-(1/2+1/6*I*15^(1/2))*x^2)^(1/2)/(-2*x^4+3
*x^2-3)^(1/2)*EllipticF(1/6*(18-6*I*15^(1/2))^(1/2)*x,1/2*(-1+I*15^(1/2))^
(1/2))+24/5/(18-6*I*15^(1/2))^(1/2)*(1-(1/2-1/6*I*15^(1/2))*x^2)^(1/2)*(1-
(1/2+1/6*I*15^(1/2))*x^2)^(1/2)/(-2*x^4+3*x^2-3)^(1/2)/(3+I*15^(1/2))*(Ell
ipticF(1/6*(18-6*I*15^(1/2))^(1/2)*x,1/2*(-1+I*15^(1/2))^(1/2))-EllipticE(
1/6*(18-6*I*15^(1/2))^(1/2)*x,1/2*(-1+I*15^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.68

$$\int \frac{1}{(-3+3x^2-2x^4)^{3/2}}\,dx = \frac{\left(\sqrt{-\frac{5}{3}}\sqrt{-3}(2\,x^4-3\,x^2+3)+\sqrt{-3}(2\,x^4-3\,x^2+3)\right)\sqrt{\frac{1}{2}\sqrt{-\frac{5}{3}}+\frac{1}{2}}E(\mathrm{arcsi}}{}$$

input
```
integrate(1/(-2*x^4+3*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/30*((sqrt(-5/3)*sqrt(-3)*(2*x^4 - 3*x^2 + 3) + sqrt(-3)*(2*x^4 - 3*x^2 +
 3))*sqrt(1/2*sqrt(-5/3) + 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(-5/3) +
1/2)), -3/4*sqrt(-5/3) - 1/4) - (3*sqrt(-5/3)*sqrt(-3)*(2*x^4 - 3*x^2 + 3)
 - sqrt(-3)*(2*x^4 - 3*x^2 + 3))*sqrt(1/2*sqrt(-5/3) + 1/2)*elliptic_f(arc
sin(x*sqrt(1/2*sqrt(-5/3) + 1/2)), -3/4*sqrt(-5/3) - 1/4) + 2*sqrt(-2*x^4
+ 3*x^2 - 3)*(2*x^3 + x))/(2*x^4 - 3*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(-3+3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+3x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+3*x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 + 3*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3+3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+3\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+3*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 3*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3+3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+3\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+3*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 3*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3+3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+3\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(3*x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(3*x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3+3x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+3x^2-3}}{4x^8-12x^6+21x^4-18x^2+9}dx$$

input
```
int(1/(-2*x^4+3*x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + 3*x**2 - 3)/(4*x**8 - 12*x**6 + 21*x**4 - 18*x**2 + 9
),x)
```

3.268 $\int \frac{1}{(-3+2x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 250

$$\begin{aligned}\int \frac{1}{(-3+2x^2-2x^4)^{3/2}}\,dx &= -\frac{x(2+x^2)}{15\sqrt{-3+2x^2-2x^4}}-\frac{x\sqrt{-3+2x^2-2x^4}}{15\left(\sqrt{6}+2x^2\right)}\\ &\quad -\frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{5\ 6^{3/4}\sqrt{-3+2x^2-2x^4}}\\ &\quad +\frac{\left(1-\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{10\ 6^{3/4}\sqrt{-3+2x^2-2x^4}}\end{aligned}$$

output
```
-1/15*x*(x^2+2)/(-2*x^4+2*x^2-3)^(1/2)-x*(-2*x^4+2*x^2-3)^(1/2)/(15*6^(1/2
)+30*x^2)-1/30*(3+6^(1/2)*x^2)*((2*x^4-2*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*E
llipticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18+3*6^(1/2))^(1/2))*6^
(1/4)/(-2*x^4+2*x^2-3)^(1/2)+1/60*(1-6^(1/2))*(3+6^(1/2)*x^2)*((2*x^4-2*x^
2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4)*3^(3/4)
*x),1/6*(18+3*6^(1/2))^(1/2))*6^(1/4)/(-2*x^4+2*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.46 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.30

$$\int \frac{1}{(-3+2x^2-2x^4)^{3/2}}\,dx = \frac{-4\sqrt{-\frac{i}{i+\sqrt{5}}}x(2+x^2)-\sqrt{2}(-i+\sqrt{5})\sqrt{\frac{i+\sqrt{5}-2ix^2}{i+\sqrt{5}}}\sqrt{\frac{-i+\sqrt{5}+2ix^2}{-i+\sqrt{5}}}E\Big(i\text{arcsinh}\Big(}{60}$$

input
```
Integrate[(-3 + 2*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(-4*Sqrt[(-I)/(I + Sqrt[5])]*x*(2 + x^2) - Sqrt[2]*(-I + Sqrt[5])*Sqrt[(I
+ Sqrt[5] - (2*I)*x^2)/(I + Sqrt[5])]*Sqrt[(-I + Sqrt[5] + (2*I)*x^2)/(-I
+ Sqrt[5])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(I + Sqrt[5])]*x], (I + Sqrt[5
])/(I - Sqrt[5])] + Sqrt[2]*(5*I + Sqrt[5])*Sqrt[(I + Sqrt[5] - (2*I)*x^2)
/(I + Sqrt[5])]*Sqrt[(-I + Sqrt[5] + (2*I)*x^2)/(-I + Sqrt[5])]*EllipticF[
I*ArcSinh[Sqrt[(-2*I)/(I + Sqrt[5])]*x], (I + Sqrt[5])/(I - Sqrt[5])])/(60
*Sqrt[(-I)/(I + Sqrt[5])]*Sqrt[-3 + 2*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+2x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$-\frac{1}{60}\int \frac{4(3-x^2)}{\sqrt{-2x^4+2x^2-3}}dx-\frac{x(x^2+2)}{15\sqrt{-2x^4+2x^2-3}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{15}\int \frac{3-x^2}{\sqrt{-2x^4+2x^2-3}}dx-\frac{x(x^2+2)}{15\sqrt{-2x^4+2x^2-3}}$$

$\big\downarrow$ 1511

$$\frac{1}{15}\left(-\frac{1}{2}\left(6-\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4+2x^2-3}}dx-\sqrt{\frac{3}{2}}\int \frac{3-\sqrt{6}x^2}{3\sqrt{-2x^4+2x^2-3}}dx\right)-\frac{x(x^2+2)}{15\sqrt{-2x^4+2x^2-3}}$$

$\big\downarrow$ 27

$$\frac{1}{15}\left(-\frac{1}{2}\left(6-\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4+2x^2-3}}dx-\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4+2x^2-3}}dx}{\sqrt{6}}\right)-\frac{x(x^2+2)}{15\sqrt{-2x^4+2x^2-3}}$$

$\big\downarrow$ 1416

$$\frac{1}{15}\left(-\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4+2x^2-3}}dx}{\sqrt{6}}-\frac{\left(6-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}(6+\sqrt{6})\right)}{4\sqrt[4]{6}\sqrt{-2x^4+2x^2-3}}\right)-\frac{x(x^2+2)}{15\sqrt{-2x^4+2x^2-3}}$$

$\big\downarrow$ 1509

$$\frac{1}{15}\left(-\frac{\left(6-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}(6+\sqrt{6})\right)}{4\sqrt[4]{6}\sqrt{-2x^4+2x^2-3}}-\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}}{\sqrt[4]{2}\sqrt{}}\right.$$

$$\frac{x(x^2+2)}{15\sqrt{-2x^4+2x^2-3}}$$

input
```
Int[(-3 + 2*x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/15*(x*(2 + x^2))/Sqrt[-3 + 2*x^2 - 2*x^4] + (-(((3*x*Sqrt[-3 + 2*x^2 -
2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2
*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6]
)/12])/(2^(1/4)*Sqrt[-3 + 2*x^2 - 2*x^4]))/Sqrt[6]) - ((6 - Sqrt[6])*(3 +
Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*Arc
Tan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(4*6^(1/4)*Sqrt[-3 + 2*x^2 - 2*x^4]
))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.75 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{x(x^2+2)}{15\sqrt{-2x^4+2x^2-3}}-\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{5\sqrt{3-3i\sqrt{5}}\,\sqrt{-2x^4+2x^2-3}}+\frac{6\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-}$
default	$\frac{-\frac{2}{15}x-\frac{1}{15}x^3}{\sqrt{-2x^4+2x^2-3}}-\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{5\sqrt{3-3i\sqrt{5}}\,\sqrt{-2x^4+2x^2-3}}+\frac{6\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}+\right.}$
elliptic	$\frac{-\frac{2}{15}x-\frac{1}{15}x^3}{\sqrt{-2x^4+2x^2-3}}-\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{5\sqrt{3-3i\sqrt{5}}\,\sqrt{-2x^4+2x^2-3}}+\frac{6\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}+\right.}$

input
```
int(1/(-2*x^4+2*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*x*(x^2+2)/(-2*x^4+2*x^2-3)^(1/2)-3/5/(3-3*I*5^(1/2))^(1/2)*(1-(1/3-1
/3*I*5^(1/2))*x^2)^(1/2)*(1-(1/3+1/3*I*5^(1/2))*x^2)^(1/2)/(-2*x^4+2*x^2-3
)^(1/2)*EllipticF(1/3*(3-3*I*5^(1/2))^(1/2)*x,1/3*(-6+3*I*5^(1/2))^(1/2))+
6/5/(3-3*I*5^(1/2))^(1/2)*(1-(1/3-1/3*I*5^(1/2))*x^2)^(1/2)*(1-(1/3+1/3*I*
5^(1/2))*x^2)^(1/2)/(-2*x^4+2*x^2-3)^(1/2)/(2+2*I*5^(1/2))*(EllipticF(1/3*
(3-3*I*5^(1/2))^(1/2)*x,1/3*(-6+3*I*5^(1/2))^(1/2))-EllipticE(1/3*(3-3*I*5
^(1/2))^(1/2)*x,1/3*(-6+3*I*5^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.66

$$\int \frac{1}{(-3+2x^2-2x^4)^{3/2}}\,dx = \frac{\sqrt{-3}\left(2\,x^4-2\,x^2+\sqrt{-5}(2\,x^4-2\,x^2+3)+3\right)\sqrt{\frac{1}{3}\sqrt{-5}+\frac{1}{3}}E\left(\arcsin\left(x\sqrt{\frac{1}{3}}\right.\right.}{}$$

input
```
integrate(1/(-2*x^4+2*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/90*(sqrt(-3)*(2*x^4 - 2*x^2 + sqrt(-5)*(2*x^4 - 2*x^2 + 3) + 3)*sqrt(1/3
*sqrt(-5) + 1/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(-5) + 1/3)), -1/3*sqrt(
-5) - 2/3) + 2*sqrt(-3)*(2*x^4 - 2*x^2 - 2*sqrt(-5)*(2*x^4 - 2*x^2 + 3) +
3)*sqrt(1/3*sqrt(-5) + 1/3)*elliptic_f(arcsin(x*sqrt(1/3*sqrt(-5) + 1/3)),
 -1/3*sqrt(-5) - 2/3) + 6*sqrt(-2*x^4 + 2*x^2 - 3)*(x^3 + 2*x))/(2*x^4 - 2
*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(-3+2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+2x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+2*x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 + 2*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3+2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+2\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+2*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 2*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3+2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+2\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+2*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 2*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3+2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+2\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(2*x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(2*x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3+2x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+2x^2-3}}{4x^8-8x^6+16x^4-12x^2+9}dx$$

input
```
int(1/(-2*x^4+2*x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + 2*x**2 - 3)/(4*x**8 - 8*x**6 + 16*x**4 - 12*x**2 + 9)
,x)
```

3.269 $\int \frac{1}{(-3+x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 249

$$\int \frac{1}{(-3+x^2-2x^4)^{3/2}}\,dx = -\frac{x(11+2x^2)}{69\sqrt{-3+x^2-2x^4}}-\frac{2x\sqrt{-3+x^2-2x^4}}{69\left(\sqrt{6}+2x^2\right)} - \frac{\sqrt[4]{2}(3+\sqrt{6}x^2)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}(12+\sqrt{6})\right)}{23\ 3^{3/4}\sqrt{-3+x^2-2x^4}} + \frac{\left(1-2\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}(12+\sqrt{6})\right)}{23\ 6^{3/4}\sqrt{-3+x^2-2x^4}}$$

output
```
-1/69*x*(2*x^2+11)/(-2*x^4+x^2-3)^(1/2)-2*x*(-2*x^4+x^2-3)^(1/2)/(69*6^(1/
2)+138*x^2)-1/69*2^(1/4)*(3+6^(1/2)*x^2)*((2*x^4-x^2+3)/(3+6^(1/2)*x^2)^2)
^(1/2)*EllipticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72+6*6^(1/2))^
(1/2))*3^(1/4)/(-2*x^4+x^2-3)^(1/2)+1/138*(1-2*6^(1/2))*(3+6^(1/2)*x^2)*((
2*x^4-x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4)
*3^(3/4)*x),1/12*(72+6*6^(1/2))^(1/2))*6^(1/4)/(-2*x^4+x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.38 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.28

$$\int \frac{1}{(-3+x^2-2x^4)^{3/2}}\,dx = \frac{-4\sqrt{-\frac{i}{i+\sqrt{23}}}x(11+2x^2)-\left(-i+\sqrt{23}\right)\sqrt{\frac{i+\sqrt{23}-4ix^2}{i+\sqrt{23}}}\sqrt{\frac{-i+\sqrt{23}+4ix^2}{-i+\sqrt{23}}}E\left(i\text{arcsinh}\right.}{}$$

input
```
Integrate[(-3 + x^2 - 2*x^4)^(-3/2),x]
```

output
```
(-4*Sqrt[(-I)/(I + Sqrt[23])]*x*(11 + 2*x^2) - (-I + Sqrt[23])*Sqrt[(I + S
qrt[23] - (4*I)*x^2)/(I + Sqrt[23])]*Sqrt[(-I + Sqrt[23] + (4*I)*x^2)/(-I
+ Sqrt[23])]*EllipticE[I*ArcSinh[2*Sqrt[(-I)/(I + Sqrt[23])]*x], (I + Sqrt
[23])/(I - Sqrt[23])] + (23*I + Sqrt[23])*Sqrt[(I + Sqrt[23] - (4*I)*x^2)/
(I + Sqrt[23])]*Sqrt[(-I + Sqrt[23] + (4*I)*x^2)/(-I + Sqrt[23])]*Elliptic
F[I*ArcSinh[2*Sqrt[(-I)/(I + Sqrt[23])]*x], (I + Sqrt[23])/(I - Sqrt[23])]
)/(276*Sqrt[(-I)/(I + Sqrt[23])]*Sqrt[-3 + x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+x^2-3)^{3/2}}dx$$

$$\Big\downarrow \; 1405$$

$$-\frac{1}{69}\int \frac{2(6-x^2)}{\sqrt{-2x^4+x^2-3}}dx-\frac{x(2x^2+11)}{69\sqrt{-2x^4+x^2-3}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2}{69}\int \frac{6-x^2}{\sqrt{-2x^4+x^2-3}}dx-\frac{x(2x^2+11)}{69\sqrt{-2x^4+x^2-3}}$$

$\Big\downarrow$ 1511

$$-\frac{2}{69}\left(\frac{1}{2}\left(12-\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4+x^2-3}}dx+\sqrt{\frac{3}{2}}\int \frac{3-\sqrt{6}x^2}{3\sqrt{-2x^4+x^2-3}}dx\right)-\frac{x(2x^2+11)}{69\sqrt{-2x^4+x^2-3}}$$

$\Big\downarrow$ 27

$$-\frac{2}{69}\left(\frac{1}{2}\left(12-\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4+x^2-3}}dx+\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4+x^2-3}}dx}{\sqrt{6}}\right)-\frac{x(2x^2+11)}{69\sqrt{-2x^4+x^2-3}}$$

$\Big\downarrow$ 1416

$$-\frac{2}{69}\left(\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4+x^2-3}}dx}{\sqrt{6}}+\frac{\left(12-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{-2x^4+x^2-3}}\right)-\frac{x(2x^2+11)}{69\sqrt{-2x^4+x^2-3}}$$

$\Big\downarrow$ 1509

$$-\frac{2}{69}\left(\frac{\left(12-\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{-2x^4+x^2-3}}+\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2}{\left(\sqrt{6}x^2+\right.}}}{\sqrt[4]{}}\right.$$
$$\frac{x(2x^2+11)}{69\sqrt{-2x^4+x^2-3}}$$

input
```
Int[(-3 + x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/69*(x*(11 + 2*x^2))/Sqrt[-3 + x^2 - 2*x^4] - (2*(((3*x*Sqrt[-3 + x^2 -
2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x
^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])
/24])/(2^(1/4)*Sqrt[-3 + x^2 - 2*x^4]))/Sqrt[6] + ((12 - Sqrt[6])*(3 + Sqr
t[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan[(
2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(4*6^(1/4)*Sqrt[-3 + x^2 - 2*x^4])))/69
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{x(2x^2+11)}{69\sqrt{-2x^4+x^2-3}}-\frac{24\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{6-6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{23\sqrt{6-6i\sqrt{23}}\sqrt{-2x^4+x^2-3}}+\frac{24\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x}}{}$
default	$\frac{-\frac{11}{69}x-\frac{2}{69}x^3}{\sqrt{-2x^4+x^2-3}}-\frac{24\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{6-6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{23\sqrt{6-6i\sqrt{23}}\sqrt{-2x^4+x^2-3}}+\frac{24\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\sqrt{1}}{}$
elliptic	$\frac{-\frac{11}{69}x-\frac{2}{69}x^3}{\sqrt{-2x^4+x^2-3}}-\frac{24\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{6-6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{23\sqrt{6-6i\sqrt{23}}\sqrt{-2x^4+x^2-3}}+\frac{24\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\sqrt{1}}{}$

input
```
int(1/(-2*x^4+x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/69*x*(2*x^2+11)/(-2*x^4+x^2-3)^(1/2)-24/23/(6-6*I*23^(1/2))^(1/2)*(1-(1
/6-1/6*I*23^(1/2))*x^2)^(1/2)*(1-(1/6+1/6*I*23^(1/2))*x^2)^(1/2)/(-2*x^4+x
^2-3)^(1/2)*EllipticF(1/6*(6-6*I*23^(1/2))^(1/2)*x,1/6*(-33+3*I*23^(1/2))^
(1/2))+24/23/(6-6*I*23^(1/2))^(1/2)*(1-(1/6-1/6*I*23^(1/2))*x^2)^(1/2)*(1-
(1/6+1/6*I*23^(1/2))*x^2)^(1/2)/(-2*x^4+x^2-3)^(1/2)/(1+I*23^(1/2))*(Ellip
ticF(1/6*(6-6*I*23^(1/2))^(1/2)*x,1/6*(-33+3*I*23^(1/2))^(1/2))-EllipticE(
1/6*(6-6*I*23^(1/2))^(1/2)*x,1/6*(-33+3*I*23^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(-3+x^2-2x^4\right)^{3/2}}\,dx = \frac{\sqrt{-3}\left(2\,x^4 - x^2 + \sqrt{-23}(2\,x^4 - x^2 + 3) + 3\right)\sqrt{\frac{1}{6}\,\sqrt{-23} + \frac{1}{6}}E(\arcsin\left(x\sqrt{\frac{1}{6}\,\sqrt{}}\right.}{}$$

input
```
integrate(1/(-2*x^4+x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/414*(sqrt(-3)*(2*x^4 - x^2 + sqrt(-23)*(2*x^4 - x^2 + 3) + 3)*sqrt(1/6*s
qrt(-23) + 1/6)*elliptic_e(arcsin(x*sqrt(1/6*sqrt(-23) + 1/6)), -1/12*sqrt
(-23) - 11/12) + sqrt(-3)*(10*x^4 - 5*x^2 - 7*sqrt(-23)*(2*x^4 - x^2 + 3)
+ 15)*sqrt(1/6*sqrt(-23) + 1/6)*elliptic_f(arcsin(x*sqrt(1/6*sqrt(-23) + 1
/6)), -1/12*sqrt(-23) - 11/12) + 6*sqrt(-2*x^4 + x^2 - 3)*(2*x^3 + 11*x))/
(2*x^4 - x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{\left(-3+x^2-2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-2x^4+x^2-3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 + x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(-3+x^2-2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-2\,x^4+x^2-3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(-3+x^2-2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-2\,x^4+x^2-3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-3+x^2-2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-2\,x^4+x^2-3\right)^{3/2}}\,dx$$

input
```
int(1/(x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(-3+x^2-2x^4\right)^{3/2}}\,dx=\int \frac{\sqrt{-2x^4+x^2-3}}{4x^8-4x^6+13x^4-6x^2+9}dx$$

input
```
int(1/(-2*x^4+x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + x**2 - 3)/(4*x**8 - 4*x**6 + 13*x**4 - 6*x**2 + 9),x)
```

3.270 $\int \frac{1}{(-3-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 89

$$\int \frac{1}{(-3-2x^4)^{3/2}}\,dx = -\frac{x}{6\sqrt{-3-2x^4}} - \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}\right)}{12\sqrt[4]{6}\sqrt{-3-2x^4}}$$

output
```
-1/6*x/(-2*x^4-3)^(1/2)-1/72*(3+6^(1/2)*x^2)*((2*x^4+3)/(3+6^(1/2)*x^2)^2)
^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x),1/2*2^(1/2))*6^(3/4
)/(-2*x^4-3)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{1}{(-3-2x^4)^{3/2}}\,dx = -\frac{x\left(3+\sqrt{9+6x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{2x^4}{3}\right)\right)}{18\sqrt{-3-2x^4}}$$

input
```
Integrate[(-3 - 2*x^4)^(-3/2),x]
```

output
```
-1/18*(x*(3 + Sqrt[9 + 6*x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, (-2*x^4)/3]
))/Sqrt[-3 - 2*x^4]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-3)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 749}$$

$$-\frac{1}{6}\int \frac{1}{\sqrt{-2x^4-3}}dx-\frac{x}{6\sqrt{-2x^4-3}}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}\right)}{12\sqrt[4]{6}\sqrt{-2x^4-3}}-\frac{x}{6\sqrt{-2x^4-3}}$$

input
```
Int[(-3 - 2*x^4)^(-3/2),x]
```

output
```
-1/6*x/Sqrt[-3 - 2*x^4] - ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]
*x^2)^2]*EllipticF[2*ArcTan[(2/3)^(1/4)*x], 1/2])/(12*6^(1/4)*Sqrt[-3 - 2*
x^4])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.21

method	result	size
meijerg	$\frac{i\sqrt{3}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],-\frac{2x^{4}}{3}\right)}{9}$	19
default	$-\frac{x}{6\sqrt{-2x^{4}-3}}-\frac{\sqrt{3}\,\sqrt{9+3i\sqrt{6}\,x^{2}}\,\sqrt{9-3i\sqrt{6}\,x^{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-i\sqrt{6}}}{3},i\right)}{54\sqrt{-i\sqrt{6}}\,\sqrt{-2x^{4}-3}}$	79
risch	$-\frac{x}{6\sqrt{-2x^{4}-3}}-\frac{\sqrt{3}\,\sqrt{9+3i\sqrt{6}\,x^{2}}\,\sqrt{9-3i\sqrt{6}\,x^{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-i\sqrt{6}}}{3},i\right)}{54\sqrt{-i\sqrt{6}}\,\sqrt{-2x^{4}-3}}$	79
elliptic	$-\frac{x}{6\sqrt{-2x^{4}-3}}-\frac{\sqrt{3}\,\sqrt{9+3i\sqrt{6}\,x^{2}}\,\sqrt{9-3i\sqrt{6}\,x^{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-i\sqrt{6}}}{3},i\right)}{54\sqrt{-i\sqrt{6}}\,\sqrt{-2x^{4}-3}}$	79

input
```
int(1/(-2*x^4-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*I*3^(1/2)*x*hypergeom([1/4,3/2],[5/4],-2/3*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{1}{(-3-2x^4)^{3/2}}\,dx = -\frac{\left(-\frac{2}{3}\right)^{\frac{3}{4}}\sqrt{-3}(2\,x^4+3)F\left(\arcsin\left(\left(-\frac{2}{3}\right)^{\frac{1}{4}}x\right)|-1\right)-2\sqrt{-2\,x^4-3}x}{12\,(2\,x^4+3)}$$

input
```
integrate(1/(-2*x^4-3)^(3/2),x, algorithm="fricas")
```

output
```
-1/12*((-2/3)^(3/4)*sqrt(-3)*(2*x^4 + 3)*elliptic_f(arcsin((-2/3)^(1/4)*x)
, -1) - 2*sqrt(-2*x^4 - 3)*x)/(2*x^4 + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.42

$$\int \frac{1}{(-3-2x^4)^{3/2}}\,dx = \frac{\sqrt{3}ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4e^{i\pi}}{3}\right)}{36\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-2*x**4-3)**(3/2),x)
```

output
```
sqrt(3)*I*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), 2*x**4*exp_polar(I*pi)/3)
/(36*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(-3-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - 3)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.38

$$\int \frac{1}{(-3-2x^4)^{3/2}}\,dx = \frac{\sqrt{3}\,x\,(2\,x^4+3)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{2\,x^4}{3}\right)}{9\,(-2\,x^4-3)^{3/2}}$$

input
```
int(1/(- 2*x^4 - 3)^(3/2),x)
```

output
```
(3^(1/2)*x*(2*x^4 + 3)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(2*x^4)/3))/(9*(-
 2*x^4 - 3)^(3/2))
```

Reduce [F]

$$\int \frac{1}{(-3-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-3}}{4x^8+12x^4+9}dx$$

input
```
int(1/(-2*x^4-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - 3)/(4*x**8 + 12*x**4 + 9),x)
```

3.271 $\int \frac{1}{(-3-x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 257

$$\int \frac{1}{(-3-x^2-2x^4)^{3/2}}\,dx = -\frac{x(11-2x^2)}{69\sqrt{-3-x^2-2x^4}}+\frac{2x\sqrt{-3-x^2-2x^4}}{69\left(\sqrt{6}+2x^2\right)} + \frac{\sqrt[4]{2}(3+\sqrt{6}x^2)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}(12-\sqrt{6})\right)}{23\ 3^{3/4}\sqrt{-3-x^2-2x^4}} - \frac{\left(1+2\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}(12-\sqrt{6})\right)}{23\ 6^{3/4}\sqrt{-3-x^2-2x^4}}$$

output
```
-1/69*x*(-2*x^2+11)/(-2*x^4-x^2-3)^(1/2)+2*x*(-2*x^4-x^2-3)^(1/2)/(69*6^(1
/2)+138*x^2)+1/69*2^(1/4)*(3+6^(1/2)*x^2)*((2*x^4+x^2+3)/(3+6^(1/2)*x^2)^2
)^(1/2)*EllipticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72-6*6^(1/2))
^(1/2))*3^(1/4)/(-2*x^4-x^2-3)^(1/2)-1/138*(1+2*6^(1/2))*(3+6^(1/2)*x^2)*(
(2*x^4+x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4
)*3^(3/4)*x),1/12*(72-6*6^(1/2))^(1/2))*6^(1/4)/(-2*x^4-x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.35 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.25

$$\int \frac{1}{(-3-x^2-2x^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{-i+\sqrt{23}}}x(-11+2x^2)+\left(i+\sqrt{23}\right)\sqrt{\frac{-i+\sqrt{23}-4ix^2}{-i+\sqrt{23}}}\sqrt{\frac{i+\sqrt{23}+4ix^2}{i+\sqrt{23}}}E\Big(i\text{arcsinh}$$

input
```
Integrate[(-3 - x^2 - 2*x^4)^(-3/2),x]
```

output
```
(4*Sqrt[(-I)/(-I + Sqrt[23])]*x*(-11 + 2*x^2) + (I + Sqrt[23])*Sqrt[(-I +
Sqrt[23] - (4*I)*x^2)/(-I + Sqrt[23])]*Sqrt[(I + Sqrt[23] + (4*I)*x^2)/(I
+ Sqrt[23])]*EllipticE[I*ArcSinh[2*Sqrt[(-I)/(-I + Sqrt[23])]*x], (I - Sqr
t[23])/(I + Sqrt[23])] - (-23*I + Sqrt[23])*Sqrt[(-I + Sqrt[23] - (4*I)*x^
2)/(-I + Sqrt[23])]*Sqrt[(I + Sqrt[23] + (4*I)*x^2)/(I + Sqrt[23])]*Ellipt
icF[I*ArcSinh[2*Sqrt[(-I)/(-I + Sqrt[23])]*x], (I - Sqrt[23])/(I + Sqrt[23
])])/(276*Sqrt[(-I)/(-I + Sqrt[23])]*Sqrt[-3 - x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-x^2-3)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$-\frac{1}{69}\int \frac{2(x^2+6)}{\sqrt{-2x^4-x^2-3}}dx-\frac{x(11-2x^2)}{69\sqrt{-2x^4-x^2-3}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2}{69}\int \frac{x^2+6}{\sqrt{-2x^4-x^2-3}}dx-\frac{x(11-2x^2)}{69\sqrt{-2x^4-x^2-3}}$$

$$\Big\downarrow \; 1511$$

$$-\frac{2}{69}\left(\frac{1}{2}\left(12+\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4-x^2-3}}dx-\sqrt{\frac{3}{2}}\int \frac{3-\sqrt{6}x^2}{3\sqrt{-2x^4-x^2-3}}dx\right)-\frac{x(11-2x^2)}{69\sqrt{-2x^4-x^2-3}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2}{69}\left(\frac{1}{2}\left(12+\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4-x^2-3}}dx-\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4-x^2-3}}dx}{\sqrt{6}}\right)-\frac{x(11-2x^2)}{69\sqrt{-2x^4-x^2-3}}$$

$$\Big\downarrow \; 1416$$

$$-\frac{2}{69}\left(\frac{\left(12+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{-2x^4-x^2-3}}-\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4-x^2-3}}dx}{\sqrt{6}}\right)-$$

$$\frac{x(11-2x^2)}{69\sqrt{-2x^4-x^2-3}}$$

$$\Big\downarrow \; 1509$$

$$-\frac{2}{69}\left(\frac{\left(12+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{-2x^4-x^2-3}}-\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2}{\left(\sqrt{6}x^2+\right.}}}{\sqrt[4]{}}\right.$$

$$\frac{x(11-2x^2)}{69\sqrt{-2x^4-x^2-3}}$$

input
```
Int[(-3 - x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/69*(x*(11 - 2*x^2))/Sqrt[-3 - x^2 - 2*x^4] - (2*(-(((3*x*Sqrt[-3 - x^2
 - 2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2
*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6
])/24])/(2^(1/4)*Sqrt[-3 - x^2 - 2*x^4]))/Sqrt[6]) + ((12 + Sqrt[6])*(3 +
Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTa
n[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(4*6^(1/4)*Sqrt[-3 - x^2 - 2*x^4])))
/69
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.92

method	result
risch	$\frac{x(2x^2-11)}{69\sqrt{-2x^4-x^2-3}} - \frac{24\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-6-6i\sqrt{23}}\,x}{6}, \frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{23\sqrt{-6-6i\sqrt{23}}\,\sqrt{-2x^4-x^2-3}} - \frac{24\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{\ }}{6}\right.}}{}$
default	$\frac{-\frac{11}{69}x+\frac{2}{69}x^3}{\sqrt{-2x^4-x^2-3}} - \frac{24\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-6-6i\sqrt{23}}\,x}{6}, \frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{23\sqrt{-6-6i\sqrt{23}}\,\sqrt{-2x^4-x^2-3}} - \frac{24\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right.}}{}$
elliptic	$\frac{-\frac{11}{69}x+\frac{2}{69}x^3}{\sqrt{-2x^4-x^2-3}} - \frac{24\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-6-6i\sqrt{23}}\,x}{6}, \frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{23\sqrt{-6-6i\sqrt{23}}\,\sqrt{-2x^4-x^2-3}} - \frac{24\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right.}}{}$

input
```
int(1/(-2*x^4-x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/69*x*(2*x^2-11)/(-2*x^4-x^2-3)^(1/2)-24/23/(-6-6*I*23^(1/2))^(1/2)*(1-(-
1/6-1/6*I*23^(1/2))*x^2)^(1/2)*(1-(-1/6+1/6*I*23^(1/2))*x^2)^(1/2)/(-2*x^4
-x^2-3)^(1/2)*EllipticF(1/6*(-6-6*I*23^(1/2))^(1/2)*x,1/6*(-33-3*I*23^(1/2
))^(1/2))-24/23/(-6-6*I*23^(1/2))^(1/2)*(1-(-1/6-1/6*I*23^(1/2))*x^2)^(1/2
)*(1-(-1/6+1/6*I*23^(1/2))*x^2)^(1/2)/(-2*x^4-x^2-3)^(1/2)/(-1+I*23^(1/2))
*(EllipticF(1/6*(-6-6*I*23^(1/2))^(1/2)*x,1/6*(-33-3*I*23^(1/2))^(1/2))-El
lipticE(1/6*(-6-6*I*23^(1/2))^(1/2)*x,1/6*(-33-3*I*23^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.62

$$\int \frac{1}{(-3-x^2-2x^4)^{3/2}}\,dx = \frac{\sqrt{-3}\left(2\,x^4+x^2-\sqrt{-23}(2\,x^4+x^2+3)+3\right)\sqrt{\frac{1}{6}\sqrt{-23}-\frac{1}{6}}E\left(\arcsin\left(x\sqrt{\frac{1}{6}\sqrt{}\right.\right.}{}$$

input
```
integrate(1/(-2*x^4-x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/414*(sqrt(-3)*(2*x^4 + x^2 - sqrt(-23)*(2*x^4 + x^2 + 3) + 3)*sqrt(1/6*s
qrt(-23) - 1/6)*elliptic_e(arcsin(x*sqrt(1/6*sqrt(-23) - 1/6)), 1/12*sqrt(
-23) - 11/12) - sqrt(-3)*(14*x^4 + 7*x^2 + 5*sqrt(-23)*(2*x^4 + x^2 + 3) +
 21)*sqrt(1/6*sqrt(-23) - 1/6)*elliptic_f(arcsin(x*sqrt(1/6*sqrt(-23) - 1/
6)), 1/12*sqrt(-23) - 11/12) - 6*sqrt(-2*x^4 - x^2 - 3)*(2*x^3 - 11*x))/(2
*x^4 + x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(-3-x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 - x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3-x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-x^2-3)^{3/2}}\,dx$$

input
```
int(1/(- x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(- x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3-x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-x^2-3}}{4x^8+4x^6+13x^4+6x^2+9}dx$$

input
```
int(1/(-2*x^4-x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - x**2 - 3)/(4*x**8 + 4*x**6 + 13*x**4 + 6*x**2 + 9),x)
```

3.272 $\int \frac{1}{(-3-2x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 254

$$\int \frac{1}{(-3-2x^2-2x^4)^{3/2}}\,dx = -\frac{x(2-x^2)}{15\sqrt{-3-2x^2-2x^4}}+\frac{x\sqrt{-3-2x^2-2x^4}}{15\left(\sqrt{6}+2x^2\right)} + \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{5\ 6^{3/4}\sqrt{-3-2x^2-2x^4}} - \frac{\left(1+\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{10\ 6^{3/4}\sqrt{-3-2x^2-2x^4}}$$

output
```
-1/15*x*(-x^2+2)/(-2*x^4-2*x^2-3)^(1/2)+x*(-2*x^4-2*x^2-3)^(1/2)/(15*6^(1/
2)+30*x^2)+1/30*(3+6^(1/2)*x^2)*((2*x^4+2*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*
EllipticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18-3*6^(1/2))^(1/2))*6
^(1/4)/(-2*x^4-2*x^2-3)^(1/2)-1/60*(1+6^(1/2))*(3+6^(1/2)*x^2)*((2*x^4+2*x
^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4)*3^(3/4
)*x),1/6*(18-3*6^(1/2))^(1/2))*6^(1/4)/(-2*x^4-2*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.28

$$\int \frac{1}{(-3-2x^2-2x^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{-i+\sqrt{5}}}x(-2+x^2)+\sqrt{2}(i+\sqrt{5})\sqrt{\frac{-i+\sqrt{5}-2ix^2}{-i+\sqrt{5}}}\sqrt{\frac{i+\sqrt{5}+2ix^2}{i+\sqrt{5}}}E\Big(i\text{arcsinh}\Big(}{6}$$

input
```
Integrate[(-3 - 2*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(4*Sqrt[(-I)/(-I + Sqrt[5])]*x*(-2 + x^2) + Sqrt[2]*(I + Sqrt[5])*Sqrt[(-I
 + Sqrt[5] - (2*I)*x^2)/(-I + Sqrt[5])]*Sqrt[(I + Sqrt[5] + (2*I)*x^2)/(I
+ Sqrt[5])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[5])]*x], (I - Sqrt[
5])/(I + Sqrt[5])] - Sqrt[2]*(-5*I + Sqrt[5])*Sqrt[(-I + Sqrt[5] - (2*I)*x
^2)/(-I + Sqrt[5])]*Sqrt[(I + Sqrt[5] + (2*I)*x^2)/(I + Sqrt[5])]*Elliptic
F[I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[5])]*x], (I - Sqrt[5])/(I + Sqrt[5])])/
(60*Sqrt[(-I)/(-I + Sqrt[5])]*Sqrt[-3 - 2*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-2x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$-\frac{1}{60}\int \frac{4(x^2+3)}{\sqrt{-2x^4-2x^2-3}}dx - \frac{x(2-x^2)}{15\sqrt{-2x^4-2x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}\int \frac{x^2+3}{\sqrt{-2x^4-2x^2-3}}dx-\frac{x\left(2-x^2\right)}{15\sqrt{-2x^4-2x^2-3}}$$

$\Big\downarrow$ 1511

$$\frac{1}{15}\left(\sqrt{\frac{3}{2}}\int \frac{3-\sqrt{6}x^2}{3\sqrt{-2x^4-2x^2-3}}dx-\frac{1}{2}\left(6+\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4-2x^2-3}}dx\right)-\frac{x\left(2-x^2\right)}{15\sqrt{-2x^4-2x^2-3}}$$

$\Big\downarrow$ 27

$$\frac{1}{15}\left(\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4-2x^2-3}}dx}{\sqrt{6}}-\frac{1}{2}\left(6+\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4-2x^2-3}}dx\right)-\frac{x\left(2-x^2\right)}{15\sqrt{-2x^4-2x^2-3}}$$

$\Big\downarrow$ 1416

$$\frac{1}{15}\left(\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4-2x^2-3}}dx}{\sqrt{6}}-\frac{\left(6+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{-2x^4-2x^2-3}}\right)-\frac{x\left(2-x^2\right)}{15\sqrt{-2x^4-2x^2-3}}$$

$\Big\downarrow$ 1509

$$\frac{1}{15}\left(\frac{\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{\sqrt[4]{2}\sqrt{-2x^4-2x^2-3}}+\frac{3\sqrt{-2x^4-2x^2-3}x}{\sqrt{6}x^2+3}}{\sqrt{6}}-\frac{\left(6+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+}{\left(\sqrt{6}x^2+3\right)}}}{4\sqrt[4]{6}}\right.$$

$$\frac{x\left(2-x^2\right)}{15\sqrt{-2x^4-2x^2-3}}$$

input
```
Int[(-3 - 2*x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/15*(x*(2 - x^2))/Sqrt[-3 - 2*x^2 - 2*x^4] + (((3*x*Sqrt[-3 - 2*x^2 - 2*
x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x
^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/
12])/(2^(1/4)*Sqrt[-3 - 2*x^2 - 2*x^4]))/Sqrt[6] - ((6 + Sqrt[6])*(3 + Sqr
t[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(4*6^(1/4)*Sqrt[-3 - 2*x^2 - 2*x^4]))/
15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.93

method	result
risch	$\frac{x(x^2-2)}{15\sqrt{-2x^4-2x^2-3}} - \frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\ \text{EllipticF}\left(\frac{\sqrt{-3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{5\sqrt{-3-3i\sqrt{5}}\,\sqrt{-2x^4-2x^2-3}} - \frac{6\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}$
default	$\frac{-\frac{2}{15}x+\frac{1}{15}x^3}{\sqrt{-2x^4-2x^2-3}} - \frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\ \text{EllipticF}\left(\frac{\sqrt{-3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{5\sqrt{-3-3i\sqrt{5}}\,\sqrt{-2x^4-2x^2-3}} - \frac{6\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}$
elliptic	$\frac{-\frac{2}{15}x+\frac{1}{15}x^3}{\sqrt{-2x^4-2x^2-3}} - \frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\ \text{EllipticF}\left(\frac{\sqrt{-3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{5\sqrt{-3-3i\sqrt{5}}\,\sqrt{-2x^4-2x^2-3}} - \frac{6\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}$

input
```
int(1/(-2*x^4-2*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(x^2-2)/(-2*x^4-2*x^2-3)^(1/2)-3/5/(-3-3*I*5^(1/2))^(1/2)*(1-(-1/3-
1/3*I*5^(1/2))*x^2)^(1/2)*(1-(-1/3+1/3*I*5^(1/2))*x^2)^(1/2)/(-2*x^4-2*x^2
-3)^(1/2)*EllipticF(1/3*(-3-3*I*5^(1/2))^(1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2
))-6/5/(-3-3*I*5^(1/2))^(1/2)*(1-(-1/3-1/3*I*5^(1/2))*x^2)^(1/2)*(1-(-1/3+
1/3*I*5^(1/2))*x^2)^(1/2)/(-2*x^4-2*x^2-3)^(1/2)/(-2+2*I*5^(1/2))*(Ellipti
cF(1/3*(-3-3*I*5^(1/2))^(1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2))-EllipticE(1/3*
(-3-3*I*5^(1/2))^(1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.65

$$\int \frac{1}{(-3-2x^2-2x^4)^{3/2}}\,dx = \frac{\sqrt{-3}\left(2\,x^4+2\,x^2-\sqrt{-5}(2\,x^4+2\,x^2+3)+3\right)\sqrt{\frac{1}{3}\,\sqrt{-5}-\frac{1}{3}}E\left(\arcsin\left(x\sqrt{\frac{1}{3}}\right.\right.}{}$$

input
```
integrate(1/(-2*x^4-2*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/90*(sqrt(-3)*(2*x^4 + 2*x^2 - sqrt(-5)*(2*x^4 + 2*x^2 + 3) + 3)*sqrt(1/3
*sqrt(-5) - 1/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(-5) - 1/3)), 1/3*sqrt(-
5) - 2/3) - 2*sqrt(-3)*(4*x^4 + 4*x^2 + sqrt(-5)*(2*x^4 + 2*x^2 + 3) + 6)*
sqrt(1/3*sqrt(-5) - 1/3)*elliptic_f(arcsin(x*sqrt(1/3*sqrt(-5) - 1/3)), 1/
3*sqrt(-5) - 2/3) - 6*sqrt(-2*x^4 - 2*x^2 - 3)*(x^3 - 2*x))/(2*x^4 + 2*x^2
 + 3)
```

Sympy [F]

$$\int \frac{1}{(-3-2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-2x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-2*x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 - 2*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-2\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-2*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - 2*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-2\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-2*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - 2*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3-2x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-2\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(- 2*x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(- 2*x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3-2x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-2x^2-3}}{4x^8+8x^6+16x^4+12x^2+9}dx$$

input
```
int(1/(-2*x^4-2*x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - 2*x**2 - 3)/(4*x**8 + 8*x**6 + 16*x**4 + 12*x**2 + 9)
,x)
```

3.273 $\int \frac{1}{(-3-3x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 261

$$\int \frac{1}{(-3-3x^2-2x^4)^{3/2}}\,dx = -\frac{x(1-2x^2)}{15\sqrt{-3-3x^2-2x^4}}+\frac{2x\sqrt{-3-3x^2-2x^4}}{15\left(\sqrt{6}+2x^2\right)}+\frac{\sqrt[4]{2}(3+\sqrt{6}x^2)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{5\ 3^{3/4}\sqrt{-3-3x^2-2x^4}}-\frac{\left(3+2\sqrt{6}\right)\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{15\ 6^{3/4}\sqrt{-3-3x^2-2x^4}}$$

output
```
-1/15*x*(-2*x^2+1)/(-2*x^4-3*x^2-3)^(1/2)+2*x*(-2*x^4-3*x^2-3)^(1/2)/(15*6
^(1/2)+30*x^2)+1/15*2^(1/4)*(3+6^(1/2)*x^2)*((2*x^4+3*x^2+3)/(3+6^(1/2)*x^
2)^2)^(1/2)*EllipticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/4*(8-2*6^(1/2
))^(1/2))*3^(1/4)/(-2*x^4-3*x^2-3)^(1/2)-1/90*(3+2*6^(1/2))*(3+6^(1/2)*x^2
)*((2*x^4+3*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2
^(1/4)*3^(3/4)*x),1/4*(8-2*6^(1/2))^(1/2))*6^(1/4)/(-2*x^4-3*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.52 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(-3-3x^2-2x^4\right)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{-3i+\sqrt{15}}}x(-1+2x^2)+\left(3i+\sqrt{15}\right)\sqrt{\frac{-3i+\sqrt{15}-4ix^2}{-3i+\sqrt{15}}}\sqrt{\frac{3i+\sqrt{15}+4ix^2}{3i+\sqrt{15}}}E\Big(i\text{arcs}}{}$$

input
```
Integrate[(-3 - 3*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(4*Sqrt[(-I)/(-3*I + Sqrt[15])]*x*(-1 + 2*x^2) + (3*I + Sqrt[15])*Sqrt[(-3
*I + Sqrt[15] - (4*I)*x^2)/(-3*I + Sqrt[15])]*Sqrt[(3*I + Sqrt[15] + (4*I)
*x^2)/(3*I + Sqrt[15])]*EllipticE[I*ArcSinh[2*Sqrt[(-I)/(-3*I + Sqrt[15])]
*x], (3*I - Sqrt[15])/(3*I + Sqrt[15])] - (-5*I + Sqrt[15])*Sqrt[(-3*I + S
qrt[15] - (4*I)*x^2)/(-3*I + Sqrt[15])]*Sqrt[(3*I + Sqrt[15] + (4*I)*x^2)/
(3*I + Sqrt[15])]*EllipticF[I*ArcSinh[2*Sqrt[(-I)/(-3*I + Sqrt[15])]*x], (
3*I - Sqrt[15])/(3*I + Sqrt[15])])/(60*Sqrt[(-I)/(-3*I + Sqrt[15])]*Sqrt[-
3 - 3*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-3x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$-\frac{1}{45}\int \frac{6(x^2+2)}{\sqrt{-2x^4-3x^2-3}}dx-\frac{x(1-2x^2)}{15\sqrt{-2x^4-3x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{15}\int \frac{x^2+2}{\sqrt{-2x^4-3x^2-3}}dx-\frac{x\left(1-2x^2\right)}{15\sqrt{-2x^4-3x^2-3}}$$

$$\Big\downarrow \text{1511}$$

$$-\frac{2}{15}\left(\frac{1}{2}\left(4+\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4-3x^2-3}}dx-\sqrt{\frac{3}{2}}\int \frac{3-\sqrt{6}x^2}{3\sqrt{-2x^4-3x^2-3}}dx\right)-\frac{x\left(1-2x^2\right)}{15\sqrt{-2x^4-3x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{15}\left(\frac{1}{2}\left(4+\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4-3x^2-3}}dx-\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4-3x^2-3}}dx}{\sqrt{6}}\right)-\frac{x\left(1-2x^2\right)}{15\sqrt{-2x^4-3x^2-3}}$$

$$\Big\downarrow \text{1416}$$

$$-\frac{2}{15}\left(\frac{\left(4+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{-2x^4-3x^2-3}}-\frac{\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4-3x^2-3}}dx}{\sqrt{6}}\right)-\frac{x\left(1-2x^2\right)}{15\sqrt{-2x^4-3x^2-3}}$$

$$\Big\downarrow \text{1509}$$

$$-\frac{2}{15}\left(\frac{\left(4+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{4\sqrt[4]{6}\sqrt{-2x^4-3x^2-3}}-\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\cdots}{\sqrt[4]{2}\sqrt{-\cdots}}\right.$$

$$\frac{x\left(1-2x^2\right)}{15\sqrt{-2x^4-3x^2-3}}$$

input
```
Int[(-3 - 3*x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/15*(x*(1 - 2*x^2))/Sqrt[-3 - 3*x^2 - 2*x^4] - (2*(-(((3*x*Sqrt[-3 - 3*x
^2 - 2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^
2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], (4 - Sq
rt[6])/8])/(2^(1/4)*Sqrt[-3 - 3*x^2 - 2*x^4]))/Sqrt[6]) + ((4 + Sqrt[6])*(
3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2
*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(4*6^(1/4)*Sqrt[-3 - 3*x^2 - 2*x
^4])))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.91

method	result
risch	$\frac{x(2x^2-1)}{15\sqrt{-2x^4-3x^2-3}} - \frac{8\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-18-6i\sqrt{15}}\,x}{6}, \frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{5\sqrt{-18-6i\sqrt{15}}\sqrt{-2x^4-3x^2-3}} - \frac{24\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right.}}{}$
default	$\frac{-\frac{1}{15}x+\frac{2}{15}x^3}{\sqrt{-2x^4-3x^2-3}} - \frac{8\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-18-6i\sqrt{15}}\,x}{6}, \frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{5\sqrt{-18-6i\sqrt{15}}\sqrt{-2x^4-3x^2-3}} - \frac{24\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)}}{}$
elliptic	$\frac{-\frac{1}{15}x+\frac{2}{15}x^3}{\sqrt{-2x^4-3x^2-3}} - \frac{8\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-18-6i\sqrt{15}}\,x}{6}, \frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{5\sqrt{-18-6i\sqrt{15}}\sqrt{-2x^4-3x^2-3}} - \frac{24\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)}}{}$

input
```
int(1/(-2*x^4-3*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(2*x^2-1)/(-2*x^4-3*x^2-3)^(1/2)-8/5/(-18-6*I*15^(1/2))^(1/2)*(1-(-
1/2-1/6*I*15^(1/2))*x^2)^(1/2)*(1-(-1/2+1/6*I*15^(1/2))*x^2)^(1/2)/(-2*x^4
-3*x^2-3)^(1/2)*EllipticF(1/6*(-18-6*I*15^(1/2))^(1/2)*x,1/2*(-1-I*15^(1/2
))^(1/2))-24/5/(-18-6*I*15^(1/2))^(1/2)*(1-(-1/2-1/6*I*15^(1/2))*x^2)^(1/2
)*(1-(-1/2+1/6*I*15^(1/2))*x^2)^(1/2)/(-2*x^4-3*x^2-3)^(1/2)/(-3+I*15^(1/2
))*(EllipticF(1/6*(-18-6*I*15^(1/2))^(1/2)*x,1/2*(-1-I*15^(1/2))^(1/2))-El
lipticE(1/6*(-18-6*I*15^(1/2))^(1/2)*x,1/2*(-1-I*15^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.67

$$\int \frac{1}{(-3-3x^2-2x^4)^{3/2}}\,dx =$$

$$-\frac{\left(\sqrt{-\frac{5}{3}}\sqrt{-3}(2\,x^4+3\,x^2+3)-\sqrt{-3}(2\,x^4+3\,x^2+3)\right)\sqrt{\frac{1}{2}\sqrt{-\frac{5}{3}}-\frac{1}{2}}E(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-\frac{5}{3}}-\frac{1}{2}}\right)\mid\frac{3}{4}\sqrt{-}}{}$$

input
```
integrate(1/(-2*x^4-3*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
-1/30*((sqrt(-5/3)*sqrt(-3)*(2*x^4 + 3*x^2 + 3) - sqrt(-3)*(2*x^4 + 3*x^2
+ 3))*sqrt(1/2*sqrt(-5/3) - 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(-5/3) -
 1/2)), 3/4*sqrt(-5/3) - 1/4) + (sqrt(-5/3)*sqrt(-3)*(2*x^4 + 3*x^2 + 3) +
 3*sqrt(-3)*(2*x^4 + 3*x^2 + 3))*sqrt(1/2*sqrt(-5/3) - 1/2)*elliptic_f(arc
sin(x*sqrt(1/2*sqrt(-5/3) - 1/2)), 3/4*sqrt(-5/3) - 1/4) + 2*sqrt(-2*x^4 -
 3*x^2 - 3)*(2*x^3 - x))/(2*x^4 + 3*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(-3-3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-3x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-3*x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 - 3*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-3\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-3*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - 3*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-3\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-3*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - 3*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3-3x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-3\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(- 3*x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(- 3*x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3-3x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-3x^2-3}}{4x^8+12x^6+21x^4+18x^2+9}dx$$

input
```
int(1/(-2*x^4-3*x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - 3*x**2 - 3)/(4*x**8 + 12*x**6 + 21*x**4 + 18*x**2 + 9
),x)
```

3.274 $\int \frac{1}{(-3-4x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 247

$$\int \frac{1}{(-3-4x^2-2x^4)^{3/2}}\,dx = \frac{x(1+2x^2)}{6\sqrt{-3-4x^2-2x^4}}+\frac{x\sqrt{-3-4x^2-2x^4}}{3\left(\sqrt{6}+2x^2\right)} + \frac{(3+\sqrt{6}x^2)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{6^{3/4}\sqrt{-3-4x^2-2x^4}} - \frac{(2+\sqrt{6})\,(3+\sqrt{6}x^2)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{4\,6^{3/4}\sqrt{-3-4x^2-2x^4}}$$

output
```
1/6*x*(2*x^2+1)/(-2*x^4-4*x^2-3)^(1/2)+x*(-2*x^4-4*x^2-3)^(1/2)/(3*6^(1/2)
+6*x^2)+1/6*(3+6^(1/2)*x^2)*((2*x^4+4*x^2+3)/(3+6^(1/2)*x^2)^2)^(1/2)*Elli
pticE(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18-6*6^(1/2))^(1/2))*6^(1/
4)/(-2*x^4-4*x^2-3)^(1/2)-1/24*(2+6^(1/2))*(3+6^(1/2)*x^2)*((2*x^4+4*x^2+3
)/(3+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*2^(1/4)*3^(3/4)*x)
,1/6*(18-6*6^(1/2))^(1/2))*6^(1/4)/(-2*x^4-4*x^2-3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.50 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.31

$$\int \frac{1}{(-3-4x^2-2x^4)^{3/2}}\,dx = \frac{2\sqrt{-\frac{i}{-2i+\sqrt{2}}}x(1+2x^2)+2(1+i\sqrt{2})\sqrt{\frac{-2i+\sqrt{2}-2ix^2}{-2i+\sqrt{2}}}\sqrt{\frac{2i+\sqrt{2}+2ix^2}{2i+\sqrt{2}}}E\Big(i\text{arcsinh}\Big(}{1}$$

input
```
Integrate[(-3 - 4*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(2*Sqrt[(-I)/(-2*I + Sqrt[2])]*x*(1 + 2*x^2) + 2*(1 + I*Sqrt[2])*Sqrt[(-2*
I + Sqrt[2] - (2*I)*x^2)/(-2*I + Sqrt[2])]*Sqrt[(2*I + Sqrt[2] + (2*I)*x^2
)/(2*I + Sqrt[2])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-2*I + Sqrt[2])]*x], (
2*I - Sqrt[2])/(2*I + Sqrt[2])] + I*(2*I + Sqrt[2])*Sqrt[(-2*I + Sqrt[2] -
 (2*I)*x^2)/(-2*I + Sqrt[2])]*Sqrt[(2*I + Sqrt[2] + (2*I)*x^2)/(2*I + Sqrt
[2])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-2*I + Sqrt[2])]*x], (2*I - Sqrt[2]
)/(2*I + Sqrt[2])])/(12*Sqrt[(-I)/(-2*I + Sqrt[2])]*Sqrt[-3 - 4*x^2 - 2*x^
4])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-4x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x(2x^2+1)}{6\sqrt{-2x^4-4x^2-3}} - \frac{1}{24}\int \frac{4(2x^2+3)}{\sqrt{-2x^4-4x^2-3}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x\left(2x^2+1\right)}{6\sqrt{-2x^4-4x^2-3}}-\frac{1}{6}\int \frac{2x^2+3}{\sqrt{-2x^4-4x^2-3}}dx$$

$$\Big\downarrow \; 1511$$

$$\frac{1}{6}\left(\sqrt{6}\int \frac{3-\sqrt{6}x^2}{3\sqrt{-2x^4-4x^2-3}}dx-\left(3+\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4-4x^2-3}}dx\right)+\frac{x\left(2x^2+1\right)}{6\sqrt{-2x^4-4x^2-3}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\left(\sqrt{\frac{2}{3}}\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4-4x^2-3}}dx-\left(3+\sqrt{6}\right)\int \frac{1}{\sqrt{-2x^4-4x^2-3}}dx\right)+\frac{x\left(2x^2+1\right)}{6\sqrt{-2x^4-4x^2-3}}$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{6}\left(\sqrt{\frac{2}{3}}\int \frac{3-\sqrt{6}x^2}{\sqrt{-2x^4-4x^2-3}}dx-\frac{\left(3+\sqrt{6}\right)\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2x^4-4x^2-3}}\right)+$$

$$\frac{x\left(2x^2+1\right)}{6\sqrt{-2x^4-4x^2-3}}$$

$$\Big\downarrow \; 1509$$

$$\frac{1}{6}\left(\sqrt{\frac{2}{3}}\left(\frac{3^{3/4}\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{\sqrt[4]{2}\sqrt{-2x^4-4x^2-3}}+\frac{3\sqrt{-2x^4-4x^2-3}x}{\sqrt{6}x^2+3}\right)-\frac{\left(3+\sqrt{6}\right)\left(\sqrt{\right.}}{}$$

$$\frac{x\left(2x^2+1\right)}{6\sqrt{-2x^4-4x^2-3}}$$

input
```
Int[(-3 - 4*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(1 + 2*x^2))/(6*Sqrt[-3 - 4*x^2 - 2*x^4]) + (Sqrt[2/3]*((3*x*Sqrt[-3 -
4*x^2 - 2*x^4])/(3 + Sqrt[6]*x^2) + (3^(3/4)*(3 + Sqrt[6]*x^2)*Sqrt[(3 + 4
*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(2/3)^(1/4)*x], 1/2
- 1/Sqrt[6]])/(2^(1/4)*Sqrt[-3 - 4*x^2 - 2*x^4])) - ((3 + Sqrt[6])*(3 + Sq
rt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTa
n[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-3 - 4*x^2 - 2*x^4]))/
6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.55 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.96

method	result
risch	$\frac{x(2x^2+1)}{6\sqrt{-2x^4-4x^2-3}} - \frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{2\sqrt{-6-3i\sqrt{2}}\sqrt{-2x^4-4x^2-3}} - \frac{6\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1}}{}$
default	$\frac{\frac{1}{3}x^3+\frac{1}{6}x}{\sqrt{-2x^4-4x^2-3}} - \frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{2\sqrt{-6-3i\sqrt{2}}\sqrt{-2x^4-4x^2-3}} - \frac{6\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-}}{}$
elliptic	$\frac{\frac{1}{3}x^3+\frac{1}{6}x}{\sqrt{-2x^4-4x^2-3}} - \frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{2\sqrt{-6-3i\sqrt{2}}\sqrt{-2x^4-4x^2-3}} - \frac{6\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-}}{}$

input
```
int(1/(-2*x^4-4*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*x*(2*x^2+1)/(-2*x^4-4*x^2-3)^(1/2)-3/2/(-6-3*I*2^(1/2))^(1/2)*(1-(-2/3
-1/3*I*2^(1/2))*x^2)^(1/2)*(1-(-2/3+1/3*I*2^(1/2))*x^2)^(1/2)/(-2*x^4-4*x^
2-3)^(1/2)*EllipticF(1/3*(-6-3*I*2^(1/2))^(1/2)*x,1/3*(3-6*I*2^(1/2))^(1/2
))-6/(-6-3*I*2^(1/2))^(1/2)*(1-(-2/3-1/3*I*2^(1/2))*x^2)^(1/2)*(1-(-2/3+1/
3*I*2^(1/2))*x^2)^(1/2)/(-2*x^4-4*x^2-3)^(1/2)/(-4+2*I*2^(1/2))*(EllipticF
(1/3*(-6-3*I*2^(1/2))^(1/2)*x,1/3*(3-6*I*2^(1/2))^(1/2))-EllipticE(1/3*(-6
-3*I*2^(1/2))^(1/2)*x,1/3*(3-6*I*2^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-3-4x^2-2x^4)^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{-2}\sqrt{-3}(2\,x^4+4\,x^2+3)-2\,\sqrt{-3}(2\,x^4+4\,x^2+3)\right)\sqrt{\frac{1}{3}\,\sqrt{-2}-\frac{2}{3}}E(\arcsin\left(x\sqrt{\frac{1}{3}\,\sqrt{-2}-\frac{2}{3}}\right)\mid\frac{2}{3}\,\sqrt{-}$$

input
```
integrate(1/(-2*x^4-4*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
-1/36*(2*(sqrt(-2)*sqrt(-3)*(2*x^4 + 4*x^2 + 3) - 2*sqrt(-3)*(2*x^4 + 4*x^
2 + 3))*sqrt(1/3*sqrt(-2) - 2/3)*elliptic_e(arcsin(x*sqrt(1/3*sqrt(-2) - 2
/3)), 2/3*sqrt(-2) + 1/3) + (sqrt(-2)*sqrt(-3)*(2*x^4 + 4*x^2 + 3) + 10*sq
rt(-3)*(2*x^4 + 4*x^2 + 3))*sqrt(1/3*sqrt(-2) - 2/3)*elliptic_f(arcsin(x*s
qrt(1/3*sqrt(-2) - 2/3)), 2/3*sqrt(-2) + 1/3) + 6*sqrt(-2*x^4 - 4*x^2 - 3)
*(2*x^3 + x))/(2*x^4 + 4*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(-3-4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-4x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-4*x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 - 4*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-4\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-4*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - 4*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-4\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-4*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - 4*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3-4x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-4\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(- 4*x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(- 4*x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3-4x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-4x^2-3}}{4x^8+16x^6+28x^4+24x^2+9}dx$$

input
```
int(1/(-2*x^4-4*x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - 4*x**2 - 3)/(4*x**8 + 16*x**6 + 28*x**4 + 24*x**2 + 9
),x)
```

3.275 $\int \frac{1}{\left(-3-5x^2-2x^4\right)^{3/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{1}{\left(-3-5x^2-2x^4\right)^{3/2}} dx = -\frac{x}{\sqrt{-3-5x^2-2x^4}} + \frac{5\sqrt{2}\sqrt{-1-x^2}E\left(\arctan\left(\sqrt{\frac{2}{3}}x\right)|-\frac{1}{2}\right)}{3\sqrt{1+x^2}} + \frac{2\sqrt{2}\sqrt{1+x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{2}{3}}x\right),-\frac{1}{2}\right)}{\sqrt{-1-x^2}}$$

output
```
-x/(-2*x^4-5*x^2-3)^(1/2)+5/3*2^(1/2)*(-x^2-1)^(1/2)*EllipticE(x*6^(1/2)/(
6*x^2+9)^(1/2),1/2*I*2^(1/2))/(x^2+1)^(1/2)+2*(x^2+1)^(1/2)*InverseJacobiA
M(arctan(1/3*x*6^(1/2)),1/2*I*2^(1/2))*2^(1/2)/(-x^2-1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.18

$$\int \frac{1}{(-3-5x^2-2x^4)^{3/2}}\,dx = -\frac{13x+10x^3+5i\sqrt{2}\sqrt{1+x^2}\sqrt{3+2x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{3}}x\right)|\frac{3}{2}\right)+i\sqrt{2}\sqrt{1+x^2}\sqrt{3+2x^2}\,\text{EllipticF}\left(i\text{arcsin}\right.}{3\sqrt{-3-5x^2-2x^4}}$$

input
```
Integrate[(-3 - 5*x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/3*(13*x + 10*x^3 + (5*I)*Sqrt[2]*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*Elliptic
E[I*ArcSinh[Sqrt[2/3]*x], 3/2] + I*Sqrt[2]*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*E
llipticF[I*ArcSinh[Sqrt[2/3]*x], 3/2])/Sqrt[-3 - 5*x^2 - 2*x^4]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.67, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1405, 27, 1494, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-5x^2-3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{3}\int \frac{2(5x^2+6)}{\sqrt{-2x^4-5x^2-3}}dx-\frac{x(10x^2+13)}{3\sqrt{-2x^4-5x^2-3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}\int \frac{5x^2+6}{\sqrt{-2x^4-5x^2-3}}dx-\frac{x(10x^2+13)}{3\sqrt{-2x^4-5x^2-3}}$$

$$\Big\downarrow \text{ 1494}$$

$$\frac{4}{3}\sqrt{2}\int \frac{5x^2+6}{2\sqrt{2}\sqrt{-x^2-1}\sqrt{2x^2+3}}dx - \frac{x(10x^2+13)}{3\sqrt{-2x^4-5x^2-3}}$$

$$\Big\downarrow 27$$

$$\frac{2}{3}\int \frac{5x^2+6}{\sqrt{-x^2-1}\sqrt{2x^2+3}}dx - \frac{x(10x^2+13)}{3\sqrt{-2x^4-5x^2-3}}$$

$$\Big\downarrow 406$$

$$\frac{2}{3}\left(6\int \frac{1}{\sqrt{-x^2-1}\sqrt{2x^2+3}}dx + 5\int \frac{x^2}{\sqrt{-x^2-1}\sqrt{2x^2+3}}dx\right) - \frac{x(10x^2+13)}{3\sqrt{-2x^4-5x^2-3}}$$

$$\Big\downarrow 320$$

$$\frac{2}{3}\left(5\int \frac{x^2}{\sqrt{-x^2-1}\sqrt{2x^2+3}}dx + \frac{2\sqrt{3}\sqrt{2x^2+3}\,\text{EllipticF}\left(\arctan(x),\frac{1}{3}\right)}{\sqrt{-x^2-1}\sqrt{\frac{2x^2+3}{x^2+1}}}\right) - \frac{x(10x^2+13)}{3\sqrt{-2x^4-5x^2-3}}$$

$$\Big\downarrow 388$$

$$\frac{2}{3}\left(5\left(\frac{1}{2}\int \frac{\sqrt{2x^2+3}}{(-x^2-1)^{3/2}}dx + \frac{\sqrt{2x^2+3}x}{2\sqrt{-x^2-1}}\right) + \frac{2\sqrt{3}\sqrt{2x^2+3}\,\text{EllipticF}\left(\arctan(x),\frac{1}{3}\right)}{\sqrt{-x^2-1}\sqrt{\frac{2x^2+3}{x^2+1}}}\right) - \frac{x(10x^2+13)}{3\sqrt{-2x^4-5x^2-3}}$$

$$\Big\downarrow 313$$

$$\frac{2}{3}\left(\frac{2\sqrt{3}\sqrt{2x^2+3}\,\text{EllipticF}\left(\arctan(x),\frac{1}{3}\right)}{\sqrt{-x^2-1}\sqrt{\frac{2x^2+3}{x^2+1}}} + 5\left(\frac{x\sqrt{2x^2+3}}{2\sqrt{-x^2-1}} - \frac{\sqrt{3}\sqrt{2x^2+3}E\left(\arctan(x)\left|\frac{1}{3}\right.\right)}{2\sqrt{-x^2-1}\sqrt{\frac{2x^2+3}{x^2+1}}}\right)\right) - \frac{x(10x^2+13)}{3\sqrt{-2x^4-5x^2-3}}$$

input
```
Int[(-3 - 5*x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/3*(x*(13 + 10*x^2))/Sqrt[-3 - 5*x^2 - 2*x^4] + (2*(5*((x*Sqrt[3 + 2*x^2
])/(2*Sqrt[-1 - x^2]) - (Sqrt[3]*Sqrt[3 + 2*x^2]*EllipticE[ArcTan[x], 1/3]
)/(2*Sqrt[-1 - x^2]*Sqrt[(3 + 2*x^2)/(1 + x^2)])) + (2*Sqrt[3]*Sqrt[3 + 2*
x^2]*EllipticF[ArcTan[x], 1/3])/(Sqrt[-1 - x^2]*Sqrt[(3 + 2*x^2)/(1 + x^2)
])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.20

method	result	s
risch	$-\frac{x\left(10x^{2}+13\right)}{3\sqrt{-2x^{4}-5x^{2}-3}}-\frac{4i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+9}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{3}\right)}{3\sqrt{-2x^{4}-5x^{2}-3}}+\frac{5i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+9}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{3}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{3}\right)\right)}{3\sqrt{-2x^{4}-5x^{2}-3}}$	
default	$\frac{-\frac{10}{3}x^{3}-\frac{13}{3}x}{\sqrt{-2x^{4}-5x^{2}-3}}-\frac{4i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+9}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{3}\right)}{3\sqrt{-2x^{4}-5x^{2}-3}}+\frac{5i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+9}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{3}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{3}\right)\right)}{3\sqrt{-2x^{4}-5x^{2}-3}}$	
elliptic	$\frac{-\frac{10}{3}x^{3}-\frac{13}{3}x}{\sqrt{-2x^{4}-5x^{2}-3}}-\frac{4i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+9}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{3}\right)}{3\sqrt{-2x^{4}-5x^{2}-3}}+\frac{5i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+9}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{3}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{3}\right)\right)}{3\sqrt{-2x^{4}-5x^{2}-3}}$	

input
```
int(1/(-2*x^4-5*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(10*x^2+13)/(-2*x^4-5*x^2-3)^(1/2)-4/3*I*(x^2+1)^(1/2)*(6*x^2+9)^(1
/2)/(-2*x^4-5*x^2-3)^(1/2)*EllipticF(I*x,1/3*6^(1/2))+5/3*I*(x^2+1)^(1/2)*
(6*x^2+9)^(1/2)/(-2*x^4-5*x^2-3)^(1/2)*(EllipticF(I*x,1/3*6^(1/2))-Ellipti
cE(I*x,1/3*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.94

$$\int \frac{1}{(-3-5x^2-2x^4)^{3/2}}\,dx =$$
$$-\frac{10\sqrt{-\frac{2}{3}}\sqrt{-3}(2\,x^4+5\,x^2+3)E(\arcsin\left(\sqrt{-\frac{2}{3}}x\right)\mid\frac{3}{2})-28\sqrt{-\frac{2}{3}}\sqrt{-3}(2\,x^4+5\,x^2+3)F(\arcsin\left(\sqrt{-\frac{2}{3}}x\right.}{9\,(2\,x^4+5\,x^2+3)}$$

input
```
integrate(1/(-2*x^4-5*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
-1/9*(10*sqrt(-2/3)*sqrt(-3)*(2*x^4 + 5*x^2 + 3)*elliptic_e(arcsin(sqrt(-2
/3)*x), 3/2) - 28*sqrt(-2/3)*sqrt(-3)*(2*x^4 + 5*x^2 + 3)*elliptic_f(arcsi
n(sqrt(-2/3)*x), 3/2) - 3*sqrt(-2*x^4 - 5*x^2 - 3)*(10*x^3 + 13*x))/(2*x^4
 + 5*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(-3-5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-5x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-5*x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 - 5*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-5\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-5*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - 5*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-5\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-5*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - 5*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3-5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-5\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(- 5*x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(- 5*x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3-5x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-5x^2-3}}{4x^8+20x^6+37x^4+30x^2+9}dx$$

input
```
int(1/(-2*x^4-5*x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - 5*x**2 - 3)/(4*x**8 + 20*x**6 + 37*x**4 + 30*x**2 + 9
),x)
```

3.276 $\int \frac{1}{(-3-6x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 200

$$\int \frac{1}{(-3-6x^2-2x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{3}\left(3-\sqrt{3}\right)\sqrt{-3-6x^2-2x^4}} + \frac{\sqrt{3-\sqrt{3}}\sqrt{-3+\sqrt{3}-2x^2}\,E\left(\arctan\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|-1-\sqrt{3}\right)}{6\sqrt{3-\sqrt{3}+2x^2}} + \frac{\sqrt{\frac{1}{6}\left(3+\sqrt{3}\right)}\sqrt{3-\sqrt{3}+2x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),-1-\sqrt{3}\right)}{3\sqrt{-3+\sqrt{3}-2x^2}}$$

output

```
-1/3*x*3^(1/2)/(3-3^(1/2))/(-2*x^4-6*x^2-3)^(1/2)+1/6*(3-3^(1/2))^(1/2)*(-
3+3^(1/2)-2*x^2)^(1/2)*EllipticE((9-3*3^(1/2))^(1/2)*x/(9+(9-3*3^(1/2))*x^
2)^(1/2),(-1-3^(1/2))^(1/2))/(3-3^(1/2)+2*x^2)^(1/2)+1/18*(18+6*3^(1/2))^(
1/2)*(3-3^(1/2)+2*x^2)^(1/2)*InverseJacobiAM(arctan(1/3*(9-3*3^(1/2))^(1/2
)*x),(-1-3^(1/2))^(1/2))/(-3+3^(1/2)-2*x^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.56 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-3-6x^2-2x^4)^{3/2}}\,dx = \frac{-4x(2+x^2)+i\sqrt{2}(-3+\sqrt{3})\sqrt{\frac{-3+\sqrt{3}-2x^2}{-3+\sqrt{3}}}\sqrt{3+\sqrt{3}+2x^2}E\Big(i\text{arcsinh}\Big(\sqrt{1}$$

input
```
Integrate[(-3 - 6*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(-4*x*(2 + x^2) + I*Sqrt[2]*(-3 + Sqrt[3])*Sqrt[(-3 + Sqrt[3] - 2*x^2)/(-3
 + Sqrt[3])]*Sqrt[3 + Sqrt[3] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - 1/Sqrt
[3]]*x], 2 + Sqrt[3]] - I*Sqrt[2]*(-1 + Sqrt[3])*Sqrt[(-3 + Sqrt[3] - 2*x^
2)/(-3 + Sqrt[3])]*Sqrt[3 + Sqrt[3] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[1 -
1/Sqrt[3]]*x], 2 + Sqrt[3]])/(12*Sqrt[-3 - 6*x^2 - 2*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.52, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1405, 27, 1494, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-6x^2-3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{36}\int \frac{12(x^2+1)}{\sqrt{-2x^4-6x^2-3}}dx-\frac{x(x^2+2)}{3\sqrt{-2x^4-6x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{x^2+1}{\sqrt{-2x^4-6x^2-3}}dx-\frac{x(x^2+2)}{3\sqrt{-2x^4-6x^2-3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{2}{3}\sqrt{2}\int \frac{x^2+1}{2\sqrt{-2x^2+\sqrt{3}-3}\sqrt{2x^2+\sqrt{3}+3}}dx-\frac{x(x^2+2)}{3\sqrt{-2x^4-6x^2-3}}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\sqrt{2}\int \frac{x^2+1}{\sqrt{-2x^2+\sqrt{3}-3}\sqrt{2x^2+\sqrt{3}+3}}dx-\frac{x(x^2+2)}{3\sqrt{-2x^4-6x^2-3}}$$

$\Big\downarrow$ 406

$$\frac{1}{3}\sqrt{2}\left(\int \frac{1}{\sqrt{-2x^2+\sqrt{3}-3}\sqrt{2x^2+\sqrt{3}+3}}dx+\int \frac{x^2}{\sqrt{-2x^2+\sqrt{3}-3}\sqrt{2x^2+\sqrt{3}+3}}dx\right)-\frac{x(x^2+2)}{3\sqrt{-2x^4-6x^2-3}}$$

$\Big\downarrow$ 320

$$\frac{1}{3}\sqrt{2}\left(\int \frac{x^2}{\sqrt{-2x^2+\sqrt{3}-3}\sqrt{2x^2+\sqrt{3}+3}}dx+\frac{\sqrt{\frac{3}{3-\sqrt{3}}}\sqrt{2x^2+\sqrt{3}+3}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right),-\right.}{\left(3+\sqrt{3}\right)\sqrt{-2x^2+\sqrt{3}-3}\sqrt{\frac{2x^2+\sqrt{3}+3}{2x^2-\sqrt{3}+3}}}\right.$$
$$\frac{x(x^2+2)}{3\sqrt{-2x^4-6x^2-3}}$$

$\Big\downarrow$ 388

$$\frac{1}{3}\sqrt{2}\left(\frac{1}{2}\left(3-\sqrt{3}\right)\int \frac{\sqrt{2x^2+\sqrt{3}+3}}{\left(-2x^2+\sqrt{3}-3\right)^{3/2}}dx+\frac{\sqrt{\frac{3}{3-\sqrt{3}}}\sqrt{2x^2+\sqrt{3}+3}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right),-\right.}{\left(3+\sqrt{3}\right)\sqrt{-2x^2+\sqrt{3}-3}\sqrt{\frac{2x^2+\sqrt{3}+3}{2x^2-\sqrt{3}+3}}}\right.$$
$$\frac{x(x^2+2)}{3\sqrt{-2x^4-6x^2-3}}$$

