Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.2-Quartic-trinomial/116-1.2.2.3-a

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**493**]. This is test number [116].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	90.26 (445)	9.74 (48)
Maple	88.44 (436)	11.56 (57)
Fricas	81.54 (402)	18.46 (91)
Rubi	81.34 (401)	18.66 (92)
Sympy	47.06 (232)	52.94 (261)
Reduce	25.76 (127)	74.24 (366)
Giac	22.31 (110)	77.69 (383)
Mupad	16.23 (80)	83.77 (413)
Maxima	8.52 (42)	91.48 (451)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

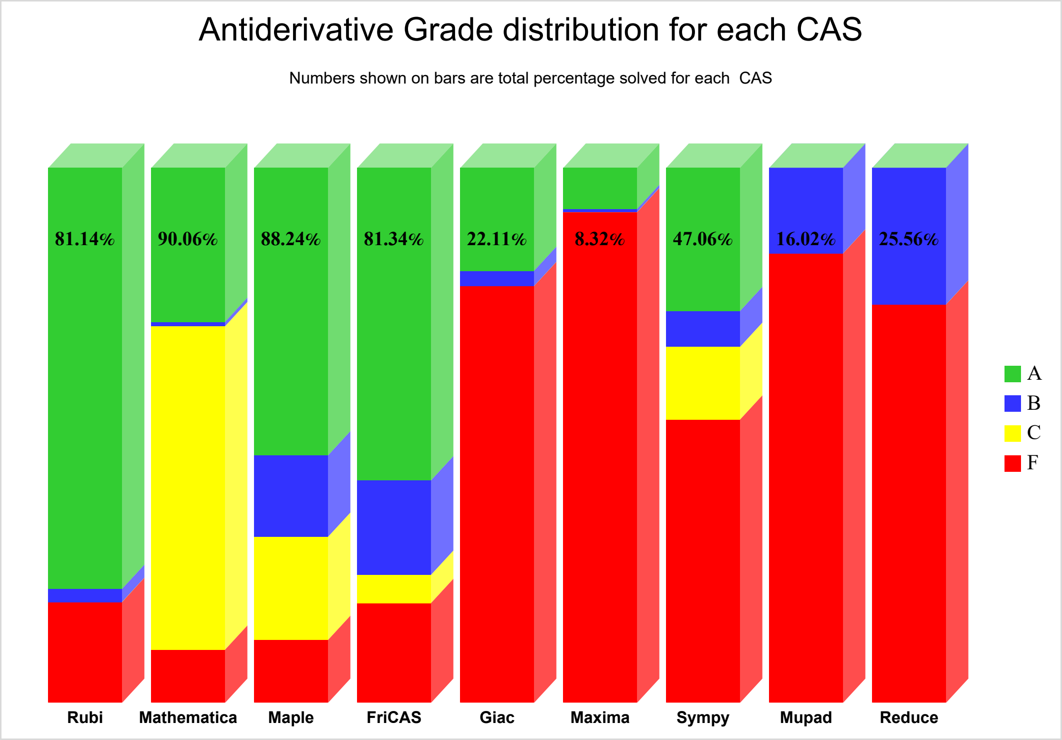
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

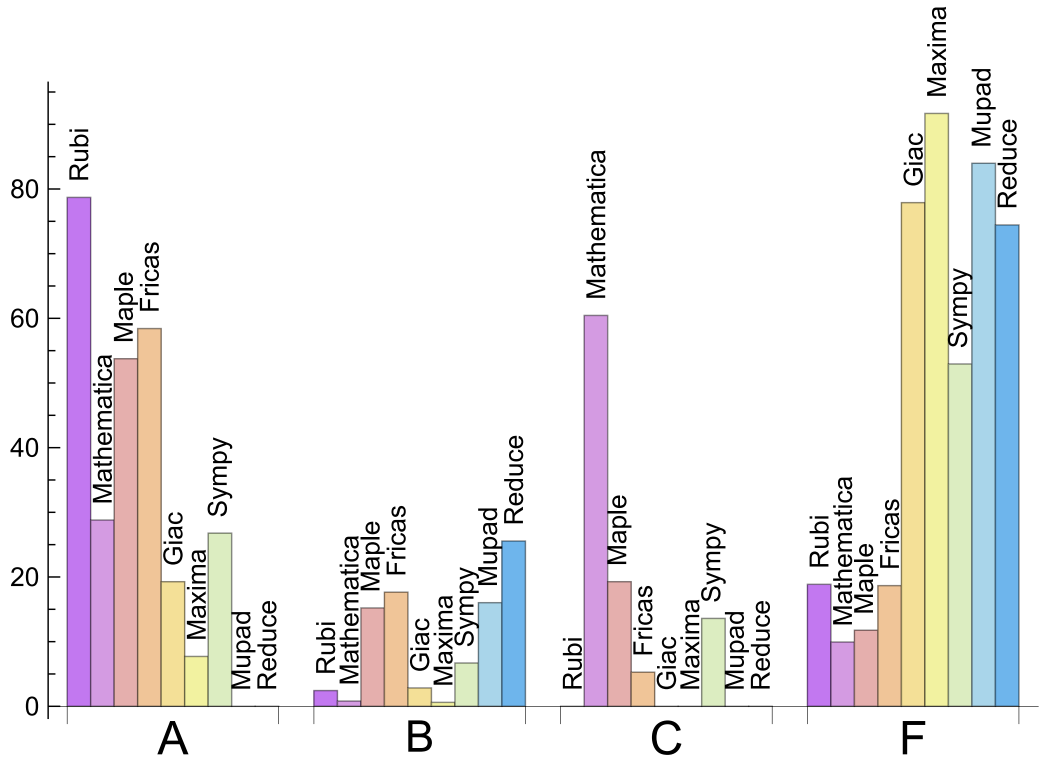
System	% A grade	% B grade	% C grade	% F grade
Rubi	78.702	2.434	0.000	18.864
Fricas	58.418	17.647	5.274	18.661
Maple	53.753	15.213	19.270	11.765
Mathematica	28.803	0.811	60.446	9.939
Sympy	26.775	6.694	13.590	52.941
Giac	19.270	2.840	0.000	77.890
Maxima	7.708	0.609	0.000	91.684
Mupad	0.000	16.024	0.000	83.976
Reduce	0.000	25.558	0.000	74.442

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	48	100.00	0.00	0.00
Maple	57	100.00	0.00	0.00
Fricas	91	74.73	25.27	0.00
Rubi	92	100.00	0.00	0.00
Sympy	261	78.93	21.07	0.00
Reduce	366	100.00	0.00	0.00
Giac	383	89.03	9.14	1.83
Mupad	413	0.00	100.00	0.00
Maxima	451	86.92	0.00	13.08

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Giac	0.16
Reduce	0.33
Rubi	0.61
Maple	1.34
Fricas	3.63
Sympy	4.50
Mathematica	5.22
Mupad	11.21

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	159.26	1.49	78.00	0.87
Giac	166.03	1.02	110.00	0.92
Maxima	170.43	1.39	141.50	1.23
Rubi	183.87	1.20	158.00	1.04
Mathematica	194.14	1.15	97.00	0.84
Maple	220.17	1.37	121.00	1.01
Reduce	296.85	2.36	165.00	1.37
Fricas	604.08	2.93	146.00	1.18
Mupad	1116.46	3.77	69.50	0.94

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

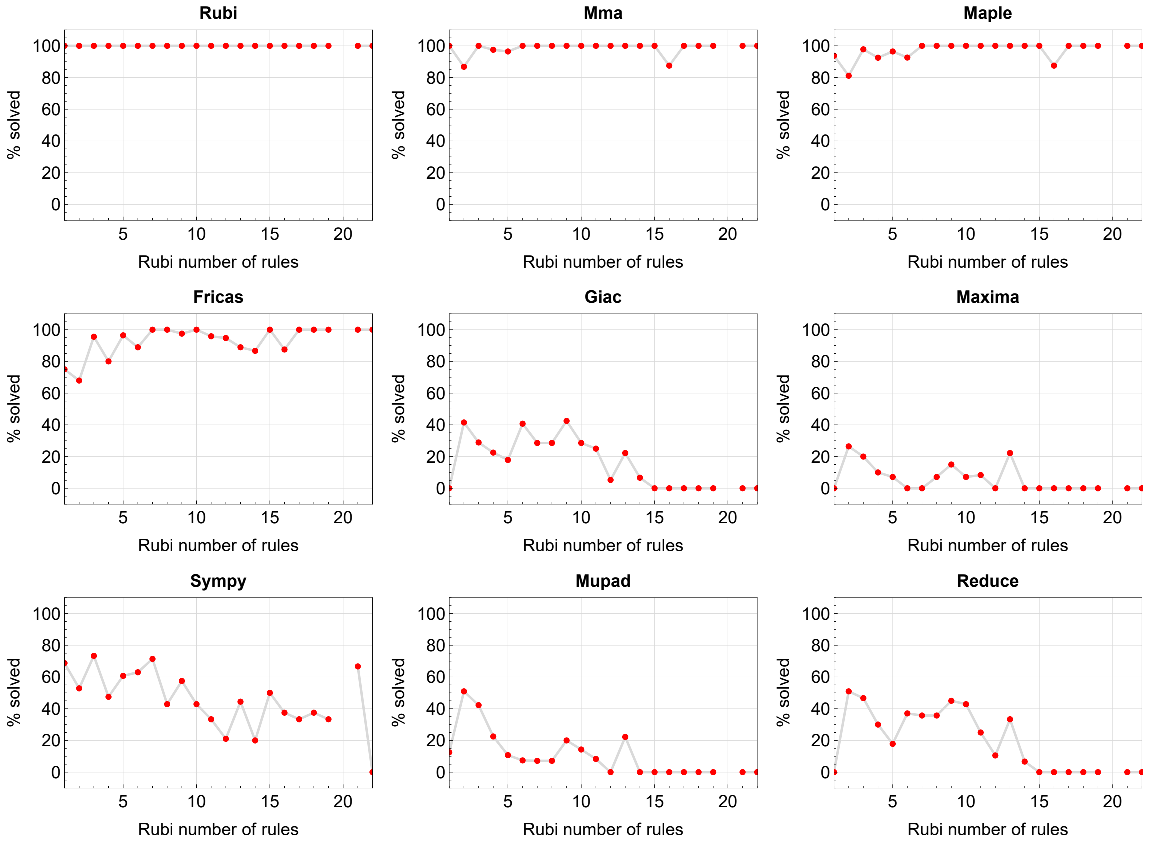


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

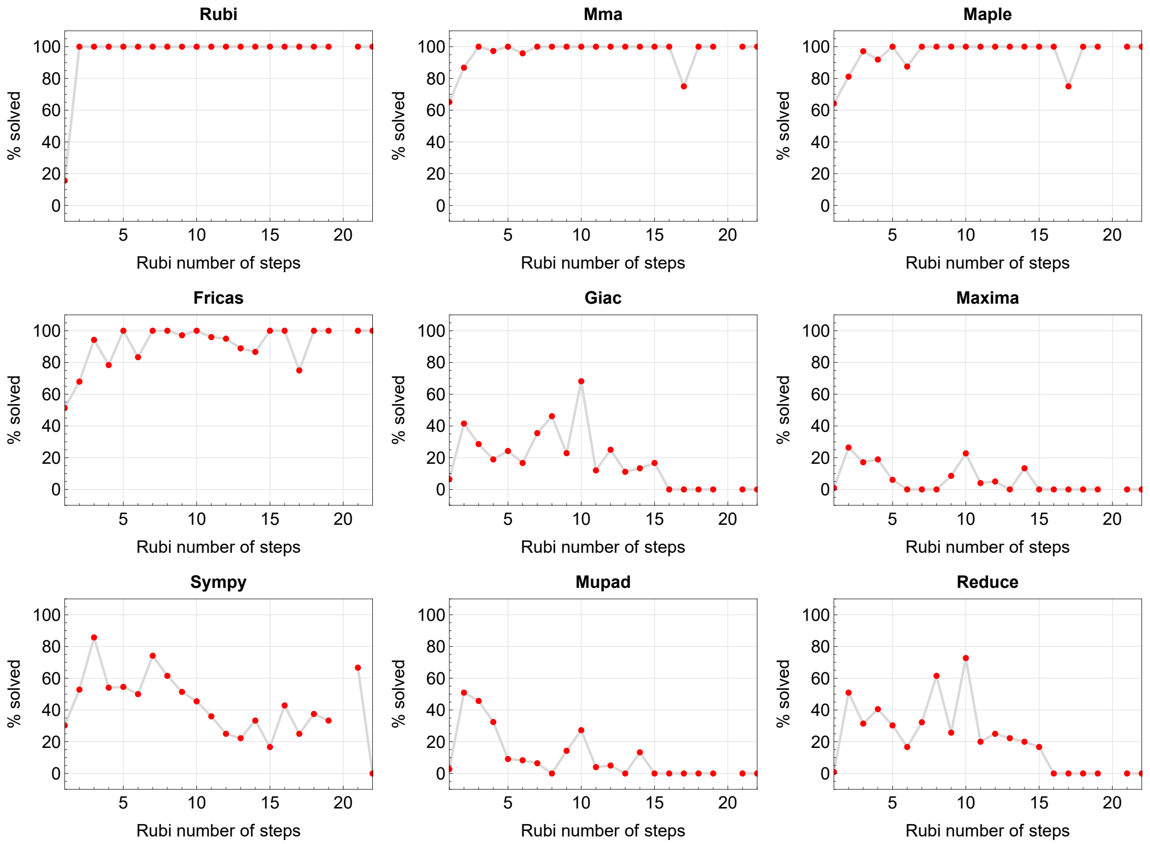


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

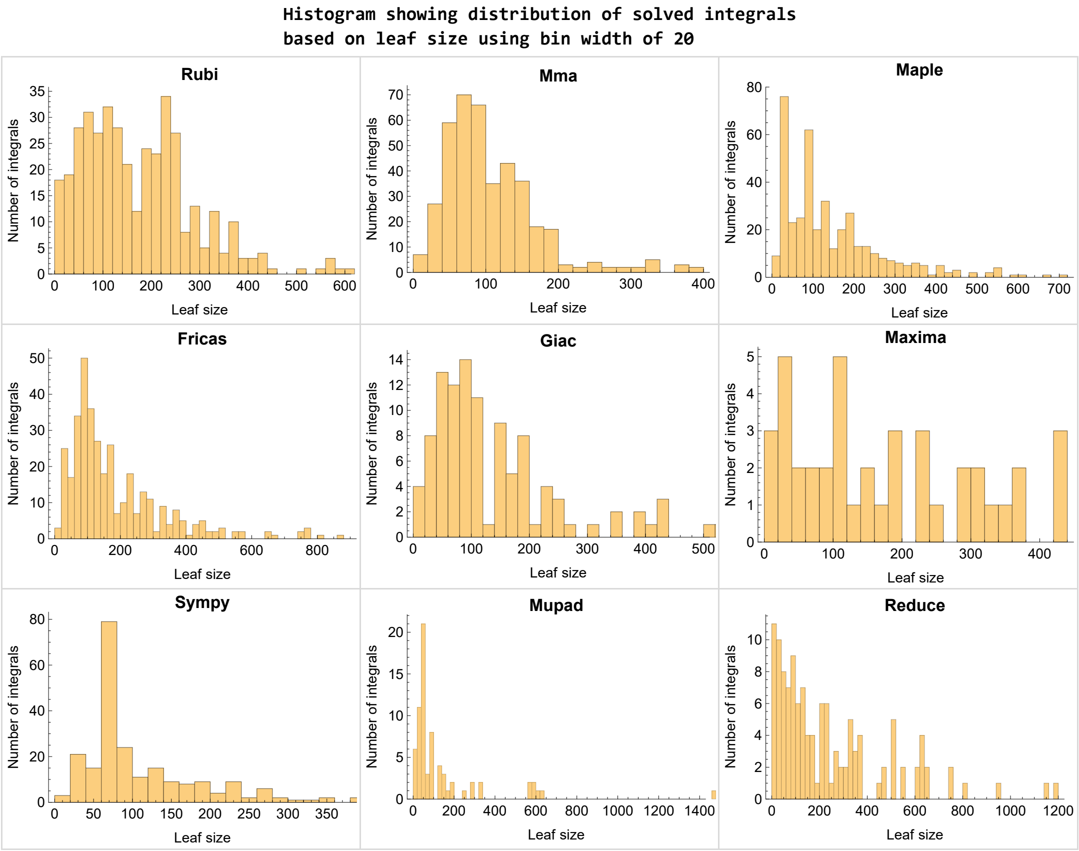


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

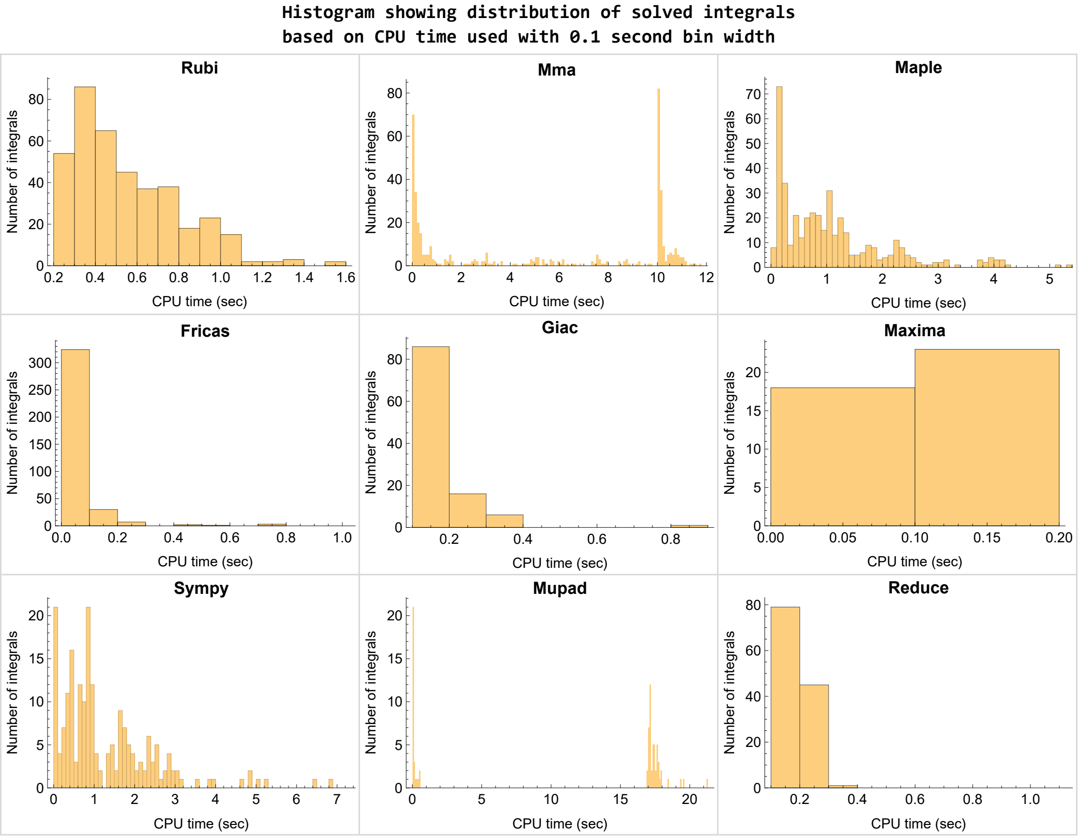


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

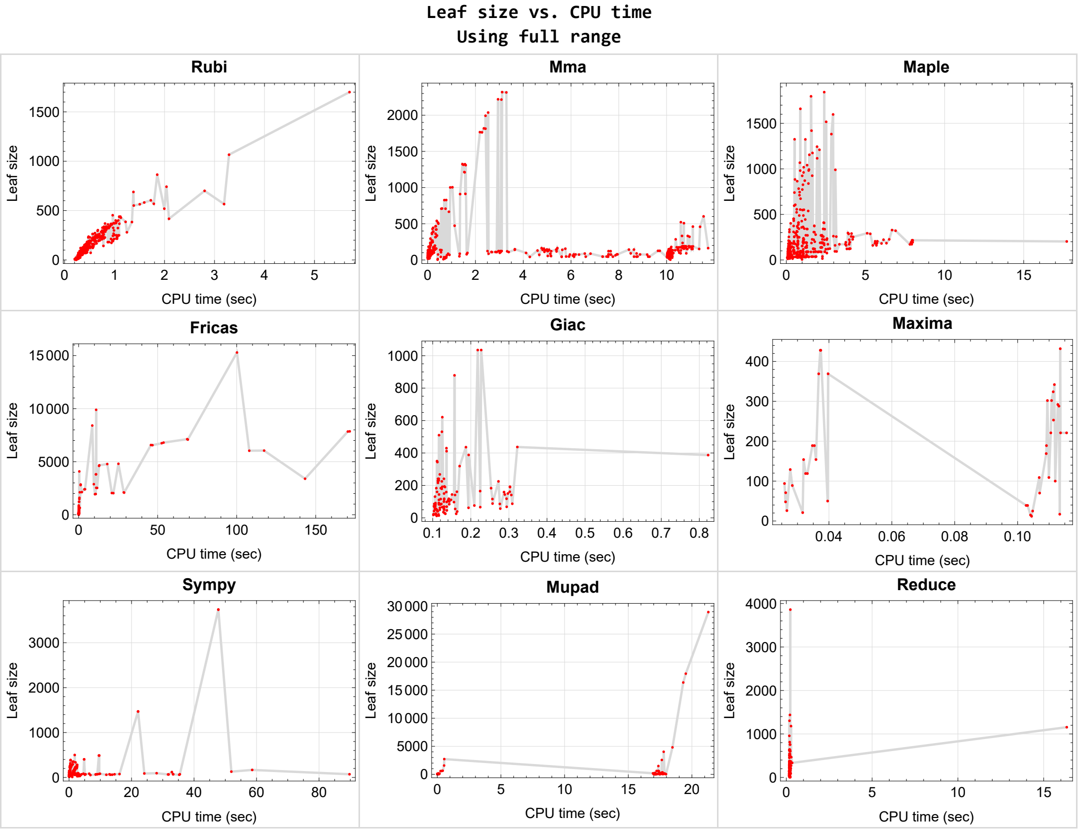


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{480}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{438\}$

Mathematica $\{51, 74, 101, 184, 427\}$

Maple $\{77, 128, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
 from sage.interfaces.maxima_lib import maxima_lib
 maxima_lib.set('extra_definite_integration_methods', '[]')
 maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

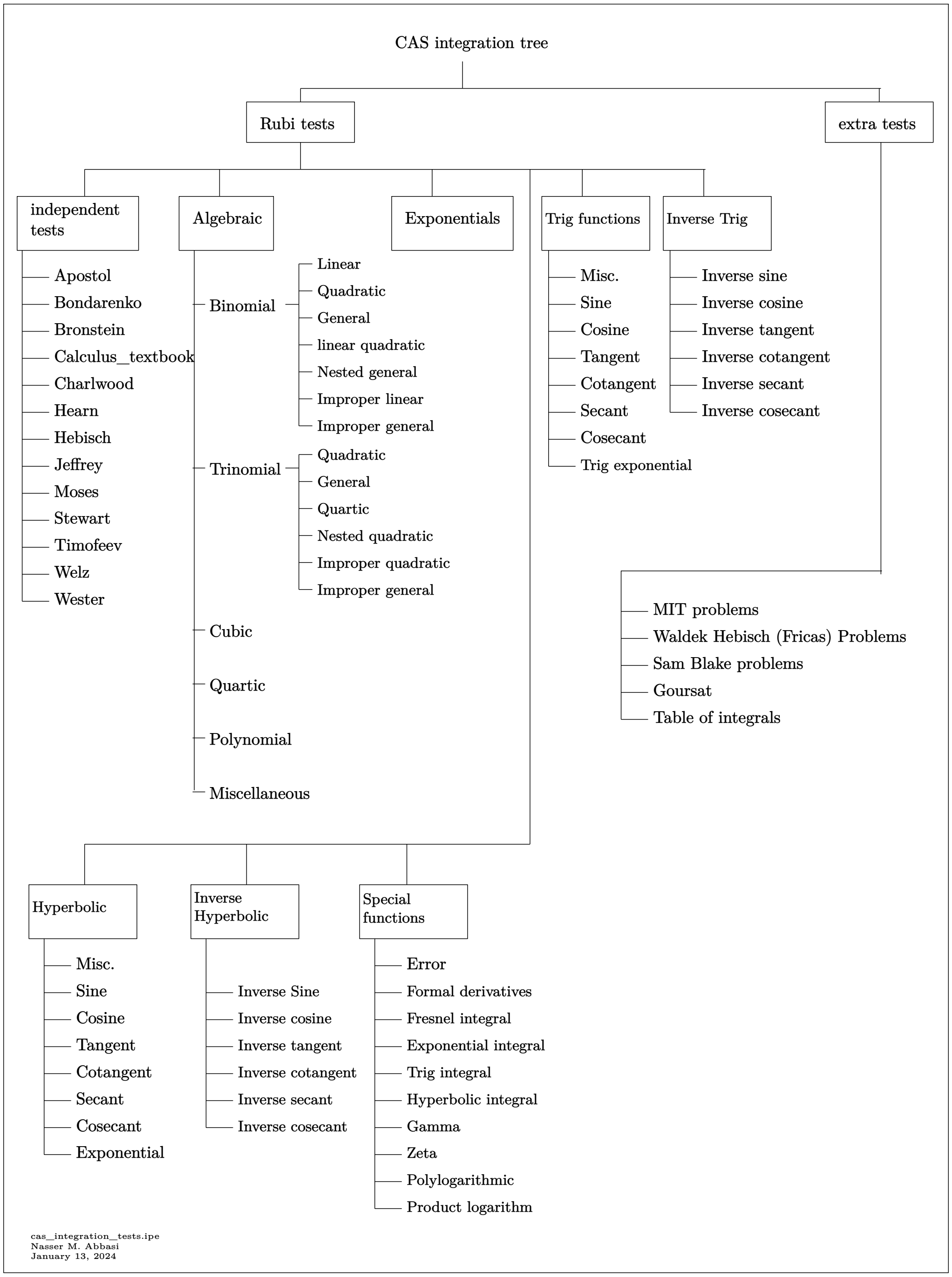
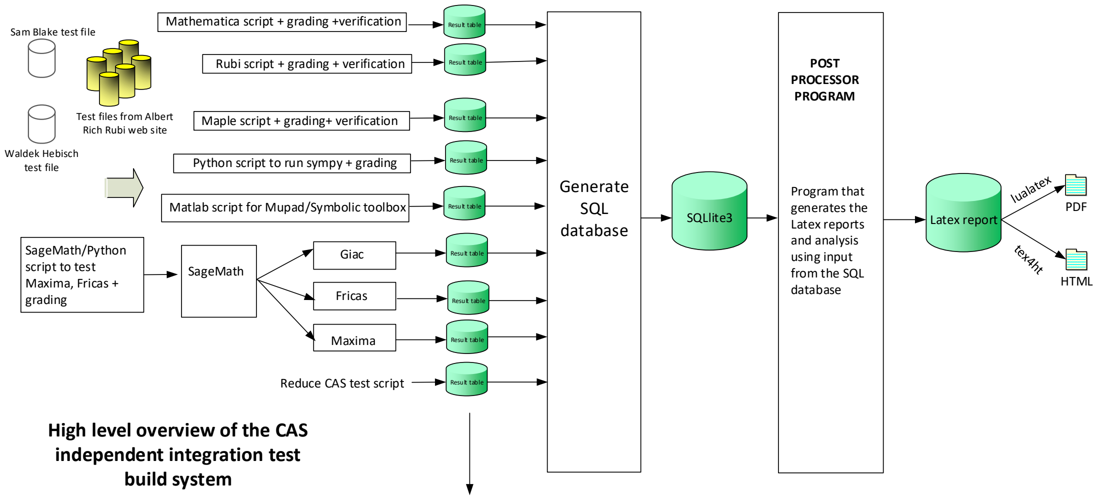


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
January 13, 2024
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 79, 80, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 269, 275, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 438, 454, 474, 475, 476, 477, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493 }

B grade { 39, 77, 78, 81, 82, 83, 113, 210, 212, 341, 342, 420 }

C grade { }

F normal fail { 10, 20, 36, 45, 63, 70, 86, 95, 114, 121, 266, 267, 268, 270, 271, 272, 273, 274, 276, 277, 278, 279, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 478, 479 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 6, 12, 16, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 76, 78, 127, 129, 135, 136, 144, 145, 146, 147, 148, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 178, 179, 180, 181, 182, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 432, 474, 475, 476, 481, 482, 483, 484, 487, 488, 489, 490 }

B grade { 149, 173, 174, 184 }

C grade { 1, 4, 5, 7, 8, 9, 10, 11, 13, 14, 15, 17, 18, 19, 20, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 130, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 150, 151, 152, 153, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 433, 434, 435 }

F normal fail { 183, 185, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 477, 478, 479, 485, 486, 491, 492, 493 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 3, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 137, 138, 139, 140, 141, 142, 148, 150, 151, 152, 153, 157, 158, 159, 160, 164, 165, 166, 167, 168, 175, 176, 177, 178, 179, 180, 181, 182, 193, 194, 196, 205, 206, 207, 208, 211, 213, 214, 215, 216, 217, 218, 219, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 264, 265, 280, 281, 282, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 316, 317, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 405, 406, 407, 408, 409, 413, 414, 415, 416, 417, 428, 429, 487, 488, 489, 490 }

B grade { 2, 5, 7, 8, 9, 10, 12, 15, 56, 58, 76, 78, 79, 80, 88, 114, 127, 134, 135, 136, 143, 144, 145, 146, 147, 149, 154, 155, 156, 161, 162, 163, 169, 170, 171, 172, 173, 174, 209, 210, 212, 341, 342, 343, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 410, 411, 412, 418, 419, 420, 432, 434 }

C grade { 1, 4, 6, 11, 14, 16, 39, 75, 77, 81, 82, 83, 113, 126, 128, 186, 187, 188, 189, 190, 191, 192, 195, 197, 198, 199, 201, 202, 203, 204, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 256, 263, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 283, 290, 311, 312, 313, 314, 315, 318, 319, 320, 321, 421, 422, 423, 424, 425, 426, 427, 430, 431, 433, 435 }

F normal fail { 183, 184, 185, 200, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 491, 492, 493 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 7, 8, 9, 10, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 178, 187, 188, 194, 195, 205, 206, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 233, 234, 235, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 287, 288, 289, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 405, 406, 407, 408, 413, 414, 415, 416, 421, 422, 423, 424, 428, 430 }

B grade { 1, 2, 3, 4, 5, 6, 11, 12, 13, 14, 15, 16, 26, 29, 30, 75, 79, 80, 126, 149, 173, 174, 176, 177, 179, 181, 201, 202, 203, 204, 207, 212, 229, 231, 232, 236, 237, 238, 239, 240, 279, 311, 312, 313, 314, 316, 317, 318, 319, 320, 322, 323, 341, 342, 343, 344, 345, 346, 347, 350, 351, 352, 353, 354, 355, 358, 359, 360, 361, 362, 382, 383, 384, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 399, 400, 401, 402 }

C grade { 180, 186, 189, 190, 191, 192, 193, 196, 197, 198, 199, 247, 248, 249, 250, 251, 252, 284, 285, 286, 290, 291, 292, 293, 315, 321 }

F normal fail { 182, 183, 184, 185, 200, 409, 417, 420, 425, 429, 431, 432, 433, 434, 435, 437, 438, 439, 440, 441, 442, 443, 444, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493 }

F(-1) timedout fail { 348, 349, 356, 357, 363, 364, 389, 396, 397, 398, 403, 404, 410, 411, 412, 418, 419, 426, 427, 436, 445, 446, 458 }

F(-2) exception fail { }

Maxima

A grade { 148, 201, 202, 203, 204, 205, 206, 208, 209, 210, 213, 294, 295, 296, 297, 302, 303, 304, 305, 311, 312, 313, 314, 315, 318, 319, 320, 321, 329, 330, 331, 339, 340, 370, 371, 372, 380, 381 }

B grade { 207, 211, 212 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493 }

F(-1) timedout fail { }

F(-2) exception fail { 8, 9, 18, 19, 21, 22, 23, 24, 25, 26, 33, 38, 47, 58, 65, 72, 83, 88, 97, 108, 116, 123, 298, 299, 300, 301, 306, 307, 308, 309, 310, 316, 317, 322, 323, 324, 325, 326, 327, 328, 332, 333, 334, 335, 336, 337, 338, 365, 366, 367, 368, 369, 373, 374, 375, 376, 377, 378, 379 }

Giac

A grade { 21, 22, 23, 24, 25, 26, 30, 130, 131, 132, 133, 137, 138, 139, 140, 142, 148, 149, 157, 158, 159, 160, 164, 165, 166, 201, 202, 205, 206, 208, 211, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 316, 319, 320, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 349, 350, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 389, 390, 397 }

B grade { 27, 28, 29, 203, 204, 207, 212, 213, 315, 317, 318, 322, 323, 357 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 134, 135, 136, 141, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 161, 162, 163, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 343, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493 }

F(-1) timedout fail { 344, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 385, 386, 387, 388, 391, 392, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 404 }

F(-2) exception fail { 209, 210, 341, 342, 382, 383, 384 }

Mupad

A grade { }

B grade { 21, 22, 23, 24, 25, 26, 37, 46, 57, 64, 71, 87, 96, 107, 115, 122, 148, 160, 167, 168, 182, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 329, 330, 331, 339, 340, 370, 371, 372, 380, 381, 408, 416, 476, 484, 490 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 324, 325, 326, 327, 328, 332, 333, 334, 335, 336, 337, 338, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 373, 374, 375, 376, 377, 378, 379, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 481, 482, 483, 485, 486, 487, 488, 489, 491, 492, 493 }

F(-2) exception fail { }

Sympy

A grade { 7, 10, 17, 20, 21, 22, 23, 34, 35, 36, 37, 43, 44, 45, 46, 47, 54, 55, 56, 57, 63, 64, 70, 71, 79, 84, 85, 86, 87, 93, 94, 95, 96, 97, 104, 105, 106, 107, 114, 115, 121, 122, 128, 148, 201, 202, 203, 204, 205, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 227, 228, 230, 231, 232, 234, 235, 237, 238, 239, 240, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 294, 295, 296, 297, 301, 302, 303, 304, 305, 308, 309, 310, 311, 312, 313, 314, 315, 318, 319, 320, 321, 324, 325, 326, 327, 332, 333, 334, 365, 366, 367, 368, 373, 374, 375, 405, 406, 407, 408, 415, 416, 428 }

B grade { 1, 3, 4, 11, 14, 24, 25, 26, 75, 77, 80, 126, 206, 207, 210, 220, 226, 229, 233, 236, 298, 299, 300, 306, 307, 328, 329, 330, 331, 369, 370, 371, 372 }

C grade { 182, 186, 187, 188, 189, 191, 193, 194, 195, 196, 198, 200, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 421, 422, 423, 424, 430, 474, 475, 476, 481, 482, 483, 484, 487, 488, 489, 490 }

F normal fail { 2, 5, 6, 8, 9, 12, 13, 15, 16, 18, 19, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 41, 42, 48, 49, 50, 51, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 72, 73, 74, 76, 78, 81, 82, 83, 88, 89, 90, 91, 92, 98, 99, 100, 101, 108, 109, 110, 111, 112, 113, 116, 117, 118, 119, 120, 123, 124, 125, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 184, 185, 190, 192, 197, 199, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 409, 410, 411, 412, 413, 414, 417, 418, 419, 425, 426, 427, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 468, 469, 470, 471, 472, 473 }

F(-1) timedout fail { 52, 53, 102, 103, 145, 146, 147, 164, 183, 316, 317, 322, 323, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 420, 449, 450, 464, 465, 466, 467, 477, 478, 479, 480, 485, 486, 491, 492, 493 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 130, 131, 132, 137, 138, 139, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 133, 134, 135, 136, 140, 141, 142, 143, 144, 145, 146, 147, 149, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	36	27	0	34	61	0	20	0
N.S.	1	1.00	9.00	6.75	0.00	8.50	15.25	0.00	5.00	0.00
time (sec)	N/A	0.227	10.019	0.723	0.000	0.081	0.651	0.000	0.236	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	70	0	34	0	0	20	0
N.S.	1	1.00	1.00	17.50	0.00	8.50	0.00	0.00	5.00	0.00
time (sec)	N/A	0.220	10.158	1.087	0.000	0.085	0.000	0.000	0.240	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	0	41	10	0	26	0
N.S.	1	1.00	1.00	1.25	0.00	10.25	2.50	0.00	6.50	0.00
time (sec)	N/A	0.207	0.416	0.995	0.000	0.076	1.020	0.000	0.206	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	47	38	0	65	71	0	27	0
N.S.	1	1.00	4.70	3.80	0.00	6.50	7.10	0.00	2.70	0.00
time (sec)	N/A	0.240	10.031	1.874	0.000	0.076	0.786	0.000	0.192	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	31	118	0	65	0	0	27	0
N.S.	1	1.00	3.10	11.80	0.00	6.50	0.00	0.00	2.70	0.00
time (sec)	N/A	0.239	10.235	2.497	0.000	0.075	0.000	0.000	0.185	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	15	0	76	0	0	37	0
N.S.	1	1.00	1.00	1.50	0.00	7.60	0.00	0.00	3.70	0.00
time (sec)	N/A	0.205	0.659	2.282	0.000	0.075	0.000	0.000	0.197	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	87	138	0	90	78	0	26	0
N.S.	1	1.00	1.47	2.34	0.00	1.53	1.32	0.00	0.44	0.00
time (sec)	N/A	0.334	10.054	2.232	0.000	0.081	0.880	0.000	0.168	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	78	67	138	0	90	0	0	26	0
N.S.	1	1.32	1.14	2.34	0.00	1.53	0.00	0.00	0.44	0.00
time (sec)	N/A	0.343	10.202	1.230	0.000	0.078	0.000	0.000	0.182	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	141	95	188	0	110	0	0	47	0
N.S.	1	1.47	0.99	1.96	0.00	1.15	0.00	0.00	0.49	0.00
time (sec)	N/A	0.427	10.282	1.278	0.000	0.083	0.000	0.000	0.187	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	0	112	188	0	110	83	0	47	0
N.S.	1	0.00	1.18	1.98	0.00	1.16	0.87	0.00	0.49	0.00
time (sec)	N/A	0.000	10.208	2.276	0.000	0.084	2.733	0.000	0.199	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	36	27	0	23	61	0	18	0
N.S.	1	1.00	2.77	2.08	0.00	1.77	4.69	0.00	1.38	0.00
time (sec)	N/A	0.276	10.038	0.903	0.000	0.077	0.691	0.000	0.180	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	69	0	23	0	0	18	0
N.S.	1	1.00	1.00	5.31	0.00	1.77	0.00	0.00	1.38	0.00
time (sec)	N/A	0.279	10.114	0.491	0.000	0.084	0.000	0.000	0.170	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	0	31	0	0	24	0
N.S.	1	1.00	0.92	1.08	0.00	2.38	0.00	0.00	1.85	0.00
time (sec)	N/A	0.263	0.423	0.398	0.000	0.078	0.000	0.000	0.185	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	47	38	0	62	71	0	25	0
N.S.	1	1.00	2.04	1.65	0.00	2.70	3.09	0.00	1.09	0.00
time (sec)	N/A	0.312	10.030	2.285	0.000	0.085	0.825	0.000	0.178	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	52	117	0	62	0	0	25	0
N.S.	1	1.00	2.26	5.09	0.00	2.70	0.00	0.00	1.09	0.00
time (sec)	N/A	0.299	10.244	1.070	0.000	0.076	0.000	0.000	0.172	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	28	0	73	0	0	35	0
N.S.	1	1.00	1.04	1.22	0.00	3.17	0.00	0.00	1.52	0.00
time (sec)	N/A	0.276	0.712	0.770	0.000	0.078	0.000	0.000	0.183	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	90	88	137	0	91	78	0	25	0
N.S.	1	0.74	0.73	1.13	0.00	0.75	0.64	0.00	0.21	0.00
time (sec)	N/A	0.420	10.046	2.181	0.000	0.080	0.854	0.000	0.170	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	175	89	137	0	91	0	0	25	0
N.S.	1	1.45	0.74	1.13	0.00	0.75	0.00	0.00	0.21	0.00
time (sec)	N/A	0.506	10.263	1.213	0.000	0.075	0.000	0.000	0.181	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	214	135	191	0	113	0	0	47	0
N.S.	1	1.37	0.87	1.22	0.00	0.72	0.00	0.00	0.30	0.00
time (sec)	N/A	0.574	10.256	1.277	0.000	0.086	0.000	0.000	0.177	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	0	112	191	0	113	83	0	47	0
N.S.	1	0.00	0.73	1.25	0.00	0.74	0.54	0.00	0.31	0.00
time (sec)	N/A	0.000	10.067	2.225	0.000	0.080	2.254	0.000	0.171	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	42	0	116	75	54	71	42
N.S.	1	1.00	1.00	0.82	0.00	2.27	1.47	1.06	1.39	0.82
time (sec)	N/A	0.323	0.026	0.161	0.000	0.080	0.090	0.106	0.171	0.102

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	0	90	58	42	56	28
N.S.	1	1.00	1.00	0.82	0.00	2.37	1.53	1.11	1.47	0.74
time (sec)	N/A	0.306	0.020	0.149	0.000	0.081	0.084	0.103	0.169	0.061

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	0	73	34	23	43	21
N.S.	1	1.00	1.00	0.76	0.00	2.52	1.17	0.79	1.48	0.72
time (sec)	N/A	0.274	0.011	0.145	0.000	0.082	0.071	0.102	0.174	17.485

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	68	46	18	38	16
N.S.	1	1.00	1.00	0.67	0.00	2.83	1.92	0.75	1.58	0.67
time (sec)	N/A	0.253	0.006	0.136	0.000	0.075	0.063	0.102	0.166	17.001

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	75	65	54	0	189	226	56	142	74
N.S.	1	1.04	0.90	0.75	0.00	2.62	3.14	0.78	1.97	1.03
time (sec)	N/A	0.351	0.041	0.282	0.000	0.089	0.206	0.109	0.162	17.174

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	101	76	64	0	278	257	66	244	96
N.S.	1	1.13	0.85	0.72	0.00	3.12	2.89	0.74	2.74	1.08
time (sec)	N/A	0.408	0.064	0.303	0.000	0.088	0.288	0.105	0.177	0.176

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	70	50	0	199	0	109	153	0
N.S.	1	1.00	1.13	0.81	0.00	3.21	0.00	1.76	2.47	0.00
time (sec)	N/A	0.363	0.124	1.025	0.000	0.092	0.000	0.141	0.175	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	50	33	0	138	0	82	127	0
N.S.	1	1.00	1.32	0.87	0.00	3.63	0.00	2.16	3.34	0.00
time (sec)	N/A	0.278	0.083	0.837	0.000	0.087	0.000	0.120	0.180	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	69	47	0	209	0	101	327	0
N.S.	1	1.00	1.13	0.77	0.00	3.43	0.00	1.66	5.36	0.00
time (sec)	N/A	0.312	0.132	0.848	0.000	0.087	0.000	0.143	0.182	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	88	80	69	0	279	0	113	542	0
N.S.	1	1.10	1.00	0.86	0.00	3.49	0.00	1.41	6.78	0.00
time (sec)	N/A	0.375	0.178	0.611	0.000	0.094	0.000	0.130	0.184	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	234	71	124	0	92	0	0	36	0
N.S.	1	1.58	0.48	0.84	0.00	0.62	0.00	0.00	0.24	0.00
time (sec)	N/A	0.718	10.045	4.000	0.000	0.077	0.000	0.000	0.188	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	234	134	124	0	92	0	0	36	0
N.S.	1	1.55	0.89	0.82	0.00	0.61	0.00	0.00	0.24	0.00
time (sec)	N/A	0.711	10.360	1.711	0.000	0.082	0.000	0.000	0.186	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	234	134	124	0	92	0	0	36	0
N.S.	1	1.55	0.89	0.82	0.00	0.61	0.00	0.00	0.24	0.00
time (sec)	N/A	0.739	10.180	2.754	0.000	0.086	0.000	0.000	0.184	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	306	143	194	0	148	184	0	159	0
N.S.	1	1.49	0.70	0.95	0.00	0.72	0.90	0.00	0.78	0.00
time (sec)	N/A	0.801	8.424	7.879	0.000	0.079	1.759	0.000	0.198	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	277	122	183	0	137	133	0	135	0
N.S.	1	1.56	0.69	1.03	0.00	0.77	0.75	0.00	0.76	0.00
time (sec)	N/A	0.752	7.570	5.599	0.000	0.085	1.379	0.000	0.195	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	0	87	172	0	126	85	0	111	0
N.S.	1	0.00	0.56	1.10	0.00	0.81	0.54	0.00	0.71	0.00
time (sec)	N/A	0.000	6.817	3.735	0.000	0.082	1.032	0.000	0.200	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	86	82	0	54	39	0	52	46
N.S.	1	1.00	1.02	0.98	0.00	0.64	0.46	0.00	0.62	0.55
time (sec)	N/A	0.315	5.135	0.940	0.000	0.079	0.463	0.000	0.186	17.139

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	175	89	137	0	91	0	0	25	0
N.S.	1	1.45	0.74	1.13	0.00	0.75	0.00	0.00	0.21	0.00
time (sec)	N/A	0.501	0.042	1.290	0.000	0.079	0.000	0.000	0.182	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	233	100	121	0	85	0	0	36	0
N.S.	1	3.28	1.41	1.70	0.00	1.20	0.00	0.00	0.51	0.00
time (sec)	N/A	0.700	10.528	1.793	0.000	0.085	0.000	0.000	0.195	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	253	172	222	0	166	0	0	47	0
N.S.	1	1.32	0.90	1.16	0.00	0.87	0.00	0.00	0.25	0.00
time (sec)	N/A	0.685	10.679	2.408	0.000	0.085	0.000	0.000	0.199	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	287	187	256	0	225	0	0	58	0
N.S.	1	1.28	0.83	1.14	0.00	1.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.767	10.823	3.160	0.000	0.083	0.000	0.000	0.217	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	331	152	290	0	280	0	0	69	0
N.S.	1	1.29	0.59	1.13	0.00	1.09	0.00	0.00	0.27	0.00
time (sec)	N/A	0.859	11.087	4.122	0.000	0.095	0.000	0.000	0.226	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	374	135	216	0	169	279	0	207	0
N.S.	1	1.56	0.56	0.90	0.00	0.71	1.17	0.00	0.87	0.00
time (sec)	N/A	0.987	10.160	7.943	0.000	0.081	2.591	0.000	0.210	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	341	126	205	0	158	180	0	183	0
N.S.	1	1.61	0.59	0.97	0.00	0.75	0.85	0.00	0.86	0.00
time (sec)	N/A	0.901	9.714	5.613	0.000	0.084	1.814	0.000	0.211	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	0	90	194	0	147	180	0	159	0
N.S.	1	0.00	0.47	1.02	0.00	0.77	0.95	0.00	0.84	0.00
time (sec)	N/A	0.000	8.744	3.760	0.000	0.088	1.757	0.000	0.203	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	113	58	96	0	70	41	0	76	46
N.S.	1	1.05	0.54	0.89	0.00	0.65	0.38	0.00	0.70	0.43
time (sec)	N/A	0.365	6.081	0.950	0.000	0.078	0.501	0.000	0.199	17.417

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	243	88	172	0	126	85	0	111	0
N.S.	1	1.56	0.56	1.10	0.00	0.81	0.54	0.00	0.71	0.00
time (sec)	N/A	0.665	6.577	3.354	0.000	0.079	1.808	0.000	0.197	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	209	142	160	0	112	0	0	59	0
N.S.	1	1.45	0.99	1.11	0.00	0.78	0.00	0.00	0.41	0.00
time (sec)	N/A	0.584	10.480	3.780	0.000	0.093	0.000	0.000	0.174	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	201	131	181	0	145	0	0	140	0
N.S.	1	1.35	0.88	1.21	0.00	0.97	0.00	0.00	0.94	0.00
time (sec)	N/A	0.590	10.598	2.385	0.000	0.085	0.000	0.000	0.201	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	138	91	93	0	83	0	0	383	0
N.S.	1	1.48	0.98	1.00	0.00	0.89	0.00	0.00	4.12	0.00
time (sec)	N/A	0.435	10.824	3.178	0.000	0.083	0.000	0.000	0.216	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	286	187	253	0	225	0	0	1023	0
N.S.	1	1.29	0.85	1.14	0.00	1.02	0.00	0.00	4.63	0.00
time (sec)	N/A	0.793	11.104	4.227	0.000	0.094	0.000	0.000	0.247	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	320	152	287	0	280	0	0	871	0
N.S.	1	1.26	0.60	1.13	0.00	1.10	0.00	0.00	3.43	0.00
time (sec)	N/A	0.852	11.362	5.306	0.000	0.098	0.000	0.000	0.254	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	359	163	321	0	336	0	0	1311	0
N.S.	1	1.25	0.57	1.12	0.00	1.17	0.00	0.00	4.57	0.00
time (sec)	N/A	0.935	11.739	6.905	0.000	0.106	0.000	0.000	0.289	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	243	141	171	0	124	172	0	111	0
N.S.	1	1.44	0.83	1.01	0.00	0.73	1.02	0.00	0.66	0.00
time (sec)	N/A	0.677	10.102	7.809	0.000	0.087	1.838	0.000	0.196	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	209	121	160	0	111	122	0	60	0
N.S.	1	1.45	0.84	1.11	0.00	0.77	0.85	0.00	0.42	0.00
time (sec)	N/A	0.594	10.079	5.464	0.000	0.084	1.424	0.000	0.190	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	87	138	0	90	78	0	26	0
N.S.	1	1.00	1.47	2.34	0.00	1.53	1.32	0.00	0.44	0.00
time (sec)	N/A	0.334	0.015	2.252	0.000	0.081	0.927	0.000	0.166	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	66	60	0	23	37	0	30	46
N.S.	1	1.00	1.12	1.02	0.00	0.39	0.63	0.00	0.51	0.78
time (sec)	N/A	0.284	10.041	0.741	0.000	0.078	0.420	0.000	0.171	17.513

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	141	95	188	0	110	0	0	47	0
N.S.	1	1.47	0.99	1.96	0.00	1.15	0.00	0.00	0.49	0.00
time (sec)	N/A	0.428	0.081	1.285	0.000	0.083	0.000	0.000	0.179	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	248	172	222	0	170	0	0	48	0
N.S.	1	1.30	0.90	1.16	0.00	0.89	0.00	0.00	0.25	0.00
time (sec)	N/A	0.678	10.791	1.730	0.000	0.086	0.000	0.000	0.194	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	292	187	256	0	225	0	0	70	0
N.S.	1	1.30	0.83	1.14	0.00	1.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.777	10.782	2.342	0.000	0.082	0.000	0.000	0.211	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	201	112	184	0	144	0	0	141	0
N.S.	1	1.37	0.76	1.25	0.00	0.98	0.00	0.00	0.96	0.00
time (sec)	N/A	0.572	10.096	6.327	0.000	0.083	0.000	0.000	0.199	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	234	71	124	0	92	0	0	36	0
N.S.	1	1.58	0.48	0.84	0.00	0.62	0.00	0.00	0.24	0.00
time (sec)	N/A	0.716	0.016	4.045	0.000	0.086	0.000	0.000	0.178	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	0	112	191	0	113	83	0	47	0
N.S.	1	0.00	0.73	1.25	0.00	0.74	0.54	0.00	0.31	0.00
time (sec)	N/A	0.000	0.021	2.300	0.000	0.082	2.329	0.000	0.169	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	64	90	0	73	39	0	40	46
N.S.	1	1.00	0.74	1.03	0.00	0.84	0.45	0.00	0.46	0.53
time (sec)	N/A	0.327	7.860	0.762	0.000	0.077	0.483	0.000	0.164	17.275

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	292	172	270	0	224	0	0	67	0
N.S.	1	1.52	0.90	1.41	0.00	1.17	0.00	0.00	0.35	0.00
time (sec)	N/A	0.779	10.479	1.341	0.000	0.079	0.000	0.000	0.205	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	326	187	304	0	238	0	0	80	0
N.S.	1	1.45	0.83	1.35	0.00	1.06	0.00	0.00	0.36	0.00
time (sec)	N/A	0.873	10.703	1.780	0.000	0.081	0.000	0.000	0.209	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	364	153	338	0	338	0	0	89	0
N.S.	1	1.41	0.59	1.31	0.00	1.31	0.00	0.00	0.34	0.00
time (sec)	N/A	0.926	10.847	2.426	0.000	0.080	0.000	0.000	0.225	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	253	155	226	0	167	0	0	48	0
N.S.	1	1.37	0.84	1.22	0.00	0.90	0.00	0.00	0.26	0.00
time (sec)	N/A	0.683	10.169	6.501	0.000	0.076	0.000	0.000	0.178	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	248	146	226	0	171	0	0	48	0
N.S.	1	1.31	0.77	1.19	0.00	0.90	0.00	0.00	0.25	0.00
time (sec)	N/A	0.664	10.106	4.194	0.000	0.081	0.000	0.000	0.179	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	0	147	271	0	226	83	0	69	0
N.S.	1	0.00	0.77	1.43	0.00	1.19	0.44	0.00	0.36	0.00
time (sec)	N/A	0.000	10.087	2.253	0.000	0.078	6.452	0.000	0.190	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	119	90	127	0	107	39	0	52	46
N.S.	1	1.07	0.81	1.14	0.00	0.96	0.35	0.00	0.47	0.41
time (sec)	N/A	0.366	10.035	0.759	0.000	0.073	0.528	0.000	0.168	17.929

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	361	163	339	0	330	0	0	91	0
N.S.	1	1.60	0.72	1.50	0.00	1.46	0.00	0.00	0.40	0.00
time (sec)	N/A	0.949	10.758	1.328	0.000	0.089	0.000	0.000	0.227	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	400	174	373	0	391	0	0	92	0
N.S.	1	1.54	0.67	1.44	0.00	1.51	0.00	0.00	0.36	0.00
time (sec)	N/A	1.076	10.950	1.770	0.000	0.087	0.000	0.000	0.232	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	439	185	407	0	440	0	0	92	0
N.S.	1	1.50	0.63	1.39	0.00	1.51	0.00	0.00	0.32	0.00
time (sec)	N/A	1.116	11.035	2.387	0.000	0.090	0.000	0.000	0.246	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	36	27	0	34	61	0	20	0
N.S.	1	1.00	9.00	6.75	0.00	8.50	15.25	0.00	5.00	0.00
time (sec)	N/A	0.243	0.003	0.687	0.000	0.070	0.694	0.000	0.180	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	34	38	88	0	31	0	0	28	0
N.S.	1	1.06	1.19	2.75	0.00	0.97	0.00	0.00	0.88	0.00
time (sec)	N/A	0.268	10.135	0.888	0.000	0.072	0.000	0.000	0.192	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	25	126	58	63	0	29	53	0	16	0
N.S.	1	5.04	2.32	2.52	0.00	1.16	2.12	0.00	0.64	0.00
time (sec)	N/A	0.362	10.046	0.867	0.000	0.069	0.655	0.000	0.177	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	121	36	93	0	32	0	0	24	0
N.S.	1	2.42	0.72	1.86	0.00	0.64	0.00	0.00	0.48	0.00
time (sec)	N/A	0.320	10.109	1.036	0.000	0.073	0.000	0.000	0.175	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	86	158	0	102	70	0	58	0
N.S.	1	1.00	1.59	2.93	0.00	1.89	1.30	0.00	1.07	0.00
time (sec)	N/A	0.357	10.055	1.298	0.000	0.085	0.891	0.000	0.190	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	85	165	0	119	76	0	63	0
N.S.	1	1.00	1.63	3.17	0.00	2.29	1.46	0.00	1.21	0.00
time (sec)	N/A	0.358	10.046	2.395	0.000	0.112	0.873	0.000	0.199	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	233	71	121	0	85	0	0	36	0
N.S.	1	3.24	0.99	1.68	0.00	1.18	0.00	0.00	0.50	0.00
time (sec)	N/A	0.758	10.071	3.968	0.000	0.077	0.000	0.000	0.200	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	233	100	121	0	85	0	0	36	0
N.S.	1	3.19	1.37	1.66	0.00	1.16	0.00	0.00	0.49	0.00
time (sec)	N/A	0.716	0.264	1.704	0.000	0.074	0.000	0.000	0.193	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	233	100	121	0	85	0	0	36	0
N.S.	1	3.24	1.39	1.68	0.00	1.18	0.00	0.00	0.50	0.00
time (sec)	N/A	0.750	10.242	2.645	0.000	0.077	0.000	0.000	0.193	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	306	143	194	0	148	184	0	159	0
N.S.	1	1.49	0.70	0.95	0.00	0.72	0.90	0.00	0.78	0.00
time (sec)	N/A	0.834	8.634	7.982	0.000	0.079	1.732	0.000	0.208	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	277	122	183	0	136	133	0	135	0
N.S.	1	1.56	0.69	1.03	0.00	0.76	0.75	0.00	0.76	0.00
time (sec)	N/A	0.766	7.657	5.727	0.000	0.073	1.461	0.000	0.201	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	0	88	172	0	126	85	0	111	0
N.S.	1	0.00	0.56	1.10	0.00	0.81	0.54	0.00	0.71	0.00
time (sec)	N/A	0.000	0.014	3.969	0.000	0.076	1.035	0.000	0.197	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	86	82	0	54	39	0	52	46
N.S.	1	1.00	1.02	0.98	0.00	0.64	0.46	0.00	0.62	0.55
time (sec)	N/A	0.324	0.158	0.870	0.000	0.069	0.451	0.000	0.176	0.002

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	78	67	138	0	90	0	0	26	0
N.S.	1	1.32	1.14	2.34	0.00	1.53	0.00	0.00	0.44	0.00
time (sec)	N/A	0.351	0.045	1.215	0.000	0.077	0.000	0.000	0.180	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	234	134	124	0	92	0	0	36	0
N.S.	1	1.58	0.91	0.84	0.00	0.62	0.00	0.00	0.24	0.00
time (sec)	N/A	0.723	0.104	1.699	0.000	0.075	0.000	0.000	0.177	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	253	175	226	0	167	0	0	48	0
N.S.	1	1.31	0.91	1.17	0.00	0.87	0.00	0.00	0.25	0.00
time (sec)	N/A	0.679	10.579	2.278	0.000	0.075	0.000	0.000	0.190	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	287	190	261	0	228	0	0	58	0
N.S.	1	1.26	0.84	1.15	0.00	1.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.778	10.779	2.976	0.000	0.079	0.000	0.000	0.265	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	331	151	296	0	281	0	0	70	0
N.S.	1	1.27	0.58	1.13	0.00	1.08	0.00	0.00	0.27	0.00
time (sec)	N/A	0.871	10.869	3.896	0.000	0.081	0.000	0.000	0.272	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	374	135	216	0	169	279	0	207	0
N.S.	1	1.56	0.56	0.90	0.00	0.71	1.17	0.00	0.87	0.00
time (sec)	N/A	1.031	10.159	7.985	0.000	0.078	2.567	0.000	0.290	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	341	126	205	0	159	180	0	183	0
N.S.	1	1.61	0.59	0.97	0.00	0.75	0.85	0.00	0.86	0.00
time (sec)	N/A	0.913	9.641	5.754	0.000	0.077	1.790	0.000	0.283	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	0	91	194	0	147	180	0	159	0
N.S.	1	0.00	0.48	1.02	0.00	0.77	0.95	0.00	0.84	0.00
time (sec)	N/A	0.000	8.500	4.009	0.000	0.076	1.763	0.000	0.268	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	113	58	96	0	70	41	0	76	46
N.S.	1	1.05	0.54	0.89	0.00	0.65	0.38	0.00	0.70	0.43
time (sec)	N/A	0.374	0.004	0.885	0.000	0.073	0.489	0.000	0.244	0.002

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	243	87	172	0	126	85	0	111	0
N.S.	1	1.56	0.56	1.10	0.00	0.81	0.54	0.00	0.71	0.00
time (sec)	N/A	0.667	0.018	2.626	0.000	0.074	2.116	0.000	0.266	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	209	142	160	0	111	0	0	60	0
N.S.	1	1.45	0.99	1.11	0.00	0.77	0.00	0.00	0.42	0.00
time (sec)	N/A	0.589	10.466	3.083	0.000	0.081	0.000	0.000	0.245	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	201	130	184	0	144	0	0	141	0
N.S.	1	1.34	0.87	1.23	0.00	0.96	0.00	0.00	0.94	0.00
time (sec)	N/A	0.567	10.489	2.272	0.000	0.078	0.000	0.000	0.266	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	138	91	94	0	83	0	0	383	0
N.S.	1	1.47	0.97	1.00	0.00	0.88	0.00	0.00	4.07	0.00
time (sec)	N/A	0.434	10.587	3.072	0.000	0.073	0.000	0.000	0.299	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	286	190	258	0	228	0	0	1012	0
N.S.	1	1.28	0.85	1.15	0.00	1.02	0.00	0.00	4.52	0.00
time (sec)	N/A	0.755	10.933	4.102	0.000	0.078	0.000	0.000	0.245	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	320	151	293	0	281	0	0	871	0
N.S.	1	1.24	0.59	1.14	0.00	1.09	0.00	0.00	3.38	0.00
time (sec)	N/A	0.845	11.047	5.108	0.000	0.079	0.000	0.000	0.247	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	359	161	328	0	338	0	0	1299	0
N.S.	1	1.23	0.55	1.12	0.00	1.16	0.00	0.00	4.45	0.00
time (sec)	N/A	0.948	11.064	6.680	0.000	0.082	0.000	0.000	0.252	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	243	141	172	0	124	172	0	111	0
N.S.	1	1.44	0.83	1.02	0.00	0.73	1.02	0.00	0.66	0.00
time (sec)	N/A	0.677	10.104	7.925	0.000	0.078	1.944	0.000	0.197	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	209	121	160	0	112	122	0	59	0
N.S.	1	1.45	0.84	1.11	0.00	0.78	0.85	0.00	0.41	0.00
time (sec)	N/A	0.590	10.079	5.694	0.000	0.077	1.480	0.000	0.193	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	90	88	137	0	91	78	0	25	0
N.S.	1	0.74	0.73	1.13	0.00	0.75	0.64	0.00	0.21	0.00
time (sec)	N/A	0.439	0.016	2.171	0.000	0.073	0.920	0.000	0.194	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	66	60	0	23	37	0	30	46
N.S.	1	1.00	1.12	1.02	0.00	0.39	0.63	0.00	0.51	0.78
time (sec)	N/A	0.283	0.027	0.688	0.000	0.068	0.422	0.000	0.194	0.002

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	214	135	191	0	113	0	0	47	0
N.S.	1	1.37	0.87	1.22	0.00	0.72	0.00	0.00	0.30	0.00
time (sec)	N/A	0.587	0.109	1.232	0.000	0.079	0.000	0.000	0.176	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	248	174	226	0	171	0	0	48	0
N.S.	1	1.28	0.90	1.17	0.00	0.89	0.00	0.00	0.25	0.00
time (sec)	N/A	0.683	10.513	1.633	0.000	0.078	0.000	0.000	0.191	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	292	190	261	0	228	0	0	69	0
N.S.	1	1.29	0.84	1.15	0.00	1.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.753	10.711	2.250	0.000	0.080	0.000	0.000	0.207	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	235	129	203	0	164	0	0	274	0
N.S.	1	1.37	0.75	1.18	0.00	0.95	0.00	0.00	1.59	0.00
time (sec)	N/A	0.656	10.120	17.716	0.000	0.083	0.000	0.000	0.225	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	201	111	181	0	145	0	0	140	0
N.S.	1	1.36	0.75	1.22	0.00	0.98	0.00	0.00	0.95	0.00
time (sec)	N/A	0.586	10.089	6.016	0.000	0.081	0.000	0.000	0.218	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	233	71	121	0	85	0	0	36	0
N.S.	1	3.28	1.00	1.70	0.00	1.20	0.00	0.00	0.51	0.00
time (sec)	N/A	0.732	0.019	3.829	0.000	0.078	0.000	0.000	0.193	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	0	112	188	0	110	83	0	47	0
N.S.	1	0.00	1.18	1.98	0.00	1.16	0.87	0.00	0.49	0.00
time (sec)	N/A	0.000	0.025	2.138	0.000	0.077	2.715	0.000	0.186	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	64	90	0	73	39	0	40	46
N.S.	1	1.00	0.74	1.03	0.00	0.84	0.45	0.00	0.46	0.53
time (sec)	N/A	0.329	0.012	0.716	0.000	0.077	0.483	0.000	0.178	0.002

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	287	175	271	0	226	0	0	69	0
N.S.	1	1.49	0.91	1.40	0.00	1.17	0.00	0.00	0.36	0.00
time (sec)	N/A	0.816	10.422	1.236	0.000	0.077	0.000	0.000	0.200	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	327	190	306	0	239	0	0	80	0
N.S.	1	1.44	0.84	1.35	0.00	1.05	0.00	0.00	0.35	0.00
time (sec)	N/A	0.835	10.656	1.737	0.000	0.088	0.000	0.000	0.199	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	361	155	341	0	336	0	0	91	0
N.S.	1	1.38	0.59	1.31	0.00	1.29	0.00	0.00	0.35	0.00
time (sec)	N/A	0.932	10.726	2.326	0.000	0.085	0.000	0.000	0.202	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	253	155	222	0	166	0	0	47	0
N.S.	1	1.36	0.83	1.19	0.00	0.89	0.00	0.00	0.25	0.00
time (sec)	N/A	0.698	10.155	6.142	0.000	0.076	0.000	0.000	0.207	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	248	172	222	0	170	0	0	48	0
N.S.	1	1.30	0.90	1.16	0.00	0.89	0.00	0.00	0.25	0.00
time (sec)	N/A	0.693	0.280	3.976	0.000	0.077	0.000	0.000	0.196	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	0	147	270	0	224	83	0	67	0
N.S.	1	0.00	0.77	1.41	0.00	1.17	0.43	0.00	0.35	0.00
time (sec)	N/A	0.000	10.091	2.189	0.000	0.082	4.616	0.000	0.205	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	119	90	127	0	107	39	0	52	46
N.S.	1	1.07	0.81	1.14	0.00	0.96	0.35	0.00	0.47	0.41
time (sec)	N/A	0.368	0.027	0.726	0.000	0.077	0.506	0.000	0.174	0.002

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	366	163	340	0	335	0	0	91	0
N.S.	1	1.61	0.72	1.50	0.00	1.48	0.00	0.00	0.40	0.00
time (sec)	N/A	0.950	10.663	1.302	0.000	0.091	0.000	0.000	0.213	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	400	235	375	0	391	0	0	92	0
N.S.	1	1.53	0.90	1.44	0.00	1.50	0.00	0.00	0.35	0.00
time (sec)	N/A	1.023	10.669	1.769	0.000	0.093	0.000	0.000	0.206	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	439	185	410	0	445	0	0	91	0
N.S.	1	1.49	0.63	1.39	0.00	1.51	0.00	0.00	0.31	0.00
time (sec)	N/A	1.095	10.967	2.358	0.000	0.091	0.000	0.000	0.208	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	36	27	0	23	61	0	18	0
N.S.	1	1.00	2.77	2.08	0.00	1.77	4.69	0.00	1.38	0.00
time (sec)	N/A	0.272	0.003	0.889	0.000	0.074	0.707	0.000	0.169	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	43	45	88	0	42	0	0	28	0
N.S.	1	1.13	1.18	2.32	0.00	1.11	0.00	0.00	0.74	0.00
time (sec)	N/A	0.316	10.204	1.556	0.000	0.090	0.000	0.000	0.177	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	53	72	58	63	0	21	51	0	18	0
N.S.	1	1.36	1.09	1.19	0.00	0.40	0.96	0.00	0.34	0.00
time (sec)	N/A	0.276	10.047	1.076	0.000	0.066	0.670	0.000	0.183	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	56	52	93	0	44	0	0	28	0
N.S.	1	0.73	0.68	1.21	0.00	0.57	0.00	0.00	0.36	0.00
time (sec)	N/A	0.273	10.150	1.855	0.000	0.076	0.000	0.000	0.182	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	173	119	138	0	298	0	83	93	0
N.S.	1	0.88	0.61	0.70	0.00	1.52	0.00	0.42	0.47	0.00
time (sec)	N/A	0.476	3.327	0.208	0.000	0.086	0.000	0.116	0.167	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	141	108	117	0	276	0	69	73	0
N.S.	1	0.90	0.69	0.75	0.00	1.77	0.00	0.44	0.47	0.00
time (sec)	N/A	0.413	2.891	0.197	0.000	0.086	0.000	0.130	0.178	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	111	97	96	0	254	0	55	53	0
N.S.	1	0.96	0.84	0.83	0.00	2.19	0.00	0.47	0.46	0.00
time (sec)	N/A	0.357	1.665	0.192	0.000	0.079	0.000	0.159	0.172	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	88	86	73	0	225	0	43	33	0
N.S.	1	1.13	1.10	0.94	0.00	2.88	0.00	0.55	0.42	0.00
time (sec)	N/A	0.330	1.345	0.187	0.000	0.085	0.000	0.112	0.186	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	104	122	180	0	279	0	0	44	0
N.S.	1	1.11	1.30	1.91	0.00	2.97	0.00	0.00	0.47	0.00
time (sec)	N/A	0.397	2.505	0.426	0.000	0.087	0.000	0.000	0.210	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	112	110	308	0	294	0	0	242	0
N.S.	1	1.20	1.18	3.31	0.00	3.16	0.00	0.00	2.60	0.00
time (sec)	N/A	0.369	2.837	0.459	0.000	0.088	0.000	0.000	0.212	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	149	123	452	0	366	0	0	452	0
N.S.	1	1.16	0.96	3.53	0.00	2.86	0.00	0.00	3.53	0.00
time (sec)	N/A	0.415	3.098	0.458	0.000	0.088	0.000	0.000	0.231	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	228	141	180	0	341	0	111	133	0
N.S.	1	0.83	0.51	0.65	0.00	1.24	0.00	0.40	0.48	0.00
time (sec)	N/A	0.570	5.081	0.246	0.000	0.087	0.000	0.148	0.175	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	196	130	159	0	319	0	97	113	0
N.S.	1	0.83	0.55	0.67	0.00	1.35	0.00	0.41	0.48	0.00
time (sec)	N/A	0.503	4.599	0.200	0.000	0.142	0.000	0.156	0.178	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	164	119	138	0	297	0	83	93	0
N.S.	1	0.84	0.61	0.70	0.00	1.52	0.00	0.42	0.47	0.00
time (sec)	N/A	0.433	3.015	0.198	0.000	0.083	0.000	0.107	0.183	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	134	108	117	0	275	0	69	143	0
N.S.	1	0.86	0.69	0.75	0.00	1.76	0.00	0.44	0.92	0.00
time (sec)	N/A	0.417	2.688	0.197	0.000	0.084	0.000	0.137	0.261	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	111	97	96	0	253	0	0	113	0
N.S.	1	0.98	0.86	0.85	0.00	2.24	0.00	0.00	1.00	0.00
time (sec)	N/A	0.356	4.119	0.198	0.000	0.084	0.000	0.000	0.243	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	128	163	204	0	401	0	165	289	0
N.S.	1	0.98	1.24	1.56	0.00	3.06	0.00	1.26	2.21	0.00
time (sec)	N/A	0.450	4.770	0.481	0.000	0.090	0.000	0.224	0.291	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	130	156	373	0	442	0	0	442	0
N.S.	1	1.06	1.27	3.03	0.00	3.59	0.00	0.00	3.59	0.00
time (sec)	N/A	0.464	4.864	0.464	0.000	0.107	0.000	0.000	0.333	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	146	122	452	0	360	0	0	451	0
N.S.	1	1.17	0.98	3.62	0.00	2.88	0.00	0.00	3.61	0.00
time (sec)	N/A	0.414	5.013	0.471	0.000	0.092	0.000	0.000	0.274	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	183	134	596	0	432	0	0	587	0
N.S.	1	1.14	0.84	3.72	0.00	2.70	0.00	0.00	3.67	0.00
time (sec)	N/A	0.454	4.918	0.498	0.000	0.094	0.000	0.000	0.295	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	217	145	740	0	498	0	0	854	0
N.S.	1	1.11	0.74	3.79	0.00	2.55	0.00	0.00	4.38	0.00
time (sec)	N/A	0.586	4.991	0.515	0.000	0.102	0.000	0.000	0.326	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	254	139	884	0	564	0	0	1011	0
N.S.	1	1.10	0.60	3.84	0.00	2.45	0.00	0.00	4.40	0.00
time (sec)	N/A	0.659	5.394	0.549	0.000	0.106	0.000	0.000	0.324	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	33	16	15	25	15	25	21	15
N.S.	1	1.00	1.57	0.76	0.71	1.19	0.71	1.19	1.00	0.71
time (sec)	N/A	0.230	0.033	0.225	0.104	0.067	0.070	0.133	0.172	0.034

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	70	40	0	120	0	25	21	0
N.S.	1	1.00	3.33	1.90	0.00	5.71	0.00	1.19	1.00	0.00
time (sec)	N/A	0.256	0.913	0.168	0.000	0.075	0.000	0.162	200.018	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	150	109	117	0	275	0	0	73	0
N.S.	1	0.96	0.70	0.75	0.00	1.76	0.00	0.00	0.47	0.00
time (sec)	N/A	0.449	3.391	0.221	0.000	0.085	0.000	0.000	0.182	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	118	98	96	0	253	0	0	53	0
N.S.	1	1.02	0.84	0.83	0.00	2.18	0.00	0.00	0.46	0.00
time (sec)	N/A	0.380	3.018	0.204	0.000	0.082	0.000	0.000	0.188	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	88	86	75	0	225	0	0	33	0
N.S.	1	1.13	1.10	0.96	0.00	2.88	0.00	0.00	0.42	0.00
time (sec)	N/A	0.332	2.549	0.200	0.000	0.086	0.000	0.000	0.212	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	65	50	54	0	123	0	0	15	0
N.S.	1	1.59	1.22	1.32	0.00	3.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.294	1.328	0.184	0.000	0.081	0.000	0.000	0.201	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	78	78	249	0	153	0	0	81	0
N.S.	1	1.44	1.44	4.61	0.00	2.83	0.00	0.00	1.50	0.00
time (sec)	N/A	0.318	1.632	0.814	0.000	0.076	0.000	0.000	0.237	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	112	111	488	0	298	0	0	208	0
N.S.	1	1.20	1.19	5.25	0.00	3.20	0.00	0.00	2.24	0.00
time (sec)	N/A	0.361	2.669	0.820	0.000	0.079	0.000	0.000	0.239	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	149	123	711	0	366	0	0	345	0
N.S.	1	1.16	0.96	5.55	0.00	2.86	0.00	0.00	2.70	0.00
time (sec)	N/A	0.447	3.051	0.835	0.000	0.087	0.000	0.000	0.216	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	140	143	96	0	296	0	62	137	0
N.S.	1	0.92	0.93	0.63	0.00	1.93	0.00	0.41	0.90	0.00
time (sec)	N/A	0.445	5.380	0.249	0.000	0.083	0.000	0.119	0.201	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	108	127	83	0	267	0	46	114	0
N.S.	1	0.96	1.13	0.74	0.00	2.38	0.00	0.41	1.02	0.00
time (sec)	N/A	0.396	5.053	0.236	0.000	0.084	0.000	0.137	0.214	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	81	112	72	0	243	0	35	90	0
N.S.	1	1.08	1.49	0.96	0.00	3.24	0.00	0.47	1.20	0.00
time (sec)	N/A	0.327	4.887	0.217	0.000	0.086	0.000	0.128	0.191	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	42	37	0	43	0	14	39	29
N.S.	1	1.00	1.27	1.12	0.00	1.30	0.00	0.42	1.18	0.88
time (sec)	N/A	0.278	4.275	0.203	0.000	0.071	0.000	0.116	0.207	17.969

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	101	108	343	0	272	0	0	327	0
N.S.	1	1.06	1.14	3.61	0.00	2.86	0.00	0.00	3.44	0.00
time (sec)	N/A	0.360	3.055	0.800	0.000	0.084	0.000	0.000	0.227	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	138	123	1068	0	368	0	0	373	0
N.S.	1	1.05	0.94	8.15	0.00	2.81	0.00	0.00	2.85	0.00
time (sec)	N/A	0.435	2.964	0.858	0.000	0.086	0.000	0.000	0.226	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	175	140	981	0	398	0	0	755	0
N.S.	1	1.05	0.84	5.87	0.00	2.38	0.00	0.00	4.52	0.00
time (sec)	N/A	0.518	4.811	0.881	0.000	0.089	0.000	0.000	0.217	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	173	155	141	0	394	0	76	212	0
N.S.	1	0.91	0.81	0.74	0.00	2.06	0.00	0.40	1.11	0.00
time (sec)	N/A	0.511	5.763	0.289	0.000	0.094	0.000	0.209	0.217	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	139	140	128	0	367	0	62	189	0
N.S.	1	0.93	0.93	0.85	0.00	2.45	0.00	0.41	1.26	0.00
time (sec)	N/A	0.437	5.634	0.274	0.000	0.090	0.000	0.194	0.199	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	117	107	0	324	0	39	151	0
N.S.	1	1.00	1.06	0.97	0.00	2.95	0.00	0.35	1.37	0.00
time (sec)	N/A	0.379	5.394	0.226	0.000	0.084	0.000	0.164	0.210	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	87	54	47	0	71	0	0	83	86
N.S.	1	1.24	0.77	0.67	0.00	1.01	0.00	0.00	1.19	1.23
time (sec)	N/A	0.329	5.162	0.222	0.000	0.069	0.000	0.000	0.196	17.330

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	79	55	48	0	74	0	0	84	89
N.S.	1	1.08	0.75	0.66	0.00	1.01	0.00	0.00	1.15	1.22
time (sec)	N/A	0.306	5.027	0.205	0.000	0.069	0.000	0.000	0.208	17.373

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	128	121	606	0	372	0	0	542	0
N.S.	1	0.98	0.92	4.63	0.00	2.84	0.00	0.00	4.14	0.00
time (sec)	N/A	0.428	5.170	0.569	0.000	0.087	0.000	0.000	0.212	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	165	140	745	0	356	0	0	755	0
N.S.	1	0.99	0.84	4.46	0.00	2.13	0.00	0.00	4.52	0.00
time (sec)	N/A	0.482	3.647	0.842	0.000	0.092	0.000	0.000	0.221	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	202	148	1660	0	504	0	0	604	0
N.S.	1	1.00	0.73	8.18	0.00	2.48	0.00	0.00	2.98	0.00
time (sec)	N/A	0.562	3.650	0.891	0.000	0.092	0.000	0.000	0.235	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	164	1325	0	568	0	0	1181	0
N.S.	1	1.00	0.69	5.54	0.00	2.38	0.00	0.00	4.94	0.00
time (sec)	N/A	0.637	5.429	0.524	0.000	0.100	0.000	0.000	0.274	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	32	29	0	28	0	0	2	0
N.S.	1	1.00	16.00	14.50	0.00	14.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.231	0.781	0.103	0.000	0.070	0.000	0.000	0.207	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	42	29	0	81	0	0	9	0
N.S.	1	1.00	21.00	14.50	0.00	40.50	0.00	0.00	4.50	0.00
time (sec)	N/A	0.233	0.771	0.102	0.000	0.072	0.000	0.000	0.238	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	46	32	39	0	28	0	0	5	0
N.S.	1	2.00	1.39	1.70	0.00	1.22	0.00	0.00	0.22	0.00
time (sec)	N/A	0.275	0.757	0.116	0.000	0.067	0.000	0.000	0.222	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	44	38	37	0	73	0	0	4	0
N.S.	1	1.76	1.52	1.48	0.00	2.92	0.00	0.00	0.16	0.00
time (sec)	N/A	0.281	0.760	0.105	0.000	0.065	0.000	0.000	0.220	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	40	34	33	0	65	0	0	9	0
N.S.	1	1.90	1.62	1.57	0.00	3.10	0.00	0.00	0.43	0.00
time (sec)	N/A	0.268	0.696	0.109	0.000	0.069	0.000	0.000	0.215	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	46	23	39	0	19	0	0	5	0
N.S.	1	2.00	1.00	1.70	0.00	0.83	0.00	0.00	0.22	0.00
time (sec)	N/A	0.279	0.723	0.112	0.000	0.066	0.000	0.000	0.207	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	30	38	25	0	73	0	0	9	0
N.S.	1	1.43	1.81	1.19	0.00	3.48	0.00	0.00	0.43	0.00
time (sec)	N/A	0.265	0.686	0.107	0.000	0.066	0.000	0.000	0.219	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	34	23	29	0	75	0	0	11	0
N.S.	1	1.48	1.00	1.26	0.00	3.26	0.00	0.00	0.48	0.00
time (sec)	N/A	0.283	0.730	0.441	0.000	0.069	0.000	0.000	0.219	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	72	71	59	0	137	0	0	21	0
N.S.	1	1.60	1.58	1.31	0.00	3.04	0.00	0.00	0.47	0.00
time (sec)	N/A	0.522	4.671	0.185	0.000	0.069	0.000	0.000	0.185	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	24	0	0	26	0	93	34
N.S.	1	1.00	1.00	1.09	0.00	0.00	1.18	0.00	4.23	1.55
time (sec)	N/A	0.229	0.081	0.146	0.000	0.000	2.629	0.000	0.187	17.102

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	0	0	0	0	0	0	23	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.05	0.00
time (sec)	N/A	0.264	0.000	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	177	0	0	0	0	0	551	0
N.S.	1	1.00	4.21	0.00	0.00	0.00	0.00	0.00	13.12	0.00
time (sec)	N/A	0.265	0.238	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	0	0	0	0	0	551	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	13.12	0.00
time (sec)	N/A	0.278	0.000	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	40	31	0	46	58	0	38	0
N.S.	1	1.00	0.67	0.52	0.00	0.77	0.97	0.00	0.63	0.00
time (sec)	N/A	0.281	10.035	0.431	0.000	0.074	0.640	0.000	0.189	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	49	40	0	77	68	0	57	0
N.S.	1	1.00	0.55	0.45	0.00	0.87	0.76	0.00	0.64	0.00
time (sec)	N/A	0.318	10.030	0.903	0.000	0.079	0.811	0.000	0.192	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	88	153	0	95	75	0	65	0
N.S.	1	1.00	0.84	1.46	0.00	0.90	0.71	0.00	0.62	0.00
time (sec)	N/A	0.334	10.052	1.009	0.000	0.074	0.885	0.000	0.200	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	40	31	0	46	58	0	38	0
N.S.	1	1.00	0.67	0.52	0.00	0.77	0.97	0.00	0.63	0.00
time (sec)	N/A	0.282	0.001	0.370	0.000	0.078	0.654	0.000	0.183	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	24	51	0	56	0	0	48	0
N.S.	1	1.00	0.34	0.73	0.00	0.80	0.00	0.00	0.69	0.00
time (sec)	N/A	0.423	10.147	0.866	0.000	0.111	0.000	0.000	0.210	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	34	0	48	61	0	40	0
N.S.	1	1.00	0.98	0.54	0.00	0.76	0.97	0.00	0.63	0.00
time (sec)	N/A	0.285	10.044	0.568	0.000	0.073	0.686	0.000	0.188	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	44	52	0	77	0	0	49	0
N.S.	1	1.00	0.59	0.70	0.00	1.04	0.00	0.00	0.66	0.00
time (sec)	N/A	0.443	10.145	1.046	0.000	0.108	0.000	0.000	0.191	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	40	31	0	28	58	0	36	0
N.S.	1	1.00	0.40	0.31	0.00	0.28	0.58	0.00	0.36	0.00
time (sec)	N/A	0.353	10.032	0.296	0.000	0.080	0.635	0.000	0.184	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	49	40	0	77	68	0	56	0
N.S.	1	1.00	0.33	0.27	0.00	0.51	0.45	0.00	0.37	0.00
time (sec)	N/A	0.411	10.027	0.688	0.000	0.074	0.797	0.000	0.184	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	87	153	0	96	75	0	64	0
N.S.	1	1.00	0.48	0.84	0.00	0.53	0.41	0.00	0.35	0.00
time (sec)	N/A	0.450	10.059	0.997	0.000	0.081	0.869	0.000	0.202	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	40	31	0	28	58	0	36	0
N.S.	1	1.00	0.40	0.31	0.00	0.28	0.58	0.00	0.36	0.00
time (sec)	N/A	0.342	0.001	0.275	0.000	0.079	0.640	0.000	0.186	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	24	51	0	32	0	0	22	0
N.S.	1	1.00	0.34	0.73	0.00	0.46	0.00	0.00	0.31	0.00
time (sec)	N/A	0.410	10.098	0.362	0.000	0.106	0.000	0.000	0.195	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	62	34	0	31	63	0	39	0
N.S.	1	1.00	0.58	0.32	0.00	0.29	0.59	0.00	0.36	0.00
time (sec)	N/A	0.367	10.039	0.402	0.000	0.063	0.693	0.000	0.186	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	44	107	0	75	0	0	25	0
N.S.	1	1.00	0.59	1.45	0.00	1.01	0.00	0.00	0.34	0.00
time (sec)	N/A	0.426	10.098	0.440	0.000	0.097	0.000	0.000	0.194	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	152	90	0	0	0	56	0	69	0
N.S.	1	1.35	0.80	0.00	0.00	0.00	0.50	0.00	0.61	0.00
time (sec)	N/A	0.451	10.181	0.000	0.000	0.000	1.910	0.000	0.203	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	230	183	34	221	767	109	241	239	599
N.S.	1	1.28	1.02	0.19	1.23	4.26	0.61	1.34	1.33	3.33
time (sec)	N/A	0.739	0.088	0.105	0.111	0.080	0.331	0.116	0.185	17.225

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	230	184	35	221	767	110	241	239	603
N.S.	1	1.27	1.02	0.19	1.22	4.24	0.61	1.33	1.32	3.33
time (sec)	N/A	0.668	0.054	0.101	0.116	0.087	0.341	0.129	0.187	17.353

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	95	36	109	755	110	230	101	579
N.S.	1	1.00	1.12	0.42	1.28	8.88	1.29	2.71	1.19	6.81
time (sec)	N/A	0.341	0.033	0.102	0.110	0.085	0.339	0.116	0.199	17.405

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	95	37	109	755	110	228	101	579
N.S.	1	1.00	1.12	0.44	1.28	8.88	1.29	2.68	1.19	6.81
time (sec)	N/A	0.341	0.026	0.100	0.107	0.086	0.340	0.113	0.192	0.260

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	33	35	39	33	41	52	31	29
N.S.	1	1.00	0.82	0.88	0.98	0.82	1.02	1.30	0.78	0.72
time (sec)	N/A	0.291	0.018	0.103	0.103	0.064	0.052	0.108	0.188	0.094

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	51	44	40	39	42	49	40	33	21
N.S.	1	1.89	1.63	1.48	1.44	1.56	1.81	1.48	1.22	0.78
time (sec)	N/A	0.320	0.016	0.095	0.103	0.062	0.046	0.113	0.184	17.165

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	32	13	25	29	32	29	23	12
N.S.	1	1.00	2.00	0.81	1.56	1.81	2.00	1.81	1.44	0.75
time (sec)	N/A	0.241	0.018	0.093	0.105	0.064	0.041	0.131	0.180	0.108

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	12	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	0.75	0.75	0.75
time (sec)	N/A	0.233	0.007	0.092	0.104	0.059	0.044	0.110	0.185	17.122

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	204	100	148	138	0	67	57
N.S.	1	1.00	0.80	2.72	1.33	1.97	1.84	0.00	0.89	0.76
time (sec)	N/A	0.341	0.025	0.200	0.112	0.095	0.175	0.000	0.192	17.667

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	106	91	204	70	151	131	0	54	43
N.S.	1	2.08	1.78	4.00	1.37	2.96	2.57	0.00	1.06	0.84
time (sec)	N/A	0.402	0.026	0.209	0.107	0.092	0.180	0.000	0.178	17.672

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	83	302	137	87	92	69	57
N.S.	1	1.00	0.80	1.11	4.03	1.83	1.16	1.23	0.92	0.76
time (sec)	N/A	0.335	0.040	0.129	0.111	0.069	0.093	0.116	0.185	17.144

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	90	75	75	302	140	80	100	50	41
N.S.	1	2.09	1.74	1.74	7.02	3.26	1.86	2.33	1.16	0.95
time (sec)	N/A	0.393	0.025	0.119	0.109	0.066	0.094	0.129	0.185	0.095

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	18	17	17	22	19	17	9
N.S.	1	1.00	1.92	1.38	1.31	1.31	1.69	1.46	1.31	0.69
time (sec)	N/A	0.240	0.010	0.112	0.113	0.069	0.068	0.108	0.182	0.042

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	225	80	224	0	162	272	0	164	0
N.S.	1	1.04	0.37	1.04	0.00	0.75	1.26	0.00	0.76	0.00
time (sec)	N/A	0.801	10.054	1.111	0.000	0.077	2.553	0.000	0.251	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	192	78	200	0	135	177	0	122	0
N.S.	1	1.03	0.42	1.08	0.00	0.73	0.95	0.00	0.66	0.00
time (sec)	N/A	0.678	10.047	1.092	0.000	0.077	1.687	0.000	0.224	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	159	77	182	0	109	85	0	84	0
N.S.	1	1.01	0.49	1.15	0.00	0.69	0.54	0.00	0.53	0.00
time (sec)	N/A	0.575	7.917	1.076	0.000	0.071	1.000	0.000	0.217	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	77	154	0	91	82	0	52	0
N.S.	1	1.00	0.63	1.25	0.00	0.74	0.67	0.00	0.42	0.00
time (sec)	N/A	0.490	10.046	0.698	0.000	0.071	0.853	0.000	0.199	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	158	100	199	0	120	82	0	72	0
N.S.	1	1.01	0.64	1.28	0.00	0.77	0.53	0.00	0.46	0.00
time (sec)	N/A	0.577	10.066	0.723	0.000	0.074	2.977	0.000	0.220	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	199	125	245	0	192	82	0	96	0
N.S.	1	1.06	0.66	1.30	0.00	1.02	0.44	0.00	0.51	0.00
time (sec)	N/A	0.689	10.130	0.725	0.000	0.081	9.889	0.000	0.236	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	135	45	34	0	96	231	0	128	0
N.S.	1	1.08	0.36	0.27	0.00	0.77	1.85	0.00	1.02	0.00
time (sec)	N/A	0.578	8.741	1.273	0.000	0.078	2.124	0.000	0.201	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	104	45	34	0	85	153	0	100	0
N.S.	1	1.05	0.45	0.34	0.00	0.86	1.55	0.00	1.01	0.00
time (sec)	N/A	0.455	7.324	1.232	0.000	0.078	1.460	0.000	0.201	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	44	33	0	72	75	0	72	0
N.S.	1	1.00	0.60	0.45	0.00	0.99	1.03	0.00	0.99	0.00
time (sec)	N/A	0.389	5.608	1.217	0.000	0.077	0.846	0.000	0.191	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	49	34	0	56	68	0	46	0
N.S.	1	1.00	1.26	0.87	0.00	1.44	1.74	0.00	1.18	0.00
time (sec)	N/A	0.315	10.025	0.763	0.000	0.074	0.721	0.000	0.185	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	34	0	80	68	0	54	0
N.S.	1	1.00	0.93	0.50	0.00	1.18	1.00	0.00	0.79	0.00
time (sec)	N/A	0.378	10.040	1.422	0.000	0.075	2.249	0.000	0.193	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	101	70	34	0	122	68	0	66	0
N.S.	1	1.05	0.73	0.35	0.00	1.27	0.71	0.00	0.69	0.00
time (sec)	N/A	0.452	10.124	1.418	0.000	0.074	3.133	0.000	0.195	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	226	45	36	0	125	236	0	166	0
N.S.	1	1.88	0.38	0.30	0.00	1.04	1.97	0.00	1.38	0.00
time (sec)	N/A	0.707	9.284	1.682	0.000	0.074	2.506	0.000	0.204	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	160	45	36	0	110	155	0	128	0
N.S.	1	1.72	0.48	0.39	0.00	1.18	1.67	0.00	1.38	0.00
time (sec)	N/A	0.558	7.528	1.646	0.000	0.084	1.663	0.000	0.209	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	101	45	36	0	93	73	0	90	0
N.S.	1	1.53	0.68	0.55	0.00	1.41	1.11	0.00	1.36	0.00
time (sec)	N/A	0.429	6.616	1.589	0.000	0.076	0.968	0.000	0.200	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	45	36	0	71	70	0	25	0
N.S.	1	1.00	2.81	2.25	0.00	4.44	4.38	0.00	1.56	0.00
time (sec)	N/A	0.253	10.031	0.951	0.000	0.068	0.810	0.000	0.194	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	68	64	36	0	81	70	0	39	0
N.S.	1	1.10	1.03	0.58	0.00	1.31	1.13	0.00	0.63	0.00
time (sec)	N/A	0.364	10.045	1.023	0.000	0.071	2.070	0.000	0.186	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	127	73	36	0	165	70	0	56	0
N.S.	1	1.38	0.79	0.39	0.00	1.79	0.76	0.00	0.61	0.00
time (sec)	N/A	0.502	10.086	0.951	0.000	0.072	5.281	0.000	0.206	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	193	81	36	0	246	70	0	71	0
N.S.	1	1.62	0.68	0.30	0.00	2.07	0.59	0.00	0.60	0.00
time (sec)	N/A	0.626	10.095	0.932	0.000	0.076	12.944	0.000	0.200	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	226	45	36	0	126	236	0	166	0
N.S.	1	1.88	0.38	0.30	0.00	1.05	1.97	0.00	1.38	0.00
time (sec)	N/A	0.706	9.230	1.862	0.000	0.078	2.466	0.000	0.219	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	160	45	36	0	109	155	0	128	0
N.S.	1	1.72	0.48	0.39	0.00	1.17	1.67	0.00	1.38	0.00
time (sec)	N/A	0.566	7.751	1.882	0.000	0.074	1.662	0.000	0.201	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	101	45	36	0	94	73	0	90	0
N.S.	1	1.53	0.68	0.55	0.00	1.42	1.11	0.00	1.36	0.00
time (sec)	N/A	0.429	6.128	1.824	0.000	0.066	0.961	0.000	0.192	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	45	36	0	69	70	0	23	0
N.S.	1	1.00	1.29	1.03	0.00	1.97	2.00	0.00	0.66	0.00
time (sec)	N/A	0.310	10.031	1.184	0.000	0.064	0.832	0.000	0.193	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	48	64	36	0	77	70	0	40	0
N.S.	1	1.02	1.36	0.77	0.00	1.64	1.49	0.00	0.85	0.00
time (sec)	N/A	0.302	10.039	1.174	0.000	0.075	2.339	0.000	0.189	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	134	73	36	0	166	70	0	53	0
N.S.	1	1.44	0.78	0.39	0.00	1.78	0.75	0.00	0.57	0.00
time (sec)	N/A	0.504	10.073	1.158	0.000	0.072	3.996	0.000	0.219	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	186	81	36	0	239	70	0	72	0
N.S.	1	1.55	0.68	0.30	0.00	1.99	0.58	0.00	0.60	0.00
time (sec)	N/A	0.605	10.074	1.171	0.000	0.079	14.560	0.000	0.243	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	259	90	36	0	326	70	0	85	0
N.S.	1	1.76	0.61	0.24	0.00	2.22	0.48	0.00	0.58	0.00
time (sec)	N/A	0.782	10.137	1.230	0.000	0.076	35.459	0.000	0.234	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	327	80	238	0	160	262	0	154	0
N.S.	1	1.03	0.25	0.75	0.00	0.50	0.82	0.00	0.48	0.00
time (sec)	N/A	0.758	10.051	1.084	0.000	0.072	2.456	0.000	0.212	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	295	78	214	0	134	170	0	114	0
N.S.	1	1.02	0.27	0.74	0.00	0.46	0.59	0.00	0.39	0.00
time (sec)	N/A	0.661	10.043	1.073	0.000	0.070	1.624	0.000	0.215	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	263	77	196	0	107	82	0	78	0
N.S.	1	1.01	0.30	0.75	0.00	0.41	0.31	0.00	0.30	0.00
time (sec)	N/A	0.560	7.947	1.048	0.000	0.072	0.918	0.000	0.201	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	228	77	169	0	89	78	0	48	0
N.S.	1	1.00	0.34	0.74	0.00	0.39	0.34	0.00	0.21	0.00
time (sec)	N/A	0.475	10.044	0.683	0.000	0.069	0.817	0.000	0.190	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	99	212	0	119	78	0	70	0
N.S.	1	1.00	0.38	0.81	0.00	0.45	0.30	0.00	0.27	0.00
time (sec)	N/A	0.568	10.064	0.735	0.000	0.073	2.375	0.000	0.207	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	302	122	256	0	195	78	0	92	0
N.S.	1	1.03	0.42	0.88	0.00	0.67	0.27	0.00	0.32	0.00
time (sec)	N/A	0.667	10.117	0.717	0.000	0.081	9.673	0.000	0.239	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	223	45	34	0	94	221	0	112	0
N.S.	1	1.06	0.21	0.16	0.00	0.45	1.05	0.00	0.53	0.00
time (sec)	N/A	0.624	8.638	1.090	0.000	0.073	1.920	0.000	0.209	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	194	45	34	0	82	146	0	88	0
N.S.	1	1.04	0.24	0.18	0.00	0.44	0.78	0.00	0.47	0.00
time (sec)	N/A	0.530	7.554	0.991	0.000	0.072	1.321	0.000	0.194	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	165	44	33	0	70	71	0	64	0
N.S.	1	1.01	0.27	0.20	0.00	0.43	0.44	0.00	0.39	0.00
time (sec)	N/A	0.463	5.609	1.016	0.000	0.077	0.784	0.000	0.197	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	134	49	34	0	54	65	0	40	0
N.S.	1	1.02	0.37	0.26	0.00	0.41	0.49	0.00	0.30	0.00
time (sec)	N/A	0.382	10.024	0.629	0.000	0.076	0.659	0.000	0.185	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	159	61	34	0	76	65	0	50	0
N.S.	1	1.01	0.39	0.22	0.00	0.48	0.41	0.00	0.32	0.00
time (sec)	N/A	0.432	10.037	1.198	0.000	0.072	1.678	0.000	0.192	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	191	68	34	0	120	65	0	60	0
N.S.	1	1.04	0.37	0.18	0.00	0.65	0.35	0.00	0.33	0.00
time (sec)	N/A	0.527	10.079	1.143	0.000	0.169	2.827	0.000	0.196	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	249	47	38	0	127	226	0	158	0
N.S.	1	1.05	0.20	0.16	0.00	0.53	0.95	0.00	0.66	0.00
time (sec)	N/A	0.650	8.785	1.359	0.000	0.075	2.315	0.000	0.209	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	218	47	38	0	111	148	0	122	0
N.S.	1	1.03	0.22	0.18	0.00	0.52	0.70	0.00	0.58	0.00
time (sec)	N/A	0.569	7.488	1.317	0.000	0.077	1.540	0.000	0.199	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	47	38	0	95	70	0	86	0
N.S.	1	1.00	0.25	0.20	0.00	0.51	0.38	0.00	0.46	0.00
time (sec)	N/A	0.478	5.917	1.332	0.000	0.068	0.899	0.000	0.195	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	47	38	0	73	66	0	55	0
N.S.	1	1.00	0.53	0.43	0.00	0.82	0.74	0.00	0.62	0.00
time (sec)	N/A	0.302	10.026	0.855	0.000	0.068	0.775	0.000	0.210	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	183	65	38	0	108	66	0	71	0
N.S.	1	0.98	0.35	0.20	0.00	0.58	0.35	0.00	0.38	0.00
time (sec)	N/A	0.481	10.044	0.898	0.000	0.069	2.340	0.000	0.193	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	218	74	38	0	162	66	0	87	0
N.S.	1	1.03	0.35	0.18	0.00	0.76	0.31	0.00	0.41	0.00
time (sec)	N/A	0.574	10.080	0.875	0.000	0.070	8.513	0.000	0.213	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	249	83	38	0	210	66	0	103	0
N.S.	1	1.05	0.35	0.16	0.00	0.88	0.28	0.00	0.43	0.00
time (sec)	N/A	0.625	10.091	0.855	0.000	0.074	31.680	0.000	0.212	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	218	47	38	0	111	148	0	122	0
N.S.	1	1.03	0.22	0.18	0.00	0.52	0.70	0.00	0.58	0.00
time (sec)	N/A	0.553	7.322	1.118	0.000	0.067	1.567	0.000	0.205	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	187	47	38	0	95	70	0	86	0
N.S.	1	1.01	0.25	0.20	0.00	0.51	0.38	0.00	0.46	0.00
time (sec)	N/A	0.480	5.793	1.123	0.000	0.073	0.927	0.000	0.193	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	47	38	0	73	66	0	54	0
N.S.	1	1.00	0.31	0.25	0.00	0.48	0.43	0.00	0.36	0.00
time (sec)	N/A	0.399	10.028	0.683	0.000	0.065	0.778	0.000	0.192	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	65	38	0	105	66	0	70	0
N.S.	1	1.00	0.55	0.32	0.00	0.88	0.55	0.00	0.59	0.00
time (sec)	N/A	0.360	10.033	0.710	0.000	0.071	2.349	0.000	0.203	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	216	74	38	0	162	66	0	86	0
N.S.	1	1.02	0.35	0.18	0.00	0.77	0.31	0.00	0.41	0.00
time (sec)	N/A	0.566	10.072	0.671	0.000	0.071	8.547	0.000	0.208	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	248	83	38	0	210	66	0	102	0
N.S.	1	1.05	0.35	0.16	0.00	0.89	0.28	0.00	0.43	0.00
time (sec)	N/A	0.643	10.080	0.661	0.000	0.070	31.862	0.000	0.231	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	171	0	74	88	0	106	206	0	158	0
N.S.	1	0.00	0.43	0.51	0.00	0.62	1.20	0.00	0.92	0.00
time (sec)	N/A	0.000	9.188	2.000	0.000	0.077	2.467	0.000	0.215	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	145	0	74	88	0	91	134	0	122	0
N.S.	1	0.00	0.51	0.61	0.00	0.63	0.92	0.00	0.84	0.00
time (sec)	N/A	0.000	7.674	1.734	0.000	0.066	1.703	0.000	0.218	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	119	0	74	88	0	74	63	0	86	0
N.S.	1	0.00	0.62	0.74	0.00	0.62	0.53	0.00	0.72	0.00
time (sec)	N/A	0.000	5.954	1.992	0.000	0.067	0.955	0.000	0.199	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	74	88	0	56	61	0	22	0
N.S.	1	1.00	1.72	2.05	0.00	1.30	1.42	0.00	0.51	0.00
time (sec)	N/A	0.300	10.031	0.980	0.000	0.067	0.803	0.000	0.188	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	116	0	90	88	0	83	60	0	38	0
N.S.	1	0.00	0.78	0.76	0.00	0.72	0.52	0.00	0.33	0.00
time (sec)	N/A	0.000	10.041	1.001	0.000	0.072	1.924	0.000	0.183	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	144	0	98	88	0	169	61	0	53	0
N.S.	1	0.00	0.68	0.61	0.00	1.17	0.42	0.00	0.37	0.00
time (sec)	N/A	0.000	10.069	1.035	0.000	0.076	5.074	0.000	0.196	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	170	0	106	88	0	249	60	0	70	0
N.S.	1	0.00	0.62	0.52	0.00	1.46	0.35	0.00	0.41	0.00
time (sec)	N/A	0.000	10.088	0.999	0.000	0.075	13.222	0.000	0.193	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	145	0	74	88	0	90	134	0	122	0
N.S.	1	0.00	0.51	0.61	0.00	0.62	0.92	0.00	0.84	0.00
time (sec)	N/A	0.000	7.653	2.023	0.000	0.068	1.680	0.000	0.199	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	119	0	74	88	0	75	63	0	86	0
N.S.	1	0.00	0.62	0.74	0.00	0.63	0.53	0.00	0.72	0.00
time (sec)	N/A	0.000	6.334	2.205	0.000	0.068	0.972	0.000	0.202	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	89	63	74	88	0	53	60	0	24	0
N.S.	1	0.71	0.83	0.99	0.00	0.60	0.67	0.00	0.27	0.00
time (sec)	N/A	0.358	10.106	1.266	0.000	0.067	0.819	0.000	0.177	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	73	0	90	88	0	81	60	0	39	0
N.S.	1	0.00	1.23	1.21	0.00	1.11	0.82	0.00	0.53	0.00
time (sec)	N/A	0.000	10.037	1.299	0.000	0.069	1.677	0.000	0.187	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	145	0	98	88	0	169	60	0	54	0
N.S.	1	0.00	0.68	0.61	0.00	1.17	0.41	0.00	0.37	0.00
time (sec)	N/A	0.000	10.056	1.264	0.000	0.075	3.819	0.000	0.217	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	171	0	106	88	0	247	60	0	71	0
N.S.	1	0.00	0.62	0.51	0.00	1.44	0.35	0.00	0.42	0.00
time (sec)	N/A	0.000	10.068	1.282	0.000	0.081	11.938	0.000	0.242	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	B	A	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	197	0	114	88	0	329	60	0	86	0
N.S.	1	0.00	0.58	0.45	0.00	1.67	0.30	0.00	0.44	0.00
time (sec)	N/A	0.000	10.109	1.328	0.000	0.077	35.263	0.000	0.255	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	244	254	76	90	0	138	236	0	161	0
N.S.	1	1.04	0.31	0.37	0.00	0.57	0.97	0.00	0.66	0.00
time (sec)	N/A	0.683	8.889	2.427	0.000	0.068	2.556	0.000	0.210	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	217	222	76	90	0	122	155	0	125	0
N.S.	1	1.02	0.35	0.41	0.00	0.56	0.71	0.00	0.58	0.00
time (sec)	N/A	0.573	7.568	2.154	0.000	0.062	1.685	0.000	0.197	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	190	189	76	90	0	106	73	0	89	0
N.S.	1	0.99	0.40	0.47	0.00	0.56	0.38	0.00	0.47	0.00
time (sec)	N/A	0.489	6.190	2.431	0.000	0.063	0.994	0.000	0.192	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	90	0	87	70	0	58	0
N.S.	1	1.00	0.84	1.00	0.00	0.97	0.78	0.00	0.64	0.00
time (sec)	N/A	0.305	10.042	1.374	0.000	0.064	0.829	0.000	0.189	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	190	187	91	90	0	115	70	0	73	0
N.S.	1	0.98	0.48	0.47	0.00	0.61	0.37	0.00	0.38	0.00
time (sec)	N/A	0.487	10.049	1.365	0.000	0.063	2.888	0.000	0.206	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	217	222	99	90	0	168	70	0	90	0
N.S.	1	1.02	0.46	0.41	0.00	0.77	0.32	0.00	0.41	0.00
time (sec)	N/A	0.574	10.054	1.339	0.000	0.070	9.015	0.000	0.206	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	244	254	107	90	0	215	70	0	105	0
N.S.	1	1.04	0.44	0.37	0.00	0.88	0.29	0.00	0.43	0.00
time (sec)	N/A	0.656	10.086	1.328	0.000	0.070	33.281	0.000	0.217	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	217	222	76	90	0	121	156	0	125	0
N.S.	1	1.02	0.35	0.41	0.00	0.56	0.72	0.00	0.58	0.00
time (sec)	N/A	0.585	7.571	1.849	0.000	0.061	1.722	0.000	0.213	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	76	90	0	105	73	0	89	0
N.S.	1	1.00	0.40	0.47	0.00	0.55	0.38	0.00	0.47	0.00
time (sec)	N/A	0.489	6.112	2.072	0.000	0.064	0.982	0.000	0.189	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	76	90	0	84	71	0	57	0
N.S.	1	1.00	0.49	0.58	0.00	0.54	0.46	0.00	0.37	0.00
time (sec)	N/A	0.416	10.034	1.028	0.000	0.063	0.847	0.000	0.190	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	122	123	91	90	0	111	70	0	72	0
N.S.	1	1.01	0.75	0.74	0.00	0.91	0.57	0.00	0.59	0.00
time (sec)	N/A	0.378	10.028	1.064	0.000	0.067	2.946	0.000	0.194	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	216	220	99	90	0	167	71	0	89	0
N.S.	1	1.02	0.46	0.42	0.00	0.77	0.33	0.00	0.41	0.00
time (sec)	N/A	0.576	10.041	1.056	0.000	0.065	9.043	0.000	0.197	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	243	253	107	90	0	216	70	0	104	0
N.S.	1	1.04	0.44	0.37	0.00	0.89	0.29	0.00	0.43	0.00
time (sec)	N/A	0.657	10.048	1.038	0.000	0.073	33.346	0.000	0.218	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	270	285	115	90	0	263	71	0	121	0
N.S.	1	1.06	0.43	0.33	0.00	0.97	0.26	0.00	0.45	0.00
time (sec)	N/A	0.731	10.070	1.076	0.000	0.068	89.756	0.000	0.236	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	96	94	94	110	98	101	95
N.S.	1	1.00	1.00	0.91	0.89	0.89	1.04	0.92	0.95	0.90
time (sec)	N/A	0.436	0.016	0.103	0.026	0.057	0.026	0.137	0.179	0.049

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	72	71	71	78	73	77	71
N.S.	1	1.00	1.00	0.91	0.90	0.90	0.99	0.92	0.97	0.90
time (sec)	N/A	0.393	0.012	0.100	0.026	0.055	0.023	0.107	0.173	0.033

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	49	48	48	56	50	53	49
N.S.	1	1.00	1.00	0.88	0.86	0.86	1.00	0.89	0.95	0.88
time (sec)	N/A	0.340	0.009	0.100	0.026	0.060	0.020	0.111	0.179	0.025

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	26	29	26
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.91	0.81
time (sec)	N/A	0.291	0.002	0.059	0.027	0.060	0.017	0.108	0.180	0.044

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	47	0	131	104	50	70	45
N.S.	1	1.00	1.00	0.85	0.00	2.38	1.89	0.91	1.27	0.82
time (sec)	N/A	0.333	0.026	0.115	0.000	0.071	0.150	0.130	0.181	17.015

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	85	78	70	0	222	138	72	139	68
N.S.	1	1.15	1.05	0.95	0.00	3.00	1.86	0.97	1.88	0.92
time (sec)	N/A	0.364	0.038	0.116	0.000	0.073	0.231	0.103	0.181	17.013

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	106	92	92	0	306	219	86	221	97
N.S.	1	1.14	0.99	0.99	0.00	3.29	2.35	0.92	2.38	1.04
time (sec)	N/A	0.395	0.045	0.113	0.000	0.081	0.357	0.104	0.179	17.019

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	131	113	113	0	424	204	111	312	129
N.S.	1	1.07	0.92	0.92	0.00	3.45	1.66	0.90	2.54	1.05
time (sec)	N/A	0.443	0.055	0.124	0.000	0.074	0.459	0.116	0.190	17.022

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	133	128	129	129	144	131	135	127
N.S.	1	1.00	1.00	0.96	0.97	0.97	1.08	0.98	1.02	0.95
time (sec)	N/A	0.519	0.016	0.112	0.028	0.056	0.027	0.106	0.175	0.061

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	90	89	89	104	91	94	89
N.S.	1	1.00	1.00	0.93	0.92	0.92	1.07	0.94	0.97	0.92
time (sec)	N/A	0.436	0.014	0.111	0.028	0.059	0.024	0.107	0.180	0.052

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	60	50	53	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.00	0.83	0.88	0.83
time (sec)	N/A	0.345	0.002	0.100	0.040	0.056	0.021	0.118	0.179	0.026

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	24	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.96	0.84
time (sec)	N/A	0.273	0.001	0.067	0.032	0.053	0.017	0.115	0.176	0.030

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	97	104	0	268	236	118	155	141
N.S.	1	1.00	0.90	0.96	0.00	2.48	2.19	1.09	1.44	1.31
time (sec)	N/A	0.435	0.052	0.141	0.000	0.078	0.238	0.130	0.182	16.954

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	142	134	129	0	394	314	144	263	183
N.S.	1	1.08	1.02	0.98	0.00	3.01	2.40	1.10	2.01	1.40
time (sec)	N/A	0.778	0.079	0.125	0.000	0.072	0.427	0.151	0.173	16.944