$\Big\downarrow$ 313

$$\frac{1}{3}\sqrt{2}\left(\frac{\sqrt{\frac{3}{3-\sqrt{3}}}\sqrt{2x^2+\sqrt{3}+3}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right),-1+\sqrt{3}\right)}{\left(3+\sqrt{3}\right)\sqrt{-2x^2+\sqrt{3}-3}\sqrt{\frac{2x^2+\sqrt{3}+3}{2x^2-\sqrt{3}+3}}}-\frac{\sqrt{\frac{3}{3-\sqrt{3}}}\sqrt{2x^2+\sqrt{3}+3}E\left(\text{ar}\right.}{2\sqrt{-2x^2+}}\right.$$
$$\frac{x(x^2+2)}{3\sqrt{-2x^4-6x^2-3}}$$

input
```
Int[(-3 - 6*x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/3*(x*(2 + x^2))/Sqrt[-3 - 6*x^2 - 2*x^4] + (Sqrt[2]*((x*Sqrt[3 + Sqrt[3
] + 2*x^2])/(2*Sqrt[-3 + Sqrt[3] - 2*x^2]) - (Sqrt[3/(3 - Sqrt[3])]*Sqrt[3
 + Sqrt[3] + 2*x^2]*EllipticE[ArcTan[Sqrt[(3 + Sqrt[3])/3]*x], -1 + Sqrt[3
]])/(2*Sqrt[-3 + Sqrt[3] - 2*x^2]*Sqrt[(3 + Sqrt[3] + 2*x^2)/(3 - Sqrt[3]
+ 2*x^2)]) + (Sqrt[3/(3 - Sqrt[3])]*Sqrt[3 + Sqrt[3] + 2*x^2]*EllipticF[Ar
cTan[Sqrt[(3 + Sqrt[3])/3]*x], -1 + Sqrt[3]])/((3 + Sqrt[3])*Sqrt[-3 + Sqr
t[3] - 2*x^2]*Sqrt[(3 + Sqrt[3] + 2*x^2)/(3 - Sqrt[3] + 2*x^2)])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{(x^2+2)x}{3\sqrt{-2x^4-6x^2-3}}+\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-9-3\sqrt{3}}\,x}{3},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{-9-3\sqrt{3}}\sqrt{-2x^4-6x^2-3}}+\frac{6\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\sqrt{1-\left(-\right.}}{}$
default	$\frac{-\frac{2}{3}x-\frac{1}{3}x^3}{\sqrt{-2x^4-6x^2-3}}+\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-9-3\sqrt{3}}\,x}{3},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{-9-3\sqrt{3}}\sqrt{-2x^4-6x^2-3}}+\frac{6\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\sqrt{1-\left(-1+\right.}}{}$
elliptic	$\frac{-\frac{2}{3}x-\frac{1}{3}x^3}{\sqrt{-2x^4-6x^2-3}}+\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{-9-3\sqrt{3}}\,x}{3},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{-9-3\sqrt{3}}\sqrt{-2x^4-6x^2-3}}+\frac{6\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\sqrt{1-\left(-1+\right.}}{}$

input
```
int(1/(-2*x^4-6*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(x^2+2)*x/(-2*x^4-6*x^2-3)^(1/2)+1/(-9-3*3^(1/2))^(1/2)*(1-(-1-1/3*3^
(1/2))*x^2)^(1/2)*(1-(-1+1/3*3^(1/2))*x^2)^(1/2)/(-2*x^4-6*x^2-3)^(1/2)*El
lipticF(1/3*(-9-3*3^(1/2))^(1/2)*x,1/2*6^(1/2)-1/2*2^(1/2))+6/(-9-3*3^(1/2
))^(1/2)*(1-(-1-1/3*3^(1/2))*x^2)^(1/2)*(1-(-1+1/3*3^(1/2))*x^2)^(1/2)/(-2
*x^4-6*x^2-3)^(1/2)/(-6+2*3^(1/2))*(EllipticF(1/3*(-9-3*3^(1/2))^(1/2)*x,1
/2*6^(1/2)-1/2*2^(1/2))-EllipticE(1/3*(-9-3*3^(1/2))^(1/2)*x,1/2*6^(1/2)-1
/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-3-6x^2-2x^4)^{3/2}}\,dx = \frac{6\sqrt{-3}(2x^4+6x^2+3)\sqrt{\frac{1}{3}\sqrt{3}-1}F\left(\arcsin\left(x\sqrt{\frac{1}{3}\sqrt{3}-1}\right)\mid\sqrt{3}+2\right)+(\sqrt{}}{}$$

input
```
integrate(1/(-2*x^4-6*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
1/18*(6*sqrt(-3)*(2*x^4 + 6*x^2 + 3)*sqrt(1/3*sqrt(3) - 1)*elliptic_f(arcs
in(x*sqrt(1/3*sqrt(3) - 1)), sqrt(3) + 2) + (sqrt(3)*sqrt(-3)*(2*x^4 + 6*x
^2 + 3) - 3*sqrt(-3)*(2*x^4 + 6*x^2 + 3))*sqrt(1/3*sqrt(3) - 1)*elliptic_e
(arcsin(x*sqrt(1/3*sqrt(3) - 1)), sqrt(3) + 2) + 6*sqrt(-2*x^4 - 6*x^2 - 3
)*(x^3 + 2*x))/(2*x^4 + 6*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(-3-6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-6x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-6*x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 - 6*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-6\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-6*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - 6*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-6\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-6*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - 6*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3-6x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-6\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(- 6*x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(- 6*x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3-6x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-6x^2-3}}{4x^8+24x^6+48x^4+36x^2+9}dx$$

input
```
int(1/(-2*x^4-6*x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - 6*x**2 - 3)/(4*x**8 + 24*x**6 + 48*x**4 + 36*x**2 + 9
),x)
```

3.277 $\int \frac{1}{(-3-7x^2-2x^4)^{3/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 94

$$\int \frac{1}{(-3-7x^2-2x^4)^{3/2}} dx = -\frac{2x}{5\sqrt{-3-7x^2-2x^4}} + \frac{7\sqrt{-1-2x^2}E\left(\arctan\left(\frac{x}{\sqrt{3}}\right)\middle|-5\right)}{75\sqrt{1+2x^2}} + \frac{4\sqrt{1+2x^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}}\right),-5\right)}{25\sqrt{-1-2x^2}}$$

output
```
-2/5*x/(-2*x^4-7*x^2-3)^(1/2)+7/75*(-2*x^2-1)^(1/2)*EllipticE(x*3^(1/2)/(3
*x^2+9)^(1/2),I*5^(1/2))/(2*x^2+1)^(1/2)+4/25*(2*x^2+1)^(1/2)*InverseJacob
iAM(arctan(1/3*x*3^(1/2)),I*5^(1/2))/(-2*x^2-1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-3-7x^2-2x^4)^{3/2}} dx = -\frac{37x+14x^3+7i\sqrt{6}\sqrt{3+x^2}\sqrt{1+2x^2}E\left(i\,\text{arcsinh}(\sqrt{2}x)\,|\frac{1}{6}\right)-5i\sqrt{6}\sqrt{3+x^2}\sqrt{1+2x^2}\,\text{EllipticF}\left(i\,\text{arcsinh}\right.}{75\sqrt{-3-7x^2-2x^4}}$$

input
```
Integrate[(-3 - 7*x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/75*(37*x + 14*x^3 + (7*I)*Sqrt[6]*Sqrt[3 + x^2]*Sqrt[1 + 2*x^2]*Ellipti
cE[I*ArcSinh[Sqrt[2]*x], 1/6] - (5*I)*Sqrt[6]*Sqrt[3 + x^2]*Sqrt[1 + 2*x^2
]*EllipticF[I*ArcSinh[Sqrt[2]*x], 1/6])/Sqrt[-3 - 7*x^2 - 2*x^4]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.77, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1405, 27, 1494, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4-7x^2-3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{75}\int \frac{2(7x^2+6)}{\sqrt{-2x^4-7x^2-3}}dx-\frac{x(14x^2+37)}{75\sqrt{-2x^4-7x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{75}\int \frac{7x^2+6}{\sqrt{-2x^4-7x^2-3}}dx-\frac{x(14x^2+37)}{75\sqrt{-2x^4-7x^2-3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{4}{75}\sqrt{2}\int \frac{7x^2+6}{2\sqrt{2}\sqrt{-2x^2-1}\sqrt{x^2+3}}dx-\frac{x(14x^2+37)}{75\sqrt{-2x^4-7x^2-3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{75}\int \frac{7x^2+6}{\sqrt{-2x^2-1}\sqrt{x^2+3}}dx-\frac{x(14x^2+37)}{75\sqrt{-2x^4-7x^2-3}}$$

$$\Big\downarrow \text{406}$$

$$\frac{2}{75}\left(6\int \frac{1}{\sqrt{-2x^2-1}\sqrt{x^2+3}}dx+7\int \frac{x^2}{\sqrt{-2x^2-1}\sqrt{x^2+3}}dx\right)-\frac{x(14x^2+37)}{75\sqrt{-2x^4-7x^2-3}}$$

$$\Big\downarrow \; 320$$

$$\frac{2}{75}\left(7\int \frac{x^2}{\sqrt{-2x^2-1}\sqrt{x^2+3}}dx-\frac{6\sqrt{-2x^2-1}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}}\right),-5\right)}{\sqrt{x^2+3}\sqrt{\frac{2x^2+1}{x^2+3}}}\right)-\frac{x(14x^2+37)}{75\sqrt{-2x^4-7x^2-3}}$$

$$\Big\downarrow \; 388$$

$$\frac{2}{75}\left(7\left(\frac{3}{2}\int \frac{\sqrt{-2x^2-1}}{(x^2+3)^{3/2}}dx-\frac{x\sqrt{-2x^2-1}}{2\sqrt{x^2+3}}\right)-\frac{6\sqrt{-2x^2-1}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}}\right),-5\right)}{\sqrt{x^2+3}\sqrt{\frac{2x^2+1}{x^2+3}}}\right)-\frac{x(14x^2+37)}{75\sqrt{-2x^4-7x^2-3}}$$

$$\Big\downarrow \; 313$$

$$\frac{2}{75}\left(7\left(\frac{\sqrt{-2x^2-1}E\left(\arctan\left(\frac{x}{\sqrt{3}}\right)\middle|-5\right)}{2\sqrt{x^2+3}\sqrt{\frac{2x^2+1}{x^2+3}}}-\frac{x\sqrt{-2x^2-1}}{2\sqrt{x^2+3}}\right)-\frac{6\sqrt{-2x^2-1}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}}\right),-5\right)}{\sqrt{x^2+3}\sqrt{\frac{2x^2+1}{x^2+3}}}\right)-\frac{x(14x^2+37)}{75\sqrt{-2x^4-7x^2-3}}$$

input
```
Int[(-3 - 7*x^2 - 2*x^4)^(-3/2),x]
```

output
```
-1/75*(x*(37 + 14*x^2))/Sqrt[-3 - 7*x^2 - 2*x^4] + (2*(7*(-1/2*(x*Sqrt[-1
- 2*x^2])/Sqrt[3 + x^2] + (Sqrt[-1 - 2*x^2]*EllipticE[ArcTan[x/Sqrt[3]], -
5])/(2*Sqrt[3 + x^2]*Sqrt[(1 + 2*x^2)/(3 + x^2)])) - (6*Sqrt[-1 - 2*x^2]*E
llipticF[ArcTan[x/Sqrt[3]], -5])/(Sqrt[3 + x^2]*Sqrt[(1 + 2*x^2)/(3 + x^2)
])))/75
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.53

method	result
risch	$-\frac{x(14x^2+37)}{75\sqrt{-2x^4-7x^2-3}}-\frac{2i\sqrt{2}\,\sqrt{2x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(ix\sqrt{2},\frac{\sqrt{6}}{6}\right)}{75\sqrt{-2x^4-7x^2-3}}+\frac{7i\sqrt{2}\,\sqrt{2x^2+1}\,\sqrt{3x^2+9}\left(\text{EllipticF}\left(ix\sqrt{2},\frac{\sqrt{6}}{6}\right)-\text{Ellipti}\right.}{75\sqrt{-2x^4-7x^2-3}}$
default	$\frac{-\frac{37}{75}x-\frac{14}{75}x^3}{\sqrt{-2x^4-7x^2-3}}-\frac{2i\sqrt{2}\,\sqrt{2x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(ix\sqrt{2},\frac{\sqrt{6}}{6}\right)}{75\sqrt{-2x^4-7x^2-3}}+\frac{7i\sqrt{2}\,\sqrt{2x^2+1}\,\sqrt{3x^2+9}\left(\text{EllipticF}\left(ix\sqrt{2},\frac{\sqrt{6}}{6}\right)-\text{EllipticE}\left(i\right.\right.}{75\sqrt{-2x^4-7x^2-3}}$
elliptic	$\frac{-\frac{37}{75}x-\frac{14}{75}x^3}{\sqrt{-2x^4-7x^2-3}}-\frac{2i\sqrt{2}\,\sqrt{2x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(ix\sqrt{2},\frac{\sqrt{6}}{6}\right)}{75\sqrt{-2x^4-7x^2-3}}+\frac{7i\sqrt{2}\,\sqrt{2x^2+1}\,\sqrt{3x^2+9}\left(\text{EllipticF}\left(ix\sqrt{2},\frac{\sqrt{6}}{6}\right)-\text{EllipticE}\left(i\right.\right.}{75\sqrt{-2x^4-7x^2-3}}$

input
```
int(1/(-2*x^4-7*x^2-3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/75*x*(14*x^2+37)/(-2*x^4-7*x^2-3)^(1/2)-2/75*I*2^(1/2)*(2*x^2+1)^(1/2)*
(3*x^2+9)^(1/2)/(-2*x^4-7*x^2-3)^(1/2)*EllipticF(I*2^(1/2)*x,1/6*6^(1/2))+
7/75*I*2^(1/2)*(2*x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-2*x^4-7*x^2-3)^(1/2)*(Ell
ipticF(I*2^(1/2)*x,1/6*6^(1/2))-EllipticE(I*2^(1/2)*x,1/6*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int \frac{1}{(-3-7x^2-2x^4)^{3/2}}\,dx =$$
$$-\frac{7\sqrt{-\frac{1}{3}}\sqrt{-3}(2\,x^4+7\,x^2+3)E(\arcsin\left(\sqrt{-\frac{1}{3}}x\right)\mid 6)-43\sqrt{-\frac{1}{3}}\sqrt{-3}(2\,x^4+7\,x^2+3)F(\arcsin\left(\sqrt{-\frac{1}{3}}x\right)}{225\,(2\,x^4+7\,x^2+3)}$$

input
```
integrate(1/(-2*x^4-7*x^2-3)^(3/2),x, algorithm="fricas")
```

output
```
-1/225*(7*sqrt(-1/3)*sqrt(-3)*(2*x^4 + 7*x^2 + 3)*elliptic_e(arcsin(sqrt(-
1/3)*x), 6) - 43*sqrt(-1/3)*sqrt(-3)*(2*x^4 + 7*x^2 + 3)*elliptic_f(arcsin
(sqrt(-1/3)*x), 6) - 3*sqrt(-2*x^4 - 7*x^2 - 3)*(14*x^3 + 37*x))/(2*x^4 +
7*x^2 + 3)
```

Sympy [F]

$$\int \frac{1}{(-3-7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4-7x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4-7*x**2-3)**(3/2),x)
```

output
```
Integral((-2*x**4 - 7*x**2 - 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-3-7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-7\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-7*x^2-3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 - 7*x^2 - 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-3-7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-7\,x^2-3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4-7*x^2-3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 - 7*x^2 - 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3-7x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4-7\,x^2-3)^{3/2}}\,dx$$

input
```
int(1/(- 7*x^2 - 2*x^4 - 3)^(3/2),x)
```

output
```
int(1/(- 7*x^2 - 2*x^4 - 3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-3-7x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4-7x^2-3}}{4x^8+28x^6+61x^4+42x^2+9}dx$$

input
```
int(1/(-2*x^4-7*x^2-3)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 - 7*x**2 - 3)/(4*x**8 + 28*x**6 + 61*x**4 + 42*x**2 + 9
),x)
```

3.278 $\int \frac{1}{(-2+7x^2-3x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{1}{(-2+7x^2-3x^4)^{3/2}} \, dx = -\frac{x(37-21x^2)}{50\sqrt{-2+7x^2-3x^4}} + \frac{7E\left(\arccos\left(\frac{x}{\sqrt{2}}\right)|\frac{6}{5}\right)}{10\sqrt{5}} - \frac{\text{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{2}}\right),\frac{6}{5}\right)}{10\sqrt{5}}$$

output
```
-1/50*x*(-21*x^2+37)/(-3*x^4+7*x^2-2)^(1/2)+7/50*EllipticE(1/2*(-2*x^2+4)^
(1/2),1/5*30^(1/2))*5^(1/2)-1/50*InverseJacobiAM(arccos(1/2*x*2^(1/2)),1/5
*30^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 6.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42

$$\int \frac{1}{(-2+7x^2-3x^4)^{3/2}} \, dx = \frac{-37x+21x^3+7\sqrt{6-18x^2}\sqrt{2-x^2}E\left(\arcsin\left(\sqrt{3}x\right)|\frac{1}{6}\right)-5\sqrt{6-18x^2}\sqrt{2-}}{50\sqrt{-2+7x^2-3x^4}}$$

input
```
Integrate[(-2 + 7*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(-37*x + 21*x^3 + 7*Sqrt[6 - 18*x^2]*Sqrt[2 - x^2]*EllipticE[ArcSin[Sqrt[3
]*x], 1/6] - 5*Sqrt[6 - 18*x^2]*Sqrt[2 - x^2]*EllipticF[ArcSin[Sqrt[3]*x],
 1/6])/(50*Sqrt[-2 + 7*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1405, 27, 1494, 27, 399, 322, 328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4+7x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{50}\int \frac{3(4-7x^2)}{\sqrt{-3x^4+7x^2-2}}dx-\frac{x(37-21x^2)}{50\sqrt{-3x^4+7x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{50}\int \frac{4-7x^2}{\sqrt{-3x^4+7x^2-2}}dx-\frac{x(37-21x^2)}{50\sqrt{-3x^4+7x^2-2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{3}{25}\sqrt{3}\int \frac{4-7x^2}{2\sqrt{3}\sqrt{2-x^2}\sqrt{3x^2-1}}dx-\frac{x(37-21x^2)}{50\sqrt{-3x^4+7x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{50}\int \frac{4-7x^2}{\sqrt{2-x^2}\sqrt{3x^2-1}}dx-\frac{x(37-21x^2)}{50\sqrt{-3x^4+7x^2-2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{3}{50}\left(\frac{5}{3}\int \frac{1}{\sqrt{2-x^2}\sqrt{3x^2-1}}dx-\frac{7}{3}\int \frac{\sqrt{3x^2-1}}{\sqrt{2-x^2}}dx\right)-\frac{x(37-21x^2)}{50\sqrt{-3x^4+7x^2-2}}$$

$$\Big\downarrow \text{322}$$

$$\frac{3}{50}\left(-\frac{7}{3}\int \frac{\sqrt{3x^2-1}}{\sqrt{2-x^2}}dx-\frac{1}{3}\sqrt{5}\,\text{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{2}}\right),\frac{6}{5}\right)\right)-\frac{x(37-21x^2)}{50\sqrt{-3x^4+7x^2-2}}$$

$$\Big\downarrow \; 328$$

$$\frac{3}{50}\left(\frac{7}{3}\sqrt{5}E\left(\arccos\left(\frac{x}{\sqrt{2}}\right)|\frac{6}{5}\right)-\frac{1}{3}\sqrt{5}\operatorname{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{2}}\right),\frac{6}{5}\right)\right)-\frac{x(37-21x^2)}{50\sqrt{-3x^4+7x^2-2}}$$

input
```
Int[(-2 + 7*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/50*(x*(37 - 21*x^2))/Sqrt[-2 + 7*x^2 - 3*x^4] + (3*((7*Sqrt[5]*Elliptic
E[ArcCos[x/Sqrt[2]], 6/5])/3 - (Sqrt[5]*EllipticF[ArcCos[x/Sqrt[2]], 6/5])
/3))/50
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(65) = 130$.

Time = 2.65 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.87

method	result
risch	$\frac{x(21x^2-37)}{50\sqrt{-3x^4+7x^2-2}}+\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{-3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},\sqrt{6}\right)}{25\sqrt{-3x^4+7x^2-2}}-\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{-3x^2+1}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},\sqrt{6}\right)-\mathrm{Ellipt}\right.}{100\sqrt{-3x^4+7x^2-2}}$
default	$\frac{-\frac{37}{50}x+\frac{21}{50}x^3}{\sqrt{-3x^4+7x^2-2}}+\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{-3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},\sqrt{6}\right)}{25\sqrt{-3x^4+7x^2-2}}-\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{-3x^2+1}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},\sqrt{6}\right)-\mathrm{Elliptic}\right.}{100\sqrt{-3x^4+7x^2-2}}$
elliptic	$\frac{-\frac{37}{50}x+\frac{21}{50}x^3}{\sqrt{-3x^4+7x^2-2}}+\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{-3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},\sqrt{6}\right)}{25\sqrt{-3x^4+7x^2-2}}-\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{-3x^2+1}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},\sqrt{6}\right)-\mathrm{Elliptic}\right.}{100\sqrt{-3x^4+7x^2-2}}$

input
```
int(1/(-3*x^4+7*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/50*x*(21*x^2-37)/(-3*x^4+7*x^2-2)^(1/2)+3/25*2^(1/2)*(-2*x^2+4)^(1/2)*(-
3*x^2+1)^(1/2)/(-3*x^4+7*x^2-2)^(1/2)*EllipticF(1/2*x*2^(1/2),6^(1/2))-7/1
00*2^(1/2)*(-2*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)/(-3*x^4+7*x^2-2)^(1/2)*(Ellip
ticF(1/2*x*2^(1/2),6^(1/2))-EllipticE(1/2*x*2^(1/2),6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int \frac{1}{(-2+7x^2-3x^4)^{3/2}}\,dx = -\frac{21\sqrt{3}\sqrt{-2}(3x^4-7x^2+2)E(\arcsin(\sqrt{3}x)\mid\frac{1}{6}) - 19\sqrt{3}\sqrt{-2}(3x^4-7x^2+2)F(\arcsin(\sqrt{3}x)\mid\frac{1}{6}) + \sqrt{-}}{50(3x^4-7x^2+2)}$$

input
```
integrate(1/(-3*x^4+7*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
-1/50*(21*sqrt(3)*sqrt(-2)*(3*x^4 - 7*x^2 + 2)*elliptic_e(arcsin(sqrt(3)*x
), 1/6) - 19*sqrt(3)*sqrt(-2)*(3*x^4 - 7*x^2 + 2)*elliptic_f(arcsin(sqrt(3
)*x), 1/6) + sqrt(-3*x^4 + 7*x^2 - 2)*(21*x^3 - 37*x))/(3*x^4 - 7*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(-2+7x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4+7x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4+7*x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 + 7*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2+7x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4+7x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+7*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + 7*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+7x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+7\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+7*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + 7*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+7x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+7\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(7*x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(7*x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+7x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4+7x^2-2}}{9x^8-42x^6+61x^4-28x^2+4}dx$$

input
```
int(1/(-3*x^4+7*x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + 7*x**2 - 2)/(9*x**8 - 42*x**6 + 61*x**4 - 28*x**2 + 4
),x)
```

3.279 $\int \frac{1}{(-2+6x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{1}{(-2+6x^2-3x^4)^{3/2}}\,dx = -\frac{x(4-3x^2)}{4\sqrt{-2+6x^2-3x^4}} + \frac{\sqrt[4]{3}E\left(\arccos\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right)|\frac{1}{2}(1+\sqrt{3})\right)}{2\sqrt{2}} + \frac{(1-\sqrt{3})\,\text{EllipticF}\left(\arccos\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right),\frac{1}{2}(1+\sqrt{3})\right)}{4\sqrt{2}\sqrt[4]{3}}$$

output

```
-1/4*x*(-3*x^2+4)/(-3*x^4+6*x^2-2)^(1/2)+1/4*3^(1/4)*EllipticE((1-3/(3+3^(
1/2))*x^2)^(1/2),1/2*(2+2*3^(1/2))^(1/2))*2^(1/2)+1/24*(1-3^(1/2))*Inverse
JacobiAM(arccos(3^(1/2)/(3+3^(1/2))^(1/2)*x),1/2*(2+2*3^(1/2))^(1/2))*2^(1
/2)*3^(3/4)
```

Mathematica [A] (warning: unable to verify)

Time = 6.64 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.47

$$\int \frac{1}{(-2+6x^2-3x^4)^{3/2}}\,dx = \frac{6x(-4+3x^2)+3\sqrt{2}(1+\sqrt{3})\sqrt{3-\sqrt{3}-3x^2}\sqrt{2+(-3+\sqrt{3})\,x^2}E\Big(\text{arcsi}}{}$$

input
```
Integrate[(-2 + 6*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(6*x*(-4 + 3*x^2) + 3*Sqrt[2]*(1 + Sqrt[3])*Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt
[2 + (-3 + Sqrt[3])*x^2]*EllipticE[ArcSin[Sqrt[(3 + Sqrt[3])/2]*x], 2 - Sq
rt[3]] - Sqrt[2]*(3 + Sqrt[3])*Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt[2 + (-3 + Sq
rt[3])*x^2]*EllipticF[ArcSin[Sqrt[(3 + Sqrt[3])/2]*x], 2 - Sqrt[3]])/(24*S
qrt[-2 + 6*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1405, 27, 1494, 27, 399, 322, 328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4+6x^2-2)^{3/2}}dx$$

$$\Big\downarrow \ 1405$$

$$\frac{1}{24}\int \frac{6(2-3x^2)}{\sqrt{-3x^4+6x^2-2}}dx-\frac{x(4-3x^2)}{4\sqrt{-3x^4+6x^2-2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\int \frac{2-3x^2}{\sqrt{-3x^4+6x^2-2}}dx-\frac{x(4-3x^2)}{4\sqrt{-3x^4+6x^2-2}}$$

$$\Big\downarrow \ 1494$$

$$\frac{1}{2}\sqrt{3}\int \frac{2-3x^2}{2\sqrt{-3x^2+\sqrt{3}+3}\sqrt{3x^2+\sqrt{3}-3}}dx - \frac{x(4-3x^2)}{4\sqrt{-3x^4+6x^2-2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\sqrt{3}\int \frac{2-3x^2}{\sqrt{-3x^2+\sqrt{3}+3}\sqrt{3x^2+\sqrt{3}-3}}dx - \frac{x(4-3x^2)}{4\sqrt{-3x^4+6x^2-2}}$$

$$\Big\downarrow \; 399$$

$$\frac{1}{4}\sqrt{3}\left(-\left(\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{-3x^2+\sqrt{3}+3}\sqrt{3x^2+\sqrt{3}-3}}dx\right) - \int \frac{\sqrt{3x^2+\sqrt{3}-3}}{\sqrt{-3x^2+\sqrt{3}+3}}dx\right) - \frac{x(4-3x^2)}{4\sqrt{-3x^4+6x^2-2}}$$

$$\Big\downarrow \; 322$$

$$\frac{1}{4}\sqrt{3}\left(\frac{(1-\sqrt{3})\,\text{EllipticF}\left(\arccos\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right),\frac{1}{2}(1+\sqrt{3})\right)}{\sqrt{2}3^{3/4}} - \int \frac{\sqrt{3x^2+\sqrt{3}-3}}{\sqrt{-3x^2+\sqrt{3}+3}}dx\right) - \frac{x(4-3x^2)}{4\sqrt{-3x^4+6x^2-2}}$$

$$\Big\downarrow \; 328$$

$$\frac{1}{4}\sqrt{3}\left(\frac{(1-\sqrt{3})\,\text{EllipticF}\left(\arccos\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right),\frac{1}{2}(1+\sqrt{3})\right)}{\sqrt{2}3^{3/4}} + \frac{\sqrt{2}E\left(\arccos\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right)|\frac{1}{2}(1+\sqrt{3})\right)}{\sqrt[4]{3}}\right) - \frac{x(4-3x^2)}{4\sqrt{-3x^4+6x^2-2}}$$

input
```
Int[(-2 + 6*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/4*(x*(4 - 3*x^2))/Sqrt[-2 + 6*x^2 - 3*x^4] + (Sqrt[3]*((Sqrt[2]*Ellipti
cE[ArcCos[Sqrt[3/(3 + Sqrt[3])]*x], (1 + Sqrt[3])/2])/3^(1/4) + ((1 - Sqrt
[3])*EllipticF[ArcCos[Sqrt[3/(3 + Sqrt[3])]*x], (1 + Sqrt[3])/2])/(Sqrt[2]
*3^(3/4))))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(101) = 202.

Time = 2.53 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.77

method	result
risch	$\frac{x(3x^2-4)}{4\sqrt{-3x^4+6x^2-2}}+\frac{\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{6-2\sqrt{3}}\,\sqrt{-3x^4+6x^2-2}}-\frac{6\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x}$
default	$\frac{\frac{3}{4}x^3-x}{\sqrt{-3x^4+6x^2-2}}+\frac{\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{6-2\sqrt{3}}\,\sqrt{-3x^4+6x^2-2}}-\frac{6\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}}{\surd}$
elliptic	$\frac{\frac{3}{4}x^3-x}{\sqrt{-3x^4+6x^2-2}}+\frac{\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{6-2\sqrt{3}}\,\sqrt{-3x^4+6x^2-2}}-\frac{6\sqrt{1-\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}}{\surd}$

input
```
int(1/(-3*x^4+6*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(3*x^2-4)/(-3*x^4+6*x^2-2)^(1/2)+1/(6-2*3^(1/2))^(1/2)*(1-(3/2-1/2*3
^(1/2))*x^2)^(1/2)*(1-(3/2+1/2*3^(1/2))*x^2)^(1/2)/(-3*x^4+6*x^2-2)^(1/2)*
EllipticF(1/2*(6-2*3^(1/2))^(1/2)*x,1/2*6^(1/2)+1/2*2^(1/2))-6/(6-2*3^(1/2
))^(1/2)*(1-(3/2-1/2*3^(1/2))*x^2)^(1/2)*(1-(3/2+1/2*3^(1/2))*x^2)^(1/2)/(
-3*x^4+6*x^2-2)^(1/2)/(6+2*3^(1/2))*(EllipticF(1/2*(6-2*3^(1/2))^(1/2)*x,1
/2*6^(1/2)+1/2*2^(1/2))-EllipticE(1/2*(6-2*3^(1/2))^(1/2)*x,1/2*6^(1/2)+1/
2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.42

$$\int \frac{1}{(-2+6x^2-3x^4)^{3/2}}\,dx =$$

$$-\frac{3\left(\sqrt{3}\sqrt{-2}(3\,x^4-6\,x^2+2)+3\,\sqrt{-2}(3\,x^4-6\,x^2+2)\right)\sqrt{\frac{1}{2}\,\sqrt{3}+\frac{3}{2}}E(\arcsin\left(x\sqrt{\frac{1}{2}\,\sqrt{3}+\frac{3}{2}}\right)\,|-\sqrt{3}+2)}{}$$

input
```
integrate(1/(-3*x^4+6*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
-1/24*(3*(sqrt(3)*sqrt(-2)*(3*x^4 - 6*x^2 + 2) + 3*sqrt(-2)*(3*x^4 - 6*x^2
 + 2))*sqrt(1/2*sqrt(3) + 3/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(3) + 3/2)
), -sqrt(3) + 2) - (5*sqrt(3)*sqrt(-2)*(3*x^4 - 6*x^2 + 2) + 3*sqrt(-2)*(3
*x^4 - 6*x^2 + 2))*sqrt(1/2*sqrt(3) + 3/2)*elliptic_f(arcsin(x*sqrt(1/2*sq
rt(3) + 3/2)), -sqrt(3) + 2) + 6*sqrt(-3*x^4 + 6*x^2 - 2)*(3*x^3 - 4*x))/(
3*x^4 - 6*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(-2+6x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4+6x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4+6*x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 + 6*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2+6x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+6\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+6*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + 6*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+6x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+6\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+6*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + 6*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+6x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+6\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(6*x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(6*x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+6x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4+6x^2-2}}{9x^8-36x^6+48x^4-24x^2+4}dx$$

input
```
int(1/(-3*x^4+6*x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + 6*x**2 - 2)/(9*x**8 - 36*x**6 + 48*x**4 - 24*x**2 + 4
),x)
```

3.280 $\int \frac{1}{(-2+5x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{1}{(-2+5x^2-3x^4)^{3/2}}\,dx = -\frac{x(13-15x^2)}{2\sqrt{-2+5x^2-3x^4}} + \frac{5E(\arccos(x)|3)}{2} - \text{EllipticF}(\arccos(x),3)$$

output
```
-1/2*x*(-15*x^2+13)/(-3*x^4+5*x^2-2)^(1/2)+5/2*EllipticE((-x^2+1)^(1/2),3^
(1/2))-InverseJacobiAM(arccos(x),3^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 105 vs. 2(43) = 86.

Time = 6.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.44

$$\int \frac{1}{(-2+5x^2-3x^4)^{3/2}}\,dx = \frac{-13x+15x^3+5\sqrt{6-9x^2}\sqrt{1-x^2}E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-\sqrt{6-9x^2}\sqrt{1-x}}{2\sqrt{-2+5x^2-3x^4}}$$

input
```
Integrate[(-2 + 5*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(-13*x + 15*x^3 + 5*Sqrt[6 - 9*x^2]*Sqrt[1 - x^2]*EllipticE[ArcSin[Sqrt[3/
2]*x], 2/3] - Sqrt[6 - 9*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x],
 2/3])/(2*Sqrt[-2 + 5*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 322, 328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4+5x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{2}\int \frac{3(4-5x^2)}{\sqrt{-3x^4+5x^2-2}}dx-\frac{x(13-15x^2)}{2\sqrt{-3x^4+5x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\int \frac{4-5x^2}{\sqrt{-3x^4+5x^2-2}}dx-\frac{x(13-15x^2)}{2\sqrt{-3x^4+5x^2-2}}$$

$$\Big\downarrow \text{1494}$$

$$3\sqrt{3}\int \frac{4-5x^2}{2\sqrt{3}\sqrt{1-x^2}\sqrt{3x^2-2}}dx-\frac{x(13-15x^2)}{2\sqrt{-3x^4+5x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\int \frac{4-5x^2}{\sqrt{1-x^2}\sqrt{3x^2-2}}dx-\frac{x(13-15x^2)}{2\sqrt{-3x^4+5x^2-2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{3}{2}\left(\frac{2}{3}\int \frac{1}{\sqrt{1-x^2}\sqrt{3x^2-2}}dx-\frac{5}{3}\int \frac{\sqrt{3x^2-2}}{\sqrt{1-x^2}}dx\right)-\frac{x(13-15x^2)}{2\sqrt{-3x^4+5x^2-2}}$$

$$\Big\downarrow \text{322}$$

$$\frac{3}{2}\left(-\frac{5}{3}\int \frac{\sqrt{3x^2-2}}{\sqrt{1-x^2}}dx-\frac{2}{3}\text{EllipticF}(\arccos(x),3)\right)-\frac{x(13-15x^2)}{2\sqrt{-3x^4+5x^2-2}}$$

$$\Big\downarrow \; 328$$

$$\frac{3}{2}\left(\frac{5E(\arccos(x)|3)}{3}-\frac{2\,\text{EllipticF}(\arccos(x),3)}{3}\right)-\frac{x(13-15x^2)}{2\sqrt{-3x^4+5x^2-2}}$$

input
```
Int[(-2 + 5*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/2*(x*(13 - 15*x^2))/Sqrt[-2 + 5*x^2 - 3*x^4] + (3*((5*EllipticE[ArcCos[
x], 3])/3 - (2*EllipticF[ArcCos[x], 3])/3))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(48) = 96$.

Time = 2.63 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.74

method	result
risch	$\frac{x(15x^2-13)}{2\sqrt{-3x^4+5x^2-2}}+\frac{3\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\ \text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{\sqrt{-3x^4+5x^2-2}}-\frac{5\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\left(\text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(x,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{-3x^4+5x^2-2}}$
default	$\frac{\frac{15}{2}x^3-\frac{13}{2}x}{\sqrt{-3x^4+5x^2-2}}+\frac{3\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\ \text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{\sqrt{-3x^4+5x^2-2}}-\frac{5\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\left(\text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(x,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{-3x^4+5x^2-2}}$
elliptic	$\frac{\frac{15}{2}x^3-\frac{13}{2}x}{\sqrt{-3x^4+5x^2-2}}+\frac{3\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\ \text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{\sqrt{-3x^4+5x^2-2}}-\frac{5\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\left(\text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(x,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{-3x^4+5x^2-2}}$

input
```
int(1/(-3*x^4+5*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(15*x^2-13)/(-3*x^4+5*x^2-2)^(1/2)+3*(-x^2+1)^(1/2)*(-6*x^2+4)^(1/2)
/(-3*x^4+5*x^2-2)^(1/2)*EllipticF(x,1/2*6^(1/2))-5/2*(-x^2+1)^(1/2)*(-6*x^
2+4)^(1/2)/(-3*x^4+5*x^2-2)^(1/2)*(EllipticF(x,1/2*6^(1/2))-EllipticE(x,1/
2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.91

$$\int \frac{1}{\left(-2+5x^2-3x^4\right)^{3/2}}\,dx =$$
$$-\frac{5\sqrt{-2}(3\,x^4-5\,x^2+2)E(\arcsin{(x)}\mid\frac{3}{2})+\sqrt{-2}(3\,x^4-5\,x^2+2)F(\arcsin{(x)}\mid\frac{3}{2})+\sqrt{-3\,x^4+5\,x^2-2}(}{2\,(3\,x^4-5\,x^2+2)}$$

input
```
integrate(1/(-3*x^4+5*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(5*sqrt(-2)*(3*x^4 - 5*x^2 + 2)*elliptic_e(arcsin(x), 3/2) + sqrt(-2)
*(3*x^4 - 5*x^2 + 2)*elliptic_f(arcsin(x), 3/2) + sqrt(-3*x^4 + 5*x^2 - 2)
*(15*x^3 - 13*x))/(3*x^4 - 5*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{\left(-2+5x^2-3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-3x^4+5x^2-2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4+5*x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 + 5*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(-2+5x^2-3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-3\,x^4+5\,x^2-2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+5*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + 5*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+5\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+5*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + 5*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+5\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(5*x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+5x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4+5x^2-2}}{9x^8-30x^6+37x^4-20x^2+4}dx$$

input
```
int(1/(-3*x^4+5*x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + 5*x**2 - 2)/(9*x**8 - 30*x**6 + 37*x**4 - 20*x**2 + 4
),x)
```

3.281 $\int \frac{1}{(-2+4x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 258

$$\int \frac{1}{(-2+4x^2-3x^4)^{3/2}}\,dx = \frac{x(1-3x^2)}{4\sqrt{-2+4x^2-3x^4}} - \frac{3x\sqrt{-2+4x^2-3x^4}}{4\left(\sqrt{6}+3x^2\right)} - \frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\,2^{3/4}\sqrt{-2+4x^2-3x^4}} + \frac{\sqrt[4]{3}(2-\sqrt{6})\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{8\,2^{3/4}\sqrt{-2+4x^2-3x^4}}$$

output
```
1/4*x*(-3*x^2+1)/(-3*x^4+4*x^2-2)^(1/2)-3*x*(-3*x^4+4*x^2-2)^(1/2)/(4*6^(1
/2)+12*x^2)-1/4*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4-4*x^2+2)/(2+6^(1/2)*x^2)^2
)^(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18+6*6^(1/2))^
(1/2))*2^(1/4)/(-3*x^4+4*x^2-2)^(1/2)+1/16*3^(1/4)*(2-6^(1/2))*(2+6^(1/2)*
x^2)*((3*x^4-4*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/
2*3^(1/4)*2^(3/4)*x),1/6*(18+6*6^(1/2))^(1/2))*2^(1/4)/(-3*x^4+4*x^2-2)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.59 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-2+4x^2-3x^4)^{3/2}}\,dx = \frac{3\sqrt{-\frac{i}{2i+\sqrt{2}}}x(1-3x^2)-\sqrt{3}(-2i+\sqrt{2})\sqrt{\frac{2i+\sqrt{2}-3ix^2}{2i+\sqrt{2}}}\sqrt{\frac{-2i+\sqrt{2}+3ix^2}{-2i+\sqrt{2}}}E\Big(i\text{arcsin}$$

input
```
Integrate[(-2 + 4*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(3*Sqrt[(-I)/(2*I + Sqrt[2])]*x*(1 - 3*x^2) - Sqrt[3]*(-2*I + Sqrt[2])*Sqr
t[(2*I + Sqrt[2] - (3*I)*x^2)/(2*I + Sqrt[2])]*Sqrt[(-2*I + Sqrt[2] + (3*I
)*x^2)/(-2*I + Sqrt[2])]*EllipticE[I*ArcSinh[Sqrt[(-3*I)/(2*I + Sqrt[2])]*
x], (2*I + Sqrt[2])/(2*I - Sqrt[2])] + Sqrt[3]*(I + Sqrt[2])*Sqrt[(2*I + S
qrt[2] - (3*I)*x^2)/(2*I + Sqrt[2])]*Sqrt[(-2*I + Sqrt[2] + (3*I)*x^2)/(-2
*I + Sqrt[2])]*EllipticF[I*ArcSinh[Sqrt[(-3*I)/(2*I + Sqrt[2])]*x], (2*I +
 Sqrt[2])/(2*I - Sqrt[2])])/(12*Sqrt[(-I)/(2*I + Sqrt[2])]*Sqrt[-2 + 4*x^2
 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4+4x^2-2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1405$$

$$\frac{x(1-3x^2)}{4\sqrt{-3x^4+4x^2-2}}-\frac{1}{16}\int \frac{12(1-x^2)}{\sqrt{-3x^4+4x^2-2}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{x(1-3x^2)}{4\sqrt{-3x^4+4x^2-2}}-\frac{3}{4}\int \frac{1-x^2}{\sqrt{-3x^4+4x^2-2}}dx$$

$\Big\downarrow$ 1511

$$\frac{x(1-3x^2)}{4\sqrt{-3x^4+4x^2-2}}-\frac{3}{4}\left(\frac{1}{3}\left(3-\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4+4x^2-2}}dx+\sqrt{\frac{2}{3}}\int \frac{2-\sqrt{6}x^2}{2\sqrt{-3x^4+4x^2-2}}dx\right)$$

$\Big\downarrow$ 27

$$\frac{x(1-3x^2)}{4\sqrt{-3x^4+4x^2-2}}-\frac{3}{4}\left(\frac{1}{3}\left(3-\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4+4x^2-2}}dx+\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4+4x^2-2}}dx}{\sqrt{6}}\right)$$

$\Big\downarrow$ 1416

$$\frac{x(1-3x^2)}{4\sqrt{-3x^4+4x^2-2}}-\frac{3}{4}\left(\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4+4x^2-2}}dx}{\sqrt{6}}+\frac{\left(3-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{6\sqrt[4]{6}\sqrt{-3x^4+4x^2-2}}\right)$$

$\Big\downarrow$ 1509

$$\frac{x(1-3x^2)}{4\sqrt{-3x^4+4x^2-2}}-\frac{3}{4}\left(\frac{\left(3-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{6\sqrt[4]{6}\sqrt{-3x^4+4x^2-2}}+\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}E\left(2\text{arc}\right.}{\sqrt[4]{3}\sqrt{-3x^4+4x^2}}\right.$$

input
```
Int[(-2 + 4*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(x*(1 - 3*x^2))/(4*Sqrt[-2 + 4*x^2 - 3*x^4]) - (3*(((2*x*Sqrt[-2 + 4*x^2 -
 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 +
3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqr
t[6]])/(3^(1/4)*Sqrt[-2 + 4*x^2 - 3*x^4]))/Sqrt[6] + ((3 - Sqrt[6])*(2 + S
qrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcT
an[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(6*6^(1/4)*Sqrt[-2 + 4*x^2 - 3*x^4]))
)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{x(3x^2-1)}{4\sqrt{-3x^4+4x^2-2}} - \frac{3\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{2\sqrt{4-2i\sqrt{2}}\,\sqrt{-3x^4+4x^2-2}} + \frac{6\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{2}}{2}\right.}}{}$
default	$\frac{\frac{1}{4}x-\frac{3}{4}x^3}{\sqrt{-3x^4+4x^2-2}} - \frac{3\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{2\sqrt{4-2i\sqrt{2}}\,\sqrt{-3x^4+4x^2-2}} + \frac{6\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right.}}{}$
elliptic	$\frac{\frac{1}{4}x-\frac{3}{4}x^3}{\sqrt{-3x^4+4x^2-2}} - \frac{3\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right)x^2}\ \mathrm{EllipticF}\left(\frac{\sqrt{4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{2\sqrt{4-2i\sqrt{2}}\,\sqrt{-3x^4+4x^2-2}} + \frac{6\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{2}}{2}+1\right.}}{}$

input
```
int(1/(-3*x^4+4*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x*(3*x^2-1)/(-3*x^4+4*x^2-2)^(1/2)-3/2/(4-2*I*2^(1/2))^(1/2)*(1-(1-1/
2*I*2^(1/2))*x^2)^(1/2)*(1-(1/2*I*2^(1/2)+1)*x^2)^(1/2)/(-3*x^4+4*x^2-2)^(
1/2)*EllipticF(1/2*(4-2*I*2^(1/2))^(1/2)*x,1/3*(3+6*I*2^(1/2))^(1/2))+6/(4
-2*I*2^(1/2))^(1/2)*(1-(1-1/2*I*2^(1/2))*x^2)^(1/2)*(1-(1/2*I*2^(1/2)+1)*x
^2)^(1/2)/(-3*x^4+4*x^2-2)^(1/2)/(4+2*I*2^(1/2))*(EllipticF(1/2*(4-2*I*2^(
1/2))^(1/2)*x,1/3*(3+6*I*2^(1/2))^(1/2))-EllipticE(1/2*(4-2*I*2^(1/2))^(1/
2)*x,1/3*(3+6*I*2^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.56

$$\int \frac{1}{(-2+4x^2-3x^4)^{3/2}}\,dx =$$
$$-\frac{\left(3\,x^4-4\,x^2-\sqrt{-2}(3\,x^4-4\,x^2+2)+2\right)\sqrt{\frac{1}{2}\,\sqrt{-2}+1}E(\arcsin\left(x\sqrt{\frac{1}{2}\,\sqrt{-2}+1}\right)\,|-\frac{2}{3}\,\sqrt{-2}+\frac{1}{3})-2\,(3}{4\,(3\,x^4-4}$$

input
```
integrate(1/(-3*x^4+4*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*((3*x^4 - 4*x^2 - sqrt(-2)*(3*x^4 - 4*x^2 + 2) + 2)*sqrt(1/2*sqrt(-2)
 + 1)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(-2) + 1)), -2/3*sqrt(-2) + 1/3) -
2*(3*x^4 - 4*x^2 + 2)*sqrt(1/2*sqrt(-2) + 1)*elliptic_f(arcsin(x*sqrt(1/2*
sqrt(-2) + 1)), -2/3*sqrt(-2) + 1/3) - sqrt(-3*x^4 + 4*x^2 - 2)*(3*x^3 - x
))/(3*x^4 - 4*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(-2+4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4+4x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4+4*x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 + 4*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2+4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+4\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+4*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + 4*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+4\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+4*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + 4*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+4\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(4*x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(4*x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+4x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4+4x^2-2}}{9x^8-24x^6+28x^4-16x^2+4}dx$$

input
```
int(1/(-3*x^4+4*x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + 4*x**2 - 2)/(9*x**8 - 24*x**6 + 28*x**4 - 16*x**2 + 4
),x)
```

3.282 $\int \frac{1}{(-2+3x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 257

$$\int \frac{1}{(-2+3x^2-3x^4)^{3/2}}\,dx = -\frac{x(1+3x^2)}{10\sqrt{-2+3x^2-3x^4}}-\frac{3x\sqrt{-2+3x^2-3x^4}}{10\left(\sqrt{6}+3x^2\right)} -\frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}(4+\sqrt{6})\right)}{5\ 2^{3/4}\sqrt{-2+3x^2-3x^4}} +\frac{\left(3-2\sqrt{6}\right)\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}(4+\sqrt{6})\right)}{10\ 6^{3/4}\sqrt{-2+3x^2-3x^4}}$$

output
```
-1/10*x*(3*x^2+1)/(-3*x^4+3*x^2-2)^(1/2)-3*x*(-3*x^4+3*x^2-2)^(1/2)/(10*6^
(1/2)+30*x^2)-1/10*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4-3*x^2+2)/(2+6^(1/2)*x^2
)^2)^(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/4*(8+2*6^(1/2)
)^(1/2))*2^(1/4)/(-3*x^4+3*x^2-2)^(1/2)+1/60*(3-2*6^(1/2))*(2+6^(1/2)*x^2)
*((3*x^4-3*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*3^
(1/4)*2^(3/4)*x),1/4*(8+2*6^(1/2))^(1/2))*6^(1/4)/(-3*x^4+3*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.77 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.34

$$\int \frac{1}{(-2+3x^2-3x^4)^{3/2}}\,dx = \frac{-12\sqrt{-\frac{i}{3i+\sqrt{15}}}x(1+3x^2)+3i\sqrt{2}(\sqrt{3}+i\sqrt{5})\sqrt{\frac{3i+\sqrt{15}-6ix^2}{3i+\sqrt{15}}}\sqrt{\frac{-3i+\sqrt{15}+6ix^2}{-3i+\sqrt{15}}}E}{}$$

input
```
Integrate[(-2 + 3*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(-12*Sqrt[(-I)/(3*I + Sqrt[15])]*x*(1 + 3*x^2) + (3*I)*Sqrt[2]*(Sqrt[3] +
I*Sqrt[5])*Sqrt[(3*I + Sqrt[15] - (6*I)*x^2)/(3*I + Sqrt[15])]*Sqrt[(-3*I
+ Sqrt[15] + (6*I)*x^2)/(-3*I + Sqrt[15])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)
/(3*I + Sqrt[15])]*x], (3*I + Sqrt[15])/(3*I - Sqrt[15])] + Sqrt[2]*((5*I)
*Sqrt[3] + 3*Sqrt[5])*Sqrt[(3*I + Sqrt[15] - (6*I)*x^2)/(3*I + Sqrt[15])]*
Sqrt[(-3*I + Sqrt[15] + (6*I)*x^2)/(-3*I + Sqrt[15])]*EllipticF[I*ArcSinh[
Sqrt[(-6*I)/(3*I + Sqrt[15])]*x], (3*I + Sqrt[15])/(3*I - Sqrt[15])])/(120
*Sqrt[(-I)/(3*I + Sqrt[15])]*Sqrt[-2 + 3*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4+3x^2-2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$-\frac{1}{30}\int \frac{3(4-3x^2)}{\sqrt{-3x^4+3x^2-2}}dx - \frac{x(3x^2+1)}{10\sqrt{-3x^4+3x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{10}\int \frac{4-3x^2}{\sqrt{-3x^4+3x^2-2}}dx-\frac{x\left(3x^2+1\right)}{10\sqrt{-3x^4+3x^2-2}}$$

$\Big\downarrow$ 1511

$$\frac{1}{10}\left(-\left(\left(4-\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4+3x^2-2}}dx\right)-\sqrt{6}\int \frac{2-\sqrt{6}x^2}{2\sqrt{-3x^4+3x^2-2}}dx\right)-\frac{x\left(3x^2+1\right)}{10\sqrt{-3x^4+3x^2-2}}$$

$\Big\downarrow$ 27

$$\frac{1}{10}\left(-\left(\left(4-\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4+3x^2-2}}dx\right)-\sqrt{\frac{3}{2}}\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4+3x^2-2}}dx\right)-\frac{x\left(3x^2+1\right)}{10\sqrt{-3x^4+3x^2-2}}$$

$\Big\downarrow$ 1416

$$\frac{1}{10}\left(-\sqrt{\frac{3}{2}}\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4+3x^2-2}}dx-\frac{\left(4-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+3x^2-2}}\right)-\frac{x\left(3x^2+1\right)}{10\sqrt{-3x^4+3x^2-2}}$$

$\Big\downarrow$ 1509

$$\frac{1}{10}\left(-\frac{\left(4-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+3x^2-2}}-\sqrt{\frac{3}{2}}\left(\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\ldots}{\ldots}\right)\right)-\frac{x\left(3x^2+1\right)}{10\sqrt{-3x^4+3x^2-2}}$$

input
```
Int[(-2 + 3*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/10*(x*(1 + 3*x^2))/Sqrt[-2 + 3*x^2 - 3*x^4] + (-(Sqrt[3/2]*((2*x*Sqrt[-
2 + 3*x^2 - 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2
 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x],
(4 + Sqrt[6])/8])/(3^(1/4)*Sqrt[-2 + 3*x^2 - 3*x^4]))) - ((4 - Sqrt[6])*(2
 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-2 + 3*x^2 - 3*x^
4]))/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{x(3x^2+1)}{10\sqrt{-3x^4+3x^2-2}}-\frac{4\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{5\sqrt{3-i\sqrt{15}}\,\sqrt{-3x^4+3x^2-2}}+\frac{12\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{}}{}$
default	$\frac{-\frac{1}{10}x-\frac{3}{10}x^3}{\sqrt{-3x^4+3x^2-2}}-\frac{4\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{5\sqrt{3-i\sqrt{15}}\,\sqrt{-3x^4+3x^2-2}}+\frac{12\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-(}}{}$
elliptic	$\frac{-\frac{1}{10}x-\frac{3}{10}x^3}{\sqrt{-3x^4+3x^2-2}}-\frac{4\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{5\sqrt{3-i\sqrt{15}}\,\sqrt{-3x^4+3x^2-2}}+\frac{12\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-(}}{}$

input
```
int(1/(-3*x^4+3*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*x*(3*x^2+1)/(-3*x^4+3*x^2-2)^(1/2)-4/5/(3-I*15^(1/2))^(1/2)*(1-(3/4-
1/4*I*15^(1/2))*x^2)^(1/2)*(1-(3/4+1/4*I*15^(1/2))*x^2)^(1/2)/(-3*x^4+3*x^
2-2)^(1/2)*EllipticF(1/2*(3-I*15^(1/2))^(1/2)*x,1/2*(-1+I*15^(1/2))^(1/2))
+12/5/(3-I*15^(1/2))^(1/2)*(1-(3/4-1/4*I*15^(1/2))*x^2)^(1/2)*(1-(3/4+1/4*
I*15^(1/2))*x^2)^(1/2)/(-3*x^4+3*x^2-2)^(1/2)/(3+I*15^(1/2))*(EllipticF(1/
2*(3-I*15^(1/2))^(1/2)*x,1/2*(-1+I*15^(1/2))^(1/2))-EllipticE(1/2*(3-I*15^
(1/2))^(1/2)*x,1/2*(-1+I*15^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.61

$$\int \frac{1}{(-2+3x^2-3x^4)^{3/2}}\,dx = \frac{3\sqrt{-2}\left(9\,x^4-9\,x^2+\sqrt{-15}(3\,x^4-3\,x^2+2)+6\right)\sqrt{\sqrt{-15}+3}E(\arcsin\left(\frac{1}{2}\,x\right.}{}$$

input
```
integrate(1/(-3*x^4+3*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/240*(3*sqrt(-2)*(9*x^4 - 9*x^2 + sqrt(-15)*(3*x^4 - 3*x^2 + 2) + 6)*sqrt
(sqrt(-15) + 3)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(-15) + 3)), -1/4*sqrt(-1
5) - 1/4) + sqrt(-2)*(9*x^4 - 9*x^2 - 7*sqrt(-15)*(3*x^4 - 3*x^2 + 2) + 6)
*sqrt(sqrt(-15) + 3)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(-15) + 3)), -1/4*sq
rt(-15) - 1/4) + 24*sqrt(-3*x^4 + 3*x^2 - 2)*(3*x^3 + x))/(3*x^4 - 3*x^2 +
 2)
```

Sympy [F]

$$\int \frac{1}{(-2+3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4+3x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4+3*x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 + 3*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2+3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+3\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+3*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + 3*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+3\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+3*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + 3*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+3\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(3*x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(3*x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+3x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4+3x^2-2}}{9x^8-18x^6+21x^4-12x^2+4}dx$$

input
```
int(1/(-3*x^4+3*x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + 3*x**2 - 2)/(9*x**8 - 18*x**6 + 21*x**4 - 12*x**2 + 4
),x)
```

3.283 $\int \frac{1}{(-2+2x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 262

$$\int \frac{1}{(-2+2x^2-3x^4)^{3/2}}\,dx = -\frac{x(4+3x^2)}{20\sqrt{-2+2x^2-3x^4}} - \frac{3x\sqrt{-2+2x^2-3x^4}}{20\left(\sqrt{6}+3x^2\right)} - \frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)\middle|\frac{1}{12}(6+\sqrt{6})\right)}{10\ 2^{3/4}\sqrt{-2+2x^2-3x^4}} + \frac{\sqrt[4]{3}(1-\sqrt{6})\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}(6+\sqrt{6})\right)}{20\ 2^{3/4}\sqrt{-2+2x^2-3x^4}}$$

output

```
-1/20*x*(3*x^2+4)/(-3*x^4+2*x^2-2)^(1/2)-3*x*(-3*x^4+2*x^2-2)^(1/2)/(20*6^
(1/2)+60*x^2)-1/20*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4-2*x^2+2)/(2+6^(1/2)*x^2
)^2)^(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18+3*6^(1/2
))^(1/2))*2^(1/4)/(-3*x^4+2*x^2-2)^(1/2)+1/40*3^(1/4)*(1-6^(1/2))*(2+6^(1/
2)*x^2)*((3*x^4-2*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan
(1/2*3^(1/4)*2^(3/4)*x),1/6*(18+3*6^(1/2))^(1/2))*2^(1/4)/(-3*x^4+2*x^2-2)
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.45 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.25

$$\int \frac{1}{(-2+2x^2-3x^4)^{3/2}}\,dx = \frac{-3\sqrt{-\frac{i}{i+\sqrt{5}}}\,x(4+3x^2)-\sqrt{3}(-i+\sqrt{5})\sqrt{\frac{i+\sqrt{5}-3ix^2}{i+\sqrt{5}}}\sqrt{\frac{-i+\sqrt{5}+3ix^2}{-i+\sqrt{5}}}\,E\Big(i\operatorname{arcsinh}}{60}$$

input
```
Integrate[(-2 + 2*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(-3*Sqrt[(-I)/(I + Sqrt[5])]*x*(4 + 3*x^2) - Sqrt[3]*(-I + Sqrt[5])*Sqrt[(
I + Sqrt[5] - (3*I)*x^2)/(I + Sqrt[5])]*Sqrt[(-I + Sqrt[5] + (3*I)*x^2)/(-
I + Sqrt[5])]*EllipticE[I*ArcSinh[Sqrt[(-3*I)/(I + Sqrt[5])]*x], (I + Sqrt
[5])/(I - Sqrt[5])] + Sqrt[3]*(5*I + Sqrt[5])*Sqrt[(I + Sqrt[5] - (3*I)*x^
2)/(I + Sqrt[5])]*Sqrt[(-I + Sqrt[5] + (3*I)*x^2)/(-I + Sqrt[5])]*Elliptic
F[I*ArcSinh[Sqrt[(-3*I)/(I + Sqrt[5])]*x], (I + Sqrt[5])/(I - Sqrt[5])])/(
60*Sqrt[(-I)/(I + Sqrt[5])]*Sqrt[-2 + 2*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4+2x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$-\frac{1}{40}\int \frac{6(2-x^2)}{\sqrt{-3x^4+2x^2-2}}dx-\frac{x(3x^2+4)}{20\sqrt{-3x^4+2x^2-2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{20}\int \frac{2-x^2}{\sqrt{-3x^4+2x^2-2}}dx-\frac{x\left(3x^2+4\right)}{20\sqrt{-3x^4+2x^2-2}}$$

$\big\downarrow$ 1511

$$-\frac{3}{20}\left(\frac{1}{3}\left(6-\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4+2x^2-2}}dx+\sqrt{\frac{2}{3}}\int \frac{2-\sqrt{6}x^2}{2\sqrt{-3x^4+2x^2-2}}dx\right)-\frac{x\left(3x^2+4\right)}{20\sqrt{-3x^4+2x^2-2}}$$

$\big\downarrow$ 27

$$-\frac{3}{20}\left(\frac{1}{3}\left(6-\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4+2x^2-2}}dx+\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4+2x^2-2}}dx}{\sqrt{6}}\right)-\frac{x\left(3x^2+4\right)}{20\sqrt{-3x^4+2x^2-2}}$$

$\big\downarrow$ 1416

$$-\frac{3}{20}\left(\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4+2x^2-2}}dx}{\sqrt{6}}+\frac{\left(6-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{-3x^4+2x^2-2}}\right)-\frac{x\left(3x^2+4\right)}{20\sqrt{-3x^4+2x^2-2}}$$

$\big\downarrow$ 1509

$$-\frac{3}{20}\left(\frac{\left(6-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{-3x^4+2x^2-2}}+\frac{\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\cdots}{\sqrt[4]{3}\cdots}}{\cdots}\right)-\frac{x\left(3x^2+4\right)}{20\sqrt{-3x^4+2x^2-2}}$$

input
```
Int[(-2 + 2*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/20*(x*(4 + 3*x^2))/Sqrt[-2 + 2*x^2 - 3*x^4] - (3*(((2*x*Sqrt[-2 + 2*x^2
 - 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2
+ 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt
[6])/12])/(3^(1/4)*Sqrt[-2 + 2*x^2 - 3*x^4]))/Sqrt[6] + ((6 - Sqrt[6])*(2
+ Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*A
rcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(6*6^(1/4)*Sqrt[-2 + 2*x^2 - 3*x^
4])))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{x(3x^2+4)}{20\sqrt{-3x^4+2x^2-2}}-\frac{3\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{2-2i\sqrt{5}}\,x}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{5\sqrt{2-2i\sqrt{5}}\,\sqrt{-3x^4+2x^2-2}}+\frac{6\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-}$
default	$\frac{-\frac{1}{5}x-\frac{3}{20}x^3}{\sqrt{-3x^4+2x^2-2}}-\frac{3\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{2-2i\sqrt{5}}\,x}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{5\sqrt{2-2i\sqrt{5}}\,\sqrt{-3x^4+2x^2-2}}+\frac{6\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\right.}$
elliptic	$\frac{-\frac{1}{5}x-\frac{3}{20}x^3}{\sqrt{-3x^4+2x^2-2}}-\frac{3\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{2-2i\sqrt{5}}\,x}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{5\sqrt{2-2i\sqrt{5}}\,\sqrt{-3x^4+2x^2-2}}+\frac{6\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}+\right.}$

input
```
int(1/(-3*x^4+2*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*x*(3*x^2+4)/(-3*x^4+2*x^2-2)^(1/2)-3/5/(2-2*I*5^(1/2))^(1/2)*(1-(1/2
-1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2+1/2*I*5^(1/2))*x^2)^(1/2)/(-3*x^4+2*x^2
-2)^(1/2)*EllipticF(1/2*(2-2*I*5^(1/2))^(1/2)*x,1/3*(-6+3*I*5^(1/2))^(1/2)
)+6/5/(2-2*I*5^(1/2))^(1/2)*(1-(1/2-1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2+1/2*
I*5^(1/2))*x^2)^(1/2)/(-3*x^4+2*x^2-2)^(1/2)/(2+2*I*5^(1/2))*(EllipticF(1/
2*(2-2*I*5^(1/2))^(1/2)*x,1/3*(-6+3*I*5^(1/2))^(1/2))-EllipticE(1/2*(2-2*I
*5^(1/2))^(1/2)*x,1/3*(-6+3*I*5^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.63

$$\int \frac{1}{\left(-2+2x^2-3x^4\right)^{3/2}}\,dx = \frac{\sqrt{-2}\left(3\,x^4-2\,x^2+\sqrt{-5}\left(3\,x^4-2\,x^2+2\right)+2\right)\sqrt{\frac{1}{2}\sqrt{-5}+\frac{1}{2}}E\left(\arcsin\left(x\sqrt{\frac{1}{2}\right.\right.}{}$$

input
```
integrate(1/(-3*x^4+2*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/40*(sqrt(-2)*(3*x^4 - 2*x^2 + sqrt(-5)*(3*x^4 - 2*x^2 + 2) + 2)*sqrt(1/2
*sqrt(-5) + 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(-5) + 1/2)), -1/3*sqrt(
-5) - 2/3) + sqrt(-2)*(3*x^4 - 2*x^2 - 3*sqrt(-5)*(3*x^4 - 2*x^2 + 2) + 2)
*sqrt(1/2*sqrt(-5) + 1/2)*elliptic_f(arcsin(x*sqrt(1/2*sqrt(-5) + 1/2)), -
1/3*sqrt(-5) - 2/3) + 2*sqrt(-3*x^4 + 2*x^2 - 2)*(3*x^3 + 4*x))/(3*x^4 - 2
*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{\left(-2+2x^2-3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-3x^4+2x^2-2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4+2*x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 + 2*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(-2+2x^2-3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-3\,x^4+2\,x^2-2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+2*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + 2*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+2x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+2\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+2*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + 2*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+2x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+2\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(2*x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(2*x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+2x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4+2x^2-2}}{9x^8-12x^6+16x^4-8x^2+4}dx$$

input
```
int(1/(-3*x^4+2*x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + 2*x**2 - 2)/(9*x**8 - 12*x**6 + 16*x**4 - 8*x**2 + 4)
,x)
```

3.284 $\int \frac{1}{(-2+x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 254

$$\int \frac{1}{(-2+x^2-3x^4)^{3/2}}\,dx = -\frac{x(11+3x^2)}{46\sqrt{-2+x^2-3x^4}} - \frac{3x\sqrt{-2+x^2-3x^4}}{46\left(\sqrt{6}+3x^2\right)} - \frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)\middle|\frac{1}{24}(12+\sqrt{6})\right)}{23\ 2^{3/4}\sqrt{-2+x^2-3x^4}} + \frac{\sqrt[4]{3}(1-2\sqrt{6})\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}(12+\sqrt{6})\right)}{46\ 2^{3/4}\sqrt{-2+x^2-3x^4}}$$

output
```
-1/46*x*(3*x^2+11)/(-3*x^4+x^2-2)^(1/2)-3*x*(-3*x^4+x^2-2)^(1/2)/(46*6^(1/
2)+138*x^2)-1/46*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4-x^2+2)/(2+6^(1/2)*x^2)^2)
^(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72+6*6^(1/2))^
(1/2))*2^(1/4)/(-3*x^4+x^2-2)^(1/2)+1/92*3^(1/4)*(1-2*6^(1/2))*(2+6^(1/2)*
x^2)*((3*x^4-x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*
3^(1/4)*2^(3/4)*x),1/12*(72+6*6^(1/2))^(1/2))*2^(1/4)/(-3*x^4+x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.76 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.28

$$\int \frac{1}{(-2+x^2-3x^4)^{3/2}}\,dx = \frac{-12\sqrt{-\frac{i}{i+\sqrt{23}}}x(11+3x^2)-\sqrt{6}(-i+\sqrt{23})\sqrt{\frac{i+\sqrt{23}-6ix^2}{i+\sqrt{23}}}\sqrt{\frac{-i+\sqrt{23}+6ix^2}{-i+\sqrt{23}}}E\Big(i\text{arc}}{}$$

input
```
Integrate[(-2 + x^2 - 3*x^4)^(-3/2),x]
```

output
```
(-12*Sqrt[(-I)/(I + Sqrt[23])]*x*(11 + 3*x^2) - Sqrt[6]*(-I + Sqrt[23])*Sq
rt[(I + Sqrt[23] - (6*I)*x^2)/(I + Sqrt[23])]*Sqrt[(-I + Sqrt[23] + (6*I)*
x^2)/(-I + Sqrt[23])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(I + Sqrt[23])]*x],
(I + Sqrt[23])/(I - Sqrt[23])] + Sqrt[6]*(23*I + Sqrt[23])*Sqrt[(I + Sqrt[
23] - (6*I)*x^2)/(I + Sqrt[23])]*Sqrt[(-I + Sqrt[23] + (6*I)*x^2)/(-I + Sq
rt[23])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(I + Sqrt[23])]*x], (I + Sqrt[23]
)/(I - Sqrt[23])])/(552*Sqrt[(-I)/(I + Sqrt[23])]*Sqrt[-2 + x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4+x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$-\frac{1}{46}\int \frac{3(4-x^2)}{\sqrt{-3x^4+x^2-2}}dx-\frac{x(3x^2+11)}{46\sqrt{-3x^4+x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{46}\int \frac{4-x^2}{\sqrt{-3x^4+x^2-2}}dx-\frac{x(3x^2+11)}{46\sqrt{-3x^4+x^2-2}}$$

$$\Big\downarrow \; 1511$$

$$-\frac{3}{46}\left(\frac{1}{3}\left(12-\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4+x^2-2}}dx+\sqrt{\frac{2}{3}}\int \frac{2-\sqrt{6}x^2}{2\sqrt{-3x^4+x^2-2}}dx\right)-\frac{x(3x^2+11)}{46\sqrt{-3x^4+x^2-2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{3}{46}\left(\frac{1}{3}\left(12-\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4+x^2-2}}dx+\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4+x^2-2}}dx}{\sqrt{6}}\right)-\frac{x(3x^2+11)}{46\sqrt{-3x^4+x^2-2}}$$

$$\Big\downarrow \; 1416$$

$$-\frac{3}{46}\left(\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4+x^2-2}}dx}{\sqrt{6}}+\frac{\left(12-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{-3x^4+x^2-2}}\right)-\frac{x(3x^2+11)}{46\sqrt{-3x^4+x^2-2}}$$

$$\Big\downarrow \; 1509$$

$$-\frac{3}{46}\left(\frac{\left(12-\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{-3x^4+x^2-2}}+\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2}{\left(\sqrt{6}x^2+\right.}}}{\sqrt[4]{}}\right.$$

$$\frac{x(3x^2+11)}{46\sqrt{-3x^4+x^2-2}}$$

input
```
Int[(-2 + x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/46*(x*(11 + 3*x^2))/Sqrt[-2 + x^2 - 3*x^4] - (3*(((2*x*Sqrt[-2 + x^2 -
3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x
^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])
/24])/(3^(1/4)*Sqrt[-2 + x^2 - 3*x^4]))/Sqrt[6] + ((12 - Sqrt[6])*(2 + Sqr
t[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan[(
3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(6*6^(1/4)*Sqrt[-2 + x^2 - 3*x^4])))/46
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{x(3x^2+11)}{46\sqrt{-3x^4+x^2-2}} - \frac{12\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{23\sqrt{1-i\sqrt{23}}\sqrt{-3x^4+x^2-2}} + \frac{12\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\cdots}{\cdots}$
default	$\frac{-\frac{11}{46}x-\frac{3}{46}x^3}{\sqrt{-3x^4+x^2-2}} - \frac{12\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{23\sqrt{1-i\sqrt{23}}\sqrt{-3x^4+x^2-2}} + \frac{12\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\cdots}}{\cdots}$
elliptic	$\frac{-\frac{11}{46}x-\frac{3}{46}x^3}{\sqrt{-3x^4+x^2-2}} - \frac{12\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{23\sqrt{1-i\sqrt{23}}\sqrt{-3x^4+x^2-2}} + \frac{12\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\cdots}}{\cdots}$

input
```
int(1/(-3*x^4+x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/46*x*(3*x^2+11)/(-3*x^4+x^2-2)^(1/2)-12/23/(1-I*23^(1/2))^(1/2)*(1-(1/4
-1/4*I*23^(1/2))*x^2)^(1/2)*(1-(1/4+1/4*I*23^(1/2))*x^2)^(1/2)/(-3*x^4+x^2
-2)^(1/2)*EllipticF(1/2*(1-I*23^(1/2))^(1/2)*x,1/6*(-33+3*I*23^(1/2))^(1/2
))+12/23/(1-I*23^(1/2))^(1/2)*(1-(1/4-1/4*I*23^(1/2))*x^2)^(1/2)*(1-(1/4+1
/4*I*23^(1/2))*x^2)^(1/2)/(-3*x^4+x^2-2)^(1/2)/(1+I*23^(1/2))*(EllipticF(1
/2*(1-I*23^(1/2))^(1/2)*x,1/6*(-33+3*I*23^(1/2))^(1/2))-EllipticE(1/2*(1-I
*23^(1/2))^(1/2)*x,1/6*(-33+3*I*23^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(-2+x^2-3x^4\right)^{3/2}}\,dx = \frac{\sqrt{-2}\left(3\,x^4-x^2+\sqrt{-23}(3\,x^4-x^2+2)+2\right)\sqrt{\sqrt{-23}+1}E(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{-\cdots}}\right.}{\cdots}$$

input
```
integrate(1/(-3*x^4+x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/368*(sqrt(-2)*(3*x^4 - x^2 + sqrt(-23)*(3*x^4 - x^2 + 2) + 2)*sqrt(sqrt(
-23) + 1)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(-23) + 1)), -1/12*sqrt(-23) -
11/12) + sqrt(-2)*(9*x^4 - 3*x^2 - 5*sqrt(-23)*(3*x^4 - x^2 + 2) + 6)*sqrt
(sqrt(-23) + 1)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(-23) + 1)), -1/12*sqrt(-
23) - 11/12) + 8*sqrt(-3*x^4 + x^2 - 2)*(3*x^3 + 11*x))/(3*x^4 - x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{\left(-2+x^2-3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-3x^4+x^2-2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4+x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 + x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(-2+x^2-3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-3\,x^4+x^2-2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2+x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+x^2-2)^{3/2}}\,dx$$

input
```
int(1/(x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2+x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4+x^2-2}}{9x^8-6x^6+13x^4-4x^2+4}dx$$

input
```
int(1/(-3*x^4+x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + x**2 - 2)/(9*x**8 - 6*x**6 + 13*x**4 - 4*x**2 + 4),x)
```

3.285 $\int \frac{1}{(-2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 89

$$\int \frac{1}{(-2-3x^4)^{3/2}}\,dx = -\frac{x}{4\sqrt{-2-3x^4}} - \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}\right)}{8\sqrt[4]{6}\sqrt{-2-3x^4}}$$

output
```
-1/4*x/(-3*x^4-2)^(1/2)-1/48*(2+6^(1/2)*x^2)*((3*x^4+2)/(2+6^(1/2)*x^2)^2)
^(1/2)*InverseJacobiAM(2*arctan(1/2*3^(1/4)*2^(3/4)*x),1/2*2^(1/2))*6^(3/4
)/(-3*x^4-2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.43 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{1}{(-2-3x^4)^{3/2}}\,dx = -\frac{x\left(2+\sqrt{4+6x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{3x^4}{2}\right)\right)}{8\sqrt{-2-3x^4}}$$

input
```
Integrate[(-2 - 3*x^4)^(-3/2),x]
```

output
```
-1/8*(x*(2 + Sqrt[4 + 6*x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, (-3*x^4)/2])
)/Sqrt[-2 - 3*x^4]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4-2)^{3/2}}\,dx$$

$$\Big\downarrow \text{749}$$

$$-\frac{1}{4}\int \frac{1}{\sqrt{-3x^4-2}}dx-\frac{x}{4\sqrt{-3x^4-2}}$$

$$\Big\downarrow \text{761}$$

$$-\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}\right)}{8\sqrt[4]{6}\sqrt{-3x^4-2}}-\frac{x}{4\sqrt{-3x^4-2}}$$

input
```
Int[(-2 - 3*x^4)^(-3/2),x]
```

output
```
-1/4*x/Sqrt[-2 - 3*x^4] - ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]
*x^2)^2]*EllipticF[2*ArcTan[(3/2)^(1/4)*x], 1/2])/(8*6^(1/4)*Sqrt[-2 - 3*x
^4])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.21

method	result	size
meijerg	$\frac{i\sqrt{2}\,x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],-\frac{3x^4}{2}\right)}{4}$	19
default	$-\frac{x}{4\sqrt{-3x^4-2}}-\frac{\sqrt{2}\,\sqrt{4+2i\sqrt{6}\,x^2}\,\sqrt{4-2i\sqrt{6}\,x^2}\,\text{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{-i\sqrt{6}}\,x}{2},i\right)}{16\sqrt{-i\sqrt{6}}\,\sqrt{-3x^4-2}}$	79
risch	$-\frac{x}{4\sqrt{-3x^4-2}}-\frac{\sqrt{2}\,\sqrt{4+2i\sqrt{6}\,x^2}\,\sqrt{4-2i\sqrt{6}\,x^2}\,\text{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{-i\sqrt{6}}\,x}{2},i\right)}{16\sqrt{-i\sqrt{6}}\,\sqrt{-3x^4-2}}$	79
elliptic	$-\frac{x}{4\sqrt{-3x^4-2}}-\frac{\sqrt{2}\,\sqrt{4+2i\sqrt{6}\,x^2}\,\sqrt{4-2i\sqrt{6}\,x^2}\,\text{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{-i\sqrt{6}}\,x}{2},i\right)}{16\sqrt{-i\sqrt{6}}\,\sqrt{-3x^4-2}}$	79

input
```
int(1/(-3*x^4-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*2^(1/2)*x*hypergeom([1/4,3/2],[5/4],-3/2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{1}{(-2-3x^4)^{3/2}}\,dx = -\frac{\sqrt{\frac{1}{2}}\sqrt{-2}(-6)^{\frac{3}{4}}\,(3\,x^4+2)F(\arcsin\left(\sqrt{\frac{1}{2}}(-6)^{\frac{1}{4}}\,x\right)\,|\,-1)-6\,\sqrt{-3\,x^4-2}x}{24\,(3\,x^4+2)}$$

input
```
integrate(1/(-3*x^4-2)^(3/2),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(1/2)*sqrt(-2)*(-6)^(3/4)*(3*x^4 + 2)*elliptic_f(arcsin(sqrt(1/
2)*(-6)^(1/4)*x), -1) - 6*sqrt(-3*x^4 - 2)*x)/(3*x^4 + 2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.42

$$\int \frac{1}{(-2-3x^4)^{3/2}}\,dx = \frac{\sqrt{2}ix\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{3x^4e^{i\pi}}{2}\right)}{16\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-3*x**4-2)**(3/2),x)
```

output
```
sqrt(2)*I*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), 3*x**4*exp_polar(I*pi)/2)
/(16*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(-2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - 2)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 17.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.38

$$\int \frac{1}{(-2-3x^4)^{3/2}}\,dx = \frac{\sqrt{2}\,x\,(3\,x^4+2)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{3\,x^4}{2}\right)}{4\,(-3\,x^4-2)^{3/2}}$$

input
```
int(1/(- 3*x^4 - 2)^(3/2),x)
```

output
```
(2^(1/2)*x*(3*x^4 + 2)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(3*x^4)/2))/(4*(-
 3*x^4 - 2)^(3/2))
```

Reduce [F]

$$\int \frac{1}{(-2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-2}}{9x^8+12x^4+4}dx$$

input
```
int(1/(-3*x^4-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - 2)/(9*x**8 + 12*x**4 + 4),x)
```

3.286 $\int \frac{1}{(-2-x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 262

$$\int \frac{1}{(-2-x^2-3x^4)^{3/2}}\,dx = -\frac{x(11-3x^2)}{46\sqrt{-2-x^2-3x^4}}+\frac{3x\sqrt{-2-x^2-3x^4}}{46\left(\sqrt{6}+3x^2\right)} + \frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}(12-\sqrt{6})\right)}{23\ 2^{3/4}\sqrt{-2-x^2-3x^4}} - \frac{\sqrt[4]{3}(1+2\sqrt{6})\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}(12-\sqrt{6})\right)}{46\ 2^{3/4}\sqrt{-2-x^2-3x^4}}$$

output
```
-1/46*x*(-3*x^2+11)/(-3*x^4-x^2-2)^(1/2)+3*x*(-3*x^4-x^2-2)^(1/2)/(46*6^(1
/2)+138*x^2)+1/46*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4+x^2+2)/(2+6^(1/2)*x^2)^2
)^(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72-6*6^(1/2))
^(1/2))*2^(1/4)/(-3*x^4-x^2-2)^(1/2)-1/92*3^(1/4)*(1+2*6^(1/2))*(2+6^(1/2)
*x^2)*((3*x^4+x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2
*3^(1/4)*2^(3/4)*x),1/12*(72-6*6^(1/2))^(1/2))*2^(1/4)/(-3*x^4-x^2-2)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.60 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.25

$$\int \frac{1}{(-2-x^2-3x^4)^{3/2}}\,dx = \frac{12\sqrt{-\frac{i}{-i+\sqrt{23}}}x(-11+3x^2)+\sqrt{6}(i+\sqrt{23})\sqrt{\frac{-i+\sqrt{23}-6ix^2}{-i+\sqrt{23}}}\sqrt{\frac{i+\sqrt{23}+6ix^2}{i+\sqrt{23}}}E\Big(i\mathrm{arc}}{}$$

input
```
Integrate[(-2 - x^2 - 3*x^4)^(-3/2),x]
```

output
```
(12*Sqrt[(-I)/(-I + Sqrt[23])]*x*(-11 + 3*x^2) + Sqrt[6]*(I + Sqrt[23])*Sq
rt[(-I + Sqrt[23] - (6*I)*x^2)/(-I + Sqrt[23])]*Sqrt[(I + Sqrt[23] + (6*I)
*x^2)/(I + Sqrt[23])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(-I + Sqrt[23])]*x],
 (I - Sqrt[23])/(I + Sqrt[23])] - Sqrt[6]*(-23*I + Sqrt[23])*Sqrt[(-I + Sq
rt[23] - (6*I)*x^2)/(-I + Sqrt[23])]*Sqrt[(I + Sqrt[23] + (6*I)*x^2)/(I +
Sqrt[23])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(-I + Sqrt[23])]*x], (I - Sqrt[
23])/(I + Sqrt[23])])/(552*Sqrt[(-I)/(-I + Sqrt[23])]*Sqrt[-2 - x^2 - 3*x^
4])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4-x^2-2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$-\frac{1}{46}\int \frac{3(x^2+4)}{\sqrt{-3x^4-x^2-2}}dx-\frac{x(11-3x^2)}{46\sqrt{-3x^4-x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{46}\int \frac{x^2+4}{\sqrt{-3x^4-x^2-2}}dx-\frac{x(11-3x^2)}{46\sqrt{-3x^4-x^2-2}}$$

$\Big\downarrow$ 1511

$$-\frac{3}{46}\left(\frac{1}{3}\left(12+\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4-x^2-2}}dx-\sqrt{\frac{2}{3}}\int \frac{2-\sqrt{6}x^2}{2\sqrt{-3x^4-x^2-2}}dx\right)-\frac{x(11-3x^2)}{46\sqrt{-3x^4-x^2-2}}$$

$\Big\downarrow$ 27

$$-\frac{3}{46}\left(\frac{1}{3}\left(12+\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4-x^2-2}}dx-\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4-x^2-2}}dx}{\sqrt{6}}\right)-\frac{x(11-3x^2)}{46\sqrt{-3x^4-x^2-2}}$$

$\Big\downarrow$ 1416

$$-\frac{3}{46}\left(\frac{\left(12+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{-3x^4-x^2-2}}-\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4-x^2-2}}dx}{\sqrt{6}}\right)-\frac{x(11-3x^2)}{46\sqrt{-3x^4-x^2-2}}$$

$\Big\downarrow$ 1509

$$-\frac{3}{46}\left(\frac{\left(12+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{-3x^4-x^2-2}}-\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2}{\left(\sqrt{6}x^2+\right.}}}{\sqrt[4]{}}\right.$$

$$\frac{x(11-3x^2)}{46\sqrt{-3x^4-x^2-2}}$$

input
```
Int[(-2 - x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/46*(x*(11 - 3*x^2))/Sqrt[-2 - x^2 - 3*x^4] - (3*(-(((2*x*Sqrt[-2 - x^2
 - 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3
*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6
])/24])/(3^(1/4)*Sqrt[-2 - x^2 - 3*x^4]))/Sqrt[6]) + ((12 + Sqrt[6])*(2 +
Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTa
n[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(6*6^(1/4)*Sqrt[-2 - x^2 - 3*x^4])))
/46
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.90

method	result
risch	$\frac{x(3x^2-11)}{46\sqrt{-3x^4-x^2-2}}-\frac{12\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{23\sqrt{-1-i\sqrt{23}}\sqrt{-3x^4-x^2-2}}-\frac{12\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right.}}{}$
default	$\frac{-\frac{11}{46}x+\frac{3}{46}x^3}{\sqrt{-3x^4-x^2-2}}-\frac{12\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{23\sqrt{-1-i\sqrt{23}}\sqrt{-3x^4-x^2-2}}-\frac{12\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)}}{}$
elliptic	$\frac{-\frac{11}{46}x+\frac{3}{46}x^3}{\sqrt{-3x^4-x^2-2}}-\frac{12\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{23\sqrt{-1-i\sqrt{23}}\sqrt{-3x^4-x^2-2}}-\frac{12\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)}}{}$

input
```
int(1/(-3*x^4-x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/46*x*(3*x^2-11)/(-3*x^4-x^2-2)^(1/2)-12/23/(-1-I*23^(1/2))^(1/2)*(1-(-1/
4-1/4*I*23^(1/2))*x^2)^(1/2)*(1-(-1/4+1/4*I*23^(1/2))*x^2)^(1/2)/(-3*x^4-x
^2-2)^(1/2)*EllipticF(1/2*(-1-I*23^(1/2))^(1/2)*x,1/6*(-33-3*I*23^(1/2))^(
1/2))-12/23/(-1-I*23^(1/2))^(1/2)*(1-(-1/4-1/4*I*23^(1/2))*x^2)^(1/2)*(1-(
-1/4+1/4*I*23^(1/2))*x^2)^(1/2)/(-3*x^4-x^2-2)^(1/2)/(-1+I*23^(1/2))*(Elli
pticF(1/2*(-1-I*23^(1/2))^(1/2)*x,1/6*(-33-3*I*23^(1/2))^(1/2))-EllipticE(
1/2*(-1-I*23^(1/2))^(1/2)*x,1/6*(-33-3*I*23^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.58

$$\int \frac{1}{(-2-x^2-3x^4)^{3/2}}\,dx = \frac{\sqrt{-2}\left(3\,x^4+x^2-\sqrt{-23}(3\,x^4+x^2+2)+2\right)\sqrt{\sqrt{-23}-1}E\left(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{-}\right.\right.}}{}$$

input
```
integrate(1/(-3*x^4-x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/368*(sqrt(-2)*(3*x^4 + x^2 - sqrt(-23)*(3*x^4 + x^2 + 2) + 2)*sqrt(sqrt(
-23) - 1)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(-23) - 1)), 1/12*sqrt(-23) - 1
1/12) - sqrt(-2)*(15*x^4 + 5*x^2 + 3*sqrt(-23)*(3*x^4 + x^2 + 2) + 10)*sqr
t(sqrt(-23) - 1)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(-23) - 1)), 1/12*sqrt(-
23) - 11/12) - 8*sqrt(-3*x^4 - x^2 - 2)*(3*x^3 - 11*x))/(3*x^4 + x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(-2-x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4-x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 - x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2-x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-x^2-2)^{3/2}}\,dx$$

input
```
int(1/(- x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(- x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-x^2-2}}{9x^8+6x^6+13x^4+4x^2+4}dx$$

input
```
int(1/(-3*x^4-x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - x**2 - 2)/(9*x**8 + 6*x**6 + 13*x**4 + 4*x**2 + 4),x)
```

3.287 $\int \frac{1}{(-2-2x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 264

$$\int \frac{1}{(-2-2x^2-3x^4)^{3/2}}\,dx = -\frac{x(4-3x^2)}{20\sqrt{-2-2x^2-3x^4}}+\frac{3x\sqrt{-2-2x^2-3x^4}}{20\left(\sqrt{6}+3x^2\right)}$$
$$+\frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)\middle|\frac{1}{12}(6-\sqrt{6})\right)}{10\ 2^{3/4}\sqrt{-2-2x^2-3x^4}}$$
$$-\frac{\sqrt[4]{3}(1+\sqrt{6})\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}(6-\sqrt{6})\right)}{20\ 2^{3/4}\sqrt{-2-2x^2-3x^4}}$$

output
```
-1/20*x*(-3*x^2+4)/(-3*x^4-2*x^2-2)^(1/2)+3*x*(-3*x^4-2*x^2-2)^(1/2)/(20*6
^(1/2)+60*x^2)+1/20*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4+2*x^2+2)/(2+6^(1/2)*x^
2)^2)^(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18-3*6^(1/
2))^(1/2))*2^(1/4)/(-3*x^4-2*x^2-2)^(1/2)-1/40*3^(1/4)*(1+6^(1/2))*(2+6^(1
/2)*x^2)*((3*x^4+2*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arcta
n(1/2*3^(1/4)*2^(3/4)*x),1/6*(18-3*6^(1/2))^(1/2))*2^(1/4)/(-3*x^4-2*x^2-2
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.68 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.24

$$\int \frac{1}{(-2-2x^2-3x^4)^{3/2}}\,dx = \frac{3\sqrt{-\frac{i}{-i+\sqrt{5}}}x(-4+3x^2)+\sqrt{3}(i+\sqrt{5})\sqrt{\frac{-i+\sqrt{5}-3ix^2}{-i+\sqrt{5}}}\sqrt{\frac{i+\sqrt{5}+3ix^2}{i+\sqrt{5}}}E\Big(i\text{arcsinh}\Big(}{6}$$

input
```
Integrate[(-2 - 2*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(3*Sqrt[(-I)/(-I + Sqrt[5])]*x*(-4 + 3*x^2) + Sqrt[3]*(I + Sqrt[5])*Sqrt[(
-I + Sqrt[5] - (3*I)*x^2)/(-I + Sqrt[5])]*Sqrt[(I + Sqrt[5] + (3*I)*x^2)/(
I + Sqrt[5])]*EllipticE[I*ArcSinh[Sqrt[(-3*I)/(-I + Sqrt[5])]*x], (I - Sqr
t[5])/(I + Sqrt[5])] - Sqrt[3]*(-5*I + Sqrt[5])*Sqrt[(-I + Sqrt[5] - (3*I)
*x^2)/(-I + Sqrt[5])]*Sqrt[(I + Sqrt[5] + (3*I)*x^2)/(I + Sqrt[5])]*Ellipt
icF[I*ArcSinh[Sqrt[(-3*I)/(-I + Sqrt[5])]*x], (I - Sqrt[5])/(I + Sqrt[5])]
)/(60*Sqrt[(-I)/(-I + Sqrt[5])]*Sqrt[-2 - 2*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4-2x^2-2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$-\frac{1}{40}\int \frac{6(x^2+2)}{\sqrt{-3x^4-2x^2-2}}dx - \frac{x(4-3x^2)}{20\sqrt{-3x^4-2x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{20}\int \frac{x^2+2}{\sqrt{-3x^4-2x^2-2}}dx-\frac{x(4-3x^2)}{20\sqrt{-3x^4-2x^2-2}}$$

$$\Big\downarrow \; 1511$$

$$-\frac{3}{20}\left(\frac{1}{3}\left(6+\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4-2x^2-2}}dx-\sqrt{\frac{2}{3}}\int \frac{2-\sqrt{6}x^2}{2\sqrt{-3x^4-2x^2-2}}dx\right)-\frac{x(4-3x^2)}{20\sqrt{-3x^4-2x^2-2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{3}{20}\left(\frac{1}{3}\left(6+\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4-2x^2-2}}dx-\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4-2x^2-2}}dx}{\sqrt{6}}\right)-\frac{x(4-3x^2)}{20\sqrt{-3x^4-2x^2-2}}$$

$$\Big\downarrow \; 1416$$

$$-\frac{3}{20}\left(\frac{\left(6+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{-3x^4-2x^2-2}}-\frac{\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4-2x^2-2}}dx}{\sqrt{6}}\right)-\frac{x(4-3x^2)}{20\sqrt{-3x^4-2x^2-2}}$$

$$\Big\downarrow \; 1509$$

$$-\frac{3}{20}\left(\frac{\left(6+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{6\sqrt[4]{6}\sqrt{-3x^4-2x^2-2}}-\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}}{\sqrt[4]{3}\sqrt{}}\right)-\frac{x(4-3x^2)}{20\sqrt{-3x^4-2x^2-2}}$$

input
```
Int[(-2 - 2*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/20*(x*(4 - 3*x^2))/Sqrt[-2 - 2*x^2 - 3*x^4] - (3*(-(((2*x*Sqrt[-2 - 2*x
^2 - 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^
2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], (6 - Sq
rt[6])/12])/(3^(1/4)*Sqrt[-2 - 2*x^2 - 3*x^4]))/Sqrt[6]) + ((6 + Sqrt[6])*
(2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(6*6^(1/4)*Sqrt[-2 - 2*x^2 - 3
*x^4])))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.90

method	result
risch	$\frac{x(3x^2-4)}{20\sqrt{-3x^4-2x^2-2}} - \frac{3\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-2-2i\sqrt{5}}\,x}{2}, \frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{5\sqrt{-2-2i\sqrt{5}}\,\sqrt{-3x^4-2x^2-2}} - \frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{}}{}$
default	$\frac{-\frac{1}{5}x+\frac{3}{20}x^3}{\sqrt{-3x^4-2x^2-2}} - \frac{3\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-2-2i\sqrt{5}}\,x}{2}, \frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{5\sqrt{-2-2i\sqrt{5}}\,\sqrt{-3x^4-2x^2-2}} - \frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-}}{}$
elliptic	$\frac{-\frac{1}{5}x+\frac{3}{20}x^3}{\sqrt{-3x^4-2x^2-2}} - \frac{3\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{\sqrt{-2-2i\sqrt{5}}\,x}{2}, \frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{5\sqrt{-2-2i\sqrt{5}}\,\sqrt{-3x^4-2x^2-2}} - \frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-}}{}$

input
```
int(1/(-3*x^4-2*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*x*(3*x^2-4)/(-3*x^4-2*x^2-2)^(1/2)-3/5/(-2-2*I*5^(1/2))^(1/2)*(1-(-1/
2-1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2*I*5^(1/2)-1/2)*x^2)^(1/2)/(-3*x^4-2*x^
2-2)^(1/2)*EllipticF(1/2*(-2-2*I*5^(1/2))^(1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/
2))-6/5/(-2-2*I*5^(1/2))^(1/2)*(1-(-1/2-1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2*
I*5^(1/2)-1/2)*x^2)^(1/2)/(-3*x^4-2*x^2-2)^(1/2)/(-2+2*I*5^(1/2))*(Ellipti
cF(1/2*(-2-2*I*5^(1/2))^(1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2))-EllipticE(1/2*
(-2-2*I*5^(1/2))^(1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.63

$$\int \frac{1}{(-2-2x^2-3x^4)^{3/2}}\,dx = \frac{\sqrt{-2}\left(3\,x^4+2\,x^2-\sqrt{-5}(3\,x^4+2\,x^2+2)+2\right)\sqrt{\frac{1}{2}\,\sqrt{-5}-\frac{1}{2}}E(\arcsin\left(x\sqrt{\frac{1}{2}}\right.}{}$$

input
```
integrate(1/(-3*x^4-2*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/40*(sqrt(-2)*(3*x^4 + 2*x^2 - sqrt(-5)*(3*x^4 + 2*x^2 + 2) + 2)*sqrt(1/2
*sqrt(-5) - 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(-5) - 1/2)), 1/3*sqrt(-
5) - 2/3) - sqrt(-2)*(9*x^4 + 6*x^2 + sqrt(-5)*(3*x^4 + 2*x^2 + 2) + 6)*sq
rt(1/2*sqrt(-5) - 1/2)*elliptic_f(arcsin(x*sqrt(1/2*sqrt(-5) - 1/2)), 1/3*
sqrt(-5) - 2/3) - 2*sqrt(-3*x^4 - 2*x^2 - 2)*(3*x^3 - 4*x))/(3*x^4 + 2*x^2
 + 2)
```

Sympy [F]

$$\int \frac{1}{(-2-2x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-2x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4-2*x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 - 2*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2-2x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-2\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-2*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - 2*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-2x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-2\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-2*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - 2*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-2x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-2\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(- 2*x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(- 2*x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-2x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-2x^2-2}}{9x^8+12x^6+16x^4+8x^2+4}dx$$

input
```
int(1/(-3*x^4-2*x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - 2*x**2 - 2)/(9*x**8 + 12*x**6 + 16*x**4 + 8*x**2 + 4)
,x)
```

3.288 $\int \frac{1}{(-2-3x^2-3x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 261

$$\int \frac{1}{(-2-3x^2-3x^4)^{3/2}} \, dx = -\frac{x(1-3x^2)}{10\sqrt{-2-3x^2-3x^4}} + \frac{3x\sqrt{-2-3x^2-3x^4}}{10\left(\sqrt{6}+3x^2\right)} + \frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}(4-\sqrt{6})\right)}{5\,2^{3/4}\sqrt{-2-3x^2-3x^4}} - \frac{\left(3+2\sqrt{6}\right)\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}(4-\sqrt{6})\right)}{10\,6^{3/4}\sqrt{-2-3x^2-3x^4}}$$

output
```
-1/10*x*(-3*x^2+1)/(-3*x^4-3*x^2-2)^(1/2)+3*x*(-3*x^4-3*x^2-2)^(1/2)/(10*6
^(1/2)+30*x^2)+1/10*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4+3*x^2+2)/(2+6^(1/2)*x^
2)^2)^(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/4*(8-2*6^(1/2
))^(1/2))*2^(1/4)/(-3*x^4-3*x^2-2)^(1/2)-1/60*(3+2*6^(1/2))*(2+6^(1/2)*x^2
)*((3*x^4+3*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*3
^(1/4)*2^(3/4)*x),1/4*(8-2*6^(1/2))^(1/2))*6^(1/4)/(-3*x^4-3*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.05 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.32

$$\int \frac{1}{(-2-3x^2-3x^4)^{3/2}}\,dx = \frac{12\sqrt{-\frac{i}{-3i+\sqrt{15}}}x(-1+3x^2)+3\sqrt{2}\left(i\sqrt{3}+\sqrt{5}\right)\sqrt{\frac{-3i+\sqrt{15}-6ix^2}{-3i+\sqrt{15}}}\sqrt{\frac{3i+\sqrt{15}+6ix^2}{3i+\sqrt{15}}}E}{}$$

input
```
Integrate[(-2 - 3*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(12*Sqrt[(-I)/(-3*I + Sqrt[15])]*x*(-1 + 3*x^2) + 3*Sqrt[2]*(I*Sqrt[3] + S
qrt[5])*Sqrt[(-3*I + Sqrt[15] - (6*I)*x^2)/(-3*I + Sqrt[15])]*Sqrt[(3*I +
Sqrt[15] + (6*I)*x^2)/(3*I + Sqrt[15])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(-
3*I + Sqrt[15])]*x], (3*I - Sqrt[15])/(3*I + Sqrt[15])] + I*Sqrt[2]*(5*Sqr
t[3] + (3*I)*Sqrt[5])*Sqrt[(-3*I + Sqrt[15] - (6*I)*x^2)/(-3*I + Sqrt[15])
]*Sqrt[(3*I + Sqrt[15] + (6*I)*x^2)/(3*I + Sqrt[15])]*EllipticF[I*ArcSinh[
Sqrt[(-6*I)/(-3*I + Sqrt[15])]*x], (3*I - Sqrt[15])/(3*I + Sqrt[15])])/(12
0*Sqrt[(-I)/(-3*I + Sqrt[15])]*Sqrt[-2 - 3*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4-3x^2-2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$-\frac{1}{30}\int \frac{3(3x^2+4)}{\sqrt{-3x^4-3x^2-2}}dx-\frac{x(1-3x^2)}{10\sqrt{-3x^4-3x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{10}\int \frac{3x^2+4}{\sqrt{-3x^4-3x^2-2}}dx-\frac{x(1-3x^2)}{10\sqrt{-3x^4-3x^2-2}}$$

$$\Big\downarrow \ 1511$$

$$\frac{1}{10}\left(\sqrt{6}\int \frac{2-\sqrt{6}x^2}{2\sqrt{-3x^4-3x^2-2}}dx-\left(4+\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4-3x^2-2}}dx\right)-\frac{x(1-3x^2)}{10\sqrt{-3x^4-3x^2-2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{10}\left(\sqrt{\frac{3}{2}}\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4-3x^2-2}}dx-\left(4+\sqrt{6}\right)\int \frac{1}{\sqrt{-3x^4-3x^2-2}}dx\right)-\frac{x(1-3x^2)}{10\sqrt{-3x^4-3x^2-2}}$$

$$\Big\downarrow \ 1416$$

$$\frac{1}{10}\left(\sqrt{\frac{3}{2}}\int \frac{2-\sqrt{6}x^2}{\sqrt{-3x^4-3x^2-2}}dx-\frac{\left(4+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right.}{2\sqrt[4]{6}\sqrt{-3x^4-3x^2-2}}\right.$$

$$\frac{x(1-3x^2)}{10\sqrt{-3x^4-3x^2-2}}$$

$$\Big\downarrow \ 1509$$

$$\frac{1}{10}\left(\sqrt{\frac{3}{2}}\left(\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{\sqrt[4]{3}\sqrt{-3x^4-3x^2-2}}+\frac{2\sqrt{-3x^4-3x^2-2}x}{\sqrt{6}x^2+2}\right)-\frac{(4+\sqrt{6}}{}\right.$$

$$\frac{x(1-3x^2)}{10\sqrt{-3x^4-3x^2-2}}$$

input
```
Int[(-2 - 3*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/10*(x*(1 - 3*x^2))/Sqrt[-2 - 3*x^2 - 3*x^4] + (Sqrt[3/2]*((2*x*Sqrt[-2
- 3*x^2 - 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 +
 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], (4
 - Sqrt[6])/8])/(3^(1/4)*Sqrt[-2 - 3*x^2 - 3*x^4])) - ((4 + Sqrt[6])*(2 +
Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*Arc
Tan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-2 - 3*x^2 - 3*x^4])
)/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.91

method	result
risch	$\frac{x(3x^2-1)}{10\sqrt{-3x^4-3x^2-2}} - \frac{4\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{5\sqrt{-3-i\sqrt{15}}\,\sqrt{-3x^4-3x^2-2}} - \frac{12\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)}}{}$
default	$\frac{-\frac{1}{10}x+\frac{3}{10}x^3}{\sqrt{-3x^4-3x^2-2}} - \frac{4\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{5\sqrt{-3-i\sqrt{15}}\,\sqrt{-3x^4-3x^2-2}} - \frac{12\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}}{}$
elliptic	$\frac{-\frac{1}{10}x+\frac{3}{10}x^3}{\sqrt{-3x^4-3x^2-2}} - \frac{4\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{5\sqrt{-3-i\sqrt{15}}\,\sqrt{-3x^4-3x^2-2}} - \frac{12\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}}{}$

input
```
int(1/(-3*x^4-3*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*x*(3*x^2-1)/(-3*x^4-3*x^2-2)^(1/2)-4/5/(-3-I*15^(1/2))^(1/2)*(1-(-3/4
-1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-3/4+1/4*I*15^(1/2))*x^2)^(1/2)/(-3*x^4-3*
x^2-2)^(1/2)*EllipticF(1/2*(-3-I*15^(1/2))^(1/2)*x,1/2*(-1-I*15^(1/2))^(1/
2))-12/5/(-3-I*15^(1/2))^(1/2)*(1-(-3/4-1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-3/
4+1/4*I*15^(1/2))*x^2)^(1/2)/(-3*x^4-3*x^2-2)^(1/2)/(-3+I*15^(1/2))*(Ellip
ticF(1/2*(-3-I*15^(1/2))^(1/2)*x,1/2*(-1-I*15^(1/2))^(1/2))-EllipticE(1/2*
(-3-I*15^(1/2))^(1/2)*x,1/2*(-1-I*15^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.62

$$\int \frac{1}{(-2-3x^2-3x^4)^{3/2}}\,dx = \frac{3\sqrt{-2}\left(9\,x^4+9\,x^2-\sqrt{-15}(3\,x^4+3\,x^2+2)+6\right)\sqrt{\sqrt{-15}-3}E(\arcsin\left(\frac{1}{2}\,x\right.}{}$$

input
```
integrate(1/(-3*x^4-3*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/240*(3*sqrt(-2)*(9*x^4 + 9*x^2 - sqrt(-15)*(3*x^4 + 3*x^2 + 2) + 6)*sqrt
(sqrt(-15) - 3)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(-15) - 3)), 1/4*sqrt(-15
) - 1/4) - sqrt(-2)*(63*x^4 + 63*x^2 + sqrt(-15)*(3*x^4 + 3*x^2 + 2) + 42)
*sqrt(sqrt(-15) - 3)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(-15) - 3)), 1/4*sqr
t(-15) - 1/4) - 24*sqrt(-3*x^4 - 3*x^2 - 2)*(3*x^3 - x))/(3*x^4 + 3*x^2 +
2)
```

Sympy [F]

$$\int \frac{1}{(-2-3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-3x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4-3*x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 - 3*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2-3x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-3\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-3*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - 3*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(-2-3x^2-3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-3\,x^4-3\,x^2-2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-3*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - 3*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-2-3x^2-3x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-3\,x^4-3\,x^2-2\right)^{3/2}}\,dx$$

input
```
int(1/(- 3*x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(- 3*x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(-2-3x^2-3x^4\right)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-3x^2-2}}{9x^8+18x^6+21x^4+12x^2+4}dx$$

input
```
int(1/(-3*x^4-3*x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - 3*x**2 - 2)/(9*x**8 + 18*x**6 + 21*x**4 + 12*x**2 + 4
),x)
```

3.289 $\int \frac{1}{(-2-4x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 260

$$\int \frac{1}{(-2-4x^2-3x^4)^{3/2}}\,dx = \frac{x(1+3x^2)}{4\sqrt{-2-4x^2-3x^4}}+\frac{3x\sqrt{-2-4x^2-3x^4}}{4\left(\sqrt{6}+3x^2\right)}$$
$$+\frac{\sqrt[4]{3}(2+\sqrt{6}x^2)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\,2^{3/4}\sqrt{-2-4x^2-3x^4}}$$
$$-\frac{\sqrt[4]{3}(2+\sqrt{6})\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{8\,2^{3/4}\sqrt{-2-4x^2-3x^4}}$$

output
```
1/4*x*(3*x^2+1)/(-3*x^4-4*x^2-2)^(1/2)+3*x*(-3*x^4-4*x^2-2)^(1/2)/(4*6^(1/
2)+12*x^2)+1/4*3^(1/4)*(2+6^(1/2)*x^2)*((3*x^4+4*x^2+2)/(2+6^(1/2)*x^2)^2)
^(1/2)*EllipticE(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18-6*6^(1/2))^(
1/2))*2^(1/4)/(-3*x^4-4*x^2-2)^(1/2)-1/16*3^(1/4)*(2+6^(1/2))*(2+6^(1/2)*x
^2)*((3*x^4+4*x^2+2)/(2+6^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2
*3^(1/4)*2^(3/4)*x),1/6*(18-6*6^(1/2))^(1/2))*2^(1/4)/(-3*x^4-4*x^2-2)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.91 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.26

$$\int \frac{1}{(-2-4x^2-3x^4)^{3/2}}\,dx = \frac{3\sqrt{-\frac{i}{-2i+\sqrt{2}}}x(1+3x^2)+\sqrt{3}(2i+\sqrt{2})\sqrt{\frac{-2i+\sqrt{2}-3ix^2}{-2i+\sqrt{2}}}\sqrt{\frac{2i+\sqrt{2}+3ix^2}{2i+\sqrt{2}}}E\Big(i\text{arcsin}$$

input
```
Integrate[(-2 - 4*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(3*Sqrt[(-I)/(-2*I + Sqrt[2])]*x*(1 + 3*x^2) + Sqrt[3]*(2*I + Sqrt[2])*Sqr
t[(-2*I + Sqrt[2] - (3*I)*x^2)/(-2*I + Sqrt[2])]*Sqrt[(2*I + Sqrt[2] + (3*
I)*x^2)/(2*I + Sqrt[2])]*EllipticE[I*ArcSinh[Sqrt[(-3*I)/(-2*I + Sqrt[2])]
*x], (2*I - Sqrt[2])/(2*I + Sqrt[2])] - Sqrt[3]*(-I + Sqrt[2])*Sqrt[(-2*I
+ Sqrt[2] - (3*I)*x^2)/(-2*I + Sqrt[2])]*Sqrt[(2*I + Sqrt[2] + (3*I)*x^2)/
(2*I + Sqrt[2])]*EllipticF[I*ArcSinh[Sqrt[(-3*I)/(-2*I + Sqrt[2])]*x], (2*
I - Sqrt[2])/(2*I + Sqrt[2])])/(12*Sqrt[(-I)/(-2*I + Sqrt[2])]*Sqrt[-2 - 4
*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4-4x^2-2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(3x^2+1)}{4\sqrt{-3x^4-4x^2-2}}-\frac{1}{16}\int \frac{12(x^2+1)}{\sqrt{-3x^4-4x^2-2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(3x^2+1)}{4\sqrt{-3x^4-4x^2-2}}-\frac{3}{4}\int\frac{x^2+1}{\sqrt{-3x^4-4x^2-2}}dx$$

$\Big\downarrow$ 1511

$$\frac{x(3x^2+1)}{4\sqrt{-3x^4-4x^2-2}}-\frac{3}{4}\left(\frac{1}{3}\left(3+\sqrt{6}\right)\int\frac{1}{\sqrt{-3x^4-4x^2-2}}dx-\sqrt{\frac{2}{3}}\int\frac{2-\sqrt{6}x^2}{2\sqrt{-3x^4-4x^2-2}}dx\right)$$

$\Big\downarrow$ 27

$$\frac{x(3x^2+1)}{4\sqrt{-3x^4-4x^2-2}}-\frac{3}{4}\left(\frac{1}{3}\left(3+\sqrt{6}\right)\int\frac{1}{\sqrt{-3x^4-4x^2-2}}dx-\frac{\int\frac{2-\sqrt{6}x^2}{\sqrt{-3x^4-4x^2-2}}dx}{\sqrt{6}}\right)$$

$\Big\downarrow$ 1416

$$\frac{x(3x^2+1)}{4\sqrt{-3x^4-4x^2-2}}-\frac{3}{4}\left(\frac{\left(3+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{6\sqrt[4]{6}\sqrt{-3x^4-4x^2-2}}-\frac{\int\frac{2-\sqrt{6}x^2}{\sqrt{-3x^4-4x^2-2}}dx}{\sqrt{6}}\right)$$

$\Big\downarrow$ 1509

$$\frac{x(3x^2+1)}{4\sqrt{-3x^4-4x^2-2}}-\frac{3}{4}\left(\frac{\left(3+\sqrt{6}\right)\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{6\sqrt[4]{6}\sqrt{-3x^4-4x^2-2}}-\frac{2^{3/4}\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}E\left(2\text{arc}\right.}{\sqrt[4]{3}\sqrt{-3x^4-4x^2}}\right.$$

input
```
Int[(-2 - 4*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(x*(1 + 3*x^2))/(4*Sqrt[-2 - 4*x^2 - 3*x^4]) - (3*(-(((2*x*Sqrt[-2 - 4*x^2
 - 3*x^4])/(2 + Sqrt[6]*x^2) + (2^(3/4)*(2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2
+ 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticE[2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/S
qrt[6]])/(3^(1/4)*Sqrt[-2 - 4*x^2 - 3*x^4]))/Sqrt[6]) + ((3 + Sqrt[6])*(2
+ Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*A
rcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(6*6^(1/4)*Sqrt[-2 - 4*x^2 - 3*x^4
])))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.91

method	result
risch	$\frac{x(3x^2+1)}{4\sqrt{-3x^4-4x^2-2}} - \frac{3\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{2\sqrt{-4-2i\sqrt{2}}\,\sqrt{-3x^4-4x^2-2}} - 6\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\cdots}$
default	$\frac{\frac{1}{4}x+\frac{3}{4}x^3}{\sqrt{-3x^4-4x^2-2}} - \frac{3\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{2\sqrt{-4-2i\sqrt{2}}\,\sqrt{-3x^4-4x^2-2}} - 6\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\cdots}$
elliptic	$\frac{\frac{1}{4}x+\frac{3}{4}x^3}{\sqrt{-3x^4-4x^2-2}} - \frac{3\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{\sqrt{-4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{2\sqrt{-4-2i\sqrt{2}}\,\sqrt{-3x^4-4x^2-2}} - 6\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\cdots}$

input
```
int(1/(-3*x^4-4*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(3*x^2+1)/(-3*x^4-4*x^2-2)^(1/2)-3/2/(-4-2*I*2^(1/2))^(1/2)*(1-(-1-1
/2*I*2^(1/2))*x^2)^(1/2)*(1-(-1+1/2*I*2^(1/2))*x^2)^(1/2)/(-3*x^4-4*x^2-2)
^(1/2)*EllipticF(1/2*(-4-2*I*2^(1/2))^(1/2)*x,1/3*(3-6*I*2^(1/2))^(1/2))-6
/(-4-2*I*2^(1/2))^(1/2)*(1-(-1-1/2*I*2^(1/2))*x^2)^(1/2)*(1-(-1+1/2*I*2^(1
/2))*x^2)^(1/2)/(-3*x^4-4*x^2-2)^(1/2)/(-4+2*I*2^(1/2))*(EllipticF(1/2*(-4
-2*I*2^(1/2))^(1/2)*x,1/3*(3-6*I*2^(1/2))^(1/2))-EllipticE(1/2*(-4-2*I*2^(
1/2))^(1/2)*x,1/3*(3-6*I*2^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.55

$$\int \frac{1}{(-2-4x^2-3x^4)^{3/2}}\,dx =$$
$$-\frac{2\sqrt{-2}(3x^4+4x^2+2)\sqrt{\frac{1}{2}\sqrt{-2}-1}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-2}-1}\right) | \frac{2}{3}\sqrt{-2}+\frac{1}{3}) - (3x^4+4x^2+\sqrt{-2}(3x}{4(3x^4+}$$

input
```
integrate(1/(-3*x^4-4*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(-2)*(3*x^4 + 4*x^2 + 2)*sqrt(1/2*sqrt(-2) - 1)*elliptic_f(arc
sin(x*sqrt(1/2*sqrt(-2) - 1)), 2/3*sqrt(-2) + 1/3) - (3*x^4 + 4*x^2 + sqrt
(-2)*(3*x^4 + 4*x^2 + 2) + 2)*sqrt(1/2*sqrt(-2) - 1)*elliptic_e(arcsin(x*s
qrt(1/2*sqrt(-2) - 1)), 2/3*sqrt(-2) + 1/3) + sqrt(-3*x^4 - 4*x^2 - 2)*(3*
x^3 + x))/(3*x^4 + 4*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(-2-4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-4x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4-4*x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 - 4*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2-4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-4\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-4*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - 4*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-4\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-4*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - 4*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-4x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-4\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(- 4*x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(- 4*x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-4x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-4x^2-2}}{9x^8+24x^6+28x^4+16x^2+4}dx$$

input
```
int(1/(-3*x^4-4*x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - 4*x**2 - 2)/(9*x**8 + 24*x**6 + 28*x**4 + 16*x**2 + 4
),x)
```

3.290 $\int \frac{1}{(-2-5x^2-3x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}} \, dx = -\frac{3x}{2\sqrt{-2-5x^2-3x^4}} + \frac{5\sqrt{-2-3x^2}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{2+3x^2}} + \frac{3\sqrt{2}\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{-2-3x^2}}$$

output
```
-3/2*x/(-3*x^4-5*x^2-2)^(1/2)+5/2*(-3*x^2-2)^(1/2)*EllipticE(x/(x^2+1)^(1/
2),1/2*I*2^(1/2))*2^(1/2)/(3*x^2+2)^(1/2)+3*(3*x^2+2)^(1/2)*InverseJacobiA
M(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(-3*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.46 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.34

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}} \, dx = -\frac{13x+15x^3+5i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(i\text{arcsin}\right.}{2\sqrt{-2-5x^2-3x^4}}$$

input
```
Integrate[(-2 - 5*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/2*(13*x + 15*x^3 + (5*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Elliptic
E[I*ArcSinh[Sqrt[3/2]*x], 2/3] - I*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*E
llipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/Sqrt[-2 - 5*x^2 - 3*x^4]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1405, 27, 1494, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4-5x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{2}\int \frac{3(5x^2+4)}{\sqrt{-3x^4-5x^2-2}}dx-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\int \frac{5x^2+4}{\sqrt{-3x^4-5x^2-2}}dx-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{1494}$$

$$3\sqrt{3}\int \frac{5x^2+4}{2\sqrt{3}\sqrt{-3x^2-2}\sqrt{x^2+1}}dx-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\int \frac{5x^2+4}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{406}$$

$$\frac{3}{2}\left(4\int \frac{1}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx+5\int \frac{x^2}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx\right)-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \ 320$$

$$\frac{3}{2}\left(5\int \frac{x^2}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx-\frac{2\sqrt{2}\sqrt{-3x^2-2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}\right)-\frac{x\left(15x^2+13\right)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \ 388$$

$$\frac{3}{2}\left(5\left(\frac{1}{3}\int \frac{\sqrt{-3x^2-2}}{\left(x^2+1\right)^{3/2}}dx-\frac{x\sqrt{-3x^2-2}}{3\sqrt{x^2+1}}\right)-\frac{2\sqrt{2}\sqrt{-3x^2-2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}\right)-\frac{x\left(15x^2+13\right)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \ 313$$

$$\frac{3}{2}\left(5\left(\frac{\sqrt{2}\sqrt{-3x^2-2}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}-\frac{x\sqrt{-3x^2-2}}{3\sqrt{x^2+1}}\right)-\frac{2\sqrt{2}\sqrt{-3x^2-2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}\right)-\frac{x\left(15x^2+13\right)}{2\sqrt{-3x^4-5x^2-2}}$$

input
```
Int[(-2 - 5*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/2*(x*(13 + 15*x^2))/Sqrt[-2 - 5*x^2 - 3*x^4] + (3*(5*(-1/3*(x*Sqrt[-2 -
 3*x^2])/Sqrt[1 + x^2] + (Sqrt[2]*Sqrt[-2 - 3*x^2]*EllipticE[ArcTan[x], -1
/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)])) - (2*Sqrt[2]*Sqrt[-2 -
 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x
^2)])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{\sqrt{-3x^4-5x^2-2}}+\frac{5i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\right.\right.}{4\sqrt{-3x^4-5x^2-2}}$
default	$\frac{-\frac{15}{2}x^3-\frac{13}{2}x}{\sqrt{-3x^4-5x^2-2}}-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{\sqrt{-3x^4-5x^2-2}}+\frac{5i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\frac{ix\sqrt{6}}{2}\right.\right.}{4\sqrt{-3x^4-5x^2-2}}$
elliptic	$\frac{-\frac{15}{2}x^3-\frac{13}{2}x}{\sqrt{-3x^4-5x^2-2}}-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{\sqrt{-3x^4-5x^2-2}}+\frac{5i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\frac{ix\sqrt{6}}{2}\right.\right.}{4\sqrt{-3x^4-5x^2-2}}$

input
```
int(1/(-3*x^4-5*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(15*x^2+13)/(-3*x^4-5*x^2-2)^(1/2)-I*6^(1/2)*(6*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(-3*x^4-5*x^2-2)^(1/2)*EllipticF(1/2*I*x*6^(1/2),1/3*6^(1/2))+5/4*
I*6^(1/2)*(6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-5*x^2-2)^(1/2)*(EllipticF(
1/2*I*x*6^(1/2),1/3*6^(1/2))-EllipticE(1/2*I*x*6^(1/2),1/3*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx = \frac{-5i\sqrt{-2}(3\,x^4+5\,x^2+2)E(\arcsin{(i\,x)}\mid\frac{3}{2})+11i\sqrt{-2}(3\,x^4+5\,x^2+2)F(\mathrm{ar}}{2\,(3\,x^4+5\,x^2+2)}$$

input
```
integrate(1/(-3*x^4-5*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(-5*I*sqrt(-2)*(3*x^4 + 5*x^2 + 2)*elliptic_e(arcsin(I*x), 3/2) + 11*I
*sqrt(-2)*(3*x^4 + 5*x^2 + 2)*elliptic_f(arcsin(I*x), 3/2) + sqrt(-3*x^4 -
 5*x^2 - 2)*(15*x^3 + 13*x))/(3*x^4 + 5*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-5x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4-5*x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 - 5*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-5\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-5*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - 5*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-5\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-5*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - 5*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-5\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(- 5*x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(- 5*x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-5x^2-2}}{9x^8+30x^6+37x^4+20x^2+4}dx$$

input
```
int(1/(-3*x^4-5*x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - 5*x**2 - 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4
),x)
```

3.291 $\int \frac{1}{(-2-6x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 198

$$\begin{aligned}\int \frac{1}{(-2-6x^2-3x^4)^{3/2}}\,dx &= -\frac{\sqrt{3}x}{2\left(3-\sqrt{3}\right)\sqrt{-2-6x^2-3x^4}} \\ &+ \frac{\sqrt{3-\sqrt{3}}\sqrt{-3+\sqrt{3}-3x^2}\,E\left(\arctan\left(\sqrt{\frac{1}{2}\left(3-\sqrt{3}\right)}x\right)\middle|-1-\sqrt{3}\right)}{4\sqrt{3-\sqrt{3}+3x^2}} \\ &+ \frac{\sqrt{3-\sqrt{3}+3x^2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right),-1-\sqrt{3}\right)}{2\sqrt{3-\sqrt{3}}\sqrt{-3+\sqrt{3}-3x^2}}\end{aligned}$$

output
```
-1/2*3^(1/2)*x/(3-3^(1/2))/(-3*x^4-6*x^2-2)^(1/2)+1/4*(3-3^(1/2))^(1/2)*(-
3+3^(1/2)-3*x^2)^(1/2)*EllipticE((6-2*3^(1/2))^(1/2)*x/(4+(6-2*3^(1/2))*x^
2)^(1/2),(-1-3^(1/2))^(1/2))/(3-3^(1/2)+3*x^2)^(1/2)+1/2*(3-3^(1/2)+3*x^2)
^(1/2)*InverseJacobiAM(arctan(3^(1/2)/(3+3^(1/2))^(1/2)*x),(-1-3^(1/2))^(1
/2))/(3-3^(1/2))^(1/2)/(-3+3^(1/2)-3*x^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.68 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.95

$$\int \frac{1}{(-2-6x^2-3x^4)^{3/2}}\,dx = \frac{-3x(4+3x^2)-3i(-1+\sqrt{3})\sqrt{\frac{-3+\sqrt{3}-3x^2}{-3+\sqrt{3}}}\sqrt{3+\sqrt{3}+3x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{}{3+}}\right.\right.}{}$$

input
```
Integrate[(-2 - 6*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(-3*x*(4 + 3*x^2) - (3*I)*(-1 + Sqrt[3])*Sqrt[(-3 + Sqrt[3] - 3*x^2)/(-3 +
 Sqrt[3])]*Sqrt[3 + Sqrt[3] + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/(3 + Sqrt[
3])]*x], 2 + Sqrt[3]] - I*Sqrt[(-3 + Sqrt[3])*(-3 + Sqrt[3] - 3*x^2)]*Sqrt
[3 + Sqrt[3] + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/(3 + Sqrt[3])]*x], 2 + Sq
rt[3]])/(12*Sqrt[-2 - 6*x^2 - 3*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.53, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1405, 27, 1494, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4-6x^2-2)^{3/2}}\,dx$$

$$\Big\downarrow \ 1405$$

$$\frac{1}{24}\int \frac{6(3x^2+2)}{\sqrt{-3x^4-6x^2-2}}dx - \frac{x(3x^2+4)}{4\sqrt{-3x^4-6x^2-2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\int \frac{3x^2+2}{\sqrt{-3x^4-6x^2-2}}dx - \frac{x(3x^2+4)}{4\sqrt{-3x^4-6x^2-2}}$$

$$\Big\downarrow \ 1494$$

$$\frac{1}{2}\sqrt{3}\int \frac{3x^2+2}{2\sqrt{-3x^2+\sqrt{3}-3}\sqrt{3x^2+\sqrt{3}+3}}dx-\frac{x(3x^2+4)}{4\sqrt{-3x^4-6x^2-2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\sqrt{3}\int \frac{3x^2+2}{\sqrt{-3x^2+\sqrt{3}-3}\sqrt{3x^2+\sqrt{3}+3}}dx-\frac{x(3x^2+4)}{4\sqrt{-3x^4-6x^2-2}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{1}{4}\sqrt{3}\left(2\int \frac{1}{\sqrt{-3x^2+\sqrt{3}-3}\sqrt{3x^2+\sqrt{3}+3}}dx+3\int \frac{x^2}{\sqrt{-3x^2+\sqrt{3}-3}\sqrt{3x^2+\sqrt{3}+3}}dx\right)-\frac{x(3x^2+4)}{4\sqrt{-3x^4-6x^2-2}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{1}{4}\sqrt{3}\left(3\int \frac{x^2}{\sqrt{-3x^2+\sqrt{3}-3}\sqrt{3x^2+\sqrt{3}+3}}dx-\frac{2\sqrt{-3x^2+\sqrt{3}-3}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right),-1-\sqrt{3}\right)}{\sqrt{3\left(3-\sqrt{3}\right)}\sqrt{\frac{3x^2-\sqrt{3}+3}{3x^2+\sqrt{3}+3}}\sqrt{3x^2+\sqrt{3}+3}}\right.$$
$$\frac{x(3x^2+4)}{4\sqrt{-3x^4-6x^2-2}}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{1}{4}\sqrt{3}\left(3\left(\frac{1}{3}\left(3+\sqrt{3}\right)\int \frac{\sqrt{-3x^2+\sqrt{3}-3}}{\left(3x^2+\sqrt{3}+3\right)^{3/2}}dx-\frac{x\sqrt{-3x^2+\sqrt{3}-3}}{3\sqrt{3x^2+\sqrt{3}+3}}\right)-\frac{2\sqrt{-3x^2+\sqrt{3}-3}\,\text{EllipticF}\left(\arctan\left(\right.\right.}{\sqrt{3\left(3-\sqrt{3}\right)}\sqrt{\frac{3x^2-\sqrt{3}+3}{3x^2+\sqrt{3}+3}}\sqrt{3}}\right.$$
$$\frac{x(3x^2+4)}{4\sqrt{-3x^4-6x^2-2}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{1}{4}\sqrt{3}\left(3\left(\frac{\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}\sqrt{-3x^2+\sqrt{3}-3}E\left(\arctan\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right)|-1-\sqrt{3}\right)}{3\sqrt{\frac{3x^2-\sqrt{3}+3}{3x^2+\sqrt{3}+3}}\sqrt{3x^2+\sqrt{3}+3}}-\frac{x\sqrt{-3x^2+\sqrt{3}-3}}{3\sqrt{3x^2+\sqrt{3}+3}}\right)-\frac{2\sqrt{-3x^2}}{}\right.$$
$$\frac{x(3x^2+4)}{4\sqrt{-3x^4-6x^2-2}}$$

input
```
Int[(-2 - 6*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/4*(x*(4 + 3*x^2))/Sqrt[-2 - 6*x^2 - 3*x^4] + (Sqrt[3]*(3*(-1/3*(x*Sqrt[
-3 + Sqrt[3] - 3*x^2])/Sqrt[3 + Sqrt[3] + 3*x^2] + (Sqrt[(3 - Sqrt[3])/3]*
Sqrt[-3 + Sqrt[3] - 3*x^2]*EllipticE[ArcTan[Sqrt[3/(3 + Sqrt[3])]*x], -1 -
 Sqrt[3]])/(3*Sqrt[(3 - Sqrt[3] + 3*x^2)/(3 + Sqrt[3] + 3*x^2)]*Sqrt[3 + S
qrt[3] + 3*x^2])) - (2*Sqrt[-3 + Sqrt[3] - 3*x^2]*EllipticF[ArcTan[Sqrt[3/
(3 + Sqrt[3])]*x], -1 - Sqrt[3]])/(Sqrt[3*(3 - Sqrt[3])]*Sqrt[(3 - Sqrt[3]
 + 3*x^2)/(3 + Sqrt[3] + 3*x^2)]*Sqrt[3 + Sqrt[3] + 3*x^2])))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{x(3x^2+4)}{4\sqrt{-3x^4-6x^2-2}}+\frac{\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6-2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{-6-2\sqrt{3}}\,\sqrt{-3x^4-6x^2-2}}+\frac{6\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-(\cdots}{\cdots}$
default	$\frac{-\frac{3}{4}x^3-x}{\sqrt{-3x^4-6x^2-2}}+\frac{\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6-2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{-6-2\sqrt{3}}\,\sqrt{-3x^4-6x^2-2}}+\frac{6\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{3}{\cdots}\right.}}{\cdots}$
elliptic	$\frac{-\frac{3}{4}x^3-x}{\sqrt{-3x^4-6x^2-2}}+\frac{\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6-2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{-6-2\sqrt{3}}\,\sqrt{-3x^4-6x^2-2}}+\frac{6\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{3}{\cdots}\right.}}{\cdots}$

input
```
int(1/(-3*x^4-6*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x*(3*x^2+4)/(-3*x^4-6*x^2-2)^(1/2)+1/(-6-2*3^(1/2))^(1/2)*(1-(-3/2-1/
2*3^(1/2))*x^2)^(1/2)*(1-(-3/2+1/2*3^(1/2))*x^2)^(1/2)/(-3*x^4-6*x^2-2)^(1
/2)*EllipticF(1/2*(-6-2*3^(1/2))^(1/2)*x,1/2*6^(1/2)-1/2*2^(1/2))+6/(-6-2*
3^(1/2))^(1/2)*(1-(-3/2-1/2*3^(1/2))*x^2)^(1/2)*(1-(-3/2+1/2*3^(1/2))*x^2)
^(1/2)/(-3*x^4-6*x^2-2)^(1/2)/(-6+2*3^(1/2))*(EllipticF(1/2*(-6-2*3^(1/2))
^(1/2)*x,1/2*6^(1/2)-1/2*2^(1/2))-EllipticE(1/2*(-6-2*3^(1/2))^(1/2)*x,1/2
*6^(1/2)-1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.88

$$\int \frac{1}{(-2-6x^2-3x^4)^{3/2}}\,dx = \frac{3\left(\sqrt{3}\sqrt{-2}(3\,x^4+6\,x^2+2)-3\,\sqrt{-2}(3\,x^4+6\,x^2+2)\right)\sqrt{\frac{1}{2}\,\sqrt{3}-\frac{3}{2}}E(\arcsin\left(\right.}{}$$

input
```
integrate(1/(-3*x^4-6*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(3*(sqrt(3)*sqrt(-2)*(3*x^4 + 6*x^2 + 2) - 3*sqrt(-2)*(3*x^4 + 6*x^2
+ 2))*sqrt(1/2*sqrt(3) - 3/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(3) - 3/2))
, sqrt(3) + 2) - (sqrt(3)*sqrt(-2)*(3*x^4 + 6*x^2 + 2) - 15*sqrt(-2)*(3*x^
4 + 6*x^2 + 2))*sqrt(1/2*sqrt(3) - 3/2)*elliptic_f(arcsin(x*sqrt(1/2*sqrt(
3) - 3/2)), sqrt(3) + 2) + 6*sqrt(-3*x^4 - 6*x^2 - 2)*(3*x^3 + 4*x))/(3*x^
4 + 6*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(-2-6x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-6x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4-6*x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 - 6*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2-6x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-6\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-6*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - 6*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-6x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-6\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-6*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - 6*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-6x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-6\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(- 6*x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(- 6*x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-6x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-6x^2-2}}{9x^8+36x^6+48x^4+24x^2+4}dx$$

input
```
int(1/(-3*x^4-6*x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - 6*x**2 - 2)/(9*x**8 + 36*x**6 + 48*x**4 + 24*x**2 + 4
),x)
```

3.292 $\int \frac{1}{(-2-7x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 94

$$\int \frac{1}{(-2-7x^2-3x^4)^{3/2}}\,dx = -\frac{3x}{5\sqrt{-2-7x^2-3x^4}} + \frac{7\sqrt{-1-3x^2}E\left(\arctan\left(\frac{x}{\sqrt{2}}\right)\middle|-5\right)}{50\sqrt{1+3x^2}} + \frac{6\sqrt{1+3x^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{2}}\right),-5\right)}{25\sqrt{-1-3x^2}}$$

output
```
-3/5*x/(-3*x^4-7*x^2-2)^(1/2)+7/50*(-3*x^2-1)^(1/2)*EllipticE(x*2^(1/2)/(2
*x^2+4)^(1/2),I*5^(1/2))/(3*x^2+1)^(1/2)+6/25*(3*x^2+1)^(1/2)*InverseJacob
iAM(arctan(1/2*x*2^(1/2)),I*5^(1/2))/(-3*x^2-1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-2-7x^2-3x^4)^{3/2}}\,dx = -\frac{37x+21x^3+7i\sqrt{6}\sqrt{2+x^2}\sqrt{1+3x^2}E\left(i\,\text{arcsinh}\left(\sqrt{3}x\right)|\frac{1}{6}\right)-5i\sqrt{6}\sqrt{2+x^2}\sqrt{1+3x^2}\,\text{EllipticF}\left(i\,\text{arcsinh}\right.}{50\sqrt{-2-7x^2-3x^4}}$$

input
```
Integrate[(-2 - 7*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/50*(37*x + 21*x^3 + (7*I)*Sqrt[6]*Sqrt[2 + x^2]*Sqrt[1 + 3*x^2]*Ellipti
cE[I*ArcSinh[Sqrt[3]*x], 1/6] - (5*I)*Sqrt[6]*Sqrt[2 + x^2]*Sqrt[1 + 3*x^2
]*EllipticF[I*ArcSinh[Sqrt[3]*x], 1/6])/Sqrt[-2 - 7*x^2 - 3*x^4]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.77, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1405, 27, 1494, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4-7x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{50}\int \frac{3(7x^2+4)}{\sqrt{-3x^4-7x^2-2}}dx-\frac{x(21x^2+37)}{50\sqrt{-3x^4-7x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{50}\int \frac{7x^2+4}{\sqrt{-3x^4-7x^2-2}}dx-\frac{x(21x^2+37)}{50\sqrt{-3x^4-7x^2-2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{3}{25}\sqrt{3}\int \frac{7x^2+4}{2\sqrt{3}\sqrt{-3x^2-1}\sqrt{x^2+2}}dx-\frac{x(21x^2+37)}{50\sqrt{-3x^4-7x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{50}\int \frac{7x^2+4}{\sqrt{-3x^2-1}\sqrt{x^2+2}}dx-\frac{x(21x^2+37)}{50\sqrt{-3x^4-7x^2-2}}$$

$$\Big\downarrow \text{406}$$

$$\frac{3}{50}\left(4\int \frac{1}{\sqrt{-3x^2-1}\sqrt{x^2+2}}dx+7\int \frac{x^2}{\sqrt{-3x^2-1}\sqrt{x^2+2}}dx\right)-\frac{x(21x^2+37)}{50\sqrt{-3x^4-7x^2-2}}$$

$$\Big\downarrow \ 320$$

$$\frac{3}{50}\left(7\int \frac{x^2}{\sqrt{-3x^2-1}\sqrt{x^2+2}}dx-\frac{4\sqrt{-3x^2-1}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{2}}\right),-5\right)}{\sqrt{x^2+2}\sqrt{\frac{3x^2+1}{x^2+2}}}\right)-\frac{x\left(21x^2+37\right)}{50\sqrt{-3x^4-7x^2-2}}$$

$$\Big\downarrow \ 388$$

$$\frac{3}{50}\left(7\left(\frac{2}{3}\int \frac{\sqrt{-3x^2-1}}{\left(x^2+2\right)^{3/2}}dx-\frac{x\sqrt{-3x^2-1}}{3\sqrt{x^2+2}}\right)-\frac{4\sqrt{-3x^2-1}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{2}}\right),-5\right)}{\sqrt{x^2+2}\sqrt{\frac{3x^2+1}{x^2+2}}}\right)-\frac{x\left(21x^2+37\right)}{50\sqrt{-3x^4-7x^2-2}}$$

$$\Big\downarrow \ 313$$

$$\frac{3}{50}\left(7\left(\frac{\sqrt{-3x^2-1}E\left(\arctan\left(\frac{x}{\sqrt{2}}\right)\middle|-5\right)}{3\sqrt{x^2+2}\sqrt{\frac{3x^2+1}{x^2+2}}}-\frac{x\sqrt{-3x^2-1}}{3\sqrt{x^2+2}}\right)-\frac{4\sqrt{-3x^2-1}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{2}}\right),-5\right)}{\sqrt{x^2+2}\sqrt{\frac{3x^2+1}{x^2+2}}}\right)-\frac{x\left(21x^2+37\right)}{50\sqrt{-3x^4-7x^2-2}}$$

input
```
Int[(-2 - 7*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/50*(x*(37 + 21*x^2))/Sqrt[-2 - 7*x^2 - 3*x^4] + (3*(7*(-1/3*(x*Sqrt[-1
- 3*x^2])/Sqrt[2 + x^2] + (Sqrt[-1 - 3*x^2]*EllipticE[ArcTan[x/Sqrt[2]], -
5])/(3*Sqrt[2 + x^2]*Sqrt[(1 + 3*x^2)/(2 + x^2)])) - (4*Sqrt[-1 - 3*x^2]*E
llipticF[ArcTan[x/Sqrt[2]], -5])/(Sqrt[2 + x^2]*Sqrt[(1 + 3*x^2)/(2 + x^2)
])))/50
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.53

method	result
risch	$-\frac{x(21x^2+37)}{50\sqrt{-3x^4-7x^2-2}}-\frac{i\sqrt{3}\sqrt{3x^2+1}\sqrt{2x^2+4}\,\text{EllipticF}\left(i\sqrt{3}\,x,\frac{\sqrt{6}}{6}\right)}{25\sqrt{-3x^4-7x^2-2}}+\frac{7i\sqrt{3}\sqrt{3x^2+1}\sqrt{2x^2+4}\left(\text{EllipticF}\left(i\sqrt{3}\,x,\frac{\sqrt{6}}{6}\right)-\text{Elliptic}\right.}{50\sqrt{-3x^4-7x^2-2}}$
default	$\frac{-\frac{37}{50}x-\frac{21}{50}x^3}{\sqrt{-3x^4-7x^2-2}}-\frac{i\sqrt{3}\sqrt{3x^2+1}\sqrt{2x^2+4}\,\text{EllipticF}\left(i\sqrt{3}\,x,\frac{\sqrt{6}}{6}\right)}{25\sqrt{-3x^4-7x^2-2}}+\frac{7i\sqrt{3}\sqrt{3x^2+1}\sqrt{2x^2+4}\left(\text{EllipticF}\left(i\sqrt{3}\,x,\frac{\sqrt{6}}{6}\right)-\text{EllipticE}\left(i\right.\right.}{50\sqrt{-3x^4-7x^2-2}}$
elliptic	$\frac{-\frac{37}{50}x-\frac{21}{50}x^3}{\sqrt{-3x^4-7x^2-2}}-\frac{i\sqrt{3}\sqrt{3x^2+1}\sqrt{2x^2+4}\,\text{EllipticF}\left(i\sqrt{3}\,x,\frac{\sqrt{6}}{6}\right)}{25\sqrt{-3x^4-7x^2-2}}+\frac{7i\sqrt{3}\sqrt{3x^2+1}\sqrt{2x^2+4}\left(\text{EllipticF}\left(i\sqrt{3}\,x,\frac{\sqrt{6}}{6}\right)-\text{EllipticE}\left(i\right.\right.}{50\sqrt{-3x^4-7x^2-2}}$

input
```
int(1/(-3*x^4-7*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/50*x*(21*x^2+37)/(-3*x^4-7*x^2-2)^(1/2)-1/25*I*3^(1/2)*(3*x^2+1)^(1/2)*
(2*x^2+4)^(1/2)/(-3*x^4-7*x^2-2)^(1/2)*EllipticF(I*3^(1/2)*x,1/6*6^(1/2))+
7/50*I*3^(1/2)*(3*x^2+1)^(1/2)*(2*x^2+4)^(1/2)/(-3*x^4-7*x^2-2)^(1/2)*(Ell
ipticF(I*3^(1/2)*x,1/6*6^(1/2))-EllipticE(I*3^(1/2)*x,1/6*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int \frac{1}{(-2-7x^2-3x^4)^{3/2}}\,dx =$$
$$-\frac{7\sqrt{-\frac{1}{2}}\sqrt{-2}(3\,x^4+7\,x^2+2)E(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)\,|\,6)-31\sqrt{-\frac{1}{2}}\sqrt{-2}(3\,x^4+7\,x^2+2)F(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)}{100\,(3\,x^4+7\,x^2+2)}$$

input
```
integrate(1/(-3*x^4-7*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
-1/100*(7*sqrt(-1/2)*sqrt(-2)*(3*x^4 + 7*x^2 + 2)*elliptic_e(arcsin(sqrt(-
1/2)*x), 6) - 31*sqrt(-1/2)*sqrt(-2)*(3*x^4 + 7*x^2 + 2)*elliptic_f(arcsin
(sqrt(-1/2)*x), 6) - 2*sqrt(-3*x^4 - 7*x^2 - 2)*(21*x^3 + 37*x))/(3*x^4 +
7*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(-2-7x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-7x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4-7*x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 - 7*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2-7x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-7\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-7*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - 7*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-7x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-7\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-7*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - 7*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-7x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-7\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(- 7*x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(- 7*x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-7x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-7x^2-2}}{9x^8+42x^6+61x^4+28x^2+4}dx$$

input
```
int(1/(-3*x^4-7*x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - 7*x**2 - 2)/(9*x**8 + 42*x**6 + 61*x**4 + 28*x**2 + 4
),x)
```

3.293 $\int \frac{1}{(-1+5x^2-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 118

$$\int \frac{1}{(-1+5x^2-x^4)^{3/2}}\,dx = -\frac{x(23-5x^2)}{21\sqrt{-1+5x^2-x^4}} + \frac{5E\left(\arccos\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)\middle|\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{21^{3/4}} - \frac{\left(5-\sqrt{21}\right)\text{EllipticF}\left(\arccos\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right),\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{2\ 21^{3/4}}$$

output
```
-1/21*x*(-5*x^2+23)/(-x^4+5*x^2-1)^(1/2)+5/21*EllipticE((1-2/(5+21^(1/2))*
x^2)^(1/2),1/42*(882+210*21^(1/2))^(1/2))*21^(1/4)-1/42*(5-21^(1/2))*Inver
seJacobiAM(arccos(2^(1/2)/(5+21^(1/2))^(1/2)*x),1/42*(882+210*21^(1/2))^(1
/2))*21^(1/4)
```

Mathematica [A] (warning: unable to verify)

Time = 5.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.57

$$\int \frac{1}{(-1+5x^2-x^4)^{3/2}}\,dx = \frac{4x(-23+5x^2)+5\left(5+\sqrt{21}\right)\sqrt{5-\sqrt{21}-2x^2}\sqrt{2+\left(-5+\sqrt{21}\right)x^2}E\left(\text{arcsi}\right.}{}$$

input
```
Integrate[(-1 + 5*x^2 - x^4)^(-3/2),x]
```

output
```
(4*x*(-23 + 5*x^2) + 5*(5 + Sqrt[21])*Sqrt[5 - Sqrt[21] - 2*x^2]*Sqrt[2 +
(-5 + Sqrt[21])*x^2]*EllipticE[ArcSin[Sqrt[(5 + Sqrt[21])/2]*x], 23/2 - (5
*Sqrt[21])/2] - (21 + 5*Sqrt[21])*Sqrt[5 - Sqrt[21] - 2*x^2]*Sqrt[2 + (-5
+ Sqrt[21])*x^2]*EllipticF[ArcSin[Sqrt[(5 + Sqrt[21])/2]*x], 23/2 - (5*Sqr
t[21])/2])/(84*Sqrt[-1 + 5*x^2 - x^4])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1405, 1494, 399, 322, 328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^4+5x^2-1)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{21}\int \frac{2-5x^2}{\sqrt{-x^4+5x^2-1}}dx-\frac{x(23-5x^2)}{21\sqrt{-x^4+5x^2-1}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{2}{21}\int \frac{2-5x^2}{\sqrt{-2x^2+\sqrt{21}+5}\sqrt{2x^2+\sqrt{21}-5}}dx-\frac{x(23-5x^2)}{21\sqrt{-x^4+5x^2-1}}$$

$$\Big\downarrow \text{399}$$

$$\frac{2}{21}\left(-\frac{1}{2}\left(21-5\sqrt{21}\right)\int \frac{1}{\sqrt{-2x^2+\sqrt{21}+5}\sqrt{2x^2+\sqrt{21}-5}}dx-\frac{5}{2}\int \frac{\sqrt{2x^2+\sqrt{21}-5}}{\sqrt{-2x^2+\sqrt{21}+5}}dx\right)-\frac{x\left(23-5x^2\right)}{21\sqrt{-x^4+5x^2-1}}$$

$$\Big\downarrow \ 322$$

$$\frac{2}{21}\left(\frac{\left(21-5\sqrt{21}\right)\text{EllipticF}\left(\arccos\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right),\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{4\sqrt[4]{21}}-\frac{5}{2}\int \frac{\sqrt{2x^2+\sqrt{21}-5}}{\sqrt{-2x^2+\sqrt{21}+5}}dx\right)-\frac{x\left(23-5x^2\right)}{21\sqrt{-x^4+5x^2-1}}$$

$$\Big\downarrow \ 328$$

$$\frac{2}{21}\left(\frac{\left(21-5\sqrt{21}\right)\text{EllipticF}\left(\arccos\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right),\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{4\sqrt[4]{21}}+\frac{5}{2}\sqrt[4]{21}E\left(\arccos\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)|\frac{1}{42}\left(21+\right.\right.\right.$$
$$\frac{x\left(23-5x^2\right)}{21\sqrt{-x^4+5x^2-1}}$$

input
```
Int[(-1 + 5*x^2 - x^4)^(-3/2),x]
```

output
```
-1/21*(x*(23 - 5*x^2))/Sqrt[-1 + 5*x^2 - x^4] + (2*((5*21^(1/4)*EllipticE[
ArcCos[Sqrt[2/(5 + Sqrt[21])]*x], (21 + 5*Sqrt[21])/42])/2 + ((21 - 5*Sqrt
[21])*EllipticF[ArcCos[Sqrt[2/(5 + Sqrt[21])]*x], (21 + 5*Sqrt[21])/42])/(
4*21^(1/4))))/21
```

Defintions of rubi rules used

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(95) = 190$.

Time = 1.15 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.87

method	result
risch	$\frac{x(5x^2-23)}{21\sqrt{-x^4+5x^2-1}}+\frac{2\sqrt{1-\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)x^2}\,\sqrt{1-\left(\frac{5}{2}+\frac{\sqrt{21}}{2}\right)x^2}\ \text{EllipticF}\left(x\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right),\frac{5}{2}+\frac{\sqrt{21}}{2}\right)}{21\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right)\sqrt{-x^4+5x^2-1}}-\frac{10\sqrt{1-\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)x^2}\,\sqrt{1-\left(\frac{5}{2}+\right.}$
default	$\frac{-\frac{23}{21}x+\frac{5}{21}x^3}{\sqrt{-x^4+5x^2-1}}+\frac{2\sqrt{1-\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)x^2}\,\sqrt{1-\left(\frac{5}{2}+\frac{\sqrt{21}}{2}\right)x^2}\ \text{EllipticF}\left(x\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right),\frac{5}{2}+\frac{\sqrt{21}}{2}\right)}{21\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right)\sqrt{-x^4+5x^2-1}}-\frac{10\sqrt{1-\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)x^2}\,\sqrt{1-\left(\frac{5}{2}+\right.}$
elliptic	$\frac{-\frac{23}{21}x+\frac{5}{21}x^3}{\sqrt{-x^4+5x^2-1}}+\frac{2\sqrt{1-\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)x^2}\,\sqrt{1-\left(\frac{5}{2}+\frac{\sqrt{21}}{2}\right)x^2}\ \text{EllipticF}\left(x\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right),\frac{5}{2}+\frac{\sqrt{21}}{2}\right)}{21\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right)\sqrt{-x^4+5x^2-1}}-\frac{10\sqrt{1-\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)x^2}\,\sqrt{1-\left(\frac{5}{2}+\right.}$

input
```
int(1/(-x^4+5*x^2-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*x*(5*x^2-23)/(-x^4+5*x^2-1)^(1/2)+2/21/(1/2*7^(1/2)-1/2*3^(1/2))*(1-(
5/2-1/2*21^(1/2))*x^2)^(1/2)*(1-(5/2+1/2*21^(1/2))*x^2)^(1/2)/(-x^4+5*x^2-
1)^(1/2)*EllipticF(x*(1/2*7^(1/2)-1/2*3^(1/2)),5/2+1/2*21^(1/2))-10/21/(1/
2*7^(1/2)-1/2*3^(1/2))*(1-(5/2-1/2*21^(1/2))*x^2)^(1/2)*(1-(5/2+1/2*21^(1/
2))*x^2)^(1/2)/(-x^4+5*x^2-1)^(1/2)/(5+21^(1/2))*(EllipticF(x*(1/2*7^(1/2)
-1/2*3^(1/2)),5/2+1/2*21^(1/2))-EllipticE(x*(1/2*7^(1/2)-1/2*3^(1/2)),5/2+
1/2*21^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

$$\int \frac{1}{(-1+5x^2-x^4)^{3/2}}\,dx =$$

$$-\frac{5\left(5i\,x^4-25i\,x^2+\sqrt{21}(i\,x^4-5i\,x^2+i)+5i\right)\sqrt{\frac{1}{2}\sqrt{21}+\frac{5}{2}}E(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{21}+\frac{5}{2}}\right)\mid-\frac{5}{2}\sqrt{21}+\frac{23}{2})-}{}$$

input
```
integrate(1/(-x^4+5*x^2-1)^(3/2),x, algorithm="fricas")
```

output
```
-1/42*(5*(5*I*x^4 - 25*I*x^2 + sqrt(21)*(I*x^4 - 5*I*x^2 + I) + 5*I)*sqrt(
1/2*sqrt(21) + 5/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(21) + 5/2)), -5/2*sq
rt(21) + 23/2) - (15*I*x^4 - 75*I*x^2 - 7*sqrt(21)*(-I*x^4 + 5*I*x^2 - I)
+ 15*I)*sqrt(1/2*sqrt(21) + 5/2)*elliptic_f(arcsin(x*sqrt(1/2*sqrt(21) + 5
/2)), -5/2*sqrt(21) + 23/2) + 2*sqrt(-x^4 + 5*x^2 - 1)*(5*x^3 - 23*x))/(x^
4 - 5*x^2 + 1)
```

Sympy [F]

$$\int \frac{1}{(-1+5x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+5x^2-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x**4+5*x**2-1)**(3/2),x)
```

output
```
Integral((-x**4 + 5*x**2 - 1)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-1+5x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+5\,x^2-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+5*x^2-1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 5*x^2 - 1)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(-1+5x^2-x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-x^4+5\,x^2-1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+5*x^2-1)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 5*x^2 - 1)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-1+5x^2-x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-x^4+5\,x^2-1\right)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - x^4 - 1)^(3/2),x)
```

output
```
int(1/(5*x^2 - x^4 - 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(-1+5x^2-x^4\right)^{3/2}}\,dx = \int \frac{\sqrt{-x^4+5x^2-1}}{x^8-10x^6+27x^4-10x^2+1}dx$$

input
```
int(1/(-x^4+5*x^2-1)^(3/2),x)
```

output
```
int(sqrt( - x**4 + 5*x**2 - 1)/(x**8 - 10*x**6 + 27*x**4 - 10*x**2 + 1),x)
```

3.294 $\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx = -\frac{3x}{2\sqrt{-2-5x^2-3x^4}} + \frac{5\sqrt{-2-3x^2}E\left(\arctan(x)\,\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2}} + \frac{3\sqrt{2}\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{-2-3x^2}}$$

output
```
-3/2*x/(-3*x^4-5*x^2-2)^(1/2)+5/2*(-3*x^2-2)^(1/2)*EllipticE(x/(x^2+1)^(1/
2),1/2*I*2^(1/2))*2^(1/2)/(3*x^2+2)^(1/2)+3*(3*x^2+2)^(1/2)*InverseJacobiA
M(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(-3*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.34

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx = -\frac{13x+15x^3+5i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\middle|\frac{2}{3}\right)-i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(i\text{arcsin}\right.}{2\sqrt{-2-5x^2-3x^4}}$$

input
```
Integrate[(-2 - 5*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/2*(13*x + 15*x^3 + (5*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Elliptic
E[I*ArcSinh[Sqrt[3/2]*x], 2/3] - I*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*E
llipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/Sqrt[-2 - 5*x^2 - 3*x^4]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1405, 27, 1494, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-3x^4-5x^2-2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{2}\int \frac{3\left(5x^2+4\right)}{\sqrt{-3x^4-5x^2-2}}dx-\frac{x\left(15x^2+13\right)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\int \frac{5x^2+4}{\sqrt{-3x^4-5x^2-2}}dx-\frac{x\left(15x^2+13\right)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{1494}$$

$$3\sqrt{3}\int \frac{5x^2+4}{2\sqrt{3}\sqrt{-3x^2-2}\sqrt{x^2+1}}dx-\frac{x\left(15x^2+13\right)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\int \frac{5x^2+4}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx-\frac{x\left(15x^2+13\right)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{406}$$

$$\frac{3}{2}\left(4\int \frac{1}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx+5\int \frac{x^2}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx\right)-\frac{x\left(15x^2+13\right)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \textbf{320}$$

$$\frac{3}{2}\left(5\int \frac{x^2}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx-\frac{2\sqrt{2}\sqrt{-3x^2-2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}\right)-\frac{x\left(15x^2+13\right)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \textbf{388}$$

$$\frac{3}{2}\left(5\left(\frac{1}{3}\int \frac{\sqrt{-3x^2-2}}{\left(x^2+1\right)^{3/2}}dx-\frac{x\sqrt{-3x^2-2}}{3\sqrt{x^2+1}}\right)-\frac{2\sqrt{2}\sqrt{-3x^2-2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}\right)-\frac{x\left(15x^2+13\right)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \textbf{313}$$

$$\frac{3}{2}\left(5\left(\frac{\sqrt{2}\sqrt{-3x^2-2}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}-\frac{x\sqrt{-3x^2-2}}{3\sqrt{x^2+1}}\right)-\frac{2\sqrt{2}\sqrt{-3x^2-2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}\right)-\frac{x\left(15x^2+13\right)}{2\sqrt{-3x^4-5x^2-2}}$$

input
```
Int[(-2 - 5*x^2 - 3*x^4)^(-3/2),x]
```

output
```
-1/2*(x*(13 + 15*x^2))/Sqrt[-2 - 5*x^2 - 3*x^4] + (3*(5*(-1/3*(x*Sqrt[-2 -
 3*x^2])/Sqrt[1 + x^2] + (Sqrt[2]*Sqrt[-2 - 3*x^2]*EllipticE[ArcTan[x], -1
/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)])) - (2*Sqrt[2]*Sqrt[-2 -
 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x
^2)])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{\sqrt{-3x^4-5x^2-2}}+\frac{5i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\frac{i}{}\right.\right.}{4\sqrt{-3x^4-5x^2-2}}$
default	$\frac{-\frac{15}{2}x^3-\frac{13}{2}x}{\sqrt{-3x^4-5x^2-2}}-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{\sqrt{-3x^4-5x^2-2}}+\frac{5i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\frac{ix\sqrt{6}}{2}\right.\right.}{4\sqrt{-3x^4-5x^2-2}}$
elliptic	$\frac{-\frac{15}{2}x^3-\frac{13}{2}x}{\sqrt{-3x^4-5x^2-2}}-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{\sqrt{-3x^4-5x^2-2}}+\frac{5i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\frac{ix\sqrt{6}}{2}\right.\right.}{4\sqrt{-3x^4-5x^2-2}}$

input
```
int(1/(-3*x^4-5*x^2-2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(15*x^2+13)/(-3*x^4-5*x^2-2)^(1/2)-I*6^(1/2)*(6*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(-3*x^4-5*x^2-2)^(1/2)*EllipticF(1/2*I*x*6^(1/2),1/3*6^(1/2))+5/4*
I*6^(1/2)*(6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-5*x^2-2)^(1/2)*(EllipticF(
1/2*I*x*6^(1/2),1/3*6^(1/2))-EllipticE(1/2*I*x*6^(1/2),1/3*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx=\frac{-5i\,\sqrt{-2}(3\,x^4+5\,x^2+2)E(\arcsin{(i\,x)}\mid\frac{3}{2})+11i\,\sqrt{-2}(3\,x^4+5\,x^2+2)F(\mathrm{ar}}{2\,(3\,x^4+5\,x^2+2)}$$

input
```
integrate(1/(-3*x^4-5*x^2-2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(-5*I*sqrt(-2)*(3*x^4 + 5*x^2 + 2)*elliptic_e(arcsin(I*x), 3/2) + 11*I
*sqrt(-2)*(3*x^4 + 5*x^2 + 2)*elliptic_f(arcsin(I*x), 3/2) + sqrt(-3*x^4 -
 5*x^2 - 2)*(15*x^3 + 13*x))/(3*x^4 + 5*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4-5x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4-5*x**2-2)**(3/2),x)
```

output
```
Integral((-3*x**4 - 5*x**2 - 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-5\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-5*x^2-2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 - 5*x^2 - 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-5\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4-5*x^2-2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 - 5*x^2 - 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4-5\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/(- 5*x^2 - 3*x^4 - 2)^(3/2),x)
```

output
```
int(1/(- 5*x^2 - 3*x^4 - 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-2-5x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-5x^2-2}}{9x^8+30x^6+37x^4+20x^2+4}dx$$

input
```
int(1/(-3*x^4-5*x^2-2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - 5*x**2 - 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4
),x)
```

3.295 $\int \frac{1}{(-2-3x^2)^{3/2}(1+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{1}{(-2-3x^2)^{3/2}(1+x^2)^{3/2}}\,dx = -\frac{3x}{2\sqrt{-2-3x^2}\sqrt{1+x^2}} - \frac{5\sqrt{2+3x^2}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{-2-3x^2}} + \frac{3\sqrt{2}\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{-2-3x^2}}$$

output
```
-3/2*x/(-3*x^2-2)^(1/2)/(x^2+1)^(1/2)-5/2*(3*x^2+2)^(1/2)*EllipticE(x/(x^2
+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)/(-3*x^2-2)^(1/2)+3*(3*x^2+2)^(1/2)*Invers
eJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(-3*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.63 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.23

$$\int \frac{1}{(-2-3x^2)^{3/2}(1+x^2)^{3/2}}\,dx = \frac{-x\sqrt{1+x^2}(13+15x^2)-5i(1+x^2)\sqrt{6+9x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\left|\frac{2}{3}\right.\right)+}{2\sqrt{-2-3x^2}\,(1+x^2}$$

input
```
Integrate[1/((-2 - 3*x^2)^(3/2)*(1 + x^2)^(3/2)),x]
```

output
```
(-(x*Sqrt[1 + x^2]*(13 + 15*x^2)) - (5*I)*(1 + x^2)*Sqrt[6 + 9*x^2]*Ellipt
icE[I*ArcSinh[Sqrt[3/2]*x], 2/3] + I*(1 + x^2)*Sqrt[6 + 9*x^2]*EllipticF[I
*ArcSinh[Sqrt[3/2]*x], 2/3])/(2*Sqrt[-2 - 3*x^2]*(1 + x^2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {316, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^2-2)^{3/2}(x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{316}$$

$$-\frac{1}{2}\int -\frac{2-3x^2}{\sqrt{-3x^2-2}(x^2+1)^{3/2}}dx-\frac{3x}{2\sqrt{-3x^2-2}\sqrt{x^2+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{2-3x^2}{\sqrt{-3x^2-2}(x^2+1)^{3/2}}dx-\frac{3x}{2\sqrt{-3x^2-2}\sqrt{x^2+1}}$$

$$\Big\downarrow \text{400}$$

$$\frac{1}{2}\left(5\int \frac{\sqrt{-3x^2-2}}{(x^2+1)^{3/2}}dx+12\int \frac{1}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx\right)-\frac{3x}{2\sqrt{-3x^2-2}\sqrt{x^2+1}}$$

$$\Big\downarrow \text{313}$$

$$\frac{1}{2}\left(12\int \frac{1}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx+\frac{5\sqrt{2}\sqrt{-3x^2-2}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}\right)-\frac{3x}{2\sqrt{-3x^2-2}\sqrt{x^2+1}}$$

$$\Big\downarrow \text{320}$$

$$\frac{1}{2}\left(\frac{5\sqrt{2}\sqrt{-3x^2-2}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}-\frac{6\sqrt{2}\sqrt{-3x^2-2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}\right)-\frac{3x}{2\sqrt{-3x^2-2}\sqrt{x^2+1}}$$

input
```
Int[1/((-2 - 3*x^2)^(3/2)*(1 + x^2)^(3/2)),x]
```

output
```
(-3*x)/(2*Sqrt[-2 - 3*x^2]*Sqrt[1 + x^2]) + ((5*Sqrt[2]*Sqrt[-2 - 3*x^2]*E
llipticE[ArcTan[x], -1/2])/(Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)]) - (
6*Sqrt[2]*Sqrt[-2 - 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[1 + x^2]*Sqrt
[(2 + 3*x^2)/(1 + x^2)]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

Maple [A] (verified)

Time = 5.99 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

method	result
default	$-\frac{\sqrt{-3x^2-2}\,\sqrt{x^2+1}\,\left(i\sqrt{3}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)\sqrt{x^2+1}\,\sqrt{3x^2+2}-5i\sqrt{3}\ \mathrm{EllipticE}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)\sqrt{x^2+1}\,\sqrt{3x^2+2}-15x^3-13x\right)}{2(3x^4+5x^2+2)}$
elliptic	$\frac{\sqrt{-(3x^2+2)(x^2+1)}\,\left(\frac{-\frac{15}{2}x^3-\frac{13}{2}x}{\sqrt{-3x^4-5x^2-2}}-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{\sqrt{-3x^4-5x^2-2}}+\frac{5i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)-\mathrm{Ellipti}\right.}{4\sqrt{-3x^4-5x^2-2}}\right.}{\sqrt{x^2+1}\,\sqrt{-3x^2-2}}$

input
```
int(1/(-3*x^2-2)^(3/2)/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-3*x^2-2)^(1/2)*(x^2+1)^(1/2)*(I*3^(1/2)*EllipticF(1/2*I*x*6^(1/2),1
/3*6^(1/2))*(x^2+1)^(1/2)*(3*x^2+2)^(1/2)-5*I*3^(1/2)*EllipticE(1/2*I*x*6^
(1/2),1/3*6^(1/2))*(x^2+1)^(1/2)*(3*x^2+2)^(1/2)-15*x^3-13*x)/(3*x^4+5*x^2
+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

$$\int \frac{1}{(-2-3x^2)^{3/2}\,(1+x^2)^{3/2}}\,dx = \frac{-5i\sqrt{-2}(3\,x^4+5\,x^2+2)E(\arcsin{(i\,x)}\mid\frac{3}{2})+11i\sqrt{-2}(3\,x^4+5\,x^2+2)}{2\,(3\,x^4+5\,x^2+2}$$

input
```
integrate(1/(-3*x^2-2)^(3/2)/(x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(-5*I*sqrt(-2)*(3*x^4 + 5*x^2 + 2)*elliptic_e(arcsin(I*x), 3/2) + 11*I
*sqrt(-2)*(3*x^4 + 5*x^2 + 2)*elliptic_f(arcsin(I*x), 3/2) + (15*x^3 + 13*
x)*sqrt(x^2 + 1)*sqrt(-3*x^2 - 2))/(3*x^4 + 5*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(-2-3x^2)^{3/2}(1+x^2)^{3/2}}\,dx = \int \frac{1}{(-3x^2-2)^{\frac{3}{2}}(x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**2-2)**(3/2)/(x**2+1)**(3/2),x)
```

output
```
Integral(1/((-3*x**2 - 2)**(3/2)*(x**2 + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(-2-3x^2)^{3/2}(1+x^2)^{3/2}}\,dx = \int \frac{1}{(x^2+1)^{\frac{3}{2}}(-3\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^2-2)^(3/2)/(x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 1)^(3/2)*(-3*x^2 - 2)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(-2-3x^2)^{3/2}(1+x^2)^{3/2}}\,dx = \int \frac{1}{(x^2+1)^{\frac{3}{2}}(-3\,x^2-2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^2-2)^(3/2)/(x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 + 1)^(3/2)*(-3*x^2 - 2)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-3x^2)^{3/2}\,(1+x^2)^{3/2}}\,dx = \int \frac{1}{(x^2+1)^{3/2}\,(-3\,x^2-2)^{3/2}}\,dx$$

input
```
int(1/((x^2 + 1)^(3/2)*(- 3*x^2 - 2)^(3/2)),x)
```

output
```
int(1/((x^2 + 1)^(3/2)*(- 3*x^2 - 2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(-2-3x^2)^{3/2}\,(1+x^2)^{3/2}}\,dx = \int \frac{\sqrt{-3x^2-2}\,\sqrt{x^2+1}}{9x^8+30x^6+37x^4+20x^2+4}dx$$

input
```
int(1/(-3*x^2-2)^(3/2)/(x^2+1)^(3/2),x)
```

output
```
int((sqrt( - 3*x**2 - 2)*sqrt(x**2 + 1))/(9*x**8 + 30*x**6 + 37*x**4 + 20*
x**2 + 4),x)
```

3.296 $\int \frac{1}{\left(\left(-1-x^2\right)\left(2+3x^2\right)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 96

$$\int \frac{1}{\left((-1-x^2)(2+3x^2)\right)^{3/2}}\,dx = -\frac{3x}{2\sqrt{-2-3x^2}\sqrt{1+x^2}} - \frac{5\sqrt{2+3x^2}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{-2-3x^2}} + \frac{3\sqrt{2}\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{-2-3x^2}}$$

output
```
-3/2*x/(-3*x^2-2)^(1/2)/(x^2+1)^(1/2)-5/2*(3*x^2+2)^(1/2)*EllipticE(x/(x^2
+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)/(-3*x^2-2)^(1/2)+3*(3*x^2+2)^(1/2)*Invers
eJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(-3*x^2-2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left((-1-x^2)(2+3x^2)\right)^{3/2}}\,dx = -\frac{13x+15x^3+5i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(i\text{arcsin}\right.}{2\sqrt{-2-5x^2-3x^4}}$$

input
```
Integrate[((-1 - x^2)*(2 + 3*x^2))^(-3/2),x]
```

output
```
-1/2*(13*x + 15*x^3 + (5*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Elliptic
E[I*ArcSinh[Sqrt[3/2]*x], 2/3] - I*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*E
llipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/Sqrt[-2 - 5*x^2 - 3*x^4]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.75, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {2048, 1405, 27, 1494, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{((-x^2-1)(3x^2+2))^{3/2}}dx$$

$$\Big\downarrow \text{ 2048}$$

$$\int \frac{1}{(-3x^4-5x^2-2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{1}{2}\int \frac{3(5x^2+4)}{\sqrt{-3x^4-5x^2-2}}dx-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{2}\int \frac{5x^2+4}{\sqrt{-3x^4-5x^2-2}}dx-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{ 1494}$$

$$3\sqrt{3}\int \frac{5x^2+4}{2\sqrt{3}\sqrt{-3x^2-2}\sqrt{x^2+1}}dx-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{2}\int \frac{5x^2+4}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{406}$$

$$\frac{3}{2}\left(4\int \frac{1}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx+5\int \frac{x^2}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx\right)-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{320}$$

$$\frac{3}{2}\left(5\int \frac{x^2}{\sqrt{-3x^2-2}\sqrt{x^2+1}}dx-\frac{2\sqrt{2}\sqrt{-3x^2-2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}\right)-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{388}$$

$$\frac{3}{2}\left(5\left(\frac{1}{3}\int \frac{\sqrt{-3x^2-2}}{(x^2+1)^{3/2}}dx-\frac{x\sqrt{-3x^2-2}}{3\sqrt{x^2+1}}\right)-\frac{2\sqrt{2}\sqrt{-3x^2-2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}\right)-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}$$

$$\Big\downarrow \text{313}$$

$$\frac{3}{2}\left(5\left(\frac{\sqrt{2}\sqrt{-3x^2-2}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}-\frac{x\sqrt{-3x^2-2}}{3\sqrt{x^2+1}}\right)-\frac{2\sqrt{2}\sqrt{-3x^2-2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}\right)-\frac{x(15x^2+13)}{2\sqrt{-3x^4-5x^2-2}}$$

input