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	173	154	152	0	516	257	160	377	164
N.S.	1	1.12	0.99	0.98	0.00	3.33	1.66	1.03	2.43	1.06
time (sec)	N/A	0.938	0.080	0.134	0.000	0.076	0.750	0.111	0.180	0.098

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	212	174	179	0	662	292	183	504	199
N.S.	1	1.13	0.93	0.96	0.00	3.54	1.56	0.98	2.70	1.06
time (sec)	N/A	0.991	0.095	0.133	0.000	0.078	1.198	0.109	0.185	17.094

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	250	200	212	0	806	335	216	631	240
N.S.	1	1.11	0.88	0.94	0.00	3.57	1.48	0.96	2.79	1.06
time (sec)	N/A	1.038	0.127	0.146	0.000	0.076	2.063	0.114	0.180	17.104

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	437	444	118	432	2878	500	510	812	4022
N.S.	1	1.24	1.26	0.33	1.22	8.15	1.42	1.44	2.30	11.39
time (sec)	N/A	1.094	0.228	0.115	0.114	9.624	1.811	0.116	0.199	17.782

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	370	360	80	342	2133	350	411	624	2712
N.S.	1	1.33	1.29	0.29	1.23	7.67	1.26	1.48	2.24	9.76
time (sec)	N/A	0.999	0.180	0.108	0.112	2.039	1.143	0.136	0.179	0.552

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	297	269	56	288	1480	238	319	452	1479
N.S.	1	1.35	1.22	0.25	1.31	6.73	1.08	1.45	2.05	6.72
time (sec)	N/A	0.753	0.174	0.109	0.113	0.463	0.719	0.171	0.182	17.367

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	230	183	34	221	767	109	241	239	599
N.S.	1	1.28	1.02	0.19	1.23	4.26	0.61	1.34	1.33	3.33
time (sec)	N/A	0.713	0.036	0.099	0.114	0.085	0.335	0.114	0.188	0.362

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	200	134	27	169	112	20	179	112	33
N.S.	1	1.49	1.00	0.20	1.26	0.84	0.15	1.34	0.84	0.25
time (sec)	N/A	0.609	0.014	0.090	0.109	0.069	0.071	0.124	0.180	0.086

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	336	234	253	0	4084	0	344	330	4802
N.S.	1	1.31	0.91	0.98	0.00	15.89	0.00	1.34	1.28	18.68
time (sec)	N/A	0.787	0.106	0.165	0.000	0.442	0.000	0.112	0.191	18.473

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	453	362	307	0	8409	0	531	1155	16369
N.S.	1	1.35	1.08	0.91	0.00	25.03	0.00	1.58	3.44	48.72
time (sec)	N/A	0.963	0.330	0.198	0.000	8.634	0.000	0.124	16.342	19.321

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	323	371	119	292	2116	352	430	1304	2560
N.S.	1	1.21	1.38	0.44	1.09	7.90	1.31	1.60	4.87	9.55
time (sec)	N/A	1.068	0.176	0.112	0.113	0.730	1.428	0.136	0.186	17.638

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	325	295	97	324	1596	275	350	957	1565
N.S.	1	1.31	1.18	0.39	1.30	6.41	1.10	1.41	3.84	6.29
time (sec)	N/A	0.973	0.117	0.112	0.111	0.505	0.922	0.111	0.178	0.537

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	264	267	67	253	873	136	268	610	637
N.S.	1	1.28	1.29	0.32	1.22	4.22	0.66	1.29	2.95	3.08
time (sec)	N/A	0.778	0.201	0.105	0.111	0.086	0.455	0.119	0.194	0.431

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	225	183	46	189	183	39	194	305	58
N.S.	1	1.49	1.21	0.30	1.25	1.21	0.26	1.28	2.02	0.38
time (sec)	N/A	0.658	0.083	0.096	0.109	0.066	0.138	0.130	0.177	17.069

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	689	429	334	0	9892	0	621	1438	17945
N.S.	1	1.86	1.16	0.90	0.00	26.66	0.00	1.67	3.88	48.37
time (sec)	N/A	1.378	0.202	0.201	0.000	11.082	0.000	0.125	0.214	19.520

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	492	864	540	402	0	15292	0	879	3860	28923
N.S.	1	1.76	1.10	0.82	0.00	31.08	0.00	1.79	7.85	58.79
time (sec)	N/A	1.854	0.380	0.277	0.000	100.166	0.000	0.157	0.237	21.289

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	139	118	102	0	236	170	116	167	0
N.S.	1	0.95	0.81	0.70	0.00	1.62	1.16	0.79	1.14	0.00
time (sec)	N/A	0.409	0.164	0.174	0.000	0.093	0.377	0.121	0.179	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	117	93	80	0	188	121	88	127	0
N.S.	1	1.03	0.82	0.70	0.00	1.65	1.06	0.77	1.11	0.00
time (sec)	N/A	0.385	0.118	0.147	0.000	0.083	0.303	0.117	0.186	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	93	74	62	0	146	100	66	91	0
N.S.	1	1.09	0.87	0.73	0.00	1.72	1.18	0.78	1.07	0.00
time (sec)	N/A	0.349	0.078	0.137	0.000	0.076	0.264	0.125	0.178	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	79	77	73	0	201	94	68	165	0
N.S.	1	1.01	0.99	0.94	0.00	2.58	1.21	0.87	2.12	0.00
time (sec)	N/A	0.340	0.113	0.157	0.000	0.081	2.885	0.129	0.194	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	98	84	88	0	273	403	84	223	0
N.S.	1	1.14	0.98	1.02	0.00	3.17	4.69	0.98	2.59	0.00
time (sec)	N/A	0.383	0.148	0.189	0.000	0.092	4.829	0.143	0.207	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	91	52	49	119	81	488	58	209	96
N.S.	1	0.99	0.57	0.53	1.29	0.88	5.30	0.63	2.27	1.04
time (sec)	N/A	0.363	0.114	0.123	0.033	0.079	9.555	0.129	0.263	17.137

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	117	74	64	154	112	1469	86	288	126
N.S.	1	0.93	0.59	0.51	1.22	0.89	11.66	0.68	2.29	1.00
time (sec)	N/A	0.399	0.136	0.141	0.032	0.095	22.087	0.123	0.241	17.116

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	147	95	83	189	146	3738	118	367	157
N.S.	1	0.92	0.59	0.52	1.18	0.91	23.36	0.74	2.29	0.98
time (sec)	N/A	0.425	0.161	0.143	0.035	0.114	47.777	0.142	0.266	17.854

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	245	197	174	0	428	389	225	335	0
N.S.	1	0.91	0.73	0.64	0.00	1.59	1.44	0.83	1.24	0.00
time (sec)	N/A	0.774	0.302	0.214	0.000	0.232	0.497	0.124	0.342	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	223	163	141	0	354	280	183	270	0
N.S.	1	1.01	0.74	0.64	0.00	1.60	1.27	0.83	1.22	0.00
time (sec)	N/A	0.738	0.241	0.201	0.000	0.138	0.417	0.121	0.281	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	198	138	113	0	282	192	144	209	0
N.S.	1	1.14	0.79	0.65	0.00	1.62	1.10	0.83	1.20	0.00
time (sec)	N/A	0.707	0.524	0.190	0.000	0.107	0.406	0.116	0.245	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	181	131	132	0	369	0	150	357	0
N.S.	1	1.14	0.82	0.83	0.00	2.32	0.00	0.94	2.25	0.00
time (sec)	N/A	0.890	0.218	0.207	0.000	0.112	0.000	0.130	0.244	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	191	146	147	0	478	0	161	466	0
N.S.	1	1.11	0.85	0.85	0.00	2.78	0.00	0.94	2.71	0.00
time (sec)	N/A	0.968	0.263	0.331	0.000	0.124	0.000	0.163	0.293	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	198	139	138	0	459	0	159	505	0
N.S.	1	1.06	0.75	0.74	0.00	2.47	0.00	0.85	2.72	0.00
time (sec)	N/A	0.960	0.273	0.222	0.000	0.128	0.000	0.135	0.213	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	235	157	183	0	557	0	192	641	0
N.S.	1	1.12	0.75	0.87	0.00	2.65	0.00	0.91	3.05	0.00
time (sec)	N/A	1.057	0.360	0.315	0.000	0.194	0.000	0.125	0.202	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	216	109	100	369	167	0	141	513	287
N.S.	1	0.94	0.47	0.43	1.60	0.72	0.00	0.61	2.22	1.24
time (sec)	N/A	1.038	0.248	0.189	0.037	0.173	0.000	0.158	0.212	17.602

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	249	142	130	428	217	0	190	636	332
N.S.	1	0.89	0.51	0.46	1.53	0.78	0.00	0.68	2.27	1.19
time (sec)	N/A	1.073	0.293	0.191	0.037	0.251	0.000	0.132	0.214	17.669

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	582	465	357	0	6742	0	0	114	0
N.S.	1	2.51	2.00	1.54	0.00	29.06	0.00	0.00	0.49	0.00
time (sec)	N/A	1.595	0.328	0.974	0.000	52.554	0.000	0.000	0.342	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	428	389	295	0	4605	0	0	82	0
N.S.	1	2.15	1.95	1.48	0.00	23.14	0.00	0.00	0.41	0.00
time (sec)	N/A	1.127	0.238	0.581	0.000	12.904	0.000	0.000	0.299	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	310	241	257	0	2793	0	0	50	0
N.S.	1	1.82	1.42	1.51	0.00	16.43	0.00	0.00	0.29	0.00
time (sec)	N/A	0.834	0.167	0.513	0.000	1.233	0.000	0.000	0.335	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	239	211	137	0	487	0	0	21	0
N.S.	1	1.65	1.46	0.94	0.00	3.36	0.00	0.00	0.14	0.00
time (sec)	N/A	0.609	0.126	0.460	0.000	0.187	0.000	0.000	0.261	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	132	102	0	1225	0	0	29	0
N.S.	1	1.00	0.98	0.76	0.00	9.07	0.00	0.00	0.21	0.00
time (sec)	N/A	0.425	0.088	0.488	0.000	0.282	0.000	0.000	0.266	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	227	247	253	0	3813	0	0	60	0
N.S.	1	1.27	1.38	1.41	0.00	21.30	0.00	0.00	0.34	0.00
time (sec)	N/A	0.807	0.213	0.500	0.000	11.028	0.000	0.000	0.293	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	367	339	315	0	6571	0	0	100	0
N.S.	1	1.65	1.53	1.42	0.00	29.60	0.00	0.00	0.45	0.00
time (sec)	N/A	1.097	0.387	0.665	0.000	45.745	0.000	0.000	37.226	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	564	473	408	0	0	0	0	140	0
N.S.	1	1.92	1.61	1.39	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	1.505	1.128	0.768	0.000	0.000	0.000	0.000	0.546	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	A	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	0	1004	426	0	0	0	436	196	0
N.S.	1	0.00	3.07	1.30	0.00	0.00	0.00	1.33	0.60	0.00
time (sec)	N/A	0.000	1.036	0.978	0.000	0.000	0.000	0.187	51.609	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	B	F	A	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	0	828	389	0	7084	0	388	154	0
N.S.	1	0.00	2.91	1.36	0.00	24.86	0.00	1.36	0.54	0.00
time (sec)	N/A	0.000	0.777	0.763	0.000	69.077	0.000	0.194	3.949	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	0	711	310	0	1935	0	0	112	0
N.S.	1	0.00	3.05	1.33	0.00	8.30	0.00	0.00	0.48	0.00
time (sec)	N/A	0.000	0.602	0.694	0.000	10.333	0.000	0.000	3.213	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	0	516	176	0	641	0	0	70	0
N.S.	1	0.00	2.93	1.00	0.00	3.64	0.00	0.00	0.40	0.00
time (sec)	N/A	0.000	0.433	0.566	0.000	0.736	0.000	0.000	1.502	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	0	421	179	0	2399	0	0	31	0
N.S.	1	0.00	2.02	0.86	0.00	11.53	0.00	0.00	0.15	0.00
time (sec)	N/A	0.000	0.367	0.527	0.000	4.055	0.000	0.000	0.573	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	0	317	294	0	4804	0	0	47	0
N.S.	1	0.00	1.43	1.33	0.00	21.74	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	0.341	0.589	0.000	25.175	0.000	0.000	0.325	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	0	708	370	0	7859	0	0	98	0
N.S.	1	0.00	2.53	1.32	0.00	28.07	0.00	0.00	0.35	0.00
time (sec)	N/A	0.000	0.747	0.690	0.000	171.476	0.000	0.000	15.663	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	0	913	488	0	0	0	0	163	0
N.S.	1	0.00	2.68	1.43	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.000	1.578	1.079	0.000	0.000	0.000	0.000	167.398	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	0	2319	538	0	0	0	1035	22	0
N.S.	1	0.00	5.52	1.28	0.00	0.00	0.00	2.46	0.05	0.00
time (sec)	N/A	0.000	3.116	1.384	0.000	0.000	0.000	0.218	200.033	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	0	2220	444	0	3392	0	0	22	0
N.S.	1	0.00	6.24	1.25	0.00	9.53	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	2.946	1.198	0.000	143.234	0.000	0.000	200.026	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	0	1818	363	0	2082	0	0	22	0
N.S.	1	0.00	6.83	1.36	0.00	7.83	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	2.354	1.060	0.000	28.791	0.000	0.000	200.031	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	0	1766	344	0	2034	0	0	22	0
N.S.	1	0.00	6.05	1.18	0.00	6.97	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	2.188	1.138	0.000	21.976	0.000	0.000	200.027	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	0	1315	274	0	2526	0	0	22	0
N.S.	1	0.00	5.28	1.10	0.00	10.14	0.00	0.00	0.09	0.00
time (sec)	N/A	0.000	1.493	0.643	0.000	11.700	0.000	0.000	200.027	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	0	1324	355	0	6046	0	0	22	0
N.S.	1	0.00	4.53	1.22	0.00	20.71	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	1.455	0.721	0.000	117.338	0.000	0.000	200.026	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	0	1214	411	0	0	0	0	22	0
N.S.	1	0.00	3.81	1.29	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	1.534	0.772	0.000	0.000	0.000	0.000	200.031	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	0	2037	548	0	0	0	0	22	0
N.S.	1	0.00	5.14	1.38	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	2.531	1.023	0.000	0.000	0.000	0.000	200.027	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	139	118	102	0	236	170	116	167	0
N.S.	1	0.97	0.82	0.71	0.00	1.64	1.18	0.81	1.16	0.00
time (sec)	N/A	0.402	0.155	0.154	0.000	0.101	0.346	0.257	0.246	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	117	93	80	0	188	121	88	127	0
N.S.	1	1.04	0.82	0.71	0.00	1.66	1.07	0.78	1.12	0.00
time (sec)	N/A	0.376	0.123	0.149	0.000	0.097	0.297	0.257	0.232	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	93	82	62	0	146	100	66	91	0
N.S.	1	1.09	0.96	0.73	0.00	1.72	1.18	0.78	1.07	0.00
time (sec)	N/A	0.357	0.192	0.137	0.000	0.086	0.258	0.225	0.257	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	75	72	0	201	94	68	165	0
N.S.	1	1.03	0.97	0.94	0.00	2.61	1.22	0.88	2.14	0.00
time (sec)	N/A	0.348	0.132	0.149	0.000	0.089	2.895	0.294	0.195	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	98	86	88	0	277	403	87	223	0
N.S.	1	1.17	1.02	1.05	0.00	3.30	4.80	1.04	2.65	0.00
time (sec)	N/A	0.386	0.154	0.174	0.000	0.097	4.830	0.309	0.169	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	52	49	119	80	488	57	209	96
N.S.	1	1.00	0.57	0.54	1.31	0.88	5.36	0.63	2.30	1.05
time (sec)	N/A	0.366	0.108	0.123	0.033	0.096	9.667	0.277	0.174	17.412

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	117	74	64	154	112	1469	86	288	127
N.S.	1	0.94	0.60	0.52	1.24	0.90	11.85	0.69	2.32	1.02
time (sec)	N/A	0.402	0.139	0.130	0.036	0.096	22.093	0.276	0.171	17.373

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	147	94	83	189	146	3738	118	367	158
N.S.	1	0.93	0.59	0.53	1.20	0.92	23.66	0.75	2.32	1.00
time (sec)	N/A	0.435	0.160	0.145	0.035	0.126	47.787	0.294	0.196	17.508

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	245	197	173	0	428	382	225	335	0
N.S.	1	0.91	0.73	0.64	0.00	1.59	1.41	0.83	1.24	0.00
time (sec)	N/A	0.752	0.301	0.233	0.000	0.230	0.471	0.273	0.250	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	223	163	141	0	354	269	183	270	0
N.S.	1	1.01	0.74	0.64	0.00	1.60	1.22	0.83	1.22	0.00
time (sec)	N/A	0.729	0.238	0.211	0.000	0.152	0.380	0.253	0.224	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	198	130	113	0	282	192	144	209	0
N.S.	1	1.14	0.75	0.65	0.00	1.62	1.10	0.83	1.20	0.00
time (sec)	N/A	0.687	0.163	0.201	0.000	0.109	0.379	0.289	0.177	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	180	131	131	0	369	0	150	357	0
N.S.	1	1.14	0.83	0.83	0.00	2.34	0.00	0.95	2.26	0.00
time (sec)	N/A	0.865	0.220	0.244	0.000	0.111	0.000	0.297	0.174	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	190	146	147	0	478	0	161	466	0
N.S.	1	1.11	0.85	0.86	0.00	2.80	0.00	0.94	2.73	0.00
time (sec)	N/A	0.960	0.256	0.355	0.000	0.131	0.000	0.302	0.191	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	197	139	138	0	459	0	159	505	0
N.S.	1	1.06	0.75	0.75	0.00	2.48	0.00	0.86	2.73	0.00
time (sec)	N/A	0.960	0.283	0.239	0.000	0.142	0.000	0.287	0.180	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	234	158	183	0	557	0	192	641	0
N.S.	1	1.12	0.76	0.88	0.00	2.68	0.00	0.92	3.08	0.00
time (sec)	N/A	1.052	0.337	0.327	0.000	0.191	0.000	0.303	0.185	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	215	109	100	369	167	0	141	513	285
N.S.	1	0.94	0.48	0.44	1.62	0.73	0.00	0.62	2.25	1.25
time (sec)	N/A	1.051	0.249	0.178	0.040	0.174	0.000	0.306	0.192	17.434

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	249	142	130	428	217	0	190	636	330
N.S.	1	0.90	0.51	0.47	1.54	0.78	0.00	0.68	2.29	1.19
time (sec)	N/A	1.090	0.310	0.192	0.037	0.257	0.000	0.303	0.200	17.701

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	742	466	990	0	6799	0	0	110	0
N.S.	1	1.34	0.84	1.78	0.00	12.25	0.00	0.00	0.20	0.00
time (sec)	N/A	2.040	0.339	3.100	0.000	53.718	0.000	0.000	0.352	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	551	391	826	0	4661	0	0	79	0
N.S.	1	1.16	0.82	1.74	0.00	9.83	0.00	0.00	0.17	0.00
time (sec)	N/A	1.385	0.229	1.053	0.000	13.096	0.000	0.000	0.284	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	390	241	783	0	2849	0	0	48	0
N.S.	1	0.88	0.55	1.77	0.00	6.45	0.00	0.00	0.11	0.00
time (sec)	N/A	1.041	0.158	0.782	0.000	1.298	0.000	0.000	0.216	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	263	211	551	0	503	0	0	20	0
N.S.	1	0.92	0.74	1.92	0.00	1.75	0.00	0.00	0.07	0.00
time (sec)	N/A	0.696	0.118	1.112	0.000	0.212	0.000	0.000	0.189	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	151	132	673	0	1217	0	0	28	0
N.S.	1	0.41	0.36	1.82	0.00	3.29	0.00	0.00	0.08	0.00
time (sec)	N/A	0.497	0.086	0.900	0.000	0.286	0.000	0.000	0.196	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	244	246	773	0	3803	0	0	58	0
N.S.	1	0.61	0.62	1.94	0.00	9.53	0.00	0.00	0.15	0.00
time (sec)	N/A	0.968	0.194	1.013	0.000	10.930	0.000	0.000	0.230	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	384	335	924	0	6557	0	0	98	0
N.S.	1	0.78	0.68	1.87	0.00	13.27	0.00	0.00	0.20	0.00
time (sec)	N/A	1.348	0.386	1.004	0.000	46.717	0.000	0.000	35.860	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	A	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	627	0	1002	1175	0	0	0	437	196	0
N.S.	1	0.00	1.60	1.87	0.00	0.00	0.00	0.70	0.31	0.00
time (sec)	N/A	0.000	0.945	1.655	0.000	0.000	0.000	0.322	53.719	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F	A	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	0	827	1155	0	7122	0	387	154	0
N.S.	1	0.00	1.44	2.02	0.00	12.43	0.00	0.68	0.27	0.00
time (sec)	N/A	0.000	0.699	1.457	0.000	68.677	0.000	0.823	4.050	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	0	708	871	0	1951	0	0	112	0
N.S.	1	0.00	1.50	1.84	0.00	4.12	0.00	0.00	0.24	0.00
time (sec)	N/A	0.000	0.571	1.168	0.000	10.824	0.000	0.000	3.523	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	0	502	553	0	647	0	0	70	0
N.S.	1	0.00	1.58	1.74	0.00	2.04	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	0.450	0.880	0.000	0.747	0.000	0.000	2.167	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	0	420	839	0	2397	0	0	31	0
N.S.	1	0.00	1.06	2.12	0.00	6.07	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.348	1.398	0.000	3.656	0.000	0.000	0.862	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	464	0	314	1041	0	4777	0	0	47	0
N.S.	1	0.00	0.68	2.24	0.00	10.30	0.00	0.00	0.10	0.00
time (sec)	N/A	0.000	0.322	1.316	0.000	18.116	0.000	0.000	0.383	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	0	666	1116	0	7841	0	0	98	0
N.S.	1	0.00	1.32	2.21	0.00	15.56	0.00	0.00	0.19	0.00
time (sec)	N/A	0.000	0.889	1.950	0.000	170.297	0.000	0.000	16.802	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	0	910	1382	0	0	0	0	163	0
N.S.	1	0.00	1.44	2.19	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.000	1.358	2.849	0.000	0.000	0.000	0.000	170.883	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	A	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	748	0	2315	1517	0	0	0	1035	21	0
N.S.	1	0.00	3.09	2.03	0.00	0.00	0.00	1.38	0.03	0.00
time (sec)	N/A	0.000	3.294	2.520	0.000	0.000	0.000	0.227	200.029	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	632	0	2216	1244	0	0	0	0	21	0
N.S.	1	0.00	3.51	1.97	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	3.086	1.943	0.000	0.000	0.000	0.000	200.032	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	0	1814	993	0	2105	0	0	21	0
N.S.	1	0.00	3.59	1.97	0.00	4.17	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	2.419	1.434	0.000	28.541	0.000	0.000	200.029	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	0	1763	982	0	2045	0	0	21	0
N.S.	1	0.00	3.44	1.92	0.00	3.99	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	2.274	1.413	0.000	21.003	0.000	0.000	200.035	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	0	1310	925	0	2534	0	0	21	0
N.S.	1	0.00	3.01	2.13	0.00	5.83	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	1.592	1.638	0.000	11.136	0.000	0.000	200.031	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	0	1323	1210	0	6035	0	0	21	0
N.S.	1	0.00	2.58	2.36	0.00	11.79	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	1.560	2.095	0.000	107.948	0.000	0.000	200.025	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	0	1211	1843	0	0	0	0	21	0
N.S.	1	0.00	2.03	3.09	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	1.549	2.396	0.000	0.000	0.000	0.000	200.032	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	668	0	1993	1598	0	0	0	0	21	0
N.S.	1	0.00	2.98	2.39	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	2.421	2.956	0.000	0.000	0.000	0.000	200.027	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	141	222	0	167	180	0	159	0
N.S.	1	1.00	0.66	1.04	0.00	0.78	0.85	0.00	0.75	0.00
time (sec)	N/A	0.930	10.135	2.559	0.000	0.085	1.827	0.000	0.218	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	167	121	186	0	115	129	0	106	0
N.S.	1	1.03	0.75	1.15	0.00	0.71	0.80	0.00	0.65	0.00
time (sec)	N/A	0.641	10.075	1.592	0.000	0.100	1.397	0.000	0.184	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	77	154	0	91	82	0	52	0
N.S.	1	1.00	0.63	1.25	0.00	0.74	0.67	0.00	0.42	0.00
time (sec)	N/A	0.514	0.011	0.681	0.000	0.100	0.848	0.000	0.180	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	72	64	0	26	37	0	22	38
N.S.	1	1.00	1.36	1.21	0.00	0.49	0.70	0.00	0.42	0.72
time (sec)	N/A	0.272	10.031	0.290	0.000	0.078	0.402	0.000	0.164	17.192

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	91	97	0	0	0	0	38	0
N.S.	1	1.00	1.26	1.35	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.344	10.160	0.467	0.000	0.000	0.000	0.000	0.177	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	280	508	523	0	0	0	0	62	0
N.S.	1	0.94	1.70	1.75	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.247	10.725	0.651	0.000	0.000	0.000	0.000	0.192	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	417	321	961	0	0	0	0	86	0
N.S.	1	0.98	0.76	2.26	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.090	10.958	1.090	0.000	0.000	0.000	0.000	0.500	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	566	458	1420	0	0	0	0	110	0
N.S.	1	1.01	0.81	2.52	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	3.191	11.401	1.594	0.000	0.000	0.000	0.000	0.651	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	252	143	278	0	271	0	0	391	0
N.S.	1	1.12	0.63	1.23	0.00	1.20	0.00	0.00	1.73	0.00
time (sec)	N/A	0.998	10.112	1.854	0.000	0.097	0.000	0.000	0.241	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	209	126	233	0	171	0	0	277	0
N.S.	1	1.17	0.71	1.31	0.00	0.96	0.00	0.00	1.56	0.00
time (sec)	N/A	0.747	10.078	1.082	0.000	0.092	0.000	0.000	0.199	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	158	100	199	0	120	82	0	72	0
N.S.	1	1.01	0.64	1.28	0.00	0.77	0.53	0.00	0.46	0.00
time (sec)	N/A	0.596	0.016	0.743	0.000	0.091	3.094	0.000	0.194	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	90	0	66	37	0	32	38
N.S.	1	1.00	0.73	1.17	0.00	0.86	0.48	0.00	0.42	0.49
time (sec)	N/A	0.308	5.153	0.302	0.000	0.087	0.463	0.000	0.169	17.175

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	266	285	431	0	0	0	0	60	0
N.S.	1	0.96	1.03	1.55	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.981	10.381	0.472	0.000	0.000	0.000	0.000	0.438	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	604	332	865	0	0	0	0	101	0
N.S.	1	1.44	0.79	2.06	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.726	10.645	0.691	0.000	0.000	0.000	0.000	0.604	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	1066	461	1324	0	0	0	0	142	0
N.S.	1	1.90	0.82	2.36	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	3.291	11.101	1.205	0.000	0.000	0.000	0.000	0.876	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	713	1700	602	1797	0	0	0	0	183	0
N.S.	1	2.38	0.84	2.52	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	5.701	11.548	1.563	0.000	0.000	0.000	0.000	1.229	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	180	286	0	204	214	0	223	0
N.S.	1	1.00	0.47	0.74	0.00	0.53	0.55	0.00	0.58	0.00
time (sec)	N/A	1.221	10.197	3.940	0.000	0.083	2.166	0.000	0.221	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	322	140	235	0	166	173	0	148	0
N.S.	1	0.98	0.43	0.71	0.00	0.50	0.53	0.00	0.45	0.00
time (sec)	N/A	0.885	10.102	2.575	0.000	0.076	1.674	0.000	0.199	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	272	120	200	0	114	124	0	99	0
N.S.	1	1.03	0.45	0.76	0.00	0.43	0.47	0.00	0.38	0.00
time (sec)	N/A	0.608	10.068	1.801	0.000	0.078	1.304	0.000	0.188	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	228	77	169	0	89	78	0	48	0
N.S.	1	1.00	0.34	0.74	0.00	0.39	0.34	0.00	0.21	0.00
time (sec)	N/A	0.509	0.009	0.677	0.000	0.076	0.793	0.000	0.166	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	360	95	107	0	0	0	0	35	0
N.S.	1	1.07	0.28	0.32	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.869	10.147	0.487	0.000	0.000	0.000	0.000	0.178	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	569	522	556	0	0	0	0	59	0
N.S.	1	0.98	0.90	0.96	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.786	10.597	0.661	0.000	0.000	0.000	0.000	0.190	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	729	700	332	1018	0	0	0	0	83	0
N.S.	1	0.96	0.46	1.40	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.802	10.884	1.099	0.000	0.000	0.000	0.000	0.775	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	78	160	0	87	73	0	56	0
N.S.	1	1.00	0.62	1.28	0.00	0.70	0.58	0.00	0.45	0.00
time (sec)	N/A	0.493	10.037	0.711	0.000	0.077	0.858	0.000	0.184	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	92	99	0	0	0	0	41	0
N.S.	1	1.00	1.26	1.36	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.359	10.168	0.486	0.000	0.000	0.000	0.000	0.185	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	236	80	175	0	97	83	0	51	0
N.S.	1	1.00	0.34	0.74	0.00	0.41	0.35	0.00	0.22	0.00
time (sec)	N/A	0.480	10.038	0.746	0.000	0.070	0.812	0.000	0.186	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	369	98	110	0	0	0	0	38	0
N.S.	1	1.07	0.28	0.32	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.810	10.151	0.471	0.000	0.000	0.000	0.000	0.185	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	79	0	0	0	0	37	0
N.S.	1	1.00	1.00	1.98	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.318	10.135	0.587	0.000	0.000	0.000	0.000	0.180	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	338	50	86	0	0	0	0	35	0
N.S.	1	1.13	0.17	0.29	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.832	10.110	0.718	0.000	0.000	0.000	0.000	0.171	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	59	78	0	0	0	0	38	0
N.S.	1	1.00	1.48	1.95	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.268	10.136	0.487	0.000	0.000	0.000	0.000	0.179	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	318	65	86	0	0	0	0	35	0
N.S.	1	1.06	0.22	0.29	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.794	10.122	0.484	0.000	0.000	0.000	0.000	0.175	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	567	0	0	0	0	0	0	0	261	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.626	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	499	0	0	0	0	0	0	0	177	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.429	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	458	519	0	0	0	0	0	0	29	0
N.S.	1	1.13	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.996	0.000	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	490	0	0	0	0	0	0	0	40	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.451	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	382	0	0	0	0	0	0	0	51	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.710	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	506	0	0	0	0	0	0	0	1641	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.24	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.198	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	623	0	0	0	0	0	0	0	73	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.925	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	716	0	0	0	0	0	0	0	559	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.100	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	634	0	0	0	0	0	0	0	441	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.815	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	580	0	0	0	0	0	0	0	327	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.680	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	597	0	0	0	0	0	0	0	719	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.148	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	598	0	0	0	0	0	0	0	111	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.402	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	639	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.315	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	580	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	4.788	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	714	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	31.604	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	518	0	0	0	0	0	0	0	300	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.671	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	465	0	0	0	0	0	0	0	68	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.300	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	134	0	0	0	0	0	0	46	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.549	0.000	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	337	0	0	0	0	0	0	0	70	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	464	0	0	0	0	0	0	0	94	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	570	0	0	0	0	0	0	0	118	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.069	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	627	0	0	0	0	0	0	0	1062	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.644	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	538	0	0	0	0	0	0	0	828	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.54	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.083	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	369	0	0	0	0	0	0	0	353	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.96	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.376	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	0	0	0	0	0	0	0	137	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.307	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	390	0	0	0	0	0	0	0	68	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.048	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	464	0	0	0	0	0	0	0	109	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.176	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	584	0	0	0	0	0	0	0	150	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.453	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	719	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	21.640	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	531	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	13.774	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	404	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	9.108	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	411	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	7.751	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	438	0	0	0	0	0	0	0	52	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.713	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	525	0	0	0	0	0	0	0	94	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.776	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	657	0	0	0	0	0	0	0	152	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	10.812	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	793	0	0	0	0	0	0	0	210	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	170.021	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	0	0	0	0	0	0	0	28	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	321	299	106	0	0	0	88	0	0	0
N.S.	1	0.93	0.33	0.00	0.00	0.00	0.27	0.00	0.00	0.00
time (sec)	N/A	0.989	0.111	0.000	0.000	0.000	24.095	0.000	0.238	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	134	75	0	0	0	53	0	929	0
N.S.	1	0.94	0.52	0.00	0.00	0.00	0.37	0.00	6.50	0.00
time (sec)	N/A	0.430	0.070	0.000	0.000	0.000	6.845	0.000	0.225	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	22	0	94	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.50	0.00	2.14	0.93
time (sec)	N/A	0.259	0.026	0.000	0.000	0.000	1.009	0.000	0.205	17.775

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	0	0	0	0	0	0	21	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.458	0.000	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	351	0	0	0	0	0	0	0	32	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.320	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	553	0	0	0	0	0	0	0	43	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.346	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	21	0	21	1249	21
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	65.74	1.11
time (sec)	N/A	0.251	0.148	0.085	0.072	0.099	0.000	0.224	0.292	17.256

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	218	136	0	0	0	167	0	0	0
N.S.	1	1.15	0.72	0.00	0.00	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.788	0.579	0.000	0.000	0.000	58.616	0.000	0.229	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	158	106	0	0	0	119	0	1509	0
N.S.	1	1.11	0.75	0.00	0.00	0.00	0.84	0.00	10.63	0.00
time (sec)	N/A	0.558	0.558	0.000	0.000	0.000	32.908	0.000	0.200	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	75	0	0	0	75	0	535	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.78	0.00	5.57	0.00
time (sec)	N/A	0.366	0.435	0.000	0.000	0.000	16.156	0.000	0.194	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	34	0	94	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	2.14	0.93
time (sec)	N/A	0.267	0.056	0.000	0.000	0.000	3.515	0.000	0.190	17.132

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	21	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.483	0.000	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0	32	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.628	0.000	0.000	0.000	0.000	0.000	0.000	0.261	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	122	86	75	0	0	129	0	0	0
N.S.	1	1.03	0.73	0.64	0.00	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.566	0.856	2.564	0.000	0.000	51.945	0.000	0.209	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	94	65	56	0	0	94	0	1320	0
N.S.	1	1.19	0.82	0.71	0.00	0.00	1.19	0.00	16.71	0.00
time (sec)	N/A	0.429	0.807	1.280	0.000	0.000	28.031	0.000	0.183	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	0	0	61	0	484	0
N.S.	1	1.00	1.00	0.88	0.00	0.00	1.45	0.00	11.52	0.00
time (sec)	N/A	0.288	0.550	0.651	0.000	0.000	13.809	0.000	0.181	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	0	0	29	0	90	15
N.S.	1	1.00	1.00	0.94	0.00	0.00	1.61	0.00	5.00	0.83
time (sec)	N/A	0.235	0.048	0.229	0.000	0.000	3.083	0.000	0.181	0.076

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	0	0	0	0	0	0	21	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.341	0.000	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	0	0	0	0	0	0	24	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.450	0.000	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	31	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.464	0.000	0.000	0.000	0.000	0.000	0.000	0.230	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [321] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	17	0.118
2	A	2	2	1.00	21	0.095
3	A	1	1	1.00	21	0.048
4	A	2	2	1.00	24	0.083
5	A	2	2	1.00	27	0.074
6	A	1	1	1.00	28	0.036
7	A	3	3	1.00	24	0.125
8	A	3	3	1.32	27	0.111
9	A	6	6	1.47	26	0.231
10	F	0	0	N/A	0.000	N/A
11	A	5	5	1.00	19	0.263
12	A	5	5	1.00	19	0.263
13	A	4	4	1.00	21	0.190
14	A	5	5	1.00	25	0.200
15	A	5	5	1.00	26	0.192
16	A	4	4	1.00	28	0.143
17	A	6	6	0.74	25	0.240
18	A	7	7	1.45	26	0.269
19	A	9	9	1.37	27	0.333
20	F	0	0	N/A	0.000	N/A
21	A	3	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	1.00	24	0.125
23	A	3	3	1.00	24	0.125
24	A	2	2	1.00	22	0.091
25	A	7	7	1.04	24	0.292
26	A	9	9	1.13	24	0.375
27	A	7	6	1.00	26	0.231
28	A	4	3	1.00	26	0.115
29	A	5	4	1.00	26	0.154
30	A	9	8	1.10	26	0.308
31	A	11	11	1.58	26	0.423
32	A	11	11	1.55	27	0.407
33	A	11	11	1.55	34	0.324
34	A	15	15	1.49	26	0.577
35	A	14	14	1.56	26	0.538
36	F	0	0	N/A	0.000	N/A
37	A	3	3	1.00	16	0.188
38	A	7	7	1.45	26	0.269
39	B	12	12	3.28	26	0.462
40	A	12	12	1.32	26	0.462
41	A	14	14	1.28	26	0.538
42	A	17	17	1.29	26	0.654
43	A	21	21	1.56	26	0.808
44	A	18	18	1.61	26	0.692
45	F	0	0	N/A	0.000	N/A
46	A	4	4	1.05	16	0.250
47	A	12	12	1.56	26	0.462
48	A	9	9	1.45	26	0.346
49	A	10	10	1.35	26	0.385
50	A	6	6	1.48	26	0.231
51	A	14	14	1.29	26	0.538
52	A	16	16	1.26	26	0.615
53	A	18	18	1.25	26	0.692

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	12	12	1.44	26	0.462
55	A	9	9	1.45	26	0.346
56	A	3	3	1.00	24	0.125
57	A	2	2	1.00	16	0.125
58	A	6	6	1.47	26	0.231
59	A	12	12	1.30	26	0.462
60	A	14	14	1.30	26	0.538
61	A	9	9	1.37	26	0.346
62	A	11	11	1.58	26	0.423
63	F	0	0	N/A	0.000	N/A
64	A	3	3	1.00	16	0.188
65	A	14	14	1.52	26	0.538
66	A	16	16	1.45	26	0.615
67	A	18	18	1.41	26	0.692
68	A	11	11	1.37	26	0.423
69	A	11	11	1.31	26	0.423
70	F	0	0	N/A	0.000	N/A
71	A	4	4	1.07	16	0.250
72	A	18	18	1.60	26	0.692
73	A	19	19	1.54	26	0.731
74	A	22	22	1.50	26	0.846
75	A	2	2	1.00	17	0.118
76	A	4	4	1.06	19	0.211
77	B	3	3	5.04	15	0.200
78	B	1	1	2.42	17	0.059
79	A	3	3	1.00	29	0.103
80	A	3	3	1.00	29	0.103
81	B	12	12	3.24	27	0.444
82	B	12	12	3.19	26	0.462
83	B	12	12	3.24	34	0.353
84	A	16	16	1.49	27	0.593
85	A	14	14	1.56	27	0.519

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	F	0	0	N/A	0.000	N/A
87	A	3	3	1.00	16	0.188
88	A	3	3	1.32	27	0.111
89	A	11	11	1.58	27	0.407
90	A	11	11	1.31	27	0.407
91	A	13	13	1.26	27	0.481
92	A	15	15	1.27	27	0.556
93	A	19	19	1.56	27	0.704
94	A	17	17	1.61	27	0.630
95	F	0	0	N/A	0.000	N/A
96	A	4	4	1.05	16	0.250
97	A	11	11	1.56	27	0.407
98	A	9	9	1.45	27	0.333
99	A	9	9	1.34	27	0.333
100	A	7	7	1.47	27	0.259
101	A	14	14	1.28	27	0.519
102	A	16	16	1.24	27	0.593
103	A	18	18	1.23	27	0.667
104	A	11	11	1.44	27	0.407
105	A	9	9	1.45	27	0.333
106	A	6	6	0.74	25	0.240
107	A	2	2	1.00	16	0.125
108	A	9	9	1.37	27	0.333
109	A	11	11	1.28	27	0.407
110	A	13	13	1.29	27	0.481
111	A	12	12	1.37	27	0.444
112	A	10	10	1.36	27	0.370
113	B	12	12	3.28	27	0.444
114	F	0	0	N/A	0.000	N/A
115	A	3	3	1.00	16	0.188
116	A	14	14	1.49	27	0.519
117	A	16	16	1.44	27	0.593

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	17	17	1.38	27	0.630
119	A	12	12	1.36	27	0.444
120	A	12	12	1.30	27	0.444
121	F	0	0	N/A	0.000	N/A
122	A	4	4	1.07	16	0.250
123	A	18	18	1.61	27	0.667
124	A	19	19	1.53	27	0.704
125	A	21	21	1.49	27	0.778
126	A	5	5	1.00	19	0.263
127	A	6	6	1.13	21	0.286
128	A	1	1	1.36	17	0.059
129	A	1	1	0.73	19	0.053
130	A	12	11	0.88	28	0.393
131	A	9	8	0.90	28	0.286
132	A	6	5	0.96	28	0.179
133	A	5	4	1.13	28	0.143
134	A	7	6	1.11	28	0.214
135	A	5	4	1.20	28	0.143
136	A	6	5	1.16	28	0.179
137	A	15	14	0.83	28	0.500
138	A	13	12	0.83	28	0.429
139	A	10	9	0.84	28	0.321
140	A	7	6	0.86	28	0.214
141	A	6	5	0.98	28	0.179
142	A	9	8	0.98	28	0.286
143	A	9	8	1.06	28	0.286
144	A	6	5	1.17	28	0.179
145	A	7	6	1.14	28	0.214
146	A	14	13	1.11	28	0.464
147	A	15	14	1.10	28	0.500
148	A	2	2	1.00	9	0.222
149	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	11	10	0.96	28	0.357
151	A	8	7	1.02	28	0.250
152	A	5	4	1.13	28	0.143
153	A	4	3	1.59	28	0.107
154	A	4	3	1.44	28	0.107
155	A	5	4	1.20	28	0.143
156	A	9	8	1.16	28	0.286
157	A	10	9	0.92	29	0.310
158	A	8	7	0.96	29	0.241
159	A	5	4	1.08	29	0.138
160	A	2	2	1.00	29	0.069
161	A	5	4	1.06	29	0.138
162	A	8	7	1.05	29	0.241
163	A	10	9	1.05	29	0.310
164	A	12	11	0.91	29	0.379
165	A	10	9	0.93	29	0.310
166	A	7	6	1.00	29	0.207
167	A	3	3	1.24	29	0.103
168	A	3	3	1.08	29	0.103
169	A	9	8	0.98	29	0.276
170	A	11	10	0.99	29	0.345
171	A	13	12	1.00	29	0.414
172	A	14	13	1.00	29	0.448
173	A	2	2	1.00	21	0.095
174	A	2	2	1.00	23	0.087
175	A	4	3	2.00	21	0.143
176	A	4	3	1.76	23	0.130
177	A	4	3	1.90	19	0.158
178	A	4	3	2.00	21	0.143
179	A	2	2	1.43	19	0.105
180	A	2	2	1.48	21	0.095
181	A	2	2	1.60	31	0.065

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	1	1	1.00	11	0.091
183	A	2	2	1.00	21	0.095
184	A	1	1	1.00	22	0.045
185	A	2	2	1.00	22	0.091
186	A	1	1	1.00	17	0.059
187	A	1	1	1.00	24	0.042
188	A	1	1	1.00	24	0.042
189	A	1	1	1.00	17	0.059
190	A	5	4	1.00	19	0.211
191	A	1	1	1.00	19	0.053
192	A	5	4	1.00	21	0.190
193	A	3	3	1.00	15	0.200
194	A	3	3	1.00	23	0.130
195	A	4	4	1.00	23	0.174
196	A	3	3	1.00	15	0.200
197	A	5	4	1.00	17	0.235
198	A	3	3	1.00	17	0.176
199	A	5	4	1.00	19	0.211
200	A	6	6	1.35	19	0.316
201	A	10	9	1.28	17	0.529
202	A	10	9	1.27	18	0.500
203	A	3	3	1.00	18	0.167
204	A	3	3	1.00	19	0.158
205	A	4	3	1.00	17	0.176
206	A	3	3	1.89	17	0.176
207	A	2	2	1.00	17	0.118
208	A	2	2	1.00	17	0.118
209	A	4	3	1.00	27	0.111
210	B	4	4	2.08	28	0.143
211	A	4	3	1.00	21	0.143
212	B	4	4	2.09	22	0.182
213	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	13	13	1.04	20	0.650
215	A	11	11	1.03	20	0.550
216	A	9	9	1.01	20	0.450
217	A	7	7	1.00	20	0.350
218	A	9	9	1.01	20	0.450
219	A	11	11	1.06	20	0.550
220	A	11	11	1.08	19	0.579
221	A	9	9	1.05	19	0.474
222	A	7	7	1.00	19	0.368
223	A	5	5	1.00	19	0.263
224	A	7	7	1.00	19	0.368
225	A	9	9	1.05	19	0.474
226	A	18	18	1.88	22	0.818
227	A	14	14	1.72	22	0.636
228	A	9	9	1.53	22	0.409
229	A	2	2	1.00	22	0.091
230	A	7	7	1.10	22	0.318
231	A	12	12	1.38	22	0.545
232	A	16	16	1.62	22	0.727
233	A	18	18	1.88	23	0.783
234	A	13	13	1.72	23	0.565
235	A	10	10	1.53	23	0.435
236	A	5	5	1.00	23	0.217
237	A	5	5	1.02	23	0.217
238	A	12	12	1.44	23	0.522
239	A	16	16	1.55	23	0.696
240	A	21	21	1.76	23	0.913
241	A	10	10	1.03	19	0.526
242	A	8	8	1.02	19	0.421
243	A	6	6	1.01	19	0.316
244	A	4	4	1.00	19	0.211
245	A	6	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	8	8	1.03	19	0.421
247	A	10	10	1.06	17	0.588
248	A	8	8	1.04	17	0.471
249	A	6	6	1.01	17	0.353
250	A	4	4	1.02	17	0.235
251	A	6	6	1.01	17	0.353
252	A	8	8	1.04	17	0.471
253	A	9	9	1.05	22	0.409
254	A	7	7	1.03	22	0.318
255	A	5	5	1.00	22	0.227
256	A	1	1	1.00	22	0.045
257	A	5	5	0.98	22	0.227
258	A	7	7	1.03	22	0.318
259	A	9	9	1.05	22	0.409
260	A	7	7	1.03	21	0.333
261	A	5	5	1.01	21	0.238
262	A	3	3	1.00	21	0.143
263	A	3	3	1.00	21	0.143
264	A	7	7	1.02	21	0.333
265	A	9	9	1.05	21	0.429
266	F	0	0	N/A	0.000	N/A
267	F	0	0	N/A	0.000	N/A
268	F	0	0	N/A	0.000	N/A
269	A	3	3	1.00	21	0.143
270	F	0	0	N/A	0.000	N/A
271	F	0	0	N/A	0.000	N/A
272	F	0	0	N/A	0.000	N/A
273	F	0	0	N/A	0.000	N/A
274	F	0	0	N/A	0.000	N/A
275	A	6	6	0.71	22	0.273
276	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
277	F	0	0	N/A	0.000	N/A
278	F	0	0	N/A	0.000	N/A
279	F	0	0	N/A	0.000	N/A
280	A	9	9	1.04	23	0.391
281	A	7	7	1.02	23	0.304
282	A	5	5	0.99	23	0.217
283	A	1	1	1.00	23	0.043
284	A	5	5	0.98	23	0.217
285	A	7	7	1.02	23	0.304
286	A	9	9	1.04	23	0.391
287	A	7	7	1.02	22	0.318
288	A	5	5	1.00	22	0.227
289	A	3	3	1.00	22	0.136
290	A	3	3	1.01	22	0.136
291	A	7	7	1.02	22	0.318
292	A	9	9	1.04	22	0.409
293	A	11	11	1.06	22	0.500
294	A	2	2	1.00	17	0.118
295	A	2	2	1.00	17	0.118
296	A	2	2	1.00	17	0.118
297	A	2	2	1.00	15	0.133
298	A	2	2	1.00	17	0.118
299	A	5	5	1.15	17	0.294
300	A	6	6	1.14	17	0.353
301	A	7	7	1.07	17	0.412
302	A	2	2	1.00	19	0.105
303	A	2	2	1.00	19	0.105
304	A	2	2	1.00	17	0.118
305	A	2	2	1.00	9	0.222
306	A	2	2	1.00	19	0.105
307	A	4	4	1.08	19	0.211
308	A	6	6	1.12	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
309	A	10	10	1.13	19	0.526
310	A	9	9	1.11	19	0.474
311	A	2	2	1.24	19	0.105
312	A	2	2	1.33	19	0.105
313	A	2	2	1.35	19	0.105
314	A	10	9	1.28	17	0.529
315	A	9	8	1.49	9	0.889
316	A	2	2	1.31	19	0.105
317	A	2	2	1.35	19	0.105
318	A	14	13	1.21	19	0.684
319	A	14	13	1.31	19	0.684
320	A	12	11	1.28	17	0.647
321	A	10	9	1.49	9	1.000
322	A	2	2	1.86	19	0.105
323	A	2	2	1.76	19	0.105
324	A	7	6	0.95	20	0.300
325	A	7	6	1.03	20	0.300
326	A	5	4	1.09	20	0.200
327	A	7	6	1.01	20	0.300
328	A	7	6	1.14	20	0.300
329	A	3	3	0.99	20	0.150
330	A	4	4	0.93	20	0.200
331	A	5	5	0.92	20	0.250
332	A	11	10	0.91	22	0.455
333	A	10	9	1.01	22	0.409
334	A	8	7	1.14	22	0.318
335	A	8	7	1.14	22	0.318
336	A	10	9	1.11	22	0.409
337	A	10	9	1.06	22	0.409
338	A	12	11	1.12	22	0.500
339	A	9	9	0.94	22	0.409
340	A	9	9	0.89	22	0.409

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	B	15	14	2.51	22	0.636
342	B	13	12	2.15	22	0.545
343	A	11	10	1.82	22	0.455
344	A	9	8	1.65	22	0.364
345	A	6	5	1.00	22	0.227
346	A	4	4	1.27	22	0.182
347	A	5	5	1.65	22	0.227
348	A	6	6	1.92	22	0.273
349	F	0	0	N/A	0.000	N/A
350	F	0	0	N/A	0.000	N/A
351	F	0	0	N/A	0.000	N/A
352	F	0	0	N/A	0.000	N/A
353	F	0	0	N/A	0.000	N/A
354	F	0	0	N/A	0.000	N/A
355	F	0	0	N/A	0.000	N/A
356	F	0	0	N/A	0.000	N/A
357	F	0	0	N/A	0.000	N/A
358	F	0	0	N/A	0.000	N/A
359	F	0	0	N/A	0.000	N/A
360	F	0	0	N/A	0.000	N/A
361	F	0	0	N/A	0.000	N/A
362	F	0	0	N/A	0.000	N/A
363	F	0	0	N/A	0.000	N/A
364	F	0	0	N/A	0.000	N/A
365	A	7	6	0.97	19	0.316
366	A	7	6	1.04	19	0.316
367	A	5	4	1.09	19	0.211
368	A	6	5	1.03	19	0.263
369	A	8	7	1.17	19	0.368
370	A	3	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
371	A	4	4	0.94	19	0.211
372	A	5	5	0.93	19	0.263
373	A	11	10	0.91	21	0.476
374	A	10	9	1.01	21	0.429
375	A	8	7	1.14	21	0.333
376	A	8	7	1.14	21	0.333
377	A	10	9	1.11	21	0.429
378	A	10	9	1.06	21	0.429
379	A	12	11	1.12	21	0.524
380	A	11	11	0.94	21	0.524
381	A	10	10	0.90	21	0.476
382	A	15	14	1.34	21	0.667
383	A	13	12	1.16	21	0.571
384	A	11	10	0.88	21	0.476
385	A	9	8	0.92	21	0.381
386	A	6	5	0.41	21	0.238
387	A	4	4	0.61	21	0.190
388	A	5	5	0.78	21	0.238
389	F	0	0	N/A	0.000	N/A
390	F	0	0	N/A	0.000	N/A
391	F	0	0	N/A	0.000	N/A
392	F	0	0	N/A	0.000	N/A
393	F	0	0	N/A	0.000	N/A
394	F	0	0	N/A	0.000	N/A
395	F	0	0	N/A	0.000	N/A
396	F	0	0	N/A	0.000	N/A
397	F	0	0	N/A	0.000	N/A
398	F	0	0	N/A	0.000	N/A
399	F	0	0	N/A	0.000	N/A
400	F	0	0	N/A	0.000	N/A
401	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
402	F	0	0	N/A	0.000	N/A
403	F	0	0	N/A	0.000	N/A
404	F	0	0	N/A	0.000	N/A
405	A	11	11	1.00	22	0.500
406	A	9	9	1.03	22	0.409
407	A	7	7	1.00	20	0.350
408	A	2	2	1.00	12	0.167
409	A	2	2	1.00	22	0.091
410	A	12	12	0.94	22	0.545
411	A	13	13	0.98	22	0.591
412	A	14	14	1.01	22	0.636
413	A	11	11	1.12	22	0.500
414	A	10	10	1.17	22	0.455
415	A	9	9	1.01	20	0.450
416	A	3	3	1.00	12	0.250
417	A	14	14	0.96	22	0.636
418	A	2	2	1.44	22	0.091
419	A	2	2	1.90	22	0.091
420	B	2	2	2.38	22	0.091
421	A	8	8	1.00	21	0.381
422	A	7	7	0.98	21	0.333
423	A	5	5	1.03	21	0.238
424	A	4	4	1.00	19	0.211
425	A	4	4	1.07	21	0.190
426	A	9	9	0.98	21	0.429
427	A	11	11	0.96	21	0.524
428	A	7	7	1.00	21	0.333
429	A	2	2	1.00	23	0.087
430	A	4	4	1.00	22	0.182
431	A	4	4	1.07	24	0.167
432	A	2	2	1.00	21	0.095
433	A	4	4	1.13	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
434	A	1	1	1.00	22	0.045
435	A	4	4	1.06	21	0.190
436	F	0	0	N/A	0.000	N/A
437	F	0	0	N/A	0.000	N/A
438	A	17	16	1.13	24	0.667
439	F	0	0	N/A	0.000	N/A
440	F	0	0	N/A	0.000	N/A
441	F	0	0	N/A	0.000	N/A
442	F	0	0	N/A	0.000	N/A
443	F	0	0	N/A	0.000	N/A
444	F	0	0	N/A	0.000	N/A
445	F	0	0	N/A	0.000	N/A
446	F	0	0	N/A	0.000	N/A
447	F	0	0	N/A	0.000	N/A
448	F	0	0	N/A	0.000	N/A
449	F	0	0	N/A	0.000	N/A
450	F	0	0	N/A	0.000	N/A
451	F	0	0	N/A	0.000	N/A
452	F	0	0	N/A	0.000	N/A
453	F	0	0	N/A	0.000	N/A
454	A	6	5	0.98	24	0.208
455	F	0	0	N/A	0.000	N/A
456	F	0	0	N/A	0.000	N/A
457	F	0	0	N/A	0.000	N/A
458	F	0	0	N/A	0.000	N/A
459	F	0	0	N/A	0.000	N/A
460	F	0	0	N/A	0.000	N/A
461	F	0	0	N/A	0.000	N/A
462	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
463	F	0	0	N/A	0.000	N/A
464	F	0	0	N/A	0.000	N/A
465	F	0	0	N/A	0.000	N/A
466	F	0	0	N/A	0.000	N/A
467	F	0	0	N/A	0.000	N/A
468	F	0	0	N/A	0.000	N/A
469	F	0	0	N/A	0.000	N/A
470	F	0	0	N/A	0.000	N/A
471	F	0	0	N/A	0.000	N/A
472	F	0	0	N/A	0.000	N/A
473	F	0	0	N/A	0.000	N/A
474	A	6	6	0.93	19	0.316
475	A	4	4	0.94	17	0.235
476	A	2	2	1.00	9	0.222
477	A	4	4	1.00	19	0.211
478	F	0	0	N/A	0.000	N/A
479	F	0	0	N/A	0.000	N/A
480	N/A	1	0	1.00	19	0.000
481	A	3	3	1.15	19	0.158
482	A	4	4	1.11	19	0.211
483	A	2	2	1.00	17	0.118
484	A	2	2	1.00	9	0.222
485	A	2	2	1.00	19	0.105
486	A	2	2	1.00	19	0.105
487	A	3	3	1.03	19	0.158
488	A	4	4	1.19	19	0.211
489	A	2	2	1.00	17	0.118
490	A	1	1	1.00	9	0.111
491	A	2	2	1.00	19	0.105
492	A	2	2	1.00	19	0.105
493	A	2	2	1.00	19	0.105

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 4

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx = E(\arcsin(x)|-1)$$

output
```
EllipticE(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 9.00

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right) + \frac{1}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^4\right)$$

input
```
Integrate[(1 + x^2)/Sqrt[1 - x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, x^4] + (x^3*Hypergeometric2F1[1/2, 3/4,
 7/4, x^4])/3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{1388, 327\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$E(\arcsin(x)|-1)$$

input
```
Int[(1 + x^2)/Sqrt[1 - x^4],x]
```

output
```
EllipticE[ArcSin[x], -1]
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 6.75

method	result	size
meijerg	$\frac{x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], x^{4}\right)$	27
default	$\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x , i\right)}{\sqrt{-x^{4}+1}}-\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(x , i\right)-\operatorname{EllipticE}\left(x , i\right)\right)}{\sqrt{-x^{4}+1}}$	70
elliptic	$\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x , i\right)}{\sqrt{-x^{4}+1}}-\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(x , i\right)-\operatorname{EllipticE}\left(x , i\right)\right)}{\sqrt{-x^{4}+1}}$	70

input
```
int((x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*hypergeom([1/2,3/4],[7/4],x^4)+x*hypergeom([1/4,1/2],[5/4],x^4)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(3) = 6.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 8.50

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx = \frac{-i\,xE\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right) + 2i\,xF\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right) - \sqrt{-x^4+1}}{x}$$

input
```
integrate((x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(-I*x*elliptic_e(arcsin(1/x), -1) + 2*I*x*elliptic_f(arcsin(1/x), -1) - sq
rt(-x^4 + 1))/x
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(2) = 4$.

Time = 0.65 (sec) , antiderivative size = 61, normalized size of antiderivative = 15.25

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((x**2+1)/(-x**4+1)**(1/2),x)
```

output
```
x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(2*I*pi))/(4*gamma
(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(2*I*pi))/(4
*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx = \int \frac{x^2+1}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx = \int \frac{x^2+1}{\sqrt{-x^4+1}}\,dx$$

input

```
integrate((x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output

```
integrate((x^2 + 1)/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx = \int \frac{x^2+1}{\sqrt{1-x^4}}\,dx$$

input

```
int((x^2 + 1)/(1 - x^4)^(1/2),x)
```

output

```
int((x^2 + 1)/(1 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4+1}}{x^2-1}dx\right)$$

input

```
int((x^2+1)/(-x^4+1)^(1/2),x)
```

output

```
 - int(sqrt( - x**4 + 1)/(x**2 - 1),x)
```

3.2 $\int \frac{\sqrt{1-x^4}}{1-x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 4

$$\int \frac{\sqrt{1-x^4}}{1-x^2}\,dx = E(\arcsin(x)|-1)$$

output
```
EllipticE(x,I)
```

Mathematica [A] (verified)

Time = 10.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-x^4}}{1-x^2}\,dx = E(\arcsin(x)|-1)$$

input
```
Integrate[Sqrt[1 - x^4]/(1 - x^2),x]
```

output
```
EllipticE[ArcSin[x], -1]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{1388, 327\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^4}}{1-x^2}\,dx$$

$$\Big\downarrow \; 1388$$

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \; 327$$

$$E(\arcsin(x)|-1)$$

input

```
Int[Sqrt[1 - x^4]/(1 - x^2),x]
```

output

```
EllipticE[ArcSin[x], -1]
```

Defintions of rubi rules used

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(4) = 8$.

Time = 1.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 17.50

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\ \mathrm{EllipticF}(x,i)}{\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i))}{\sqrt{-x^4+1}}$	70
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\ \mathrm{EllipticF}(x,i)}{\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i))}{\sqrt{-x^4+1}}$	70

input
```
int((-x^4+1)^(1/2)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*EllipticF(x,I)-(-x^2+1)^(1/2)*
(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(EllipticF(x,I)-EllipticE(x,I))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(3) = 6$.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 8.50

$$\int \frac{\sqrt{1-x^4}}{1-x^2}\,dx = \frac{-i\,xE\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right)+2i\,xF\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right)-\sqrt{-x^4+1}}{x}$$

input
```
integrate((-x^4+1)^(1/2)/(-x^2+1),x, algorithm="fricas")
```

output
```
(-I*x*elliptic_e(arcsin(1/x), -1) + 2*I*x*elliptic_f(arcsin(1/x), -1) - sq
rt(-x^4 + 1))/x
```

Sympy [F]

$$\int \frac{\sqrt{1-x^4}}{1-x^2}\,dx = -\int \frac{\sqrt{1-x^4}}{x^2-1}\,dx$$

input
```
integrate((-x**4+1)**(1/2)/(-x**2+1),x)
```

output
```
-Integral(sqrt(1 - x**4)/(x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^4}}{1-x^2}\,dx = \int -\frac{\sqrt{-x^4+1}}{x^2-1}\,dx$$

input
```
integrate((-x^4+1)^(1/2)/(-x^2+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-x^4 + 1)/(x^2 - 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^4}}{1-x^2}\,dx = \int -\frac{\sqrt{-x^4+1}}{x^2-1}\,dx$$

input
```
integrate((-x^4+1)^(1/2)/(-x^2+1),x, algorithm="giac")
```

output
```
integrate(-sqrt(-x^4 + 1)/(x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^4}}{1-x^2}\,dx = -\int \frac{\sqrt{1-x^4}}{x^2-1}\,dx$$

input

```
int(-(1 - x^4)^(1/2)/(x^2 - 1),x)
```

output

```
-int((1 - x^4)^(1/2)/(x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-x^4}}{1-x^2}\,dx = -\left(\int \frac{\sqrt{-x^4+1}}{x^2-1}dx\right)$$

input

```
int((-x^4+1)^(1/2)/(-x^2+1),x)
```

output

```
 - int(sqrt( - x**4 + 1)/(x**2 - 1),x)
```

3.3 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 4

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = E(\arcsin(x)|-1)$$

output

```
EllipticE(x,I)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = E(\arcsin(x)|-1)$$

input

```
Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^2],x]
```

output

```
EllipticE[ArcSin[x], -1]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{327\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}\,dx$$
$$\Big\downarrow \; 327$$
$$E(\arcsin(x)|-1)$$

input
```
Int[Sqrt[1 + x^2]/Sqrt[1 - x^2],x]
```

output
```
EllipticE[ArcSin[x], -1]
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$\operatorname{EllipticE}\left(x, i\right)$	5
elliptic	$\frac{\sqrt{-x^{4}+1}\,\left(\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}\left(x,i\right)}{\sqrt{-x^{4}+1}}-\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\left(\operatorname{EllipticF}\left(x,i\right)-\operatorname{EllipticE}\left(x,i\right)\right)}{\sqrt{-x^{4}+1}}\right)}{\sqrt{x^{2}+1}\,\sqrt{-x^{2}+1}}$	96

input
```
int((x^2+1)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticE(x,I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(3) = 6.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 10.25

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \frac{-i\,xE\left(\arcsin\left(\frac{1}{x}\right)\mid -1\right) + 2i\,xF\left(\arcsin\left(\frac{1}{x}\right)\mid -1\right) - \sqrt{x^2+1}\sqrt{-x^2+1}}{x}$$

input
```
integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(-I*x*elliptic_e(arcsin(1/x), -1) + 2*I*x*elliptic_f(arcsin(1/x), -1) - sq
rt(x^2 + 1)*sqrt(-x^2 + 1))/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. 2(2) = 4.

Time = 1.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \begin{cases} E(\operatorname{asin}(x)|-1) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

input
```
integrate((x**2+1)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Piecewise((elliptic_e(asin(x), -1), (x > -1) & (x < 1)))
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-x^2+1}}\,dx$$

input

```
integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output

```
integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-x^2+1}}\,dx$$

input

```
integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output

```
integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}\,dx$$

input

```
int((x^2 + 1)^(1/2)/(1 - x^2)^(1/2),x)
```

output

```
int((x^2 + 1)^(1/2)/(1 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = -\left(\int \frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}}{x^2-1}dx\right)$$

input

```
int((x^2+1)^(1/2)/(-x^2+1)^(1/2),x)
```

output

```
 - int((sqrt( - x**2 + 1)*sqrt(x**2 + 1))/(x**2 - 1),x)
```

3.4 $\int \frac{1+e^2x^2}{\sqrt{1-e^4x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 10

$$\int \frac{1+e^2x^2}{\sqrt{1-e^4x^4}}\,dx=\frac{E(\arcsin(ex)|-1)}{e}$$

output
```
EllipticE(e*x,I)/e
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 4.70

$$\int \frac{1+e^2x^2}{\sqrt{1-e^4x^4}}\,dx=x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},e^4x^4\right)+\frac{1}{3}e^2x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e^4x^4\right)$$

input
```
Integrate[(1 + e^2*x^2)/Sqrt[1 - e^4*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, e^4*x^4] + (e^2*x^3*Hypergeometric2F1[1
/2, 3/4, 7/4, e^4*x^4])/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1388, 327\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^2x^2+1}{\sqrt{1-e^4x^4}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{\sqrt{e^2x^2+1}}{\sqrt{1-e^2x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$\frac{E(\arcsin(ex)|-1)}{e}$$

input
```
Int[(1 + e^2*x^2)/Sqrt[1 - e^4*x^4],x]
```

output
```
EllipticE[ArcSin[e*x], -1]/e
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.87 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.80

method	result	size
meijerg	$\frac{e^2x^3 \text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],e^4x^4\right)}{3}+x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],e^4x^4\right)$	38
default	$\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\,\text{EllipticF}\left(x\sqrt{e^2},i\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}-\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\,\left(\text{EllipticF}\left(x\sqrt{e^2},i\right)-\text{EllipticE}\left(x\sqrt{e^2},i\right)\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}$	118
elliptic	$\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\,\text{EllipticF}\left(x\sqrt{e^2},i\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}-\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\,\left(\text{EllipticF}\left(x\sqrt{e^2},i\right)-\text{EllipticE}\left(x\sqrt{e^2},i\right)\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}$	118

input
```
int((e^2*x^2+1)/(-e^4*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*e^2*x^3*hypergeom([1/2,3/4],[7/4],e^4*x^4)+x*hypergeom([1/4,1/2],[5/4]
,e^4*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(9) = 18.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 6.50

$$\int \frac{1+e^2x^2}{\sqrt{1-e^4x^4}}\,dx$$
$$=-\frac{\sqrt{-e^4x^4+1}e^3-\sqrt{-e^4}\left((e^2+1)xF\left(\arcsin\left(\frac{1}{ex}\right)|-1\right)-xE\left(\arcsin\left(\frac{1}{ex}\right)|-1\right)\right)}{e^5x}$$

input
```
integrate((e^2*x^2+1)/(-e^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-e^4*x^4 + 1)*e^3 - sqrt(-e^4)*((e^2 + 1)*x*elliptic_f(arcsin(1/(e*
x)), -1) - x*elliptic_e(arcsin(1/(e*x)), -1)))/(e^5*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(5) = 10$.

Time = 0.79 (sec) , antiderivative size = 71, normalized size of antiderivative = 7.10

$$\int \frac{1+e^2x^2}{\sqrt{1-e^4x^4}}\,dx = \frac{e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|e^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|e^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((e**2*x**2+1)/(-e**4*x**4+1)**(1/2),x)
```

output
```
e**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**4*x**4*exp_polar(2*I*pi)
)/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**4*x**4*exp_po
lar(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+e^2x^2}{\sqrt{1-e^4x^4}}\,dx = \int \frac{e^2x^2+1}{\sqrt{-e^4x^4+1}}\,dx$$

input
```
integrate((e^2*x^2+1)/(-e^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e^2*x^2 + 1)/sqrt(-e^4*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+e^2x^2}{\sqrt{1-e^4x^4}}\,dx=\int \frac{e^2x^2+1}{\sqrt{-e^4x^4+1}}\,dx$$

input
```
integrate((e^2*x^2+1)/(-e^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((e^2*x^2 + 1)/sqrt(-e^4*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+e^2x^2}{\sqrt{1-e^4x^4}}\,dx=\int \frac{e^2\,x^2+1}{\sqrt{1-e^4\,x^4}}\,dx$$

input
```
int((e^2*x^2 + 1)/(1 - e^4*x^4)^(1/2),x)
```

output
```
int((e^2*x^2 + 1)/(1 - e^4*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+e^2x^2}{\sqrt{1-e^4x^4}}\,dx=-\left(\int \frac{\sqrt{-e^4x^4+1}}{e^2x^2-1}dx\right)$$

input
```
int((e^2*x^2+1)/(-e^4*x^4+1)^(1/2),x)
```

output
```
- int(sqrt( - e**4*x**4 + 1)/(e**2*x**2 - 1),x)
```

3.5 $\int \frac{\sqrt{1-e^4x^4}}{1-e^2x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 10

$$\int \frac{\sqrt{1-e^4x^4}}{1-e^2x^2}\,dx = \frac{E(\arcsin(ex)|-1)}{e}$$

output
```
EllipticE(e*x,I)/e
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.10

$$\int \frac{\sqrt{1-e^4x^4}}{1-e^2x^2}\,dx = -\frac{iE\left(i\operatorname{arcsinh}\left(\sqrt{-e^2}x\right)\middle|-1\right)}{\sqrt{-e^2}}$$

input
```
Integrate[Sqrt[1 - e^4*x^4]/(1 - e^2*x^2),x]
```

output
```
((-I)*EllipticE[I*ArcSinh[Sqrt[-e^2]*x], -1])/Sqrt[-e^2]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-e^4x^4}}{1-e^2x^2}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{\sqrt{e^2x^2+1}}{\sqrt{1-e^2x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$\frac{E(\arcsin(ex)|-1)}{e}$$

input
```
Int[Sqrt[1 - e^4*x^4]/(1 - e^2*x^2),x]
```

output
```
EllipticE[ArcSin[e*x], -1]/e
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(10) = 20$.

Time = 2.50 (sec) , antiderivative size = 118, normalized size of antiderivative = 11.80

method	result	size
default	$\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{e^2},i\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}-\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{e^2},i\right)-\mathrm{EllipticE}\left(x\sqrt{e^2},i\right)\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}$	118
elliptic	$\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{e^2},i\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}-\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{e^2},i\right)-\mathrm{EllipticE}\left(x\sqrt{e^2},i\right)\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}$	118

input

```
int((-e^4*x^4+1)^(1/2)/(-e^2*x^2+1),x,method=_RETURNVERBOSE)
```

output

```
1/(e^2)^(1/2)*(-e^2*x^2+1)^(1/2)*(e^2*x^2+1)^(1/2)/(-e^4*x^4+1)^(1/2)*Elli
pticF(x*(e^2)^(1/2),I)-1/(e^2)^(1/2)*(-e^2*x^2+1)^(1/2)*(e^2*x^2+1)^(1/2)/
(-e^4*x^4+1)^(1/2)*(EllipticF(x*(e^2)^(1/2),I)-EllipticE(x*(e^2)^(1/2),I))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(9) = 18$.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 6.50

$$\int \frac{\sqrt{1-e^4x^4}}{1-e^2x^2}\,dx$$
$$= -\frac{\sqrt{-e^4x^4+1}e^3 - \sqrt{-e^4}\left((e^2+1)xF\left(\arcsin\left(\frac{1}{ex}\right)|-1\right) - xE\left(\arcsin\left(\frac{1}{ex}\right)|-1\right)\right)}{e^5x}$$

input

```
integrate((-e^4*x^4+1)^(1/2)/(-e^2*x^2+1),x, algorithm="fricas")
```

output

```
-(sqrt(-e^4*x^4 + 1)*e^3 - sqrt(-e^4)*((e^2 + 1)*x*elliptic_f(arcsin(1/(e*
x)), -1) - x*elliptic_e(arcsin(1/(e*x)), -1)))/(e^5*x)
```

Sympy [F]

$$\int \frac{\sqrt{1-e^4x^4}}{1-e^2x^2}\,dx = -\int \frac{\sqrt{-e^4x^4+1}}{e^2x^2-1}\,dx$$

input

```
integrate((-e**4*x**4+1)**(1/2)/(-e**2*x**2+1),x)
```

output

```
-Integral(sqrt(-e**4*x**4 + 1)/(e**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-e^4x^4}}{1-e^2x^2}\,dx = \int -\frac{\sqrt{-e^4x^4+1}}{e^2x^2-1}\,dx$$

input

```
integrate((-e^4*x^4+1)^(1/2)/(-e^2*x^2+1),x, algorithm="maxima")
```

output

```
-integrate(sqrt(-e^4*x^4 + 1)/(e^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-e^4x^4}}{1-e^2x^2}\,dx = \int -\frac{\sqrt{-e^4x^4+1}}{e^2x^2-1}\,dx$$

input

```
integrate((-e^4*x^4+1)^(1/2)/(-e^2*x^2+1),x, algorithm="giac")
```

output

```
integrate(-sqrt(-e^4*x^4 + 1)/(e^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-e^4x^4}}{1-e^2x^2}\,dx = -\int \frac{\sqrt{1-e^4\,x^4}}{e^2\,x^2-1}\,dx$$

input
```
int(-(1 - e^4*x^4)^(1/2)/(e^2*x^2 - 1),x)
```

output
```
-int((1 - e^4*x^4)^(1/2)/(e^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-e^4x^4}}{1-e^2x^2}\,dx = -\left(\int \frac{\sqrt{-e^4x^4+1}}{e^2x^2-1}dx\right)$$

input
```
int((-e^4*x^4+1)^(1/2)/(-e^2*x^2+1),x)
```

output
```
 - int(sqrt( - e**4*x**4 + 1)/(e**2*x**2 - 1),x)
```

3.6 $\int \frac{\sqrt{1+e^2x^2}}{\sqrt{1-e^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 10

$$\int \frac{\sqrt{1+e^2x^2}}{\sqrt{1-e^2x^2}}\,dx = \frac{E(\arcsin(ex)|-1)}{e}$$

output

```
EllipticE(e*x,I)/e
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+e^2x^2}}{\sqrt{1-e^2x^2}}\,dx = \frac{E(\arcsin(ex)|-1)}{e}$$

input

```
Integrate[Sqrt[1 + e^2*x^2]/Sqrt[1 - e^2*x^2],x]
```

output

```
EllipticE[ArcSin[e*x], -1]/e
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e^2x^2+1}}{\sqrt{1-e^2x^2}}\,dx$$
$$\Big\downarrow \; 327$$
$$\frac{E(\arcsin(ex)|-1)}{e}$$

input
```
Int[Sqrt[1 + e^2*x^2]/Sqrt[1 - e^2*x^2],x]
```

output
```
EllipticE[ArcSin[e*x], -1]/e
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\operatorname{EllipticE}(x\ \operatorname{csgn}(e)e,i)\operatorname{csgn}(e)}{e}$	15
elliptic	$\frac{\sqrt{-e^4x^4+1}\left(\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\ \operatorname{EllipticF}\left(x\sqrt{e^2},i\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}-\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\left(\operatorname{EllipticF}\left(x\sqrt{e^2},i\right)-\operatorname{EllipticE}\left(x\sqrt{e^2},i\right)\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}\right)}{\sqrt{e^2x^2+1}\,\sqrt{-e^2x^2+1}}$	154

input
```
int((e^2*x^2+1)^(1/2)/(-e^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticE(x*csgn(e)*e,I)*csgn(e)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(9) = 18$.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 7.60

$$\int \frac{\sqrt{1+e^2x^2}}{\sqrt{1-e^2x^2}}\,dx = -\frac{\sqrt{e^2x^2+1}\sqrt{-e^2x^2+1}e^3 - \sqrt{-e^4}\left((e^2+1)xF\left(\arcsin\left(\frac{1}{ex}\right)|-1\right) - xE\left(\arcsin\left(\frac{1}{ex}\right)|-1\right)\right)}{e^5x}$$

input
```
integrate((e^2*x^2+1)^(1/2)/(-e^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(e^2*x^2 + 1)*sqrt(-e^2*x^2 + 1)*e^3 - sqrt(-e^4)*((e^2 + 1)*x*ellip
tic_f(arcsin(1/(e*x)), -1) - x*elliptic_e(arcsin(1/(e*x)), -1)))/(e^5*x)
```

Sympy [F]

$$\int \frac{\sqrt{1+e^2x^2}}{\sqrt{1-e^2x^2}}\,dx = \int \frac{\sqrt{e^2x^2+1}}{\sqrt{-(ex-1)(ex+1)}}\,dx$$

input
```
integrate((e**2*x**2+1)**(1/2)/(-e**2*x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(e**2*x**2 + 1)/sqrt(-(e*x - 1)*(e*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+e^2x^2}}{\sqrt{1-e^2x^2}}\,dx = \int \frac{\sqrt{e^2x^2+1}}{\sqrt{-e^2x^2+1}}\,dx$$

input
```
integrate((e^2*x^2+1)^(1/2)/(-e^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e^2*x^2 + 1)/sqrt(-e^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+e^2x^2}}{\sqrt{1-e^2x^2}}\,dx = \int \frac{\sqrt{e^2x^2+1}}{\sqrt{-e^2x^2+1}}\,dx$$

input
```
integrate((e^2*x^2+1)^(1/2)/(-e^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e^2*x^2 + 1)/sqrt(-e^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+e^2x^2}}{\sqrt{1-e^2x^2}}\,dx = \int \frac{\sqrt{e^2\,x^2+1}}{\sqrt{1-e^2\,x^2}}\,dx$$

input
```
int((e^2*x^2 + 1)^(1/2)/(1 - e^2*x^2)^(1/2),x)
```

output
```
int((e^2*x^2 + 1)^(1/2)/(1 - e^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1+e^2x^2}}{\sqrt{1-e^2x^2}}\,dx = -\left(\int \frac{\sqrt{e^2x^2+1}\,\sqrt{-e^2x^2+1}}{e^2x^2-1}dx\right)$$

input

```
int((e^2*x^2+1)^(1/2)/(-e^2*x^2+1)^(1/2),x)
```

output

```
 - int((sqrt(e**2*x**2 + 1)*sqrt( - e**2*x**2 + 1))/(e**2*x**2 - 1),x)
```

3.7 $\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 59

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2
*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx$$
$$=\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{e^2x^4}{d^2}\right)\right)}{3\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[d^2 - e^2*x^4],x]
```

output

```
(Sqrt[1 - (e^2*x^4)/d^2]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (e^2*x^4)
/d^2] + e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(3*Sqrt[d^
2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{ 1390}$$

$$\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{ex^2+d}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d + e*x^2)/Sqrt[d^2 - e^2*x^4],x]
```

output

```
(d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1
])/(Sqrt[e]*Sqrt[d^2 - e^2*x^4])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(47) = 94$.

Time = 2.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.34

method	result	size
default	$\frac{d\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}} - \frac{d\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$	138
elliptic	$\frac{d\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}} - \frac{d\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$	138

input
```
int((e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*Ell
ipticF(x*(e/d)^(1/2),I)-d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/
(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx = -\frac{\sqrt{-e^2}dx\sqrt{\frac{d}{e}}E\left(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1\right)-\sqrt{-e^2}(d+e)x\sqrt{\frac{d}{e}}F\left(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1\right)+\sqrt{-e^2x^4+d^2}e}{e^2x}$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-e^2)*d*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - sqrt(-e^2
)*(d + e)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1) + sqrt(-e^2*x^4
+ d^2)*e)/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**2*x**4*exp_
polar(2*I*pi)/d**2)/(4*d*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{d^2-e^2\,x^4}}\,dx$$

input
```
int((d + e*x^2)/(d^2 - e^2*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(d^2 - e^2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{-e\,x^2+d}dx$$

input

```
int((e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d - e*x**2),x)
```

3.8 $\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 59

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}\,dx=\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2
*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}\,dx=\frac{ie\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)}{\left(-\frac{e}{d}\right)^{3/2}\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d - e*x^2),x]
```

output
```
(I*e*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1])/((-
(e/d))^(3/2)*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1396, 329, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}\,dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 329}$$

$$\frac{d\sqrt{d^2-e^2x^4}\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{(d-ex^2)(d+ex^2)}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{d^{3/2}\sqrt{d^2-e^2x^4}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}(d-ex^2)(d+ex^2)}$$

input
```
Int[Sqrt[d^2 - e^2*x^4]/(d - e*x^2),x]
```

output
```
(d^(3/2)*Sqrt[d^2 - e^2*x^4]*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqr
t[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*(d - e*x^2)*(d + e*x^2))
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(47) = 94$.

Time = 1.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.34

method	result	size
default	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	138
elliptic	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	138

input
```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d),x,method=_RETURNVERBOSE)
```

output

```
d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*Ell
ipticF(x*(e/d)^(1/2),I)-d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/
(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}\,dx = -\frac{\sqrt{-e^2}dx\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)-\sqrt{-e^2}(d+e)x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)+\sqrt{-e^2x^4+d^2}e}{e^2x}$$

input

```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d),x, algorithm="fricas")
```

output

```
-(sqrt(-e^2)*d*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - sqrt(-e^2
)*(d + e)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1) + sqrt(-e^2*x^4
+ d^2)*e)/(e^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}\,dx = -\int \frac{\sqrt{d^2-e^2x^4}}{-d+ex^2}\,dx$$

input

```
integrate((-e**2*x**4+d**2)**(1/2)/(-e*x**2+d),x)
```

output

```
-Integral(sqrt(d**2 - e**2*x**4)/(-d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d^2 - e^2x^4}}{d - ex^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{d^2 - e^2x^4}}{d - ex^2} \, dx = \int -\frac{\sqrt{-e^2x^4 + d^2}}{ex^2 - d} \, dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d),x, algorithm="giac")
```

output
```
integrate(-sqrt(-e^2*x^4 + d^2)/(e*x^2 - d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2 - e^2x^4}}{d - ex^2} \, dx = \int \frac{\sqrt{d^2 - e^2 x^4}}{d - e x^2} \, dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2),x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{-e\,x^2+d}dx$$

input
```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d - e*x**2),x)
```

3.9 $\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \frac{x\sqrt{d^2-e^2x^4}}{2d^2(d+ex^2)} + \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
1/2*x*(-e^2*x^4+d^2)^(1/2)/d^2/(e*x^2+d)+1/2*(1-e^2*x^4/d^2)^(1/2)*Ellipti
cE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \frac{\sqrt{-\frac{e}{d}}\,x(d-ex^2) - id\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(i\,\text{arcsinh}\left(\sqrt{-\frac{e}{d}}\,x\right)\middle|-1\right)}{2d^2\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[1/((d + e*x^2)*Sqrt[d^2 - e^2*x^4]),x]
```

output

```
(Sqrt[-(e/d)]*x*(d - e*x^2) - I*d*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcS
inh[Sqrt[-(e/d)]*x], -1])/(2*d^2*Sqrt[-(e/d)]*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.47, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1396, 316, 25, 27, 329, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \ 1396$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 316$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d-ex^2}}{2d^2\sqrt{d+ex^2}}-\frac{\int -\frac{e\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d^2e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d^2e}+\frac{x\sqrt{d-ex^2}}{2d^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d^2}+\frac{x\sqrt{d-ex^2}}{2d^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{2d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{2d^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{2d^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[1/((d + e*x^2)*Sqrt[d^2 - e^2*x^4]),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d - e*x^2])/(2*d^2*Sqrt[d + e*x^
2]) + (Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])
/(2*Sqrt[d]*Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(78) = 156$.

Time = 1.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.96

method	result
default	$\frac{\left(-e^{2} x^{2}+d e\right) x}{2 d^{2} e \sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2} x^{2}+d e\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)}{2 d \sqrt{\frac{e}{d}}\,\sqrt{-e^{2} x^{4}+d^{2}}}-\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}(\right.}{2 d \sqrt{\frac{e}{d}}\,\sqrt{-e^{2} x^{4}+d^{2}}}$
elliptic	$\frac{\left(-e^{2} x^{2}+d e\right) x}{2 d^{2} e \sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2} x^{2}+d e\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)}{2 d \sqrt{\frac{e}{d}}\,\sqrt{-e^{2} x^{4}+d^{2}}}-\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}(\right.}{2 d \sqrt{\frac{e}{d}}\,\sqrt{-e^{2} x^{4}+d^{2}}}$

input
```
int(1/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e^2*x^2+d*e)/d^2*x/e/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/2/d/(e/d)^(1
/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(
e/d)^(1/2),I)-1/2/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*
x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx$$
$$= \frac{\sqrt{-e^2x^4+d^2}\,ex + (e^2x^2+de)\sqrt{\frac{e}{d}}E(\arcsin(x\sqrt{\frac{e}{d}})\,|-1) + ((de-e^2)x^2+d^2-de)\sqrt{\frac{e}{d}}F(\arcsin(x\sqrt{\frac{e}{d}})}{2\,(d^2e^2x^2+d^3e)}$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-e^2*x^4 + d^2)*e*x + (e^2*x^2 + d*e)*sqrt(e/d)*elliptic_e(arcsi
n(x*sqrt(e/d)), -1) + ((d*e - e^2)*x^2 + d^2 - d*e)*sqrt(e/d)*elliptic_f(a
rcsin(x*sqrt(e/d)), -1))/(d^2*e^2*x^2 + d^3*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-(-d+ex^2)(d+ex^2)}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x**2)*(d + e*x**2))*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2-e^2\,x^4}\,(e\,x^2+d)}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^3x^6-d\,e^2x^4+d^2e\,x^2+d^3}dx$$

input
```
int(1/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**3 + d**2*e*x**2 - d*e**2*x**4 - e**3*x**6),
x)
```

3.10 $\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{x\left(d-ex^2\right)}{2d^2\sqrt{d^2-e^2x^4}}+\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
1/2*x*(-e*x^2+d)/d^2/(-e^2*x^4+d^2)^(1/2)+1/2*(1-e^2*x^4/d^2)^(1/2)*Ellipt
icE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{3dx+3dx\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-2ex^3\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hyperge}}{6d^2\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[(d - e*x^2)/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(3*d*x + 3*d*x*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (e
^2*x^4)/d^2] - 2*e*x^3*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4, 3/2,
 7/4, (e^2*x^4)/d^2])/(6*d^2*Sqrt[d^2 - e^2*x^4])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{3/2}}dx$$

input
```
Int[(d - e*x^2)/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(77) = 154.

Time = 2.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.98

method	result
elliptic	$\frac{(-e^2x^2+de)x}{2d^2e\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{2d\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\right.}{2d\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$
default	$d\left(\frac{x}{2d^2\sqrt{-\left(x^4-\frac{d^2}{e^2}\right)e^2}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{2d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)-e\left(\frac{x^3}{2d^2\sqrt{-\left(x^4-\frac{d^2}{e^2}\right)e^2}}+\right.$

input
```
int((-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e^2*x^2+d*e)/d^2*x/e/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/2/d/(e/d)^(1
/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(
e/d)^(1/2),I)-1/2/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*
x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{\sqrt{-e^2x^4+d^2}ex+(e^2x^2+de)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\mid -1\right)+((de-e^2)x^2+d^2-de)}{2\left(d^2e^2x^2+d^3e\right)}$$

input
```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-e^2*x^4 + d^2)*e*x + (e^2*x^2 + d*e)*sqrt(e/d)*elliptic_e(arcsi
n(x*sqrt(e/d)), -1) + ((d*e - e^2)*x^2 + d^2 - d*e)*sqrt(e/d)*elliptic_f(a
rcsin(x*sqrt(e/d)), -1))/(d^2*e^2*x^2 + d^3*e)
```

Sympy [A] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{d-ex^2}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^2\Gamma\left(\frac{5}{4}\right)} - \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-e*x**2+d)/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*d**2*gamma(5/4)) - e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), e**2*x**4
*exp_polar(2*I*pi)/d**2)/(4*d**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{d-ex^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int -\frac{ex^2-d}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 - d)/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \frac{d-ex^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int -\frac{ex^2-d}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(e*x^2 - d)/(-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d-ex^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{d-e\,x^2}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input

```
int((d - e*x^2)/(d^2 - e^2*x^4)^(3/2),x)
```

output

```
int((d - e*x^2)/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{d-ex^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^3x^6-d\,e^2x^4+d^2e\,x^2+d^3}dx$$

input

```
int((-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**3 + d**2*e*x**2 - d*e**2*x**4 - e**3*x**6),
x)
```

3.11 $\int \frac{1-x^2}{\sqrt{1-x^4}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{1-x^2}{\sqrt{1-x^4}} \, dx = -E(\arcsin(x)|-1) + 2 \operatorname{EllipticF}(\arcsin(x), -1)$$

output
```
-EllipticE(x,I)+2*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.77

$$\begin{aligned}\int \frac{1-x^2}{\sqrt{1-x^4}} \, dx = {} & x \operatorname{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, x^4\right) \\ & - \frac{1}{3} x^3 \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, x^4\right)\end{aligned}$$

input
```
Integrate[(1 - x^2)/Sqrt[1 - x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, x^4] - (x^3*Hypergeometric2F1[1/2, 3/4,
 7/4, x^4])/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1388, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \; 1388$$

$$\int \frac{\sqrt{1-x^2}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \; 326$$

$$2\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+1}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \; 284$$

$$2\int \frac{1}{\sqrt{1-x^4}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \; 327$$

$$2\int \frac{1}{\sqrt{1-x^4}}dx-E(\arcsin(x)|-1)$$

$$\Big\downarrow \; 762$$

$$2\,\text{EllipticF}(\arcsin(x),-1)-E(\arcsin(x)|-1)$$

input
```
Int[(1 - x^2)/Sqrt[1 - x^4],x]
```

output
```
-EllipticE[ArcSin[x], -1] + 2*EllipticF[ArcSin[x], -1]
```

Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

method	result	size
meijerg	$-\frac{x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], x^{4}\right)$	27
default	$\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(x,i\right)-\operatorname{EllipticE}\left(x,i\right)\right)}{\sqrt{-x^{4}+1}}+\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x,i\right)}{\sqrt{-x^{4}+1}}$	69
elliptic	$\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(x,i\right)-\operatorname{EllipticE}\left(x,i\right)\right)}{\sqrt{-x^{4}+1}}+\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x,i\right)}{\sqrt{-x^{4}+1}}$	69

input
```
int((-x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3*hypergeom([1/2,3/4],[7/4],x^4)+x*hypergeom([1/4,1/2],[5/4],x^4)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = \frac{i\,xE\left(\arcsin\left(\frac{1}{x}\right)\mid -1\right)+\sqrt{-x^4+1}}{x}$$

input
```
integrate((-x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(I*x*elliptic_e(arcsin(1/x), -1) + sqrt(-x^4 + 1))/x
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(7) = 14$.

Time = 0.69 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.69

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = -\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-x**2+1)/(-x**4+1)**(1/2),x)
```

output
```
-x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(2*I*pi))/(4*gamm
a(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(2*I*pi))/(
4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{-x^4+1}}\,dx$$

input

```
integrate((-x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output

```
-integrate((x^2 - 1)/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{-x^4+1}}\,dx$$

input

```
integrate((-x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output

```
integrate(-(x^2 - 1)/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = -\int \frac{x^2-1}{\sqrt{1-x^4}}\,dx$$

input

```
int(-(x^2 - 1)/(1 - x^4)^(1/2),x)
```

output

```
-int((x^2 - 1)/(1 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-x^4+1}}{x^2+1}dx$$

input

```
int((-x^2+1)/(-x^4+1)^(1/2),x)
```

output

```
int(sqrt( - x**4 + 1)/(x**2 + 1),x)
```

3.12 $\int \frac{\sqrt{1-x^4}}{1+x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{\sqrt{1-x^4}}{1+x^2}\,dx = -E(\arcsin(x)|-1) + 2\,\mathrm{EllipticF}(\arcsin(x),-1)$$

output
```
-EllipticE(x,I)+2*EllipticF(x,I)
```

Mathematica [A] (verified)

Time = 10.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-x^4}}{1+x^2}\,dx = -E(\arcsin(x)|-1) + 2\,\mathrm{EllipticF}(\arcsin(x),-1)$$

input
```
Integrate[Sqrt[1 - x^4]/(1 + x^2),x]
```

output
```
-EllipticE[ArcSin[x], -1] + 2*EllipticF[ArcSin[x], -1]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = $\{1388, 326, 284, 327, 762\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^4}}{x^2+1}dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{\sqrt{1-x^2}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 326}$$

$$2\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+1}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 284}$$

$$2\int \frac{1}{\sqrt{1-x^4}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$2\int \frac{1}{\sqrt{1-x^4}}dx-E(\arcsin(x)|-1)$$

$$\Big\downarrow \text{ 762}$$

$$2\operatorname{EllipticF}(\arcsin(x),-1)-E(\arcsin(x)|-1)$$

input
```
Int[Sqrt[1 - x^4]/(1 + x^2),x]
```

output
```
-EllipticE[ArcSin[x], -1] + 2*EllipticF[ArcSin[x], -1]
```

Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(13) = 26$.

Time $= 0.49$ (sec) , antiderivative size $= 69$, normalized size of antiderivative $= 5.31$

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i))}{\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\ \mathrm{EllipticF}(x,i)}{\sqrt{-x^4+1}}$	69
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i))}{\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\ \mathrm{EllipticF}(x,i)}{\sqrt{-x^4+1}}$	69

input

```
int((-x^4+1)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output

```
(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(EllipticF(x,I)-EllipticE(x,I)
)+(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*EllipticF(x,I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{1-x^4}}{1+x^2}\,dx = \frac{i\,xE\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right)+\sqrt{-x^4+1}}{x}$$

input

```
integrate((-x^4+1)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output

```
(I*x*elliptic_e(arcsin(1/x), -1) + sqrt(-x^4 + 1))/x
```

Sympy [F]

$$\int \frac{\sqrt{1-x^4}}{1+x^2}\,dx = \int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}}{x^2+1}\,dx$$

input

```
integrate((-x**4+1)**(1/2)/(x**2+1),x)
```

output

```
Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^4}}{1+x^2}\,dx = \int \frac{\sqrt{-x^4+1}}{x^2+1}\,dx$$

input

```
integrate((-x^4+1)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output

```
integrate(sqrt(-x^4 + 1)/(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^4}}{1+x^2}\,dx = \int \frac{\sqrt{-x^4+1}}{x^2+1}\,dx$$

input

```
integrate((-x^4+1)^(1/2)/(x^2+1),x, algorithm="giac")
```

output

```
integrate(sqrt(-x^4 + 1)/(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^4}}{1+x^2}\,dx = \int \frac{\sqrt{1-x^4}}{x^2+1}\,dx$$

input

```
int((1 - x^4)^(1/2)/(x^2 + 1),x)
```

output

```
int((1 - x^4)^(1/2)/(x^2 + 1), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-x^4}}{1+x^2}\, dx = \int \frac{\sqrt{-x^4+1}}{x^2+1} dx$$

input

```
int((-x^4+1)^(1/2)/(x^2+1),x)
```

output

```
int(sqrt( - x**4 + 1)/(x**2 + 1),x)
```

3.13 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = -E(\arcsin(x)|-1) + 2\,\text{EllipticF}(\arcsin(x),-1)$$

output
```
-EllipticE(x,I)+2*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = -iE(i\text{arcsinh}(x)|-1)$$

input
```
Integrate[Sqrt[1 - x^2]/Sqrt[1 + x^2],x]
```

output
```
(-I)*EllipticE[I*ArcSinh[x], -1]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \; 326$$

$$2\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+1}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \; 284$$

$$2\int \frac{1}{\sqrt{1-x^4}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \; 327$$

$$2\int \frac{1}{\sqrt{1-x^4}}dx-E(\arcsin(x)|-1)$$

$$\Big\downarrow \; 762$$

$$2\,\text{EllipticF}(\arcsin(x),-1)-E(\arcsin(x)|-1)$$

input
```
Int[Sqrt[1 - x^2]/Sqrt[1 + x^2],x]
```

output
```
-EllipticE[ArcSin[x], -1] + 2*EllipticF[ArcSin[x], -1]
```

Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\operatorname{EllipticE}\left(x, i\right)+2 \operatorname{EllipticF}\left(x, i\right)$	14
elliptic	$\frac{\sqrt{-x^{4}+1}\, \left(\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(x, i\right)-\operatorname{EllipticE}\left(x, i\right)\right)}{\sqrt{-x^{4}+1}}+\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x, i\right)}{\sqrt{-x^{4}+1}}\right)}{\sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}}$	95

input
```
int((-x^2+1)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-EllipticE(x,I)+2*EllipticF(x,I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(11) = 22$.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \frac{i\,xE\left(\arcsin\left(\frac{1}{x}\right) \mid -1\right) + \sqrt{x^2+1}\sqrt{-x^2+1}}{x}$$

input

```
integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output

```
(I*x*elliptic_e(arcsin(1/x), -1) + sqrt(x^2 + 1)*sqrt(-x^2 + 1))/x
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{x^2+1}}\,dx$$

input

```
integrate((-x**2+1)**(1/2)/(x**2+1)**(1/2),x)
```

output

```
Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{x^2+1}}\,dx$$

input

```
integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output

```
integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{1-x^2}}{\sqrt{x^2+1}}\,dx$$

input
```
int((1 - x^2)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((1 - x^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}}{x^2+1}dx$$

input
```
int((-x^2+1)^(1/2)/(x^2+1)^(1/2),x)
```

output
```
int((sqrt( - x**2 + 1)*sqrt(x**2 + 1))/(x**2 + 1),x)
```

3.14 $\int \frac{1-e^2x^2}{\sqrt{1-e^4x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 23

$$\int \frac{1-e^2x^2}{\sqrt{1-e^4x^4}}\,dx=-\frac{E(\arcsin(ex)|-1)}{e}+\frac{2\,\text{EllipticF}(\arcsin(ex),-1)}{e}$$

output

```
-EllipticE(e*x,I)/e+2*EllipticF(e*x,I)/e
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\begin{aligned}\int \frac{1-e^2x^2}{\sqrt{1-e^4x^4}}\,dx&=x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},e^4x^4\right)\\&\quad-\frac{1}{3}e^2x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e^4x^4\right)\end{aligned}$$

input

```
Integrate[(1 - e^2*x^2)/Sqrt[1 - e^4*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, e^4*x^4] - (e^2*x^3*Hypergeometric2F1[1
/2, 3/4, 7/4, e^4*x^4])/3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{1388, 326, 284, 327, 762\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-e^2x^2}{\sqrt{1-e^4x^4}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{\sqrt{1-e^2x^2}}{\sqrt{e^2x^2+1}}dx$$

$$\Big\downarrow \text{326}$$

$$2\int \frac{1}{\sqrt{1-e^2x^2}\sqrt{e^2x^2+1}}dx-\int \frac{\sqrt{e^2x^2+1}}{\sqrt{1-e^2x^2}}dx$$

$$\Big\downarrow \text{284}$$

$$2\int \frac{1}{\sqrt{1-e^4x^4}}dx-\int \frac{\sqrt{e^2x^2+1}}{\sqrt{1-e^2x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$2\int \frac{1}{\sqrt{1-e^4x^4}}dx-\frac{E(\arcsin(ex)|-1)}{e}$$

$$\Big\downarrow \text{762}$$

$$\frac{2\,\text{EllipticF}(\arcsin(ex),-1)}{e}-\frac{E(\arcsin(ex)|-1)}{e}$$

input
```
Int[(1 - e^2*x^2)/Sqrt[1 - e^4*x^4],x]
```

output

```
-(EllipticE[ArcSin[e*x], -1]/e) + (2*EllipticF[ArcSin[e*x], -1])/e
```

Defintions of rubi rules used

rule 284

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

method	result	size
meijerg	$-\frac{e^{2} x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right],\left[\frac{7}{4}\right], e^{4} x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{5}{4}\right], e^{4} x^{4}\right)$	38
default	$\frac{\sqrt{-e^{2} x^{2}+1}\, \sqrt{e^{2} x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{e^{2}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{e^{2}}, i\right)\right)}{\sqrt{e^{2}}\, \sqrt{-e^{4} x^{4}+1}}+\frac{\sqrt{-e^{2} x^{2}+1}\, \sqrt{e^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{e^{2}}, i\right)}{\sqrt{e^{2}}\, \sqrt{-e^{4} x^{4}+1}}$	117
elliptic	$\frac{\sqrt{-e^{2} x^{2}+1}\, \sqrt{e^{2} x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{e^{2}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{e^{2}}, i\right)\right)}{\sqrt{e^{2}}\, \sqrt{-e^{4} x^{4}+1}}+\frac{\sqrt{-e^{2} x^{2}+1}\, \sqrt{e^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{e^{2}}, i\right)}{\sqrt{e^{2}}\, \sqrt{-e^{4} x^{4}+1}}$	117

input
```
int((-e^2*x^2+1)/(-e^4*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*e^2*x^3*hypergeom([1/2,3/4],[7/4],e^4*x^4)+x*hypergeom([1/4,1/2],[5/4
],e^4*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(21) = 42$.

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.70

$$\int \frac{1-e^2x^2}{\sqrt{1-e^4x^4}}\,dx$$
$$= \frac{\sqrt{-e^4x^4+1}e^3+\sqrt{-e^4}\left((e^2-1)xF(\arcsin\left(\frac{1}{ex}\right)\mid -1)+xE(\arcsin\left(\frac{1}{ex}\right)\mid -1)\right)}{e^5x}$$

input
```
integrate((-e^2*x^2+1)/(-e^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-e^4*x^4 + 1)*e^3 + sqrt(-e^4)*((e^2 - 1)*x*elliptic_f(arcsin(1/(e*x
)), -1) + x*elliptic_e(arcsin(1/(e*x)), -1)))/(e^5*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(14) = 28.