```
Int[((-1 - x^2)*(2 + 3*x^2))^(-3/2),x]
```

output

```
-1/2*(x*(13 + 15*x^2))/Sqrt[-2 - 5*x^2 - 3*x^4] + (3*(5*(-1/3*(x*Sqrt[-2 -
 3*x^2])/Sqrt[1 + x^2] + (Sqrt[2]*Sqrt[-2 - 3*x^2]*EllipticE[ArcTan[x], -1
/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)])) - (2*Sqrt[2]*Sqrt[-2 -
 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x
^2)])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.47

method	result
default	$\frac{-\frac{15}{2}x^3-\frac{13}{2}x}{\sqrt{-3x^4-5x^2-2}}-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{\sqrt{-3x^4-5x^2-2}}+\frac{5i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)-\text{EllipticE}\left(\frac{ix\sqrt{6}}{2}\right.\right.}{4\sqrt{-3x^4-5x^2-2}}$
elliptic	$\frac{-\frac{15}{2}x^3-\frac{13}{2}x}{\sqrt{-3x^4-5x^2-2}}-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{\sqrt{-3x^4-5x^2-2}}+\frac{5i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)-\text{EllipticE}\left(\frac{ix\sqrt{6}}{2}\right.\right.}{4\sqrt{-3x^4-5x^2-2}}$

input
```
int(1/((-x^2-1)*(3*x^2+2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
6*(-5/4*x^3-13/12*x)/(-3*x^4-5*x^2-2)^(1/2)-I*6^(1/2)*(6*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(-3*x^4-5*x^2-2)^(1/2)*EllipticF(1/2*I*x*6^(1/2),1/3*6^(1/2))+5/
4*I*6^(1/2)*(6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-5*x^2-2)^(1/2)*(Elliptic
F(1/2*I*x*6^(1/2),1/3*6^(1/2))-EllipticE(1/2*I*x*6^(1/2),1/3*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left((-1-x^2)\left(2+3x^2\right)\right)^{3/2}}\,dx = \frac{-5i\sqrt{-2}(3\,x^4+5\,x^2+2)E(\arcsin\left(i\,x\right)\mid\frac{3}{2})+11i\sqrt{-2}(3\,x^4+5\,x^2+2)I}{2\left(3\,x^4+5\,x^2+2\right)}$$

input
```
integrate(1/((-x^2-1)*(3*x^2+2))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(-5*I*sqrt(-2)*(3*x^4 + 5*x^2 + 2)*elliptic_e(arcsin(I*x), 3/2) + 11*I
*sqrt(-2)*(3*x^4 + 5*x^2 + 2)*elliptic_f(arcsin(I*x), 3/2) + sqrt(-3*x^4 -
 5*x^2 - 2)*(15*x^3 + 13*x))/(3*x^4 + 5*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{\left((-1-x^2)\left(2+3x^2\right)\right)^{3/2}}\,dx = \int \frac{1}{\left((-x^2-1)\left(3x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((-x**2-1)*(3*x**2+2))**(3/2),x)
```

output
```
Integral(((-x**2 - 1)*(3*x**2 + 2))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left((-1-x^2)\left(2+3x^2\right)\right)^{3/2}}\,dx = \int \frac{1}{\left(-(3\,x^2+2)(x^2+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((-x^2-1)*(3*x^2+2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-(3*x^2 + 2)*(x^2 + 1))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left((-1-x^2)\left(2+3x^2\right)\right)^{3/2}}\,dx = \int \frac{1}{\left(-(3\,x^2+2)(x^2+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((-x^2-1)*(3*x^2+2))^(3/2),x, algorithm="giac")
```

output
```
integrate((-(3*x^2 + 2)*(x^2 + 1))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{((-1-x^2)(2+3x^2))^{3/2}}\,dx = \int \frac{1}{\left(-\left(x^2+1\right)\left(3\,x^2+2\right)\right)^{3/2}}\,dx$$

input
```
int(1/(-(x^2 + 1)*(3*x^2 + 2))^(3/2),x)
```

output
```
int(1/(-(x^2 + 1)*(3*x^2 + 2))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{((-1-x^2)(2+3x^2))^{3/2}}\,dx = \int \frac{\sqrt{-3x^4-5x^2-2}}{9x^8+30x^6+37x^4+20x^2+4}dx$$

input
```
int(1/((-x^2-1)*(3*x^2+2))^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 - 5*x**2 - 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4
),x)
```

3.297 $\int \frac{1}{(2+5x^2+5x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 261

$$\int \frac{1}{(2+5x^2+5x^4)^{3/2}}\,dx = -\frac{x(1+5x^2)}{6\sqrt{2+5x^2+5x^4}}+\frac{5x\sqrt{2+5x^2+5x^4}}{6\left(\sqrt{10}+5x^2\right)} - \frac{\sqrt[4]{5}(2+\sqrt{10}x^2)\sqrt{\frac{2+5x^2+5x^4}{\left(2+\sqrt{10}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{\frac{5}{2}}x\right)|\frac{1}{8}(4-\sqrt{10})\right)}{3\,2^{3/4}\sqrt{2+5x^2+5x^4}} + \frac{\left(5+2\sqrt{10}\right)\left(2+\sqrt{10}x^2\right)\sqrt{\frac{2+5x^2+5x^4}{\left(2+\sqrt{10}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{5}{2}}x\right),\frac{1}{8}(4-\sqrt{10})\right)}{6\,10^{3/4}\sqrt{2+5x^2+5x^4}}$$

output

```
-1/6*x*(5*x^2+1)/(5*x^4+5*x^2+2)^(1/2)+5*x*(5*x^4+5*x^2+2)^(1/2)/(6*10^(1/
2)+30*x^2)-1/6*5^(1/4)*(2+10^(1/2)*x^2)*((5*x^4+5*x^2+2)/(2+10^(1/2)*x^2)^
2)^(1/2)*EllipticE(sin(2*arctan(1/2*5^(1/4)*2^(3/4)*x)),1/4*(8-2*10^(1/2))
^(1/2))*2^(1/4)/(5*x^4+5*x^2+2)^(1/2)+1/60*(5+2*10^(1/2))*(2+10^(1/2)*x^2)
*((5*x^4+5*x^2+2)/(2+10^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*5
^(1/4)*2^(3/4)*x),1/4*(8-2*10^(1/2))^(1/2))*10^(1/4)/(5*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.47 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.31

$$\int \frac{1}{(2+5x^2+5x^4)^{3/2}}\,dx = \frac{-20\sqrt{-\frac{i}{-5i+\sqrt{15}}}x(1+5x^2)-5\sqrt{2}(\sqrt{3}+i\sqrt{5})\sqrt{\frac{-5i+\sqrt{15}-10ix^2}{-5i+\sqrt{15}}}\sqrt{\frac{5i+\sqrt{15}+10ix^2}{5i+\sqrt{15}}}E}{}$$

input
```
Integrate[(2 + 5*x^2 + 5*x^4)^(-3/2),x]
```

output
```
(-20*Sqrt[(-I)/(-5*I + Sqrt[15])]*x*(1 + 5*x^2) - 5*Sqrt[2]*(Sqrt[3] + I*S
qrt[5])*Sqrt[(-5*I + Sqrt[15] - (10*I)*x^2)/(-5*I + Sqrt[15])]*Sqrt[(5*I +
 Sqrt[15] + (10*I)*x^2)/(5*I + Sqrt[15])]*EllipticE[I*ArcSinh[Sqrt[(-10*I)
/(-5*I + Sqrt[15])]*x], (5*I - Sqrt[15])/(5*I + Sqrt[15])] + Sqrt[2]*(5*Sq
rt[3] - (3*I)*Sqrt[5])*Sqrt[(-5*I + Sqrt[15] - (10*I)*x^2)/(-5*I + Sqrt[15
])]*Sqrt[(5*I + Sqrt[15] + (10*I)*x^2)/(5*I + Sqrt[15])]*EllipticF[I*ArcSi
nh[Sqrt[(-10*I)/(-5*I + Sqrt[15])]*x], (5*I - Sqrt[15])/(5*I + Sqrt[15])])
/(120*Sqrt[(-I)/(-5*I + Sqrt[15])]*Sqrt[2 + 5*x^2 + 5*x^4])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^4+5x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1405$$

$$\frac{1}{30}\int \frac{5(5x^2+4)}{\sqrt{5x^4+5x^2+2}}dx - \frac{x(5x^2+1)}{6\sqrt{5x^4+5x^2+2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\int \frac{5x^2+4}{\sqrt{5x^4+5x^2+2}}dx-\frac{x(5x^2+1)}{6\sqrt{5x^4+5x^2+2}}$$

$$\Big\downarrow \; 1511$$

$$\frac{1}{6}\left(\left(4+\sqrt{10}\right)\int \frac{1}{\sqrt{5x^4+5x^2+2}}dx-\sqrt{10}\int \frac{2-\sqrt{10}x^2}{2\sqrt{5x^4+5x^2+2}}dx\right)-\frac{x(5x^2+1)}{6\sqrt{5x^4+5x^2+2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\left(\left(4+\sqrt{10}\right)\int \frac{1}{\sqrt{5x^4+5x^2+2}}dx-\sqrt{\frac{5}{2}}\int \frac{2-\sqrt{10}x^2}{\sqrt{5x^4+5x^2+2}}dx\right)-\frac{x(5x^2+1)}{6\sqrt{5x^4+5x^2+2}}$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{6}\left(\frac{(4+\sqrt{10})(\sqrt{10}x^2+2)\sqrt{\frac{5x^4+5x^2+2}{(\sqrt{10}x^2+2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{5}{2}}x\right),\frac{1}{8}(4-\sqrt{10})\right)}{2\sqrt[4]{10}\sqrt{5x^4+5x^2+2}}-\sqrt{\frac{5}{2}}\int \frac{2-\sqrt{10}x^2}{\sqrt{5x^4+5x^2+}}\right.$$

$$\frac{x(5x^2+1)}{6\sqrt{5x^4+5x^2+2}}$$

$$\Big\downarrow \; 1509$$

$$\frac{1}{6}\left(\frac{(4+\sqrt{10})(\sqrt{10}x^2+2)\sqrt{\frac{5x^4+5x^2+2}{(\sqrt{10}x^2+2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{\frac{5}{2}}x\right),\frac{1}{8}(4-\sqrt{10})\right)}{2\sqrt[4]{10}\sqrt{5x^4+5x^2+2}}-\sqrt{\frac{5}{2}}\left(\frac{2^{3/4}(\sqrt{10}x^2+2)}{}\right.\right.$$

$$\frac{x(5x^2+1)}{6\sqrt{5x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 + 5*x^4)^(-3/2),x]
```

output
```
-1/6*(x*(1 + 5*x^2))/Sqrt[2 + 5*x^2 + 5*x^4] + (-(Sqrt[5/2]*((-2*x*Sqrt[2
+ 5*x^2 + 5*x^4])/(2 + Sqrt[10]*x^2) + (2^(3/4)*(2 + Sqrt[10]*x^2)*Sqrt[(2
 + 5*x^2 + 5*x^4)/(2 + Sqrt[10]*x^2)^2]*EllipticE[2*ArcTan[(5/2)^(1/4)*x],
 (4 - Sqrt[10])/8])/(5^(1/4)*Sqrt[2 + 5*x^2 + 5*x^4]))) + ((4 + Sqrt[10])*
(2 + Sqrt[10]*x^2)*Sqrt[(2 + 5*x^2 + 5*x^4)/(2 + Sqrt[10]*x^2)^2]*Elliptic
F[2*ArcTan[(5/2)^(1/4)*x], (4 - Sqrt[10])/8])/(2*10^(1/4)*Sqrt[2 + 5*x^2 +
 5*x^4]))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec), antiderivative size = 237, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{x(5x^2+1)}{6\sqrt{5x^4+5x^2+2}}+\frac{4\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{15}}}{2},\frac{\sqrt{1+i\sqrt{15}}}{2}\right)}{3\sqrt{-5+i\sqrt{15}}\,\sqrt{5x^4+5x^2+2}}-\frac{20\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{15}}{4}\right)x^2}}{}$
default	$-\frac{10\left(\frac{1}{60}x+\frac{1}{12}x^3\right)}{\sqrt{5x^4+5x^2+2}}+\frac{4\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{15}}}{2},\frac{\sqrt{1+i\sqrt{15}}}{2}\right)}{3\sqrt{-5+i\sqrt{15}}\,\sqrt{5x^4+5x^2+2}}-\frac{20\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{15}}{4}\right)x^2}}{}$
elliptic	$-\frac{10\left(\frac{1}{60}x+\frac{1}{12}x^3\right)}{\sqrt{5x^4+5x^2+2}}+\frac{4\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{15}}}{2},\frac{\sqrt{1+i\sqrt{15}}}{2}\right)}{3\sqrt{-5+i\sqrt{15}}\,\sqrt{5x^4+5x^2+2}}-\frac{20\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{15}}{4}\right)x^2}}{}$

input
```
int(1/(5*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x*(5*x^2+1)/(5*x^4+5*x^2+2)^(1/2)+4/3/(-5+I*15^(1/2))^(1/2)*(1-(-5/4+
1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*I*15^(1/2))*x^2)^(1/2)/(5*x^4+5*x^
2+2)^(1/2)*EllipticF(1/2*x*(-5+I*15^(1/2))^(1/2),1/2*(1+I*15^(1/2))^(1/2))
-20/3/(-5+I*15^(1/2))^(1/2)*(1-(-5/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-5/4-1
/4*I*15^(1/2))*x^2)^(1/2)/(5*x^4+5*x^2+2)^(1/2)/(5+I*15^(1/2))*(EllipticF(
1/2*x*(-5+I*15^(1/2))^(1/2),1/2*(1+I*15^(1/2))^(1/2))-EllipticE(1/2*x*(-5+
I*15^(1/2))^(1/2),1/2*(1+I*15^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.61

$$\int \frac{1}{(2+5x^2+5x^4)^{3/2}}\,dx = \frac{5\sqrt{2}\left(25\,x^4+25\,x^2-\sqrt{-15}(5\,x^4+5\,x^2+2)+10\right)\sqrt{\sqrt{-15}-5}E\left(\arcsin\left(\frac{1}{2}\,x\right.\right.}{}$$

input
```
integrate(1/(5*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/240*(5*sqrt(2)*(25*x^4 + 25*x^2 - sqrt(-15)*(5*x^4 + 5*x^2 + 2) + 10)*sq
rt(sqrt(-15) - 5)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(-15) - 5)), 1/4*sqrt(-
15) + 1/4) - sqrt(2)*(225*x^4 + 225*x^2 - sqrt(-15)*(5*x^4 + 5*x^2 + 2) +
90)*sqrt(sqrt(-15) - 5)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(-15) - 5)), 1/4*
sqrt(-15) + 1/4) - 40*sqrt(5*x^4 + 5*x^2 + 2)*(5*x^3 + x))/(5*x^4 + 5*x^2
+ 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2+5x^4)^{3/2}}\,dx = \int \frac{1}{(5x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(5*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((5*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2+5x^4)^{3/2}}\,dx = \int \frac{1}{(5\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(5*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2+5x^4)^{3/2}}\,dx = \int \frac{1}{(5\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(5*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2+5x^4)^{3/2}}\,dx = \int \frac{1}{(5\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 + 5*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 + 5*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2+5x^4)^{3/2}}\,dx = \int \frac{\sqrt{5x^4+5x^2+2}}{25x^8+50x^6+45x^4+20x^2+4}dx$$

input
```
int(1/(5*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt(5*x**4 + 5*x**2 + 2)/(25*x**8 + 50*x**6 + 45*x**4 + 20*x**2 + 4),
x)
```

3.298 $\int \frac{1}{(2+5x^2+4x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 257

$$\int \frac{1}{(2+5x^2+4x^4)^{3/2}}\,dx = -\frac{x(9+20x^2)}{14\sqrt{2+5x^2+4x^4}} + \frac{5x\sqrt{2+5x^2+4x^4}}{7\sqrt{2}\left(1+\sqrt{2}x^2\right)} - \frac{5\left(1+\sqrt{2}x^2\right)\sqrt{\frac{2+5x^2+4x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{7\sqrt[4]{2}\sqrt{2+5x^2+4x^4}} + \frac{\left(5+4\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{2+5x^2+4x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{14\sqrt[4]{2}\sqrt{2+5x^2+4x^4}}$$

output
```
-1/14*x*(20*x^2+9)/(4*x^4+5*x^2+2)^(1/2)+5/14*x*(4*x^4+5*x^2+2)^(1/2)*2^(1
/2)/(1+x^2*2^(1/2))-5/14*(1+x^2*2^(1/2))*((4*x^4+5*x^2+2)/(1+x^2*2^(1/2))^
2)^(1/2)*EllipticE(sin(2*arctan(2^(1/4)*x)),1/4*(8-5*2^(1/2))^(1/2))*2^(3/
4)/(4*x^4+5*x^2+2)^(1/2)+1/28*(5+4*2^(1/2))*(1+x^2*2^(1/2))*((4*x^4+5*x^2+
2)/(1+x^2*2^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(2^(1/4)*x),1/4*(8-5*2
^(1/2))^(1/2))*2^(3/4)/(4*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.46 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.29

$$\int \frac{1}{(2+5x^2+4x^4)^{3/2}}\,dx = \frac{-8\sqrt{-\frac{i}{-5i+\sqrt{7}}}x(9+20x^2)-5\sqrt{2}(5i+\sqrt{7})\sqrt{\frac{-5i+\sqrt{7}-8ix^2}{-5i+\sqrt{7}}}\sqrt{\frac{5i+\sqrt{7}+8ix^2}{5i+\sqrt{7}}}E\left(i\text{arcs}\right.}{}$$

input
```
Integrate[(2 + 5*x^2 + 4*x^4)^(-3/2),x]
```

output
```
(-8*Sqrt[(-I)/(-5*I + Sqrt[7])]*x*(9 + 20*x^2) - 5*Sqrt[2]*(5*I + Sqrt[7])
*Sqrt[(-5*I + Sqrt[7] - (8*I)*x^2)/(-5*I + Sqrt[7])]*Sqrt[(5*I + Sqrt[7] +
 (8*I)*x^2)/(5*I + Sqrt[7])]*EllipticE[I*ArcSinh[2*Sqrt[(-2*I)/(-5*I + Sqr
t[7])]*x], (5*I - Sqrt[7])/(5*I + Sqrt[7])] + Sqrt[2]*(-7*I + 5*Sqrt[7])*S
qrt[(-5*I + Sqrt[7] - (8*I)*x^2)/(-5*I + Sqrt[7])]*Sqrt[(5*I + Sqrt[7] + (
8*I)*x^2)/(5*I + Sqrt[7])]*EllipticF[I*ArcSinh[2*Sqrt[(-2*I)/(-5*I + Sqrt[
7])]*x], (5*I - Sqrt[7])/(5*I + Sqrt[7])])/(112*Sqrt[(-I)/(-5*I + Sqrt[7])
]*Sqrt[2 + 5*x^2 + 4*x^4])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1405, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{14}\int \frac{4(5x^2+4)}{\sqrt{4x^4+5x^2+2}}dx-\frac{x(20x^2+9)}{14\sqrt{4x^4+5x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{7}\int \frac{5x^2+4}{\sqrt{4x^4+5x^2+2}}dx-\frac{x(20x^2+9)}{14\sqrt{4x^4+5x^2+2}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{2}{7}\left(\frac{1}{2}\left(8+5\sqrt{2}\right)\int \frac{1}{\sqrt{4x^4+5x^2+2}}dx-\frac{5\int \frac{1-\sqrt{2}x^2}{\sqrt{4x^4+5x^2+2}}dx}{\sqrt{2}}\right)-\frac{x(20x^2+9)}{14\sqrt{4x^4+5x^2+2}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{2}{7}\left(\frac{\left(8+5\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{4x^4+5x^2+2}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{4\ 2^{3/4}\sqrt{4x^4+5x^2+2}}-\frac{5\int \frac{1-\sqrt{2}x^2}{\sqrt{4x^4+5x^2+2}}dx}{\sqrt{2}}\right)-\frac{x(20x^2+9)}{14\sqrt{4x^4+5x^2+2}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{2}{7}\left(\frac{\left(8+5\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{4x^4+5x^2+2}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{4\ 2^{3/4}\sqrt{4x^4+5x^2+2}}-\frac{5\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{4x^4+5x^2+2}{\left(\sqrt{2}x^2+1\right)^2}}E\left(\cdots\right.}{2^{3/4}\sqrt{4\cdots}}\right.}{\cdots}\right.$$

$$\frac{x(20x^2+9)}{14\sqrt{4x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 + 4*x^4)^(-3/2),x]
```

output
```
-1/14*(x*(9 + 20*x^2))/Sqrt[2 + 5*x^2 + 4*x^4] + (2*((-5*(-1/2*(x*Sqrt[2 +
 5*x^2 + 4*x^4])/(1 + Sqrt[2]*x^2) + ((1 + Sqrt[2]*x^2)*Sqrt[(2 + 5*x^2 +
4*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (8 - 5*Sqrt[2])
/16])/(2^(3/4)*Sqrt[2 + 5*x^2 + 4*x^4])))/Sqrt[2] + ((8 + 5*Sqrt[2])*(1 +
Sqrt[2]*x^2)*Sqrt[(2 + 5*x^2 + 4*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*Arc
Tan[2^(1/4)*x], (8 - 5*Sqrt[2])/16])/(4*2^(3/4)*Sqrt[2 + 5*x^2 + 4*x^4])))
/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{x(20x^2+9)}{14\sqrt{4x^4+5x^2+2}}+\frac{16\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{7}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{7}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{7}}}{2},\frac{\sqrt{9+5i\sqrt{7}}}{4}\right)}{7\sqrt{-5+i\sqrt{7}}\,\sqrt{4x^4+5x^2+2}}-\frac{80\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{7}}{4}\right)x^2}\,\sqrt{}$
default	$-\frac{8\left(\frac{9}{112}x+\frac{5}{28}x^3\right)}{\sqrt{4x^4+5x^2+2}}+\frac{16\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{7}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{7}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{7}}}{2},\frac{\sqrt{9+5i\sqrt{7}}}{4}\right)}{7\sqrt{-5+i\sqrt{7}}\,\sqrt{4x^4+5x^2+2}}-\frac{80\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{7}}{4}\right)x^2}\,\sqrt{}$
elliptic	$-\frac{8\left(\frac{9}{112}x+\frac{5}{28}x^3\right)}{\sqrt{4x^4+5x^2+2}}+\frac{16\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{7}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{7}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{7}}}{2},\frac{\sqrt{9+5i\sqrt{7}}}{4}\right)}{7\sqrt{-5+i\sqrt{7}}\,\sqrt{4x^4+5x^2+2}}-\frac{80\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{7}}{4}\right)x^2}\,\sqrt{}$

input
```
int(1/(4*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/14*x*(20*x^2+9)/(4*x^4+5*x^2+2)^(1/2)+16/7/(-5+I*7^(1/2))^(1/2)*(1-(-5/
4+1/4*I*7^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*I*7^(1/2))*x^2)^(1/2)/(4*x^4+5*x^
2+2)^(1/2)*EllipticF(1/2*x*(-5+I*7^(1/2))^(1/2),1/4*(9+5*I*7^(1/2))^(1/2))
-80/7/(-5+I*7^(1/2))^(1/2)*(1-(-5/4+1/4*I*7^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4
*I*7^(1/2))*x^2)^(1/2)/(4*x^4+5*x^2+2)^(1/2)/(I*7^(1/2)+5)*(EllipticF(1/2*
x*(-5+I*7^(1/2))^(1/2),1/4*(9+5*I*7^(1/2))^(1/2))-EllipticE(1/2*x*(-5+I*7^
(1/2))^(1/2),1/4*(9+5*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.63

$$\int \frac{1}{(2+5x^2+4x^4)^{3/2}}\,dx = \frac{5\sqrt{2}\left(20\,x^4+25\,x^2-\sqrt{-7}(4\,x^4+5\,x^2+2)+10\right)\sqrt{\sqrt{-7}-5}\,E\left(\arcsin\left(\frac{1}{2}\,x\sqrt{\right.\right.}}{}$$

input
```
integrate(1/(4*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/112*(5*sqrt(2)*(20*x^4 + 25*x^2 - sqrt(-7)*(4*x^4 + 5*x^2 + 2) + 10)*sqr
t(sqrt(-7) - 5)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(-7) - 5)), 5/16*sqrt(-7)
 + 9/16) - sqrt(2)*(180*x^4 + 225*x^2 - sqrt(-7)*(4*x^4 + 5*x^2 + 2) + 90)
*sqrt(sqrt(-7) - 5)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(-7) - 5)), 5/16*sqrt
(-7) + 9/16) - 8*sqrt(4*x^4 + 5*x^2 + 2)*(20*x^3 + 9*x))/(4*x^4 + 5*x^2 +
2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2+4x^4)^{3/2}}\,dx=\int \frac{1}{(4x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((4*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2+4x^4)^{3/2}}\,dx=\int \frac{1}{(4\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((4*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((4*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 + 4*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 + 4*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2+4x^4)^{3/2}}\,dx = \int \frac{\sqrt{4x^4+5x^2+2}}{16x^8+40x^6+41x^4+20x^2+4}dx$$

input
```
int(1/(4*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt(4*x**4 + 5*x**2 + 2)/(16*x**8 + 40*x**6 + 41*x**4 + 20*x**2 + 4),
x)
```

3.299 $\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 120

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{3x}{2\sqrt{2+5x^2+3x^4}} + \frac{5\sqrt{2+5x^2+3x^4}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{2+3x^2}} - \frac{3\sqrt{2}\sqrt{1+x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2+5x^2+3x^4}}$$

output
```
3/2*x/(3*x^4+5*x^2+2)^(1/2)+5/2*(3*x^4+5*x^2+2)^(1/2)*EllipticE(x/(x^2+1)^
(1/2),1/2*I*2^(1/2))*2^(1/2)/(x^2+1)^(1/2)/(3*x^2+2)^(1/2)-3*(x^2+1)^(1/2)
*(3*x^2+2)^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5
*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{13x+15x^3+5i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\left|\frac{2}{3}\right.\right) - i\sqrt{3}\sqrt{1+x^2}\sqrt{2}}{2\sqrt{2+5x^2+3x^4}}$$

input
```
Integrate[(2 + 5*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(13*x + 15*x^3 + (5*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*A
rcSinh[Sqrt[3/2]*x], 2/3] - I*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Ellipt
icF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(2*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1405, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(15x^2+13)}{2\sqrt{3x^4+5x^2+2}}-\frac{1}{2}\int \frac{3(5x^2+4)}{\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(15x^2+13)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\int \frac{5x^2+4}{\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{1503}$$

$$\frac{x(15x^2+13)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\left(4\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+5\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)$$

$$\Big\downarrow \text{1413}$$

$$\frac{x(15x^2+13)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\left(5\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{2\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right)$$

$$\Big\downarrow \text{1456}$$

$$\frac{x(15x^2+13)}{2\sqrt{3x^4+5x^2+2}} - \frac{3}{2}\left(\frac{2\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}} + 5\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}} - \frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E(\arctan(x)}{3\sqrt{3x^4+5x^2+2}}\right.\right.$$

input
```
Int[(2 + 5*x^2 + 3*x^4)^(-3/2),x]
```

output
```
(x*(13 + 15*x^2))/(2*Sqrt[2 + 5*x^2 + 3*x^4]) - (3*(5*((x*(2 + 3*x^2))/(3*
Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*
EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (2*Sqrt[2]*(1 +
 x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x
^2 + 3*x^4]))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04

method	result	si
risch	$\frac{x\left(15x^{2}+13\right)}{2\sqrt{3x^{4}+5x^{2}+2}}+\frac{3i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^{4}+5x^{2}+2}}-\frac{5i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{3x^{4}+5x^{2}+2}}$	12
default	$-\frac{6\left(-\frac{5}{4}x^{3}-\frac{13}{12}x\right)}{\sqrt{3x^{4}+5x^{2}+2}}+\frac{3i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^{4}+5x^{2}+2}}-\frac{5i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{3x^{4}+5x^{2}+2}}$	12
elliptic	$-\frac{6\left(-\frac{5}{4}x^{3}-\frac{13}{12}x\right)}{\sqrt{3x^{4}+5x^{2}+2}}+\frac{3i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^{4}+5x^{2}+2}}-\frac{5i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{3x^{4}+5x^{2}+2}}$	12

input
```
int(1/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(15*x^2+13)/(3*x^4+5*x^2+2)^(1/2)+3*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/
(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))-5/2*I*(x^2+1)^(1/2)*(6*x^
2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x
,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx =$$
$$-\frac{5\sqrt{2}(3i\,x^4+5i\,x^2+2i)E(\arcsin(i\,x)\mid\frac{3}{2})+11\sqrt{2}(-3i\,x^4-5i\,x^2-2i)F(\arcsin(i\,x)\mid\frac{3}{2})-\sqrt{3\,x^4+5\,x}}{2\,(3\,x^4+5\,x^2+2)}$$

input
```
integrate(1/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(5*sqrt(2)*(3*I*x^4 + 5*I*x^2 + 2*I)*elliptic_e(arcsin(I*x), 3/2) + 1
1*sqrt(2)*(-3*I*x^4 - 5*I*x^2 - 2*I)*elliptic_f(arcsin(I*x), 3/2) - sqrt(3
*x^4 + 5*x^2 + 2)*(15*x^3 + 13*x))/(3*x^4 + 5*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((3*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+5x^2+2}}{9x^8+30x^6+37x^4+20x^2+4}dx$$

input
```
int(1/(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x
)
```

3.300 $\int \frac{1}{(2+5x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 122

$$\int \frac{1}{(2+5x^2+2x^4)^{3/2}}\,dx = \frac{2x}{3\sqrt{2+5x^2+2x^4}} + \frac{5\sqrt{2+5x^2+2x^4}E\left(\arctan\left(\frac{x}{\sqrt{2}}\right)\middle|-3\right)}{18\sqrt{2+x^2}\sqrt{1+2x^2}} - \frac{4\sqrt{2+x^2}\sqrt{1+2x^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{2}}\right),-3\right)}{9\sqrt{2+5x^2+2x^4}}$$

output
```
2/3*x/(2*x^4+5*x^2+2)^(1/2)+5/18*(2*x^4+5*x^2+2)^(1/2)*EllipticE(x*2^(1/2)
/(2*x^2+4)^(1/2),I*3^(1/2))/(x^2+2)^(1/2)/(2*x^2+1)^(1/2)-4/9*(x^2+2)^(1/2
)*(2*x^2+1)^(1/2)*InverseJacobiAM(arctan(1/2*x*2^(1/2)),I*3^(1/2))/(2*x^4+
5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.67 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

$$\int \frac{1}{(2+5x^2+2x^4)^{3/2}}\,dx = \frac{17x+10x^3+10i\sqrt{2+x^2}\sqrt{1+2x^2}E\left(i\text{arcsinh}\left(\sqrt{2}x\right)|\frac{1}{4}\right)-6i\sqrt{2+x^2}\sqrt{1+2x}}{18\sqrt{2+5x^2+2x^4}}$$

input
```
Integrate[(2 + 5*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(17*x + 10*x^3 + (10*I)*Sqrt[2 + x^2]*Sqrt[1 + 2*x^2]*EllipticE[I*ArcSinh[
Sqrt[2]*x], 1/4] - (6*I)*Sqrt[2 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh
[Sqrt[2]*x], 1/4])/(18*Sqrt[2 + 5*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1405, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(10x^2+17)}{18\sqrt{2x^4+5x^2+2}}-\frac{1}{18}\int \frac{2(5x^2+4)}{\sqrt{2x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(10x^2+17)}{18\sqrt{2x^4+5x^2+2}}-\frac{1}{9}\int \frac{5x^2+4}{\sqrt{2x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{9}\left(-4\int \frac{1}{\sqrt{2x^4+5x^2+2}}dx-5\int \frac{x^2}{\sqrt{2x^4+5x^2+2}}dx\right)+\frac{x(10x^2+17)}{18\sqrt{2x^4+5x^2+2}}$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{9}\left(-5\int \frac{x^2}{\sqrt{2x^4+5x^2+2}}dx-\frac{2\sqrt{\frac{x^2+2}{2x^2+1}}\left(2x^2+1\right)\text{EllipticF}\left(\arctan\left(\sqrt{2}x\right),\frac{3}{4}\right)}{\sqrt{2x^4+5x^2+2}}\right)+\frac{x\left(10x^2+17\right)}{18\sqrt{2x^4+5x^2+2}}$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{9}\left(-\frac{2\sqrt{\frac{x^2+2}{2x^2+1}}\left(2x^2+1\right)\text{EllipticF}\left(\arctan\left(\sqrt{2}x\right),\frac{3}{4}\right)}{\sqrt{2x^4+5x^2+2}}-5\left(\frac{x\left(x^2+2\right)}{\sqrt{2x^4+5x^2+2}}-\frac{\sqrt{\frac{x^2+2}{2x^2+1}}\left(2x^2+1\right)E\left(\arctan\left(\sqrt{2}x\right)\right.}{\sqrt{2x^4+5x^2+2}}\right.\right.$$

$$\frac{x\left(10x^2+17\right)}{18\sqrt{2x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 + 2*x^4)^(-3/2),x]
```

output
```
(x*(17 + 10*x^2))/(18*Sqrt[2 + 5*x^2 + 2*x^4]) + (-5*((x*(2 + x^2))/Sqrt[2
 + 5*x^2 + 2*x^4] - (Sqrt[(2 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticE[Arc
Tan[Sqrt[2]*x], 3/4])/Sqrt[2 + 5*x^2 + 2*x^4]) - (2*Sqrt[(2 + x^2)/(1 + 2*
x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]*x], 3/4])/Sqrt[2 + 5*x^2 + 2*x^
4])/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08

method	result
risch	$\frac{x(10x^2+17)}{18\sqrt{2x^4+5x^2+2}}+\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{2x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{2}}{2},2\right)}{9\sqrt{2x^4+5x^2+2}}-\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{2x^2+1}\ \left(\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},2\right)-\text{EllipticE}\left(\frac{ix\sqrt{2}}{2}\right.\right.}{36\sqrt{2x^4+5x^2+2}}$
default	$-\frac{4\left(-\frac{17}{72}x-\frac{5}{36}x^3\right)}{\sqrt{2x^4+5x^2+2}}+\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{2x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{2}}{2},2\right)}{9\sqrt{2x^4+5x^2+2}}-\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{2x^2+1}\ \left(\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},2\right)-\text{EllipticE}\left(\frac{ix}{2}\right.\right.}{36\sqrt{2x^4+5x^2+2}}$
elliptic	$-\frac{4\left(-\frac{17}{72}x-\frac{5}{36}x^3\right)}{\sqrt{2x^4+5x^2+2}}+\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{2x^2+1}\ \text{EllipticF}\left(\frac{ix\sqrt{2}}{2},2\right)}{9\sqrt{2x^4+5x^2+2}}-\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{2x^2+1}\ \left(\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},2\right)-\text{EllipticE}\left(\frac{ix}{2}\right.\right.}{36\sqrt{2x^4+5x^2+2}}$

input
```
int(1/(2*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*x*(10*x^2+17)/(2*x^4+5*x^2+2)^(1/2)+2/9*I*2^(1/2)*(2*x^2+4)^(1/2)*(2*
x^2+1)^(1/2)/(2*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2)-5/36*I*2^(
1/2)*(2*x^2+4)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+5*x^2+2)^(1/2)*(EllipticF(1/2*
I*x*2^(1/2),2)-EllipticE(1/2*I*x*2^(1/2),2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \frac{1}{(2+5x^2+2x^4)^{3/2}}\,dx =$$
$$-\frac{5\sqrt{2}\sqrt{-\frac{1}{2}}(2\,x^4+5\,x^2+2)E(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)\,|\,4)-21\sqrt{2}\sqrt{-\frac{1}{2}}(2\,x^4+5\,x^2+2)F(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)\,|\,4)}{36\,(2\,x^4+5\,x^2+2)}$$

input
```
integrate(1/(2*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/36*(5*sqrt(2)*sqrt(-1/2)*(2*x^4 + 5*x^2 + 2)*elliptic_e(arcsin(sqrt(-1/
2)*x), 4) - 21*sqrt(2)*sqrt(-1/2)*(2*x^4 + 5*x^2 + 2)*elliptic_f(arcsin(sq
rt(-1/2)*x), 4) - 2*sqrt(2*x^4 + 5*x^2 + 2)*(10*x^3 + 17*x))/(2*x^4 + 5*x^
2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((2*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((2*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 + 2*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 + 2*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2+2x^4)^{3/2}}\,dx = \int \frac{\sqrt{2x^4+5x^2+2}}{4x^8+20x^6+33x^4+20x^2+4}dx$$

input
```
int(1/(2*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt(2*x**4 + 5*x**2 + 2)/(4*x**8 + 20*x**6 + 33*x**4 + 20*x**2 + 4),x
)
```

3.301 $\int \frac{1}{(2+5x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 234

$$\begin{aligned}\int \frac{1}{(2+5x^2+x^4)^{3/2}}\,dx &= \frac{2x}{\sqrt{17}\left(5-\sqrt{17}\right)\sqrt{2+5x^2+x^4}}\\ &+ \frac{5\sqrt{\frac{2}{5+\sqrt{17}}}\sqrt{2+5x^2+x^4}\,E\left(\arctan\left(\sqrt{\frac{2}{5+\sqrt{17}}}x\right)|\frac{1}{4}\left(-17-5\sqrt{17}\right)\right)}{17\sqrt{\frac{4}{5+\sqrt{17}}+x^2}\sqrt{5+\sqrt{17}+2x^2}}\\ &- \frac{\sqrt{4+\left(5-\sqrt{17}\right)x^2}\sqrt{4+\left(5+\sqrt{17}\right)x^2}\,\text{EllipticF}\left(\arctan\left(\frac{1}{2}\sqrt{5-\sqrt{17}}x\right),\frac{1}{4}\left(-17-5\sqrt{17}\right)\right)}{17\sqrt{5-\sqrt{17}}\sqrt{2+5x^2+x^4}}\end{aligned}$$

output

```
2/17*x*17^(1/2)/(5-17^(1/2))/(x^4+5*x^2+2)^(1/2)+5/17*2^(1/2)/(5+17^(1/2))
^(1/2)*(x^4+5*x^2+2)^(1/2)*EllipticE(2^(1/2)/(5+17^(1/2))^(1/2)*x/(1+2/(5+
17^(1/2))*x^2)^(1/2),1/2*(-17-5*17^(1/2))^(1/2))/(4/(5+17^(1/2))+x^2)^(1/2
)/(5+17^(1/2)+2*x^2)^(1/2)-1/17*(4+(5-17^(1/2))*x^2)^(1/2)*(4+(5+17^(1/2))
*x^2)^(1/2)*InverseJacobiAM(arctan(1/2*(5-17^(1/2))^(1/2)*x),1/2*(-17-5*17
^(1/2))^(1/2))/(5-17^(1/2))^(1/2)/(x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.53 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.94

$$\int \frac{1}{(2+5x^2+x^4)^{3/2}}\,dx = \frac{4x(21+5x^2)-5i\sqrt{2}(-5+\sqrt{17})\sqrt{\frac{-5+\sqrt{17}-2x^2}{-5+\sqrt{17}}}\sqrt{5+\sqrt{17}+2x^2}\,E\Big(i\text{arcsinh}\Big(\sqrt{}$$

input
```
Integrate[(2 + 5*x^2 + x^4)^(-3/2),x]
```

output
```
(4*x*(21 + 5*x^2) - (5*I)*Sqrt[2]*(-5 + Sqrt[17])*Sqrt[(-5 + Sqrt[17] - 2*
x^2)/(-5 + Sqrt[17])]*Sqrt[5 + Sqrt[17] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[
2/(5 + Sqrt[17])]*x], 21/4 + (5*Sqrt[17])/4] + I*Sqrt[2]*(-17 + 5*Sqrt[17]
)*Sqrt[(-5 + Sqrt[17] - 2*x^2)/(-5 + Sqrt[17])]*Sqrt[5 + Sqrt[17] + 2*x^2]
*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[17])]*x], 21/4 + (5*Sqrt[17])/4])/(1
36*Sqrt[2 + 5*x^2 + x^4])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1405, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^4+5x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1405$$

$$\frac{x(5x^2+21)}{34\sqrt{x^4+5x^2+2}} - \frac{1}{34}\int \frac{5x^2+4}{\sqrt{x^4+5x^2+2}}dx$$

$$\Big\downarrow \; 1503$$

$$\frac{1}{34}\left(-4\int \frac{1}{\sqrt{x^4+5x^2+2}}dx - 5\int \frac{x^2}{\sqrt{x^4+5x^2+2}}dx\right) + \frac{x(5x^2+21)}{34\sqrt{x^4+5x^2+2}}$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{34}\left(-5\int \frac{x^2}{\sqrt{x^4+5x^2+2}}dx-\frac{2\sqrt{\frac{\left(5-\sqrt{17}\right)x^2+4}{\left(5+\sqrt{17}\right)x^2+4}}\left(\left(5+\sqrt{17}\right)x^2+4\right)\text{EllipticF}\left(\arctan\left(\frac{1}{2}\sqrt{5+\sqrt{17}}x\right),\frac{1}{4}\left(-17+\right.\right.}{\sqrt{5+\sqrt{17}}\sqrt{x^4+5x^2+2}}\right.$$

$$\frac{x\left(5x^2+21\right)}{34\sqrt{x^4+5x^2+2}}$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{34}\left(-\frac{2\sqrt{\frac{\left(5-\sqrt{17}\right)x^2+4}{\left(5+\sqrt{17}\right)x^2+4}}\left(\left(5+\sqrt{17}\right)x^2+4\right)\text{EllipticF}\left(\arctan\left(\frac{1}{2}\sqrt{5+\sqrt{17}}x\right),\frac{1}{4}\left(-17+5\sqrt{17}\right)\right)}{\sqrt{5+\sqrt{17}}\sqrt{x^4+5x^2+2}}-5\left(\frac{x\left(2x^2+\right.}{2\sqrt{x^4+}}\right.\right.$$

$$\frac{x\left(5x^2+21\right)}{34\sqrt{x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 + x^4)^(-3/2),x]
```

output
```
(x*(21 + 5*x^2))/(34*Sqrt[2 + 5*x^2 + x^4]) + (-5*((x*(5 + Sqrt[17] + 2*x^
2))/(2*Sqrt[2 + 5*x^2 + x^4]) - (Sqrt[5 + Sqrt[17]]*Sqrt[(4 + (5 - Sqrt[17
])*x^2)/(4 + (5 + Sqrt[17])*x^2)]*(4 + (5 + Sqrt[17])*x^2)*EllipticE[ArcTa
n[(Sqrt[5 + Sqrt[17]]*x)/2], (-17 + 5*Sqrt[17])/4])/(4*Sqrt[2 + 5*x^2 + x^
4])) - (2*Sqrt[(4 + (5 - Sqrt[17])*x^2)/(4 + (5 + Sqrt[17])*x^2)]*(4 + (5
+ Sqrt[17])*x^2)*EllipticF[ArcTan[(Sqrt[5 + Sqrt[17]]*x)/2], (-17 + 5*Sqrt
[17])/4])/(Sqrt[5 + Sqrt[17]]*Sqrt[2 + 5*x^2 + x^4]))/34
```

Defintions of rubi rules used

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.88

method	result
risch	$\frac{x(5x^2+21)}{34\sqrt{x^4+5x^2+2}}-\frac{4\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{17}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{17}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{17}}}{2},\frac{5\sqrt{2}}{4}+\frac{\sqrt{34}}{4}\right)}{17\sqrt{-5+\sqrt{17}}\,\sqrt{x^4+5x^2+2}}+\frac{20\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{17}}{4}\right)x^2}\,\sqrt{1}$
default	$-\frac{2\left(-\frac{21}{68}x-\frac{5}{68}x^3\right)}{\sqrt{x^4+5x^2+2}}-\frac{4\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{17}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{17}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{17}}}{2},\frac{5\sqrt{2}}{4}+\frac{\sqrt{34}}{4}\right)}{17\sqrt{-5+\sqrt{17}}\,\sqrt{x^4+5x^2+2}}+\frac{20\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{17}}{4}\right)x^2}}{}$
elliptic	$-\frac{2\left(-\frac{21}{68}x-\frac{5}{68}x^3\right)}{\sqrt{x^4+5x^2+2}}-\frac{4\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{17}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{17}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{17}}}{2},\frac{5\sqrt{2}}{4}+\frac{\sqrt{34}}{4}\right)}{17\sqrt{-5+\sqrt{17}}\,\sqrt{x^4+5x^2+2}}+\frac{20\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{17}}{4}\right)x^2}}{}$

input
```
int(1/(x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/34*x*(5*x^2+21)/(x^4+5*x^2+2)^(1/2)-4/17/(-5+17^(1/2))^(1/2)*(1-(-5/4+1/
4*17^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*17^(1/2))*x^2)^(1/2)/(x^4+5*x^2+2)^(1/
2)*EllipticF(1/2*x*(-5+17^(1/2))^(1/2),5/4*2^(1/2)+1/4*34^(1/2))+20/17/(-5
+17^(1/2))^(1/2)*(1-(-5/4+1/4*17^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*17^(1/2))*
x^2)^(1/2)/(x^4+5*x^2+2)^(1/2)/(5+17^(1/2))*(EllipticF(1/2*x*(-5+17^(1/2))
^(1/2),5/4*2^(1/2)+1/4*34^(1/2))-EllipticE(1/2*x*(-5+17^(1/2))^(1/2),5/4*2
^(1/2)+1/4*34^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.68

$$\int \frac{1}{(2+5x^2+x^4)^{3/2}}\,dx = \frac{5\left(\sqrt{17}\sqrt{2}(x^4+5\,x^2+2)-5\,\sqrt{2}(x^4+5\,x^2+2)\right)\sqrt{\sqrt{17}-5}E(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{1}\right.}$$

input
```
integrate(1/(x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/272*(5*(sqrt(17)*sqrt(2)*(x^4 + 5*x^2 + 2) - 5*sqrt(2)*(x^4 + 5*x^2 + 2)
)*sqrt(sqrt(17) - 5)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(17) - 5)), 5/4*sqrt
(17) + 21/4) - (sqrt(17)*sqrt(2)*(x^4 + 5*x^2 + 2) - 45*sqrt(2)*(x^4 + 5*x
^2 + 2))*sqrt(sqrt(17) - 5)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(17) - 5)), 5
/4*sqrt(17) + 21/4) + 8*sqrt(x^4 + 5*x^2 + 2)*(5*x^3 + 21*x))/(x^4 + 5*x^2
 + 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 + x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 + x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2+x^4)^{3/2}}\,dx = \int \frac{\sqrt{x^4+5x^2+2}}{x^8+10x^6+29x^4+20x^2+4}dx$$

input
```
int(1/(x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt(x**4 + 5*x**2 + 2)/(x**8 + 10*x**6 + 29*x**4 + 20*x**2 + 4),x)
```

3.302 $\int \frac{1}{(2+5x^2-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 131

$$\int \frac{1}{(2+5x^2-x^4)^{3/2}}\,dx = \frac{x(29-5x^2)}{66\sqrt{2+5x^2-x^4}} + \frac{5}{66}\sqrt{\frac{1}{2}\left(-5+\sqrt{33}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{5+\sqrt{33}}}x\right)|\frac{1}{4}\left(-29-5\sqrt{33}\right)\right) + \frac{1}{2}\sqrt{\frac{1}{66}\left(-5+\sqrt{33}\right)}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{5+\sqrt{33}}}x\right),\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)$$

output
```
1/66*x*(-5*x^2+29)/(-x^4+5*x^2+2)^(1/2)+5/132*(-10+2*33^(1/2))^(1/2)*Ellip
ticE(2^(1/2)/(5+33^(1/2))^(1/2)*x,5/4*I*2^(1/2)+1/4*I*66^(1/2))+1/132*(-33
0+66*33^(1/2))^(1/2)*EllipticF(2^(1/2)/(5+33^(1/2))^(1/2)*x,5/4*I*2^(1/2)+
1/4*I*66^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.46 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.34

$$\int \frac{1}{(2+5x^2-x^4)^{3/2}}\,dx = \frac{4x(29-5x^2)+10i\sqrt{2\left(5+\sqrt{33}\right)}\sqrt{2+5x^2-x^4}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{-5+\sqrt{33}}}x\right)|-\frac{29}{4}\right.}{264\sqrt{2+5x^2}}$$

input
```
Integrate[(2 + 5*x^2 - x^4)^(-3/2),x]
```

output
```
(4*x*(29 - 5*x^2) + (10*I)*Sqrt[2*(5 + Sqrt[33])]*Sqrt[2 + 5*x^2 - x^4]*El
lipticE[I*ArcSinh[Sqrt[2/(-5 + Sqrt[33])]*x], -29/4 + (5*Sqrt[33])/4] - ((
2*I)*(33 + 5*Sqrt[33])*Sqrt[4 + 10*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2
/(-5 + Sqrt[33])]*x], -29/4 + (5*Sqrt[33])/4])/Sqrt[5 + Sqrt[33]])/(264*Sq
rt[2 + 5*x^2 - x^4])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 25, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x(29-5x^2)}{66\sqrt{-x^4+5x^2+2}}-\frac{1}{66}\int -\frac{5x^2+4}{\sqrt{-x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{66}\int \frac{5x^2+4}{\sqrt{-x^4+5x^2+2}}dx+\frac{x(29-5x^2)}{66\sqrt{-x^4+5x^2+2}}$$

$$\Big\downarrow \; 1494$$

$$\frac{1}{33}\int \frac{5x^2+4}{\sqrt{-2x^2+\sqrt{33}+5}\sqrt{2x^2+\sqrt{33}-5}}dx+\frac{x(29-5x^2)}{66\sqrt{-x^4+5x^2+2}}$$

$$\Big\downarrow \; 399$$

$$\frac{1}{33}\left(\frac{1}{2}\left(33-5\sqrt{33}\right)\int \frac{1}{\sqrt{-2x^2+\sqrt{33}+5}\sqrt{2x^2+\sqrt{33}-5}}dx+\frac{5}{2}\int \frac{\sqrt{2x^2+\sqrt{33}-5}}{\sqrt{-2x^2+\sqrt{33}+5}}dx\right)+\frac{x(29-5x^2)}{66\sqrt{-x^4+5x^2+2}}$$

$$\Big\downarrow \; 321$$

$$\frac{1}{33}\left(\frac{5}{2}\int \frac{\sqrt{2x^2+\sqrt{33}-5}}{\sqrt{-2x^2+\sqrt{33}+5}}dx+\frac{\left(33-5\sqrt{33}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{5+\sqrt{33}}}x\right),\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)}{2\sqrt{2\left(\sqrt{33}-5\right)}}\right)+\frac{x(29-5x^2)}{66\sqrt{-x^4+5x^2+2}}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{33}\left(\frac{\left(33-5\sqrt{33}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{5+\sqrt{33}}}x\right),\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)}{2\sqrt{2\left(\sqrt{33}-5\right)}}+\frac{5}{2}\sqrt{\frac{1}{2}\left(\sqrt{33}-5\right)}E\left(\arcsin\left(\sqrt{\frac{2}{5+\sqrt{33}}}x\right.\right.\right.$$
$$\frac{x(29-5x^2)}{66\sqrt{-x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 - x^4)^(-3/2),x]
```

output
```
(x*(29 - 5*x^2))/(66*Sqrt[2 + 5*x^2 - x^4]) + ((5*Sqrt[(-5 + Sqrt[33])/2]*
EllipticE[ArcSin[Sqrt[2/(5 + Sqrt[33])]*x], (-29 - 5*Sqrt[33])/4])/2 + ((3
3 - 5*Sqrt[33])*EllipticF[ArcSin[Sqrt[2/(5 + Sqrt[33])]*x], (-29 - 5*Sqrt[
33])/4])/(2*Sqrt[2*(-5 + Sqrt[33])]))/33
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(99) = 198.

Time = 1.98 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.66

method	result
risch	$-\frac{x(5x^2-29)}{66\sqrt{-x^4+5x^2+2}}+\frac{4\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{33}}}{2},\frac{5i\sqrt{2}}{4}+\frac{i\sqrt{66}}{4}\right)}{33\sqrt{-5+\sqrt{33}}\,\sqrt{-x^4+5x^2+2}}-\frac{20\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{33}}{4}\right)x}}{}$
default	$\frac{\frac{29}{66}x-\frac{5}{66}x^3}{\sqrt{-x^4+5x^2+2}}+\frac{4\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{33}}}{2},\frac{5i\sqrt{2}}{4}+\frac{i\sqrt{66}}{4}\right)}{33\sqrt{-5+\sqrt{33}}\,\sqrt{-x^4+5x^2+2}}-\frac{20\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{}}{}$
elliptic	$\frac{\frac{29}{66}x-\frac{5}{66}x^3}{\sqrt{-x^4+5x^2+2}}+\frac{4\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{33}}}{2},\frac{5i\sqrt{2}}{4}+\frac{i\sqrt{66}}{4}\right)}{33\sqrt{-5+\sqrt{33}}\,\sqrt{-x^4+5x^2+2}}-\frac{20\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{}}{}$

input
```
int(1/(-x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/66*x*(5*x^2-29)/(-x^4+5*x^2+2)^(1/2)+4/33/(-5+33^(1/2))^(1/2)*(1-(-5/4+
1/4*33^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*33^(1/2))*x^2)^(1/2)/(-x^4+5*x^2+2)^
(1/2)*EllipticF(1/2*x*(-5+33^(1/2))^(1/2),5/4*I*2^(1/2)+1/4*I*66^(1/2))-20
/33/(-5+33^(1/2))^(1/2)*(1-(-5/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*33^
(1/2))*x^2)^(1/2)/(-x^4+5*x^2+2)^(1/2)/(33^(1/2)+5)*(EllipticF(1/2*x*(-5+3
3^(1/2))^(1/2),5/4*I*2^(1/2)+1/4*I*66^(1/2))-EllipticE(1/2*x*(-5+33^(1/2))
^(1/2),5/4*I*2^(1/2)+1/4*I*66^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.24

$$\int \frac{1}{(2+5x^2-x^4)^{3/2}}\,dx=\frac{5\left(\sqrt{33}\sqrt{2}(x^4-5\,x^2-2)-5\,\sqrt{2}(x^4-5\,x^2-2)\right)\sqrt{\sqrt{33}-5}E(\arcsin\left(\frac{1}{2}\,x\sqrt{\sqrt{3}}\right.}{}$$

input
```
integrate(1/(-x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/528*(5*(sqrt(33)*sqrt(2)*(x^4 - 5*x^2 - 2) - 5*sqrt(2)*(x^4 - 5*x^2 - 2)
)*sqrt(sqrt(33) - 5)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(33) - 5)), -5/4*sqr
t(33) - 29/4) - (sqrt(33)*sqrt(2)*(x^4 - 5*x^2 - 2) - 45*sqrt(2)*(x^4 - 5*
x^2 - 2))*sqrt(sqrt(33) - 5)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(33) - 5)),
-5/4*sqrt(33) - 29/4) + 8*sqrt(-x^4 + 5*x^2 + 2)*(5*x^3 - 29*x))/(x^4 - 5*
x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((-x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 - x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2-x^4)^{3/2}}\,dx = \int \frac{\sqrt{-x^4+5x^2+2}}{x^8-10x^6+21x^4+20x^2+4}dx$$

input
```
int(1/(-x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - x**4 + 5*x**2 + 2)/(x**8 - 10*x**6 + 21*x**4 + 20*x**2 + 4),x)
```

3.303 $\int \frac{1}{(2+5x^2-2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{(2+5x^2-2x^4)^{3/2}}\,dx = \frac{x(33-10x^2)}{82\sqrt{2+5x^2-2x^4}} + \frac{5}{82}\sqrt{\frac{1}{2}\left(-5+\sqrt{41}\right)}E\left(\arcsin\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right)|\frac{1}{8}\left(-33-5\sqrt{41}\right)\right) + \frac{1}{2}\sqrt{\frac{1}{82}\left(-5+\sqrt{41}\right)}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right),\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

output
```
1/82*x*(-10*x^2+33)/(-2*x^4+5*x^2+2)^(1/2)+5/164*(-10+2*41^(1/2))^(1/2)*El
lipticE(2*x/(5+41^(1/2))^(1/2),5/4*I+1/4*I*41^(1/2))+1/164*(-410+82*41^(1/
2))^(1/2)*EllipticF(2*x/(5+41^(1/2))^(1/2),5/4*I+1/4*I*41^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.34

$$\int \frac{1}{(2+5x^2-2x^4)^{3/2}}\,dx = \frac{132x-40x^3+10i\sqrt{2\left(5+\sqrt{41}\right)}\sqrt{2+5x^2-2x^4}E\left(i\operatorname{arcsinh}\left(\frac{2x}{\sqrt{-5+\sqrt{41}}}\right)|-\frac{33}{8}\cdots\right.}{328\sqrt{2+5x^2-\cdots}}$$

input
```
Integrate[(2 + 5*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(132*x - 40*x^3 + (10*I)*Sqrt[2*(5 + Sqrt[41])]*Sqrt[2 + 5*x^2 - 2*x^4]*El
lipticE[I*ArcSinh[(2*x)/Sqrt[-5 + Sqrt[41]]], -33/8 + (5*Sqrt[41])/8] - ((
2*I)*(41 + 5*Sqrt[41])*Sqrt[4 + 10*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/
Sqrt[-5 + Sqrt[41]]], -33/8 + (5*Sqrt[41])/8])/Sqrt[5 + Sqrt[41]])/(328*Sq
rt[2 + 5*x^2 - 2*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(33-10x^2)}{82\sqrt{-2x^4+5x^2+2}}-\frac{1}{82}\int -\frac{2(5x^2+4)}{\sqrt{-2x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{41}\int \frac{5x^2+4}{\sqrt{-2x^4+5x^2+2}}dx+\frac{x(33-10x^2)}{82\sqrt{-2x^4+5x^2+2}}$$

$$\Big\downarrow \ 1494$$

$$\frac{2}{41}\sqrt{2}\int \frac{5x^2+4}{\sqrt{-4x^2+\sqrt{41}+5}\sqrt{4x^2+\sqrt{41}-5}}dx+\frac{x(33-10x^2)}{82\sqrt{-2x^4+5x^2+2}}$$

$$\Big\downarrow \ 399$$

$$\frac{2}{41}\sqrt{2}\left(\frac{1}{4}\left(41-5\sqrt{41}\right)\int \frac{1}{\sqrt{-4x^2+\sqrt{41}+5}\sqrt{4x^2+\sqrt{41}-5}}dx+\frac{5}{4}\int \frac{\sqrt{4x^2+\sqrt{41}-5}}{\sqrt{-4x^2+\sqrt{41}+5}}dx\right)+\frac{x(33-10x^2)}{82\sqrt{-2x^4+5x^2+2}}$$

$$\Big\downarrow \ 321$$

$$\frac{2}{41}\sqrt{2}\left(\frac{5}{4}\int \frac{\sqrt{4x^2+\sqrt{41}-5}}{\sqrt{-4x^2+\sqrt{41}+5}}dx+\frac{\left(41-5\sqrt{41}\right)\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right),\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)}{8\sqrt{\sqrt{41}-5}}\right)+\frac{x(33-10x^2)}{82\sqrt{-2x^4+5x^2+2}}$$

$$\Big\downarrow \ 327$$

$$\frac{2}{41}\sqrt{2}\left(\frac{\left(41-5\sqrt{41}\right)\text{EllipticF}\left(\arcsin\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right),\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)}{8\sqrt{\sqrt{41}-5}}+\frac{5}{8}\sqrt{\sqrt{41}-5}E\left(\arcsin\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right)|\frac{1}{8}\right.\right.$$
$$\frac{x(33-10x^2)}{82\sqrt{-2x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 - 2*x^4)^(-3/2),x]
```

output
```
(x*(33 - 10*x^2))/(82*Sqrt[2 + 5*x^2 - 2*x^4]) + (2*Sqrt[2]*((5*Sqrt[-5 +
Sqrt[41]]*EllipticE[ArcSin[(2*x)/Sqrt[5 + Sqrt[41]]], (-33 - 5*Sqrt[41])/8
])/8 + ((41 - 5*Sqrt[41])*EllipticF[ArcSin[(2*x)/Sqrt[5 + Sqrt[41]]], (-33
 - 5*Sqrt[41])/8])/(8*Sqrt[-5 + Sqrt[41]])))/41
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(87) = 174.

Time = 1.73 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.65

method	result
risch	$-\frac{x(10x^2-33)}{82\sqrt{-2x^4+5x^2+2}}+\frac{8\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{41}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{41}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{41}}}{2},\frac{5i}{4}+\frac{i\sqrt{41}}{4}\right)}{41\sqrt{-5+\sqrt{41}}\,\sqrt{-2x^4+5x^2+2}}-\frac{40\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{41}}{4}\right)x^2}}{}$
default	$\frac{\frac{33}{82}x-\frac{5}{41}x^3}{\sqrt{-2x^4+5x^2+2}}+\frac{8\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{41}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{41}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{41}}}{2},\frac{5i}{4}+\frac{i\sqrt{41}}{4}\right)}{41\sqrt{-5+\sqrt{41}}\,\sqrt{-2x^4+5x^2+2}}-\frac{40\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{41}}{4}\right)x^2}\,\sqrt{1-}}{}$
elliptic	$\frac{\frac{33}{82}x-\frac{5}{41}x^3}{\sqrt{-2x^4+5x^2+2}}+\frac{8\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{41}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{41}}{4}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{41}}}{2},\frac{5i}{4}+\frac{i\sqrt{41}}{4}\right)}{41\sqrt{-5+\sqrt{41}}\,\sqrt{-2x^4+5x^2+2}}-\frac{40\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{41}}{4}\right)x^2}\,\sqrt{1-}}{}$

input
```
int(1/(-2*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/82*x*(10*x^2-33)/(-2*x^4+5*x^2+2)^(1/2)+8/41/(-5+41^(1/2))^(1/2)*(1-(-5
/4+1/4*41^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*41^(1/2))*x^2)^(1/2)/(-2*x^4+5*x^
2+2)^(1/2)*EllipticF(1/2*x*(-5+41^(1/2))^(1/2),5/4*I+1/4*I*41^(1/2))-40/41
/(-5+41^(1/2))^(1/2)*(1-(-5/4+1/4*41^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*41^(1/
2))*x^2)^(1/2)/(-2*x^4+5*x^2+2)^(1/2)/(5+41^(1/2))*(EllipticF(1/2*x*(-5+41
^(1/2))^(1/2),5/4*I+1/4*I*41^(1/2))-EllipticE(1/2*x*(-5+41^(1/2))^(1/2),5/
4*I+1/4*I*41^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(83) = 166.

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.38

$$\int \frac{1}{(2+5x^2-2x^4)^{3/2}}\,dx = \frac{5\left(\sqrt{41}\sqrt{2}(2\,x^4-5\,x^2-2)-5\,\sqrt{2}(2\,x^4-5\,x^2-2)\right)\sqrt{\sqrt{41}-5}E\left(\arcsin\left(\frac{1}{2}\,x\sqrt{\right.\right.}{}$$

input
```
integrate(1/(-2*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/656*(5*(sqrt(41)*sqrt(2)*(2*x^4 - 5*x^2 - 2) - 5*sqrt(2)*(2*x^4 - 5*x^2
- 2))*sqrt(sqrt(41) - 5)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(41) - 5)), -5/8
*sqrt(41) - 33/8) - (sqrt(41)*sqrt(2)*(2*x^4 - 5*x^2 - 2) - 45*sqrt(2)*(2*
x^4 - 5*x^2 - 2))*sqrt(sqrt(41) - 5)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(41)
 - 5)), -5/8*sqrt(41) - 33/8) + 8*sqrt(-2*x^4 + 5*x^2 + 2)*(10*x^3 - 33*x)
)/(2*x^4 - 5*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((-2*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-2*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-2*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2-2x^4)^{3/2}}\,dx = \int \frac{1}{(-2\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - 2*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 - 2*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2-2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-2x^4+5x^2+2}}{4x^8-20x^6+17x^4+20x^2+4}dx$$

input
```
int(1/(-2*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 2*x**4 + 5*x**2 + 2)/(4*x**8 - 20*x**6 + 17*x**4 + 20*x**2 + 4
),x)
```

3.304 $\int \frac{1}{(2+5x^2-3x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{(2+5x^2-3x^4)^{3/2}} \, dx = \frac{x(37-15x^2)}{98\sqrt{2+5x^2-3x^4}} + \frac{5}{98}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right) + \frac{1}{14}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-6\right)$$

output
```
1/98*x*(-15*x^2+37)/(-3*x^4+5*x^2+2)^(1/2)+5/98*EllipticE(1/2*x*2^(1/2),I*
6^(1/2))+1/14*EllipticF(1/2*x*2^(1/2),I*6^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.16

$$\int \frac{1}{(2+5x^2-3x^4)^{3/2}} \, dx = \frac{37x-15x^3+5i\sqrt{6}\sqrt{2-x^2}\sqrt{1+3x^2}E\left(i\text{arcsinh}\left(\sqrt{3}x\right)\middle|-\frac{1}{6}\right)-7i\sqrt{6}\sqrt{2-x^2}\sqrt{}}{98\sqrt{2+5x^2-3x^4}}$$

input
```
Integrate[(2 + 5*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(37*x - 15*x^3 + (5*I)*Sqrt[6]*Sqrt[2 - x^2]*Sqrt[1 + 3*x^2]*EllipticE[I*A
rcSinh[Sqrt[3]*x], -1/6] - (7*I)*Sqrt[6]*Sqrt[2 - x^2]*Sqrt[1 + 3*x^2]*Ell
ipticF[I*ArcSinh[Sqrt[3]*x], -1/6])/(98*Sqrt[2 + 5*x^2 - 3*x^4])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}-\frac{1}{98}\int -\frac{3(5x^2+4)}{\sqrt{-3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{98}\int \frac{5x^2+4}{\sqrt{-3x^4+5x^2+2}}dx+\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{3}{49}\sqrt{3}\int \frac{5x^2+4}{2\sqrt{3}\sqrt{2-x^2}\sqrt{3x^2+1}}dx+\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{98}\int \frac{5x^2+4}{\sqrt{2-x^2}\sqrt{3x^2+1}}dx+\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{3}{98}\left(\frac{7}{3}\int \frac{1}{\sqrt{2-x^2}\sqrt{3x^2+1}}dx+\frac{5}{3}\int \frac{\sqrt{3x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{3}{98}\left(\frac{5}{3}\int \frac{\sqrt{3x^2+1}}{\sqrt{2-x^2}}dx+\frac{7}{3}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-6\right)\right)+\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{3}{98}\left(\frac{7}{3}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-6\right)+\frac{5}{3}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)\right)+\frac{x(37-15x^2)}{98\sqrt{-3x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 - 3*x^4)^(-3/2),x]
```

output
```
(x*(37 - 15*x^2))/(98*Sqrt[2 + 5*x^2 - 3*x^4]) + (3*((5*EllipticE[ArcSin[x
/Sqrt[2]], -6])/3 + (7*EllipticF[ArcSin[x/Sqrt[2]], -6])/3))/98
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(55) = 110$.

Time = 2.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.49

method	result
risch	$-\frac{x(15x^2-37)}{98\sqrt{-3x^4+5x^2+2}}+\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)}{49\sqrt{-3x^4+5x^2+2}}-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)-\mathrm{Ellip}\right.}{196\sqrt{-3x^4+5x^2+2}}$
default	$\frac{\frac{37}{98}x-\frac{15}{98}x^3}{\sqrt{-3x^4+5x^2+2}}+\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)}{49\sqrt{-3x^4+5x^2+2}}-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)-\mathrm{EllipticE}\right.}{196\sqrt{-3x^4+5x^2+2}}$
elliptic	$\frac{\frac{37}{98}x-\frac{15}{98}x^3}{\sqrt{-3x^4+5x^2+2}}+\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)}{49\sqrt{-3x^4+5x^2+2}}-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)-\mathrm{EllipticE}\right.}{196\sqrt{-3x^4+5x^2+2}}$

input
```
int(1/(-3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/98*x*(15*x^2-37)/(-3*x^4+5*x^2+2)^(1/2)+3/49*2^(1/2)*(-2*x^2+4)^(1/2)*(
3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*6^(1/2))-5
/196*2^(1/2)*(-2*x^2+4)^(1/2)*(3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/2)*(Elli
pticF(1/2*x*2^(1/2),I*6^(1/2))-EllipticE(1/2*x*2^(1/2),I*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.54

$$\int \frac{1}{(2+5x^2-3x^4)^{3/2}}\,dx = \frac{5\,(3\,x^4-5\,x^2-2)E(\arcsin\left(\frac{1}{2}\,\sqrt{2}x\right)\,|-6)+19\,(3\,x^4-5\,x^2-2)F(\arcsin\left(\frac{1}{2}\,\sqrt{\right.}}{196\,(3\,x^4-5\,x^2-2)}$$

input
```
integrate(1/(-3*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/196*(5*(3*x^4 - 5*x^2 - 2)*elliptic_e(arcsin(1/2*sqrt(2)*x), -6) + 19*(3
*x^4 - 5*x^2 - 2)*elliptic_f(arcsin(1/2*sqrt(2)*x), -6) + 2*sqrt(-3*x^4 +
5*x^2 + 2)*(15*x^3 - 37*x))/(3*x^4 - 5*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((-3*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-3*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2-3x^4)^{3/2}}\,dx = \int \frac{1}{(-3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - 3*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 - 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2-3x^4)^{3/2}}\,dx = \int \frac{\sqrt{-3x^4+5x^2+2}}{9x^8-30x^6+13x^4+20x^2+4}dx$$

input
```
int(1/(-3*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 3*x**4 + 5*x**2 + 2)/(9*x**8 - 30*x**6 + 13*x**4 + 20*x**2 + 4
),x)
```

3.305 $\int \frac{1}{(2+5x^2-4x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{1}{(2+5x^2-4x^4)^{3/2}}\,dx=\frac{x(41-20x^2)}{114\sqrt{2+5x^2-4x^4}}+\frac{5}{114}\sqrt{\frac{1}{2}\left(-5+\sqrt{57}\right)}E\left(\arcsin\left(2\sqrt{\frac{2}{5+\sqrt{57}}}x\right)|\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)+\frac{1}{2}\sqrt{\frac{1}{114}\left(-5+\sqrt{57}\right)}\,\text{EllipticF}\left(\arcsin\left(2\sqrt{\frac{2}{5+\sqrt{57}}}x\right),\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)$$

output
```
1/114*x*(-20*x^2+41)/(-4*x^4+5*x^2+2)^(1/2)+5/228*(-10+2*57^(1/2))^(1/2)*E
llipticE(2*2^(1/2)/(5+57^(1/2))^(1/2)*x,5/8*I*2^(1/2)+1/8*I*114^(1/2))+1/2
28*(-570+114*57^(1/2))^(1/2)*EllipticF(2*2^(1/2)/(5+57^(1/2))^(1/2)*x,5/8*
I*2^(1/2)+1/8*I*114^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.42 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.32

$$\int \frac{1}{(2+5x^2-4x^4)^{3/2}}\,dx = \frac{-8x(-41+20x^2)+20i\sqrt{5+\sqrt{57}}\sqrt{4+10x^2-8x^4}E\left(i\text{arcsinh}\left(2\sqrt{\frac{2}{-5+\sqrt{57}}}x\right)\right.}{912\sqrt{2}}$$

input
```
Integrate[(2 + 5*x^2 - 4*x^4)^(-3/2),x]
```

output
```
(-8*x*(-41 + 20*x^2) + (20*I)*Sqrt[5 + Sqrt[57]]*Sqrt[4 + 10*x^2 - 8*x^4]*
EllipticE[I*ArcSinh[2*Sqrt[2/(-5 + Sqrt[57])]*x], (-41 + 5*Sqrt[57])/16] -
 ((4*I)*(57 + 5*Sqrt[57])*Sqrt[4 + 10*x^2 - 8*x^4]*EllipticF[I*ArcSinh[2*S
qrt[2/(-5 + Sqrt[57])]*x], (-41 + 5*Sqrt[57])/16])/Sqrt[5 + Sqrt[57]])/(91
2*Sqrt[2 + 5*x^2 - 4*x^4])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-4x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(41-20x^2)}{114\sqrt{-4x^4+5x^2+2}}-\frac{1}{114}\int -\frac{4(5x^2+4)}{\sqrt{-4x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{57}\int \frac{5x^2+4}{\sqrt{-4x^4+5x^2+2}}dx+\frac{x(41-20x^2)}{114\sqrt{-4x^4+5x^2+2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{8}{57}\int \frac{5x^2+4}{\sqrt{-8x^2+\sqrt{57}+5}\sqrt{8x^2+\sqrt{57}-5}}dx+\frac{x(41-20x^2)}{114\sqrt{-4x^4+5x^2+2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{8}{57}\left(\frac{1}{8}\left(57-5\sqrt{57}\right)\int \frac{1}{\sqrt{-8x^2+\sqrt{57}+5}\sqrt{8x^2+\sqrt{57}-5}}dx+\frac{5}{8}\int \frac{\sqrt{8x^2+\sqrt{57}-5}}{\sqrt{-8x^2+\sqrt{57}+5}}dx\right)+\frac{x(41-20x^2)}{114\sqrt{-4x^4+5x^2+2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{8}{57}\left(\frac{5}{8}\int \frac{\sqrt{8x^2+\sqrt{57}-5}}{\sqrt{-8x^2+\sqrt{57}+5}}dx+\frac{\left(57-5\sqrt{57}\right)\text{EllipticF}\left(\arcsin\left(2\sqrt{\frac{2}{5+\sqrt{57}}}x\right),\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)}{16\sqrt{2\left(\sqrt{57}-5\right)}}\right)+\frac{x(41-20x^2)}{114\sqrt{-4x^4+5x^2+2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{8}{57}\left(\frac{\left(57-5\sqrt{57}\right)\text{EllipticF}\left(\arcsin\left(2\sqrt{\frac{2}{5+\sqrt{57}}}x\right),\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)}{16\sqrt{2\left(\sqrt{57}-5\right)}}+\frac{5}{16}\sqrt{\frac{1}{2}\left(\sqrt{57}-5\right)}E\left(\arcsin\left(2\sqrt{\frac{2}{5+\sqrt{}}}\right.\right.\right.$$

$$\frac{x(41-20x^2)}{114\sqrt{-4x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 - 4*x^4)^(-3/2),x]
```

output
```
(x*(41 - 20*x^2))/(114*Sqrt[2 + 5*x^2 - 4*x^4]) + (8*((5*Sqrt[(-5 + Sqrt[5
7])/2]*EllipticE[ArcSin[2*Sqrt[2/(5 + Sqrt[57])]*x], (-41 - 5*Sqrt[57])/16
])/16 + ((57 - 5*Sqrt[57])*EllipticF[ArcSin[2*Sqrt[2/(5 + Sqrt[57])]*x], (
-41 - 5*Sqrt[57])/16])/(16*Sqrt[2*(-5 + Sqrt[57])])))/57
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(101) = 202$.

Time = 2.03 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.64

method	result
risch	$-\frac{x(20x^2-41)}{114\sqrt{-4x^4+5x^2+2}}+\frac{16\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{57}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{57}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{57}}}{2},\frac{5i\sqrt{2}}{8}+\frac{i\sqrt{114}}{8}\right)}{57\sqrt{-5+\sqrt{57}}\,\sqrt{-4x^4+5x^2+2}}-\frac{80\sqrt{1-\left(-\frac{5}{4}+\right.}}{}$
default	$\frac{\frac{41}{114}x-\frac{10}{57}x^3}{\sqrt{-4x^4+5x^2+2}}+\frac{16\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{57}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{57}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{57}}}{2},\frac{5i\sqrt{2}}{8}+\frac{i\sqrt{114}}{8}\right)}{57\sqrt{-5+\sqrt{57}}\,\sqrt{-4x^4+5x^2+2}}-\frac{80\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{57}}{4}\right)x^2}}{}$
elliptic	$\frac{\frac{41}{114}x-\frac{10}{57}x^3}{\sqrt{-4x^4+5x^2+2}}+\frac{16\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{57}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{57}}{4}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{57}}}{2},\frac{5i\sqrt{2}}{8}+\frac{i\sqrt{114}}{8}\right)}{57\sqrt{-5+\sqrt{57}}\,\sqrt{-4x^4+5x^2+2}}-\frac{80\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{57}}{4}\right)x^2}}{}$

input
```
int(1/(-4*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/114*x*(20*x^2-41)/(-4*x^4+5*x^2+2)^(1/2)+16/57/(-5+57^(1/2))^(1/2)*(1-(
-5/4+1/4*57^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*57^(1/2))*x^2)^(1/2)/(-4*x^4+5*
x^2+2)^(1/2)*EllipticF(1/2*x*(-5+57^(1/2))^(1/2),5/8*I*2^(1/2)+1/8*I*114^(
1/2))-80/57/(-5+57^(1/2))^(1/2)*(1-(-5/4+1/4*57^(1/2))*x^2)^(1/2)*(1-(-5/4
-1/4*57^(1/2))*x^2)^(1/2)/(-4*x^4+5*x^2+2)^(1/2)/(5+57^(1/2))*(EllipticF(1
/2*x*(-5+57^(1/2))^(1/2),5/8*I*2^(1/2)+1/8*I*114^(1/2))-EllipticE(1/2*x*(-
5+57^(1/2))^(1/2),5/8*I*2^(1/2)+1/8*I*114^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

$$\int \frac{1}{(2+5x^2-4x^4)^{3/2}}\,dx=\frac{5\left(\sqrt{57}\sqrt{2}(4\,x^4-5\,x^2-2)-5\,\sqrt{2}(4\,x^4-5\,x^2-2)\right)\sqrt{\sqrt{57}-5}\,E(\arcsin\left(\frac{1}{2}\,x\sqrt{\right.}}{}$$

input
```
integrate(1/(-4*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/912*(5*(sqrt(57)*sqrt(2)*(4*x^4 - 5*x^2 - 2) - 5*sqrt(2)*(4*x^4 - 5*x^2
- 2))*sqrt(sqrt(57) - 5)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(57) - 5)), -5/1
6*sqrt(57) - 41/16) - (sqrt(57)*sqrt(2)*(4*x^4 - 5*x^2 - 2) - 45*sqrt(2)*(
4*x^4 - 5*x^2 - 2))*sqrt(sqrt(57) - 5)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(5
7) - 5)), -5/16*sqrt(57) - 41/16) + 8*sqrt(-4*x^4 + 5*x^2 + 2)*(20*x^3 - 4
1*x))/(4*x^4 - 5*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2-4x^4)^{3/2}}\,dx = \int \frac{1}{(-4x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-4*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((-4*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2-4x^4)^{3/2}}\,dx = \int \frac{1}{(-4\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-4*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-4*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2-4x^4)^{3/2}}\,dx = \int \frac{1}{(-4\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-4*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-4*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2-4x^4)^{3/2}}\,dx = \int \frac{1}{(-4\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - 4*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 - 4*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2-4x^4)^{3/2}}\,dx = \int \frac{\sqrt{-4x^4+5x^2+2}}{16x^8-40x^6+9x^4+20x^2+4}dx$$

input
```
int(1/(-4*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 4*x**4 + 5*x**2 + 2)/(16*x**8 - 40*x**6 + 9*x**4 + 20*x**2 + 4
),x)
```

3.306 $\int \frac{1}{(2+5x^2-5x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 131

$$\int \frac{1}{(2+5x^2-5x^4)^{3/2}}\,dx = \frac{x(9-5x^2)}{26\sqrt{2+5x^2-5x^4}} + \frac{1}{26}\sqrt{\frac{1}{2}\left(-5+\sqrt{65}\right)}E\left(\arcsin\left(\sqrt{\frac{10}{5+\sqrt{65}}}x\right)|\frac{1}{4}\left(-9-\sqrt{65}\right)\right) + \frac{1}{2}\sqrt{\frac{1}{130}\left(-5+\sqrt{65}\right)}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{10}{5+\sqrt{65}}}x\right),\frac{1}{4}\left(-9-\sqrt{65}\right)\right)$$

output
```
1/26*x*(-5*x^2+9)/(-5*x^4+5*x^2+2)^(1/2)+1/52*(-10+2*65^(1/2))^(1/2)*Ellip
ticE(10^(1/2)/(5+65^(1/2))^(1/2)*x,1/4*I*10^(1/2)+1/4*I*26^(1/2))+1/260*(-
650+130*65^(1/2))^(1/2)*EllipticF(10^(1/2)/(5+65^(1/2))^(1/2)*x,1/4*I*10^(
1/2)+1/4*I*26^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.97 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

$$\int \frac{1}{(2+5x^2-5x^4)^{3/2}}\,dx = \frac{1}{52}\left(\frac{2x(9-5x^2)}{\sqrt{2+5x^2-5x^4}}\right.$$
$$+i\sqrt{2\left(5+\sqrt{65}\right)}E\left(i\text{arcsinh}\left(\frac{1}{2}\sqrt{5+\sqrt{65}}x\right)|\frac{1}{4}\left(-9+\sqrt{65}\right)\right)-i\sqrt{\frac{2}{5+\sqrt{65}}}\left(13+\sqrt{65}\right)\text{EllipticF}\left(i\text{arcs}\right.$$

input
```
Integrate[(2 + 5*x^2 - 5*x^4)^(-3/2),x]
```

output
```
((2*x*(9 - 5*x^2))/Sqrt[2 + 5*x^2 - 5*x^4] + I*Sqrt[2*(5 + Sqrt[65])]*Elli
pticE[I*ArcSinh[(Sqrt[5 + Sqrt[65]]*x)/2], (-9 + Sqrt[65])/4] - I*Sqrt[2/(
5 + Sqrt[65])]*(13 + Sqrt[65])*EllipticF[I*ArcSinh[(Sqrt[5 + Sqrt[65]]*x)/
2], (-9 + Sqrt[65])/4])/52
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-5x^4+5x^2+2)^{3/2}}dx$$
$$\Big\downarrow \; 1405$$
$$\frac{x(9-5x^2)}{26\sqrt{-5x^4+5x^2+2}}-\frac{1}{130}\int -\frac{5(5x^2+4)}{\sqrt{-5x^4+5x^2+2}}dx$$
$$\Big\downarrow \; 27$$
$$\frac{1}{26}\int \frac{5x^2+4}{\sqrt{-5x^4+5x^2+2}}dx+\frac{x(9-5x^2)}{26\sqrt{-5x^4+5x^2+2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{13}\sqrt{5}\int \frac{5x^2+4}{\sqrt{-10x^2+\sqrt{65}+5}\sqrt{10x^2+\sqrt{65}-5}}dx+\frac{x\left(9-5x^2\right)}{26\sqrt{-5x^4+5x^2+2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{13}\sqrt{5}\left(\frac{1}{2}\left(13-\sqrt{65}\right)\int \frac{1}{\sqrt{-10x^2+\sqrt{65}+5}\sqrt{10x^2+\sqrt{65}-5}}dx+\frac{1}{2}\int \frac{\sqrt{10x^2+\sqrt{65}-5}}{\sqrt{-10x^2+\sqrt{65}+5}}dx\right)+\frac{x\left(9-5x^2\right)}{26\sqrt{-5x^4+5x^2+2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{13}\sqrt{5}\left(\frac{1}{2}\int \frac{\sqrt{10x^2+\sqrt{65}-5}}{\sqrt{-10x^2+\sqrt{65}+5}}dx+\frac{\left(13-\sqrt{65}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{10}{5+\sqrt{65}}}x\right),\frac{1}{4}\left(-9-\sqrt{65}\right)\right)}{2\sqrt{10\left(\sqrt{65}-5\right)}}\right)+\frac{x\left(9-5x^2\right)}{26\sqrt{-5x^4+5x^2+2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{13}\sqrt{5}\left(\frac{\left(13-\sqrt{65}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{10}{5+\sqrt{65}}}x\right),\frac{1}{4}\left(-9-\sqrt{65}\right)\right)}{2\sqrt{10\left(\sqrt{65}-5\right)}}+\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{65}-5\right)}E\left(\arcsin\left(\sqrt{\frac{10}{5+\sqrt{65}}}\right.\right.\right.$$
$$\frac{x\left(9-5x^2\right)}{26\sqrt{-5x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 - 5*x^4)^(-3/2),x]
```