Time = 0.82 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{1-e^2x^2}{\sqrt{1-e^4x^4}}\,dx = -\frac{e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right| e^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right| e^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-e**2*x**2+1)/(-e**4*x**4+1)**(1/2),x)
```

output
```
-e**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**4*x**4*exp_polar(2*I*pi
))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**4*x**4*exp_p
olar(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-e^2x^2}{\sqrt{1-e^4x^4}}\,dx = \int -\frac{e^2x^2-1}{\sqrt{-e^4x^4+1}}\,dx$$

input
```
integrate((-e^2*x^2+1)/(-e^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((e^2*x^2 - 1)/sqrt(-e^4*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-e^2x^2}{\sqrt{1-e^4x^4}}\,dx = \int -\frac{e^2x^2-1}{\sqrt{-e^4x^4+1}}\,dx$$

input

```
integrate((-e^2*x^2+1)/(-e^4*x^4+1)^(1/2),x, algorithm="giac")
```

output

```
integrate(-(e^2*x^2 - 1)/sqrt(-e^4*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-e^2x^2}{\sqrt{1-e^4x^4}}\,dx = -\int \frac{e^2\,x^2-1}{\sqrt{1-e^4\,x^4}}\,dx$$

input

```
int(-(e^2*x^2 - 1)/(1 - e^4*x^4)^(1/2),x)
```

output

```
-int((e^2*x^2 - 1)/(1 - e^4*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-e^2x^2}{\sqrt{1-e^4x^4}}\,dx = \int \frac{\sqrt{-e^4x^4+1}}{e^2x^2+1}dx$$

input

```
int((-e^2*x^2+1)/(-e^4*x^4+1)^(1/2),x)
```

output

```
int(sqrt( - e**4*x**4 + 1)/(e**2*x**2 + 1),x)
```

3.15 $\int \frac{\sqrt{1-e^4x^4}}{1+e^2x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 23

$$\int \frac{\sqrt{1-e^4x^4}}{1+e^2x^2}\,dx = -\frac{E(\arcsin(ex)|-1)}{e} + \frac{2\,\text{EllipticF}(\arcsin(ex),-1)}{e}$$

output
```
-EllipticE(e*x,I)/e+2*EllipticF(e*x,I)/e
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.26

$$\int \frac{\sqrt{1-e^4x^4}}{1+e^2x^2}\,dx = \frac{i\left(E\left(i\text{arcsinh}\left(\sqrt{-e^2}x\right)\middle|-1\right)-2\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-e^2}x\right),-1\right)\right)}{\sqrt{-e^2}}$$

input
```
Integrate[Sqrt[1 - e^4*x^4]/(1 + e^2*x^2),x]
```

output
```
(I*(EllipticE[I*ArcSinh[Sqrt[-e^2]*x], -1] - 2*EllipticF[I*ArcSinh[Sqrt[-e
^2]*x], -1]))/Sqrt[-e^2]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1388, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-e^4x^4}}{e^2x^2+1}dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{\sqrt{1-e^2x^2}}{\sqrt{e^2x^2+1}}dx$$

$$\Big\downarrow \text{ 326}$$

$$2\int \frac{1}{\sqrt{1-e^2x^2}\sqrt{e^2x^2+1}}dx-\int \frac{\sqrt{e^2x^2+1}}{\sqrt{1-e^2x^2}}dx$$

$$\Big\downarrow \text{ 284}$$

$$2\int \frac{1}{\sqrt{1-e^4x^4}}dx-\int \frac{\sqrt{e^2x^2+1}}{\sqrt{1-e^2x^2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$2\int \frac{1}{\sqrt{1-e^4x^4}}dx-\frac{E(\arcsin(ex)|-1)}{e}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{2\,\text{EllipticF}(\arcsin(ex),-1)}{e}-\frac{E(\arcsin(ex)|-1)}{e}$$

input
```
Int[Sqrt[1 - e^4*x^4]/(1 + e^2*x^2),x]
```

output
```
-(EllipticE[ArcSin[e*x], -1]/e) + (2*EllipticF[ArcSin[e*x], -1])/e
```

Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(23) = 46$.

Time = 1.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 5.09

method	result	size
default	$\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{e^2},i\right)-\operatorname{EllipticE}\left(x\sqrt{e^2},i\right)\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}+\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{e^2},i\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}$	117
elliptic	$\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{e^2},i\right)-\operatorname{EllipticE}\left(x\sqrt{e^2},i\right)\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}+\frac{\sqrt{-e^2x^2+1}\,\sqrt{e^2x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{e^2},i\right)}{\sqrt{e^2}\,\sqrt{-e^4x^4+1}}$	117

input
```
int((-e^4*x^4+1)^(1/2)/(e^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/(e^2)^(1/2)*(-e^2*x^2+1)^(1/2)*(e^2*x^2+1)^(1/2)/(-e^4*x^4+1)^(1/2)*(Ell
ipticF(x*(e^2)^(1/2),I)-EllipticE(x*(e^2)^(1/2),I))+1/(e^2)^(1/2)*(-e^2*x^
2+1)^(1/2)*(e^2*x^2+1)^(1/2)/(-e^4*x^4+1)^(1/2)*EllipticF(x*(e^2)^(1/2),I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.70

$$\int \frac{\sqrt{1-e^4x^4}}{1+e^2x^2}\,dx$$
$$=\frac{\sqrt{-e^4x^4+1}e^3+\sqrt{-e^4}\left((e^2-1)xF\left(\arcsin\left(\frac{1}{ex}\right)|-1\right)+xE\left(\arcsin\left(\frac{1}{ex}\right)|-1\right)\right)}{e^5x}$$

input
```
integrate((-e^4*x^4+1)^(1/2)/(e^2*x^2+1),x, algorithm="fricas")
```

output
```
(sqrt(-e^4*x^4 + 1)*e^3 + sqrt(-e^4)*((e^2 - 1)*x*elliptic_f(arcsin(1/(e*x
)), -1) + x*elliptic_e(arcsin(1/(e*x)), -1)))/(e^5*x)
```

Sympy [F]

$$\int \frac{\sqrt{1-e^4x^4}}{1+e^2x^2}\,dx=\int \frac{\sqrt{-(ex-1)(ex+1)(e^2x^2+1)}}{e^2x^2+1}\,dx$$

input
```
integrate((-e**4*x**4+1)**(1/2)/(e**2*x**2+1),x)
```

output
```
Integral(sqrt(-(e*x - 1)*(e*x + 1)*(e**2*x**2 + 1))/(e**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-e^4x^4}}{1+e^2x^2}\,dx=\int \frac{\sqrt{-e^4x^4+1}}{e^2x^2+1}\,dx$$

input
```
integrate((-e^4*x^4+1)^(1/2)/(e^2*x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^4*x^4 + 1)/(e^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-e^4x^4}}{1+e^2x^2}\,dx=\int \frac{\sqrt{-e^4x^4+1}}{e^2x^2+1}\,dx$$

input
```
integrate((-e^4*x^4+1)^(1/2)/(e^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(sqrt(-e^4*x^4 + 1)/(e^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-e^4x^4}}{1+e^2x^2}\,dx=\int \frac{\sqrt{1-e^4\,x^4}}{e^2\,x^2+1}\,dx$$

input
```
int((1 - e^4*x^4)^(1/2)/(e^2*x^2 + 1),x)
```

output
```
int((1 - e^4*x^4)^(1/2)/(e^2*x^2 + 1), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-e^4x^4}}{1+e^2x^2}\,dx=\int \frac{\sqrt{-e^4x^4+1}}{e^2x^2+1}dx$$

input

```
int((-e^4*x^4+1)^(1/2)/(e^2*x^2+1),x)
```

output

```
int(sqrt( - e**4*x**4 + 1)/(e**2*x**2 + 1),x)
```

3.16 $\int \frac{\sqrt{1-e^2x^2}}{\sqrt{1+e^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 23

$$\int \frac{\sqrt{1-e^2x^2}}{\sqrt{1+e^2x^2}}\,dx = -\frac{E(\arcsin(ex)|-1)}{e} + \frac{2\,\text{EllipticF}(\arcsin(ex),-1)}{e}$$

output
```
-EllipticE(e*x,I)/e+2*EllipticF(e*x,I)/e
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{1-e^2x^2}}{\sqrt{1+e^2x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{-e^2}x\right)\middle|-1\right)}{\sqrt{-e^2}}$$

input
```
Integrate[Sqrt[1 - e^2*x^2]/Sqrt[1 + e^2*x^2],x]
```

output
```
EllipticE[ArcSin[Sqrt[-e^2]*x], -1]/Sqrt[-e^2]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-e^2x^2}}{\sqrt{e^2x^2+1}}dx$$

$$\Big\downarrow \text{ 326}$$

$$2\int \frac{1}{\sqrt{1-e^2x^2}\sqrt{e^2x^2+1}}dx-\int \frac{\sqrt{e^2x^2+1}}{\sqrt{1-e^2x^2}}dx$$

$$\Big\downarrow \text{ 284}$$

$$2\int \frac{1}{\sqrt{1-e^4x^4}}dx-\int \frac{\sqrt{e^2x^2+1}}{\sqrt{1-e^2x^2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$2\int \frac{1}{\sqrt{1-e^4x^4}}dx-\frac{E(\arcsin(ex)|-1)}{e}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{2\,\text{EllipticF}(\arcsin(ex),-1)}{e}-\frac{E(\arcsin(ex)|-1)}{e}$$

input
```
Int[Sqrt[1 - e^2*x^2]/Sqrt[1 + e^2*x^2],x]
```

output
```
-(EllipticE[ArcSin[e*x], -1]/e) + (2*EllipticF[ArcSin[e*x], -1])/e
```

Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.77 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\left(2\operatorname{EllipticF}\left(x\operatorname{csgn}\left(e\right)e,i\right)-\operatorname{EllipticE}\left(x\operatorname{csgn}\left(e\right)e,i\right)\right)\operatorname{csgn}\left(e\right)}{e}$	28
elliptic	$\frac{\sqrt{-e^{4}x^{4}+1}\left(\frac{\sqrt{-e^{2}x^{2}+1}\,\sqrt{e^{2}x^{2}+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{e^{2}},i\right)-\operatorname{EllipticE}\left(x\sqrt{e^{2}},i\right)\right)}{\sqrt{e^{2}}\,\sqrt{-e^{4}x^{4}+1}}+\frac{\sqrt{-e^{2}x^{2}+1}\,\sqrt{e^{2}x^{2}+1}\,\operatorname{EllipticF}\left(x\sqrt{e^{2}},i\right)}{\sqrt{e^{2}}\,\sqrt{-e^{4}x^{4}+1}}\right)}{\sqrt{e^{2}x^{2}+1}\,\sqrt{-e^{2}x^{2}+1}}$	153

input
```
int((-e^2*x^2+1)^(1/2)/(e^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2*EllipticF(x*csgn(e)*e,I)-EllipticE(x*csgn(e)*e,I))*csgn(e)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.17

$$\int \frac{\sqrt{1-e^2x^2}}{\sqrt{1+e^2x^2}}\,dx = \frac{\sqrt{e^2x^2+1}\sqrt{-e^2x^2+1}e^3+\sqrt{-e^4}\left((e^2-1)xF\left(\arcsin\left(\frac{1}{ex}\right)|-1\right)+xE\left(\arcsin\left(\frac{1}{ex}\right)|-1\right)\right)}{e^5x}$$

input

```
integrate((-e^2*x^2+1)^(1/2)/(e^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output

```
(sqrt(e^2*x^2 + 1)*sqrt(-e^2*x^2 + 1)*e^3 + sqrt(-e^4)*((e^2 - 1)*x*ellipt
ic_f(arcsin(1/(e*x)), -1) + x*elliptic_e(arcsin(1/(e*x)), -1)))/(e^5*x)
```

Sympy [F]

$$\int \frac{\sqrt{1-e^2x^2}}{\sqrt{1+e^2x^2}}\,dx = \int \frac{\sqrt{-(ex-1)(ex+1)}}{\sqrt{e^2x^2+1}}\,dx$$

input

```
integrate((-e**2*x**2+1)**(1/2)/(e**2*x**2+1)**(1/2),x)
```

output

```
Integral(sqrt(-(e*x - 1)*(e*x + 1))/sqrt(e**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-e^2x^2}}{\sqrt{1+e^2x^2}}\,dx = \int \frac{\sqrt{-e^2x^2+1}}{\sqrt{e^2x^2+1}}\,dx$$

input

```
integrate((-e^2*x^2+1)^(1/2)/(e^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output

```
integrate(sqrt(-e^2*x^2 + 1)/sqrt(e^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-e^2x^2}}{\sqrt{1+e^2x^2}}\,dx = \int \frac{\sqrt{-e^2x^2+1}}{\sqrt{e^2x^2+1}}\,dx$$

input

```
integrate((-e^2*x^2+1)^(1/2)/(e^2*x^2+1)^(1/2),x, algorithm="giac")
```

output

```
integrate(sqrt(-e^2*x^2 + 1)/sqrt(e^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-e^2x^2}}{\sqrt{1+e^2x^2}}\,dx = \int \frac{\sqrt{1-e^2\,x^2}}{\sqrt{e^2\,x^2+1}}\,dx$$

input

```
int((1 - e^2*x^2)^(1/2)/(e^2*x^2 + 1)^(1/2),x)
```

output

```
int((1 - e^2*x^2)^(1/2)/(e^2*x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-e^2x^2}}{\sqrt{1+e^2x^2}}\,dx = \int \frac{\sqrt{e^2x^2+1}\,\sqrt{-e^2x^2+1}}{e^2x^2+1}\,dx$$

input

```
int((-e^2*x^2+1)^(1/2)/(e^2*x^2+1)^(1/2),x)
```

output

```
int((sqrt(e**2*x**2 + 1)*sqrt( - e**2*x**2 + 1))/(e**2*x**2 + 1),x)
```

3.17 $\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx = -\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
-d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^
2*x^4+d^2)^(1/2)+2*d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/
2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx = \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{e^2x^4}{d^2}\right)\right)}{3\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[(d - e*x^2)/Sqrt[d^2 - e^2*x^4],x]
```

output

```
(Sqrt[1 - (e^2*x^4)/d^2]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (e^2*x^4)
/d^2] - e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(3*Sqrt[d^
2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1390, 1389, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{ 1390}$$

$$\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{d-ex^2}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{1-\frac{ex^2}{d}}}{\sqrt{\frac{ex^2}{d}+1}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 326}$$

$$\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\left(2\int \frac{1}{\sqrt{1-\frac{ex^2}{d}}\sqrt{\frac{ex^2}{d}+1}}dx-\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 284}$$

$$\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\left(2\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx-\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\left(2\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx-\frac{\sqrt{d}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\left(\frac{2\sqrt{d}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}}-\frac{\sqrt{d}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d - e*x^2)/Sqrt[d^2 - e^2*x^4],x]
```

output

```
(d*Sqrt[1 - (e^2*x^4)/d^2]*(-((Sqrt[d]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d
]], -1])/Sqrt[e]) + (2*Sqrt[d]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])
/Sqrt[e]))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 284

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	137
elliptic	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	137

input
```
int((-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*Ell
ipticF(x*(e/d)^(1/2),I)+d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/
(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx$$
$$= \frac{\sqrt{-e^2}dx\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) - \sqrt{-e^2}(d-e)x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) + \sqrt{-e^2x^4+d^2}e}{e^2x}$$

input
```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-e^2)*d*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - sqrt(-e^2)
*(d - e)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1) + sqrt(-e^2*x^4 +
 d^2)*e)/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-e*x**2+d)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*gamma(5/4)) - e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**2*x**4*exp_
polar(2*I*pi)/d**2)/(4*d*gamma(7/4))
```

Maxima [F]

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int -\frac{ex^2-d}{\sqrt{-e^2x^4+d^2}}\,dx$$

input

```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output

```
-integrate((e*x^2 - d)/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int -\frac{ex^2-d}{\sqrt{-e^2x^4+d^2}}\,dx$$

input

```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output

```
integrate(-(e*x^2 - d)/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{d-e\,x^2}{\sqrt{d^2-e^2\,x^4}}\,dx$$

input

```
int((d - e*x^2)/(d^2 - e^2*x^4)^(1/2),x)
```

output

```
int((d - e*x^2)/(d^2 - e^2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e\,x^2+d}dx$$

input
```
int((-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d + e*x**2),x)
```

3.18 $\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = -\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
-d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^
2*x^4+d^2)^(1/2)+2*d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/
2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = \frac{id\sqrt{1-\frac{e^2x^4}{d^2}}\left(E\left(i\,\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-2\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)\right)}{\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d + e*x^2),x]
```

output
```
(I*d*Sqrt[1 - (e^2*x^4)/d^2]*(EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - 2
*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1]))/(Sqrt[-(e/d)]*Sqrt[d^2 - e^2*x
^4])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.45, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1396, 326, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{326}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(2d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{329}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 327$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[Sqrt[d^2 - e^2*x^4]/(d + e*x^2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(-((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[
(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])) + (2
*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1
])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/(Sqrt[d - e*x^2]*Sqrt[d + e
*x^2])
```

Defintions of rubi rules used

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 326

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	137
elliptic	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	137

input
```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*Ell
ipticF(x*(e/d)^(1/2),I)+d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/
(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I
))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = \frac{\sqrt{-e^2}dx\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)-\sqrt{-e^2}(d-e)x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)+\sqrt{-e^2x^4+d^2}e}{e^2x}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
(sqrt(-e^2)*d*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - sqrt(-e^2)
*(d - e)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1) + sqrt(-e^2*x^4 +
 d^2)*e)/(e^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = \int \frac{\sqrt{-(-d+ex^2)(d+ex^2)}}{d+ex^2}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(e*x**2+d),x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{ex^2+d}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = \int \frac{\sqrt{d^2-e^2\,x^4}}{e\,x^2+d}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2),x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e\,x^2+d}dx$$

input

```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d + e*x**2),x)
```

3.19 $\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 156

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = \frac{x\sqrt{d^2-e^2x^4}}{2d^2(d-ex^2)} - \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
1/2*x*(-e^2*x^4+d^2)^(1/2)/d^2/(-e*x^2+d)-1/2*(1-e^2*x^4/d^2)^(1/2)*Ellipt
icE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+(1-e^2*x^4/d
^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx$$
$$= \frac{\sqrt{-\frac{e}{d}}x(d+ex^2)+id\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-2id\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\right.}{2d^2\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[1/((d - e*x^2)*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[-(e/d)]*x*(d + e*x^2) + I*d*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcS
inh[Sqrt[-(e/d)]*x], -1] - (2*I)*d*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*Arc
Sinh[Sqrt[-(e/d)]*x], -1])/(2*d^2*Sqrt[-(e/d)]*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1396, 316, 27, 326, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}dx$$
$$\Big\downarrow \text{1396}$$
$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{3/2}\sqrt{ex^2+d}}dx}{\sqrt{d^2-e^2x^4}}$$
$$\Big\downarrow \text{316}$$
$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx}{2d^2e}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$
$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{326}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{329}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{2d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[1/((d - e*x^2)*Sqrt[d^2 - e^2*x^4]),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d + e*x^2])/(2*d^2*Sqrt[d - e*x^
2]) + (-((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqr
t[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])) + (2*d^(3/2)*Sqrt[1
 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqr
t[d - e*x^2]*Sqrt[d + e*x^2]))/(2*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\left(-e^{2} x^{2}-d e\right) x}{2 d^{2} e \sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2} x^{2}-d e\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d \sqrt{\frac{e}{d}}\,\sqrt{-e^{2} x^{4}+d^{2}}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\text{Elliptic}}{2 d \sqrt{\frac{e}{d}}\,\sqrt{-e^{2} x^{4}+d^{2}}}$
elliptic	$-\frac{\left(-e^{2} x^{2}-d e\right) x}{2 d^{2} e \sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2} x^{2}-d e\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d \sqrt{\frac{e}{d}}\,\sqrt{-e^{2} x^{4}+d^{2}}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\text{Elliptic}}{2 d \sqrt{\frac{e}{d}}\,\sqrt{-e^{2} x^{4}+d^{2}}}$

input
```
int(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2-d*e)/d^2*x/e/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/2/d/(e/d)^(
1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*
(e/d)^(1/2),I)+1/2/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2
*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(d-e x^{2}\right) \sqrt{d^{2}-e^{2} x^{4}}}\, dx = -\frac{\sqrt{-e^{2} x^{4}+d^{2}} e x+\left(e^{2} x^{2}-d e\right) \sqrt{\frac{e}{d}} E(\arcsin \left(x \sqrt{\frac{e}{d}}\right) \mid -1)-\left(\left(d e+e^{2}\right) x^{2}-d^{2}-d e\right) \sqrt{\frac{e}{d}} F(\arcsin (x}{2\left(d^{2} e^{2} x^{2}-d^{3} e\right)}$$

input
```
integrate(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-e^2*x^4 + d^2)*e*x + (e^2*x^2 - d*e)*sqrt(e/d)*elliptic_e(arcs
in(x*sqrt(e/d)), -1) - ((d*e + e^2)*x^2 - d^2 - d*e)*sqrt(e/d)*elliptic_f(
arcsin(x*sqrt(e/d)), -1))/(d^2*e^2*x^2 - d^3*e)
```

Sympy [F]

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = -\int \frac{1}{-d\sqrt{d^2-e^2x^4}+ex^2\sqrt{d^2-e^2x^4}}\,dx$$

input
```
integrate(1/(-e*x**2+d)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
-Integral(1/(-d*sqrt(d**2 - e**2*x**4) + e*x**2*sqrt(d**2 - e**2*x**4)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int -\frac{1}{\sqrt{-e^2x^4+d^2}(ex^2-d)}\,dx$$

input
```
integrate(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 - d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx=\int \frac{1}{\sqrt{d^2-e^2\,x^4}\,\left(d-e\,x^2\right)}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^3x^6-d\,e^2x^4-d^2e\,x^2+d^3}dx$$

input
```
int(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**3 - d**2*e*x**2 - d*e**2*x**4 + e**3*x**6),
x)
```

3.20 $\int \frac{d+ex^2}{(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 153

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{x(d+ex^2)}{2d^2\sqrt{d^2-e^2x^4}} - \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
1/2*x*(e*x^2+d)/d^2/(-e^2*x^4+d^2)^(1/2)-1/2*(1-e^2*x^4/d^2)^(1/2)*Ellipti
cE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+(1-e^2*x^4/d^
2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{3dx+3dx\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)+2ex^3\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hyperge}}{6d^2\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d + e*x^2)/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(3*d*x + 3*d*x*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (e
^2*x^4)/d^2] + 2*e*x^3*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4, 3/2,
 7/4, (e^2*x^4)/d^2])/(6*d^2*Sqrt[d^2 - e^2*x^4])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \ 1571$$

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{3/2}}dx$$

input
```
Int[(d + e*x^2)/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.25

method	result
elliptic	$-\frac{(-e^2x^2-de)x}{2d^2e\sqrt{\left(x^2-\frac{d}{e}\right)(-e^2x^2-de)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \text{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{2d\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ (\text{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\text{Elliptic}}{2d\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$
default	$d\left(\frac{x}{2d^2\sqrt{-\left(x^4-\frac{d^2}{e^2}\right)e^2}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \text{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{2d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)+e\left(\frac{x^3}{2d^2\sqrt{-\left(x^4-\frac{d^2}{e^2}\right)e^2}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ (\text{Elli}}{2d\sqrt{}}\right.$

input
```
int((e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2-d*e)/d^2*x/e/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/2/d/(e/d)^(
1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*
(e/d)^(1/2),I)+1/2/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2
*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.74

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{3/2}}\,dx =$$

$$-\frac{\sqrt{-e^2x^4+d^2}ex+(e^2x^2-de)\sqrt{\frac{e}{d}}E(\arcsin(x\sqrt{\frac{e}{d}})\mid -1)-((de+e^2)x^2-d^2-de)\sqrt{\frac{e}{d}}F(\arcsin(x\sqrt{\frac{e}{d}})}{2(d^2e^2x^2-d^3e)}$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-e^2*x^4 + d^2)*e*x + (e^2*x^2 - d*e)*sqrt(e/d)*elliptic_e(arcs
in(x*sqrt(e/d)), -1) - ((d*e + e^2)*x^2 - d^2 - d*e)*sqrt(e/d)*elliptic_f(
arcsin(x*sqrt(e/d)), -1))/(d^2*e^2*x^2 - d^3*e)
```

Sympy [A] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.54

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^2\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*d**2*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), e**2*x**4
*exp_polar(2*I*pi)/d**2)/(4*d**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{ex^2+d}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{e\,x^2+d}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input
```
int((d + e*x^2)/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
int((d + e*x^2)/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e^3x^6-d\,e^2x^4-d^2e\,x^2+d^3}dx$$

input
```
int((e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**3 - d**2*e*x**2 - d*e**2*x**4 + e**3*x**6),
x)
```

3.21 $\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -7d^2x - \frac{4}{3}dex^3 - \frac{e^2x^5}{5} + \frac{8d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

output
```
-7*d^2*x-4/3*d*e*x^3-1/5*e^2*x^5+8*d^(5/2)*arctanh(e^(1/2)*x/d^(1/2))/e^(1
/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -7d^2x - \frac{4}{3}dex^3 - \frac{e^2x^5}{5} + \frac{8d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
```

output
```
-7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sq
rt[d]])/Sqrt[e]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1388, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}dx$$
$$\Big\downarrow \text{1388}$$
$$\int \frac{(d+ex^2)^3}{d-ex^2}dx$$
$$\Big\downarrow \text{300}$$
$$\int \left(\frac{8d^3}{d-ex^2}-7d^2-4dex^2-e^2x^4\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{8d^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-7d^2x-\frac{4}{3}dex^3-\frac{1}{5}e^2x^5$$

input
```
Int[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
```

output
```
-7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sq
rt[d]])/Sqrt[e]
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{e^{2} x^{5}}{5}-\frac{4 d e \,x^{3}}{3}-7 d^{2} x +\frac{8 d^{3} \operatorname{arctanh}\left(\frac{e x}{\sqrt{d e}}\right)}{\sqrt{d e}}$	42
risch	$-\frac{e^{2} x^{5}}{5}-\frac{4 d e \,x^{3}}{3}-7 d^{2} x -\frac{4 \sqrt{d e}\, d^{2} \ln \left(\sqrt{d e}\, x -d \right)}{e}+\frac{4 \sqrt{d e}\, d^{2} \ln \left(-\sqrt{d e}\, x -d \right)}{e}$	74

input
```
int((e*x^2+d)^4/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*e^2*x^5-4/3*d*e*x^3-7*d^2*x+8*d^3/(d*e)^(1/2)*arctanh(e*x/(d*e)^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.27

$$\int \frac{\left(d+e x^2\right)^4}{d^2-e^2 x^4}\,dx=\left[-\frac{1}{5}\,e^{2} x^{5}-\frac{4}{3}\,d e x^{3}+4\,d^{2} \sqrt{\frac{d}{e}} \log\left(\frac{e x^{2}+2\,e x \sqrt{\frac{d}{e}}+d}{e x^{2}-d}\right)-7\,d^{2} x,\ -\frac{1}{5}\,e^{2} x^{5}-\frac{4}{3}\,d e x^{3}-8\,d^{2} \sqrt{-\frac{d}{e}} \arctan\left(\frac{e x \sqrt{-\frac{d}{e}}}{d}\right)-7\,d^{2} x\right]$$

input
```
integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[-1/5*e^2*x^5 - 4/3*d*e*x^3 + 4*d^2*sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e)
 + d)/(e*x^2 - d)) - 7*d^2*x, -1/5*e^2*x^5 - 4/3*d*e*x^3 - 8*d^2*sqrt(-d/e
)*arctan(e*x*sqrt(-d/e)/d) - 7*d^2*x]
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -7d^2x - \frac{4dex^3}{3} - \frac{e^2x^5}{5} - 4\sqrt{\frac{d^5}{e}}\log\left(x - \frac{\sqrt{\frac{d^5}{e}}}{d^2}\right) + 4\sqrt{\frac{d^5}{e}}\log\left(x + \frac{\sqrt{\frac{d^5}{e}}}{d^2}\right)$$

input
```
integrate((e*x**2+d)**4/(-e**2*x**4+d**2),x)
```

output
```
-7*d**2*x - 4*d*e*x**3/3 - e**2*x**5/5 - 4*sqrt(d**5/e)*log(x - sqrt(d**5/
e)/d**2) + 4*sqrt(d**5/e)*log(x + sqrt(d**5/e)/d**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -\frac{8\,d^3\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{\sqrt{-de}} - \frac{3\,e^7x^5+20\,de^6x^3+105\,d^2e^5x}{15\,e^5}$$

input
```
integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
-8*d^3*arctan(e*x/sqrt(-d*e))/sqrt(-d*e) - 1/15*(3*e^7*x^5 + 20*d*e^6*x^3
+ 105*d^2*e^5*x)/e^5
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -7\,d^2\,x-\frac{e^2\,x^5}{5}-\frac{4\,d\,e\,x^3}{3}-\frac{d^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x\,1\mathrm{i}}{\sqrt{d}}\right)\,8\mathrm{i}}{\sqrt{e}}$$

input
```
int((d + e*x^2)^4/(d^2 - e^2*x^4),x)
```

output
```
- 7*d^2*x - (e^2*x^5)/5 - (d^(5/2)*atan((e^(1/2)*x*1i)/d^(1/2))*8i)/e^(1/2
) - (4*d*e*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx$$

$$= \frac{60\sqrt{e}\,\sqrt{d}\log\left(-\sqrt{e}\,\sqrt{d}-ex\right)d^2-60\sqrt{e}\,\sqrt{d}\log\left(\sqrt{e}\,\sqrt{d}-ex\right)d^2-105d^2ex-20d\,e^2x^3-3e^3x^5}{15e}$$

input
```
int((e*x^2+d)^4/(-e^2*x^4+d^2),x)
```

output

```
(60*sqrt(e)*sqrt(d)*log( - sqrt(e)*sqrt(d) - e*x)*d**2 - 60*sqrt(e)*sqrt(d
)*log(sqrt(e)*sqrt(d) - e*x)*d**2 - 105*d**2*e*x - 20*d*e**2*x**3 - 3*e**3
*x**5)/(15*e)
```

3.22 $\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = -3dx - \frac{ex^3}{3} + \frac{4d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

output
```
-3*d*x-1/3*e*x^3+4*d^(3/2)*arctanh(e^(1/2)*x/d^(1/2))/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = -3dx - \frac{ex^3}{3} + \frac{4d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
```

output
```
-3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1388, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^3}{d^2-e^2x^4}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{\left(d+ex^2\right)^2}{d-ex^2}dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{4d^2}{d-ex^2}-3d-ex^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4d^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-3dx-\frac{ex^3}{3}$$

input

```
Int[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
```

output

```
-3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Defintions of rubi rules used

rule 300

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{e\,x^{3}}{3}-3dx+\frac{4d^{2}\operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$	31
risch	$-\frac{e\,x^{3}}{3}-3dx+\frac{2\sqrt{de}\,d\ln\left(\sqrt{de}\,x+d\right)}{e}-\frac{2\sqrt{de}\,d\ln\left(-\sqrt{de}\,x+d\right)}{e}$	55

input
```
int((e*x^2+d)^3/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*e*x^3-3*d*x+4*d^2/(d*e)^(1/2)*arctanh(e*x/(d*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.37

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = \left[-\frac{1}{3}\,ex^3+2\,d\sqrt{\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{\frac{d}{e}}+d}{ex^2-d}\right)-3\,dx,-\frac{1}{3}\,ex^3 - 4\,d\sqrt{-\frac{d}{e}}\arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right)-3\,dx\right]$$

input
```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[-1/3*e*x^3 + 2*d*sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d))
 - 3*d*x, -1/3*e*x^3 - 4*d*sqrt(-d/e)*arctan(e*x*sqrt(-d/e)/d) - 3*d*x]
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = -3dx - \frac{ex^3}{3} - 2\sqrt{\frac{d^3}{e}}\log\left(x - \frac{\sqrt{\frac{d^3}{e}}}{d}\right) + 2\sqrt{\frac{d^3}{e}}\log\left(x + \frac{\sqrt{\frac{d^3}{e}}}{d}\right)$$

input
```
integrate((e*x**2+d)**3/(-e**2*x**4+d**2),x)
```

output
```
-3*d*x - e*x**3/3 - 2*sqrt(d**3/e)*log(x - sqrt(d**3/e)/d) + 2*sqrt(d**3/e
)*log(x + sqrt(d**3/e)/d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = -\frac{4\,d^2\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{\sqrt{-de}} - \frac{e^4x^3+9\,de^3x}{3\,e^3}$$

input
```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
-4*d^2*arctan(e*x/sqrt(-d*e))/sqrt(-d*e) - 1/3*(e^4*x^3 + 9*d*e^3*x)/e^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = \frac{4\,d^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{e\,x^3}{3} - 3\,d\,x$$

input
```
int((d + e*x^2)^3/(d^2 - e^2*x^4),x)
```

output
```
(4*d^(3/2)*atanh((e^(1/2)*x)/d^(1/2)))/e^(1/2) - (e*x^3)/3 - 3*d*x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = \frac{6\sqrt{e}\,\sqrt{d}\log\left(-\sqrt{e}\,\sqrt{d}-ex\right)d-6\sqrt{e}\,\sqrt{d}\log\left(\sqrt{e}\,\sqrt{d}-ex\right)d-9dex-e^2x^3}{3e}$$

input
```
int((e*x^2+d)^3/(-e^2*x^4+d^2),x)
```

output
```
(6*sqrt(e)*sqrt(d)*log( - sqrt(e)*sqrt(d) - e*x)*d - 6*sqrt(e)*sqrt(d)*log
(sqrt(e)*sqrt(d) - e*x)*d - 9*d*e*x - e**2*x**3)/(3*e)
```

3.23 $\int \frac{\left(d+ex^2\right)^2}{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 29

$$\int \frac{\left(d+ex^2\right)^2}{d^2-e^2x^4}\,dx=-x+\frac{2\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

output
```
-x+2*d^(1/2)*arctanh(e^(1/2)*x/d^(1/2))/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+ex^2\right)^2}{d^2-e^2x^4}\,dx=-x+\frac{2\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)^2/(d^2 - e^2*x^4),x]
```

output
```
-x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1388, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{d+ex^2}{d-ex^2}dx$$

$$\Big\downarrow \text{299}$$

$$2d\int \frac{1}{d-ex^2}dx-x$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

input
```
Int[(d + e*x^2)^2/(d^2 - e^2*x^4),x]
```

output
```
-x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
default	$-x+\frac{2d\,\operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$	22
risch	$-x-\frac{\sqrt{de}\,\ln\left(\sqrt{de}\,x-d\right)}{e}+\frac{\sqrt{de}\,\ln\left(-\sqrt{de}\,x-d\right)}{e}$	49

input
```
int((e*x^2+d)^2/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
-x+2*d/(d*e)^(1/2)*arctanh(e*x/(d*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.52

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx$$
$$=\left[\sqrt{\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{\frac{d}{e}}+d}{ex^2-d}\right)-x,-2\sqrt{-\frac{d}{e}}\arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right)-x\right]$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d)) - x, -2*sqrt(-d/
e)*arctan(e*x*sqrt(-d/e)/d) - x]
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx=-x-\sqrt{\frac{d}{e}}\log\left(x-\sqrt{\frac{d}{e}}\right)+\sqrt{\frac{d}{e}}\log\left(x+\sqrt{\frac{d}{e}}\right)$$

input
```
integrate((e*x**2+d)**2/(-e**2*x**4+d**2),x)
```

output
```
-x - sqrt(d/e)*log(x - sqrt(d/e)) + sqrt(d/e)*log(x + sqrt(d/e))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = -\frac{2\,d\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{\sqrt{-de}} - x$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
-2*d*arctan(e*x/sqrt(-d*e))/sqrt(-d*e) - x
```

Mupad [B] (verification not implemented)

Time = 17.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = \frac{2\,\sqrt{d}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} - x$$

input
```
int((d + e*x^2)^2/(d^2 - e^2*x^4),x)
```

output
```
(2*d^(1/2)*atanh((e^(1/2)*x)/d^(1/2)))/e^(1/2) - x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = \frac{\sqrt{e}\,\sqrt{d}\log\left(-\sqrt{e}\,\sqrt{d}-ex\right)-\sqrt{e}\,\sqrt{d}\log\left(\sqrt{e}\,\sqrt{d}-ex\right)-ex}{e}$$

input
```
int((e*x^2+d)^2/(-e^2*x^4+d^2),x)
```

output
```
(sqrt(e)*sqrt(d)*log( - sqrt(e)*sqrt(d) - e*x) - sqrt(e)*sqrt(d)*log(sqrt(
e)*sqrt(d) - e*x) - e*x)/e
```

3.24 $\int \frac{d+ex^2}{d^2-e^2x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{d+ex^2}{d^2-e^2x^4} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

output
```
arctanh(e^(1/2)*x/d^(1/2))/d^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{d+ex^2}{d^2-e^2x^4} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)/(d^2 - e^2*x^4),x]
```

output
```
ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1388, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{d-ex^2}dx$$

$$\Big\downarrow \text{221}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

input
```
Int[(d + e*x^2)/(d^2 - e^2*x^4),x]
```

output
```
ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [A] (verified)

Time $= 0.14$ (sec) , antiderivative size $= 16$, normalized size of antiderivative $= 0.67$

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$	16
risch	$\frac{\ln\left(ex+\sqrt{de}\right)}{2\sqrt{de}}-\frac{\ln\left(-ex+\sqrt{de}\right)}{2\sqrt{de}}$	37

input

```
int((e*x^2+d)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output

```
1/(d*e)^(1/2)*arctanh(e*x/(d*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time $= 0.07$ (sec) , antiderivative size $= 68$, normalized size of antiderivative $= 2.83$

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx = \left[\frac{\sqrt{de}\log\left(\frac{ex^2+2\sqrt{de}x+d}{ex^2-d}\right)}{2\,de}, -\frac{\sqrt{-de}\arctan\left(\frac{\sqrt{-de}x}{d}\right)}{de}\right]$$

input

```
integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output

```
[1/2*sqrt(d*e)*log((e*x^2 + 2*sqrt(d*e)*x + d)/(e*x^2 - d))/(d*e), -sqrt(-
d*e)*arctan(sqrt(-d*e)*x/d)/(d*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time $= 0.06$ (sec) , antiderivative size $= 46$, normalized size of antiderivative $= 1.92$

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx = -\frac{\sqrt{\frac{1}{de}}\log\left(-d\sqrt{\frac{1}{de}}+x\right)}{2}+\frac{\sqrt{\frac{1}{de}}\log\left(d\sqrt{\frac{1}{de}}+x\right)}{2}$$

input
```
integrate((e*x**2+d)/(-e**2*x**4+d**2),x)
```

output
```
-sqrt(1/(d*e))*log(-d*sqrt(1/(d*e)) + x)/2 + sqrt(1/(d*e))*log(d*sqrt(1/(d
*e)) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx = -\frac{\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{\sqrt{-de}}$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
-arctan(e*x/sqrt(-d*e))/sqrt(-d*e)
```

Mupad [B] (verification not implemented)

Time = 17.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx=\frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}$$

input
```
int((d + e*x^2)/(d^2 - e^2*x^4),x)
```

output
```
atanh((e^(1/2)*x)/d^(1/2))/(d^(1/2)*e^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx=\frac{\sqrt{e}\,\sqrt{d}\,\left(\log\left(-\sqrt{e}\,\sqrt{d}-ex\right)-\log\left(\sqrt{e}\,\sqrt{d}-ex\right)\right)}{2de}$$

input
```
int((e*x^2+d)/(-e^2*x^4+d^2),x)
```

output
```
(sqrt(e)*sqrt(d)*(log( - sqrt(e)*sqrt(d) - e*x) - log(sqrt(e)*sqrt(d) - e*
x)))/(2*d*e)
```

3.25 $\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \frac{x}{4d^2(d+ex^2)} + \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}$$

output

```
1/4*x/d^2/(e*x^2+d)+1/2*arctan(e^(1/2)*x/d^(1/2))/d^(5/2)/e^(1/2)+1/4*arct
anh(e^(1/2)*x/d^(1/2))/d^(5/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \frac{\frac{\sqrt{d}x}{d+ex^2} + \frac{2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{4d^{5/2}}$$

input

```
Integrate[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

output

```
((Sqrt[d]*x)/(d + e*x^2) + (2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ArcTa
nh[(Sqrt[e]*x)/Sqrt[d]]/Sqrt[e])/(4*d^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = $\{1388, 316, 25, 27, 397, 218, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{(d-ex^2)(d+ex^2)^2}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{x}{4d^2(d+ex^2)}-\frac{\int -\frac{e(3d-ex^2)}{(d-ex^2)(ex^2+d)}dx}{4d^2e}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{e(3d-ex^2)}{(d-ex^2)(ex^2+d)}dx}{4d^2e}+\frac{x}{4d^2(d+ex^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3d-ex^2}{(d-ex^2)(ex^2+d)}dx}{4d^2}+\frac{x}{4d^2(d+ex^2)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\int \frac{1}{d-ex^2}dx+2\int \frac{1}{ex^2+d}dx}{4d^2}+\frac{x}{4d^2(d+ex^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\int \frac{1}{d-ex^2}dx+\frac{2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}}{4d^2}+\frac{x}{4d^2(d+ex^2)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}}{4d^2}+\frac{x}{4d^2(d+ex^2)}$$

input

```
Int[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

output

```
x/(4*d^2*(d + e*x^2)) + ((2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e])
 + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e]))/(4*d^2)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397

```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 1388

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\frac{x}{e\,x^{2}+d}+\frac{2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}}{4d^{2}}+\frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{4d^{2}\sqrt{de}}$	54
risch	$\frac{x}{4d^{2}\left(e\,x^{2}+d\right)}-\frac{\ln\left(-ex-\sqrt{-de}\right)}{4\sqrt{-de}\,d^{2}}+\frac{\ln\left(ex-\sqrt{-de}\right)}{4\sqrt{-de}\,d^{2}}+\frac{\ln\left(ex+\sqrt{de}\right)}{8\sqrt{de}\,d^{2}}-\frac{\ln\left(-ex+\sqrt{de}\right)}{8\sqrt{de}\,d^{2}}$	107

input

```
int(1/(e*x^2+d)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output

```
1/4/d^2*(x/(e*x^2+d)+2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))+1/4/d^2/(d*e)^
(1/2)*arctanh(e*x/(d*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.62

$$\int \frac{1}{\left(d+ex^{2}\right)\left(d^{2}-e^{2}x^{4}\right)}\,dx$$
$$=\left[\frac{2\,dex+4\left(ex^{2}+d\right)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)+\left(ex^{2}+d\right)\sqrt{de}\log\left(\frac{ex^{2}+2\sqrt{de}x+d}{ex^{2}-d}\right)}{8\left(d^{3}e^{2}x^{2}+d^{4}e\right)},\frac{dex-\left(ex^{2}+d\right)\sqrt{-de}\,\mathrm{arcta}}{}\right.$$

input

```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/8*(2*d*e*x + 4*(e*x^2 + d)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (e*x^2 + d
)*sqrt(d*e)*log((e*x^2 + 2*sqrt(d*e)*x + d)/(e*x^2 - d)))/(d^3*e^2*x^2 + d
^4*e), 1/4*(d*e*x - (e*x^2 + d)*sqrt(-d*e)*arctan(sqrt(-d*e)*x/d) - (e*x^2
 + d)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^3*e^2*x
^2 + d^4*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(63) = 126$.

Time = 0.21 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.14

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \frac{x}{4d^3+4d^2ex^2} - \frac{\sqrt{\frac{1}{d^5e}}\log\left(-\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10} - \frac{9d^3\sqrt{\frac{1}{d^5e}}}{10} + x\right)}{8} + \frac{\sqrt{\frac{1}{d^5e}}\log\left(\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10} + \frac{9d^3\sqrt{\frac{1}{d^5e}}}{10} + x\right)}{8} - \frac{\sqrt{-\frac{1}{d^5e}}\log\left(-\frac{4d^8e\left(-\frac{1}{d^5e}\right)^{\frac{3}{2}}}{5} - \frac{9d^3\sqrt{-\frac{1}{d^5e}}}{5} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^5e}}\log\left(\frac{4d^8e\left(-\frac{1}{d^5e}\right)^{\frac{3}{2}}}{5} + \frac{9d^3\sqrt{-\frac{1}{d^5e}}}{5} + x\right)}{4}$$

input
```
integrate(1/(e*x**2+d)/(-e**2*x**4+d**2),x)
```

output
```
x/(4*d**3 + 4*d**2*e*x**2) - sqrt(1/(d**5*e))*log(-d**8*e*(1/(d**5*e))**(3
/2)/10 - 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 + sqrt(1/(d**5*e))*log(d**8*e*(
1/(d**5*e))**(3/2)/10 + 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 - sqrt(-1/(d**5*
e))*log(-4*d**8*e*(-1/(d**5*e))**(3/2)/5 - 9*d**3*sqrt(-1/(d**5*e))/5 + x)
/4 + sqrt(-1/(d**5*e))*log(4*d**8*e*(-1/(d**5*e))**(3/2)/5 + 9*d**3*sqrt(-
1/(d**5*e))/5 + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^2} - \frac{\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{4\sqrt{-de}d^2} + \frac{x}{4(ex^2+d)d^2}$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/2*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2) - 1/4*arctan(e*x/sqrt(-d*e))/(sq
rt(-d*e)*d^2) + 1/4*x/((e*x^2 + d)*d^2)
```

Mupad [B] (verification not implemented)

Time = 17.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \frac{x}{4\,d^2\,\left(e\,x^2+d\right)} + \frac{\mathrm{atanh}\left(\frac{x\,\sqrt{d^5\,e}}{d^3}\right)\,\sqrt{d^5\,e}}{4\,d^5\,e} - \frac{\mathrm{atanh}\left(\frac{x\,\sqrt{-d^5\,e}}{d^3}\right)\,\sqrt{-d^5\,e}}{2\,d^5\,e}$$

input
```
int(1/((d^2 - e^2*x^4)*(d + e*x^2)),x)
```

output
```
x/(4*d^2*(d + e*x^2)) + (atanh((x*(d^5*e)^(1/2))/d^3)*(d^5*e)^(1/2))/(4*d^
5*e) - (atanh((x*(-d^5*e)^(1/2))/d^3)*(-d^5*e)^(1/2))/(2*d^5*e)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.97

$$\int \frac{1}{(d+ex^2)\,(d^2-e^2x^4)}\,dx = \frac{4\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)d + 4\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)e\,x^2 + \sqrt{e}\,\sqrt{d}\log\left(-\sqrt{e}\,\sqrt{d}-ex\right)d + \sqrt{e}\,\sqrt{d}\log\left(-\sqrt{e}\right.}{8d^3e\,(e\,x^2+d)}$$

input
```
int(1/(e*x^2+d)/(-e^2*x^4+d^2),x)
```

output
```
(4*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d + 4*sqrt(e)*sqrt(d)*ata
n((e*x)/(sqrt(e)*sqrt(d)))*e*x**2 + sqrt(e)*sqrt(d)*log( - sqrt(e)*sqrt(d)
 - e*x)*d + sqrt(e)*sqrt(d)*log( - sqrt(e)*sqrt(d) - e*x)*e*x**2 - sqrt(e)
*sqrt(d)*log(sqrt(e)*sqrt(d) - e*x)*d - sqrt(e)*sqrt(d)*log(sqrt(e)*sqrt(d
) - e*x)*e*x**2 + 2*d*e*x)/(8*d**3*e*(d + e*x**2))
```

3.26 $\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx = \frac{x}{8d^2(d+ex^2)^2}+\frac{5x}{16d^3(d+ex^2)} + \frac{7\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}$$

output

```
1/8*x/d^2/(e*x^2+d)^2+5/16*x/d^3/(e*x^2+d)+7/16*arctan(e^(1/2)*x/d^(1/2))/
d^(7/2)/e^(1/2)+1/8*arctanh(e^(1/2)*x/d^(1/2))/d^(7/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx = \frac{\frac{\sqrt{d}x(7d+5ex^2)}{(d+ex^2)^2}+\frac{7\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{16d^{7/2}}$$

input

```
Integrate[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

output
```
((Sqrt[d]*x*(7*d + 5*e*x^2))/(d + e*x^2)^2 + (7*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/Sqrt[e] + (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e])/(16*d^(7/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1388, 316, 25, 27, 402, 27, 397, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{(d-ex^2)(d+ex^2)^3}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{x}{8d^2(d+ex^2)^2}-\frac{\int -\frac{e(7d-3ex^2)}{(d-ex^2)(ex^2+d)^2}dx}{8d^2e}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{e(7d-3ex^2)}{(d-ex^2)(ex^2+d)^2}dx}{8d^2e}+\frac{x}{8d^2(d+ex^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7d-3ex^2}{(d-ex^2)(ex^2+d)^2}dx}{8d^2}+\frac{x}{8d^2(d+ex^2)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{5x}{2d(d+ex^2)}-\frac{\int -\frac{2de\left(9d-5ex^2\right)}{\left(d-ex^2\right)\left(ex^2+d\right)}dx}{4d^2e}}{8d^2}+\frac{x}{8d^2(d+ex^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{9d-5ex^2}{\left(d-ex^2\right)\left(ex^2+d\right)}dx}{2d}+\frac{5x}{2d(d+ex^2)}}{8d^2}+\frac{x}{8d^2\left(d+ex^2\right)^2}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{2\int \frac{1}{d-ex^2}dx+7\int \frac{1}{ex^2+d}dx}{2d}+\frac{5x}{2d(d+ex^2)}}{8d^2}+\frac{x}{8d^2\left(d+ex^2\right)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{2\int \frac{1}{d-ex^2}dx+\frac{7\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}}{2d}+\frac{5x}{2d(d+ex^2)}}{8d^2}+\frac{x}{8d^2\left(d+ex^2\right)^2}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{7\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}}{2d}+\frac{5x}{2d(d+ex^2)}}{8d^2}+\frac{x}{8d^2\left(d+ex^2\right)^2}$$

input
```
Int[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

output
```
x/(8*d^2*(d + e*x^2)^2) + ((5*x)/(2*d*(d + e*x^2)) + ((7*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]) + (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*
Sqrt[e]))/(2*d))/(8*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397

```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1388

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\frac{\frac{5}{2} e\,x^{3}+\frac{7}{2} d x}{\left(e\,x^{2}+d \right)^{2}}+\frac{7 \arctan \left(\frac{e x}{\sqrt{d e}}\right)}{2 \sqrt{d e}}}{8 d^{3}}+\frac{\operatorname{arctanh}\left(\frac{e x}{\sqrt{d e}}\right)}{8 d^{3} \sqrt{d e}}$	64
risch	$\frac{\frac{5 e\,x^{3}}{16 d^{3}}+\frac{7 x}{16 d^{2}}}{\left(e\,x^{2}+d \right)^{2}}+\frac{\ln \left(e x +\sqrt{d e}\right)}{16 \sqrt{d e}\, d^{3}}-\frac{\ln \left(-e x +\sqrt{d e}\right)}{16 \sqrt{d e}\, d^{3}}-\frac{7 \ln \left(-e x -\sqrt{-d e}\right)}{32 \sqrt{-d e}\, d^{3}}+\frac{7 \ln \left(e x -\sqrt{-d e}\right)}{32 \sqrt{-d e}\, d^{3}}$	118

input

```
int(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output

```
1/8/d^3*((5/2*e*x^3+7/2*d*x)/(e*x^2+d)^2+7/2/(d*e)^(1/2)*arctan(e*x/(d*e)^
(1/2)))+1/8/d^3/(d*e)^(1/2)*arctanh(e*x/(d*e)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(65) = 130.

Time = 0.09 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.12

$$\int \frac{1}{(d+ex^2)^2 (d^2-e^2x^4)} \, dx$$
$$= \left[\frac{5\,de^2x^3 + 7\,d^2ex + 7\,(e^2x^4 + 2\,dex^2 + d^2)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right) + (e^2x^4 + 2\,dex^2 + d^2)\sqrt{de}\log\bigg(\frac{ex^2+2\,\sqrt{de}}{ex^2-d}}{16\,(d^4e^3x^4 + 2\,d^5e^2x^2 + d^6e)}\right.$$

input

```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output

```
[1/16*(5*d*e^2*x^3 + 7*d^2*e*x + 7*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(d*e)*a
rctan(sqrt(d*e)*x/d) + (e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(d*e)*log((e*x^2 +
2*sqrt(d*e)*x + d)/(e*x^2 - d)))/(d^4*e^3*x^4 + 2*d^5*e^2*x^2 + d^6*e), 1/
32*(10*d*e^2*x^3 + 14*d^2*e*x - 4*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-d*e)*a
rctan(sqrt(-d*e)*x/d) - 7*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-d*e)*log((e*x^
2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^4*e^3*x^4 + 2*d^5*e^2*x^2 + d^6*e
)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(82) = 164$.

Time = 0.29 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.89

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx = &-\frac{\sqrt{\frac{1}{d^7e}}\log\left(-\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371}-\frac{351d^4\sqrt{\frac{1}{d^7e}}}{371}+x\right)}{16} \\ &+\frac{\sqrt{\frac{1}{d^7e}}\log\left(\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371}+\frac{351d^4\sqrt{\frac{1}{d^7e}}}{371}+x\right)}{16} \\ &-\frac{7\sqrt{-\frac{1}{d^7e}}\log\left(-\frac{245d^{11}e\left(-\frac{1}{d^7e}\right)^{\frac{3}{2}}}{106}-\frac{351d^4\sqrt{-\frac{1}{d^7e}}}{106}+x\right)}{32} \\ &+\frac{7\sqrt{-\frac{1}{d^7e}}\log\left(\frac{245d^{11}e\left(-\frac{1}{d^7e}\right)^{\frac{3}{2}}}{106}+\frac{351d^4\sqrt{-\frac{1}{d^7e}}}{106}+x\right)}{32} \\ &-\frac{-7dx-5ex^3}{16d^5+32d^4ex^2+16d^3e^2x^4}\end{aligned}$$

input

```
integrate(1/(e*x**2+d)**2/(-e**2*x**4+d**2),x)
```

output

```
-sqrt(1/(d**7*e))*log(-20*d**11*e*(1/(d**7*e))**(3/2)/371 - 351*d**4*sqrt(
1/(d**7*e))/371 + x)/16 + sqrt(1/(d**7*e))*log(20*d**11*e*(1/(d**7*e))**(3
/2)/371 + 351*d**4*sqrt(1/(d**7*e))/371 + x)/16 - 7*sqrt(-1/(d**7*e))*log(
-245*d**11*e*(-1/(d**7*e))**(3/2)/106 - 351*d**4*sqrt(-1/(d**7*e))/106 + x
)/32 + 7*sqrt(-1/(d**7*e))*log(245*d**11*e*(-1/(d**7*e))**(3/2)/106 + 351*
d**4*sqrt(-1/(d**7*e))/106 + x)/32 - (-7*d*x - 5*e*x**3)/(16*d**5 + 32*d**
4*e*x**2 + 16*d**3*e**2*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)}\,dx = \frac{7\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de}d^3} - \frac{\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{8\sqrt{-de}d^3} + \frac{5ex^3+7dx}{16\left(ex^2+d\right)^2d^3}$$

input
```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
7/16*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3) - 1/8*arctan(e*x/sqrt(-d*e))/(s
qrt(-d*e)*d^3) + 1/16*(5*e*x^3 + 7*d*x)/((e*x^2 + d)^2*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)}\,dx = \frac{\frac{7x}{16d^2}+\frac{5ex^3}{16d^3}}{d^2+2dex^2+e^2x^4} + \frac{\operatorname{atanh}\left(\frac{x\sqrt{d^7e}}{d^4}\right)\sqrt{d^7e}}{8d^7e} - \frac{7\operatorname{atanh}\left(\frac{x\sqrt{-d^7e}}{d^4}\right)\sqrt{-d^7e}}{16d^7e}$$

input
```
int(1/((d^2 - e^2*x^4)*(d + e*x^2)^2),x)
```

output
```
((7*x)/(16*d^2) + (5*e*x^3)/(16*d^3))/(d^2 + e^2*x^4 + 2*d*e*x^2) + (atanh
((x*(d^7*e)^(1/2))/d^4)*(d^7*e)^(1/2))/(8*d^7*e) - (7*atanh((x*(-d^7*e)^(1
/2))/d^4)*(-d^7*e)^(1/2))/(16*d^7*e)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.74

$$\int \frac{1}{(d+ex^2)^2\,(d^2-e^2x^4)}\,dx = \frac{7\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)d^2+14\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)de\,x^2+7\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)e^2x^4+\sqrt{e}\,\sqrt{d}\log\left(-\sqrt{e}\right.}{}$$

input
```
int(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x)
```

output
```
(7*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2 + 14*sqrt(e)*sqrt(d)
*atan((e*x)/(sqrt(e)*sqrt(d)))*d*e*x**2 + 7*sqrt(e)*sqrt(d)*atan((e*x)/(sq
rt(e)*sqrt(d)))*e**2*x**4 + sqrt(e)*sqrt(d)*log( - sqrt(e)*sqrt(d) - e*x)*
d**2 + 2*sqrt(e)*sqrt(d)*log( - sqrt(e)*sqrt(d) - e*x)*d*e*x**2 + sqrt(e)*
sqrt(d)*log( - sqrt(e)*sqrt(d) - e*x)*e**2*x**4 - sqrt(e)*sqrt(d)*log(sqrt
(e)*sqrt(d) - e*x)*d**2 - 2*sqrt(e)*sqrt(d)*log(sqrt(e)*sqrt(d) - e*x)*d*e
*x**2 - sqrt(e)*sqrt(d)*log(sqrt(e)*sqrt(d) - e*x)*e**2*x**4 + 7*d**2*e*x
+ 5*d*e**2*x**3)/(16*d**4*e*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.27 $\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 62

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} + \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

output
```
-arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(1/2)+2^(1/2)*arctanh(2^(1/2)*e^(1/2
)*x/(e*x^2+d)^(1/2))/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{d-ex^2+\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{2}d}\right) + \log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

output
```
(Sqrt[2]*ArcTanh[(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2])/(Sqrt[2]*d)] + Lo
g[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/Sqrt[e]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1388, 301, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}}{d^2-e^2x^4}\,dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{\sqrt{d+ex^2}}{d-ex^2}dx$$

$$\Big\downarrow \text{ 301}$$

$$2d\int \frac{1}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx-\int \frac{1}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{ 224}$$

$$2d\int \frac{1}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx-\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}$$

$$\Big\downarrow \text{ 219}$$

$$2d\int \frac{1}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

$$\Big\downarrow \text{ 291}$$

$$2d\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

input

```
Int[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

output
```
-(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[e]) + (Sqrt[2]*ArcTanh[(Sqrt[2
]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}\,\sqrt{2}}{2x\sqrt{e}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}}{x\sqrt{e}}\right)}{\sqrt{e}}$	50
default	Expression too large to display	1356

input
```
int((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
(2^(1/2)*arctanh(1/2*(e*x^2+d)^(1/2)/x*2^(1/2)/e^(1/2))-arctanh((e*x^2+d)^
(1/2)/x/e^(1/2)))/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.21

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = \left[\frac{\sqrt{2}\sqrt{e}\log\left(\frac{17\,e^2x^4+14\,dex^2+d^2+\frac{4\sqrt{2}\left(3\,e^2x^3+dex\right)\sqrt{ex^2+d}}{\sqrt{e}}}{e^2x^4-2\,dex^2+d^2}\right)+2\sqrt{e}\log\left(-2\,ex^2+2\sqrt{ex^2+d}\right.}{4\,e}\right.$$

$$-\frac{\sqrt{2}e\sqrt{-\frac{1}{e}}\arctan\left(\frac{\sqrt{2}\left(3\,ex^2+d\right)\sqrt{ex^2+d}\sqrt{-\frac{1}{e}}}{4\,(ex^3+dx)}\right)-2\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2\,e}\right]$$

input
```
integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*sqrt(e)*log((17*e^2*x^4 + 14*d*e*x^2 + d^2 + 4*sqrt(2)*(3*e^
2*x^3 + d*e*x)*sqrt(e*x^2 + d)/sqrt(e))/(e^2*x^4 - 2*d*e*x^2 + d^2)) + 2*s
qrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/e, -1/2*(sqrt(2)*e
*sqrt(-1/e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-1/e)/(e
*x^3 + d*x)) - 2*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e]
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = -\int \frac{\sqrt{d+ex^2}}{-d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

output
```
-Integral(sqrt(d + e*x**2)/(-d + e*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = \int -\frac{(ex^2+d)^{\frac{3}{2}}}{e^2x^4-d^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 + d)^(3/2)/(e^2*x^4 - d^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(46) = 92$.

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = \frac{\sqrt{2}d\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\sqrt{2}|d|-6d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\sqrt{2}|d|-6d\right|}\right)}{2\sqrt{e}|d|} + \frac{\log\left(\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2\right)}{2\sqrt{e}}$$

input
```
integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*d*log(abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d)
 - 6*d)/abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(
sqrt(e)*abs(d)) + 1/2*log((sqrt(e)*x - sqrt(e*x^2 + d))^2)/sqrt(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx=\int \frac{{\left(e\,x^2+d\right)}^{3/2}}{d^2-e^2\,x^4}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x)
```

output
```
int((d + e*x^2)^(3/2)/(d^2 - e^2*x^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.47

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx=\frac{\sqrt{e}\,\left(-\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)+\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)+\sqrt{2}\log\left(\right.\right.}{2e}$$

input
```
int((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x)
```

output
```
(sqrt(e)*( - sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + s
qrt(e)*x)/sqrt(d)) + sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqr
t(d) + sqrt(e)*x)/sqrt(d)) + sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(
2) - sqrt(d) + sqrt(e)*x)/sqrt(d)) - sqrt(2)*log((sqrt(d + e*x**2) + sqrt(
d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d)) - 2*log((sqrt(d + e*x**2) + sqr
t(e)*x)/sqrt(d))))/(2*e)
```

3.28 $\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

output
```
1/2*arctanh(2^(1/2)*e^(1/2)*x/(e*x^2+d)^(1/2))*2^(1/2)/d/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{d-ex^2+\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{2}d}\right)}{\sqrt{2}d\sqrt{e}}$$

input
```
Integrate[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]
```

output
```
ArcTanh[(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2])/(Sqrt[2]*d)]/(Sqrt[2]*d*Sq
rt[e])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1388, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}dx$$

$$\Big\downarrow \; 1388$$

$$\int \frac{1}{(d-ex^2)\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \; 291$$

$$\int \frac{1}{d-\frac{2dex^2}{d+ex^2}}d\frac{x}{\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

input

```
Int[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]
```

output

```
ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}\,\sqrt{2}}{2x\sqrt{e}}\right)}{2d\sqrt{e}}$
default	$-\frac{e\left(\sqrt{\left(x-\frac{\sqrt{de}}{e}\right)^{2}e+2\sqrt{de}\left(x-\frac{\sqrt{de}}{e}\right)+2d}+\frac{\sqrt{de}\,\ln\left(\frac{\sqrt{de}+e\left(x-\frac{\sqrt{de}}{e}\right)}{\sqrt{e}}+\sqrt{\left(x-\frac{\sqrt{de}}{e}\right)^{2}e+2\sqrt{de}\left(x-\frac{\sqrt{de}}{e}\right)+2d}\right)}{\sqrt{e}}-\sqrt{d}\sqrt{2}\,\ln\left(\right.\right.}{2\left(\sqrt{de}-\sqrt{-de}\right)\left(\sqrt{de}+\sqrt{-de}\right)\sqrt{de}}$

input
```
int((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*2^(1/2)/e^(1/2)*arctanh(1/2*(e*x^2+d)^(1/2)/x*2^(1/2)/e^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.63

$$\int\frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx=\left[\frac{\sqrt{2}\log\left(\frac{17\,e^2x^4+14\,dex^2+4\,\sqrt{2}(3\,ex^3+dx)\sqrt{ex^2+d}\sqrt{e}+d^2}{e^2x^4-2\,dex^2+d^2}\right)}{8\,d\sqrt{e}},\ -\frac{\sqrt{2}\sqrt{-e}\arctan\left(\frac{\sqrt{2}(3\,ex^2+d)\sqrt{ex^2+d}\sqrt{-e}}{4\,(e^2x^3+dex)}\right)}{4\,de}\right]$$

input
```
integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/8*sqrt(2)*log((17*e^2*x^4 + 14*d*e*x^2 + 4*sqrt(2)*(3*e*x^3 + d*x)*sqrt
(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2))/(d*sqrt(e)), -1/4*
sqrt(2)*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)
/(e^2*x^3 + d*e*x))/(d*e)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = -\int \frac{1}{-d\sqrt{d+ex^2}+ex^2\sqrt{d+ex^2}}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

output
```
-Integral(1/(-d*sqrt(d + e*x**2) + e*x**2*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \int -\frac{\sqrt{ex^2+d}}{e^2x^4-d^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(e*x^2 + d)/(e^2*x^4 - d^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(29) = 58$.

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\sqrt{2}|d|-6d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\sqrt{2}|d|-6d\right|}\right)}{4\sqrt{e}|d|}$$

input
```
integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d) -
 6*d)/abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(sq
rt(e)*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \int \frac{\sqrt{e\,x^2+d}}{d^2-e^2\,x^4}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(d^2 - e^2*x^4),x)
```

output
```
int((d + e*x^2)^(1/2)/(d^2 - e^2*x^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.34

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \frac{\sqrt{e}\,\sqrt{2}\left(-\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)+\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)+\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)-\mathrm{lo}}{4de}$$

input
```
int((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x)
```

output
```
(sqrt(e)*sqrt(2)*( - log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + s
qrt(e)*x)/sqrt(d)) + log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + s
qrt(e)*x)/sqrt(d)) + log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + s
qrt(e)*x)/sqrt(d)) - log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + s
qrt(e)*x)/sqrt(d))))/(4*d*e)
```

3.29 $\int \frac{1}{\sqrt{d+ex^2}\left(d^2-e^2x^4\right)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{1}{\sqrt{d+ex^2}\left(d^2-e^2x^4\right)}\,dx=\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

output
```
1/2*x/d^2/(e*x^2+d)^(1/2)+1/4*arctanh(2^(1/2)*e^(1/2)*x/(e*x^2+d)^(1/2))*2
^(1/2)/d^2/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sqrt{d+ex^2}\left(d^2-e^2x^4\right)}\,dx=\frac{\frac{2x}{\sqrt{d+ex^2}}+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{d-ex^2+\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{2}d}\right)}{\sqrt{e}}}{4d^2}$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
```

output
```
((2*x)/Sqrt[d + e*x^2] + (Sqrt[2]*ArcTanh[(d - e*x^2 + Sqrt[e]*x*Sqrt[d +
e*x^2])/(Sqrt[2]*d)])/Sqrt[e])/(4*d^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1388, 296, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex^2}\left(d^2-e^2x^4\right)}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{296}$$

$$\frac{\int \frac{1}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx}{2d}+\frac{x}{2d^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2d}+\frac{x}{2d^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}+\frac{x}{2d^2\sqrt{d+ex^2}}$$

input

```
Int[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
```

output

```
x/(2*d^2*Sqrt[d + e*x^2]) + ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(
2*Sqrt[2]*d^2*Sqrt[e])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}\,\sqrt{2}}{2x\sqrt{e}}\right)}{2\sqrt{e}}+\frac{x}{\sqrt{e\,x^{2}+d}}}{2d^{2}}$
default	$-\frac{e\sqrt{2}\,\ln\left(\frac{4d+2\sqrt{de}\left(x-\frac{\sqrt{de}}{e}\right)+2\sqrt{2}\,\sqrt{d}\,\sqrt{\left(x-\frac{\sqrt{de}}{e}\right)^{2}e+2\sqrt{de}\left(x-\frac{\sqrt{de}}{e}\right)+2d}}{x-\frac{\sqrt{de}}{e}}\right)}{4\left(-\sqrt{de}+\sqrt{-de}\right)\left(\sqrt{de}+\sqrt{-de}\right)\sqrt{de}\,\sqrt{d}}+\frac{e\sqrt{2}\,\ln\left(\frac{4d-2\sqrt{de}\left(x+\frac{\sqrt{de}}{e}\right)+2\sqrt{2}\,\sqrt{d}\,\sqrt{(x}}{x+}\right.}{4\left(-\sqrt{de}+\sqrt{-de}\right)(\sqrt{de}-}$

input
```
int(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(1/2*2^(1/2)/e^(1/2)*arctanh(1/2*(e*x^2+d)^(1/2)/x*2^(1/2)/e^(1/2))+1/
(e*x^2+d)^(1/2)*x)/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.43

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx = \left[\frac{\sqrt{2}(ex^2+d)\sqrt{e}\log\left(\frac{17\,e^2x^4+14\,dex^2+4\,\sqrt{2}(3\,ex^3+dx)\sqrt{ex^2+d}\sqrt{e}+d^2}{e^2x^4-2\,dex^2+d^2}\right)+8\,\sqrt{ex^2+d}ex}{16\,(d^2e^2x^2+d^3e)}, -\frac{\sqrt{2}(ex^2+d)\sqrt{-e}\arctan\left(\frac{\sqrt{2}(3\,ex^2+d)\sqrt{ex^2+d}\sqrt{-e}}{4\,(e^2x^3+dex)}\right)-4\,\sqrt{ex^2+d}ex}{8\,(d^2e^2x^2+d^3e)}\right]$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/16*(sqrt(2)*(e*x^2 + d)*sqrt(e)*log((17*e^2*x^4 + 14*d*e*x^2 + 4*sqrt(2
)*(3*e*x^3 + d*x)*sqrt(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4 - 2*d*e*x^2 + d^
2)) + 8*sqrt(e*x^2 + d)*e*x)/(d^2*e^2*x^2 + d^3*e), -1/8*(sqrt(2)*(e*x^2 +
 d)*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e^
2*x^3 + d*e*x)) - 4*sqrt(e*x^2 + d)*e*x)/(d^2*e^2*x^2 + d^3*e)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx = -\int \frac{1}{-d^2\sqrt{d+ex^2}+e^2x^4\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

output
```
-Integral(1/(-d**2*sqrt(d + e*x**2) + e**2*x**4*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx=\int -\frac{1}{(e^2x^4-d^2)\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
-integrate(1/((e^2*x^4 - d^2)*sqrt(e*x^2 + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(45)=90$.

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.66

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx=\frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\sqrt{2}|d|-6d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\sqrt{2}|d|-6d\right|}\right)}{8d\sqrt{e}|d|}+\frac{x}{2\sqrt{ex^2+d}d^2}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*log(abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d) -
 6*d)/abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(d*
sqrt(e)*abs(d)) + 1/2*x/(sqrt(e*x^2 + d)*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx=\int \frac{1}{(d^2-e^2\,x^4)\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 327, normalized size of antiderivative = 5.36

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx$$

$$= \frac{4\sqrt{e\,x^2+d}\,ex-\sqrt{e}\,\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)d-\sqrt{e}\,\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)e\,x^2+\sqrt{e}\,\sqrt{}}{}$$

input
```
int(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x)
```

output
```
(4*sqrt(d + e*x**2)*e*x - sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*
sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d - sqrt(e)*sqrt(2)*log((sqrt(d +
e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e*x**2 + sqrt(e)
*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sq
rt(d))*d + sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(
d) + sqrt(e)*x)/sqrt(d))*e*x**2 + sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) +
sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d + sqrt(e)*sqrt(2)*log((s
qrt(d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e*x**2 -
 sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(
e)*x)/sqrt(d))*d - sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2)
 + sqrt(d) + sqrt(e)*x)/sqrt(d))*e*x**2 + 4*sqrt(e)*d + 4*sqrt(e)*e*x**2)/
(8*d**2*e*(d + e*x**2))
```

3.30 $\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx=\frac{x}{6d^2(d+ex^2)^{3/2}}+\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}$$

output
```
1/6*x/d^2/(e*x^2+d)^(3/2)+7/12*x/d^3/(e*x^2+d)^(1/2)+1/8*arctanh(2^(1/2)*e
^(1/2)*x/(e*x^2+d)^(1/2))*2^(1/2)/d^3/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx=\frac{\frac{2(9dx+7ex^3)}{(d+ex^2)^{3/2}}+\frac{3\sqrt{2}\operatorname{arctanh}\left(\frac{d-ex^2+\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{2}d}\right)}{\sqrt{e}}}{24d^3}$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

output
```
((2*(9*d*x + 7*e*x^3))/(d + e*x^2)^(3/2) + (3*Sqrt[2]*ArcTanh[(d - e*x^2 +
 Sqrt[e]*x*Sqrt[d + e*x^2])/(Sqrt[2]*d)])/Sqrt[e])/(24*d^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1388, 316, 25, 27, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{(d-ex^2)(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{x}{6d^2(d+ex^2)^{3/2}}-\frac{\int -\frac{e(5d-2ex^2)}{(d-ex^2)(ex^2+d)^{3/2}}dx}{6d^2e}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{e(5d-2ex^2)}{(d-ex^2)(ex^2+d)^{3/2}}dx}{6d^2e}+\frac{x}{6d^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5d-2ex^2}{(d-ex^2)(ex^2+d)^{3/2}}dx}{6d^2}+\frac{x}{6d^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{7x}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{3d^2e}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx}{2d^2e}}{6d^2}+\frac{x}{6d^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3}{2}\int \frac{1}{(d-ex^2)\sqrt{ex^2+d}}dx+\frac{7x}{2d\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{3}{2}\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{7x}{2d\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d\sqrt{e}}+\frac{7x}{2d\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}$$

input

```
Int[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

output

```
x/(6*d^2*(d + e*x^2)^(3/2)) + ((7*x)/(2*d*Sqrt[d + e*x^2]) + (3*ArcTanh[(S
qrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[2]*d*Sqrt[e]))/(6*d^2)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{14e^{\frac{3}{2}}x^{3}+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}\,\sqrt{2}}{2x\sqrt{e}}\right)\left(e\,x^{2}+d\right)^{\frac{3}{2}}+18\sqrt{e}\,dx}{24\sqrt{e}\,\left(e\,x^{2}+d\right)^{\frac{3}{2}}d^{3}}$
default	$\frac{e\left(\frac{1}{2d\sqrt{\left(x-\frac{\sqrt{de}}{e}\right)^{2}e+2\sqrt{de}\,\left(x-\frac{\sqrt{de}}{e}\right)+2d}}-\frac{\sqrt{de}\,\left(2e\left(x-\frac{\sqrt{de}}{e}\right)+2\sqrt{de}\right)}{4d^{2}e\sqrt{\left(x-\frac{\sqrt{de}}{e}\right)^{2}e+2\sqrt{de}\,\left(x-\frac{\sqrt{de}}{e}\right)+2d}}-\frac{\sqrt{2}\,\ln\left(\frac{4d+2\sqrt{de}\,\left(x-\frac{\sqrt{de}}{e}\right)+2\sqrt{2}\,\sqrt{d}}{x-}\right)}{4d^{\frac{3}{2}}}\right)}{2\left(-\sqrt{de}+\sqrt{-de}\right)\left(\sqrt{de}+\sqrt{-de}\right)\sqrt{de}}$

input
```
int(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(14*e^(3/2)*x^3+3*2^(1/2)*arctanh(1/2*(e*x^2+d)^(1/2)/x*2^(1/2)/e^(1/
2))*(e*x^2+d)^(3/2)+18*e^(1/2)*d*x)/e^(1/2)/(e*x^2+d)^(3/2)/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(60) = 120$.

Time = 0.09 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.49

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx = \left[\frac{3\sqrt{2}(e^2x^4+2\,dex^2+d^2)\sqrt{e}\log\left(\frac{17\,e^2x^4+14\,dex^2+4\,\sqrt{2}(3\,ex^3+dx)\sqrt{ex^2+d}\sqrt{e}+d^2}{e^2x^4-2\,dex^2+d^2}\right.}{96\,(d^3e^3x^4+2\,d^4e^2x^2+d^5e)}\right.$$

$$\left.-\frac{3\sqrt{2}(e^2x^4+2\,dex^2+d^2)\sqrt{-e}\arctan\left(\frac{\sqrt{2}(3\,ex^2+d)\sqrt{ex^2+d}\sqrt{-e}}{4\,(e^2x^3+dex)}\right)-4\,(7\,e^2x^3+9\,dex)\sqrt{ex^2+d}}{48\,(d^3e^3x^4+2\,d^4e^2x^2+d^5e)}\right]$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/96*(3*sqrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e)*log((17*e^2*x^4 + 14*
d*e*x^2 + 4*sqrt(2)*(3*e*x^3 + d*x)*sqrt(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^
4 - 2*d*e*x^2 + d^2)) + 8*(7*e^2*x^3 + 9*d*e*x)*sqrt(e*x^2 + d))/(d^3*e^3*
x^4 + 2*d^4*e^2*x^2 + d^5*e), -1/48*(3*sqrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)
*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e^2*x
^3 + d*e*x)) - 4*(7*e^2*x^3 + 9*d*e*x)*sqrt(e*x^2 + d))/(d^3*e^3*x^4 + 2*d
^4*e^2*x^2 + d^5*e)]
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx = -\int \frac{1}{-d^3\sqrt{d+ex^2}-d^2ex^2\sqrt{d+ex^2}+de^2x^4\sqrt{d+ex^2}+e^3x^6\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

output
```
-Integral(1/(-d**3*sqrt(d + e*x**2) - d**2*e*x**2*sqrt(d + e*x**2) + d*e**
2*x**4*sqrt(d + e*x**2) + e**3*x**6*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\,(d^2-e^2x^4)}\,dx = \int -\frac{1}{(e^2x^4-d^2)(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
-integrate(1/((e^2*x^4 - d^2)*(e*x^2 + d)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{1}{(d+ex^2)^{3/2}\,(d^2-e^2x^4)}\,dx = \frac{x\left(\frac{7\,ex^2}{d^3}+\frac{9}{d^2}\right)}{12\,(ex^2+d)^{\frac{3}{2}}} + \frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\sqrt{2}|d|-6\,d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\sqrt{2}|d|-6\,d\right|}\right)}{16\,d^2\sqrt{e}|d|}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/12*x*(7*e*x^2/d^3 + 9/d^2)/(e*x^2 + d)^(3/2) + 1/16*sqrt(2)*log(abs(2*(s
qrt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d) - 6*d)/abs(2*(sqrt(e)*x -
 sqrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(d^2*sqrt(e)*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx = \int \frac{1}{(d^2-e^2\,x^4)\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 542, normalized size of antiderivative = 6.78

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx = \frac{36\sqrt{e\,x^2+d}\,dex+28\sqrt{e\,x^2+d}\,e^2x^3-3\sqrt{e}\,\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right.}{}$$

input
```
int(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x)
```

output

```
(36*sqrt(d + e*x**2)*d*e*x + 28*sqrt(d + e*x**2)*e**2*x**3 - 3*sqrt(e)*sqr
t(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d
))*d**2 - 6*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt
(d) + sqrt(e)*x)/sqrt(d))*d*e*x**2 - 3*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**
2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e**2*x**4 + 3*sqrt(e)
*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sq
rt(d))*d**2 + 6*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) +
sqrt(d) + sqrt(e)*x)/sqrt(d))*d*e*x**2 + 3*sqrt(e)*sqrt(2)*log((sqrt(d + e
*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*e**2*x**4 + 3*sqr
t(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x
)/sqrt(d))*d**2 + 6*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2
) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d*e*x**2 + 3*sqrt(e)*sqrt(2)*log((sqrt(d
 + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e**2*x**4 - 3
*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(
e)*x)/sqrt(d))*d**2 - 6*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sq
rt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*d*e*x**2 - 3*sqrt(e)*sqrt(2)*log((sq
rt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*e**2*x**4
 - 20*sqrt(e)*d**2 - 40*sqrt(e)*d*e*x**2 - 20*sqrt(e)*e**2*x**4)/(48*d**3*
e*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.31 $\int \frac{\left(d+ex^2\right)^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 148

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{x(d+ex^2)}{d\sqrt{d^2-e^2x^4}} - \frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
x*(e*x^2+d)/d/(-e^2*x^4+d^2)^(1/2)-d^(1/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE
(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+d^(1/2)*(1-e^2*x^4/d^2)
^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{3dx+2ex^3\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},\frac{e^2x^4}{d^2}\right)}{3d\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d + e*x^2)^2/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(3*d*x + 2*e*x^3*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4, 3/2, 7/4,
(e^2*x^4)/d^2])/(3*d*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.58, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1396, 314, 27, 344, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^2}{\left(d^2-e^2x^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\sqrt{ex^2+d}}{\left(d-ex^2\right)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{\int \frac{ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\int \frac{x^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{344}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\int \frac{x^2}{\sqrt{d^2-e^2x^4}}dx}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{836}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\int\frac{ex^2+d}{d\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int\frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int\frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int\frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 1390$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{ex^2+d}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d + e*x^2)^2/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d + e*x^2])/(d*Sqrt[d - e*x^2])
- (e*Sqrt[d^2 - e^2*x^4]*((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSi
n[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^2*x^4]) - (d^(3/2)*Sqrt
[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*S
qrt[d^2 - e^2*x^4])))/(d*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/Sqrt[d^2 - e^2
*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 344
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c
+ b*d*x^4)^FracPart[p])   Int[(e*x)^m*(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a
, b, c, d, e, m, p}, x] && EqQ[b*c + a*d, 0] &&  !IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

method	result
elliptic	$-\frac{\left(-e^{2} x^{2}-d e\right) x}{e d \sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2} x^{2}-d e\right)}}+\frac{\sqrt{1-\frac{e x^{2}}{d}}\, \sqrt{1+\frac{e x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$
default	$d^{2} \left(\frac{x}{2 d^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}+\frac{\sqrt{1-\frac{e x^{2}}{d}}\, \sqrt{1+\frac{e x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d^{2} \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}\right)+e^{2} \left(\frac{x}{2 e^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}-\frac{\sqrt{1-\frac{e x^{2}}{d}}\, \sqrt{1+\frac{e x^{2}}{d}}\, \mathrm{E}}{2 e^{2} \sqrt{\frac{e}{d}}\, \sqrt{-e}}\right.$

input

```
int((e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-(-e^2*x^2-d*e)/e/d*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/(e/d)^(1/2)*(1-e*
x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/
2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = -\frac{(ex^2-d)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1\right)-(ex^2-d)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1\right)+\sqrt{-e^2x^4+d^2}\,x}{dex^2-d^2}$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-((e*x^2 - d)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - (e*x^2 - d)*
sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*x)/(d
*e*x^2 - d^2)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(d+ex^2)^2}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**2/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral((d + e*x**2)**2/(-(-d + e*x**2)*(d + e*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(ex^2+d)^2}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(ex^2+d)^2}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/(-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(e\,x^2+d)^2}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input
```
int((d + e*x^2)^2/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
int((d + e*x^2)^2/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e^2x^4-2de\,x^2+d^2}dx$$

input
```
int((e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**2 - 2*d*e*x**2 + e**2*x**4),x)
```

3.32 $\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 151

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^2}\,dx = \frac{x\sqrt{d^2-e^2x^4}}{d(d-ex^2)} - \frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
x*(-e^2*x^4+d^2)^(1/2)/d/(-e*x^2+d)-d^(1/2)*(1-e^2*x^4/d^2)^(1/2)*Elliptic
E(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+d^(1/2)*(1-e^2*x^4/d^2
)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^2}\,dx = -\frac{x\sqrt{d^2-e^2x^4}}{d(-d+ex^2)} + \frac{i\sqrt{1-\frac{ex^2}{d}}\sqrt{1+\frac{ex^2}{d}}\left(E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)\right)}{\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[Sqrt[d^2 - e^2*x^4]/(d - e*x^2)^2,x]
```

output

```
-((x*Sqrt[d^2 - e^2*x^4])/(d*(-d + e*x^2))) + (I*Sqrt[1 - (e*x^2)/d]*Sqrt[
1 + (e*x^2)/d]*(EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - EllipticF[I*Arc
Sinh[Sqrt[-(e/d)]*x], -1]))/(Sqrt[-(e/d)]*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.55, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {1396, 314, 27, 344, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{ex^2+d}}{\left(d-ex^2\right)^{3/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 314}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{\int \frac{ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\int \frac{x^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 344}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\int \frac{x^2}{\sqrt{d^2-e^2x^4}}dx}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 836$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\int \frac{ex^2+d}{d\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 1390$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{ex^2+d}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[Sqrt[d^2 - e^2*x^4]/(d - e*x^2)^2,x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d + e*x^2])/(d*Sqrt[d - e*x^2]) - (e*Sqrt[d^
2 - e^2*x^4]*((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x
)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^2*x^4]) - (d^(3/2)*Sqrt[1 - (e^2*x^
4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^
2*x^4])))/(d*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/(Sqrt[d - e*x^2]*Sqrt[d +
e*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 344
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c
+ b*d*x^4)^FracPart[p])   Int[(e*x)^m*(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a
, b, c, d, e, m, p}, x] && EqQ[b*c + a*d, 0] &&  !IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\left(-e^{2} x^{2}-d e \right) x}{e d \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	124
elliptic	$-\frac{\left(-e^{2} x^{2}-d e \right) x}{e d \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	124

input
```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2-d*e)/e/d*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/(e/d)^(1/2)*(1-e*
x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/
2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^2}\,dx = -\frac{(ex^2-d)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)-(ex^2-d)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)+\sqrt{-e^2x^4+d^2}\,x}{dex^2-d^2}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^2,x, algorithm="fricas")
```

output
```
-((e*x^2 - d)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - (e*x^2 - d)*
sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*x)/(d
*e*x^2 - d^2)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^2}\,dx = \int \frac{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}}{\left(-d+ex^2\right)^2}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(-e*x**2+d)**2,x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/(-d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^2}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{\left(ex^2-d\right)^2}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 - d)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^2}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{(ex^2-d)^2}\,dx$$

input

```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^2,x, algorithm="giac")
```

output

```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 - d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^2}\,dx = \int \frac{\sqrt{d^2-e^2\,x^4}}{(d-e\,x^2)^2}\,dx$$

input

```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2)^2,x)
```

output

```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^2}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e^2x^4-2de\,x^2+d^2}dx$$

input

```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^2,x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**2 - 2*d*e*x**2 + e**2*x**4),x)
```

3.33 $\int \frac{d+ex^2}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 151

$$\int \frac{d+ex^2}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = \frac{x\sqrt{d^2-e^2x^4}}{d(d-ex^2)} - \frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
x*(-e^2*x^4+d^2)^(1/2)/d/(-e*x^2+d)-d^(1/2)*(1-e^2*x^4/d^2)^(1/2)*Elliptic
E(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+d^(1/2)*(1-e^2*x^4/d^2
)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89

$$\int \frac{d+ex^2}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = -\frac{x\sqrt{d^2-e^2x^4}}{d(-d+ex^2)} + \frac{i\sqrt{1-\frac{ex^2}{d}}\sqrt{1+\frac{ex^2}{d}}\left(E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)\right)}{\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d + e*x^2)/((d - e*x^2)*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
-((x*Sqrt[d^2 - e^2*x^4])/(d*(-d + e*x^2))) + (I*Sqrt[1 - (e*x^2)/d]*Sqrt[
1 + (e*x^2)/d]*(EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - EllipticF[I*Arc
Sinh[Sqrt[-(e/d)]*x], -1]))/(Sqrt[-(e/d)]*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.55, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {1396, 314, 27, 344, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(d-ex^2)\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\sqrt{ex^2+d}}{(d-ex^2)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{\int \frac{ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\int \frac{x^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{344}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\int \frac{x^2}{\sqrt{d^2-e^2x^4}}dx}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 836$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\int\frac{ex^2+d}{d\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int\frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int\frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int\frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 1390$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{ex^2+d}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d + e*x^2)/((d - e*x^2)*Sqrt[d^2 - e^2*x^4]),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d + e*x^2])/(d*Sqrt[d - e*x^2])
- (e*Sqrt[d^2 - e^2*x^4]*((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSi
n[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^2*x^4]) - (d^(3/2)*Sqrt
[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*S
qrt[d^2 - e^2*x^4])))/(d*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/Sqrt[d^2 - e^2
*x^4]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 344

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c
+ b*d*x^4)^FracPart[p])   Int[(e*x)^m*(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a
, b, c, d, e, m, p}, x] && EqQ[b*c + a*d, 0] &&  !IntegerQ[p]
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.82

method	result
elliptic	$-\frac{\left(-e^{2} x^{2}-d e \right) x}{e d \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$
default	$-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}+2 d \left(-\frac{\left(-e^{2} x^{2}-d e \right) x}{2 d^{2} e \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}\right.$

input
```
int((e*x^2+d)/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2-d*e)/e/d*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/(e/d)^(1/2)*(1-e*
x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/
2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.61

$$\int \frac{d+ex^2}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = -\frac{(ex^2-d)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)-(ex^2-d)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)+\sqrt{-e^2x^4+d^2}\,x}{dex^2-d^2}$$

input

```
integrate((e*x^2+d)/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output

```
-((e*x^2 - d)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - (e*x^2 - d)*
sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*x)/(d
*e*x^2 - d^2)
```

Sympy [F]

$$\int \frac{d+ex^2}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = -\int \frac{d}{-d\sqrt{d^2-e^2x^4}+ex^2\sqrt{d^2-e^2x^4}}\,dx - \int \frac{ex^2}{-d\sqrt{d^2-e^2x^4}+ex^2\sqrt{d^2-e^2x^4}}\,dx$$

input

```
integrate((e*x**2+d)/(-e*x**2+d)/(-e**2*x**4+d**2)**(1/2),x)
```

output

```
-Integral(d/(-d*sqrt(d**2 - e**2*x**4) + e*x**2*sqrt(d**2 - e**2*x**4)), x
) - Integral(e*x**2/(-d*sqrt(d**2 - e**2*x**4) + e*x**2*sqrt(d**2 - e**2*x
**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{d+ex^2}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int -\frac{ex^2+d}{\sqrt{-e^2x^4+d^2}(ex^2-d)}\,dx$$

input
```
integrate((e*x^2+d)/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(e*x^2 + d)/(sqrt(-e^2*x^4 + d^2)*(e*x^2 - d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{d^2-e^2\,x^4}\,\left(d-e\,x^2\right)}\,dx$$

input
```
int((d + e*x^2)/((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)),x)
```

output
```
int((d + e*x^2)/((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^2x^4-2de\,x^2+d^2}dx$$

input
```
int((e*x^2+d)/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**2 - 2*d*e*x**2 + e**2*x**4),x)
```

3.34 $\int \left(d+ex^2\right)^3 \sqrt{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 205

$$\begin{aligned}\int \left(d+ex^2\right)^3 \sqrt{d^2-e^2x^4}\,dx = {} & \frac{2}{21}d^2x\left(5d+7ex^2\right)\sqrt{d^2-e^2x^4} \\ & -\frac{3}{7}dx\left(d^2-e^2x^4\right)^{3/2}-\frac{1}{9}ex^3\left(d^2-e^2x^4\right)^{3/2} \\ & +\frac{4d^{11/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}} \\ & -\frac{8d^{11/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{21\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output
```
2/21*d^2*x*(7*e*x^2+5*d)*(-e^2*x^4+d^2)^(1/2)-3/7*d*x*(-e^2*x^4+d^2)^(3/2)
-1/9*e*x^3*(-e^2*x^4+d^2)^(3/2)+4/3*d^(11/2)*(1-e^2*x^4/d^2)^(1/2)*Ellipti
cE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-8/21*d^(11/2)*(1-e^2*
x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.42 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.70

$$\int \left(d+ex^2\right)^3\sqrt{d^2-e^2x^4}\,dx$$
$$=\frac{x\sqrt{d^2-e^2x^4}\left(\sqrt{1-\frac{e^2x^4}{d^2}}\left(-27d^3-7d^2ex^2+27de^2x^4+7e^3x^6\right)+90d^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{e^2}{d^2}\right.\right.}{63\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input
```
Integrate[(d + e*x^2)^3*Sqrt[d^2 - e^2*x^4],x]
```

output
```
(x*Sqrt[d^2 - e^2*x^4]*(Sqrt[1 - (e^2*x^4)/d^2]*(-27*d^3 - 7*d^2*e*x^2 + 2
7*d*e^2*x^4 + 7*e^3*x^6) + 90*d^3*Hypergeometric2F1[-1/2, 1/4, 5/4, (e^2*x
^4)/d^2] + 70*d^2*e*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, (e^2*x^4)/d^2]))
/(63*Sqrt[1 - (e^2*x^4)/d^2])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.49, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {1396, 318, 27, 403, 27, 403, 27, 403, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3\sqrt{d^2-e^2x^4}\,dx$$
$$\Big\downarrow \text{1396}$$
$$\frac{\sqrt{d^2-e^2x^4}\int \sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$
$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{\int -10de\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}\left(2ex^2+d\right)dx}{9e}-\frac{1}{9}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{5/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\int \sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}\left(2ex^2+d\right)dx-\frac{1}{9}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{5/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\Big(\frac{10}{9}d\Big(-\frac{\int -3de\sqrt{d-ex^2}\sqrt{ex^2+d}\left(7ex^2+3d\right)dx}{7e}-\frac{2}{7}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}\Big)-\frac{1}{9}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{5/}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\Big(\frac{10}{9}d\Big(\frac{3}{7}d\int \sqrt{d-ex^2}\sqrt{ex^2+d}\left(7ex^2+3d\right)dx-\frac{2}{7}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}\Big)-\frac{1}{9}x\left(d-ex^2\right)^{3/2}(d-}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{10}{9}d\Bigg(\frac{3}{7}d\Bigg(-\frac{\int -\frac{2de\sqrt{d-ex^2}\left(18ex^2+11d\right)}{\sqrt{ex^2+d}}dx}{5e}-\frac{7}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\Bigg)-\frac{2}{7}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}\Bigg)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\Big(\frac{10}{9}d\Big(\frac{3}{7}d\Big(\frac{2}{5}d\int \frac{\sqrt{d-ex^2}\left(18ex^2+11d\right)}{\sqrt{ex^2+d}}dx-\frac{7}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\Big)-\frac{2}{7}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}\Big)-}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{10}{9}d\Bigg(\frac{3}{7}d\Bigg(\frac{2}{5}d\Bigg(\frac{\int \frac{3de\left(7ex^2+5d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}+6x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)-\frac{7}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\Bigg)-\frac{2}{7}x(d-}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\Big(\frac{10}{9}d\Big(\frac{3}{7}d\Big(\frac{2}{5}d\Big(d\int \frac{7ex^2+5d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+6x\sqrt{d-ex^2}\sqrt{d+ex^2}\Big)-\frac{7}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\Big)-\frac{2}{7}x(d-}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\left(\frac{3}{7}d\left(\frac{2}{5}d\left(d\left(7\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-2d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx\right)+6x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{7}{5}x\left(d-ex^2\right)^{3/2}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\left(\frac{3}{7}d\left(\frac{2}{5}d\left(d\left(7\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{2d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+6x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{7}{5}x(d-ex^2}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{329}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\left(\frac{3}{7}d\left(\frac{2}{5}d\left(d\left(\frac{7d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+6x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{7}{5}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\left(\frac{3}{7}d\left(\frac{2}{5}d\left(d\left(\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+6x\sqrt{d-ex^2}\sqrt{d+ex}}{\sqrt{d-ex^2}\sqrt{d+}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\left(\frac{3}{7}d\left(\frac{2}{5}d\left(d\left(\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+6x\sqrt{d-ex^2}\sqrt{d+e}}{\sqrt{d-ex^2}\sqrt{d+}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\left(\frac{3}{7}d\left(\frac{2}{5}d\left(d\left(\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+6x\sqrt{d-}}{\sqrt{d-ex^2}}$$

input

```
Int[(d + e*x^2)^3*Sqrt[d^2 - e^2*x^4],x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(-1/9*(x*(d - e*x^2)^(3/2)*(d + e*x^2)^(5/2)) + (10*d
*((-2*x*(d - e*x^2)^(3/2)*(d + e*x^2)^(3/2))/7 + (3*d*((-7*x*(d - e*x^2)^(
3/2)*Sqrt[d + e*x^2])/5 + (2*d*(6*x*Sqrt[d - e*x^2]*Sqrt[d + e*x^2] + d*((
7*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -
1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) - (2*d^(3/2)*Sqrt[1 - (e^2*x
^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x
^2]*Sqrt[d + e*x^2]))))/5))/7))/9))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 7.88 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95

method	result
risch	$\frac{x\left(7e^3x^6+27d\,e^2x^4+35d^2e\,x^2+3d^3\right)\sqrt{-e^2x^4+d^2}}{63}+\frac{4d^4\left(\frac{5d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{7d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\left(\operatorname{EllipticF}\left(x\right.\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x}}\right.}{21}$
elliptic	$\frac{e^3x^7\sqrt{-e^2x^4+d^2}}{9}+\frac{3d\,e^2x^5\sqrt{-e^2x^4+d^2}}{7}+\frac{5d^2e\,x^3\sqrt{-e^2x^4+d^2}}{9}+\frac{d^3x\sqrt{-e^2x^4+d^2}}{21}+\frac{20d^5\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\right.}{21\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$
default	$d^3\left(\frac{x\sqrt{-e^2x^4+d^2}}{3}+\frac{2d^2\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)+e^3\left(\frac{x^7\sqrt{-e^2x^4+d^2}}{9}-\frac{2d^2x^3\sqrt{-e^2x^4+d^2}}{45e^2}-\frac{2d^5\sqrt{1}}{}\right.$

input
```
int((e*x^2+d)^3*(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/63*x*(7*e^3*x^6+27*d*e^2*x^4+35*d^2*e*x^2+3*d^3)*(-e^2*x^4+d^2)^(1/2)+4/
21*d^4*(5*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)
^(1/2)*EllipticF(x*(e/d)^(1/2),I)-7*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x
^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(
e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.72

$$\int \left(d+ex^2\right)^3\sqrt{d^2-e^2x^4}\,dx =$$

$$-\frac{84\sqrt{-e^2}d^5x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)-12\,(7\,d^5+5\,d^4e)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)-(7\,e^5x^8}{63\,e^2x}$$

input
```
integrate((e*x^2+d)^3*(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/63*(84*sqrt(-e^2)*d^5*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) -
 12*(7*d^5 + 5*d^4*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x
), -1) - (7*e^5*x^8 + 27*d*e^4*x^6 + 35*d^2*e^3*x^4 + 3*d^3*e^2*x^2 - 84*d
^4*e)*sqrt(-e^2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \left(d+ex^2\right)^3\sqrt{d^2-e^2x^4}\,dx = {} & \frac{d^4x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} \\ & + \frac{3d^3ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{7}{4}\right)} \\ & + \frac{3d^2e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{9}{4}\right)} \\ & + \frac{de^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{11}{4}\right)}\end{aligned}$$

input
```
integrate((e*x**2+d)**3*(-e**2*x**4+d**2)**(1/2),x)
```

output
```
d**4*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) + 3*d**3*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,),
e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(7/4)) + 3*d**2*e**2*x**5*gamma(
5/4)*hyper((-1/2, 5/4), (9/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma
(9/4)) + d*e**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), e**2*x**4*exp_
polar(2*I*pi)/d**2)/(4*gamma(11/4))
```

Maxima [F]

$$\int \left(d+ex^2\right)^3 \sqrt{d^2-e^2x^4}\,dx = \int \sqrt{-e^2x^4+d^2}\left(ex^2+d\right)^3\,dx$$

input
```
integrate((e*x^2+d)^3*(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^3, x)
```

Giac [F]

$$\int \left(d+ex^2\right)^3 \sqrt{d^2-e^2x^4}\,dx = \int \sqrt{-e^2x^4+d^2}\left(ex^2+d\right)^3\,dx$$

input
```
integrate((e*x^2+d)^3*(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^3 \sqrt{d^2-e^2x^4}\,dx = \int \sqrt{d^2-e^2\,x^4}\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^3,x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^3, x)
```

Reduce [F]

$$\begin{aligned}\int \left(d+ex^2\right)^3 \sqrt{d^2-e^2x^4}\,dx &= \frac{\sqrt{-e^2x^4+d^2}\,d^3x}{21}+\frac{5\sqrt{-e^2x^4+d^2}\,d^2e\,x^3}{9}\\ &\quad+\frac{3\sqrt{-e^2x^4+d^2}\,d\,e^2x^5}{7}+\frac{\sqrt{-e^2x^4+d^2}\,e^3x^7}{9}\\ &\quad+\frac{20\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^5}{21}+\frac{4\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^4e}{3}\end{aligned}$$

input

```
int((e*x^2+d)^3*(-e^2*x^4+d^2)^(1/2),x)
```

output

```
(3*sqrt(d**2 - e**2*x**4)*d**3*x + 35*sqrt(d**2 - e**2*x**4)*d**2*e*x**3 +
 27*sqrt(d**2 - e**2*x**4)*d*e**2*x**5 + 7*sqrt(d**2 - e**2*x**4)*e**3*x**
7 + 60*int(sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)*d**5 + 84*int((sqr
t(d**2 - e**2*x**4)*x**2)/(d**2 - e**2*x**4),x)*d**4*e)/63
```

3.35 $\int \left(d+ex^2\right)^2 \sqrt{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 178

$$\int \left(d+ex^2\right)^2 \sqrt{d^2-e^2x^4}\,dx = \frac{2}{105}dx\left(20d+21ex^2\right)\sqrt{d^2-e^2x^4}-\frac{1}{7}x\left(d^2-e^2x^4\right)^{3/2} + \frac{4d^{9/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)\middle|-1\right)}{5\sqrt{e}\sqrt{d^2-e^2x^4}} - \frac{4d^{9/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{ex}}{\sqrt{d}}\right),-1\right)}{105\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
2/105*d*x*(21*e*x^2+20*d)*(-e^2*x^4+d^2)^(1/2)-1/7*x*(-e^2*x^4+d^2)^(3/2)+
4/5*d^(9/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(
-e^2*x^4+d^2)^(1/2)-4/105*d^(9/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*
x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.57 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.69

$$\int \left(d+ex^2\right)^2\sqrt{d^2-e^2x^4}\,dx$$
$$=\frac{x\sqrt{d^2-e^2x^4}\left(-3(d^2-e^2x^4)\sqrt{1-\frac{e^2x^4}{d^2}}+24d^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)+14dex^2\,\text{Hypergeo}\right.}{21\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input

```
Integrate[(d + e*x^2)^2*Sqrt[d^2 - e^2*x^4],x]
```

output

```
(x*Sqrt[d^2 - e^2*x^4]*(-3*(d^2 - e^2*x^4)*Sqrt[1 - (e^2*x^4)/d^2] + 24*d^
2*Hypergeometric2F1[-1/2, 1/4, 5/4, (e^2*x^4)/d^2] + 14*d*e*x^2*Hypergeome
tric2F1[-1/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(21*Sqrt[1 - (e^2*x^4)/d^2])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.56, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1396, 318, 27, 403, 25, 27, 403, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2\sqrt{d^2-e^2x^4}\,dx$$
$$\Big\downarrow \text{1396}$$
$$\frac{\sqrt{d^2-e^2x^4}\int \sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$
$$\Big\downarrow \text{318}$$
$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{\int -2de\sqrt{d-ex^2}\sqrt{ex^2+d}\left(7ex^2+4d\right)dx}{7e}-\frac{1}{7}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{7}d\int \sqrt{d-ex^2}\sqrt{ex^2+d}(7ex^2+4d)\,dx-\frac{1}{7}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{7}d\left(-\frac{\int -\frac{de\sqrt{d-ex^2}\left(41ex^2+27d\right)}{\sqrt{ex^2+d}}dx}{5e}-\frac{7}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)-\frac{1}{7}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{7}d\left(\frac{\int \frac{de\sqrt{d-ex^2}\left(41ex^2+27d\right)}{\sqrt{ex^2+d}}dx}{5e}-\frac{7}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\right)-\frac{1}{7}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{7}d\left(\frac{1}{5}d\int \frac{\sqrt{d-ex^2}\left(41ex^2+27d\right)}{\sqrt{ex^2+d}}dx-\frac{7}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\right)-\frac{1}{7}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 403$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{7}d\Bigg(\frac{1}{5}d\Bigg(\frac{\int \frac{2de\left(21ex^2+20d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}+\frac{41}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)-\frac{7}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\Bigg)-\frac{1}{7}x\big(d-ex^2}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{7}d\left(\frac{1}{5}d\left(\frac{2}{3}d\int \frac{21ex^2+20d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+\frac{41}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{7}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\right)-\frac{1}{7}x\big(d-e\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 399$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{7}d\left(\frac{1}{5}d\left(\frac{2}{3}d\left(21\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx\right)+\frac{41}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{7}{5}x\left(d-ex^2\right)^{3/2}\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 289$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{7}d\Bigg(\frac{1}{5}d\Bigg(\frac{2}{3}d\Bigg(21\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+\frac{41}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)-\frac{7}{5}x(d-ex^2)^{3/}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 329$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{7}d\Bigg(\frac{1}{5}d\Bigg(\frac{2}{3}d\Bigg(\frac{21d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+\frac{41}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)-\frac{7}{5}x(}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{7}d\Bigg(\frac{1}{5}d\Bigg(\frac{2}{3}d\Bigg(\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+\frac{41}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 765$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{7}d\Bigg(\frac{1}{5}d\Bigg(\frac{2}{3}d\Bigg(\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+\frac{41}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 762$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{7}d\Bigg(\frac{1}{5}d\Bigg(\frac{2}{3}d\Bigg(\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+\frac{41}{3}x\sqrt{d-e}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[(d + e*x^2)^2*Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/7*(x*(d - e*x^2)^(3/2)*(d + e*x^2)^(3/2)) + (2*d*
((-7*x*(d - e*x^2)^(3/2)*Sqrt[d + e*x^2])/5 + (d*((41*x*Sqrt[d - e*x^2]*Sq
rt[d + e*x^2])/3 + (2*d*((21*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[Arc
Sin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) -
 (d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -
1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/3))/5))/7))/(Sqrt[d - e*x^
2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 5.60 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.03

method	result
risch	$\frac{x(15e^2x^4+42de\,x^2+25d^2)\sqrt{-e^2x^4+d^2}}{105}+\frac{4d^3\left(\frac{20d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{21d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right.}{105}$
elliptic	$\frac{e^2x^5\sqrt{-e^2x^4+d^2}}{7}+\frac{2de\,x^3\sqrt{-e^2x^4+d^2}}{5}+\frac{5d^2x\sqrt{-e^2x^4+d^2}}{21}+\frac{16d^4\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{21\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-4d^4\sqrt{1-\frac{e\,x^2}{d}}$
default	$d^2\left(\frac{x\sqrt{-e^2x^4+d^2}}{3}+\frac{2d^2\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)+e^2\left(\frac{x^5\sqrt{-e^2x^4+d^2}}{7}-\frac{2d^2x\sqrt{-e^2x^4+d^2}}{21e^2}+2d^4\sqrt{1-}\right.$

input
```
int((e*x^2+d)^2*(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*x*(15*e^2*x^4+42*d*e*x^2+25*d^2)*(-e^2*x^4+d^2)^(1/2)+4/105*d^3*(20*
d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*Ell
ipticF(x*(e/d)^(1/2),I)-21*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/
2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2
),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.77

$$\int \left(d+ex^2\right)^2\sqrt{d^2-e^2x^4}\,dx =$$

$$-\frac{84\sqrt{-e^2}d^4x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)-4\left(21\,d^4+20\,d^3e\right)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)-(15\,e^4x}{105\,e^2x}$$

input
```
integrate((e*x^2+d)^2*(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/105*(84*sqrt(-e^2)*d^4*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1)
- 4*(21*d^4 + 20*d^3*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)
/x), -1) - (15*e^4*x^6 + 42*d*e^3*x^4 + 25*d^2*e^2*x^2 - 84*d^3*e)*sqrt(-e
^2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.75

$$\int \left(d+ex^2\right)^2\sqrt{d^2-e^2x^4}\,dx=\frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{7}{4}\right)}+\frac{de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((e*x**2+d)**2*(-e**2*x**4+d**2)**(1/2),x)
```

output
```
d**3*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) + d**2*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), e*
*2*x**4*exp_polar(2*I*pi)/d**2)/(2*gamma(7/4)) + d*e**2*x**5*gamma(5/4)*hy
per((-1/2, 5/4), (9/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(9/4))
```

Maxima [F]

$$\int \left(d+ex^2\right)^2 \sqrt{d^2-e^2x^4}\,dx = \int \sqrt{-e^2x^4+d^2}\left(ex^2+d\right)^2\,dx$$

input

```
integrate((e*x^2+d)^2*(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output

```
integrate(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^2, x)
```

Giac [F]

$$\int \left(d+ex^2\right)^2 \sqrt{d^2-e^2x^4}\,dx = \int \sqrt{-e^2x^4+d^2}\left(ex^2+d\right)^2\,dx$$

input

```
integrate((e*x^2+d)^2*(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output

```
integrate(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 \sqrt{d^2-e^2x^4}\,dx = \int \sqrt{d^2-e^2\,x^4}\,\left(e\,x^2+d\right)^2\,dx$$

input

```
int((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^2,x)
```

output

```
int((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^2 \sqrt{d^2-e^2x^4}\,dx = \frac{5\sqrt{-e^2x^4+d^2}\,d^2x}{21}+\frac{2\sqrt{-e^2x^4+d^2}\,de\,x^3}{5} + \frac{\sqrt{-e^2x^4+d^2}\,e^2x^5}{7}+\frac{16\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^4}{21} + \frac{4\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^3e}{5}$$

input

```
int((e*x^2+d)^2*(-e^2*x^4+d^2)^(1/2),x)
```

output

```
(25*sqrt(d**2 - e**2*x**4)*d**2*x + 42*sqrt(d**2 - e**2*x**4)*d*e*x**3 + 1
5*sqrt(d**2 - e**2*x**4)*e**2*x**5 + 80*int(sqrt(d**2 - e**2*x**4)/(d**2 -
 e**2*x**4),x)*d**4 + 84*int((sqrt(d**2 - e**2*x**4)*x**2)/(d**2 - e**2*x*
*4),x)*d**3*e)/105
```

3.36 $\int (d+ex^2)\sqrt{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\begin{aligned}\int (d+ex^2)\sqrt{d^2-e^2x^4}\,dx &= \frac{1}{15}x(5d+3ex^2)\sqrt{d^2-e^2x^4} \\ &\quad + \frac{2d^{7/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{5\sqrt{e}\sqrt{d^2-e^2x^4}} \\ &\quad + \frac{4d^{7/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{15\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/15*x*(3*e*x^2+5*d)*(-e^2*x^4+d^2)^(1/2)+2/5*d^(7/2)*(1-e^2*x^4/d^2)^(1/2
)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+4/15*d^(7/2)
*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.82 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.56

$$\int (d+ex^2)\sqrt{d^2-e^2x^4}\,dx = \frac{\sqrt{d^2-e^2x^4}\left(3dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)+ex^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{e^2x^4}{d^2}\right)\right)}{3\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input

```
Integrate[(d + e*x^2)*Sqrt[d^2 - e^2*x^4],x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(3*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, (e^2*x^4)/d^
2] + e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(3*Sqrt[1 -
(e^2*x^4)/d^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)\sqrt{d^2-e^2x^4}\,dx$$

$$\Big\downarrow \text{ 1571}$$

$$\int (d+ex^2)\sqrt{d^2-e^2x^4}dx$$

input

```
Int[(d + e*x^2)*Sqrt[d^2 - e^2*x^4],x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10

method	result
risch	$\frac{x(3e x^2+5d)\sqrt{-e^2x^4+d^2}}{15} + \frac{2d^2\bigg(\frac{5d\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}} - \frac{3d\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\ \left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right.\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}}{15}$
elliptic	$\frac{e x^3\sqrt{-e^2x^4+d^2}}{5}+\frac{dx\sqrt{-e^2x^4+d^2}}{3}+\frac{2d^3\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{2d^3\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\ \bigg(\mathrm{EllipticF}\bigg(x\sqrt{\frac{e}{}}}{5\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d}}$
default	$d\left(\frac{x\sqrt{-e^2x^4+d^2}}{3}+\frac{2d^2\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)+e\bigg(\frac{x^3\sqrt{-e^2x^4+d^2}}{5}-\frac{2d^3\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\ \bigg(\mathrm{Elliptic}}{5\sqrt{\frac{e}{d}}\,\sqrt{-}}$

input
```
int((e*x^2+d)*(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*e*x^2+5*d)*(-e^2*x^4+d^2)^(1/2)+2/15*d^2*(5*d/(e/d)^(1/2)*(1-e*x
^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2)
,I)-3*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/
2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int \left(d+ex^2\right)\sqrt{d^2-e^2x^4}\,dx =$$

$$-\frac{6\sqrt{-e^2}d^3x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)-2\left(3\,d^3+5\,d^2e\right)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)-(3\,e^3x^4+}{15\,e^2x}$$

input
```
integrate((e*x^2+d)*(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(6*sqrt(-e^2)*d^3*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) -
2*(3*d^3 + 5*d^2*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x),
 -1) - (3*e^3*x^4 + 5*d*e^2*x^2 - 6*d^2*e)*sqrt(-e^2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.54

$$\int (d+ex^2)\sqrt{d^2-e^2x^4}\,dx = \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)*(-e**2*x**4+d**2)**(1/2),x)
```

output
```
d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), e**2*
x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(7/4))
```

Maxima [F]

$$\int (d+ex^2)\sqrt{d^2-e^2x^4}\,dx = \int \sqrt{-e^2x^4+d^2}\,(ex^2+d)\,dx$$

input
```
integrate((e*x^2+d)*(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d), x)
```

Giac [F]

$$\int (d+ex^2)\sqrt{d^2-e^2x^4}\,dx=\int \sqrt{-e^2x^4+d^2}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\sqrt{d^2-e^2x^4}\,dx=\int \sqrt{d^2-e^2\,x^4}\,\left(e\,x^2+d\right)\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)*(d + e*x^2),x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2)\sqrt{d^2-e^2x^4}\,dx=\frac{\sqrt{-e^2x^4+d^2}\,dx}{3}+\frac{\sqrt{-e^2x^4+d^2}\,e\,x^3}{5}+\frac{2\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^3}{3}+\frac{2\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^2e}{5}$$

input
```
int((e*x^2+d)*(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(5*sqrt(d**2 - e**2*x**4)*d*x + 3*sqrt(d**2 - e**2*x**4)*e*x**3 + 10*int(s
qrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)*d**3 + 6*int((sqrt(d**2 - e**2
*x**4)*x**2)/(d**2 - e**2*x**4),x)*d**2*e)/15
```

3.37 $\int \sqrt{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int \sqrt{d^2-e^2x^4}\,dx=\frac{1}{3}x\sqrt{d^2-e^2x^4}+\frac{2d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
1/3*x*(-e^2*x^4+d^2)^(1/2)+2/3*d^(5/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(
1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \sqrt{d^2-e^2x^4}\,dx=\frac{d^2x-e^2x^5+\frac{2ide\sqrt{1-\frac{e^2x^4}{d^2}}\,\mathrm{EllipticF}\left(i\,\mathrm{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)}{\left(-\frac{e}{d}\right)^{3/2}}}{3\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4],x]
```

output

```
(d^2*x - e^2*x^5 + ((2*I)*d*e*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[
Sqrt[-(e/d)]*x], -1])/(-(e/d))^(3/2))/(3*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {748, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d^2-e^2x^4}\,dx$$

$$\Big\downarrow \text{748}$$

$$\frac{2}{3}d^2\int \frac{1}{\sqrt{d^2-e^2x^4}}dx+\frac{1}{3}x\sqrt{d^2-e^2x^4}$$

$$\Big\downarrow \text{765}$$

$$\frac{2d^2\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{3\sqrt{d^2-e^2x^4}}+\frac{1}{3}x\sqrt{d^2-e^2x^4}$$

$$\Big\downarrow \text{762}$$

$$\frac{2d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{1}{3}x\sqrt{d^2-e^2x^4}$$

input

```
Int[Sqrt[d^2 - e^2*x^4],x]
```

output

```
(x*Sqrt[d^2 - e^2*x^4])/3 + (2*d^(5/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[A
rcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(3*Sqrt[e]*Sqrt[d^2 - e^2*x^4])
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{2d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	82
risch	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{2d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	82
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{2d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	82

input
```
int((-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(-e^2*x^4+d^2)^(1/2)+2/3*d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/
d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int \sqrt{d^2-e^2x^4}\,dx=\frac{2\,\sqrt{-e^2}d\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\mid -1)+\sqrt{-e^2x^4+d^2}ex}{3\,e}$$

input

```
integrate((-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output

```
1/3*(2*sqrt(-e^2)*d*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1) + sqrt(-
e^2*x^4 + d^2)*e*x)/e
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.46

$$\int \sqrt{d^2-e^2x^4}\,dx=\frac{dx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((-e**2*x**4+d**2)**(1/2),x)
```

output

```
d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2
)/(4*gamma(5/4))
```

Maxima [F]

$$\int \sqrt{d^2-e^2x^4}\,dx=\int \sqrt{-e^2x^4+d^2}\,dx$$

input

```
integrate((-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \sqrt{d^2-e^2x^4}\,dx = \int \sqrt{-e^2x^4+d^2}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2), x)
```

Mupad [B] (verification not implemented)

Time = 17.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \sqrt{d^2-e^2x^4}\,dx = \frac{x\,\sqrt{d^2-e^2\,x^4}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ \frac{e^2\,x^4}{d^2}\right)}{\sqrt{1-\frac{e^2\,x^4}{d^2}}}$$

input
```
int((d^2 - e^2*x^4)^(1/2),x)
```

output
```
(x*(d^2 - e^2*x^4)^(1/2)*hypergeom([-1/2, 1/4], 5/4, (e^2*x^4)/d^2))/(1 -
(e^2*x^4)/d^2)^(1/2)
```

Reduce [F]

$$\int \sqrt{d^2-e^2x^4}\,dx = \frac{\sqrt{-e^2x^4+d^2}\,x}{3} + \frac{2\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^2}{3}$$

input
```
int((-e^2*x^4+d^2)^(1/2),x)
```

output

```
(sqrt(d**2 - e**2*x**4)*x + 2*int(sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4
),x)*d**2)/3
```

3.38 $\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = -\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
-d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^
2*x^4+d^2)^(1/2)+2*d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/
2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = \frac{id\sqrt{1-\frac{e^2x^4}{d^2}}\left(E\left(i\,\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-2\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)\right)}{\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[Sqrt[d^2 - e^2*x^4]/(d + e*x^2),x]
```

output

```
(I*d*Sqrt[1 - (e^2*x^4)/d^2]*(EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - 2
*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1]))/(Sqrt[-(e/d)]*Sqrt[d^2 - e^2*x
^4])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.45, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1396, 326, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{326}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(2d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{329}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[Sqrt[d^2 - e^2*x^4]/(d + e*x^2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(-((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[
(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])) + (2
*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1
])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/(Sqrt[d - e*x^2]*Sqrt[d + e
*x^2])
```

Defintions of rubi rules used

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 326

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	137
elliptic	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	137

input
```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*Ell
ipticF(x*(e/d)^(1/2),I)+d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/
(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = \frac{\sqrt{-e^2}dx\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)-\sqrt{-e^2}(d-e)x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)+\sqrt{-e^2x^4+d^2}e}{e^2x}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
(sqrt(-e^2)*d*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - sqrt(-e^2)
*(d - e)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1) + sqrt(-e^2*x^4 +
 d^2)*e)/(e^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = \int \frac{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}}{d+ex^2}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(e*x**2+d),x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{ex^2+d}\,dx$$

input

```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output

```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = \int \frac{\sqrt{d^2-e^2\,x^4}}{e\,x^2+d}\,dx$$

input

```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2),x)
```

output

```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{d+ex^2}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e\,x^2+d}dx$$

input

```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d + e*x**2),x)
```

3.39 $\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^2}\,dx=\frac{\sqrt{\frac{d-ex^2}{d+ex^2}}\,(d+ex^2)\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|2\right)}{\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
((-e*x^2+d)/(e*x^2+d))^(1/2)*(e*x^2+d)*EllipticE(e^(1/2)*x/d^(1/2)/(1+e*x^
2/d)^(1/2),2^(1/2))/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.53 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^2}\,dx=\frac{x-\frac{ex^3}{d}-\frac{i\sqrt{1-\frac{e^2x^4}{d^2}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-\text{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)\right)}{\sqrt{-\frac{e}{d}}}}{\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^2,x]
```

output
```
(x - (e*x^3)/d - (I*Sqrt[1 - (e^2*x^4)/d^2]*(EllipticE[I*ArcSinh[Sqrt[-(e/
d)]*x], -1] - EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1]))/Sqrt[-(e/d)])/Sqr
t[d^2 - e^2*x^4]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. $2(71) = 142$.

Time $= 0.70$ (sec) , antiderivative size $= 233$, normalized size of antiderivative $= 3.28$, number of steps used $= 12$, number of rules used $= 12$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used $= \{1396, 314, 25, 27, 344, 836, 27, 765, 762, 1390, 1389, 327\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^2}dx$$

$$\Big\downarrow \; 1396$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{d-ex^2}}{(ex^2+d)^{3/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 314$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\int -\frac{ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\int \frac{x^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 344$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\int \frac{x^2}{\sqrt{d^2-e^2x^4}}dx}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 836$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\int \frac{ex^2+d}{d\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 1390$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{ex^2+d}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 1389$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^2,x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d - e*x^2])/(d*Sqrt[d + e*x^2]) + (e*Sqrt[d^
2 - e^2*x^4]*((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x
)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^2*x^4]) - (d^(3/2)*Sqrt[1 - (e^2*x^
4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^
2*x^4])))/(d*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/(Sqrt[d - e*x^2]*Sqrt[d +
e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 344
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c
+ b*d*x^4)^FracPart[p])   Int[(e*x)^m*(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a
, b, c, d, e, m, p}, x] && EqQ[b*c + a*d, 0] &&  !IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

method	result	size
default	$\frac{\left(-e^{2} x^{2}+d e \right) x}{d e \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	121
elliptic	$\frac{\left(-e^{2} x^{2}+d e \right) x}{d e \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	121

input

```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
(-e^2*x^2+d*e)/d*x/e/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)-1/(e/d)^(1/2)*(1-e*x
^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2
),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^2}\,dx = \frac{(ex^2+d)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)-(ex^2+d)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)+\sqrt{-e^2x^4+d^2}x}{dex^2+d^2}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
((e*x^2 + d)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - (e*x^2 + d)*s
qrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*x)/(d*
e*x^2 + d^2)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^2}\,dx = \int \frac{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}}{(d+ex^2)^2}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(e*x**2+d)**2,x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\sqrt{d^2-e^2\,x^4}}{\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^2,x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^2x^4+2de\,x^2+d^2}dx$$

input

```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^2,x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)
```

3.40 $\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 191

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^3}\,dx=\frac{x\sqrt{d^2-e^2x^4}}{3d\left(d+ex^2\right)^2}+\frac{x\sqrt{d^2-e^2x^4}}{2d^2\left(d+ex^2\right)}+\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}-\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
1/3*x*(-e^2*x^4+d^2)^(1/2)/d/(e*x^2+d)^2+1/2*x*(-e^2*x^4+d^2)^(1/2)/d^2/(e
*x^2+d)+1/2*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e
^(1/2)/(-e^2*x^4+d^2)^(1/2)-1/3*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/
d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.68 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^3}\,dx$$
$$=\frac{\sqrt{-\frac{e}{d}}x(5d^2-2dex^2-3e^2x^4)-3id(d+ex^2)\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)+2id(d+ex^2)\sqrt{1-}}{6d^2\sqrt{-\frac{e}{d}}\,(d+ex^2)\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^3,x]
```

output
```
(Sqrt[-(e/d)]*x*(5*d^2 - 2*d*e*x^2 - 3*e^2*x^4) - (3*I)*d*(d + e*x^2)*Sqrt
[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] + (2*I)*d*(d
+ e*x^2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1])
/(6*d^2*Sqrt[-(e/d)]*(d + e*x^2)*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.32, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1396, 314, 25, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^3}\,dx$$
$$\Big\downarrow \; 1396$$
$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{d-ex^2}}{(ex^2+d)^{5/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$
$$\Big\downarrow \; 314$$
$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}-\frac{\int -\frac{2d-ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{3d}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{2d-ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{de\left(3ex^2+d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\int \frac{de\left(3ex^2+d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\int \frac{3ex^2+d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-2d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 289$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 329$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^3,x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d - e*x^2])/(3*d*(d + e*x^2)^(3/2)) + ((3*x*
Sqrt[d - e*x^2])/(2*d*Sqrt[d + e*x^2]) + ((3*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^
2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sq
rt[d + e*x^2]) - (2*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt
[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/(2*d))/(3
*d)))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 314

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.16

method	result
default	$\frac{x\sqrt{-e^2x^4+d^2}}{3d\,e^2\left(x^2+\frac{d}{e}\right)^2}+\frac{(-e^2x^2+de)x}{2d^2e\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ (\mathrm{EllipticF}(x}{2d\sqrt{\frac{e}{d}}\,\sqrt{-e}}$
elliptic	$\frac{x\sqrt{-e^2x^4+d^2}}{3d\,e^2\left(x^2+\frac{d}{e}\right)^2}+\frac{(-e^2x^2+de)x}{2d^2e\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ (\mathrm{EllipticF}(x}{2d\sqrt{\frac{e}{d}}\,\sqrt{-e}}$

input
```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/d*x/e^2*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+1/2*(-e^2*x^2+d*e)/d^2*x/e/((
x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/6/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^
2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-1/2/d/(e/d)^(1/
2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(
e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^3}\,dx = \frac{3\,(e^3x^4+2\,de^2x^2+d^2e)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)+((de^2-3\,e^3)x^4+d^3-3\,d^2e+2\,(d^2e-3\,de^2)x^2)\sqrt{}}{6\,(d^2e^3x^4+2\,d^3e^2x^2+d^4e)}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/6*(3*(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(
e/d)), -1) + ((d*e^2 - 3*e^3)*x^4 + d^3 - 3*d^2*e + 2*(d^2*e - 3*d*e^2)*x^
2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*(3
*e^2*x^3 + 5*d*e*x))/(d^2*e^3*x^4 + 2*d^3*e^2*x^2 + d^4*e)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^3}\,dx=\int \frac{\sqrt{-(-d+ex^2)\,(d+ex^2)}}{(d+ex^2)^3}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(e*x**2+d)**3,x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/(d + e*x**2)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^3}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{(ex^2+d)^3}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^3}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{(ex^2+d)^3}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^3}\,dx=\int \frac{\sqrt{d^2-e^2\,x^4}}{(e\,x^2+d)^3}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^3,x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^3}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx$$

input
```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^3,x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x*
*6),x)
```

3.41 $\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 224

$$\begin{aligned}\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^4}\,dx &= \frac{x\sqrt{d^2-e^2x^4}}{5d\,(d+ex^2)^3}+\frac{7x\sqrt{d^2-e^2x^4}}{30d^2\,(d+ex^2)^2}+\frac{2x\sqrt{d^2-e^2x^4}}{5d^3\,(d+ex^2)}\\ &\quad+\frac{2\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{5d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\\ &\quad-\frac{7\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{30d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/5*x*(-e^2*x^4+d^2)^(1/2)/d/(e*x^2+d)^3+7/30*x*(-e^2*x^4+d^2)^(1/2)/d^2/(
e*x^2+d)^2+2/5*x*(-e^2*x^4+d^2)^(1/2)/d^3/(e*x^2+d)+2/5*(1-e^2*x^4/d^2)^(1
/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(3/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-7/
30*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(3/2)/e^(1/2)/(-
e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.82 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^4}\,dx$$
$$= \frac{\sqrt{-\frac{e}{d}}x(25d^3+6d^2ex^2-19de^2x^4-12e^3x^6)-12id(d+ex^2)^2\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)+7i}{30d^3\sqrt{-\frac{e}{d}}\,(d+ex^2)^2\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^4,x]
```

output
```
(Sqrt[-(e/d)]*x*(25*d^3 + 6*d^2*e*x^2 - 19*d*e^2*x^4 - 12*e^3*x^6) - (12*I
)*d*(d + e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]
*x], -1] + (7*I)*d*(d + e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSi
nh[Sqrt[-(e/d)]*x], -1])/(30*d^3*Sqrt[-(e/d)]*(d + e*x^2)^2*Sqrt[d^2 - e^2
*x^4])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1396, 314, 25, 402, 25, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^4}\,dx$$
$$\Big\downarrow \text{1396}$$
$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{d-ex^2}}{(ex^2+d)^{7/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$
$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}-\frac{\int -\frac{4d-3ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{5d}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{4d-3ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{5d}+\frac{x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{7x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{de\left(17d-7ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}}{5d}+\frac{x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\int \frac{de\left(17d-7ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}+\frac{7x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\int \frac{17d-7ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d}+\frac{7x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{12x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\int -\frac{2de\left(12ex^2+5d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{6d}+\frac{7x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\int \frac{12ex^2+5d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{12x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{7x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{12\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-7d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{12x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{7x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 289$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{12\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{7d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{12x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{7x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 329$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{12d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{12x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{7x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{12d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{12x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{7x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{12d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{12x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{7x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{12d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{12x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{7x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d-ex}}{5d(d+ex^2)}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^4,x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d - e*x^2])/(5*d*(d + e*x^2)^(5/2)) + ((7*x*
Sqrt[d - e*x^2])/(6*d*(d + e*x^2)^(3/2)) + ((12*x*Sqrt[d - e*x^2])/(d*Sqrt
[d + e*x^2]) + ((12*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt
[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) - (7*d^(3/
2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sq
rt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/d)/(6*d))/(5*d)))/(Sqrt[d - e*x^2]
*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.14

method	result
default	$\frac{x\sqrt{-e^2x^4+d^2}}{5d\,e^3\left(x^2+\frac{d}{e}\right)^3}+\frac{7x\sqrt{-e^2x^4+d^2}}{30e^2d^2\left(x^2+\frac{d}{e}\right)^2}+\frac{2\left(-e^2x^2+de\right)x}{5e\,d^3\sqrt{\left(x^2+\frac{d}{e}\right)\left(-e^2x^2+de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{2\sqrt{1-\frac{e\,x^2}{d}}}{}$
elliptic	$\frac{x\sqrt{-e^2x^4+d^2}}{5d\,e^3\left(x^2+\frac{d}{e}\right)^3}+\frac{7x\sqrt{-e^2x^4+d^2}}{30e^2d^2\left(x^2+\frac{d}{e}\right)^2}+\frac{2\left(-e^2x^2+de\right)x}{5e\,d^3\sqrt{\left(x^2+\frac{d}{e}\right)\left(-e^2x^2+de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{2\sqrt{1-\frac{e\,x^2}{d}}}{}$

input

```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

output

```
1/5/d*x/e^3*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^3+7/30/e^2/d^2*x*(-e^2*x^4+d^2)
^(1/2)/(x^2+d/e)^2+2/5*(-e^2*x^2+d*e)/e/d^3*x/((x^2+d/e)*(-e^2*x^2+d*e))^(
1/2)+1/6/d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2
)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-2/5/d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(
1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-Elliptic
E(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^4}\,dx$$

$$=\frac{12\left(e^4x^6+3\,de^3x^4+3\,d^2e^2x^2+d^3e\right)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|\,-1\right)+\left(\left(5\,de^3-12\,e^4\right)x^6+3\left(5\,d^2e^2-12\,de^3\right)\right.}{30\left(d^3e^4x^6+3\right.}$$

input

```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^4,x, algorithm="fricas")
```

output
```
1/30*(12*(e^4*x^6 + 3*d*e^3*x^4 + 3*d^2*e^2*x^2 + d^3*e)*sqrt(e/d)*ellipti
c_e(arcsin(x*sqrt(e/d)), -1) + ((5*d*e^3 - 12*e^4)*x^6 + 3*(5*d^2*e^2 - 12
*d*e^3)*x^4 + 5*d^4 - 12*d^3*e + 3*(5*d^3*e - 12*d^2*e^2)*x^2)*sqrt(e/d)*e
lliptic_f(arcsin(x*sqrt(e/d)), -1) + (12*e^3*x^5 + 31*d*e^2*x^3 + 25*d^2*e
*x)*sqrt(-e^2*x^4 + d^2))/(d^3*e^4*x^6 + 3*d^4*e^3*x^4 + 3*d^5*e^2*x^2 + d
^6*e)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^4}\,dx=\int \frac{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}}{\left(d+ex^2\right)^4}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(e*x**2+d)**4,x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/(d + e*x**2)**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^4}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{\left(ex^2+d\right)^4}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^4, x)
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^4}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{(ex^2+d)^4}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^4}\,dx=\int \frac{\sqrt{d^2-e^2\,x^4}}{(e\,x^2+d)^4}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^4,x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^4}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^4x^8+4d\,e^3x^6+6d^2e^2x^4+4d^3e\,x^2+d^4}dx$$

input
```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^4,x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**4 + 4*d**3*e*x**2 + 6*d**2*e**2*x**4 + 4*d*
e**3*x**6 + e**4*x**8),x)
```

3.42 $\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 257

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^5}\,dx = \frac{x\sqrt{d^2-e^2x^4}}{7d\,(d+ex^2)^4} + \frac{11x\sqrt{d^2-e^2x^4}}{70d^2\,(d+ex^2)^3} + \frac{41x\sqrt{d^2-e^2x^4}}{210d^3\,(d+ex^2)^2} + \frac{7x\sqrt{d^2-e^2x^4}}{20d^4\,(d+ex^2)} + \frac{7\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{20d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}} - \frac{41\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{210d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
1/7*x*(-e^2*x^4+d^2)^(1/2)/d/(e*x^2+d)^4+11/70*x*(-e^2*x^4+d^2)^(1/2)/d^2/
(e*x^2+d)^3+41/210*x*(-e^2*x^4+d^2)^(1/2)/d^3/(e*x^2+d)^2+7/20*x*(-e^2*x^4
+d^2)^(1/2)/d^4/(e*x^2+d)+7/20*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d
^(1/2),I)/d^(5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-41/210*(1-e^2*x^4/d^2)^(1/2
)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^5}\,dx$$
$$= \frac{-\frac{x(-d+ex^2)(355d^3+671d^2ex^2+523de^2x^4+147e^3x^6)}{(d+ex^2)^3}+\frac{ie\sqrt{1-\frac{e^2x^4}{d^2}}\Big(147E\Big(i\text{arcsinh}\Big(\sqrt{-\frac{e}{d}}x\Big)\Big|-1\Big)-82\,\text{EllipticF}\Big(i\text{arcsinh}\Big(\sqrt{-\frac{e}{d}}x\Big)}{(-\frac{e}{d})^{3/2}}}{420d^4\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^5,x]
```

output
```
(-((x*(-d + e*x^2)*(355*d^3 + 671*d^2*e*x^2 + 523*d*e^2*x^4 + 147*e^3*x^6)
)/(d + e*x^2)^3) + (I*e*Sqrt[1 - (e^2*x^4)/d^2]*(147*EllipticE[I*ArcSinh[S
qrt[-(e/d)]*x], -1] - 82*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1]))/(-(e/d
))^(3/2))/(420*d^4*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.29, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.654$, Rules used = {1396, 314, 25, 402, 25, 27, 402, 27, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^5}\,dx$$
$$\Big\downarrow \text{ 1396}$$
$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{d-ex^2}}{(ex^2+d)^{9/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$
$$\Big\downarrow \text{ 314}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}-\frac{\int -\frac{6d-5ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{7d}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{6d-5ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{7d}+\frac{x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{11x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{\int -\frac{de\left(49d-33ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d^2e}}{7d}+\frac{x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\int \frac{de\left(49d-33ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d^2e}+\frac{11x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\int \frac{49d-33ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d}+\frac{11x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{41x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{2de\left(106d-41ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}}{10d}+\frac{11x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\int \frac{106d-41ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{41x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{147x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{de\left(147ex^2+65d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{3d}+\frac{41x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{\int \frac{de\left(147ex^2+65d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{147x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{41x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{\int \frac{147ex^2+65d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{147x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{41x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \textcolor{blue}{399}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{147\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-82d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{147x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{41x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \textcolor{blue}{289}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{147\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{82d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{147x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{41x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \textcolor{blue}{329}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{\frac{147d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{82d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{147x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{41x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{\frac{147d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{82d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{147x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{41x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{}}{7d(d}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{\frac{147d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{82d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{147x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{41x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{}}{7d(d-}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{\frac{147d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{82d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{147x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{41x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d-}}{10d\left(d+ex\right.}}{7d}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^5,x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d - e*x^2])/(7*d*(d + e*x^2)^(7/2)) + ((11*x
*Sqrt[d - e*x^2])/(10*d*(d + e*x^2)^(5/2)) + ((41*x*Sqrt[d - e*x^2])/(3*d*
(d + e*x^2)^(3/2)) + ((147*x*Sqrt[d - e*x^2])/(2*d*Sqrt[d + e*x^2]) + ((14
7*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -
1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) - (82*d^(3/2)*Sqrt[1 - (e^2*
x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*
x^2]*Sqrt[d + e*x^2]))/(2*d))/(3*d))/(10*d))/(7*d)))/(Sqrt[d - e*x^2]*Sqrt
[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.13

method	result
default	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{7d\,e^{4}\left(x^{2}+\frac{d}{e}\right)^{4}}+\frac{11x\sqrt{-e^{2}x^{4}+d^{2}}}{70d^{2}e^{3}\left(x^{2}+\frac{d}{e}\right)^{3}}+\frac{41x\sqrt{-e^{2}x^{4}+d^{2}}}{210d^{3}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{7\left(-e^{2}x^{2}+de\right)x}{20e\,d^{4}\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}+\frac{13\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\ \mathrm{Elliptic}}{84d^{3}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{7d\,e^{4}\left(x^{2}+\frac{d}{e}\right)^{4}}+\frac{11x\sqrt{-e^{2}x^{4}+d^{2}}}{70d^{2}e^{3}\left(x^{2}+\frac{d}{e}\right)^{3}}+\frac{41x\sqrt{-e^{2}x^{4}+d^{2}}}{210d^{3}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{7\left(-e^{2}x^{2}+de\right)x}{20e\,d^{4}\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}+\frac{13\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\ \mathrm{Elliptic}}{84d^{3}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$

input
```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^5,x,method=_RETURNVERBOSE)
```

output
```
1/7/d*x/e^4*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^4+11/70/d^2/e^3*x*(-e^2*x^4+d^2
)^(1/2)/(x^2+d/e)^3+41/210/d^3/e^2*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+7/20
*(-e^2*x^2+d*e)/e/d^4*x/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+13/84/d^3/(e/d)^(
1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*
(e/d)^(1/2),I)-7/20/d^3/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-
e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^5}\,dx$$
$$=\frac{147\left(e^5x^8+4\,de^4x^6+6\,d^2e^3x^4+4\,d^3e^2x^2+d^4e\right)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|\,-1\right)+\left((65\,de^4-147\,e^5)x^8+4\,(6\right.}{}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^5,x, algorithm="fricas")
```

output
```
1/420*(147*(e^5*x^8 + 4*d*e^4*x^6 + 6*d^2*e^3*x^4 + 4*d^3*e^2*x^2 + d^4*e)
*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) + ((65*d*e^4 - 147*e^5)*x^8
 + 4*(65*d^2*e^3 - 147*d*e^4)*x^6 + 65*d^5 - 147*d^4*e + 6*(65*d^3*e^2 - 1
47*d^2*e^3)*x^4 + 4*(65*d^4*e - 147*d^3*e^2)*x^2)*sqrt(e/d)*elliptic_f(arc
sin(x*sqrt(e/d)), -1) + (147*e^4*x^7 + 523*d*e^3*x^5 + 671*d^2*e^2*x^3 + 3
55*d^3*e*x)*sqrt(-e^2*x^4 + d^2))/(d^4*e^5*x^8 + 4*d^5*e^4*x^6 + 6*d^6*e^3
*x^4 + 4*d^7*e^2*x^2 + d^8*e)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^5}\,dx=\int \frac{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}}{(d+ex^2)^5}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(e*x**2+d)**5,x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/(d + e*x**2)**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^5}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{(ex^2+d)^5}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^5, x)
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^5}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{(ex^2+d)^5}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^5,x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^5}\,dx=\int \frac{\sqrt{d^2-e^2\,x^4}}{(e\,x^2+d)^5}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^5,x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2 - e^2x^4}}{\left(d + ex^2\right)^5}\,dx = \int \frac{\sqrt{-e^2x^4 + d^2}}{e^5x^{10} + 5d\,e^4x^8 + 10d^2e^3x^6 + 10d^3e^2x^4 + 5d^4e\,x^2 + d^5}dx$$

input

```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^5,x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**5 + 5*d**4*e*x**2 + 10*d**3*e**2*x**4 + 10*
d**2*e**3*x**6 + 5*d*e**4*x**8 + e**5*x**10),x)
```

3.43 $\int (d+ex^2)^3 (d^2-e^2x^4)^{3/2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 239

$$\int (d+ex^2)^3 (d^2-e^2x^4)^{3/2} \, dx = \frac{4}{715} d^4 x (65d+77ex^2) \sqrt{d^2-e^2x^4}$$

$$+ \frac{2}{429} d^2 x (39d+77ex^2) (d^2-e^2x^4)^{3/2} - \frac{3}{11} dx (d^2-e^2x^4)^{5/2}$$

$$- \frac{1}{13} ex^3 (d^2-e^2x^4)^{5/2} + \frac{56 d^{15/2} \sqrt{1-\frac{e^2x^4}{d^2}} E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle| -1\right)}{65\sqrt{e}\sqrt{d^2-e^2x^4}} - \frac{96 d^{15/2} \sqrt{1-\frac{e^2x^4}{d^2}} \, \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}}{\sqrt{}}\right.\right.}{715\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
4/715*d^4*x*(77*e*x^2+65*d)*(-e^2*x^4+d^2)^(1/2)+2/429*d^2*x*(77*e*x^2+39*
d)*(-e^2*x^4+d^2)^(3/2)-3/11*d*x*(-e^2*x^4+d^2)^(5/2)-1/13*e*x^3*(-e^2*x^4
+d^2)^(5/2)+56/65*d^(15/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/
2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-96/715*d^(15/2)*(1-e^2*x^4/d^2)^(1/2)*E
llipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.56

$$\int \left(d+ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2}\,dx = -\frac{x\sqrt{d^2-e^2x^4}\left(\left(39d+11ex^2\right)\left(d^2-e^2x^4\right)^2\sqrt{1-\frac{e^2x^4}{d^2}}-182d^5\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-154\right.}{143\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input
```
Integrate[(d + e*x^2)^3*(d^2 - e^2*x^4)^(3/2),x]
```

output
```
-1/143*(x*Sqrt[d^2 - e^2*x^4]*((39*d + 11*e*x^2)*(d^2 - e^2*x^4)^2*Sqrt[1
- (e^2*x^4)/d^2] - 182*d^5*Hypergeometric2F1[-3/2, 1/4, 5/4, (e^2*x^4)/d^2
] - 154*d^4*e*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, (e^2*x^4)/d^2]))/Sqrt[
1 - (e^2*x^4)/d^2]
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.56, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.808$, Rules used = {1396, 318, 27, 403, 25, 27, 403, 27, 403, 25, 27, 403, 27, 403, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{9/2}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{\int -14de\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{5/2}\left(2ex^2+d\right)dx}{13e}-\frac{1}{13}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{7/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{14}{13}d\int \left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{5/2}\left(2ex^2+d\right)dx-\frac{1}{13}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{7/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{14}{13}d\Bigg(-\frac{\int -de\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{3/2}\left(33ex^2+13d\right)dx}{11e}-\frac{2}{11}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{5/2}\Bigg)-\frac{1}{13}x\left(d-ex^2\right)^{5/2}\big(d}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{14}{13}d\Bigg(\frac{\int de\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{3/2}\left(33ex^2+13d\right)dx}{11e}-\frac{2}{11}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{5/2}\Bigg)-\frac{1}{13}x\left(d-ex^2\right)^{5/2}\big(d+}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\Big(\frac{14}{13}d\Big(\frac{1}{11}d\int \left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{3/2}\left(33ex^2+13d\right)dx-\frac{2}{11}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{5/2}\Big)-\frac{1}{13}x\big(d-}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{14}{13}d\Bigg(\frac{1}{11}d\Bigg(-\frac{\int -6de\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}\left(58ex^2+25d\right)dx}{9e}-\frac{11}{3}x\left(d+ex^2\right)^{3/2}\left(d-ex^2\right)^{5/2}\Bigg)-\frac{2}{11}x\left(d-ex^2\right)^{5/}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\Big(\frac{14}{13}d\Big(\frac{1}{11}d\Big(\frac{2}{3}d\int \left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}\left(58ex^2+25d\right)dx-\frac{11}{3}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}\Big)-\frac{2}{11}x\big(d-}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{14}{13}d\left(\frac{1}{11}d\left(\frac{2}{3}d\left(-\frac{\int -\frac{de\left(d-ex^2\right)^{3/2}\left(349ex^2+233d\right)}{\sqrt{ex^2+d}}dx}{7e}-\frac{58}{7}x\sqrt{d+ex^2}\left(d-ex^2\right)^{5/2}\right)-\frac{11}{3}x\left(d-ex^2\right)^{5/2}\big(d\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{14}{13}d\left(\frac{1}{11}d\left(\frac{2}{3}d\left(\frac{\int\frac{de\left(d-ex^2\right)^{3/2}\left(349ex^2+233d\right)}{\sqrt{ex^2+d}}dx}{7e}-\frac{58}{7}x\left(d-ex^2\right)^{5/2}\sqrt{d+ex^2}\right)-\frac{11}{3}x\left(d-ex^2\right)^{5/2}(d+\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{14}{13}d\left(\frac{1}{11}d\left(\frac{2}{3}d\left(\frac{1}{7}d\int\frac{\left(d-ex^2\right)^{3/2}\left(349ex^2+233d\right)}{\sqrt{ex^2+d}}dx-\frac{58}{7}x\left(d-ex^2\right)^{5/2}\sqrt{d+ex^2}\right)-\frac{11}{3}x\left(d-ex^2\right)^{5/2}(d-\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \textcolor{blue}{403}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{14}{13}d\left(\frac{1}{11}d\left(\frac{2}{3}d\left(\frac{1}{7}d\left(\frac{\int\frac{6de\sqrt{d-ex^2}\left(213ex^2+136d\right)}{\sqrt{ex^2+d}}dx}{5e}+\frac{349}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)-\frac{58}{7}x\left(d-ex^2\right)^{5/2}\sqrt{}\right.\right.\right.\right.}{\sqrt{d-ex^2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{14}{13}d\left(\frac{1}{11}d\left(\frac{2}{3}d\left(\frac{1}{7}d\left(\frac{6}{5}d\int\frac{\sqrt{d-ex^2}\left(213ex^2+136d\right)}{\sqrt{ex^2+d}}dx+\frac{349}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)-\frac{58}{7}x\left(d-ex^2\right)^{5/2}\sqrt{d}\right.\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{}}$$

$$\Big\downarrow \textcolor{blue}{403}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{14}{13}d\left(\frac{1}{11}d\left(\frac{2}{3}d\left(\frac{1}{7}d\left(\frac{6}{5}d\left(\frac{\int\frac{3de\left(77ex^2+65d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}+71x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{349}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/}\right.\right.\right.\right.\right.}{}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{14}{13}d\left(\frac{1}{11}d\left(\frac{2}{3}d\left(\frac{1}{7}d\left(\frac{6}{5}d\left(d\int\frac{77ex^2+65d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+71x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{349}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right.\right.\right.\right.\right.}{}$$

$$\Big\downarrow \textcolor{blue}{399}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{14}{13}d\left(\frac{1}{11}d\left(\frac{2}{3}d\left(\frac{1}{7}d\left(\frac{6}{5}d\left(d\left(77\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-12d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx\right)+71x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{349}{5}\right.\right.\right.\right.\right.}{}$$

$$\Big\downarrow 289$$

$$\sqrt{d^2-e^2x^4}\Bigg(\frac{14}{13}d\Bigg(\frac{1}{11}d\Bigg(\frac{2}{3}d\Bigg(\frac{1}{7}d\Bigg(\frac{6}{5}d\Bigg(d\Bigg(77\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{12d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+71x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)+$$

$$\Big\downarrow 329$$

$$\sqrt{d^2-e^2x^4}\Bigg(\frac{14}{13}d\Bigg(\frac{1}{11}d\Bigg(\frac{2}{3}d\Bigg(\frac{1}{7}d\Bigg(\frac{6}{5}d\Bigg(d\Bigg(\frac{77d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+71x\sqrt{d-ex^2}$$

$$\Big\downarrow 327$$

$$\sqrt{d^2-e^2x^4}\Bigg(\frac{14}{13}d\Bigg(\frac{1}{11}d\Bigg(\frac{2}{3}d\Bigg(\frac{1}{7}d\Bigg(\frac{6}{5}d\Bigg(d\Bigg(\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+71x\sqrt{d-$$

$$\Big\downarrow 765$$

$$\sqrt{d^2-e^2x^4}\Bigg(\frac{14}{13}d\Bigg(\frac{1}{11}d\Bigg(\frac{2}{3}d\Bigg(\frac{1}{7}d\Bigg(\frac{6}{5}d\Bigg(d\Bigg(\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+71x\sqrt{d$$

$$\Big\downarrow 762$$

$$\sqrt{d^2-e^2x^4}\Bigg(\frac{14}{13}d\Bigg(\frac{1}{11}d\Bigg(\frac{2}{3}d\Bigg(\frac{1}{7}d\Bigg(\frac{6}{5}d\Bigg(d\Bigg(\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[(d + e*x^2)^3*(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(-1/13*(x*(d - e*x^2)^(5/2)*(d + e*x^2)^(7/2)) + (14*
d*((-2*x*(d - e*x^2)^(5/2)*(d + e*x^2)^(5/2))/11 + (d*((-11*x*(d - e*x^2)^
(5/2)*(d + e*x^2)^(3/2))/3 + (2*d*((-58*x*(d - e*x^2)^(5/2)*Sqrt[d + e*x^2
])/7 + (d*((349*x*(d - e*x^2)^(3/2)*Sqrt[d + e*x^2])/5 + (6*d*(71*x*Sqrt[d
 - e*x^2]*Sqrt[d + e*x^2] + d*((77*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*Ellipti
cE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x
^2]) - (12*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sq
rt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))))/5))/7))/3))/11))
/13))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 7.94 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.90

method	result
risch	$\frac{x\left(-165e^5x^{10}-585d\,e^4x^8-440d^2e^3x^6+780d^3e^2x^4+1529d^4e\,x^2+585d^5\right)\sqrt{-e^2x^4+d^2}}{2145}+\frac{8d^6\left(\frac{65d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left(x\sqrt{\frac{e}{d}}\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right.}{}$
elliptic	$-\frac{e^5x^{11}\sqrt{-e^2x^4+d^2}}{13}-\frac{3e^4d\,x^9\sqrt{-e^2x^4+d^2}}{11}-\frac{8d^2e^3x^7\sqrt{-e^2x^4+d^2}}{39}+\frac{4d^3e^2x^5\sqrt{-e^2x^4+d^2}}{11}+\frac{139d^4e\,x^3\sqrt{-e^2x^4+d^2}}{195}+$
default	$d^3\left(-\frac{e^2x^5\sqrt{-e^2x^4+d^2}}{7}+\frac{3d^2x\sqrt{-e^2x^4+d^2}}{7}+\frac{4d^4\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{7\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)+e^3\left(-\frac{e^2x^{11}\sqrt{-e^2x^4+d^2}}{13}\right.$

input

```
int((e*x^2+d)^3*(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/2145*x*(-165*e^5*x^10-585*d*e^4*x^8-440*d^2*e^3*x^6+780*d^3*e^2*x^4+1529
*d^4*e*x^2+585*d^5)*(-e^2*x^4+d^2)^(1/2)+8/715*d^6*(65*d/(e/d)^(1/2)*(1-e*
x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2
),I)-77*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(
1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.71

$$\int \left(d+ex^2\right)^3\left(d^2-e^2x^4\right)^{3/2}\,dx =$$

$$-\frac{1848\sqrt{-e^2}d^7x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\mid -1)-24\left(77\,d^7+65\,d^6e\right)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\mid -1)+(165\,e}{2145\,e^2}$$

input

```
integrate((e*x^2+d)^3*(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2145*(1848*sqrt(-e^2)*d^7*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -
1) - 24*(77*d^7 + 65*d^6*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(
d/e)/x), -1) + (165*e^7*x^12 + 585*d*e^6*x^10 + 440*d^2*e^5*x^8 - 780*d^3*
e^4*x^6 - 1529*d^4*e^3*x^4 - 585*d^5*e^2*x^2 + 1848*d^6*e)*sqrt(-e^2*x^4 +
 d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.17

$$\int \left(d+ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2}\,dx = \frac{d^6x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{3d^5ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{d^4e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{9}{4}\right)} - \frac{d^3e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{11}{4}\right)} - \frac{3d^2e^4x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{13}{4}\right)} - \frac{de^5x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((e*x**2+d)**3*(-e**2*x**4+d**2)**(3/2),x)
```

output

```
d**6*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) + 3*d**5*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,),
e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(7/4)) + d**4*e**2*x**5*gamma(5/
4)*hyper((-1/2, 5/4), (9/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(2*gamma(9
/4)) - d**3*e**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), e**2*x**4*exp
_polar(2*I*pi)/d**2)/(2*gamma(11/4)) - 3*d**2*e**4*x**9*gamma(9/4)*hyper((
-1/2, 9/4), (13/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(13/4)) - d
*e**5*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), e**2*x**4*exp_polar(2
*I*pi)/d**2)/(4*gamma(15/4))
```

Maxima [F]

$$\int \left(d+ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2} \, dx = \int \left(-e^2x^4+d^2\right)^{\frac{3}{2}} \left(ex^2+d\right)^3 \, dx$$

input

```
integrate((e*x^2+d)^3*(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output

```
integrate((-e^2*x^4 + d^2)^(3/2)*(e*x^2 + d)^3, x)
```

Giac [F]

$$\int \left(d+ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2} \, dx = \int \left(-e^2x^4+d^2\right)^{\frac{3}{2}} \left(ex^2+d\right)^3 \, dx$$

input

```
integrate((e*x^2+d)^3*(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output

```
integrate((-e^2*x^4 + d^2)^(3/2)*(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(d^2-e^2\,x^4\right)^{3/2}\left(e\,x^2+d\right)^3\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^3,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{3\sqrt{-e^2x^4+d^2}\,d^5x}{11}+\frac{139\sqrt{-e^2x^4+d^2}\,d^4e\,x^3}{195}+\frac{4\sqrt{-e^2x^4+d^2}\,d^3e^2x^5}{11}-\frac{8\sqrt{-e^2x^4+d^2}\,d^2e^3x^7}{39}-\frac{3\sqrt{-e^2x^4+d^2}\,d\,e^4x^9}{11}-\frac{\sqrt{-e^2x^4+d^2}\,e^5x^{11}}{13}+\frac{8\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^7}{11}+\frac{56\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^6e}{65}$$

input
```
int((e*x^2+d)^3*(-e^2*x^4+d^2)^(3/2),x)
```

output
```
(585*sqrt(d**2 - e**2*x**4)*d**5*x + 1529*sqrt(d**2 - e**2*x**4)*d**4*e*x*
*3 + 780*sqrt(d**2 - e**2*x**4)*d**3*e**2*x**5 - 440*sqrt(d**2 - e**2*x**4
)*d**2*e**3*x**7 - 585*sqrt(d**2 - e**2*x**4)*d*e**4*x**9 - 165*sqrt(d**2
- e**2*x**4)*e**5*x**11 + 1560*int(sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**
4),x)*d**7 + 1848*int((sqrt(d**2 - e**2*x**4)*x**2)/(d**2 - e**2*x**4),x)*
d**6*e)/2145
```

3.44 $\int \left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 212

$$\begin{aligned}\int \left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2} \, dx &= \frac{4d^3x(90d+77ex^2)\sqrt{d^2-e^2x^4}}{1155} \\ &+ \frac{2}{693}dx\left(54d+77ex^2\right)\left(d^2-e^2x^4\right)^{3/2} \\ &- \frac{1}{11}x\left(d^2-e^2x^4\right)^{5/2} + \frac{8d^{13/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{15\sqrt{e}\sqrt{d^2-e^2x^4}} \\ &+ \frac{104d^{13/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{1155\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
4/1155*d^3*x*(77*e*x^2+90*d)*(-e^2*x^4+d^2)^(1/2)+2/693*d*x*(77*e*x^2+54*d
)*(-e^2*x^4+d^2)^(3/2)-1/11*x*(-e^2*x^4+d^2)^(5/2)+8/15*d^(13/2)*(1-e^2*x^
4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+1
04/1155*d^(13/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1
/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.71 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.59

$$\int \left(d+ex^2\right)^2 \left(d^2 -e^2x^4\right)^{3/2} \, dx = \frac{x\sqrt{d^2-e^2x^4}\left(-3(d^2-e^2x^4)^2\sqrt{1-\frac{e^2x^4}{d^2}}+36d^4\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)+22}{33\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input
```
Integrate[(d + e*x^2)^2*(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(x*Sqrt[d^2 - e^2*x^4]*(-3*(d^2 - e^2*x^4)^2*Sqrt[1 - (e^2*x^4)/d^2] + 36*
d^4*Hypergeometric2F1[-3/2, 1/4, 5/4, (e^2*x^4)/d^2] + 22*d^3*e*x^2*Hyperg
eometric2F1[-3/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(33*Sqrt[1 - (e^2*x^4)/d^2])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.61, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {1396, 318, 27, 403, 25, 27, 403, 27, 403, 27, 403, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{7/2}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{\int -2de(d-ex^2)^{3/2}(ex^2+d)^{3/2}(11ex^2+6d)dx}{11e}-\frac{1}{11}x(d-ex^2)^{5/2}(d+ex^2)^{5/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\int (d-ex^2)^{3/2}(ex^2+d)^{3/2}(11ex^2+6d)\,dx-\frac{1}{11}x(d-ex^2)^{5/2}(d+ex^2)^{5/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 403$$

$$\sqrt{d^2-e^2x^4}\Big(\frac{2}{11}d\Big(-\frac{\int -de(d-ex^2)^{3/2}\sqrt{ex^2+d}(131ex^2+65d)dx}{9e}-\frac{11}{9}x(d+ex^2)^{3/2}(d-ex^2)^{5/2}\Big)-\frac{1}{11}x(d-ex^2)^{5/2}(d$$
$$\overline{\qquad\qquad\sqrt{d-ex^2}\sqrt{d+ex^2}\qquad\qquad}$$

$$\Big\downarrow \; 25$$

$$\sqrt{d^2-e^2x^4}\Big(\frac{2}{11}d\Big(\frac{\int de(d-ex^2)^{3/2}\sqrt{ex^2+d}(131ex^2+65d)dx}{9e}-\frac{11}{9}x(d-ex^2)^{5/2}(d+ex^2)^{3/2}\Big)-\frac{1}{11}x(d-ex^2)^{5/2}(d+e$$
$$\overline{\qquad\qquad\sqrt{d-ex^2}\sqrt{d+ex^2}\qquad\qquad}$$

$$\Big\downarrow \; 27$$

$$\sqrt{d^2-e^2x^4}\Big(\frac{2}{11}d\Big(\frac{1}{9}d\int (d-ex^2)^{3/2}\sqrt{ex^2+d}(131ex^2+65d)\,dx-\frac{11}{9}x(d-ex^2)^{5/2}(d+ex^2)^{3/2}\Big)-\frac{1}{11}x(d-ex$$
$$\overline{\qquad\qquad\sqrt{d-ex^2}\sqrt{d+ex^2}\qquad\qquad}$$

$$\Big\downarrow \; 403$$

$$\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{11}d\Bigg(\frac{1}{9}d\Bigg(-\frac{\int -\frac{2de\left(d-ex^2\right)^{3/2}\left(424ex^2+293d\right)}{\sqrt{ex^2+d}}dx}{7e}-\frac{131}{7}x\sqrt{d+ex^2}(d-ex^2)^{5/2}\Bigg)-\frac{11}{9}x(d-ex^2)^{5/2}(d+e$$
$$\overline{\qquad\qquad\sqrt{d-ex^2}\sqrt{d+ex^2}\qquad\qquad}$$

$$\Big\downarrow \; 27$$

$$\sqrt{d^2-e^2x^4}\Big(\frac{2}{11}d\Big(\frac{1}{9}d\Big(\frac{2}{7}d\int \frac{(d-ex^2)^{3/2}(424ex^2+293d)}{\sqrt{ex^2+d}}dx-\frac{131}{7}x(d-ex^2)^{5/2}\sqrt{d+ex^2}\Big)-\frac{11}{9}x(d-ex^2)^{5/2}(d+ex^2$$
$$\overline{\qquad\qquad\sqrt{d-ex^2}\sqrt{d+ex^2}\qquad\qquad}$$

$$\Big\downarrow \; 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{\int \frac{3de\sqrt{d-ex^2}\left(501ex^2+347d\right)}{\sqrt{ex^2+d}}dx}{5e}+\frac{424}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)-\frac{131}{7}x\left(d-ex^2\right)^{5/2}\sqrt{d+e}\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\int \frac{\sqrt{d-ex^2}\left(501ex^2+347d\right)}{\sqrt{ex^2+d}}dx+\frac{424}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)-\frac{131}{7}x\left(d-ex^2\right)^{5/2}\sqrt{d+e}\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\left(\frac{\int \frac{6de\left(77ex^2+90d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}+167x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{424}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)-\right.\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+e}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\left(2d\int \frac{77ex^2+90d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+167x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{424}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)-\right.\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+e}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\left(2d\left(13d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+77\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)+167x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{424}{5}x\sqrt{}\right.\right.\right.\right.}{\sqrt{d-}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\left(2d\left(\frac{13d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+77\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)+167x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{42}{5}\right.\right.\right.\right.}{\sqrt{}}$$

$$\Big\downarrow \text{329}$$

$$\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\left(2d\left(\frac{77d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{13d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+167x\sqrt{d-ex^2}\sqrt{d+}\right.\right.\right.\right.\right.$$

$$\Big\downarrow 327$$

$$\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{11}d\Bigg(\frac{1}{9}d\Bigg(\frac{2}{7}d\Bigg(\frac{3}{5}d\Bigg(2d\Bigg(\frac{13d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+167x\sqrt{d-ex$$

$$\Big\downarrow 765$$

$$\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{11}d\Bigg(\frac{1}{9}d\Bigg(\frac{2}{7}d\Bigg(\frac{3}{5}d\Bigg(2d\Bigg(\frac{13d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+167x\sqrt{d-e$$

$$\Big\downarrow 762$$

$$\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{11}d\Bigg(\frac{1}{9}d\Bigg(\frac{2}{7}d\Bigg(\frac{3}{5}d\Bigg(2d\Bigg(\frac{13d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+$$

input
```
Int[(d + e*x^2)^2*(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/11*(x*(d - e*x^2)^(5/2)*(d + e*x^2)^(5/2)) + (2*d
*((-11*x*(d - e*x^2)^(5/2)*(d + e*x^2)^(3/2))/9 + (d*((-131*x*(d - e*x^2)^
(5/2)*Sqrt[d + e*x^2])/7 + (2*d*((424*x*(d - e*x^2)^(3/2)*Sqrt[d + e*x^2])
/5 + (3*d*(167*x*Sqrt[d - e*x^2]*Sqrt[d + e*x^2] + 2*d*((77*d^(3/2)*Sqrt[1
 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqr
t[d - e*x^2]*Sqrt[d + e*x^2]) + (13*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*Ellipt
icF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*
x^2]))))/5))/7))/9))/11))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 5.61 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.97

method	result
risch	$\frac{x\left(-315e^4x^8-770d\,e^3x^6+90d^2e^2x^4+1694d^3e\,x^2+1305d^4\right)\sqrt{-e^2x^4+d^2}}{3465}+\frac{8d^5\left(\frac{90d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-77d\sqrt{1-}\right.}{115}$
elliptic	$-\frac{e^4x^9\sqrt{-e^2x^4+d^2}}{11}-\frac{2d\,e^3x^7\sqrt{-e^2x^4+d^2}}{9}+\frac{2d^2e^2x^5\sqrt{-e^2x^4+d^2}}{77}+\frac{22d^3e\,x^3\sqrt{-e^2x^4+d^2}}{45}+\frac{29d^4x\sqrt{-e^2x^4+d^2}}{77}+48d^6$
default	$d^2\left(-\frac{e^2x^5\sqrt{-e^2x^4+d^2}}{7}+\frac{3d^2x\sqrt{-e^2x^4+d^2}}{7}+\frac{4d^4\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{7\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)+e^2\left(-\frac{e^2x^9\sqrt{-e^2x^4+d^2}}{11}\right.$

input
```
int((e*x^2+d)^2*(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3465*x*(-315*e^4*x^8-770*d*e^3*x^6+90*d^2*e^2*x^4+1694*d^3*e*x^2+1305*d^
4)*(-e^2*x^4+d^2)^(1/2)+8/1155*d^5*(90*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+
e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-77*d/(e/d)^
(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(
x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.75

$$\int \left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2}\,dx =$$

$$-\frac{1848\,\sqrt{-e^2}d^6x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) - 24\,(77\,d^6+90\,d^5e)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) + (315\,e}{3465\,e^2x}$$

input
```
integrate((e*x^2+d)^2*(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3465*(1848*sqrt(-e^2)*d^6*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -
1) - 24*(77*d^6 + 90*d^5*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(
d/e)/x), -1) + (315*e^6*x^10 + 770*d*e^5*x^8 - 90*d^2*e^4*x^6 - 1694*d^3*e
^3*x^4 - 1305*d^4*e^2*x^2 + 1848*d^5*e)*sqrt(-e^2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.85

$$\int \left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2}\,dx = \frac{d^5x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{d^4ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{7}{4}\right)} - \frac{d^2e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{11}{4}\right)} - \frac{de^4x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input

```
integrate((e*x**2+d)**2*(-e**2*x**4+d**2)**(3/2),x)
```

output

```
d**5*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) + d**4*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), e*
*2*x**4*exp_polar(2*I*pi)/d**2)/(2*gamma(7/4)) - d**2*e**3*x**7*gamma(7/4)
*hyper((-1/2, 7/4), (11/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(2*gamma(11
/4)) - d*e**4*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), e**2*x**4*exp_po
lar(2*I*pi)/d**2)/(4*gamma(13/4))
```

Maxima [F]

$$\int \left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2}\,dx = \int \left(-e^2x^4+d^2\right)^{\frac{3}{2}}\left(ex^2+d\right)^2\,dx$$

input

```
integrate((e*x^2+d)^2*(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output

```
integrate((-e^2*x^4 + d^2)^(3/2)*(e*x^2 + d)^2, x)
```

Giac [F]

$$\int \left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2} \, dx = \int \left(-e^2x^4+d^2\right)^{\frac{3}{2}} \left(ex^2+d\right)^2 dx$$

input
```
integrate((e*x^2+d)^2*(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)*(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2} \, dx = \int \left(d^2-e^2\,x^4\right)^{3/2} \left(e\,x^2+d\right)^2 dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^2,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^2, x)
```

Reduce [F]

$$\begin{aligned}\int \left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2} \, dx &= \frac{29\sqrt{-e^2x^4+d^2}\,d^4x}{77} \\ &+ \frac{22\sqrt{-e^2x^4+d^2}\,d^3e\,x^3}{45} + \frac{2\sqrt{-e^2x^4+d^2}\,d^2e^2x^5}{77} - \frac{2\sqrt{-e^2x^4+d^2}\,d\,e^3x^7}{9} \\ &- \frac{\sqrt{-e^2x^4+d^2}\,e^4x^9}{11} + \frac{48\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^6}{77} + \frac{8\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^5e}{15}\end{aligned}$$

input
```
int((e*x^2+d)^2*(-e^2*x^4+d^2)^(3/2),x)
```

output

```
(1305*sqrt(d**2 - e**2*x**4)*d**4*x + 1694*sqrt(d**2 - e**2*x**4)*d**3*e*x
**3 + 90*sqrt(d**2 - e**2*x**4)*d**2*e**2*x**5 - 770*sqrt(d**2 - e**2*x**4
)*d*e**3*x**7 - 315*sqrt(d**2 - e**2*x**4)*e**4*x**9 + 2160*int(sqrt(d**2
- e**2*x**4)/(d**2 - e**2*x**4),x)*d**6 + 1848*int((sqrt(d**2 - e**2*x**4)
*x**2)/(d**2 - e**2*x**4),x)*d**5*e)/3465
```

3.45 $\int \left(d+ex^2\right) \left(d^2-e^2x^4\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 190

$$\int \left(d+ex^2\right) \left(d^2-e^2x^4\right)^{3/2} \, dx = \frac{2}{105}d^2x\left(15d+7ex^2\right)\sqrt{d^2-e^2x^4} + \frac{1}{63}x\left(9d+7ex^2\right)\left(d^2-e^2x^4\right)^{3/2} + \frac{4d^{11/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{15\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{32d^{11/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{105\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
2/105*d^2*x*(7*e*x^2+15*d)*(-e^2*x^4+d^2)^(1/2)+1/63*x*(7*e*x^2+9*d)*(-e^2
*x^4+d^2)^(3/2)+4/15*d^(11/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^
(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+32/105*d^(11/2)*(1-e^2*x^4/d^2)^(1/2
)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.74 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$\int \left(d+ex^2\right) \left(d^2 -e^2x^4\right)^{3/2}\,dx = \frac{d^2x\sqrt{d^2-e^2x^4}\left(3d\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)+ex^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\right.\right.}{3\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input

```
Integrate[(d + e*x^2)*(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(d^2*x*Sqrt[d^2 - e^2*x^4]*(3*d*Hypergeometric2F1[-3/2, 1/4, 5/4, (e^2*x^4
)/d^2] + e*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(3*Sqrt[
1 - (e^2*x^4)/d^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right) \left(d^2-e^2x^4\right)^{3/2}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \left(d+ex^2\right) \left(d^2-e^2x^4\right)^{3/2}\,dx$$

input

```
Int[(d + e*x^2)*(d^2 - e^2*x^4)^(3/2),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.02

method	result
risch	$\frac{x\left(-35e^3x^6-45d\,e^2x^4+77d^2e\,x^2+135d^3\right)\sqrt{-e^2x^4+d^2}}{315}+\frac{4d^4\Bigg(\frac{15d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{7d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\Big(\operatorname{Elli}}{\sqrt{\frac{e}{d}}}}{105}$
elliptic	$-\frac{e^3x^7\sqrt{-e^2x^4+d^2}}{9}-\frac{d\,e^2x^5\sqrt{-e^2x^4+d^2}}{7}+\frac{11d^2e\,x^3\sqrt{-e^2x^4+d^2}}{45}+\frac{3d^3x\sqrt{-e^2x^4+d^2}}{7}+\frac{4d^5\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}}{7\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$
default	$d\left(-\frac{e^2x^5\sqrt{-e^2x^4+d^2}}{7}+\frac{3d^2x\sqrt{-e^2x^4+d^2}}{7}+\frac{4d^4\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{7\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)+e\Bigg(-\frac{e^2x^7\sqrt{-e^2x^4+d^2}}{9}+$

input
```
int((e*x^2+d)*(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*x*(-35*e^3*x^6-45*d*e^2*x^4+77*d^2*e*x^2+135*d^3)*(-e^2*x^4+d^2)^(1/
2)+4/105*d^4*(15*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x
^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-7*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)
*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-Ellipt
icE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.77

$$\int \left(d+ex^2\right)\left(d^2-e^2x^4\right)^{3/2}\,dx=$$

$$-\frac{84\sqrt{-e^2}d^5x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)-12\,(7\,d^5+15\,d^4e)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)+(35\,e^5x^8+}{315\,e^2x}$$

input
```
integrate((e*x^2+d)*(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/315*(84*sqrt(-e^2)*d^5*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1)
- 12*(7*d^5 + 15*d^4*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)
/x), -1) + (35*e^5*x^8 + 45*d*e^4*x^6 - 77*d^2*e^3*x^4 - 135*d^3*e^2*x^2 +
 84*d^4*e)*sqrt(-e^2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)\left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{d^4x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{d^3ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{d^2e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{de^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x**2+d)*(-e**2*x**4+d**2)**(3/2),x)
```

output
```
d**4*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) + d**3*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), e*
*2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(7/4)) - d**2*e**2*x**5*gamma(5/4)
*hyper((-1/2, 5/4), (9/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(9/4
)) - d*e**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), e**2*x**4*exp_pola
r(2*I*pi)/d**2)/(4*gamma(11/4))
```

Maxima [F]

$$\int (d+ex^2)\left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(-e^2x^4+d^2\right)^{\frac{3}{2}}(ex^2+d)\,dx$$

input
```
integrate((e*x^2+d)*(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)*(e*x^2 + d), x)
```

Giac [F]

$$\int (d+ex^2)\left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(-e^2x^4+d^2\right)^{\frac{3}{2}}(ex^2+d)\,dx$$

input
```
integrate((e*x^2+d)*(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(d^2-e^2\,x^4\right)^{3/2}\left(e\,x^2+d\right)\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)*(d + e*x^2),x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)\left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{3\sqrt{-e^2x^4+d^2}\,d^3x}{7}+\frac{11\sqrt{-e^2x^4+d^2}\,d^2e\,x^3}{45}-\frac{\sqrt{-e^2x^4+d^2}\,d\,e^2x^5}{7}-\frac{\sqrt{-e^2x^4+d^2}\,e^3x^7}{9}+\frac{4\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^5}{7}+\frac{4\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^4e}{15}$$

input

```
int((e*x^2+d)*(-e^2*x^4+d^2)^(3/2),x)
```

output

```
(135*sqrt(d**2 - e**2*x**4)*d**3*x + 77*sqrt(d**2 - e**2*x**4)*d**2*e*x**3
 - 45*sqrt(d**2 - e**2*x**4)*d*e**2*x**5 - 35*sqrt(d**2 - e**2*x**4)*e**3*
x**7 + 180*int(sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)*d**5 + 84*int(
(sqrt(d**2 - e**2*x**4)*x**2)/(d**2 - e**2*x**4),x)*d**4*e)/315
```

3.46 $\int \left(d^2-e^2x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{2}{7}d^2x\sqrt{d^2-e^2x^4}+\frac{1}{7}x\left(d^2-e^2x^4\right)^{3/2}+\frac{4d^{9/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{7\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
2/7*d^2*x*(-e^2*x^4+d^2)^(1/2)+1/7*x*(-e^2*x^4+d^2)^(3/2)+4/7*d^(9/2)*(1-e
^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{d^2x\sqrt{d^2-e^2x^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)}{\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(d^2*x*Sqrt[d^2 - e^2*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, (e^2*x^4)/d^2
])/Sqrt[1 - (e^2*x^4)/d^2]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {748, 748, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d^2-e^2x^4\right)^{3/2}dx$$

$$\Big\downarrow 748$$

$$\frac{6}{7}d^2\int \sqrt{d^2-e^2x^4}dx+\frac{1}{7}x\left(d^2-e^2x^4\right)^{3/2}$$

$$\Big\downarrow 748$$

$$\frac{6}{7}d^2\left(\frac{2}{3}d^2\int \frac{1}{\sqrt{d^2-e^2x^4}}dx+\frac{1}{3}x\sqrt{d^2-e^2x^4}\right)+\frac{1}{7}x\left(d^2-e^2x^4\right)^{3/2}$$

$$\Big\downarrow 765$$

$$\frac{6}{7}d^2\left(\frac{2d^2\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{3\sqrt{d^2-e^2x^4}}+\frac{1}{3}x\sqrt{d^2-e^2x^4}\right)+\frac{1}{7}x\left(d^2-e^2x^4\right)^{3/2}$$

$$\Big\downarrow 762$$

$$\frac{6}{7}d^2\left(\frac{2d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{1}{3}x\sqrt{d^2-e^2x^4}\right)+\frac{1}{7}x\left(d^2-e^2x^4\right)^{3/2}$$

input

```
Int[(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(x*(d^2 - e^2*x^4)^(3/2))/7 + (6*d^2*((x*Sqrt[d^2 - e^2*x^4])/3 + (2*d^(5/
2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(3*
Sqrt[e]*Sqrt[d^2 - e^2*x^4])))/7
```

Defintions of rubi rules used

rule 748

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x\left(-e^{2} x^{4}+3 d^{2}\right) \sqrt{-e^{2} x^{4}+d^{2}}}{7}+\frac{4 d^{4} \sqrt{1-\frac{e\,x^{2}}{d}}\, \sqrt{1+\frac{e\,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{7 \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	96
default	$-\frac{e^{2} x^{5} \sqrt{-e^{2} x^{4}+d^{2}}}{7}+\frac{3 d^{2} x \sqrt{-e^{2} x^{4}+d^{2}}}{7}+\frac{4 d^{4} \sqrt{1-\frac{e\,x^{2}}{d}}\, \sqrt{1+\frac{e\,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{7 \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	107
elliptic	$-\frac{e^{2} x^{5} \sqrt{-e^{2} x^{4}+d^{2}}}{7}+\frac{3 d^{2} x \sqrt{-e^{2} x^{4}+d^{2}}}{7}+\frac{4 d^{4} \sqrt{1-\frac{e\,x^{2}}{d}}\, \sqrt{1+\frac{e\,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{7 \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	107

input

```
int((-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/7*x*(-e^2*x^4+3*d^2)*(-e^2*x^4+d^2)^(1/2)+4/7*d^4/(e/d)^(1/2)*(1-e*x^2/d
)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.65

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{4\sqrt{-e^2}d^3\sqrt{\frac{d}{e}}F\left(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|\,-1\right)-\left(e^3x^5-3\,d^2ex\right)\sqrt{-e^2x^4+d^2}}{7\,e}$$

input

```
integrate((-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output

```
1/7*(4*sqrt(-e^2)*d^3*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1) - (e^3
*x^5 - 3*d^2*e*x)*sqrt(-e^2*x^4 + d^2))/e
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.38

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((-e**2*x**4+d**2)**(3/2),x)
```

output

```
d**3*x*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(-e^2x^4+d^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(-e^2x^4+d^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 17.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.43

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{x\,\left(d^2-e^2\,x^4\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{4};\ \frac{5}{4};\ \frac{e^2\,x^4}{d^2}\right)}{\left(1-\frac{e^2\,x^4}{d^2}\right)^{3/2}}$$

input
```
int((d^2 - e^2*x^4)^(3/2),x)
```

output
```
(x*(d^2 - e^2*x^4)^(3/2)*hypergeom([-3/2, 1/4], 5/4, (e^2*x^4)/d^2))/(1 -
(e^2*x^4)/d^2)^(3/2)
```

Reduce [F]

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{3\sqrt{-e^2x^4+d^2}\,d^2x}{7}-\frac{\sqrt{-e^2x^4+d^2}\,e^2x^5}{7}+\frac{4\left(\int\frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^4}{7}$$

input
```
int((-e^2*x^4+d^2)^(3/2),x)
```

output
```
(3*sqrt(d**2 - e**2*x**4)*d**2*x - sqrt(d**2 - e**2*x**4)*e**2*x**5 + 4*in
t(sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)*d**4)/7
```

3.47 $\int \frac{(d^2-e^2x^4)^{3/2}}{d+ex^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 156

$$\begin{aligned}\int \frac{(d^2-e^2x^4)^{3/2}}{d+ex^2}\,dx = {} & \frac{1}{15}x(5d-3ex^2)\sqrt{d^2-e^2x^4} \\ & - \frac{2d^{7/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{5\sqrt{e}\sqrt{d^2-e^2x^4}} \\ & + \frac{16d^{7/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{15\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output
```
1/15*x*(-3*e*x^2+5*d)*(-e^2*x^4+d^2)^(1/2)-2/5*d^(7/2)*(1-e^2*x^4/d^2)^(1/
2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+16/15*d^(7/
2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+
d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.58 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.56

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{d+ex^2}\,dx=\frac{\sqrt{d^2-e^2x^4}\Big(3dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-ex^3\,\text{Hypergeometric2F1}\Big(}{3\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input

```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(3*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, (e^2*x^4)/d^
2] - e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(3*Sqrt[1 -
(e^2*x^4)/d^2])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.56, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1396, 318, 27, 403, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{d+ex^2}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{2de\left(3d-4ex^2\right)\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{5e}-\frac{1}{5}x\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{5}d\int\frac{\left(3d-4ex^2\right)\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{1}{5}x\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{5}d\left(\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}-\frac{\int-\frac{de\left(5d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}\right)-\frac{1}{5}x\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{5}d\left(\frac{\int\frac{de\left(5d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}x\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{5}d\left(\frac{1}{3}d\int\frac{5d-3ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}x\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d^2-e^2x^4}\Big(\frac{2}{5}d\Big(\frac{1}{3}d\Big(8d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-3\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\Big)+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Big)-\frac{1}{5}x\sqrt{d-ex^2}\big(d+ex^2\big)^{3}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d^2-e^2x^4}\Big(\frac{2}{5}d\Big(\frac{1}{3}d\Big(\frac{8d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-3\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\Big)+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Big)-\frac{1}{5}x\sqrt{d-ex^2}\big(d+ex}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{329}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{5}d\Bigg(\frac{1}{3}d\Bigg(\frac{8d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)-\frac{1}{5}x\sqrt{d-e}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{5}d\Bigg(\frac{1}{3}d\Bigg(\frac{8d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\Big(\arcsin\Big(\frac{\sqrt{e}x}{\sqrt{d}}\Big)\Big|-1\Big)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)-\frac{1}{5}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 765$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{5}d\Bigg(\frac{1}{3}d\Bigg(\frac{8d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\Big(\arcsin\Big(\frac{\sqrt{e}x}{\sqrt{d}}\Big)\Big|-1\Big)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)-\frac{1}{5}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{5}d\Bigg(\frac{1}{3}d\Bigg(\frac{8d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\Big(\arcsin\Big(\frac{\sqrt{e}x}{\sqrt{d}}\Big),-1\Big)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\Big(\arcsin\Big(\frac{\sqrt{e}x}{\sqrt{d}}\Big)\Big|-1\Big)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/5*(x*Sqrt[d - e*x^2]*(d + e*x^2)^(3/2)) + (2*d*((
4*x*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])/3 + (d*((-3*d^(3/2)*Sqrt[1 - (e^2*x^4
)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2
]*Sqrt[d + e*x^2]) + (8*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(
Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/3))/
5))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10

method	result
risch	$\frac{x\left(-3e\,x^{2}+5d\right)\sqrt{-e^{2}x^{4}+d^{2}}}{15}+\frac{2d^{2}\left(\frac{5d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{3d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}}\right.\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right.}{15}$
default	$-\frac{e\,x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{5}+\frac{dx\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{2d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{2d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{}\right.\right.}{5\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}}}$
elliptic	$-\frac{e\,x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{5}+\frac{dx\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{2d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{2d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{}\right.\right.}{5\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}}}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(-3*e*x^2+5*d)*(-e^2*x^4+d^2)^(1/2)+2/15*d^2*(5*d/(e/d)^(1/2)*(1-e*
x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2
),I)+3*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1
/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{d+ex^2}\,dx=\frac{6\sqrt{-e^2}d^3x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\mid-1)-2\left(3\,d^3-5\,d^2e\right)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)}{15\,e^2x}$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
1/15*(6*sqrt(-e^2)*d^3*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - 2
*(3*d^3 - 5*d^2*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x),
-1) - (3*e^3*x^4 - 5*d*e^2*x^2 - 6*d^2*e)*sqrt(-e^2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.54

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{d+ex^2}\,dx=\frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}-\frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d),x)
```

output
```
d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) - d*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), e**2*
x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(7/4))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{d+ex^2}\,dx = \int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{ex^2+d}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{d+ex^2}\,dx = \int \frac{(d^2-e^2\,x^4)^{3/2}}{e\,x^2+d}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2),x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{d+ex^2}\,dx = \frac{\sqrt{-e^2x^4+d^2}\,dx}{3} - \frac{\sqrt{-e^2x^4+d^2}\,e\,x^3}{5} + \frac{2\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^3}{3} - \frac{2\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^2e}{5}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d),x)
```

output
```
(5*sqrt(d**2 - e**2*x**4)*d*x - 3*sqrt(d**2 - e**2*x**4)*e*x**3 + 10*int(s
qrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)*d**3 - 6*int((sqrt(d**2 - e**2
*x**4)*x**2)/(d**2 - e**2*x**4),x)*d**2*e)/15
```

3.48 $\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 144

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx=-\frac{1}{3}x\sqrt{d^2-e^2x^4}-\frac{2d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{10d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
-1/3*x*(-e^2*x^4+d^2)^(1/2)-2*d^(5/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1
/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+10/3*d^(5/2)*(1-e^2*x^4/d^2)
^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.48 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx=\frac{\sqrt{-\frac{e}{d}}x\left(-d^2+e^2x^4\right)+6id^2\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-10id^2\sqrt{1-\frac{e^2x^4}{d^2}}}{3\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^2,x]
```

output

```
(Sqrt[-(e/d)]*x*(-d^2 + e^2*x^4) + (6*I)*d^2*Sqrt[1 - (e^2*x^4)/d^2]*Ellip
ticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - (10*I)*d^2*Sqrt[1 - (e^2*x^4)/d^2]*E
llipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1])/(3*Sqrt[-(e/d)]*Sqrt[d^2 - e^2*x^
4])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.45, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1396, 318, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(d-ex^2\right)^{3/2}}{\sqrt{ex^2+d}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{2de\left(2d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{3}d\int \frac{2d-3ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{3}d\left(5d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{3}d\left(\frac{5d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{329}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{3}d\left(\frac{5d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{3}d\left(\frac{5d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{3}d\left(\frac{5d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{3}d\left(\frac{5d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^2,x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(-1/3*(x*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (2*d*((-3
*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1
])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (5*d^(3/2)*Sqrt[1 - (e^2*x^
4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^
2]*Sqrt[d + e*x^2])))/3))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.11

method	result
default	$-\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{4d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{2d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}}\right.\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$
risch	$-\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{2d\left(\frac{2d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{3d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right)}{3}$
elliptic	$-\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{4d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{2d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}}\right.\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(-e^2*x^4+d^2)^(1/2)+4/3*d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2
/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+2*d^2/(e/d)^(1/2
)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e
/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^2}\,dx = \frac{6\sqrt{-e^2}d^2x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) - 2\,(3\,d^2-2\,de)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|}{3\,e^2x}$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
1/3*(6*sqrt(-e^2)*d^2*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - 2*
(3*d^2 - 2*d*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1)
 - sqrt(-e^2*x^4 + d^2)*(e^2*x^2 - 6*d*e))/(e^2*x)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^2}\,dx = \int \frac{(-(-d+ex^2)\,(d+ex^2))^{\frac{3}{2}}}{(d+ex^2)^2}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**2,x)
```

output
```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^2}\,dx=\int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^2}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^2, x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^2}\,dx=\int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^2}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^2}\,dx=\int \frac{(d^2-e^2\,x^4)^{3/2}}{(e\,x^2+d)^2}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^2,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx=\left(\int \frac{\sqrt{-e^2x^4+d^2}}{e\,x^2+d}dx\right)d-\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{e\,x^2+d}dx\right)e$$

input

```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^2,x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d + e*x**2),x)*d - int((sqrt(d**2 - e**2*x**4)
*x**2)/(d + e*x**2),x)*e
```

3.49 $\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 149

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^3}\,dx = \frac{2x\sqrt{d^2-e^2x^4}}{d+ex^2} + \frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}} - \frac{4d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
2*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)+3*d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*Ellipti
cE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-4*d^(3/2)*(1-e^2*x^4/
d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.60 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.88

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^3}\,dx = \frac{2\sqrt{-\frac{e}{d}}x(d-ex^2) - 3id\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right) + 4id\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Ellip}}{\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^3,x]
```

output
```
(2*Sqrt[-(e/d)]*x*(d - e*x^2) - (3*I)*d*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[
I*ArcSinh[Sqrt[-(e/d)]*x], -1] + (4*I)*d*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF
[I*ArcSinh[Sqrt[-(e/d)]*x], -1])/(Sqrt[-(e/d)]*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.35, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1396, 315, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(d-ex^2\right)^{3/2}}{\left(ex^2+d\right)^{3/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int -\frac{de\left(d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{de}+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}-\frac{\int \frac{de\left(d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{de}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}-\int \frac{d-3ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 399$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-4d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 289$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 329$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 327$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{4d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{4d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^3,x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((2*x*Sqrt[d - e*x^2])/Sqrt[d + e*x^2] + (3*d^(3/2)*S
qrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e
]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) - (4*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*El
lipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d
+ e*x^2])))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 315

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.21

method	result
default	$\frac{2\left(-e^{2} x^{2}+d e \right) x}{e \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}-\frac{d \sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}-\frac{3 d \sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\bigl(\right.}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$
elliptic	$\frac{2\left(-e^{2} x^{2}+d e \right) x}{e \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}-\frac{d \sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}-\frac{3 d \sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\bigl(\right.}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```
2*(-e^2*x^2+d*e)*x/e/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)-d/(e/d)^(1/2)*(1-e*x
^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2)
,I)-3*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/
2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^3}\,dx =$$

$$-\frac{3\left(dex^3+d^2x\right)\sqrt{-e^2}\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)-\left(\left(3\,de-e^2\right)x^3+\left(3\,d^2-de\right)x\right)\sqrt{-e^2}\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right.}{e^3x^3+de^2x}$$

input

```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^3,x, algorithm="fricas")
```

output

```
-(3*(d*e*x^3 + d^2*x)*sqrt(-e^2)*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x),
 -1) - ((3*d*e - e^2)*x^3 + (3*d^2 - d*e)*x)*sqrt(-e^2)*sqrt(d/e)*elliptic
_f(arcsin(sqrt(d/e)/x), -1) + sqrt(-e^2*x^4 + d^2)*(e^2*x^2 + 3*d*e))/(e^3
*x^3 + d*e^2*x)
```

Sympy [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^3}\,dx = \int \frac{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}}{\left(d+ex^2\right)^3}\,dx$$

input

```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**3,x)
```

output

```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)/(d + e*x**2)**3, x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^3}\,dx = \int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^3}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^3, x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^3}\,dx = \int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^3}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^3}\,dx = \int \frac{(d^2-e^2\,x^4)^{3/2}}{(e\,x^2+d)^3}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^3,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^3}\,dx = \frac{\sqrt{-e^2x^4+d^2}\,x+2\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^4}{-e^3x^6-d\,e^2x^4+d^2e\,x^2+d^3}dx\right)d\,e^2+2\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^4}{-e^3x^6-d\,e^2x^4+d^2e\,x^2+d^3}dx\right)}{e\,x^2+d}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^3,x)
```

output
```
(sqrt(d**2 - e**2*x**4)*x + 2*int((sqrt(d**2 - e**2*x**4)*x**4)/(d**3 + d*
*2*e*x**2 - d*e**2*x**4 - e**3*x**6),x)*d*e**2 + 2*int((sqrt(d**2 - e**2*x
**4)*x**4)/(d**3 + d**2*e*x**2 - d*e**2*x**4 - e**3*x**6),x)*e**3*x**2)/(d
 + e*x**2)
```

3.50 $\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^4} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 93

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^4} \, dx = \frac{2x\sqrt{d^2-e^2x^4}}{3(d+ex^2)^2} + \frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
2/3*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^2+1/3*d^(1/2)*(1-e^2*x^4/d^2)^(1/2)*E
llipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.82 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^4} \, dx = \frac{\frac{2x(d-ex^2)}{d+ex^2} - \frac{i\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)}{\sqrt{-\frac{e}{d}}}}{3\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^4,x]
```

output
```
((2*x*(d - e*x^2))/(d + e*x^2) - (I*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*Ar
cSinh[Sqrt[-(e/d)]*x], -1])/Sqrt[-(e/d)])/(3*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.48, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1396, 315, 27, 289, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^4}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(d-ex^2\right)^{3/2}}{\left(ex^2+d\right)^{5/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{de}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3de}+\frac{2x\sqrt{d-ex^2}}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{3}\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+\frac{2x\sqrt{d-ex^2}}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{3\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d-ex^2}}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{3\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d-ex^2}}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d-ex^2}}{3(d+ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^4,x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((2*x*Sqrt[d - e*x^2])/(3*(d + e*x^2)^(3/2)) + (Sqrt[
d]*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(3*
Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2
])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2x\sqrt{-e^2x^4+d^2}}{3e^2\left(x^2+\frac{d}{e}\right)^2}+\frac{\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$	93
elliptic	$\frac{2x\sqrt{-e^2x^4+d^2}}{3e^2\left(x^2+\frac{d}{e}\right)^2}+\frac{\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$	93

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

output
```
2/3*x/e^2*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+1/3/(e/d)^(1/2)*(1-e*x^2/d)^(1/
2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^4}\,dx = \frac{2\sqrt{-e^2x^4+d^2}ex+(e^2x^4+2\,dex^2+d^2)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|\,-1\right)}{3\left(e^3x^4+2\,de^2x^2+d^2e\right)}$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^4,x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(-e^2*x^4 + d^2)*e*x + (e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e/d)*el
liptic_f(arcsin(x*sqrt(e/d)), -1))/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^4}\,dx=\int \frac{(-(-d+ex^2)\,(d+ex^2))^{\frac{3}{2}}}{(d+ex^2)^4}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**4,x)
```

output
```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)/(d + e*x**2)**4, x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^4}\,dx=\int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^4}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^4, x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^4}\,dx=\int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^4}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^4,x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^4)^{3/2}}{(d + ex^2)^4} \, dx = \int \frac{(d^2 - e^2\,x^4)^{3/2}}{(e\,x^2 + d)^4} \, dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^4,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^4, x)
```

Reduce [F]

$$\int \frac{(d^2 - e^2x^4)^{3/2}}{(d + ex^2)^4} \, dx = \frac{\sqrt{-e^2x^4 + d^2}\, x + \left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^4x^8-2d\,e^3x^6+2d^3e\,x^2+d^4}dx\right) d^4 + 2\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^4x^8-2d\,e^3x^6+2d^3e\,x^2+d^4}dx\right)}{}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^4,x)
```

output
```
(sqrt(d**2 - e**2*x**4)*x + int(sqrt(d**2 - e**2*x**4)/(d**4 + 2*d**3*e*x*
*2 - 2*d*e**3*x**6 - e**4*x**8),x)*d**4 + 2*int(sqrt(d**2 - e**2*x**4)/(d*
*4 + 2*d**3*e*x**2 - 2*d*e**3*x**6 - e**4*x**8),x)*d**3*e*x**2 + int(sqrt(
d**2 - e**2*x**4)/(d**4 + 2*d**3*e*x**2 - 2*d*e**3*x**6 - e**4*x**8),x)*d*
*2*e**2*x**4 + int((sqrt(d**2 - e**2*x**4)*x**4)/(d**4 + 2*d**3*e*x**2 - 2
*d*e**3*x**6 - e**4*x**8),x)*d**2*e**2 + 2*int((sqrt(d**2 - e**2*x**4)*x**
4)/(d**4 + 2*d**3*e*x**2 - 2*d*e**3*x**6 - e**4*x**8),x)*d*e**3*x**2 + int
((sqrt(d**2 - e**2*x**4)*x**4)/(d**4 + 2*d**3*e*x**2 - 2*d*e**3*x**6 - e**
4*x**8),x)*e**4*x**4)/(2*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.51 $\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 221

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^5}\,dx = \frac{2x\sqrt{d^2-e^2x^4}}{5\left(d+ex^2\right)^3}+\frac{2x\sqrt{d^2-e^2x^4}}{15d\left(d+ex^2\right)^2}+\frac{3x\sqrt{d^2-e^2x^4}}{10d^2\left(d+ex^2\right)} + \frac{3\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{10\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}-\frac{2\sqrt{1-\frac{e^2x^4}{d^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{15\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
2/5*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^3+2/15*x*(-e^2*x^4+d^2)^(1/2)/d/(e*x^
2+d)^2+3/10*x*(-e^2*x^4+d^2)^(1/2)/d^2/(e*x^2+d)+3/10*(1-e^2*x^4/d^2)^(1/2
)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-2/15
*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^
2*x^4+d^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^5}\,dx=\frac{\sqrt{-\frac{e}{d}}x(25d^3-3d^2ex^2-13de^2x^4-9e^3x^6)-9id(d+ex^2)^2\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\operatorname{arcsinh}\left(\right.\right.}{30d^2\sqrt{-\frac{e}{d}}\left(d+ex^2\right)^2}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^5,x]
```

output
```
(Sqrt[-(e/d)]*x*(25*d^3 - 3*d^2*e*x^2 - 13*d*e^2*x^4 - 9*e^3*x^6) - (9*I)*
d*(d + e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x
], -1] + (4*I)*d*(d + e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh
[Sqrt[-(e/d)]*x], -1])/(30*d^2*Sqrt[-(e/d)]*(d + e*x^2)^2*Sqrt[d^2 - e^2*x
^4])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.29, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {1396, 315, 27, 402, 27, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^5}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(d-ex^2\right)^{3/2}}{\left(ex^2+d\right)^{7/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{de\left(3d-ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{5de}+\frac{2x\sqrt{d-ex^2}}{5(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\int \frac{3d-ex^2}{\sqrt{d-ex^2}(ex^2+d)^{5/2}}dx+\frac{2x\sqrt{d-ex^2}}{5(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{2x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}-\frac{\int -\frac{2de\left(7d-2ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}\right)+\frac{2x\sqrt{d-ex^2}}{5(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\int \frac{7d-2ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{2x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)+\frac{2x\sqrt{d-ex^2}}{5(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{de\left(9ex^2+5d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)+\frac{2x\sqrt{d-ex^2}}{5(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{\int \frac{de\left(9ex^2+5d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)+\frac{2x\sqrt{d-ex^2}}{5(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{\int \frac{9ex^2+5d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)+\frac{2x\sqrt{d-ex^2}}{5(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 399$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{9\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-4d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)+\frac{2x\sqrt{d-ex^2}}{5(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 289$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{9\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{4d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)+\frac{2x\sqrt{d-ex^2}}{5(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 329$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{\frac{9d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)+\frac{2x\sqrt{d-ex^2}}{5(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)+\frac{2x\sqrt{d-ex^2}}{5(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)+\frac{2x\sqrt{d-ex^2}}{5(d+ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)+\frac{2x}{5(d}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^5,x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((2*x*Sqrt[d - e*x^2])/(5*(d + e*x^2)^(5/2)) + ((2*x*
Sqrt[d - e*x^2])/(3*d*(d + e*x^2)^(3/2)) + ((9*x*Sqrt[d - e*x^2])/(2*d*Sqr
t[d + e*x^2]) + ((9*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt
[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) - (4*d^(3/
2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sq
rt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/(2*d))/(3*d))/5))/(Sqrt[d - e*x^2]
*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.14

method	result
default	$\frac{2x\sqrt{-e^2x^4+d^2}}{5e^3\left(x^2+\frac{d}{e}\right)^3}+\frac{2x\sqrt{-e^2x^4+d^2}}{15d\,e^2\left(x^2+\frac{d}{e}\right)^2}+\frac{3\left(-e^2x^2+de\right)x}{10d^2e\sqrt{\left(x^2+\frac{d}{e}\right)\left(-e^2x^2+de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{3\sqrt{1-\frac{e\,x^2}{d}}}{}$
elliptic	$\frac{2x\sqrt{-e^2x^4+d^2}}{5e^3\left(x^2+\frac{d}{e}\right)^3}+\frac{2x\sqrt{-e^2x^4+d^2}}{15d\,e^2\left(x^2+\frac{d}{e}\right)^2}+\frac{3\left(-e^2x^2+de\right)x}{10d^2e\sqrt{\left(x^2+\frac{d}{e}\right)\left(-e^2x^2+de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{3\sqrt{1-\frac{e\,x^2}{d}}}{}$

input

```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^5,x,method=_RETURNVERBOSE)
```

output

```
2/5*x/e^3*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^3+2/15/d*x/e^2*(-e^2*x^4+d^2)^(1/
2)/(x^2+d/e)^2+3/10*(-e^2*x^2+d*e)/d^2*x/e/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2
)+1/6/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/
2)*EllipticF(x*(e/d)^(1/2),I)-3/10/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^
2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e
/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.02

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^5}\,dx = \frac{9\,(e^4x^6+3\,de^3x^4+3\,d^2e^2x^2+d^3e)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)+((5\,de^3-9\,e^4)x^6+}{}$$

input

```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^5,x, algorithm="fricas")
```

output

```
1/30*(9*(e^4*x^6 + 3*d*e^3*x^4 + 3*d^2*e^2*x^2 + d^3*e)*sqrt(e/d)*elliptic
_e(arcsin(x*sqrt(e/d)), -1) + ((5*d*e^3 - 9*e^4)*x^6 + 3*(5*d^2*e^2 - 9*d*
e^3)*x^4 + 5*d^4 - 9*d^3*e + 3*(5*d^3*e - 9*d^2*e^2)*x^2)*sqrt(e/d)*ellipt
ic_f(arcsin(x*sqrt(e/d)), -1) + (9*e^3*x^5 + 22*d*e^2*x^3 + 25*d^2*e*x)*sq
rt(-e^2*x^4 + d^2))/(d^2*e^4*x^6 + 3*d^3*e^3*x^4 + 3*d^4*e^2*x^2 + d^5*e)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^5}\,dx = \int \frac{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}}{\left(d+ex^2\right)^5}\,dx$$

input

```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**5,x)
```

output

```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)/(d + e*x**2)**5, x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^5}\,dx = \int \frac{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^5}\,dx$$

input

```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^5,x, algorithm="maxima")
```

output

```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^5, x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^5}\,dx = \int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^5}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^5,x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^5}\,dx = \int \frac{(d^2-e^2\,x^4)^{3/2}}{(e\,x^2+d)^5}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^5,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^5, x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^5}\,dx = \text{Too large to display}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^5,x)
```

output

```
(5*sqrt(d**2 - e**2*x**4)*x + int(sqrt(d**2 - e**2*x**4)/(d**5 + 3*d**4*e*
x**2 + 2*d**3*e**2*x**4 - 2*d**2*e**3*x**6 - 3*d*e**4*x**8 - e**5*x**10),x
)*d**5 + 3*int(sqrt(d**2 - e**2*x**4)/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*
x**4 - 2*d**2*e**3*x**6 - 3*d*e**4*x**8 - e**5*x**10),x)*d**4*e*x**2 + 3*i
nt(sqrt(d**2 - e**2*x**4)/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2*d**
2*e**3*x**6 - 3*d*e**4*x**8 - e**5*x**10),x)*d**3*e**2*x**4 + int(sqrt(d**
2 - e**2*x**4)/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2*d**2*e**3*x**6
 - 3*d*e**4*x**8 - e**5*x**10),x)*d**2*e**3*x**6 - 9*int((sqrt(d**2 - e**2
*x**4)*x**4)/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2*d**2*e**3*x**6 -
 3*d*e**4*x**8 - e**5*x**10),x)*d**3*e**2 - 27*int((sqrt(d**2 - e**2*x**4)
*x**4)/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2*d**2*e**3*x**6 - 3*d*e
**4*x**8 - e**5*x**10),x)*d**2*e**3*x**2 - 27*int((sqrt(d**2 - e**2*x**4)*
x**4)/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2*d**2*e**3*x**6 - 3*d*e*
*4*x**8 - e**5*x**10),x)*d*e**4*x**4 - 9*int((sqrt(d**2 - e**2*x**4)*x**4)
/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2*d**2*e**3*x**6 - 3*d*e**4*x*
*8 - e**5*x**10),x)*e**5*x**6 + 18*int((sqrt(d**2 - e**2*x**4)*x**2)/(d**5
 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2*d**2*e**3*x**6 - 3*d*e**4*x**8 - e
**5*x**10),x)*d**4*e + 54*int((sqrt(d**2 - e**2*x**4)*x**2)/(d**5 + 3*d**4
*e*x**2 + 2*d**3*e**2*x**4 - 2*d**2*e**3*x**6 - 3*d*e**4*x**8 - e**5*x**10
),x)*d**3*e**2*x**2 + 54*int((sqrt(d**2 - e**2*x**4)*x**2)/(d**5 + 3*d*...
```

3.52 $\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 254

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^6}\,dx = \frac{2x\sqrt{d^2-e^2x^4}}{7\left(d+ex^2\right)^4}+\frac{4x\sqrt{d^2-e^2x^4}}{35d\left(d+ex^2\right)^3}+\frac{11x\sqrt{d^2-e^2x^4}}{70d^2\left(d+ex^2\right)^2} + \frac{3x\sqrt{d^2-e^2x^4}}{10d^3\left(d+ex^2\right)} + \frac{3\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{10d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}} - \frac{11\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{70d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
2/7*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^4+4/35*x*(-e^2*x^4+d^2)^(1/2)/d/(e*x^
2+d)^3+11/70*x*(-e^2*x^4+d^2)^(1/2)/d^2/(e*x^2+d)^2+3/10*x*(-e^2*x^4+d^2)^
(1/2)/d^3/(e*x^2+d)+3/10*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2)
,I)/d^(3/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-11/70*(1-e^2*x^4/d^2)^(1/2)*Ellip
ticF(e^(1/2)*x/d^(1/2),I)/d^(3/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.60

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^6}\,dx = \frac{-\frac{x(-d+ex^2)(60d^3+93d^2ex^2+74de^2x^4+21e^3x^6)}{(d+ex^2)^3} + \frac{ie\sqrt{1-\frac{e^2x^4}{d^2}}\left(21E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-11\,\text{Elliptic}}{\left(-\frac{e}{d}\right)^{3/2}}}{70d^3\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^6,x]
```

output
```
(-((x*(-d + e*x^2)*(60*d^3 + 93*d^2*e*x^2 + 74*d*e^2*x^4 + 21*e^3*x^6))/(d
 + e*x^2)^3) + (I*e*Sqrt[1 - (e^2*x^4)/d^2]*(21*EllipticE[I*ArcSinh[Sqrt[-
(e/d)]*x], -1] - 11*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1]))/(-(e/d))^(3
/2))/(70*d^3*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.26, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {1396, 315, 27, 402, 27, 402, 25, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^6}dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{(d-ex^2)^{3/2}}{(ex^2+d)^{9/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{de\left(5d-3ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{7de}+\frac{2x\sqrt{d-ex^2}}{7(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\int \frac{5d-3ex^2}{\sqrt{d-ex^2}(ex^2+d)^{7/2}}dx+\frac{2x\sqrt{d-ex^2}}{7(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{4x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}-\frac{\int -\frac{6de\left(7d-4ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d^2e}\right)+\frac{2x\sqrt{d-ex^2}}{7(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\int \frac{7d-4ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{5d}+\frac{4x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)+\frac{2x\sqrt{d-ex^2}}{7(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{11x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{de\left(31d-11ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}\right)}{5d}+\frac{4x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)+\frac{2x\sqrt{d-ex^2}}{7(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\int \frac{de\left(31d-11ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}+\frac{11x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)+\frac{2x\sqrt{d-ex^2}}{7(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\int \frac{31d-11ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d}+\frac{11x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)+\frac{2x\sqrt{d-ex^2}}{7(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\int -\frac{2de\left(21ex^2+10d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{6d}+\frac{11x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)+\frac{2x\sqrt{d-ex^2}}{7(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{\int \frac{21ex^2+10d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{11x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)+\frac{2x\sqrt{d-ex^2}}{7(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 399$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{21\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-11d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{11x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)+\frac{2x\sqrt{d-ex^2}}{7(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 289$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{21\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{11d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{11x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)+\frac{2x\sqrt{d-ex^2}}{7(d+ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 329}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{\frac{21d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{11d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{11x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right)+\frac{2x}{7(d}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{11d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{11x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{11d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{11x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d-ex^2}}{5d(d+ex^2)^{5/2}}\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{11d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{11x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^6,x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((2*x*Sqrt[d - e*x^2])/(7*(d + e*x^2)^(7/2)) + ((4*x*
Sqrt[d - e*x^2])/(5*d*(d + e*x^2)^(5/2)) + (3*((11*x*Sqrt[d - e*x^2])/(6*d
*(d + e*x^2)^(3/2)) + ((21*x*Sqrt[d - e*x^2])/(d*Sqrt[d + e*x^2]) + ((21*d
^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])
/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) - (11*d^(3/2)*Sqrt[1 - (e^2*x^4
)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2
]*Sqrt[d + e*x^2]))/d)/(6*d)))/(5*d))/7))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 5.31 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

method	result
default	$\frac{2x\sqrt{-e^2x^4+d^2}}{7e^4\left(x^2+\frac{d}{e}\right)^4}+\frac{4x\sqrt{-e^2x^4+d^2}}{35d\,e^3\left(x^2+\frac{d}{e}\right)^3}+\frac{11x\sqrt{-e^2x^4+d^2}}{70e^2d^2\left(x^2+\frac{d}{e}\right)^2}+\frac{3\left(-e^2x^2+de\right)x}{10e\,d^3\sqrt{\left(x^2+\frac{d}{e}\right)\left(-e^2x^2+de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\big(x}{7d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$
elliptic	$\frac{2x\sqrt{-e^2x^4+d^2}}{7e^4\left(x^2+\frac{d}{e}\right)^4}+\frac{4x\sqrt{-e^2x^4+d^2}}{35d\,e^3\left(x^2+\frac{d}{e}\right)^3}+\frac{11x\sqrt{-e^2x^4+d^2}}{70e^2d^2\left(x^2+\frac{d}{e}\right)^2}+\frac{3\left(-e^2x^2+de\right)x}{10e\,d^3\sqrt{\left(x^2+\frac{d}{e}\right)\left(-e^2x^2+de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\big(x}{7d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^6,x,method=_RETURNVERBOSE)
```

output
```
2/7*x/e^4*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^4+4/35/d*x/e^3*(-e^2*x^4+d^2)^(1/
2)/(x^2+d/e)^3+11/70/e^2/d^2*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+3/10*(-e^2
*x^2+d*e)/e/d^3*x/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/7/d^2/(e/d)^(1/2)*(1-
e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1
/2),I)-3/10/d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+
d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.10

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^6}\,dx = \frac{21\left(e^5x^8+4\,de^4x^6+6\,d^2e^3x^4+4\,d^3e^2x^2+d^4e\right)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)+((10\,de}{}$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^6,x, algorithm="fricas")
```

output
```
1/70*(21*(e^5*x^8 + 4*d*e^4*x^6 + 6*d^2*e^3*x^4 + 4*d^3*e^2*x^2 + d^4*e)*s
qrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) + ((10*d*e^4 - 21*e^5)*x^8 +
4*(10*d^2*e^3 - 21*d*e^4)*x^6 + 10*d^5 - 21*d^4*e + 6*(10*d^3*e^2 - 21*d^2
*e^3)*x^4 + 4*(10*d^4*e - 21*d^3*e^2)*x^2)*sqrt(e/d)*elliptic_f(arcsin(x*s
qrt(e/d)), -1) + (21*e^4*x^7 + 74*d*e^3*x^5 + 93*d^2*e^2*x^3 + 60*d^3*e*x)
*sqrt(-e^2*x^4 + d^2))/(d^3*e^5*x^8 + 4*d^4*e^4*x^6 + 6*d^5*e^3*x^4 + 4*d^
6*e^2*x^2 + d^7*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^6}\,dx = \text{Timed out}$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^6}\,dx=\int \frac{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^6}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^6,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^6, x)
```

Giac [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^6}\,dx=\int \frac{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^6}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^6,x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^6}\,dx=\int \frac{\left(d^2-e^2\,x^4\right)^{3/2}}{\left(e\,x^2+d\right)^6}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^6,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^6, x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^6}\,dx = \frac{\sqrt{-e^2x^4+d^2}\,x+3\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^6x^{12}-4d\,e^5x^{10}-5d^2e^4x^8+5d^4e^2x^4+4d^5e\,x^2+d^6}dx\right)d^6+12\left(\int \frac{}{-e^6x^{12}-}\right.}{}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^6,x)
```

output
```
(sqrt(d**2 - e**2*x**4)*x + 3*int(sqrt(d**2 - e**2*x**4)/(d**6 + 4*d**5*e*
x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 - 4*d*e**5*x**10 - e**6*x**12),
x)*d**6 + 12*int(sqrt(d**2 - e**2*x**4)/(d**6 + 4*d**5*e*x**2 + 5*d**4*e**
2*x**4 - 5*d**2*e**4*x**8 - 4*d*e**5*x**10 - e**6*x**12),x)*d**5*e*x**2 +
18*int(sqrt(d**2 - e**2*x**4)/(d**6 + 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5
*d**2*e**4*x**8 - 4*d*e**5*x**10 - e**6*x**12),x)*d**4*e**2*x**4 + 12*int(
sqrt(d**2 - e**2*x**4)/(d**6 + 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e
**4*x**8 - 4*d*e**5*x**10 - e**6*x**12),x)*d**3*e**3*x**6 + 3*int(sqrt(d**
2 - e**2*x**4)/(d**6 + 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8
 - 4*d*e**5*x**10 - e**6*x**12),x)*d**2*e**4*x**8 - int((sqrt(d**2 - e**2*
x**4)*x**4)/(d**6 + 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 -
4*d*e**5*x**10 - e**6*x**12),x)*d**4*e**2 - 4*int((sqrt(d**2 - e**2*x**4)*
x**4)/(d**6 + 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 - 4*d*e*
*5*x**10 - e**6*x**12),x)*d**3*e**3*x**2 - 6*int((sqrt(d**2 - e**2*x**4)*x
**4)/(d**6 + 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 - 4*d*e**
5*x**10 - e**6*x**12),x)*d**2*e**4*x**4 - 4*int((sqrt(d**2 - e**2*x**4)*x*
*4)/(d**6 + 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 - 4*d*e**5
*x**10 - e**6*x**12),x)*d*e**5*x**6 - int((sqrt(d**2 - e**2*x**4)*x**4)/(d
**6 + 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 - 4*d*e**5*x**10
 - e**6*x**12),x)*e**6*x**8)/(4*(d**4 + 4*d**3*e*x**2 + 6*d**2*e**2*x**...
```

3.53 $\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 287

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^7}\,dx=\frac{2x\sqrt{d^2-e^2x^4}}{9\left(d+ex^2\right)^5}+\frac{2x\sqrt{d^2-e^2x^4}}{21d\left(d+ex^2\right)^4}+\frac{73x\sqrt{d^2-e^2x^4}}{630d^2\left(d+ex^2\right)^3}+\frac{16x\sqrt{d^2-e^2x^4}}{105d^3\left(d+ex^2\right)^2}+\frac{17x\sqrt{d^2-e^2x^4}}{60d^4\left(d+ex^2\right)}+\frac{17\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{60d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}-\frac{16\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{105d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
2/9*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^5+2/21*x*(-e^2*x^4+d^2)^(1/2)/d/(e*x^
2+d)^4+73/630*x*(-e^2*x^4+d^2)^(1/2)/d^2/(e*x^2+d)^3+16/105*x*(-e^2*x^4+d^
2)^(1/2)/d^3/(e*x^2+d)^2+17/60*x*(-e^2*x^4+d^2)^(1/2)/d^4/(e*x^2+d)+17/60*
(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(5/2)/e^(1/2)/(-e^2
*x^4+d^2)^(1/2)-16/105*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I
)/d^(5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.74 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.57

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^7}\,dx = \frac{-\frac{x\left(-d+ex^2\right)\left(1095d^4+2416d^3ex^2+2864d^2e^2x^4+1620de^3x^6+357e^4x^8\right)}{\left(d+ex^2\right)^4}+\frac{3ie\sqrt{1-\frac{e^2x^4}{d^2}}\left(119E\left(i\operatorname{arcsinh}\left(\sqrt{\ }\right.\right.\right.}{}}{1260d^4\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^7,x]
```

output
```
(-((x*(-d + e*x^2)*(1095*d^4 + 2416*d^3*e*x^2 + 2864*d^2*e^2*x^4 + 1620*d*
e^3*x^6 + 357*e^4*x^8))/(d + e*x^2)^4) + ((3*I)*e*Sqrt[1 - (e^2*x^4)/d^2]*
(119*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - 64*EllipticF[I*ArcSinh[Sqr
t[-(e/d)]*x], -1]))/(-(e/d))^(3/2))/(1260*d^4*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.25, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {1396, 315, 27, 402, 27, 402, 27, 402, 27, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^7}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(d-ex^2\right)^{3/2}}{\left(ex^2+d\right)^{11/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{de\left(7d-5ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{9/2}}dx}{9de}+\frac{2x\sqrt{d-ex^2}}{9(d+ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\int \frac{7d-5ex^2}{\sqrt{d-ex^2}(ex^2+d)^{9/2}}dx+\frac{2x\sqrt{d-ex^2}}{9(d+ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{6x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}-\frac{\int -\frac{2de\left(43d-30ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{14d^2e}\right)+\frac{2x\sqrt{d-ex^2}}{9(d+ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\int \frac{43d-30ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{7d}+\frac{6x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)+\frac{2x\sqrt{d-ex^2}}{9(d+ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{73x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{\int -\frac{3de\left(119d-73ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d^2e}}{7d}+\frac{6x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)+\frac{2x\sqrt{d-ex^2}}{9(d+ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\int \frac{119d-73ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d}+\frac{73x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)+\frac{2x\sqrt{d-ex^2}}{9(d+ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{32x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{6de\left(87d-32ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}\right)}{10d}+\frac{73x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)+\frac{2x\sqrt{d-ex^2}}{9(d+ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\int \frac{87d-32ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{d}+\frac{32x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)+\frac{2x\sqrt{d-ex^2}}{9(d+ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{119x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{de\left(119ex^2+55d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{d}+\frac{32x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)+\frac{2x\sqrt{d-ex^2}}{9(d+ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{\int \frac{de\left(119ex^2+55d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{119x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{32x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)+\frac{2x\sqrt{d-ex^2}}{9(d+ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{\int \frac{119ex^2+55d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{119x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{32x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)+\frac{2x\sqrt{d-ex^2}}{9(d+ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{119\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-64d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{119x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{32x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)+\frac{2x}{9(d}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 289$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{119\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{64d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{119x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{32x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d-ex^2}}{7d(d+ex^2)^{7/2}}\right)-\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 329$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{\frac{119d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{64d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{119x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{32x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d}}{7d(d+e}\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{\frac{119d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{64d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{119x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{32x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{\frac{119d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{64d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{119x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{32x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}-\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{\frac{119d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{64d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{119x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{32x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}+}{7d}\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^7,x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((2*x*Sqrt[d - e*x^2])/(9*(d + e*x^2)^(9/2)) + ((6*x*
Sqrt[d - e*x^2])/(7*d*(d + e*x^2)^(7/2)) + ((73*x*Sqrt[d - e*x^2])/(10*d*(
d + e*x^2)^(5/2)) + (3*((32*x*Sqrt[d - e*x^2])/(d*(d + e*x^2)^(3/2)) + ((1
19*x*Sqrt[d - e*x^2])/(2*d*Sqrt[d + e*x^2]) + ((119*d^(3/2)*Sqrt[1 - (e^2*
x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*
x^2]*Sqrt[d + e*x^2]) - (64*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcS
in[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/(
2*d))/d))/(10*d))/(7*d))/9))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 6.90 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.12

method	result
default	$\frac{2x\sqrt{-e^2x^4+d^2}}{9e^5\left(x^2+\frac{d}{e}\right)^5}+\frac{2x\sqrt{-e^2x^4+d^2}}{21d\,e^4\left(x^2+\frac{d}{e}\right)^4}+\frac{73x\sqrt{-e^2x^4+d^2}}{630d^2e^3\left(x^2+\frac{d}{e}\right)^3}+\frac{16x\sqrt{-e^2x^4+d^2}}{105d^3e^2\left(x^2+\frac{d}{e}\right)^2}+\frac{17(-e^2x^2+de)x}{60e\,d^4\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}+$ $11\sqrt{1-}$
elliptic	$\frac{2x\sqrt{-e^2x^4+d^2}}{9e^5\left(x^2+\frac{d}{e}\right)^5}+\frac{2x\sqrt{-e^2x^4+d^2}}{21d\,e^4\left(x^2+\frac{d}{e}\right)^4}+\frac{73x\sqrt{-e^2x^4+d^2}}{630d^2e^3\left(x^2+\frac{d}{e}\right)^3}+\frac{16x\sqrt{-e^2x^4+d^2}}{105d^3e^2\left(x^2+\frac{d}{e}\right)^2}+\frac{17(-e^2x^2+de)x}{60e\,d^4\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}+$ $11\sqrt{1-}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^7,x,method=_RETURNVERBOSE)
```

output
```
2/9*x/e^5*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^5+2/21/d*x/e^4*(-e^2*x^4+d^2)^(1/
2)/(x^2+d/e)^4+73/630/d^2/e^3*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^3+16/105/d^
3/e^2*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+17/60*(-e^2*x^2+d*e)/e/d^4*x/((x^
2+d/e)*(-e^2*x^2+d*e))^(1/2)+11/84/d^3/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*
x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-17/60/d^3/(e/
d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(Ellipti
cF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.17

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^7}\,dx = \frac{357\,(e^6x^{10}+5\,de^5x^8+10\,d^2e^4x^6+10\,d^3e^3x^4+5\,d^4e^2x^2+d^5e)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)}{}$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^7,x, algorithm="fricas")
```

output
```
1/1260*(357*(e^6*x^10 + 5*d*e^5*x^8 + 10*d^2*e^4*x^6 + 10*d^3*e^3*x^4 + 5*
d^4*e^2*x^2 + d^5*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) + 3*((5
5*d*e^5 - 119*e^6)*x^10 + 5*(55*d^2*e^4 - 119*d*e^5)*x^8 + 10*(55*d^3*e^3
- 119*d^2*e^4)*x^6 + 55*d^6 - 119*d^5*e + 10*(55*d^4*e^2 - 119*d^3*e^3)*x^
4 + 5*(55*d^5*e - 119*d^4*e^2)*x^2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d
)), -1) + (357*e^5*x^9 + 1620*d*e^4*x^7 + 2864*d^2*e^3*x^5 + 2416*d^3*e^2*
x^3 + 1095*d^4*e*x)*sqrt(-e^2*x^4 + d^2))/(d^4*e^6*x^10 + 5*d^5*e^5*x^8 +
10*d^6*e^4*x^6 + 10*d^7*e^3*x^4 + 5*d^8*e^2*x^2 + d^9*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^7}\,dx = \text{Timed out}$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**7,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^7}\,dx=\int \frac{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^7}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^7,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^7, x)
```

Giac [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^7}\,dx=\int \frac{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^7}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^7,x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^7}\,dx=\int \frac{\left(d^2-e^2\,x^4\right)^{3/2}}{\left(e\,x^2+d\right)^7}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^7,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^7, x)
```

Reduce [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^7}\,dx = \text{Too large to display}$$

input

```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^7,x)
```

output

```
(sqrt(d**2 - e**2*x**4)*x + 4*int(sqrt(d**2 - e**2*x**4)/(d**7 + 5*d**6*e*
x**2 + 9*d**5*e**2*x**4 + 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 - 9*d**2*e**
5*x**10 - 5*d*e**6*x**12 - e**7*x**14),x)*d**7 + 20*int(sqrt(d**2 - e**2*x
**4)/(d**7 + 5*d**6*e*x**2 + 9*d**5*e**2*x**4 + 5*d**4*e**3*x**6 - 5*d**3*
e**4*x**8 - 9*d**2*e**5*x**10 - 5*d*e**6*x**12 - e**7*x**14),x)*d**6*e*x**
2 + 40*int(sqrt(d**2 - e**2*x**4)/(d**7 + 5*d**6*e*x**2 + 9*d**5*e**2*x**4
 + 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 - 9*d**2*e**5*x**10 - 5*d*e**6*x**1
2 - e**7*x**14),x)*d**5*e**2*x**4 + 40*int(sqrt(d**2 - e**2*x**4)/(d**7 +
5*d**6*e*x**2 + 9*d**5*e**2*x**4 + 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 - 9
*d**2*e**5*x**10 - 5*d*e**6*x**12 - e**7*x**14),x)*d**4*e**3*x**6 + 20*int
(sqrt(d**2 - e**2*x**4)/(d**7 + 5*d**6*e*x**2 + 9*d**5*e**2*x**4 + 5*d**4*
e**3*x**6 - 5*d**3*e**4*x**8 - 9*d**2*e**5*x**10 - 5*d*e**6*x**12 - e**7*x
**14),x)*d**3*e**4*x**8 + 4*int(sqrt(d**2 - e**2*x**4)/(d**7 + 5*d**6*e*x*
*2 + 9*d**5*e**2*x**4 + 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 - 9*d**2*e**5*
x**10 - 5*d*e**6*x**12 - e**7*x**14),x)*d**2*e**5*x**10 - 2*int((sqrt(d**2
 - e**2*x**4)*x**4)/(d**7 + 5*d**6*e*x**2 + 9*d**5*e**2*x**4 + 5*d**4*e**3
*x**6 - 5*d**3*e**4*x**8 - 9*d**2*e**5*x**10 - 5*d*e**6*x**12 - e**7*x**14
),x)*d**5*e**2 - 10*int((sqrt(d**2 - e**2*x**4)*x**4)/(d**7 + 5*d**6*e*x**
2 + 9*d**5*e**2*x**4 + 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 - 9*d**2*e**5*x
**10 - 5*d*e**6*x**12 - e**7*x**14),x)*d**4*e**3*x**2 - 20*int((sqrt(d*...
```

3.54 $\int \frac{\left(d+ex^2\right)^3}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 169

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^3}{\sqrt{d^2-e^2x^4}}\,dx &= -dx\sqrt{d^2-e^2x^4}-\frac{1}{5}ex^3\sqrt{d^2-e^2x^4}\\ &\quad+\frac{18d^{7/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{5\sqrt{e}\sqrt{d^2-e^2x^4}}\\ &\quad-\frac{8d^{7/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{5\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
-d*x*(-e^2*x^4+d^2)^(1/2)-1/5*e*x^3*(-e^2*x^4+d^2)^(1/2)+18/5*d^(7/2)*(1-e
^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1
/2)-8/5*d^(7/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/
2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx = \frac{-5d^3x-d^2ex^3+5de^2x^5+e^3x^7+10d^3x\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)+6d^2ex^3\sqrt{1-}}{5\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[(d + e*x^2)^3/Sqrt[d^2 - e^2*x^4],x]
```

output

```
(-5*d^3*x - d^2*e*x^3 + 5*d*e^2*x^5 + e^3*x^7 + 10*d^3*x*Sqrt[1 - (e^2*x^4
)/d^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (e^2*x^4)/d^2] + 6*d^2*e*x^3*Sqrt[
1 - (e^2*x^4)/d^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (e^2*x^4)/d^2])/(5*Sqr
t[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.44, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1396, 318, 27, 403, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{(ex^2+d)^{5/2}}{\sqrt{d-ex^2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{\int -\frac{6de\sqrt{ex^2+d}\left(2ex^2+d\right)}{\sqrt{d-ex^2}}dx}{5e}-\frac{1}{5}x\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{6}{5}d\int \frac{\sqrt{ex^2+d}(2ex^2+d)}{\sqrt{d-ex^2}}dx-\frac{1}{5}x\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 403$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{6}{5}d\left(-\frac{\int -\frac{de\left(9ex^2+5d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}x\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{6}{5}d\left(\frac{\int \frac{de\left(9ex^2+5d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}x\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{6}{5}d\left(\frac{1}{3}d\int \frac{9ex^2+5d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}x\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{6}{5}d\left(\frac{1}{3}d\left(9\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-4d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx\right)-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}x\sqrt{d-ex^2}\left(d\right.\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{6}{5}d\left(\frac{1}{3}d\left(9\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}x\sqrt{d-ex^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{6}{5}d\Bigg(\frac{1}{3}d\Bigg(\frac{9d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)-\frac{1}{5}x}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{6}{5}d\Bigg(\frac{1}{3}d\Bigg(\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{6}{5}d\Bigg(\frac{1}{3}d\Bigg(\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{6}{5}d\Bigg(\frac{1}{3}d\Bigg(\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-\frac{2}{3}x\sqrt{d-}}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d + e*x^2)^3/Sqrt[d^2 - e^2*x^4],x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(-1/5*(x*Sqrt[d - e*x^2]*(d + e*x^2)^(3/2
)) + (6*d*((-2*x*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])/3 + (d*((9*d^(3/2)*Sqrt[
1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sq
rt[d - e*x^2]*Sqrt[d + e*x^2]) - (4*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*Ellipt
icF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*
x^2])))/3))/5))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] && !IntegerQ[p] && !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 7.81 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{x\left(e x^2+5d\right)\sqrt{-e^2x^4+d^2}}{5}+\frac{2d^2\Bigg(\frac{5d\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{9d\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\,\Big(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\operatorname{EllipticE}\Big(x\sqrt{\frac{e}{d}}}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}}{5}$
elliptic	$-\frac{e x^3\sqrt{-e^2x^4+d^2}}{5}-dx\sqrt{-e^2x^4+d^2}+\frac{2d^3\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{18d^3\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\,\Big(\text{Ellipti}}{5\sqrt{\frac{e}{d}}\,\sqrt{-}}$
default	$\frac{d^3\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}+e^3\Bigg(-\frac{x^3\sqrt{-e^2x^4+d^2}}{5e^2}-\frac{3d^3\sqrt{1-\frac{e x^2}{d}}\,\sqrt{1+\frac{e x^2}{d}}\,\Big(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\operatorname{EllipticE}\Big(}{5e^3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$

input
```
int((e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*x*(e*x^2+5*d)*(-e^2*x^4+d^2)^(1/2)+2/5*d^2*(5*d/(e/d)^(1/2)*(1-e*x^2/
d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)
-9*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*
(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx = -\frac{18\sqrt{-e^2}d^3x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) - 2\,(9\,d^3+5\,d^2e)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) + (e^3x^4+5}{5\,e^2x}$$

input
```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(18*sqrt(-e^2)*d^3*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) -
2*(9*d^3 + 5*d^2*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x),
 -1) + (e^3*x^4 + 5*d*e^2*x^2 + 18*d^2*e)*sqrt(-e^2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx = \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{3dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{3e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x**2+d)**3/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d*
*2)/(4*gamma(5/4)) + 3*d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**2*
x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(7/4)) + 3*e**2*x**5*gamma(5/4)*hyper
((1/2, 5/4), (9/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(9/4)) + e*
*3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), e**2*x**4*exp_polar(2*I*pi)/
d**2)/(4*d*gamma(11/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^3/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(e\,x^2+d)^3}{\sqrt{d^2-e^2\,x^4}}\,dx$$

input
```
int((d + e*x^2)^3/(d^2 - e^2*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^3/(d^2 - e^2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx = -\sqrt{-e^2x^4+d^2}\,dx - \frac{\sqrt{-e^2x^4+d^2}\,e\,x^3}{5} + 2\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^3 + \frac{18\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^2e}{5}$$

input
```
int((e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
( - 5*sqrt(d**2 - e**2*x**4)*d*x - sqrt(d**2 - e**2*x**4)*e*x**3 + 10*int(
sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)*d**3 + 18*int((sqrt(d**2 - e*
*2*x**4)*x**2)/(d**2 - e**2*x**4),x)*d**2*e)/5
```

3.55 $\int \frac{(d+ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 144

$$\int \frac{(d+ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx = -\frac{1}{3}x\sqrt{d^2-e^2x^4} + \frac{2d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}} - \frac{2d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
-1/3*x*(-e^2*x^4+d^2)^(1/2)+2*d^(5/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1
/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-2/3*d^(5/2)*(1-e^2*x^4/d^2)^
(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx$$

$$= \frac{-d^2x + e^2x^5 + 4d^2x\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right) + 2dex^3\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F}}{3\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d + e*x^2)^2/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(-(d^2*x) + e^2*x^5 + 4*d^2*x*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[1/
4, 1/2, 5/4, (e^2*x^4)/d^2] + 2*d*e*x^3*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeome
tric2F1[1/2, 3/4, 7/4, (e^2*x^4)/d^2])/(3*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.45, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1396, 318, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\left(ex^2+d\right)^{3/2}}{\sqrt{d-ex^2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{\int -\frac{2de\left(3ex^2+2d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\int \frac{3ex^2+2d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\left(3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\left(3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 329}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\left(\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\left(\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\left(\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\left(\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d + e*x^2)^2/Sqrt[d^2 - e^2*x^4],x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(-1/3*(x*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
 + (2*d*((3*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/S
qrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) - (d^(3/2)*Sqrt[1
- (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt
[d - e*x^2]*Sqrt[d + e*x^2])))/3))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 5.46 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{2d\left(\frac{2d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-\frac{3d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right)}{3}$
elliptic	$-\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{4d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-\frac{2d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\right.\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$
default	$\frac{d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+e^{2}\left(-\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3e^{2}}+\frac{d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3e^{2}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right)-2d^{2}$

input
```
int((e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(-e^2*x^4+d^2)^(1/2)+2/3*d*(2*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*
x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-3*d/(e/d)^(1/
2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(
e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx =$$

$$-\frac{6\sqrt{-e^2}d^2x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) - 2\,(3\,d^2+2\,de)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) + \sqrt{-e^2x^4+}}{3\,e^2x}$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(6*sqrt(-e^2)*d^2*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - 2
*(3*d^2 + 2*d*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1
) + sqrt(-e^2*x^4 + d^2)*(e^2*x^2 + 6*d*e))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((e*x**2+d)**2/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)
/(4*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**2*x**4*ex
p_polar(2*I*pi)/d**2)/(2*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1/2, 5/
4), (9/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*d*gamma(9/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(e\,x^2+d)^2}{\sqrt{d^2-e^2\,x^4}}\,dx$$

input
```
int((d + e*x^2)^2/(d^2 - e^2*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^2/(d^2 - e^2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx=\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e\,x^2+d}dx\right)d+\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e\,x^2+d}dx\right)e$$

input

```
int((e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d - e*x**2),x)*d + int((sqrt(d**2 - e**2*x**4)
*x**2)/(d - e*x**2),x)*e
```

3.56 $\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 59

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2
*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\begin{aligned}&\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx\\&=\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{e^2x^4}{d^2}\right)\right)}{3\sqrt{d^2-e^2x^4}}\end{aligned}$$

input
```
Integrate[(d + e*x^2)/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[1 - (e^2*x^4)/d^2]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (e^2*x^4)
/d^2] + e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(3*Sqrt[d^
2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{ 1390}$$

$$\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{ex^2+d}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d + e*x^2)/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1
])/(Sqrt[e]*Sqrt[d^2 - e^2*x^4])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(47) = 94$.