output
```
(x*(9 - 5*x^2))/(26*Sqrt[2 + 5*x^2 - 5*x^4]) + (Sqrt[5]*((Sqrt[(-5 + Sqrt[
65])/10]*EllipticE[ArcSin[Sqrt[10/(5 + Sqrt[65])]*x], (-9 - Sqrt[65])/4])/
2 + ((13 - Sqrt[65])*EllipticF[ArcSin[Sqrt[10/(5 + Sqrt[65])]*x], (-9 - Sq
rt[65])/4])/(2*Sqrt[10*(-5 + Sqrt[65])])))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(99) = 198.

Time = 2.36 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.66

method	result
risch	$-\frac{x(5x^2-9)}{26\sqrt{-5x^4+5x^2+2}}+\frac{4\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{65}}{4}\right)x^2}\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{65}}{4}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{65}}}{2},\frac{i\sqrt{10}}{4}+\frac{i\sqrt{26}}{4}\right)}{13\sqrt{-5+\sqrt{65}}\sqrt{-5x^4+5x^2+2}}-\frac{20\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{65}}{4}\right)}}{}$
default	$\frac{\frac{9}{26}x-\frac{5}{26}x^3}{\sqrt{-5x^4+5x^2+2}}+\frac{4\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{65}}{4}\right)x^2}\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{65}}{4}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{65}}}{2},\frac{i\sqrt{10}}{4}+\frac{i\sqrt{26}}{4}\right)}{13\sqrt{-5+\sqrt{65}}\sqrt{-5x^4+5x^2+2}}-\frac{20\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{65}}{4}\right)x^2}}{}$
elliptic	$\frac{\frac{9}{26}x-\frac{5}{26}x^3}{\sqrt{-5x^4+5x^2+2}}+\frac{4\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{65}}{4}\right)x^2}\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{65}}{4}\right)x^2}\ \text{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{65}}}{2},\frac{i\sqrt{10}}{4}+\frac{i\sqrt{26}}{4}\right)}{13\sqrt{-5+\sqrt{65}}\sqrt{-5x^4+5x^2+2}}-\frac{20\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{65}}{4}\right)x^2}}{}$

input
```
int(1/(-5*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/26*x*(5*x^2-9)/(-5*x^4+5*x^2+2)^(1/2)+4/13/(-5+65^(1/2))^(1/2)*(1-(-5/4
+1/4*65^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*65^(1/2))*x^2)^(1/2)/(-5*x^4+5*x^2+
2)^(1/2)*EllipticF(1/2*x*(-5+65^(1/2))^(1/2),1/4*I*10^(1/2)+1/4*I*26^(1/2)
)-20/13/(-5+65^(1/2))^(1/2)*(1-(-5/4+1/4*65^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4
*65^(1/2))*x^2)^(1/2)/(-5*x^4+5*x^2+2)^(1/2)/(5+65^(1/2))*(EllipticF(1/2*x
*(-5+65^(1/2))^(1/2),1/4*I*10^(1/2)+1/4*I*26^(1/2))-EllipticE(1/2*x*(-5+65
^(1/2))^(1/2),1/4*I*10^(1/2)+1/4*I*26^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.31

$$\int \frac{1}{(2+5x^2-5x^4)^{3/2}}\,dx = \frac{5\left(\sqrt{65}\sqrt{2}(5\,x^4-5\,x^2-2)-5\,\sqrt{2}(5\,x^4-5\,x^2-2)\right)\sqrt{\sqrt{65}-5}E(\arcsin\left(\frac{1}{2}\,x\sqrt{\right.}}{}$$

input
```
integrate(1/(-5*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/1040*(5*(sqrt(65)*sqrt(2)*(5*x^4 - 5*x^2 - 2) - 5*sqrt(2)*(5*x^4 - 5*x^2
 - 2))*sqrt(sqrt(65) - 5)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(65) - 5)), -1/
4*sqrt(65) - 9/4) - (sqrt(65)*sqrt(2)*(5*x^4 - 5*x^2 - 2) - 45*sqrt(2)*(5*
x^4 - 5*x^2 - 2))*sqrt(sqrt(65) - 5)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(65)
 - 5)), -1/4*sqrt(65) - 9/4) + 40*sqrt(-5*x^4 + 5*x^2 + 2)*(5*x^3 - 9*x))/
(5*x^4 - 5*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2-5x^4)^{3/2}}\,dx = \int \frac{1}{(-5x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-5*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((-5*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2-5x^4)^{3/2}}\,dx = \int \frac{1}{(-5\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-5*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-5*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2-5x^4)^{3/2}}\,dx = \int \frac{1}{(-5\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-5*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-5*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2-5x^4)^{3/2}}\,dx = \int \frac{1}{(-5\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - 5*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 - 5*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2-5x^4)^{3/2}}\,dx = \int \frac{\sqrt{-5x^4+5x^2+2}}{25x^8-50x^6+5x^4+20x^2+4}dx$$

input
```
int(1/(-5*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 5*x**4 + 5*x**2 + 2)/(25*x**8 - 50*x**6 + 5*x**4 + 20*x**2 + 4
),x)
```

3.307 $\int \frac{1}{(2+5x^2-6x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{1}{(2+5x^2-6x^4)^{3/2}}\,dx = \frac{x(49-30x^2)}{146\sqrt{2+5x^2-6x^4}} + \frac{5}{146}\sqrt{\frac{1}{2}\left(-5+\sqrt{73}\right)}E\left(\arcsin\left(2\sqrt{\frac{3}{5+\sqrt{73}}}x\right)|\frac{1}{24}\left(-49-5\sqrt{73}\right)\right) + \frac{1}{2}\sqrt{\frac{1}{146}\left(-5+\sqrt{73}\right)}\,\text{EllipticF}\left(\arcsin\left(2\sqrt{\frac{3}{5+\sqrt{73}}}x\right),\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)$$

output
```
1/146*x*(-30*x^2+49)/(-6*x^4+5*x^2+2)^(1/2)+5/292*(-10+2*73^(1/2))^(1/2)*E
llipticE(2*3^(1/2)/(5+73^(1/2))^(1/2)*x,5/12*I*3^(1/2)+1/12*I*219^(1/2))+1
/292*(-730+146*73^(1/2))^(1/2)*EllipticF(2*3^(1/2)/(5+73^(1/2))^(1/2)*x,5/
12*I*3^(1/2)+1/12*I*219^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.66 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.23

$$\int \frac{1}{(2+5x^2-6x^4)^{3/2}}\,dx = \frac{1}{292}\left(\frac{98x}{\sqrt{2+5x^2-6x^4}} - \frac{60x^3}{\sqrt{2+5x^2-6x^4}} + 5i\sqrt{2\left(5+\sqrt{73}\right)}E\left(i\text{arcsinh}\left(2\sqrt{\frac{3}{-5+\sqrt{73}}}x\right)|\frac{1}{24}\left(-49+5\sqrt{73}\right)\right) - i\sqrt{\frac{2}{5+\sqrt{73}}}\left(73+5\sqrt{73}\right)\text{Elliptic}$$

input
```
Integrate[(2 + 5*x^2 - 6*x^4)^(-3/2),x]
```

output
```
((98*x)/Sqrt[2 + 5*x^2 - 6*x^4] - (60*x^3)/Sqrt[2 + 5*x^2 - 6*x^4] + (5*I)
*Sqrt[2*(5 + Sqrt[73])]*EllipticE[I*ArcSinh[2*Sqrt[3/(-5 + Sqrt[73])]*x],
(-49 + 5*Sqrt[73])/24] - I*Sqrt[2/(5 + Sqrt[73])]*(73 + 5*Sqrt[73])*Ellipt
icF[I*ArcSinh[2*Sqrt[3/(-5 + Sqrt[73])]*x], (-49 + 5*Sqrt[73])/24])/292
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-6x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(49-30x^2)}{146\sqrt{-6x^4+5x^2+2}} - \frac{1}{146}\int -\frac{6(5x^2+4)}{\sqrt{-6x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{73}\int \frac{5x^2+4}{\sqrt{-6x^4+5x^2+2}}dx + \frac{x(49-30x^2)}{146\sqrt{-6x^4+5x^2+2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{6}{73}\sqrt{6}\int \frac{5x^2+4}{\sqrt{-12x^2+\sqrt{73}+5}\sqrt{12x^2+\sqrt{73}-5}}dx+\frac{x(49-30x^2)}{146\sqrt{-6x^4+5x^2+2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{6}{73}\sqrt{6}\left(\frac{1}{12}\left(73-5\sqrt{73}\right)\int \frac{1}{\sqrt{-12x^2+\sqrt{73}+5}\sqrt{12x^2+\sqrt{73}-5}}dx+\frac{5}{12}\int \frac{\sqrt{12x^2+\sqrt{73}-5}}{\sqrt{-12x^2+\sqrt{73}+5}}dx\right)+\frac{x(49-30x^2)}{146\sqrt{-6x^4+5x^2+2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{6}{73}\sqrt{6}\left(\frac{5}{12}\int \frac{\sqrt{12x^2+\sqrt{73}-5}}{\sqrt{-12x^2+\sqrt{73}+5}}dx+\frac{\left(73-5\sqrt{73}\right)\text{EllipticF}\left(\arcsin\left(2\sqrt{\frac{3}{5+\sqrt{73}}}x\right),\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)}{24\sqrt{3\left(\sqrt{73}-5\right)}}\right)+\frac{x(49-30x^2)}{146\sqrt{-6x^4+5x^2+2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{6}{73}\sqrt{6}\left(\frac{\left(73-5\sqrt{73}\right)\text{EllipticF}\left(\arcsin\left(2\sqrt{\frac{3}{5+\sqrt{73}}}x\right),\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)}{24\sqrt{3\left(\sqrt{73}-5\right)}}+\frac{5}{24}\sqrt{\frac{1}{3}\left(\sqrt{73}-5\right)}E\left(\arcsin\left(2\sqrt{\frac{3}{5+\dots}}\right.\right.\right.$$
$$\frac{x(49-30x^2)}{146\sqrt{-6x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 - 6*x^4)^(-3/2),x]
```

output
```
(x*(49 - 30*x^2))/(146*Sqrt[2 + 5*x^2 - 6*x^4]) + (6*Sqrt[6]*((5*Sqrt[(-5
+ Sqrt[73])/3]*EllipticE[ArcSin[2*Sqrt[3/(5 + Sqrt[73])]*x], (-49 - 5*Sqrt
[73])/24])/24 + ((73 - 5*Sqrt[73])*EllipticF[ArcSin[2*Sqrt[3/(5 + Sqrt[73]
)]*x], (-49 - 5*Sqrt[73])/24])/(24*Sqrt[3*(-5 + Sqrt[73])])))/73
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(101) = 202$.

Time = 2.38 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.64

method	result
risch	$-\frac{x(30x^2-49)}{146\sqrt{-6x^4+5x^2+2}}+\frac{24\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{73}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{73}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{73}}}{2},\frac{5i\sqrt{3}}{12}+\frac{i\sqrt{219}}{12}\right)}{73\sqrt{-5+\sqrt{73}}\,\sqrt{-6x^4+5x^2+2}}-\frac{120\sqrt{1-\left(-\frac{5}{4}+\right.}}{}$
default	$\frac{\frac{49}{146}x-\frac{15}{73}x^3}{\sqrt{-6x^4+5x^2+2}}+\frac{24\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{73}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{73}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{73}}}{2},\frac{5i\sqrt{3}}{12}+\frac{i\sqrt{219}}{12}\right)}{73\sqrt{-5+\sqrt{73}}\,\sqrt{-6x^4+5x^2+2}}-\frac{120\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{73}}{4}\right)x}}{}$
elliptic	$\frac{\frac{49}{146}x-\frac{15}{73}x^3}{\sqrt{-6x^4+5x^2+2}}+\frac{24\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{73}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{73}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{73}}}{2},\frac{5i\sqrt{3}}{12}+\frac{i\sqrt{219}}{12}\right)}{73\sqrt{-5+\sqrt{73}}\,\sqrt{-6x^4+5x^2+2}}-\frac{120\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{73}}{4}\right)x}}{}$

input
```
int(1/(-6*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/146*x*(30*x^2-49)/(-6*x^4+5*x^2+2)^(1/2)+24/73/(-5+73^(1/2))^(1/2)*(1-(
-5/4+1/4*73^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*73^(1/2))*x^2)^(1/2)/(-6*x^4+5*
x^2+2)^(1/2)*EllipticF(1/2*x*(-5+73^(1/2))^(1/2),5/12*I*3^(1/2)+1/12*I*219
^(1/2))-120/73/(-5+73^(1/2))^(1/2)*(1-(-5/4+1/4*73^(1/2))*x^2)^(1/2)*(1-(-
5/4-1/4*73^(1/2))*x^2)^(1/2)/(-6*x^4+5*x^2+2)^(1/2)/(5+73^(1/2))*(Elliptic
F(1/2*x*(-5+73^(1/2))^(1/2),5/12*I*3^(1/2)+1/12*I*219^(1/2))-EllipticE(1/2
*x*(-5+73^(1/2))^(1/2),5/12*I*3^(1/2)+1/12*I*219^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

$$\int \frac{1}{(2+5x^2-6x^4)^{3/2}}\,dx=\frac{5\left(\sqrt{73}\sqrt{2}(6\,x^4-5\,x^2-2)-5\,\sqrt{2}(6\,x^4-5\,x^2-2)\right)\sqrt{\sqrt{73}-5}\,E(\arcsin\left(\frac{1}{2}\,x\sqrt{\right.}}{}$$

input
```
integrate(1/(-6*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/1168*(5*(sqrt(73)*sqrt(2)*(6*x^4 - 5*x^2 - 2) - 5*sqrt(2)*(6*x^4 - 5*x^2
 - 2))*sqrt(sqrt(73) - 5)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(73) - 5)), -5/
24*sqrt(73) - 49/24) - (sqrt(73)*sqrt(2)*(6*x^4 - 5*x^2 - 2) - 45*sqrt(2)*
(6*x^4 - 5*x^2 - 2))*sqrt(sqrt(73) - 5)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(
73) - 5)), -5/24*sqrt(73) - 49/24) + 8*sqrt(-6*x^4 + 5*x^2 + 2)*(30*x^3 -
49*x))/(6*x^4 - 5*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2-6x^4)^{3/2}}\,dx = \int \frac{1}{(-6x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-6*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((-6*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2-6x^4)^{3/2}}\,dx = \int \frac{1}{(-6\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-6*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-6*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2-6x^4)^{3/2}}\,dx = \int \frac{1}{(-6\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-6*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-6*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2-6x^4)^{3/2}}\,dx = \int \frac{1}{(-6\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - 6*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 - 6*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2-6x^4)^{3/2}}\,dx = \int \frac{\sqrt{-6x^4+5x^2+2}}{36x^8-60x^6+x^4+20x^2+4}dx$$

input
```
int(1/(-6*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 6*x**4 + 5*x**2 + 2)/(36*x**8 - 60*x**6 + x**4 + 20*x**2 + 4),
x)
```

3.308 $\int \frac{1}{(2+5x^2-7x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{1}{(2+5x^2-7x^4)^{3/2}}\,dx = \frac{x(53-35x^2)}{162\sqrt{2+5x^2-7x^4}} + \frac{5E\left(\arcsin(x)\,\middle|\,-\frac{7}{2}\right)}{81\sqrt{2}} + \frac{\text{EllipticF}\left(\arcsin(x),-\frac{7}{2}\right)}{9\sqrt{2}}$$

output
```
1/162*x*(-35*x^2+53)/(-7*x^4+5*x^2+2)^(1/2)+5/162*EllipticE(x,1/2*I*14^(1/
2))*2^(1/2)+1/18*EllipticF(x,1/2*I*14^(1/2))*2^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.55 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

$$\int \frac{1}{(2+5x^2-7x^4)^{3/2}}\,dx = \frac{1}{162}\left(\frac{53x}{\sqrt{2+5x^2-7x^4}} - \frac{35x^3}{\sqrt{2+5x^2-7x^4}} + 5i\sqrt{7}E\left(i\text{arcsinh}\left(\sqrt{\frac{7}{2}}x\right)\,\middle|\,-\frac{2}{7}\right) - 9i\sqrt{7}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{7}{2}}x\right),-\frac{2}{7}\right)\right)$$

input
```
Integrate[(2 + 5*x^2 - 7*x^4)^(-3/2),x]
```

output
```
((53*x)/Sqrt[2 + 5*x^2 - 7*x^4] - (35*x^3)/Sqrt[2 + 5*x^2 - 7*x^4] + (5*I)
*Sqrt[7]*EllipticE[I*ArcSinh[Sqrt[7/2]*x], -2/7] - (9*I)*Sqrt[7]*EllipticF
[I*ArcSinh[Sqrt[7/2]*x], -2/7])/162
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-7x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(53-35x^2)}{162\sqrt{-7x^4+5x^2+2}}-\frac{1}{162}\int -\frac{7(5x^2+4)}{\sqrt{-7x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{162}\int \frac{5x^2+4}{\sqrt{-7x^4+5x^2+2}}dx+\frac{x(53-35x^2)}{162\sqrt{-7x^4+5x^2+2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{7}{81}\sqrt{7}\int \frac{5x^2+4}{2\sqrt{7}\sqrt{1-x^2}\sqrt{7x^2+2}}dx+\frac{x(53-35x^2)}{162\sqrt{-7x^4+5x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{162}\int \frac{5x^2+4}{\sqrt{1-x^2}\sqrt{7x^2+2}}dx+\frac{x(53-35x^2)}{162\sqrt{-7x^4+5x^2+2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{7}{162}\left(\frac{18}{7}\int \frac{1}{\sqrt{1-x^2}\sqrt{7x^2+2}}dx+\frac{5}{7}\int \frac{\sqrt{7x^2+2}}{\sqrt{1-x^2}}dx\right)+\frac{x(53-35x^2)}{162\sqrt{-7x^4+5x^2+2}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{7}{162}\left(\frac{5}{7}\int \frac{\sqrt{7x^2+2}}{\sqrt{1-x^2}}dx+\frac{9}{7}\sqrt{2}\,\text{EllipticF}\left(\arcsin(x),-\frac{7}{2}\right)\right)+\frac{x(53-35x^2)}{162\sqrt{-7x^4+5x^2+2}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{7}{162}\left(\frac{9}{7}\sqrt{2}\,\text{EllipticF}\left(\arcsin(x),-\frac{7}{2}\right)+\frac{5}{7}\sqrt{2}E\left(\arcsin(x)\left|-\frac{7}{2}\right.\right)\right)+\frac{x(53-35x^2)}{162\sqrt{-7x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 - 7*x^4)^(-3/2),x]
```

output
```
(x*(53 - 35*x^2))/(162*Sqrt[2 + 5*x^2 - 7*x^4]) + (7*((5*Sqrt[2]*EllipticE
[ArcSin[x], -7/2])/7 + (9*Sqrt[2]*EllipticF[ArcSin[x], -7/2])/7))/162
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(51) = 102$.

Time = 2.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{x(35x^2-53)}{162\sqrt{-7x^4+5x^2+2}}+\frac{7\sqrt{-x^2+1}\,\sqrt{14x^2+4}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{14}}{2}\right)}{81\sqrt{-7x^4+5x^2+2}}-\frac{5\sqrt{-x^2+1}\,\sqrt{14x^2+4}\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{14}}{2}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{14}}{2}\right)\right)}{162\sqrt{-7x^4+5x^2+2}}$
default	$\frac{\frac{53}{162}x-\frac{35}{162}x^3}{\sqrt{-7x^4+5x^2+2}}+\frac{7\sqrt{-x^2+1}\,\sqrt{14x^2+4}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{14}}{2}\right)}{81\sqrt{-7x^4+5x^2+2}}-\frac{5\sqrt{-x^2+1}\,\sqrt{14x^2+4}\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{14}}{2}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{14}}{2}\right)\right)}{162\sqrt{-7x^4+5x^2+2}}$
elliptic	$\frac{\frac{53}{162}x-\frac{35}{162}x^3}{\sqrt{-7x^4+5x^2+2}}+\frac{7\sqrt{-x^2+1}\,\sqrt{14x^2+4}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{14}}{2}\right)}{81\sqrt{-7x^4+5x^2+2}}-\frac{5\sqrt{-x^2+1}\,\sqrt{14x^2+4}\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{14}}{2}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{14}}{2}\right)\right)}{162\sqrt{-7x^4+5x^2+2}}$

input
```
int(1/(-7*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/162*x*(35*x^2-53)/(-7*x^4+5*x^2+2)^(1/2)+7/81*(-x^2+1)^(1/2)*(14*x^2+4)
^(1/2)/(-7*x^4+5*x^2+2)^(1/2)*EllipticF(x,1/2*I*14^(1/2))-5/162*(-x^2+1)^(
1/2)*(14*x^2+4)^(1/2)/(-7*x^4+5*x^2+2)^(1/2)*(EllipticF(x,1/2*I*14^(1/2))-
EllipticE(x,1/2*I*14^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

$$\int \frac{1}{(2+5x^2-7x^4)^{3/2}}\,dx = \frac{5\sqrt{2}(7\,x^4-5\,x^2-2)E(\arcsin{(x)}\mid-\frac{7}{2})+9\sqrt{2}(7\,x^4-5\,x^2-2)F(\arcsin{(x)}\mid}{162\,(7\,x^4-5\,x^2-2)}$$

input
```
integrate(1/(-7*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/162*(5*sqrt(2)*(7*x^4 - 5*x^2 - 2)*elliptic_e(arcsin(x), -7/2) + 9*sqrt(
2)*(7*x^4 - 5*x^2 - 2)*elliptic_f(arcsin(x), -7/2) + sqrt(-7*x^4 + 5*x^2 +
 2)*(35*x^3 - 53*x))/(7*x^4 - 5*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2-7x^4)^{3/2}}\,dx = \int \frac{1}{(-7x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-7*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((-7*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2-7x^4)^{3/2}}\,dx = \int \frac{1}{(-7\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-7*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-7*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(2+5x^2-7x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-7\,x^4+5\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-7*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-7*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2+5x^2-7x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-7\,x^4+5\,x^2+2\right)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - 7*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 - 7*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(2+5x^2-7x^4\right)^{3/2}}\,dx = \int \frac{\sqrt{-7x^4+5x^2+2}}{49x^8-70x^6-3x^4+20x^2+4}dx$$

input
```
int(1/(-7*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 7*x**4 + 5*x**2 + 2)/(49*x**8 - 70*x**6 - 3*x**4 + 20*x**2 + 4
),x)
```

3.309 $\int \frac{1}{(2+5x^2-8x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{(2+5x^2-8x^4)^{3/2}}\,dx = \frac{x(57-40x^2)}{178\sqrt{2+5x^2-8x^4}} + \frac{5}{178}\sqrt{\frac{1}{2}\left(-5+\sqrt{89}\right)}E\left(\arcsin\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right)|\frac{1}{32}\left(-57-5\sqrt{89}\right)\right) + \frac{1}{2}\sqrt{\frac{1}{178}\left(-5+\sqrt{89}\right)}\,\text{EllipticF}\left(\arcsin\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right),\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

output
```
1/178*x*(-40*x^2+57)/(-8*x^4+5*x^2+2)^(1/2)+5/356*(-10+2*89^(1/2))^(1/2)*E
llipticE(4*x/(5+89^(1/2))^(1/2),5/8*I+1/8*I*89^(1/2))+1/356*(-890+178*89^(
1/2))^(1/2)*EllipticF(4*x/(5+89^(1/2))^(1/2),5/8*I+1/8*I*89^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.25

$$\int \frac{1}{(2+5x^2-8x^4)^{3/2}}\,dx = \frac{1}{356}\left(\frac{114x}{\sqrt{2+5x^2-8x^4}}-\frac{80x^3}{\sqrt{2+5x^2-8x^4}}\right.$$
$$+5i\sqrt{2\left(5+\sqrt{89}\right)}E\left(i\operatorname{arcsinh}\left(\frac{4x}{\sqrt{-5+\sqrt{89}}}\right)|\frac{1}{32}\left(-57+5\sqrt{89}\right)\right)-i\sqrt{\frac{2}{5+\sqrt{89}}}\left(89+5\sqrt{89}\right)\text{EllipticF}\left(\right.$$

input
```
Integrate[(2 + 5*x^2 - 8*x^4)^(-3/2),x]
```

output
```
((114*x)/Sqrt[2 + 5*x^2 - 8*x^4] - (80*x^3)/Sqrt[2 + 5*x^2 - 8*x^4] + (5*I
)*Sqrt[2*(5 + Sqrt[89])]*EllipticE[I*ArcSinh[(4*x)/Sqrt[-5 + Sqrt[89]]], (
-57 + 5*Sqrt[89])/32] - I*Sqrt[2/(5 + Sqrt[89])]*(89 + 5*Sqrt[89])*Ellipti
cF[I*ArcSinh[(4*x)/Sqrt[-5 + Sqrt[89]]], (-57 + 5*Sqrt[89])/32])/356
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-8x^4+5x^2+2)^{3/2}}dx$$
$$\Big\downarrow \text{ 1405}$$
$$\frac{x(57-40x^2)}{178\sqrt{-8x^4+5x^2+2}}-\frac{1}{178}\int -\frac{8(5x^2+4)}{\sqrt{-8x^4+5x^2+2}}dx$$
$$\Big\downarrow \text{ 27}$$
$$\frac{4}{89}\int \frac{5x^2+4}{\sqrt{-8x^4+5x^2+2}}dx+\frac{x(57-40x^2)}{178\sqrt{-8x^4+5x^2+2}}$$

$$\Big\downarrow \; 1494$$

$$\frac{16}{89}\sqrt{2}\int \frac{5x^2+4}{\sqrt{-16x^2+\sqrt{89}+5}\sqrt{16x^2+\sqrt{89}-5}}dx+\frac{x(57-40x^2)}{178\sqrt{-8x^4+5x^2+2}}$$

$$\Big\downarrow \; 399$$

$$\frac{16}{89}\sqrt{2}\left(\frac{1}{16}\left(89-5\sqrt{89}\right)\int \frac{1}{\sqrt{-16x^2+\sqrt{89}+5}\sqrt{16x^2+\sqrt{89}-5}}dx+\frac{5}{16}\int \frac{\sqrt{16x^2+\sqrt{89}-5}}{\sqrt{-16x^2+\sqrt{89}+5}}dx\right)+\frac{x(57-40x^2)}{178\sqrt{-8x^4+5x^2+2}}$$

$$\Big\downarrow \; 321$$

$$\frac{16}{89}\sqrt{2}\left(\frac{5}{16}\int \frac{\sqrt{16x^2+\sqrt{89}-5}}{\sqrt{-16x^2+\sqrt{89}+5}}dx+\frac{\left(89-5\sqrt{89}\right)\text{EllipticF}\left(\arcsin\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right),\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)}{64\sqrt{\sqrt{89}-5}}\right)+\frac{x(57-40x^2)}{178\sqrt{-8x^4+5x^2+2}}$$

$$\Big\downarrow \; 327$$

$$\frac{16}{89}\sqrt{2}\left(\frac{\left(89-5\sqrt{89}\right)\text{EllipticF}\left(\arcsin\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right),\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)}{64\sqrt{\sqrt{89}-5}}+\frac{5}{64}\sqrt{\sqrt{89}-5}E\left(\arcsin\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right)\right.\right.$$

$$\frac{x(57-40x^2)}{178\sqrt{-8x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 - 8*x^4)^(-3/2),x]
```

output
```
(x*(57 - 40*x^2))/(178*Sqrt[2 + 5*x^2 - 8*x^4]) + (16*Sqrt[2]*((5*Sqrt[-5
+ Sqrt[89]]*EllipticE[ArcSin[(4*x)/Sqrt[5 + Sqrt[89]]], (-57 - 5*Sqrt[89])
/32])/64 + ((89 - 5*Sqrt[89])*EllipticF[ArcSin[(4*x)/Sqrt[5 + Sqrt[89]]],
(-57 - 5*Sqrt[89])/32])/(64*Sqrt[-5 + Sqrt[89]])))/89
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(87) = 174.

Time = 1.78 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.65

method	result
risch	$-\frac{x(40x^2-57)}{178\sqrt{-8x^4+5x^2+2}}+\frac{32\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{89}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{89}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{89}}}{2},\frac{5i}{8}+\frac{i\sqrt{89}}{8}\right)}{89\sqrt{-5+\sqrt{89}}\,\sqrt{-8x^4+5x^2+2}}-\frac{160\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{89}}{4}\right.}}{}$
default	$\frac{\frac{57}{178}x-\frac{20}{89}x^3}{\sqrt{-8x^4+5x^2+2}}+\frac{32\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{89}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{89}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{89}}}{2},\frac{5i}{8}+\frac{i\sqrt{89}}{8}\right)}{89\sqrt{-5+\sqrt{89}}\,\sqrt{-8x^4+5x^2+2}}-\frac{160\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{89}}{4}\right)x^2}\,\sqrt{}}{}$
elliptic	$\frac{\frac{57}{178}x-\frac{20}{89}x^3}{\sqrt{-8x^4+5x^2+2}}+\frac{32\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{89}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{89}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{89}}}{2},\frac{5i}{8}+\frac{i\sqrt{89}}{8}\right)}{89\sqrt{-5+\sqrt{89}}\,\sqrt{-8x^4+5x^2+2}}-\frac{160\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{89}}{4}\right)x^2}\,\sqrt{}}{}$

input
```
int(1/(-8*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/178*x*(40*x^2-57)/(-8*x^4+5*x^2+2)^(1/2)+32/89/(-5+89^(1/2))^(1/2)*(1-(
-5/4+1/4*89^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*89^(1/2))*x^2)^(1/2)/(-8*x^4+5*
x^2+2)^(1/2)*EllipticF(1/2*x*(-5+89^(1/2))^(1/2),5/8*I+1/8*I*89^(1/2))-160
/89/(-5+89^(1/2))^(1/2)*(1-(-5/4+1/4*89^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*89^
(1/2))*x^2)^(1/2)/(-8*x^4+5*x^2+2)^(1/2)/(5+89^(1/2))*(EllipticF(1/2*x*(-5
+89^(1/2))^(1/2),5/8*I+1/8*I*89^(1/2))-EllipticE(1/2*x*(-5+89^(1/2))^(1/2)
,5/8*I+1/8*I*89^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(83) = 166.

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.38

$$\int \frac{1}{(2+5x^2-8x^4)^{3/2}}\,dx=\frac{5\left(\sqrt{89}\sqrt{2}(8\,x^4-5\,x^2-2)-5\,\sqrt{2}(8\,x^4-5\,x^2-2)\right)\sqrt{\sqrt{89}-5}E(\arcsin\left(\frac{1}{2}\,x\sqrt{}\right.}{}$$

input
```
integrate(1/(-8*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/1424*(5*(sqrt(89)*sqrt(2)*(8*x^4 - 5*x^2 - 2) - 5*sqrt(2)*(8*x^4 - 5*x^2
 - 2))*sqrt(sqrt(89) - 5)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(89) - 5)), -5/
32*sqrt(89) - 57/32) - (sqrt(89)*sqrt(2)*(8*x^4 - 5*x^2 - 2) - 45*sqrt(2)*
(8*x^4 - 5*x^2 - 2))*sqrt(sqrt(89) - 5)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(
89) - 5)), -5/32*sqrt(89) - 57/32) + 8*sqrt(-8*x^4 + 5*x^2 + 2)*(40*x^3 -
57*x))/(8*x^4 - 5*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2-8x^4)^{3/2}}\,dx = \int \frac{1}{(-8x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-8*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((-8*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2-8x^4)^{3/2}}\,dx = \int \frac{1}{(-8\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-8*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-8*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2-8x^4)^{3/2}}\,dx=\int \frac{1}{(-8\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-8*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-8*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2-8x^4)^{3/2}}\,dx=\int \frac{1}{(-8\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - 8*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 - 8*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2-8x^4)^{3/2}}\,dx=\int \frac{\sqrt{-8x^4+5x^2+2}}{64x^8-80x^6-7x^4+20x^2+4}dx$$

input
```
int(1/(-8*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 8*x**4 + 5*x**2 + 2)/(64*x**8 - 80*x**6 - 7*x**4 + 20*x**2 + 4
),x)
```

3.310 $\int \frac{1}{(2+5x^2-9x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{1}{(2+5x^2-9x^4)^{3/2}}\,dx = \frac{x(61-45x^2)}{194\sqrt{2+5x^2-9x^4}} + \frac{5}{194}\sqrt{\frac{1}{2}\left(-5+\sqrt{97}\right)}E\left(\arcsin\left(3\sqrt{\frac{2}{5+\sqrt{97}}}x\right)|\frac{1}{36}\left(-61-5\sqrt{97}\right)\right) + \frac{1}{2}\sqrt{\frac{1}{194}\left(-5+\sqrt{97}\right)}\,\text{EllipticF}\left(\arcsin\left(3\sqrt{\frac{2}{5+\sqrt{97}}}x\right),\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)$$

output
```
1/194*x*(-45*x^2+61)/(-9*x^4+5*x^2+2)^(1/2)+5/388*(-10+2*97^(1/2))^(1/2)*E
llipticE(3*2^(1/2)/(5+97^(1/2))^(1/2)*x,5/12*I*2^(1/2)+1/12*I*194^(1/2))+1
/388*(-970+194*97^(1/2))^(1/2)*EllipticF(3*2^(1/2)/(5+97^(1/2))^(1/2)*x,5/
12*I*2^(1/2)+1/12*I*194^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.50 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.23

$$\int \frac{1}{(2+5x^2-9x^4)^{3/2}}\,dx = \frac{1}{388}\left(\frac{122x}{\sqrt{2+5x^2-9x^4}} - \frac{90x^3}{\sqrt{2+5x^2-9x^4}} + 5i\sqrt{2\left(5+\sqrt{97}\right)}E\left(i\operatorname{arcsinh}\left(3\sqrt{\frac{2}{-5+\sqrt{97}}}x\right)|\frac{1}{36}\left(-61+5\sqrt{97}\right)\right) - i\sqrt{\frac{2}{5+\sqrt{97}}}\left(97+5\sqrt{97}\right)\text{Elliptic}$$

input
```
Integrate[(2 + 5*x^2 - 9*x^4)^(-3/2),x]
```

output
```
((122*x)/Sqrt[2 + 5*x^2 - 9*x^4] - (90*x^3)/Sqrt[2 + 5*x^2 - 9*x^4] + (5*I
)*Sqrt[2*(5 + Sqrt[97])]*EllipticE[I*ArcSinh[3*Sqrt[2/(-5 + Sqrt[97])]*x],
 (-61 + 5*Sqrt[97])/36] - I*Sqrt[2/(5 + Sqrt[97])]*(97 + 5*Sqrt[97])*Ellip
ticF[I*ArcSinh[3*Sqrt[2/(-5 + Sqrt[97])]*x], (-61 + 5*Sqrt[97])/36])/388
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-9x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x(61-45x^2)}{194\sqrt{-9x^4+5x^2+2}} - \frac{1}{194}\int -\frac{9(5x^2+4)}{\sqrt{-9x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{9}{194}\int \frac{5x^2+4}{\sqrt{-9x^4+5x^2+2}}dx + \frac{x(61-45x^2)}{194\sqrt{-9x^4+5x^2+2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{27}{97}\int \frac{5x^2+4}{\sqrt{-18x^2+\sqrt{97}+5}\sqrt{18x^2+\sqrt{97}-5}}dx+\frac{x(61-45x^2)}{194\sqrt{-9x^4+5x^2+2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{27}{97}\left(\frac{1}{18}\left(97-5\sqrt{97}\right)\int \frac{1}{\sqrt{-18x^2+\sqrt{97}+5}\sqrt{18x^2+\sqrt{97}-5}}dx+\frac{5}{18}\int \frac{\sqrt{18x^2+\sqrt{97}-5}}{\sqrt{-18x^2+\sqrt{97}+5}}dx\right)+\frac{x(61-45x^2)}{194\sqrt{-9x^4+5x^2+2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{27}{97}\left(\frac{5}{18}\int \frac{\sqrt{18x^2+\sqrt{97}-5}}{\sqrt{-18x^2+\sqrt{97}+5}}dx+\frac{(97-5\sqrt{97})\,\text{EllipticF}\left(\arcsin\left(3\sqrt{\frac{2}{5+\sqrt{97}}}x\right),\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)}{54\sqrt{2\left(\sqrt{97}-5\right)}}\right)+\frac{x(61-45x^2)}{194\sqrt{-9x^4+5x^2+2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{27}{97}\left(\frac{(97-5\sqrt{97})\,\text{EllipticF}\left(\arcsin\left(3\sqrt{\frac{2}{5+\sqrt{97}}}x\right),\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)}{54\sqrt{2\left(\sqrt{97}-5\right)}}+\frac{5}{54}\sqrt{\frac{1}{2}\left(\sqrt{97}-5\right)}E\left(\arcsin\left(3\sqrt{\frac{2}{5+\sqrt{\quad}}}\right.\right.\right.$$

$$\frac{x(61-45x^2)}{194\sqrt{-9x^4+5x^2+2}}$$

input
```
Int[(2 + 5*x^2 - 9*x^4)^(-3/2),x]
```

output
```
(x*(61 - 45*x^2))/(194*Sqrt[2 + 5*x^2 - 9*x^4]) + (27*((5*Sqrt[(-5 + Sqrt[
97])/2]*EllipticE[ArcSin[3*Sqrt[2/(5 + Sqrt[97])]*x], (-61 - 5*Sqrt[97])/3
6])/54 + ((97 - 5*Sqrt[97])*EllipticF[ArcSin[3*Sqrt[2/(5 + Sqrt[97])]*x],
(-61 - 5*Sqrt[97])/36])/(54*Sqrt[2*(-5 + Sqrt[97])])))/97
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(101) = 202$.

Time = 2.10 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.64

method	result
risch	$-\frac{x(45x^2-61)}{194\sqrt{-9x^4+5x^2+2}}+\frac{36\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{97}}{4}\right)x^2}\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{97}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{97}}}{2},\frac{5i\sqrt{2}}{12}+\frac{i\sqrt{194}}{12}\right)}{97\sqrt{-5+\sqrt{97}}\sqrt{-9x^4+5x^2+2}}-\frac{180\sqrt{1-\left(-\frac{5}{4}+\right.}}{}$
default	$\frac{\frac{61}{194}x-\frac{45}{194}x^3}{\sqrt{-9x^4+5x^2+2}}+\frac{36\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{97}}{4}\right)x^2}\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{97}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{97}}}{2},\frac{5i\sqrt{2}}{12}+\frac{i\sqrt{194}}{12}\right)}{97\sqrt{-5+\sqrt{97}}\sqrt{-9x^4+5x^2+2}}-\frac{180\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{97}}{4}\right)x}}{}$
elliptic	$\frac{\frac{61}{194}x-\frac{45}{194}x^3}{\sqrt{-9x^4+5x^2+2}}+\frac{36\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{97}}{4}\right)x^2}\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{97}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{97}}}{2},\frac{5i\sqrt{2}}{12}+\frac{i\sqrt{194}}{12}\right)}{97\sqrt{-5+\sqrt{97}}\sqrt{-9x^4+5x^2+2}}-\frac{180\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{97}}{4}\right)x}}{}$

input
```
int(1/(-9*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/194*x*(45*x^2-61)/(-9*x^4+5*x^2+2)^(1/2)+36/97/(-5+97^(1/2))^(1/2)*(1-(
-5/4+1/4*97^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*97^(1/2))*x^2)^(1/2)/(-9*x^4+5*
x^2+2)^(1/2)*EllipticF(1/2*x*(-5+97^(1/2))^(1/2),5/12*I*2^(1/2)+1/12*I*194
^(1/2))-180/97/(-5+97^(1/2))^(1/2)*(1-(-5/4+1/4*97^(1/2))*x^2)^(1/2)*(1-(-
5/4-1/4*97^(1/2))*x^2)^(1/2)/(-9*x^4+5*x^2+2)^(1/2)/(5+97^(1/2))*(Elliptic
F(1/2*x*(-5+97^(1/2))^(1/2),5/12*I*2^(1/2)+1/12*I*194^(1/2))-EllipticE(1/2
*x*(-5+97^(1/2))^(1/2),5/12*I*2^(1/2)+1/12*I*194^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

$$\int \frac{1}{(2+5x^2-9x^4)^{3/2}}\,dx = \frac{5\left(\sqrt{97}\sqrt{2}(9\,x^4-5\,x^2-2)-5\,\sqrt{2}(9\,x^4-5\,x^2-2)\right)\sqrt{\sqrt{97}-5}E(\arcsin\left(\frac{1}{2}\,x\sqrt{\right.}}{}$$

input
```
integrate(1/(-9*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/1552*(5*(sqrt(97)*sqrt(2)*(9*x^4 - 5*x^2 - 2) - 5*sqrt(2)*(9*x^4 - 5*x^2
 - 2))*sqrt(sqrt(97) - 5)*elliptic_e(arcsin(1/2*x*sqrt(sqrt(97) - 5)), -5/
36*sqrt(97) - 61/36) - (sqrt(97)*sqrt(2)*(9*x^4 - 5*x^2 - 2) - 45*sqrt(2)*
(9*x^4 - 5*x^2 - 2))*sqrt(sqrt(97) - 5)*elliptic_f(arcsin(1/2*x*sqrt(sqrt(
97) - 5)), -5/36*sqrt(97) - 61/36) + 8*sqrt(-9*x^4 + 5*x^2 + 2)*(45*x^3 -
61*x))/(9*x^4 - 5*x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+5x^2-9x^4)^{3/2}}\,dx = \int \frac{1}{(-9x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-9*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((-9*x**4 + 5*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+5x^2-9x^4)^{3/2}}\,dx = \int \frac{1}{(-9\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-9*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-9*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+5x^2-9x^4)^{3/2}}\,dx = \int \frac{1}{(-9\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-9*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-9*x^4 + 5*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+5x^2-9x^4)^{3/2}}\,dx = \int \frac{1}{(-9\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(5*x^2 - 9*x^4 + 2)^(3/2),x)
```

output
```
int(1/(5*x^2 - 9*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(2+5x^2-9x^4)^{3/2}}\,dx = \int \frac{\sqrt{-9x^4+5x^2+2}}{81x^8-90x^6-11x^4+20x^2+4}dx$$

input
```
int(1/(-9*x^4+5*x^2+2)^(3/2),x)
```

output
```
int(sqrt( - 9*x**4 + 5*x**2 + 2)/(81*x**8 - 90*x**6 - 11*x**4 + 20*x**2 +
4),x)
```

3.311 $\int \frac{1}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 353

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{x\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}$$

output
```
x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*c^(1/2)*x*(c*
x^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)+b*c^(1/4)*(a^(1/2)
+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(-
4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4
+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*
x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/
(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.63 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx =$$
$$-\frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(b^2-2ac+bcx^2)+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\Big(i\text{arcsinh}}{}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(-3/2),x]
```

output
```
-1/4*(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*c*x^2) + I*b*(
-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
 + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[
b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}dx$$
$$\Big\downarrow \; 1405$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\int \frac{c\left(bx^2+2a\right)}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1511$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1416$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1509$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-$$

$$\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.}{\sqrt{}}\right.}{a\left(b^2-4ac\right)}$$

input
```
Int[(a + b*x^2 + c*x^4)^(-3/2),x]
```

output
```
(x*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (c
*(-((b*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(2*Sqrt[a] + b/Sqrt
[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/
4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.36

method	result
default	$-\frac{2c\left(\frac{b x^3}{2a\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^2}{a\left(4ac-b^2\right)}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{}}{}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c x^4+b x^2+a}}$
elliptic	$-\frac{2c\left(\frac{b x^3}{2a\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^2}{a\left(4ac-b^2\right)}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{}}{}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c x^4+b x^2+a}}$

input
```
int(1/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(1/2*b/a/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4+1/c
*b*x^2+1/c*a)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*
(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(b^2cx^4+b^3x^2+ab^2-\left(abcx^4+ab^2x^2+a^2b\right)\sqrt{\frac{b^2-4\,ac}{a^2}}\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}E(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-}{a}}\right.}{}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(b^2*c*x^4 + b^3*x^2 + a*b^2 - (a*b*c*x^4 + a*b^2*x^2 + a^
2*b)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)
/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a))
, 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*a
*b + b^2)*c*x^4 + 2*a^2*b + a*b^2 + (2*a*b^2 + b^3)*x^2 + ((2*a^2 - a*b)*c
*x^4 + 2*a^3 - a^2*b + (2*a^2*b - a*b^2)*x^2)*sqrt((b^2 - 4*a*c)/a^2))*sqr
t(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x
*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2
) + b^2 - 2*a*c)/(a*c)) - 2*(a*b*c*x^3 + (a*b^2 - 2*a^2*c)*x)*sqrt(c*x^4 +
 b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 -
 4*a^3*b*c)*x^2)
```

Sympy [F]

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{1}{(c\,x^4 + b\,x^2 + a)^{3/2}} \, dx$$

input
```
int(1/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(1/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int(1/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4
+ 2*b*c*x**6 + c**2*x**8),x)
```

3.312 $\int \frac{1}{(a-bx^2+cx^4)^{3/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 358

$$\int \frac{1}{(a-bx^2+cx^4)^{3/2}} dx = \frac{x(b^2-2ac-bcx^2)}{a(b^2-4ac)\sqrt{a-bx^2+cx^4}} + \frac{b\sqrt{c}x\sqrt{a-bx^2+cx^4}}{a(b^2-4ac)(\sqrt{a}+\sqrt{c}x^2)} - \frac{b\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)\sqrt{a-bx^2+cx^4}} + \frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}(b+2\sqrt{a}\sqrt{c})\sqrt{a-bx^2+cx^4}}$$

output
```
x*(-b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4-b*x^2+a)^(1/2)+b*c^(1/2)*x*(c
*x^4-b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)-b*c^(1/4)*(a^(1/2
)+c^(1/2)*x^2)*((c*x^4-b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(s
in(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(
-4*a*c+b^2)/(c*x^4-b*x^2+a)^(1/2)+1/2*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^
4-b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)
*x/a^(1/4)),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(b+2*a^(1/2)*c^(1/2))
/(c*x^4-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a-bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}x(-b^2+2ac+bcx^2)-i\sqrt{2}b\left(b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}-2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{-b+\sqrt{b^2-4ac}+2cx^2}{-b+\sqrt{b^2-4ac}}}E\left(i\text{arcsin}\right.}{}$$

input
```
Integrate[(a - b*x^2 + c*x^4)^(-3/2),x]
```

output
```
-1/4*(4*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x*(-b^2 + 2*a*c + b*c*x^2) - I*Sq
rt[2]*b*(b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*Sqrt[(-b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]
*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])] + I*Sqrt[2]*(b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])]*Sqrt[(-b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 -
 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x],
 (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*Sqrt[c
/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[a - b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \ 1405$$

$$\frac{x\left(-2ac+b^2-bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a-bx^2+cx^4}}-\frac{\int \frac{c\left(2a-bx^2\right)}{\sqrt{cx^4-bx^2+a}}dx}{a\left(b^2-4ac\right)}$$

$\downarrow$ 27

$$\frac{x\left(-2ac+b^2-bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a-bx^2+cx^4}}-\frac{c\int \frac{2a-bx^2}{\sqrt{cx^4-bx^2+a}}dx}{a\left(b^2-4ac\right)}$$

$\downarrow$ 1511

$$\frac{x\left(-2ac+b^2-bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a-bx^2+cx^4}}-\frac{c\left(\sqrt{a}\left(2\sqrt{a}-\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4-bx^2+a}}dx+\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4-bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)}$$

$\downarrow$ 27

$$\frac{x\left(-2ac+b^2-bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a-bx^2+cx^4}}-\frac{c\left(\sqrt{a}\left(2\sqrt{a}-\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4-bx^2+a}}dx+\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4-bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)}$$

$\downarrow$ 1416

$$\frac{x\left(-2ac+b^2-bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a-bx^2+cx^4}}-\frac{c\left(\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4-bx^2+a}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\left(2\sqrt{a}-\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{c}\sqrt{a-bx^2+cx^4}}\right)}{a\left(b^2-4ac\right)}$$

$\downarrow$ 1509

$$\frac{x\left(-2ac+b^2-bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a-bx^2+cx^4}}-\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}-\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{c}\sqrt{a-bx^2+cx^4}}+\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{c}\sqrt{a-bx^2+cx^4}}\right.}{\sqrt{}}\right)}{a\left(b^2-4ac\right)}$$

input
```
Int[(a - b*x^2 + c*x^4)^(-3/2),x]
```

output
```
(x*(b^2 - 2*a*c - b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a - b*x^2 + c*x^4]) - (c
*((b*(-((x*Sqrt[a - b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(c^
(1/4)*Sqrt[a - b*x^2 + c*x^4])))/Sqrt[c] + (a^(1/4)*(2*Sqrt[a] - b/Sqrt[c]
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])
/(2*c^(1/4)*Sqrt[a - b*x^2 + c*x^4])))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.35

method	result
default	$-\frac{2c\left(-\frac{b\,x^{3}}{2a\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)x}{2a\left(4ac-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}-\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^{2}}{a\left(4ac-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}}{}\right.}{4\sqrt{\frac{b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}-b\,x^{2}+a}}$
elliptic	$-\frac{2c\left(-\frac{b\,x^{3}}{2a\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)x}{2a\left(4ac-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}-\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^{2}}{a\left(4ac-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}}{}\right.}{4\sqrt{\frac{b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}-b\,x^{2}+a}}$

input
```
int(1/(c*x^4-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/2*b/a/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4-1/
c*b*x^2+1/c*a)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(
-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4-b*x^2+a)^(1/2)*EllipticF(1/2*x*
2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(-b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2))+1/2*b/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4-b*x^2+a)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*
2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(-b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
2*(-4-2*b*(-b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a-bx^2+cx^4)^{3/2}}\,dx =$$
$$-\frac{\sqrt{\frac{1}{2}}\left(b^2cx^4-b^3x^2+ab^2+(abcx^4-ab^2x^2+a^2b)\sqrt{\frac{b^2-4\,ac}{a^2}}\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}+b}{a}}\,E(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}+}{a}}\right.}{}$$

input
```
integrate(1/(c*x^4-b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(b^2*c*x^4 - b^3*x^2 + a*b^2 + (a*b*c*x^4 - a*b^2*x^2 + a^
2*b)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) + b)
/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) + b)/a))
, -1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) - b^2 + 2*a*c)/(a*c)) + sqrt(1/2)*((2*
a*b - b^2)*c*x^4 + 2*a^2*b - a*b^2 - (2*a*b^2 - b^3)*x^2 - ((2*a^2 + a*b)*
c*x^4 + 2*a^3 + a^2*b - (2*a^2*b + a*b^2)*x^2)*sqrt((b^2 - 4*a*c)/a^2))*sq
rt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) + b)/a)*elliptic_f(arcsin(sqrt(1/2)*
x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) + b)/a)), -1/2*(a*b*sqrt((b^2 - 4*a*c)/a
^2) - b^2 + 2*a*c)/(a*c)) + 2*(a*b*c*x^3 - (a*b^2 - 2*a^2*c)*x)*sqrt(c*x^4
 - b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 - (a^2*b^3
 - 4*a^3*b*c)*x^2)
```

Sympy [F]

$$\int \frac{1}{(a-bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(a-bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**4-b*x**2+a)**(3/2),x)
```

output
```
Integral((a - b*x**2 + c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2+cx^4)^{3/2}}\,dx=\int \frac{1}{(cx^4-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4-b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 - b*x^2 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2+cx^4)^{3/2}}\,dx=\int \frac{1}{(cx^4-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4-b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 - b*x^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2+cx^4)^{3/2}}\,dx=\int \frac{1}{(c\,x^4-b\,x^2+a)^{3/2}}\,dx$$

input
```
int(1/(a - b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(1/(a - b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^4-b\,x^2+a}}{c^2x^8-2bc\,x^6+2ac\,x^4+b^2x^4-2ab\,x^2+a^2}dx$$

input
```
int(1/(c*x^4-b*x^2+a)^(3/2),x)
```

output
```
int(sqrt(a - b*x**2 + c*x**4)/(a**2 - 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4
- 2*b*c*x**6 + c**2*x**8),x)
```

3.313 $\int \frac{1}{(-a+bx^2-cx^4)^{3/2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 366

$$\begin{aligned}\int \frac{1}{(-a+bx^2-cx^4)^{3/2}}\,dx = & -\frac{x(b^2-2ac-bcx^2)}{a(b^2-4ac)\sqrt{-a+bx^2-cx^4}}+\frac{b\sqrt{c}x\sqrt{-a+bx^2-cx^4}}{a(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)} \\ & +\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)\sqrt{-a+bx^2-cx^4}} \\ & -\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b+2\sqrt{a}\sqrt{c}\right)\sqrt{-a+bx^2-cx^4}}\end{aligned}$$

output
```
-x*(-b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(-c*x^4+b*x^2-a)^(1/2)+b*c^(1/2)*x*
(-c*x^4+b*x^2-a)^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)+b*c^(1/4)*(a^(
1/2)+c^(1/2)*x^2)*((c*x^4-b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Elliptic
E(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4
)/(-4*a*c+b^2)/(-c*x^4+b*x^2-a)^(1/2)-1/2*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((
c*x^4-b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(
1/4)*x/a^(1/4)),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(b+2*a^(1/2)*c^(1
/2))/(-c*x^4+b*x^2-a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.52 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.28

$$\int \frac{1}{(-a+bx^2-cx^4)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}x(b^2-2ac-bcx^2)-i\sqrt{2}b\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}-2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{-b+\sqrt{b^2-4ac}+2cx^2}{-b+\sqrt{b^2-4ac}}}E\left(i\text{arcsi}\right.}{}$$

input
```
Integrate[(-a + b*x^2 - c*x^4)^(-3/2),x]
```

output
```
-1/4*(4*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x*(b^2 - 2*a*c - b*c*x^2) - I*S
qrt[2]*b*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*Sqrt[(-b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c
]))]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*Sqrt[2]*(-b^
2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[(-b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]
))]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)
*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[-a + b*x^2 - c*x^4])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-a+bx^2-cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{\int \frac{c(2a-bx^2)}{\sqrt{-cx^4+bx^2-a}}dx}{a(b^2-4ac)}-\frac{x(-2ac+b^2-bcx^2)}{a(b^2-4ac)\sqrt{-a+bx^2-cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{2a-bx^2}{\sqrt{-cx^4+bx^2-a}}dx}{a(b^2-4ac)}-\frac{x(-2ac+b^2-bcx^2)}{a(b^2-4ac)\sqrt{-a+bx^2-cx^4}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{c\left(\sqrt{a}\left(2\sqrt{a}-\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-cx^4+bx^2-a}}dx+\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{-cx^4+bx^2-a}}dx}{\sqrt{c}}\right)}{a(b^2-4ac)}-\frac{x(-2ac+b^2-bcx^2)}{a(b^2-4ac)\sqrt{-a+bx^2-cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\left(\sqrt{a}\left(2\sqrt{a}-\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-cx^4+bx^2-a}}dx+\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{-cx^4+bx^2-a}}dx}{\sqrt{c}}\right)}{a(b^2-4ac)}-\frac{x(-2ac+b^2-bcx^2)}{a(b^2-4ac)\sqrt{-a+bx^2-cx^4}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{c\left(\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{-cx^4+bx^2-a}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\left(2\sqrt{a}-\frac{b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{c}\sqrt{-a+bx^2-cx^4}}\right)}{a(b^2-4ac)}-\frac{x(-2ac+b^2-bcx^2)}{a(b^2-4ac)\sqrt{-a+bx^2-cx^4}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}-\frac{b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{c}\sqrt{-a+bx^2-cx^4}}+\frac{b\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\right.}{\sqrt[4]{c}\sqrt{-a+bx^2-cx^4}}\right.}{\sqrt{}}\right)}{a(b^2-4ac)}-\frac{x(-2ac+b^2-bcx^2)}{a(b^2-4ac)\sqrt{-a+bx^2-cx^4}}$$

input
```
Int[(-a + b*x^2 - c*x^4)^(-3/2),x]
```

output
```
-((x*(b^2 - 2*a*c - b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[-a + b*x^2 - c*x^4]))
+ (c*((b*((x*Sqrt[-a + b*x^2 - c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) + (a^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(1/4)*Sqrt[-a + b*x^2 - c*x^4])))/Sqrt[c] + (a^(1/4)*(2*Sqrt[a] - b/Sqrt
[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/
4])/(2*c^(1/4)*Sqrt[-a + b*x^2 - c*x^4])))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.31

method	result
default	$\frac{2c\left(-\frac{b\,x^3}{2a\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)c}\right)}{\sqrt{-\left(x^4-\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(-\frac{1}{a}+\frac{2ac-b^2}{a\left(4ac-b^2\right)}\right)\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-\frac{2(-}{}}}{}\right.}{2\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}\,\sqrt{-c\,x^4+b\,x^2-a}}$
elliptic	$\frac{2c\left(-\frac{b\,x^3}{2a\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)c}\right)}{\sqrt{-\left(x^4-\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(-\frac{1}{a}+\frac{2ac-b^2}{a\left(4ac-b^2\right)}\right)\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-\frac{2(-}{}}}{}\right.}{2\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}\,\sqrt{-c\,x^4+b\,x^2-a}}$

input
```
int(1/(-c*x^4+b*x^2-a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(-1/2*b/a/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/(-(x^4-1/
c*b*x^2+1/c*a)*c)^(1/2)+1/2*(-1/a+(2*a*c-b^2)/a/(4*a*c-b^2))/(-2*(-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*(b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2-a)^(1/2)*EllipticF(1/2*x*(-
2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2))+b/(4*a*c-b^2)*c/(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*
x^4+b*x^2-a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Ellip
ticE(1/2*x*(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(-a+bx^2-cx^4\right)^{3/2}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(b^2cx^4-b^3x^2+ab^2+\left(abcx^4-ab^2x^2+a^2b\right)\sqrt{\frac{b^2-4\,ac}{a^2}}\right)\sqrt{-a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}+b}{a}}E\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,a}{a^2}}}{a}}\right.\right.}{}$$

input
```
integrate(1/(-c*x^4+b*x^2-a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(b^2*c*x^4 - b^3*x^2 + a*b^2 + (a*b*c*x^4 - a*b^2*x^2 + a^
2*b)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(-a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) + b
)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) + b)/a)
), -1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) - b^2 + 2*a*c)/(a*c)) + sqrt(1/2)*((2
*a*b - b^2)*c*x^4 + 2*a^2*b - a*b^2 - (2*a*b^2 - b^3)*x^2 - ((2*a^2 + a*b)
*c*x^4 + 2*a^3 + a^2*b - (2*a^2*b + a*b^2)*x^2)*sqrt((b^2 - 4*a*c)/a^2))*s
qrt(-a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) + b)/a)*elliptic_f(arcsin(sqrt(1/2
)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) + b)/a)), -1/2*(a*b*sqrt((b^2 - 4*a*c)
/a^2) - b^2 + 2*a*c)/(a*c)) + 2*(a*b*c*x^3 - (a*b^2 - 2*a^2*c)*x)*sqrt(-c*
x^4 + b*x^2 - a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 - (a^2*
b^3 - 4*a^3*b*c)*x^2)
```

Sympy [F]

$$\int \frac{1}{\left(-a+bx^2-cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-a+bx^2-cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*x**4+b*x**2-a)**(3/2),x)
```

output
```
Integral((-a + b*x**2 - c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-a+bx^2-cx^4)^{3/2}}\,dx = \int \frac{1}{(-cx^4+bx^2-a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*x^4+b*x^2-a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 + b*x^2 - a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-a+bx^2-cx^4)^{3/2}}\,dx = \int \frac{1}{(-cx^4+bx^2-a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*x^4+b*x^2-a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + b*x^2 - a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-a+bx^2-cx^4)^{3/2}}\,dx = \int \frac{1}{(-c\,x^4+b\,x^2-a)^{3/2}}\,dx$$

input
```
int(1/(b*x^2 - a - c*x^4)^(3/2),x)
```

output
```
int(1/(b*x^2 - a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2-cx^4)^{3/2}}\,dx = \int \frac{\sqrt{-c\,x^4+b\,x^2-a}}{c^2x^8-2bc\,x^6+2ac\,x^4+b^2x^4-2ab\,x^2+a^2}dx$$

input
```
int(1/(-c*x^4+b*x^2-a)^(3/2),x)
```

output
```
int(sqrt( - a + b*x**2 - c*x**4)/(a**2 - 2*a*b*x**2 + 2*a*c*x**4 + b**2*x*
*4 - 2*b*c*x**6 + c**2*x**8),x)
```

3.314 $\int \frac{1}{(-a-bx^2-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 371

$$\int \frac{1}{(-a-bx^2-cx^4)^{3/2}}\,dx = -\frac{x(b^2-2ac+bcx^2)}{a(b^2-4ac)\sqrt{-a-bx^2-cx^4}} - \frac{b\sqrt{c}x\sqrt{-a-bx^2-cx^4}}{a(b^2-4ac)(\sqrt{a}+\sqrt{c}x^2)} - \frac{b\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)\sqrt{-a-bx^2-cx^4}} + \frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}(b-2\sqrt{a}\sqrt{c})\sqrt{-a-bx^2-cx^4}}$$

output
```
-x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(-c*x^4-b*x^2-a)^(1/2)-b*c^(1/2)*x*(
-c*x^4-b*x^2-a)^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)-b*c^(1/4)*(a^(1
/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE
(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)
/(-4*a*c+b^2)/(-c*x^4-b*x^2-a)^(1/2)+1/2*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c
*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1
/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(b-2*a^(1/2)*c^(1/
2))/(-c*x^4-b*x^2-a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.25

$$\int \frac{1}{(-a-bx^2-cx^4)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{\frac{c}{b-\sqrt{b^2-4ac}}}x(b^2-2ac+bcx^2)+ib\left(b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\left(\sqrt{2}\right.\right.}{4a}$$

input
```
Integrate[(-a - b*x^2 - c*x^4)^(-3/2),x]
```

output
```
-1/4*(4*Sqrt[c/(b - Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*c*x^2) + I*b*(b
 + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b - Sqrt[b^2 - 4*a*c])]*x], (b -
 Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])] - I*(b^2 - 4*a*c + b*Sqrt[b^2
 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b - Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 -
 4*a*c])/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*Sqrt[c/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[-a - x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-a-bx^2-cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{\int \frac{c(bx^2+2a)}{\sqrt{-cx^4-bx^2-a}}dx}{a(b^2-4ac)}-\frac{x(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{-a-bx^2-cx^4}}$$

$\Big\downarrow$ 27

$$\frac{c\int \frac{bx^2+2a}{\sqrt{-cx^4-bx^2-a}}dx}{a(b^2-4ac)}-\frac{x(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{-a-bx^2-cx^4}}$$

$\Big\downarrow$ 1511

$$\frac{c\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-cx^4-bx^2-a}}dx-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{-cx^4-bx^2-a}}dx}{\sqrt{c}}\right)}{a(b^2-4ac)}-\frac{x(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{-a-bx^2-cx^4}}$$

$\Big\downarrow$ 27

$$\frac{c\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-cx^4-bx^2-a}}dx-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{-cx^4-bx^2-a}}dx}{\sqrt{c}}\right)}{a(b^2-4ac)}-\frac{x(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{-a-bx^2-cx^4}}$$

$\Big\downarrow$ 1416

$$\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{-a-bx^2-cx^4}}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{-cx^4-bx^2-a}}dx}{\sqrt{c}}\right)}{a(b^2-4ac)}-\frac{x(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{-a-bx^2-cx^4}}$$

$\Big\downarrow$ 1509

$$\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{-a-bx^2-cx^4}}-\frac{b\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\right.}{\sqrt[4]{c}\sqrt{-a-bx^2-cx}}\right.}{\sqrt{}}\right.}{a(b^2-4ac)}$$

$$\frac{x(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{-a-bx^2-cx^4}}$$

input
```
Int[(-a - b*x^2 - c*x^4)^(-3/2),x]
```

output
```
-((x*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[-a - b*x^2 - c*x^4]))
+ (c*(-((b*((x*Sqrt[-a - b*x^2 - c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) + (a^(1/4
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])
/(c^(1/4)*Sqrt[-a - b*x^2 - c*x^4])))/Sqrt[c]) + (a^(1/4)*(2*Sqrt[a] + b/S
qrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]
))/4])/(2*c^(1/4)*Sqrt[-a - b*x^2 - c*x^4])))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.29

method	result
default	$\frac{2c\left(\frac{b\,x^3}{2a\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)c}\right)}{\sqrt{-\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(-\frac{1}{a}+\frac{2ac-b^2}{a\left(4ac-b^2\right)}\right)\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(b+\right.}{}}}{}\right.}{2\sqrt{-\frac{2\left(b+\sqrt{-4ac+b^2}\right)}{a}}\,\sqrt{-c\,x^4-b\,x^2-a}}$
elliptic	$\frac{2c\left(\frac{b\,x^3}{2a\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)c}\right)}{\sqrt{-\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(-\frac{1}{a}+\frac{2ac-b^2}{a\left(4ac-b^2\right)}\right)\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(b+\right.}{}}}{}\right.}{2\sqrt{-\frac{2\left(b+\sqrt{-4ac+b^2}\right)}{a}}\,\sqrt{-c\,x^4-b\,x^2-a}}$

input
```
int(1/(-c*x^4-b*x^2-a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(1/2*b/a/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/(-(x^4+1/c
*b*x^2+1/c*a)*c)^(1/2)+1/2*(-1/a+(2*a*c-b^2)/a/(4*a*c-b^2))/(-2*(b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*(-b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4-b*x^2-a)^(1/2)*EllipticF(1/2*x*(-2*
(b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(-b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2))-b/(4*a*c-b^2)*c/(-2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4
-b*x^2-a)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(-b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Ellipti
cE(1/2*x*(-2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(-b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.24

$$\int \frac{1}{(-a-bx^2-cx^4)^{3/2}}\,dx = -\frac{\sqrt{\frac{1}{2}}\left(b^2cx^4+b^3x^2+ab^2-(abcx^4+ab^2x^2+a^2b)\sqrt{\frac{b^2-4\,ac}{a^2}}\right)\sqrt{-a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}E\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}}{a}}\right.\right.}{}$$

input
```
integrate(1/(-c*x^4-b*x^2-a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(b^2*c*x^4 + b^3*x^2 + a*b^2 - (a*b*c*x^4 + a*b^2*x^2 + a^
2*b)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(-a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b
)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)
), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*
a*b + b^2)*c*x^4 + 2*a^2*b + a*b^2 + (2*a*b^2 + b^3)*x^2 + ((2*a^2 - a*b)*
c*x^4 + 2*a^3 - a^2*b + (2*a^2*b - a*b^2)*x^2)*sqrt((b^2 - 4*a*c)/a^2))*sq
rt(-a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)
*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a
^2) + b^2 - 2*a*c)/(a*c)) - 2*(a*b*c*x^3 + (a*b^2 - 2*a^2*c)*x)*sqrt(-c*x^
4 - b*x^2 - a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^
3 - 4*a^3*b*c)*x^2)
```

Sympy [F]

$$\int \frac{1}{(-a-bx^2-cx^4)^{3/2}}\,dx = \int \frac{1}{(-a-bx^2-cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*x**4-b*x**2-a)**(3/2),x)
```

output
```
Integral((-a - b*x**2 - c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-a-bx^2-cx^4)^{3/2}}\,dx=\int \frac{1}{(-cx^4-bx^2-a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*x^4-b*x^2-a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 - b*x^2 - a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-a-bx^2-cx^4)^{3/2}}\,dx=\int \frac{1}{(-cx^4-bx^2-a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*x^4-b*x^2-a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 - b*x^2 - a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-a-bx^2-cx^4)^{3/2}}\,dx=\int \frac{1}{(-c\,x^4-b\,x^2-a)^{3/2}}\,dx$$

input
```
int(1/(- a - b*x^2 - c*x^4)^(3/2),x)
```

output
```
int(1/(- a - b*x^2 - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-a-bx^2-cx^4)^{3/2}}\,dx = \left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx\right)i$$

input
```
int(1/(-c*x^4-b*x^2-a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4
+ 2*b*c*x**6 + c**2*x**8),x)*i
```

3.315 $\int \frac{1}{\left(\mathbf{a1}+\mathbf{a2}+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 405

$$\int \frac{1}{(\mathrm{a1}+\mathrm{a2}+bx^2+cx^4)^{3/2}}\,dx = \frac{x(b^2-2(\mathrm{a1}+\mathrm{a2})c+bcx^2)}{(\mathrm{a1}+\mathrm{a2})\left(b^2-4(\mathrm{a1}+\mathrm{a2})c\right)\sqrt{\mathrm{a1}+\mathrm{a2}+bx^2+cx^4}}$$
$$-\frac{b\sqrt{c}x\sqrt{\mathrm{a1}+\mathrm{a2}+bx^2+cx^4}}{(\mathrm{a1}+\mathrm{a2})\left(b^2-4(\mathrm{a1}+\mathrm{a2})c\right)\left(\sqrt{\mathrm{a1}+\mathrm{a2}}+\sqrt{c}x^2\right)}$$
$$+\frac{b\sqrt[4]{c}\left(\sqrt{\mathrm{a1}+\mathrm{a2}}+\sqrt{c}x^2\right)\sqrt{\frac{\mathrm{a1}+\mathrm{a2}+bx^2+cx^4}{\left(\sqrt{\mathrm{a1}+\mathrm{a2}}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{\mathrm{a1}+\mathrm{a2}}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{\mathrm{a1}+\mathrm{a2}}\sqrt{c}}\right)\right)}{(\mathrm{a1}+\mathrm{a2})^{3/4}\left(b^2-4(\mathrm{a1}+\mathrm{a2})c\right)\sqrt{\mathrm{a1}+\mathrm{a2}+bx^2+cx^4}}$$
$$-\frac{\left(b+2\sqrt{\mathrm{a1}+\mathrm{a2}}\sqrt{c}\right)\sqrt[4]{c}\left(\sqrt{\mathrm{a1}+\mathrm{a2}}+\sqrt{c}x^2\right)\sqrt{\frac{\mathrm{a1}+\mathrm{a2}+bx^2+cx^4}{\left(\sqrt{\mathrm{a1}+\mathrm{a2}}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{\mathrm{a1}+\mathrm{a2}}}\right),\frac{1}{4}\left(2-\right.\right.}{2(\mathrm{a1}+\mathrm{a2})^{3/4}\left(b^2-4(\mathrm{a1}+\mathrm{a2})c\right)\sqrt{\mathrm{a1}+\mathrm{a2}+bx^2+cx^4}}$$

output
```
x*(b^2-2*(a1+a2)*c+b*c*x^2)/(a1+a2)/(b^2-4*(a1+a2)*c)/(c*x^4+b*x^2+a1+a2)^
(1/2)-b*c^(1/2)*x*(c*x^4+b*x^2+a1+a2)^(1/2)/(a1+a2)/(b^2-4*(a1+a2)*c)/(c^(
1/2)*x^2+(a1+a2)^(1/2))+b*c^(1/4)*(c^(1/2)*x^2+(a1+a2)^(1/2))*((c*x^4+b*x^
2+a1+a2)/(c^(1/2)*x^2+(a1+a2)^(1/2))^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/
4)*x/(a1+a2)^(1/4))),1/2*(2-b/(a1+a2)^(1/2)/c^(1/2))^(1/2))/(a1+a2)^(3/4)/
(b^2-4*(a1+a2)*c)/(c*x^4+b*x^2+a1+a2)^(1/2)-1/2*(b+2*(a1+a2)^(1/2)*c^(1/2)
)*c^(1/4)*(c^(1/2)*x^2+(a1+a2)^(1/2))*((c*x^4+b*x^2+a1+a2)/(c^(1/2)*x^2+(a
1+a2)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/(a1+a2)^(1/4)),1/
2*(2-b/(a1+a2)^(1/2)/c^(1/2))^(1/2))/(a1+a2)^(3/4)/(b^2-4*(a1+a2)*c)/(c*x^
4+b*x^2+a1+a2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.07 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(\mathrm{a1}+\mathrm{a2}+bx^2+cx^4\right)^{3/2}}\,dx =$$

$$-\frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4(\mathrm{a1}+\mathrm{a2})c}}}x\left(b^2-2(\mathrm{a1}+\mathrm{a2})c+bcx^2\right)+i\sqrt{2}b\left(-b+\sqrt{b^2-4(\mathrm{a1}+\mathrm{a2})c}\right)\sqrt{\frac{-b+\sqrt{b^2-4(\mathrm{a1}+\mathrm{a2})c}}{-b+\sqrt{b^2-4(\mathrm{a1}+\mathrm{a}}}}}{}$$

input
```
Integrate[(a1 + a2 + b*x^2 + c*x^4)^(-3/2),x]
```

output
```
-1/4*(-4*Sqrt[c/(b + Sqrt[b^2 - 4*(a1 + a2)*c])]*x*(b^2 - 2*(a1 + a2)*c +
b*c*x^2) + I*Sqrt[2]*b*(-b + Sqrt[b^2 - 4*(a1 + a2)*c])*Sqrt[(-b + Sqrt[b^
2 - 4*(a1 + a2)*c] - 2*c*x^2)/(-b + Sqrt[b^2 - 4*(a1 + a2)*c])]*Sqrt[(b +
Sqrt[b^2 - 4*(a1 + a2)*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*(a1 + a2)*c])]*Elli
pticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*(a1 + a2)*c])]*x], -((b +
 Sqrt[b^2 - 4*(a1 + a2)*c])/(-b + Sqrt[b^2 - 4*(a1 + a2)*c]))] - I*Sqrt[2]
*(-b^2 + 4*(a1 + a2)*c + b*Sqrt[b^2 - 4*(a1 + a2)*c])*Sqrt[(-b + Sqrt[b^2
- 4*(a1 + a2)*c] - 2*c*x^2)/(-b + Sqrt[b^2 - 4*(a1 + a2)*c])]*Sqrt[(b + Sq
rt[b^2 - 4*(a1 + a2)*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*(a1 + a2)*c])]*Ellipt
icF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*(a1 + a2)*c])]*x], -((b + S
qrt[b^2 - 4*(a1 + a2)*c])/(-b + Sqrt[b^2 - 4*(a1 + a2)*c]))])/((a1 + a2)*(
b^2 - 4*(a1 + a2)*c)*Sqrt[c/(b + Sqrt[b^2 - 4*(a1 + a2)*c])]*Sqrt[a1 + a2
+ b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1405, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\text{a1}+\text{a2}+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x\left(-2c(\text{a1}+\text{a2})+b^2+bcx^2\right)}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)\sqrt{\text{a1}+\text{a2}+bx^2+cx^4}}-\frac{\int \frac{c\left(bx^2+2(\text{a1}+\text{a2})\right)}{\sqrt{cx^4+bx^2+\text{a1}+\text{a2}}}dx}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\left(-2c(\text{a1}+\text{a2})+b^2+bcx^2\right)}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)\sqrt{\text{a1}+\text{a2}+bx^2+cx^4}}-\frac{c\int \frac{bx^2+2(\text{a1}+\text{a2})}{\sqrt{cx^4+bx^2+\text{a1}+\text{a2}}}dx}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)}$$

$$\Big\downarrow \text{1511}$$

$$\frac{x\left(-2c(\text{a1}+\text{a2})+b^2+bcx^2\right)}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)\sqrt{\text{a1}+\text{a2}+bx^2+cx^4}}-\frac{c\left(\sqrt{\text{a1}+\text{a2}}\left(2\sqrt{\text{a1}+\text{a2}}+\frac{b}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^4+bx^2+\text{a1}+\text{a2}}}dx-\frac{b\sqrt{\text{a1}+\text{a2}}\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{\text{a1}+\text{a2}}}}{\sqrt{cx^4+bx^2+\text{a1}+\text{a2}}}dx}{\sqrt{c}}\right)}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)}$$

$$\Big\downarrow \text{1416}$$

$$\frac{x\left(-2c(\text{a1}+\text{a2})+b^2+bcx^2\right)}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)\sqrt{\text{a1}+\text{a2}+bx^2+cx^4}}-\frac{c\left(\frac{(\text{a1}+\text{a2})^{3/4}\left(2\sqrt{\text{a1}+\text{a2}}+\frac{b}{\sqrt{c}}\right)\left(\frac{\sqrt{c}x^2}{\sqrt{\text{a1}+\text{a2}}}+1\right)\sqrt{\frac{\text{a1}+\text{a2}+bx^2+cx^4}{(\text{a1}+\text{a2})\left(\frac{\sqrt{c}x^2}{\sqrt{\text{a1}+\text{a2}}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{\text{a1}+\text{a2}}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{\text{a1}+\text{a2}}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{\text{a1}+\text{a2}+bx^2+cx^4}}\right.}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)}$$

$$\Big\downarrow \text{1509}$$

$$\frac{x\left(-2c(\text{a1}+\text{a2})+b^2+bcx^2\right)}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)\sqrt{\text{a1}+\text{a2}+bx^2+cx^4}}-\frac{c\left(\frac{(\text{a1}+\text{a2})^{3/4}\left(2\sqrt{\text{a1}+\text{a2}}+\frac{b}{\sqrt{c}}\right)\left(\frac{\sqrt{c}x^2}{\sqrt{\text{a1}+\text{a2}}}+1\right)\sqrt{\frac{\text{a1}+\text{a2}+bx^2+cx^4}{(\text{a1}+\text{a2})\left(\frac{\sqrt{c}x^2}{\sqrt{\text{a1}+\text{a2}}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{\text{a1}+\text{a2}}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{\text{a1}+\text{a2}}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{\text{a1}+\text{a2}+bx^2+cx^4}}\right.}{(\text{a1}+\text{a2})}$$

input
```
Int[(a1 + a2 + b*x^2 + c*x^4)^(-3/2),x]
```

output
```
(x*(b^2 - 2*(a1 + a2)*c + b*c*x^2))/((a1 + a2)*(b^2 - 4*(a1 + a2)*c)*Sqrt[
a1 + a2 + b*x^2 + c*x^4]) - (c*(-((Sqrt[a1 + a2]*b*(-((x*Sqrt[a1 + a2 + b*
x^2 + c*x^4])/((a1 + a2)*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2]))) + ((a1 + a2)^
(1/4)*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2])*Sqrt[(a1 + a2 + b*x^2 + c*x^4)/((a
1 + a2)*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*
x)/(a1 + a2)^(1/4)], (2 - b/(Sqrt[a1 + a2]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a1
+ a2 + b*x^2 + c*x^4])))/Sqrt[c]) + ((a1 + a2)^(3/4)*(2*Sqrt[a1 + a2] + b/
Sqrt[c])*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2])*Sqrt[(a1 + a2 + b*x^2 + c*x^4)/
((a1 + a2)*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2])^2)]*EllipticF[2*ArcTan[(c^(1/
4)*x)/(a1 + a2)^(1/4)], (2 - b/(Sqrt[a1 + a2]*Sqrt[c]))/4])/(2*c^(1/4)*Sqr
t[a1 + a2 + b*x^2 + c*x^4])))/((a1 + a2)*(b^2 - 4*(a1 + a2)*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.46

method	result
default	$-\frac{2c\left(\frac{b\,x^{3}}{2(\mathrm{a1}+\mathrm{a2})\left(4\,\mathrm{a1}\,c+4c\,\mathrm{a2}-b^{2}\right)}-\frac{\left(2\,\mathrm{a1}\,c+2c\,\mathrm{a2}-b^{2}\right)x}{2(\mathrm{a1}+\mathrm{a2})\left(4\,\mathrm{a1}\,c+4c\,\mathrm{a2}-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{\mathrm{a1}+\mathrm{a2}}{c}\right)c}}+\frac{\left(\frac{1}{\mathrm{a1}+\mathrm{a2}}-\frac{2\,\mathrm{a1}\,c+2c\,\mathrm{a2}-b^{2}}{(\mathrm{a1}+\mathrm{a2})\left(4\,\mathrm{a1}\,c+4c\,\mathrm{a2}-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4\,\mathrm{a1}\,\ldots}\right.}{\mathrm{a1}\ldots}}}{\ldots}$
elliptic	$-\frac{2c\left(\frac{b\,x^{3}}{2(\mathrm{a1}+\mathrm{a2})\left(4\,\mathrm{a1}\,c+4c\,\mathrm{a2}-b^{2}\right)}-\frac{\left(2\,\mathrm{a1}\,c+2c\,\mathrm{a2}-b^{2}\right)x}{2(\mathrm{a1}+\mathrm{a2})\left(4\,\mathrm{a1}\,c+4c\,\mathrm{a2}-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{\mathrm{a1}+\mathrm{a2}}{c}\right)c}}+\frac{\left(\frac{1}{\mathrm{a1}+\mathrm{a2}}-\frac{2\,\mathrm{a1}\,c+2c\,\mathrm{a2}-b^{2}}{(\mathrm{a1}+\mathrm{a2})\left(4\,\mathrm{a1}\,c+4c\,\mathrm{a2}-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4\,\mathrm{a1}\,\ldots}\right.}{\mathrm{a1}\ldots}}}{\ldots}$

input
```
int(1/(c*x^4+b*x^2+a1+a2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(1/2/(a1+a2)*b/(4*a1*c+4*a2*c-b^2)*x^3-1/2*(2*a1*c+2*a2*c-b^2)/(a1+a2
)/(4*a1*c+4*a2*c-b^2)/c*x)/((x^4+1/c*b*x^2+(a1+a2)/c)*c)^(1/2)+1/4*(1/(a1+
a2)-(2*a1*c+2*a2*c-b^2)/(a1+a2)/(4*a1*c+4*a2*c-b^2))*2^(1/2)/((-b+(-4*a1*c
-4*a2*c+b^2)^(1/2))/(a1+a2))^(1/2)*(4-2*(-b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a
1+a2)*x^2)^(1/2)*(4+2*(b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2)*x^2)^(1/2)/(c
*x^4+b*x^2+a1+a2)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a1*c-4*a2*c+b^2)^
(1/2))/(a1+a2))^(1/2),1/2*(-4+2*b*(b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2)/c
)^(1/2))-1/2*b/(4*a1*c+4*a2*c-b^2)*c*2^(1/2)/((-b+(-4*a1*c-4*a2*c+b^2)^(1/
2))/(a1+a2))^(1/2)*(4-2*(-b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2)*x^2)^(1/2)
*(4+2*(b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2)*x^2)^(1/2)/(c*x^4+b*x^2+a1+a2
)^(1/2)/(b+(-4*a1*c-4*a2*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a
1*c-4*a2*c+b^2)^(1/2))/(a1+a2))^(1/2),1/2*(-4+2*b*(b+(-4*a1*c-4*a2*c+b^2)^
(1/2))/(a1+a2)/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a1*c-4*a2*c+b^2)
^(1/2))/(a1+a2))^(1/2),1/2*(-4+2*b*(b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2)/
c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(342) = 684$.

Time = 0.09 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.86

$$\int \frac{1}{(\mathrm{a1} + \mathrm{a2} + bx^2 + cx^4)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a1+a2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(b^2*c*x^4 + b^3*x^2 + (a1 + a2)*b^2 - ((a1 + a2)*b*c*x^4
+ (a1 + a2)*b^2*x^2 + (a1^2 + 2*a1*a2 + a2^2)*b)*sqrt((b^2 - 4*(a1 + a2)*c
)/(a1^2 + 2*a1*a2 + a2^2)))*sqrt(a1 + a2)*sqrt(((a1 + a2)*sqrt((b^2 - 4*(a
1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2)) - b)/(a1 + a2))*elliptic_e(arcsin(sqrt
(1/2)*x*sqrt(((a1 + a2)*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2)
) - b)/(a1 + a2))), 1/2*((a1 + a2)*b*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2*
a1*a2 + a2^2)) + b^2 - 2*(a1 + a2)*c)/((a1 + a2)*c)) - sqrt(1/2)*((2*(a1 +
 a2)*b + b^2)*c*x^4 + (a1 + a2)*b^2 + (2*(a1 + a2)*b^2 + b^3)*x^2 + 2*(a1^
2 + 2*a1*a2 + a2^2)*b + ((2*a1^2 + 4*a1*a2 + 2*a2^2 - (a1 + a2)*b)*c*x^4 +
 2*a1^3 + 6*a1^2*a2 + 6*a1*a2^2 + 2*a2^3 - ((a1 + a2)*b^2 - 2*(a1^2 + 2*a1
*a2 + a2^2)*b)*x^2 - (a1^2 + 2*a1*a2 + a2^2)*b)*sqrt((b^2 - 4*(a1 + a2)*c)
/(a1^2 + 2*a1*a2 + a2^2)))*sqrt(a1 + a2)*sqrt(((a1 + a2)*sqrt((b^2 - 4*(a1
 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2)) - b)/(a1 + a2))*elliptic_f(arcsin(sqrt(
1/2)*x*sqrt(((a1 + a2)*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2))
 - b)/(a1 + a2))), 1/2*((a1 + a2)*b*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2*a
1*a2 + a2^2)) + b^2 - 2*(a1 + a2)*c)/((a1 + a2)*c)) - 2*((a1 + a2)*b*c*x^3
 + ((a1 + a2)*b^2 - 2*(a1^2 + 2*a1*a2 + a2^2)*c)*x)*sqrt(c*x^4 + b*x^2 + a
1 + a2))/(((a1^2 + 2*a1*a2 + a2^2)*b^2*c - 4*(a1^3 + 3*a1^2*a2 + 3*a1*a2^2
 + a2^3)*c^2)*x^4 + (a1^3 + 3*a1^2*a2 + 3*a1*a2^2 + a2^3)*b^2 + ((a1^2 + 2
*a1*a2 + a2^2)*b^3 - 4*(a1^3 + 3*a1^2*a2 + 3*a1*a2^2 + a2^3)*b*c)*x^2 -...
```

Sympy [F]

$$\int \frac{1}{(\mathrm{a1} + \mathrm{a2} + bx^2 + cx^4)^{3/2}}\, dx = \int \frac{1}{(a_1 + a_2 + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(c*x**4+b*x**2+a1+a2)**(3/2),x)
```

output
```
Integral((a1 + a2 + b*x**2 + c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(\mathrm{a1} + \mathrm{a2} + bx^2 + cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4 + bx^2 + a_1 + a_2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a1+a2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a1 + a2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(\mathrm{a1} + \mathrm{a2} + bx^2 + cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4 + bx^2 + a_1 + a_2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a1+a2)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a1 + a2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(\mathrm{a1} + \mathrm{a2} + bx^2 + cx^4)^{3/2}}\,dx = \int \frac{1}{(c\,x^4 + b\,x^2 + a_1 + a_2)^{3/2}}\,dx$$

input
```
int(1/(a1 + a2 + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(1/(a1 + a2 + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(\mathrm{a1} + \mathrm{a2} + bx^2 + cx^4)^{3/2}}\, dx = \int \frac{\sqrt{c\,x^4 + b\,x^2 + a1 + a2}}{c^2x^8 + 2bc\,x^6 + 2a1\,c\,x^4 + 2a2\,c\,x^4 + b^2x^4 + 2a1\,b\,x^2 + 2a2\,b\,x^2 + a1^2}$$

input
```
int(1/(c*x^4+b*x^2+a1+a2)^(3/2),x)
```

output
```
int(sqrt(a1 + a2 + b*x**2 + c*x**4)/(a1**2 + 2*a1*a2 + 2*a1*b*x**2 + 2*a1*
c*x**4 + a2**2 + 2*a2*b*x**2 + 2*a2*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2
*x**8),x)
```

3.316 $\int \frac{1}{\left(\mathbf{a1}+\mathbf{a2}-bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 410

$$\int \frac{1}{(\mathrm{a1}+\mathrm{a2}-bx^2+cx^4)^{3/2}}\,dx = \frac{x(b^2-2(\mathrm{a1}+\mathrm{a2})c-bcx^2)}{(\mathrm{a1}+\mathrm{a2})\left(b^2-4(\mathrm{a1}+\mathrm{a2})c\right)\sqrt{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}}$$
$$+\frac{b\sqrt{c}x\sqrt{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}}{(\mathrm{a1}+\mathrm{a2})\left(b^2-4(\mathrm{a1}+\mathrm{a2})c\right)\left(\sqrt{\mathrm{a1}+\mathrm{a2}}+\sqrt{c}x^2\right)}$$
$$-\frac{b\sqrt[4]{c}\left(\sqrt{\mathrm{a1}+\mathrm{a2}}+\sqrt{c}x^2\right)\sqrt{\frac{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}{\left(\sqrt{\mathrm{a1}+\mathrm{a2}}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{\mathrm{a1}+\mathrm{a2}}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{\mathrm{a1}+\mathrm{a2}}\sqrt{c}}\right)\right)}{(\mathrm{a1}+\mathrm{a2})^{3/4}\left(b^2-4(\mathrm{a1}+\mathrm{a2})c\right)\sqrt{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}}$$
$$+\frac{\left(b-2\sqrt{\mathrm{a1}+\mathrm{a2}}\sqrt{c}\right)\sqrt[4]{c}\left(\sqrt{\mathrm{a1}+\mathrm{a2}}+\sqrt{c}x^2\right)\sqrt{\frac{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}{\left(\sqrt{\mathrm{a1}+\mathrm{a2}}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{\mathrm{a1}+\mathrm{a2}}}\right),\frac{1}{4}\left(2+\right.\right.}{2(\mathrm{a1}+\mathrm{a2})^{3/4}\left(b^2-4(\mathrm{a1}+\mathrm{a2})c\right)\sqrt{\mathrm{a1}+\mathrm{a2}-bx^2+cx^4}}$$

output
```
x*(b^2-2*(a1+a2)*c-b*c*x^2)/(a1+a2)/(b^2-4*(a1+a2)*c)/(c*x^4-b*x^2+a1+a2)^
(1/2)+b*c^(1/2)*x*(c*x^4-b*x^2+a1+a2)^(1/2)/(a1+a2)/(b^2-4*(a1+a2)*c)/(c^(
1/2)*x^2+(a1+a2)^(1/2))-b*c^(1/4)*(c^(1/2)*x^2+(a1+a2)^(1/2))*((c*x^4-b*x^
2+a1+a2)/(c^(1/2)*x^2+(a1+a2)^(1/2))^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/
4)*x/(a1+a2)^(1/4))),1/2*(2+b/(a1+a2)^(1/2)/c^(1/2))^(1/2))/(a1+a2)^(3/4)/
(b^2-4*(a1+a2)*c)/(c*x^4-b*x^2+a1+a2)^(1/2)+1/2*(b-2*(a1+a2)^(1/2)*c^(1/2)
)*c^(1/4)*(c^(1/2)*x^2+(a1+a2)^(1/2))*((c*x^4-b*x^2+a1+a2)/(c^(1/2)*x^2+(a
1+a2)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/(a1+a2)^(1/4)),1/
2*(2+b/(a1+a2)^(1/2)/c^(1/2))^(1/2))/(a1+a2)^(3/4)/(b^2-4*(a1+a2)*c)/(c*x^
4-b*x^2+a1+a2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.94 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.25

$$\int \frac{1}{(\mathrm{a1}+\mathrm{a2}-bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{\frac{c}{-b+\sqrt{b^2-4(\mathrm{a1}+\mathrm{a2})c}}}\,x(-b^2+2(\mathrm{a1}+\mathrm{a2})c+bcx^2)-i\sqrt{2}b\left(b+\sqrt{b^2-4(\mathrm{a1}+\mathrm{a2})c}\right)\sqrt{\frac{b+\sqrt{b^2-4(\mathrm{a1}+\mathrm{a2})c}-2}{b+\sqrt{b^2-4(\mathrm{a1}+\mathrm{a2})}}}}{}$$

input
```
Integrate[(a1 + a2 - b*x^2 + c*x^4)^(-3/2),x]
```

output
```
-1/4*(4*Sqrt[c/(-b + Sqrt[b^2 - 4*(a1 + a2)*c])]*x*(-b^2 + 2*(a1 + a2)*c +
 b*c*x^2) - I*Sqrt[2]*b*(b + Sqrt[b^2 - 4*(a1 + a2)*c])*Sqrt[(b + Sqrt[b^2
 - 4*(a1 + a2)*c] - 2*c*x^2)/(b + Sqrt[b^2 - 4*(a1 + a2)*c])]*Sqrt[(-b + S
qrt[b^2 - 4*(a1 + a2)*c] + 2*c*x^2)/(-b + Sqrt[b^2 - 4*(a1 + a2)*c])]*Elli
pticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*(a1 + a2)*c])]*x], (b -
Sqrt[b^2 - 4*(a1 + a2)*c])/(b + Sqrt[b^2 - 4*(a1 + a2)*c])] + I*Sqrt[2]*(b
^2 - 4*(a1 + a2)*c + b*Sqrt[b^2 - 4*(a1 + a2)*c])*Sqrt[(b + Sqrt[b^2 - 4*(
a1 + a2)*c] - 2*c*x^2)/(b + Sqrt[b^2 - 4*(a1 + a2)*c])]*Sqrt[(-b + Sqrt[b^
2 - 4*(a1 + a2)*c] + 2*c*x^2)/(-b + Sqrt[b^2 - 4*(a1 + a2)*c])]*EllipticF[
I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*(a1 + a2)*c])]*x], (b - Sqrt[b
^2 - 4*(a1 + a2)*c])/(b + Sqrt[b^2 - 4*(a1 + a2)*c])])/((a1 + a2)*(b^2 - 4
*(a1 + a2)*c)*Sqrt[c/(-b + Sqrt[b^2 - 4*(a1 + a2)*c])]*Sqrt[a1 + a2 - b*x^
2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1405, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\text{a1}+\text{a2}-bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(-2c(\text{a1}+\text{a2})+b^2-bcx^2)}{(\text{a1}+\text{a2})(b^2-4c(\text{a1}+\text{a2}))\sqrt{\text{a1}+\text{a2}-bx^2+cx^4}}-\frac{\int \frac{c(2(\text{a1}+\text{a2})-bx^2)}{\sqrt{cx^4-bx^2+\text{a1}+\text{a2}}}dx}{(\text{a1}+\text{a2})(b^2-4c(\text{a1}+\text{a2}))}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(-2c(\text{a1}+\text{a2})+b^2-bcx^2)}{(\text{a1}+\text{a2})(b^2-4c(\text{a1}+\text{a2}))\sqrt{\text{a1}+\text{a2}-bx^2+cx^4}}-\frac{c\int \frac{2(\text{a1}+\text{a2})-bx^2}{\sqrt{cx^4-bx^2+\text{a1}+\text{a2}}}dx}{(\text{a1}+\text{a2})(b^2-4c(\text{a1}+\text{a2}))}$$

$$\Big\downarrow \text{1511}$$

$$\frac{x\left(-2c(\text{a1}+\text{a2})+b^2-bcx^2\right)}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)\sqrt{\text{a1}+\text{a2}-bx^2+cx^4}}-\frac{c\left(\sqrt{\text{a1}+\text{a2}}\left(2\sqrt{\text{a1}+\text{a2}}-\frac{b}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^4-bx^2+\text{a1}+\text{a2}}}dx+\frac{b\sqrt{\text{a1}+\text{a2}}\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{\text{a1}+\text{a2}}}}{\sqrt{cx^4-bx^2+\text{a1}+\text{a2}}}dx}{\sqrt{c}}\right)}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)}$$

$\Big\downarrow$ 1416

$$\frac{x\left(-2c(\text{a1}+\text{a2})+b^2-bcx^2\right)}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)\sqrt{\text{a1}+\text{a2}-bx^2+cx^4}}-\frac{c\left(\frac{b\sqrt{\text{a1}+\text{a2}}\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{\text{a1}+\text{a2}}}}{\sqrt{cx^4-bx^2+\text{a1}+\text{a2}}}dx}{\sqrt{c}}+\frac{(\text{a1}+\text{a2})^{3/4}\left(2\sqrt{\text{a1}+\text{a2}}-\frac{b}{\sqrt{c}}\right)\left(\frac{\sqrt{c}x^2}{\sqrt{\text{a1}+\text{a2}}}+1\right)\sqrt{\frac{\text{a1}+\text{a2}-bx^2+cx^4}{(\text{a1}+\text{a2})\left(\frac{\sqrt{c}x^2}{\sqrt{\text{a1}+\text{a2}}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\right.\right.}{2\sqrt[4]{c}\sqrt{\text{a1}+\text{a2}-bx^2+cx^4}}\right.}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)}$$

$\Big\downarrow$ 1509

$$\frac{x\left(-2c(\text{a1}+\text{a2})+b^2-bcx^2\right)}{(\text{a1}+\text{a2})\left(b^2-4c(\text{a1}+\text{a2})\right)\sqrt{\text{a1}+\text{a2}-bx^2+cx^4}}-\frac{c\left(\frac{(\text{a1}+\text{a2})^{3/4}\left(2\sqrt{\text{a1}+\text{a2}}-\frac{b}{\sqrt{c}}\right)\left(\frac{\sqrt{c}x^2}{\sqrt{\text{a1}+\text{a2}}}+1\right)\sqrt{\frac{\text{a1}+\text{a2}-bx^2+cx^4}{(\text{a1}+\text{a2})\left(\frac{\sqrt{c}x^2}{\sqrt{\text{a1}+\text{a2}}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{\text{a1}+\text{a2}}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{\text{a1}+\text{a2}}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{c}\sqrt{\text{a1}+\text{a2}-bx^2+cx^4}}\right.}{(\text{a1}+\text{a2}}$$

input
```
Int[(a1 + a2 - b*x^2 + c*x^4)^(-3/2),x]
```

output
```
(x*(b^2 - 2*(a1 + a2)*c - b*c*x^2))/((a1 + a2)*(b^2 - 4*(a1 + a2)*c)*Sqrt[
a1 + a2 - b*x^2 + c*x^4]) - (c*((Sqrt[a1 + a2]*b*(-((x*Sqrt[a1 + a2 - b*x^
2 + c*x^4])/((a1 + a2)*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2]))) + ((a1 + a2)^(1
/4)*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2])*Sqrt[(a1 + a2 - b*x^2 + c*x^4)/((a1
+ a2)*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*x)
/(a1 + a2)^(1/4)], (2 + b/(Sqrt[a1 + a2]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a1 +
a2 - b*x^2 + c*x^4])))/Sqrt[c] + ((a1 + a2)^(3/4)*(2*Sqrt[a1 + a2] - b/Sqr
t[c])*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2])*Sqrt[(a1 + a2 - b*x^2 + c*x^4)/((a
1 + a2)*(1 + (Sqrt[c]*x^2)/Sqrt[a1 + a2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*
x)/(a1 + a2)^(1/4)], (2 + b/(Sqrt[a1 + a2]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a
1 + a2 - b*x^2 + c*x^4])))/((a1 + a2)*(b^2 - 4*(a1 + a2)*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.45

method	result
default	$-\frac{2c\left(-\frac{b\,x^{3}}{2(\mathrm{a1}+\mathrm{a2})\left(4\,\mathrm{a1}\,c+4c\,\mathrm{a2}-b^{2}\right)}-\frac{\left(2\,\mathrm{a1}\,c+2c\,\mathrm{a2}-b^{2}\right)x}{2(\mathrm{a1}+\mathrm{a2})\left(4\,\mathrm{a1}\,c+4c\,\mathrm{a2}-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}-\frac{b\,x^{2}}{c}+\frac{\mathrm{a1}+\mathrm{a2}}{c}\right)c}}+\frac{\left(\frac{1}{\mathrm{a1}+\mathrm{a2}}-\frac{2\,\mathrm{a1}\,c+2c\,\mathrm{a2}-b^{2}}{(\mathrm{a1}+\mathrm{a2})\left(4\,\mathrm{a1}\,c+4c\,\mathrm{a2}-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(b+\sqrt{-4\,\mathrm{a1}\right.}{\mathrm{a1}}}}{}$
elliptic	$-\frac{2c\left(-\frac{b\,x^{3}}{2(\mathrm{a1}+\mathrm{a2})\left(4\,\mathrm{a1}\,c+4c\,\mathrm{a2}-b^{2}\right)}-\frac{\left(2\,\mathrm{a1}\,c+2c\,\mathrm{a2}-b^{2}\right)x}{2(\mathrm{a1}+\mathrm{a2})\left(4\,\mathrm{a1}\,c+4c\,\mathrm{a2}-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}-\frac{b\,x^{2}}{c}+\frac{\mathrm{a1}+\mathrm{a2}}{c}\right)c}}+\frac{\left(\frac{1}{\mathrm{a1}+\mathrm{a2}}-\frac{2\,\mathrm{a1}\,c+2c\,\mathrm{a2}-b^{2}}{(\mathrm{a1}+\mathrm{a2})\left(4\,\mathrm{a1}\,c+4c\,\mathrm{a2}-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(b+\sqrt{-4\,\mathrm{a1}\right.}{\mathrm{a1}}}}{}$

input
```
int(1/(c*x^4-b*x^2+a1+a2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/2/(a1+a2)*b/(4*a1*c+4*a2*c-b^2)*x^3-1/2*(2*a1*c+2*a2*c-b^2)/(a1+a
2)/(4*a1*c+4*a2*c-b^2)/c*x)/((x^4-1/c*b*x^2+(a1+a2)/c)*c)^(1/2)+1/4*(1/(a1
+a2)-(2*a1*c+2*a2*c-b^2)/(a1+a2)/(4*a1*c+4*a2*c-b^2))*2^(1/2)/((b+(-4*a1*c
-4*a2*c+b^2)^(1/2))/(a1+a2))^(1/2)*(4-2*(b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1
+a2)*x^2)^(1/2)*(4+2*(-b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2)*x^2)^(1/2)/(c
*x^4-b*x^2+a1+a2)^(1/2)*EllipticF(1/2*x*2^(1/2)*((b+(-4*a1*c-4*a2*c+b^2)^(
1/2))/(a1+a2))^(1/2),1/2*(-4-2*b*(-b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2)/c
)^(1/2))+1/2*b/(4*a1*c+4*a2*c-b^2)*c*2^(1/2)/((b+(-4*a1*c-4*a2*c+b^2)^(1/2
))/(a1+a2))^(1/2)*(4-2*(b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2)*x^2)^(1/2)*(
4+2*(-b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2)*x^2)^(1/2)/(c*x^4-b*x^2+a1+a2)
^(1/2)/(-b+(-4*a1*c-4*a2*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((b+(-4*a1
*c-4*a2*c+b^2)^(1/2))/(a1+a2))^(1/2),1/2*(-4-2*b*(-b+(-4*a1*c-4*a2*c+b^2)^
(1/2))/(a1+a2)/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((b+(-4*a1*c-4*a2*c+b^2)^
(1/2))/(a1+a2))^(1/2),1/2*(-4-2*b*(-b+(-4*a1*c-4*a2*c+b^2)^(1/2))/(a1+a2)/
c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(349) = 698$.

Time = 0.09 (sec) , antiderivative size = 759, normalized size of antiderivative = 1.85

$$\int \frac{1}{(\mathrm{a1} + \mathrm{a2} - bx^2 + cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4-b*x^2+a1+a2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(b^2*c*x^4 - b^3*x^2 + (a1 + a2)*b^2 + ((a1 + a2)*b*c*x^4
- (a1 + a2)*b^2*x^2 + (a1^2 + 2*a1*a2 + a2^2)*b)*sqrt((b^2 - 4*(a1 + a2)*c
)/(a1^2 + 2*a1*a2 + a2^2)))*sqrt(a1 + a2)*sqrt(((a1 + a2)*sqrt((b^2 - 4*(a
1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2)) + b)/(a1 + a2))*elliptic_e(arcsin(sqrt
(1/2)*x*sqrt(((a1 + a2)*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2)
) + b)/(a1 + a2))), -1/2*((a1 + a2)*b*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2
*a1*a2 + a2^2)) - b^2 + 2*(a1 + a2)*c)/((a1 + a2)*c)) + sqrt(1/2)*((2*(a1
+ a2)*b - b^2)*c*x^4 - (a1 + a2)*b^2 - (2*(a1 + a2)*b^2 - b^3)*x^2 + 2*(a1
^2 + 2*a1*a2 + a2^2)*b - ((2*a1^2 + 4*a1*a2 + 2*a2^2 + (a1 + a2)*b)*c*x^4
+ 2*a1^3 + 6*a1^2*a2 + 6*a1*a2^2 + 2*a2^3 - ((a1 + a2)*b^2 + 2*(a1^2 + 2*a
1*a2 + a2^2)*b)*x^2 + (a1^2 + 2*a1*a2 + a2^2)*b)*sqrt((b^2 - 4*(a1 + a2)*c
)/(a1^2 + 2*a1*a2 + a2^2)))*sqrt(a1 + a2)*sqrt(((a1 + a2)*sqrt((b^2 - 4*(a
1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2)) + b)/(a1 + a2))*elliptic_f(arcsin(sqrt
(1/2)*x*sqrt(((a1 + a2)*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2*a1*a2 + a2^2)
) + b)/(a1 + a2))), -1/2*((a1 + a2)*b*sqrt((b^2 - 4*(a1 + a2)*c)/(a1^2 + 2
*a1*a2 + a2^2)) - b^2 + 2*(a1 + a2)*c)/((a1 + a2)*c)) + 2*((a1 + a2)*b*c*x
^3 - ((a1 + a2)*b^2 - 2*(a1^2 + 2*a1*a2 + a2^2)*c)*x)*sqrt(c*x^4 - b*x^2 +
 a1 + a2))/(((a1^2 + 2*a1*a2 + a2^2)*b^2*c - 4*(a1^3 + 3*a1^2*a2 + 3*a1*a2
^2 + a2^3)*c^2)*x^4 + (a1^3 + 3*a1^2*a2 + 3*a1*a2^2 + a2^3)*b^2 - ((a1^2 +
 2*a1*a2 + a2^2)*b^3 - 4*(a1^3 + 3*a1^2*a2 + 3*a1*a2^2 + a2^3)*b*c)*x^2...
```

Sympy [F]

$$\int \frac{1}{\left(\mathrm{a1}+\mathrm{a2}-bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(a_1+a_2-bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**4-b*x**2+a1+a2)**(3/2),x)
```

output
```
Integral((a1 + a2 - b*x**2 + c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(\mathrm{a1}+\mathrm{a2}-bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4-bx^2+a_1+a_2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4-b*x^2+a1+a2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 - b*x^2 + a1 + a2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(\mathrm{a1}+\mathrm{a2}-bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4-bx^2+a_1+a_2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4-b*x^2+a1+a2)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 - b*x^2 + a1 + a2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(\mathrm{a1}+\mathrm{a2}-bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(c\,x^4-b\,x^2+a_1+a_2)^{3/2}}\,dx$$

input
```
int(1/(a1 + a2 - b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(1/(a1 + a2 - b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(\mathrm{a1}+\mathrm{a2}-bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^4-b\,x^2+a1+a2}}{c^2x^8-2bc\,x^6+2a1\,c\,x^4+2a2\,c\,x^4+b^2x^4-2a1\,b\,x^2-2a2\,b\,x^2+a1^2}$$

input
```
int(1/(c*x^4-b*x^2+a1+a2)^(3/2),x)
```

output
```
int(sqrt(a1 + a2 - b*x**2 + c*x**4)/(a1**2 + 2*a1*a2 - 2*a1*b*x**2 + 2*a1*
c*x**4 + a2**2 - 2*a2*b*x**2 + 2*a2*c*x**4 + b**2*x**4 - 2*b*c*x**6 + c**2
*x**8),x)
```

3.317 $\int \frac{1}{(a+bx^2-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 400

$$\int \frac{1}{(a+bx^2-cx^4)^{3/2}}\,dx = \frac{x(b^2+2ac-bcx^2)}{a(b^2+4ac)\sqrt{a+bx^2-cx^4}} - \frac{b\sqrt{c}x\sqrt{a+bx^2-cx^4}}{a(b^2+4ac)\left(\sqrt{-a}+\sqrt{c}x^2\right)}$$
$$+\frac{b\sqrt[4]{c}\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)\sqrt{\frac{a+bx^2-cx^4}{a\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{-a}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{-a}\sqrt{c}}\right)\right)}{\sqrt[4]{-a}(b^2+4ac)\sqrt{a+bx^2-cx^4}}$$
$$-\frac{\left(b-2\sqrt{-a}\sqrt{c}\right)\sqrt[4]{c}\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)\sqrt{\frac{a+bx^2-cx^4}{a\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{-a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{-a}\sqrt{c}}\right)\right)}{2\sqrt[4]{-a}(b^2+4ac)\sqrt{a+bx^2-cx^4}}$$

output
```
x*(-b*c*x^2+2*a*c+b^2)/a/(4*a*c+b^2)/(-c*x^4+b*x^2+a)^(1/2)-b*c^(1/2)*x*(-
c*x^4+b*x^2+a)^(1/2)/a/(4*a*c+b^2)/(c^(1/2)*x^2+(-a)^(1/2))+b*c^(1/4)*(1+c
^(1/2)*x^2/(-a)^(1/2))*((-c*x^4+b*x^2+a)/a/(1+c^(1/2)*x^2/(-a)^(1/2))^2)^(
1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/(-a)^(1/4))),1/2*(2+b/(-a)^(1/2)/c^(
1/2))^(1/2))/(-a)^(1/4)/(4*a*c+b^2)/(-c*x^4+b*x^2+a)^(1/2)-1/2*(b-2*(-a)^(
1/2)*c^(1/2))*c^(1/4)*(1+c^(1/2)*x^2/(-a)^(1/2))*((-c*x^4+b*x^2+a)/a/(1+c^
(1/2)*x^2/(-a)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/(-a)^(1/
4)),1/2*(2+b/(-a)^(1/2)/c^(1/2))^(1/2))/(-a)^(1/4)/(4*a*c+b^2)/(-c*x^4+b*x
^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a+bx^2-cx^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x(b^2+2ac-bcx^2)-i\sqrt{2}b\left(-b+\sqrt{b^2+4ac}\right)\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{}$$

input
```
Integrate[(a + b*x^2 - c*x^4)^(-3/2),x]
```

output
```
(4*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x*(b^2 + 2*a*c - b*c*x^2) - I*Sqrt[2
]*b*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + S
qrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2
+ 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*
x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-b^2 - 4
*a*c + b*Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sq
rt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 +
 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x
], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(4*a*(b^2 + 4*a*c)*Sq
rt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1405, 25, 27, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2-cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{a+bx^2-cx^4}}-\frac{\int -\frac{c\left(bx^2+2a\right)}{\sqrt{-cx^4+bx^2+a}}dx}{a\left(4ac+b^2\right)}$$

$\downarrow$ 25

$$\frac{\int \frac{c\left(bx^2+2a\right)}{\sqrt{-cx^4+bx^2+a}}dx}{a\left(4ac+b^2\right)}+\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{a+bx^2-cx^4}}$$

$\downarrow$ 27

$$\frac{c\int \frac{bx^2+2a}{\sqrt{-cx^4+bx^2+a}}dx}{a\left(4ac+b^2\right)}+\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{a+bx^2-cx^4}}$$

$\downarrow$ 1514

$$\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int \frac{bx^2+2a}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{a\left(4ac+b^2\right)\sqrt{a+bx^2-cx^4}}+\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{a+bx^2-cx^4}}$$

$\downarrow$ 399

$$\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(-b\sqrt{4ac+b^2}+4ac+b^2\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{2c}-\frac{b\left(b-\sqrt{4ac+b^2}\right)\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+}}}}}{2c}\right.}{a\left(4ac+b^2\right)\sqrt{a+bx^2-cx^4}}$$

$$\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{a+bx^2-cx^4}}$$

$\downarrow$ 321

$$\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+4ac+b^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}}-\frac{b\left(b-\right.}{}\right.}{a\left(4ac+b^2\right)\sqrt{a+bx^2-cx^4}}$$

$$\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{a+bx^2-cx^4}}$$

$$\big\downarrow \ 327$$

$$\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+4ac+b^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}}-\frac{b\left(b-\cdots\right.}{\cdots}\right.}{a\left(4ac+b^2\right)\sqrt{a+bx^2-cx^4}}$$

$$\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{a+bx^2-cx^4}}$$

input
```
Int[(a + b*x^2 - c*x^4)^(-3/2),x]
```

output
```
(x*(b^2 + 2*a*c - b*c*x^2))/(a*(b^2 + 4*a*c)*Sqrt[a + b*x^2 - c*x^4]) + (c
*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*(-1/2*(b*(b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]
]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b +
Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2)) + (Sqrt[b +
 Sqrt[b^2 + 4*a*c]]*(b^2 + 4*a*c - b*Sqrt[b^2 + 4*a*c])*EllipticF[ArcSin[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(
b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2))))/(a*(b^2 + 4*a*c)*Sqrt[a + b
*x^2 - c*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.18

method	result
default	$\frac{2c\left(-\frac{b\,x^{3}}{2a\left(4ac+b^{2}\right)}+\frac{\left(2ac+b^{2}\right)x}{2a\left(4ac+b^{2}\right)c}\right)}{\sqrt{-\left(x^{4}-\frac{b\,x^{2}}{c}-\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac+b^{2}}{a\left(4ac+b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\cdots}{\cdots}}}{2\cdots}\right.}{4\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}\,\sqrt{-c\,x^{4}+b\,x^{2}+a}}$
elliptic	$\frac{2c\left(-\frac{b\,x^{3}}{2a\left(4ac+b^{2}\right)}+\frac{\left(2ac+b^{2}\right)x}{2a\left(4ac+b^{2}\right)c}\right)}{\sqrt{-\left(x^{4}-\frac{b\,x^{2}}{c}-\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac+b^{2}}{a\left(4ac+b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\cdots}{\cdots}}}{2\cdots}\right.}{4\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}\,\sqrt{-c\,x^{4}+b\,x^{2}+a}}$

input
```
int(1/(-c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(-1/2*b/a/(4*a*c+b^2)*x^3+1/2*(2*a*c+b^2)/a/(4*a*c+b^2)/c*x)/(-(x^4-1/
c*b*x^2-1/c*a)*c)^(1/2)+1/4*(1/a-(2*a*c+b^2)/a/(4*a*c+b^2))*2^(1/2)/((-b+(
4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(
b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2
^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/
a/c)^(1/2))-1/2*b/(4*a*c+b^2)*c*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(
4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2
)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^
(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*
b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+bx^2-cx^4)^{3/2}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left((abcx^4-ab^2x^2-a^2b)\sqrt{a}\sqrt{\frac{b^2+4\,ac}{a^2}}-(b^2cx^4-b^3x^2-ab^2)\sqrt{a}\right)\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}-b}{a}}E\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{b^2}}{}}\right.\right.}{}$$

input
```
integrate(1/(-c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*((a*b*c*x^4 - a*b^2*x^2 - a^2*b)*sqrt(a)*sqrt((b^2 + 4*a*c
)/a^2) - (b^2*c*x^4 - b^3*x^2 - a*b^2)*sqrt(a))*sqrt((a*sqrt((b^2 + 4*a*c)
/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 + 4*a*c)/a^2
) - b)/a)), -1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) + b^2 + 2*a*c)/(a*c)) + sqrt
(1/2)*(((2*a^2 - a*b)*c*x^4 - 2*a^3 + a^2*b - (2*a^2*b - a*b^2)*x^2)*sqrt(
a)*sqrt((b^2 + 4*a*c)/a^2) + ((2*a*b + b^2)*c*x^4 - 2*a^2*b - a*b^2 - (2*a
*b^2 + b^3)*x^2)*sqrt(a))*sqrt((a*sqrt((b^2 + 4*a*c)/a^2) - b)/a)*elliptic
_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 + 4*a*c)/a^2) - b)/a)), -1/2*(a*b*
sqrt((b^2 + 4*a*c)/a^2) + b^2 + 2*a*c)/(a*c)) + 2*(a*b*c*x^3 - (a*b^2 + 2*
a^2*c)*x)*sqrt(-c*x^4 + b*x^2 + a))/(a^3*b^2 + 4*a^4*c - (a^2*b^2*c + 4*a^
3*c^2)*x^4 + (a^2*b^3 + 4*a^3*b*c)*x^2)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2-cx^4)^{3/2}}\,dx = \int \frac{1}{(a+bx^2-cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((a + b*x**2 - c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2-cx^4)^{3/2}}\,dx = \int \frac{1}{(-cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 + b*x^2 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2-cx^4)^{3/2}}\,dx = \int \frac{1}{(-cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + b*x^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2-cx^4)^{3/2}}\,dx = \int \frac{1}{(-c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int(1/(a + b*x^2 - c*x^4)^(3/2),x)
```

output
```
int(1/(a + b*x^2 - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2-cx^4)^{3/2}}\,dx = \int \frac{\sqrt{-c\,x^4+b\,x^2+a}}{c^2x^8-2bc\,x^6-2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int(1/(-c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**2 - c*x**4)/(a**2 + 2*a*b*x**2 - 2*a*c*x**4 + b**2*x**4
- 2*b*c*x**6 + c**2*x**8),x)
```

3.318 $\int \frac{1}{(a-bx^2-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 407

$$\int \frac{1}{(a-bx^2-cx^4)^{3/2}}\,dx = \frac{x(b^2+2ac+bcx^2)}{a(b^2+4ac)\sqrt{a-bx^2-cx^4}}+\frac{b\sqrt{c}x\sqrt{a-bx^2-cx^4}}{a(b^2+4ac)\left(\sqrt{-a}+\sqrt{c}x^2\right)}$$
$$-\frac{b\sqrt[4]{c}\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)\sqrt{\frac{a-bx^2-cx^4}{a\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{-a}}\right)|\frac{1}{4}\left(2+\frac{ab}{(-a)^{3/2}\sqrt{c}}\right)\right)}{\sqrt[4]{-a}(b^2+4ac)\sqrt{a-bx^2-cx^4}}$$
$$+\frac{\left(b+2\sqrt{-a}\sqrt{c}\right)\sqrt[4]{c}\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)\sqrt{\frac{a-bx^2-cx^4}{a\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{-a}}\right),\frac{1}{4}\left(2+\frac{ab}{(-a)^{3/2}\sqrt{c}}\right)\right)}{2\sqrt[4]{-a}(b^2+4ac)\sqrt{a-bx^2-cx^4}}$$

output
```
x*(b*c*x^2+2*a*c+b^2)/a/(4*a*c+b^2)/(-c*x^4-b*x^2+a)^(1/2)+b*c^(1/2)*x*(-c
*x^4-b*x^2+a)^(1/2)/a/(4*a*c+b^2)/(c^(1/2)*x^2+(-a)^(1/2))-b*c^(1/4)*(1+c^
(1/2)*x^2/(-a)^(1/2))*((-c*x^4-b*x^2+a)/a/(1+c^(1/2)*x^2/(-a)^(1/2))^2)^(1
/2)*EllipticE(sin(2*arctan(c^(1/4)*x/(-a)^(1/4))),1/2*(2+a*b/(-a)^(3/2)/c^
(1/2))^(1/2))/(-a)^(1/4)/(4*a*c+b^2)/(-c*x^4-b*x^2+a)^(1/2)+1/2*(b+2*(-a)^
(1/2)*c^(1/2))*c^(1/4)*(1+c^(1/2)*x^2/(-a)^(1/2))*((-c*x^4-b*x^2+a)/a/(1+c
^(1/2)*x^2/(-a)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/(-a)^(1
/4)),1/2*(2+a*b/(-a)^(3/2)/c^(1/2))^(1/2))/(-a)^(1/4)/(4*a*c+b^2)/(-c*x^4-
b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.67 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a-bx^2-cx^4)^{3/2}}\,dx = \frac{4\sqrt{\frac{c}{b-\sqrt{b^2+4ac}}}x(b^2+2ac+bcx^2)+ib\left(b+\sqrt{b^2+4ac}\right)\sqrt{\frac{b+\sqrt{b^2+4ac}+2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{\frac{2b-2\sqrt{b^2}}{b-\sqrt{}}}}{}$$

input
```
Integrate[(a - b*x^2 - c*x^4)^(-3/2),x]
```

output
```
(4*Sqrt[c/(b - Sqrt[b^2 + 4*a*c])]*x*(b^2 + 2*a*c + b*c*x^2) + I*b*(b + Sq
rt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 + 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]
*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b - Sqrt[b^2 + 4*a*c])]*x], (b - Sqrt
[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])] - I*(b^2 + 4*a*c + b*Sqrt[b^2 + 4*
a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt
[(2*b - 2*Sqrt[b^2 + 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*EllipticF[
I*ArcSinh[Sqrt[2]*Sqrt[c/(b - Sqrt[b^2 + 4*a*c])]*x], (b - Sqrt[b^2 + 4*a*
c])/(b + Sqrt[b^2 + 4*a*c])])/(4*a*(b^2 + 4*a*c)*Sqrt[c/(b - Sqrt[b^2 + 4*
a*c])]*Sqrt[a - x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 662, normalized size of antiderivative = 1.63, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1405, 25, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2-cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x\left(2ac+b^2+bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{a-bx^2-cx^4}}-\frac{\int -\frac{c\left(2a-bx^2\right)}{\sqrt{-cx^4-bx^2+a}}dx}{a\left(4ac+b^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{c\left(2a-bx^2\right)}{\sqrt{-cx^4-bx^2+a}}dx}{a\left(4ac+b^2\right)}+\frac{x\left(2ac+b^2+bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{a-bx^2-cx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{2a-bx^2}{\sqrt{-cx^4-bx^2+a}}dx}{a\left(4ac+b^2\right)}+\frac{x\left(2ac+b^2+bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{a-bx^2-cx^4}}$$

$$\Big\downarrow \; 1514$$

$$\frac{c\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\int \frac{2a-bx^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx}{a\left(4ac+b^2\right)\sqrt{a-bx^2-cx^4}}+\frac{x\left(2ac+b^2+bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{a-bx^2-cx^4}}$$

$$\Big\downarrow \; 406$$

$$\frac{c\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\left(2a\int \frac{1}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx-b\int \frac{x^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx\right)}{a\left(4ac+b^2\right)\sqrt{a-bx^2-cx^4}}+\frac{x\left(2ac+b^2+bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{a-bx^2-cx^4}}$$

$$\Big\downarrow \; 320$$

$$\frac{c\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\left(\frac{\sqrt{2}a\sqrt{\sqrt{4ac+b^2}+b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}-b\int \cdots\right)}{a\left(4ac+b^2\right)\sqrt{a-bx^2-cx^4}}+\frac{x\left(2ac+b^2+bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{a-bx^2-cx^4}}$$

$$\Big\downarrow \; 388$$

$$\frac{c\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\left(\frac{\sqrt{2}a\sqrt{\sqrt{4ac+b^2}+b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}-b\left(\frac{x(\ldots}{\ldots}\right.\right.}{a\,(4ac+b^2)\sqrt{a-bx^2-cx^4}}$$

$$\frac{x\left(2ac+b^2+bcx^2\right)}{a\,(4ac+b^2)\sqrt{a-bx^2-cx^4}}$$

$\downarrow$ 313

$$\frac{c\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\left(\frac{\sqrt{2}a\sqrt{\sqrt{4ac+b^2}+b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}-b\left(\frac{x(\ldots}{\ldots}\right.\right.}{a\,(4ac+b^2)\sqrt{a-b\ldots}}$$

$$\frac{x\left(2ac+b^2+bcx^2\right)}{a\,(4ac+b^2)\sqrt{a-bx^2-cx^4}}$$

input

```
Int[(a - b*x^2 - c*x^4)^(-3/2),x]
```

output

```
(x*(b^2 + 2*a*c + b*c*x^2))/(a*(b^2 + 4*a*c)*Sqrt[a - b*x^2 - c*x^4]) + (c
*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*(-(b*(((b - Sqrt[b^2 + 4*a*c])*x*Sqrt[1 + (2*c*x^2)/(b - Sq
rt[b^2 + 4*a*c])])/(2*c*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]) - ((b
 - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b -
Sqrt[b^2 + 4*a*c])]*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 + 4*a*c]]], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c
^(3/2)*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sq
rt[b^2 + 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]))) + (Sqrt[
2]*a*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]
)]*EllipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*
Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[c]*Sqrt[(1 + (2*c*x^2)/
(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])])))/(a*(b^2 + 4*a*c)*Sqrt[a - b*x^2 - c
*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.16

method	result
default	$\frac{2c\left(\frac{b\,x^3}{2a\left(4ac+b^2\right)}+\frac{\left(2ac+b^2\right)x}{2a\left(4ac+b^2\right)c}\right)}{\sqrt{-\left(x^4+\frac{b\,x^2}{c}-\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac+b^2}{a\left(4ac+b^2\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{b+\sqrt{4}}{}}}{2}\right.}{4\sqrt{\frac{b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4-b\,x^2+a}}$
elliptic	$\frac{2c\left(\frac{b\,x^3}{2a\left(4ac+b^2\right)}+\frac{\left(2ac+b^2\right)x}{2a\left(4ac+b^2\right)c}\right)}{\sqrt{-\left(x^4+\frac{b\,x^2}{c}-\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac+b^2}{a\left(4ac+b^2\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{b+\sqrt{4}}{}}}{2}\right.}{4\sqrt{\frac{b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4-b\,x^2+a}}$

input
```
int(1/(-c*x^4-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(1/2*b/a/(4*a*c+b^2)*x^3+1/2*(2*a*c+b^2)/a/(4*a*c+b^2)/c*x)/(-(x^4+1/c
*b*x^2-1/c*a)*c)^(1/2)+1/4*(1/a-(2*a*c+b^2)/a/(4*a*c+b^2))*2^(1/2)/((b+(4*
a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(-b+
(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4-b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(
1/2)*((b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(-b+(4*a*c+b^2)^(1/2))/a/
c)^(1/2))+1/2*b/(4*a*c+b^2)*c*2^(1/2)/((b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2
*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(-c*x^4-b*x^2+a)^(1/2)/(-b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*
((b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(-b+(4*a*c+b^2)^(1/2))/a/c)^(1
/2))-EllipticE(1/2*x*2^(1/2)*((b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
-b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a-bx^2-cx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(\left(abcx^4+ab^2x^2-a^2b\right)\sqrt{a}\sqrt{\frac{b^2+4\,ac}{a^2}}+\left(b^2cx^4+b^3x^2-ab^2\right)\sqrt{a}\right)\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}}{a}}}{}$$

input
```
integrate(1/(-c*x^4-b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*((a*b*c*x^4 + a*b^2*x^2 - a^2*b)*sqrt(a)*sqrt((b^2 + 4*a*c)
/a^2) + (b^2*c*x^4 + b^3*x^2 - a*b^2)*sqrt(a))*sqrt((a*sqrt((b^2 + 4*a*c)/
a^2) + b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 + 4*a*c)/a^2)
 + b)/a)), 1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) - b^2 - 2*a*c)/(a*c)) - sqrt(1
/2)*(((2*a^2 + a*b)*c*x^4 - 2*a^3 - a^2*b + (2*a^2*b + a*b^2)*x^2)*sqrt(a)
*sqrt((b^2 + 4*a*c)/a^2) - ((2*a*b - b^2)*c*x^4 - 2*a^2*b + a*b^2 + (2*a*b
^2 - b^3)*x^2)*sqrt(a))*sqrt((a*sqrt((b^2 + 4*a*c)/a^2) + b)/a)*elliptic_f
(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 + 4*a*c)/a^2) + b)/a)), 1/2*(a*b*sqr
t((b^2 + 4*a*c)/a^2) - b^2 - 2*a*c)/(a*c)) + 2*(a*b*c*x^3 + (a*b^2 + 2*a^2
*c)*x)*sqrt(-c*x^4 - b*x^2 + a))/(a^3*b^2 + 4*a^4*c - (a^2*b^2*c + 4*a^3*c
^2)*x^4 - (a^2*b^3 + 4*a^3*b*c)*x^2)
```

Sympy [F]

$$\int \frac{1}{(a-bx^2-cx^4)^{3/2}}\,dx = \int \frac{1}{(a-bx^2-cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*x**4-b*x**2+a)**(3/2),x)
```

output
```
Integral((a - b*x**2 - c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2-cx^4)^{3/2}}\,dx = \int \frac{1}{(-cx^4-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*x^4-b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 - b*x^2 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2-cx^4)^{3/2}}\,dx = \int \frac{1}{(-cx^4-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*x^4-b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 - b*x^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2-cx^4)^{3/2}}\,dx = \int \frac{1}{(-c\,x^4-b\,x^2+a)^{3/2}}\,dx$$

input
```
int(1/(a - b*x^2 - c*x^4)^(3/2),x)
```

output
```
int(1/(a - b*x^2 - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2-cx^4)^{3/2}}\,dx = \int \frac{\sqrt{-c\,x^4-b\,x^2+a}}{c^2x^8+2bc\,x^6-2ac\,x^4+b^2x^4-2ab\,x^2+a^2}dx$$

input
```
int(1/(-c*x^4-b*x^2+a)^(3/2),x)
```

output
```
int(sqrt(a - b*x**2 - c*x**4)/(a**2 - 2*a*b*x**2 - 2*a*c*x**4 + b**2*x**4
+ 2*b*c*x**6 + c**2*x**8),x)
```

3.319 $\int \frac{1}{(-a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 407

$$\begin{aligned}\int \frac{1}{(-a+bx^2+cx^4)^{3/2}}\,dx &= -\frac{x(b^2+2ac+bcx^2)}{a(b^2+4ac)\sqrt{-a+bx^2+cx^4}} \\ &+ \frac{b\sqrt{c}x\sqrt{-a+bx^2+cx^4}}{a(b^2+4ac)\left(\sqrt{-a}+\sqrt{c}x^2\right)} \\ &+ \frac{b\sqrt[4]{c}\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)\sqrt{\frac{a-bx^2-cx^4}{a\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{-a}}\right)|\frac{1}{4}\left(2+\frac{ab}{(-a)^{3/2}\sqrt{c}}\right)\right)}{\sqrt[4]{-a}(b^2+4ac)\sqrt{-a+bx^2+cx^4}} \\ &- \frac{\left(b+2\sqrt{-a}\sqrt{c}\right)\sqrt[4]{c}\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)\sqrt{\frac{a-bx^2-cx^4}{a\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{-a}}\right),\frac{1}{4}\left(2+\frac{ab}{(-a)^{3/2}\sqrt{c}}\right)\right)}{2\sqrt[4]{-a}(b^2+4ac)\sqrt{-a+bx^2+cx^4}}\end{aligned}$$

output
```
-x*(b*c*x^2+2*a*c+b^2)/a/(4*a*c+b^2)/(c*x^4+b*x^2-a)^(1/2)+b*c^(1/2)*x*(c*
x^4+b*x^2-a)^(1/2)/a/(4*a*c+b^2)/(c^(1/2)*x^2+(-a)^(1/2))+b*c^(1/4)*(1+c^(
1/2)*x^2/(-a)^(1/2))*((-c*x^4-b*x^2+a)/a/(1+c^(1/2)*x^2/(-a)^(1/2))^2)^(1/
2)*EllipticE(sin(2*arctan(c^(1/4)*x/(-a)^(1/4))),1/2*(2+a*b/(-a)^(3/2)/c^(
1/2))^(1/2))/(-a)^(1/4)/(4*a*c+b^2)/(c*x^4+b*x^2-a)^(1/2)-1/2*(b+2*(-a)^(1
/2)*c^(1/2))*c^(1/4)*(1+c^(1/2)*x^2/(-a)^(1/2))*((-c*x^4-b*x^2+a)/a/(1+c^(
1/2)*x^2/(-a)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/(-a)^(1/4
)),1/2*(2+a*b/(-a)^(3/2)/c^(1/2))^(1/2))/(-a)^(1/4)/(4*a*c+b^2)/(c*x^4+b*x
^2-a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.66 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.13

$$\int \frac{1}{(-a+bx^2+cx^4)^{3/2}}\,dx = \frac{-4\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}x(b^2+2ac+bcx^2)+ib\left(-b+\sqrt{b^2+4ac}\right)\sqrt{\frac{b+\sqrt{b^2+4ac}+2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{2\cdots}}{\cdots}$$

input
```
Integrate[(-a + b*x^2 + c*x^4)^(-3/2),x]
```

output
```
(-4*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x*(b^2 + 2*a*c + b*c*x^2) + I*b*(-b +
Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 +
4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 + 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 + 4*a*c]
)]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sq
rt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] - I*(-b^2 - 4*a*c + b*Sqrt[b^2 +
 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*S
qrt[(2*b - 2*Sqrt[b^2 + 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Ellipti
cF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 + 4
*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(4*a*(b^2 + 4*a*c)*Sqrt[c/(b + Sqrt[b^2 +
 4*a*c])]*Sqrt[-a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.63, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1405, 25, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{\int -\frac{c(2a-bx^2)}{\sqrt{cx^4+bx^2-a}}dx}{a(4ac+b^2)}-\frac{x(2ac+b^2+bcx^2)}{a(4ac+b^2)\sqrt{-a+bx^2+cx^4}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{c(2a-bx^2)}{\sqrt{cx^4+bx^2-a}}dx}{a(4ac+b^2)}-\frac{x(2ac+b^2+bcx^2)}{a(4ac+b^2)\sqrt{-a+bx^2+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\int \frac{2a-bx^2}{\sqrt{cx^4+bx^2-a}}dx}{a(4ac+b^2)}-\frac{x(2ac+b^2+bcx^2)}{a(4ac+b^2)\sqrt{-a+bx^2+cx^4}}$$

$$\Big\downarrow \text{1514}$$

$$-\frac{c\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\int \frac{2a-bx^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx}{a(4ac+b^2)\sqrt{-a+bx^2+cx^4}}-\frac{x(2ac+b^2+bcx^2)}{a(4ac+b^2)\sqrt{-a+bx^2+cx^4}}$$

$$\Big\downarrow \text{406}$$

$$-\frac{c\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\left(2a\int \frac{1}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx-b\int \frac{x^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx\right)}{a(4ac+b^2)\sqrt{-a+bx^2+cx^4}}-\frac{x(2ac+b^2+bcx^2)}{a(4ac+b^2)\sqrt{-a+bx^2+cx^4}}$$

$$\Big\downarrow \text{320}$$

$$-\frac{c\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\left(\frac{\sqrt{2}a\sqrt{\sqrt{4ac+b^2}+b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}-b\int\right.}{a\left(4ac+b^2\right)\sqrt{-a+bx^2+cx^4}}$$

$$\frac{x\left(2ac+b^2+bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{-a+bx^2+cx^4}}$$

$$\Big\downarrow \text{388}$$

$$-\frac{c\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\left(\frac{\sqrt{2}a\sqrt{\sqrt{4ac+b^2}+b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}-b\left(\right.\right.}{a\left(4ac+b^2\right)\sqrt{-a+bx^2+cx^4}}$$

$$\frac{x\left(2ac+b^2+bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{-a+bx^2+cx^4}}$$

$$\Big\downarrow \text{313}$$

$$-\frac{c\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\left(\frac{\sqrt{2}a\sqrt{\sqrt{4ac+b^2}+b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}-b\left(\right.\right.}{a\left(4ac+b^2\right)\sqrt{-a}}$$

$$\frac{x\left(2ac+b^2+bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{-a+bx^2+cx^4}}$$

input

```
Int[(-a + b*x^2 + c*x^4)^(-3/2),x]
```

output
```
-((x*(b^2 + 2*a*c + b*c*x^2))/(a*(b^2 + 4*a*c)*Sqrt[-a + b*x^2 + c*x^4]))
- (c*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 + 4*a*c])]*(-(b*(((b - Sqrt[b^2 + 4*a*c])*x*Sqrt[1 + (2*c*x^2)/(b
- Sqrt[b^2 + 4*a*c])])/(2*c*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]) -
 ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(
b - Sqrt[b^2 + 4*a*c])]*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 + 4*a*c]]], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[
2]*c^(3/2)*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b
+ Sqrt[b^2 + 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]))) + (S
qrt[2]*a*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*
a*c])]*EllipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]],
(-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[c]*Sqrt[(1 + (2*c*x
^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])])))/(a*(b^2 + 4*a*c)*Sqrt[-a + b*x^
2 + c*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.14

method	result
default	$-\frac{2c\left(\frac{b\,x^3}{2a\left(4ac+b^2\right)}+\frac{\left(2ac+b^2\right)x}{2a\left(4ac+b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}-\frac{a}{c}\right)c}}+\frac{\left(-\frac{1}{a}+\frac{2ac+b^2}{a\left(4ac+b^2\right)}\right)\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\right.}{}}}{2}\right.}{2\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4+b\,x^2-a}}$
elliptic	$-\frac{2c\left(\frac{b\,x^3}{2a\left(4ac+b^2\right)}+\frac{\left(2ac+b^2\right)x}{2a\left(4ac+b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}-\frac{a}{c}\right)c}}+\frac{\left(-\frac{1}{a}+\frac{2ac+b^2}{a\left(4ac+b^2\right)}\right)\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\right.}{}}}{2}\right.}{2\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4+b\,x^2-a}}$

input
```
int(1/(c*x^4+b*x^2-a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(1/2*b/a/(4*a*c+b^2)*x^3+1/2*(2*a*c+b^2)/a/(4*a*c+b^2)/c*x)/((x^4+1/c
*b*x^2-1/c*a)*c)^(1/2)+1/2*(-1/a+(2*a*c+b^2)/a/(4*a*c+b^2))/(-2*(-b+(4*a*c
+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*(b+(4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*EllipticF(1/2*x*(-2*(-b
+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))
+b/(4*a*c+b^2)*c/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a
)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/
a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*(-2
*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.16

$$\int \frac{1}{(-a+bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(\left(abcx^4+ab^2x^2-a^2b\right)\sqrt{-a}\sqrt{\frac{b^2+4\,ac}{a^2}}+\left(b^2cx^4+b^3x^2-ab^2\right)\sqrt{-a}\right)\sqrt{\frac{a\sqrt{}}{}}}{}$$

input
```
integrate(1/(c*x^4+b*x^2-a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*((a*b*c*x^4 + a*b^2*x^2 - a^2*b)*sqrt(-a)*sqrt((b^2 + 4*a*c
)/a^2) + (b^2*c*x^4 + b^3*x^2 - a*b^2)*sqrt(-a))*sqrt((a*sqrt((b^2 + 4*a*c
)/a^2) + b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 + 4*a*c)/a^
2) + b)/a)), 1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) - b^2 - 2*a*c)/(a*c)) - sqrt
(1/2)*(((2*a^2 + a*b)*c*x^4 - 2*a^3 - a^2*b + (2*a^2*b + a*b^2)*x^2)*sqrt(
-a)*sqrt((b^2 + 4*a*c)/a^2) - ((2*a*b - b^2)*c*x^4 - 2*a^2*b + a*b^2 + (2*
a*b^2 - b^3)*x^2)*sqrt(-a))*sqrt((a*sqrt((b^2 + 4*a*c)/a^2) + b)/a)*ellipt
ic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 + 4*a*c)/a^2) + b)/a)), 1/2*(a*b
*sqrt((b^2 + 4*a*c)/a^2) - b^2 - 2*a*c)/(a*c)) + 2*(a*b*c*x^3 + (a*b^2 + 2
*a^2*c)*x)*sqrt(c*x^4 + b*x^2 - a))/(a^3*b^2 + 4*a^4*c - (a^2*b^2*c + 4*a^
3*c^2)*x^4 - (a^2*b^3 + 4*a^3*b*c)*x^2)
```

Sympy [F]

$$\int \frac{1}{(-a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(-a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**2-a)**(3/2),x)
```

output
```
Integral((-a + b*x**2 + c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^2-a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2-a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 - a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^2-a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2-a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 - a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(c\,x^4+b\,x^2-a)^{3/2}}\,dx$$

input
```
int(1/(b*x^2 - a + c*x^4)^(3/2),x)
```

output
```
int(1/(b*x^2 - a + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2-a}}{c^2x^8+2bc\,x^6-2ac\,x^4+b^2x^4-2ab\,x^2+a^2}dx$$

input
```
int(1/(c*x^4+b*x^2-a)^(3/2),x)
```

output
```
int(sqrt( - a + b*x**2 + c*x**4)/(a**2 - 2*a*b*x**2 - 2*a*c*x**4 + b**2*x*
*4 + 2*b*c*x**6 + c**2*x**8),x)
```

3.320 $\int \frac{1}{(-a-bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 410

$$\begin{aligned}\int \frac{1}{(-a-bx^2+cx^4)^{3/2}}\,dx = &-\frac{x(b^2+2ac-bcx^2)}{a(b^2+4ac)\sqrt{-a-bx^2+cx^4}}\\ &-\frac{b\sqrt{c}x\sqrt{-a-bx^2+cx^4}}{a(b^2+4ac)\left(\sqrt{-a}+\sqrt{c}x^2\right)}\\ &-\frac{b\sqrt[4]{c}\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)\sqrt{\frac{a+bx^2-cx^4}{a\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{-a}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{-a}\sqrt{c}}\right)\right)}{\sqrt[4]{-a}(b^2+4ac)\sqrt{-a-bx^2+cx^4}}\\ &+\frac{\left(b-2\sqrt{-a}\sqrt{c}\right)\sqrt[4]{c}\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)\sqrt{\frac{a+bx^2-cx^4}{a\left(1+\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{-a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{-a}\sqrt{c}}\right)\right)}{2\sqrt[4]{-a}(b^2+4ac)\sqrt{-a-bx^2+cx^4}}\end{aligned}$$

output
```
-x*(-b*c*x^2+2*a*c+b^2)/a/(4*a*c+b^2)/(c*x^4-b*x^2-a)^(1/2)-b*c^(1/2)*x*(c
*x^4-b*x^2-a)^(1/2)/a/(4*a*c+b^2)/(c^(1/2)*x^2+(-a)^(1/2))-b*c^(1/4)*(1+c^
(1/2)*x^2/(-a)^(1/2))*((-c*x^4+b*x^2+a)/a/(1+c^(1/2)*x^2/(-a)^(1/2))^2)^(1
/2)*EllipticE(sin(2*arctan(c^(1/4)*x/(-a)^(1/4))),1/2*(2+b/(-a)^(1/2)/c^(1
/2))^(1/2))/(-a)^(1/4)/(4*a*c+b^2)/(c*x^4-b*x^2-a)^(1/2)+1/2*(b-2*(-a)^(1/
2)*c^(1/2))*c^(1/4)*(1+c^(1/2)*x^2/(-a)^(1/2))*((-c*x^4+b*x^2+a)/a/(1+c^(1
/2)*x^2/(-a)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/(-a)^(1/4)
),1/2*(2+b/(-a)^(1/2)/c^(1/2))^(1/2))/(-a)^(1/4)/(4*a*c+b^2)/(c*x^4-b*x^2-
a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.53 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.15

$$\int \frac{1}{(-a-bx^2+cx^4)^{3/2}}\,dx = \frac{4\sqrt{\frac{c}{-b+\sqrt{b^2+4ac}}}x(-b^2-2ac+bcx^2)-i\sqrt{2}b\left(b+\sqrt{b^2+4ac}\right)\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}}{}$$

input
```
Integrate[(-a - b*x^2 + c*x^4)^(-3/2),x]
```

output
```
(4*Sqrt[c/(-b + Sqrt[b^2 + 4*a*c])]*x*(-b^2 - 2*a*c + b*c*x^2) - I*Sqrt[2]
*b*(b + Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqr
t[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 +
4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 + 4*a*c])]*x],
(b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(b^2 + 4*a*c
+ b*Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^
2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*
c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 + 4*a*c])]*x], (b -
 Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])])/(4*a*(b^2 + 4*a*c)*Sqrt[c/(-
b + Sqrt[b^2 + 4*a*c])]*Sqrt[-a - b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1405, 25, 27, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-a-bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{\int -\frac{c\left(bx^2+2a\right)}{\sqrt{cx^4-bx^2-a}}dx}{a\left(4ac+b^2\right)}-\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{-a-bx^2+cx^4}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{c\left(bx^2+2a\right)}{\sqrt{cx^4-bx^2-a}}dx}{a\left(4ac+b^2\right)}-\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{-a-bx^2+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\int \frac{bx^2+2a}{\sqrt{cx^4-bx^2-a}}dx}{a\left(4ac+b^2\right)}-\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{-a-bx^2+cx^4}}$$

$$\Big\downarrow \text{1514}$$

$$-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int \frac{bx^2+2a}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{a\left(4ac+b^2\right)\sqrt{-a-bx^2+cx^4}}-\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{-a-bx^2+cx^4}}$$

$$\Big\downarrow \text{399}$$

$$-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(-b\sqrt{4ac+b^2}+4ac+b^2\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{2c}-\frac{b\left(b-\sqrt{4ac+b^2}\right)\int \frac{\sqrt{1-\frac{2c\cdots}{b-\sqrt{b\cdots}}}}{\sqrt{1-\frac{2c\cdots}{b+\sqrt{b\cdots}}}}\cdots}{2c}\cdots\right.}{a\left(4ac+b^2\right)\sqrt{-a-bx^2+cx^4}}$$

$$\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{-a-bx^2+cx^4}}$$

$\Big\downarrow$ 321

$$-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+4ac+b^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}}-\frac{b(b\cdots}{\cdots}\cdots\right.}{a\left(4ac+b^2\right)\sqrt{-a-bx^2+cx^4}}$$

$$\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{-a-bx^2+cx^4}}$$

$\Big\downarrow$ 327

$$-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+4ac+b^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}}-\frac{b(b\cdots}{\cdots}\cdots\right.}{a\left(4ac+b^2\right)\sqrt{-a-bx^2+cx^4}}$$

$$\frac{x\left(2ac+b^2-bcx^2\right)}{a\left(4ac+b^2\right)\sqrt{-a-bx^2+cx^4}}$$

input
```
Int[(-a - b*x^2 + c*x^4)^(-3/2),x]
```

output
```
-((x*(b^2 + 2*a*c - b*c*x^2))/(a*(b^2 + 4*a*c)*Sqrt[-a - b*x^2 + c*x^4]))
- (c*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + S
qrt[b^2 + 4*a*c])]*(-1/2*(b*(b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*
a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (
b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2)) + (Sqrt
[b + Sqrt[b^2 + 4*a*c]]*(b^2 + 4*a*c - b*Sqrt[b^2 + 4*a*c])*EllipticF[ArcS
in[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c
])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2))))/(a*(b^2 + 4*a*c)*Sqrt[-
a - b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.14

method	result
default	$-\frac{2c\left(-\frac{b\,x^3}{2a\left(4ac+b^2\right)}+\frac{\left(2ac+b^2\right)x}{2a\left(4ac+b^2\right)c}\right)}{\sqrt{\left(x^4-\frac{b\,x^2}{c}-\frac{a}{c}\right)c}}+\frac{\left(-\frac{1}{a}+\frac{2ac+b^2}{a\left(4ac+b^2\right)}\right)\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-\frac{2(b+}{}}}{}\right.}{2\sqrt{-\frac{2\left(b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4-b\,x^2-a}}$
elliptic	$-\frac{2c\left(-\frac{b\,x^3}{2a\left(4ac+b^2\right)}+\frac{\left(2ac+b^2\right)x}{2a\left(4ac+b^2\right)c}\right)}{\sqrt{\left(x^4-\frac{b\,x^2}{c}-\frac{a}{c}\right)c}}+\frac{\left(-\frac{1}{a}+\frac{2ac+b^2}{a\left(4ac+b^2\right)}\right)\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-\frac{2(b+}{}}}{}\right.}{2\sqrt{-\frac{2\left(b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4-b\,x^2-a}}$

input
```
int(1/(c*x^4-b*x^2-a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/2*b/a/(4*a*c+b^2)*x^3+1/2*(2*a*c+b^2)/a/(4*a*c+b^2)/c*x)/((x^4-1/
c*b*x^2-1/c*a)*c)^(1/2)+1/2*(-1/a+(2*a*c+b^2)/a/(4*a*c+b^2))/(-2*(b+(4*a*c
+b^2)^(1/2))/a)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*(-b+(4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4-b*x^2-a)^(1/2)*EllipticF(1/2*x*(-2*(b+
(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(-b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))
-b/(4*a*c+b^2)*c/(-2*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4-b*x^2-a)
^(1/2)/(-b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(b+(4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(-b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*(-2
*(b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(-b+(4*a*c+b^2)^(1/2))/a/c)^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.17

$$\int \frac{1}{(-a-bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(\left(abcx^4-ab^2x^2-a^2b\right)\sqrt{-a}\sqrt{\frac{b^2+4\,ac}{a^2}}-\left(b^2cx^4-b^3x^2-ab^2\right)\sqrt{-a}\right)\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}-b}{a}}\,E\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\cdots}\right)\right)\cdots}{\cdots}$$

input
```
integrate(1/(c*x^4-b*x^2-a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*((a*b*c*x^4 - a*b^2*x^2 - a^2*b)*sqrt(-a)*sqrt((b^2 + 4*a*
c)/a^2) - (b^2*c*x^4 - b^3*x^2 - a*b^2)*sqrt(-a))*sqrt((a*sqrt((b^2 + 4*a*
c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 + 4*a*c)/a
^2) - b)/a)), -1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) + b^2 + 2*a*c)/(a*c)) + sq
rt(1/2)*(((2*a^2 - a*b)*c*x^4 - 2*a^3 + a^2*b - (2*a^2*b - a*b^2)*x^2)*sqr
t(-a)*sqrt((b^2 + 4*a*c)/a^2) + ((2*a*b + b^2)*c*x^4 - 2*a^2*b - a*b^2 - (
2*a*b^2 + b^3)*x^2)*sqrt(-a))*sqrt((a*sqrt((b^2 + 4*a*c)/a^2) - b)/a)*elli
ptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 + 4*a*c)/a^2) - b)/a)), -1/2*(
a*b*sqrt((b^2 + 4*a*c)/a^2) + b^2 + 2*a*c)/(a*c)) + 2*(a*b*c*x^3 - (a*b^2
+ 2*a^2*c)*x)*sqrt(c*x^4 - b*x^2 - a))/(a^3*b^2 + 4*a^4*c - (a^2*b^2*c + 4
*a^3*c^2)*x^4 + (a^2*b^3 + 4*a^3*b*c)*x^2)
```

Sympy [F]

$$\int \frac{1}{(-a-bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(-a-bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**4-b*x**2-a)**(3/2),x)
```

output
```
Integral((-a - b*x**2 + c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(-a-bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4-bx^2-a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4-b*x^2-a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 - b*x^2 - a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(-a-bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4-bx^2-a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4-b*x^2-a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 - b*x^2 - a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-a-bx^2+cx^4)^{3/2}}\,dx=\int \frac{1}{(c\,x^4-b\,x^2-a)^{3/2}}\,dx$$

input
```
int(1/(c*x^4 - b*x^2 - a)^(3/2),x)
```

output
```
int(1/(c*x^4 - b*x^2 - a)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(-a-bx^2+cx^4)^{3/2}}\,dx=\int \frac{\sqrt{c\,x^4-b\,x^2-a}}{c^2x^8-2bc\,x^6-2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int(1/(c*x^4-b*x^2-a)^(3/2),x)
```

output
```
int(sqrt( - a - b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 - 2*a*c*x**4 + b**2*x*
*4 - 2*b*c*x**6 + c**2*x**8),x)
```

3.321 $\int (a+bx^2+cx^4)^3 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 81

$$\int (a+bx^2+cx^4)^3 \, dx = a^3x+a^2bx^3+\frac{3}{5}a(b^2+ac)\,x^5+\frac{1}{7}b(b^2+6ac)\,x^7 +\frac{1}{3}c(b^2+ac)\,x^9+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

output
```
a^3*x+a^2*b*x^3+3/5*a*(a*c+b^2)*x^5+1/7*b*(6*a*c+b^2)*x^7+1/3*c*(a*c+b^2)*
x^9+3/11*b*c^2*x^11+1/13*c^3*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int (a+bx^2+cx^4)^3 \, dx = a^3x+a^2bx^3+\frac{3}{5}a(b^2+ac)\,x^5+\frac{1}{7}b(b^2+6ac)\,x^7 +\frac{1}{3}c(b^2+ac)\,x^9+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^3,x]
```

output
```
a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^7)/7 + (c
*(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^3 \, dx$$

$$\Big\downarrow \text{1403}$$

$$\int \left(a^3+3a^2bx^2+3b^2cx^8\left(\frac{ac}{b^2}+1\right)+3ab^2x^4\left(\frac{ac}{b^2}+1\right)+b^3x^6\left(\frac{6ac}{b^2}+1\right)+3bc^2x^{10}+c^3x^{12}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^3x+a^2bx^3+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

input
```
Int[(a + b*x^2 + c*x^4)^3,x]
```

output

```
a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^7)/7 + (c
*(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

method	result
norman	$\frac{c^3 x^{13}}{13}+\frac{3b\,c^2 x^{11}}{11}+\left(\frac{1}{3}c^2 a+\frac{1}{3}b^2 c\right)x^9+\left(\frac{6}{7}abc+\frac{1}{7}b^3\right)x^7+\left(\frac{3}{5}c\,a^2+\frac{3}{5}b^2 a\right)x^5+a^2 b\,x^3+a^3 x$
gosper	$\frac{1}{13}c^3 x^{13}+\frac{3}{11}b\,c^2 x^{11}+\frac{1}{3}x^9 c^2 a+\frac{1}{3}x^9 b^2 c+\frac{6}{7}x^7 abc+\frac{1}{7}x^7 b^3+\frac{3}{5}x^5 c\,a^2+\frac{3}{5}x^5 b^2 a+a^2 b\,x^3+a^3$
risch	$\frac{1}{13}c^3 x^{13}+\frac{3}{11}b\,c^2 x^{11}+\frac{1}{3}x^9 c^2 a+\frac{1}{3}x^9 b^2 c+\frac{6}{7}x^7 abc+\frac{1}{7}x^7 b^3+\frac{3}{5}x^5 c\,a^2+\frac{3}{5}x^5 b^2 a+a^2 b\,x^3+a^3$
parallelrisch	$\frac{1}{13}c^3 x^{13}+\frac{3}{11}b\,c^2 x^{11}+\frac{1}{3}x^9 c^2 a+\frac{1}{3}x^9 b^2 c+\frac{6}{7}x^7 abc+\frac{1}{7}x^7 b^3+\frac{3}{5}x^5 c\,a^2+\frac{3}{5}x^5 b^2 a+a^2 b\,x^3+a^3$
orering	$\frac{x(1155c^3x^{12}+4095b\,c^2x^{10}+5005a\,c^2x^8+5005b^2c\,x^8+12870abc\,x^6+2145b^3x^6+9009a^2c\,x^4+9009b^2x^4a+15015a^2b\,x^2+15015}{15015}$
default	$\frac{c^3 x^{13}}{13}+\frac{3b\,c^2 x^{11}}{11}+\frac{(c^2a+2b^2c+c(2ac+b^2))x^9}{9}+\frac{(4abc+b(2ac+b^2))x^7}{7}+\frac{(a(2ac+b^2)+2b^2a+c\,a^2)x^5}{5}+a^2b\,x^3+$

input
```
int((c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*c^3*x^13+3/11*b*c^2*x^11+(1/3*c^2*a+1/3*b^2*c)*x^9+(6/7*a*b*c+1/7*b^3
)*x^7+(3/5*c*a^2+3/5*b^2*a)*x^5+a^2*b*x^3+a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{13}\,c^3x^{13}+\frac{3}{11}\,bc^2x^{11}+\frac{1}{3}\,(b^2c+ac^2)x^9+\frac{1}{7}\,(b^3+6\,abc)x^7+a^2bx^3+\frac{3}{5}\,(ab^2+a^2c)x^5+a^3x$$

input
```
integrate((c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*(b^2*c + a*c^2)*x^9 + 1/7*(b^3 + 6*a
*b*c)*x^7 + a^2*b*x^3 + 3/5*(a*b^2 + a^2*c)*x^5 + a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \left(a + bx^2 + cx^4\right)^3 \, dx = a^3x + a^2bx^3 + \frac{3bc^2x^{11}}{11} + \frac{c^3x^{13}}{13} + x^9\left(\frac{ac^2}{3} + \frac{b^2c}{3}\right) + x^7 \cdot \left(\frac{6abc}{7} + \frac{b^3}{7}\right) + x^5 \cdot \left(\frac{3a^2c}{5} + \frac{3ab^2}{5}\right)$$

input
```
integrate((c*x**4+b*x**2+a)**3,x)
```

output
```
a**3*x + a**2*b*x**3 + 3*b*c**2*x**11/11 + c**3*x**13/13 + x**9*(a*c**2/3
+ b**2*c/3) + x**7*(6*a*b*c/7 + b**3/7) + x**5*(3*a**2*c/5 + 3*a*b**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \left(a + bx^2 + cx^4\right)^3 \, dx = \frac{1}{13}\, c^3x^{13} + \frac{3}{11}\, bc^2x^{11} + \frac{1}{3}\, b^2cx^9 + \frac{1}{7}\, b^3x^7 + a^3x + \frac{1}{5}\left(3\, cx^5 + 5\, bx^3\right)a^2 + \frac{1}{105}\left(35\, c^2x^9 + 90\, bcx^7 + 63\, b^2x^5\right)a$$

input
```
integrate((c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/7*b^3*x^7 + a^3*x + 1/
5*(3*c*x^5 + 5*b*x^3)*a^2 + 1/105*(35*c^2*x^9 + 90*b*c*x^7 + 63*b^2*x^5)*a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int \left(a+bx^2+cx^4\right)^3 \, dx = \frac{1}{13}c^3x^{13} + \frac{3}{11}bc^2x^{11} + \frac{1}{3}b^2cx^9 + \frac{1}{3}ac^2x^9 + \frac{1}{7}b^3x^7 + \frac{6}{7}abcx^7 + \frac{3}{5}ab^2x^5 + \frac{3}{5}a^2cx^5 + a^2bx^3 + a^3x$$

input
```
integrate((c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/3*a*c^2*x^9 + 1/7*b^3*
x^7 + 6/7*a*b*c*x^7 + 3/5*a*b^2*x^5 + 3/5*a^2*c*x^5 + a^2*b*x^3 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 18.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2+cx^4\right)^3 \, dx = x^7 \left(\frac{b^3}{7} + \frac{6\,a\,c\,b}{7}\right) + a^3\,x + \frac{c^3\,x^{13}}{13} + a^2\,b\,x^3 + \frac{3\,b\,c^2\,x^{11}}{11} + \frac{3\,a\,x^5\,\left(b^2+a\,c\right)}{5} + \frac{c\,x^9\,\left(b^2+a\,c\right)}{3}$$

input
```
int((a + b*x^2 + c*x^4)^3,x)
```

output
```
x^7*(b^3/7 + (6*a*b*c)/7) + a^3*x + (c^3*x^13)/13 + a^2*b*x^3 + (3*b*c^2*x
^11)/11 + (3*a*x^5*(a*c + b^2))/5 + (c*x^9*(a*c + b^2))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \left(a + bx^2 + cx^4\right)^3 \, dx$$
$$= \frac{x(1155c^3x^{12} + 4095b\,c^2x^{10} + 5005a\,c^2x^8 + 5005b^2c\,x^8 + 12870abc\,x^6 + 2145b^3x^6 + 9009a^2c\,x^4 + 9009a\,b}{15015}$$

input
```
int((c*x^4+b*x^2+a)^3,x)
```

output
```
(x*(15015*a**3 + 15015*a**2*b*x**2 + 9009*a**2*c*x**4 + 9009*a*b**2*x**4 +
 12870*a*b*c*x**6 + 5005*a*c**2*x**8 + 2145*b**3*x**6 + 5005*b**2*c*x**8 +
 4095*b*c**2*x**10 + 1155*c**3*x**12))/15015
```

3.322 $\int (a+bx^2+cx^4)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int (a+bx^2+cx^4)^2\,dx = a^2x+\frac{2}{3}abx^3+\frac{1}{5}(b^2+2ac)\,x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

output
```
a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int (a+bx^2+cx^4)^2\,dx = a^2x+\frac{2}{3}abx^3+\frac{1}{5}(b^2+2ac)\,x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/
9
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^2 dx$$

$$\Big\downarrow \text{1403}$$

$$\int \left(a^2+b^2x^4\left(\frac{2ac}{b^2}+1\right)+2abx^2+2bcx^6+c^2x^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

input
```
Int[(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/
9
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
default	$a^2x+\frac{2ab\,x^3}{3}+\frac{(2ac+b^2)x^5}{5}+\frac{2bc\,x^7}{7}+\frac{c^2x^9}{9}$	42
norman	$\frac{c^2x^9}{9}+\frac{2bc\,x^7}{7}+\left(\frac{2ac}{5}+\frac{b^2}{5}\right)x^5+\frac{2ab\,x^3}{3}+a^2x$	43
gosper	$\frac{1}{9}c^2x^9+\frac{2}{7}bc\,x^7+\frac{2}{5}x^5ac+\frac{1}{5}x^5b^2+\frac{2}{3}ab\,x^3+a^2x$	44
risch	$\frac{1}{9}c^2x^9+\frac{2}{7}bc\,x^7+\frac{2}{5}x^5ac+\frac{1}{5}x^5b^2+\frac{2}{3}ab\,x^3+a^2x$	44
parallelrisch	$\frac{1}{9}c^2x^9+\frac{2}{7}bc\,x^7+\frac{2}{5}x^5ac+\frac{1}{5}x^5b^2+\frac{2}{3}ab\,x^3+a^2x$	44
orering	$\frac{x(35c^2x^8+90bc\,x^6+126ac\,x^4+63b^2x^4+210ab\,x^2+315a^2)}{315}$	47

input
```
int((c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,(b^2+2\,ac)x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \left(a+bx^2+cx^4\right)^2 \, dx = a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5\cdot\left(\frac{2ac}{5}+\frac{b^2}{5}\right)$$

input
```
integrate((c*x**4+b*x**2+a)**2,x)
```

output
```
a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/
5)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{9}\,c^2x^9 + \frac{2}{7}\,bcx^7 + \frac{1}{5}\,b^2x^5 + a^2x + \frac{2}{15}\,\left(3\,cx^5+5\,bx^3\right)a$$

input
```
integrate((c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 + 5*b*x^3)
*a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{9}\,c^2x^9 + \frac{2}{7}\,bcx^7 + \frac{1}{5}\,b^2x^5 + \frac{2}{5}\,acx^5 + \frac{2}{3}\,abx^3 + a^2x$$

input
```
integrate((c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*
x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2+cx^4\right)^2\,dx = a^2\,x+x^5\,\left(\frac{b^2}{5}+\frac{2\,a\,c}{5}\right)+\frac{c^2\,x^9}{9}+\frac{2\,a\,b\,x^3}{3}+\frac{2\,b\,c\,x^7}{7}$$

input
```
int((a + b*x^2 + c*x^4)^2,x)
```

output
```
a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7
)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \left(a+bx^2+cx^4\right)^2\,dx = \frac{x(35c^2x^8+90bc\,x^6+126ac\,x^4+63b^2x^4+210ab\,x^2+315a^2)}{315}$$

input
```
int((c*x^4+b*x^2+a)^2,x)
```

output
```
(x*(315*a**2 + 210*a*b*x**2 + 126*a*c*x**4 + 63*b**2*x**4 + 90*b*c*x**6 +
35*c**2*x**8))/315
```

3.323 $\int (a + bx^2 + cx^4) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int (a + bx^2 + cx^4) \, dx = ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

output
```
a*x+1/3*b*x^3+1/5*c*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a + bx^2 + cx^4) \, dx = ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

input
```
Integrate[a + b*x^2 + c*x^4,x]
```

output
```
a*x + (b*x^3)/3 + (c*x^5)/5
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

input
```
Int[a + b*x^2 + c*x^4,x]
```

output
```
a*x + (b*x^3)/3 + (c*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$ax + \frac{1}{3}b\,x^{3} + \frac{1}{5}c\,x^{5}$	17
default	$ax + \frac{1}{3}b\,x^{3} + \frac{1}{5}c\,x^{5}$	17
norman	$ax + \frac{1}{3}b\,x^{3} + \frac{1}{5}c\,x^{5}$	17
risch	$ax + \frac{1}{3}b\,x^{3} + \frac{1}{5}c\,x^{5}$	17
parallelrisch	$ax + \frac{1}{3}b\,x^{3} + \frac{1}{5}c\,x^{5}$	17
parts	$ax + \frac{1}{3}b\,x^{3} + \frac{1}{5}c\,x^{5}$	17
orering	$\frac{x\left(3c\,x^{4}+5b\,x^{2}+15a\right)}{15}$	20

input
```
int(c*x^4+b*x^2+a,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/3*b*x^3+1/5*c*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2+cx^4\right)\,dx=\frac{1}{5}\,cx^{5}+\frac{1}{3}\,bx^{3}+ax$$

input
```
integrate(c*x^4+b*x^2+a,x, algorithm="fricas")
```

output
```
1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \left(a+bx^2+cx^4\right)\,dx = ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

input
```
integrate(c*x**4+b*x**2+a,x)
```

output
```
a*x + b*x**3/3 + c*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2+cx^4\right)\,dx = \frac{1}{5}\,cx^5+\frac{1}{3}\,bx^3+ax$$

input
```
integrate(c*x^4+b*x^2+a,x, algorithm="maxima")
```

output
```
1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2+cx^4\right)\,dx = \frac{1}{5}\,cx^5+\frac{1}{3}\,bx^3+ax$$

input
```
integrate(c*x^4+b*x^2+a,x, algorithm="giac")
```

output
```
1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (a + bx^2 + cx^4) \, dx = \frac{c\,x^5}{5} + \frac{b\,x^3}{3} + a\,x$$

input
```
int(a + b*x^2 + c*x^4,x)
```

output
```
a*x + (b*x^3)/3 + (c*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (a + bx^2 + cx^4) \, dx = \frac{x(3c\,x^4 + 5b\,x^2 + 15a)}{15}$$

input
```
int(c*x^4+b*x^2+a,x)
```

output
```
(x*(15*a + 5*b*x**2 + 3*c*x**4))/15
```

3.324 $\int 1\,dx$

Optimal result

Integrand size = 1, antiderivative size = 1

$$\int 1\,dx = x$$

output
```
x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1\,dx = x$$

input
```
Integrate[1,x]
```

output
```
x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 1\,dx$$
$$\Big\downarrow \; 24$$
$$x$$

input
```
Int[1,x]
```

output
```
x
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	x	2
norman	x	2
risch	x	2
parallelrisch	x	2
orering	x	2

input
```
int(1,x,method=_RETURNVERBOSE)
```

output
```
x
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1\,dx = x$$

input
```
integrate(1,x, algorithm="fricas")
```

output
```
x
```

Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 0, normalized size of antiderivative = 0.00

$$\int 1\,dx = x$$

input
```
integrate(1,x)
```

output
```
x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1\,dx = x$$

input
```
integrate(1,x, algorithm="maxima")
```

output
```
x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1\,dx = x$$

input
```
integrate(1,x, algorithm="giac")
```

output
```
x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1\,dx = x$$

input
```
int(1,x)
```

output
```
x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1\,dx = x$$

input
```
int(1,x)
```

output
```
x
```

3.325 $\int \frac{1}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{a+bx^2+cx^4}\,dx=\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
2^(1/2)*c^(1/2)*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))/(-4
*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2^(1/2)*c^(1/2)*arctan(2^(1/2
)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+bx^2+cx^4}\,dx=\frac{\sqrt{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/
Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1406, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^2+cx^4} \, dx$$

$$\Big\downarrow \text{1406}$$

$$\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

input
```
Int[(a + b*x^2 + c*x^4)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.25

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 c+b\_Z^2+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^3 c+\_R b}\right)}{2}$	38
default	$4c\left(-\frac{\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$	117

input
```
int(1/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(114) = 228$.

Time = 0.09 (sec) , antiderivative size = 613, normalized size of antiderivative = 4.09

$$\int \frac{1}{a+bx^2+cx^4}\,dx = -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx + \sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx - \sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx + \sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx - \sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

input
```
integrate(1/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^
2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqr
t(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)
)/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^
2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c -
(a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/
sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a
*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sq
rt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-
(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*
sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4
*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2
*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*
b^2 - 4*a^2*c)))
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.58

$$\int \frac{1}{a+bx^2+cx^4}\,dx$$
$$=\operatorname{RootSum}\left(t^4\cdot\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3a\right.\right.\right.$$

input
```
integrate(1/(c*x**4+b*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a
*b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b
**3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Maxima [F]

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \int \frac{1}{cx^4+bx^2+a}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(114) = 228$.

Time = 0.38 (sec) , antiderivative size = 1026, normalized size of antiderivative = 6.84

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2
 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a
*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*
a^2*c^3)*abs(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2...
```

Mupad [B] (verification not implemented)

Time = 18.38 (sec) , antiderivative size = 763, normalized size of antiderivative = 5.09

$$\int \frac{1}{a+bx^2+cx^4}\,dx =$$

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2}{4\,a\,b^4\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right.$$

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}-b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2}{4\,a\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-}\right.$$

input
```
int(1/(a + b*x^2 + c*x^4),x)
```

output
```
- atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 -
64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))
^(1/2) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(
b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8
*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*
b^2*c*x*8i)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^
3*c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a
*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6
- 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b
^2*c))^(1/2)*2i
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.34

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \frac{2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)b-4\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)a-2\sqrt{a}\,\sqrt{2\sqrt{}}}{}$$

input
```
int(1/(c*x^4+b*x^2+a),x)
```

output
```
(2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b - 4*sqrt(c)*sqrt(2*sqrt(c)*sq
rt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)
*sqrt(a) + b))*a - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt
(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b + 4*sqrt(c)
*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)
*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a - sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*l
og( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b + sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) +
sqrt(c)*x**2)*b - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt
(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a + 2*sqrt(c)*sqrt(2*sqrt(c)*
sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a
)/(4*a*(4*a*c - b**2))
```

3.326 $\int \frac{1}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 250

$$\int \frac{1}{(a+bx^2+cx^4)^2} dx = \frac{x(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(b+\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*c^(1/2)*(b+(-
12*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(
1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*c^(1/
2)*(b-(-12*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*
c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \frac{\frac{2x(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

output
```
((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[
b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1405, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 1405$$

$$\frac{x(-2ac+b^2+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\,(b^2-4ac)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{1}{2}c\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2a\,(b^2-4ac)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\,(b^2-4ac)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

input
```
Int[(a + b*x^2 + c*x^4)^(-2),x]
```

output
```
(x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((Sq
rt[c]*(b + (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
c]*(b - (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2
- 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60

method	result
risch	$\frac{-\frac{b c x^3}{2a(4ac-b^2)}+\frac{(2ac-b^2)x}{2a(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 c+b\_Z^2+a)}\frac{\left(-\frac{bc\_R^2}{4ac-b^2}+\frac{6ac-b^2}{4ac-b^2}\right)\ln(x-\_R)}{2\_R^3 c+\_R b}}{4a}$
default	$16c^2\left(-\frac{-\frac{\left(b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16ac^2\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}+\frac{\left(b\sqrt{-4ac+b^2}+12ac-b^2\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{16ac\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{4\sqrt{-4ac+b^2}\,(4ac-b^2)}-\frac{-\frac{\left(-b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16ac^2\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}\cdots}{}\right.$

input
```
int(1/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*b/a/(4*a*c-b^2)*c*x^3+1/2*(2*a*c-b^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+
a)+1/4/a*sum((-b*c/(4*a*c-b^2)*_R^2+(6*a*c-b^2)/(4*a*c-b^2))/(2*_R^3*c+_R*
b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2309 vs. $2(208) = 416$.