Time = 2.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.34

method	result	size
default	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	138
elliptic	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	138

input
```
int((e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*Ell
ipticF(x*(e/d)^(1/2),I)-d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/
(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx = -\frac{\sqrt{-e^2}dx\sqrt{\frac{d}{e}}E\left(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\mid -1\right)-\sqrt{-e^2}(d+e)x\sqrt{\frac{d}{e}}F\left(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\mid -1\right)+\sqrt{-e^2x^4+d^2}e}{e^2x}$$

input

```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output

```
-(sqrt(-e^2)*d*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - sqrt(-e^2
)*(d + e)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1) + sqrt(-e^2*x^4
+ d^2)*e)/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d\Gamma\left(\frac{7}{4}\right)}$$

input

```
integrate((e*x**2+d)/(-e**2*x**4+d**2)**(1/2),x)
```

output

```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**2*x**4*exp_
polar(2*I*pi)/d**2)/(4*d*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{d^2-e^2\,x^4}}\,dx$$

input
```
int((d + e*x^2)/(d^2 - e^2*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(d^2 - e^2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{-e\,x^2+d}dx$$

input
```
int((e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d - e*x**2),x)
```

3.57 $\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx=\frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
d^(1/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2
*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx=-\frac{i\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)}{\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[1/Sqrt[d^2 - e^2*x^4],x]
```

output
```
((-I)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1])/(S
qrt[-(e/d)]*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[d]*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1
])/(Sqrt[e]*Sqrt[d^2 - e^2*x^4])
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	60
elliptic	$\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	60

input
```
int(1/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*Ell
ipticF(x*(e/d)^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx=\frac{\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|\,-1\right)}{e}$$

input
```
integrate(1/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1)/e
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*d*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate(1/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}}\,dx$$

input

```
integrate(1/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output

```
integrate(1/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [B] (verification not implemented)

Time = 17.51 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx = \frac{x\,\sqrt{1-\frac{e^2\,x^4}{d^2}}\;{}_2\mathrm{F}_1\!\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ \frac{e^2\,x^4}{d^2}\right)}{\sqrt{d^2-e^2\,x^4}}$$

input

```
int(1/(d^2 - e^2*x^4)^(1/2),x)
```

output

```
(x*(1 - (e^2*x^4)/d^2)^(1/2)*hypergeom([1/4, 1/2], 5/4, (e^2*x^4)/d^2))/(d
^2 - e^2*x^4)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx$$

input

```
int(1/(-e^2*x^4+d^2)^(1/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)
```

3.58 $\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \frac{x\sqrt{d^2-e^2x^4}}{2d^2(d+ex^2)} + \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
1/2*x*(-e^2*x^4+d^2)^(1/2)/d^2/(e*x^2+d)+1/2*(1-e^2*x^4/d^2)^(1/2)*Ellipti
cE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \frac{\sqrt{-\frac{e}{d}}x(d-ex^2) - id\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)}{2d^2\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[1/((d + e*x^2)*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[-(e/d)]*x*(d - e*x^2) - I*d*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcS
inh[Sqrt[-(e/d)]*x], -1])/(2*d^2*Sqrt[-(e/d)]*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.47, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1396, 316, 25, 27, 329, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d-ex^2}}{2d^2\sqrt{d+ex^2}}-\frac{\int -\frac{e\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d^2e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d^2e}+\frac{x\sqrt{d-ex^2}}{2d^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d^2}+\frac{x\sqrt{d-ex^2}}{2d^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{329}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{2d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{2d^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{2d^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/((d + e*x^2)*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d - e*x^2])/(2*d^2*Sqrt[d + e*x^
2]) + (Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])
/(2*Sqrt[d]*Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(78) = 156.

Time = 1.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.96

method	result
default	$\frac{\left(-e^{2} x^{2}+d e \right) x}{2 d^{2} e \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, (\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}(}{2 d \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$
elliptic	$\frac{\left(-e^{2} x^{2}+d e \right) x}{2 d^{2} e \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, (\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}(}{2 d \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$

input
```
int(1/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e^2*x^2+d*e)/d^2*x/e/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/2/d/(e/d)^(1
/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(
e/d)^(1/2),I)-1/2/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*
x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \frac{\sqrt{-e^2x^4+d^2}ex + (e^2x^2+de)\sqrt{\frac{e}{d}}E(\arcsin(x\sqrt{\frac{e}{d}})\,|-1) + ((de-e^2)x^2+d^2-de)\sqrt{\frac{e}{d}}F(\arcsin(x\sqrt{\frac{e}{d}})}{2(d^2e^2x^2+d^3e)}$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-e^2*x^4 + d^2)*e*x + (e^2*x^2 + d*e)*sqrt(e/d)*elliptic_e(arcsi
n(x*sqrt(e/d)), -1) + ((d*e - e^2)*x^2 + d^2 - d*e)*sqrt(e/d)*elliptic_f(a
rcsin(x*sqrt(e/d)), -1))/(d^2*e^2*x^2 + d^3*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-(-d+ex^2)(d+ex^2)}(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x**2)*(d + e*x**2))*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2-e^2\,x^4}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^3x^6-d\,e^2x^4+d^2e\,x^2+d^3}dx$$

input

```
int(1/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**3 + d**2*e*x**2 - d*e**2*x**4 - e**3*x**6),
x)
```

3.59 $\int \frac{1}{(d+ex^2)^2\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 191

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2\sqrt{d^2-e^2x^4}}\,dx &= \frac{x\sqrt{d^2-e^2x^4}}{6d^2(d+ex^2)^2}+\frac{x\sqrt{d^2-e^2x^4}}{2d^3(d+ex^2)} \\ &\quad+\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}} \\ &\quad-\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{6d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/6*x*(-e^2*x^4+d^2)^(1/2)/d^2/(e*x^2+d)^2+1/2*x*(-e^2*x^4+d^2)^(1/2)/d^3/
(e*x^2+d)+1/2*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(3/2)
/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-1/6*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*
x/d^(1/2),I)/d^(3/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.79 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex^2)^2\sqrt{d^2-e^2x^4}}\,dx$$
$$= \frac{\sqrt{-\frac{e}{d}}x(4d^2-dex^2-3e^2x^4)-3id(d+ex^2)\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)+id(d+ex^2)\sqrt{1-}}{6d^3\sqrt{-\frac{e}{d}}\,(d+ex^2)\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[1/((d + e*x^2)^2*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[-(e/d)]*x*(4*d^2 - d*e*x^2 - 3*e^2*x^4) - (3*I)*d*(d + e*x^2)*Sqrt[1
 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] + I*d*(d + e*x^
2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1])/(6*d^
3*Sqrt[-(e/d)]*(d + e*x^2)*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.30, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1396, 316, 25, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2\sqrt{d^2-e^2x^4}}\,dx$$
$$\Big\downarrow \text{ 1396}$$
$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$
$$\Big\downarrow \text{ 316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}-\frac{\int -\frac{e\left(5d-ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(5d-ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}+\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{5d-ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \color{blue}{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\int -\frac{2de\left(3ex^2+2d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{3ex^2+2d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \color{blue}{399}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \color{blue}{289}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[1/((d + e*x^2)^2*Sqrt[d^2 - e^2*x^4]),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d - e*x^2])/(6*d^2*(d + e*x^2)^(
3/2)) + ((3*x*Sqrt[d - e*x^2])/(d*Sqrt[d + e*x^2]) + ((3*d^(3/2)*Sqrt[1 -
(e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d
 - e*x^2]*Sqrt[d + e*x^2]) - (d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[Ar
cSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))
/d)/(6*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.16

method	result
default	$\frac{x\sqrt{-e^2x^4+d^2}}{6e^2d^2\left(x^2+\frac{d}{e}\right)^2}+\frac{(-e^2x^2+de)x}{2e\,d^3\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \left(\mathrm{EllipticF}\right.}{2d^2\sqrt{\frac{e}{d}}\,\sqrt{-}}$
elliptic	$\frac{x\sqrt{-e^2x^4+d^2}}{6e^2d^2\left(x^2+\frac{d}{e}\right)^2}+\frac{(-e^2x^2+de)x}{2e\,d^3\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \left(\mathrm{EllipticF}\right.}{2d^2\sqrt{\frac{e}{d}}\,\sqrt{-}}$

input
```
int(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/e^2/d^2*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+1/2*(-e^2*x^2+d*e)/e/d^3*x/
((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/3/d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+
e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-1/2/d^2/(e/
d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(Ellipti
cF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.89

$$\int \frac{1}{(d+ex^2)^2\sqrt{d^2-e^2x^4}}\,dx$$
$$=\frac{3\left(e^3x^4+2\,de^2x^2+d^2e\right)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)+((2\,de^2-3\,e^3)x^4+2\,d^3-3\,d^2e+2\,(2\,d^2e-3\,de^2)}{6\left(d^3e^3x^4+2\,d^4e^2x^2+d^5e\right)}$$

input
```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(
e/d)), -1) + ((2*d*e^2 - 3*e^3)*x^4 + 2*d^3 - 3*d^2*e + 2*(2*d^2*e - 3*d*e
^2)*x^2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d
^2)*(3*e^2*x^3 + 4*d*e*x))/(d^3*e^3*x^4 + 2*d^4*e^2*x^2 + d^5*e)
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}\left(d+ex^2\right)^2}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x**2)*(d + e*x**2))*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}\left(ex^2+d\right)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}\left(ex^2+d\right)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{d^2-e^2x^4}}\,dx=\int \frac{1}{\sqrt{d^2-e^2\,x^4}\,{\left(e\,x^2+d\right)}^2}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{d^2-e^2x^4}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{-e^4x^8-2d\,e^3x^6+2d^3e\,x^2+d^4}dx$$

input
```
int(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**4 + 2*d**3*e*x**2 - 2*d*e**3*x**6 - e**4*x*
*8),x)
```

3.60 $\int \frac{1}{\left(d+ex^2\right)^3\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 224

$$\begin{aligned}\int \frac{1}{\left(d+ex^2\right)^3\sqrt{d^2-e^2x^4}}\,dx &= \frac{x\sqrt{d^2-e^2x^4}}{10d^2\left(d+ex^2\right)^3}+\frac{x\sqrt{d^2-e^2x^4}}{5d^3\left(d+ex^2\right)^2}+\frac{9x\sqrt{d^2-e^2x^4}}{20d^4\left(d+ex^2\right)}\\ &\quad+\frac{9\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{20d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\\ &\quad-\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{5d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/10*x*(-e^2*x^4+d^2)^(1/2)/d^2/(e*x^2+d)^3+1/5*x*(-e^2*x^4+d^2)^(1/2)/d^3
/(e*x^2+d)^2+9/20*x*(-e^2*x^4+d^2)^(1/2)/d^4/(e*x^2+d)+9/20*(1-e^2*x^4/d^2
)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2
)-1/5*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(5/2)/e^(1/2)
/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.78 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex^2)^3\sqrt{d^2-e^2x^4}}\,dx$$

$$= \frac{\sqrt{-\frac{e}{d}}x(15d^3+7d^2ex^2-13de^2x^4-9e^3x^6)-9id(d+ex^2)^2\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)+4id(}{20d^4\sqrt{-\frac{e}{d}}\left(d+ex^2\right)^2\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[1/((d + e*x^2)^3*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[-(e/d)]*x*(15*d^3 + 7*d^2*e*x^2 - 13*d*e^2*x^4 - 9*e^3*x^6) - (9*I)*
d*(d + e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x
], -1] + (4*I)*d*(d + e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh
[Sqrt[-(e/d)]*x], -1])/(20*d^4*Sqrt[-(e/d)]*(d + e*x^2)^2*Sqrt[d^2 - e^2*x
^4])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.30, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1396, 316, 27, 402, 27, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^3\sqrt{d^2-e^2x^4}}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)^{7/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d-ex^2}}{10d^2(d+ex^2)^{5/2}}-\frac{\int -\frac{3e\left(3d-ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d^2e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\int \frac{3d-ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d^2}+\frac{x\sqrt{d-ex^2}}{10d^2(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{2x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{2de\left(7d-2ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}\right)}{10d^2}+\frac{x\sqrt{d-ex^2}}{10d^2(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\int \frac{7d-2ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{2x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d-ex^2}}{10d^2(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{de\left(9ex^2+5d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d-ex^2}}{10d^2(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{\int \frac{de\left(9ex^2+5d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d-ex^2}}{10d^2(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{\int \frac{9ex^2+5d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d-ex^2}}{10d^2(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{9\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-4d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d-ex^2}}{10d^2(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{9\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d-ex^2}}{10d^2(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{\frac{9d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d-ex^2}}{10d^2(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d-ex}}{10d^2(d+ex^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d-ex^2}}{10d^2(d+ex^2)}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{2x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}\right)}{10d^2}+\right.}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[1/((d + e*x^2)^3*Sqrt[d^2 - e^2*x^4]),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d - e*x^2])/(10*d^2*(d + e*x^2)^
(5/2)) + (3*((2*x*Sqrt[d - e*x^2])/(3*d*(d + e*x^2)^(3/2)) + ((9*x*Sqrt[d
- e*x^2])/(2*d*Sqrt[d + e*x^2]) + ((9*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*Elli
pticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d +
e*x^2]) - (4*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/
Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/(2*d))/(3*d)))/(
10*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.14

method	result
default	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{10d^{2}e^{3}\left(x^{2}+\frac{d}{e}\right)^{3}}+\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{5d^{3}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{9\left(-e^{2}x^{2}+de\right)x}{20e\,d^{4}\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{4d^{3}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-$
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{10d^{2}e^{3}\left(x^{2}+\frac{d}{e}\right)^{3}}+\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{5d^{3}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{9\left(-e^{2}x^{2}+de\right)x}{20e\,d^{4}\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{4d^{3}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-$

input
```
int(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/10/d^2/e^3*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^3+1/5/d^3/e^2*x*(-e^2*x^4+d^
2)^(1/2)/(x^2+d/e)^2+9/20*(-e^2*x^2+d*e)/e/d^4*x/((x^2+d/e)*(-e^2*x^2+d*e)
)^(1/2)+1/4/d^3/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+
d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-9/20/d^3/(e/d)^(1/2)*(1-e*x^2/d)^(1/
2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-Elli
pticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^3\sqrt{d^2-e^2x^4}}\,dx$$
$$= \frac{9\,(e^4x^6+3\,de^3x^4+3\,d^2e^2x^2+d^3e)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1)+((5\,de^3-9\,e^4)x^6+3\,(5\,d^2e^2-9\,de^3)x^4}{20\,(d^4e^4x^6+3\,d^5}$$

input
```
integrate(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/20*(9*(e^4*x^6 + 3*d*e^3*x^4 + 3*d^2*e^2*x^2 + d^3*e)*sqrt(e/d)*elliptic
_e(arcsin(x*sqrt(e/d)), -1) + ((5*d*e^3 - 9*e^4)*x^6 + 3*(5*d^2*e^2 - 9*d*
e^3)*x^4 + 5*d^4 - 9*d^3*e + 3*(5*d^3*e - 9*d^2*e^2)*x^2)*sqrt(e/d)*ellipt
ic_f(arcsin(x*sqrt(e/d)), -1) + (9*e^3*x^5 + 22*d*e^2*x^3 + 15*d^2*e*x)*sq
rt(-e^2*x^4 + d^2))/(d^4*e^4*x^6 + 3*d^5*e^3*x^4 + 3*d^6*e^2*x^2 + d^7*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^3\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-(-d+ex^2)(d+ex^2)}\,(d+ex^2)^3}\,dx$$

input
```
integrate(1/(e*x**2+d)**3/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x**2)*(d + e*x**2))*(d + e*x**2)**3), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^3\sqrt{d^2-e^2x^4}}\,dx=\int \frac{1}{\sqrt{-e^2x^4+d^2}(ex^2+d)^3}\,dx$$

input
```
integrate(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^3\sqrt{d^2-e^2x^4}}\,dx=\int \frac{1}{\sqrt{-e^2x^4+d^2}(ex^2+d)^3}\,dx$$

input
```
integrate(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^3\sqrt{d^2-e^2x^4}}\,dx=\int \frac{1}{\sqrt{d^2-e^2\,x^4}\,(e\,x^2+d)^3}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^3),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^3\sqrt{d^2-e^2x^4}}\,dx$$
$$=\int \frac{\sqrt{-e^2x^4+d^2}}{-e^5x^{10}-3d\,e^4x^8-2d^2e^3x^6+2d^3e^2x^4+3d^4e\,x^2+d^5}dx$$

input
```
int(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2*d*
*2*e**3*x**6 - 3*d*e**4*x**8 - e**5*x**10),x)
```

3.61 $\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{2x(d+ex^2)}{\sqrt{d^2-e^2x^4}} - \frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
2*x*(e*x^2+d)/(-e^2*x^4+d^2)^(1/2)-3*d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*Ellipti
cE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+2*d^(3/2)*(1-e^2*x^4/
d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{2dx - ex^3 - dx\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right) + 2ex^3\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hy}}{\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d + e*x^2)^3/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(2*d*x - e*x^3 - d*x*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[1/4, 1/2, 5
/4, (e^2*x^4)/d^2] + 2*e*x^3*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4
, 3/2, 7/4, (e^2*x^4)/d^2])/Sqrt[d^2 - e^2*x^4]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1396, 315, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^3}{\left(d^2-e^2x^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\left(ex^2+d\right)^{3/2}}{\left(d-ex^2\right)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}-\frac{\int \frac{de\left(3ex^2+d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{de}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}-\int \frac{3ex^2+d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(2d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx+\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx+\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d + e*x^2)^3/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((2*x*Sqrt[d + e*x^2])/Sqrt[d - e*x^2] -
(3*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]],
-1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (2*d^(3/2)*Sqrt[1 - (e^2*
x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*
x^2]*Sqrt[d + e*x^2])))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 6.33 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.25

method	result
elliptic	$-\frac{2\left(-e^{2} x^{2}-d e\right) x}{e \sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2} x^{2}-d e\right)}}-\frac{d \sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2} x^{4}+d^{2}}}+\frac{3 d \sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\mathrm{Elliptic}\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2} x^{4}+d^{2}}}$
default	$d^{3}\left(\frac{x}{2 d^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d^{2} \sqrt{\frac{e}{d}}\,\sqrt{-e^{2} x^{4}+d^{2}}}\right)+e^{3}\left(\frac{x^{3}}{2 e^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}+\frac{3 d \sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}}{}\right.$

input
```
int((e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-e^2*x^2-d*e)*x/e/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)-d/(e/d)^(1/2)*(1-e*
x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2
),I)+3*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1
/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{3\,(dex^3-d^2x)\sqrt{-e^2}\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)-((3\,de+e^2)x^3-(3\,d^2+de)x)\sqrt{-}}{e^3x^3-de^2x}$$

input

```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output

```
(3*(d*e*x^3 - d^2*x)*sqrt(-e^2)*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x),
-1) - ((3*d*e + e^2)*x^3 - (3*d^2 + d*e)*x)*sqrt(-e^2)*sqrt(d/e)*elliptic_
f(arcsin(sqrt(d/e)/x), -1) + sqrt(-e^2*x^4 + d^2)*(e^2*x^2 - 3*d*e))/(e^3*
x^3 - d*e^2*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(d+ex^2)^3}{(-(-d+ex^2)\,(d+ex^2))^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x**2+d)**3/(-e**2*x**4+d**2)**(3/2),x)
```

output

```
Integral((d + e*x**2)**3/(-(-d + e*x**2)*(d + e*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(ex^2+d)^3}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output

```
integrate((e*x^2 + d)^3/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \frac{\left(d+ex^2\right)^3}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{\left(ex^2+d\right)^3}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3/(-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^3}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{\left(e\,x^2+d\right)^3}{\left(d^2-e^2\,x^4\right)^{3/2}}\,dx$$

input
```
int((d + e*x^2)^3/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
int((d + e*x^2)^3/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{\left(d+ex^2\right)^3}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{\sqrt{-e^2x^4+d^2}\,x+2\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^4}{e^3x^6-d\,e^2x^4-d^2e\,x^2+d^3}dx\right)d\,e^2-2\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^4}{e^3x^6-d\,e^2x^4-d^2e\,x^2+d^3}dx\right)e^3}{-e\,x^2+d}$$

input
```
int((e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
(sqrt(d**2 - e**2*x**4)*x + 2*int((sqrt(d**2 - e**2*x**4)*x**4)/(d**3 - d*
*2*e*x**2 - d*e**2*x**4 + e**3*x**6),x)*d*e**2 - 2*int((sqrt(d**2 - e**2*x
**4)*x**4)/(d**3 - d**2*e*x**2 - d*e**2*x**4 + e**3*x**6),x)*e**3*x**2)/(d
 - e*x**2)
```

3.62 $\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 148

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{x(d+ex^2)}{d\sqrt{d^2-e^2x^4}} - \frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
x*(e*x^2+d)/d/(-e^2*x^4+d^2)^(1/2)-d^(1/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE
(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+d^(1/2)*(1-e^2*x^4/d^2)
^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{3dx+2ex^3\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},\frac{e^2x^4}{d^2}\right)}{3d\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[(d + e*x^2)^2/(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(3*d*x + 2*e*x^3*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4, 3/2, 7/4,
(e^2*x^4)/d^2])/(3*d*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.58, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1396, 314, 27, 344, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\sqrt{ex^2+d}}{(d-ex^2)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 314}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{\int \frac{ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\int \frac{x^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 344}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\int \frac{x^2}{\sqrt{d^2-e^2x^4}}dx}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 836}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\int\frac{ex^2+d}{d\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow\ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int\frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow\ 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int\frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow\ 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int\frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow\ 1390$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{ex^2+d}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d + e*x^2)^2/(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d + e*x^2])/(d*Sqrt[d - e*x^2])
- (e*Sqrt[d^2 - e^2*x^4]*((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSi
n[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^2*x^4]) - (d^(3/2)*Sqrt
[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*S
qrt[d^2 - e^2*x^4])))/(d*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/Sqrt[d^2 - e^2
*x^4]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 344
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c
+ b*d*x^4)^FracPart[p])   Int[(e*x)^m*(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a
, b, c, d, e, m, p}, x] && EqQ[b*c + a*d, 0] &&  !IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

method	result
elliptic	$-\frac{\left(-e^{2} x^{2}-d e \right) x}{e d \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$
default	$d^{2} \left(\frac{x}{2 d^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d^{2} \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}\right)+e^{2} \left(\frac{x}{2 e^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \mathrm{E}}{2 e^{2} \sqrt{\frac{e}{d}}\, \sqrt{-e}}\right.$

input
```
int((e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2-d*e)/e/d*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/(e/d)^(1/2)*(1-e*
x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/
2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = -\frac{(ex^2-d)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1\right)-(ex^2-d)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1\right)+\sqrt{-e^2x^4+d^2}\,x}{dex^2-d^2}$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-((e*x^2 - d)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - (e*x^2 - d)*
sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*x)/(d
*e*x^2 - d^2)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(d+ex^2)^2}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**2/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral((d + e*x**2)**2/(-(-d + e*x**2)*(d + e*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(ex^2+d)^2}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{(ex^2+d)^2}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/(-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{(e\,x^2+d)^2}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input
```
int((d + e*x^2)^2/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
int((d + e*x^2)^2/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^2x^4-2de\,x^2+d^2}dx$$

input
```
int((e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**2 - 2*d*e*x**2 + e**2*x**4),x)
```

3.63 $\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 153

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx = \frac{x\left(d+ex^2\right)}{2d^2\sqrt{d^2-e^2x^4}} - \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
1/2*x*(e*x^2+d)/d^2/(-e^2*x^4+d^2)^(1/2)-1/2*(1-e^2*x^4/d^2)^(1/2)*Ellipti
cE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+(1-e^2*x^4/d^
2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx = \frac{3dx+3dx\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)+2ex^3\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hyperge}}{6d^2\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[(d + e*x^2)/(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(3*d*x + 3*d*x*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (e
^2*x^4)/d^2] + 2*e*x^3*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4, 3/2,
 7/4, (e^2*x^4)/d^2])/(6*d^2*Sqrt[d^2 - e^2*x^4])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{3/2}}dx$$

input

```
Int[(d + e*x^2)/(d^2 - e^2*x^4)^(3/2),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.25

method	result
elliptic	$-\frac{\left(-e^{2} x^{2}-d e \right) x}{2 d^{2} e \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{Elliptic}\right.}{2 d \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$
default	$d \left(\frac{x}{2 d^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d^{2} \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}\right)+e \left(\frac{x^{3}}{2 d^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\mathrm{Elli}\right.}{2 d}\right.$

input
```
int((e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2-d*e)/d^2*x/e/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/2/d/(e/d)^(
1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*
(e/d)^(1/2),I)+1/2/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2
*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.74

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx =$$
$$-\frac{\sqrt{-e^{2} x^{4} + d^{2}} e x + {\left(e^{2} x^{2} - d e\right)} \sqrt{\frac{e}{d}} E(\arcsin\left(x \sqrt{\frac{e}{d}}\right)\,|\,-1) - {\left({\left(d e + e^{2}\right)} x^{2} - d^{2} - d e\right)} \sqrt{\frac{e}{d}} F(\arcsin\left(x \sqrt{\frac{e}{d}}\right)}{2 \, {\left(d^{2} e^{2} x^{2} - d^{3} e\right)}}$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output

```
-1/2*(sqrt(-e^2*x^4 + d^2)*e*x + (e^2*x^2 - d*e)*sqrt(e/d)*elliptic_e(arcs
in(x*sqrt(e/d)), -1) - ((d*e + e^2)*x^2 - d^2 - d*e)*sqrt(e/d)*elliptic_f(
arcsin(x*sqrt(e/d)), -1))/(d^2*e^2*x^2 - d^3*e)
```

Sympy [A] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.54

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^2\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^3\Gamma\left(\frac{7}{4}\right)}$$

input

```
integrate((e*x**2+d)/(-e**2*x**4+d**2)**(3/2),x)
```

output

```
x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*d**2*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), e**2*x**4
*exp_polar(2*I*pi)/d**2)/(4*d**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{ex^2+d}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output

```
integrate((e*x^2 + d)/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{ex^2+d}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output

```
integrate((e*x^2 + d)/(-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{e\,x^2+d}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input

```
int((d + e*x^2)/(d^2 - e^2*x^4)^(3/2),x)
```

output

```
int((d + e*x^2)/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^3x^6-d\,e^2x^4-d^2e\,x^2+d^3}dx$$

input

```
int((e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**3 - d**2*e*x**2 - d*e**2*x**4 + e**3*x**6),
x)
```

3.64 $\int \frac{1}{(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{1}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{x}{2d^2\sqrt{d^2-e^2x^4}} + \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{2d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
1/2*x/d^2/(-e^2*x^4+d^2)^(1/2)+1/2*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)
*x/d^(1/2),I)/d^(3/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.86 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{1}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{x+x\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)}{2d^2\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(-3/2),x]
```

output
```
(x + x*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (e^2*x^4)/
d^2])/(2*d^2*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = $\{749, 765, 762\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d^2-e^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{749}$$

$$\frac{\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{2d^2}+\frac{x}{2d^2\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{2d^2\sqrt{d^2-e^2x^4}}+\frac{x}{2d^2\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{2d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{x}{2d^2\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d^2 - e^2*x^4)^(-3/2),x]
```

output
```
x/(2*d^2*Sqrt[d^2 - e^2*x^4]) + (Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[
(Sqrt[e]*x)/Sqrt[d]], -1])/(2*d^(3/2)*Sqrt[e]*Sqrt[d^2 - e^2*x^4])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{x}{2d^{2}\sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right)e^{2}}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{2d^{2}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	90
elliptic	$\frac{x}{2d^{2}\sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right)e^{2}}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{2d^{2}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	90

input
```
int(1/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d^2*x/(-(x^4-d^2/e^2)*e^2)^(1/2)+1/2/d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)
*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{3/2}}\,dx = -\frac{\sqrt{-e^2x^4+d^2}ex-\left(e^2x^4-d^2\right)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\middle|-1\right)}{2\left(d^2e^3x^4-d^4e\right)}$$

input
```
integrate(1/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-e^2*x^4 + d^2)*e*x - (e^2*x^4 - d^2)*sqrt(e/d)*elliptic_f(arcs
in(x*sqrt(e/d)), -1))/(d^2*e^3*x^4 - d^4*e)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{3/2}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^3\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*d**3*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 17.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{x\left(1-\frac{e^2\,x^4}{d^2}\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ \frac{e^2\,x^4}{d^2}\right)}{\left(d^2-e^2\,x^4\right)^{3/2}}$$

input
```
int(1/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
(x*(1 - (e^2*x^4)/d^2)^(3/2)*hypergeom([1/4, 3/2], 5/4, (e^2*x^4)/d^2))/(d
^2 - e^2*x^4)^(3/2)
```

Reduce [F]

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^4x^8-2d^2e^2x^4+d^4}dx$$

input
```
int(1/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**4 - 2*d**2*e**2*x**4 + e**4*x**8),x)
```

3.65 $\int \frac{1}{(d+ex^2)(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 192

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)^{3/2}}\,dx=\frac{x(5d-3ex^2)}{12d^4\sqrt{d^2-e^2x^4}}+\frac{x}{6d^2(d+ex^2)\sqrt{d^2-e^2x^4}}+\frac{\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{4d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{6d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
1/12*x*(-3*e*x^2+5*d)/d^4/(-e^2*x^4+d^2)^(1/2)+1/6*x/d^2/(e*x^2+d)/(-e^2*x
^4+d^2)^(1/2)+1/4*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(
5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+1/6*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1
/2)*x/d^(1/2),I)/d^(5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.48 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)^{3/2}}\,dx=\frac{\sqrt{-\frac{e}{d}}x(7d^2+2dex^2-3e^2x^4)-3id(d+ex^2)\sqrt{1-\frac{e^2x^4}{d^2}}E(i\text{arcsinh}(\sqrt{-}}{12d^4\sqrt{-\frac{e}{d}}(d+ex}$$

input
```
Integrate[1/((d + e*x^2)*(d^2 - e^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[-(e/d)]*x*(7*d^2 + 2*d*e*x^2 - 3*e^2*x^4) - (3*I)*d*(d + e*x^2)*Sqrt
[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - (2*I)*d*(d
+ e*x^2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1])
/(12*d^4*Sqrt[-(e/d)]*(d + e*x^2)*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.52, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1396, 316, 27, 402, 27, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)^{3/2}}dx$$

$$\Big\downarrow 1396$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{3/2}(ex^2+d)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 316$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(3ex^2+d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{2d^2e}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{3ex^2+d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{-\frac{\int -\frac{2de\left(ex^2+4d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}-\frac{x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{ex^2+4d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{3d}-\frac{x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{de\left(3ex^2+5d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{3d}-\frac{x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\int \frac{de\left(3ex^2+5d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}-\frac{x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\int \frac{3ex^2+5d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}-\frac{x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{2d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}-\frac{x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}-\frac{x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}-\frac{x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}-\frac{x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+e}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{2d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}-\frac{x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}-\frac{x\sqrt{d-ex^2}}{3d\left(d+ex^2\right)^{3/2}}}{2d^2}+\frac{}{2d}\right.}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/((d + e*x^2)*(d^2 - e^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(2*d^2*Sqrt[d - e*x^2]*(d + e*x^2)^(3/
2)) + (-1/3*(x*Sqrt[d - e*x^2])/(d*(d + e*x^2)^(3/2)) + ((3*x*Sqrt[d - e*x
^2])/(2*d*Sqrt[d + e*x^2]) + ((3*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE
[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2
]) + (2*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[
d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/(2*d))/(3*d))/(2*d^2)
))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.41

method	result
default	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{12d^{3}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{3\left(-e^{2}x^{2}+de\right)x}{8e\,d^{4}\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}-\frac{\left(-e^{2}x^{2}-de\right)x}{8d^{4}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{5\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{12d^{3}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{12d^{3}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{3\left(-e^{2}x^{2}+de\right)x}{8e\,d^{4}\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}-\frac{\left(-e^{2}x^{2}-de\right)x}{8d^{4}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{5\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{12d^{3}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$

input
```
int(1/(e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/d^3/e^2*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+3/8*(-e^2*x^2+d*e)/e/d^4*x
/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)-1/8*(-e^2*x^2-d*e)/d^4*x/e/((x^2-d/e)*(-
e^2*x^2-d*e))^(1/2)+5/12/d^3/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/
2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-1/4/d^3/(e/d)^(1/2)*(1-
e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(
1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.17

$$\int \frac{1}{(d+ex^2)\,(d^2-e^2x^4)^{3/2}}\,dx = \frac{3\,(e^4x^6+de^3x^4-d^2e^2x^2-d^3e)\sqrt{\frac{e}{d}}E(\arcsin\big(x\sqrt{\frac{e}{d}}\big)\,|-1)+((5\,de^3-3\,e}{}$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/12*(3*(e^4*x^6 + d*e^3*x^4 - d^2*e^2*x^2 - d^3*e)*sqrt(e/d)*elliptic_e(a
rcsin(x*sqrt(e/d)), -1) + ((5*d*e^3 - 3*e^4)*x^6 + (5*d^2*e^2 - 3*d*e^3)*x
^4 - 5*d^4 + 3*d^3*e - (5*d^3*e - 3*d^2*e^2)*x^2)*sqrt(e/d)*elliptic_f(arc
sin(x*sqrt(e/d)), -1) + (3*e^3*x^5 - 2*d*e^2*x^3 - 7*d^2*e*x)*sqrt(-e^2*x^
4 + d^2))/(d^4*e^4*x^6 + d^5*e^3*x^4 - d^6*e^2*x^2 - d^7*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\,(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}\left(d+ex^2\right)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral(1/((-(-d + e*x**2)*(d + e*x**2))**(3/2)*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\left(d^2-e^2x^4\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\left(d^2-e^2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}(ex^2+d)}\,dx$$

input

```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output

```
integrate(1/((-e^2*x^4 + d^2)^(3/2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\left(d^2-e^2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(d^2-e^2\,x^4\right)^{3/2}\,\left(e\,x^2+d\right)}\,dx$$

input

```
int(1/((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)),x)
```

output

```
int(1/((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)\left(d^2-e^2x^4\right)^{3/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e^5x^{10}+d\,e^4x^8-2d^2e^3x^6-2d^3e^2x^4+d^4e\,x^2+d^5}dx$$

input

```
int(1/(e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**5 + d**4*e*x**2 - 2*d**3*e**2*x**4 - 2*d**2
*e**3*x**6 + d*e**4*x**8 + e**5*x**10),x)
```

3.66 $\int \frac{1}{\left(d+ex^2\right)^2\left(d^2-e^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 225

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2\,(d^2-e^2x^4)^{3/2}}\,dx &= \frac{x(20d-21ex^2)}{60d^5\sqrt{d^2-e^2x^4}}\\ &+\frac{x}{10d^2\,(d+ex^2)^2\,\sqrt{d^2-e^2x^4}}+\frac{7x}{30d^3\,(d+ex^2)\,\sqrt{d^2-e^2x^4}}\\ &+\frac{7\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{20d^{7/2}\sqrt{e}\sqrt{d^2-e^2x^4}}-\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{60d^{7/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/60*x*(-21*e*x^2+20*d)/d^5/(-e^2*x^4+d^2)^(1/2)+1/10*x/d^2/(e*x^2+d)^2/(-
e^2*x^4+d^2)^(1/2)+7/30*x/d^3/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2)+7/20*(1-e^2*x
^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(7/2)/e^(1/2)/(-e^2*x^4+d^2
)^(1/2)-1/60*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(7/2)/
e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.70 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)^{3/2}}\,dx = \frac{\sqrt{-\frac{e}{d}}x(40d^3+33d^2ex^2-22de^2x^4-21e^3x^6)-21id(d+ex^2)^2\sqrt{1-\frac{e^2x}{d^2}}}{60d^5\sqrt{-}}$$

input

```
Integrate[1/((d + e*x^2)^2*(d^2 - e^2*x^4)^(3/2)),x]
```

output

```
(Sqrt[-(e/d)]*x*(40*d^3 + 33*d^2*e*x^2 - 22*d*e^2*x^4 - 21*e^3*x^6) - (21*
I)*d*(d + e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)
]*x], -1] + I*d*(d + e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[
Sqrt[-(e/d)]*x], -1])/(60*d^5*Sqrt[-(e/d)]*(d + e*x^2)^2*Sqrt[d^2 - e^2*x^
4])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.45, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {1396, 316, 27, 402, 27, 402, 25, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{3/2}(ex^2+d)^{7/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(5ex^2+d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{2d^2e}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{5ex^2+d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{-\frac{\int -\frac{2de\left(6ex^2+7d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d^2e}-\frac{2x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{6ex^2+7d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{5d}-\frac{2x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{de\left(41d-ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}}{5d}-\frac{2x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\int \frac{de\left(41d-ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}+\frac{x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}-\frac{2x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\int \frac{41d-ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d}+\frac{x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}-\frac{2x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\int -\frac{2de\left(21ex^2+20d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{6d}+\frac{x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}-\frac{2x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{\int \frac{21ex^2+20d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}-\frac{2x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{21\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}-\frac{2x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{21\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}-\frac{2x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{5/}}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{\frac{21d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}-\frac{2x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{2d^2}+\frac{}{2d^2\sqrt{d-e}}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}-\frac{2x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{2d^2}+\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \textcolor{blue}{765}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}-\frac{2x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{2d^2}+\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \textcolor{blue}{762}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d}+\frac{x\sqrt{d-ex^2}}{6d\left(d+ex^2\right)^{3/2}}}{5d}-\right.}{2d^2}}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[1/((d + e*x^2)^2*(d^2 - e^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(2*d^2*Sqrt[d - e*x^2]*(d + e*x^2)^(5/
2)) + ((-2*x*Sqrt[d - e*x^2])/(5*d*(d + e*x^2)^(5/2)) + ((x*Sqrt[d - e*x^2
])/(6*d*(d + e*x^2)^(3/2)) + ((21*x*Sqrt[d - e*x^2])/(d*Sqrt[d + e*x^2]) +
 ((21*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]
], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) - (d^(3/2)*Sqrt[1 - (e^2
*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e
*x^2]*Sqrt[d + e*x^2]))/d)/(6*d))/(5*d))/(2*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.35

method	result
default	$\frac{x\sqrt{-e^2x^4+d^2}}{20d^3e^3\left(x^2+\frac{d}{e}\right)^3}+\frac{17x\sqrt{-e^2x^4+d^2}}{120e^2d^4\left(x^2+\frac{d}{e}\right)^2}+\frac{33\left(-e^2x^2+de\right)x}{80e\,d^5\sqrt{\left(x^2+\frac{d}{e}\right)\left(-e^2x^2+de\right)}}-\frac{\left(-e^2x^2-de\right)x}{16e\,d^5\sqrt{\left(x^2-\frac{d}{e}\right)\left(-e^2x^2-de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+}}{3d^4\sqrt{}}$
elliptic	$\frac{x\sqrt{-e^2x^4+d^2}}{20d^3e^3\left(x^2+\frac{d}{e}\right)^3}+\frac{17x\sqrt{-e^2x^4+d^2}}{120e^2d^4\left(x^2+\frac{d}{e}\right)^2}+\frac{33\left(-e^2x^2+de\right)x}{80e\,d^5\sqrt{\left(x^2+\frac{d}{e}\right)\left(-e^2x^2+de\right)}}-\frac{\left(-e^2x^2-de\right)x}{16e\,d^5\sqrt{\left(x^2-\frac{d}{e}\right)\left(-e^2x^2-de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+}}{3d^4\sqrt{}}$

input
```
int(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20/d^3*x/e^3*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^3+17/120/e^2/d^4*x*(-e^2*x^4
+d^2)^(1/2)/(x^2+d/e)^2+33/80*(-e^2*x^2+d*e)/e/d^5*x/((x^2+d/e)*(-e^2*x^2+
d*e))^(1/2)-1/16*(-e^2*x^2-d*e)/e/d^5*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1
/3/d^4/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2
)*EllipticF(x*(e/d)^(1/2),I)-7/20/d^4/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x
^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(
e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(d+ex^2\right)^2\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{21\left(e^5x^8+2\,de^4x^6-2\,d^3e^2x^2-d^4e\right)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)+((20\,de}{}$$

input
```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/60*(21*(e^5*x^8 + 2*d*e^4*x^6 - 2*d^3*e^2*x^2 - d^4*e)*sqrt(e/d)*ellipti
c_e(arcsin(x*sqrt(e/d)), -1) + ((20*d*e^4 - 21*e^5)*x^8 + 2*(20*d^2*e^3 -
21*d*e^4)*x^6 - 20*d^5 + 21*d^4*e - 2*(20*d^4*e - 21*d^3*e^2)*x^2)*sqrt(e/
d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + (21*e^4*x^7 + 22*d*e^3*x^5 - 33*d
^2*e^2*x^3 - 40*d^3*e*x)*sqrt(-e^2*x^4 + d^2))/(d^5*e^5*x^8 + 2*d^6*e^4*x^
6 - 2*d^8*e^2*x^2 - d^9*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^2 (d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}\left(d+ex^2\right)^2}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral(1/((-(-d + e*x**2)*(d + e*x**2))**(3/2)*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^2 (d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{(-e^2x^4+d^2)^{\frac{3}{2}}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(3/2)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^2 (d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{(-e^2x^4+d^2)^{\frac{3}{2}}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(3/2)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(d^2-e^2\,x^4\right)^{3/2}\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^6x^{12}+2d\,e^5x^{10}-d^2e^4x^8-4d^3e^3x^6-d^4e^2x^4+2d^5e\,x^2+d^6}dx$$

input
```
int(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**6 + 2*d**5*e*x**2 - d**4*e**2*x**4 - 4*d**3
*e**3*x**6 - d**2*e**4*x**8 + 2*d*e**5*x**10 + e**6*x**12),x)
```

3.67 $\int \frac{1}{(d+ex^2)^3(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 258

$$\int \frac{1}{(d+ex^2)^3(d^2-e^2x^4)^{3/2}}\,dx = \frac{3x(5d-7ex^2)}{56d^6\sqrt{d^2-e^2x^4}} + \frac{x}{14d^2(d+ex^2)^3\sqrt{d^2-e^2x^4}} + \frac{x}{7d^3(d+ex^2)^2\sqrt{d^2-e^2x^4}} + \frac{x}{4d^4(d+ex^2)\sqrt{d^2-e^2x^4}} + \frac{3\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{8d^{9/2}\sqrt{e}\sqrt{d^2-e^2x^4}} - \frac{3\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{28d^{9/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
3/56*x*(-7*e*x^2+5*d)/d^6/(-e^2*x^4+d^2)^(1/2)+1/14*x/d^2/(e*x^2+d)^3/(-e^
2*x^4+d^2)^(1/2)+1/7*x/d^3/(e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2)+1/4*x/d^4/(e*x
^2+d)/(-e^2*x^4+d^2)^(1/2)+3/8*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d
^(1/2),I)/d^(9/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-3/28*(1-e^2*x^4/d^2)^(1/2)*
EllipticF(e^(1/2)*x/d^(1/2),I)/d^(9/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.85 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.59

$$\int \frac{1}{(d+ex^2)^3 (d^2-e^2x^4)^{3/2}}\,dx = \frac{\frac{41d^4x+60d^3ex^3-4d^2e^2x^5-48de^3x^7-21e^4x^9}{(d+ex^2)^3} + \frac{3ie\sqrt{1-\frac{e^2x^4}{d^2}}(7E(i\operatorname{arcsinh}(\sqrt{-\frac{e}{d}}x)|-1)-2}{(-\frac{e}{d})^{3/2}}}{56d^6\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[1/((d + e*x^2)^3*(d^2 - e^2*x^4)^(3/2)),x]
```

output
```
((41*d^4*x + 60*d^3*e*x^3 - 4*d^2*e^2*x^5 - 48*d*e^3*x^7 - 21*e^4*x^9)/(d
+ e*x^2)^3 + ((3*I)*e*Sqrt[1 - (e^2*x^4)/d^2]*(7*EllipticE[I*ArcSinh[Sqrt[
-(e/d)]*x], -1] - 2*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1]))/(-(e/d))^(3
/2))/(56*d^6*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.41, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {1396, 316, 27, 402, 27, 402, 27, 402, 27, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^3 (d^2-e^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{3/2}(ex^2+d)^{9/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(7ex^2+d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{9/2}}dx}{2d^2e}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{7ex^2+d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{9/2}}dx}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{-\frac{\int -\frac{10de\left(3ex^2+2d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{14d^2e}-\frac{3x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5\int \frac{3ex^2+2d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{7d}-\frac{3x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5\left(-\frac{\int -\frac{3de\left(ex^2+7d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d^2e}-\frac{x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}\right)}{7d}-\frac{3x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5\left(\frac{3\int \frac{ex^2+7d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d}-\frac{x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}\right)}{7d}-\frac{3x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5\left(\frac{3\left(\frac{x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{6de\left(6d-ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}\right)}{10d}-\frac{x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}\right)}{7d}-\frac{3x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5\left(\frac{3\left(\frac{\int \frac{6d-ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{d}+\frac{x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}-\frac{x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}\right)}{7d}-\frac{3x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5\left(\frac{3\left(\frac{\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{de\left(7ex^2+5d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{d}+\frac{x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}-\frac{x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}\right)}{7d}-\frac{3x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5\left(\frac{3\left(\frac{\frac{\int \frac{de\left(7ex^2+5d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}-\frac{x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}\right)}{7d}-\frac{3x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5\left(\frac{3\left(\frac{\frac{\int \frac{7ex^2+5d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}-\frac{x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}\right)}{7d}-\frac{3x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}}{2d^2}+\frac{x}{2d^2\sqrt{d-ex^2}(d+ex^2)}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5\left(\frac{3\left(\frac{\frac{7\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-2d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}-\frac{x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}\right)}{7d}-\frac{3x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}}{2d^2}+\frac{}{2d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5\left(\frac{3\left(\frac{\frac{7\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{2d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}-\frac{x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}\right)}{7d}-\frac{3x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}}{2d^2}+\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{329}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5\left(\frac{3\left(\frac{\frac{\frac{7d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}-\frac{x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}\right)}{7d}-\frac{3x}{7d\left(d\right.}}{2d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{\frac{5\bigg(\frac{3\left(\frac{\frac{\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}-\frac{x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/}}}{7d}}{2d^2}}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{\frac{5\bigg(\frac{3\left(\frac{\frac{\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}-\frac{x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/}}}{7d}}{2d^2}}{\sqrt{d^2-e^2x^4}}$$

$\Big\downarrow$ 762

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{5\left(\frac{3\left(\frac{\frac{\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}\right.}{7d}}{2d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/((d + e*x^2)^3*(d^2 - e^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(2*d^2*Sqrt[d - e*x^2]*(d + e*x^2)^(7/
2)) + ((-3*x*Sqrt[d - e*x^2])/(7*d*(d + e*x^2)^(7/2)) + (5*(-1/10*(x*Sqrt[
d - e*x^2])/(d*(d + e*x^2)^(5/2)) + (3*((x*Sqrt[d - e*x^2])/(d*(d + e*x^2)
^(3/2)) + ((7*x*Sqrt[d - e*x^2])/(2*d*Sqrt[d + e*x^2]) + ((7*d^(3/2)*Sqrt[
1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sq
rt[d - e*x^2]*Sqrt[d + e*x^2]) - (2*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*Ellipt
icF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*
x^2]))/(2*d))/d))/(10*d)))/(7*d))/(2*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.31

method	result
default	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{28d^{3}e^{4}\left(x^{2}+\frac{d}{e}\right)^{4}}+\frac{5x\sqrt{-e^{2}x^{4}+d^{2}}}{56d^{4}e^{3}\left(x^{2}+\frac{d}{e}\right)^{3}}+\frac{19x\sqrt{-e^{2}x^{4}+d^{2}}}{112d^{5}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{13(-e^{2}x^{2}+de)x}{32e\,d^{6}\sqrt{\left(x^{2}+\frac{d}{e}\right)(-e^{2}x^{2}+de)}}-\frac{(-e^{2}x^{2}-de)x}{32e\,d^{6}\sqrt{\left(x^{2}-\frac{d}{e}\right)(-e^{2}x^{2}-de}}$
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{28d^{3}e^{4}\left(x^{2}+\frac{d}{e}\right)^{4}}+\frac{5x\sqrt{-e^{2}x^{4}+d^{2}}}{56d^{4}e^{3}\left(x^{2}+\frac{d}{e}\right)^{3}}+\frac{19x\sqrt{-e^{2}x^{4}+d^{2}}}{112d^{5}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{13(-e^{2}x^{2}+de)x}{32e\,d^{6}\sqrt{\left(x^{2}+\frac{d}{e}\right)(-e^{2}x^{2}+de)}}-\frac{(-e^{2}x^{2}-de)x}{32e\,d^{6}\sqrt{\left(x^{2}-\frac{d}{e}\right)(-e^{2}x^{2}-de}}$

input
```
int(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/28/d^3*x/e^4*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^4+5/56/d^4/e^3*x*(-e^2*x^4+d
^2)^(1/2)/(x^2+d/e)^3+19/112/d^5/e^2*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+13
/32*(-e^2*x^2+d*e)/e/d^6*x/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)-1/32*(-e^2*x^2
-d*e)/e/d^6*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+15/56/d^5/(e/d)^(1/2)*(1-e*
x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2
),I)-3/8/d^5/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2
)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(d+ex^2\right)^3\left(d^2-e^2x^4\right)^{3/2}}\,dx = \frac{21\left(e^6x^{10}+3\,de^5x^8+2\,d^2e^4x^6-2\,d^3e^3x^4-3\,d^4e^2x^2-d^5e\right)\sqrt{\frac{e}{d}}E(\arcsin}{}$$

input
```
integrate(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/56*(21*(e^6*x^10 + 3*d*e^5*x^8 + 2*d^2*e^4*x^6 - 2*d^3*e^3*x^4 - 3*d^4*e
^2*x^2 - d^5*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) + 3*((5*d*e^
5 - 7*e^6)*x^10 + 3*(5*d^2*e^4 - 7*d*e^5)*x^8 + 2*(5*d^3*e^3 - 7*d^2*e^4)*
x^6 - 5*d^6 + 7*d^5*e - 2*(5*d^4*e^2 - 7*d^3*e^3)*x^4 - 3*(5*d^5*e - 7*d^4
*e^2)*x^2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + (21*e^5*x^9 + 4
8*d*e^4*x^7 + 4*d^2*e^3*x^5 - 60*d^3*e^2*x^3 - 41*d^4*e*x)*sqrt(-e^2*x^4 +
 d^2))/(d^6*e^6*x^10 + 3*d^7*e^5*x^8 + 2*d^8*e^4*x^6 - 2*d^9*e^3*x^4 - 3*d
^10*e^2*x^2 - d^11*e)
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^3\left(d^2-e^2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}\left(d+ex^2\right)^3}\,dx$$

input
```
integrate(1/(e*x**2+d)**3/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral(1/((-(-d + e*x**2)*(d + e*x**2))**(3/2)*(d + e*x**2)**3), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^3\,(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{(-e^2x^4+d^2)^{\frac{3}{2}}(ex^2+d)^3}\,dx$$

input

```
integrate(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output

```
integrate(1/((-e^2*x^4 + d^2)^(3/2)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^3\,(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{(-e^2x^4+d^2)^{\frac{3}{2}}(ex^2+d)^3}\,dx$$

input

```
integrate(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output

```
integrate(1/((-e^2*x^4 + d^2)^(3/2)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^3\,(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{(d^2-e^2\,x^4)^{3/2}\,(e\,x^2+d)^3}\,dx$$

input

```
int(1/((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^3),x)
```

output

```
int(1/((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^3\,(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e^7x^{14}+3d\,e^6x^{12}+d^2e^5x^{10}-5d^3e^4x^8-5d^4e^3x^6+d^5e^2x^4+3d^6e\,x^2+}$$

input

```
int(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**7 + 3*d**6*e*x**2 + d**5*e**2*x**4 - 5*d**4
*e**3*x**6 - 5*d**3*e**4*x**8 + d**2*e**5*x**10 + 3*d*e**6*x**12 + e**7*x*
*14),x)
```

3.68 $\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 185

$$\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{2x(d+ex^2)}{3\left(d^2-e^2x^4\right)^{3/2}} + \frac{x(d+3ex^2)}{6d^2\sqrt{d^2-e^2x^4}} - \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{2\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
2/3*x*(e*x^2+d)/(-e^2*x^4+d^2)^(3/2)+1/6*x*(3*e*x^2+d)/d^2/(-e^2*x^4+d^2)^
(1/2)-1/2*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(
1/2)/(-e^2*x^4+d^2)^(1/2)+2/3*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^
(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{dx(5d^2+2dex^2-e^2x^4)+dx(d^2-e^2x^4)\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\right.}{6d^2\left(d^2-e^2x^4\right)^{3}}$$

input
```
Integrate[(d + e*x^2)^3/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(d*x*(5*d^2 + 2*d*e*x^2 - e^2*x^4) + d*x*(d^2 - e^2*x^4)*Sqrt[1 - (e^2*x^4
)/d^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (e^2*x^4)/d^2] + 4*e*x^3*(d^2 - e^
2*x^4)*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4, 5/2, 7/4, (e^2*x^4)/
d^2])/(6*d^2*(d^2 - e^2*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.37, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1396, 314, 25, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^3}{\left(d^2-e^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\sqrt{ex^2+d}}{\left(d-ex^2\right)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}-\frac{\int -\frac{ex^2+2d}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{3d}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{ex^2+2d}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{3d}+\frac{x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{de\left(d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{d-3ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{4d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{4d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{4d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{4d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d + e*x^2)^3/(d^2 - e^2*x^4)^(5/2),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d + e*x^2])/(3*d*(d - e*x^2)^(3/
2)) + ((3*x*Sqrt[d + e*x^2])/(2*d*Sqrt[d - e*x^2]) + ((-3*d^(3/2)*Sqrt[1 -
 (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[
d - e*x^2]*Sqrt[d + e*x^2]) + (4*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF
[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2
]))/(2*d))/(3*d)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 6.50 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3d\,e^{2}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{\left(-e^{2}x^{2}-de\right)x}{2d^{2}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\right.\right.}{2d\sqrt{\frac{e}{d}}\,\sqrt{-e}}$
default	$d^{3}\left(\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{6d^{2}e^{4}\left(x^{4}-\frac{d^{2}}{e^{2}}\right)^{2}}+\frac{5x}{12d^{4}\sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right)e^{2}}}+\frac{5\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{12d^{4}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right)+e^{3}\left(\frac{x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{6e^{6}\left(x^{4}-\frac{d^{2}}{e^{2}}\right)^{2}}-\right.$

input
```
int((e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*x/e^2*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-1/2*(-e^2*x^2-d*e)/d^2*x/e/((
x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/6/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^
2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+1/2/d/(e/d)^(1/
2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(
e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx =$$
$$-\frac{3\,(e^3x^4-2\,de^2x^2+d^2e)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1\right)-((de^2+3\,e^3)x^4+d^3+3\,d^2e-2\,(d^2e+3\,de^2)x^2)\sqrt{}}{6\,(d^2e^3x^4-2\,d^3e^2x^2+d^4e)}$$

input
```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(3*(e^3*x^4 - 2*d*e^2*x^2 + d^2*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt
(e/d)), -1) - ((d*e^2 + 3*e^3)*x^4 + d^3 + 3*d^2*e - 2*(d^2*e + 3*d*e^2)*x
^2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*(
3*e^2*x^3 - 5*d*e*x))/(d^2*e^3*x^4 - 2*d^3*e^2*x^2 + d^4*e)
```

Sympy [F]

$$\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(d+ex^2)^3}{(-(-d+ex^2)\,(d+ex^2))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**3/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
Integral((d + e*x**2)**3/(-(-d + e*x**2)*(d + e*x**2))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(ex^2+d)^3}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^3/(-e^2*x^4 + d^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(ex^2+d)^3}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3/(-e^2*x^4 + d^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(e\,x^2+d)^3}{(d^2-e^2\,x^4)^{5/2}}\,dx$$

input
```
int((d + e*x^2)^3/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
int((d + e*x^2)^3/(d^2 - e^2*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^3x^6+3d\,e^2x^4-3d^2e\,x^2+d^3}dx$$

input

```
int((e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**3 - 3*d**2*e*x**2 + 3*d*e**2*x**4 - e**3*x*
*6),x)
```

3.69 $\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 190

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{x(d+ex^2)}{3d(d^2-e^2x^4)^{3/2}} + \frac{x(2d+3ex^2)}{6d^3\sqrt{d^2-e^2x^4}} - \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{5\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{6d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
1/3*x*(e*x^2+d)/d/(-e^2*x^4+d^2)^(3/2)+1/6*x*(3*e*x^2+2*d)/d^3/(-e^2*x^4+d
^2)^(1/2)-1/2*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(3/2)
/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+5/6*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*
x/d^(1/2),I)/d^(3/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{2d^3x - de^2x^5 + dx(d^2-e^2x^4)\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right) + 2ex}{3d^3(d^2-e^2x^4)^{3/2}}$$

input
```
Integrate[(d + e*x^2)^2/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(2*d^3*x - d*e^2*x^5 + d*x*(d^2 - e^2*x^4)*Sqrt[1 - (e^2*x^4)/d^2]*Hyperge
ometric2F1[1/4, 1/2, 5/4, (e^2*x^4)/d^2] + 2*e*x^3*(d^2 - e^2*x^4)*Sqrt[1
- (e^2*x^4)/d^2]*Hypergeometric2F1[3/4, 5/2, 7/4, (e^2*x^4)/d^2])/(3*d^3*(
d^2 - e^2*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1396, 316, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{5/2}\sqrt{ex^2+d}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e(ex^2+5d)}{(d-ex^2)^{3/2}\sqrt{ex^2+d}}dx}{6d^2e}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{ex^2+5d}{(d-ex^2)^{3/2}\sqrt{ex^2+d}}dx}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{2de\left(2d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{2d-3ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{5d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{5d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{5d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{5d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{5d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d + e*x^2)^2/(d^2 - e^2*x^4)^(5/2),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d + e*x^2])/(6*d^2*(d - e*x^2)^(
3/2)) + ((3*x*Sqrt[d + e*x^2])/(d*Sqrt[d - e*x^2]) + ((-3*d^(3/2)*Sqrt[1 -
 (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[
d - e*x^2]*Sqrt[d + e*x^2]) + (5*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF
[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2
]))/d)/(6*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{x\sqrt{-e^2x^4+d^2}}{6d^2e^2\left(x^2-\frac{d}{e}\right)^2}-\frac{\left(-e^2x^2-de\right)x}{2e\,d^3\sqrt{\left(x^2-\frac{d}{e}\right)\left(-e^2x^2-de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}+$
default	$d^2\left(\frac{x\sqrt{-e^2x^4+d^2}}{6d^2e^4\left(x^4-\frac{d^2}{e^2}\right)^2}+\frac{5x}{12d^4\sqrt{-\left(x^4-\frac{d^2}{e^2}\right)e^2}}+\frac{5\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{12d^4\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)+e^2\left(\frac{x\sqrt{-e^2x^4+d^2}}{6e^6\left(x^4-\frac{d^2}{e^2}\right)^2}-\right.$

input
```
int((e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/d^2*x/e^2*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-1/2*(-e^2*x^2-d*e)/e/d^3*x/
((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/3/d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+
e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+1/2/d^2/(e/
d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(Elliptic
cF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx =$$

$$-\frac{3\,(e^3x^4-2\,de^2x^2+d^2e)\sqrt{\frac{e}{d}}E\big(\arcsin\big(x\sqrt{\frac{e}{d}}\big)\,|\,-1\big)-((2\,de^2+3\,e^3)x^4+2\,d^3+3\,d^2e-2\,(2\,d^2e+3\,de^2)x}{6\,(d^3e^3x^4-2\,d^4e^2x^2+d^5e)}$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(3*(e^3*x^4 - 2*d*e^2*x^2 + d^2*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt
(e/d)), -1) - ((2*d*e^2 + 3*e^3)*x^4 + 2*d^3 + 3*d^2*e - 2*(2*d^2*e + 3*d*
e^2)*x^2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 +
d^2)*(3*e^2*x^3 - 4*d*e*x))/(d^3*e^3*x^4 - 2*d^4*e^2*x^2 + d^5*e)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(d+ex^2)^2}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**2/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
Integral((d + e*x**2)**2/(-(-d + e*x**2)*(d + e*x**2))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(ex^2+d)^2}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/(-e^2*x^4 + d^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(ex^2+d)^2}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/(-e^2*x^4 + d^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(e\,x^2+d)^2}{(d^2-e^2\,x^4)^{5/2}}\,dx$$

input
```
int((d + e*x^2)^2/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
int((d + e*x^2)^2/(d^2 - e^2*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{\left(d+ex^2\right)^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^4x^8+2d\,e^3x^6-2d^3e\,x^2+d^4}dx$$

input

```
int((e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**4 - 2*d**3*e*x**2 + 2*d*e**3*x**6 - e**4*x*
*8),x)
```

3.70 $\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 190

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{x\left(d+ex^2\right)}{6d^2\left(d^2-e^2x^4\right)^{3/2}}+\frac{x\left(5d+3ex^2\right)}{12d^4\sqrt{d^2-e^2x^4}}-\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{4d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{2\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
1/6*x*(e*x^2+d)/d^2/(-e^2*x^4+d^2)^(3/2)+1/12*x*(3*e*x^2+5*d)/d^4/(-e^2*x^
4+d^2)^(1/2)-1/4*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(5
/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+2/3*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/
2)*x/d^(1/2),I)/d^(5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.77

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{7d^3x-5de^2x^5+5dx\left(d^2-e^2x^4\right)\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)+4e}{12d^4\left(d^2-e^2x^4\right)^{3/2}}$$

input
```
Integrate[(d + e*x^2)/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(7*d^3*x - 5*d*e^2*x^5 + 5*d*x*(d^2 - e^2*x^4)*Sqrt[1 - (e^2*x^4)/d^2]*Hyp
ergeometric2F1[1/4, 1/2, 5/4, (e^2*x^4)/d^2] + 4*e*x^3*(d^2 - e^2*x^4)*Sqr
t[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4, 5/2, 7/4, (e^2*x^4)/d^2])/(12*
d^4*(d^2 - e^2*x^4)^(3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 1571}$$
$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{5/2}}dx$$

input
```
Int[(d + e*x^2)/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.43

method	result
elliptic	$\frac{(-e^2x^2+de)x}{8e\,d^4\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}+\frac{x\sqrt{-e^2x^4+d^2}}{12e^2d^3\left(x^2-\frac{d}{e}\right)^2}-\frac{3(-e^2x^2-de)x}{8d^4e\sqrt{\left(x^2-\frac{d}{e}\right)(-e^2x^2-de)}}+\frac{5\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{12d^3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$
default	$d\left(\frac{x\sqrt{-e^2x^4+d^2}}{6d^2e^4\left(x^4-\frac{d^2}{e^2}\right)^2}+\frac{5x}{12d^4\sqrt{-\left(x^4-\frac{d^2}{e^2}\right)e^2}}+\frac{5\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{12d^4\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)+e\left(\frac{x^3\sqrt{-e^2x^4+d^2}}{6d^2e^4\left(x^4-\frac{d^2}{e^2}\right)^2}+\frac{}{4d^4}\right.$

input
```
int((e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-e^2*x^2+d*e)/e/d^4*x/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/12/e^2/d^3*x
*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-3/8*(-e^2*x^2-d*e)/d^4*x/e/((x^2-d/e)*(-
e^2*x^2-d*e))^(1/2)+5/12/d^3/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/
2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+1/4/d^3/(e/d)^(1/2)*(1-
e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(
1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.19

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{5/2}}\,dx =$$
$$-\frac{3\,(e^4x^6-de^3x^4-d^2e^2x^2+d^3e)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)-((5\,de^3+3\,e^4)x^6-(5\,d^2e^2+3\,de^3)x^4+5\,d}{12\,(d^4e^4x^6-d^5e^3x^4}$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*(e^4*x^6 - d*e^3*x^4 - d^2*e^2*x^2 + d^3*e)*sqrt(e/d)*elliptic_e(
arcsin(x*sqrt(e/d)), -1) - ((5*d*e^3 + 3*e^4)*x^6 - (5*d^2*e^2 + 3*d*e^3)*
x^4 + 5*d^4 + 3*d^3*e - (5*d^3*e + 3*d^2*e^2)*x^2)*sqrt(e/d)*elliptic_f(ar
csin(x*sqrt(e/d)), -1) + (3*e^3*x^5 + 2*d*e^2*x^3 - 7*d^2*e*x)*sqrt(-e^2*x
^4 + d^2))/(d^4*e^4*x^6 - d^5*e^3*x^4 - d^6*e^2*x^2 + d^7*e)
```

Sympy [A] (verification not implemented)

Time = 6.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.44

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^4\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^5\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*d**4*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), e**2*x**4
*exp_polar(2*I*pi)/d**2)/(4*d**5*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\int \frac{ex^2+d}{\left(-e^2x^4+d^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(-e^2*x^4 + d^2)^(5/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{ex^2+d}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(-e^2*x^4 + d^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{e\,x^2+d}{(d^2-e^2\,x^4)^{5/2}}\,dx$$

input
```
int((d + e*x^2)/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
int((d + e*x^2)/(d^2 - e^2*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^5x^{10}+d\,e^4x^8+2d^2e^3x^6-2d^3e^2x^4-d^4e\,x^2+d^5}dx$$

input
```
int((e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**5 - d**4*e*x**2 - 2*d**3*e**2*x**4 + 2*d**2
*e**3*x**6 + d*e**4*x**8 - e**5*x**10),x)
```

3.71 $\int \frac{1}{\left(d^2-e^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 111

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{x}{6d^2\left(d^2-e^2x^4\right)^{3/2}}+\frac{5x}{12d^4\sqrt{d^2-e^2x^4}}+\frac{5\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{12d^{7/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
1/6*x/d^2/(-e^2*x^4+d^2)^(3/2)+5/12*x/d^4/(-e^2*x^4+d^2)^(1/2)+5/12*(1-e^2
*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(7/2)/e^(1/2)/(-e^2*x^4+d
^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{7d^2x-5e^2x^5+5x(d^2-e^2x^4)\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)}{12d^4\left(d^2-e^2x^4\right)^{3/2}}$$

input

```
Integrate[(d^2 - e^2*x^4)^(-5/2),x]
```

output

```
(7*d^2*x - 5*e^2*x^5 + 5*x*(d^2 - e^2*x^4)*Sqrt[1 - (e^2*x^4)/d^2]*Hyperge
ometric2F1[1/4, 1/2, 5/4, (e^2*x^4)/d^2])/(12*d^4*(d^2 - e^2*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {749, 749, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \; 749$$

$$\frac{5\int \frac{1}{\left(d^2-e^2x^4\right)^{3/2}}dx}{6d^2}+\frac{x}{6d^2\left(d^2-e^2x^4\right)^{3/2}}$$

$$\Big\downarrow \; 749$$

$$\frac{5\left(\frac{\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{2d^2}+\frac{x}{2d^2\sqrt{d^2-e^2x^4}}\right)}{6d^2}+\frac{x}{6d^2\left(d^2-e^2x^4\right)^{3/2}}$$

$$\Big\downarrow \; 765$$

$$\frac{5\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{2d^2\sqrt{d^2-e^2x^4}}+\frac{x}{2d^2\sqrt{d^2-e^2x^4}}\right)}{6d^2}+\frac{x}{6d^2\left(d^2-e^2x^4\right)^{3/2}}$$

$$\Big\downarrow \; 762$$

$$\frac{5\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{2d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{x}{2d^2\sqrt{d^2-e^2x^4}}\right)}{6d^2}+\frac{x}{6d^2\left(d^2-e^2x^4\right)^{3/2}}$$

input
```
Int[(d^2 - e^2*x^4)^(-5/2),x]
```

output
```
x/(6*d^2*(d^2 - e^2*x^4)^(3/2)) + (5*(x/(2*d^2*Sqrt[d^2 - e^2*x^4]) + (Sqr
t[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(2*d^(3/2
)*Sqrt[e]*Sqrt[d^2 - e^2*x^4])))/(6*d^2)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{x\sqrt{-e^{2} x^{4}+d^{2}}}{6 d^{2} e^{4} \left(x^{4}-\frac{d^{2}}{e^{2}}\right)^{2}}+\frac{5 x}{12 d^{4} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}+\frac{5 \sqrt{1-\frac{e\,x^{2}}{d}}\, \sqrt{1+\frac{e\,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{12 d^{4} \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	127
elliptic	$\frac{x\sqrt{-e^{2} x^{4}+d^{2}}}{6 d^{2} e^{4} \left(x^{4}-\frac{d^{2}}{e^{2}}\right)^{2}}+\frac{5 x}{12 d^{4} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}+\frac{5 \sqrt{1-\frac{e\,x^{2}}{d}}\, \sqrt{1+\frac{e\,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{12 d^{4} \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	127

input
```
int(1/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/d^2*x/e^4*(-e^2*x^4+d^2)^(1/2)/(x^4-d^2/e^2)^2+5/12/d^4*x/(-(x^4-d^2/e
^2)*e^2)^(1/2)+5/12/d^4/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-
e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{5\left(e^4x^8-2\,d^2e^2x^4+d^4\right)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1\right)-\left(5\,e^3x^5-7\,d^2ex\right)\sqrt{-e^2x^4+d^2}}{12\left(d^4e^5x^8-2\,d^6e^3x^4+d^8e\right)}$$

input
```
integrate(1/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(5*(e^4*x^8 - 2*d^2*e^2*x^4 + d^4)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt
(e/d)), -1) - (5*e^3*x^5 - 7*d^2*e*x)*sqrt(-e^2*x^4 + d^2))/(d^4*e^5*x^8 -
 2*d^6*e^3*x^4 + d^8*e)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.35

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^5\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*d**5*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(d^2-e^2x^4)^{5/2}}\,dx=\int \frac{1}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(d^2-e^2x^4)^{5/2}}\,dx=\int \frac{1}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(-5/2), x)
```

Mupad [B] (verification not implemented)

Time = 17.93 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.41

$$\int \frac{1}{(d^2-e^2x^4)^{5/2}}\,dx=\frac{x\left(1-\frac{e^2x^4}{d^2}\right)^{5/2}{}_2F_1\left(\frac{1}{4},\frac{5}{2};\ \frac{5}{4};\ \frac{e^2x^4}{d^2}\right)}{(d^2-e^2x^4)^{5/2}}$$

input
```
int(1/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
(x*(1 - (e^2*x^4)/d^2)^(5/2)*hypergeom([1/4, 5/2], 5/4, (e^2*x^4)/d^2))/(d
^2 - e^2*x^4)^(5/2)
```

Reduce [F]

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{-e^6x^{12}+3d^2e^4x^8-3d^4e^2x^4+d^6}dx$$

input
```
int(1/(-e^2*x^4+d^2)^(5/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**6 - 3*d**4*e**2*x**4 + 3*d**2*e**4*x**8 - e
**6*x**12),x)
```

3.72 $\int \frac{1}{\left(d+ex^2\right)\left(d^2-e^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 226

$$\begin{aligned}\int \frac{1}{\left(d+ex^2\right)\left(d^2-e^2x^4\right)^{5/2}}\,dx &= \frac{x(9d-7ex^2)}{60d^4\left(d^2-e^2x^4\right)^{3/2}} \\ &+ \frac{x}{10d^2\left(d+ex^2\right)\left(d^2-e^2x^4\right)^{3/2}} + \frac{x(15d-7ex^2)}{40d^6\sqrt{d^2-e^2x^4}} \\ &+ \frac{7\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{40d^{9/2}\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{5d^{9/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/60*x*(-7*e*x^2+9*d)/d^4/(-e^2*x^4+d^2)^(3/2)+1/10*x/d^2/(e*x^2+d)/(-e^2*
x^4+d^2)^(3/2)+1/40*x*(-7*e*x^2+15*d)/d^6/(-e^2*x^4+d^2)^(1/2)+7/40*(1-e^2
*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(9/2)/e^(1/2)/(-e^2*x^4+d
^2)^(1/2)+1/5*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(9/2)
/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.76 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.72

$$\int \frac{1}{(d+ex^2)\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{\frac{x(75d^4+28d^3ex^2-80d^2e^2x^4-24de^3x^6+21e^4x^8)}{(d-ex^2)(d+ex^2)^2}+\frac{3ie\sqrt{1-\frac{e^2x^4}{d^2}}\big(7E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)+}{\left(-\frac{e}{d}\right)^{3/}}}{120d^6\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[1/((d + e*x^2)*(d^2 - e^2*x^4)^(5/2)),x]
```

output
```
((x*(75*d^4 + 28*d^3*e*x^2 - 80*d^2*e^2*x^4 - 24*d*e^3*x^6 + 21*e^4*x^8))/
((d - e*x^2)*(d + e*x^2)^2) + ((3*I)*e*Sqrt[1 - (e^2*x^4)/d^2]*(7*Elliptic
E[I*ArcSinh[Sqrt[-(e/d)]*x], -1] + 8*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x],
-1]))/(-(e/d))^(3/2))/(120*d^6*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.60, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {1396, 316, 27, 402, 27, 402, 27, 402, 27, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\left(d^2-e^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{5/2}(ex^2+d)^{7/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(7ex^2+5d\right)}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{7/2}}dx}{6d^2e}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{7ex^2+5d}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{7/2}}dx}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int -\frac{2de\left(d-30ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{2d^2e}+\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\int \frac{d-30ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\frac{31x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{\int \frac{3de\left(31ex^2+7d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d^2e}}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\frac{31x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{3\int\frac{31ex^2+7d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d}}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\frac{31x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{3\left(-\frac{\int-\frac{6de\left(4ex^2+11d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}-\frac{4x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{5}}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\frac{31x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{3\left(\frac{\int\frac{4ex^2+11d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{d}-\frac{4x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\frac{31x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{3\left(\frac{\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{de\left(7ex^2+15d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{d}-\frac{4x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\frac{31x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{3\left(\frac{\frac{\int \frac{de\left(7ex^2+15d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}-\frac{4x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\frac{31x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{3\left(\frac{\frac{\int\frac{7ex^2+15d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}-\frac{4x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\frac{31x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{3\left(\frac{\frac{8d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+7\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}-\frac{4x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}}{d}}{6d^2}+\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\frac{31x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{3\left(\frac{\frac{\frac{8d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+7\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}-\frac{4x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{329}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\frac{31x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{3\left(\frac{\frac{\frac{7d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{8d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}-\frac{4x}{d\left(d\right.}\right.}{10d}}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\frac{31x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{3\left(\frac{\frac{\frac{8d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{7x\sqrt{d}}{2d\sqrt{d}}}{d}\right.}{10d}}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\frac{31x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{3\left(\frac{\frac{\frac{8d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{7x\sqrt{d-}}{2d\sqrt{d+}}}{d}\right.}{10d}}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{6x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{5/2}}-\frac{\frac{31x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}-\frac{3\left(\frac{\frac{\frac{8d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right.\right.}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}}{d}\right.}{10d}}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[1/((d + e*x^2)*(d^2 - e^2*x^4)^(5/2)),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(6*d^2*(d - e*x^2)^(3/2)*(d + e*x^2)^(
5/2)) + ((6*x)/(d*Sqrt[d - e*x^2]*(d + e*x^2)^(5/2)) - ((31*x*Sqrt[d - e*x
^2])/(10*d*(d + e*x^2)^(5/2)) - (3*((-4*x*Sqrt[d - e*x^2])/(d*(d + e*x^2)^
(3/2)) + ((7*x*Sqrt[d - e*x^2])/(2*d*Sqrt[d + e*x^2]) + ((7*d^(3/2)*Sqrt[1
 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqr
t[d - e*x^2]*Sqrt[d + e*x^2]) + (8*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*Ellipti
cF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x
^2]))/(2*d))/d))/(10*d))/d)/(6*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.50

method	result
default	$\frac{x\sqrt{-e^2x^4+d^2}}{40d^4e^3\left(x^2+\frac{d}{e}\right)^3}+\frac{11x\sqrt{-e^2x^4+d^2}}{120d^5e^2\left(x^2+\frac{d}{e}\right)^2}+\frac{53(-e^2x^2+de)x}{160e\,d^6\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}+\frac{x\sqrt{-e^2x^4+d^2}}{48e^2d^5\left(x^2-\frac{d}{e}\right)^2}-\frac{5(-e^2x^2-de)x}{32e\,d^6\sqrt{\left(x^2-\frac{d}{e}\right)(-e^2x^2-d}}$
elliptic	$\frac{x\sqrt{-e^2x^4+d^2}}{40d^4e^3\left(x^2+\frac{d}{e}\right)^3}+\frac{11x\sqrt{-e^2x^4+d^2}}{120d^5e^2\left(x^2+\frac{d}{e}\right)^2}+\frac{53(-e^2x^2+de)x}{160e\,d^6\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}+\frac{x\sqrt{-e^2x^4+d^2}}{48e^2d^5\left(x^2-\frac{d}{e}\right)^2}-\frac{5(-e^2x^2-de)x}{32e\,d^6\sqrt{\left(x^2-\frac{d}{e}\right)(-e^2x^2-d}}$

input

```
int(1/(e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/40/d^4/e^3*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^3+11/120/d^5/e^2*x*(-e^2*x^4
+d^2)^(1/2)/(x^2+d/e)^2+53/160*(-e^2*x^2+d*e)/e/d^6*x/((x^2+d/e)*(-e^2*x^2
+d*e))^(1/2)+1/48/e^2/d^5*x*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-5/32*(-e^2*x^
2-d*e)/e/d^6*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+3/8/d^5/(e/d)^(1/2)*(1-e*x
^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2)
,I)-7/40/d^5/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2
)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.46

$$\int \frac{1}{(d+ex^2)\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{21\left(e^6x^{10}+de^5x^8-2\,d^2e^4x^6-2\,d^3e^3x^4+d^4e^2x^2+d^5e\right)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\right.}\right.}{}$$

input

```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/120*(21*(e^6*x^10 + d*e^5*x^8 - 2*d^2*e^4*x^6 - 2*d^3*e^3*x^4 + d^4*e^2*
x^2 + d^5*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) + 3*((15*d*e^5
- 7*e^6)*x^10 + (15*d^2*e^4 - 7*d*e^5)*x^8 - 2*(15*d^3*e^3 - 7*d^2*e^4)*x^
6 + 15*d^6 - 7*d^5*e - 2*(15*d^4*e^2 - 7*d^3*e^3)*x^4 + (15*d^5*e - 7*d^4*
e^2)*x^2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + (21*e^5*x^9 - 24
*d*e^4*x^7 - 80*d^2*e^3*x^5 + 28*d^3*e^2*x^3 + 75*d^4*e*x)*sqrt(-e^2*x^4 +
 d^2))/(d^6*e^6*x^10 + d^7*e^5*x^8 - 2*d^8*e^4*x^6 - 2*d^9*e^3*x^4 + d^10*
e^2*x^2 + d^11*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{(-(-d+ex^2)(d+ex^2))^{\frac{5}{2}}(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
Integral(1/((-(-d + e*x**2)*(d + e*x**2))**(5/2)*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\,(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{(-e^2x^4+d^2)^{\frac{5}{2}}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(5/2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\,(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{(d^2-e^2\,x^4)^{5/2}\,(e\,x^2+d)}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d + e*x^2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)\,(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^7x^{14}-d\,e^6x^{12}+3d^2e^5x^{10}+3d^3e^4x^8-3d^4e^3x^6-3d^5e^2x^4+d^6e\,x^2}$$

input
```
int(1/(e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**7 + d**6*e*x**2 - 3*d**5*e**2*x**4 - 3*d**4
*e**3*x**6 + 3*d**3*e**4*x**8 + 3*d**2*e**5*x**10 - d*e**6*x**12 - e**7*x*
*14),x)
```

3.73 $\int \frac{1}{\left(d+ex^2\right)^2\left(d^2-e^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 259

$$\begin{aligned}\int \frac{1}{\left(d+ex^2\right)^2\left(d^2-e^2x^4\right)^{5/2}}\,dx &= \frac{x(54d-77ex^2)}{420d^5\left(d^2-e^2x^4\right)^{3/2}} \\ &+ \frac{x}{14d^2\left(d+ex^2\right)^2\left(d^2-e^2x^4\right)^{3/2}} + \frac{11x}{70d^3\left(d+ex^2\right)\left(d^2-e^2x^4\right)^{3/2}} \\ &+ \frac{x(90d-77ex^2)}{280d^7\sqrt{d^2-e^2x^4}} + \frac{11\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{40d^{11/2}\sqrt{e}\sqrt{d^2-e^2x^4}} \\ &+ \frac{13\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{280d^{11/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/420*x*(-77*e*x^2+54*d)/d^5/(-e^2*x^4+d^2)^(3/2)+1/14*x/d^2/(e*x^2+d)^2/(
-e^2*x^4+d^2)^(3/2)+11/70*x/d^3/(e*x^2+d)/(-e^2*x^4+d^2)^(3/2)+1/280*x*(-7
7*e*x^2+90*d)/d^7/(-e^2*x^4+d^2)^(1/2)+11/40*(1-e^2*x^4/d^2)^(1/2)*Ellipti
cE(e^(1/2)*x/d^(1/2),I)/d^(11/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+13/280*(1-e^
2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(11/2)/e^(1/2)/(-e^2*x^4
+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.95 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.67

$$\int \frac{1}{(d+ex^2)^2 (d^2-e^2x^4)^{5/2}}\,dx = \frac{\frac{x(570d^5+503d^4ex^2-662d^3e^2x^4-694d^2e^3x^6+192de^4x^8+231e^5x^{10})}{(d-ex^2)(d+ex^2)^3} + \frac{3ie\sqrt{1-\frac{e^2x^4}{d^2}}\,(77E(i\mathrm{arcs}}{}}{840d^7\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[1/((d + e*x^2)^2*(d^2 - e^2*x^4)^(5/2)),x]
```

output

```
((x*(570*d^5 + 503*d^4*e*x^2 - 662*d^3*e^2*x^4 - 694*d^2*e^3*x^6 + 192*d*e
^4*x^8 + 231*e^5*x^10))/((d - e*x^2)*(d + e*x^2)^3) + ((3*I)*e*Sqrt[1 - (e
^2*x^4)/d^2]*(77*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] + 13*EllipticF[I
*ArcSinh[Sqrt[-(e/d)]*x], -1]))/(-(e/d))^(3/2))/(840*d^7*Sqrt[d^2 - e^2*x^
4])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.54, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.731$, Rules used = {1396, 316, 27, 402, 27, 402, 27, 402, 27, 402, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 (d^2-e^2x^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{5/2}(ex^2+d)^{9/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(9ex^2+5d\right)}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{9/2}}dx}{6d^2e}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{9ex^2+5d}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{9/2}}dx}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int -\frac{2de\left(2d-49ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{9/2}}dx}{2d^2e}+\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\int \frac{2d-49ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{9/2}}dx}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{\int \frac{de\left(255ex^2+23d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{14d^2e}}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{\int \frac{255ex^2+23d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{14d}}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{-\frac{\int -\frac{6de\left(116ex^2+77d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d^2e}-\frac{116x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{14d}}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{\frac{3\int \frac{116ex^2+77d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{5d}-\frac{116x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{14d}}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{7/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{\frac{3\left(-\frac{\int -\frac{3de\left(13ex^2+167d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}-\frac{13x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}\right)}{5d}-\frac{116x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{14d}}{d}}{6d^2}+\frac{}{6d^2(d-}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{\frac{3\left(\frac{\int \frac{13ex^2+167d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{2d}-\frac{13x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}\right)}{5d}-\frac{116x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{14d}}{d}}{6d^2}+\frac{}{6d^2(d-ex}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{\frac{3\left(\frac{\frac{77x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\int -\frac{2de\left(77ex^2+90d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{2d}-\frac{13x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}\right)}{5d}-\frac{116x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{14d}}{d}}{6d^2}+\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{\frac{3\left(\frac{\frac{\int \frac{77ex^2+90d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{77x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{2d}-\frac{13x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}\right)}{5d}-\frac{116x\sqrt{d-ex^2}}{5d\left(d+ex^2\right)^{5/2}}}{14d}}{d}}{6d^2}+\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{\frac{3\left(\frac{\frac{13d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+77\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}+\frac{77x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{2d}-\frac{13x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}\right)}{5d}-}{14d}}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 289}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{\frac{3\left(\frac{\frac{\frac{13d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+77\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}+\frac{77x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{2d}-\frac{13x\sqrt{d-ex}}{2d\left(d+ex^2\right)^{3}}\right.}{5d}}{14d}}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 329}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{\frac{3\left(\frac{\frac{\frac{77d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{13d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}}{2d}+\frac{77x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\right.}{5d}}{14d}}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\downarrow 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{\frac{3\left(\frac{\frac{\frac{13d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}}{2d}+\frac{77x}{d}\right.}{5d}}{14d}}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\downarrow 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{\frac{3\left(\frac{\frac{\frac{13d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{77x\sqrt{}}{d\sqrt{}}}{2d}\right.}{5d}}{14d}}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}-\frac{\frac{51x\sqrt{d-ex^2}}{14d\left(d+ex^2\right)^{7/2}}-\frac{\frac{3\left(\frac{\frac{\frac{13d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\right.\right.}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}}{2d}\right.}{5d}}{14d}}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/((d + e*x^2)^2*(d^2 - e^2*x^4)^(5/2)),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(6*d^2*(d - e*x^2)^(3/2)*(d + e*x^2)^(
7/2)) + ((7*x)/(d*Sqrt[d - e*x^2]*(d + e*x^2)^(7/2)) - ((51*x*Sqrt[d - e*x
^2])/(14*d*(d + e*x^2)^(7/2)) - ((-116*x*Sqrt[d - e*x^2])/(5*d*(d + e*x^2)
^(5/2)) + (3*((-13*x*Sqrt[d - e*x^2])/(2*d*(d + e*x^2)^(3/2)) + ((77*x*Sqr
t[d - e*x^2])/(d*Sqrt[d + e*x^2]) + ((77*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*E
llipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d
 + e*x^2]) + (13*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]
*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/d)/(2*d)))/(
5*d))/(14*d))/d)/(6*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.44

method	result
default	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{56d^{4}e^{4}\left(x^{2}+\frac{d}{e}\right)^{4}}+\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{35d^{5}e^{3}\left(x^{2}+\frac{d}{e}\right)^{3}}+\frac{439x\sqrt{-e^{2}x^{4}+d^{2}}}{3360d^{6}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{59\left(-e^{2}x^{2}+de\right)x}{160e\,d^{7}\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}+\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{96d^{6}e^{2}\left(x^{2}-\frac{d}{e}\right)^{2}}-$
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{56d^{4}e^{4}\left(x^{2}+\frac{d}{e}\right)^{4}}+\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{35d^{5}e^{3}\left(x^{2}+\frac{d}{e}\right)^{3}}+\frac{439x\sqrt{-e^{2}x^{4}+d^{2}}}{3360d^{6}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{59\left(-e^{2}x^{2}+de\right)x}{160e\,d^{7}\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}+\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{96d^{6}e^{2}\left(x^{2}-\frac{d}{e}\right)^{2}}-$

input

```
int(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/56/d^4*x/e^4*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^4+2/35/d^5/e^3*x*(-e^2*x^4+d
^2)^(1/2)/(x^2+d/e)^3+439/3360/d^6/e^2*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+
59/160*(-e^2*x^2+d*e)/e/d^7*x/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/96/d^6/e^
2*x*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-3/32*(-e^2*x^2-d*e)/e/d^7*x/((x^2-d/e
)*(-e^2*x^2-d*e))^(1/2)+9/28/d^6/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)
^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-11/40/d^6/(e/d)^(1/
2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(
e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.51

$$\int \frac{1}{\left(d+ex^{2}\right)^{2}\left(d^{2}-e^{2}x^{4}\right)^{5/2}}\,dx = \frac{231\left(e^{7}x^{12}+2\,de^{6}x^{10}-d^{2}e^{5}x^{8}-4\,d^{3}e^{4}x^{6}-d^{4}e^{3}x^{4}+2\,d^{5}e^{2}x^{2}+d^{6}e\right)}{}$$

input

```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/840*(231*(e^7*x^12 + 2*d*e^6*x^10 - d^2*e^5*x^8 - 4*d^3*e^4*x^6 - d^4*e^
3*x^4 + 2*d^5*e^2*x^2 + d^6*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -
1) + 3*((90*d*e^6 - 77*e^7)*x^12 + 2*(90*d^2*e^5 - 77*d*e^6)*x^10 - (90*d^
3*e^4 - 77*d^2*e^5)*x^8 + 90*d^7 - 77*d^6*e - 4*(90*d^4*e^3 - 77*d^3*e^4)*
x^6 - (90*d^5*e^2 - 77*d^4*e^3)*x^4 + 2*(90*d^6*e - 77*d^5*e^2)*x^2)*sqrt(
e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + (231*e^6*x^11 + 192*d*e^5*x^9 -
 694*d^2*e^4*x^7 - 662*d^3*e^3*x^5 + 503*d^4*e^2*x^3 + 570*d^5*e*x)*sqrt(-
e^2*x^4 + d^2))/(d^7*e^7*x^12 + 2*d^8*e^6*x^10 - d^9*e^5*x^8 - 4*d^10*e^4*
x^6 - d^11*e^3*x^4 + 2*d^12*e^2*x^2 + d^13*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^2\,(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{5}{2}}\left(d+ex^2\right)^2}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
Integral(1/((-(-d + e*x**2)*(d + e*x**2))**(5/2)*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^2\,(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^4+d^2\right)^{\frac{5}{2}}\left(ex^2+d\right)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(5/2)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^2\,(d^2-e^2x^4)^{5/2}}\,dx=\int \frac{1}{(-e^2x^4+d^2)^{\frac{5}{2}}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(5/2)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\,(d^2-e^2x^4)^{5/2}}\,dx=\int \frac{1}{(d^2-e^2\,x^4)^{5/2}\,(e\,x^2+d)^2}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^2\,(d^2-e^2x^4)^{5/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{-e^8x^{16}-2d\,e^7x^{14}+2d^2e^6x^{12}+6d^3e^5x^{10}-6d^5e^3x^6-2d^6e^2x^4+2d^7e}$$

input
```
int(1/(e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**8 + 2*d**7*e*x**2 - 2*d**6*e**2*x**4 - 6*d*
*5*e**3*x**6 + 6*d**3*e**5*x**10 + 2*d**2*e**6*x**12 - 2*d*e**7*x**14 - e*
*8*x**16),x)
```

3.74 $\int \frac{1}{(d+ex^2)^3 (d^2-e^2x^4)^{5/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 292

$$\begin{aligned}\int \frac{1}{(d+ex^2)^3 (d^2-e^2x^4)^{5/2}} \, dx &= \frac{x(39d-77ex^2)}{360d^6 (d^2-e^2x^4)^{3/2}} \\ &+ \frac{x}{18d^2 (d+ex^2)^3 (d^2-e^2x^4)^{3/2}} + \frac{x}{9d^3 (d+ex^2)^2 (d^2-e^2x^4)^{3/2}} \\ &+ \frac{11x}{60d^4 (d+ex^2) (d^2-e^2x^4)^{3/2}} + \frac{x(65d-77ex^2)}{240d^8\sqrt{d^2-e^2x^4}} \\ &+ \frac{77\sqrt{1-\frac{e^2x^4}{d^2}}\, E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{240d^{13/2}\sqrt{e}\sqrt{d^2-e^2x^4}} - \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\, \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{20d^{13/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/360*x*(-77*e*x^2+39*d)/d^6/(-e^2*x^4+d^2)^(3/2)+1/18*x/d^2/(e*x^2+d)^3/(
-e^2*x^4+d^2)^(3/2)+1/9*x/d^3/(e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2)+11/60*x/d^4
/(e*x^2+d)/(-e^2*x^4+d^2)^(3/2)+1/240*x*(-77*e*x^2+65*d)/d^8/(-e^2*x^4+d^2
)^(1/2)+77/240*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(13/
2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-1/20*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/
2)*x/d^(1/2),I)/d^(13/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.03 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.63

$$\int \frac{1}{(d+ex^2)^3\,(d^2-e^2x^4)^{5/2}}\,dx = \frac{\frac{x(525d^6+778d^5ex^2-399d^4e^2x^4-1236d^3e^3x^6-277d^2e^4x^8+498de^5x^{10}+231e^6x^{12})}{(d-ex^2)(d+ex^2)^4}+\frac{3ie\sqrt{1-\frac{e^2x}{d^2}}}{}}{720d^8\sqrt{d^2-e^2x}}$$

input

```
Integrate[1/((d + e*x^2)^3*(d^2 - e^2*x^4)^(5/2)),x]
```

output

```
((x*(525*d^6 + 778*d^5*e*x^2 - 399*d^4*e^2*x^4 - 1236*d^3*e^3*x^6 - 277*d^
2*e^4*x^8 + 498*d*e^5*x^10 + 231*e^6*x^12))/((d - e*x^2)*(d + e*x^2)^4) +
((3*I)*e*Sqrt[1 - (e^2*x^4)/d^2]*(77*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x],
-1] - 12*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1]))/(-(e/d))^(3/2))/(720*d
^8*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.50, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {1396, 316, 27, 402, 27, 402, 27, 402, 27, 402, 27, 402, 27, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^3\,(d^2-e^2x^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{5/2}(ex^2+d)^{11/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(11ex^2+5d\right)}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{11/2}}dx}{6d^2e}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{9/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{11ex^2+5d}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{11/2}}dx}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{9/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int -\frac{6de\left(d-24ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{11/2}}dx}{2d^2e}+\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{9/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\int \frac{d-24ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{11/2}}dx}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{9/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{\int \frac{7de\left(25ex^2+d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{9/2}}dx}{18d^2e}\right)}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{9/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\int\frac{25ex^2+d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{9/2}}dx}{18d}\right)}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2)^{9/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(-\frac{\int-\frac{2de\left(60ex^2+19d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{14d^2e}-\frac{12x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}\right)}{18d}\right)}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{\int\frac{60ex^2+19d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{7/2}}dx}{7d}-\frac{12x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}\right)}{18d}\right)}{d}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}(d+ex^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{-\frac{\int-\frac{3de\left(41ex^2+77d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d^2e}-\frac{41x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}-\frac{12x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}\right)}{18d}\right)}{d}}{6d^2}+\frac{\;}{6}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{\frac{3\int\frac{41ex^2+77d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{10d}-\frac{41x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}-\frac{12x\sqrt{d-ex^2}}{7d\left(d+ex^2\right)^{7/2}}\right)}{18d}\right)}{d}}{6d^2}+\frac{}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{\frac{3\left(\frac{6x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{6de\left(71d-6ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}\right)}{10d}-\frac{41x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}-\frac{12x\sqrt{}}{7d\left(d+\right.}\right.}{18d}\right.}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{3\left(\frac{\int\frac{71d-6ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{d}+\frac{6x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}-\frac{41x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}\right)}{7d}-\frac{12x\sqrt{d}}{7d(d+e}}{18d}\right)}{d}}{6d^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$\downarrow$ 402

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{3\left(\frac{\frac{77x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{de\left(77ex^2+65d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}\,dx}{2d^2e}}{d}+\frac{6x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}-\frac{41x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5}}\right.}{7d}\right.}{18d}}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

↓ 25

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{3\left(\frac{\frac{\int\frac{de\left(77ex^2+65d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{d}+\frac{77x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}+\frac{6x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}-\frac{41x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}}{18d}}{d}}{6d^2}}{\sqrt{d^2-e^2x^4}}$$

$$\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{\frac{3\left(\frac{\frac{\int\frac{77ex^2+65d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{77x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{6x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{10d}-\frac{41x\sqrt{d-ex^2}}{10d\left(d+ex^2\right)^{5/2}}}{7d}\right.}{18d}\right.}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\downarrow 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{\frac{3\left(\frac{\frac{77\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-12d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{77x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{6x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/}}\right.}{10d}}{7d}\right.}{18d}\right.}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$\downarrow$ 289

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{\frac{3\left(\frac{\frac{77\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{12d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{77x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{6x\sqrt{d}}{d\left(d+e\right.}\right.}{10d}}{7d}\right.}{18d}\right.}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

↓ 329

$$\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{3\left(\frac{\frac{\frac{77d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{77x\sqrt{d-}}{2d\sqrt{d+}}}{d}\right.}{10d}}{7d}\right.}{18d}\right.}{d}}{6d^2}\right.$$

↓ 327

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{3\left(\frac{\frac{\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}}{d}\right.}{10d}}{7d}\right.}{18d}\right.}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

↓ 765

$$\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{\frac{3\left(\frac{\frac{\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}}{d}\right.}{10d}}{7d}\right.}{18d}\right.}{d}}{6d^2}\right.$$

↓ 762

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{8x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}-\frac{3\left(\frac{25x\sqrt{d-ex^2}}{18d\left(d+ex^2\right)^{9/2}}-\frac{7\left(\frac{3\left(\frac{\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\mathrm{EllipticF}\left(\mathrm{arc}\right.}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+e}}}{2d}}{d}\right.}{10d}\right.}{7d}\right.}{d}}{6d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[1/((d + e*x^2)^3*(d^2 - e^2*x^4)^(5/2)),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(6*d^2*(d - e*x^2)^(3/2)*(d + e*x^2)^(
9/2)) + ((8*x)/(d*Sqrt[d - e*x^2]*(d + e*x^2)^(9/2)) - (3*((25*x*Sqrt[d -
e*x^2])/(18*d*(d + e*x^2)^(9/2)) - (7*((-12*x*Sqrt[d - e*x^2])/(7*d*(d + e
*x^2)^(7/2)) + ((-41*x*Sqrt[d - e*x^2])/(10*d*(d + e*x^2)^(5/2)) + (3*((6*
x*Sqrt[d - e*x^2])/(d*(d + e*x^2)^(3/2)) + ((77*x*Sqrt[d - e*x^2])/(2*d*Sq
rt[d + e*x^2]) + ((77*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sq
rt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) - (12*d^
(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/
(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/(2*d))/d))/(10*d))/(7*d)))/(18*
d)))/d)/(6*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.39

method	result
default	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{72d^{4}e^{5}\left(x^{2}+\frac{d}{e}\right)^{5}}+\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{24e^{4}d^{5}\left(x^{2}+\frac{d}{e}\right)^{4}}+\frac{121x\sqrt{-e^{2}x^{4}+d^{2}}}{1440e^{3}d^{6}\left(x^{2}+\frac{d}{e}\right)^{3}}+\frac{37x\sqrt{-e^{2}x^{4}+d^{2}}}{240d^{7}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{721\left(-e^{2}x^{2}+de\right)x}{1920e\,d^{8}\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}+$
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{72d^{4}e^{5}\left(x^{2}+\frac{d}{e}\right)^{5}}+\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{24e^{4}d^{5}\left(x^{2}+\frac{d}{e}\right)^{4}}+\frac{121x\sqrt{-e^{2}x^{4}+d^{2}}}{1440e^{3}d^{6}\left(x^{2}+\frac{d}{e}\right)^{3}}+\frac{37x\sqrt{-e^{2}x^{4}+d^{2}}}{240d^{7}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{721\left(-e^{2}x^{2}+de\right)x}{1920e\,d^{8}\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}+$

input
```
int(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/72/d^4*x/e^5*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^5+1/24/e^4/d^5*x*(-e^2*x^4+d
^2)^(1/2)/(x^2+d/e)^4+121/1440/e^3/d^6*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^3+
37/240/d^7/e^2*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+721/1920*(-e^2*x^2+d*e)/
e/d^8*x/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/192/d^7/e^2*x*(-e^2*x^4+d^2)^(1
/2)/(x^2-d/e)^2-7/128*(-e^2*x^2-d*e)/e/d^8*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1
/2)+13/48/d^7/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^
2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-77/240/d^7/(e/d)^(1/2)*(1-e*x^2/d)^(1/
2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-Elli
pticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.51

$$\int \frac{1}{\left(d+ex^2\right)^3\left(d^2-e^2x^4\right)^{5/2}}\,dx = \frac{231\left(e^8x^{14}+3\,de^7x^{12}+d^2e^6x^{10}-5\,d^3e^5x^8-5\,d^4e^4x^6+d^5e^3x^4+3\,d^6e^2x\right.}{}$$

input
```
integrate(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/720*(231*(e^8*x^14 + 3*d*e^7*x^12 + d^2*e^6*x^10 - 5*d^3*e^5*x^8 - 5*d^4
*e^4*x^6 + d^5*e^3*x^4 + 3*d^6*e^2*x^2 + d^7*e)*sqrt(e/d)*elliptic_e(arcsi
n(x*sqrt(e/d)), -1) + 3*((65*d*e^7 - 77*e^8)*x^14 + 3*(65*d^2*e^6 - 77*d*e
^7)*x^12 + (65*d^3*e^5 - 77*d^2*e^6)*x^10 - 5*(65*d^4*e^4 - 77*d^3*e^5)*x^
8 + 65*d^8 - 77*d^7*e - 5*(65*d^5*e^3 - 77*d^4*e^4)*x^6 + (65*d^6*e^2 - 77
*d^5*e^3)*x^4 + 3*(65*d^7*e - 77*d^6*e^2)*x^2)*sqrt(e/d)*elliptic_f(arcsin
(x*sqrt(e/d)), -1) + (231*e^7*x^13 + 498*d*e^6*x^11 - 277*d^2*e^5*x^9 - 12
36*d^3*e^4*x^7 - 399*d^4*e^3*x^5 + 778*d^5*e^2*x^3 + 525*d^6*e*x)*sqrt(-e^
2*x^4 + d^2))/(d^8*e^8*x^14 + 3*d^9*e^7*x^12 + d^10*e^6*x^10 - 5*d^11*e^5*
x^8 - 5*d^12*e^4*x^6 + d^13*e^3*x^4 + 3*d^14*e^2*x^2 + d^15*e)
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^3\left(d^2-e^2x^4\right)^{5/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{5}{2}}\left(d+ex^2\right)^3}\,dx$$

input
```
integrate(1/(e*x**2+d)**3/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
Integral(1/((-(-d + e*x**2)*(d + e*x**2))**(5/2)*(d + e*x**2)**3), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^3\left(d^2-e^2x^4\right)^{5/2}}\,dx=\int \frac{1}{\left(-e^2x^4+d^2\right)^{\frac{5}{2}}\left(ex^2+d\right)^3}\,dx$$

input
```
integrate(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(5/2)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex^2\right)^3\left(d^2-e^2x^4\right)^{5/2}}\,dx=\int \frac{1}{\left(-e^2x^4+d^2\right)^{\frac{5}{2}}\left(ex^2+d\right)^3}\,dx$$

input
```
integrate(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(5/2)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^3\left(d^2-e^2x^4\right)^{5/2}}\,dx=\int \frac{1}{\left(d^2-e^2\,x^4\right)^{5/2}\left(e\,x^2+d\right)^3}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d + e*x^2)^3),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d + e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^3\,(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^9x^{18}-3d\,e^8x^{16}+8d^3e^6x^{12}+6d^4e^5x^{10}-6d^5e^4x^8-8d^6e^3x^6+3d^8e}$$

input

```
int(1/(e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**9 + 3*d**8*e*x**2 - 8*d**6*e**3*x**6 - 6*d*
*5*e**4*x**8 + 6*d**4*e**5*x**10 + 8*d**3*e**6*x**12 - 3*d*e**8*x**16 - e*
*9*x**18),x)
```

3.75 $\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 4

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx = E(\arcsin(x)|-1)$$

output

```
EllipticE(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 9.00

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx = x \,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right) + \frac{1}{3}x^3 \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^4\right)$$

input

```
Integrate[(1 + x^2)/Sqrt[1 - x^4],x]
```

output

```
x*Hypergeometric2F1[1/4, 1/2, 5/4, x^4] + (x^3*Hypergeometric2F1[1/2, 3/4,
 7/4, x^4])/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{1388, 327\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$E(\arcsin(x)|-1)$$

input

```
Int[(1 + x^2)/Sqrt[1 - x^4],x]
```

output

```
EllipticE[ArcSin[x], -1]
```

Defintions of rubi rules used

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.69 (sec) , antiderivative size = 27, normalized size of antiderivative = 6.75

method	result	size
meijerg	$\frac{x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],x^{4}\right)$	27
default	$\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\ \operatorname{EllipticF}(x,i)}{\sqrt{-x^{4}+1}}-\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{\sqrt{-x^{4}+1}}$	70
elliptic	$\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\ \operatorname{EllipticF}(x,i)}{\sqrt{-x^{4}+1}}-\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{\sqrt{-x^{4}+1}}$	70

input
```
int((x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*hypergeom([1/2,3/4],[7/4],x^4)+x*hypergeom([1/4,1/2],[5/4],x^4)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(3) = 6.

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 8.50

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx=\frac{-i\,xE\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right)+2i\,xF\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right)-\sqrt{-x^4+1}}{x}$$

input
```
integrate((x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(-I*x*elliptic_e(arcsin(1/x), -1) + 2*I*x*elliptic_f(arcsin(1/x), -1) - sq
rt(-x^4 + 1))/x
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(2) = 4$.

Time = 0.69 (sec) , antiderivative size = 61, normalized size of antiderivative = 15.25

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((x**2+1)/(-x**4+1)**(1/2),x)
```

output
```
x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(2*I*pi))/(4*gamma
(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(2*I*pi))/(4
*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx = \int \frac{x^2+1}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx=\int \frac{x^2+1}{\sqrt{-x^4+1}}\,dx$$

input

```
integrate((x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output

```
integrate((x^2 + 1)/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx=\int \frac{x^2+1}{\sqrt{1-x^4}}\,dx$$

input

```
int((x^2 + 1)/(1 - x^4)^(1/2),x)
```

output

```
int((x^2 + 1)/(1 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx=-\left(\int \frac{\sqrt{-x^4+1}}{x^2-1}dx\right)$$

input

```
int((x^2+1)/(-x^4+1)^(1/2),x)
```

output

```
 - int(sqrt( - x**4 + 1)/(x**2 - 1),x)
```

3.76 $\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{x\sqrt{1-x^4}}{2(1+x^2)} + \frac{1}{2}E(\arcsin(x)|-1)$$

output

```
x*(-x^4+1)^(1/2)/(2*x^2+2)+1/2*EllipticE(x,I)
```

Mathematica [A] (verified)

Time = 10.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{x-x^3+\sqrt{1-x^4}E(\arcsin(x)|-1)}{2\sqrt{1-x^4}}$$

input

```
Integrate[1/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output

```
(x - x^3 + Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1])/(2*Sqrt[1 - x^4])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1388, 316, 25, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{\sqrt{1-x^2}\,(x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{x\sqrt{1-x^2}}{2\sqrt{x^2+1}}-\frac{1}{2}\int -\frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx+\frac{\sqrt{1-x^2}x}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{2}E(\arcsin(x)|-1)+\frac{\sqrt{1-x^2}x}{2\sqrt{x^2+1}}$$

input

```
Int[1/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output

```
(x*Sqrt[1 - x^2])/(2*Sqrt[1 + x^2]) + EllipticE[ArcSin[x], -1]/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(27) = 54$.

Time = 0.89 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.75

method	result	size
risch	$-\frac{x(x^2-1)}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\mathrm{EllipticF}(x,i)}{2\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$	88
default	$\frac{(-x^2+1)x}{2\sqrt{(x^2+1)(-x^2+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\mathrm{EllipticF}(x,i)}{2\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$	96
elliptic	$\frac{(-x^2+1)x}{2\sqrt{(x^2+1)(-x^2+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\mathrm{EllipticF}(x,i)}{2\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$	96

input
```
int(1/(x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(x^2-1)/(-x^4+1)^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1
/2)*EllipticF(x,I)-1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(Ellipt
icF(x,I)-EllipticE(x,I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{(x^2+1)E(\arcsin{(x)}\,|\,-1)+\sqrt{-x^4+1}x}{2\,(x^2+1)}$$

input
```
integrate(1/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*elliptic_e(arcsin(x), -1) + sqrt(-x^4 + 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-(x-1)(x+1)(x^2+1)}\,(x^2+1)}\,dx$$

input
```
integrate(1/(x**2+1)/(-x**4+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate(1/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate(1/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{1}{(x^2+1)\,\sqrt{1-x^4}}\,dx$$

input
```
int(1/((x^2 + 1)*(1 - x^4)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)*(1 - x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4+1}}{x^6+x^4-x^2-1}dx\right)$$

input
```
int(1/(x^2+1)/(-x^4+1)^(1/2),x)
```

output
```
- int(sqrt( - x**4 + 1)/(x**6 + x**4 - x**2 - 1),x)
```

3.77 $\int \frac{1+x^2}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1+x^2}{\sqrt{-1+x^4}}\,dx=\frac{\sqrt{1-x^4}E(\arcsin (x)|-1)}{\sqrt{-1+x^4}}$$

output
```
(-x^4+1)^(1/2)*EllipticE(x,I)/(x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{1+x^2}{\sqrt{-1+x^4}}\,dx =\frac{\sqrt{1-x^4}\left(3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right)+x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^4\right)\right)}{3\sqrt{-1+x^4}}$$

input
```
Integrate[(1 + x^2)/Sqrt[-1 + x^4],x]
```

output
```
(Sqrt[1 - x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, x^4] + x^3*Hypergeome
tric2F1[1/2, 3/4, 7/4, x^4]))/(3*Sqrt[-1 + x^4])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 126 vs. $2(25) = 50$.