Time = 0.17 (sec) , antiderivative size = 2309, normalized size of antiderivative = 9.24

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
+ (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b
^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c +
81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*
b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b
^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^
2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*
c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2
)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15
*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a
^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*
b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 -
12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^
2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 6
72*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 -
 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/...
```

Sympy [A] (verification not implemented)

Time = 103.57 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.58

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \frac{-bcx^3+x(2ac-b^2)}{8a^3c-2a^2b^2+x^4\cdot(8a^2c^2-2ab^2c)+x^2\cdot(8a^2bc-2ab^3)}$$
$$+\mathrm{RootSum}\left(t^4\cdot\left(1048576a^9c^6-1572864a^8b^2c^5+983040a^7b^4c^4-327680a^6b^6c^3+61440a^5b^8c^2-6144a\right.\right.$$

input
```
integrate(1/(c*x**4+b*x**2+a)**2,x)
```

output
```
(-b*c*x**3 + x*(2*a*c - b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2
 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + RootSum(_t**4*(1048576*a*
*9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**
6*c**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + _t**
2*(-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608
*a**2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c
**4 + 25*b**4*c**3, Lambda(_t, _t*log(x + (32768*_t**3*a**7*b*c**4 - 28672
*_t**3*a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**4*b**7*c
 + 64*_t**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*b**2*c**3 + 740*_
t*a**2*b**4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(324*a**2*c**4 - 81*a*b**2*
c**3 + 5*b**4*c**2))))
```

Maxima [F]

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \int \frac{1}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a
^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c
*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. $2(208) = 416$.

Time = 0.37 (sec) , antiderivative size = 2682, normalized size of antiderivative = 10.73

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) -
1/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 -
 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*
b*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2
 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 - 2*(sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 6...
```

Mupad [B] (verification not implemented)

Time = 19.66 (sec) , antiderivative size = 6404, normalized size of antiderivative = 25.62

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
 + b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3
 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 384
0*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a
*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24
*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144
*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 -
 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2
*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^
5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^
6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^
4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 +
 b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*
c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5
)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^...
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 2409, normalized size of antiderivative = 9.64

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(c*x^4+b*x^2+a)^2,x)
```

output

```
(16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c - 2*sqrt(a)*sqrt(2*sq
rt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2
*sqrt(c)*sqrt(a) + b))*a*b**3 + 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a*b**2*c*x**2 + 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**2*x**4
- 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*x**2 - 2*sqrt(a)*sqrt(2*s
qrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*b**3*c*x**4 - 24*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) +
b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
 + b))*a**3*c + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2 - 24*s
qrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c*x**2 - 24*sqrt(c)*sqrt(2*s
qrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*a**2*c**2*x**4 + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a)
+ b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a*b**3*x**2 + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt...
```

3.327 $\int \frac{1}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 355

$$
\begin{aligned}
\int \frac{1}{(a+bx^2+cx^4)^3}\,dx &= \frac{x(b^2-2ac+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x((b^2-7ac)(3b^2-4ac)+3bc(b^2-8ac)x^2)}{8a^2(b^2-4ac)^2(a+bx^2+cx^4)} \\
&+ \frac{3\sqrt{c}\left(b^4-10ab^2c+56a^2c^2+b(b^2-8ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&+ \frac{3\sqrt{c}\left(b^3-8abc-\frac{b^4-10ab^2c+56a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

output

```
1/4*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/8*x*((-7*a*c+
b^2)*(-4*a*c+3*b^2)+3*b*c*(-8*a*c+b^2)*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^
2+a)+3/16*c^(1/2)*(b^4-10*a*b^2*c+56*c^2*a^2+b*(-8*a*c+b^2)*(-4*a*c+b^2)^(
1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(
-4*a*c+b^2)^(5/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/16*c^(1/2)*(b^3-8*a*b*c-(
56*a^2*c^2-10*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b
+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^2/(b+(-4*a*c+b^2)^(1/
2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.05

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{\frac{4ax(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x(3b^4-25ab^2c+28a^2c^2+3b^3cx^2-24abc^2x^2)}{(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\left(b^4-10ab^2c+56a^2c^2+b^3\sqrt{b^2-4ac}-8abc\sqrt{b^2-4ac}\right)\text{arcta}}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}}{16a^2}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(-3),x]
```

output
```
((4*a*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (
2*x*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*a*b*c^2*x^2))/((b^
2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c +
 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^
2 - b^3*Sqrt[b^2 - 4*a*c] + 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]))/(16*a^2)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1405, 25, 1492, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx$$
$$\Big\downarrow \; 1405$$
$$\frac{x(-2ac+b^2+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\int -\frac{3b^2+5cx^2b-14ac}{(cx^4+bx^2+a)^2}dx}{4a(b^2-4ac)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{3b^2+5cx^2b-14ac}{(cx^4+bx^2+a)^2}dx}{4a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$\Big\downarrow$ 1492

$$\frac{\frac{x(3bcx^2(b^2-8ac)+(b^2-7ac)(3b^2-4ac))}{2a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{3\left(b^4-9acb^2+c\left(b^2-8ac\right)x^2b+28a^2c^2\right)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}}{4a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{3\int \frac{b^4-9acb^2+c\left(b^2-8ac\right)x^2b+28a^2c^2}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{x(3bcx^2(b^2-8ac)+(b^2-7ac)(3b^2-4ac))}{2a(b^2-4ac)(a+bx^2+cx^4)}}{4a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$\Big\downarrow$ 1480

$$\frac{\frac{3\left(\frac{1}{2}c\left(\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}+b\left(b^2-8ac\right)\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(b\left(b^2-8ac\right)-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx\right)}{2a(b^2-4ac)}+\frac{x(3bcx^2\cdots}{\cdots}}{4a\left(b^2-4ac\right)}$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{3\left(\frac{\sqrt{c}\left(\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}+b\left(b^2-8ac\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b\left(b^2-8ac\right)-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2a(b^2-4ac)}+\frac{x(3bcx^2(b^2-8\cdots}{2a(b^2-\cdots}}{4a\left(b^2-4ac\right)}$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

input

```
Int[(a + b*x^2 + c*x^4)^(-3),x]
```

output
```
(x*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((
x*((b^2 - 7*a*c)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^2))/(2*a*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) + (3*((Sqrt[c]*(b*(b^2 - 8*a*c) + (b^4 - 10*a*b
^2*c + 56*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*(
b^2 - 8*a*c) - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^
2 - 4*a*c]])))/(2*a*(b^2 - 4*a*c)))/(4*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.31 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.93

method	result
risch	$\frac{-\frac{3b\,c^2\left(8ac-b^2\right)x^7}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{c\left(28a^2c^2-49a\,b^2c+6b^4\right)x^5}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{b\left(4a^2c^2+20a\,b^2c-3b^4\right)x^3}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(44a^2c^2-37a\,b^2c+5b^4\right)x}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4c\right.}\right.}{}$
default	$64c^3\left(-\frac{\frac{3\left(128\sqrt{-4ac+b^2}\,a^3b\,c^3-80\sqrt{-4ac+b^2}\,a^2b^3c^2+16\sqrt{-4ac+b^2}\,a\,b^5c-\sqrt{-4ac+b^2}\,b^7+384a^4c^4-352a^3b^2c^3+120a^2b^4c^2-18a\,b^6c+b^8\right)}{64a^2c^3}}{\left(x^2+\frac{\sqrt{-4a}}{2c}\right.}\right.$

input
```
int(1/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/8*b*c^2*(8*a*c-b^2)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8/a^2*c*(28*a
^2*c^2-49*a*b^2*c+6*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*b*(4*a^2*c^2+2
0*a*b^2*c-3*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/8*(44*a^2*c^2-37*a*b
^2*c+5*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x)/(c*x^4+b*x^2+a)^2+3/16/a^2*sum
((-b*c*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+(28*a^2*c^2-9*a*b^2*c+b
^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+
_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4323 vs. $2(309) = 618$.

Time = 0.56 (sec) , antiderivative size = 4323, normalized size of antiderivative = 12.18

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx = \int \frac{1}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(3*(b^3*c^2 - 8*a*b*c^3)*x^7 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*x
^5 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*x^3 + (5*a*b^4 - 37*a^2*b^2*c + 44
*a^3*c^2)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8
*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6
 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 1
6*a^5*b*c^2)*x^2) - 3/8*integrate(-(b^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^3*c
- 8*a*b*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*
c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2707 vs. $2(309) = 618$.

Time = 0.89 (sec) , antiderivative size = 2707, normalized size of antiderivative = 7.63

$$\int \frac{1}{(a + bx^2 + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^7*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 +
 26*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
4*c^3 - 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4
 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^
4 + 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5
- 896*a^4*c^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^6*c - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c^2 - 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
*c^2 + 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
*c^3 + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
2*c^3 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*...
```

Mupad [B] (verification not implemented)

Time = 21.79 (sec) , antiderivative size = 10979, normalized size of antiderivative = 30.93

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((x*(5*b^4 + 44*a^2*c^2 - 37*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
) + (x^5*(6*b^4*c + 28*a^2*c^3 - 49*a*b^2*c^2))/(8*a^2*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c)) - (x^3*(4*a^2*b*c^2 - 3*b^5 + 20*a*b^3*c))/(8*a^2*(b^4 + 16*a
^2*c^2 - 8*a*b^2*c)) + (3*c*x^7*(b^3*c - 8*a*b*c^2))/(8*a^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*
x^6) - atan(((((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3
- 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*
a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a
^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a
^9*b^2*c^5)) - (x*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9
*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 31686
4*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^
3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-
(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18
*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^
10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^
4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2
 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*
b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256
*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^...
```

Reduce [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 6794, normalized size of antiderivative = 19.14

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(c*x^4+b*x^2+a)^3,x)
```

output

```
(264*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b*c**2 - 66*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**3*c + 528*sqrt(a)*sqrt(2*sqrt(c)*sqrt(
a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sq
rt(a) + b))*a**3*b**2*c**2*x**2 + 528*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*
atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
b))*a**3*b*c**3*x**4 + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**5
 - 132*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**4*c*x**2 + 132*sqrt
(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt
(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c**2*x**4 + 528*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**3*x**6 + 264*sqrt(a)*sqrt(2*sqrt(
c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sq
rt(c)*sqrt(a) + b))*a**2*b*c**4*x**8 + 12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a
) + b))*a*b**6*x**2 - 54*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**5...
```

3.328 $\int \frac{1}{a^2+b+2ax^2+x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 227

$$\int \frac{1}{a^2+b+2ax^2+x^4}\,dx = -\frac{\arctan\left(\frac{\sqrt{-a+\sqrt{a^2+b}}-\sqrt{2}x}{\sqrt{a+\sqrt{a^2+b}}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{a+\sqrt{a^2+b}}}+\frac{\arctan\left(\frac{\sqrt{-a+\sqrt{a^2+b}}+\sqrt{2}x}{\sqrt{a+\sqrt{a^2+b}}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{a+\sqrt{a^2+b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}x}{\sqrt{a^2+b}+x^2}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}}$$

output
```
-1/4*arctan(((-a+(a^2+b)^(1/2))^(1/2)-x*2^(1/2))/(a+(a^2+b)^(1/2))^(1/2))*
2^(1/2)/(a^2+b)^(1/2)/(a+(a^2+b)^(1/2))^(1/2)+1/4*arctan(((-a+(a^2+b)^(1/2
))^(1/2)+x*2^(1/2))/(a+(a^2+b)^(1/2))^(1/2))*2^(1/2)/(a^2+b)^(1/2)/(a+(a^2
+b)^(1/2))^(1/2)+1/4*arctanh(2^(1/2)*(-a+(a^2+b)^(1/2))^(1/2)*x/((a^2+b)^(
1/2)+x^2))*2^(1/2)/(a^2+b)^(1/2)/(-a+(a^2+b)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.36

$$\int \frac{1}{a^2+b+2ax^2+x^4}\,dx = -\frac{i\left(\frac{\arctan\left(\frac{x}{\sqrt{a-i\sqrt{b}}}\right)}{\sqrt{a-i\sqrt{b}}}-\frac{\arctan\left(\frac{x}{\sqrt{a+i\sqrt{b}}}\right)}{\sqrt{a+i\sqrt{b}}}\right)}{2\sqrt{b}}$$

input
```
Integrate[(a^2 + b + 2*a*x^2 + x^4)^(-1),x]
```

output
```
((-1/2*I)*(ArcTan[x/Sqrt[a - I*Sqrt[b]]]/Sqrt[a - I*Sqrt[b]] - ArcTan[x/Sq
rt[a + I*Sqrt[b]]]/Sqrt[a + I*Sqrt[b]]))/Sqrt[b]
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.42, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1407, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+2ax^2+b+x^4}dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{a^2+b}-a}-x}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}+\frac{\int \frac{x+\sqrt{2}\sqrt{\sqrt{a^2+b}-a}}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\sqrt{\sqrt{a^2+b}-a}\int \frac{1}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{\sqrt{2}}-\frac{1}{2}\int -\frac{\sqrt{2}\left(\sqrt{\sqrt{a^2+b}-a}-\sqrt{2}x\right)}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}+\frac{\frac{\sqrt{\sqrt{a^2+b}-a}\int \frac{1}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{2}x+\sqrt{\sqrt{a^2+b}-a}\right)}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{\sqrt{\sqrt{a^2+b}-a}\int \frac{1}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{\sqrt{a^2+b}-a}-\sqrt{2}x\right)}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}+\frac{\frac{\sqrt{\sqrt{a^2+b}-a}\int \frac{1}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{2}x+\sqrt{\sqrt{a^2+b}-a}\right)}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{\sqrt{\sqrt{a^2+b}-a}\int \frac{1}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{\sqrt{2}}+\frac{\int \frac{\sqrt{\sqrt{a^2+b}-a}-\sqrt{2}x}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{\sqrt{2}}}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}+\frac{\frac{\sqrt{\sqrt{a^2+b}-a}\int \frac{1}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{\sqrt{2}}+\frac{\int \frac{\sqrt{2}x+\sqrt{\sqrt{a^2+b}-a}}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{\sqrt{2}}}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}$$

$$\Big\downarrow \textbf{1083}$$

$$\frac{\frac{\int \frac{\sqrt{\sqrt{a^2+b}-a}-\sqrt{2}x}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}\int \frac{1}{-\left(2x-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}\right)^2-2\left(a+\sqrt{a^2+b}\right)}d\left(2x-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}+\frac{\frac{\int \frac{\sqrt{2}x+\sqrt{\sqrt{a^2+b}-a}}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}\int \frac{1}{-\left(2x+\sqrt{2}\sqrt{\sqrt{a^2+b}-a}\right)^2-2\left(a+\sqrt{a^2+b}\right)}d\left(2x+\sqrt{2}\sqrt{\sqrt{a^2+b}-a}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{\frac{\int \frac{\sqrt{\sqrt{a^2+b}-a}-\sqrt{2}x}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{\sqrt{2}}+\frac{\sqrt{\sqrt{a^2+b}-a}\arctan\left(\frac{2x-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}}{\sqrt{2}\sqrt{\sqrt{a^2+b}+a}}\right)}{\sqrt{\sqrt{a^2+b}+a}}}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}+\frac{\frac{\int \frac{\sqrt{2}x+\sqrt{\sqrt{a^2+b}-a}}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+b}-a}x+\sqrt{a^2+b}}dx}{\sqrt{2}}+\frac{\sqrt{\sqrt{a^2+b}-a}\arctan\left(\frac{\sqrt{2}\sqrt{\sqrt{a^2+b}-a}+2x}{\sqrt{2}\sqrt{\sqrt{a^2+b}+a}}\right)}{\sqrt{\sqrt{a^2+b}+a}}}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\sqrt{\sqrt{a^2+b}-a}\arctan\left(\frac{2x-\sqrt{2}\sqrt{\sqrt{a^2+b}-a}}{\sqrt{2}\sqrt{\sqrt{a^2+b}+a}}\right)}{\sqrt{\sqrt{a^2+b}+a}}-\frac{1}{2}\log\left(-\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}+\frac{\frac{\sqrt{\sqrt{a^2+b}-a}\arctan\left(\frac{\sqrt{2}\sqrt{\sqrt{a^2+b}-a}+2x}{\sqrt{2}\sqrt{\sqrt{a^2+b}+a}}\right)}{\sqrt{\sqrt{a^2+b}+a}}+\frac{1}{2}\log\left(\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}$$

input
```
Int[(a^2 + b + 2*a*x^2 + x^4)^(-1),x]
```

output
```
((Sqrt[-a + Sqrt[a^2 + b]]*ArcTan[(-(Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]]) + 2
*x)/(Sqrt[2]*Sqrt[a + Sqrt[a^2 + b]])])/Sqrt[a + Sqrt[a^2 + b]] - Log[Sqrt
[a^2 + b] - Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]]*x + x^2]/2)/(2*Sqrt[2]*Sqrt[a
^2 + b]*Sqrt[-a + Sqrt[a^2 + b]]) + ((Sqrt[-a + Sqrt[a^2 + b]]*ArcTan[(Sqr
t[2]*Sqrt[-a + Sqrt[a^2 + b]] + 2*x)/(Sqrt[2]*Sqrt[a + Sqrt[a^2 + b]])])/S
qrt[a + Sqrt[a^2 + b]] + Log[Sqrt[a^2 + b] + Sqrt[2]*Sqrt[-a + Sqrt[a^2 +
b]]*x + x^2]/2)/(2*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[-a + Sqrt[a^2 + b]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+2a\_Z^2+a^2+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^3+\_Ra}\right)}{4}$
default	$\frac{\frac{\left(\sqrt{a^2+b}\,\sqrt{2\sqrt{a^2+b}-2a}\,a^2+\sqrt{2\sqrt{a^2+b}-2a}\,a^3+\sqrt{a^2+b}\,\sqrt{2\sqrt{a^2+b}-2a}\,b+\sqrt{2\sqrt{a^2+b}-2a}\,ab\right)\ln\left(x^2+x\sqrt{2\sqrt{a^2+b}-2a}+\sqrt{a^2+b}\right)}{2}+\frac{2\left(2a^2b+2b^2-\right.}{}}{4b(a^2+}$

input
```
int(1/(x^4+2*a*x^2+a^2+b),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(_R^3+_R*a)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2*a+a^2+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(171) = 342$.

Time = 0.10 (sec) , antiderivative size = 583, normalized size of antiderivative = 2.57

$$\int \frac{1}{a^2+b+2ax^2+x^4}\,dx$$

$$= \frac{1}{4}\sqrt{\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}+a}{a^2b+b^2}}\log\left(\left(\left(a^3b+ab^2\right)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}+b\right)\sqrt{\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2}}}{a^2b+b^2}}\right.$$
$$\left. +x\right)$$
$$-\frac{1}{4}\sqrt{\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}+a}{a^2b+b^2}}\log\left(-\left(\left(a^3b+ab^2\right)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}+b\right)\sqrt{\frac{(a^2b+b^2)\sqrt{-}}{a^2b}}\right.$$
$$\left. +x\right)$$
$$-\frac{1}{4}\sqrt{-\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}-a}{a^2b+b^2}}\log\left(\left(\left(a^3b+ab^2\right)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}-b\right)\sqrt{-\frac{(a^2b+b^2)\sqrt{-}}{a^2}}\right.$$
$$\left. +x\right)$$
$$+\frac{1}{4}\sqrt{-\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}-a}{a^2b+b^2}}\log\left(-\left(\left(a^3b+ab^2\right)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}-b\right)\sqrt{-\frac{(a^2b+b^2)}{}}\right.$$
$$\left. +x\right)$$

input
```
integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="fricas")
```

output
```
1/4*sqrt(((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b
^2))*log(((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + b)*sqrt(((a
^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2)) + x) -
1/4*sqrt(((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b
^2))*log(-((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + b)*sqrt(((
a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2)) + x) -
 1/4*sqrt(-((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b +
 b^2))*log(((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - b)*sqrt(-
((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2)) + x)
 + 1/4*sqrt(-((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b
 + b^2))*log(-((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - b)*sqr
t(-((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2)) +
 x)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.28

$$\int \frac{1}{a^2+b+2ax^2+x^4}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(256a^2b^2+256b^3\right)-32t^2ab+1,\left(t\mapsto t\log\left(64t^3a^3b+64t^3ab^2-4ta^2+4tb+x\right)\right)\right)$$

input
```
integrate(1/(x**4+2*a*x**2+a**2+b),x)
```

output
```
RootSum(_t**4*(256*a**2*b**2 + 256*b**3) - 32*_t**2*a*b + 1, Lambda(_t, _t
*log(64*_t**3*a**3*b + 64*_t**3*a*b**2 - 4*_t*a**2 + 4*_t*b + x)))
```

Maxima [F]

$$\int \frac{1}{a^2+b+2ax^2+x^4}\,dx=\int \frac{1}{x^4+2\,ax^2+a^2+b}\,dx$$

input
```
integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="maxima")
```

output
```
integrate(1/(x^4 + 2*a*x^2 + a^2 + b), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.33

$$\int \frac{1}{a^2+b+2ax^2+x^4}\,dx = -\frac{\sqrt{a+\sqrt{-b}}\arctan\left(\frac{x}{\sqrt{a+\sqrt{-b}}}\right)}{2\left(a\sqrt{-b}-b\right)} + \frac{\sqrt{a-\sqrt{-b}}\arctan\left(\frac{x}{\sqrt{a-\sqrt{-b}}}\right)}{2\left(a\sqrt{-b}+b\right)}$$

input
```
integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="giac")
```

output
```
-1/2*sqrt(a + sqrt(-b))*arctan(x/sqrt(a + sqrt(-b)))/(a*sqrt(-b) - b) + 1/
2*sqrt(a - sqrt(-b))*arctan(x/sqrt(a - sqrt(-b)))/(a*sqrt(-b) + b)
```

Mupad [B] (verification not implemented)

Time = 17.43 (sec) , antiderivative size = 872, normalized size of antiderivative = 3.84

$$\int \frac{1}{a^2+b+2ax^2+x^4}\,dx = -2\,\mathrm{atanh}\left(\frac{8\,x\,\sqrt{\frac{a\,b}{16\,(a^2\,b^2+b^3)}-\frac{\sqrt{-b^3}}{16\,(a^2\,b^2+b^3)}}}{\frac{2\,b\,\sqrt{-b^3}}{a^2\,b^2+b^3}-\frac{2\,a\,b^2}{a^2\,b^2+b^3}} - \frac{8\,a^2\,b^2\,x\,\sqrt{\frac{a\,b}{16\,(a^2\,b^2+b^3)}-\frac{\sqrt{-b^3}}{16\,(a^2\,b^2+b^3)}}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}-\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}-\frac{2\,a\,b^5}{a^2\,b^2+b^3}+\frac{2\,a^2\,b^3\,\sqrt{-b^3}}{a^2\,b^2+b^3}} + \frac{8\,a\,b\,x\,\sqrt{\frac{a\,b}{16\,(a^2\,b^2+b^3)}-\frac{\sqrt{-b^3}}{16\,(a^2\,b^2+b^3)}}\,\sqrt{-b^3}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}-\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}-\frac{2\,a\,b^5}{a^2\,b^2+b^3}+\frac{2\,a^2\,b^3\,\sqrt{-b^3}}{a^2\,b^2+b^3}}\right)\sqrt{\frac{a\,b-\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}} - 2\,\mathrm{atanh}\left(\frac{8\,a^2\,b^2\,x\,\sqrt{\frac{\sqrt{-b^3}}{16\,(a^2\,b^2+b^3)}+\frac{a\,b}{16\,(a^2\,b^2+b^3)}}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}+\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}+\frac{2\,a\,b^5}{a^2\,b^2+b^3}+\frac{2\,a^2\,b^3\,\sqrt{-b^3}}{a^2\,b^2+b^3}} - \frac{8\,x\,\sqrt{\frac{\sqrt{-b^3}}{16\,(a^2\,b^2+b^3)}+\frac{a\,b}{16\,(a^2\,b^2+b^3)}}}{\frac{2\,b\,\sqrt{-b^3}}{a^2\,b^2+b^3}+\frac{2\,a\,b^2}{a^2\,b^2+b^3}} + \frac{8\,a\,b\,x\,\sqrt{\frac{\sqrt{-b^3}}{16\,(a^2\,b^2+b^3)}+\frac{a\,b}{16\,(a^2\,b^2+b^3)}}\,\sqrt{-b^3}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}+\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}+\frac{2\,a\,b^5}{a^2\,b^2+b^3}+\frac{2\,a^2\,b^3\,\sqrt{-b^3}}{a^2\,b^2+b^3}}\right)\sqrt{\frac{a\,b+\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}}$$

input
```
int(1/(b + 2*a*x^2 + a^2 + x^4),x)
```

output
```
- 2*atanh((8*x*((a*b)/(16*(b^3 + a^2*b^2)) - (-b^3)^(1/2)/(16*(b^3 + a^2*b
^2)))^(1/2))/((2*b*(-b^3)^(1/2))/(b^3 + a^2*b^2) - (2*a*b^2)/(b^3 + a^2*b^
2)) - (8*a^2*b^2*x*((a*b)/(16*(b^3 + a^2*b^2)) - (-b^3)^(1/2)/(16*(b^3 + a
^2*b^2)))^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) - (2*a^3*b^4)/(b^3
+ a^2*b^2) - (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b^3 + a
^2*b^2)) + (8*a*b*x*((a*b)/(16*(b^3 + a^2*b^2)) - (-b^3)^(1/2)/(16*(b^3 +
a^2*b^2)))^(1/2)*(-b^3)^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) - (2*
a^3*b^4)/(b^3 + a^2*b^2) - (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(
1/2))/(b^3 + a^2*b^2)))*((a*b - (-b^3)^(1/2))/(16*(b^3 + a^2*b^2)))^(1/2)
- 2*atanh((8*a^2*b^2*x*((-b^3)^(1/2)/(16*(b^3 + a^2*b^2)) + (a*b)/(16*(b^3
 + a^2*b^2)))^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) + (2*a^3*b^4)/(
b^3 + a^2*b^2) + (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b^3
 + a^2*b^2)) - (8*x*((-b^3)^(1/2)/(16*(b^3 + a^2*b^2)) + (a*b)/(16*(b^3 +
a^2*b^2)))^(1/2))/((2*b*(-b^3)^(1/2))/(b^3 + a^2*b^2) + (2*a*b^2)/(b^3 + a
^2*b^2)) + (8*a*b*x*((-b^3)^(1/2)/(16*(b^3 + a^2*b^2)) + (a*b)/(16*(b^3 +
a^2*b^2)))^(1/2)*(-b^3)^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) + (2*
a^3*b^4)/(b^3 + a^2*b^2) + (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(
1/2))/(b^3 + a^2*b^2)))*((a*b + (-b^3)^(1/2))/(16*(b^3 + a^2*b^2)))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 569, normalized size of antiderivative = 2.51

$$\int \frac{1}{a^2+b+2ax^2+x^4}\,dx = \frac{\sqrt{2}\left(2\sqrt{\sqrt{a^2+b}+a}\,\sqrt{a^2+b}\,atan\left(\frac{\sqrt{\sqrt{a^2+b}-a}\,\sqrt{2}-2x}{\sqrt{\sqrt{a^2+b}+a}\,\sqrt{2}}\right)a-2\sqrt{\sqrt{a^2+b}+a}\,atan\left(\frac{\sqrt{\sqrt{a^2+b}-a}\,\sqrt{2}-2x}{\sqrt{\sqrt{a^2+b}+a}\,\sqrt{2}}\right)a^2-2\cdots\right.}{}$$

input
```
int(1/(x^4+2*a*x^2+a^2+b),x)
```

output

```
(sqrt(2)*(2*sqrt(sqrt(a**2 + b) + a)*sqrt(a**2 + b)*atan((sqrt(sqrt(a**2 +
 b) - a)*sqrt(2) - 2*x)/(sqrt(sqrt(a**2 + b) + a)*sqrt(2)))*a - 2*sqrt(sqr
t(a**2 + b) + a)*atan((sqrt(sqrt(a**2 + b) - a)*sqrt(2) - 2*x)/(sqrt(sqrt(
a**2 + b) + a)*sqrt(2)))*a**2 - 2*sqrt(sqrt(a**2 + b) + a)*atan((sqrt(sqrt
(a**2 + b) - a)*sqrt(2) - 2*x)/(sqrt(sqrt(a**2 + b) + a)*sqrt(2)))*b - 2*s
qrt(sqrt(a**2 + b) + a)*sqrt(a**2 + b)*atan((sqrt(sqrt(a**2 + b) - a)*sqrt
(2) + 2*x)/(sqrt(sqrt(a**2 + b) + a)*sqrt(2)))*a + 2*sqrt(sqrt(a**2 + b) +
 a)*atan((sqrt(sqrt(a**2 + b) - a)*sqrt(2) + 2*x)/(sqrt(sqrt(a**2 + b) + a
)*sqrt(2)))*a**2 + 2*sqrt(sqrt(a**2 + b) + a)*atan((sqrt(sqrt(a**2 + b) -
a)*sqrt(2) + 2*x)/(sqrt(sqrt(a**2 + b) + a)*sqrt(2)))*b - sqrt(sqrt(a**2 +
 b) - a)*sqrt(a**2 + b)*log(sqrt(a**2 + b) - sqrt(sqrt(a**2 + b) - a)*sqrt
(2)*x + x**2)*a + sqrt(sqrt(a**2 + b) - a)*sqrt(a**2 + b)*log(sqrt(a**2 +
b) + sqrt(sqrt(a**2 + b) - a)*sqrt(2)*x + x**2)*a - sqrt(sqrt(a**2 + b) -
a)*log(sqrt(a**2 + b) - sqrt(sqrt(a**2 + b) - a)*sqrt(2)*x + x**2)*a**2 -
sqrt(sqrt(a**2 + b) - a)*log(sqrt(a**2 + b) - sqrt(sqrt(a**2 + b) - a)*sqr
t(2)*x + x**2)*b + sqrt(sqrt(a**2 + b) - a)*log(sqrt(a**2 + b) + sqrt(sqrt
(a**2 + b) - a)*sqrt(2)*x + x**2)*a**2 + sqrt(sqrt(a**2 + b) - a)*log(sqrt
(a**2 + b) + sqrt(sqrt(a**2 + b) - a)*sqrt(2)*x + x**2)*b))/(8*b*(a**2 + b
))
```

3.329 $\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 47

$$\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx = -\frac{\arctan\left(\frac{x}{\sqrt{1+a}}\right)}{2\sqrt{1+a}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

output
```
-1/2*arctan(x/(1+a)^(1/2))/(1+a)^(1/2)-1/2*arctanh(x/(1-a)^(1/2))/(1-a)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx = \frac{\arctan\left(\frac{x}{\sqrt{-1+a}}\right)}{2\sqrt{-1+a}} - \frac{\arctan\left(\frac{x}{\sqrt{1+a}}\right)}{2\sqrt{1+a}}$$

input
```
Integrate[(-1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

output
```
ArcTan[x/Sqrt[-1 + a]]/(2*Sqrt[-1 + a]) - ArcTan[x/Sqrt[1 + a]]/(2*Sqrt[1
+ a])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+2ax^2+x^4-1}dx$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{2}\int \frac{1}{x^2+a-1}dx-\frac{1}{2}\int \frac{1}{x^2+a+1}dx$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\int \frac{1}{x^2+a-1}dx-\frac{\arctan\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}$$

$$\Big\downarrow \text{220}$$

$$-\frac{\arctan\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

input
```
Int[(-1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

output
```
-1/2*ArcTan[x/Sqrt[1 + a]]/Sqrt[1 + a] - ArcTanh[x/Sqrt[1 - a]]/(2*Sqrt[1
- a])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{\arctan\left(\frac{x}{\sqrt{1+a}}\right)}{2\sqrt{1+a}}+\frac{\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2\sqrt{a-1}}$	32
risch	$-\frac{\ln\left(x\sqrt{1-a}-a+1\right)}{4\sqrt{1-a}}+\frac{\ln\left(x\sqrt{1-a}+a-1\right)}{4\sqrt{1-a}}-\frac{\ln\left(-x\sqrt{-1-a}-a-1\right)}{4\sqrt{-1-a}}+\frac{\ln\left(-x\sqrt{-1-a}+a+1\right)}{4\sqrt{-1-a}}$	96

input
```
int(1/(x^4+2*a*x^2+a^2-1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(x/(1+a)^(1/2))/(1+a)^(1/2)+1/2/(a-1)^(1/2)*arctan(x/(a-1)^(1/2
))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 269, normalized size of antiderivative = 5.72

$$\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx$$

$$= \left[-\frac{(a-1)\sqrt{-a-1}\log\left(\frac{x^2+2\sqrt{-a-1}x-a-1}{x^2+a+1}\right)+(a+1)\sqrt{-a+1}\log\left(\frac{x^2-2\sqrt{-a+1}x-a+1}{x^2+a-1}\right)}{4(a^2-1)}, \frac{2(a+1)\sqrt{a-1}\,\mathrm{a}\right.$$

$$-\frac{2\sqrt{a+1}(a-1)\arctan\left(\frac{x}{\sqrt{a+1}}\right)+(a+1)\sqrt{-a+1}\log\left(\frac{x^2-2\sqrt{-a+1}x-a+1}{x^2+a-1}\right)}{4(a^2-1)},$$

$$\left.-\frac{\sqrt{a+1}(a-1)\arctan\left(\frac{x}{\sqrt{a+1}}\right)-(a+1)\sqrt{a-1}\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2(a^2-1)}\right]$$

input
```
integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="fricas")
```

output
```
[-1/4*((a - 1)*sqrt(-a - 1)*log((x^2 + 2*sqrt(-a - 1)*x - a - 1)/(x^2 + a
+ 1)) + (a + 1)*sqrt(-a + 1)*log((x^2 - 2*sqrt(-a + 1)*x - a + 1)/(x^2 + a
 - 1)))/(a^2 - 1), 1/4*(2*(a + 1)*sqrt(a - 1)*arctan(x/sqrt(a - 1)) - (a -
 1)*sqrt(-a - 1)*log((x^2 + 2*sqrt(-a - 1)*x - a - 1)/(x^2 + a + 1)))/(a^2
 - 1), -1/4*(2*sqrt(a + 1)*(a - 1)*arctan(x/sqrt(a + 1)) + (a + 1)*sqrt(-a
 + 1)*log((x^2 - 2*sqrt(-a + 1)*x - a + 1)/(x^2 + a - 1)))/(a^2 - 1), -1/2
*(sqrt(a + 1)*(a - 1)*arctan(x/sqrt(a + 1)) - (a + 1)*sqrt(a - 1)*arctan(x
/sqrt(a - 1)))/(a^2 - 1)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(37) = 74.

Time = 0.32 (sec) , antiderivative size = 257, normalized size of antiderivative = 5.47

$$\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx = \frac{\sqrt{-\frac{1}{a-1}}\log\left(-a^3\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}-a^2\sqrt{-\frac{1}{a-1}}+a\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+x-\sqrt{-\frac{1}{a-1}}\right)}{4} - \frac{\sqrt{-\frac{1}{a-1}}\log\left(a^3\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+a^2\sqrt{-\frac{1}{a-1}}-a\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+x+\sqrt{-\frac{1}{a-1}}\right)}{4} + \frac{\sqrt{-\frac{1}{a+1}}\log\left(-a^3\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}-a^2\sqrt{-\frac{1}{a+1}}+a\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}+x-\sqrt{-\frac{1}{a+1}}\right)}{4} - \frac{\sqrt{-\frac{1}{a+1}}\log\left(a^3\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}+a^2\sqrt{-\frac{1}{a+1}}-a\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}+x+\sqrt{-\frac{1}{a+1}}\right)}{4}$$

input
```
integrate(1/(x**4+2*a*x**2+a**2-1),x)
```

output
```
sqrt(-1/(a - 1))*log(-a**3*(-1/(a - 1))**(3/2) - a**2*sqrt(-1/(a - 1)) + a
*(-1/(a - 1))**(3/2) + x - sqrt(-1/(a - 1)))/4 - sqrt(-1/(a - 1))*log(a**3
*(-1/(a - 1))**(3/2) + a**2*sqrt(-1/(a - 1)) - a*(-1/(a - 1))**(3/2) + x +
 sqrt(-1/(a - 1)))/4 + sqrt(-1/(a + 1))*log(-a**3*(-1/(a + 1))**(3/2) - a*
*2*sqrt(-1/(a + 1)) + a*(-1/(a + 1))**(3/2) + x - sqrt(-1/(a + 1)))/4 - sq
rt(-1/(a + 1))*log(a**3*(-1/(a + 1))**(3/2) + a**2*sqrt(-1/(a + 1)) - a*(-
1/(a + 1))**(3/2) + x + sqrt(-1/(a + 1)))/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-1.0>0)', see `assume?` for mor
e details)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx = -\frac{\arctan\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}+\frac{\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2\sqrt{a-1}}$$

input
```
integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="giac")
```

output
```
-1/2*arctan(x/sqrt(a + 1))/sqrt(a + 1) + 1/2*arctan(x/sqrt(a - 1))/sqrt(a
- 1)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.81

$$\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx = \frac{\operatorname{atanh}\left(\frac{2x\left(\frac{a}{2}-\frac{1}{2}\right)}{\sqrt{1-a}}+\frac{2ax\left(\frac{a}{2}-\frac{1}{2}\right)}{(1-a)^{3/2}}\right)}{2\sqrt{1-a}}+\frac{\operatorname{atanh}\left(\frac{2x\left(\frac{a}{2}+\frac{1}{2}\right)}{\sqrt{-a-1}}+\frac{2ax\left(\frac{a}{2}+\frac{1}{2}\right)}{(-a-1)^{3/2}}\right)}{2\sqrt{-a-1}}$$

input
```
int(1/(2*a*x^2 + a^2 + x^4 - 1),x)
```

output
```
atanh((2*x*(a/2 - 1/2))/(1 - a)^(1/2) + (2*a*x*(a/2 - 1/2))/(1 - a)^(3/2))
/(2*(1 - a)^(1/2)) + atanh((2*x*(a/2 + 1/2))/(- a - 1)^(1/2) + (2*a*x*(a/2
 + 1/2))/(- a - 1)^(3/2))/(2*(- a - 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx = \frac{\sqrt{a-1}\,atan\left(\frac{x}{\sqrt{a-1}}\right)a+\sqrt{a-1}\,atan\left(\frac{x}{\sqrt{a-1}}\right)-\sqrt{a+1}\,atan\left(\frac{x}{\sqrt{a+1}}\right)a+\sqrt{a+1}\,atan\left(\frac{x}{\sqrt{a+1}}\right)}{2a^2-2}$$

input
```
int(1/(x^4+2*a*x^2+a^2-1),x)
```

output
```
(sqrt(a - 1)*atan(x/sqrt(a - 1))*a + sqrt(a - 1)*atan(x/sqrt(a - 1)) - sqr
t(a + 1)*atan(x/sqrt(a + 1))*a + sqrt(a + 1)*atan(x/sqrt(a + 1)))/(2*(a**2
 - 1))
```

3.330 $\int \frac{1}{1+a^2+2ax^2+x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 227

$$\int \frac{1}{1+a^2+2ax^2+x^4}\,dx = -\frac{\arctan\left(\frac{\sqrt{-a+\sqrt{1+a^2}}-\sqrt{2}x}{\sqrt{a+\sqrt{1+a^2}}}\right)}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}}+\frac{\arctan\left(\frac{\sqrt{-a+\sqrt{1+a^2}}+\sqrt{2}x}{\sqrt{a+\sqrt{1+a^2}}}\right)}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}x}{\sqrt{1+a^2}+x^2}\right)}{2\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}}$$

output

```
-1/4*arctan(((-a+(a^2+1)^(1/2))^(1/2)-x*2^(1/2))/(a+(a^2+1)^(1/2))^(1/2))*
2^(1/2)/(a^2+1)^(1/2)/(a+(a^2+1)^(1/2))^(1/2)+1/4*arctan(((-a+(a^2+1)^(1/2
))^(1/2)+x*2^(1/2))/(a+(a^2+1)^(1/2))^(1/2))*2^(1/2)/(a^2+1)^(1/2)/(a+(a^2
+1)^(1/2))^(1/2)+1/4*arctanh(2^(1/2)*(-a+(a^2+1)^(1/2))^(1/2)*x/((a^2+1)^(
1/2)+x^2))*2^(1/2)/(a^2+1)^(1/2)/(-a+(a^2+1)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.23

$$\int \frac{1}{1+a^2+2ax^2+x^4}\,dx=-\frac{1}{2}i\left(\frac{\arctan\left(\frac{x}{\sqrt{-i+a}}\right)}{\sqrt{-i+a}}-\frac{\arctan\left(\frac{x}{\sqrt{i+a}}\right)}{\sqrt{i+a}}\right)$$

input
```
Integrate[(1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

output
```
(-1/2*I)*(ArcTan[x/Sqrt[-I + a]]/Sqrt[-I + a] - ArcTan[x/Sqrt[I + a]]/Sqrt
[I + a])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.42, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.438$, Rules used = {1407, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+2ax^2+x^4+1}\,dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{a^2+1}-a}-x}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\int \frac{x+\sqrt{2}\sqrt{\sqrt{a^2+1}-a}}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\sqrt{\sqrt{a^2+1}-a}\int \frac{1}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{\sqrt{2}}-\frac{1}{2}\int -\frac{\sqrt{2}\left(\sqrt{\sqrt{a^2+1}-a}-\sqrt{2}x\right)}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\frac{\sqrt{\sqrt{a^2+1}-a}\int \frac{1}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{2}x+\sqrt{\sqrt{a^2+1}-a}\right)}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\sqrt{\sqrt{a^2+1}-a}\int \frac{1}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{\sqrt{a^2+1}-a}-\sqrt{2}x\right)}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\frac{\sqrt{\sqrt{a^2+1}-a}\int \frac{1}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{2}x+\sqrt{\sqrt{a^2+1}-a}\right)}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\sqrt{\sqrt{a^2+1}-a}\int \frac{1}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{\sqrt{2}}+\frac{\int \frac{\sqrt{\sqrt{a^2+1}-a}-\sqrt{2}x}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{\sqrt{2}}}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\frac{\sqrt{\sqrt{a^2+1}-a}\int \frac{1}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{\sqrt{2}}+\frac{\int \frac{\sqrt{2}x+\sqrt{\sqrt{a^2+1}-a}}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{\sqrt{2}}}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}$$

$$\Big\downarrow \ 1083$$

$$\frac{\frac{\int \frac{\sqrt{\sqrt{a^2+1}-a}-\sqrt{2}x}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\int \frac{1}{-\left(2x-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\right)^2-2\left(a+\sqrt{a^2+1}\right)}d\left(2x-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\frac{\int \frac{\sqrt{2}x+\sqrt{\sqrt{a^2+1}-a}}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\int \frac{1}{-\left(2x+\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\right)^2-2\left(a+\sqrt{a^2+1}\right)}d\left(2x+\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{\int \frac{\sqrt{\sqrt{a^2+1}-a}-\sqrt{2}x}{x^2-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{\sqrt{2}}+\frac{\sqrt{\sqrt{a^2+1}-a}\arctan\left(\frac{2x-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}}{\sqrt{2}\sqrt{\sqrt{a^2+1}+a}}\right)}{\sqrt{\sqrt{a^2+1}+a}}}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\frac{\int \frac{\sqrt{2}x+\sqrt{\sqrt{a^2+1}-a}}{x^2+\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}}dx}{\sqrt{2}}+\frac{\sqrt{\sqrt{a^2+1}-a}\arctan\left(\frac{\sqrt{2}\sqrt{\sqrt{a^2+1}-a}+2x}{\sqrt{2}\sqrt{\sqrt{a^2+1}+a}}\right)}{\sqrt{\sqrt{a^2+1}+a}}}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\sqrt{\sqrt{a^2+1}-a}\arctan\left(\frac{2x-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}}{\sqrt{2}\sqrt{\sqrt{a^2+1}+a}}\right)}{\sqrt{\sqrt{a^2+1}+a}}-\frac{1}{2}\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}+x^2\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\frac{\sqrt{\sqrt{a^2+1}-a}\arctan\left(\frac{\sqrt{2}\sqrt{\sqrt{a^2+1}-a}+2x}{\sqrt{2}\sqrt{\sqrt{a^2+1}+a}}\right)}{\sqrt{\sqrt{a^2+1}+a}}+\frac{1}{2}\log\left(\sqrt{2}\sqrt{\sqrt{a^2+1}-a}x+\sqrt{a^2+1}+x^2\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}$$

input
```
Int[(1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

output
```
((Sqrt[-a + Sqrt[1 + a^2]]*ArcTan[(-(Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]) + 2
*x)/(Sqrt[2]*Sqrt[a + Sqrt[1 + a^2]])])/Sqrt[a + Sqrt[1 + a^2]] - Log[Sqrt
[1 + a^2] - Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]*x + x^2]/2)/(2*Sqrt[2]*Sqrt[1
 + a^2]*Sqrt[-a + Sqrt[1 + a^2]]) + ((Sqrt[-a + Sqrt[1 + a^2]]*ArcTan[(Sqr
t[2]*Sqrt[-a + Sqrt[1 + a^2]] + 2*x)/(Sqrt[2]*Sqrt[a + Sqrt[1 + a^2]])])/S
qrt[a + Sqrt[1 + a^2]] + Log[Sqrt[1 + a^2] + Sqrt[2]*Sqrt[-a + Sqrt[1 + a^
2]]*x + x^2]/2)/(2*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[-a + Sqrt[1 + a^2]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}+2a\_Z^{2}+a^{2}+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}+\_Ra}\right)}{4}$
default	$\frac{\frac{\left(-\sqrt{a^{2}+1}\,\sqrt{2\sqrt{a^{2}+1}-2a}\,a^{2}-\sqrt{2\sqrt{a^{2}+1}-2a}\,a^{3}-\sqrt{a^{2}+1}\,\sqrt{2\sqrt{a^{2}+1}-2a}-\sqrt{2\sqrt{a^{2}+1}-2a}\,a\right)\ln\left(x^{2}-x\sqrt{2\sqrt{a^{2}+1}-2a}+\sqrt{a^{2}+1}\right)}{2}+\frac{2\left(2a^{2}+2+\ldots\right.}{\ldots}}{4(a^{2}+1)\ldots}$

input
```
int(1/(x^4+2*a*x^2+a^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(_R^3+_R*a)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2*a+a^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(171) = 342$.

Time = 0.09 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.79

$$\int \frac{1}{1+a^2+2ax^2+x^4}\,dx$$
$$= \frac{1}{4}\sqrt{\frac{(a^2+1)\sqrt{-\frac{1}{a^4+2\,a^2+1}}+a}{a^2+1}}\log\left(\left((a^3+a)\sqrt{-\frac{1}{a^4+2\,a^2+1}}+1\right)\sqrt{\frac{(a^2+1)\sqrt{-\frac{1}{a^4+2\,a^2+1}}+a}{a^2+1}}+x\right)$$
$$-\frac{1}{4}\sqrt{\frac{(a^2+1)\sqrt{-\frac{1}{a^4+2\,a^2+1}}+a}{a^2+1}}\log\left(-\left((a^3+a)\sqrt{-\frac{1}{a^4+2\,a^2+1}}+1\right)\sqrt{\frac{(a^2+1)\sqrt{-\frac{1}{a^4+2\,a^2+1}}+a}{a^2+1}}+x\right)$$
$$-\frac{1}{4}\sqrt{-\frac{(a^2+1)\sqrt{-\frac{1}{a^4+2\,a^2+1}}-a}{a^2+1}}\log\left(\left((a^3+a)\sqrt{-\frac{1}{a^4+2\,a^2+1}}-1\right)\sqrt{-\frac{(a^2+1)\sqrt{-\frac{1}{a^4+2\,a^2+1}}-a}{a^2+1}}+x\right)$$
$$+\frac{1}{4}\sqrt{-\frac{(a^2+1)\sqrt{-\frac{1}{a^4+2\,a^2+1}}-a}{a^2+1}}\log\left(-\left((a^3+a)\sqrt{-\frac{1}{a^4+2\,a^2+1}}-1\right)\sqrt{-\frac{(a^2+1)\sqrt{-\frac{1}{a^4+2\,a^2+1}}-a}{a^2+1}}+x\right)$$

input
```
integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(((a^2 + 1)*sqrt(-1/(a^4 + 2*a^2 + 1)) + a)/(a^2 + 1))*log(((a^3 +
 a)*sqrt(-1/(a^4 + 2*a^2 + 1)) + 1)*sqrt(((a^2 + 1)*sqrt(-1/(a^4 + 2*a^2 +
 1)) + a)/(a^2 + 1)) + x) - 1/4*sqrt(((a^2 + 1)*sqrt(-1/(a^4 + 2*a^2 + 1))
 + a)/(a^2 + 1))*log(-((a^3 + a)*sqrt(-1/(a^4 + 2*a^2 + 1)) + 1)*sqrt(((a^
2 + 1)*sqrt(-1/(a^4 + 2*a^2 + 1)) + a)/(a^2 + 1)) + x) - 1/4*sqrt(-((a^2 +
 1)*sqrt(-1/(a^4 + 2*a^2 + 1)) - a)/(a^2 + 1))*log(((a^3 + a)*sqrt(-1/(a^4
 + 2*a^2 + 1)) - 1)*sqrt(-((a^2 + 1)*sqrt(-1/(a^4 + 2*a^2 + 1)) - a)/(a^2
+ 1)) + x) + 1/4*sqrt(-((a^2 + 1)*sqrt(-1/(a^4 + 2*a^2 + 1)) - a)/(a^2 + 1
))*log(-((a^3 + a)*sqrt(-1/(a^4 + 2*a^2 + 1)) - 1)*sqrt(-((a^2 + 1)*sqrt(-
1/(a^4 + 2*a^2 + 1)) - a)/(a^2 + 1)) + x)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.21

$$\int \frac{1}{1+a^2+2ax^2+x^4}\,dx$$
$$=\operatorname{RootSum}\left(t^4\cdot\left(256a^2+256\right)-32t^2a+1,\left(t\mapsto t\log\left(64t^3a^3+64t^3a-4ta^2+4t+x\right)\right)\right)$$

input
```
integrate(1/(x**4+2*a*x**2+a**2+1),x)
```

output
```
RootSum(_t**4*(256*a**2 + 256) - 32*_t**2*a + 1, Lambda(_t, _t*log(64*_t**
3*a**3 + 64*_t**3*a - 4*_t*a**2 + 4*_t + x)))
```

Maxima [F]

$$\int \frac{1}{1+a^2+2ax^2+x^4}\,dx=\int \frac{1}{x^4+2\,ax^2+a^2+1}\,dx$$

input
```
integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^4 + 2*a*x^2 + a^2 + 1), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{1+a^2+2ax^2+x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 17.38 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.07

$$\int \frac{1}{1+a^2+2ax^2+x^4}\,dx = -\frac{\operatorname{atanh}\left(-\frac{2\,x\,\sqrt{\frac{a}{a^2+1}+\frac{1{}\mathrm{i}}{a^2+1}}}{\frac{2\,a}{a^2+1}+\frac{2{}\mathrm{i}}{a^2+1}}+\frac{a\,x\,\sqrt{\frac{a}{a^2+1}+\frac{1{}\mathrm{i}}{a^2+1}}\,2{}\mathrm{i}}{\frac{2\,a}{a^2+1}+\frac{2\,a^3}{a^2+1}+\frac{2{}\mathrm{i}}{a^2+1}+\frac{a^2\,2{}\mathrm{i}}{a^2+1}}+\frac{2\,a^2\,x\,\sqrt{\frac{a}{a^2+1}+\frac{1{}\mathrm{i}}{a^2+1}}}{\frac{2\,a}{a^2+1}+\frac{2\,a^3}{a^2+1}+\frac{2{}\mathrm{i}}{a^2+1}+\frac{a^2\,2{}\mathrm{i}}{a^2+1}}\right)\,\sqrt{\frac{a+1{}\mathrm{i}}{a^2+1}}}{2} + 2\operatorname{atanh}\left(\frac{8\,x\,\sqrt{\frac{a}{16\,a^2+16}-\frac{1{}\mathrm{i}}{16\,a^2+16}}}{\frac{32\,a}{16\,a^2+16}-\frac{32{}\mathrm{i}}{16\,a^2+16}}+\frac{a\,x\,\sqrt{\frac{a}{16\,a^2+16}-\frac{1{}\mathrm{i}}{16\,a^2+16}}\,128{}\mathrm{i}}{\frac{512\,a}{16\,a^2+16}+\frac{512\,a^3}{16\,a^2+16}-\frac{512{}\mathrm{i}}{16\,a^2+16}-\frac{a^2\,512{}\mathrm{i}}{16\,a^2+16}} - \frac{128\,a^2\,x\,\sqrt{\frac{a}{16\,a^2+16}-\frac{1{}\mathrm{i}}{16\,a^2+16}}}{\frac{512\,a}{16\,a^2+16}+\frac{512\,a^3}{16\,a^2+16}-\frac{512{}\mathrm{i}}{16\,a^2+16}-\frac{a^2\,512{}\mathrm{i}}{16\,a^2+16}}\right)\,\sqrt{\frac{a-\mathrm{i}}{16\,a^2+16}}$$

input
```
int(1/(2*a*x^2 + a^2 + x^4 + 1),x)
```

output
```
2*atanh((8*x*(a/(16*a^2 + 16) - 1i/(16*a^2 + 16))^(1/2))/((32*a)/(16*a^2 +
 16) - 32i/(16*a^2 + 16)) + (a*x*(a/(16*a^2 + 16) - 1i/(16*a^2 + 16))^(1/2
)*128i)/((512*a)/(16*a^2 + 16) - 512i/(16*a^2 + 16) - (a^2*512i)/(16*a^2 +
 16) + (512*a^3)/(16*a^2 + 16)) - (128*a^2*x*(a/(16*a^2 + 16) - 1i/(16*a^2
 + 16))^(1/2))/((512*a)/(16*a^2 + 16) - 512i/(16*a^2 + 16) - (a^2*512i)/(1
6*a^2 + 16) + (512*a^3)/(16*a^2 + 16)))*((a - 1i)/(16*a^2 + 16))^(1/2) - (
atanh((a*x*(a/(a^2 + 1) + 1i/(a^2 + 1))^(1/2)*2i)/((2*a)/(a^2 + 1) + 2i/(a
^2 + 1) + (a^2*2i)/(a^2 + 1) + (2*a^3)/(a^2 + 1)) - (2*x*(a/(a^2 + 1) + 1i
/(a^2 + 1))^(1/2))/((2*a)/(a^2 + 1) + 2i/(a^2 + 1)) + (2*a^2*x*(a/(a^2 + 1
) + 1i/(a^2 + 1))^(1/2))/((2*a)/(a^2 + 1) + 2i/(a^2 + 1) + (a^2*2i)/(a^2 +
 1) + (2*a^3)/(a^2 + 1)))*((a + 1i)/(a^2 + 1))^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 563, normalized size of antiderivative = 2.48

$$\int \frac{1}{1+a^2+2ax^2+x^4}\,dx$$
$$= \frac{\sqrt{2}\left(2\sqrt{\sqrt{a^2+1}+a}\,\sqrt{a^2+1}\,atan\left(\frac{\sqrt{\sqrt{a^2+1}-a}\,\sqrt{2}-2x}{\sqrt{\sqrt{a^2+1}+a}\,\sqrt{2}}\right)a-2\sqrt{\sqrt{a^2+1}+a}\,atan\left(\frac{\sqrt{\sqrt{a^2+1}-a}\,\sqrt{2}-2x}{\sqrt{\sqrt{a^2+1}+a}\,\sqrt{2}}\right)a^2-2}{}$$

input
```
int(1/(x^4+2*a*x^2+a^2+1),x)
```

output
```
(sqrt(2)*(2*sqrt(sqrt(a**2 + 1) + a)*sqrt(a**2 + 1)*atan((sqrt(sqrt(a**2 +
 1) - a)*sqrt(2) - 2*x)/(sqrt(sqrt(a**2 + 1) + a)*sqrt(2)))*a - 2*sqrt(sqr
t(a**2 + 1) + a)*atan((sqrt(sqrt(a**2 + 1) - a)*sqrt(2) - 2*x)/(sqrt(sqrt(
a**2 + 1) + a)*sqrt(2)))*a**2 - 2*sqrt(sqrt(a**2 + 1) + a)*atan((sqrt(sqrt
(a**2 + 1) - a)*sqrt(2) - 2*x)/(sqrt(sqrt(a**2 + 1) + a)*sqrt(2))) - 2*sqr
t(sqrt(a**2 + 1) + a)*sqrt(a**2 + 1)*atan((sqrt(sqrt(a**2 + 1) - a)*sqrt(2
) + 2*x)/(sqrt(sqrt(a**2 + 1) + a)*sqrt(2)))*a + 2*sqrt(sqrt(a**2 + 1) + a
)*atan((sqrt(sqrt(a**2 + 1) - a)*sqrt(2) + 2*x)/(sqrt(sqrt(a**2 + 1) + a)*
sqrt(2)))*a**2 + 2*sqrt(sqrt(a**2 + 1) + a)*atan((sqrt(sqrt(a**2 + 1) - a)
*sqrt(2) + 2*x)/(sqrt(sqrt(a**2 + 1) + a)*sqrt(2))) - sqrt(sqrt(a**2 + 1)
- a)*sqrt(a**2 + 1)*log(sqrt(a**2 + 1) - sqrt(sqrt(a**2 + 1) - a)*sqrt(2)*
x + x**2)*a + sqrt(sqrt(a**2 + 1) - a)*sqrt(a**2 + 1)*log(sqrt(a**2 + 1) +
 sqrt(sqrt(a**2 + 1) - a)*sqrt(2)*x + x**2)*a - sqrt(sqrt(a**2 + 1) - a)*l
og(sqrt(a**2 + 1) - sqrt(sqrt(a**2 + 1) - a)*sqrt(2)*x + x**2)*a**2 - sqrt
(sqrt(a**2 + 1) - a)*log(sqrt(a**2 + 1) - sqrt(sqrt(a**2 + 1) - a)*sqrt(2)
*x + x**2) + sqrt(sqrt(a**2 + 1) - a)*log(sqrt(a**2 + 1) + sqrt(sqrt(a**2
+ 1) - a)*sqrt(2)*x + x**2)*a**2 + sqrt(sqrt(a**2 + 1) - a)*log(sqrt(a**2
+ 1) + sqrt(sqrt(a**2 + 1) - a)*sqrt(2)*x + x**2)))/(8*(a**2 + 1))
```

3.331 $\int \frac{1}{4-5x^2+x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \frac{1}{4-5x^2+x^4}\,dx = -\frac{1}{6}\operatorname{arctanh}\left(\frac{x}{2}\right)+\frac{\operatorname{arctanh}(x)}{3}$$

output
```
-1/6*arctanh(1/2*x)+1/3*arctanh(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{1}{4-5x^2+x^4}\,dx = -\frac{1}{6}\log(1-x)+\frac{1}{12}\log(2-x)+\frac{1}{6}\log(1+x)-\frac{1}{12}\log(2+x)$$

input
```
Integrate[(4 - 5*x^2 + x^4)^(-1),x]
```

output
```
-1/6*Log[1 - x] + Log[2 - x]/12 + Log[1 + x]/6 - Log[2 + x]/12
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-5x^2+4}\,dx$$
$$\Big\downarrow \text{1406}$$
$$\frac{1}{3}\int \frac{1}{x^2-4}dx-\frac{1}{3}\int \frac{1}{x^2-1}dx$$
$$\Big\downarrow \text{220}$$
$$\frac{\text{arctanh}(x)}{3}-\frac{1}{6}\text{arctanh}\left(\frac{x}{2}\right)$$

input
```
Int[(4 - 5*x^2 + x^4)^(-1),x]
```

output
```
-1/6*ArcTanh[x/2] + ArcTanh[x]/3
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

method	result	size
default	$\frac{\ln(x-2)}{12}+\frac{\ln(1+x)}{6}-\frac{\ln(x-1)}{6}-\frac{\ln(x+2)}{12}$	26
norman	$\frac{\ln(x-2)}{12}+\frac{\ln(1+x)}{6}-\frac{\ln(x-1)}{6}-\frac{\ln(x+2)}{12}$	26
risch	$\frac{\ln(x-2)}{12}+\frac{\ln(1+x)}{6}-\frac{\ln(x-1)}{6}-\frac{\ln(x+2)}{12}$	26
parallelrisch	$\frac{\ln(x-2)}{12}+\frac{\ln(1+x)}{6}-\frac{\ln(x-1)}{6}-\frac{\ln(x+2)}{12}$	26

input
```
int(1/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(x-2)+1/6*ln(1+x)-1/6*ln(x-1)-1/12*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{1}{4-5x^2+x^4}\,dx=-\frac{1}{12}\log(x+2)+\frac{1}{6}\log(x+1)-\frac{1}{6}\log(x-1)+\frac{1}{12}\log(x-2)$$

input
```
integrate(1/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
-1/12*log(x + 2) + 1/6*log(x + 1) - 1/6*log(x - 1) + 1/12*log(x - 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{1}{4-5x^2+x^4}\,dx = \frac{\log(x-2)}{12} - \frac{\log(x-1)}{6} + \frac{\log(x+1)}{6} - \frac{\log(x+2)}{12}$$

input
```
integrate(1/(x**4-5*x**2+4),x)
```

output
```
log(x - 2)/12 - log(x - 1)/6 + log(x + 1)/6 - log(x + 2)/12
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{1}{4-5x^2+x^4}\,dx = -\frac{1}{12}\log(x+2) + \frac{1}{6}\log(x+1) - \frac{1}{6}\log(x-1) + \frac{1}{12}\log(x-2)$$

input
```
integrate(1/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
-1/12*log(x + 2) + 1/6*log(x + 1) - 1/6*log(x - 1) + 1/12*log(x - 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(11) = 22$.

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{1}{4-5x^2+x^4}\,dx = -\frac{1}{12}\log(|x+2|) + \frac{1}{6}\log(|x+1|) - \frac{1}{6}\log(|x-1|) + \frac{1}{12}\log(|x-2|)$$

input
```
integrate(1/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
-1/12*log(abs(x + 2)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1)) + 1/12*l
og(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{4-5x^2+x^4}\,dx = \frac{\operatorname{atanh}(x)}{3} - \frac{\operatorname{atanh}\left(\frac{x}{2}\right)}{6}$$

input
```
int(1/(x^4 - 5*x^2 + 4),x)
```

output
```
atanh(x)/3 - atanh(x/2)/6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{1}{4-5x^2+x^4}\,dx = \frac{\log(x-2)}{12} - \frac{\log(x-1)}{6} - \frac{\log(x+2)}{12} + \frac{\log(x+1)}{6}$$

input
```
int(1/(x^4-5*x^2+4),x)
```

output
```
(log(x - 2) - 2*log(x - 1) - log(x + 2) + 2*log(x + 1))/12
```

3.332 $\int \frac{1}{3+4x^2+x^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 24

$$\int \frac{1}{3+4x^2+x^4} dx = \frac{\arctan(x)}{2} - \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
1/2*arctan(x)-1/6*arctan(1/3*x*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{3+4x^2+x^4} dx = \frac{\arctan(x)}{2} - \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Integrate[(3 + 4*x^2 + x^4)^(-1),x]
```

output
```
ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1406, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+4x^2+3}dx$$

$$\Big\downarrow \text{ 1406}$$

$$\frac{1}{2}\int \frac{1}{x^2+1}dx-\frac{1}{2}\int \frac{1}{x^2+3}dx$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan(x)}{2}-\frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[(3 + 4*x^2 + x^4)^(-1),x]
```

output
```
ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\arctan(x)}{2}-\frac{\arctan\left(\frac{\sqrt{3}\,x}{3}\right)\sqrt{3}}{6}$	18
risch	$\frac{\arctan(x)}{2}-\frac{\arctan\left(\frac{\sqrt{3}\,x}{3}\right)\sqrt{3}}{6}$	18

input
```
int(1/(x^4+4*x^2+3),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)-1/6*arctan(1/3*3^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1}{3+4x^2+x^4}\,dx=-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right)+\frac{1}{2}\,\arctan\left(x\right)$$

input
```
integrate(1/(x^4+4*x^2+3),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{3+4x^2+x^4}\,dx=\frac{\operatorname{atan}\left(x\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

input
```
integrate(1/(x**4+4*x**2+3),x)
```

output
```
atan(x)/2 - sqrt(3)*atan(sqrt(3)*x/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1}{3+4x^2+x^4}\,dx=-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{2}\arctan\left(x\right)$$

input
```
integrate(1/(x^4+4*x^2+3),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1}{3+4x^2+x^4}\,dx=-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{2}\arctan\left(x\right)$$

input
```
integrate(1/(x^4+4*x^2+3),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1}{3+4x^2+x^4}\,dx=\frac{\mathrm{atan}(x)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)}{6}$$

input
```
int(1/(4*x^2 + x^4 + 3),x)
```

output
```
atan(x)/2 - (3^(1/2)*atan((3^(1/2)*x)/3))/6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{3+4x^2+x^4}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{x}{\sqrt{3}}\right)}{6} + \frac{atan(x)}{2}$$

input
```
int(1/(x^4+4*x^2+3),x)
```

output
```
( - sqrt(3)*atan(x/sqrt(3)) + 3*atan(x))/6
```

3.333 $\int \frac{1}{9+5x^2+x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{1}{9+5x^2+x^4}\,dx=-\frac{\arctan\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{1}{6}\text{arctanh}\left(\frac{x}{3+x^2}\right)$$

output
```
-1/66*arctan(1/11*(1-2*x)*11^(1/2))*11^(1/2)+1/66*arctan(1/11*(1+2*x)*11^(
1/2))*11^(1/2)+1/6*arctanh(x/(x^2+3))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.60

$$\int \frac{1}{9+5x^2+x^4}\,dx=-\frac{i\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(5-i\sqrt{11}\right)}}\right)}{\sqrt{\frac{11}{2}\left(5-i\sqrt{11}\right)}}+\frac{i\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(5+i\sqrt{11}\right)}}\right)}{\sqrt{\frac{11}{2}\left(5+i\sqrt{11}\right)}}$$

input
```
Integrate[(9 + 5*x^2 + x^4)^(-1),x]
```

output
```
((-I)*ArcTan[x/Sqrt[(5 - I*Sqrt[11])/2]])/Sqrt[(11*(5 - I*Sqrt[11]))/2] +
(I*ArcTan[x/Sqrt[(5 + I*Sqrt[11])/2]])/Sqrt[(11*(5 + I*Sqrt[11]))/2]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+5x^2+9}dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{6}\int \frac{1-x}{x^2-x+3}dx+\frac{1}{6}\int \frac{x+1}{x^2+x+3}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{x^2-x+3}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+3}dx\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{x^2+x+3}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+3}dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{x^2-x+3}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+3}dx\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{x^2+x+3}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+3}dx\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+3}dx-\int \frac{1}{-(2x-1)^2-11}d(2x-1)\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+3}dx-\int \frac{1}{-(2x+1)^2-11}d(2x+1)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+3}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{11}}\right)}{\sqrt{11}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+3}dx+\frac{\arctan\left(\frac{2x+1}{\sqrt{11}}\right)}{\sqrt{11}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{6}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{11}}\right)}{\sqrt{11}}-\frac{1}{2}\log\left(x^2-x+3\right)\right)+\frac{1}{6}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{11}}\right)}{\sqrt{11}}+\frac{1}{2}\log\left(x^2+x+3\right)\right)$$

input
```
Int[(9 + 5*x^2 + x^4)^(-1),x]
```

output
```
(ArcTan[(-1 + 2*x)/Sqrt[11]]/Sqrt[11] - Log[3 - x + x^2]/2)/6 + (ArcTan[(1
 + 2*x)/Sqrt[11]]/Sqrt[11] + Log[3 + x + x^2]/2)/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\ln\left(x^{2}-x+3\right)}{12}+\frac{\sqrt{11}\,\arctan\left(\frac{(2x-1)\sqrt{11}}{11}\right)}{66}+\frac{\ln\left(x^{2}+x+3\right)}{12}+\frac{\arctan\left(\frac{(1+2x)\sqrt{11}}{11}\right)\sqrt{11}}{66}$	54
risch	$-\frac{\ln\left(4x^{2}-4x+12\right)}{12}+\frac{\sqrt{11}\,\arctan\left(\frac{(2x-1)\sqrt{11}}{11}\right)}{66}+\frac{\ln\left(4x^{2}+4x+12\right)}{12}+\frac{\arctan\left(\frac{(1+2x)\sqrt{11}}{11}\right)\sqrt{11}}{66}$	60

input
```
int(1/(x^4+5*x^2+9),x,method=_RETURNVERBOSE)
```

output
```
-1/12*ln(x^2-x+3)+1/66*11^(1/2)*arctan(1/11*(2*x-1)*11^(1/2))+1/12*ln(x^2+
x+3)+1/66*arctan(1/11*(1+2*x)*11^(1/2))*11^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{9+5x^2+x^4}\,dx = \frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right) + \frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right) + \frac{1}{12}\log\left(x^2+x+3\right) - \frac{1}{12}\log\left(x^2-x+3\right)$$

input
```
integrate(1/(x^4+5*x^2+9),x, algorithm="fricas")
```

output
```
1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*
sqrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{1}{9+5x^2+x^4}\,dx = -\frac{\log\left(x^2-x+3\right)}{12}+\frac{\log\left(x^2+x+3\right)}{12}+\frac{\sqrt{11}\,\mathrm{atan}\left(\frac{2\sqrt{11}x}{11}-\frac{\sqrt{11}}{11}\right)}{66}+\frac{\sqrt{11}\,\mathrm{atan}\left(\frac{2\sqrt{11}x}{11}+\frac{\sqrt{11}}{11}\right)}{66}$$

input
```
integrate(1/(x**4+5*x**2+9),x)
```

output
```
-log(x**2 - x + 3)/12 + log(x**2 + x + 3)/12 + sqrt(11)*atan(2*sqrt(11)*x/
11 - sqrt(11)/11)/66 + sqrt(11)*atan(2*sqrt(11)*x/11 + sqrt(11)/11)/66
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{9+5x^2+x^4}\,dx = \frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right)$$

input
```
integrate(1/(x^4+5*x^2+9),x, algorithm="maxima")
```

output
```
1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*
sqrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{9+5x^2+x^4}\,dx = \frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right) + \frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right) + \frac{1}{12}\log\left(x^2+x+3\right) - \frac{1}{12}\log\left(x^2-x+3\right)$$

input
```
integrate(1/(x^4+5*x^2+9),x, algorithm="giac")
```

output
```
1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*
sqrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

$$\int \frac{1}{9+5x^2+x^4}\,dx = \operatorname{atan}\left(\frac{x\,8\mathrm{i}}{27\left(-\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)} - \frac{2\sqrt{11}\,x}{27\left(-\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}\right)\left(\frac{\sqrt{11}}{66}+\frac{1}{6}\mathrm{i}\right) + \operatorname{atan}\left(\frac{x\,8\mathrm{i}}{27\left(\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)} + \frac{2\sqrt{11}\,x}{27\left(\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}\right)\left(\frac{\sqrt{11}}{66}-\frac{1}{6}\mathrm{i}\right)$$

input
```
int(1/(5*x^2 + x^4 + 9),x)
```

output
```
atan((x*8i)/(27*((11^(1/2)*1i)/9 - 5/9)) - (2*11^(1/2)*x)/(27*((11^(1/2)*1
i)/9 - 5/9)))*(11^(1/2)/66 + 1i/6) + atan((x*8i)/(27*((11^(1/2)*1i)/9 + 5/
9)) + (2*11^(1/2)*x)/(27*((11^(1/2)*1i)/9 + 5/9)))*(11^(1/2)/66 - 1i/6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{9+5x^2+x^4}\,dx = \frac{\sqrt{11}\,atan\left(\frac{2x-1}{\sqrt{11}}\right)}{66} + \frac{\sqrt{11}\,atan\left(\frac{2x+1}{\sqrt{11}}\right)}{66} - \frac{\log(x^2-x+3)}{12} + \frac{\log(x^2+x+3)}{12}$$

input
```
int(1/(x^4+5*x^2+9),x)
```

output
```
(2*sqrt(11)*atan((2*x - 1)/sqrt(11)) + 2*sqrt(11)*atan((2*x + 1)/sqrt(11))
 - 11*log(x**2 - x + 3) + 11*log(x**2 + x + 3))/132
```

3.334 $\int \frac{1}{2+2x^2+x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 137

$$\int \frac{1}{2+2x^2+x^4}\,dx = -\frac{1}{4}\sqrt{-1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{4}\sqrt{-1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}+2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}x}{\sqrt{2}+x^2}\right)}{4\sqrt{-1+\sqrt{2}}}$$

output
```
-1/4*(2^(1/2)-1)^(1/2)*arctan(((-2+2*2^(1/2))^(1/2)-2*x)/(2+2*2^(1/2))^(1/
2))+1/4*(2^(1/2)-1)^(1/2)*arctan(((-2+2*2^(1/2))^(1/2)+2*x)/(2+2*2^(1/2))^
(1/2))+1/4*arctanh((-2+2*2^(1/2))^(1/2)*x/(x^2+2^(1/2)))/(2^(1/2)-1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.30

$$\int \frac{1}{2+2x^2+x^4}\,dx = \frac{1}{4}\left((1-i)^{3/2}\arctan\left(\frac{x}{\sqrt{1-i}}\right)+(1+i)^{3/2}\arctan\left(\frac{x}{\sqrt{1+i}}\right)\right)$$

input
```
Integrate[(2 + 2*x^2 + x^4)^(-1),x]
```

output
```
((1 - I)^(3/2)*ArcTan[x/Sqrt[1 - I]] + (1 + I)^(3/2)*ArcTan[x/Sqrt[1 + I]]
)/4
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.47, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+2x^2+2}dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}-x}{x^2-\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx}{4\sqrt{\sqrt{2}-1}}+\frac{\int \frac{x+\sqrt{2\left(-1+\sqrt{2}\right)}}{x^2+\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx}{4\sqrt{\sqrt{2}-1}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx-\frac{1}{2}\int -\frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx}{4\sqrt{\sqrt{2}-1}}+\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\int \frac{1}{x^2+\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx+\frac{1}{2}\int \frac{2x+\sqrt{2\left(-1+\sqrt{2}\right)}}{x^2+\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx}{4\sqrt{\sqrt{2}-1}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx+\frac{1}{2}\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx}{4\sqrt{\sqrt{2}-1}}+\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\int \frac{1}{x^2+\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx+\frac{1}{2}\int \frac{2x+\sqrt{2\left(-1+\sqrt{2}\right)}}{x^2+\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx}{4\sqrt{\sqrt{2}-1}}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx-\sqrt{2\left(\sqrt{2}-1\right)}\int \frac{1}{-\left(2x-\sqrt{2\left(-1+\sqrt{2}\right)}\right)^2-2\left(1+\sqrt{2}\right)}d\left(2x-\sqrt{2\left(-1+\sqrt{2}\right)}\right)}{4\sqrt{\sqrt{2}-1}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2\left(-1+\sqrt{2}\right)}}{x^2+\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx-\sqrt{2\left(\sqrt{2}-1\right)}\int \frac{1}{-\left(2x+\sqrt{2\left(-1+\sqrt{2}\right)}\right)^2-2\left(1+\sqrt{2}\right)}d\left(2x+\sqrt{2\left(-1+\sqrt{2}\right)}\right)}{4\sqrt{\sqrt{2}-1}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx+\sqrt{\frac{\sqrt{2}-1}{1+\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{2}-1\right)}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)}{4\sqrt{\sqrt{2}-1}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2\left(-1+\sqrt{2}\right)}}{x^2+\sqrt{2\left(-1+\sqrt{2}\right)}x+\sqrt{2}}dx+\sqrt{\frac{\sqrt{2}-1}{1+\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{2}-1\right)}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)}{4\sqrt{\sqrt{2}-1}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\sqrt{\frac{\sqrt{2}-1}{1+\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{2}-1\right)}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{2\left(\sqrt{2}-1\right)}x+\sqrt{2}\right)}{4\sqrt{\sqrt{2}-1}}+\frac{\sqrt{\frac{\sqrt{2}-1}{1+\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{2}-1\right)}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{2}\log\left(x^2+\sqrt{2\left(\sqrt{2}-1\right)}x+\sqrt{2}\right)}{4\sqrt{\sqrt{2}-1}}$$

input

```
Int[(2 + 2*x^2 + x^4)^(-1),x]
```

output

```
(Sqrt[(-1 + Sqrt[2])/(1 + Sqrt[2])]*ArcTan[(-Sqrt[2*(-1 + Sqrt[2])] + 2*x)
/Sqrt[2*(1 + Sqrt[2])]] - Log[Sqrt[2] - Sqrt[2*(-1 + Sqrt[2])]*x + x^2]/2)
/(4*Sqrt[-1 + Sqrt[2]]) + (Sqrt[(-1 + Sqrt[2])/(1 + Sqrt[2])]*ArcTan[(Sqrt
[2*(-1 + Sqrt[2])] + 2*x)/Sqrt[2*(1 + Sqrt[2])]] + Log[Sqrt[2] + Sqrt[2*(-
1 + Sqrt[2])]*x + x^2]/2)/(4*Sqrt[-1 + Sqrt[2]])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+2\right)}\frac{\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{4}$
default	$\frac{\left(-\sqrt{-2+2\sqrt{2}}\,\sqrt{2}-2\sqrt{-2+2\sqrt{2}}\right)\ln\left(x^2-x\sqrt{-2+2\sqrt{2}}+\sqrt{2}\right)}{16}+\frac{\left(2\sqrt{2}+\frac{\left(-\sqrt{-2+2\sqrt{2}}\,\sqrt{2}-2\sqrt{-2+2\sqrt{2}}\right)\sqrt{-2+2\sqrt{2}}}{2}\right)\arctan\left(\frac{2x-\sqrt{\ldots}}{\sqrt{2\ldots}}\right.}{4\sqrt{2+2\sqrt{2}}}$

input
```
int(1/(x^4+2*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

$$\int \frac{1}{2+2x^2+x^4}\,dx = \frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\sqrt{2}x\sqrt{\sqrt{2}-1}+\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)^{\frac{3}{2}}\right) - \frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(-\sqrt{2}x\sqrt{\sqrt{2}-1}+\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)^{\frac{3}{2}}\right) - \frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(x^2+\left(\sqrt{2}x-2\,x\right)\sqrt{\sqrt{2}+1}+\sqrt{2}\right) + \frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(x^2-\left(\sqrt{2}x-2\,x\right)\sqrt{\sqrt{2}+1}+\sqrt{2}\right)$$

input
```
integrate(1/(x^4+2*x^2+2),x, algorithm="fricas")
```

output
```
1/4*sqrt(sqrt(2) - 1)*arctan(sqrt(2)*x*sqrt(sqrt(2) - 1) + sqrt(sqrt(2) +
1)*(sqrt(2) - 1)^(3/2)) - 1/4*sqrt(sqrt(2) - 1)*arctan(-sqrt(2)*x*sqrt(sqr
t(2) - 1) + sqrt(sqrt(2) + 1)*(sqrt(2) - 1)^(3/2)) - 1/8*sqrt(sqrt(2) + 1)
*log(x^2 + (sqrt(2)*x - 2*x)*sqrt(sqrt(2) + 1) + sqrt(2)) + 1/8*sqrt(sqrt(
2) + 1)*log(x^2 - (sqrt(2)*x - 2*x)*sqrt(sqrt(2) + 1) + sqrt(2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(116) = 232$.