Time = 0.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 5.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1502, 763, 1499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{1502}$$

$$2\int \frac{1}{\sqrt{x^4-1}}dx-\int \frac{1-x^2}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{763}$$

$$\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{x^4-1}}-\int \frac{1-x^2}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{1499}$$

$$\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{x^4-1}}-\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{x^2+1}E\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}+\frac{x\left(x^2+1\right)}{\sqrt{x^4-1}}$$

input
```
Int[(1 + x^2)/Sqrt[-1 + x^4],x]
```

output

```
(x*(1 + x^2))/Sqrt[-1 + x^4] - (Sqrt[2]*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*Ellip
ticE[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/Sqrt[-1 + x^4] + (Sqrt[2]*S
qrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]],
1/2])/Sqrt[-1 + x^4]
```

Defintions of rubi rules used

rule 763

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 1499

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[e*x*((q + c*x^2)/(c*Sqrt[a + c*x^4])), x] - Simp[Sqrt
[2]*e*q*Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[-a]*c*Sqrt[a + c*x^4]))
*EllipticE[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

rule 1502

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[(c*d + e*q)/c   Int[1/Sqrt[a + c*x^4], x], x] - Simp[
e/c   Int[(q - c*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[c*d + e*q, 0]] /; Free
Q[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.87 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

method	result	size
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(x^{4}-1\right)}\, x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], x^{4}\right)}{3 \sqrt{\operatorname{signum}\left(x^{4}-1\right)}}+\frac{\sqrt{-\operatorname{signum}\left(x^{4}-1\right)}\, x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], x^{4}\right)}{\sqrt{\operatorname{signum}\left(x^{4}-1\right)}}$	63
default	$-\frac{i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\, \operatorname{EllipticF}\left(i x, i\right)}{\sqrt{x^{4}-1}}-\frac{i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\, \left(\operatorname{EllipticF}\left(i x, i\right)-\operatorname{EllipticE}\left(i x, i\right)\right)}{\sqrt{x^{4}-1}}$	78
elliptic	$-\frac{i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\, \operatorname{EllipticF}\left(i x, i\right)}{\sqrt{x^{4}-1}}-\frac{i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\, \left(\operatorname{EllipticF}\left(i x, i\right)-\operatorname{EllipticE}\left(i x, i\right)\right)}{\sqrt{x^{4}-1}}$	78

input

```
int((x^2+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/3/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)*x^3*hypergeom([1/2,3/4],[7/
4],x^4)+1/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)*x*hypergeom([1/4,1/2]
,[5/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{1+x^2}{\sqrt{-1+x^4}}\,dx = \frac{xE\left(\arcsin\left(\frac{1}{x}\right)\middle|-1\right) - 2\,xF\left(\arcsin\left(\frac{1}{x}\right)\middle|-1\right) + \sqrt{x^4-1}}{x}$$

input

```
integrate((x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output

```
(x*elliptic_e(arcsin(1/x), -1) - 2*x*elliptic_f(arcsin(1/x), -1) + sqrt(x^
4 - 1))/x
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(19) = 38.

Time = 0.65 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{1+x^2}{\sqrt{-1+x^4}}\,dx = -\frac{ix^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|x^4\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((x**2+1)/(x**4-1)**(1/2),x)
```

output

```
-I*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4)/(4*gamma(7/4)) - I*x*ga
mma(1/4)*hyper((1/4, 1/2), (5/4,), x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+x^2}{\sqrt{-1+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{1+x^2}{\sqrt{-1+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + 1)/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{\sqrt{-1+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{x^4-1}}\,dx$$

input
```
int((x^2 + 1)/(x^4 - 1)^(1/2),x)
```

output
```
int((x^2 + 1)/(x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+x^2}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^4-1}}{x^2-1}dx$$

input
```
int((x^2+1)/(x^4-1)^(1/2),x)
```

output
```
int(sqrt(x**4 - 1)/(x**2 - 1),x)
```

3.78 $\int \frac{1}{(1+x^2)\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int \frac{1}{(1+x^2)\sqrt{-1+x^4}}\,dx = -\frac{x\sqrt{-1+x^4}}{2(1+x^2)} + \frac{\sqrt{1-x^4}E(\arcsin(x)|-1)}{2\sqrt{-1+x^4}}$$

output

```
-1/2*x*(x^4-1)^(1/2)/(x^2+1)+1/2*(-x^4+1)^(1/2)*EllipticE(x,I)/(x^4-1)^(1/
2)
```

Mathematica [A] (verified)

Time = 10.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

$$\int \frac{1}{(1+x^2)\sqrt{-1+x^4}}\,dx = \frac{x-x^3+\sqrt{1-x^4}E(\arcsin(x)|-1)}{2\sqrt{-1+x^4}}$$

input

```
Integrate[1/((1 + x^2)*Sqrt[-1 + x^4]),x]
```

output

```
(x - x^3 + Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 121 vs. 2(50) = 100.

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.42, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1391}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{1391}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\sqrt{x^2+1}E\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}+\frac{x}{\sqrt{x^4-1}}$$

input

```
Int[1/((1 + x^2)*Sqrt[-1 + x^4]),x]
```

output

```
x/Sqrt[-1 + x^4] - (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticE[ArcSin[(Sqrt[2]
*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4]) + (Sqrt[-1 + x^2]*Sqrt
[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2]*Sqr
t[-1 + x^4])
```

Defintions of rubi rules used

rule 1391

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
x/(d*Sqrt[a + c*x^4]), x] + (-Simp[(Sqrt[-1 + (e/d)*x^2]*Sqrt[1 + (e/d)*x^2
]*EllipticE[ArcSin[(Sqrt[2]*Rt[e/d, 2]*x)/Sqrt[-1 + (e/d)*x^2]], 1/2])/(Sqr
t[2]*d*Rt[e/d, 2]*Sqrt[a + c*x^4]), x] + Simp[(Sqrt[-1 + (e/d)*x^2]*Sqrt[1
+ (e/d)*x^2]*EllipticF[ArcSin[(Sqrt[2]*Rt[e/d, 2]*x)/Sqrt[-1 + (e/d)*x^2]],
 1/2])/(Sqrt[2]*d*Rt[e/d, 2]*Sqrt[a + c*x^4]), x]) /; FreeQ[{a, c, d, e}, x
] && EqQ[c*d^2 + a*e^2, 0] && LtQ[a, 0] && GtQ[c, 0] && PosQ[e/d]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(40) = 80$.

Time = 1.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.86

method	result	size
risch	$-\frac{x(x^2-1)}{2\sqrt{x^4-1}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\ \mathrm{EllipticF}(ix,i)}{2\sqrt{x^4-1}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-\mathrm{EllipticE}(ix,i))}{2\sqrt{x^4-1}}$	93
default	$-\frac{(x^2-1)x}{2\sqrt{(x^2-1)(x^2+1)}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\ \mathrm{EllipticF}(ix,i)}{2\sqrt{x^4-1}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-\mathrm{EllipticE}(ix,i))}{2\sqrt{x^4-1}}$	99
elliptic	$-\frac{(x^2-1)x}{2\sqrt{(x^2-1)(x^2+1)}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\ \mathrm{EllipticF}(ix,i)}{2\sqrt{x^4-1}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-\mathrm{EllipticE}(ix,i))}{2\sqrt{x^4-1}}$	99

input

```
int(1/(x^2+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*x*(x^2-1)/(x^4-1)^(1/2)-1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1
/2)*EllipticF(I*x,I)-1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*(Ell
ipticF(I*x,I)-EllipticE(I*x,I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int \frac{1}{(1+x^2)\sqrt{-1+x^4}}\,dx = \frac{(-i\,x^2-i)E(\arcsin{(x)}\mid-1)-\sqrt{x^4-1}x}{2\,(x^2+1)}$$

input

```
integrate(1/(x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output

```
1/2*((-I*x^2 - I)*elliptic_e(arcsin(x), -1) - sqrt(x^4 - 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{1}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}\,(x^2+1)}\,dx$$

input

```
integrate(1/(x**2+1)/(x**4-1)**(1/2),x)
```

output

```
Integral(1/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4-1}(x^2+1)}\,dx$$

input

```
integrate(1/(x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output

```
integrate(1/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4-1}(x^2+1)}\,dx$$

input

```
integrate(1/(x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

output

```
integrate(1/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{1}{(x^2+1)\,\sqrt{x^4-1}}\,dx$$

input
```
int(1/((x^2 + 1)*(x^4 - 1)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)*(x^4 - 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^4-1}}{x^6+x^4-x^2-1}dx$$

input
```
int(1/(x^2+1)/(x^4-1)^(1/2),x)
```

output
```
int(sqrt(x**4 - 1)/(x**6 + x**4 - x**2 - 1),x)
```

3.79 $\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 54

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx=\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{-a+cx^4}}$$

output

```
a^(3/4)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(1/4)/(c*x^4-a)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx$$
$$=\frac{\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+\sqrt{c}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{-a+cx^4}}$$

input

```
Integrate[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```

output
```
(Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)
/a] + Sqrt[c]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a
 + c*x^4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{cx^4-a}}dx$$

$$\Big\downarrow \text{ 1390}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

input
```
Int[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```

output
```
(a^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(
c^(1/4)*Sqrt[-a + c*x^4])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(42) = 84$.

Time = 1.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.93

method	result
default	$\frac{\sqrt{a}\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}+\frac{\sqrt{a}\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right.\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}$
elliptic	$\frac{\left(\sqrt{a}+\sqrt{c}\,x^{2}\right)\sqrt{-c\left(-c\,x^{4}+a\right)}\,\sqrt{-\left(-c\,x^{4}+a\right)a}\,\left(\frac{\sqrt{c}\,\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c^{2}x^{4}-a c}}+\frac{a\sqrt{1+}}{}\right.}{\sqrt{c\,x^{4}-a}\,\left(c\,x^{2}\sqrt{-\left(-c\,x^{4}+a\right)a}+a\sqrt{-c\left(-c\,x^{4}+a\right)}\right)}$

input
```
int((a^(1/2)+c^(1/2)*x^2)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^
(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^
(1/2),I)+a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2
)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*(EllipticF(x*(-1/a^(1/2)
*c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(41) = 82$.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = \frac{acx\left(\frac{a}{c}\right)^{\frac{3}{4}} E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right) - \left(acx\sqrt{\frac{a}{c}}+\sqrt{a}c^{\frac{3}{2}}x\sqrt{\frac{a}{c}}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}} F\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right) + \sqrt{cx^4-a}\,a}{acx}$$

input
```
integrate((a^(1/2)+c^(1/2)*x^2)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
(a*c*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - (a*c*x*sqrt(a/c
) + sqrt(a)*c^(3/2)*x*sqrt(a/c))*(a/c)^(1/4)*elliptic_f(arcsin((a/c)^(1/4)
/x), -1) + sqrt(c*x^4 - a)*a*sqrt(c))/(a*c*x)
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = -\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{i\sqrt{c}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((a**(1/2)+c**(1/2)*x**2)/(c*x**4-a)**(1/2),x)
```

output
```
-I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*gamma(5/4)) - I*sqr
t(c)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*gamma(
7/4))
```

Maxima [F]

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}}\,dx$$

input
```
integrate((a^(1/2)+c^(1/2)*x^2)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

Giac [F]

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}}\,dx$$

input
```
integrate((a^(1/2)+c^(1/2)*x^2)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{c\,x^4-a}}\,dx$$

input
```
int((a^(1/2) + c^(1/2)*x^2)/(c*x^4 - a)^(1/2),x)
```

output
```
int((a^(1/2) + c^(1/2)*x^2)/(c*x^4 - a)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = -\sqrt{a}\left(\int \frac{\sqrt{c\,x^4-a}}{-c\,x^4+a}dx\right)-\sqrt{c}\left(\int \frac{\sqrt{c\,x^4-a}\,x^2}{-c\,x^4+a}dx\right)$$

input

```
int((a^(1/2)+c^(1/2)*x^2)/(c*x^4-a)^(1/2),x)
```

output

```
- (sqrt(a)*int(sqrt( - a + c*x**4)/(a - c*x**4),x) + sqrt(c)*int((sqrt( -
a + c*x**4)*x**2)/(a - c*x**4),x))
```

3.80 $\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 52

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx=\frac{\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{-a+cx^4}}$$

output

```
(1-c*x^4/a)^(1/2)*EllipticE((c/a)^(1/4)*x,I)/(c/a)^(1/4)/(c*x^4-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx$$
$$=\frac{\sqrt{1-\frac{cx^4}{a}}\left(3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+\sqrt{\frac{c}{a}}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{-a+cx^4}}$$

input

```
Integrate[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

output
```
(Sqrt[1 - (c*x^4)/a]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + Sq
rt[c/a]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x
^4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1390, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}}dx$$

$$\Big\downarrow \text{1390}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{c}{a}}x^2+1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{1388}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\sqrt{\frac{c}{a}}x^2+1}}{\sqrt{1-\frac{cx^2}{a\sqrt{\frac{c}{a}}}}}dx}{\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{cx^4-a}}$$

input
```
Int[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

output
```
(Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c/a)^(1/4)*x], -1])/((c/a)^(1/4)*Sq
rt[-a + c*x^4])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(44) = 88$.

Time = 2.40 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.17

method	result
default	$\frac{\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}+\frac{\sqrt{\frac{c}{a}}\,\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},\right.\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}\,\sqrt{c}}$
elliptic	$\frac{\left(1+\sqrt{\frac{c}{a}}\,x^{2}\right)a\sqrt{-\frac{c\left(-c\,x^{4}+a\right)}{a}}\left(\frac{\sqrt{c}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{\frac{c^{2}x^{4}}{a}-c}}+\frac{\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\text{Ellip}}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}}}\right.}{c\,x^{2}\sqrt{c\,x^{4}-a}+a\sqrt{-\frac{c\left(-c\,x^{4}+a\right)}{a}}}$

input
```
int((1+(c/a)^(1/2)*x^2)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),
I)+(c/a)^(1/2)*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2
)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF
(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(43) = 86$.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.29

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx$$
$$= \frac{a\sqrt{c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}\sqrt{\frac{c}{a}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)-\left(ax\sqrt{\frac{a}{c}}\sqrt{\frac{c}{a}}+cx\sqrt{\frac{a}{c}}\right)\sqrt{c}\left(\frac{a}{c}\right)^{\frac{1}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)+\sqrt{cx^4 -}}{acx}$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
(a*sqrt(c)*x*(a/c)^(3/4)*sqrt(c/a)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) -
 (a*x*sqrt(a/c)*sqrt(c/a) + c*x*sqrt(a/c))*sqrt(c)*(a/c)^(1/4)*elliptic_f(
arcsin((a/c)^(1/4)/x), -1) + sqrt(c*x^4 - a)*a*sqrt(c/a))/(a*c*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(37) = 74$.

Time = 0.87 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.46

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = -\frac{ix^3\sqrt{\frac{c}{a}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((1+(c/a)**(1/2)*x**2)/(c*x**4-a)**(1/2),x)
```

output
```
-I*x**3*sqrt(c/a)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a
)*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt
(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}}\,dx$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

Giac [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}}\,dx$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx=\int \frac{x^2\,\sqrt{\frac{c}{a}}+1}{\sqrt{c\,x^4-a}}\,dx$$

input

```
int((x^2*(c/a)^(1/2) + 1)/(c*x^4 - a)^(1/2),x)
```

output

```
int((x^2*(c/a)^(1/2) + 1)/(c*x^4 - a)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx=\frac{-\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{c\,x^4-a}\,x^2}{-c\,x^4+a}dx\right)-\left(\int \frac{\sqrt{c\,x^4-a}}{-c\,x^4+a}dx\right)a}{a}$$

input

```
int((1+(c/a)^(1/2)*x^2)/(c*x^4-a)^(1/2),x)
```

output

```
( - (sqrt(c)*sqrt(a)*int((sqrt( - a + c*x**4)*x**2)/(a - c*x**4),x) + int(
sqrt( - a + c*x**4)/(a - c*x**4),x)*a))/a
```

3.81 $\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 72

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx=\frac{(d-ex^2)\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|2\right)}{\sqrt{d}\sqrt{e}\sqrt{\frac{d-ex^2}{d+ex^2}}\sqrt{d^2-e^2x^4}}$$

output
```
(-e*x^2+d)*EllipticE(e^(1/2)*x/d^(1/2)/(1+e*x^2/d)^(1/2),2^(1/2))/d^(1/2)/
e^(1/2)/((-e*x^2+d)/(e*x^2+d))^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx=\frac{3dx-2ex^3\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},\frac{e^2x^4}{d^2}\right)}{3d\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d - e*x^2)^2/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(3*d*x - 2*e*x^3*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4, 3/2, 7/4,
(e^2*x^4)/d^2])/(3*d*Sqrt[d^2 - e^2*x^4])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(72) = 144.

Time = 0.76 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.24, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1396, 314, 25, 27, 344, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-e x^2\right)^2}{\left(d^2-e^2 x^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-e x^2}\sqrt{d+e x^2}\int \frac{\sqrt{d-e x^2}}{\left(e x^2+d\right)^{3/2}}dx}{\sqrt{d^2-e^2 x^4}}$$

$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{d-e x^2}\sqrt{d+e x^2}\left(\frac{x\sqrt{d-e x^2}}{d\sqrt{d+e x^2}}-\frac{\int -\frac{e x^2}{\sqrt{d-e x^2}\sqrt{e x^2+d}}dx}{d}\right)}{\sqrt{d^2-e^2 x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-e x^2}\sqrt{d+e x^2}\left(\frac{\int \frac{e x^2}{\sqrt{d-e x^2}\sqrt{e x^2+d}}dx}{d}+\frac{x\sqrt{d-e x^2}}{d\sqrt{d+e x^2}}\right)}{\sqrt{d^2-e^2 x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-e x^2}\sqrt{d+e x^2}\left(\frac{e\int \frac{x^2}{\sqrt{d-e x^2}\sqrt{e x^2+d}}dx}{d}+\frac{x\sqrt{d-e x^2}}{d\sqrt{d+e x^2}}\right)}{\sqrt{d^2-e^2 x^4}}$$

$$\Big\downarrow \text{344}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\int \frac{x^2}{\sqrt{d^2-e^2x^4}}dx}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 836$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\int \frac{ex^2+d}{d\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 1390$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{ex^2+d}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d - e*x^2)^2/(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d - e*x^2])/(d*Sqrt[d + e*x^2])
+ (e*Sqrt[d^2 - e^2*x^4]*((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSi
n[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^2*x^4]) - (d^(3/2)*Sqrt
[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*S
qrt[d^2 - e^2*x^4])))/(d*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/Sqrt[d^2 - e^2
*x^4]
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 344

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c
+ b*d*x^4)^FracPart[p])   Int[(e*x)^m*(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a
, b, c, d, e, m, p}, x] && EqQ[b*c + a*d, 0] &&  !IntegerQ[p]
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.97 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.68

method	result
elliptic	$\frac{\left(-e^{2} x^{2}+d e \right) x}{d e \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$
default	$d^{2} \left(\frac{x}{2 d^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d^{2} \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}\right)+e^{2} \Bigg(\frac{x}{2 e^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \mathrm{E}}{2 e^{2} \sqrt{\frac{e}{d}}\, \sqrt{-e}}$

input
```
int((-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-e^2*x^2+d*e)/d*x/e/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)-1/(e/d)^(1/2)*(1-e*x
^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2
),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{(ex^2+d)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)-(ex^2+d)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)+\sqrt{-e^2}}{dex^2+d^2}$$

input
```
integrate((-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
((e*x^2 + d)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - (e*x^2 + d)*s
qrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*x)/(d*
e*x^2 + d^2)
```

Sympy [F]

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(-d+ex^2)^2}{\left(-(-d+ex^2)(d+ex^2)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x**2+d)**2/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral((-d + e*x**2)**2/(-(-d + e*x**2)*(d + e*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{(ex^2-d)^2}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 - d)^2/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{(ex^2-d)^2}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 - d)^2/(-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{(d-e\,x^2)^2}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input
```
int((d - e*x^2)^2/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
int((d - e*x^2)^2/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e^2x^4+2de\,x^2+d^2}dx$$

input
```
int((-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)
```

3.82 $\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^2}\,dx=\frac{\sqrt{d^2-e^2x^4}\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|2\right)}{\sqrt{d}\sqrt{e}\sqrt{\frac{d-ex^2}{d+ex^2}}\left(d+ex^2\right)}$$

output
```
(-e^2*x^4+d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2)/(1+e*x^2/d)^(1/2),2^(1/2)
)/d^(1/2)/e^(1/2)/((-e*x^2+d)/(e*x^2+d))^(1/2)/(e*x^2+d)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^2}\,dx=\frac{x-\frac{ex^3}{d}-\frac{i\sqrt{1-\frac{e^2x^4}{d^2}}\left(E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)\right)}{\sqrt{-\frac{e}{d}}}}{\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^2,x]
```

output
```
(x - (e*x^3)/d - (I*Sqrt[1 - (e^2*x^4)/d^2]*(EllipticE[I*ArcSinh[Sqrt[-(e/
d)]*x], -1] - EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1]))/Sqrt[-(e/d)])/Sqr
t[d^2 - e^2*x^4]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. $2(73) = 146$.

Time $= 0.72$ (sec) , antiderivative size $= 233$, normalized size of antiderivative $= 3.19$, number of steps used $= 12$, number of rules used $= 12$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used $= \{1396, 314, 25, 27, 344, 836, 27, 765, 762, 1390, 1389, 327\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{d-ex^2}}{(ex^2+d)^{3/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\int -\frac{ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\int \frac{x^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 344$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\int \frac{x^2}{\sqrt{d^2-e^2x^4}}dx}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 836$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\int \frac{ex^2+d}{d\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{ex^2+d}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^2,x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d - e*x^2])/(d*Sqrt[d + e*x^2]) + (e*Sqrt[d^
2 - e^2*x^4]*((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x
)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^2*x^4]) - (d^(3/2)*Sqrt[1 - (e^2*x^
4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^
2*x^4])))/(d*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/(Sqrt[d - e*x^2]*Sqrt[d +
e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 344
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c
+ b*d*x^4)^FracPart[p])   Int[(e*x)^m*(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a
, b, c, d, e, m, p}, x] && EqQ[b*c + a*d, 0] &&  !IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.70 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.66

method	result	size
default	$\frac{\left(-e^{2} x^{2}+d e \right) x}{d e \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	121
elliptic	$\frac{\left(-e^{2} x^{2}+d e \right) x}{d e \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	121

input

```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
(-e^2*x^2+d*e)/d*x/e/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)-1/(e/d)^(1/2)*(1-e*x
^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2
),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^2}\,dx = \frac{(ex^2+d)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1\right)-(ex^2+d)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1\right)+\sqrt{-e^2x^4+d^2}x}{dex^2+d^2}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
((e*x^2 + d)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - (e*x^2 + d)*s
qrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*x)/(d*
e*x^2 + d^2)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^2}\,dx = \int \frac{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}}{\left(d+ex^2\right)^2}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(e*x**2+d)**2,x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\sqrt{d^2-e^2\,x^4}}{\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^2,x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^2x^4+2de\,x^2+d^2}dx$$

input

```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^2,x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)
```

3.83 $\int \frac{d-ex^2}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 72

$$\int \frac{d-ex^2}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx=\frac{(d-ex^2)\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|2\right)}{\sqrt{d}\sqrt{e}\sqrt{\frac{d-ex^2}{d+ex^2}}\sqrt{d^2-e^2x^4}}$$

output
```
(-e*x^2+d)*EllipticE(e^(1/2)*x/d^(1/2)/(1+e*x^2/d)^(1/2),2^(1/2))/d^(1/2)/
e^(1/2)/((-e*x^2+d)/(e*x^2+d))^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \frac{d-ex^2}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx$$
$$=\frac{x-\frac{ex^3}{d}-\frac{i\sqrt{1-\frac{e^2x^4}{d^2}}\left(E\left(i\,\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)\right)}{\sqrt{-\frac{e}{d}}}}{\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d - e*x^2)/((d + e*x^2)*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(x - (e*x^3)/d - (I*Sqrt[1 - (e^2*x^4)/d^2]*(EllipticE[I*ArcSinh[Sqrt[-(e/
d)]*x], -1] - EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1]))/Sqrt[-(e/d)])/Sqr
t[d^2 - e^2*x^4]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. $2(72) = 144$.

Time = 0.75 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.24, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = $\{1396, 314, 25, 27, 344, 836, 27, 765, 762, 1390, 1389, 327\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{\left(d+ex^2\right)\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\sqrt{d-ex^2}}{\left(ex^2+d\right)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\int -\frac{ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\int \frac{x^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{344}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\int \frac{x^2}{\sqrt{d^2-e^2x^4}}dx}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{836}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\int \frac{ex^2+d}{d\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{ex^2+d}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d - e*x^2)/((d + e*x^2)*Sqrt[d^2 - e^2*x^4]),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d - e*x^2])/(d*Sqrt[d + e*x^2])
 + (e*Sqrt[d^2 - e^2*x^4]*((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSi
n[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^2*x^4]) - (d^(3/2)*Sqrt
[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*S
qrt[d^2 - e^2*x^4])))/(d*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/Sqrt[d^2 - e^2
*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 344
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c
+ b*d*x^4)^FracPart[p])   Int[(e*x)^m*(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a
, b, c, d, e, m, p}, x] && EqQ[b*c + a*d, 0] &&  !IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.68

method	result
elliptic	$\frac{\left(-e^{2} x^{2}+d e \right) x}{d e \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$
default	$-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}+2 d \left(\frac{\left(-e^{2} x^{2}+d e \right) x}{2 d^{2} e \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}-\right.$

input
```
int((-e*x^2+d)/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-e^2*x^2+d*e)/d*x/e/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)-1/(e/d)^(1/2)*(1-e*x
^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2
),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int \frac{d-ex^2}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx$$
$$=\frac{(ex^2+d)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)-(ex^2+d)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)+\sqrt{-e^2x^4+d^2}x}{dex^2+d^2}$$

input
```
integrate((-e*x^2+d)/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
((e*x^2 + d)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - (e*x^2 + d)*s
qrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*x)/(d*
e*x^2 + d^2)
```

Sympy [F]

$$\int \frac{d-ex^2}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx=-\int\left(-\frac{d}{d\sqrt{d^2-e^2x^4}+ex^2\sqrt{d^2-e^2x^4}}\right)dx-\int\frac{ex^2}{d\sqrt{d^2-e^2x^4}+ex^2\sqrt{d^2-e^2x^4}}\,dx$$

input
```
integrate((-e*x**2+d)/(e*x**2+d)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
-Integral(-d/(d*sqrt(d**2 - e**2*x**4) + e*x**2*sqrt(d**2 - e**2*x**4)), x
) - Integral(e*x**2/(d*sqrt(d**2 - e**2*x**4) + e*x**2*sqrt(d**2 - e**2*x*
*4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d-ex^2}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-e*x^2+d)/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{d-ex^2}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int -\frac{ex^2-d}{\sqrt{-e^2x^4+d^2}(ex^2+d)}\,dx$$

input
```
integrate((-e*x^2+d)/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(e*x^2 - d)/(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d-ex^2}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int \frac{d-e\,x^2}{\sqrt{d^2-e^2\,x^4}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((d - e*x^2)/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int((d - e*x^2)/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{d-ex^2}{(d+ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e^2x^4+2de\,x^2+d^2}dx$$

input
```
int((-e*x^2+d)/(e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)
```

3.84 $\int \left(d-ex^2\right)^3 \sqrt{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 205

$$\begin{aligned}\int \left(d-ex^2\right)^3 \sqrt{d^2-e^2x^4}\,dx &= \frac{2}{21}d^2x\left(5d-7ex^2\right)\sqrt{d^2-e^2x^4} \\ &\quad -\frac{3}{7}dx\left(d^2-e^2x^4\right)^{3/2}+\frac{1}{9}ex^3\left(d^2-e^2x^4\right)^{3/2} \\ &\quad -\frac{4d^{11/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}} \\ &\quad +\frac{16d^{11/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{7\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
2/21*d^2*x*(-7*e*x^2+5*d)*(-e^2*x^4+d^2)^(1/2)-3/7*d*x*(-e^2*x^4+d^2)^(3/2
)+1/9*e*x^3*(-e^2*x^4+d^2)^(3/2)-4/3*d^(11/2)*(1-e^2*x^4/d^2)^(1/2)*Ellipt
icE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+16/7*d^(11/2)*(1-e^2
*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.63 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.70

$$\int \left(d-ex^2\right)^3\sqrt{d^2-e^2x^4}\,dx$$

$$=\frac{x\sqrt{d^2-e^2x^4}\left(\sqrt{1-\frac{e^2x^4}{d^2}}\left(-27d^3+7d^2ex^2+27de^2x^4-7e^3x^6\right)+90d^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\right.\right.}{63\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input
```
Integrate[(d - e*x^2)^3*Sqrt[d^2 - e^2*x^4],x]
```

output
```
(x*Sqrt[d^2 - e^2*x^4]*(Sqrt[1 - (e^2*x^4)/d^2]*(-27*d^3 + 7*d^2*e*x^2 + 2
7*d*e^2*x^4 - 7*e^3*x^6) + 90*d^3*Hypergeometric2F1[-1/2, 1/4, 5/4, (e^2*x
^4)/d^2] - 70*d^2*e*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, (e^2*x^4)/d^2]))
/(63*Sqrt[1 - (e^2*x^4)/d^2])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.49, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.593, Rules used = {1396, 318, 27, 403, 25, 27, 403, 27, 403, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-ex^2\right)^3\sqrt{d^2-e^2x^4}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \left(d-ex^2\right)^{7/2}\sqrt{ex^2+d}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int 10de\left(d-2ex^2\right)\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}dx}{9e}-\frac{1}{9}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\int\left(d-2ex^2\right)\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}dx-\frac{1}{9}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\left(\frac{2}{7}x\left(d-ex^2\right)^{5/2}\sqrt{d+ex^2}-\frac{\int-\frac{de\left(d-ex^2\right)^{3/2}\left(ex^2+5d\right)}{\sqrt{ex^2+d}}dx}{7e}\right)-\frac{1}{9}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\left(\frac{\int\frac{de\left(d-ex^2\right)^{3/2}\left(ex^2+5d\right)}{\sqrt{ex^2+d}}dx}{7e}+\frac{2}{7}x\sqrt{d+ex^2}\left(d-ex^2\right)^{5/2}\right)-\frac{1}{9}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\left(\frac{1}{7}d\int\frac{\left(d-ex^2\right)^{3/2}\left(ex^2+5d\right)}{\sqrt{ex^2+d}}dx+\frac{2}{7}x\sqrt{d+ex^2}\left(d-ex^2\right)^{5/2}\right)-\frac{1}{9}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\left(\frac{1}{7}d\left(\frac{\int\frac{6de\left(4d-3ex^2\right)\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx}{5e}+\frac{1}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)+\frac{2}{7}x\sqrt{d+ex^2}\left(d-ex^2\right)^{5/2}\right)-\frac{1}{9}x(\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\left(\frac{1}{7}d\left(\frac{6}{5}d\int\frac{\left(4d-3ex^2\right)\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx+\frac{1}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)+\frac{2}{7}x\sqrt{d+ex^2}\left(d-ex^2\right)^{5/2}\right)-\frac{1}{9}x(d\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 403$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{10}{9}d\Bigg(\frac{1}{7}d\Bigg(\frac{6}{5}d\Bigg(\frac{\int\frac{3de\left(5d-7ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}-x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)+\frac{1}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\Bigg)+\frac{2}{7}x\sqrt{d+}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\bigg(\frac{10}{9}d\bigg(\frac{1}{7}d\bigg(\frac{6}{5}d\bigg(d\int\frac{5d-7ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-x\sqrt{d-ex^2}\sqrt{d+ex^2}\bigg)+\frac{1}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\bigg)+\frac{2}{7}x\sqrt{d+}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 399$$

$$\frac{\sqrt{d^2-e^2x^4}\bigg(\frac{10}{9}d\bigg(\frac{1}{7}d\bigg(\frac{6}{5}d\bigg(d\bigg(12d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-7\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\bigg)-x\sqrt{d-ex^2}\sqrt{d+ex^2}\bigg)+\frac{1}{5}x\sqrt{d+ex^2}(d-}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 289$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{10}{9}d\Bigg(\frac{1}{7}d\Bigg(\frac{6}{5}d\Bigg(d\Bigg(\frac{12d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-7\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\Bigg)-x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)+\frac{1}{5}x\sqrt{d+ex^2}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 329$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{10}{9}d\Bigg(\frac{1}{7}d\Bigg(\frac{6}{5}d\Bigg(d\Bigg(\frac{12d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)+\frac{1}{5}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{10}{9}d\Bigg(\frac{1}{7}d\Bigg(\frac{6}{5}d\Bigg(d\Bigg(\frac{12d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-x\sqrt{d-ex^2}\sqrt{d+ex}}{\sqrt{d-ex^2}\sqrt{d+ex}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{10}{9}d\Bigg(\frac{1}{7}d\Bigg(\frac{6}{5}d\Bigg(d\Bigg(\frac{12d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-x\sqrt{d-ex^2}\sqrt{d+ex}}{\sqrt{d-ex^2}\sqrt{d+e}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{10}{9}d\left(\frac{1}{7}d\left(\frac{6}{5}d\left(d\left(\frac{12d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-x\sqrt{d-}\right.\right.\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{}}$$

input
```
Int[(d - e*x^2)^3*Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/9*(x*(d - e*x^2)^(5/2)*(d + e*x^2)^(3/2)) + (10*d
*((2*x*(d - e*x^2)^(5/2)*Sqrt[d + e*x^2])/7 + (d*((x*(d - e*x^2)^(3/2)*Sqr
t[d + e*x^2])/5 + (6*d*(-(x*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + d*((-7*d^(3
/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(S
qrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (12*d^(3/2)*Sqrt[1 - (e^2*x^4)/d
^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*S
qrt[d + e*x^2]))))/5))/7))/9))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 7.98 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95

method	result
risch	$\frac{x\left(-7e^{3}x^{6}+27d\,e^{2}x^{4}-35d^{2}e\,x^{2}+3d^{3}\right)\sqrt{-e^{2}x^{4}+d^{2}}}{63}+\frac{4d^{4}\left(\frac{5d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{7d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}}}\right.}{21}$
elliptic	$-\frac{e^{3}x^{7}\sqrt{-e^{2}x^{4}+d^{2}}}{9}+\frac{3d\,e^{2}x^{5}\sqrt{-e^{2}x^{4}+d^{2}}}{7}-\frac{5d^{2}e\,x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{9}+\frac{d^{3}x\sqrt{-e^{2}x^{4}+d^{2}}}{21}+\frac{20d^{5}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}}{21\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$
default	$d^{3}\left(\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{2d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right)-e^{3}\left(\frac{x^{7}\sqrt{-e^{2}x^{4}+d^{2}}}{9}-\frac{2d^{2}x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{45e^{2}}-\frac{2d^{5}\sqrt{1}}{}\right.$

input
```
int((-e*x^2+d)^3*(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/63*x*(-7*e^3*x^6+27*d*e^2*x^4-35*d^2*e*x^2+3*d^3)*(-e^2*x^4+d^2)^(1/2)+4
/21*d^4*(5*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2
)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+7*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*
x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*
(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.72

$$\int \left(d-ex^2\right)^3\sqrt{d^2-e^2x^4}\,dx$$
$$=\frac{84\sqrt{-e^2}d^5x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)-12\,(7\,d^5-5\,d^4e)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)-(7\,e^5x^8-2}{63\,e^2x}$$

input
```
integrate((-e*x^2+d)^3*(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/63*(84*sqrt(-e^2)*d^5*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) -
12*(7*d^5 - 5*d^4*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x)
, -1) - (7*e^5*x^8 - 27*d*e^4*x^6 + 35*d^2*e^3*x^4 - 3*d^3*e^2*x^2 - 84*d^
4*e)*sqrt(-e^2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.90

$$\int \left(d-ex^2\right)^3\sqrt{d^2-e^2x^4}\,dx=\frac{d^4x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}-\frac{3d^3ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{3d^2e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{de^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((-e*x**2+d)**3*(-e**2*x**4+d**2)**(1/2),x)
```

output
```
d**4*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) - 3*d**3*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,),
e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(7/4)) + 3*d**2*e**2*x**5*gamma(
5/4)*hyper((-1/2, 5/4), (9/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma
(9/4)) - d*e**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), e**2*x**4*exp_
polar(2*I*pi)/d**2)/(4*gamma(11/4))
```

Maxima [F]

$$\int \left(d-ex^2\right)^3\sqrt{d^2-e^2x^4}\,dx=\int -\sqrt{-e^2x^4+d^2}\left(ex^2-d\right)^3\,dx$$

input
```
integrate((-e*x^2+d)^3*(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-e^2*x^4 + d^2)*(e*x^2 - d)^3, x)
```

Giac [F]

$$\int \left(d-ex^2\right)^3\sqrt{d^2-e^2x^4}\,dx=\int -\sqrt{-e^2x^4+d^2}\left(ex^2-d\right)^3\,dx$$

input
```
integrate((-e*x^2+d)^3*(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(-e^2*x^4 + d^2)*(e*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-ex^2\right)^3\sqrt{d^2-e^2x^4}\,dx=\int \sqrt{d^2-e^2\,x^4}\,\left(d-e\,x^2\right)^3\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)^3,x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)^3, x)
```

Reduce [F]

$$\begin{aligned}\int \left(d-ex^2\right)^3\sqrt{d^2-e^2x^4}\,dx&=\frac{\sqrt{-e^2x^4+d^2}\,d^3x}{21}-\frac{5\sqrt{-e^2x^4+d^2}\,d^2e\,x^3}{9}\\&\quad+\frac{3\sqrt{-e^2x^4+d^2}\,d\,e^2x^5}{7}-\frac{\sqrt{-e^2x^4+d^2}\,e^3x^7}{9}\\&\quad+\frac{20\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^5}{21}-\frac{4\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^4e}{3}\end{aligned}$$

input
```
int((-e*x^2+d)^3*(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(3*sqrt(d**2 - e**2*x**4)*d**3*x - 35*sqrt(d**2 - e**2*x**4)*d**2*e*x**3 +
 27*sqrt(d**2 - e**2*x**4)*d*e**2*x**5 - 7*sqrt(d**2 - e**2*x**4)*e**3*x**
7 + 60*int(sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)*d**5 - 84*int((sqr
t(d**2 - e**2*x**4)*x**2)/(d**2 - e**2*x**4),x)*d**4*e)/63
```

3.85 $\int \left(d-ex^2\right)^2 \sqrt{d^2-e^2x^4}\, dx$

Optimal result

Integrand size = 27, antiderivative size = 178

$$\begin{aligned}\int \left(d-ex^2\right)^2 \sqrt{d^2-e^2x^4}\, dx = {} & \frac{2}{105}dx\left(20d-21ex^2\right)\sqrt{d^2-e^2x^4}-\frac{1}{7}x\left(d^2-e^2x^4\right)^{3/2} \\ & -\frac{4d^{9/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{5\sqrt{e}\sqrt{d^2-e^2x^4}} \\ & +\frac{164d^{9/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{105\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
2/105*d*x*(-21*e*x^2+20*d)*(-e^2*x^4+d^2)^(1/2)-1/7*x*(-e^2*x^4+d^2)^(3/2)
-4/5*d^(9/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/
(-e^2*x^4+d^2)^(1/2)+164/105*d^(9/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/
2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.66 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.69

$$\int (d-ex^2)^2 \sqrt{d^2-e^2x^4}\,dx$$
$$= \frac{x\sqrt{d^2-e^2x^4}\left(-3(d^2-e^2x^4)\sqrt{1-\frac{e^2x^4}{d^2}}+24d^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-14dex^2\,\text{Hypergeo}\right.}{21\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input
```
Integrate[(d - e*x^2)^2*Sqrt[d^2 - e^2*x^4],x]
```

output
```
(x*Sqrt[d^2 - e^2*x^4]*(-3*(d^2 - e^2*x^4)*Sqrt[1 - (e^2*x^4)/d^2] + 24*d^
2*Hypergeometric2F1[-1/2, 1/4, 5/4, (e^2*x^4)/d^2] - 14*d*e*x^2*Hypergeome
tric2F1[-1/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(21*Sqrt[1 - (e^2*x^4)/d^2])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.56, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {1396, 318, 27, 403, 25, 27, 403, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d-ex^2)^2 \sqrt{d^2-e^2x^4}\,dx$$
$$\Big\downarrow \text{1396}$$
$$\frac{\sqrt{d^2-e^2x^4}\int (d-ex^2)^{5/2}\sqrt{ex^2+d}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$
$$\Big\downarrow \text{318}$$
$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int 2de(4d-7ex^2)\sqrt{d-ex^2}\sqrt{ex^2+d}dx}{7e}-\frac{1}{7}x(d-ex^2)^{3/2}(d+ex^2)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{7}d\int \left(4d-7ex^2\right)\sqrt{d-ex^2}\sqrt{ex^2+d}dx-\frac{1}{7}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{7}d\left(\frac{7}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}-\frac{\int -\frac{de\sqrt{d-ex^2}\left(13d-ex^2\right)}{\sqrt{ex^2+d}}dx}{5e}\right)-\frac{1}{7}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{7}d\left(\frac{\int \frac{de\sqrt{d-ex^2}\left(13d-ex^2\right)}{\sqrt{ex^2+d}}dx}{5e}+\frac{7}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)-\frac{1}{7}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{7}d\left(\frac{1}{5}d\int \frac{\sqrt{d-ex^2}\left(13d-ex^2\right)}{\sqrt{ex^2+d}}dx+\frac{7}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)-\frac{1}{7}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 403$$

$$\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{7}d\Bigg(\frac{1}{5}d\Bigg(\frac{\int \frac{2de\left(20d-21ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)+\frac{7}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\Bigg)-\frac{1}{7}x\left(d-ex^2\right)$$

$$\sqrt{d-ex^2}\sqrt{d+ex^2}$$

$$\Big\downarrow \ 27$$

$$\sqrt{d^2-e^2x^4}\Big(\frac{2}{7}d\Big(\frac{1}{5}d\Big(\frac{2}{3}d\int \frac{20d-21ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Big)+\frac{7}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\Big)-\frac{1}{7}x\big(d-ex^2$$

$$\sqrt{d-ex^2}\sqrt{d+ex^2}$$

$$\Big\downarrow \ 399$$

$$\sqrt{d^2-e^2x^4}\Big(\frac{2}{7}d\Big(\frac{1}{5}d\Big(\frac{2}{3}d\Big(41d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-21\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\Big)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Big)+\frac{7}{5}x\sqrt{d+ex^2}\big(d-$$

$$\sqrt{d-ex^2}\sqrt{d+ex^2}$$

$$\Big\downarrow \ 289$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{7}d\Bigg(\frac{1}{5}d\Bigg(\frac{2}{3}d\Bigg(\frac{41d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-21\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\Bigg)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)+\frac{7}{5}x\sqrt{d+ex^2}(}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 329}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{7}d\Bigg(\frac{1}{5}d\Bigg(\frac{2}{3}d\Bigg(\frac{41d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{21d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)+\frac{7}{5}x}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{7}d\Bigg(\frac{1}{5}d\Bigg(\frac{2}{3}d\Bigg(\frac{41d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{7}d\Bigg(\frac{1}{5}d\Bigg(\frac{2}{3}d\Bigg(\frac{41d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{7}d\Bigg(\frac{1}{5}d\Bigg(\frac{2}{3}d\Bigg(\frac{41d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-\frac{1}{3}x\sqrt{d-e}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[(d - e*x^2)^2*Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/7*(x*(d - e*x^2)^(3/2)*(d + e*x^2)^(3/2)) + (2*d*
((7*x*(d - e*x^2)^(3/2)*Sqrt[d + e*x^2])/5 + (d*(-1/3*(x*Sqrt[d - e*x^2]*S
qrt[d + e*x^2]) + (2*d*((-21*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[Arc
Sin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) +
 (41*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]]
, -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/3))/5))/7))/(Sqrt[d - e
*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 5.73 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.03

method	result
risch	$\frac{x\left(15e^2x^4-42de\,x^2+25d^2\right)\sqrt{-e^2x^4+d^2}}{105}+\frac{4d^3\left(\frac{20d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}+\frac{21d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right.}{105}$
elliptic	$\frac{e^2x^5\sqrt{-e^2x^4+d^2}}{7}-\frac{2de\,x^3\sqrt{-e^2x^4+d^2}}{5}+\frac{5d^2x\sqrt{-e^2x^4+d^2}}{21}+\frac{16d^4\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{21\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}+\frac{4d^4\sqrt{1-\frac{e\,x^2}{d}}}{}$
default	$d^2\left(\frac{x\sqrt{-e^2x^4+d^2}}{3}+\frac{2d^2\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)+e^2\left(\frac{x^5\sqrt{-e^2x^4+d^2}}{7}-\frac{2d^2x\sqrt{-e^2x^4+d^2}}{21e^2}+\frac{2d^4\sqrt{1-}}{}\right.$

input
```
int((-e*x^2+d)^2*(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*x*(15*e^2*x^4-42*d*e*x^2+25*d^2)*(-e^2*x^4+d^2)^(1/2)+4/105*d^3*(20*
d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*Ell
ipticF(x*(e/d)^(1/2),I)+21*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/
2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2
),I)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.76

$$\int \left(d-ex^2\right)^2\sqrt{d^2-e^2x^4}\,dx$$
$$=\frac{84\sqrt{-e^2}d^4x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)-4\,(21\,d^4-20\,d^3e)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)+(15\,e^4x^6\,-}{105\,e^2x}$$

input
```
integrate((-e*x^2+d)^2*(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output

```
1/105*(84*sqrt(-e^2)*d^4*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) -
 4*(21*d^4 - 20*d^3*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/
x), -1) + (15*e^4*x^6 - 42*d*e^3*x^4 + 25*d^2*e^2*x^2 + 84*d^3*e)*sqrt(-e^
2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.75

$$\int \left(d-ex^2\right)^2\sqrt{d^2-e^2x^4}\,dx = \frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input

```
integrate((-e*x**2+d)**2*(-e**2*x**4+d**2)**(1/2),x)
```

output

```
d**3*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) - d**2*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), e*
*2*x**4*exp_polar(2*I*pi)/d**2)/(2*gamma(7/4)) + d*e**2*x**5*gamma(5/4)*hy
per((-1/2, 5/4), (9/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(9/4))
```

Maxima [F]

$$\int \left(d-ex^2\right)^2 \sqrt{d^2-e^2x^4}\,dx=\int \sqrt{-e^2x^4+d^2}\left(ex^2-d\right)^2\,dx$$

input
```
integrate((-e*x^2+d)^2*(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)*(e*x^2 - d)^2, x)
```

Giac [F]

$$\int \left(d-ex^2\right)^2 \sqrt{d^2-e^2x^4}\,dx=\int \sqrt{-e^2x^4+d^2}\left(ex^2-d\right)^2\,dx$$

input
```
integrate((-e*x^2+d)^2*(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)*(e*x^2 - d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-ex^2\right)^2 \sqrt{d^2-e^2x^4}\,dx=\int \sqrt{d^2-e^2\,x^4}\,\left(d-e\,x^2\right)^2\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)^2,x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)^2, x)
```

Reduce [F]

$$\int \left(d-ex^2\right)^2\sqrt{d^2-e^2x^4}\,dx=\frac{5\sqrt{-e^2x^4+d^2}\,d^2x}{21}-\frac{2\sqrt{-e^2x^4+d^2}\,de\,x^3}{5}+\frac{\sqrt{-e^2x^4+d^2}\,e^2x^5}{7}+\frac{16\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^4}{21}-\frac{4\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^3e}{5}$$

input
```
int((-e*x^2+d)^2*(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(25*sqrt(d**2 - e**2*x**4)*d**2*x - 42*sqrt(d**2 - e**2*x**4)*d*e*x**3 + 1
5*sqrt(d**2 - e**2*x**4)*e**2*x**5 + 80*int(sqrt(d**2 - e**2*x**4)/(d**2 -
 e**2*x**4),x)*d**4 - 84*int((sqrt(d**2 - e**2*x**4)*x**2)/(d**2 - e**2*x*
*4),x)*d**3*e)/105
```

3.86 $\int \left(d-ex^2\right)\sqrt{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 156

$$\begin{aligned}\int \left(d-ex^2\right)\sqrt{d^2-e^2x^4}\,dx &= \frac{1}{15}x\left(5d-3ex^2\right)\sqrt{d^2-e^2x^4}\\ &\quad -\frac{2d^{7/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{5\sqrt{e}\sqrt{d^2-e^2x^4}}\\ &\quad +\frac{16d^{7/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{15\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/15*x*(-3*e*x^2+5*d)*(-e^2*x^4+d^2)^(1/2)-2/5*d^(7/2)*(1-e^2*x^4/d^2)^(1/
2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+16/15*d^(7/
2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+
d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.56

$$\int \left(d-ex^2\right)\sqrt{d^2-e^2x^4}\,dx$$
$$=\frac{\sqrt{d^2-e^2x^4}\left(3dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-ex^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{e^2x^4}{d^2}\right)\right)}{3\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input
```
Integrate[(d - e*x^2)*Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(3*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, (e^2*x^4)/d^
2] - e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(3*Sqrt[1 -
(e^2*x^4)/d^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-ex^2\right)\sqrt{d^2-e^2x^4}\,dx$$
$$\Big\downarrow \text{1571}$$
$$\int \left(d-ex^2\right)\sqrt{d^2-e^2x^4}dx$$

input
```
Int[(d - e*x^2)*Sqrt[d^2 - e^2*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10

method	result
risch	$\frac{x\left(-3e\,x^{2}+5d\right)\sqrt{-e^{2}x^{4}+d^{2}}}{15}+\frac{2d^{2}\left(\frac{5d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{3d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}}\right.\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right.}{15}$
elliptic	$-\frac{e\,x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{5}+\frac{dx\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{2d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{2d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{}\right.\right.}{5\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}}}$
default	$d\left(\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{2d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right)-e\left(\frac{x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{5}-\frac{2d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{Elliptic}\right.}{5\sqrt{\frac{e}{d}}\,\sqrt{-}}\right.$

input
```
int((-e*x^2+d)*(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(-3*e*x^2+5*d)*(-e^2*x^4+d^2)^(1/2)+2/15*d^2*(5*d/(e/d)^(1/2)*(1-e*
x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2
),I)+3*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1
/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int \left(d-ex^{2}\right)\sqrt{d^{2}-e^{2}x^{4}}\,dx$$

$$=\frac{6\sqrt{-e^{2}}d^{3}x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)-2\left(3\,d^{3}-5\,d^{2}e\right)\sqrt{-e^{2}}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)-(3\,e^{3}x^{4}-5\,d}{15\,e^{2}x}$$

input
```
integrate((-e*x^2+d)*(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(6*sqrt(-e^2)*d^3*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - 2
*(3*d^3 - 5*d^2*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x),
-1) - (3*e^3*x^4 - 5*d*e^2*x^2 - 6*d^2*e)*sqrt(-e^2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.54

$$\int (d-ex^2)\sqrt{d^2-e^2x^4}\,dx = \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-e*x**2+d)*(-e**2*x**4+d**2)**(1/2),x)
```

output
```
d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) - d*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), e**2*
x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(7/4))
```

Maxima [F]

$$\int (d-ex^2)\sqrt{d^2-e^2x^4}\,dx = \int -\sqrt{-e^2x^4+d^2}\,(ex^2-d)\,dx$$

input
```
integrate((-e*x^2+d)*(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-e^2*x^4 + d^2)*(e*x^2 - d), x)
```

Giac [F]

$$\int (d-ex^2)\sqrt{d^2-e^2x^4}\,dx = \int -\sqrt{-e^2x^4+d^2}\,(ex^2-d)\,dx$$

input
```
integrate((-e*x^2+d)*(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(-e^2*x^4 + d^2)*(e*x^2 - d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d-ex^2)\sqrt{d^2-e^2x^4}\,dx = \int \sqrt{d^2-e^2\,x^4}\,\left(d-e\,x^2\right)\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)*(d - e*x^2),x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)*(d - e*x^2), x)
```

Reduce [F]

$$\int (d-ex^2)\sqrt{d^2-e^2x^4}\,dx = \frac{\sqrt{-e^2x^4+d^2}\,dx}{3} - \frac{\sqrt{-e^2x^4+d^2}\,e\,x^3}{5} + \frac{2\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^3}{3} - \frac{2\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^2e}{5}$$

input
```
int((-e*x^2+d)*(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(5*sqrt(d**2 - e**2*x**4)*d*x - 3*sqrt(d**2 - e**2*x**4)*e*x**3 + 10*int(s
qrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)*d**3 - 6*int((sqrt(d**2 - e**2
*x**4)*x**2)/(d**2 - e**2*x**4),x)*d**2*e)/15
```

3.87 $\int \sqrt{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int \sqrt{d^2-e^2x^4}\,dx=\frac{1}{3}x\sqrt{d^2-e^2x^4}+\frac{2d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
1/3*x*(-e^2*x^4+d^2)^(1/2)+2/3*d^(5/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(
1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \sqrt{d^2-e^2x^4}\,dx=\frac{d^2x-e^2x^5+\frac{2ide\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)}{\left(-\frac{e}{d}\right)^{3/2}}}{3\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4],x]
```

output
```
(d^2*x - e^2*x^5 + ((2*I)*d*e*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[
Sqrt[-(e/d)]*x], -1])/(-(e/d))^(3/2))/(3*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {748, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d^2-e^2x^4}\,dx$$

$$\Big\downarrow \text{748}$$

$$\frac{2}{3}d^2\int \frac{1}{\sqrt{d^2-e^2x^4}}dx+\frac{1}{3}x\sqrt{d^2-e^2x^4}$$

$$\Big\downarrow \text{765}$$

$$\frac{2d^2\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{3\sqrt{d^2-e^2x^4}}+\frac{1}{3}x\sqrt{d^2-e^2x^4}$$

$$\Big\downarrow \text{762}$$

$$\frac{2d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{1}{3}x\sqrt{d^2-e^2x^4}$$

input
```
Int[Sqrt[d^2 - e^2*x^4],x]
```

output
```
(x*Sqrt[d^2 - e^2*x^4])/3 + (2*d^(5/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[A
rcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(3*Sqrt[e]*Sqrt[d^2 - e^2*x^4])
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{x\sqrt{-e^2x^4+d^2}}{3}+\frac{2d^2\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$	82
risch	$\frac{x\sqrt{-e^2x^4+d^2}}{3}+\frac{2d^2\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$	82
elliptic	$\frac{x\sqrt{-e^2x^4+d^2}}{3}+\frac{2d^2\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$	82

input
```
int((-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(-e^2*x^4+d^2)^(1/2)+2/3*d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/
d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int \sqrt{d^2-e^2x^4}\,dx = \frac{2\sqrt{-e^2}d\sqrt{\frac{d}{e}}F\left(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1\right)+\sqrt{-e^2x^4+d^2}ex}{3\,e}$$

input

```
integrate((-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output

```
1/3*(2*sqrt(-e^2)*d*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1) + sqrt(-
e^2*x^4 + d^2)*e*x)/e
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.46

$$\int \sqrt{d^2-e^2x^4}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((-e**2*x**4+d**2)**(1/2),x)
```

output

```
d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2
)/(4*gamma(5/4))
```

Maxima [F]

$$\int \sqrt{d^2-e^2x^4}\,dx = \int \sqrt{-e^2x^4+d^2}\,dx$$

input

```
integrate((-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \sqrt{d^2-e^2x^4}\,dx=\int \sqrt{-e^2x^4+d^2}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \sqrt{d^2-e^2x^4}\,dx=\frac{x\,\sqrt{d^2-e^2\,x^4}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ \frac{e^2\,x^4}{d^2}\right)}{\sqrt{1-\frac{e^2\,x^4}{d^2}}}$$

input
```
int((d^2 - e^2*x^4)^(1/2),x)
```

output
```
(x*(d^2 - e^2*x^4)^(1/2)*hypergeom([-1/2, 1/4], 5/4, (e^2*x^4)/d^2))/(1 -
(e^2*x^4)/d^2)^(1/2)
```

Reduce [F]

$$\int \sqrt{d^2-e^2x^4}\,dx=\frac{\sqrt{-e^2x^4+d^2}\,x}{3}+\frac{2\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^2}{3}$$

input
```
int((-e^2*x^4+d^2)^(1/2),x)
```

output

```
(sqrt(d**2 - e**2*x**4)*x + 2*int(sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4
),x)*d**2)/3
```

3.88 $\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 59

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2} \, dx = \frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2
*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2} \, dx = \frac{ie\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)}{\left(-\frac{e}{d}\right)^{3/2}\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d - e*x^2),x]
```

output
```
(I*e*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1])/((-
(e/d))^(3/2)*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1396, 329, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 329}$$

$$\frac{d\sqrt{d^2-e^2x^4}\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{(d-ex^2)(d+ex^2)}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{d^{3/2}\sqrt{d^2-e^2x^4}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}(d-ex^2)(d+ex^2)}$$

input
```
Int[Sqrt[d^2 - e^2*x^4]/(d - e*x^2),x]
```

output
```
(d^(3/2)*Sqrt[d^2 - e^2*x^4]*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqr
t[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*(d - e*x^2)*(d + e*x^2))
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(47) = 94$.

Time = 1.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.34

method	result	size
default	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	138
elliptic	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	138

input
```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*Ell
ipticF(x*(e/d)^(1/2),I)-d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/
(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}\,dx = -\frac{\sqrt{-e^2}dx\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) - \sqrt{-e^2}(d+e)x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) + \sqrt{-e^2x^4+d^2}e}{e^2x}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d),x, algorithm="fricas")
```

output
```
-(sqrt(-e^2)*d*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - sqrt(-e^2
)*(d + e)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1) + sqrt(-e^2*x^4
+ d^2)*e)/(e^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}\,dx = -\int \frac{\sqrt{d^2-e^2x^4}}{-d+ex^2}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(-e*x**2+d),x)
```

output
```
-Integral(sqrt(d**2 - e**2*x**4)/(-d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}\,dx = \int -\frac{\sqrt{-e^2x^4+d^2}}{ex^2-d}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d),x, algorithm="giac")
```

output
```
integrate(-sqrt(-e^2*x^4 + d^2)/(e*x^2 - d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}\,dx = \int \frac{\sqrt{d^2-e^2\,x^4}}{d-e\,x^2}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2),x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{d-ex^2}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{-e\,x^2+d}dx$$

input

```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d - e*x**2),x)
```

3.89 $\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^2}\,dx = \frac{x(d+ex^2)}{d\sqrt{d^2-e^2x^4}} - \frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
x*(e*x^2+d)/d/(-e^2*x^4+d^2)^(1/2)-d^(1/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE
(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+d^(1/2)*(1-e^2*x^4/d^2)
^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^2}\,dx = -\frac{x\sqrt{d^2-e^2x^4}}{d\left(-d+ex^2\right)} + \frac{i\sqrt{1-\frac{ex^2}{d}}\sqrt{1+\frac{ex^2}{d}}\left(E\left(i\,\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)\right)}{\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d - e*x^2)^2,x]
```

output
```
-((x*Sqrt[d^2 - e^2*x^4])/(d*(-d + e*x^2))) + (I*Sqrt[1 - (e*x^2)/d]*Sqrt[
1 + (e*x^2)/d]*(EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - EllipticF[I*Arc
Sinh[Sqrt[-(e/d)]*x], -1]))/(Sqrt[-(e/d)]*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.58, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {1396, 314, 27, 344, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^2}dx$$

$$\Big\downarrow \; 1396$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{ex^2+d}}{(d-ex^2)^{3/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 314$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{\int \frac{ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\int \frac{x^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 344$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\int \frac{x^2}{\sqrt{d^2-e^2x^4}}dx}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 836$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\int \frac{ex^2+d}{d\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 1390$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{ex^2+d}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}-\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[Sqrt[d^2 - e^2*x^4]/(d - e*x^2)^2,x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d + e*x^2])/(d*Sqrt[d - e*x^2]) - (e*Sqrt[d^
2 - e^2*x^4]*((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x
)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^2*x^4]) - (d^(3/2)*Sqrt[1 - (e^2*x^
4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^
2*x^4])))/(d*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/(Sqrt[d - e*x^2]*Sqrt[d +
e*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 344
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c
+ b*d*x^4)^FracPart[p])   Int[(e*x)^m*(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a
, b, c, d, e, m, p}, x] && EqQ[b*c + a*d, 0] &&  !IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\left(-e^{2} x^{2}-d e \right) x}{e d \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	124
elliptic	$-\frac{\left(-e^{2} x^{2}-d e \right) x}{e d \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	124

input
```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2-d*e)/e/d*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/(e/d)^(1/2)*(1-e*
x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/
2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^2}\,dx = -\frac{\left(ex^2-d\right)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1\right)-\left(ex^2-d\right)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1\right)+\sqrt{-e^2x^4+d^2}\,x}{dex^2-d^2}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^2,x, algorithm="fricas")
```

output
```
-((e*x^2 - d)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - (e*x^2 - d)*
sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*x)/(d
*e*x^2 - d^2)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^2}\,dx = \int \frac{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}}{\left(-d+ex^2\right)^2}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(-e*x**2+d)**2,x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/(-d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^2}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{\left(ex^2-d\right)^2}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 - d)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^2}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{(ex^2-d)^2}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 - d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^2}\,dx=\int \frac{\sqrt{d^2-e^2\,x^4}}{(d-e\,x^2)^2}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2)^2,x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^2}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^2x^4-2de\,x^2+d^2}dx$$

input
```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^2,x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**2 - 2*d*e*x**2 + e**2*x**4),x)
```

3.90 $\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 193

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^3}\,dx = \frac{x\sqrt{d^2-e^2x^4}}{3d\left(d-ex^2\right)^2} + \frac{x\sqrt{d^2-e^2x^4}}{2d^2\left(d-ex^2\right)} - \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{2\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
1/3*x*(-e^2*x^4+d^2)^(1/2)/d/(-e*x^2+d)^2+1/2*x*(-e^2*x^4+d^2)^(1/2)/d^2/(
-e*x^2+d)-1/2*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(1/2)
/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+2/3*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*
x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.58 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^3}\,dx$$
$$=\frac{\sqrt{-\frac{e}{d}}x(5d^2+2dex^2-3e^2x^4)+3id(d-ex^2)\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-4id(d-ex^2)\sqrt{1-}}{6d^2\sqrt{-\frac{e}{d}}\,(d-ex^2)\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d - e*x^2)^3,x]
```

output
```
(Sqrt[-(e/d)]*x*(5*d^2 + 2*d*e*x^2 - 3*e^2*x^4) + (3*I)*d*(d - e*x^2)*Sqrt
[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - (4*I)*d*(d
- e*x^2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1])
/(6*d^2*Sqrt[-(e/d)]*(d - e*x^2)*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {1396, 314, 25, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^3}\,dx$$
$$\Big\downarrow \text{ 1396}$$
$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{ex^2+d}}{(d-ex^2)^{5/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$
$$\Big\downarrow \text{ 314}$$
$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}-\frac{\int -\frac{ex^2+2d}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{3d}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+2d}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\int \frac{de\left(d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\int \frac{d-3ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 399$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{4d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 289$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 329$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{4d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{4d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[Sqrt[d^2 - e^2*x^4]/(d - e*x^2)^3,x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d + e*x^2])/(3*d*(d - e*x^2)^(3/2)) + ((3*x*
Sqrt[d + e*x^2])/(2*d*Sqrt[d - e*x^2]) + ((-3*d^(3/2)*Sqrt[1 - (e^2*x^4)/d
^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*S
qrt[d + e*x^2]) + (4*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqr
t[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/(2*d))/(
3*d)))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.17

method	result
default	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3d\,e^{2}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{\left(-e^{2}x^{2}-de\right)x}{2d^{2}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\right.\right.}{2d\sqrt{\frac{e}{d}}\,\sqrt{-e}}$
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3d\,e^{2}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{\left(-e^{2}x^{2}-de\right)x}{2d^{2}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\right.\right.}{2d\sqrt{\frac{e}{d}}\,\sqrt{-e}}$

input
```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/d*x/e^2*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-1/2*(-e^2*x^2-d*e)/d^2*x/e/((
x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/6/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^
2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+1/2/d/(e/d)^(1/
2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(
e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^3}\,dx = -\frac{3\,(e^3x^4-2\,de^2x^2+d^2e)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)-((de^2+3\,e^3)x^4+d^3+3\,d^2e-2\,(d^2e+3\,de^2)x^2}{6\,(d^2e^3x^4-2\,d^3e^2x^2+d^4e)}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^3,x, algorithm="fricas")
```

output
```
-1/6*(3*(e^3*x^4 - 2*d*e^2*x^2 + d^2*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt
(e/d)), -1) - ((d*e^2 + 3*e^3)*x^4 + d^3 + 3*d^2*e - 2*(d^2*e + 3*d*e^2)*x
^2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*(
3*e^2*x^3 - 5*d*e*x))/(d^2*e^3*x^4 - 2*d^3*e^2*x^2 + d^4*e)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^3}\,dx = -\int \frac{\sqrt{d^2-e^2x^4}}{-d^3+3d^2ex^2-3de^2x^4+e^3x^6}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(-e*x**2+d)**3,x)
```

output
```
-Integral(sqrt(d**2 - e**2*x**4)/(-d**3 + 3*d**2*e*x**2 - 3*d*e**2*x**4 +
e**3*x**6), x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^3}\,dx=\int -\frac{\sqrt{-e^2x^4+d^2}}{(ex^2-d)^3}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 - d)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^3}\,dx=\int -\frac{\sqrt{-e^2x^4+d^2}}{(ex^2-d)^3}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-sqrt(-e^2*x^4 + d^2)/(e*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^3}\,dx=\int \frac{\sqrt{d^2-e^2\,x^4}}{(d-e\,x^2)^3}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2)^3,x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^3}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{-e^3x^6+3d\,e^2x^4-3d^2e\,x^2+d^3}dx$$

input

```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^3,x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**3 - 3*d**2*e*x**2 + 3*d*e**2*x**4 - e**3*x*
*6),x)
```

3.91 $\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\begin{aligned}\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^4}\,dx &= \frac{x\sqrt{d^2-e^2x^4}}{5d\left(d-ex^2\right)^3}+\frac{7x\sqrt{d^2-e^2x^4}}{30d^2\left(d-ex^2\right)^2}+\frac{2x\sqrt{d^2-e^2x^4}}{5d^3\left(d-ex^2\right)} \\ &\quad -\frac{2\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{5d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}} \\ &\quad +\frac{17\sqrt{1-\frac{e^2x^4}{d^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{30d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/5*x*(-e^2*x^4+d^2)^(1/2)/d/(-e*x^2+d)^3+7/30*x*(-e^2*x^4+d^2)^(1/2)/d^2/
(-e*x^2+d)^2+2/5*x*(-e^2*x^4+d^2)^(1/2)/d^3/(-e*x^2+d)-2/5*(1-e^2*x^4/d^2)
^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(3/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
+17/30*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(3/2)/e^(1/2
)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.78 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^4}\,dx$$
$$=\frac{\sqrt{-\frac{e}{d}}x(25d^3-6d^2ex^2-19de^2x^4+12e^3x^6)+12id(d-ex^2)^2\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-17}{30d^3\sqrt{-\frac{e}{d}}\left(d-ex^2\right)^2\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[Sqrt[d^2 - e^2*x^4]/(d - e*x^2)^4,x]
```

output

```
(Sqrt[-(e/d)]*x*(25*d^3 - 6*d^2*e*x^2 - 19*d*e^2*x^4 + 12*e^3*x^6) + (12*I
)*d*(d - e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]
*x], -1] - (17*I)*d*(d - e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcS
inh[Sqrt[-(e/d)]*x], -1])/(30*d^3*Sqrt[-(e/d)]*(d - e*x^2)^2*Sqrt[d^2 - e^
2*x^4])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.26, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {1396, 314, 25, 402, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^4}\,dx$$
$$\Big\downarrow \text{1396}$$
$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{ex^2+d}}{(d-ex^2)^{7/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$
$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}-\frac{\int -\frac{3ex^2+4d}{\left(d-ex^2\right)^{5/2}\sqrt{ex^2+d}}dx}{5d}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{3ex^2+4d}{\left(d-ex^2\right)^{5/2}\sqrt{ex^2+d}}dx}{5d}+\frac{x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\int \frac{de\left(7ex^2+17d\right)}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{6d^2e}+\frac{7x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\int \frac{7ex^2+17d}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{6d}+\frac{7x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\int \frac{2de\left(5d-12ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{12x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{7x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\int \frac{5d-12ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{12x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{7x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{17d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-12\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}+\frac{12x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{7x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 289}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{17d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-12\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}+\frac{12x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{7x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 329}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{17d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{12x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{7x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{17d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{12x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{7x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{17d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{12x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{7x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{17d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{12x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{7x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}}{5d}+\frac{x\sqrt{d+e}}{5d(d-ex}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[Sqrt[d^2 - e^2*x^4]/(d - e*x^2)^4,x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d + e*x^2])/(5*d*(d - e*x^2)^(5/2)) + ((7*x*
Sqrt[d + e*x^2])/(6*d*(d - e*x^2)^(3/2)) + ((12*x*Sqrt[d + e*x^2])/(d*Sqrt
[d - e*x^2]) + ((-12*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqr
t[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (17*d^(
3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(
Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/d)/(6*d))/(5*d)))/(Sqrt[d - e*x^
2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 314

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.15

method	result
default	$-\frac{x\sqrt{-e^2x^4+d^2}}{5d\,e^3\left(x^2-\frac{d}{e}\right)^3}+\frac{7x\sqrt{-e^2x^4+d^2}}{30d^2e^2\left(x^2-\frac{d}{e}\right)^2}-\frac{2\left(-e^2x^2-de\right)x}{5e\,d^3\sqrt{\left(x^2-\frac{d}{e}\right)\left(-e^2x^2-de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}+2$
elliptic	$-\frac{x\sqrt{-e^2x^4+d^2}}{5d\,e^3\left(x^2-\frac{d}{e}\right)^3}+\frac{7x\sqrt{-e^2x^4+d^2}}{30d^2e^2\left(x^2-\frac{d}{e}\right)^2}-\frac{2\left(-e^2x^2-de\right)x}{5e\,d^3\sqrt{\left(x^2-\frac{d}{e}\right)\left(-e^2x^2-de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}+2$

input
```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/5/d*x/e^3*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^3+7/30/d^2*x/e^2*(-e^2*x^4+d^2
)^(1/2)/(x^2-d/e)^2-2/5*(-e^2*x^2-d*e)/e/d^3*x/((x^2-d/e)*(-e^2*x^2-d*e))^
(1/2)+1/6/d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^
2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+2/5/d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*
(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-Ellipti
cE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^4}\,dx =$$

$$-\frac{12\left(e^4x^6-3\,de^3x^4+3\,d^2e^2x^2-d^3e\right)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)-((5\,de^3+12\,e^4)x^6-3\,(5\,d^2e^2+12\,d}{30\,(d^3e^4x^6}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^4,x, algorithm="fricas")
```

output
```
-1/30*(12*(e^4*x^6 - 3*d*e^3*x^4 + 3*d^2*e^2*x^2 - d^3*e)*sqrt(e/d)*ellipt
ic_e(arcsin(x*sqrt(e/d)), -1) - ((5*d*e^3 + 12*e^4)*x^6 - 3*(5*d^2*e^2 + 1
2*d*e^3)*x^4 - 5*d^4 - 12*d^3*e + 3*(5*d^3*e + 12*d^2*e^2)*x^2)*sqrt(e/d)*
elliptic_f(arcsin(x*sqrt(e/d)), -1) + (12*e^3*x^5 - 31*d*e^2*x^3 + 25*d^2*
e*x)*sqrt(-e^2*x^4 + d^2))/(d^3*e^4*x^6 - 3*d^4*e^3*x^4 + 3*d^5*e^2*x^2 -
d^6*e)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^4}\,dx=\int \frac{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}}{\left(-d+ex^2\right)^4}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(-e*x**2+d)**4,x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/(-d + e*x**2)**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^4}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{\left(ex^2-d\right)^4}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 - d)^4, x)
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^4}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{\left(ex^2-d\right)^4}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 - d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^4}\,dx=\int \frac{\sqrt{d^2-e^2\,x^4}}{\left(d-e\,x^2\right)^4}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2)^4,x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2)^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^4}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^4x^8-4d\,e^3x^6+6d^2e^2x^4-4d^3e\,x^2+d^4}dx$$

input
```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^4,x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**4 - 4*d**3*e*x**2 + 6*d**2*e**2*x**4 - 4*d*
e**3*x**6 + e**4*x**8),x)
```

3.92 $\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 261

$$
\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^5}\,dx &= \frac{x\sqrt{d^2-e^2x^4}}{7d\left(d-ex^2\right)^4}+\frac{11x\sqrt{d^2-e^2x^4}}{70d^2\left(d-ex^2\right)^3}+\frac{41x\sqrt{d^2-e^2x^4}}{210d^3\left(d-ex^2\right)^2} \\
&\quad+\frac{7x\sqrt{d^2-e^2x^4}}{20d^4\left(d-ex^2\right)}-\frac{7\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{20d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}} \\
&\quad+\frac{53\sqrt{1-\frac{e^2x^4}{d^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{105d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}
\end{aligned}
$$

output

```
1/7*x*(-e^2*x^4+d^2)^(1/2)/d/(-e*x^2+d)^4+11/70*x*(-e^2*x^4+d^2)^(1/2)/d^2
/(-e*x^2+d)^3+41/210*x*(-e^2*x^4+d^2)^(1/2)/d^3/(-e*x^2+d)^2+7/20*x*(-e^2*
x^4+d^2)^(1/2)/d^4/(-e*x^2+d)-7/20*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)
*x/d^(1/2),I)/d^(5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+53/105*(1-e^2*x^4/d^2)^
(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.87 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^5}\,dx = \frac{-\frac{x\left(d+ex^2\right)\left(-355d^3+671d^2ex^2-523de^2x^4+147e^3x^6\right)}{\left(d-ex^2\right)^3}+\frac{id\sqrt{1-\frac{e^2x^4}{d^2}}\,(147E(i\text{arcsinh}(\sqrt{-\frac{e}{d}}x)|-1)-212\,\text{EllipticF}(i\text{arcsinh}(\sqrt{-\frac{e}{d}}x}{\sqrt{-\frac{e}{d}}}}{420d^4\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d - e*x^2)^5,x]
```

output
```
(-((x*(d + e*x^2)*(-355*d^3 + 671*d^2*e*x^2 - 523*d*e^2*x^4 + 147*e^3*x^6)
)/(d - e*x^2)^3) + (I*d*Sqrt[1 - (e^2*x^4)/d^2]*(147*EllipticE[I*ArcSinh[S
qrt[-(e/d)]*x], -1] - 212*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1]))/Sqrt[
-(e/d)])/(420*d^4*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.27, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1396, 314, 25, 402, 27, 402, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^5}\,dx$$

$$\Big\downarrow \ 1396$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{ex^2+d}}{\left(d-ex^2\right)^{9/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 314$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}-\frac{\int -\frac{5ex^2+6d}{\left(d-ex^2\right)^{7/2}\sqrt{ex^2+d}}dx}{7d}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{5ex^2+6d}{\left(d-ex^2\right)^{7/2}\sqrt{ex^2+d}}dx}{7d}+\frac{x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\int \frac{de\left(33ex^2+49d\right)}{\left(d-ex^2\right)^{5/2}\sqrt{ex^2+d}}dx}{10d^2e}+\frac{11x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\int \frac{33ex^2+49d}{\left(d-ex^2\right)^{5/2}\sqrt{ex^2+d}}dx}{10d}+\frac{11x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\int \frac{2de\left(41ex^2+106d\right)}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{6d^2e}+\frac{41x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\int \frac{41ex^2+106d}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{3d}+\frac{41x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{\int \frac{de\left(65d-147ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{147x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{41x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{\int \frac{65d-147ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{147x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{41x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{212d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-147\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{147x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{41x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{\frac{212d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-147\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{147x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{41x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 329$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{\frac{212d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{147d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{147x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{41x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 327$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\frac{\frac{\frac{\frac{212d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{147d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{147x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{41x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{x}{7d(}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 765$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{\frac{\frac{\frac{\frac{212d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{147d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{147x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{41x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{x}{7d(d}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \textcolor{blue}{762}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{\frac{\frac{\frac{\frac{212d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{147d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{147x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{41x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}}{10d}+\frac{11x\sqrt{d}}{10d(d-e}}{7d}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[Sqrt[d^2 - e^2*x^4]/(d - e*x^2)^5,x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d + e*x^2])/(7*d*(d - e*x^2)^(7/2)) + ((11*x
*Sqrt[d + e*x^2])/(10*d*(d - e*x^2)^(5/2)) + ((41*x*Sqrt[d + e*x^2])/(3*d*
(d - e*x^2)^(3/2)) + ((147*x*Sqrt[d + e*x^2])/(2*d*Sqrt[d - e*x^2]) + ((-1
47*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]],
-1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (212*d^(3/2)*Sqrt[1 - (e^
2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d -
e*x^2]*Sqrt[d + e*x^2]))/(2*d))/(3*d))/(10*d))/(7*d)))/(Sqrt[d - e*x^2]*Sq
rt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 3.90 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.13

method	result
default	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{7d\,e^{4}\left(x^{2}-\frac{d}{e}\right)^{4}}-\frac{11x\sqrt{-e^{2}x^{4}+d^{2}}}{70d^{2}e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+\frac{41x\sqrt{-e^{2}x^{4}+d^{2}}}{210e^{2}d^{3}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{7\left(-e^{2}x^{2}-de\right)x}{20d^{4}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{13\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\ \mathrm{Elliptic}}{84d^{3}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{7d\,e^{4}\left(x^{2}-\frac{d}{e}\right)^{4}}-\frac{11x\sqrt{-e^{2}x^{4}+d^{2}}}{70d^{2}e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+\frac{41x\sqrt{-e^{2}x^{4}+d^{2}}}{210e^{2}d^{3}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{7\left(-e^{2}x^{2}-de\right)x}{20d^{4}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{13\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\ \mathrm{Elliptic}}{84d^{3}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$

input
```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^5,x,method=_RETURNVERBOSE)
```

output
```
1/7/d*x/e^4*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^4-11/70/d^2/e^3*x*(-e^2*x^4+d^2
)^(1/2)/(x^2-d/e)^3+41/210/e^2/d^3*x*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-7/20
*(-e^2*x^2-d*e)/d^4*x/e/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+13/84/d^3/(e/d)^(
1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*
(e/d)^(1/2),I)+7/20/d^3/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-
e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^5}\,dx =$$
$$-\frac{147\left(e^5x^8-4\,de^4x^6+6\,d^2e^3x^4-4\,d^3e^2x^2+d^4e\right)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1\right)-\left(\left(65\,de^4+147\,e^5\right)x^8-\right.}{}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^5,x, algorithm="fricas")
```

output
```
-1/420*(147*(e^5*x^8 - 4*d*e^4*x^6 + 6*d^2*e^3*x^4 - 4*d^3*e^2*x^2 + d^4*e
)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - ((65*d*e^4 + 147*e^5)*x^
8 - 4*(65*d^2*e^3 + 147*d*e^4)*x^6 + 65*d^5 + 147*d^4*e + 6*(65*d^3*e^2 +
147*d^2*e^3)*x^4 - 4*(65*d^4*e + 147*d^3*e^2)*x^2)*sqrt(e/d)*elliptic_f(ar
csin(x*sqrt(e/d)), -1) + (147*e^4*x^7 - 523*d*e^3*x^5 + 671*d^2*e^2*x^3 -
355*d^3*e*x)*sqrt(-e^2*x^4 + d^2))/(d^4*e^5*x^8 - 4*d^5*e^4*x^6 + 6*d^6*e^
3*x^4 - 4*d^7*e^2*x^2 + d^8*e)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d-ex^2\right)^5}\,dx = -\int \frac{\sqrt{d^2-e^2x^4}}{-d^5+5d^4ex^2-10d^3e^2x^4+10d^2e^3x^6-5de^4x^8+e^5x^{10}}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(-e*x**2+d)**5,x)
```

output
```
-Integral(sqrt(d**2 - e**2*x**4)/(-d**5 + 5*d**4*e*x**2 - 10*d**3*e**2*x**
4 + 10*d**2*e**3*x**6 - 5*d*e**4*x**8 + e**5*x**10), x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^5}\,dx = \int -\frac{\sqrt{-e^2x^4+d^2}}{(ex^2-d)^5}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^5,x, algorithm="maxima")
```

output
```
-integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 - d)^5, x)
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^5}\,dx = \int -\frac{\sqrt{-e^2x^4+d^2}}{(ex^2-d)^5}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^5,x, algorithm="giac")
```

output
```
integrate(-sqrt(-e^2*x^4 + d^2)/(e*x^2 - d)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^5}\,dx = \int \frac{\sqrt{d^2-e^2\,x^4}}{(d-e\,x^2)^5}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2)^5,x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d - e*x^2)^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d-ex^2)^5}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^5x^{10}+5d\,e^4x^8-10d^2e^3x^6+10d^3e^2x^4-5d^4e\,x^2+d^5}dx$$

input

```
int((-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^5,x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**5 - 5*d**4*e*x**2 + 10*d**3*e**2*x**4 - 10*
d**2*e**3*x**6 + 5*d*e**4*x**8 - e**5*x**10),x)
```

3.93 $\int (d-ex^2)^3 (d^2-e^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 239

$$\int (d-ex^2)^3 (d^2-e^2x^4)^{3/2}\,dx = \frac{4}{715}d^4x(65d-77ex^2)\sqrt{d^2-e^2x^4}$$
$$+\frac{2}{429}d^2x(39d-77ex^2)\left(d^2-e^2x^4\right)^{3/2}-\frac{3}{11}dx\left(d^2-e^2x^4\right)^{5/2}$$
$$+\frac{1}{13}ex^3\left(d^2-e^2x^4\right)^{5/2}-\frac{56d^{15/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{65\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{1136d^{15/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\Big(\arcsin\Big(}{715\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
4/715*d^4*x*(-77*e*x^2+65*d)*(-e^2*x^4+d^2)^(1/2)+2/429*d^2*x*(-77*e*x^2+3
9*d)*(-e^2*x^4+d^2)^(3/2)-3/11*d*x*(-e^2*x^4+d^2)^(5/2)+1/13*e*x^3*(-e^2*x
^4+d^2)^(5/2)-56/65*d^(15/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(
1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+1136/715*d^(15/2)*(1-e^2*x^4/d^2)^(1/
2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.56

$$\int \left(d-ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2}\,dx =$$

$$-\frac{x\sqrt{d^2-e^2x^4}\left(\left(39d-11ex^2\right)\left(d^2-e^2x^4\right)^2\sqrt{1-\frac{e^2x^4}{d^2}}-182d^5\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)+154\right.}{143\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input
```
Integrate[(d - e*x^2)^3*(d^2 - e^2*x^4)^(3/2),x]
```

output
```
-1/143*(x*Sqrt[d^2 - e^2*x^4]*((39*d - 11*e*x^2)*(d^2 - e^2*x^4)^2*Sqrt[1
- (e^2*x^4)/d^2] - 182*d^5*Hypergeometric2F1[-3/2, 1/4, 5/4, (e^2*x^4)/d^2
] + 154*d^4*e*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, (e^2*x^4)/d^2]))/Sqrt[
1 - (e^2*x^4)/d^2]
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.56, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.704$, Rules used = {1396, 318, 27, 403, 27, 403, 27, 403, 27, 403, 27, 403, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \left(d-ex^2\right)^{9/2}\left(ex^2+d\right)^{3/2}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{\int -\frac{2de\left(d-ex^2\right)^{9/2}\left(8ex^2+7d\right)}{\sqrt{ex^2+d}}dx}{13e}-\frac{1}{13}x\sqrt{d+ex^2}\left(d-ex^2\right)^{11/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow\ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\int \frac{\left(d-ex^2\right)^{9/2}\left(8ex^2+7d\right)}{\sqrt{ex^2+d}}dx-\frac{1}{13}x\left(d-ex^2\right)^{11/2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow\ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\left(\frac{\int \frac{3de\left(d-ex^2\right)^{7/2}\left(25ex^2+23d\right)}{\sqrt{ex^2+d}}dx}{11e}+\frac{8}{11}x\sqrt{d+ex^2}\left(d-ex^2\right)^{9/2}\right)-\frac{1}{13}x\left(d-ex^2\right)^{11/2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow\ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\left(\frac{3}{11}d\int \frac{\left(d-ex^2\right)^{7/2}\left(25ex^2+23d\right)}{\sqrt{ex^2+d}}dx+\frac{8}{11}x\sqrt{d+ex^2}\left(d-ex^2\right)^{9/2}\right)-\frac{1}{13}x\left(d-ex^2\right)^{11/2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow\ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{13}d\Bigg(\frac{3}{11}d\left(\frac{\int \frac{14de\left(d-ex^2\right)^{5/2}\left(12ex^2+13d\right)}{\sqrt{ex^2+d}}dx}{9e}+\frac{25}{9}x\sqrt{d+ex^2}\left(d-ex^2\right)^{7/2}\right)+\frac{8}{11}x\sqrt{d+ex^2}\left(d-ex^2\right)^{9/2}\Bigg)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow\ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{13}d\Bigg(\frac{3}{11}d\left(\frac{14}{9}d\int \frac{\left(d-ex^2\right)^{5/2}\left(12ex^2+13d\right)}{\sqrt{ex^2+d}}dx+\frac{25}{9}x\sqrt{d+ex^2}\left(d-ex^2\right)^{7/2}\right)+\frac{8}{11}x\sqrt{d+ex^2}\left(d-ex^2\right)^{9/2}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow\ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{13}d\Bigg(\frac{3}{11}d\Bigg(\frac{14}{9}d\left(\frac{\int \frac{de\left(d-ex^2\right)^{3/2}\left(41ex^2+79d\right)}{\sqrt{ex^2+d}}dx}{7e}+\frac{12}{7}x\sqrt{d+ex^2}\left(d-ex^2\right)^{5/2}\right)+\frac{25}{9}x\sqrt{d+ex^2}\left(d-ex^2\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\left(\frac{3}{11}d\left(\frac{14}{9}d\left(\frac{1}{7}d\int\frac{(d-ex^2)^{3/2}(41ex^2+79d)}{\sqrt{ex^2+d}}dx+\frac{12}{7}x\sqrt{d+ex^2}(d-ex^2)^{5/2}\right)+\frac{25}{9}x\sqrt{d+ex^2}(d-ex^{2}\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\left(\frac{3}{11}d\left(\frac{14}{9}d\left(\frac{1}{7}d\left(\frac{\int\frac{6de(59d-18ex^2)\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx}{5e}+\frac{41}{5}x\sqrt{d+ex^2}(d-ex^2)^{3/2}\right)+\frac{12}{7}x\sqrt{d+ex^2}(d-e\right.\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\left(\frac{3}{11}d\left(\frac{14}{9}d\left(\frac{1}{7}d\left(\frac{6}{5}d\int\frac{(59d-18ex^2)\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx+\frac{41}{5}x\sqrt{d+ex^2}(d-ex^2)^{3/2}\right)+\frac{12}{7}x\sqrt{d+ex^2}(d-ex^{2}\right.\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+e}}$$

$$\Big\downarrow 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\left(\frac{3}{11}d\left(\frac{14}{9}d\left(\frac{1}{7}d\left(\frac{6}{5}d\left(\frac{\int\frac{3de(65d-77ex^2)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}-6x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{41}{5}x\sqrt{d+ex^2}(d-ex^2)^{3/2}\right.\right.\right.\right.\right.}{\sqrt{}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\left(\frac{3}{11}d\left(\frac{14}{9}d\left(\frac{1}{7}d\left(\frac{6}{5}d\left(d\int\frac{65d-77ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-6x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{41}{5}x\sqrt{d+ex^2}(d-ex^2)^{3/2}\right)\right.\right.\right.\right.}{\sqrt{d-}}$$

$$\Big\downarrow 399$$

$$\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\left(\frac{3}{11}d\left(\frac{14}{9}d\left(\frac{1}{7}d\left(\frac{6}{5}d\left(d\left(142d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-77\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)-6x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{41}{5}\right.\right.\right.\right.\right.$$

$$\Big\downarrow 289$$

$$\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\left(\frac{3}{11}d\left(\frac{14}{9}d\left(\frac{1}{7}d\left(\frac{6}{5}d\left(d\left(\frac{142d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-77\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)-6x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)\right.\right.\right.\right.\right.$$

$$\Big\downarrow 329$$

$$\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\left(\frac{3}{11}d\left(\frac{14}{9}d\left(\frac{1}{7}d\left(\frac{6}{5}d\left(d\left(\frac{142d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-6x\sqrt{d-ex^2}\sqrt{\right.\right.\right.\right.\right.\right.$$

$$\Big\downarrow \textbf{327}$$

$$\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\left(\frac{3}{11}d\left(\frac{14}{9}d\left(\frac{1}{7}d\left(\frac{6}{5}d\left(d\left(\frac{142d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-6x\sqrt{d\right.\right.\right.\right.\right.\right.$$

$$\Big\downarrow \textbf{765}$$

$$\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\left(\frac{3}{11}d\left(\frac{14}{9}d\left(\frac{1}{7}d\left(\frac{6}{5}d\left(d\left(\frac{142d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-6x\sqrt{d\right.\right.\right.\right.\right.\right.$$

$$\Big\downarrow \textbf{762}$$

$$\sqrt{d^2-e^2x^4}\left(\frac{2}{13}d\left(\frac{3}{11}d\left(\frac{14}{9}d\left(\frac{1}{7}d\left(\frac{6}{5}d\left(d\left(\frac{142d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right.}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right.\right.\right.\right.\right.\right.\right.$$

input
```
Int[(d - e*x^2)^3*(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/13*(x*(d - e*x^2)^(11/2)*Sqrt[d + e*x^2]) + (2*d*
((8*x*(d - e*x^2)^(9/2)*Sqrt[d + e*x^2])/11 + (3*d*((25*x*(d - e*x^2)^(7/2
)*Sqrt[d + e*x^2])/9 + (14*d*((12*x*(d - e*x^2)^(5/2)*Sqrt[d + e*x^2])/7 +
 (d*((41*x*(d - e*x^2)^(3/2)*Sqrt[d + e*x^2])/5 + (6*d*(-6*x*Sqrt[d - e*x^
2]*Sqrt[d + e*x^2] + d*((-77*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[Arc
Sin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) +
 (142*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]
], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))))/5))/7))/9))/11))/13))
/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 7.98 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.90

method	result
risch	$\frac{x\left(165e^5x^{10}-585d\,e^4x^8+440d^2e^3x^6+780d^3e^2x^4-1529d^4e\,x^2+585d^5\right)\sqrt{-e^2x^4+d^2}}{2145}+\frac{8d^6\left(\frac{65d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right.}{}$
elliptic	$\frac{e^5x^{11}\sqrt{-e^2x^4+d^2}}{13}-\frac{3e^4d\,x^9\sqrt{-e^2x^4+d^2}}{11}+\frac{8d^2e^3x^7\sqrt{-e^2x^4+d^2}}{39}+\frac{4d^3e^2x^5\sqrt{-e^2x^4+d^2}}{11}-\frac{139d^4e\,x^3\sqrt{-e^2x^4+d^2}}{195}+3d^5$
default	$d^3\left(-\frac{e^2x^5\sqrt{-e^2x^4+d^2}}{7}+\frac{3d^2x\sqrt{-e^2x^4+d^2}}{7}+\frac{4d^4\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{7\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)-e^3\left(-\frac{e^2x^{11}\sqrt{-e^2x^4+d^2}}{13}\right.$

input
```
int((-e*x^2+d)^3*(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2145*x*(165*e^5*x^10-585*d*e^4*x^8+440*d^2*e^3*x^6+780*d^3*e^2*x^4-1529*
d^4*e*x^2+585*d^5)*(-e^2*x^4+d^2)^(1/2)+8/715*d^6*(65*d/(e/d)^(1/2)*(1-e*x
^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2)
,I)+77*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1
/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.71

$$\int \left(d-ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2}\,dx = \frac{1848\sqrt{-e^2}d^7x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)-24\left(77d^7-65d^6e\right)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)}{}$$

input
```
integrate((-e*x^2+d)^3*(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2145*(1848*sqrt(-e^2)*d^7*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1
) - 24*(77*d^7 - 65*d^6*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d
/e)/x), -1) + (165*e^7*x^12 - 585*d*e^6*x^10 + 440*d^2*e^5*x^8 + 780*d^3*e
^4*x^6 - 1529*d^4*e^3*x^4 + 585*d^5*e^2*x^2 + 1848*d^6*e)*sqrt(-e^2*x^4 +
d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.17

$$\int \left(d-ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2}\,dx = \frac{d^6x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix} -\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{3d^5ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix} -\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{d^4e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix} -\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{d^3e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix} -\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{11}{4}\right)} - \frac{3d^2e^4x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix} -\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{13}{4}\right)} + \frac{de^5x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix} -\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((-e*x**2+d)**3*(-e**2*x**4+d**2)**(3/2),x)
```

output
```
d**6*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) - 3*d**5*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,),
e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(7/4)) + d**4*e**2*x**5*gamma(5/
4)*hyper((-1/2, 5/4), (9/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(2*gamma(9
/4)) + d**3*e**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), e**2*x**4*exp
_polar(2*I*pi)/d**2)/(2*gamma(11/4)) - 3*d**2*e**4*x**9*gamma(9/4)*hyper((
-1/2, 9/4), (13/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(13/4)) + d
*e**5*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), e**2*x**4*exp_polar(2
*I*pi)/d**2)/(4*gamma(15/4))
```

Maxima [F]

$$\int \left(d-ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2}\,dx=\int -\left(-e^2x^4+d^2\right)^{\frac{3}{2}}\left(ex^2-d\right)^3\,dx$$

input
```
integrate((-e*x^2+d)^3*(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((-e^2*x^4 + d^2)^(3/2)*(e*x^2 - d)^3, x)
```

Giac [F]

$$\int \left(d-ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2}\,dx=\int -\left(-e^2x^4+d^2\right)^{\frac{3}{2}}\left(ex^2-d\right)^3\,dx$$

input
```
integrate((-e*x^2+d)^3*(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(-e^2*x^4 + d^2)^(3/2)*(e*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(d^2-e^2\,x^4\right)^{3/2}\left(d-e\,x^2\right)^3\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)^3,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)^3, x)
```

Reduce [F]

$$\int \left(d-ex^2\right)^3 \left(d^2-e^2x^4\right)^{3/2}\,dx = \frac{3\sqrt{-e^2x^4+d^2}\,d^5x}{11} - \frac{139\sqrt{-e^2x^4+d^2}\,d^4e\,x^3}{195} + \frac{4\sqrt{-e^2x^4+d^2}\,d^3e^2x^5}{11} + \frac{8\sqrt{-e^2x^4+d^2}\,d^2e^3x^7}{39} - \frac{3\sqrt{-e^2x^4+d^2}\,d\,e^4x^9}{11} + \frac{\sqrt{-e^2x^4+d^2}\,e^5x^{11}}{13} + \frac{8\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^7}{11} - \frac{56\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^6e}{65}$$

input
```
int((-e*x^2+d)^3*(-e^2*x^4+d^2)^(3/2),x)
```

output
```
(585*sqrt(d**2 - e**2*x**4)*d**5*x - 1529*sqrt(d**2 - e**2*x**4)*d**4*e*x*
*3 + 780*sqrt(d**2 - e**2*x**4)*d**3*e**2*x**5 + 440*sqrt(d**2 - e**2*x**4
)*d**2*e**3*x**7 - 585*sqrt(d**2 - e**2*x**4)*d*e**4*x**9 + 165*sqrt(d**2
- e**2*x**4)*e**5*x**11 + 1560*int(sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**
4),x)*d**7 - 1848*int((sqrt(d**2 - e**2*x**4)*x**2)/(d**2 - e**2*x**4),x)*
d**6*e)/2145
```

3.94 $\int (d-ex^2)^2 (d^2-e^2x^4)^{3/2}\, dx$

Optimal result

Integrand size = 27, antiderivative size = 212

$$\begin{aligned}\int (d-ex^2)^2 (d^2-e^2x^4)^{3/2}\, dx &= \frac{4d^3x(90d-77ex^2)\sqrt{d^2-e^2x^4}}{1155} \\ &+ \frac{2}{693}dx(54d-77ex^2)\left(d^2-e^2x^4\right)^{3/2} \\ &- \frac{1}{11}x\left(d^2-e^2x^4\right)^{5/2} - \frac{8d^{13/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{15\sqrt{e}\sqrt{d^2-e^2x^4}} \\ &+ \frac{1336d^{13/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{1155\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
4/1155*d^3*x*(-77*e*x^2+90*d)*(-e^2*x^4+d^2)^(1/2)+2/693*d*x*(-77*e*x^2+54
*d)*(-e^2*x^4+d^2)^(3/2)-1/11*x*(-e^2*x^4+d^2)^(5/2)-8/15*d^(13/2)*(1-e^2*
x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
+1336/1155*d^(13/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e
^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.64 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.59

$$\int \left(d-ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2}\,dx = \frac{x\sqrt{d^2-e^2x^4}\left(-3(d^2-e^2x^4)^2\sqrt{1-\frac{e^2x^4}{d^2}}+36d^4\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-22\right.}{33\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input
```
Integrate[(d - e*x^2)^2*(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(x*Sqrt[d^2 - e^2*x^4]*(-3*(d^2 - e^2*x^4)^2*Sqrt[1 - (e^2*x^4)/d^2] + 36*
d^4*Hypergeometric2F1[-3/2, 1/4, 5/4, (e^2*x^4)/d^2] - 22*d^3*e*x^2*Hyperg
eometric2F1[-3/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(33*Sqrt[1 - (e^2*x^4)/d^2])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.61, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {1396, 318, 27, 403, 27, 403, 27, 403, 27, 403, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \left(d-ex^2\right)^{7/2}\left(ex^2+d\right)^{3/2}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{\int -\frac{2de\left(d-ex^2\right)^{7/2}\left(7ex^2+6d\right)}{\sqrt{ex^2+d}}dx}{11e}-\frac{1}{11}x\sqrt{d+ex^2}\left(d-ex^2\right)^{9/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\int \frac{\left(d-ex^2\right)^{7/2}\left(7ex^2+6d\right)}{\sqrt{ex^2+d}}dx-\frac{1}{11}x\left(d-ex^2\right)^{9/2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{\int \frac{de\left(d-ex^2\right)^{5/2}\left(51ex^2+47d\right)}{\sqrt{ex^2+d}}dx}{9e}+\frac{7}{9}x\sqrt{d+ex^2}\left(d-ex^2\right)^{7/2}\right)-\frac{1}{11}x\left(d-ex^2\right)^{9/2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\int \frac{\left(d-ex^2\right)^{5/2}\left(51ex^2+47d\right)}{\sqrt{ex^2+d}}dx+\frac{7}{9}x\sqrt{d+ex^2}\left(d-ex^2\right)^{7/2}\right)-\frac{1}{11}x\left(d-ex^2\right)^{9/2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{\int \frac{2de\left(d-ex^2\right)^{3/2}\left(116ex^2+139d\right)}{\sqrt{ex^2+d}}dx}{7e}+\frac{51}{7}x\sqrt{d+ex^2}\left(d-ex^2\right)^{5/2}\right)+\frac{7}{9}x\sqrt{d+ex^2}\left(d-ex^2\right)^{7/2}\right)\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\int \frac{\left(d-ex^2\right)^{3/2}\left(116ex^2+139d\right)}{\sqrt{ex^2+d}}dx+\frac{51}{7}x\sqrt{d+ex^2}\left(d-ex^2\right)^{5/2}\right)+\frac{7}{9}x\sqrt{d+ex^2}\left(d-ex^2\right)^{7/2}\right)\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{\int \frac{3de\sqrt{d-ex^2}\left(39ex^2+193d\right)}{\sqrt{ex^2+d}}dx}{5e}+\frac{116}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)+\frac{51}{7}x\sqrt{d+ex^2}\left(d-ex^2\right)^{5/2}\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\int \frac{\sqrt{d-ex^2}\left(39ex^2+193d\right)}{\sqrt{ex^2+d}}dx+\frac{116}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)+\frac{51}{7}x\sqrt{d+ex^2}\left(d-ex^2\right)^{5/2}\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\left(\frac{\int \frac{6de\left(90d-77ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}+13x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{116}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)+\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\left(2d\int \frac{90d-77ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+13x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{116}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)+\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\left(2d\left(167d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-77\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)+13x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{116}{5}x\right.\right.\right.\right.}{\sqrt{d-ex^2}}$$

$$\Big\downarrow \; 289$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\left(2d\left(\frac{167d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-77\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)+13x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)+\frac{11}{5}\right.\right.\right.\right.}{\sqrt{d-e}}$$

$$\Big\downarrow \; 329$$

$$\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\left(2d\left(\frac{167d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+13x\sqrt{d-ex^2}\sqrt{d+\right.\right.\right.\right.\right.$$

$$\Big\downarrow \; 327$$

$$\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\left(2d\left(\frac{167d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+13x\sqrt{d-ex\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{765}$$

$$\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\left(2d\left(\frac{167d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+13x\sqrt{d-e\right.\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{762}$$

$$\sqrt{d^2-e^2x^4}\left(\frac{2}{11}d\left(\frac{1}{9}d\left(\frac{2}{7}d\left(\frac{3}{5}d\left(2d\left(\frac{167d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+\right.\right.\right.\right.\right.\right.$$

input
```
Int[(d - e*x^2)^2*(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/11*(x*(d - e*x^2)^(9/2)*Sqrt[d + e*x^2]) + (2*d*(
(7*x*(d - e*x^2)^(7/2)*Sqrt[d + e*x^2])/9 + (d*((51*x*(d - e*x^2)^(5/2)*Sq
rt[d + e*x^2])/7 + (2*d*((116*x*(d - e*x^2)^(3/2)*Sqrt[d + e*x^2])/5 + (3*
d*(13*x*Sqrt[d - e*x^2]*Sqrt[d + e*x^2] + 2*d*((-77*d^(3/2)*Sqrt[1 - (e^2*
x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*
x^2]*Sqrt[d + e*x^2]) + (167*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[Arc
Sin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))
)/5))/7))/9))/11))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 5.75 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.97

method	result
risch	$\frac{x\left(-315e^4x^8+770d\,e^3x^6+90d^2e^2x^4-1694d^3e\,x^2+1305d^4\right)\sqrt{-e^2x^4+d^2}}{3465}+\frac{8d^5\left(\frac{90d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}+\frac{77d\sqrt{1-}}{}\right.}{115}$
elliptic	$-\frac{e^4x^9\sqrt{-e^2x^4+d^2}}{11}+\frac{2d\,e^3x^7\sqrt{-e^2x^4+d^2}}{9}+\frac{2d^2e^2x^5\sqrt{-e^2x^4+d^2}}{77}-\frac{22d^3e\,x^3\sqrt{-e^2x^4+d^2}}{45}+\frac{29d^4x\sqrt{-e^2x^4+d^2}}{77}+\frac{48d^6}{}$
default	$d^2\left(-\frac{e^2x^5\sqrt{-e^2x^4+d^2}}{7}+\frac{3d^2x\sqrt{-e^2x^4+d^2}}{7}+\frac{4d^4\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{7\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)+e^2\left(-\frac{e^2x^9\sqrt{-e^2x^4+d^2}}{11}\right.$

input

```
int((-e*x^2+d)^2*(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/3465*x*(-315*e^4*x^8+770*d*e^3*x^6+90*d^2*e^2*x^4-1694*d^3*e*x^2+1305*d^
4)*(-e^2*x^4+d^2)^(1/2)+8/1155*d^5*(90*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+
e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+77*d/(e/d)^
(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(
x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.75

$$\int \left(d-ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2}\,dx = \frac{1848\sqrt{-e^2}d^6x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)-24\left(77\,d^6-90\,d^5e\right)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)}{}$$

input
```
integrate((-e*x^2+d)^2*(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3465*(1848*sqrt(-e^2)*d^6*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1
) - 24*(77*d^6 - 90*d^5*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d
/e)/x), -1) - (315*e^6*x^10 - 770*d*e^5*x^8 - 90*d^2*e^4*x^6 + 1694*d^3*e^
3*x^4 - 1305*d^4*e^2*x^2 - 1848*d^5*e)*sqrt(-e^2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \left(d-ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2}\,dx &= \frac{d^5x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} \\ &- \frac{d^4ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{d^2e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{11}{4}\right)} \\ &- \frac{de^4x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{13}{4}\right)}\end{aligned}$$

input
```
integrate((-e*x**2+d)**2*(-e**2*x**4+d**2)**(3/2),x)
```

output
```
d**5*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) - d**4*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), e*
*2*x**4*exp_polar(2*I*pi)/d**2)/(2*gamma(7/4)) + d**2*e**3*x**7*gamma(7/4)
*hyper((-1/2, 7/4), (11/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(2*gamma(11
/4)) - d*e**4*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), e**2*x**4*exp_po
lar(2*I*pi)/d**2)/(4*gamma(13/4))
```

Maxima [F]

$$\int (d-ex^2)^2 (d^2-e^2x^4)^{3/2}\,dx = \int (-e^2x^4+d^2)^{\frac{3}{2}}(ex^2-d)^2\,dx$$

input
```
integrate((-e*x^2+d)^2*(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)*(e*x^2 - d)^2, x)
```

Giac [F]

$$\int (d-ex^2)^2 (d^2-e^2x^4)^{3/2}\,dx = \int (-e^2x^4+d^2)^{\frac{3}{2}}(ex^2-d)^2\,dx$$

input
```
integrate((-e*x^2+d)^2*(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)*(e*x^2 - d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(d^2-e^2\,x^4\right)^{3/2}\left(d-e\,x^2\right)^2\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)^2,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)^2, x)
```

Reduce [F]

$$\begin{aligned}\int \left(d-ex^2\right)^2 \left(d^2-e^2x^4\right)^{3/2}\,dx&=\frac{29\sqrt{-e^2x^4+d^2}\,d^4x}{77}\\&-\frac{22\sqrt{-e^2x^4+d^2}\,d^3e\,x^3}{45}+\frac{2\sqrt{-e^2x^4+d^2}\,d^2e^2x^5}{77}+\frac{2\sqrt{-e^2x^4+d^2}\,d\,e^3x^7}{9}\\&-\frac{\sqrt{-e^2x^4+d^2}\,e^4x^9}{11}+\frac{48\left(\int\frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^6}{77}-\frac{8\left(\int\frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^5e}{15}\end{aligned}$$

input
```
int((-e*x^2+d)^2*(-e^2*x^4+d^2)^(3/2),x)
```

output
```
(1305*sqrt(d**2 - e**2*x**4)*d**4*x - 1694*sqrt(d**2 - e**2*x**4)*d**3*e*x
**3 + 90*sqrt(d**2 - e**2*x**4)*d**2*e**2*x**5 + 770*sqrt(d**2 - e**2*x**4
)*d*e**3*x**7 - 315*sqrt(d**2 - e**2*x**4)*e**4*x**9 + 2160*int(sqrt(d**2
- e**2*x**4)/(d**2 - e**2*x**4),x)*d**6 - 1848*int((sqrt(d**2 - e**2*x**4)
*x**2)/(d**2 - e**2*x**4),x)*d**5*e)/3465
```

3.95 $\int (d-ex^2)\left(d^2-e^2x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 190

$$\begin{aligned}\int (d-ex^2)\left(d^2-e^2x^4\right)^{3/2}\,dx &= \frac{2}{105}d^2x\left(15d-7ex^2\right)\sqrt{d^2-e^2x^4}\\ &+\frac{1}{63}x\left(9d-7ex^2\right)\left(d^2-e^2x^4\right)^{3/2}-\frac{4d^{11/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{15\sqrt{e}\sqrt{d^2-e^2x^4}}\\ &+\frac{88d^{11/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{105\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output
```
2/105*d^2*x*(-7*e*x^2+15*d)*(-e^2*x^4+d^2)^(1/2)+1/63*x*(-7*e*x^2+9*d)*(-e
^2*x^4+d^2)^(3/2)-4/15*d^(11/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/
d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+88/105*d^(11/2)*(1-e^2*x^4/d^2)^(1
/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.48

$$\int (d-ex^2)\left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{d^2x\sqrt{d^2-e^2x^4}\left(3d\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-ex^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\right.\right.}{3\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input
```
Integrate[(d - e*x^2)*(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(d^2*x*Sqrt[d^2 - e^2*x^4]*(3*d*Hypergeometric2F1[-3/2, 1/4, 5/4, (e^2*x^4
)/d^2] - e*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(3*Sqrt[
1 - (e^2*x^4)/d^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d-ex^2)\left(d^2-e^2x^4\right)^{3/2}dx$$

$$\Big\downarrow \text{1571}$$

$$\int (d-ex^2)\left(d^2-e^2x^4\right)^{3/2}dx$$

input
```
Int[(d - e*x^2)*(d^2 - e^2*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.02

method	result
risch	$\frac{x\left(35e^{3}x^{6}-45d\,e^{2}x^{4}-77d^{2}e\,x^{2}+135d^{3}\right)\sqrt{-e^{2}x^{4}+d^{2}}}{315}+\frac{4d^{4}\Bigg(\frac{15d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\;\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{7d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\;(\mathrm{Ellipti}}{\sqrt{\frac{e}{d}}\,\sqrt{-}}}{105}$
elliptic	$\frac{e^{3}x^{7}\sqrt{-e^{2}x^{4}+d^{2}}}{9}-\frac{d\,e^{2}x^{5}\sqrt{-e^{2}x^{4}+d^{2}}}{7}-\frac{11d^{2}e\,x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{45}+\frac{3d^{3}x\sqrt{-e^{2}x^{4}+d^{2}}}{7}+\frac{4d^{5}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\;\operatorname{EllipticF}\Big(x}{7\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$
default	$d\left(-\frac{e^{2}x^{5}\sqrt{-e^{2}x^{4}+d^{2}}}{7}+\frac{3d^{2}x\sqrt{-e^{2}x^{4}+d^{2}}}{7}+\frac{4d^{4}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\;\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{7\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right)-e\Bigg(-\frac{e^{2}x^{7}\sqrt{-e^{2}x^{4}+d^{2}}}{9}+$

input
```
int((-e*x^2+d)*(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*x*(35*e^3*x^6-45*d*e^2*x^4-77*d^2*e*x^2+135*d^3)*(-e^2*x^4+d^2)^(1/2
)+4/105*d^4*(15*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^
4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+7*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*
(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-Ellipti
cE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.77

$$\int \left(d-ex^{2}\right)\left(d^{2}-e^{2}x^{4}\right)^{3/2}\,dx=\frac{84\sqrt{-e^{2}}d^{5}x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)-12\left(7\,d^{5}-15\,d^{4}e\right)\sqrt{-e^{2}}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-}{315\,e^{2}x}$$

input
```
integrate((-e*x^2+d)*(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(84*sqrt(-e^2)*d^5*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) -
 12*(7*d^5 - 15*d^4*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/
x), -1) + (35*e^5*x^8 - 45*d*e^4*x^6 - 77*d^2*e^3*x^4 + 135*d^3*e^2*x^2 +
84*d^4*e)*sqrt(-e^2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.95

$$\int \left(d-ex^2\right)\left(d^2-e^2x^4\right)^{3/2}\,dx = \frac{d^4x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{d^3ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{d^2e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{de^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((-e*x**2+d)*(-e**2*x**4+d**2)**(3/2),x)
```

output
```
d**4*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) - d**3*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), e*
*2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(7/4)) - d**2*e**2*x**5*gamma(5/4)
*hyper((-1/2, 5/4), (9/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(9/4
)) + d*e**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), e**2*x**4*exp_pola
r(2*I*pi)/d**2)/(4*gamma(11/4))
```

Maxima [F]

$$\int (d-ex^2)\left(d^2-e^2x^4\right)^{3/2}\,dx=\int -\left(-e^2x^4+d^2\right)^{\frac{3}{2}}\left(ex^2-d\right)dx$$

input

```
integrate((-e*x^2+d)*(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output

```
-integrate((-e^2*x^4 + d^2)^(3/2)*(e*x^2 - d), x)
```

Giac [F]

$$\int (d-ex^2)\left(d^2-e^2x^4\right)^{3/2}\,dx=\int -\left(-e^2x^4+d^2\right)^{\frac{3}{2}}\left(ex^2-d\right)dx$$

input

```
integrate((-e*x^2+d)*(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output

```
integrate(-(-e^2*x^4 + d^2)^(3/2)*(e*x^2 - d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d-ex^2)\left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(d^2-e^2\,x^4\right)^{3/2}\left(d-e\,x^2\right)\,dx$$

input

```
int((d^2 - e^2*x^4)^(3/2)*(d - e*x^2),x)
```

output

```
int((d^2 - e^2*x^4)^(3/2)*(d - e*x^2), x)
```

Reduce [F]

$$\int \left(d-e x^2\right)\left(d^2-e^2 x^4\right)^{3/2}\,dx=\frac{3\sqrt{-e^2x^4+d^2}\,d^3x}{7}-\frac{11\sqrt{-e^2x^4+d^2}\,d^2e\,x^3}{45}-\frac{\sqrt{-e^2x^4+d^2}\,d\,e^2x^5}{7}+\frac{\sqrt{-e^2x^4+d^2}\,e^3x^7}{9}+\frac{4\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^5}{7}-\frac{4\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^4e}{15}$$

input

```
int((-e*x^2+d)*(-e^2*x^4+d^2)^(3/2),x)
```

output

```
(135*sqrt(d**2 - e**2*x**4)*d**3*x - 77*sqrt(d**2 - e**2*x**4)*d**2*e*x**3
 - 45*sqrt(d**2 - e**2*x**4)*d*e**2*x**5 + 35*sqrt(d**2 - e**2*x**4)*e**3*
x**7 + 180*int(sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)*d**5 - 84*int(
(sqrt(d**2 - e**2*x**4)*x**2)/(d**2 - e**2*x**4),x)*d**4*e)/315
```

3.96 $\int \left(d^2-e^2x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{2}{7}d^2x\sqrt{d^2-e^2x^4}+\frac{1}{7}x\left(d^2-e^2x^4\right)^{3/2}+\frac{4d^{9/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{7\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
2/7*d^2*x*(-e^2*x^4+d^2)^(1/2)+1/7*x*(-e^2*x^4+d^2)^(3/2)+4/7*d^(9/2)*(1-e
^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{d^2x\sqrt{d^2-e^2x^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)}{\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(d^2*x*Sqrt[d^2 - e^2*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, (e^2*x^4)/d^2
])/Sqrt[1 - (e^2*x^4)/d^2]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {748, 748, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d^2-e^2x^4\right)^{3/2} dx$$

$$\Big\downarrow \text{748}$$

$$\frac{6}{7}d^2\int \sqrt{d^2-e^2x^4}dx+\frac{1}{7}x\left(d^2-e^2x^4\right)^{3/2}$$

$$\Big\downarrow \text{748}$$

$$\frac{6}{7}d^2\left(\frac{2}{3}d^2\int \frac{1}{\sqrt{d^2-e^2x^4}}dx+\frac{1}{3}x\sqrt{d^2-e^2x^4}\right)+\frac{1}{7}x\left(d^2-e^2x^4\right)^{3/2}$$

$$\Big\downarrow \text{765}$$

$$\frac{6}{7}d^2\left(\frac{2d^2\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{3\sqrt{d^2-e^2x^4}}+\frac{1}{3}x\sqrt{d^2-e^2x^4}\right)+\frac{1}{7}x\left(d^2-e^2x^4\right)^{3/2}$$

$$\Big\downarrow \text{762}$$

$$\frac{6}{7}d^2\left(\frac{2d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{1}{3}x\sqrt{d^2-e^2x^4}\right)+\frac{1}{7}x\left(d^2-e^2x^4\right)^{3/2}$$

input

```
Int[(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(x*(d^2 - e^2*x^4)^(3/2))/7 + (6*d^2*((x*Sqrt[d^2 - e^2*x^4])/3 + (2*d^(5/
2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(3*
Sqrt[e]*Sqrt[d^2 - e^2*x^4])))/7
```

Defintions of rubi rules used

rule 748

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
 + 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x\left(-e^{2} x^{4}+3 d^{2}\right) \sqrt{-e^{2} x^{4}+d^{2}}}{7}+\frac{4 d^{4} \sqrt{1-\frac{e\,x^{2}}{d}}\, \sqrt{1+\frac{e\,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{7 \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	96
default	$-\frac{e^{2} x^{5} \sqrt{-e^{2} x^{4}+d^{2}}}{7}+\frac{3 d^{2} x \sqrt{-e^{2} x^{4}+d^{2}}}{7}+\frac{4 d^{4} \sqrt{1-\frac{e\,x^{2}}{d}}\, \sqrt{1+\frac{e\,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{7 \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	107
elliptic	$-\frac{e^{2} x^{5} \sqrt{-e^{2} x^{4}+d^{2}}}{7}+\frac{3 d^{2} x \sqrt{-e^{2} x^{4}+d^{2}}}{7}+\frac{4 d^{4} \sqrt{1-\frac{e\,x^{2}}{d}}\, \sqrt{1+\frac{e\,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{7 \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	107

input

```
int((-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x*(-e^2*x^4+3*d^2)*(-e^2*x^4+d^2)^(1/2)+4/7*d^4/(e/d)^(1/2)*(1-e*x^2/d
)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.65

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{4\sqrt{-e^2}d^3\sqrt{\frac{d}{e}}F\left(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1\right)-\left(e^3x^5-3\,d^2ex\right)\sqrt{-e^2x^4+d^2}}{7\,e}$$

input
```
integrate((-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/7*(4*sqrt(-e^2)*d^3*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1) - (e^3
*x^5 - 3*d^2*e*x)*sqrt(-e^2*x^4 + d^2))/e
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.38

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-e**2*x**4+d**2)**(3/2),x)
```

output
```
d**3*x*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(-e^2x^4+d^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(-e^2x^4+d^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.43

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{x\left(d^2-e^2\,x^4\right)^{3/2}\,{}_2\mathrm{F}_1\!\left(-\frac{3}{2},\frac{1}{4};\ \frac{5}{4};\ \frac{e^2\,x^4}{d^2}\right)}{\left(1-\frac{e^2\,x^4}{d^2}\right)^{3/2}}$$

input
```
int((d^2 - e^2*x^4)^(3/2),x)
```

output
```
(x*(d^2 - e^2*x^4)^(3/2)*hypergeom([-3/2, 1/4], 5/4, (e^2*x^4)/d^2))/(1 -
(e^2*x^4)/d^2)^(3/2)
```

Reduce [F]

$$\int \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{3\sqrt{-e^2x^4+d^2}\,d^2x}{7}-\frac{\sqrt{-e^2x^4+d^2}\,e^2x^5}{7}+\frac{4\left(\int\frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^4}{7}$$

input
```
int((-e^2*x^4+d^2)^(3/2),x)
```

output
```
(3*sqrt(d**2 - e**2*x**4)*d**2*x - sqrt(d**2 - e**2*x**4)*e**2*x**5 + 4*in
t(sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)*d**4)/7
```

3.97 $\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{d-ex^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 156

$$\begin{aligned}\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{d-ex^2} \, dx = {} & \frac{1}{15}x\left(5d+3ex^2\right)\sqrt{d^2-e^2x^4} \\ & + \frac{2d^{7/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{5\sqrt{e}\sqrt{d^2-e^2x^4}} \\ & + \frac{4d^{7/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{15\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output
```
1/15*x*(3*e*x^2+5*d)*(-e^2*x^4+d^2)^(1/2)+2/5*d^(7/2)*(1-e^2*x^4/d^2)^(1/2
)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+4/15*d^(7/2)
*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.56

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{d-ex^2}\,dx = \frac{\sqrt{d^2-e^2x^4}\Big(3dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{e^2x^4}{d^2}\right)+ex^3\,\text{Hypergeometric2F1}\Big(-}{3\sqrt{1-\frac{e^2x^4}{d^2}}}$$

input

```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(3*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, (e^2*x^4)/d^
2] + e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(3*Sqrt[1 -
(e^2*x^4)/d^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.56, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {1396, 318, 27, 403, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{d-ex^2}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{\int -\frac{2de\sqrt{d-ex^2}\left(4ex^2+3d\right)}{\sqrt{ex^2+d}}dx}{5e}-\frac{1}{5}x\sqrt{d+ex^2}\left(d-ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{5}d\int\frac{\sqrt{d-ex^2}\left(4ex^2+3d\right)}{\sqrt{ex^2+d}}dx-\frac{1}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\big\downarrow 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{5}d\left(\frac{\int\frac{de\left(3ex^2+5d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{5}d\left(\frac{1}{3}d\int\frac{3ex^2+5d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\big\downarrow 399$$

$$\frac{\sqrt{d^2-e^2x^4}\Big(\frac{2}{5}d\Big(\frac{1}{3}d\Big(2d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+3\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\Big)+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Big)-\frac{1}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+e}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\big\downarrow 289$$

$$\frac{\sqrt{d^2-e^2x^4}\Big(\frac{2}{5}d\Big(\frac{1}{3}d\Big(\frac{2d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+3\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\Big)+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Big)-\frac{1}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\big\downarrow 329$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{5}d\Bigg(\frac{1}{3}d\Bigg(\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)-\frac{1}{5}x(d-e}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\big\downarrow 327$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{2}{5}d\Bigg(\frac{1}{3}d\Bigg(\frac{2d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg)-\frac{1}{5}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\big\downarrow 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{5}d\left(\frac{1}{3}d\left(\frac{2d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \color{blue}{762}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{5}d\left(\frac{1}{3}d\left(\frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+\frac{4}{3}x\sqrt{d-ex^2}\sqrt{d\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/5*(x*(d - e*x^2)^(3/2)*Sqrt[d + e*x^2]) + (2*d*((
4*x*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])/3 + (d*((3*d^(3/2)*Sqrt[1 - (e^2*x^4)
/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]
*Sqrt[d + e*x^2]) + (2*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(S
qrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/3))/5
))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10

method	result
risch	$\frac{x\left(3e\,x^{2}+5d\right)\sqrt{-e^{2}x^{4}+d^{2}}}{15}+\frac{2d^{2}\left(\frac{5d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-\frac{3d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}},\right.\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right.}{15}$
default	$\frac{e\,x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{5}+\frac{dx\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{2d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-\frac{2d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}}\right.\right.}{5\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d}}$
elliptic	$\frac{e\,x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{5}+\frac{dx\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{2d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-\frac{2d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}}\right.\right.}{5\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d}}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*e*x^2+5*d)*(-e^2*x^4+d^2)^(1/2)+2/15*d^2*(5*d/(e/d)^(1/2)*(1-e*x
^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2)
,I)-3*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/
2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{d-ex^2}\,dx =$$
$$-\frac{6\sqrt{-e^2}d^3x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)-2\left(3d^3+5d^2e\right)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)-\left(3e^3x^4+5de\right.}{15e^2x}$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d),x, algorithm="fricas")
```

output
```
-1/15*(6*sqrt(-e^2)*d^3*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) -
2*(3*d^3 + 5*d^2*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x),
 -1) - (3*e^3*x^4 + 5*d*e^2*x^2 - 6*d^2*e)*sqrt(-e^2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.54

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{d-ex^2}\,dx = \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(-e*x**2+d),x)
```

output
```
d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d
**2)/(4*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), e**2*
x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(7/4))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{d-ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{d-ex^2}\,dx = \int -\frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{ex^2-d}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(-e^2*x^4 + d^2)^(3/2)/(e*x^2 - d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{d-ex^2}\,dx = \int \frac{(d^2-e^2\,x^4)^{3/2}}{d-e\,x^2}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2),x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2), x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{d-ex^2}\,dx = \frac{\sqrt{-e^2x^4+d^2}\,dx}{3}+\frac{\sqrt{-e^2x^4+d^2}\,e\,x^3}{5}+\frac{2\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^3}{3}+\frac{2\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^2e}{5}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d),x)
```

output
```
(5*sqrt(d**2 - e**2*x**4)*d*x + 3*sqrt(d**2 - e**2*x**4)*e*x**3 + 10*int(s
qrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)*d**3 + 6*int((sqrt(d**2 - e**2
*x**4)*x**2)/(d**2 - e**2*x**4),x)*d**2*e)/15
```

3.98 $\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 144

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^2}\,dx = -\frac{1}{3}x\sqrt{d^2-e^2x^4} + \frac{2d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}} - \frac{2d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
-1/3*x*(-e^2*x^4+d^2)^(1/2)+2*d^(5/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1
/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-2/3*d^(5/2)*(1-e^2*x^4/d^2)^
(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.47 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^2}\,dx = \frac{\sqrt{-\frac{e}{d}}x(-d^2+e^2x^4) - 6id^2\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right) + 2id^2\sqrt{1-\frac{e^2x^4}{d^2}}}{3\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^2,x]
```

output

```
(Sqrt[-(e/d)]*x*(-d^2 + e^2*x^4) - (6*I)*d^2*Sqrt[1 - (e^2*x^4)/d^2]*Ellip
ticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] + (2*I)*d^2*Sqrt[1 - (e^2*x^4)/d^2]*El
lipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1])/(3*Sqrt[-(e/d)]*Sqrt[d^2 - e^2*x^4
])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.45, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1396, 318, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^2}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(ex^2+d\right)^{3/2}}{\sqrt{d-ex^2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{\int -\frac{2de\left(3ex^2+2d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{3}d\int \frac{3ex^2+2d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{3}d\left(3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 289$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{3}d\left(3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 329$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{3}d\left(\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 327$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{3}d\left(\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{3}d\left(\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2}{3}d\left(\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^2,x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(-1/3*(x*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (2*d*((3*
d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1]
)/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) - (d^(3/2)*Sqrt[1 - (e^2*x^4)/
d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*
Sqrt[d + e*x^2])))/3))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.11

method	result
default	$-\frac{x\sqrt{-e^2x^4+d^2}}{3}+\frac{4d^2\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{2d^2\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}}\right.\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$
risch	$-\frac{x\sqrt{-e^2x^4+d^2}}{3}+\frac{2d\left(\frac{2d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{3d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)}{3}$
elliptic	$-\frac{x\sqrt{-e^2x^4+d^2}}{3}+\frac{4d^2\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{2d^2\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}}\right.\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(-e^2*x^4+d^2)^(1/2)+4/3*d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2
/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-2*d^2/(e/d)^(1/2
)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e
/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^2}\,dx =$$

$$-\frac{6\sqrt{-e^2}d^2x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)-2\,(3\,d^2+2\,de)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)+\sqrt{-e^2x^4+d^2}(}{3\,e^2x}$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/3*(6*sqrt(-e^2)*d^2*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - 2
*(3*d^2 + 2*d*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1
) + sqrt(-e^2*x^4 + d^2)*(e^2*x^2 + 6*d*e))/(e^2*x)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^2}\,dx = \int \frac{(-(-d+ex^2)\,(d+ex^2))^{\frac{3}{2}}}{(-d+ex^2)^2}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(-e*x**2+d)**2,x)
```

output
```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)/(-d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^2}\,dx = \int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2-d)^2}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 - d)^2, x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^2}\,dx=\int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2-d)^2}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 - d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^2}\,dx=\int \frac{(d^2-e^2\,x^4)^{3/2}}{(d-e\,x^2)^2}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^2,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^2}\,dx=\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e\,x^2+d}dx\right)d+\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e\,x^2+d}dx\right)e$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^2,x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d - e*x**2),x)*d + int((sqrt(d**2 - e**2*x**4)
*x**2)/(d - e*x**2),x)*e
```

3.99 $\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 150

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^3}\,dx=\frac{2x\sqrt{d^2-e^2x^4}}{d-ex^2}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
2*x*(-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)-3*d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*Ellipt
icE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+2*d^(3/2)*(1-e^2*x^4
/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.49 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^3}\,dx=\frac{2\sqrt{-\frac{e}{d}}x\left(d+ex^2\right)+3id\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-2id\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Ellip}}{\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^3,x]
```

output

```
(2*Sqrt[-(e/d)]*x*(d + e*x^2) + (3*I)*d*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[
I*ArcSinh[Sqrt[-(e/d)]*x], -1] - (2*I)*d*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF
[I*ArcSinh[Sqrt[-(e/d)]*x], -1])/(Sqrt[-(e/d)]*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.34, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1396, 315, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^3}dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(ex^2+d\right)^{3/2}}{\left(d-ex^2\right)^{3/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}-\frac{\int \frac{de\left(3ex^2+d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{de}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}-\int \frac{3ex^2+d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(2d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx+\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 289}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx+\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 329$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^3,x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((2*x*Sqrt[d + e*x^2])/Sqrt[d - e*x^2] - (3*d^(3/2)*S
qrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e
]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (2*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*El
lipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d
+ e*x^2])))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.23

method	result
default	$-\frac{2\left(-e^{2} x^{2}-d e \right) x}{e \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}-\frac{d \sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \text{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{3 d \sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\text{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\text{Elliptic}\right.}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$
elliptic	$-\frac{2\left(-e^{2} x^{2}-d e \right) x}{e \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}-\frac{d \sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \text{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{3 d \sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\text{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\text{Elliptic}\right.}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$

input

```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```
-2*(-e^2*x^2-d*e)*x/e/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)-d/(e/d)^(1/2)*(1-e*
x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2
),I)+3*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1
/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.96

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^3}\,dx = \frac{3\,(dex^3-d^2x)\sqrt{-e^2}\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|-1)-((3\,de+e^2)x^3-(3\,d^2+de)x)\sqrt{-}}{e^3x^3-de^2x}$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^3,x, algorithm="fricas")
```

output
```
(3*(d*e*x^3 - d^2*x)*sqrt(-e^2)*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x),
-1) - ((3*d*e + e^2)*x^3 - (3*d^2 + d*e)*x)*sqrt(-e^2)*sqrt(d/e)*elliptic_
f(arcsin(sqrt(d/e)/x), -1) + sqrt(-e^2*x^4 + d^2)*(e^2*x^2 - 3*d*e))/(e^3*
x^3 - d*e^2*x)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^3}\,dx = -\int \frac{d^2\sqrt{d^2-e^2x^4}}{-d^3+3d^2ex^2-3de^2x^4+e^3x^6}\,dx - \int \left(-\frac{e^2x^4\sqrt{d^2-e^2x^4}}{-d^3+3d^2ex^2-3de^2x^4+e^3x^6}\right)dx$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(-e*x**2+d)**3,x)
```

output
```
-Integral(d**2*sqrt(d**2 - e**2*x**4)/(-d**3 + 3*d**2*e*x**2 - 3*d*e**2*x*
*4 + e**3*x**6), x) - Integral(-e**2*x**4*sqrt(d**2 - e**2*x**4)/(-d**3 +
3*d**2*e*x**2 - 3*d*e**2*x**4 + e**3*x**6), x)
```

Maxima [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^3}\,dx=\int -\frac{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}{\left(ex^2-d\right)^3}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 - d)^3, x)
```

Giac [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^3}\,dx=\int -\frac{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}{\left(ex^2-d\right)^3}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(-e^2*x^4 + d^2)^(3/2)/(e*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^3}\,dx=\int \frac{\left(d^2-e^2\,x^4\right)^{3/2}}{\left(d-e\,x^2\right)^3}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^3,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^3}\,dx = \frac{\sqrt{-e^2x^4+d^2}\,x+2\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^4}{e^3x^6-d\,e^2x^4-d^2e\,x^2+d^3}dx\right)d\,e^2-2\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^4}{e^3x^6-d\,e^2x^4-d^2e\,x^2+d^3}dx\right)e^3}{-e\,x^2+d}$$

input

```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^3,x)
```

output

```
(sqrt(d**2 - e**2*x**4)*x + 2*int((sqrt(d**2 - e**2*x**4)*x**4)/(d**3 - d*
*2*e*x**2 - d*e**2*x**4 + e**3*x**6),x)*d*e**2 - 2*int((sqrt(d**2 - e**2*x
**4)*x**4)/(d**3 - d**2*e*x**2 - d*e**2*x**4 + e**3*x**6),x)*e**3*x**2)/(d
 - e*x**2)
```

3.100 $\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 94

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^4}\,dx = \frac{2x\sqrt{d^2-e^2x^4}}{3(d-ex^2)^2} + \frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
2/3*x*(-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^2+1/3*d^(1/2)*(1-e^2*x^4/d^2)^(1/2)*
EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.59 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^4}\,dx = \frac{\frac{2x(d+ex^2)}{d-ex^2} - \frac{i\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)}{\sqrt{-\frac{e}{d}}}}{3\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^4,x]
```

output
```
((2*x*(d + e*x^2))/(d - e*x^2) - (I*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*Ar
cSinh[Sqrt[-(e/d)]*x], -1])/Sqrt[-(e/d)])/(3*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.47, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1396, 315, 25, 27, 289, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^4}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(ex^2+d\right)^{3/2}}{\left(d-ex^2\right)^{5/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2x\sqrt{d+ex^2}}{3\left(d-ex^2\right)^{3/2}}-\frac{\int -\frac{de}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3de}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{de}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3de}+\frac{2x\sqrt{d+ex^2}}{3\left(d-ex^2\right)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{3}\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+\frac{2x\sqrt{d+ex^2}}{3\left(d-ex^2\right)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{3\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d+ex^2}}{3\left(d-ex^2\right)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{3\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d+ex^2}}{3(d-ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d+ex^2}}{3(d-ex^2)^{3/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^4,x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((2*x*Sqrt[d + e*x^2])/(3*(d - e*x^2)^(3/2)) + (Sqrt[
d]*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(3*
Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2
])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{3e^{2}\left(x^{2}-\frac{d}{e}\right)^{2}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	94
elliptic	$\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{3e^{2}\left(x^{2}-\frac{d}{e}\right)^{2}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	94

input
```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

output
```
2/3*x/e^2*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2+1/3/(e/d)^(1/2)*(1-e*x^2/d)^(1/
2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^4}\,dx = \frac{2\sqrt{-e^2x^4+d^2}ex + (e^2x^4-2\,dex^2+d^2)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\middle|-1\right)}{3\,(e^3x^4-2\,de^2x^2+d^2e)}$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^4,x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(-e^2*x^4 + d^2)*e*x + (e^2*x^4 - 2*d*e*x^2 + d^2)*sqrt(e/d)*el
liptic_f(arcsin(x*sqrt(e/d)), -1))/(e^3*x^4 - 2*d*e^2*x^2 + d^2*e)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^4}\,dx = \int \frac{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}}{\left(-d+ex^2\right)^4}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(-e*x**2+d)**4,x)
```

output
```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)/(-d + e*x**2)**4, x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^4}\,dx = \int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2-d)^4}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 - d)^4, x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^4}\,dx = \int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2-d)^4}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^4,x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 - d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^4}\,dx = \int \frac{(d^2-e^2\,x^4)^{3/2}}{(d-e\,x^2)^4}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^4,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^4, x)
```

Reduce [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^4}\,dx = \frac{\sqrt{-e^2x^4+d^2}\,x + \left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^4x^8+2d\,e^3x^6-2d^3e\,x^2+d^4}dx\right)d^4 - 2\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^4x^8+2d\,e^3x^6-2d^3e\,x^2+d^4}dx\right)}{}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^4,x)
```

output
```
(sqrt(d**2 - e**2*x**4)*x + int(sqrt(d**2 - e**2*x**4)/(d**4 - 2*d**3*e*x*
*2 + 2*d*e**3*x**6 - e**4*x**8),x)*d**4 - 2*int(sqrt(d**2 - e**2*x**4)/(d*
*4 - 2*d**3*e*x**2 + 2*d*e**3*x**6 - e**4*x**8),x)*d**3*e*x**2 + int(sqrt(
d**2 - e**2*x**4)/(d**4 - 2*d**3*e*x**2 + 2*d*e**3*x**6 - e**4*x**8),x)*d*
*2*e**2*x**4 + int((sqrt(d**2 - e**2*x**4)*x**4)/(d**4 - 2*d**3*e*x**2 + 2
*d*e**3*x**6 - e**4*x**8),x)*d**2*e**2 - 2*int((sqrt(d**2 - e**2*x**4)*x**
4)/(d**4 - 2*d**3*e*x**2 + 2*d*e**3*x**6 - e**4*x**8),x)*d*e**3*x**2 + int
((sqrt(d**2 - e**2*x**4)*x**4)/(d**4 - 2*d**3*e*x**2 + 2*d*e**3*x**6 - e**
4*x**8),x)*e**4*x**4)/(2*(d**2 - 2*d*e*x**2 + e**2*x**4))
```

3.101 $\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 224

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^5}\,dx=\frac{2x\sqrt{d^2-e^2x^4}}{5\left(d-ex^2\right)^3}+\frac{2x\sqrt{d^2-e^2x^4}}{15d\left(d-ex^2\right)^2}+\frac{3x\sqrt{d^2-e^2x^4}}{10d^2\left(d-ex^2\right)}-\frac{3\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{10\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{7\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{15\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
2/5*x*(-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^3+2/15*x*(-e^2*x^4+d^2)^(1/2)/d/(-e*
x^2+d)^2+3/10*x*(-e^2*x^4+d^2)^(1/2)/d^2/(-e*x^2+d)-3/10*(1-e^2*x^4/d^2)^(
1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+7
/15*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(
-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.93 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.85

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^5}\,dx = \frac{\sqrt{-\frac{e}{d}}x(25d^3+3d^2ex^2-13de^2x^4+9e^3x^6)+9id(d-ex^2)^2\sqrt{1-\frac{e^2x^4}{d^2}}E\big(i\operatorname{arcsinh}\big(}{30d^2\sqrt{-\frac{e}{d}}\,(d-ex^2)}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^5,x]
```

output
```
(Sqrt[-(e/d)]*x*(25*d^3 + 3*d^2*e*x^2 - 13*d*e^2*x^4 + 9*e^3*x^6) + (9*I)*
d*(d - e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x
], -1] - (14*I)*d*(d - e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSin
h[Sqrt[-(e/d)]*x], -1])/(30*d^2*Sqrt[-(e/d)]*(d - e*x^2)^2*Sqrt[d^2 - e^2*
x^4])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {1396, 315, 25, 27, 402, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^5}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{(ex^2+d)^{3/2}}{(d-ex^2)^{7/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2x\sqrt{d+ex^2}}{5(d-ex^2)^{5/2}}-\frac{\int -\frac{de\left(ex^2+3d\right)}{\left(d-ex^2\right)^{5/2}\sqrt{ex^2+d}}dx}{5de}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{de\left(ex^2+3d\right)}{\left(d-ex^2\right)^{5/2}\sqrt{ex^2+d}}dx}{5de}+\frac{2x\sqrt{d+ex^2}}{5(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\int \frac{ex^2+3d}{(d-ex^2)^{5/2}\sqrt{ex^2+d}}dx+\frac{2x\sqrt{d+ex^2}}{5(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\int \frac{2de\left(2ex^2+7d\right)}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{6d^2e}+\frac{2x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)+\frac{2x\sqrt{d+ex^2}}{5(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\int \frac{2ex^2+7d}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{3d}+\frac{2x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)+\frac{2x\sqrt{d+ex^2}}{5(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{\int \frac{de\left(5d-9ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)+\frac{2x\sqrt{d+ex^2}}{5(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{\int \frac{5d-9ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)+\frac{2x\sqrt{d+ex^2}}{5(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 399$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{14d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-9\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)+\frac{2x\sqrt{d+ex^2}}{5(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 289$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{\frac{14d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-9\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)+\frac{2x\sqrt{d+ex^2}}{5(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 329$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{\frac{14d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{9d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)+\frac{2x\sqrt{d+ex^2}}{5(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{\frac{14d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)+\frac{2x\sqrt{d+ex^2}}{5(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{\frac{14d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)+\frac{2x\sqrt{d+ex^2}}{5(d-ex^2)^{5/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{\frac{\frac{14d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d(d-ex^2)^{3/2}}\right)+\frac{2}{5(}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^5,x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((2*x*Sqrt[d + e*x^2])/(5*(d - e*x^2)^(5/2)) + ((2*x*
Sqrt[d + e*x^2])/(3*d*(d - e*x^2)^(3/2)) + ((9*x*Sqrt[d + e*x^2])/(2*d*Sqr
t[d - e*x^2]) + ((-9*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqr
t[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (14*d^(
3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(
Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/(2*d))/(3*d))/5))/(Sqrt[d - e*x^
2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 315

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.15

method	result
default	$-\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{5e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{15d\,e^{2}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{3\left(-e^{2}x^{2}-de\right)x}{10d^{2}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+3\sqrt{1-e}$
elliptic	$-\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{5e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{15d\,e^{2}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{3\left(-e^{2}x^{2}-de\right)x}{10d^{2}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+3\sqrt{1-e}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-2/5*x/e^3*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^3+2/15/d*x/e^2*(-e^2*x^4+d^2)^(1
/2)/(x^2-d/e)^2-3/10*(-e^2*x^2-d*e)/d^2*x/e/((x^2-d/e)*(-e^2*x^2-d*e))^(1/
2)+1/6/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1
/2)*EllipticF(x*(e/d)^(1/2),I)+3/10/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x
^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(
e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.02

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^5}\,dx =$$
$$-\frac{9\left(e^4x^6-3\,de^3x^4+3\,d^2e^2x^2-d^3e\right)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)-((5\,de^3+9\,e^4)x^6-3\left(5\,d^2e^2+9\,de^3\right)x^4}{30\left(d^2e^4x^6-3\,d^3\right.}$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^5,x, algorithm="fricas")
```

output
```
-1/30*(9*(e^4*x^6 - 3*d*e^3*x^4 + 3*d^2*e^2*x^2 - d^3*e)*sqrt(e/d)*ellipti
c_e(arcsin(x*sqrt(e/d)), -1) - ((5*d*e^3 + 9*e^4)*x^6 - 3*(5*d^2*e^2 + 9*d
*e^3)*x^4 - 5*d^4 - 9*d^3*e + 3*(5*d^3*e + 9*d^2*e^2)*x^2)*sqrt(e/d)*ellip
tic_f(arcsin(x*sqrt(e/d)), -1) + (9*e^3*x^5 - 22*d*e^2*x^3 + 25*d^2*e*x)*s
qrt(-e^2*x^4 + d^2))/(d^2*e^4*x^6 - 3*d^3*e^3*x^4 + 3*d^4*e^2*x^2 - d^5*e)
```

Sympy [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^5}\,dx =$$
$$-\int \frac{d^2\sqrt{d^2-e^2x^4}}{-d^5+5d^4ex^2-10d^3e^2x^4+10d^2e^3x^6-5de^4x^8+e^5x^{10}}\,dx$$
$$-\int \left(-\frac{e^2x^4\sqrt{d^2-e^2x^4}}{-d^5+5d^4ex^2-10d^3e^2x^4+10d^2e^3x^6-5de^4x^8+e^5x^{10}}\right)dx$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(-e*x**2+d)**5,x)
```

output
```
-Integral(d**2*sqrt(d**2 - e**2*x**4)/(-d**5 + 5*d**4*e*x**2 - 10*d**3*e**
2*x**4 + 10*d**2*e**3*x**6 - 5*d*e**4*x**8 + e**5*x**10), x) - Integral(-e
**2*x**4*sqrt(d**2 - e**2*x**4)/(-d**5 + 5*d**4*e*x**2 - 10*d**3*e**2*x**4
 + 10*d**2*e**3*x**6 - 5*d*e**4*x**8 + e**5*x**10), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^5}\,dx = \int -\frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2-d)^5}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^5,x, algorithm="maxima")
```

output
```
-integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 - d)^5, x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^5}\,dx = \int -\frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2-d)^5}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^5,x, algorithm="giac")
```

output
```
integrate(-(-e^2*x^4 + d^2)^(3/2)/(e*x^2 - d)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^5}\,dx = \int \frac{(d^2-e^2\,x^4)^{3/2}}{(d-e\,x^2)^5}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^5,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^5, x)
```

Reduce [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^5}\,dx = \text{Too large to display}$$

input

```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^5,x)
```

output

```
(5*sqrt(d**2 - e**2*x**4)*x + int(sqrt(d**2 - e**2*x**4)/(d**5 - 3*d**4*e*
x**2 + 2*d**3*e**2*x**4 + 2*d**2*e**3*x**6 - 3*d*e**4*x**8 + e**5*x**10),x
)*d**5 - 3*int(sqrt(d**2 - e**2*x**4)/(d**5 - 3*d**4*e*x**2 + 2*d**3*e**2*
x**4 + 2*d**2*e**3*x**6 - 3*d*e**4*x**8 + e**5*x**10),x)*d**4*e*x**2 + 3*i
nt(sqrt(d**2 - e**2*x**4)/(d**5 - 3*d**4*e*x**2 + 2*d**3*e**2*x**4 + 2*d**
2*e**3*x**6 - 3*d*e**4*x**8 + e**5*x**10),x)*d**3*e**2*x**4 - int(sqrt(d**
2 - e**2*x**4)/(d**5 - 3*d**4*e*x**2 + 2*d**3*e**2*x**4 + 2*d**2*e**3*x**6
 - 3*d*e**4*x**8 + e**5*x**10),x)*d**2*e**3*x**6 - 9*int((sqrt(d**2 - e**2
*x**4)*x**4)/(d**5 - 3*d**4*e*x**2 + 2*d**3*e**2*x**4 + 2*d**2*e**3*x**6 -
 3*d*e**4*x**8 + e**5*x**10),x)*d**3*e**2 + 27*int((sqrt(d**2 - e**2*x**4)
*x**4)/(d**5 - 3*d**4*e*x**2 + 2*d**3*e**2*x**4 + 2*d**2*e**3*x**6 - 3*d*e
**4*x**8 + e**5*x**10),x)*d**2*e**3*x**2 - 27*int((sqrt(d**2 - e**2*x**4)*
x**4)/(d**5 - 3*d**4*e*x**2 + 2*d**3*e**2*x**4 + 2*d**2*e**3*x**6 - 3*d*e*
*4*x**8 + e**5*x**10),x)*d*e**4*x**4 + 9*int((sqrt(d**2 - e**2*x**4)*x**4)
/(d**5 - 3*d**4*e*x**2 + 2*d**3*e**2*x**4 + 2*d**2*e**3*x**6 - 3*d*e**4*x*
*8 + e**5*x**10),x)*e**5*x**6 - 18*int((sqrt(d**2 - e**2*x**4)*x**2)/(d**5
 - 3*d**4*e*x**2 + 2*d**3*e**2*x**4 + 2*d**2*e**3*x**6 - 3*d*e**4*x**8 + e
**5*x**10),x)*d**4*e + 54*int((sqrt(d**2 - e**2*x**4)*x**2)/(d**5 - 3*d**4
*e*x**2 + 2*d**3*e**2*x**4 + 2*d**2*e**3*x**6 - 3*d*e**4*x**8 + e**5*x**10
),x)*d**3*e**2*x**2 - 54*int((sqrt(d**2 - e**2*x**4)*x**2)/(d**5 - 3*d*...
```

3.102 $\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 258

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^6}\,dx = \frac{2x\sqrt{d^2-e^2x^4}}{7(d-ex^2)^4} + \frac{4x\sqrt{d^2-e^2x^4}}{35d(d-ex^2)^3} + \frac{11x\sqrt{d^2-e^2x^4}}{70d^2(d-ex^2)^2} + \frac{3x\sqrt{d^2-e^2x^4}}{10d^3(d-ex^2)} - \frac{3\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{10d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{31\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{70d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
2/7*x*(-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^4+4/35*x*(-e^2*x^4+d^2)^(1/2)/d/(-e*
x^2+d)^3+11/70*x*(-e^2*x^4+d^2)^(1/2)/d^2/(-e*x^2+d)^2+3/10*x*(-e^2*x^4+d^
2)^(1/2)/d^3/(-e*x^2+d)-3/10*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(
1/2),I)/d^(3/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+31/70*(1-e^2*x^4/d^2)^(1/2)*E
llipticF(e^(1/2)*x/d^(1/2),I)/d^(3/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.59

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^6}\,dx=\frac{-\frac{x\left(d+ex^2\right)\left(-60d^3+93d^2ex^2-74de^2x^4+21e^3x^6\right)}{\left(d-ex^2\right)^3}+\frac{id\sqrt{1-\frac{e^2x^4}{d^2}}\left(21E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-31\,\text{Elliptic}}{\sqrt{-\frac{e}{d}}}}{70d^3\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^6,x]
```

output
```
(-((x*(d + e*x^2)*(-60*d^3 + 93*d^2*e*x^2 - 74*d*e^2*x^4 + 21*e^3*x^6))/(d
 - e*x^2)^3) + (I*d*Sqrt[1 - (e^2*x^4)/d^2]*(21*EllipticE[I*ArcSinh[Sqrt[-
(e/d)]*x], -1] - 31*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1]))/Sqrt[-(e/d)
])/(70*d^3*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.24, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.593$, Rules used = {1396, 315, 25, 27, 402, 27, 402, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^6}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(ex^2+d\right)^{3/2}}{\left(d-ex^2\right)^{9/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2x\sqrt{d+ex^2}}{7(d-ex^2)^{7/2}}-\frac{\int -\frac{de\left(3ex^2+5d\right)}{\left(d-ex^2\right)^{7/2}\sqrt{ex^2+d}}dx}{7de}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{de\left(3ex^2+5d\right)}{\left(d-ex^2\right)^{7/2}\sqrt{ex^2+d}}dx}{7de}+\frac{2x\sqrt{d+ex^2}}{7(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\int \frac{3ex^2+5d}{(d-ex^2)^{7/2}\sqrt{ex^2+d}}dx+\frac{2x\sqrt{d+ex^2}}{7(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{\int \frac{6de\left(4ex^2+7d\right)}{\left(d-ex^2\right)^{5/2}\sqrt{ex^2+d}}dx}{10d^2e}+\frac{4x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)+\frac{2x\sqrt{d+ex^2}}{7(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\int \frac{4ex^2+7d}{\left(d-ex^2\right)^{5/2}\sqrt{ex^2+d}}dx}{5d}+\frac{4x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)+\frac{2x\sqrt{d+ex^2}}{7(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\int \frac{de\left(11ex^2+31d\right)}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{6d^2e}+\frac{11x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)+\frac{2x\sqrt{d+ex^2}}{7(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\int \frac{11ex^2+31d}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{6d}+\frac{11x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)+\frac{2x\sqrt{d+ex^2}}{7(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{\int \frac{2de\left(10d-21ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{21x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{11x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)+\frac{2x\sqrt{d+ex^2}}{7(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{\int \frac{10d-21ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{21x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{11x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)+\frac{2x\sqrt{d+ex^2}}{7(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{31d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-21\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}+\frac{21x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{11x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)+\frac{2x\sqrt{d+ex^2}}{7(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 289}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{\frac{31d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-21\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}+\frac{21x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{11x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)+\frac{2x\sqrt{d+ex^2}}{7(d-ex^2)^{7/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{329}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{\frac{31d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{21d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{11x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right)+\frac{2x}{7(d}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{\frac{31d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{11x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{\frac{31d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{11x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}\right)}{5d}+\frac{4x\sqrt{d+ex^2}}{5d(d-ex^2)^{5/2}}\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{7}\left(\frac{3\left(\frac{\frac{\frac{31d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{21x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d}+\frac{11x\sqrt{d+ex^2}}{6d\left(d-ex^2\right)^{3/2}}\right)}{5d}+\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^6,x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((2*x*Sqrt[d + e*x^2])/(7*(d - e*x^2)^(7/2)) + ((4*x*
Sqrt[d + e*x^2])/(5*d*(d - e*x^2)^(5/2)) + (3*((11*x*Sqrt[d + e*x^2])/(6*d
*(d - e*x^2)^(3/2)) + ((21*x*Sqrt[d + e*x^2])/(d*Sqrt[d - e*x^2]) + ((-21*
d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1]
)/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (31*d^(3/2)*Sqrt[1 - (e^2*x^
4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^
2]*Sqrt[d + e*x^2]))/d)/(6*d)))/(5*d))/7))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2
])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.14

method	result
default	$\frac{2x\sqrt{-e^2x^4+d^2}}{7e^4\left(x^2-\frac{d}{e}\right)^4}-\frac{4x\sqrt{-e^2x^4+d^2}}{35d\,e^3\left(x^2-\frac{d}{e}\right)^3}+\frac{11x\sqrt{-e^2x^4+d^2}}{70d^2e^2\left(x^2-\frac{d}{e}\right)^2}-\frac{3\left(-e^2x^2-de\right)x}{10e\,d^3\sqrt{\left(x^2-\frac{d}{e}\right)\left(-e^2x^2-de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}(x}{7d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$
elliptic	$\frac{2x\sqrt{-e^2x^4+d^2}}{7e^4\left(x^2-\frac{d}{e}\right)^4}-\frac{4x\sqrt{-e^2x^4+d^2}}{35d\,e^3\left(x^2-\frac{d}{e}\right)^3}+\frac{11x\sqrt{-e^2x^4+d^2}}{70d^2e^2\left(x^2-\frac{d}{e}\right)^2}-\frac{3\left(-e^2x^2-de\right)x}{10e\,d^3\sqrt{\left(x^2-\frac{d}{e}\right)\left(-e^2x^2-de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}(x}{7d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^6,x,method=_RETURNVERBOSE)
```

output
```
2/7*x/e^4*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^4-4/35/d*x/e^3*(-e^2*x^4+d^2)^(1/
2)/(x^2-d/e)^3+11/70/d^2*x/e^2*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-3/10*(-e^2
*x^2-d*e)/e/d^3*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/7/d^2/(e/d)^(1/2)*(1-
e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1
/2),I)+3/10/d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+
d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.09

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^6}\,dx =$$

$$-\ 21\left(e^5x^8-4\,de^4x^6+6\,d^2e^3x^4-4\,d^3e^2x^2+d^4e\right)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1\right)-\left(\left(10\,de^4+21\,e^5\right)x^8-4\,(10$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^6,x, algorithm="fricas")
```

output
```
-1/70*(21*(e^5*x^8 - 4*d*e^4*x^6 + 6*d^2*e^3*x^4 - 4*d^3*e^2*x^2 + d^4*e)*
sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - ((10*d*e^4 + 21*e^5)*x^8 -
 4*(10*d^2*e^3 + 21*d*e^4)*x^6 + 10*d^5 + 21*d^4*e + 6*(10*d^3*e^2 + 21*d^
2*e^3)*x^4 - 4*(10*d^4*e + 21*d^3*e^2)*x^2)*sqrt(e/d)*elliptic_f(arcsin(x*
sqrt(e/d)), -1) + (21*e^4*x^7 - 74*d*e^3*x^5 + 93*d^2*e^2*x^3 - 60*d^3*e*x
)*sqrt(-e^2*x^4 + d^2))/(d^3*e^5*x^8 - 4*d^4*e^4*x^6 + 6*d^5*e^3*x^4 - 4*d
^6*e^2*x^2 + d^7*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^6}\,dx = \text{Timed out}$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(-e*x**2+d)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^6}\,dx=\int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2-d)^6}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^6,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 - d)^6, x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^6}\,dx=\int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2-d)^6}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^6,x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 - d)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^6}\,dx=\int \frac{(d^2-e^2\,x^4)^{3/2}}{(d-e\,x^2)^6}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^6,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^6, x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^6}\,dx = \frac{\sqrt{-e^2x^4+d^2}\,x+3\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^6x^{12}+4d\,e^5x^{10}-5d^2e^4x^8+5d^4e^2x^4-4d^5e\,x^2+d^6}dx\right)d^6-12\left(\int \frac{}{-e^6x^{12}+}\right.}{}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^6,x)
```

output
```
(sqrt(d**2 - e**2*x**4)*x + 3*int(sqrt(d**2 - e**2*x**4)/(d**6 - 4*d**5*e*
x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 + 4*d*e**5*x**10 - e**6*x**12),
x)*d**6 - 12*int(sqrt(d**2 - e**2*x**4)/(d**6 - 4*d**5*e*x**2 + 5*d**4*e**
2*x**4 - 5*d**2*e**4*x**8 + 4*d*e**5*x**10 - e**6*x**12),x)*d**5*e*x**2 +
18*int(sqrt(d**2 - e**2*x**4)/(d**6 - 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5
*d**2*e**4*x**8 + 4*d*e**5*x**10 - e**6*x**12),x)*d**4*e**2*x**4 - 12*int(
sqrt(d**2 - e**2*x**4)/(d**6 - 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e
**4*x**8 + 4*d*e**5*x**10 - e**6*x**12),x)*d**3*e**3*x**6 + 3*int(sqrt(d**
2 - e**2*x**4)/(d**6 - 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8
 + 4*d*e**5*x**10 - e**6*x**12),x)*d**2*e**4*x**8 - int((sqrt(d**2 - e**2*
x**4)*x**4)/(d**6 - 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 +
4*d*e**5*x**10 - e**6*x**12),x)*d**4*e**2 + 4*int((sqrt(d**2 - e**2*x**4)*
x**4)/(d**6 - 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 + 4*d*e*
*5*x**10 - e**6*x**12),x)*d**3*e**3*x**2 - 6*int((sqrt(d**2 - e**2*x**4)*x
**4)/(d**6 - 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 + 4*d*e**
5*x**10 - e**6*x**12),x)*d**2*e**4*x**4 + 4*int((sqrt(d**2 - e**2*x**4)*x*
*4)/(d**6 - 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 + 4*d*e**5
*x**10 - e**6*x**12),x)*d*e**5*x**6 - int((sqrt(d**2 - e**2*x**4)*x**4)/(d
**6 - 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 + 4*d*e**5*x**10
 - e**6*x**12),x)*e**6*x**8)/(4*(d**4 - 4*d**3*e*x**2 + 6*d**2*e**2*x**...
```

3.103 $\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^7}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 292

$$\begin{aligned}\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^7}\,dx &= \frac{2x\sqrt{d^2-e^2x^4}}{9\,(d-ex^2)^5}+\frac{2x\sqrt{d^2-e^2x^4}}{21d\,(d-ex^2)^4}+\frac{73x\sqrt{d^2-e^2x^4}}{630d^2\,(d-ex^2)^3}\\ &+\frac{16x\sqrt{d^2-e^2x^4}}{105d^3\,(d-ex^2)^2}+\frac{17x\sqrt{d^2-e^2x^4}}{60d^4\,(d-ex^2)}-\frac{17\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{60d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\\ &+\frac{29\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{70d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output
```
2/9*x*(-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^5+2/21*x*(-e^2*x^4+d^2)^(1/2)/d/(-e*
x^2+d)^4+73/630*x*(-e^2*x^4+d^2)^(1/2)/d^2/(-e*x^2+d)^3+16/105*x*(-e^2*x^4
+d^2)^(1/2)/d^3/(-e*x^2+d)^2+17/60*x*(-e^2*x^4+d^2)^(1/2)/d^4/(-e*x^2+d)-1
7/60*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(5/2)/e^(1/2)/
(-e^2*x^4+d^2)^(1/2)+29/70*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/
2),I)/d^(5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.55

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^7}\,dx = \frac{\frac{x(d+ex^2)(1095d^4-2416d^3ex^2+2864d^2e^2x^4-1620de^3x^6+357e^4x^8)}{(d-ex^2)^4} + \frac{3id\sqrt{1-\frac{e^2x^4}{d^2}}(119E(i\text{arcsinh}(\sqrt{-\frac{e}{d}}x}{}}{1260d^4\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^7,x]
```

output
```
((x*(d + e*x^2)*(1095*d^4 - 2416*d^3*e*x^2 + 2864*d^2*e^2*x^4 - 1620*d*e^3
*x^6 + 357*e^4*x^8))/(d - e*x^2)^4 + ((3*I)*d*Sqrt[1 - (e^2*x^4)/d^2]*(119
*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - 174*EllipticF[I*ArcSinh[Sqrt[-
(e/d)]*x], -1]))/Sqrt[-(e/d)])/(1260*d^4*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.23, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1396, 315, 25, 27, 402, 27, 402, 27, 402, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^7}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{(ex^2+d)^{3/2}}{(d-ex^2)^{11/2}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{2x\sqrt{d+ex^2}}{9(d-ex^2)^{9/2}}-\frac{\int -\frac{de\left(5ex^2+7d\right)}{\left(d-ex^2\right)^{9/2}\sqrt{ex^2+d}}dx}{9de}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{de\left(5ex^2+7d\right)}{\left(d-ex^2\right)^{9/2}\sqrt{ex^2+d}}dx}{9de}+\frac{2x\sqrt{d+ex^2}}{9(d-ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\int \frac{5ex^2+7d}{(d-ex^2)^{9/2}\sqrt{ex^2+d}}dx+\frac{2x\sqrt{d+ex^2}}{9(d-ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\int \frac{2de\left(30ex^2+43d\right)}{\left(d-ex^2\right)^{7/2}\sqrt{ex^2+d}}dx}{14d^2e}+\frac{6x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)+\frac{2x\sqrt{d+ex^2}}{9(d-ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\int \frac{30ex^2+43d}{\left(d-ex^2\right)^{7/2}\sqrt{ex^2+d}}dx}{7d}+\frac{6x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)+\frac{2x\sqrt{d+ex^2}}{9(d-ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{\int \frac{3de\left(73ex^2+119d\right)}{\left(d-ex^2\right)^{5/2}\sqrt{ex^2+d}}dx}{10d^2e}+\frac{73x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)+\frac{2x\sqrt{d+ex^2}}{9(d-ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\int \frac{73ex^2+119d}{\left(d-ex^2\right)^{5/2}\sqrt{ex^2+d}}dx}{10d}+\frac{73x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)+\frac{2x\sqrt{d+ex^2}}{9(d-ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\int \frac{6de\left(32ex^2+87d\right)}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{6d^2e}+\frac{32x\sqrt{d+ex^2}}{d\left(d-ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)+\frac{2x\sqrt{d+ex^2}}{9(d-ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\int \frac{32ex^2+87d}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{d}+\frac{32x\sqrt{d+ex^2}}{d\left(d-ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)+\frac{2x\sqrt{d+ex^2}}{9(d-ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{\int \frac{de\left(55d-119ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{119x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{d}+\frac{32x\sqrt{d+ex^2}}{d\left(d-ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)+\frac{2x\sqrt{d+ex^2}}{9(d-ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{\int \frac{55d-119ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{119x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{d}+\frac{32x\sqrt{d+ex^2}}{d\left(d-ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)+\frac{2x\sqrt{d+ex^2}}{9(d-ex^2)^{9/2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{174d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-119\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{119x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{d}+\frac{32x\sqrt{d+ex^2}}{d\left(d-ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)+\frac{2}{9(}\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{\frac{174d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-119\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{119x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{d}+\frac{32x\sqrt{d+ex^2}}{d\left(d-ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{d+ex^2}}{7d(d-ex^2)^{7/2}}\right)\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 329$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{\frac{174d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{119d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{119x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{d}+\frac{32x\sqrt{d+ex^2}}{d\left(d-ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}+\frac{6x\sqrt{}}{7d(d-}\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 327$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{\frac{174d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{119d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{119x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{d}+\frac{32x\sqrt{d+ex^2}}{d\left(d-ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{\frac{174d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{119d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{119x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{d}+\frac{32x\sqrt{d+ex^2}}{d\left(d-ex^2\right)^{3/2}}\right)}{10d}+\frac{73x\sqrt{d+ex^2}}{10d\left(d-ex^2\right)^{5/2}}}{7d}\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{9}\left(\frac{\frac{3\left(\frac{\frac{\frac{174d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{119d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{119x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{d}+\frac{32x\sqrt{d+ex^2}}{d\left(d-ex^2\right)^{3/2}}\right)}{10d}+}{7d}\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^7,x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((2*x*Sqrt[d + e*x^2])/(9*(d - e*x^2)^(9/2)) + ((6*x*
Sqrt[d + e*x^2])/(7*d*(d - e*x^2)^(7/2)) + ((73*x*Sqrt[d + e*x^2])/(10*d*(
d - e*x^2)^(5/2)) + (3*((32*x*Sqrt[d + e*x^2])/(d*(d - e*x^2)^(3/2)) + ((1
19*x*Sqrt[d + e*x^2])/(2*d*Sqrt[d - e*x^2]) + ((-119*d^(3/2)*Sqrt[1 - (e^2
*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e
*x^2]*Sqrt[d + e*x^2]) + (174*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[Ar
cSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))
/(2*d))/d))/(10*d))/(7*d))/9))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 6.68 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.12

method	result
default	$-\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{9e^{5}\left(x^{2}-\frac{d}{e}\right)^{5}}+\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{21d\,e^{4}\left(x^{2}-\frac{d}{e}\right)^{4}}-\frac{73x\sqrt{-e^{2}x^{4}+d^{2}}}{630d^{2}e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+\frac{16x\sqrt{-e^{2}x^{4}+d^{2}}}{105e^{2}d^{3}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{17\left(-e^{2}x^{2}-de\right)x}{60d^{4}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+ 11\sqrt{1-}$
elliptic	$-\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{9e^{5}\left(x^{2}-\frac{d}{e}\right)^{5}}+\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{21d\,e^{4}\left(x^{2}-\frac{d}{e}\right)^{4}}-\frac{73x\sqrt{-e^{2}x^{4}+d^{2}}}{630d^{2}e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+\frac{16x\sqrt{-e^{2}x^{4}+d^{2}}}{105e^{2}d^{3}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{17\left(-e^{2}x^{2}-de\right)x}{60d^{4}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+ 11\sqrt{1-}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^7,x,method=_RETURNVERBOSE)
```

output
```
-2/9*x/e^5*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^5+2/21/d*x/e^4*(-e^2*x^4+d^2)^(1
/2)/(x^2-d/e)^4-73/630/d^2/e^3*x*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^3+16/105/e
^2/d^3*x*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-17/60*(-e^2*x^2-d*e)/d^4*x/e/((x
^2-d/e)*(-e^2*x^2-d*e))^(1/2)+11/84/d^3/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e
*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+17/60/d^3/(e
/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(Ellipt
icF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.16

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^7}\,dx =$$

$$-\frac{357\left(e^6x^{10}-5\,de^5x^8+10\,d^2e^4x^6-10\,d^3e^3x^4+5\,d^4e^2x^2-d^5e\right)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)-3\left(\left(55\,de^5+\right.\right.}{}$$

input

```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^7,x, algorithm="fricas")
```

output

```
-1/1260*(357*(e^6*x^10 - 5*d*e^5*x^8 + 10*d^2*e^4*x^6 - 10*d^3*e^3*x^4 + 5
*d^4*e^2*x^2 - d^5*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - 3*((
55*d*e^5 + 119*e^6)*x^10 - 5*(55*d^2*e^4 + 119*d*e^5)*x^8 + 10*(55*d^3*e^3
 + 119*d^2*e^4)*x^6 - 55*d^6 - 119*d^5*e - 10*(55*d^4*e^2 + 119*d^3*e^3)*x
^4 + 5*(55*d^5*e + 119*d^4*e^2)*x^2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/
d)), -1) + (357*e^5*x^9 - 1620*d*e^4*x^7 + 2864*d^2*e^3*x^5 - 2416*d^3*e^2
*x^3 + 1095*d^4*e*x)*sqrt(-e^2*x^4 + d^2))/(d^4*e^6*x^10 - 5*d^5*e^5*x^8 +
 10*d^6*e^4*x^6 - 10*d^7*e^3*x^4 + 5*d^8*e^2*x^2 - d^9*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^7}\,dx = \text{Timed out}$$

input

```
integrate((-e**2*x**4+d**2)**(3/2)/(-e*x**2+d)**7,x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^7}\,dx=\int -\frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2-d)^7}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^7,x, algorithm="maxima")
```

output
```
-integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 - d)^7, x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^7}\,dx=\int -\frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2-d)^7}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^7,x, algorithm="giac")
```

output
```
integrate(-(-e^2*x^4 + d^2)^(3/2)/(e*x^2 - d)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d-ex^2)^7}\,dx=\int \frac{(d^2-e^2\,x^4)^{3/2}}{(d-e\,x^2)^7}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^7,x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d - e*x^2)^7, x)
```

Reduce [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d-ex^2\right)^7}\,dx=\text{Too large to display}$$

input

```
int((-e^2*x^4+d^2)^(3/2)/(-e*x^2+d)^7,x)
```

output

```
(sqrt(d**2 - e**2*x**4)*x + 4*int(sqrt(d**2 - e**2*x**4)/(d**7 - 5*d**6*e*
x**2 + 9*d**5*e**2*x**4 - 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 + 9*d**2*e**
5*x**10 - 5*d*e**6*x**12 + e**7*x**14),x)*d**7 - 20*int(sqrt(d**2 - e**2*x
**4)/(d**7 - 5*d**6*e*x**2 + 9*d**5*e**2*x**4 - 5*d**4*e**3*x**6 - 5*d**3*
e**4*x**8 + 9*d**2*e**5*x**10 - 5*d*e**6*x**12 + e**7*x**14),x)*d**6*e*x**
2 + 40*int(sqrt(d**2 - e**2*x**4)/(d**7 - 5*d**6*e*x**2 + 9*d**5*e**2*x**4
 - 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 + 9*d**2*e**5*x**10 - 5*d*e**6*x**1
2 + e**7*x**14),x)*d**5*e**2*x**4 - 40*int(sqrt(d**2 - e**2*x**4)/(d**7 -
5*d**6*e*x**2 + 9*d**5*e**2*x**4 - 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 + 9
*d**2*e**5*x**10 - 5*d*e**6*x**12 + e**7*x**14),x)*d**4*e**3*x**6 + 20*int
(sqrt(d**2 - e**2*x**4)/(d**7 - 5*d**6*e*x**2 + 9*d**5*e**2*x**4 - 5*d**4*
e**3*x**6 - 5*d**3*e**4*x**8 + 9*d**2*e**5*x**10 - 5*d*e**6*x**12 + e**7*x
**14),x)*d**3*e**4*x**8 - 4*int(sqrt(d**2 - e**2*x**4)/(d**7 - 5*d**6*e*x*
*2 + 9*d**5*e**2*x**4 - 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 + 9*d**2*e**5*
x**10 - 5*d*e**6*x**12 + e**7*x**14),x)*d**2*e**5*x**10 - 2*int((sqrt(d**2
 - e**2*x**4)*x**4)/(d**7 - 5*d**6*e*x**2 + 9*d**5*e**2*x**4 - 5*d**4*e**3
*x**6 - 5*d**3*e**4*x**8 + 9*d**2*e**5*x**10 - 5*d*e**6*x**12 + e**7*x**14
),x)*d**5*e**2 + 10*int((sqrt(d**2 - e**2*x**4)*x**4)/(d**7 - 5*d**6*e*x**
2 + 9*d**5*e**2*x**4 - 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 + 9*d**2*e**5*x
**10 - 5*d*e**6*x**12 + e**7*x**14),x)*d**4*e**3*x**2 - 20*int((sqrt(d*...
```

3.104 $\int \frac{(d-ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 169

$$\int \frac{(d-ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx = -dx\sqrt{d^2-e^2x^4}+\frac{1}{5}ex^3\sqrt{d^2-e^2x^4} - \frac{18d^{7/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{5\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{28d^{7/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{5\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
-d*x*(-e^2*x^4+d^2)^(1/2)+1/5*e*x^3*(-e^2*x^4+d^2)^(1/2)-18/5*d^(7/2)*(1-e
^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1
/2)+28/5*d^(7/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1
/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.83

$$\int \frac{(d-ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx$$
$$= \frac{-5d^3x+d^2ex^3+5de^2x^5-e^3x^7+10d^3x\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-6d^2ex^3\sqrt{1-}}{5\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d - e*x^2)^3/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(-5*d^3*x + d^2*e*x^3 + 5*d*e^2*x^5 - e^3*x^7 + 10*d^3*x*Sqrt[1 - (e^2*x^4
)/d^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (e^2*x^4)/d^2] - 6*d^2*e*x^3*Sqrt[
1 - (e^2*x^4)/d^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (e^2*x^4)/d^2])/(5*Sqr
t[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.44, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {1396, 318, 27, 403, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{(d-ex^2)^{5/2}}{\sqrt{ex^2+d}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{6de\left(d-2ex^2\right)\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx}{5e}-\frac{1}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{6}{5}d\int \frac{\left(d-2ex^2\right)\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx-\frac{1}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 403$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{6}{5}d\left(\frac{\int \frac{de\left(5d-9ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{6}{5}d\left(\frac{1}{3}d\int \frac{5d-9ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}x\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{6}{5}d\left(\frac{1}{3}d\left(14d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-9\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}x\left(d-ex^2\right)^{3/}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{6}{5}d\left(\frac{1}{3}d\left(\frac{14d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-9\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}x\left(d-ex^2\right.\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{6}{5}d\left(\frac{1}{3}d\left(\frac{14d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{9d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)-\frac{1}{5}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{6}{5}d\Bigg(\frac{1}{3}d\Bigg(\frac{14d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex}}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{6}{5}d\Bigg(\frac{1}{3}d\Bigg(\frac{14d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-\frac{2}{3}x\sqrt{d-ex^2}\sqrt{d+ex}}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{6}{5}d\Bigg(\frac{1}{3}d\Bigg(\frac{14d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\Bigg)-\frac{2}{3}x\sqrt{d-}}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d - e*x^2)^3/Sqrt[d^2 - e^2*x^4],x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(-1/5*(x*(d - e*x^2)^(3/2)*Sqrt[d + e*x^2
]) + (6*d*((-2*x*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])/3 + (d*((-9*d^(3/2)*Sqrt
[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*S
qrt[d - e*x^2]*Sqrt[d + e*x^2]) + (14*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*Elli
pticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d +
e*x^2])))/3))/5))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 7.92 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{x\left(-e\,x^{2}+5d\right)\sqrt{-e^{2}x^{4}+d^{2}}}{5}+\frac{2d^{2}\left(\frac{5d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{9d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\operatorname{EllipticE}\left(x\sqrt{}\right.\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right.}{5}$
elliptic	$\frac{e\,x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{5}-dx\sqrt{-e^{2}x^{4}+d^{2}}+\frac{2d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{18d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\right.}{5\sqrt{\frac{e}{d}}\,\sqrt{-e}}$
default	$\frac{d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-e^{3}\left(-\frac{x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{5e^{2}}-\frac{3d^{3}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\operatorname{EllipticE}\left(\right.\right.}{5e^{3}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right.$

input
```
int((-e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*x*(-e*x^2+5*d)*(-e^2*x^4+d^2)^(1/2)+2/5*d^2*(5*d/(e/d)^(1/2)*(1-e*x^2
/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I
)+9*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)
*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{(d-ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx = \frac{18\sqrt{-e^2}d^3x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) - 2\,(9\,d^3-5\,d^2e)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) + (e^3x^4-5\,de}{5\,e^2x}$$

input
```
integrate((-e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(18*sqrt(-e^2)*d^3*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - 2
*(9*d^3 - 5*d^2*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x),
-1) + (e^3*x^4 - 5*d*e^2*x^2 + 18*d^2*e)*sqrt(-e^2*x^4 + d^2))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02

$$\int \frac{(d-ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx = \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{3dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{3e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((-e*x**2+d)**3/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d*
*2)/(4*gamma(5/4)) - 3*d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**2*
x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(7/4)) + 3*e**2*x**5*gamma(5/4)*hyper
((1/2, 5/4), (9/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*gamma(9/4)) - e*
*3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), e**2*x**4*exp_polar(2*I*pi)/
d**2)/(4*d*gamma(11/4))
```

Maxima [F]

$$\int \frac{(d-ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx=\int -\frac{(ex^2-d)^3}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((-e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 - d)^3/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{(d-ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx=\int -\frac{(ex^2-d)^3}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((-e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(e*x^2 - d)^3/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{(d-e\,x^2)^3}{\sqrt{d^2-e^2\,x^4}}\,dx$$

input
```
int((d - e*x^2)^3/(d^2 - e^2*x^4)^(1/2),x)
```

output
```
int((d - e*x^2)^3/(d^2 - e^2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d-ex^2)^3}{\sqrt{d^2-e^2x^4}}\,dx = -\sqrt{-e^2x^4+d^2}\,dx + \frac{\sqrt{-e^2x^4+d^2}\,e\,x^3}{5} + 2\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^3 - \frac{18\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^2e}{5}$$

input

```
int((-e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x)
```

output

```
( - 5*sqrt(d**2 - e**2*x**4)*d*x + sqrt(d**2 - e**2*x**4)*e*x**3 + 10*int(
sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)*d**3 - 18*int((sqrt(d**2 - e*
*2*x**4)*x**2)/(d**2 - e**2*x**4),x)*d**2*e)/5
```

3.105 $\int \frac{(d-ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 144

$$\int \frac{(d-ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx = -\frac{1}{3}x\sqrt{d^2-e^2x^4}-\frac{2d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{10d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
-1/3*x*(-e^2*x^4+d^2)^(1/2)-2*d^(5/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1
/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+10/3*d^(5/2)*(1-e^2*x^4/d^2)
^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84

$$\int \frac{(d-ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx = \frac{-d^2x+e^2x^5+4d^2x\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-2dex^3\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2}}{3\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d - e*x^2)^2/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(-(d^2*x) + e^2*x^5 + 4*d^2*x*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[1/
4, 1/2, 5/4, (e^2*x^4)/d^2] - 2*d*e*x^3*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeome
tric2F1[1/2, 3/4, 7/4, (e^2*x^4)/d^2])/(3*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.45, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1396, 318, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-ex^2\right)^2}{\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\left(d-ex^2\right)^{3/2}}{\sqrt{ex^2+d}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{2de\left(2d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\int \frac{2d-3ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\left(5d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\left(\frac{5d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-3\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\left(\frac{5d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\left(\frac{5d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\left(\frac{5d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}d\left(\frac{5d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)-\frac{1}{3}x\sqrt{d-ex^2}\right.}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d - e*x^2)^2/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(-1/3*(x*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
 + (2*d*((-3*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/
Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (5*d^(3/2)*Sqrt
[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*S
qrt[d - e*x^2]*Sqrt[d + e*x^2])))/3))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 5.69 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{2d\left(\frac{2d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{3d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right)}{3}$
elliptic	$-\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3}+\frac{4d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+$
default	$\frac{d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+e^{2}\left(-\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3e^{2}}+\frac{d^{2}\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3e^{2}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right)+$

input
```
int((-e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(-e^2*x^4+d^2)^(1/2)+2/3*d*(2*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*
x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+3*d/(e/d)^(1/
2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(
e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{(d-ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx$$
$$= \frac{6\sqrt{-e^2}d^2x\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) - 2\,(3\,d^2-2\,de)\sqrt{-e^2}x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1) - \sqrt{-e^2x^4+d^2}}{3\,e^2x}$$

input
```
integrate((-e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(6*sqrt(-e^2)*d^2*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - 2*
(3*d^2 - 2*d*e)*sqrt(-e^2)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1)
 - sqrt(-e^2*x^4 + d^2)*(e^2*x^2 - 6*d*e))/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.85

$$\int \frac{(d-ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((-e*x**2+d)**2/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)
/(4*gamma(5/4)) - e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**2*x**4*ex
p_polar(2*I*pi)/d**2)/(2*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1/2, 5/
4), (9/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(4*d*gamma(9/4))
```

Maxima [F]

$$\int \frac{(d-ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{(ex^2-d)^2}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((-e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 - d)^2/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{(d-ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{(ex^2-d)^2}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((-e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 - d)^2/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{(d-e\,x^2)^2}{\sqrt{d^2-e^2\,x^4}}\,dx$$

input
```
int((d - e*x^2)^2/(d^2 - e^2*x^4)^(1/2),x)
```

output
```
int((d - e*x^2)^2/(d^2 - e^2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d-ex^2)^2}{\sqrt{d^2-e^2x^4}}\,dx = \left(\int \frac{\sqrt{-e^2x^4+d^2}}{e\,x^2+d}dx\right)d-\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^2}{e\,x^2+d}dx\right)e$$

input
```
int((-e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d + e*x**2),x)*d - int((sqrt(d**2 - e**2*x**4)
*x**2)/(d + e*x**2),x)*e
```

3.106 $\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx = -\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
-d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^
2*x^4+d^2)^(1/2)+2*d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/
2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx = \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{e^2x^4}{d^2}\right)\right)}{3\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d - e*x^2)/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[1 - (e^2*x^4)/d^2]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (e^2*x^4)
/d^2] - e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (e^2*x^4)/d^2]))/(3*Sqrt[d^
2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1390, 1389, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1390}$$

$$\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{d-ex^2}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{1-\frac{ex^2}{d}}}{\sqrt{\frac{ex^2}{d}+1}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{326}$$

$$\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\left(2\int \frac{1}{\sqrt{1-\frac{ex^2}{d}}\sqrt{\frac{ex^2}{d}+1}}dx-\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{284}$$

$$\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\left(2\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx-\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 327$$

$$\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\left(2\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx-\frac{\sqrt{d}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 762$$

$$\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\left(\frac{2\sqrt{d}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}}-\frac{\sqrt{d}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d - e*x^2)/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(d*Sqrt[1 - (e^2*x^4)/d^2]*(-((Sqrt[d]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d
]], -1])/Sqrt[e]) + (2*Sqrt[d]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])
/Sqrt[e]))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	137
elliptic	$\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{d\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	137

input
```
int((-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*Ell
ipticF(x*(e/d)^(1/2),I)+d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/
(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I
))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx = \frac{\sqrt{-e^2}dx\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right) |-1) - \sqrt{-e^2}(d-e)x\sqrt{\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right) |-1) + \sqrt{-e^2x^4+d^2}e}{e^2x}$$

input
```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-e^2)*d*x*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x), -1) - sqrt(-e^2)
*(d - e)*x*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1) + sqrt(-e^2*x^4 +
 d^2)*e)/(e^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{ex^3\Gamma\left(\frac{3}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-e*x**2+d)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*gamma(5/4)) - e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**2*x**4*exp_
polar(2*I*pi)/d**2)/(4*d*gamma(7/4))
```

Maxima [F]

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int -\frac{ex^2-d}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 - d)/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int -\frac{ex^2-d}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(e*x^2 - d)/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{d-e\,x^2}{\sqrt{d^2-e^2\,x^4}}\,dx$$

input
```
int((d - e*x^2)/(d^2 - e^2*x^4)^(1/2),x)
```

output
```
int((d - e*x^2)/(d^2 - e^2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d-ex^2}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e\,x^2+d}dx$$

input
```
int((-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d + e*x**2),x)
```

3.107 $\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx=\frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
d^(1/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2
*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx=-\frac{i\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right),-1\right)}{\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[1/Sqrt[d^2 - e^2*x^4],x]
```

output
```
((-I)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1])/(S
qrt[-(e/d)]*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 762$$

$$\frac{\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[d]*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1
])/(Sqrt[e]*Sqrt[d^2 - e^2*x^4])
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\ \operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	60
elliptic	$\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\ \operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	60

input
```
int(1/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*Ell
ipticF(x*(e/d)^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx=\frac{\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|\,-1\right)}{e}$$

input
```
integrate(1/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1)/e
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx=\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*d*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{1}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate(1/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{1}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate(1/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx=\frac{x\,\sqrt{1-\frac{e^2\,x^4}{d^2}}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ \frac{e^2\,x^4}{d^2}\right)}{\sqrt{d^2-e^2\,x^4}}$$

input
```
int(1/(d^2 - e^2*x^4)^(1/2),x)
```

output
```
(x*(1 - (e^2*x^4)/d^2)^(1/2)*hypergeom([1/4, 1/2], 5/4, (e^2*x^4)/d^2))/(d
^2 - e^2*x^4)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx$$

input
```
int(1/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**2 - e**2*x**4),x)
```

3.108 $\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 156

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx=\frac{x\sqrt{d^2-e^2x^4}}{2d^2(d-ex^2)}-\frac{\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
1/2*x*(-e^2*x^4+d^2)^(1/2)/d^2/(-e*x^2+d)-1/2*(1-e^2*x^4/d^2)^(1/2)*Ellipt
icE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+(1-e^2*x^4/d
^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx$$
$$=\frac{\sqrt{-\frac{e}{d}}x(d+ex^2)+id\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-2id\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\right.}{2d^2\sqrt{-\frac{e}{d}}\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[1/((d - e*x^2)*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[-(e/d)]*x*(d + e*x^2) + I*d*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcS
inh[Sqrt[-(e/d)]*x], -1] - (2*I)*d*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*Arc
Sinh[Sqrt[-(e/d)]*x], -1])/(2*d^2*Sqrt[-(e/d)]*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1396, 316, 27, 326, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{3/2}\sqrt{ex^2+d}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx}{2d^2e}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 326$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{2d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d^2}+\frac{x\sqrt{d+ex^2}}{2d^2\sqrt{d-ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[1/((d - e*x^2)*Sqrt[d^2 - e^2*x^4]),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d + e*x^2])/(2*d^2*Sqrt[d - e*x^
2]) + (-((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqr
t[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])) + (2*d^(3/2)*Sqrt[1
 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqr
t[d - e*x^2]*Sqrt[d + e*x^2]))/(2*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\left(-e^{2} x^{2}-d e \right) x}{2 d^{2} e \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{Elliptic}}{2 d \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$
elliptic	$-\frac{\left(-e^{2} x^{2}-d e \right) x}{2 d^{2} e \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{Elliptic}}{2 d \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$

input
```
int(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2-d*e)/d^2*x/e/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/2/d/(e/d)^(
1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*
(e/d)^(1/2),I)+1/2/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2
*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.72

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx =$$
$$-\frac{\sqrt{-e^2x^4+d^2}ex+(e^2x^2-de)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\mid -1)-((de+e^2)x^2-d^2-de)\sqrt{\frac{e}{d}}F(\arcsin\left(x\right.}{2\left(d^2e^2x^2-d^3e\right)}$$

input
```
integrate(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-e^2*x^4 + d^2)*e*x + (e^2*x^2 - d*e)*sqrt(e/d)*elliptic_e(arcs
in(x*sqrt(e/d)), -1) - ((d*e + e^2)*x^2 - d^2 - d*e)*sqrt(e/d)*elliptic_f(
arcsin(x*sqrt(e/d)), -1))/(d^2*e^2*x^2 - d^3*e)
```

Sympy [F]

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = -\int \frac{1}{-d\sqrt{d^2-e^2x^4}+ex^2\sqrt{d^2-e^2x^4}}\,dx$$

input
```
integrate(1/(-e*x**2+d)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
-Integral(1/(-d*sqrt(d**2 - e**2*x**4) + e*x**2*sqrt(d**2 - e**2*x**4)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int -\frac{1}{\sqrt{-e^2x^4+d^2}(ex^2-d)}\,dx$$

input
```
integrate(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 - d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2-e^2\,x^4}\,\left(d-e\,x^2\right)}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(d-ex^2)\sqrt{d^2-e^2x^4}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e^3x^6-d\,e^2x^4-d^2e\,x^2+d^3}dx$$

input
```
int(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**3 - d**2*e*x**2 - d*e**2*x**4 + e**3*x**6),
x)
```

3.109 $\int \frac{1}{(d-ex^2)^2\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 193

$$\begin{aligned}\int \frac{1}{(d-ex^2)^2\sqrt{d^2-e^2x^4}}\,dx &= \frac{x\sqrt{d^2-e^2x^4}}{6d^2(d-ex^2)^2}+\frac{x\sqrt{d^2-e^2x^4}}{2d^3(d-ex^2)} \\ &\quad -\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}} \\ &\quad +\frac{5\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{6d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/6*x*(-e^2*x^4+d^2)^(1/2)/d^2/(-e*x^2+d)^2+1/2*x*(-e^2*x^4+d^2)^(1/2)/d^3
/(-e*x^2+d)-1/2*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(3/
2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+5/6*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2
)*x/d^(1/2),I)/d^(3/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.51 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(d-ex^2\right)^2\sqrt{d^2-e^2x^4}}\,dx$$
$$= \frac{\sqrt{-\frac{e}{d}}x(4d^2+dex^2-3e^2x^4)+3id(d-ex^2)\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-5id(d-ex^2)\sqrt{1-}}{6d^3\sqrt{-\frac{e}{d}}\left(d-ex^2\right)\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[1/((d - e*x^2)^2*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[-(e/d)]*x*(4*d^2 + d*e*x^2 - 3*e^2*x^4) + (3*I)*d*(d - e*x^2)*Sqrt[1
 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - (5*I)*d*(d -
e*x^2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1])/(
6*d^3*Sqrt[-(e/d)]*(d - e*x^2)*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.28, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {1396, 316, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(d-ex^2\right)^2\sqrt{d^2-e^2x^4}}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{5/2}\sqrt{ex^2+d}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(ex^2+5d\right)}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{6d^2e}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{ex^2+5d}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{2de\left(2d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{2d-3ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{5d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{5d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{5d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{5d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{5d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d+ex^2}}{d\sqrt{d-ex^2}}}{6d^2}+\frac{x\sqrt{d+ex^2}}{6d^2(d-ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[1/((d - e*x^2)^2*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d + e*x^2])/(6*d^2*(d - e*x^2)^(
3/2)) + ((3*x*Sqrt[d + e*x^2])/(d*Sqrt[d - e*x^2]) + ((-3*d^(3/2)*Sqrt[1 -
 (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[
d - e*x^2]*Sqrt[d + e*x^2]) + (5*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF
[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2
]))/d)/(6*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.17

method	result
default	$\frac{x\sqrt{-e^2x^4+d^2}}{6d^2e^2\left(x^2-\frac{d}{e}\right)^2}-\frac{\left(-e^2x^2-de\right)x}{2e\,d^3\sqrt{\left(x^2-\frac{d}{e}\right)\left(-e^2x^2-de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\left(\operatorname{EllipticF}\right(}{2d^2\sqrt{\frac{e}{d}}\,\sqrt{-}}$
elliptic	$\frac{x\sqrt{-e^2x^4+d^2}}{6d^2e^2\left(x^2-\frac{d}{e}\right)^2}-\frac{\left(-e^2x^2-de\right)x}{2e\,d^3\sqrt{\left(x^2-\frac{d}{e}\right)\left(-e^2x^2-de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\left(\operatorname{EllipticF}\right(}{2d^2\sqrt{\frac{e}{d}}\,\sqrt{-}}$

input
```
int(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/d^2*x/e^2*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-1/2*(-e^2*x^2-d*e)/e/d^3*x/
((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/3/d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+
e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+1/2/d^2/(e/
d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(Ellipti
cF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(d-ex^2\right)^2\sqrt{d^2-e^2x^4}}\,dx = -\frac{3\left(e^3x^4-2\,de^2x^2+d^2e\right)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)-\left(\left(2\,de^2+3\,e^3\right)x^4+2\,d^3+3\,d^2e-2\left(2\,d^2e+3\,de\right.\right.}{6\left(d^3e^3x^4-2\,d^4e^2x^2+d^5e\right)}$$

input
```
integrate(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(3*(e^3*x^4 - 2*d*e^2*x^2 + d^2*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt
(e/d)), -1) - ((2*d*e^2 + 3*e^3)*x^4 + 2*d^3 + 3*d^2*e - 2*(2*d^2*e + 3*d*
e^2)*x^2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 +
d^2)*(3*e^2*x^3 - 4*d*e*x))/(d^3*e^3*x^4 - 2*d^4*e^2*x^2 + d^5*e)
```

Sympy [F]

$$\int \frac{1}{(d-ex^2)^2\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}\left(-d+ex^2\right)^2}\,dx$$

input
```
integrate(1/(-e*x**2+d)**2/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x**2)*(d + e*x**2))*(-d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d-ex^2)^2\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}(ex^2-d)^2}\,dx$$

input
```
integrate(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 - d)^2), x)
```

Giac [F]

$$\int \frac{1}{(d-ex^2)^2\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}(ex^2-d)^2}\,dx$$

input
```
integrate(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 - d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d-ex^2)^2\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2-e^2\,x^4}\,(d-e\,x^2)^2}\,dx$$

input

```
int(1/((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)^2),x)
```

output

```
int(1/((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(d-ex^2)^2\sqrt{d^2-e^2x^4}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^4x^8+2d\,e^3x^6-2d^3e\,x^2+d^4}dx$$

input

```
int(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**4 - 2*d**3*e*x**2 + 2*d*e**3*x**6 - e**4*x*
*8),x)
```

3.110 $\int \frac{1}{(d-ex^2)^3\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\int \frac{1}{(d-ex^2)^3\sqrt{d^2-e^2x^4}}\,dx = \frac{x\sqrt{d^2-e^2x^4}}{10d^2(d-ex^2)^3}+\frac{x\sqrt{d^2-e^2x^4}}{5d^3(d-ex^2)^2}+\frac{9x\sqrt{d^2-e^2x^4}}{20d^4(d-ex^2)} - \frac{9\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{20d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{7\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{10d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
1/10*x*(-e^2*x^4+d^2)^(1/2)/d^2/(-e*x^2+d)^3+1/5*x*(-e^2*x^4+d^2)^(1/2)/d^
3/(-e*x^2+d)^2+9/20*x*(-e^2*x^4+d^2)^(1/2)/d^4/(-e*x^2+d)-9/20*(1-e^2*x^4/
d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(5/2)/e^(1/2)/(-e^2*x^4+d^2)^(
1/2)+7/10*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(5/2)/e^(
1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.71 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.84

$$\int \frac{1}{(d-ex^2)^3\sqrt{d^2-e^2x^4}}\,dx$$
$$=\frac{\sqrt{-\frac{e}{d}}x(15d^3-7d^2ex^2-13de^2x^4+9e^3x^6)+9id(d-ex^2)^2\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-14id}{20d^4\sqrt{-\frac{e}{d}}\,(d-ex^2)^2\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[1/((d - e*x^2)^3*Sqrt[d^2 - e^2*x^4]),x]
```

output

```
(Sqrt[-(e/d)]*x*(15*d^3 - 7*d^2*e*x^2 - 13*d*e^2*x^4 + 9*e^3*x^6) + (9*I)*
d*(d - e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x
], -1] - (14*I)*d*(d - e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSin
h[Sqrt[-(e/d)]*x], -1])/(20*d^4*Sqrt[-(e/d)]*(d - e*x^2)^2*Sqrt[d^2 - e^2*
x^4])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.29, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {1396, 316, 27, 402, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-ex^2)^3\sqrt{d^2-e^2x^4}}dx$$
$$\Big\downarrow \text{1396}$$
$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{7/2}\sqrt{ex^2+d}}dx}{\sqrt{d^2-e^2x^4}}$$
$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{3e\left(ex^2+3d\right)}{\left(d-ex^2\right)^{5/2}\sqrt{ex^2+d}}dx}{10d^2e}+\frac{x\sqrt{d+ex^2}}{10d^2(d-ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\int \frac{ex^2+3d}{\left(d-ex^2\right)^{5/2}\sqrt{ex^2+d}}dx}{10d^2}+\frac{x\sqrt{d+ex^2}}{10d^2(d-ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\int \frac{2de\left(2ex^2+7d\right)}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{6d^2e}+\frac{2x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d+ex^2}}{10d^2(d-ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\int \frac{2ex^2+7d}{\left(d-ex^2\right)^{3/2}\sqrt{ex^2+d}}dx}{3d}+\frac{2x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d+ex^2}}{10d^2(d-ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{\int \frac{de\left(5d-9ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d+ex^2}}{10d^2(d-ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{\int \frac{5d-9ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d+ex^2}}{10d^2(d-ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{14d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-9\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d+ex^2}}{10d^2(d-ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{\frac{14d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-9\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d+ex^2}}{10d^2(d-ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{\frac{14d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{9d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d+ex^2}}{10d^2(d-ex^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{\frac{14d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d+e}}{10d^2(d-e}}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{\frac{14d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}\right)}{10d^2}+\frac{x\sqrt{d+ex}}{10d^2(d-ex^2}}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\left(\frac{\frac{\frac{14d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{9d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{9x\sqrt{d+ex^2}}{2d\sqrt{d-ex^2}}}{3d}+\frac{2x\sqrt{d+ex^2}}{3d\left(d-ex^2\right)^{3/2}}\right)}{10d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[1/((d - e*x^2)^3*Sqrt[d^2 - e^2*x^4]),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d + e*x^2])/(10*d^2*(d - e*x^2)^
(5/2)) + (3*((2*x*Sqrt[d + e*x^2])/(3*d*(d - e*x^2)^(3/2)) + ((9*x*Sqrt[d
+ e*x^2])/(2*d*Sqrt[d - e*x^2]) + ((-9*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*Ell
ipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d +
 e*x^2]) + (14*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x
)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/(2*d))/(3*d)))
/(10*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.15

method	result
default	$-\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{10d^{2}e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{5e^{2}d^{3}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{9\left(-e^{2}x^{2}-de\right)x}{20d^{4}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{4d^{3}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\ 9\sqrt{1-}$
elliptic	$-\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{10d^{2}e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{5e^{2}d^{3}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{9\left(-e^{2}x^{2}-de\right)x}{20d^{4}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{4d^{3}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}+\ 9\sqrt{1-}$

input

```
int(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/10/d^2/e^3*x*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^3+1/5/e^2/d^3*x*(-e^2*x^4+d
^2)^(1/2)/(x^2-d/e)^2-9/20*(-e^2*x^2-d*e)/d^4*x/e/((x^2-d/e)*(-e^2*x^2-d*e
))^(1/2)+1/4/d^3/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4
+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+9/20/d^3/(e/d)^(1/2)*(1-e*x^2/d)^(1
/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-Ell
ipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d-ex^2)^3\sqrt{d^2-e^2x^4}}\,dx =$$

$$-\frac{9\left(e^4x^6-3\,de^3x^4+3\,d^2e^2x^2-d^3e\right)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1)-((5\,de^3+9\,e^4)x^6-3\,(5\,d^2e^2+9\,de^3)}{20\,(d^4e^4x^6-3}$$

input
```
integrate(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/20*(9*(e^4*x^6 - 3*d*e^3*x^4 + 3*d^2*e^2*x^2 - d^3*e)*sqrt(e/d)*ellipti
c_e(arcsin(x*sqrt(e/d)), -1) - ((5*d*e^3 + 9*e^4)*x^6 - 3*(5*d^2*e^2 + 9*d
*e^3)*x^4 - 5*d^4 - 9*d^3*e + 3*(5*d^3*e + 9*d^2*e^2)*x^2)*sqrt(e/d)*ellip
tic_f(arcsin(x*sqrt(e/d)), -1) + (9*e^3*x^5 - 22*d*e^2*x^3 + 15*d^2*e*x)*s
qrt(-e^2*x^4 + d^2))/(d^4*e^4*x^6 - 3*d^5*e^3*x^4 + 3*d^6*e^2*x^2 - d^7*e)
```

Sympy [F]

$$\int \frac{1}{(d-ex^2)^3\sqrt{d^2-e^2x^4}}\,dx$$

$$= -\int \frac{1}{-d^3\sqrt{d^2-e^2x^4}+3d^2ex^2\sqrt{d^2-e^2x^4}-3de^2x^4\sqrt{d^2-e^2x^4}+e^3x^6\sqrt{d^2-e^2x^4}}\,dx$$

input
```
integrate(1/(-e*x**2+d)**3/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
-Integral(1/(-d**3*sqrt(d**2 - e**2*x**4) + 3*d**2*e*x**2*sqrt(d**2 - e**2
*x**4) - 3*d*e**2*x**4*sqrt(d**2 - e**2*x**4) + e**3*x**6*sqrt(d**2 - e**2
*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(d-ex^2)^3\sqrt{d^2-e^2x^4}}\,dx=\int -\frac{1}{\sqrt{-e^2x^4+d^2}(ex^2-d)^3}\,dx$$

input
```
integrate(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 - d)^3), x)
```

Giac [F]

$$\int \frac{1}{(d-ex^2)^3\sqrt{d^2-e^2x^4}}\,dx=\int -\frac{1}{\sqrt{-e^2x^4+d^2}(ex^2-d)^3}\,dx$$

input
```
integrate(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 - d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d-ex^2)^3\sqrt{d^2-e^2x^4}}\,dx=\int \frac{1}{\sqrt{d^2-e^2\,x^4}\,(d-e\,x^2)^3}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)^3),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(d-ex^2)^3\sqrt{d^2-e^2x^4}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e^5x^{10}-3d\,e^4x^8+2d^2e^3x^6+2d^3e^2x^4-3d^4e\,x^2+d^5}dx$$

input
```
int(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**5 - 3*d**4*e*x**2 + 2*d**3*e**2*x**4 + 2*d*
*2*e**3*x**6 - 3*d*e**4*x**8 + e**5*x**10),x)
```

3.111 $\int \frac{(d-ex^2)^4}{(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 172

$$\begin{aligned}\int \frac{(d-ex^2)^4}{(d^2-e^2x^4)^{3/2}}\,dx &= \frac{4dx(d-ex^2)}{\sqrt{d^2-e^2x^4}}+\frac{1}{3}x\sqrt{d^2-e^2x^4}\\ &+\frac{8d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}\\ &-\frac{34d^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
4*d*x*(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2)+1/3*x*(-e^2*x^4+d^2)^(1/2)+8*d^(5/2)
*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^
2)^(1/2)-34/3*d^(5/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)
/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.75

$$\int \frac{(d-ex^2)^4}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{13d^2x+12dex^3-e^2x^5-10d^2x\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-16}{3\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d - e*x^2)^4/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(13*d^2*x + 12*d*e*x^3 - e^2*x^5 - 10*d^2*x*Sqrt[1 - (e^2*x^4)/d^2]*Hyperg
eometric2F1[1/4, 1/2, 5/4, (e^2*x^4)/d^2] - 16*d*e*x^3*Sqrt[1 - (e^2*x^4)/
d^2]*Hypergeometric2F1[3/4, 3/2, 7/4, (e^2*x^4)/d^2])/(3*Sqrt[d^2 - e^2*x^
4])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.37, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1396, 315, 25, 27, 403, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-ex^2)^4}{(d^2-e^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{(d-ex^2)^{5/2}}{(ex^2+d)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int -\frac{de\left(d-7ex^2\right)\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx}{de}+\frac{2x(d-ex^2)^{3/2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2x(d-ex^2)^{3/2}}{\sqrt{d+ex^2}}-\frac{\int \frac{de\left(d-7ex^2\right)\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx}{de}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2x(d-ex^2)^{3/2}}{\sqrt{d+ex^2}}-\int \frac{(d-7ex^2)\sqrt{d-ex^2}}{\sqrt{ex^2+d}}dx\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{\int \frac{2de\left(5d-12ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{3e}+\frac{2x(d-ex^2)^{3/2}}{\sqrt{d+ex^2}}+\frac{7}{3}x\sqrt{d+ex^2}\sqrt{d-ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{2}{3}d\int \frac{5d-12ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+\frac{2x(d-ex^2)^{3/2}}{\sqrt{d+ex^2}}+\frac{7}{3}x\sqrt{d+ex^2}\sqrt{d-ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{2}{3}d\left(17d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-12\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)+\frac{2x(d-ex^2)^{3/2}}{\sqrt{d+ex^2}}+\frac{7}{3}x\sqrt{d+ex^2}\sqrt{d-ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{2}{3}d\left(\frac{17d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-12\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx\right)+\frac{2x(d-ex^2)^{3/2}}{\sqrt{d+ex^2}}+\frac{7}{3}x\sqrt{d+ex^2}\sqrt{d-ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{329}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{2}{3}d\left(\frac{17d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+\frac{2x(d-ex^2)^{3/2}}{\sqrt{d+ex^2}}+\frac{7}{3}x\sqrt{d+ex^2}\sqrt{d}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{2}{3}d\left(\frac{17d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+\frac{2x\left(d-ex^2\right)^{3/2}}{\sqrt{d+ex^2}}+\frac{7}{3}x\sqrt{d+}}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{2}{3}d\left(\frac{17d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+\frac{2x\left(d-ex^2\right)^{3/2}}{\sqrt{d+ex^2}}+\frac{7}{3}x\sqrt{d+}}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{2}{3}d\left(\frac{17d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{12d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)+\frac{2x\left(d-ex^2\right)^{3}}{\sqrt{d+ex^2}}}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d - e*x^2)^4/(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((2*x*(d - e*x^2)^(3/2))/Sqrt[d + e*x^2]
+ (7*x*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])/3 - (2*d*((-12*d^(3/2)*Sqrt[1 - (e
^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d -
 e*x^2]*Sqrt[d + e*x^2]) + (17*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[A
rcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
))/3))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 17.72 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.18

method	result
elliptic	$\frac{4(-e^2x^2+de)dx}{e\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}+\frac{x\sqrt{-e^2x^4+d^2}}{3}-\frac{10d^2\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{8d^2\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\left(\mathrm{Elliptic}\right.}{\sqrt{\frac{e}{d}}\,\sqrt{-}}$
risch	$\frac{x\sqrt{-e^2x^4+d^2}}{3}+\frac{2d\left(-\frac{11d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{6d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{e}{d}},i\right)\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}+12d^2\right.}{}$
default	$d^4\left(\frac{x}{2d^2\sqrt{-\left(x^4-\frac{d^2}{e^2}\right)e^2}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{2d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)+e^4\left(\frac{d^2x}{2e^4\sqrt{-\left(x^4-\frac{d^2}{e^2}\right)e^2}}+\frac{x\sqrt{-e^2x^4+d^2}}{3e^4}-\frac{5d^2}{}\right.$

input
```
int((-e*x^2+d)^4/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4*(-e^2*x^2+d*e)*d/e*x/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/3*x*(-e^2*x^4+d^
2)^(1/2)-10/3*d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^
4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-8*d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2
)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-Ellip
ticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.95

$$\int \frac{(d-ex^2)^4}{(d^2-e^2x^4)^{3/2}}\,dx =$$

$$-\frac{24\,(d^2ex^3+d^3x)\sqrt{-e^2}\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)|-1)-2\,((12\,d^2e-5\,de^2)x^3+(12\,d^3-5\,d^2e)x)\sqrt{-e^2}\sqrt{\frac{d}{e}}F(}{3\,(e^3x^3+de^2x)}$$

input
```
integrate((-e*x^2+d)^4/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(24*(d^2*e*x^3 + d^3*x)*sqrt(-e^2)*sqrt(d/e)*elliptic_e(arcsin(sqrt(d
/e)/x), -1) - 2*((12*d^2*e - 5*d*e^2)*x^3 + (12*d^3 - 5*d^2*e)*x)*sqrt(-e^
2)*sqrt(d/e)*elliptic_f(arcsin(sqrt(d/e)/x), -1) - (e^3*x^4 - 11*d*e^2*x^2
 - 24*d^2*e)*sqrt(-e^2*x^4 + d^2))/(e^3*x^3 + d*e^2*x)
```

Sympy [F]

$$\int \frac{(d-ex^2)^4}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(-d+ex^2)^4}{(-(-d+ex^2)\,(d+ex^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x**2+d)**4/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral((-d + e*x**2)**4/(-(-d + e*x**2)*(d + e*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^4}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(ex^2-d)^4}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^4/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 - d)^4/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(d-ex^2)^4}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(ex^2-d)^4}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^4/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 - d)^4/(-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^4}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(d-e\,x^2)^4}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input
```
int((d - e*x^2)^4/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
int((d - e*x^2)^4/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d-ex^2)^4}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{6\sqrt{-e^2x^4+d^2}\,dx+\sqrt{-e^2x^4+d^2}\,e\,x^3-3\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^3x^6-d\,e^2x^4+d^2e\,x^2+d^3}dx\right)d^4-3\Big(\int \frac{}{-e^3}}{}$$

input
```
int((-e*x^2+d)^4/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
(6*sqrt(d**2 - e**2*x**4)*d*x + sqrt(d**2 - e**2*x**4)*e*x**3 - 3*int(sqrt
(d**2 - e**2*x**4)/(d**3 + d**2*e*x**2 - d*e**2*x**4 - e**3*x**6),x)*d**4
- 3*int(sqrt(d**2 - e**2*x**4)/(d**3 + d**2*e*x**2 - d*e**2*x**4 - e**3*x*
*6),x)*d**3*e*x**2 + 17*int((sqrt(d**2 - e**2*x**4)*x**4)/(d**3 + d**2*e*x
**2 - d*e**2*x**4 - e**3*x**6),x)*d**2*e**2 + 17*int((sqrt(d**2 - e**2*x**
4)*x**4)/(d**3 + d**2*e*x**2 - d*e**2*x**4 - e**3*x**6),x)*d*e**3*x**2)/(3
*(d + e*x**2))
```

3.112 $\int \frac{\left(d-ex^2\right)^3}{\left(d^2-e^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{\left(d-ex^2\right)^3}{\left(d^2-e^2x^4\right)^{3/2}}\,dx = \frac{2x\left(d-ex^2\right)}{\sqrt{d^2-e^2x^4}}+\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}-\frac{4d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
2*x*(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2)+3*d^(3/2)*(1-e^2*x^4/d^2)^(1/2)*Ellipt
icE(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-4*d^(3/2)*(1-e^2*x^4
/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.75

$$\int \frac{\left(d-ex^2\right)^3}{\left(d^2-e^2x^4\right)^{3/2}}\,dx = \frac{2dx+ex^3-dx\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-2ex^3\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hyp}}{\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d - e*x^2)^3/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(2*d*x + e*x^3 - d*x*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[1/4, 1/2, 5
/4, (e^2*x^4)/d^2] - 2*e*x^3*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4
, 3/2, 7/4, (e^2*x^4)/d^2])/Sqrt[d^2 - e^2*x^4]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {1396, 315, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-ex^2\right)^3}{\left(d^2-e^2x^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\left(d-ex^2\right)^{3/2}}{\left(ex^2+d\right)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int -\frac{de\left(d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{de}+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}-\frac{\int \frac{de\left(d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{de}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}-\int \frac{d-3ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-4d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx+3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{4d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{4d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{4d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{2x\sqrt{d-ex^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d - e*x^2)^3/(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((2*x*Sqrt[d - e*x^2])/Sqrt[d + e*x^2] +
(3*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]],
-1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) - (4*d^(3/2)*Sqrt[1 - (e^2*
x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*
x^2]*Sqrt[d + e*x^2])))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 315

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 6.02 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{2(-e^2x^2+de)x}{e\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}-\frac{d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{3d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,(\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\operatorname{EllipticE}(}{\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$
default	$d^3\left(\frac{x}{2d^2\sqrt{-\left(x^4-\frac{d^2}{e^2}\right)e^2}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{2d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)-e^3(\frac{x^3}{2e^2\sqrt{-\left(x^4-\frac{d^2}{e^2}\right)e^2}}+\frac{3d\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}}{}$

input
```
int((-e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-e^2*x^2+d*e)*x/e/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)-d/(e/d)^(1/2)*(1-e*x
^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2)
,I)-3*d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/
2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.98

$$\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{3/2}}\,dx =$$

$$-\frac{3\,(dex^3+d^2x)\sqrt{-e^2}\sqrt{\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{\frac{d}{e}}}{x}\right)\,|\,-1)-((3\,de-e^2)x^3+(3\,d^2-de)x)\sqrt{-e^2}\sqrt{\frac{d}{e}}F(\arcsin\Bigg(\frac{\sqrt{\frac{d}{e}}}{x}}{e^3x^3+de^2x}$$

input
```
integrate((-e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-(3*(d*e*x^3 + d^2*x)*sqrt(-e^2)*sqrt(d/e)*elliptic_e(arcsin(sqrt(d/e)/x),
 -1) - ((3*d*e - e^2)*x^3 + (3*d^2 - d*e)*x)*sqrt(-e^2)*sqrt(d/e)*elliptic
_f(arcsin(sqrt(d/e)/x), -1) + sqrt(-e^2*x^4 + d^2)*(e^2*x^2 + 3*d*e))/(e^3
*x^3 + d*e^2*x)
```

Sympy [F]

$$\begin{aligned}\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{3/2}}\,dx = &-\int\left(-\frac{d^3}{d^2\sqrt{d^2-e^2x^4}-e^2x^4\sqrt{d^2-e^2x^4}}\right)dx\\ &-\int\frac{e^3x^6}{d^2\sqrt{d^2-e^2x^4}-e^2x^4\sqrt{d^2-e^2x^4}}\,dx\\ &-\int\left(-\frac{3de^2x^4}{d^2\sqrt{d^2-e^2x^4}-e^2x^4\sqrt{d^2-e^2x^4}}\right)dx\\ &-\int\frac{3d^2ex^2}{d^2\sqrt{d^2-e^2x^4}-e^2x^4\sqrt{d^2-e^2x^4}}\,dx\end{aligned}$$

input
```
integrate((-e*x**2+d)**3/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
-Integral(-d**3/(d**2*sqrt(d**2 - e**2*x**4) - e**2*x**4*sqrt(d**2 - e**2*
x**4)), x) - Integral(e**3*x**6/(d**2*sqrt(d**2 - e**2*x**4) - e**2*x**4*s
qrt(d**2 - e**2*x**4)), x) - Integral(-3*d*e**2*x**4/(d**2*sqrt(d**2 - e**
2*x**4) - e**2*x**4*sqrt(d**2 - e**2*x**4)), x) - Integral(3*d**2*e*x**2/(
d**2*sqrt(d**2 - e**2*x**4) - e**2*x**4*sqrt(d**2 - e**2*x**4)), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{3/2}}\,dx = \int -\frac{(ex^2-d)^3}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 - d)^3/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{3/2}}\,dx = \int -\frac{(ex^2-d)^3}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(e*x^2 - d)^3/(-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(d-e\,x^2)^3}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input
```
int((d - e*x^2)^3/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
int((d - e*x^2)^3/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{\sqrt{-e^2x^4+d^2}\,x+2\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^4}{-e^3x^6-d\,e^2x^4+d^2e\,x^2+d^3}dx\right)d\,e^2+2\left(\int \frac{\sqrt{-e^2x^4+d^2}\,x^4}{-e^3x^6-d\,e^2x^4+d^2e\,x^2+d^3}dx\right)}{e\,x^2+d}$$

input
```
int((-e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
(sqrt(d**2 - e**2*x**4)*x + 2*int((sqrt(d**2 - e**2*x**4)*x**4)/(d**3 + d*
*2*e*x**2 - d*e**2*x**4 - e**3*x**6),x)*d*e**2 + 2*int((sqrt(d**2 - e**2*x
**4)*x**4)/(d**3 + d**2*e*x**2 - d*e**2*x**4 - e**3*x**6),x)*e**3*x**2)/(d
 + e*x**2)
```

3.113 $\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 71

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{\sqrt{\frac{d-ex^2}{d+ex^2}}\,(d+ex^2)\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|\,2\right)}{\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
((-e*x^2+d)/(e*x^2+d))^(1/2)*(e*x^2+d)*EllipticE(e^(1/2)*x/d^(1/2)/(1+e*x^
2/d)^(1/2),2^(1/2))/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{3dx-2ex^3\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},\frac{e^2x^4}{d^2}\right)}{3d\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[(d - e*x^2)^2/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(3*d*x - 2*e*x^3*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4, 3/2, 7/4,
(e^2*x^4)/d^2])/(3*d*Sqrt[d^2 - e^2*x^4])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. $2(71) = 142$.