Time = 0.53 (sec) , antiderivative size = 899, normalized size of antiderivative = 6.56

$$\int \frac{1}{2+2x^2+x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(x**4+2*x**2+2),x)
```

output
```
sqrt(1/64 + sqrt(2)/64)*log(x**2 + x*(-4*sqrt(2)*sqrt(1 + sqrt(2)) - sqrt(
1 + sqrt(2)) + 3*sqrt(1 + sqrt(2))*sqrt(2*sqrt(2) + 3)) - 15*sqrt(2*sqrt(2
) + 3) - 7*sqrt(2)*sqrt(2*sqrt(2) + 3) + 29 + 23*sqrt(2)) - sqrt(1/64 + sq
rt(2)/64)*log(x**2 + x*(-3*sqrt(1 + sqrt(2))*sqrt(2*sqrt(2) + 3) + sqrt(1
+ sqrt(2)) + 4*sqrt(2)*sqrt(1 + sqrt(2))) - 15*sqrt(2*sqrt(2) + 3) - 7*sqr
t(2)*sqrt(2*sqrt(2) + 3) + 29 + 23*sqrt(2)) + 2*sqrt(-sqrt(2*sqrt(2) + 3)/
32 + 1/64 + 3*sqrt(2)/64)*atan(2*x/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sq
rt(2)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)))
 - 4*sqrt(2)*sqrt(1 + sqrt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2
)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) - s
qrt(1 + sqrt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) + sqrt(2*sq
rt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) + 3*sqrt(1 + sqrt
(2))*sqrt(2*sqrt(2) + 3)/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) + s
qrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)))) + 2*sqrt
(-sqrt(2*sqrt(2) + 3)/32 + 1/64 + 3*sqrt(2)/64)*atan(2*x/(sqrt(-2*sqrt(2*s
qrt(2) + 3) + 1 + 3*sqrt(2)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2)
+ 3) + 1 + 3*sqrt(2))) - 3*sqrt(1 + sqrt(2))*sqrt(2*sqrt(2) + 3)/(sqrt(-2*
sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*
sqrt(2) + 3) + 1 + 3*sqrt(2))) + sqrt(1 + sqrt(2))/(sqrt(-2*sqrt(2*sqrt(2)
 + 3) + 1 + 3*sqrt(2)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3...
```

Maxima [F]

$$\int \frac{1}{2+2x^2+x^4}\,dx=\int \frac{1}{x^4+2\,x^2+2}\,dx$$

input
```
integrate(1/(x^4+2*x^2+2),x, algorithm="maxima")
```

output
```
integrate(1/(x^4 + 2*x^2 + 2), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int \frac{1}{2+2x^2+x^4}\,dx = \frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2\,x+2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\right)}{2\sqrt{\sqrt{2}+2}}\right) + \frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2\,x-2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\right)}{2\sqrt{\sqrt{2}+2}}\right) + \frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(x^2+2^{\frac{1}{4}}x\sqrt{-\sqrt{2}+2}+\sqrt{2}\right) - \frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(x^2-2^{\frac{1}{4}}x\sqrt{-\sqrt{2}+2}+\sqrt{2}\right)$$

input
```
integrate(1/(x^4+2*x^2+2),x, algorithm="giac")
```

output
```
1/4*sqrt(sqrt(2) - 1)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(-sqrt(2) + 2)
)/sqrt(sqrt(2) + 2)) + 1/4*sqrt(sqrt(2) - 1)*arctan(1/2*2^(3/4)*(2*x - 2^(
1/4)*sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*sqrt(sqrt(2) + 1)*log(x^
2 + 2^(1/4)*x*sqrt(-sqrt(2) + 2) + sqrt(2)) - 1/8*sqrt(sqrt(2) + 1)*log(x^
2 - 2^(1/4)*x*sqrt(-sqrt(2) + 2) + sqrt(2))
```

Mupad [B] (verification not implemented)

Time = 17.25 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.53

$$\int \frac{1}{2+2x^2+x^4}\,dx = \operatorname{atanh}\left(\frac{4\sqrt{2}\,x\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1} + \frac{4\sqrt{2}\,x\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}} - 2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right) - \operatorname{atanh}\left(\frac{4\sqrt{2}\,x\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}+1} - \frac{4\sqrt{2}\,x\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}+1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}} + 2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right)$$

input
```
int(1/(2*x^2 + x^4 + 2),x)
```

output
```
atanh((4*2^(1/2)*x*(1/64 - 2^(1/2)/64)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2
)*(2^(1/2)/64 + 1/64)^(1/2) - 1) + (4*2^(1/2)*x*(2^(1/2)/64 + 1/64)^(1/2))
/(64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2) - 1))*(2*(1/64 -
2^(1/2)/64)^(1/2) - 2*(2^(1/2)/64 + 1/64)^(1/2)) - atanh((4*2^(1/2)*x*(1/6
4 - 2^(1/2)/64)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(
1/2) + 1) - (4*2^(1/2)*x*(2^(1/2)/64 + 1/64)^(1/2))/(64*(1/64 - 2^(1/2)/64
)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2) + 1))*(2*(1/64 - 2^(1/2)/64)^(1/2) + 2*(
2^(1/2)/64 + 1/64)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.69

$$\int \frac{1}{2+2x^2+x^4}\,dx = -\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{2}+1}\,\sqrt{2}}\right)}{4} + \frac{\sqrt{\sqrt{2}+1}\,atan\left(\frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{2}+1}\,\sqrt{2}}\right)}{4} + \frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}+2x}{\sqrt{\sqrt{2}+1}\,\sqrt{2}}\right)}{4} - \frac{\sqrt{\sqrt{2}+1}\,atan\left(\frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}+2x}{\sqrt{\sqrt{2}+1}\,\sqrt{2}}\right)}{4} - \frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{2}-1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)}{8} + \frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{2}-1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)}{8} - \frac{\sqrt{\sqrt{2}-1}\log\left(-\sqrt{\sqrt{2}-1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)}{8} + \frac{\sqrt{\sqrt{2}-1}\log\left(\sqrt{\sqrt{2}-1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)}{8}$$

input
```
int(1/(x^4+2*x^2+2),x)
```

output
```
( - 2*sqrt(sqrt(2) + 1)*sqrt(2)*atan((sqrt(sqrt(2) - 1)*sqrt(2) - 2*x)/(sq
rt(sqrt(2) + 1)*sqrt(2))) + 2*sqrt(sqrt(2) + 1)*atan((sqrt(sqrt(2) - 1)*sq
rt(2) - 2*x)/(sqrt(sqrt(2) + 1)*sqrt(2))) + 2*sqrt(sqrt(2) + 1)*sqrt(2)*at
an((sqrt(sqrt(2) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(2) + 1)*sqrt(2))) - 2*sqrt
(sqrt(2) + 1)*atan((sqrt(sqrt(2) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(2) + 1)*sq
rt(2))) - sqrt(sqrt(2) - 1)*sqrt(2)*log( - sqrt(sqrt(2) - 1)*sqrt(2)*x + s
qrt(2) + x**2) + sqrt(sqrt(2) - 1)*sqrt(2)*log(sqrt(sqrt(2) - 1)*sqrt(2)*x
 + sqrt(2) + x**2) - sqrt(sqrt(2) - 1)*log( - sqrt(sqrt(2) - 1)*sqrt(2)*x
+ sqrt(2) + x**2) + sqrt(sqrt(2) - 1)*log(sqrt(sqrt(2) - 1)*sqrt(2)*x + sq
rt(2) + x**2))/8
```

3.335 $\int \frac{1}{cd^2-bde+ae^2-(2cdf-bef-bdg+2aeg)x^2+(cf^2-bfg+ag^2)x^4}\,dx$

Optimal result

Integrand size = 64, antiderivative size = 270

$$\int \frac{1}{cd^2-bde+ae^2-(2cdf-bef-bdg+2aeg)x^2+(cf^2-bfg+ag^2)\,x^4}\,dx = -\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,\mathrm{arctanh}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,x}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,(ef-dg)} + \frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\,\mathrm{arctanh}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\,x}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,(ef-dg)}$$

output
```
-(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)*arctanh((2*c*f-(b-(-4*a*c+b^2)^(1/
2))*g)^(1/2)*x/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2))/(-4*a*c+b^2)^(1/2)/
(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)/(-d*g+e*f)+(2*c*f-(b+(-4*a*c+b^2)^(
1/2))*g)^(1/2)*arctanh((2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)^(1/2)*x/(2*c*d-(b+
(-4*a*c+b^2)^(1/2))*e)^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1
/2))*e)^(1/2)/(-d*g+e*f)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.20

$$\int \frac{1}{cd^2-bde+ae^2-(2cdf-bef-bdg+2aeg)x^2+(cf^2-bfg+ag^2)\,x^4}\,dx$$
$$=\frac{\sqrt{2}\sqrt{cf^2+g(-bf+ag)}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{cf^2-bfg+ag^2}x}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4ac}dg-2aeg}}\right)}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4ac}dg-2aeg}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{cf^2-bfg+ag^2}x}{\sqrt{-2cdf+bef-\sqrt{b^2-4ac}ef+bdg+\sqrt{b^2}}}\right)}{\sqrt{-2cdf+bef-\sqrt{b^2-4ac}ef+bdg+\sqrt{b^2-}}}\right.}{\sqrt{b^2-4ac}(-ef+dg)}$$

input
```
Integrate[(c*d^2 - b*d*e + a*e^2 - (2*c*d*f - b*e*f - b*d*g + 2*a*e*g)*x^2
 + (c*f^2 - b*f*g + a*g^2)*x^4)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*(ArcTan[(Sqrt[2]*Sqrt[c*f^2 - b*f*
g + a*g^2]*x)/Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqrt
[b^2 - 4*a*c]*d*g - 2*a*e*g]]/Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*
f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g] - ArcTan[(Sqrt[2]*Sqrt[c*f^2
- b*f*g + a*g^2]*x)/Sqrt[-2*c*d*f + b*e*f - Sqrt[b^2 - 4*a*c]*e*f + b*d*g
+ Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]]/Sqrt[-2*c*d*f + b*e*f - Sqrt[b^2 - 4*a
*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]))/(Sqrt[b^2 - 4*a*c]*(-
(e*f) + d*g))
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {1406, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-x^2(2aeg-bdg-bef+2cdf)+x^4\left(ag^2-bfg+cf^2\right)+ae^2-bde+cd^2}\,dx$$
$$\Big\downarrow \text{1406}$$

$$\frac{(ag^2-bfg+cf^2)\int \frac{1}{(cf^2-bgf+ag^2)x^2+\frac{1}{2}\left(-2cdf+bef+bdg-2aeg-\sqrt{b^2-4ac}(ef-dg)\right)}dx}{\sqrt{b^2-4ac}(ef-dg)}-\frac{(ag^2-bfg+cf^2)\int \frac{1}{(cf^2-bgf+ag^2)x^2+\frac{1}{2}\left(-2cdf+bef+bdg-2aeg+\sqrt{b^2-4ac}(ef-dg)\right)}dx}{\sqrt{b^2-4ac}(ef-dg)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{2}\sqrt{ag^2-bfg+cf^2}\text{arctanh}\left(\frac{\sqrt{2}x\sqrt{ag^2-bfg+cf^2}}{\sqrt{-\sqrt{b^2-4ac}(ef-dg)+2aeg-bdg-bef+2cdf}}\right)}{\sqrt{b^2-4ac}(ef-dg)\sqrt{dg\sqrt{b^2-4ac}-ef\sqrt{b^2-4ac}+2aeg-b(dg+ef)+2cdf}}-\frac{\sqrt{2}\sqrt{ag^2-bfg+cf^2}\text{arctanh}\left(\frac{\sqrt{2}x\sqrt{ag^2-bfg+cf^2}}{\sqrt{\sqrt{b^2-4ac}(ef-dg)+2aeg-bdg-bef+2cdf}}\right)}{\sqrt{b^2-4ac}(ef-dg)\sqrt{-dg\sqrt{b^2-4ac}+ef\sqrt{b^2-4ac}+2aeg-b(dg+ef)+2cdf}}$$

input
```
Int[(c*d^2 - b*d*e + a*e^2 - (2*c*d*f - b*e*f - b*d*g + 2*a*e*g)*x^2 + (c*
f^2 - b*f*g + a*g^2)*x^4)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(Sqrt[2]*Sqrt[c*f^2 - b*f*g +
 a*g^2]*x)/Sqrt[2*c*d*f - b*e*f - b*d*g + 2*a*e*g - Sqrt[b^2 - 4*a*c]*(e*f
 - d*g)]])/(Sqrt[b^2 - 4*a*c]*(e*f - d*g)*Sqrt[2*c*d*f - Sqrt[b^2 - 4*a*c]
*e*f + Sqrt[b^2 - 4*a*c]*d*g + 2*a*e*g - b*(e*f + d*g)]) - (Sqrt[2]*Sqrt[c
*f^2 - b*f*g + a*g^2]*ArcTanh[(Sqrt[2]*Sqrt[c*f^2 - b*f*g + a*g^2]*x)/Sqrt
[2*c*d*f - b*e*f - b*d*g + 2*a*e*g + Sqrt[b^2 - 4*a*c]*(e*f - d*g)]])/(Sqr
t[b^2 - 4*a*c]*(e*f - d*g)*Sqrt[2*c*d*f + Sqrt[b^2 - 4*a*c]*e*f - Sqrt[b^2
 - 4*a*c]*d*g + 2*a*e*g - b*(e*f + d*g)])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.46

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a\,g^{2}-bfg+c\,f^{2}\right)\_Z^{4}+\left(-2aeg+bdg+bef-2cdf\right)\_Z^{2}+a\,e^{2}-bde+c\,d^{2}\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{3}a\,g^{2}-2\_R^{3}bfg+2\_R^{3}c\,f^{2}-2\_R aeg+\_R}\right.}{2}$
default	$\left(4a\,g^{2}-4bfg+4c\,f^{2}\right)\left(-\frac{\sqrt{2}\,\arctan\left(\frac{\left(2a\,g^{2}-2bfg+2c\,f^{2}\right)x\sqrt{2}}{2\sqrt{\left(a\,g^{2}-bfg+c\,f^{2}\right)\left(-2aeg+bdg+bef-2cdf+\sqrt{-\left(dg-ef\right)^{2}\left(4ac-b^{2}\right)}\right)}}\right)}{4\sqrt{-\left(dg-ef\right)^{2}\left(4ac-b^{2}\right)}\sqrt{\left(a\,g^{2}-bfg+c\,f^{2}\right)\left(-2aeg+bdg+bef-2cdf+\sqrt{-\left(dg-ef\right)^{2}\left(4ac-b^{2}\right.}\right.}}\right.$

input
```
int(1/(c*d^2-b*d*e+a*e^2-(2*a*e*g-b*d*g-b*e*f+2*c*d*f)*x^2+(a*g^2-b*f*g+c*
f^2)*x^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^3*a*g^2-2*_R^3*b*f*g+2*_R^3*c*f^2-2*_R*a*e*g+_R*b*d*g+_R*b
*e*f-2*_R*c*d*f)*ln(x-_R),_R=RootOf((a*g^2-b*f*g+c*f^2)*_Z^4+(-2*a*e*g+b*d
*g+b*e*f-2*c*d*f)*_Z^2+a*e^2-b*d*e+c*d^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8845 vs. $2(234) = 468$.

Time = 0.35 (sec) , antiderivative size = 8845, normalized size of antiderivative = 32.76

$$\int \frac{1}{cd^2-bde+ae^2-(2cdf-bef-bdg+2aeg)x^2+(cf^2-bfg+ag^2)\,x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*d^2-b*d*e+a*e^2-(2*a*e*g-b*d*g-b*e*f+2*c*d*f)*x^2+(a*g^2-b*
f*g+c*f^2)*x^4),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{cd^2-bde+ae^2-(2cdf-bef-bdg+2aeg)x^2+(cf^2-bfg+ag^2)\,x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*d**2-b*d*e+a*e**2-(2*a*e*g-b*d*g-b*e*f+2*c*d*f)*x**2+(a*g**
2-b*f*g+c*f**2)*x**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{cd^2-bde+ae^2-(2cdf-bef-bdg+2aeg)x^2+(cf^2-bfg+ag^2)\,x^4}\,dx = \int \frac{1}{(cf^2-bfg+ag^2)x^4+cd^2-bde+ae^2-(2\,cdf-bef-bdg+2\,aeg)x^2}\,dx$$

input
```
integrate(1/(c*d^2-b*d*e+a*e^2-(2*a*e*g-b*d*g-b*e*f+2*c*d*f)*x^2+(a*g^2-b*
f*g+c*f^2)*x^4),x, algorithm="maxima")
```

output
```
integrate(1/((c*f^2 - b*f*g + a*g^2)*x^4 + c*d^2 - b*d*e + a*e^2 - (2*c*d*
f - b*e*f - b*d*g + 2*a*e*g)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{cd^2 - bde + ae^2 - (2cdf - bef - bdg + 2aeg)x^2 + (cf^2 - bfg + ag^2)\,x^4}\,dx$$
$$= \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*d^2-b*d*e+a*e^2-(2*a*e*g-b*d*g-b*e*f+2*c*d*f)*x^2+(a*g^2-b*
f*g+c*f^2)*x^4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{-4,[0,0,2]%%%},[1,0,3,0]%%%}+%%%{%%{poly1[%%%{2,[0,0,1
]%%%},%%%
```

Mupad [B] (verification not implemented)

Time = 23.57 (sec) , antiderivative size = 25137, normalized size of antiderivative = 93.10

$$\int \frac{1}{cd^2 - bde + ae^2 - (2cdf - bef - bdg + 2aeg)x^2 + (cf^2 - bfg + ag^2)\,x^4}\,dx$$
$$= \text{Too large to display}$$

input
```
int(1/(x^4*(a*g^2 + c*f^2 - b*f*g) + a*e^2 + c*d^2 - x^2*(2*a*e*g - b*d*g
- b*e*f + 2*c*d*f) - b*d*e),x)
```

output
```
atan(((x*(4*a^3*g^6 + 4*c^3*f^6 - 4*b^3*f^3*g^3 + 12*a*b^2*f^2*g^4 + 12*a*
c^2*f^4*g^2 + 12*a^2*c*f^2*g^4 + 12*b^2*c*f^4*g^2 - 12*a^2*b*f*g^5 - 12*b*
c^2*f^5*g - 24*a*b*c*f^3*g^3) + (-(b^3*d*g + b^3*e*f + d*g*(-(4*a*c - b^2)
^3)^(1/2) - e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d*f - 2*a*b^2*e*g - 2*b
^2*c*d*f + 8*a^2*c*e*g - 4*a*b*c*d*g - 4*a*b*c*e*f)/(8*(a*b^4*e^4*f^2 + b^
4*c*d^4*g^2 - b^5*d*e^3*f^2 - b^5*d^3*e*g^2 + 16*a^2*c^3*d^4*g^2 + 16*a^3*
c^2*e^4*f^2 + 16*a^2*c^3*d^2*e^2*f^2 + 16*a^3*c^2*d^2*e^2*g^2 + 2*b^5*d^2*
e^2*f*g - 8*a*b^2*c^2*d^4*g^2 - 8*a^2*b^2*c*e^4*f^2 + a*b^4*d^2*e^2*g^2 +
b^4*c*d^2*e^2*f^2 - 16*a^2*b*c^2*d*e^3*f^2 - 16*a^2*b*c^2*d^3*e*g^2 - 2*a*
b^4*d*e^3*f*g - 2*b^4*c*d^3*e*f*g - 8*a*b^2*c^2*d^2*e^2*f^2 - 8*a^2*b^2*c*
d^2*e^2*g^2 + 8*a*b^3*c*d*e^3*f^2 + 8*a*b^3*c*d^3*e*g^2 - 32*a^2*c^3*d^3*e
*f*g - 32*a^3*c^2*d*e^3*f*g + 16*a*b^2*c^2*d^3*e*f*g - 16*a*b^3*c*d^2*e^2*
f*g + 16*a^2*b^2*c*d*e^3*f*g + 32*a^2*b*c^2*d^2*e^2*f*g)))^(1/2)*(x*(-(b^3
*d*g + b^3*e*f + d*g*(-(4*a*c - b^2)^3)^(1/2) - e*f*(-(4*a*c - b^2)^3)^(1/
2) + 8*a*c^2*d*f - 2*a*b^2*e*g - 2*b^2*c*d*f + 8*a^2*c*e*g - 4*a*b*c*d*g -
 4*a*b*c*e*f)/(8*(a*b^4*e^4*f^2 + b^4*c*d^4*g^2 - b^5*d*e^3*f^2 - b^5*d^3*
e*g^2 + 16*a^2*c^3*d^4*g^2 + 16*a^3*c^2*e^4*f^2 + 16*a^2*c^3*d^2*e^2*f^2 +
 16*a^3*c^2*d^2*e^2*g^2 + 2*b^5*d^2*e^2*f*g - 8*a*b^2*c^2*d^4*g^2 - 8*a^2*
b^2*c*e^4*f^2 + a*b^4*d^2*e^2*g^2 + b^4*c*d^2*e^2*f^2 - 16*a^2*b*c^2*d*e^3
*f^2 - 16*a^2*b*c^2*d^3*e*g^2 - 2*a*b^4*d*e^3*f*g - 2*b^4*c*d^3*e*f*g -...
```

Reduce [F]

$$\int \frac{1}{cd^2 - bde + ae^2 - (2cdf - bef - bdg + 2aeg)x^2 + (cf^2 - bfg + ag^2)x^4}\,dx$$
$$= \int \frac{1}{c\,d^2 - bde + a\,e^2 - (2aeg - bdg - bef + 2cdf)\,x^2 + (a\,g^2 - bfg + c\,f^2)\,x^4}dx$$

input
```
int(1/(c*d^2-b*d*e+a*e^2-(2*a*e*g-b*d*g-b*e*f+2*c*d*f)*x^2+(a*g^2-b*f*g+c*
f^2)*x^4),x)
```

output
```
int(1/(c*d^2-b*d*e+a*e^2-(2*a*e*g-b*d*g-b*e*f+2*c*d*f)*x^2+(a*g^2-b*f*g+c*
f^2)*x^4),x)
```

3.336 $\int (3-2x^2-x^4)^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int (3-2x^2-x^4)^{5/2}\,dx = \frac{16}{99}x(11-6x^2)\sqrt{3-2x^2-x^4} + \frac{10}{99}x(3-x^2)(3-2x^2-x^4)^{3/2} + \frac{1}{11}x(3-2x^2-x^4)^{5/2} - \frac{1312E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)}{33\sqrt{3}} + \frac{1856\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{33\sqrt{3}}$$

output
```
16/99*x*(-6*x^2+11)*(-x^4-2*x^2+3)^(1/2)+10/99*x*(-x^2+3)*(-x^4-2*x^2+3)^(
3/2)+1/11*x*(-x^4-2*x^2+3)^(5/2)-1312/99*EllipticE(x,1/3*I*3^(1/2))*3^(1/2
)+1856/99*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.91 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int (3-2x^2-x^4)^{5/2}\,dx = \frac{1041x-1576x^3+157x^5+488x^7-37x^9-64x^{11}-9x^{13}-1312i\sqrt{3-2x^2-x^4}E\left(i\text{arcsinh}\right.}{99\sqrt{3-2x^2-x^4}}$$

input
```
Integrate[(3 - 2*x^2 - x^4)^(5/2),x]
```

output
```
(1041*x - 1576*x^3 + 157*x^5 + 488*x^7 - 37*x^9 - 64*x^11 - 9*x^13 - (1312
*I)*Sqrt[3 - 2*x^2 - x^4]*EllipticE[I*ArcSinh[x/Sqrt[3]], -3] - (320*I)*Sq
rt[3 - 2*x^2 - x^4]*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/(99*Sqrt[3 - 2*x^
2 - x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1404, 27, 1490, 27, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-x^4-2x^2+3\right)^{5/2}dx$$

$$\Big\downarrow \text{1404}$$

$$\frac{5}{11}\int 2\left(3-x^2\right)\left(-x^4-2x^2+3\right)^{3/2}dx+\frac{1}{11}x\left(-x^4-2x^2+3\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{10}{11}\int \left(3-x^2\right)\left(-x^4-2x^2+3\right)^{3/2}dx+\frac{1}{11}x\left(-x^4-2x^2+3\right)^{5/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{10}{11}\left(\frac{1}{9}x\left(3-x^2\right)\left(-x^4-2x^2+3\right)^{3/2}-\frac{1}{21}\int -56\left(3-2x^2\right)\sqrt{-x^4-2x^2+3}dx\right)+\frac{1}{11}x\left(-x^4-2x^2+3\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{10}{11}\left(\frac{8}{3}\int \left(3-2x^2\right)\sqrt{-x^4-2x^2+3}dx+\frac{1}{9}x\left(3-x^2\right)\left(-x^4-2x^2+3\right)^{3/2}\right)+\frac{1}{11}x\left(-x^4-2x^2+3\right)^{5/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{10}{11}\left(\frac{8}{3}\left(\frac{1}{15}x\left(11-6x^2\right)\sqrt{-x^4-2x^2+3}-\frac{1}{15}\int -\frac{2\left(51-41x^2\right)}{\sqrt{-x^4-2x^2+3}}dx\right)+\frac{1}{9}x\left(3-x^2\right)\left(-x^4-2x^2+3\right)^{3/2}\right)+\frac{1}{11}x\left(-x^4-2x^2+3\right)^{5/2}$$

$\Big\downarrow$ 27

$$\frac{10}{11}\left(\frac{8}{3}\left(\frac{2}{15}\int \frac{51-41x^2}{\sqrt{-x^4-2x^2+3}}dx+\frac{1}{15}x\sqrt{-x^4-2x^2+3}\left(11-6x^2\right)\right)+\frac{1}{9}x\left(3-x^2\right)\left(-x^4-2x^2+3\right)^{3/2}\right)+\frac{1}{11}x\left(-x^4-2x^2+3\right)^{5/2}$$

$\Big\downarrow$ 1494

$$\frac{10}{11}\left(\frac{8}{3}\left(\frac{4}{15}\int \frac{51-41x^2}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{1}{15}x\sqrt{-x^4-2x^2+3}\left(11-6x^2\right)\right)+\frac{1}{9}x\left(3-x^2\right)\left(-x^4-2x^2+3\right)^{3/2}\right)+\frac{1}{11}x\left(-x^4-2x^2+3\right)^{5/2}$$

$\Big\downarrow$ 27

$$\frac{10}{11}\left(\frac{8}{3}\left(\frac{2}{15}\int \frac{51-41x^2}{\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{1}{15}x\sqrt{-x^4-2x^2+3}\left(11-6x^2\right)\right)+\frac{1}{9}x\left(3-x^2\right)\left(-x^4-2x^2+3\right)^{3/2}\right)+\frac{1}{11}x\left(-x^4-2x^2+3\right)^{5/2}$$

$\Big\downarrow$ 399

$$\frac{10}{11}\left(\frac{8}{3}\left(\frac{2}{15}\left(174\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx-41\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{1}{15}x\sqrt{-x^4-2x^2+3}\left(11-6x^2\right)\right)+\frac{1}{9}x(3-x^2\right)$$
$$\frac{1}{11}x\left(-x^4-2x^2+3\right)^{5/2}$$

$\Big\downarrow$ 321

$$\frac{10}{11}\left(\frac{8}{3}\left(\frac{2}{15}\left(58\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-41\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{1}{15}x\sqrt{-x^4-2x^2+3}\left(11-6x^2\right)\right)+\frac{1}{9}x(3\right.$$
$$\frac{1}{11}x\left(-x^4-2x^2+3\right)^{5/2}$$

$\Big\downarrow$ 327

$$\frac{10}{11}\left(\frac{8}{3}\left(\frac{2}{15}\left(58\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-41\sqrt{3}E\left(\arcsin(x)\middle|-\frac{1}{3}\right)\right)+\frac{1}{15}x\sqrt{-x^4-2x^2+3}(11-6x^2)\right)\right.$$
$$\frac{1}{11}x\left(-x^4-2x^2+3\right)^{5/2}$$

input
```
Int[(3 - 2*x^2 - x^4)^(5/2),x]
```

output
```
(x*(3 - 2*x^2 - x^4)^(5/2))/11 + (10*((x*(3 - x^2)*(3 - 2*x^2 - x^4)^(3/2)
)/9 + (8*((x*(11 - 6*x^2)*Sqrt[3 - 2*x^2 - x^4])/15 + (2*(-41*Sqrt[3]*Elli
pticE[ArcSin[x], -1/3] + 58*Sqrt[3]*EllipticF[ArcSin[x], -1/3]))/15))/3))/
11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.35

method	result
risch	$-\frac{x\left(9x^{8}+46x^{6}-28x^{4}-294x^{2}+347\right)\left(x^{4}+2x^{2}-3\right)}{99\sqrt{-x^{4}-2x^{2}+3}}+\frac{544\sqrt{-x^{2}+1}\,\sqrt{3x^{2}+9}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{99\sqrt{-x^{4}-2x^{2}+3}}+\frac{1312\sqrt{-x^{2}+1}\,\sqrt{3x^{2}+9}\ \left(\mathrm{Ellipt}\right.}{99\sqrt{-x}}$
default	$\frac{x^{9}\sqrt{-x^{4}-2x^{2}+3}}{11}+\frac{46x^{7}\sqrt{-x^{4}-2x^{2}+3}}{99}-\frac{28x^{5}\sqrt{-x^{4}-2x^{2}+3}}{99}-\frac{98x^{3}\sqrt{-x^{4}-2x^{2}+3}}{33}+\frac{347x\sqrt{-x^{4}-2x^{2}+3}}{99}+\frac{544\sqrt{-x^{2}+}}{}$
elliptic	$\frac{x^{9}\sqrt{-x^{4}-2x^{2}+3}}{11}+\frac{46x^{7}\sqrt{-x^{4}-2x^{2}+3}}{99}-\frac{28x^{5}\sqrt{-x^{4}-2x^{2}+3}}{99}-\frac{98x^{3}\sqrt{-x^{4}-2x^{2}+3}}{33}+\frac{347x\sqrt{-x^{4}-2x^{2}+3}}{99}+\frac{544\sqrt{-x^{2}+}}{}$

input
```
int((-x^4-2*x^2+3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/99*x*(9*x^8+46*x^6-28*x^4-294*x^2+347)*(x^4+2*x^2-3)/(-x^4-2*x^2+3)^(1/
2)+544/99*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,
1/3*I*3^(1/2))+1312/99*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)
*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

$$\int (3-2x^2-x^4)^{5/2}\,dx = \frac{1312i\,xE\left(\arcsin\left(\frac{1}{x}\right)\,|-3\right)+320i\,xF\left(\arcsin\left(\frac{1}{x}\right)\,|-3\right)+\left(9\,x^{10}+46\,x^8-28\,x^6-294\,x^4+3\right.}{99\,x}$$

input
```
integrate((-x^4-2*x^2+3)^(5/2),x, algorithm="fricas")
```

output
```
1/99*(1312*I*x*elliptic_e(arcsin(1/x), -3) + 320*I*x*elliptic_f(arcsin(1/x
), -3) + (9*x^10 + 46*x^8 - 28*x^6 - 294*x^4 + 347*x^2 + 1312)*sqrt(-x^4 -
 2*x^2 + 3))/x
```

Sympy [F]

$$\int (3-2x^2-x^4)^{5/2}\,dx = \int \left(-x^4-2x^2+3\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((-x**4-2*x**2+3)**(5/2),x)
```

output
```
Integral((-x**4 - 2*x**2 + 3)**(5/2), x)
```

Maxima [F]

$$\int \left(3-2x^2-x^4\right)^{5/2}\,dx=\int \left(-x^4-2\,x^2+3\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((-x^4-2*x^2+3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 - 2*x^2 + 3)^(5/2), x)
```

Giac [F]

$$\int \left(3-2x^2-x^4\right)^{5/2}\,dx=\int \left(-x^4-2\,x^2+3\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((-x^4-2*x^2+3)^(5/2),x, algorithm="giac")
```

output
```
integrate((-x^4 - 2*x^2 + 3)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3-2x^2-x^4\right)^{5/2}\,dx=\int \left(-x^4-2\,x^2+3\right)^{5/2}\,dx$$

input
```
int((3 - x^4 - 2*x^2)^(5/2),x)
```

output
```
int((3 - x^4 - 2*x^2)^(5/2), x)
```

Reduce [F]

$$\int \left(3-2x^2-x^4\right)^{5/2}\,dx=\frac{\sqrt{-x^4-2x^2+3}\,x^9}{11}+\frac{46\sqrt{-x^4-2x^2+3}\,x^7}{99}-\frac{28\sqrt{-x^4-2x^2+3}\,x^5}{99}-\frac{98\sqrt{-x^4-2x^2+3}\,x^3}{33}+\frac{347\sqrt{-x^4-2x^2+3}\,x}{99}-\frac{544\left(\int \frac{\sqrt{-x^4-2x^2+3}}{x^4+2x^2-3}dx\right)}{33}+\frac{1312\left(\int \frac{\sqrt{-x^4-2x^2+3}\,x^2}{x^4+2x^2-3}dx\right)}{99}$$

input
```
int((-x^4-2*x^2+3)^(5/2),x)
```

output
```
(9*sqrt( - x**4 - 2*x**2 + 3)*x**9 + 46*sqrt( - x**4 - 2*x**2 + 3)*x**7 -
28*sqrt( - x**4 - 2*x**2 + 3)*x**5 - 294*sqrt( - x**4 - 2*x**2 + 3)*x**3 +
 347*sqrt( - x**4 - 2*x**2 + 3)*x - 1632*int(sqrt( - x**4 - 2*x**2 + 3)/(x
**4 + 2*x**2 - 3),x) + 1312*int((sqrt( - x**4 - 2*x**2 + 3)*x**2)/(x**4 +
2*x**2 - 3),x))/99
```

3.337 $\int \left(3-2x^2-x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 80

$$\int \left(3-2x^2-x^4\right)^{3/2}\,dx=\frac{2}{35}x\left(13-3x^2\right)\sqrt{3-2x^2-x^4}+\frac{1}{7}x\left(3-2x^2-x^4\right)^{3/2}-\frac{16}{5}\sqrt{3}E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)+\frac{176}{35}\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)$$

output
```
2/35*x*(-3*x^2+13)*(-x^4-2*x^2+3)^(1/2)+1/7*x*(-x^4-2*x^2+3)^(3/2)-16/5*El
lipticE(x,1/3*I*3^(1/2))*3^(1/2)+176/35*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.65 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.40

$$\int \left(3-2x^2-x^4\right)^{3/2}\,dx=\frac{123x-130x^3-24x^5+26x^7+5x^9-112i\sqrt{3-2x^2-x^4}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-80i\sqrt{3-}}{35\sqrt{3-2x^2-x^4}}$$

input
```
Integrate[(3 - 2*x^2 - x^4)^(3/2),x]
```

output
```
(123*x - 130*x^3 - 24*x^5 + 26*x^7 + 5*x^9 - (112*I)*Sqrt[3 - 2*x^2 - x^4]
*EllipticE[I*ArcSinh[x/Sqrt[3]], -3] - (80*I)*Sqrt[3 - 2*x^2 - x^4]*Ellipt
icF[I*ArcSinh[x/Sqrt[3]], -3])/(35*Sqrt[3 - 2*x^2 - x^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1404, 27, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-x^4-2x^2+3\right)^{3/2}dx$$

$$\Big\downarrow \text{1404}$$

$$\frac{3}{7}\int 2\left(3-x^2\right)\sqrt{-x^4-2x^2+3}dx+\frac{1}{7}x\left(-x^4-2x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\int \left(3-x^2\right)\sqrt{-x^4-2x^2+3}dx+\frac{1}{7}x\left(-x^4-2x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{6}{7}\left(\frac{1}{15}x\left(13-3x^2\right)\sqrt{-x^4-2x^2+3}-\frac{1}{15}\int -\frac{8\left(12-7x^2\right)}{\sqrt{-x^4-2x^2+3}}dx\right)+\frac{1}{7}x\left(-x^4-2x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{8}{15}\int \frac{12-7x^2}{\sqrt{-x^4-2x^2+3}}dx+\frac{1}{15}x\sqrt{-x^4-2x^2+3}\left(13-3x^2\right)\right)+\frac{1}{7}x\left(-x^4-2x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1494}$$

$$\frac{6}{7}\left(\frac{16}{15}\int \frac{12-7x^2}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{1}{15}x\sqrt{-x^4-2x^2+3}\left(13-3x^2\right)\right)+\frac{1}{7}x\left(-x^4-2x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{8}{15}\int \frac{12-7x^2}{\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{1}{15}x\sqrt{-x^4-2x^2+3}\left(13-3x^2\right)\right)+\frac{1}{7}x\left(-x^4-2x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{399}$$

$$\frac{6}{7}\left(\frac{8}{15}\left(33\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx-7\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{1}{15}x\sqrt{-x^4-2x^2+3}(13-3x^2)\right)+\frac{1}{7}x\left(-x^4-2x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{321}$$

$$\frac{6}{7}\left(\frac{8}{15}\left(11\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-7\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{1}{15}x\sqrt{-x^4-2x^2+3}(13-3x^2)\right)+\frac{1}{7}x\left(-x^4-2x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{327}$$

$$\frac{6}{7}\left(\frac{8}{15}\left(11\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-7\sqrt{3}E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)\right)+\frac{1}{15}x\sqrt{-x^4-2x^2+3}(13-3x^2)\right)+\frac{1}{7}x\left(-x^4-2x^2+3\right)^{3/2}$$

input
```
Int[(3 - 2*x^2 - x^4)^(3/2),x]
```

output
```
(x*(3 - 2*x^2 - x^4)^(3/2))/7 + (6*((x*(13 - 3*x^2)*Sqrt[3 - 2*x^2 - x^4])
/15 + (8*(-7*Sqrt[3]*EllipticE[ArcSin[x], -1/3] + 11*Sqrt[3]*EllipticF[Arc
Sin[x], -1/3]))/15))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.70

method	result
risch	$\frac{x(5x^4+16x^2-41)(x^4+2x^2-3)}{35\sqrt{-x^4-2x^2+3}}+\frac{64\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{35\sqrt{-x^4-2x^2+3}}+\frac{16\sqrt{-x^2+1}\,\sqrt{3x^2+9}\left(\text{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\text{Elliptic}\right.}{5\sqrt{-x^4-2x^2+3}}$
default	$-\frac{x^5\sqrt{-x^4-2x^2+3}}{7}-\frac{16x^3\sqrt{-x^4-2x^2+3}}{35}+\frac{41x\sqrt{-x^4-2x^2+3}}{35}+\frac{64\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{35\sqrt{-x^4-2x^2+3}}+\frac{16\sqrt{-x^2+1}\,\sqrt{}}{}$
elliptic	$-\frac{x^5\sqrt{-x^4-2x^2+3}}{7}-\frac{16x^3\sqrt{-x^4-2x^2+3}}{35}+\frac{41x\sqrt{-x^4-2x^2+3}}{35}+\frac{64\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{35\sqrt{-x^4-2x^2+3}}+\frac{16\sqrt{-x^2+1}\,\sqrt{}}{}$

input
```
int((-x^4-2*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*x*(5*x^4+16*x^2-41)*(x^4+2*x^2-3)/(-x^4-2*x^2+3)^(1/2)+64/35*(-x^2+1)
^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+16/
5*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3
^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int (3-2x^2 -x^4)^{3/2}\,dx = \frac{112i\,xE(\arcsin\left(\frac{1}{x}\right)\,|-3)+80i\,xF(\arcsin\left(\frac{1}{x}\right)\,|-3)-(5\,x^6+16\,x^4-41\,x^2-112)\sqrt{-x^4-}}{35\,x}$$

input
```
integrate((-x^4-2*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/35*(112*I*x*elliptic_e(arcsin(1/x), -3) + 80*I*x*elliptic_f(arcsin(1/x),
 -3) - (5*x^6 + 16*x^4 - 41*x^2 - 112)*sqrt(-x^4 - 2*x^2 + 3))/x
```

Sympy [F]

$$\int \left(3-2x^2-x^4\right)^{3/2} \, dx = \int \left(-x^{4} - 2x^{2} + 3\right)^{\frac{3}{2}}\, dx$$

input
```
integrate((-x**4-2*x**2+3)**(3/2),x)
```

output
```
Integral((-x**4 - 2*x**2 + 3)**(3/2), x)
```

Maxima [F]

$$\int \left(3-2x^2-x^4\right)^{3/2} \, dx = \int \left(-x^{4} - 2\,x^{2} + 3\right)^{\frac{3}{2}}\, dx$$

input
```
integrate((-x^4-2*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 - 2*x^2 + 3)^(3/2), x)
```

Giac [F]

$$\int \left(3-2x^2-x^4\right)^{3/2} \, dx = \int \left(-x^{4} - 2\,x^{2} + 3\right)^{\frac{3}{2}}\, dx$$

input
```
integrate((-x^4-2*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 - 2*x^2 + 3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3-2x^2-x^4\right)^{3/2} dx = \int \left(-x^4-2\,x^2+3\right)^{3/2} dx$$

input
```
int((3 - x^4 - 2*x^2)^(3/2),x)
```

output
```
int((3 - x^4 - 2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(3-2x^2-x^4\right)^{3/2} dx = -\frac{\sqrt{-x^4-2x^2+3}\,x^5}{7} - \frac{16\sqrt{-x^4-2x^2+3}\,x^3}{35} + \frac{41\sqrt{-x^4-2x^2+3}\,x}{35} - \frac{192\left(\int \frac{\sqrt{-x^4-2x^2+3}}{x^4+2x^2-3}dx\right)}{35} + \frac{16\left(\int \frac{\sqrt{-x^4-2x^2+3}\,x^2}{x^4+2x^2-3}dx\right)}{5}$$

input
```
int((-x^4-2*x^2+3)^(3/2),x)
```

output
```
( - 5*sqrt( - x**4 - 2*x**2 + 3)*x**5 - 16*sqrt( - x**4 - 2*x**2 + 3)*x**3
 + 41*sqrt( - x**4 - 2*x**2 + 3)*x - 192*int(sqrt( - x**4 - 2*x**2 + 3)/(x
**4 + 2*x**2 - 3),x) + 112*int((sqrt( - x**4 - 2*x**2 + 3)*x**2)/(x**4 + 2
*x**2 - 3),x))/35
```

3.338 $\int \sqrt{3-2x^2-x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \sqrt{3-2x^2-x^4}\,dx=\frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)}{\sqrt{3}}+\frac{4\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
1/3*x*(-x^4-2*x^2+3)^(1/2)-2/3*EllipticE(x,1/3*I*3^(1/2))*3^(1/2)+4/3*Elli
pticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \sqrt{3-2x^2-x^4}\,dx=\frac{1}{3}\left(x\sqrt{3-2x^2-x^4}-2iE\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-4i\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right),-3\right)\right)$$

input
```
Integrate[Sqrt[3 - 2*x^2 - x^4],x]
```

output
```
(x*Sqrt[3 - 2*x^2 - x^4] - (2*I)*EllipticE[I*ArcSinh[x/Sqrt[3]], -3] - (4*
I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1404, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-x^4-2x^2+3}\,dx$$

$$\Big\downarrow \text{1404}$$

$$\frac{1}{3}\int \frac{2(3-x^2)}{\sqrt{-x^4-2x^2+3}}dx+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{3-x^2}{\sqrt{-x^4-2x^2+3}}dx+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \text{1494}$$

$$\frac{4}{3}\int \frac{3-x^2}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{3-x^2}{\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \text{399}$$

$$\frac{2}{3}\left(6\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx-\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \text{321}$$

$$\frac{2}{3}\left(2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow 327$$

$$\frac{2}{3}\left(2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-\sqrt{3}E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)\right)+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

input
```
Int[Sqrt[3 - 2*x^2 - x^4],x]
```

output
```
(x*Sqrt[3 - 2*x^2 - x^4])/3 + (2*(-(Sqrt[3]*EllipticE[ArcSin[x], -1/3]) +
2*Sqrt[3]*EllipticF[ArcSin[x], -1/3]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(44) = 88$.

Time = 1.97 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.38

method	result
default	$\frac{x\sqrt{-x^4-2x^2+3}}{3}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{3\sqrt{-x^4-2x^2+3}}$
elliptic	$\frac{x\sqrt{-x^4-2x^2+3}}{3}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{3\sqrt{-x^4-2x^2+3}}$
risch	$-\frac{x(x^4+2x^2-3)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{3\sqrt{-x^4-2x^2+3}}$

input
```
int((-x^4-2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(-x^4-2*x^2+3)^(1/2)+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+
3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x
^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \sqrt{3-2x^2-x^4}\,dx = \frac{2i\,xE\left(\arcsin\left(\frac{1}{x}\right)\,|-3\right)+4i\,xF\left(\arcsin\left(\frac{1}{x}\right)\,|-3\right)+\sqrt{-x^4-2\,x^2+3}\,(x^2+2)}{3\,x}$$

input
```
integrate((-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*I*x*elliptic_e(arcsin(1/x), -3) + 4*I*x*elliptic_f(arcsin(1/x), -3)
 + sqrt(-x^4 - 2*x^2 + 3)*(x^2 + 2))/x
```

Sympy [F]

$$\int \sqrt{3-2x^2-x^4}\,dx = \int \sqrt{-x^4-2x^2+3}\,dx$$

input
```
integrate((-x**4-2*x**2+3)**(1/2),x)
```

output
```
Integral(sqrt(-x**4 - 2*x**2 + 3), x)
```

Maxima [F]

$$\int \sqrt{3-2x^2-x^4}\,dx = \int \sqrt{-x^4-2\,x^2+3}\,dx$$

input
```
integrate((-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 - 2*x^2 + 3), x)
```

Giac [F]

$$\int \sqrt{3-2x^2-x^4}\,dx=\int \sqrt{-x^4-2\,x^2+3}\,dx$$

input
```
integrate((-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 - 2*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3-2x^2-x^4}\,dx=\int \sqrt{-x^4-2\,x^2+3}\,dx$$

input
```
int((3 - x^4 - 2*x^2)^(1/2),x)
```

output
```
int((3 - x^4 - 2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{3-2x^2-x^4}\,dx=\frac{\sqrt{-x^4-2x^2+3}\,x}{3}-2\left(\int \frac{\sqrt{-x^4-2x^2+3}}{x^4+2x^2-3}dx\right)+\frac{2\left(\int \frac{\sqrt{-x^4-2x^2+3}\,x^2}{x^4+2x^2-3}dx\right)}{3}$$

input
```
int((-x^4-2*x^2+3)^(1/2),x)
```

output
```
(sqrt( - x**4 - 2*x**2 + 3)*x - 6*int(sqrt( - x**4 - 2*x**2 + 3)/(x**4 + 2
*x**2 - 3),x) + 2*int((sqrt( - x**4 - 2*x**2 + 3)*x**2)/(x**4 + 2*x**2 - 3
),x))/3
```

3.339 $\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = -i\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right),-3\right)$$

input
```
Integrate[1/Sqrt[3 - 2*x^2 - x^4],x]
```

output
```
(-I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^4-2x^2+3}}dx$$

$$\Big\downarrow \text{ 1408}$$

$$2\int \frac{1}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[3 - 2*x^2 - x^4],x]
```

output
```
EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(13) = 26$.

Time = 0.49 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(x, \frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(x, \frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43

input
```
int(1/(-x^4-2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*
3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = \frac{1}{3}\,\sqrt{3}F(\arcsin{(x)}\mid -\frac{1}{3})$$

input
```
integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*elliptic_f(arcsin(x), -1/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4-2x^2+3}}\,dx$$

input
```
integrate(1/(-x**4-2*x**2+3)**(1/2),x)
```

output
```
Integral(1/sqrt(-x**4 - 2*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

input
```
integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

input
```
integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

input
```
int(1/(3 - x^4 - 2*x^2)^(1/2),x)
```

output
```
int(1/(3 - x^4 - 2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4-2x^2+3}}{x^4+2x^2-3}dx\right)$$

input
```
int(1/(-x^4-2*x^2+3)^(1/2),x)
```

output
```
- int(sqrt( - x**4 - 2*x**2 + 3)/(x**4 + 2*x**2 - 3),x)
```

3.340 $\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx=\frac{x(5+x^2)}{24\sqrt{3-2x^2-x^4}}-\frac{E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)}{8\sqrt{3}}+\frac{\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{4\sqrt{3}}$$

output
```
1/24*x*(x^2+5)/(-x^4-2*x^2+3)^(1/2)-1/24*EllipticE(x,1/3*I*3^(1/2))*3^(1/2
)+1/12*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx=\frac{1}{24}\left(\frac{5x}{\sqrt{3-2x^2-x^4}}+\frac{x^3}{\sqrt{3-2x^2-x^4}}-iE\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-2i\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right),-3\right)\right)$$

input
```
Integrate[(3 - 2*x^2 - x^4)^(-3/2),x]
```

output
```
((5*x)/Sqrt[3 - 2*x^2 - x^4] + x^3/Sqrt[3 - 2*x^2 - x^4] - I*EllipticE[I*A
rcSinh[x/Sqrt[3]], -3] - (2*I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/24
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^4-2x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}-\frac{1}{48}\int -\frac{2(3-x^2)}{\sqrt{-x^4-2x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\int \frac{3-x^2}{\sqrt{-x^4-2x^2+3}}dx+\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{12}\int \frac{3-x^2}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\int \frac{3-x^2}{\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{24}\left(6\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx-\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{24}\left(2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-\int\frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{1}{24}\left(2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-\sqrt{3}E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)\right)+\frac{x(x^2+5)}{24\sqrt{-x^4-2x^2+3}}$$

input
```
Int[(3 - 2*x^2 - x^4)^(-3/2),x]
```

output
```
(x*(5 + x^2))/(24*Sqrt[3 - 2*x^2 - x^4]) + (-(Sqrt[3]*EllipticE[ArcSin[x],
 -1/3]) + 2*Sqrt[3]*EllipticF[ArcSin[x], -1/3])/24
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(49) = 98$.

Time = 1.89 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.09

method	result	siz
risch	$\frac{x\left(x^{2}+5\right)}{24\sqrt{-x^{4}-2x^{2}+3}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{3x^{2}+9}\,\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{24\sqrt{-x^{4}-2x^{2}+3}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{3x^{2}+9}\,\left(\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\operatorname{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{24\sqrt{-x^{4}-2x^{2}+3}}$	11
default	$\frac{\frac{5}{24}x+\frac{1}{24}x^{3}}{\sqrt{-x^{4}-2x^{2}+3}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{3x^{2}+9}\,\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{24\sqrt{-x^{4}-2x^{2}+3}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{3x^{2}+9}\,\left(\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\operatorname{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{24\sqrt{-x^{4}-2x^{2}+3}}$	12
elliptic	$\frac{\frac{5}{24}x+\frac{1}{24}x^{3}}{\sqrt{-x^{4}-2x^{2}+3}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{3x^{2}+9}\,\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{24\sqrt{-x^{4}-2x^{2}+3}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{3x^{2}+9}\,\left(\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\operatorname{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{24\sqrt{-x^{4}-2x^{2}+3}}$	12

input
```
int(1/(-x^4-2*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*x*(x^2+5)/(-x^4-2*x^2+3)^(1/2)+1/24*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-
x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+1/24*(-x^2+1)^(1/2)*(3*x^2+9
)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I
*3^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx =$$
$$-\frac{\sqrt{3}(x^4+2\,x^2-3)E(\arcsin{(x)}\,|-\tfrac{1}{3})-2\,\sqrt{3}(x^4+2\,x^2-3)F(\arcsin{(x)}\,|-\tfrac{1}{3})+\sqrt{-x^4-2\,x^2+3}(x^3+}{24\,(x^4+2\,x^2-3)}$$

input
```
integrate(1/(-x^4-2*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(3)*(x^4 + 2*x^2 - 3)*elliptic_e(arcsin(x), -1/3) - 2*sqrt(3)*(
x^4 + 2*x^2 - 3)*elliptic_f(arcsin(x), -1/3) + sqrt(-x^4 - 2*x^2 + 3)*(x^3
 + 5*x))/(x^4 + 2*x^2 - 3)
```

Sympy [F]

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4-2x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x**4-2*x**2+3)**(3/2),x)
```

output
```
Integral((-x**4 - 2*x**2 + 3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4-2\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4-2*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 - 2*x^2 + 3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4-2\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4-2*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 - 2*x^2 + 3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4-2\,x^2+3)^{3/2}}\,dx$$

input
```
int(1/(3 - x^4 - 2*x^2)^(3/2),x)
```

output
```
int(1/(3 - x^4 - 2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx = \frac{-\sqrt{-x^4-2x^2+3}\,x+3\left(\int \frac{\sqrt{-x^4-2x^2+3}}{x^6+x^4-5x^2+3}dx\right)x^4+6\left(\int \frac{\sqrt{-x^4-2x^2+3}}{x^6+x^4-5x^2+3}dx\right)x^2-9\left(\int\right.}{12}$$

input
```
int(1/(-x^4-2*x^2+3)^(3/2),x)
```

output

```
( - sqrt( - x**4 - 2*x**2 + 3)*x + 3*int(sqrt( - x**4 - 2*x**2 + 3)/(x**6
+ x**4 - 5*x**2 + 3),x)*x**4 + 6*int(sqrt( - x**4 - 2*x**2 + 3)/(x**6 + x*
*4 - 5*x**2 + 3),x)*x**2 - 9*int(sqrt( - x**4 - 2*x**2 + 3)/(x**6 + x**4 -
 5*x**2 + 3),x) - int((sqrt( - x**4 - 2*x**2 + 3)*x**2)/(x**6 + x**4 - 5*x
**2 + 3),x)*x**4 - 2*int((sqrt( - x**4 - 2*x**2 + 3)*x**2)/(x**6 + x**4 -
5*x**2 + 3),x)*x**2 + 3*int((sqrt( - x**4 - 2*x**2 + 3)*x**2)/(x**6 + x**4
 - 5*x**2 + 3),x))/(12*(x**4 + 2*x**2 - 3))
```

3.341 $\int \frac{1}{(3-2x^2-x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{1}{(3-2x^2-x^4)^{5/2}}\,dx = \frac{x(5+x^2)}{72(3-2x^2-x^4)^{3/2}} + \frac{x(26+7x^2)}{432\sqrt{3-2x^2-x^4}} - \frac{7E\left(\arcsin(x)\,\middle|\,-\frac{1}{3}\right)}{144\sqrt{3}} + \frac{11\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{144\sqrt{3}}$$

output

```
1/72*x*(x^2+5)/(-x^4-2*x^2+3)^(3/2)+1/432*x*(7*x^2+26)/(-x^4-2*x^2+3)^(1/2
)-7/432*EllipticE(x,1/3*I*3^(1/2))*3^(1/2)+11/432*EllipticF(x,1/3*I*3^(1/2
))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.26

$$\int \frac{1}{(3-2x^2-x^4)^{5/2}}\,dx = -\frac{-108x+25x^3+40x^5+7x^7+7i(3-2x^2-x^4)^{3/2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)+5i(3-2x^2-x^4)^{3/2}\,\text{Ellipti}}{432(3-2x^2-x^4)^{3/2}}$$

input
```
Integrate[(3 - 2*x^2 - x^4)^(-5/2),x]
```

output
```
-1/432*(-108*x + 25*x^3 + 40*x^5 + 7*x^7 + (7*I)*(3 - 2*x^2 - x^4)^(3/2)*E
llipticE[I*ArcSinh[x/Sqrt[3]], -3] + (5*I)*(3 - 2*x^2 - x^4)^(3/2)*Ellipti
cF[I*ArcSinh[x/Sqrt[3]], -3])/(3 - 2*x^2 - x^4)^(3/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1405, 27, 1492, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^4-2x^2+3)^{5/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x(x^2+5)}{72(-x^4-2x^2+3)^{3/2}}-\frac{1}{144}\int -\frac{2(3x^2+19)}{(-x^4-2x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{72}\int \frac{3x^2+19}{(-x^4-2x^2+3)^{3/2}}dx+\frac{x(x^2+5)}{72(-x^4-2x^2+3)^{3/2}}$$

$$\Big\downarrow \text{ 1492}$$

$$\frac{1}{72}\left(\frac{x(7x^2+26)}{6\sqrt{-x^4-2x^2+3}}-\frac{1}{48}\int -\frac{8(12-7x^2)}{\sqrt{-x^4-2x^2+3}}dx\right)+\frac{x(x^2+5)}{72(-x^4-2x^2+3)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{72}\left(\frac{1}{6}\int \frac{12-7x^2}{\sqrt{-x^4-2x^2+3}}dx+\frac{x(7x^2+26)}{6\sqrt{-x^4-2x^2+3}}\right)+\frac{x(x^2+5)}{72(-x^4-2x^2+3)^{3/2}}$$

$$\Big\downarrow \text{ 1494}$$

$$\frac{1}{72}\left(\frac{1}{3}\int \frac{12-7x^2}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{x(7x^2+26)}{6\sqrt{-x^4-2x^2+3}}\right)+\frac{x(x^2+5)}{72(-x^4-2x^2+3)^{3/2}}$$

$$\Big\downarrow \, 27$$

$$\frac{1}{72}\left(\frac{1}{6}\int \frac{12-7x^2}{\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{x(7x^2+26)}{6\sqrt{-x^4-2x^2+3}}\right)+\frac{x(x^2+5)}{72\left(-x^4-2x^2+3\right)^{3/2}}$$

$$\Big\downarrow \, 399$$

$$\frac{1}{72}\left(\frac{1}{6}\left(33\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx-7\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{x(7x^2+26)}{6\sqrt{-x^4-2x^2+3}}\right)+\frac{x(x^2+5)}{72\left(-x^4-2x^2+3\right)^{3/2}}$$

$$\Big\downarrow \, 321$$

$$\frac{1}{72}\left(\frac{1}{6}\left(11\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-7\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{x(7x^2+26)}{6\sqrt{-x^4-2x^2+3}}\right)+\frac{x(x^2+5)}{72\left(-x^4-2x^2+3\right)^{3/2}}$$

$$\Big\downarrow \, 327$$

$$\frac{1}{72}\left(\frac{1}{6}\left(11\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-7\sqrt{3}E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)\right)+\frac{x(7x^2+26)}{6\sqrt{-x^4-2x^2+3}}\right)+\frac{x(x^2+5)}{72\left(-x^4-2x^2+3\right)^{3/2}}$$

input
```
Int[(3 - 2*x^2 - x^4)^(-5/2),x]
```

output
```
(x*(5 + x^2))/(72*(3 - 2*x^2 - x^4)^(3/2)) + ((x*(26 + 7*x^2))/(6*Sqrt[3 -
 2*x^2 - x^4]) + (-7*Sqrt[3]*EllipticE[ArcSin[x], -1/3] + 11*Sqrt[3]*Ellip
ticF[ArcSin[x], -1/3])/6)/72
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.68

method	result
risch	$\frac{x(7x^6+40x^4+25x^2-108)}{432(x^4+2x^2-3)\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{108\sqrt{-x^4-2x^2+3}}+\frac{7\sqrt{-x^2+1}\,\sqrt{3x^2+9}\left(\text{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\text{EllipticE}\left(x\right.\right.}{432\sqrt{-x^4-2x^2+3}}$
default	$\frac{\left(\frac{5}{72}x+\frac{1}{72}x^3\right)\sqrt{-x^4-2x^2+3}}{\left(x^4+2x^2-3\right)^2}+\frac{\frac{7}{432}x^3+\frac{13}{216}x}{\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{108\sqrt{-x^4-2x^2+3}}+\frac{7\sqrt{-x^2+1}\,\sqrt{3x^2+9}\left(\text{EllipticF}\left(x,\frac{i\sqrt{}}{3}\right.\right.}{432\sqrt{-x^4-2x^2}}$
elliptic	$\frac{\left(\frac{5}{72}x+\frac{1}{72}x^3\right)\sqrt{-x^4-2x^2+3}}{\left(x^4+2x^2-3\right)^2}+\frac{\frac{7}{432}x^3+\frac{13}{216}x}{\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{108\sqrt{-x^4-2x^2+3}}+\frac{7\sqrt{-x^2+1}\,\sqrt{3x^2+9}\left(\text{EllipticF}\left(x,\frac{i\sqrt{}}{3}\right.\right.}{432\sqrt{-x^4-2x^2}}$

input
```
int(1/(-x^4-2*x^2+3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/432*x*(7*x^6+40*x^4+25*x^2-108)/(x^4+2*x^2-3)/(-x^4-2*x^2+3)^(1/2)+1/108
*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3^(
1/2))+7/432*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF
(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.38

$$\int \frac{1}{(3-2x^2-x^4)^{5/2}}\,dx =$$
$$-\frac{7\sqrt{3}(x^8+4\,x^6-2\,x^4-12\,x^2+9)E(\arcsin{(x)}\mid-\frac{1}{3})-11\sqrt{3}(x^8+4\,x^6-2\,x^4-12\,x^2+9)F(\arcsin{(x)}}{432\,(x^8+4\,x^6-2\,x^4-12\,x^2+9)}$$

input
```
integrate(1/(-x^4-2*x^2+3)^(5/2),x, algorithm="fricas")
```

output
```
-1/432*(7*sqrt(3)*(x^8 + 4*x^6 - 2*x^4 - 12*x^2 + 9)*elliptic_e(arcsin(x),
 -1/3) - 11*sqrt(3)*(x^8 + 4*x^6 - 2*x^4 - 12*x^2 + 9)*elliptic_f(arcsin(x
), -1/3) + (7*x^7 + 40*x^5 + 25*x^3 - 108*x)*sqrt(-x^4 - 2*x^2 + 3))/(x^8
+ 4*x^6 - 2*x^4 - 12*x^2 + 9)
```

Sympy [F]

$$\int \frac{1}{(3-2x^2-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4-2x^2+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x**4-2*x**2+3)**(5/2),x)
```

output
```
Integral((-x**4 - 2*x**2 + 3)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-2x^2-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4-2\,x^2+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x^4-2*x^2+3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 - 2*x^2 + 3)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(3-2x^2-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4-2\,x^2+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x^4-2*x^2+3)^(5/2),x, algorithm="giac")
```

output
```
integrate((-x^4 - 2*x^2 + 3)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x^2-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4-2\,x^2+3)^{5/2}}\,dx$$

input
```
int(1/(3 - x^4 - 2*x^2)^(5/2),x)
```

output
```
int(1/(3 - x^4 - 2*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-2x^2-x^4)^{5/2}}\,dx = -\left(\int \frac{\sqrt{-x^4-2x^2+3}}{x^{12}+6x^{10}+3x^8-28x^6-9x^4+54x^2-27}dx\right)$$

input
```
int(1/(-x^4-2*x^2+3)^(5/2),x)
```

output
```
- int(sqrt( - x**4 - 2*x**2 + 3)/(x**12 + 6*x**10 + 3*x**8 - 28*x**6 - 9*
x**4 + 54*x**2 - 27),x)
```

3.342 $\int \frac{1}{\left(3-2x^2-x^4\right)^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{1}{\left(3-2x^2-x^4\right)^{7/2}}\,dx=\frac{x(5+x^2)}{120\left(3-2x^2-x^4\right)^{5/2}}+\frac{7x(7+2x^2)}{2160\left(3-2x^2-x^4\right)^{3/2}}+\frac{7x(133+41x^2)}{51840\sqrt{3-2x^2-x^4}}-\frac{287E\left(\arcsin(x)\,\middle|\,-\frac{1}{3}\right)}{17280\sqrt{3}}+\frac{203\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{8640\sqrt{3}}$$

output
```
1/120*x*(x^2+5)/(-x^4-2*x^2+3)^(5/2)+7/2160*x*(2*x^2+7)/(-x^4-2*x^2+3)^(3/
2)+7/51840*x*(41*x^2+133)/(-x^4-2*x^2+3)^(1/2)-287/51840*EllipticE(x,1/3*I
*3^(1/2))*3^(1/2)+203/25920*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(3-2x^2-x^4\right)^{7/2}}\,dx=\frac{14067x-9501x^3-7154x^5+2814x^7+2079x^9+287x^{11}-287i\left(3-2x^2-x^4\right)^{5}\cdots}{51840\left(3-\cdots\right.}$$

input
```
Integrate[(3 - 2*x^2 - x^4)^(-7/2),x]
```

output
```
(14067*x - 9501*x^3 - 7154*x^5 + 2814*x^7 + 2079*x^9 + 287*x^11 - (287*I)*
(3 - 2*x^2 - x^4)^(5/2)*EllipticE[I*ArcSinh[x/Sqrt[3]], -3] - (70*I)*(3 -
2*x^2 - x^4)^(5/2)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/(51840*(3 - 2*x^2
- x^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1405, 27, 1492, 27, 1492, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^4-2x^2+3)^{7/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(x^2+5)}{120(-x^4-2x^2+3)^{5/2}}-\frac{1}{240}\int -\frac{14(x^2+5)}{(-x^4-2x^2+3)^{5/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{120}\int \frac{x^2+5}{(-x^4-2x^2+3)^{5/2}}dx+\frac{x(x^2+5)}{120(-x^4-2x^2+3)^{5/2}}$$

$$\Big\downarrow \text{1492}$$

$$\frac{7}{120}\left(\frac{x(2x^2+7)}{18(-x^4-2x^2+3)^{3/2}}-\frac{1}{144}\int -\frac{8(6x^2+23)}{(-x^4-2x^2+3)^{3/2}}dx\right)+\frac{x(x^2+5)}{120(-x^4-2x^2+3)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{120}\left(\frac{1}{18}\int \frac{6x^2+23}{(-x^4-2x^2+3)^{3/2}}dx+\frac{x(2x^2+7)}{18(-x^4-2x^2+3)^{3/2}}\right)+\frac{x(x^2+5)}{120(-x^4-2x^2+3)^{5/2}}$$

$$\Big\downarrow \text{1492}$$

$$\frac{7}{120}\left(\frac{1}{18}\left(\frac{x(41x^2+133)}{24\sqrt{-x^4-2x^2+3}}-\frac{1}{48}\int -\frac{2(51-41x^2)}{\sqrt{-x^4-2x^2+3}}dx\right)+\frac{x(2x^2+7)}{18(-x^4-2x^2+3)^{3/2}}\right)+\frac{x(x^2+5)}{120(-x^4-2x^2+3)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{120}\left(\frac{1}{18}\left(\frac{1}{24}\int \frac{51-41x^2}{\sqrt{-x^4-2x^2+3}}dx+\frac{x\left(41x^2+133\right)}{24\sqrt{-x^4-2x^2+3}}\right)+\frac{x\left(2x^2+7\right)}{18\left(-x^4-2x^2+3\right)^{3/2}}\right)+\frac{x\left(x^2+5\right)}{120\left(-x^4-2x^2+3\right)^{5/2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{7}{120}\left(\frac{1}{18}\left(\frac{1}{12}\int \frac{51-41x^2}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{x\left(41x^2+133\right)}{24\sqrt{-x^4-2x^2+3}}\right)+\frac{x\left(2x^2+7\right)}{18\left(-x^4-2x^2+3\right)^{3/2}}\right)+\frac{x\left(x^2+5\right)}{120\left(-x^4-2x^2+3\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{120}\left(\frac{1}{18}\left(\frac{1}{24}\int \frac{51-41x^2}{\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{x\left(41x^2+133\right)}{24\sqrt{-x^4-2x^2+3}}\right)+\frac{x\left(2x^2+7\right)}{18\left(-x^4-2x^2+3\right)^{3/2}}\right)+\frac{x\left(x^2+5\right)}{120\left(-x^4-2x^2+3\right)^{5/2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{7}{120}\left(\frac{1}{18}\left(\frac{1}{24}\left(174\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx-41\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{x\left(41x^2+133\right)}{24\sqrt{-x^4-2x^2+3}}\right)+\frac{x\left(2x^2+7\right)}{18\left(-x^4-2x^2+3\right)^{3/}}\right.$$
$$\frac{x\left(x^2+5\right)}{120\left(-x^4-2x^2+3\right)^{5/2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{7}{120}\left(\frac{1}{18}\left(\frac{1}{24}\left(58\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-41\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{x\left(41x^2+133\right)}{24\sqrt{-x^4-2x^2+3}}\right)+\frac{x\left(2x^2+7\right.}{18\left(-x^4-2x^2\right.}\right.$$
$$\frac{x\left(x^2+5\right)}{120\left(-x^4-2x^2+3\right)^{5/2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{7}{120}\left(\frac{1}{18}\left(\frac{1}{24}\left(58\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-41\sqrt{3}E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)\right)+\frac{x\left(41x^2+133\right)}{24\sqrt{-x^4-2x^2+3}}\right)+\frac{x(}{18\left(-x^4\right.}\right.$$
$$\frac{x\left(x^2+5\right)}{120\left(-x^4-2x^2+3\right)^{5/2}}$$

input
```
Int[(3 - 2*x^2 - x^4)^(-7/2),x]
```

output
```
(x*(5 + x^2))/(120*(3 - 2*x^2 - x^4)^(5/2)) + (7*((x*(7 + 2*x^2))/(18*(3 -
 2*x^2 - x^4)^(3/2)) + ((x*(133 + 41*x^2))/(24*Sqrt[3 - 2*x^2 - x^4]) + (-
41*Sqrt[3]*EllipticE[ArcSin[x], -1/3] + 58*Sqrt[3]*EllipticF[ArcSin[x], -1
/3])/24)/18))/120
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.35

method	result
risch	$\frac{x\left(287x^{10}+2079x^8+2814x^6-7154x^4-9501x^2+14067\right)}{51840\left(x^4+2x^2-3\right)^2\sqrt{-x^4-2x^2+3}}+\frac{119\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{51840\sqrt{-x^4-2x^2+3}}+\frac{287\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ (\text{El}}{5184}$
default	$\frac{\left(-\frac{1}{24}x-\frac{1}{120}x^3\right)\sqrt{-x^4-2x^2+3}}{\left(x^4+2x^2-3\right)^3}+\frac{\left(\frac{7}{1080}x^3+\frac{49}{2160}x\right)\sqrt{-x^4-2x^2+3}}{\left(x^4+2x^2-3\right)^2}+\frac{\frac{287}{51840}x^3+\frac{931}{51840}x}{\sqrt{-x^4-2x^2+3}}+\frac{119\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(x,\right.}{51840\sqrt{-x^4-2x^2+3}}$
elliptic	$\frac{\left(-\frac{1}{24}x-\frac{1}{120}x^3\right)\sqrt{-x^4-2x^2+3}}{\left(x^4+2x^2-3\right)^3}+\frac{\left(\frac{7}{1080}x^3+\frac{49}{2160}x\right)\sqrt{-x^4-2x^2+3}}{\left(x^4+2x^2-3\right)^2}+\frac{\frac{287}{51840}x^3+\frac{931}{51840}x}{\sqrt{-x^4-2x^2+3}}+\frac{119\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(x,\right.}{51840\sqrt{-x^4-2x^2+3}}$

input
```
int(1/(-x^4-2*x^2+3)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/51840*x*(287*x^10+2079*x^8+2814*x^6-7154*x^4-9501*x^2+14067)/(x^4+2*x^2-
3)^2/(-x^4-2*x^2+3)^(1/2)+119/51840*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2
*x^2+3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+287/51840*(-x^2+1)^(1/2)*(3*x^2+9
)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I
*3^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.39

$$\int \frac{1}{(3-2x^2-x^4)^{7/2}}\,dx = -\frac{287\,\sqrt{3}(x^{12}+6\,x^{10}+3\,x^8-28\,x^6-9\,x^4+54\,x^2-27)E(\arcsin{(x)}\mid-\frac{1}{3})-406\,\sqrt{3}(x^{12}+6\,x^{10}+3\,x^8-}{51840\,(x^{12}+6\,x}$$

input
```
integrate(1/(-x^4-2*x^2+3)^(7/2),x, algorithm="fricas")
```

output
```
-1/51840*(287*sqrt(3)*(x^12 + 6*x^10 + 3*x^8 - 28*x^6 - 9*x^4 + 54*x^2 - 2
7)*elliptic_e(arcsin(x), -1/3) - 406*sqrt(3)*(x^12 + 6*x^10 + 3*x^8 - 28*x
^6 - 9*x^4 + 54*x^2 - 27)*elliptic_f(arcsin(x), -1/3) + (287*x^11 + 2079*x
^9 + 2814*x^7 - 7154*x^5 - 9501*x^3 + 14067*x)*sqrt(-x^4 - 2*x^2 + 3))/(x^
12 + 6*x^10 + 3*x^8 - 28*x^6 - 9*x^4 + 54*x^2 - 27)
```

Sympy [F]

$$\int \frac{1}{(3-2x^2-x^4)^{7/2}}\,dx = \int \frac{1}{(-x^4-2x^2+3)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(-x**4-2*x**2+3)**(7/2),x)
```

output
```
Integral((-x**4 - 2*x**2 + 3)**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(3-2x^2-x^4)^{7/2}}\,dx = \int \frac{1}{(-x^4-2\,x^2+3)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(-x^4-2*x^2+3)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 - 2*x^2 + 3)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(3-2x^2-x^4)^{7/2}}\,dx = \int \frac{1}{(-x^4-2\,x^2+3)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(-x^4-2*x^2+3)^(7/2),x, algorithm="giac")
```

output
```
integrate((-x^4 - 2*x^2 + 3)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x^2-x^4)^{7/2}}\,dx = \int \frac{1}{(-x^4-2\,x^2+3)^{7/2}}\,dx$$

input
```
int(1/(3 - x^4 - 2*x^2)^(7/2),x)
```

output
```
int(1/(3 - x^4 - 2*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(3-2x^2-x^4)^{7/2}}\,dx = \int \frac{\sqrt{-x^4-2x^2+3}}{x^{16}+8x^{14}+12x^{12}-40x^{10}-74x^8+120x^6+108x^4-216x^2+81}dx$$

input
```
int(1/(-x^4-2*x^2+3)^(7/2),x)
```

output
```
int(sqrt( - x**4 - 2*x**2 + 3)/(x**16 + 8*x**14 + 12*x**12 - 40*x**10 - 74
*x**8 + 120*x**6 + 108*x**4 - 216*x**2 + 81),x)
```

3.343 $\int \sqrt{(1-x^2)(3+x^2)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \frac{1}{3}x\sqrt{1-x^2}\sqrt{3+x^2}-\frac{2E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)}{\sqrt{3}}+\frac{4\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
1/3*x*(-x^2+1)^(1/2)*(x^2+3)^(1/2)-2/3*EllipticE(x,1/3*I*3^(1/2))*3^(1/2)+
4/3*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \frac{1}{3}\left(x\sqrt{3-2x^2-x^4}-2iE\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-4i\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right),-3\right)\right)$$

input
```
Integrate[Sqrt[(1 - x^2)*(3 + x^2)],x]
```

output
```
(x*Sqrt[3 - 2*x^2 - x^4] - (2*I)*EllipticE[I*ArcSinh[x/Sqrt[3]], -3] - (4*
I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2048, 1404, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{(1-x^2)(x^2+3)}\,dx$$

$$\Big\downarrow \ 2048$$

$$\int \sqrt{-x^4-2x^2+3}dx$$

$$\Big\downarrow \ 1404$$

$$\frac{1}{3}\int \frac{2(3-x^2)}{\sqrt{-x^4-2x^2+3}}dx+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \ 27$$

$$\frac{2}{3}\int \frac{3-x^2}{\sqrt{-x^4-2x^2+3}}dx+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \ 1494$$

$$\frac{4}{3}\int \frac{3-x^2}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \ 27$$

$$\frac{2}{3}\int \frac{3-x^2}{\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \ 399$$

$$\frac{2}{3}\left(6\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx-\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \ 321$$

$$\frac{2}{3}\left(2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-\int\frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \text{327}$$

$$\frac{2}{3}\left(2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-\sqrt{3}E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)\right)+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

input
```
Int[Sqrt[(1 - x^2)*(3 + x^2)],x]
```

output
```
(x*Sqrt[3 - 2*x^2 - x^4])/3 + (2*(-(Sqrt[3]*EllipticE[ArcSin[x], -1/3]) +
2*Sqrt[3]*EllipticF[ArcSin[x], -1/3]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(46) = 92$.

Time = 1.57 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.19

method	result
default	$\frac{x\sqrt{-x^4-2x^2+3}}{3}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{3\sqrt{-x^4-2x^2+3}}$
elliptic	$\frac{x\sqrt{-x^4-2x^2+3}}{3}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{3\sqrt{-x^4-2x^2+3}}$
risch	$-\frac{x(x^2-1)(x^2+3)}{3\sqrt{-(x^2-1)(x^2+3)}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$

input
```
int(((-x^2+1)*(x^2+3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(-x^4-2*x^2+3)^(1/2)+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+
3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x
^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \frac{2i\,xE\left(\arcsin\left(\frac{1}{x}\right)\,|-3\right)+4i\,xF\left(\arcsin\left(\frac{1}{x}\right)\,|-3\right)+\sqrt{-x^4-2\,x^2+3}(x^2+2)}{3\,x}$$

input
```
integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*I*x*elliptic_e(arcsin(1/x), -3) + 4*I*x*elliptic_f(arcsin(1/x), -3)
 + sqrt(-x^4 - 2*x^2 + 3)*(x^2 + 2))/x
```

Sympy [F]

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \int \sqrt{(1-x^2)(x^2+3)}\,dx$$

input
```
integrate(((-x**2+1)*(x**2+3))**(1/2),x)
```

output
```
Integral(sqrt((1 - x**2)*(x**2 + 3)), x)
```

Maxima [F]

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \int \sqrt{-(x^2+3)(x^2-1)}\,dx$$

input
```
integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Giac [F]

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx=\int \sqrt{-(x^2+3)(x^2-1)}\,dx$$

input
```
integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx=\int \sqrt{-(x^2-1)(x^2+3)}\,dx$$

input
```
int((-(x^2 - 1)*(x^2 + 3))^(1/2),x)
```

output
```
int((-(x^2 - 1)*(x^2 + 3))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx=\frac{\sqrt{-x^4-2x^2+3}\,x}{3}-2\left(\int \frac{\sqrt{-x^4-2x^2+3}}{x^4+2x^2-3}dx\right)+\frac{2\left(\int \frac{\sqrt{-x^4-2x^2+3}\,x^2}{x^4+2x^2-3}dx\right)}{3}$$

input
```
int(((-x^2+1)*(x^2+3))^(1/2),x)
```

output
```
(sqrt( - x**4 - 2*x**2 + 3)*x - 6*int(sqrt( - x**4 - 2*x**2 + 3)/(x**4 + 2
*x**2 - 3),x) + 2*int((sqrt( - x**4 - 2*x**2 + 3)*x**2)/(x**4 + 2*x**2 - 3
),x))/3
```

3.344 $\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}} dx$

Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}} dx = \frac{\text{EllipticF}\left(\arcsin(x), -\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}} dx = -i\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right), -3\right)$$

input
```
Integrate[1/Sqrt[(1 - x^2)*(3 + x^2)],x]
```

output
```
(-I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2048, 1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(1-x^2)(x^2+3)}}dx$$

$$\Big\downarrow \text{ 2048}$$

$$\int \frac{1}{\sqrt{-x^4-2x^2+3}}dx$$

$$\Big\downarrow \text{ 1408}$$

$$2\int \frac{1}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[(1 - x^2)*(3 + x^2)],x]
```

output
```
EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(13) = 26$.

Time = 0.50 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(x, \frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\ \text{EllipticF}\left(x, \frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43

input
```
int(1/((-x^2+1)*(x^2+3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*
3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \frac{1}{3}\sqrt{3}F(\arcsin{(x)}\,|-\frac{1}{3})$$

input
```
integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*elliptic_f(arcsin(x), -1/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \int \frac{1}{\sqrt{(1-x^2)(x^2+3)}}\,dx$$

input
```
integrate(1/((-x**2+1)*(x**2+3))**(1/2),x)
```

output
```
Integral(1/sqrt((1 - x**2)*(x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \int \frac{1}{\sqrt{-(x^2+3)(x^2-1)}}\,dx$$

input
```
integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \int \frac{1}{\sqrt{-(x^2+3)(x^2-1)}}\,dx$$

input
```
integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \int \frac{1}{\sqrt{-(x^2-1)(x^2+3)}}\,dx$$

input
```
int(1/(-(x^2 - 1)*(x^2 + 3))^(1/2),x)
```

output
```
int(1/(-(x^2 - 1)*(x^2 + 3))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = -\left(\int \frac{\sqrt{-x^4-2x^2+3}}{x^4+2x^2-3}dx\right)$$

input
```
int(1/((-x^2+1)*(x^2+3))^(1/2),x)
```

output
```
- int(sqrt( - x**4 - 2*x**2 + 3)/(x**4 + 2*x**2 - 3),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*   "F" if the result fails to integrate an expression that*)
(*       is integrable*)
(*   "C" if result involves higher level functions than necessary*)
(*   "B" if result is more than twice the size of the optimal*)
(*       antiderivative*)
(*   "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file