Time = 0.73 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.28, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1396, 314, 25, 27, 344, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-ex^2\right)^2}{\left(d^2-e^2x^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\sqrt{d-ex^2}}{\left(ex^2+d\right)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\int -\frac{ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\int \frac{x^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{344}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\int \frac{x^2}{\sqrt{d^2-e^2x^4}}dx}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 836$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\int \frac{ex^2+d}{d\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{e}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{ex^2+d}{\sqrt{d^2-e^2x^4}}dx}{e}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 1390$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{ex^2+d}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{e\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{e\sqrt{d^2-e^2x^4}\left(\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{d^2-e^2x^4}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d - e*x^2)^2/(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d - e*x^2])/(d*Sqrt[d + e*x^2])
+ (e*Sqrt[d^2 - e^2*x^4]*((d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSi
n[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*Sqrt[d^2 - e^2*x^4]) - (d^(3/2)*Sqrt
[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(e^(3/2)*S
qrt[d^2 - e^2*x^4])))/(d*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])))/Sqrt[d^2 - e^2
*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 344
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c
+ b*d*x^4)^FracPart[p])   Int[(e*x)^m*(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a
, b, c, d, e, m, p}, x] && EqQ[b*c + a*d, 0] &&  !IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.83 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

method	result
elliptic	$\frac{\left(-e^{2} x^{2}+d e \right) x}{d e \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{e}{d}}, i\right)\right)}{\sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$
default	$d^{2} \left(\frac{x}{2 d^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}+\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d^{2} \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}\right)+e^{2} \left(\frac{x}{2 e^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}-\frac{\sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \mathrm{E}}{2 e^{2} \sqrt{\frac{e}{d}}\, \sqrt{-e}}\right.$

input
```
int((-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-e^2*x^2+d*e)/d*x/e/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)-1/(e/d)^(1/2)*(1-e*x
^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2
),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{(ex^2+d)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)-(ex^2+d)\sqrt{\frac{e}{d}}F(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)+\sqrt{-e^2}}{dex^2+d^2}$$

input
```
integrate((-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
((e*x^2 + d)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - (e*x^2 + d)*s
qrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*x)/(d*
e*x^2 + d^2)
```

Sympy [F]

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(-d+ex^2)^2}{(-(-d+ex^2)(d+ex^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x**2+d)**2/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral((-d + e*x**2)**2/(-(-d + e*x**2)*(d + e*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(ex^2-d)^2}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 - d)^2/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(ex^2-d)^2}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 - d)^2/(-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(d-e\,x^2)^2}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input
```
int((d - e*x^2)^2/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
int((d - e*x^2)^2/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^2x^4+2de\,x^2+d^2}dx$$

input
```
int((-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)
```

3.114 $\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{x\left(d-ex^2\right)}{2d^2\sqrt{d^2-e^2x^4}}+\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
1/2*x*(-e*x^2+d)/d^2/(-e^2*x^4+d^2)^(1/2)+1/2*(1-e^2*x^4/d^2)^(1/2)*Ellipt
icE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{3dx+3dx\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-2ex^3\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hyperge}}{6d^2\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[(d - e*x^2)/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(3*d*x + 3*d*x*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (e
^2*x^4)/d^2] - 2*e*x^3*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4, 3/2,
 7/4, (e^2*x^4)/d^2])/(6*d^2*Sqrt[d^2 - e^2*x^4])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{(d^2-e^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{d-ex^2}{(d^2-e^2x^4)^{3/2}}dx$$

input
```
Int[(d - e*x^2)/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(77) = 154.

Time = 2.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.98

method	result
elliptic	$\frac{(-e^2x^2+de)x}{2d^2e\sqrt{(x^2+\frac{d}{e})(-e^2x^2+de)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}(x\sqrt{\frac{e}{d}},i)}{2d\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,(\mathrm{EllipticF}(x\sqrt{\frac{e}{d}},i)-\mathrm{EllipticE}(}{2d\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}$
default	$d\left(\frac{x}{2d^2\sqrt{-(x^4-\frac{d^2}{e^2})e^2}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}(x\sqrt{\frac{e}{d}},i)}{2d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)-e\left(\frac{x^3}{2d^2\sqrt{-(x^4-\frac{d^2}{e^2})e^2}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,(\mathrm{Ell}}{2d\sqrt{}}\right.$

input
```
int((-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e^2*x^2+d*e)/d^2*x/e/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/2/d/(e/d)^(1
/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(
e/d)^(1/2),I)-1/2/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*
x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

$$\int \frac{d-ex^2}{(d^2-e^2x^4)^{3/2}}\,dx=\frac{\sqrt{-e^2x^4+d^2}ex+(e^2x^2+de)\sqrt{\frac{e}{d}}E(\arcsin(x\sqrt{\frac{e}{d}})\mid-1)+((de-e^2)x^2+d^2-de)}{2(d^2e^2x^2+d^3e)}$$

input
```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-e^2*x^4 + d^2)*e*x + (e^2*x^2 + d*e)*sqrt(e/d)*elliptic_e(arcsi
n(x*sqrt(e/d)), -1) + ((d*e - e^2)*x^2 + d^2 - d*e)*sqrt(e/d)*elliptic_f(a
rcsin(x*sqrt(e/d)), -1))/(d^2*e^2*x^2 + d^3*e)
```

Sympy [A] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^2\Gamma\left(\frac{5}{4}\right)} - \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-e*x**2+d)/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*d**2*gamma(5/4)) - e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), e**2*x**4
*exp_polar(2*I*pi)/d**2)/(4*d**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx = \int -\frac{ex^2-d}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 - d)/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{3/2}}\,dx = \int -\frac{ex^2-d}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(e*x^2 - d)/(-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d-ex^2}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{d-e\,x^2}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input
```
int((d - e*x^2)/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
int((d - e*x^2)/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{d-ex^2}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{-e^3x^6-d\,e^2x^4+d^2e\,x^2+d^3}dx$$

input
```
int((-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**3 + d**2*e*x**2 - d*e**2*x**4 - e**3*x**6),
x)
```

3.115 $\int \frac{1}{(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{1}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{x}{2d^2\sqrt{d^2-e^2x^4}} + \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{2d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
1/2*x/d^2/(-e^2*x^4+d^2)^(1/2)+1/2*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)
*x/d^(1/2),I)/d^(3/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{1}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{x+x\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)}{2d^2\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[(d^2 - e^2*x^4)^(-3/2),x]
```

output
```
(x + x*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (e^2*x^4)/
d^2])/(2*d^2*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {749, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d^2-e^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 749}$$

$$\frac{\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{2d^2}+\frac{x}{2d^2\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{2d^2\sqrt{d^2-e^2x^4}}+\frac{x}{2d^2\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{2d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{x}{2d^2\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d^2 - e^2*x^4)^(-3/2),x]
```

output
```
x/(2*d^2*Sqrt[d^2 - e^2*x^4]) + (Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[
(Sqrt[e]*x)/Sqrt[d]], -1])/(2*d^(3/2)*Sqrt[e]*Sqrt[d^2 - e^2*x^4])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{x}{2 d^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}+\frac{\sqrt{1-\frac{e x^{2}}{d}}\, \sqrt{1+\frac{e x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d^{2} \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	90
elliptic	$\frac{x}{2 d^{2} \sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right) e^{2}}}+\frac{\sqrt{1-\frac{e x^{2}}{d}}\, \sqrt{1+\frac{e x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{2 d^{2} \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	90

input
```
int(1/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d^2*x/(-(x^4-d^2/e^2)*e^2)^(1/2)+1/2/d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)
*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=-\frac{\sqrt{-e^2x^4+d^2}ex-\left(e^2x^4-d^2\right)\sqrt{\frac{e}{d}}F\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)}{2\left(d^2e^3x^4-d^4e\right)}$$

input
```
integrate(1/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-e^2*x^4 + d^2)*e*x - (e^2*x^4 - d^2)*sqrt(e/d)*elliptic_f(arcs
in(x*sqrt(e/d)), -1))/(d^2*e^3*x^4 - d^4*e)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^3\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*d**3*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{x\left(1-\frac{e^2x^4}{d^2}\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ \frac{e^2x^4}{d^2}\right)}{\left(d^2-e^2\,x^4\right)^{3/2}}$$

input
```
int(1/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
(x*(1 - (e^2*x^4)/d^2)^(3/2)*hypergeom([1/4, 3/2], 5/4, (e^2*x^4)/d^2))/(d
^2 - e^2*x^4)^(3/2)
```

Reduce [F]

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^4x^8-2d^2e^2x^4+d^4}dx$$

input
```
int(1/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**4 - 2*d**2*e**2*x**4 + e**4*x**8),x)
```

3.116 $\int \frac{1}{(d-ex^2)(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 193

$$\int \frac{1}{(d-ex^2)\left(d^2-e^2x^4\right)^{3/2}}\,dx = \frac{x}{6d^2\left(d-ex^2\right)\sqrt{d^2-e^2x^4}}+\frac{x(5d+3ex^2)}{12d^4\sqrt{d^2-e^2x^4}} - \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{4d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{2\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
1/6*x/d^2/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2)+1/12*x*(3*e*x^2+5*d)/d^4/(-e^2*x
^4+d^2)^(1/2)-1/4*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(
5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+2/3*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1
/2)*x/d^(1/2),I)/d^(5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d-ex^2)\left(d^2-e^2x^4\right)^{3/2}}\,dx = \frac{\sqrt{-\frac{e}{d}}x(7d^2-2dex^2-3e^2x^4)+3id(d-ex^2)\sqrt{1-\frac{e^2x^4}{d^2}}E\bigl(i\text{arcsinh}\bigl(\sqrt{-}}{12d^4\sqrt{-\frac{e}{d}}\,(d-ex}$$

input
```
Integrate[1/((d - e*x^2)*(d^2 - e^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[-(e/d)]*x*(7*d^2 - 2*d*e*x^2 - 3*e^2*x^4) + (3*I)*d*(d - e*x^2)*Sqrt
[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - (8*I)*d*(d
- e*x^2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1])
/(12*d^4*Sqrt[-(e/d)]*(d - e*x^2)*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.49, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {1396, 316, 27, 402, 27, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-ex^2)(d^2-e^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{5/2}(ex^2+d)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(3ex^2+5d\right)}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}+\frac{x}{6d^2(d-ex^2)^{3/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{3ex^2+5d}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{3/2}}dx}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{2de\left(4ex^2+d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{2d^2e}+\frac{4x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{4ex^2+d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{d}+\frac{4x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{-\frac{\int -\frac{de\left(5d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}-\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{4x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\int \frac{de\left(5d-3ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}-\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{4x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\int \frac{5d-3ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}-\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{4x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{8d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}-\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{4x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{8d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}-\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{4x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{8d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}-\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{4x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}\sqrt{d+ex^2}}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{8d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}-\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{4x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/}}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{8d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}-\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{4x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2(d-ex^2)^{3/2}}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \color{blue}{762}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{8d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}-\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}+\frac{4x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}}{6d^2}+\right.}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/((d - e*x^2)*(d^2 - e^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(6*d^2*(d - e*x^2)^(3/2)*Sqrt[d + e*x^
2]) + ((4*x)/(d*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + ((-3*x*Sqrt[d - e*x^2])
/(2*d*Sqrt[d + e*x^2]) + ((-3*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[Ar
cSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
+ (8*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]]
, -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/(2*d))/d)/(6*d^2)))/Sqrt
[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.40

method	result
default	$\frac{\left(-e^{2} x^{2}+d e \right) x}{8 e \,d^{4} \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}+\frac{x \sqrt{-e^{2} x^{4}+d^{2}}}{12 e^{2} d^{3} \left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{3 \left(-e^{2} x^{2}-d e \right) x}{8 d^{4} e \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}+\frac{5 \sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{12 d^{3} \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$
elliptic	$\frac{\left(-e^{2} x^{2}+d e \right) x}{8 e \,d^{4} \sqrt{\left(x^{2}+\frac{d}{e}\right) \left(-e^{2} x^{2}+d e \right)}}+\frac{x \sqrt{-e^{2} x^{4}+d^{2}}}{12 e^{2} d^{3} \left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{3 \left(-e^{2} x^{2}-d e \right) x}{8 d^{4} e \sqrt{\left(x^{2}-\frac{d}{e}\right) \left(-e^{2} x^{2}-d e \right)}}+\frac{5 \sqrt{1-\frac{e \,x^{2}}{d}}\, \sqrt{1+\frac{e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{e}{d}}, i\right)}{12 d^{3} \sqrt{\frac{e}{d}}\, \sqrt{-e^{2} x^{4}+d^{2}}}$

input
```
int(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-e^2*x^2+d*e)/e/d^4*x/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/12/e^2/d^3*x
*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-3/8*(-e^2*x^2-d*e)/d^4*x/e/((x^2-d/e)*(-
e^2*x^2-d*e))^(1/2)+5/12/d^3/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/
2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+1/4/d^3/(e/d)^(1/2)*(1-
e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(
1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.17

$$\int \frac{1}{(d-ex^2)\left(d^2-e^2x^4\right)^{3/2}}\,dx =$$
$$-\frac{3\left(e^4x^6-de^3x^4-d^2e^2x^2+d^3e\right)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1\right)-\left(\left(5\,de^3+3\,e^4\right)x^6-\left(5\,d^2e^2+3\,de^3\right)x^4+5\,d}{12\left(d^4e^4x^6-d^5e^3x^4\right.}$$

input
```
integrate(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/12*(3*(e^4*x^6 - d*e^3*x^4 - d^2*e^2*x^2 + d^3*e)*sqrt(e/d)*elliptic_e(
arcsin(x*sqrt(e/d)), -1) - ((5*d*e^3 + 3*e^4)*x^6 - (5*d^2*e^2 + 3*d*e^3)*
x^4 + 5*d^4 + 3*d^3*e - (5*d^3*e + 3*d^2*e^2)*x^2)*sqrt(e/d)*elliptic_f(ar
csin(x*sqrt(e/d)), -1) + (3*e^3*x^5 + 2*d*e^2*x^3 - 7*d^2*e*x)*sqrt(-e^2*x
^4 + d^2))/(d^4*e^4*x^6 - d^5*e^3*x^4 - d^6*e^2*x^2 + d^7*e)
```

Sympy [F]

$$\int \frac{1}{(d-ex^2)\left(d^2-e^2x^4\right)^{3/2}}\,dx =$$
$$-\int \frac{1}{-d^3\sqrt{d^2-e^2x^4}+d^2ex^2\sqrt{d^2-e^2x^4}+de^2x^4\sqrt{d^2-e^2x^4}-e^3x^6\sqrt{d^2-e^2x^4}}\,dx$$

input
```
integrate(1/(-e*x**2+d)/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
-Integral(1/(-d**3*sqrt(d**2 - e**2*x**4) + d**2*e*x**2*sqrt(d**2 - e**2*x
**4) + d*e**2*x**4*sqrt(d**2 - e**2*x**4) - e**3*x**6*sqrt(d**2 - e**2*x**
4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d-ex^2)(d^2-e^2x^4)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{1}{(d-ex^2)(d^2-e^2x^4)^{3/2}}\,dx = \int -\frac{1}{(-e^2x^4+d^2)^{\frac{3}{2}}(ex^2-d)}\,dx$$

input
```
integrate(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((-e^2*x^4 + d^2)^(3/2)*(e*x^2 - d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d-ex^2)(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{(d^2-e^2\,x^4)^{3/2}\,(d-e\,x^2)}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(d-ex^2)\left(d^2-e^2x^4\right)^{3/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^5x^{10}+d\,e^4x^8+2d^2e^3x^6-2d^3e^2x^4-d^4e\,x^2+d^5}dx$$

input

```
int(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(3/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**5 - d**4*e*x**2 - 2*d**3*e**2*x**4 + 2*d**2
*e**3*x**6 + d*e**4*x**8 - e**5*x**10),x)
```

3.117 $\int \frac{1}{\left(d-ex^2\right)^2\left(d^2-e^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\begin{aligned}\int \frac{1}{(d-ex^2)^2(d^2-e^2x^4)^{3/2}}\,dx &= \frac{x}{10d^2(d-ex^2)^2\sqrt{d^2-e^2x^4}} \\ &+ \frac{7x}{30d^3(d-ex^2)\sqrt{d^2-e^2x^4}} + \frac{x(20d+21ex^2)}{60d^5\sqrt{d^2-e^2x^4}} \\ &- \frac{7\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{20d^{7/2}\sqrt{e}\sqrt{d^2-e^2x^4}} \\ &+ \frac{41\sqrt{1-\frac{e^2x^4}{d^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{60d^{7/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/10*x/d^2/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(1/2)+7/30*x/d^3/(-e*x^2+d)/(-e^2*x
^4+d^2)^(1/2)+1/60*x*(21*e*x^2+20*d)/d^5/(-e^2*x^4+d^2)^(1/2)-7/20*(1-e^2*
x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(7/2)/e^(1/2)/(-e^2*x^4+d^
2)^(1/2)+41/60*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(7/2
)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.66 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.84

$$\int \frac{1}{(d-ex^2)^2(d^2-e^2x^4)^{3/2}}\,dx = \frac{\sqrt{-\frac{e}{d}}x(40d^3-33d^2ex^2-22de^2x^4+21e^3x^6)+21id(d-ex^2)^2\sqrt{1-\frac{e^2x}{d^2}}}{60d^5\sqrt{-}}$$

input
```
Integrate[1/((d - e*x^2)^2*(d^2 - e^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[-(e/d)]*x*(40*d^3 - 33*d^2*e*x^2 - 22*d*e^2*x^4 + 21*e^3*x^6) + (21*
I)*d*(d - e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)
]*x], -1] - (41*I)*d*(d - e*x^2)^2*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*Arc
Sinh[Sqrt[-(e/d)]*x], -1])/(60*d^5*Sqrt[-(e/d)]*(d - e*x^2)^2*Sqrt[d^2 - e
^2*x^4])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.44, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {1396, 316, 27, 402, 27, 402, 25, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-ex^2)^2(d^2-e^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{7/2}(ex^2+d)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(5ex^2+9d\right)}{\left(d-ex^2\right)^{5/2}\left(ex^2+d\right)^{3/2}}dx}{10d^2e}+\frac{x}{10d^2(d-ex^2)^{5/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{5ex^2+9d}{\left(d-ex^2\right)^{5/2}\left(ex^2+d\right)^{3/2}}dx}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{2de\left(21ex^2+20d\right)}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}+\frac{7x}{3d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{21ex^2+20d}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{7x}{3d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\int -\frac{de\left(d-41ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{2d^2e}+\frac{41x}{2d\sqrt{d-ex^2}\sqrt{d+ex^2}}}{3d}+\frac{7x}{3d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{41x}{2d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\int \frac{de\left(d-41ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{2d^2e}}{3d}+\frac{7x}{3d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{41x}{2d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\int \frac{d-41ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{2d}}{3d}+\frac{7x}{3d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{41x}{2d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\int \frac{2de\left(20d-21ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{2d}}{3d}+\frac{7x}{3d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{41x}{2d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\int \frac{20d-21ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}}{2d}}{3d}+\frac{7x}{3d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{41x}{2d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{41d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-21\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}}{2d}}{3d}+\frac{7x}{3d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{41x}{2d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\frac{41d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-21\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}}{2d}}{3d}+\frac{7x}{3d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{10d^2}+\frac{}{10d^2(d-}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{41x}{2d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\frac{41d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{21d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}}{2d}}{3d}+\frac{7x}{3d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{10d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{41x}{2d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\frac{41d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}}{2d}}{3d}+\frac{7x}{3d\left(d-ex^2\right)^{3}}}{10d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{41x}{2d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\frac{41d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}}{2d}}{3d}+\frac{7x}{3d\left(d-ex^2\right)^{3/}}}{10d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{41x}{2d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{21x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\frac{41d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{21d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}}{2d}}{3d}}{10d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/((d - e*x^2)^2*(d^2 - e^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(10*d^2*(d - e*x^2)^(5/2)*Sqrt[d + e*x
^2]) + ((7*x)/(3*d*(d - e*x^2)^(3/2)*Sqrt[d + e*x^2]) + ((41*x)/(2*d*Sqrt[
d - e*x^2]*Sqrt[d + e*x^2]) - ((21*x*Sqrt[d - e*x^2])/(d*Sqrt[d + e*x^2])
- ((-21*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[
d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (41*d^(3/2)*Sqrt[1 -
 (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[
d - e*x^2]*Sqrt[d + e*x^2]))/d)/(2*d))/(3*d))/(10*d^2)))/Sqrt[d^2 - e^2*x^
4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.35

method	result
default	$\frac{\left(-e^{2} x^{2}+d e\right) x}{16 e\, d^{5} \sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2} x^{2}+d e\right)}}-\frac{x \sqrt{-e^{2} x^{4}+d^{2}}}{20 d^{3} e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+\frac{17 x \sqrt{-e^{2} x^{4}+d^{2}}}{120 d^{4} e^{2}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{33\left(-e^{2} x^{2}-d e\right) x}{80 e\, d^{5} \sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2} x^{2}-d e\right)}}+\frac{\sqrt{1-\frac{e\, x^{2}}{d}}\, \sqrt{1+}}{3 d^{4} \sqrt{}}$
elliptic	$\frac{\left(-e^{2} x^{2}+d e\right) x}{16 e\, d^{5} \sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2} x^{2}+d e\right)}}-\frac{x \sqrt{-e^{2} x^{4}+d^{2}}}{20 d^{3} e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+\frac{17 x \sqrt{-e^{2} x^{4}+d^{2}}}{120 d^{4} e^{2}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{33\left(-e^{2} x^{2}-d e\right) x}{80 e\, d^{5} \sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2} x^{2}-d e\right)}}+\frac{\sqrt{1-\frac{e\, x^{2}}{d}}\, \sqrt{1+}}{3 d^{4} \sqrt{}}$

input
```
int(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(-e^2*x^2+d*e)/e/d^5*x/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)-1/20/d^3*x/e^
3*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^3+17/120/d^4/e^2*x*(-e^2*x^4+d^2)^(1/2)/(
x^2-d/e)^2-33/80*(-e^2*x^2-d*e)/e/d^5*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1
/3/d^4/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2
)*EllipticF(x*(e/d)^(1/2),I)+7/20/d^4/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x
^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(
e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(d-e x^{2}\right)^{2}\left(d^{2}-e^{2} x^{4}\right)^{3/2}}\, dx =$$
$$-\frac{21\,(e^{5} x^{8}-2\, d e^{4} x^{6}+2\, d^{3} e^{2} x^{2}-d^{4} e) \sqrt{\frac{e}{d}} E(\arcsin\left(x \sqrt{\frac{e}{d}}\right) \mid -1)-((20\, d e^{4}+21\, e^{5}) x^{8}-2\,(20\, d^{2} e^{3}+21\, d}{60\,(d^{5}}$$

input
```
integrate(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/60*(21*(e^5*x^8 - 2*d*e^4*x^6 + 2*d^3*e^2*x^2 - d^4*e)*sqrt(e/d)*ellipt
ic_e(arcsin(x*sqrt(e/d)), -1) - ((20*d*e^4 + 21*e^5)*x^8 - 2*(20*d^2*e^3 +
 21*d*e^4)*x^6 - 20*d^5 - 21*d^4*e + 2*(20*d^4*e + 21*d^3*e^2)*x^2)*sqrt(e
/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + (21*e^4*x^7 - 22*d*e^3*x^5 - 33*
d^2*e^2*x^3 + 40*d^3*e*x)*sqrt(-e^2*x^4 + d^2))/(d^5*e^5*x^8 - 2*d^6*e^4*x
^6 + 2*d^8*e^2*x^2 - d^9*e)
```

Sympy [F]

$$\int \frac{1}{\left(d-ex^2\right)^2\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}\left(-d+ex^2\right)^2}\,dx$$

input
```
integrate(1/(-e*x**2+d)**2/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral(1/((-(-d + e*x**2)*(d + e*x**2))**(3/2)*(-d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(d-ex^2\right)^2\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}\left(ex^2-d\right)^2}\,dx$$

input
```
integrate(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(3/2)*(e*x^2 - d)^2), x)
```

Giac [F]

$$\int \frac{1}{(d-ex^2)^2 (d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{(-e^2x^4+d^2)^{\frac{3}{2}}(ex^2-d)^2}\,dx$$

input
```
integrate(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(3/2)*(e*x^2 - d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d-ex^2)^2 (d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{(d^2-e^2\,x^4)^{3/2}\,(d-e\,x^2)^2}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)^2),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(d-ex^2)^2 (d^2-e^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e^6x^{12}-2d\,e^5x^{10}-d^2e^4x^8+4d^3e^3x^6-d^4e^2x^4-2d^5e\,x^2+d^6}\,dx$$

input
```
int(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**6 - 2*d**5*e*x**2 - d**4*e**2*x**4 + 4*d**3
*e**3*x**6 - d**2*e**4*x**8 - 2*d*e**5*x**10 + e**6*x**12),x)
```

3.118 $\int \frac{1}{(d-ex^2)^3(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 261

$$\int \frac{1}{(d-ex^2)^3(d^2-e^2x^4)^{3/2}}\,dx = \frac{x}{14d^2(d-ex^2)^3\sqrt{d^2-e^2x^4}} + \frac{x}{7d^3(d-ex^2)^2\sqrt{d^2-e^2x^4}} + \frac{x}{4d^4(d-ex^2)\sqrt{d^2-e^2x^4}} + \frac{3x(5d+7ex^2)}{56d^6\sqrt{d^2-e^2x^4}} - \frac{3\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{8d^{9/2}\sqrt{e}\sqrt{d^2-e^2x^4}} + \frac{9\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{14d^{9/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
1/14*x/d^2/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(1/2)+1/7*x/d^3/(-e*x^2+d)^2/(-e^2*
x^4+d^2)^(1/2)+1/4*x/d^4/(-e*x^2+d)/(-e^2*x^4+d^2)^(1/2)+3/56*x*(7*e*x^2+5
*d)/d^6/(-e^2*x^4+d^2)^(1/2)-3/8*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x
/d^(1/2),I)/d^(9/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+9/14*(1-e^2*x^4/d^2)^(1/2
)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(9/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.73 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.59

$$\int \frac{1}{(d-ex^2)^3 (d^2-e^2x^4)^{3/2}}\,dx = \frac{-\frac{4x(-41d^4+60d^3ex^2+4d^2e^2x^4-48de^3x^6+21e^4x^8)}{(d-ex^2)^3} + \frac{12id\sqrt{1-\frac{e^2x^4}{d^2}}(7E(i\text{arcsinh}(\sqrt{-\frac{e}{d}}x)}{}}{224d^6\sqrt{d^2-e^2x^4}}$$

input
```
Integrate[1/((d - e*x^2)^3*(d^2 - e^2*x^4)^(3/2)),x]
```

output
```
((-4*x*(-41*d^4 + 60*d^3*e*x^2 + 4*d^2*e^2*x^4 - 48*d*e^3*x^6 + 21*e^4*x^8
))/(d - e*x^2)^3 + ((12*I)*d*Sqrt[1 - (e^2*x^4)/d^2]*(7*EllipticE[I*ArcSin
h[Sqrt[-(e/d)]*x], -1] - 12*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1]))/Sqr
t[-(e/d)])/(224*d^6*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.38, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {1396, 316, 27, 402, 27, 402, 27, 402, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-ex^2)^3 (d^2-e^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{9/2}(ex^2+d)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(7ex^2+13d\right)}{\left(d-ex^2\right)^{7/2}\left(ex^2+d\right)^{3/2}}dx}{14d^2e}+\frac{x}{14d^2(d-ex^2)^{7/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{7ex^2+13d}{\left(d-ex^2\right)^{7/2}\left(ex^2+d\right)^{3/2}}dx}{14d^2}+\frac{x}{14d^2(d-ex^2)^{7/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{10de\left(10ex^2+11d\right)}{\left(d-ex^2\right)^{5/2}\left(ex^2+d\right)^{3/2}}dx}{10d^2e}+\frac{2x}{d\left(d-ex^2\right)^{5/2}\sqrt{d+ex^2}}}{14d^2}+\frac{x}{14d^2(d-ex^2)^{7/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{10ex^2+11d}{\left(d-ex^2\right)^{5/2}\left(ex^2+d\right)^{3/2}}dx}{d}+\frac{2x}{d\left(d-ex^2\right)^{5/2}\sqrt{d+ex^2}}}{14d^2}+\frac{x}{14d^2(d-ex^2)^{7/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\int \frac{9de\left(7ex^2+5d\right)}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}+\frac{7x}{2d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{d}+\frac{2x}{d\left(d-ex^2\right)^{5/2}\sqrt{d+ex^2}}}{14d^2}+\frac{x}{14d^2(d-ex^2)^{7/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3\int \frac{7ex^2+5d}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{3/2}}dx}{2d}+\frac{7x}{2d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{d}+\frac{2x}{d\left(d-ex^2\right)^{5/2}\sqrt{d+ex^2}}}{14d^2}+\frac{x}{14d^2(d-ex^2)^{7/2}\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3\left(\frac{\int -\frac{2de\left(d-6ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{2d^2e}+\frac{6x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}\right)}{2d}+\frac{7x}{2d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{d}+\frac{2x}{d\left(d-ex^2\right)^{5/2}\sqrt{d+ex^2}}}{14d^2}+\frac{x}{14d^2(d-ex^2)^{7/}}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3\left(\frac{6x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\int \frac{d-6ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{d}\right)}{2d}+\frac{7x}{2d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{d}+\frac{2x}{d\left(d-ex^2\right)^{5/2}\sqrt{d+ex^2}}}{14d^2}+\frac{x}{14d^2(d-ex^2)^{7/2}}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3\left(\frac{6x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int\frac{de\left(5d-7ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{d}\right)}{2d}+\frac{7x}{2d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{d}+\frac{2x}{d\left(d-ex^2\right)^{5/2}\sqrt{d+ex^2}}}{14d^2}+\frac{}{14d^2(d-}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3\left(\frac{6x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int\frac{5d-7ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}}{d}\right)}{2d}+\frac{7x}{2d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{d}+\frac{2x}{d\left(d-ex^2\right)^{5/2}\sqrt{d+ex^2}}}{14d^2}+\frac{}{14d^2(d-}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3\left(\frac{6x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{12d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-7\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}}{d}\right)}{2d}+\frac{7x}{2d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{d}+\frac{2x}{d\left(d-ex^2\right)^{5/2}}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3\left(\frac{6x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\frac{12d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-7\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}}{d}\right)}{2d}+\frac{7x}{2d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{d}+\frac{}{d\left(d-ex^2\right.}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3\left(\frac{6x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\frac{12d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}}{d}\right)}{2d}+\frac{7x}{2d\left(d-ex^2\right)^{3/2}\sqrt{d+ex^2}}}{d}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3\left(\frac{6x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\frac{12d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}}{d}\right)}{2d}+\frac{7}{2d\left(d-ex^2\right)}}{d}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3\left(\frac{6x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\frac{12d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}}{d}\right)}{2d}+\frac{7x}{2d\left(d-ex^2\right)^{3}}}{d}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3\left(\frac{6x}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\frac{12d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}}{d}\right.}{2d}}{d}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/((d - e*x^2)^3*(d^2 - e^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(14*d^2*(d - e*x^2)^(7/2)*Sqrt[d + e*x
^2]) + ((2*x)/(d*(d - e*x^2)^(5/2)*Sqrt[d + e*x^2]) + ((7*x)/(2*d*(d - e*x
^2)^(3/2)*Sqrt[d + e*x^2]) + (3*((6*x)/(d*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
 - ((7*x*Sqrt[d - e*x^2])/(2*d*Sqrt[d + e*x^2]) - ((-7*d^(3/2)*Sqrt[1 - (e
^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d -
 e*x^2]*Sqrt[d + e*x^2]) + (12*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[A
rcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
)/(2*d))/d))/(2*d))/d)/(14*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.31

method	result
default	$\frac{(-e^2x^2+de)x}{32e\,d^6\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}+\frac{x\sqrt{-e^2x^4+d^2}}{28d^3e^4\left(x^2-\frac{d}{e}\right)^4}-\frac{5x\sqrt{-e^2x^4+d^2}}{56d^4e^3\left(x^2-\frac{d}{e}\right)^3}+\frac{19x\sqrt{-e^2x^4+d^2}}{112e^2d^5\left(x^2-\frac{d}{e}\right)^2}-\frac{13(-e^2x^2-de)x}{32e\,d^6\sqrt{\left(x^2-\frac{d}{e}\right)(-e^2x^2-de}}$
elliptic	$\frac{(-e^2x^2+de)x}{32e\,d^6\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}+\frac{x\sqrt{-e^2x^4+d^2}}{28d^3e^4\left(x^2-\frac{d}{e}\right)^4}-\frac{5x\sqrt{-e^2x^4+d^2}}{56d^4e^3\left(x^2-\frac{d}{e}\right)^3}+\frac{19x\sqrt{-e^2x^4+d^2}}{112e^2d^5\left(x^2-\frac{d}{e}\right)^2}-\frac{13(-e^2x^2-de)x}{32e\,d^6\sqrt{\left(x^2-\frac{d}{e}\right)(-e^2x^2-de}}$

input

```
int(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/32*(-e^2*x^2+d*e)/e/d^6*x/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/28/d^3*x/e^
4*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^4-5/56/d^4/e^3*x*(-e^2*x^4+d^2)^(1/2)/(x^
2-d/e)^3+19/112/e^2/d^5*x*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-13/32*(-e^2*x^2
-d*e)/e/d^6*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+15/56/d^5/(e/d)^(1/2)*(1-e*
x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2
),I)+3/8/d^5/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2
)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.29

$$\int \frac{1}{(d-ex^2)^3 (d^2-e^2x^4)^{3/2}}\,dx =$$

$$-\frac{21\,(e^6x^{10}-3\,de^5x^8+2\,d^2e^4x^6+2\,d^3e^3x^4-3\,d^4e^2x^2+d^5e)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)-3\left((5\,de^5+7\,e^6\right)}{}$$

input
```
integrate(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/56*(21*(e^6*x^10 - 3*d*e^5*x^8 + 2*d^2*e^4*x^6 + 2*d^3*e^3*x^4 - 3*d^4*
e^2*x^2 + d^5*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - 3*((5*d*e
^5 + 7*e^6)*x^10 - 3*(5*d^2*e^4 + 7*d*e^5)*x^8 + 2*(5*d^3*e^3 + 7*d^2*e^4)
*x^6 + 5*d^6 + 7*d^5*e + 2*(5*d^4*e^2 + 7*d^3*e^3)*x^4 - 3*(5*d^5*e + 7*d^
4*e^2)*x^2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + (21*e^5*x^9 -
48*d*e^4*x^7 + 4*d^2*e^3*x^5 + 60*d^3*e^2*x^3 - 41*d^4*e*x)*sqrt(-e^2*x^4
+ d^2))/(d^6*e^6*x^10 - 3*d^7*e^5*x^8 + 2*d^8*e^4*x^6 + 2*d^9*e^3*x^4 - 3*
d^10*e^2*x^2 + d^11*e)
```

Sympy [F]

$$\int \frac{1}{(d-ex^2)^3 (d^2-e^2x^4)^{3/2}}\,dx =$$

$$-\int \frac{1}{-d^5\sqrt{d^2-e^2x^4}+3d^4ex^2\sqrt{d^2-e^2x^4}-2d^3e^2x^4\sqrt{d^2-e^2x^4}-2d^2e^3x^6\sqrt{d^2-e^2x^4}+3de^4x^8\sqrt{d^2-e^2x}}$$

input
```
integrate(1/(-e*x**2+d)**3/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
-Integral(1/(-d**5*sqrt(d**2 - e**2*x**4) + 3*d**4*e*x**2*sqrt(d**2 - e**2
*x**4) - 2*d**3*e**2*x**4*sqrt(d**2 - e**2*x**4) - 2*d**2*e**3*x**6*sqrt(d
**2 - e**2*x**4) + 3*d*e**4*x**8*sqrt(d**2 - e**2*x**4) - e**5*x**10*sqrt(
d**2 - e**2*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\left(d-ex^2\right)^3\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int -\frac{1}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}\left(ex^2-d\right)^3}\,dx$$

input
```
integrate(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((-e^2*x^4 + d^2)^(3/2)*(e*x^2 - d)^3), x)
```

Giac [F]

$$\int \frac{1}{\left(d-ex^2\right)^3\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int -\frac{1}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}\left(ex^2-d\right)^3}\,dx$$

input
```
integrate(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((-e^2*x^4 + d^2)^(3/2)*(e*x^2 - d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d-ex^2\right)^3\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(d^2-e^2\,x^4\right)^{3/2}\,\left(d-e\,x^2\right)^3}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)^3),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(d-ex^2)^3\,(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^7x^{14}+3d\,e^6x^{12}-d^2e^5x^{10}-5d^3e^4x^8+5d^4e^3x^6+d^5e^2x^4-3d^6e\,x^2}$$

input

```
int(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**7 - 3*d**6*e*x**2 + d**5*e**2*x**4 + 5*d**4
*e**3*x**6 - 5*d**3*e**4*x**8 - d**2*e**5*x**10 + 3*d*e**6*x**12 - e**7*x*
*14),x)
```

3.119 $\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 186

$$\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx=\frac{2x(d-ex^2)}{3(d^2-e^2x^4)^{3/2}}+\frac{x(d-3ex^2)}{6d^2\sqrt{d^2-e^2x^4}}+\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}-\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{3\sqrt{d}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output

```
2/3*x*(-e*x^2+d)/(-e^2*x^4+d^2)^(3/2)+1/6*x*(-3*e*x^2+d)/d^2/(-e^2*x^4+d^2
)^(1/2)+1/2*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e
^(1/2)/(-e^2*x^4+d^2)^(1/2)-1/3*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/
d^(1/2),I)/d^(1/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.83

$$\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx=\frac{dx(5d^2-2dex^2-e^2x^4)+dx(d^2-e^2x^4)\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\right.}{6d^2(d^2-e^2x^4)^{3}}$$

input

```
Integrate[(d - e*x^2)^3/(d^2 - e^2*x^4)^(5/2),x]
```

output

```
(d*x*(5*d^2 - 2*d*e*x^2 - e^2*x^4) + d*x*(d^2 - e^2*x^4)*Sqrt[1 - (e^2*x^4
)/d^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (e^2*x^4)/d^2] - 4*e*x^3*(d^2 - e^
2*x^4)*Sqrt[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4, 5/2, 7/4, (e^2*x^4)/
d^2])/(6*d^2*(d^2 - e^2*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1396, 314, 25, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\sqrt{d-ex^2}}{(ex^2+d)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}-\frac{\int -\frac{2d-ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{3d}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{2d-ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{de\left(3ex^2+d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{de\left(3ex^2+d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{3ex^2+d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-2d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 289}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 329}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{2d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}+\frac{3x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{3d}+\frac{x\sqrt{d-ex^2}}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d - e*x^2)^3/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d - e*x^2])/(3*d*(d + e*x^2)^(3/
2)) + ((3*x*Sqrt[d - e*x^2])/(2*d*Sqrt[d + e*x^2]) + ((3*d^(3/2)*Sqrt[1 -
(e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d
 - e*x^2]*Sqrt[d + e*x^2]) - (2*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[
ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]
))/(2*d))/(3*d)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 6.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{3d\,e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{\left(-e^{2}x^{2}+de\right)x}{2d^{2}e\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}+\frac{\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{6d\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}-$
default	$d^{3}\left(\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{6d^{2}e^{4}\left(x^{4}-\frac{d^{2}}{e^{2}}\right)^{2}}+\frac{5x}{12d^{4}\sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right)e^{2}}}+\frac{5\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{12d^{4}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right)-e^{3}\left(\frac{x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{6e^{6}\left(x^{4}-\frac{d^{2}}{e^{2}}\right)^{2}}-\right.$

input
```
int((-e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*x/e^2*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+1/2*(-e^2*x^2+d*e)/d^2*x/e/((
x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/6/d/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^
2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-1/2/d/(e/d)^(1/
2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(
e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.89

$$\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{3\left(e^3x^4+2\,de^2x^2+d^2e\right)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1)+((de^2-3\,e^3)x^4+d^3-3\,d^2e+}{6\left(d^2e^3x^4+2\,d^3e}$$

input
```
integrate((-e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(3*(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(
e/d)), -1) + ((d*e^2 - 3*e^3)*x^4 + d^3 - 3*d^2*e + 2*(d^2*e - 3*d*e^2)*x^
2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d^2)*(3
*e^2*x^3 + 5*d*e*x))/(d^2*e^3*x^4 + 2*d^3*e^2*x^2 + d^4*e)
```

Sympy [F]

$$\begin{aligned}\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx =& \\ &-\int\left(-\frac{d^3}{d^4\sqrt{d^2-e^2x^4}-2d^2e^2x^4\sqrt{d^2-e^2x^4}+e^4x^8\sqrt{d^2-e^2x^4}}\right)dx \\ &-\int\frac{e^3x^6}{d^4\sqrt{d^2-e^2x^4}-2d^2e^2x^4\sqrt{d^2-e^2x^4}+e^4x^8\sqrt{d^2-e^2x^4}}\,dx \\ &-\int\left(-\frac{3de^2x^4}{d^4\sqrt{d^2-e^2x^4}-2d^2e^2x^4\sqrt{d^2-e^2x^4}+e^4x^8\sqrt{d^2-e^2x^4}}\right)dx \\ &-\int\frac{3d^2ex^2}{d^4\sqrt{d^2-e^2x^4}-2d^2e^2x^4\sqrt{d^2-e^2x^4}+e^4x^8\sqrt{d^2-e^2x^4}}\,dx\end{aligned}$$

input
```
integrate((-e*x**2+d)**3/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
-Integral(-d**3/(d**4*sqrt(d**2 - e**2*x**4) - 2*d**2*e**2*x**4*sqrt(d**2
- e**2*x**4) + e**4*x**8*sqrt(d**2 - e**2*x**4)), x) - Integral(e**3*x**6/
(d**4*sqrt(d**2 - e**2*x**4) - 2*d**2*e**2*x**4*sqrt(d**2 - e**2*x**4) + e
**4*x**8*sqrt(d**2 - e**2*x**4)), x) - Integral(-3*d*e**2*x**4/(d**4*sqrt(
d**2 - e**2*x**4) - 2*d**2*e**2*x**4*sqrt(d**2 - e**2*x**4) + e**4*x**8*sq
rt(d**2 - e**2*x**4)), x) - Integral(3*d**2*e*x**2/(d**4*sqrt(d**2 - e**2*
x**4) - 2*d**2*e**2*x**4*sqrt(d**2 - e**2*x**4) + e**4*x**8*sqrt(d**2 - e*
*2*x**4)), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx = \int -\frac{(ex^2-d)^3}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 - d)^3/(-e^2*x^4 + d^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx = \int -\frac{(ex^2-d)^3}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(e*x^2 - d)^3/(-e^2*x^4 + d^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(d-e\,x^2)^3}{(d^2-e^2\,x^4)^{5/2}}\,dx$$

input
```
int((d - e*x^2)^3/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
int((d - e*x^2)^3/(d^2 - e^2*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d-ex^2)^3}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx$$

input
```
int((-e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x)
```

output
```
int(sqrt(d**2 - e**2*x**4)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**
*6),x)
```

3.120 $\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 191

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{x(d-ex^2)}{3d\,(d^2-e^2x^4)^{3/2}} + \frac{x(2d-3ex^2)}{6d^3\sqrt{d^2-e^2x^4}} + \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}} - \frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{6d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
1/3*x*(-e*x^2+d)/d/(-e^2*x^4+d^2)^(3/2)+1/6*x*(-3*e*x^2+2*d)/d^3/(-e^2*x^4
+d^2)^(1/2)+1/2*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(3/
2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)-1/6*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2
)*x/d^(1/2),I)/d^(3/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.90

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{\sqrt{-\frac{e}{d}}x(4d^2-dex^2-3e^2x^4)-3id(d+ex^2)\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)+}{6d^3\sqrt{-\frac{e}{d}}\,(d+ex^2)\sqrt{d^2-e^2}}$$

input

```
Integrate[(d - e*x^2)^2/(d^2 - e^2*x^4)^(5/2),x]
```

output

```
(Sqrt[-(e/d)]*x*(4*d^2 - d*e*x^2 - 3*e^2*x^4) - (3*I)*d*(d + e*x^2)*Sqrt[1
 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] + I*d*(d + e*x^
2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1])/(6*d^
3*Sqrt[-(e/d)]*(d + e*x^2)*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.30, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1396, 316, 25, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-ex^2\right)^2}{\left(d^2-e^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d-ex^2}}{6d^2\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{e\left(5d-ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(5d-ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}+\frac{x\sqrt{d-ex^2}}{6d^2\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{5d-ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}-\frac{\int -\frac{2de\left(3ex^2+2d\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{3ex^2+2d}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-d\int \frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx-\frac{d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 329$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3d\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 765$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{3d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}+\frac{3x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{6d^2}+\frac{x\sqrt{d-ex^2}}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d - e*x^2)^2/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d - e*x^2])/(6*d^2*(d + e*x^2)^(
3/2)) + ((3*x*Sqrt[d - e*x^2])/(d*Sqrt[d + e*x^2]) + ((3*d^(3/2)*Sqrt[1 -
(e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d
 - e*x^2]*Sqrt[d + e*x^2]) - (d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[Ar
cSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))
/d)/(6*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.16

method	result
elliptic	$\frac{x\sqrt{-e^2x^4+d^2}}{6e^2d^2\left(x^2+\frac{d}{e}\right)^2}+\frac{\left(-e^2x^2+de\right)x}{2e\,d^3\sqrt{\left(x^2+\frac{d}{e}\right)\left(-e^2x^2+de\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{3d^2\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}-\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\left(\operatorname{EllipticF}\left(\right.\right.}{2d^2\sqrt{\frac{e}{d}}\,\sqrt{-}}$
default	$d^2\left(\frac{x\sqrt{-e^2x^4+d^2}}{6d^2e^4\left(x^4-\frac{d^2}{e^2}\right)^2}+\frac{5x}{12d^4\sqrt{-\left(x^4-\frac{d^2}{e^2}\right)e^2}}+\frac{5\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{12d^4\sqrt{\frac{e}{d}}\,\sqrt{-e^2x^4+d^2}}\right)+e^2\left(\frac{x\sqrt{-e^2x^4+d^2}}{6e^6\left(x^4-\frac{d^2}{e^2}\right)^2}-\frac{}{12e}\right.$

input
```
int((-e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/6/e^2/d^2*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+1/2*(-e^2*x^2+d*e)/e/d^3*x/
((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)+1/3/d^2/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+
e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-1/2/d^2/(e/
d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(Ellipti
cF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.89

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{3\,(e^3x^4+2\,de^2x^2+d^2e)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)+((2\,de^2-3\,e^3)x^4+2\,d^3-3\,d^2}{6\,(d^3e^3x^4+2}$$

input

```
integrate((-e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output

```
1/6*(3*(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(
e/d)), -1) + ((2*d*e^2 - 3*e^3)*x^4 + 2*d^3 - 3*d^2*e + 2*(2*d^2*e - 3*d*e
^2)*x^2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + sqrt(-e^2*x^4 + d
^2)*(3*e^2*x^3 + 4*d*e*x))/(d^3*e^3*x^4 + 2*d^4*e^2*x^2 + d^5*e)
```

Sympy [F]

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(-d+ex^2)^2}{(-(-d+ex^2)\,(d+ex^2))^{\frac{5}{2}}}\,dx$$

input

```
integrate((-e*x**2+d)**2/(-e**2*x**4+d**2)**(5/2),x)
```

output

```
Integral((-d + e*x**2)**2/(-(-d + e*x**2)*(d + e*x**2))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(ex^2-d)^2}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 - d)^2/(-e^2*x^4 + d^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(ex^2-d)^2}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 - d)^2/(-e^2*x^4 + d^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^2}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(d-e\,x^2)^2}{(d^2-e^2\,x^4)^{5/2}}\,dx$$

input
```
int((d - e*x^2)^2/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
int((d - e*x^2)^2/(d^2 - e^2*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d - ex^2)^2}{(d^2 - e^2x^4)^{5/2}}\, dx = \int \frac{\sqrt{-e^2x^4 + d^2}}{-e^4x^8 - 2d\, e^3x^6 + 2d^3e\, x^2 + d^4} dx$$

input

```
int((-e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**4 + 2*d**3*e*x**2 - 2*d*e**3*x**6 - e**4*x*
*8),x)
```

3.121 $\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 191

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{x(d-ex^2)}{6d^2\left(d^2-e^2x^4\right)^{3/2}}+\frac{x(5d-3ex^2)}{12d^4\sqrt{d^2-e^2x^4}}+\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{4d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{6d^{5/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
1/6*x*(-e*x^2+d)/d^2/(-e^2*x^4+d^2)^(3/2)+1/12*x*(-3*e*x^2+5*d)/d^4/(-e^2*
x^4+d^2)^(1/2)+1/4*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^
(5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+1/6*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(
1/2)*x/d^(1/2),I)/d^(5/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.77

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{7d^3x-5de^2x^5+5dx(d^2-e^2x^4)\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)-4e}{12d^4\left(d^2-e^2x^4\right)^{3/2}}$$

input
```
Integrate[(d - e*x^2)/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(7*d^3*x - 5*d*e^2*x^5 + 5*d*x*(d^2 - e^2*x^4)*Sqrt[1 - (e^2*x^4)/d^2]*Hyp
ergeometric2F1[1/4, 1/2, 5/4, (e^2*x^4)/d^2] - 4*e*x^3*(d^2 - e^2*x^4)*Sqr
t[1 - (e^2*x^4)/d^2]*Hypergeometric2F1[3/4, 5/2, 7/4, (e^2*x^4)/d^2])/(12*
d^4*(d^2 - e^2*x^4)^(3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{5/2}}dx$$

input
```
Int[(d - e*x^2)/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.41

method	result
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{12d^{3}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{3\left(-e^{2}x^{2}+de\right)x}{8e\,d^{4}\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}-\frac{\left(-e^{2}x^{2}-de\right)x}{8d^{4}e\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{5\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{12d^{3}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$
default	$d\left(\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{6d^{2}e^{4}\left(x^{4}-\frac{d^{2}}{e^{2}}\right)^{2}}+\frac{5x}{12d^{4}\sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right)e^{2}}}+\frac{5\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{12d^{4}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}\right)-e\left(\frac{x^{3}\sqrt{-e^{2}x^{4}+d^{2}}}{6d^{2}e^{4}\left(x^{4}-\frac{d^{2}}{e^{2}}\right)^{2}}+\frac{}{4d^{4}}\right.$

input
```
int((-e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/d^3/e^2*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+3/8*(-e^2*x^2+d*e)/e/d^4*x
/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)-1/8*(-e^2*x^2-d*e)/d^4*x/e/((x^2-d/e)*(-
e^2*x^2-d*e))^(1/2)+5/12/d^3/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/
2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)-1/4/d^3/(e/d)^(1/2)*(1-
e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(
1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.17

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx = \frac{3\left(e^4x^6+de^3x^4-d^2e^2x^2-d^3e\right)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)+((5\,de^3-3\,e^4)x^6+(5}{}$$

input
```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(3*(e^4*x^6 + d*e^3*x^4 - d^2*e^2*x^2 - d^3*e)*sqrt(e/d)*elliptic_e(a
rcsin(x*sqrt(e/d)), -1) + ((5*d*e^3 - 3*e^4)*x^6 + (5*d^2*e^2 - 3*d*e^3)*x
^4 - 5*d^4 + 3*d^3*e - (5*d^3*e - 3*d^2*e^2)*x^2)*sqrt(e/d)*elliptic_f(arc
sin(x*sqrt(e/d)), -1) + (3*e^3*x^5 - 2*d*e^2*x^3 - 7*d^2*e*x)*sqrt(-e^2*x^
4 + d^2))/(d^4*e^4*x^6 + d^5*e^3*x^4 - d^6*e^2*x^2 - d^7*e)
```

Sympy [A] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.43

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^4\Gamma\left(\frac{5}{4}\right)} - \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\right|\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^5\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-e*x**2+d)/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*d**4*gamma(5/4)) - e*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), e**2*x**4
*exp_polar(2*I*pi)/d**2)/(4*d**5*gamma(7/4))
```

Maxima [F]

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx = \int -\frac{ex^2-d}{\left(-e^2x^4+d^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 - d)/(-e^2*x^4 + d^2)^(5/2), x)
```

Giac [F]

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx = \int -\frac{ex^2-d}{\left(-e^2x^4+d^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(e*x^2 - d)/(-e^2*x^4 + d^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\int \frac{d-e\,x^2}{\left(d^2-e^2\,x^4\right)^{5/2}}\,dx$$

input

```
int((d - e*x^2)/(d^2 - e^2*x^4)^(5/2),x)
```

output

```
int((d - e*x^2)/(d^2 - e^2*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{d-ex^2}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^5x^{10}+d\,e^4x^8-2d^2e^3x^6-2d^3e^2x^4+d^4e\,x^2+d^5}dx$$

input

```
int((-e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**5 + d**4*e*x**2 - 2*d**3*e**2*x**4 - 2*d**2
*e**3*x**6 + d*e**4*x**8 + e**5*x**10),x)
```

3.122 $\int \frac{1}{\left(d^2-e^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 111

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{x}{6d^2\left(d^2-e^2x^4\right)^{3/2}}+\frac{5x}{12d^4\sqrt{d^2-e^2x^4}}+\frac{5\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{12d^{7/2}\sqrt{e}\sqrt{d^2-e^2x^4}}$$

output
```
1/6*x/d^2/(-e^2*x^4+d^2)^(3/2)+5/12*x/d^4/(-e^2*x^4+d^2)^(1/2)+5/12*(1-e^2
*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(7/2)/e^(1/2)/(-e^2*x^4+d
^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{7d^2x-5e^2x^5+5x(d^2-e^2x^4)\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{e^2x^4}{d^2}\right)}{12d^4\left(d^2-e^2x^4\right)^{3/2}}$$

input

```
Integrate[(d^2 - e^2*x^4)^(-5/2),x]
```

output

```
(7*d^2*x - 5*e^2*x^5 + 5*x*(d^2 - e^2*x^4)*Sqrt[1 - (e^2*x^4)/d^2]*Hyperge
ometric2F1[1/4, 1/2, 5/4, (e^2*x^4)/d^2])/(12*d^4*(d^2 - e^2*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {749, 749, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d^2-e^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{749}$$

$$\frac{5\int \frac{1}{(d^2-e^2x^4)^{3/2}}dx}{6d^2}+\frac{x}{6d^2\left(d^2-e^2x^4\right)^{3/2}}$$

$$\Big\downarrow \text{749}$$

$$\frac{5\left(\frac{\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{2d^2}+\frac{x}{2d^2\sqrt{d^2-e^2x^4}}\right)}{6d^2}+\frac{x}{6d^2\left(d^2-e^2x^4\right)^{3/2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{5\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{2d^2\sqrt{d^2-e^2x^4}}+\frac{x}{2d^2\sqrt{d^2-e^2x^4}}\right)}{6d^2}+\frac{x}{6d^2\left(d^2-e^2x^4\right)^{3/2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{5\left(\frac{\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{2d^{3/2}\sqrt{e}\sqrt{d^2-e^2x^4}}+\frac{x}{2d^2\sqrt{d^2-e^2x^4}}\right)}{6d^2}+\frac{x}{6d^2\left(d^2-e^2x^4\right)^{3/2}}$$

input
```
Int[(d^2 - e^2*x^4)^(-5/2),x]
```

output
```
x/(6*d^2*(d^2 - e^2*x^4)^(3/2)) + (5*(x/(2*d^2*Sqrt[d^2 - e^2*x^4]) + (Sqr
t[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(2*d^(3/2
)*Sqrt[e]*Sqrt[d^2 - e^2*x^4])))/(6*d^2)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{6d^{2}e^{4}\left(x^{4}-\frac{d^{2}}{e^{2}}\right)^{2}}+\frac{5x}{12d^{4}\sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right)e^{2}}}+\frac{5\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{12d^{4}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	127
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{6d^{2}e^{4}\left(x^{4}-\frac{d^{2}}{e^{2}}\right)^{2}}+\frac{5x}{12d^{4}\sqrt{-\left(x^{4}-\frac{d^{2}}{e^{2}}\right)e^{2}}}+\frac{5\sqrt{1-\frac{e\,x^{2}}{d}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{12d^{4}\sqrt{\frac{e}{d}}\,\sqrt{-e^{2}x^{4}+d^{2}}}$	127

input
```
int(1/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/d^2*x/e^4*(-e^2*x^4+d^2)^(1/2)/(x^4-d^2/e^2)^2+5/12/d^4*x/(-(x^4-d^2/e
^2)*e^2)^(1/2)+5/12/d^4/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-
e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{5/2}}\,dx = \frac{5\left(e^4x^8-2\,d^2e^2x^4+d^4\right)\sqrt{\frac{e}{d}}F(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)-\left(5\,e^3x^5-7\,d^2ex\right)\sqrt{-e^2x^4+d^2}}{12\left(d^4e^5x^8-2\,d^6e^3x^4+d^8e\right)}$$

input
```
integrate(1/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(5*(e^4*x^8 - 2*d^2*e^2*x^4 + d^4)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt
(e/d)), -1) - (5*e^3*x^5 - 7*d^2*e*x)*sqrt(-e^2*x^4 + d^2))/(d^4*e^5*x^8 -
 2*d^6*e^3*x^4 + d^8*e)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.35

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{5/2}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{e^2x^4e^{2i\pi}}{d^2}\right)}{4d^5\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), e**2*x**4*exp_polar(2*I*pi)/d**2)/(
4*d**5*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(d^2-e^2x^4)^{5/2}}\,dx=\int \frac{1}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(d^2-e^2x^4)^{5/2}}\,dx=\int \frac{1}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(-5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.41

$$\int \frac{1}{(d^2-e^2x^4)^{5/2}}\,dx=\frac{x\left(1-\frac{e^2\,x^4}{d^2}\right)^{5/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{5}{2};\ \frac{5}{4};\ \frac{e^2\,x^4}{d^2}\right)}{(d^2-e^2\,x^4)^{5/2}}$$

input
```
int(1/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
(x*(1 - (e^2*x^4)/d^2)^(5/2)*hypergeom([1/4, 5/2], 5/4, (e^2*x^4)/d^2))/(d
^2 - e^2*x^4)^(5/2)
```

Reduce [F]

$$\int \frac{1}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{-e^6x^{12}+3d^2e^4x^8-3d^4e^2x^4+d^6}dx$$

input

```
int(1/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**6 - 3*d**4*e**2*x**4 + 3*d**2*e**4*x**8 - e
**6*x**12),x)
```

3.123 $\int \frac{1}{(d-ex^2)(d^2-e^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\begin{aligned}\int \frac{1}{(d-ex^2)\left(d^2-e^2x^4\right)^{5/2}}\,dx &= \frac{x}{10d^2\left(d-ex^2\right)\left(d^2-e^2x^4\right)^{3/2}}\\ &+ \frac{x(9d+7ex^2)}{60d^4\left(d^2-e^2x^4\right)^{3/2}} + \frac{x(15d+7ex^2)}{40d^6\sqrt{d^2-e^2x^4}} - \frac{7\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{40d^{9/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\\ &+ \frac{11\sqrt{1-\frac{e^2x^4}{d^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{20d^{9/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/10*x/d^2/(-e*x^2+d)/(-e^2*x^4+d^2)^(3/2)+1/60*x*(7*e*x^2+9*d)/d^4/(-e^2*
x^4+d^2)^(3/2)+1/40*x*(7*e*x^2+15*d)/d^6/(-e^2*x^4+d^2)^(1/2)-7/40*(1-e^2*
x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(9/2)/e^(1/2)/(-e^2*x^4+d^
2)^(1/2)+11/20*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(9/2
)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.66 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.72

$$\int \frac{1}{(d-ex^2)(d^2-e^2x^4)^{5/2}}\,dx = \frac{\frac{x(75d^4-28d^3ex^2-80d^2e^2x^4+24de^3x^6+21e^4x^8)}{(d-ex^2)^2(d+ex^2)} + \frac{3id\sqrt{1-\frac{e^2x^4}{d^2}}\left(7E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-}{\sqrt{-\frac{e}{d}}}}{120d^6\sqrt{d^2-e^2x^4}}$$

input

```
Integrate[1/((d - e*x^2)*(d^2 - e^2*x^4)^(5/2)),x]
```

output

```
((x*(75*d^4 - 28*d^3*e*x^2 - 80*d^2*e^2*x^4 + 24*d*e^3*x^6 + 21*e^4*x^8))/
((d - e*x^2)^2*(d + e*x^2)) + ((3*I)*d*Sqrt[1 - (e^2*x^4)/d^2]*(7*Elliptic
E[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - 22*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x],
 -1]))/Sqrt[-(e/d)])/(120*d^6*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.61, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1396, 316, 27, 402, 27, 402, 27, 402, 27, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-ex^2)(d^2-e^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{7/2}(ex^2+d)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(7ex^2+9d\right)}{\left(d-ex^2\right)^{5/2}\left(ex^2+d\right)^{5/2}}dx}{10d^2e}+\frac{x}{10d^2(d-ex^2)^{5/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{7ex^2+9d}{\left(d-ex^2\right)^{5/2}\left(ex^2+d\right)^{5/2}}dx}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{2de\left(40ex^2+19d\right)}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{5/2}}dx}{6d^2e}+\frac{8x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{40ex^2+19d}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{5/2}}dx}{3d}+\frac{8x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\int -\frac{3de\left(7d-59ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{2d^2e}+\frac{59x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}}{3d}+\frac{8x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{59x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\int \frac{7d-59ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{2d}}{3d}+\frac{8x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{59x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{11x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\int \frac{6de\left(11ex^2+4d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}\right)}{2d}}{3d}+\frac{8x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{59x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{11x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\int \frac{11ex^2+4d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{d}\right)}{2d}}{3d}+\frac{8x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d^2}+\frac{x}{10d^2(d-ex^2)^{5/2}}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{59x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{11x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{-\frac{\int -\frac{de\left(15d-7ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}-\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}\right)}{2d}}{3d}+\frac{8x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d^2}+\frac{}{10d}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{59x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{11x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\frac{\int \frac{de\left(15d-7ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}-\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}\right)}{2d}}{3d}+\frac{8x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d^2}+\frac{}{10d^2(d}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{59x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{11x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\frac{\int\frac{15d-7ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}-\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}\right)}{2d}}{3d}+\frac{8x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d^2}+\frac{}{10d^2(d}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{59x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{11x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\frac{22d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-7\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}-\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}\right)}{2d}}{3d}+\frac{8x}{3d\left(d-ex^2\right)^{3/2}(d+e}}{10d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 289$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{59x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{11x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\frac{\frac{22d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-7\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}-\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}\right)}{2d}}{3d}+\frac{8}{3d\left(d-ex^2\right)^{3/}}}{10d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{329}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{59x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{11x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\frac{\frac{22d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}-\frac{7x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}}{d}\right)}{2d}}{3d}+}{10d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{59x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{11x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\frac{\frac{22d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}-\frac{7x\sqrt{d}}{2d\sqrt{d}}}{d}\right.}{2d}}{3d}}{10d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{59x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{11x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\frac{\frac{22d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}-\frac{7x\sqrt{d-}}{2d\sqrt{d+}}}{d}\right.}{2d}}{3d}}{10d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{59x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{11x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\frac{\frac{22d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{7d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right.\right.}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}}{d}\right.}{2d}}{3d}}{10d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/((d - e*x^2)*(d^2 - e^2*x^4)^(5/2)),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(10*d^2*(d - e*x^2)^(5/2)*(d + e*x^2)^
(3/2)) + ((8*x)/(3*d*(d - e*x^2)^(3/2)*(d + e*x^2)^(3/2)) + ((59*x)/(2*d*S
qrt[d - e*x^2]*(d + e*x^2)^(3/2)) - (3*((11*x*Sqrt[d - e*x^2])/(d*(d + e*x
^2)^(3/2)) - ((-7*x*Sqrt[d - e*x^2])/(2*d*Sqrt[d + e*x^2]) + ((-7*d^(3/2)*
Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[
e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (22*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*
EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[
d + e*x^2]))/(2*d))/d))/(2*d))/(3*d))/(10*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.50

method	result
default	$\frac{x\sqrt{-e^2x^4+d^2}}{48d^5e^2\left(x^2+\frac{d}{e}\right)^2}+\frac{5(-e^2x^2+de)x}{32e\,d^6\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}-\frac{x\sqrt{-e^2x^4+d^2}}{40d^4e^3\left(x^2-\frac{d}{e}\right)^3}+\frac{11x\sqrt{-e^2x^4+d^2}}{120e^2d^5\left(x^2-\frac{d}{e}\right)^2}-\frac{53(-e^2x^2-de)x}{160e\,d^6\sqrt{\left(x^2-\frac{d}{e}\right)(-e^2x^2-d}}$
elliptic	$\frac{x\sqrt{-e^2x^4+d^2}}{48d^5e^2\left(x^2+\frac{d}{e}\right)^2}+\frac{5(-e^2x^2+de)x}{32e\,d^6\sqrt{\left(x^2+\frac{d}{e}\right)(-e^2x^2+de)}}-\frac{x\sqrt{-e^2x^4+d^2}}{40d^4e^3\left(x^2-\frac{d}{e}\right)^3}+\frac{11x\sqrt{-e^2x^4+d^2}}{120e^2d^5\left(x^2-\frac{d}{e}\right)^2}-\frac{53(-e^2x^2-de)x}{160e\,d^6\sqrt{\left(x^2-\frac{d}{e}\right)(-e^2x^2-d}}$

input
```
int(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d^5/e^2*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+5/32*(-e^2*x^2+d*e)/e/d^6*
x/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)-1/40/d^4/e^3*x*(-e^2*x^4+d^2)^(1/2)/(x^
2-d/e)^3+11/120/e^2/d^5*x*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-53/160*(-e^2*x^
2-d*e)/e/d^6*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+3/8/d^5/(e/d)^(1/2)*(1-e*x
^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2)
,I)+7/40/d^5/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2
)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.48

$$\int \frac{1}{(d-ex^2)\,(d^2-e^2x^4)^{5/2}}\,dx =$$

$$-\frac{21\,(e^6x^{10}-de^5x^8-2\,d^2e^4x^6+2\,d^3e^3x^4+d^4e^2x^2-d^5e)\sqrt{\frac{e}{d}}E(\arcsin\,(x\sqrt{\frac{e}{d}})\;|-1)-3\,((15\,de^5+7\,e^6)x}{}$$

input
```
integrate(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/120*(21*(e^6*x^10 - d*e^5*x^8 - 2*d^2*e^4*x^6 + 2*d^3*e^3*x^4 + d^4*e^2
*x^2 - d^5*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)), -1) - 3*((15*d*e^5
 + 7*e^6)*x^10 - (15*d^2*e^4 + 7*d*e^5)*x^8 - 2*(15*d^3*e^3 + 7*d^2*e^4)*x
^6 - 15*d^6 - 7*d^5*e + 2*(15*d^4*e^2 + 7*d^3*e^3)*x^4 + (15*d^5*e + 7*d^4
*e^2)*x^2)*sqrt(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + (21*e^5*x^9 + 2
4*d*e^4*x^7 - 80*d^2*e^3*x^5 - 28*d^3*e^2*x^3 + 75*d^4*e*x)*sqrt(-e^2*x^4
+ d^2))/(d^6*e^6*x^10 - d^7*e^5*x^8 - 2*d^8*e^4*x^6 + 2*d^9*e^3*x^4 + d^10
*e^2*x^2 - d^11*e)
```

Sympy [F]

$$\int \frac{1}{(d-ex^2)\left(d^2-e^2x^4\right)^{5/2}}\,dx =$$

$$-\int \frac{1}{-d^5\sqrt{d^2-e^2x^4}+d^4ex^2\sqrt{d^2-e^2x^4}+2d^3e^2x^4\sqrt{d^2-e^2x^4}-2d^2e^3x^6\sqrt{d^2-e^2x^4}-de^4x^8\sqrt{d^2-e^2x^4}}$$

input
```
integrate(1/(-e*x**2+d)/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
-Integral(1/(-d**5*sqrt(d**2 - e**2*x**4) + d**4*e*x**2*sqrt(d**2 - e**2*x
**4) + 2*d**3*e**2*x**4*sqrt(d**2 - e**2*x**4) - 2*d**2*e**3*x**6*sqrt(d**
2 - e**2*x**4) - d*e**4*x**8*sqrt(d**2 - e**2*x**4) + e**5*x**10*sqrt(d**2
 - e**2*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d-ex^2)\left(d^2-e^2x^4\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{1}{(d-ex^2)\left(d^2-e^2x^4\right)^{5/2}}\,dx=\int -\frac{1}{\left(-e^2x^4+d^2\right)^{\frac{5}{2}}(ex^2-d)}\,dx$$

input
```
integrate(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-1/((-e^2*x^4 + d^2)^(5/2)*(e*x^2 - d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d-ex^2)\left(d^2-e^2x^4\right)^{5/2}}\,dx=\int \frac{1}{\left(d^2-e^2\,x^4\right)^{5/2}\,\left(d-e\,x^2\right)}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d - e*x^2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d - e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(d-ex^2)\left(d^2-e^2x^4\right)^{5/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{e^7x^{14}-d\,e^6x^{12}-3d^2e^5x^{10}+3d^3e^4x^8+3d^4e^3x^6-3d^5e^2x^4-d^6e\,x^2+}$$

input

```
int(1/(-e*x^2+d)/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**7 - d**6*e*x**2 - 3*d**5*e**2*x**4 + 3*d**4
*e**3*x**6 + 3*d**3*e**4*x**8 - 3*d**2*e**5*x**10 - d*e**6*x**12 + e**7*x*
*14),x)
```

3.124 $\int \frac{1}{\left(d-ex^2\right)^2\left(d^2-e^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 261

$$\begin{aligned}\int \frac{1}{\left(d-ex^2\right)^2\left(d^2-e^2x^4\right)^{5/2}}\,dx &= \frac{x}{14d^2\left(d-ex^2\right)^2\left(d^2-e^2x^4\right)^{3/2}} \\ &+ \frac{11x}{70d^3\left(d-ex^2\right)\left(d^2-e^2x^4\right)^{3/2}} + \frac{x\left(54d+77ex^2\right)}{420d^5\left(d^2-e^2x^4\right)^{3/2}} \\ &+ \frac{x\left(90d+77ex^2\right)}{280d^7\sqrt{d^2-e^2x^4}} - \frac{11\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{40d^{11/2}\sqrt{e}\sqrt{d^2-e^2x^4}} \\ &+ \frac{167\sqrt{1-\frac{e^2x^4}{d^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{280d^{11/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output

```
1/14*x/d^2/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(3/2)+11/70*x/d^3/(-e*x^2+d)/(-e^2*
x^4+d^2)^(3/2)+1/420*x*(77*e*x^2+54*d)/d^5/(-e^2*x^4+d^2)^(3/2)+1/280*x*(7
7*e*x^2+90*d)/d^7/(-e^2*x^4+d^2)^(1/2)-11/40*(1-e^2*x^4/d^2)^(1/2)*Ellipti
cE(e^(1/2)*x/d^(1/2),I)/d^(11/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+167/280*(1-e
^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)/d^(11/2)/e^(1/2)/(-e^2*x^
4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.67 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d-ex^2)^2\,(d^2-e^2x^4)^{5/2}}\,dx = \frac{\sqrt{-\frac{e}{d}}x(570d^5-503d^4ex^2-662d^3e^2x^4+694d^2e^3x^6+192de^4x^8-231e^5}{}$$

input
```
Integrate[1/((d - e*x^2)^2*(d^2 - e^2*x^4)^(5/2)),x]
```

output
```
(Sqrt[-(e/d)]*x*(570*d^5 - 503*d^4*e*x^2 - 662*d^3*e^2*x^4 + 694*d^2*e^3*x
^6 + 192*d*e^4*x^8 - 231*e^5*x^10) + (231*I)*d*(d - e*x^2)^3*(d + e*x^2)*S
qrt[1 - (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - (501*I)*
d*(d - e*x^2)^3*(d + e*x^2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[Sq
rt[-(e/d)]*x], -1])/(840*d^7*Sqrt[-(e/d)]*(d - e*x^2)^3*(d + e*x^2)*Sqrt[d
^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.53, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.704$, Rules used = {1396, 316, 27, 402, 27, 402, 27, 402, 27, 402, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-ex^2)^2\,(d^2-e^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{9/2}(ex^2+d)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(9ex^2+13d\right)}{\left(d-ex^2\right)^{7/2}\left(ex^2+d\right)^{5/2}}dx}{14d^2e}+\frac{x}{14d^2(d-ex^2)^{7/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{9ex^2+13d}{\left(d-ex^2\right)^{7/2}\left(ex^2+d\right)^{5/2}}dx}{14d^2}+\frac{x}{14d^2(d-ex^2)^{7/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{2de\left(77ex^2+54d\right)}{\left(d-ex^2\right)^{5/2}\left(ex^2+d\right)^{5/2}}dx}{10d^2e}+\frac{11x}{5d\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}}}{14d^2}+\frac{x}{14d^2(d-ex^2)^{7/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{77ex^2+54d}{\left(d-ex^2\right)^{5/2}\left(ex^2+d\right)^{5/2}}dx}{5d}+\frac{11x}{5d\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}}}{14d^2}+\frac{x}{14d^2(d-ex^2)^{7/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\int \frac{de\left(655ex^2+193d\right)}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{5/2}}dx}{6d^2e}+\frac{131x}{6d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{5d}+\frac{11x}{5d\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}}}{14d^2}+\frac{x}{14d^2(d-ex^2)^{7/2}(d+ex^2)^{3/2}}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\int \frac{655ex^2+193d}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{5/2}}dx}{6d}+\frac{131x}{6d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{5d}+\frac{11x}{5d\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}}}{14d^2}+\frac{x}{14d^2(d-ex^2)^{7/2}(d+ex^2)^{3/2}}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{\int -\frac{6de\left(77d-424ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{2d^2e}+\frac{424x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}}{6d}+\frac{131x}{6d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{5d}+\frac{11x}{5d\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}}}{14d^2}+\frac{}{14d^2(d-}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{424x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\int \frac{77d-424ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{d}}{6d}+\frac{131x}{6d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{5d}+\frac{11x}{5d\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}}}{14d^2}+\frac{}{14d^2(d-}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{424x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{167x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}-\frac{\int\frac{3de\left(167ex^2+13d\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}\right)}{d}}{6d}+\frac{131x}{6d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{5d}+\frac{11x}{5d\left(d-ex^2\right)^{5/2}\left(d+e\right.}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{424x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{167x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}-\frac{\int\frac{167ex^2+13d}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{2d}\right)}{d}}{6d}+\frac{131x}{6d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{5d}+\frac{11x}{5d\left(d-ex^2\right)^{5/2}\left(d+e\right.}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{424x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{167x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}-\frac{-\frac{\int -\frac{2de\left(90d-77ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}-\frac{77x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{2d}\right)}{d}}{6d}+\frac{131x}{6d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{5d}+\frac{}{5d(d}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{424x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{167x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}-\frac{\frac{\int \frac{90d-77ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{d}-\frac{77x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{2d}\right)}{d}}{6d}+\frac{131x}{6d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{5d}+\frac{}{5d(d-ex}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 399$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{424x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{167x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}-\frac{\frac{167d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-77\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}-\frac{77x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{2d}\right)}{d}}{6d}+\frac{131x}{6d\left(d-ex^2\right)^{3/2}(}}{5d}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{289}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{424x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{167x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}-\frac{\frac{\frac{167d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-77\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{d}-\frac{77x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{2d}\right)}{d}}{6d}+\frac{}{6d\left(d-ex^2\right.}}{5d}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{329}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{424x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{167x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}-\frac{\frac{\frac{167d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}-\frac{77x\sqrt{d-ex^2}}{d\sqrt{d+ex^2}}}{2d}\right)}{d}}{6d}}{5d}}{14d^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{424x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{167x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}-\frac{\frac{\frac{167d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}-\frac{77x}{d}}{2d}\right)}{d}}{6d}}{5d}}{14d^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$\Big\downarrow$ 765

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{424x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{167x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}-\frac{\frac{\frac{167d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}}{2d}-\frac{77x}{d\sqrt{}}\right.}{d}}{6d}}{5d}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 762$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{424x}{d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{167x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)^{3/2}}-\frac{\frac{\frac{167d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\right.\right.}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{d}}{2d}\right.}{d}}{6d}}{5d}}{14d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/((d - e*x^2)^2*(d^2 - e^2*x^4)^(5/2)),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(14*d^2*(d - e*x^2)^(7/2)*(d + e*x^2)^
(3/2)) + ((11*x)/(5*d*(d - e*x^2)^(5/2)*(d + e*x^2)^(3/2)) + ((131*x)/(6*d
*(d - e*x^2)^(3/2)*(d + e*x^2)^(3/2)) + ((424*x)/(d*Sqrt[d - e*x^2]*(d + e
*x^2)^(3/2)) - (3*((167*x*Sqrt[d - e*x^2])/(2*d*(d + e*x^2)^(3/2)) - ((-77
*x*Sqrt[d - e*x^2])/(d*Sqrt[d + e*x^2]) + ((-77*d^(3/2)*Sqrt[1 - (e^2*x^4)
/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]
*Sqrt[d + e*x^2]) + (167*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[
(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/d)/(
2*d)))/d)/(6*d))/(5*d))/(14*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.44

method	result
default	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{56d^{4}e^{4}\left(x^{2}-\frac{d}{e}\right)^{4}}-\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{35d^{5}e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+\frac{439x\sqrt{-e^{2}x^{4}+d^{2}}}{3360d^{6}e^{2}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{59\left(-e^{2}x^{2}-de\right)x}{160e\,d^{7}\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{96d^{6}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{}{32e\,d}$
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{56d^{4}e^{4}\left(x^{2}-\frac{d}{e}\right)^{4}}-\frac{2x\sqrt{-e^{2}x^{4}+d^{2}}}{35d^{5}e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+\frac{439x\sqrt{-e^{2}x^{4}+d^{2}}}{3360d^{6}e^{2}\left(x^{2}-\frac{d}{e}\right)^{2}}-\frac{59\left(-e^{2}x^{2}-de\right)x}{160e\,d^{7}\sqrt{\left(x^{2}-\frac{d}{e}\right)\left(-e^{2}x^{2}-de\right)}}+\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{96d^{6}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{}{32e\,d}$

input
```
int(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/56/d^4*x/e^4*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^4-2/35/d^5/e^3*x*(-e^2*x^4+d
^2)^(1/2)/(x^2-d/e)^3+439/3360/d^6/e^2*x*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^2-
59/160*(-e^2*x^2-d*e)/e/d^7*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1/2)+1/96/d^6/e^
2*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+3/32*(-e^2*x^2+d*e)/e/d^7*x/((x^2+d/e
)*(-e^2*x^2+d*e))^(1/2)+9/28/d^6/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)
^(1/2)/(-e^2*x^4+d^2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+11/40/d^6/(e/d)^(1/
2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(
e/d)^(1/2),I)-EllipticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.50

$$\int \frac{1}{\left(d-ex^2\right)^2\left(d^2-e^2x^4\right)^{5/2}}\,dx =$$
$$-\frac{231\left(e^7x^{12}-2\,de^6x^{10}-d^2e^5x^8+4\,d^3e^4x^6-d^4e^3x^4-2\,d^5e^2x^2+d^6e\right)\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,|-1)-3\left(\left(90\right.\right.}{}$$

input
```
integrate(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output

```
-1/840*(231*(e^7*x^12 - 2*d*e^6*x^10 - d^2*e^5*x^8 + 4*d^3*e^4*x^6 - d^4*e
^3*x^4 - 2*d^5*e^2*x^2 + d^6*e)*sqrt(e/d)*elliptic_e(arcsin(x*sqrt(e/d)),
-1) - 3*((90*d*e^6 + 77*e^7)*x^12 - 2*(90*d^2*e^5 + 77*d*e^6)*x^10 - (90*d
^3*e^4 + 77*d^2*e^5)*x^8 + 90*d^7 + 77*d^6*e + 4*(90*d^4*e^3 + 77*d^3*e^4)
*x^6 - (90*d^5*e^2 + 77*d^4*e^3)*x^4 - 2*(90*d^6*e + 77*d^5*e^2)*x^2)*sqrt
(e/d)*elliptic_f(arcsin(x*sqrt(e/d)), -1) + (231*e^6*x^11 - 192*d*e^5*x^9
- 694*d^2*e^4*x^7 + 662*d^3*e^3*x^5 + 503*d^4*e^2*x^3 - 570*d^5*e*x)*sqrt(
-e^2*x^4 + d^2))/(d^7*e^7*x^12 - 2*d^8*e^6*x^10 - d^9*e^5*x^8 + 4*d^10*e^4
*x^6 - d^11*e^3*x^4 - 2*d^12*e^2*x^2 + d^13*e)
```

Sympy [F]

$$\int \frac{1}{\left(d-ex^2\right)^2 \left(d^2-e^2x^4\right)^{5/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{5}{2}}\left(-d+ex^2\right)^2}\,dx$$

input

```
integrate(1/(-e*x**2+d)**2/(-e**2*x**4+d**2)**(5/2),x)
```

output

```
Integral(1/((-(-d + e*x**2)*(d + e*x**2))**(5/2)*(-d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(d-ex^2\right)^2 \left(d^2-e^2x^4\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^4+d^2\right)^{\frac{5}{2}}\left(ex^2-d\right)^2}\,dx$$

input

```
integrate(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output

```
integrate(1/((-e^2*x^4 + d^2)^(5/2)*(e*x^2 - d)^2), x)
```

Giac [F]

$$\int \frac{1}{(d-ex^2)^2(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{(-e^2x^4+d^2)^{\frac{5}{2}}(ex^2-d)^2}\,dx$$

input

```
integrate(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output

```
integrate(1/((-e^2*x^4 + d^2)^(5/2)*(e*x^2 - d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d-ex^2)^2(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{(d^2-e^2\,x^4)^{5/2}\,(d-e\,x^2)^2}\,dx$$

input

```
int(1/((d^2 - e^2*x^4)^(5/2)*(d - e*x^2)^2),x)
```

output

```
int(1/((d^2 - e^2*x^4)^(5/2)*(d - e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(d-ex^2)^2(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{-e^8x^{16}+2d\,e^7x^{14}+2d^2e^6x^{12}-6d^3e^5x^{10}+6d^5e^3x^6-2d^6e^2x^4-2d^7e}$$

input

```
int(1/(-e*x^2+d)^2/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**8 - 2*d**7*e*x**2 - 2*d**6*e**2*x**4 + 6*d*
*5*e**3*x**6 - 6*d**3*e**5*x**10 + 2*d**2*e**6*x**12 + 2*d*e**7*x**14 - e*
*8*x**16),x)
```

3.125 $\int \frac{1}{(d-ex^2)^3(d^2-e^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 295

$$\begin{aligned}\int \frac{1}{(d-ex^2)^3\,(d^2-e^2x^4)^{5/2}}\,dx &= \frac{x}{18d^2\,(d-ex^2)^3\,(d^2-e^2x^4)^{3/2}}\\ &+ \frac{x}{9d^3\,(d-ex^2)^2\,(d^2-e^2x^4)^{3/2}} + \frac{11x}{60d^4\,(d-ex^2)\,(d^2-e^2x^4)^{3/2}}\\ &+ \frac{x(39d+77ex^2)}{360d^6\,(d^2-e^2x^4)^{3/2}} + \frac{x(65d+77ex^2)}{240d^8\sqrt{d^2-e^2x^4}} - \frac{77\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{240d^{13/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\\ &+ \frac{71\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{120d^{13/2}\sqrt{e}\sqrt{d^2-e^2x^4}}\end{aligned}$$

output
```
1/18*x/d^2/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(3/2)+1/9*x/d^3/(-e*x^2+d)^2/(-e^2*
x^4+d^2)^(3/2)+11/60*x/d^4/(-e*x^2+d)/(-e^2*x^4+d^2)^(3/2)+1/360*x*(77*e*x
^2+39*d)/d^6/(-e^2*x^4+d^2)^(3/2)+1/240*x*(77*e*x^2+65*d)/d^8/(-e^2*x^4+d^
2)^(1/2)-77/240*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(13
/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)+71/120*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^
(1/2)*x/d^(1/2),I)/d^(13/2)/e^(1/2)/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.97 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.63

$$\int \frac{1}{(d-ex^2)^3(d^2-e^2x^4)^{5/2}}\,dx = \frac{\frac{x(525d^6-778d^5ex^2-399d^4e^2x^4+1236d^3e^3x^6-277d^2e^4x^8-498de^5x^{10}+231e^6x^{12})}{(d-ex^2)^4(d+ex^2)} + \frac{3id\sqrt{1-\frac{e^2x}{d^2}}}{}}{720d^8\sqrt{d^2-e^2x}}$$

input
```
Integrate[1/((d - e*x^2)^3*(d^2 - e^2*x^4)^(5/2)),x]
```

output
```
((x*(525*d^6 - 778*d^5*e*x^2 - 399*d^4*e^2*x^4 + 1236*d^3*e^3*x^6 - 277*d^
2*e^4*x^8 - 498*d*e^5*x^10 + 231*e^6*x^12))/((d - e*x^2)^4*(d + e*x^2)) +
((3*I)*d*Sqrt[1 - (e^2*x^4)/d^2]*(77*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x],
-1] - 142*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1]))/Sqrt[-(e/d)])/(720*d^
8*Sqrt[d^2 - e^2*x^4])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.49, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {1396, 316, 27, 402, 27, 402, 27, 402, 27, 402, 27, 402, 27, 402, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-ex^2)^3(d^2-e^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^{11/2}(ex^2+d)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(11ex^2+17d\right)}{\left(d-ex^2\right)^{9/2}\left(ex^2+d\right)^{5/2}}dx}{18d^2e}+\frac{x}{18d^2(d-ex^2)^{9/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{11ex^2+17d}{\left(d-ex^2\right)^{9/2}\left(ex^2+d\right)^{5/2}}dx}{18d^2}+\frac{x}{18d^2(d-ex^2)^{9/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{42de\left(6ex^2+5d\right)}{\left(d-ex^2\right)^{7/2}\left(ex^2+d\right)^{5/2}}dx}{14d^2e}+\frac{2x}{d\left(d-ex^2\right)^{7/2}\left(d+ex^2\right)^{3/2}}}{18d^2}+\frac{x}{18d^2(d-ex^2)^{9/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\int \frac{6ex^2+5d}{\left(d-ex^2\right)^{7/2}\left(ex^2+d\right)^{5/2}}dx}{d}+\frac{2x}{d\left(d-ex^2\right)^{7/2}\left(d+ex^2\right)^{3/2}}}{18d^2}+\frac{x}{18d^2(d-ex^2)^{9/2}(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\int \frac{de\left(77ex^2+39d\right)}{\left(d-ex^2\right)^{5/2}\left(ex^2+d\right)^{5/2}}dx}{10d^2e}+\frac{11x}{10d\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}}\right)}{d}+\frac{2x}{d\left(d-ex^2\right)^{7/2}\left(d+ex^2\right)^{3/2}}}{18d^2}+\frac{x}{18d^2(d-ex^2)^{9/2}(d+ex^2)}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\int \frac{77ex^2+39d}{\left(d-ex^2\right)^{5/2}\left(ex^2+d\right)^{5/2}}dx}{10d}+\frac{11x}{10d\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}}\right)}{d}+\frac{2x}{d\left(d-ex^2\right)^{7/2}\left(d+ex^2\right)^{3/2}}}{18d^2}+\frac{x}{18d^2(d-ex^2)^{9/2}(d+ex^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{\int \frac{2de\left(290ex^2+59d\right)}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{5/2}}dx}{6d^2e}+\frac{58x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x}{10d\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}}\right)}{d}+\frac{2x}{d\left(d-ex^2\right)^{7/2}\left(d+ex^2\right)^{3/2}}}{18d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{\int \frac{290ex^2+59d}{\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^{5/2}}dx}{3d}+\frac{58x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x}{10d\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}}\right)}{d}+\frac{2x}{d\left(d-ex^2\right)^{7/2}\left(d+ex^2\right)^{3/2}}}{18d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{\frac{3\left(\frac{\frac{\frac{\int -\frac{3de\left(77d-349ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{2d^2e}+\frac{349x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}}{3d}+\frac{58x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x}{10d\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}}\right)}{d}+}{18d^2}}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{\frac{3\left(\frac{\frac{\frac{349x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\int \frac{77d-349ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{5/2}}dx}{2d}}{3d}+\frac{58x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x}{10d\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{3/2}}\right)}{d}+}{18d^2}}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{349x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{71x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{6de\left(6d-71ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{6d^2e}\right)}{2d}}{3d}+\frac{58x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11}{10d\left(d-ex^2\right)^{5/}}\right.}{d}}{18d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$\downarrow$ 27

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{\frac{349x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{\int \frac{6d-71ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^{3/2}}dx}{d}+\frac{71x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{2d}}{3d}+\frac{58x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d}+\frac{11x}{10d\left(d-ex^2\right)^{5/2}(}\right.}{d}}{18d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$\downarrow$ 402

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{\Big(3\Big(\frac{\frac{\frac{349x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{\frac{77x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int\frac{de\left(65d-77ex^2\right)}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d^2e}}{d}+\frac{71x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{2d}}{3d}+\frac{58x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d}+\frac{}{10d(d}}{d}}{18d^2}}{\sqrt{d^2-e^2x^4}}$$

$\downarrow$ 27

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{\frac{349x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{\frac{77x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\int\frac{65d-77ex^2}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx}{2d}}{d}+\frac{71x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{2d}}{3d}+\frac{58x}{3d\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}}{10d}+\frac{}{10d(d}\right.}{d}}{18d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{349x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{\frac{77x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{142d\int\frac{1}{\sqrt{d-ex^2}\sqrt{ex^2+d}}dx-77\int\frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}}{d}+\frac{71x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{2d}}{3d}+\frac{5}{3d\left(d-ex^2\right)^{3/}}\right.}{10d}}{d}}{18d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$\downarrow$ 289

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{3\left(\frac{\frac{349x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{\frac{77x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\frac{142d\sqrt{d^2-e^2x^4}\int \frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-77\int \frac{\sqrt{ex^2+d}}{\sqrt{d-ex^2}}dx}{2d}}{d}+\frac{71x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right)}{2d}}{3d}+\frac{}{3d\left(d-e\right.}}{10d}}{d}}{18d^2}}{\sqrt{d^2-e^2x^4}}$$

$\Big\downarrow$ 329

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{\frac{349x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{\frac{77x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\frac{142d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}}{d}+\frac{71x\sqrt{d-ex^2}}{d\left(d+ex^2\right)^{3/2}}\right.}{2d}}{3d}}{10d}\right.}{d}}{18d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

↓ 327

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{349x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{\frac{77x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\frac{142d\sqrt{d^2-e^2x^4}\int\frac{1}{\sqrt{d^2-e^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}}{d}+\right.}{2d}}{3d}}{10d}\right.}{d}}{18d^2}\right.}{\sqrt{d^2-e^2}}$$

↓ 765

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{349x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{\frac{77x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\frac{142d\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}}{2d}}{d}+\frac{71}{d(}\right.}{2d}}{3d}\right.}{10d}}{d}}{18d^2}\right.}{\sqrt{d^2-e^2x}}$$

↓ 762

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{\frac{349x}{2d\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}-\frac{3\left(\frac{\frac{77x\sqrt{d-ex^2}}{2d\sqrt{d+ex^2}}-\frac{\frac{142d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}-\frac{77d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\right.\right.}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex}}}{2d}}{d}\right.}{2d}}{3d}}{10d}\right.}{d}}{18d^2}\right.}{\sqrt{d^2}}$$

input
```
Int[1/((d - e*x^2)^3*(d^2 - e^2*x^4)^(5/2)),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(18*d^2*(d - e*x^2)^(9/2)*(d + e*x^2)^
(3/2)) + ((2*x)/(d*(d - e*x^2)^(7/2)*(d + e*x^2)^(3/2)) + (3*((11*x)/(10*d
*(d - e*x^2)^(5/2)*(d + e*x^2)^(3/2)) + ((58*x)/(3*d*(d - e*x^2)^(3/2)*(d
+ e*x^2)^(3/2)) + ((349*x)/(2*d*Sqrt[d - e*x^2]*(d + e*x^2)^(3/2)) - (3*((
71*x*Sqrt[d - e*x^2])/(d*(d + e*x^2)^(3/2)) + ((77*x*Sqrt[d - e*x^2])/(2*d
*Sqrt[d + e*x^2]) - ((-77*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticE[ArcSin
[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (1
42*d^(3/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[ArcSin[(Sqrt[e]*x)/Sqrt[d]],
-1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]))/(2*d))/d))/(2*d))/(3*d))/(
10*d)))/d)/(18*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.39

method	result
default	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{192d^{7}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{7\left(-e^{2}x^{2}+de\right)x}{128e\,d^{8}\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}-\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{72d^{4}e^{5}\left(x^{2}-\frac{d}{e}\right)^{5}}+\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{24d^{5}e^{4}\left(x^{2}-\frac{d}{e}\right)^{4}}-\frac{121x\sqrt{-e^{2}x^{4}+d^{2}}}{1440d^{6}e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+$
elliptic	$\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{192d^{7}e^{2}\left(x^{2}+\frac{d}{e}\right)^{2}}+\frac{7\left(-e^{2}x^{2}+de\right)x}{128e\,d^{8}\sqrt{\left(x^{2}+\frac{d}{e}\right)\left(-e^{2}x^{2}+de\right)}}-\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{72d^{4}e^{5}\left(x^{2}-\frac{d}{e}\right)^{5}}+\frac{x\sqrt{-e^{2}x^{4}+d^{2}}}{24d^{5}e^{4}\left(x^{2}-\frac{d}{e}\right)^{4}}-\frac{121x\sqrt{-e^{2}x^{4}+d^{2}}}{1440d^{6}e^{3}\left(x^{2}-\frac{d}{e}\right)^{3}}+$

input
```
int(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/192/d^7/e^2*x*(-e^2*x^4+d^2)^(1/2)/(x^2+d/e)^2+7/128*(-e^2*x^2+d*e)/e/d^
8*x/((x^2+d/e)*(-e^2*x^2+d*e))^(1/2)-1/72/d^4*x/e^5*(-e^2*x^4+d^2)^(1/2)/(
x^2-d/e)^5+1/24/d^5/e^4*x*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^4-121/1440/d^6/e^
3*x*(-e^2*x^4+d^2)^(1/2)/(x^2-d/e)^3+37/240/d^7/e^2*x*(-e^2*x^4+d^2)^(1/2)
/(x^2-d/e)^2-721/1920*(-e^2*x^2-d*e)/e/d^8*x/((x^2-d/e)*(-e^2*x^2-d*e))^(1
/2)+13/48/d^7/(e/d)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^
2)^(1/2)*EllipticF(x*(e/d)^(1/2),I)+77/240/d^7/(e/d)^(1/2)*(1-e*x^2/d)^(1/
2)*(1+e*x^2/d)^(1/2)/(-e^2*x^4+d^2)^(1/2)*(EllipticF(x*(e/d)^(1/2),I)-Elli
pticE(x*(e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.51

$$\int \frac{1}{\left(d-ex^{2}\right)^{3}\left(d^{2}-e^{2}x^{4}\right)^{5/2}}\,dx =$$

$$-\frac{231\left(e^{8}x^{14}-3\,de^{7}x^{12}+d^{2}e^{6}x^{10}+5\,d^{3}e^{5}x^{8}-5\,d^{4}e^{4}x^{6}-d^{5}e^{3}x^{4}+3\,d^{6}e^{2}x^{2}-d^{7}e\right)\sqrt{\frac{e}{d}}E\left(\arcsin\left(x\sqrt{\frac{e}{d}}\right)\,\middle|\right.}{}$$

input
```
integrate(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output

```
-1/720*(231*(e^8*x^14 - 3*d*e^7*x^12 + d^2*e^6*x^10 + 5*d^3*e^5*x^8 - 5*d^
4*e^4*x^6 - d^5*e^3*x^4 + 3*d^6*e^2*x^2 - d^7*e)*sqrt(e/d)*elliptic_e(arcs
in(x*sqrt(e/d)), -1) - 3*((65*d*e^7 + 77*e^8)*x^14 - 3*(65*d^2*e^6 + 77*d*
e^7)*x^12 + (65*d^3*e^5 + 77*d^2*e^6)*x^10 + 5*(65*d^4*e^4 + 77*d^3*e^5)*x
^8 - 65*d^8 - 77*d^7*e - 5*(65*d^5*e^3 + 77*d^4*e^4)*x^6 - (65*d^6*e^2 + 7
7*d^5*e^3)*x^4 + 3*(65*d^7*e + 77*d^6*e^2)*x^2)*sqrt(e/d)*elliptic_f(arcsi
n(x*sqrt(e/d)), -1) + (231*e^7*x^13 - 498*d*e^6*x^11 - 277*d^2*e^5*x^9 + 1
236*d^3*e^4*x^7 - 399*d^4*e^3*x^5 - 778*d^5*e^2*x^3 + 525*d^6*e*x)*sqrt(-e
^2*x^4 + d^2))/(d^8*e^8*x^14 - 3*d^9*e^7*x^12 + d^10*e^6*x^10 + 5*d^11*e^5
*x^8 - 5*d^12*e^4*x^6 - d^13*e^3*x^4 + 3*d^14*e^2*x^2 - d^15*e)
```

Sympy [F]

$$\int \frac{1}{(d-ex^2)^3\,(d^2-e^2x^4)^{5/2}}\,dx =$$
$$-\int \frac{1}{-d^7\sqrt{d^2-e^2x^4}+3d^6ex^2\sqrt{d^2-e^2x^4}-d^5e^2x^4\sqrt{d^2-e^2x^4}-5d^4e^3x^6\sqrt{d^2-e^2x^4}+5d^3e^4x^8\sqrt{d^2-e^2x}}$$

input

```
integrate(1/(-e*x**2+d)**3/(-e**2*x**4+d**2)**(5/2),x)
```

output

```
-Integral(1/(-d**7*sqrt(d**2 - e**2*x**4) + 3*d**6*e*x**2*sqrt(d**2 - e**2
*x**4) - d**5*e**2*x**4*sqrt(d**2 - e**2*x**4) - 5*d**4*e**3*x**6*sqrt(d**
2 - e**2*x**4) + 5*d**3*e**4*x**8*sqrt(d**2 - e**2*x**4) + d**2*e**5*x**10
*sqrt(d**2 - e**2*x**4) - 3*d*e**6*x**12*sqrt(d**2 - e**2*x**4) + e**7*x**
14*sqrt(d**2 - e**2*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(d-ex^2)^3\,(d^2-e^2x^4)^{5/2}}\,dx = \int -\frac{1}{(-e^2x^4+d^2)^{\frac{5}{2}}(ex^2-d)^3}\,dx$$

input

```
integrate(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate(1/((-e^2*x^4 + d^2)^(5/2)*(e*x^2 - d)^3), x)
```

Giac [F]

$$\int \frac{1}{(d-ex^2)^3(d^2-e^2x^4)^{5/2}}\,dx = \int -\frac{1}{(-e^2x^4+d^2)^{\frac{5}{2}}(ex^2-d)^3}\,dx$$

input
```
integrate(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-1/((-e^2*x^4 + d^2)^(5/2)*(e*x^2 - d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d-ex^2)^3(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{(d^2-e^2\,x^4)^{5/2}\,(d-e\,x^2)^3}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d - e*x^2)^3),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d - e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(d-ex^2)^3(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{\sqrt{-e^2x^4+d^2}}{e^9x^{18}-3d\,e^8x^{16}+8d^3e^6x^{12}-6d^4e^5x^{10}-6d^5e^4x^8+8d^6e^3x^6-3d^8e\,x}$$

input
```
int(1/(-e*x^2+d)^3/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
int(sqrt(d**2 - e**2*x**4)/(d**9 - 3*d**8*e*x**2 + 8*d**6*e**3*x**6 - 6*d*
*5*e**4*x**8 - 6*d**4*e**5*x**10 + 8*d**3*e**6*x**12 - 3*d*e**8*x**16 + e*
*9*x**18),x)
```

3.126 $\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = -E(\arcsin(x)|-1) + 2\,\text{EllipticF}(\arcsin(x),-1)$$

output
```
-EllipticE(x,I)+2*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.77

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right) - \frac{1}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^4\right)$$

input
```
Integrate[(1 - x^2)/Sqrt[1 - x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, x^4] - (x^3*Hypergeometric2F1[1/2, 3/4,
 7/4, x^4])/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1388, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{\sqrt{1-x^2}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{326}$$

$$2\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+1}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{284}$$

$$2\int \frac{1}{\sqrt{1-x^4}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$2\int \frac{1}{\sqrt{1-x^4}}dx-E(\arcsin(x)|-1)$$

$$\Big\downarrow \text{762}$$

$$2\,\text{EllipticF}(\arcsin(x),-1)-E(\arcsin(x)|-1)$$

input
```
Int[(1 - x^2)/Sqrt[1 - x^4],x]
```

output
```
-EllipticE[ArcSin[x], -1] + 2*EllipticF[ArcSin[x], -1]
```

Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

method	result	size
meijerg	$-\frac{x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], x^{4}\right)$	27
default	$\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(x , i\right)-\operatorname{EllipticE}\left(x , i\right)\right)}{\sqrt{-x^{4}+1}}+\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x , i\right)}{\sqrt{-x^{4}+1}}$	69
elliptic	$\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(x , i\right)-\operatorname{EllipticE}\left(x , i\right)\right)}{\sqrt{-x^{4}+1}}+\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x , i\right)}{\sqrt{-x^{4}+1}}$	69

input
```
int((-x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3*hypergeom([1/2,3/4],[7/4],x^4)+x*hypergeom([1/4,1/2],[5/4],x^4)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = \frac{i\,xE\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right)+\sqrt{-x^4+1}}{x}$$

input
```
integrate((-x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(I*x*elliptic_e(arcsin(1/x), -1) + sqrt(-x^4 + 1))/x
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(7) = 14$.

Time = 0.71 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.69

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = -\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-x**2+1)/(-x**4+1)**(1/2),x)
```

output
```
-x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(2*I*pi))/(4*gamm
a(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(2*I*pi))/(
4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((-x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((-x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = -\int \frac{x^2-1}{\sqrt{1-x^4}}\,dx$$

input
```
int(-(x^2 - 1)/(1 - x^4)^(1/2),x)
```

output
```
-int((x^2 - 1)/(1 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-x^2}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-x^4+1}}{x^2+1}dx$$

input

```
int((-x^2+1)/(-x^4+1)^(1/2),x)
```

output

```
int(sqrt( - x**4 + 1)/(x**2 + 1),x)
```

3.127 $\int \frac{1}{(1-x^2)\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 38

$$\int \frac{1}{(1-x^2)\sqrt{1-x^4}}\,dx = \frac{x\sqrt{1-x^4}}{2(1-x^2)} - \frac{1}{2}E(\arcsin(x)|-1) + \text{EllipticF}(\arcsin(x),-1)$$

output
```
x*(-x^4+1)^(1/2)/(-2*x^2+2)-1/2*EllipticE(x,I)+EllipticF(x,I)
```

Mathematica [A] (verified)

Time = 10.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{1}{(1-x^2)\sqrt{1-x^4}}\,dx = \frac{1}{2}\left(-E(\arcsin(x)|-1) + \frac{x+x^3+2\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{\sqrt{1-x^4}}\right)$$

input
```
Integrate[1/((1 - x^2)*Sqrt[1 - x^4]),x]
```

output
```
(-EllipticE[ArcSin[x], -1] + (x + x^3 + 2*Sqrt[1 - x^4]*EllipticF[ArcSin[x
], -1])/Sqrt[1 - x^4])/2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1388, 316, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x^2)\sqrt{1-x^4}}\,dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{(1-x^2)^{3/2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{1}{2}\int \frac{\sqrt{1-x^2}}{\sqrt{x^2+1}}dx+\frac{\sqrt{x^2+1}x}{2\sqrt{1-x^2}}$$

$$\Big\downarrow \text{326}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+1}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx\right)+\frac{\sqrt{x^2+1}x}{2\sqrt{1-x^2}}$$

$$\Big\downarrow \text{284}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-x^4}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx\right)+\frac{\sqrt{x^2+1}x}{2\sqrt{1-x^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-x^4}}dx-E(\arcsin(x)|-1)\right)+\frac{\sqrt{x^2+1}x}{2\sqrt{1-x^2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{2}(2\,\text{EllipticF}(\arcsin(x),-1)-E(\arcsin(x)|-1))+\frac{\sqrt{x^2+1}x}{2\sqrt{1-x^2}}$$

input

```
Int[1/((1 - x^2)*Sqrt[1 - x^4]),x]
```

output
```
(x*Sqrt[1 + x^2])/(2*Sqrt[1 - x^2]) + (-EllipticE[ArcSin[x], -1] + 2*Ellip
ticF[ArcSin[x], -1])/2
```

Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(31) = 62$.

Time = 1.56 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.32

method	result
risch	$\frac{x(x^2+1)}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\ \mathrm{EllipticF}(x,i)}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$
elliptic	$-\frac{(-x^2-1)x}{2\sqrt{(x^2-1)(-x^2-1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\ \mathrm{EllipticF}(x,i)}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$
default	$-\frac{-x^3+x^2-x+1}{4\sqrt{(1+x)(-x^3+x^2-x+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\ \mathrm{EllipticF}(x,i)}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i))}{2\sqrt{-x^4+1}}-\frac{-}{4\sqrt{(x-}}$

input

```
int(1/(-x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/2*x*(x^2+1)/(-x^4+1)^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/
2)*EllipticF(x,I)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(Ellipti
cF(x,I)-EllipticE(x,I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{1}{(1-x^2)\sqrt{1-x^4}}\,dx$$
$$=-\frac{(x^2-1)E(\arcsin(x)\mid-1)-2\,(x^2-1)F(\arcsin(x)\mid-1)+\sqrt{-x^4+1}x}{2\,(x^2-1)}$$

input

```
integrate(1/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((x^2 - 1)*elliptic_e(arcsin(x), -1) - 2*(x^2 - 1)*elliptic_f(arcsin(
x), -1) + sqrt(-x^4 + 1)*x)/(x^2 - 1)
```

Sympy [F]

$$\int \frac{1}{(1-x^2)\sqrt{1-x^4}}\,dx = -\int \frac{1}{x^2\sqrt{1-x^4}-\sqrt{1-x^4}}\,dx$$

input
```
integrate(1/(-x**2+1)/(-x**4+1)**(1/2),x)
```

output
```
-Integral(1/(x**2*sqrt(1 - x**4) - sqrt(1 - x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(1-x^2)\sqrt{1-x^4}}\,dx = \int -\frac{1}{\sqrt{-x^4+1}(x^2-1)}\,dx$$

input
```
integrate(1/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(-x^4 + 1)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{(1-x^2)\sqrt{1-x^4}}\,dx = \int -\frac{1}{\sqrt{-x^4+1}(x^2-1)}\,dx$$

input
```
integrate(1/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(-x^4 + 1)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-x^2)\sqrt{1-x^4}}\,dx = -\int \frac{1}{(x^2-1)\sqrt{1-x^4}}\,dx$$

input

```
int(-1/((x^2 - 1)*(1 - x^4)^(1/2)),x)
```

output

```
-int(1/((x^2 - 1)*(1 - x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-x^2)\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-x^4+1}}{x^6-x^4-x^2+1}dx$$

input

```
int(1/(-x^2+1)/(-x^4+1)^(1/2),x)
```

output

```
int(sqrt( - x**4 + 1)/(x**6 - x**4 - x**2 + 1),x)
```

3.128 $\int \frac{1-x^2}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{1-x^2}{\sqrt{-1+x^4}}\,dx=-\frac{\sqrt{1-x^4}E(\arcsin(x)|-1)}{\sqrt{-1+x^4}}+\frac{2\sqrt{1-x^4}\operatorname{EllipticF}(\arcsin(x),-1)}{\sqrt{-1+x^4}}$$

output
```
-(-x^4+1)^(1/2)*EllipticE(x,I)/(x^4-1)^(1/2)+2*(-x^4+1)^(1/2)*EllipticF(x,
I)/(x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{1-x^2}{\sqrt{-1+x^4}}\,dx = -\frac{\sqrt{1-x^4}\left(-3x\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right)+x^3\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^4\right)\right)}{3\sqrt{-1+x^4}}$$

input
```
Integrate[(1 - x^2)/Sqrt[-1 + x^4],x]
```

output
```
-1/3*(Sqrt[1 - x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, x^4] + x^3*Hype
rgeometric2F1[1/2, 3/4, 7/4, x^4]))/Sqrt[-1 + x^4]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{1499\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{\sqrt{x^4-1}}dx$$
$$\Big\downarrow \text{1499}$$
$$\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{x^2+1}E\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}-\frac{x\left(x^2+1\right)}{\sqrt{x^4-1}}$$

input
```
Int[(1 - x^2)/Sqrt[-1 + x^4],x]
```

output
```
-((x*(1 + x^2))/Sqrt[-1 + x^4]) + (Sqrt[2]*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*El
lipticE[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/Sqrt[-1 + x^4]
```

Defintions of rubi rules used

rule 1499
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[e*x*((q + c*x^2)/(c*Sqrt[a + c*x^4])), x] - Simp[Sqrt
[2]*e*q*Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[-a]*c*Sqrt[a + c*x^4]))
*EllipticE[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

method	result	size
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^4-1)}\,x^3\,\operatorname{hypergeom}([\frac{1}{2},\frac{3}{4}],[\frac{7}{4}],x^4)}{3\sqrt{\operatorname{signum}(x^4-1)}}+\frac{\sqrt{-\operatorname{signum}(x^4-1)}\,x\,\operatorname{hypergeom}([\frac{1}{4},\frac{1}{2}],[\frac{5}{4}],x^4)}{\sqrt{\operatorname{signum}(x^4-1)}}$	63
default	$\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\operatorname{EllipticF}(ix,i)-\operatorname{EllipticE}(ix,i))}{\sqrt{x^4-1}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}(ix,i)}{\sqrt{x^4-1}}$	78
elliptic	$\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\operatorname{EllipticF}(ix,i)-\operatorname{EllipticE}(ix,i))}{\sqrt{x^4-1}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}(ix,i)}{\sqrt{x^4-1}}$	78

input
```
int((-x^2+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)*x^3*hypergeom([1/2,3/4],[7
/4],x^4)+1/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)*x*hypergeom([1/4,1/2
],[5/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.40

$$\int \frac{1-x^2}{\sqrt{-1+x^4}}\,dx=-\frac{xE\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right)+\sqrt{x^4-1}}{x}$$

input
```
integrate((-x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-(x*elliptic_e(arcsin(1/x), -1) + sqrt(x^4 - 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{1-x^2}{\sqrt{-1+x^4}}\,dx = \frac{ix^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|x^4\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((-x**2+1)/(x**4-1)**(1/2),x)
```

output

```
I*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4)/(4*gamma(7/4)) - I*x*gam
ma(1/4)*hyper((1/4, 1/2), (5/4,), x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-x^2}{\sqrt{-1+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4-1}}\,dx$$

input

```
integrate((-x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output

```
-integrate((x^2 - 1)/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{1-x^2}{\sqrt{-1+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4-1}}\,dx$$

input

```
integrate((-x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

output

```
integrate(-(x^2 - 1)/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{\sqrt{-1+x^4}}\,dx=-\int \frac{x^2-1}{\sqrt{x^4-1}}\,dx$$

input
```
int(-(x^2 - 1)/(x^4 - 1)^(1/2),x)
```

output
```
-int((x^2 - 1)/(x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-x^2}{\sqrt{-1+x^4}}\,dx=-\left(\int \frac{\sqrt{x^4-1}}{x^2+1}dx\right)$$

input
```
int((-x^2+1)/(x^4-1)^(1/2),x)
```

output
```
 - int(sqrt(x**4 - 1)/(x**2 + 1),x)
```

3.129 $\int \frac{1}{(1-x^2)\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{1}{(1-x^2)\sqrt{-1+x^4}}\,dx = -\frac{x\sqrt{-1+x^4}}{2(1-x^2)} - \frac{\sqrt{1-x^4}\,E(\arcsin(x)|-1)}{2\sqrt{-1+x^4}} + \frac{\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{\sqrt{-1+x^4}}$$

output

```
-1/2*x*(x^4-1)^(1/2)/(-x^2+1)-1/2*(-x^4+1)^(1/2)*EllipticE(x,I)/(x^4-1)^(1
/2)+(-x^4+1)^(1/2)*EllipticF(x,I)/(x^4-1)^(1/2)
```

Mathematica [A] (verified)

Time = 10.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{1}{(1-x^2)\sqrt{-1+x^4}}\,dx = \frac{x+x^3-\sqrt{1-x^4}\,E(\arcsin(x)|-1)+2\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{2\sqrt{-1+x^4}}$$

input

```
Integrate[1/((1 - x^2)*Sqrt[-1 + x^4]),x]
```

output
```
(x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1] + 2*Sqrt[1 - x^4]*Ellipt
icF[ArcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.73, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1392}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x^2)\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{1392}$$

$$\frac{x}{\sqrt{x^4-1}}-\frac{xE\left(\arcsin\left(\frac{\sqrt{2}\sqrt{x^2-1}}{\sqrt{x^4-1}}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2}}$$

input
```
Int[1/((1 - x^2)*Sqrt[-1 + x^4]),x]
```

output
```
x/Sqrt[-1 + x^4] - (x*EllipticE[ArcSin[(Sqrt[2]*Sqrt[-1 + x^2])/Sqrt[-1 +
x^4]], 1/2])/(Sqrt[2]*Sqrt[x^2])
```

Defintions of rubi rules used

rule 1392
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
x/(d*Sqrt[a + c*x^4]), x] - Simp[(x/(d*Sqrt[-2*a]*Sqrt[(-e/d)*x^2]))*Ellipt
icE[ArcSin[Sqrt[-2*a]*(Sqrt[-1 - (e/d)*x^2]/Sqrt[a + c*x^4])], 1/2], x] /;
FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[a, 0] && GtQ[c, 0] &
& NegQ[e/d]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21

method	result
risch	$\frac{x(x^2+1)}{2\sqrt{x^4-1}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\ \mathrm{EllipticF}(ix,i)}{2\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-\mathrm{EllipticE}(ix,i))}{2\sqrt{x^4-1}}$
elliptic	$\frac{(x^2+1)x}{2\sqrt{(x^2-1)(x^2+1)}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\ \mathrm{EllipticF}(ix,i)}{2\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-\mathrm{EllipticE}(ix,i))}{2\sqrt{x^4-1}}$
default	$\frac{x^3-x^2+x-1}{4\sqrt{(1+x)(x^3-x^2+x-1)}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\ \mathrm{EllipticF}(ix,i)}{2\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-\mathrm{EllipticE}(ix,i))}{2\sqrt{x^4-1}}+\frac{x}{4\sqrt{(x-}}$

input
```
int(1/(-x^2+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^2+1)/(x^4-1)^(1/2)-1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/
2)*EllipticF(I*x,I)+1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*(Elli
pticF(I*x,I)-EllipticE(I*x,I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.57

$$\int \frac{1}{(1-x^2)\sqrt{-1+x^4}}\,dx = \frac{(i\,x^2-i)E(\arcsin{(x)}\mid -1)-2\,(i\,x^2-i)F(\arcsin{(x)}\mid -1)+\sqrt{x^4-1}x}{2\,(x^2-1)}$$

input
```
integrate(1/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((I*x^2 - I)*elliptic_e(arcsin(x), -1) - 2*(I*x^2 - I)*elliptic_f(arcs
in(x), -1) + sqrt(x^4 - 1)*x)/(x^2 - 1)
```

Sympy [F]

$$\int \frac{1}{(1-x^2)\sqrt{-1+x^4}}\,dx = -\int \frac{1}{x^2\sqrt{x^4-1}-\sqrt{x^4-1}}\,dx$$

input
```
integrate(1/(-x**2+1)/(x**4-1)**(1/2),x)
```

output
```
-Integral(1/(x**2*sqrt(x**4 - 1) - sqrt(x**4 - 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1-x^2)\sqrt{-1+x^4}}\,dx = \int -\frac{1}{\sqrt{x^4-1}(x^2-1)}\,dx$$

input
```
integrate(1/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(x^4 - 1)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{(1-x^2)\sqrt{-1+x^4}}\,dx = \int -\frac{1}{\sqrt{x^4-1}(x^2-1)}\,dx$$

input
```
integrate(1/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(x^4 - 1)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-x^2)\sqrt{-1+x^4}}\,dx = -\int \frac{1}{(x^2-1)\sqrt{x^4-1}}\,dx$$

input

```
int(-1/((x^2 - 1)*(x^4 - 1)^(1/2)),x)
```

output

```
-int(1/((x^2 - 1)*(x^4 - 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-x^2)\sqrt{-1+x^4}}\,dx = -\left(\int \frac{\sqrt{x^4-1}}{x^6-x^4-x^2+1}dx\right)$$

input

```
int(1/(-x^2+1)/(x^4-1)^(1/2),x)
```

output

```
 - int(sqrt(x**4 - 1)/(x**6 - x**4 - x**2 + 1),x)
```

3.130 $\int (d+ex^2)^{5/2}\sqrt{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 196

$$\int (d+ex^2)^{5/2}\sqrt{d^2-e^2x^4}\,dx=-\frac{13d^3x\sqrt{d^2-e^2x^4}}{128\sqrt{d+ex^2}}+\frac{115d^2ex^3\sqrt{d^2-e^2x^4}}{192\sqrt{d+ex^2}}+\frac{23de^2x^5\sqrt{d^2-e^2x^4}}{48\sqrt{d+ex^2}}+\frac{e^3x^7\sqrt{d^2-e^2x^4}}{8\sqrt{d+ex^2}}+\frac{141d^4\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{128\sqrt{e}}$$

output
```
-13/128*d^3*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+115/192*d^2*e*x^3*(-e^2
*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+23/48*d*e^2*x^5*(-e^2*x^4+d^2)^(1/2)/(e*x^
2+d)^(1/2)+1/8*e^3*x^7*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+141/128*d^4*ar
ctan(e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.61

$$\int (d + ex^2)^{5/2} \sqrt{d^2 - e^2x^4}\, dx = \frac{1}{384}\left(\frac{x\sqrt{d^2 - e^2x^4}(-39d^3 + 230d^2ex^2 + 184de^2x^4 + 48e^3x^6)}{\sqrt{d + ex^2}} + \frac{423id^4 \log\left(-2i\sqrt{e}x + \frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}\right)$$

input

```
Integrate[(d + e*x^2)^(5/2)*Sqrt[d^2 - e^2*x^4],x]
```

output

```
((x*Sqrt[d^2 - e^2*x^4]*(-39*d^3 + 230*d^2*e*x^2 + 184*d*e^2*x^4 + 48*e^3*
x^6))/Sqrt[d + e*x^2] + ((423*I)*d^4*Log[(-2*I)*Sqrt[e]*x + (2*Sqrt[d^2 -
e^2*x^4])/Sqrt[d + e*x^2]])/Sqrt[e])/384
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1396, 318, 25, 27, 403, 25, 27, 299, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex^2)^{5/2} \sqrt{d^2 - e^2x^4}\, dx$$

$$\Big\downarrow \; 1396$$

$$\frac{\sqrt{d^2 - e^2x^4}\int \sqrt{d - ex^2}\,(ex^2 + d)^3\, dx}{\sqrt{d - ex^2}\sqrt{d + ex^2}}$$

$$\Big\downarrow \; 318$$

$$\frac{\sqrt{d^2 - e^2x^4}\left(-\frac{\int -de\sqrt{d-ex^2}\,(ex^2+d)\,(17ex^2+9d)dx}{8e} - \frac{1}{8}x\,(d - ex^2)^{3/2}\,(d + ex^2)^2\right)}{\sqrt{d - ex^2}\sqrt{d + ex^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int de\sqrt{d-ex^2}\left(ex^2+d\right)\left(17ex^2+9d\right)dx}{8e}-\frac{1}{8}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^2\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{8}d\int\sqrt{d-ex^2}\left(ex^2+d\right)\left(17ex^2+9d\right)dx-\frac{1}{8}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^2\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{8}d\left(-\frac{\int -de\sqrt{d-ex^2}\left(139ex^2+71d\right)dx}{6e}-\frac{17}{6}x\left(d+ex^2\right)\left(d-ex^2\right)^{3/2}\right)-\frac{1}{8}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^2\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{8}d\left(\frac{\int de\sqrt{d-ex^2}\left(139ex^2+71d\right)dx}{6e}-\frac{17}{6}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)\right)-\frac{1}{8}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^2\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{8}d\left(\frac{1}{6}d\int\sqrt{d-ex^2}\left(139ex^2+71d\right)dx-\frac{17}{6}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)\right)-\frac{1}{8}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^2\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 299$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{1}{8}d\Bigg(\frac{1}{6}d\Bigg(\frac{423}{4}d\int\sqrt{d-ex^2}dx-\frac{139}{4}x\left(d-ex^2\right)^{3/2}\Bigg)-\frac{17}{6}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)\Bigg)-\frac{1}{8}x\left(d-ex^2\right)^{3/2}\Big(}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 211$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{1}{8}d\Bigg(\frac{1}{6}d\Bigg(\frac{423}{4}d\Bigg(\frac{1}{2}d\int\frac{1}{\sqrt{d-ex^2}}dx+\frac{1}{2}x\sqrt{d-ex^2}\Bigg)-\frac{139}{4}x\left(d-ex^2\right)^{3/2}\Bigg)-\frac{17}{6}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)\Bigg)-}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 224$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{1}{8}d\Bigg(\frac{1}{6}d\Bigg(\frac{423}{4}d\Bigg(\frac{1}{2}d\int\frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}+\frac{1}{2}x\sqrt{d-ex^2}\Bigg)-\frac{139}{4}x\left(d-ex^2\right)^{3/2}\Bigg)-\frac{17}{6}x\left(d-ex^2\right)^{3/2}\big(d+}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 216$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{1}{8}d\Bigg(\frac{1}{6}d\Bigg(\frac{423}{4}d\Bigg(\frac{d\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d-ex^2}\Bigg)-\frac{139}{4}x\left(d-ex^2\right)^{3/2}\Bigg)-\frac{17}{6}x\left(d-ex^2\right)^{3/2}(d+ex}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d + e*x^2)^(5/2)*Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/8*(x*(d - e*x^2)^(3/2)*(d + e*x^2)^2) + (d*((-17*
x*(d - e*x^2)^(3/2)*(d + e*x^2))/6 + (d*((-139*x*(d - e*x^2)^(3/2))/4 + (4
23*d*((x*Sqrt[d - e*x^2])/2 + (d*ArcTan[(Sqrt[e]*x)/Sqrt[d - e*x^2]])/(2*S
qrt[e])))/4))/6))/8))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.70

method	result
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\,\left(48 e^{\frac{7}{2}} x^{7} \sqrt{-e\,x^{2}+d}+184 d\,e^{\frac{5}{2}} x^{5} \sqrt{-e\,x^{2}+d}+230 d^{2} e^{\frac{3}{2}} x^{3} \sqrt{-e\,x^{2}+d}-39 \sqrt{-e\,x^{2}+d}\,\sqrt{e}\,d^{3} x+423 \arctan\left(\frac{\sqrt{e}\,x}{\sqrt{-e\,x^{2}+d}}\right)\right.}{384 \sqrt{e\,x^{2}+d}\,\sqrt{-e\,x^{2}+d}\,\sqrt{e}}$
risch	$-\frac{x \left(-48 e^{3} x^{6}-184 d\,e^{2} x^{4}-230 d^{2} e\,x^{2}+39 d^{3}\right) \sqrt{-e\,x^{2}+d}\,\sqrt{\frac{-e^{2} x^{4}+d^{2}}{e\,x^{2}+d}}\,\sqrt{e\,x^{2}+d}}{384 \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{141 d^{4} \arctan\left(\frac{\sqrt{e}\,x}{\sqrt{-e\,x^{2}+d}}\right) \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e\,x^{2}+d}}\,\sqrt{e\,x^{2}+}}{128 \sqrt{e}\,\sqrt{-e^{2} x^{4}+d^{2}}}$

input
```
int((e*x^2+d)^(5/2)*(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*(-e^2*x^4+d^2)^(1/2)*(48*e^(7/2)*x^7*(-e*x^2+d)^(1/2)+184*d*e^(5/2)*
x^5*(-e*x^2+d)^(1/2)+230*d^2*e^(3/2)*x^3*(-e*x^2+d)^(1/2)-39*(-e*x^2+d)^(1
/2)*e^(1/2)*d^3*x+423*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*d^4)/(e*x^2+d)^(1
/2)/(-e*x^2+d)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.52

$$\int (d + ex^2)^{5/2} \sqrt{d^2 - e^2x^4}\, dx = \left[-\frac{423\,(d^4ex^2 + d^5)\sqrt{-e}\,\log\left(-\frac{2\,e^2x^4+dex^2-2\,\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{-e}x-d^2}{ex^2+d}\right) - 2\,(48\,e^4x}{768\,(e^2x^2+de)} \right.$$

$$-\frac{423\,(d^4ex^2+d^5)\sqrt{e}\arctan\left(\frac{\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{e}x}{e^2x^4-d^2}\right) - (48\,e^4x^7+184\,de^3x^5+230\,d^2e^2x^3-39\,d^3ex)\sqrt{-e^2x}}{384\,(e^2x^2+de)}$$

input
```
integrate((e*x^2+d)^(5/2)*(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/768*(423*(d^4*e*x^2 + d^5)*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt
(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)) - 2*(48*e^
4*x^7 + 184*d*e^3*x^5 + 230*d^2*e^2*x^3 - 39*d^3*e*x)*sqrt(-e^2*x^4 + d^2)
*sqrt(e*x^2 + d))/(e^2*x^2 + d*e), -1/384*(423*(d^4*e*x^2 + d^5)*sqrt(e)*a
rctan(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)) - (4
8*e^4*x^7 + 184*d*e^3*x^5 + 230*d^2*e^2*x^3 - 39*d^3*e*x)*sqrt(-e^2*x^4 +
d^2)*sqrt(e*x^2 + d))/(e^2*x^2 + d*e)]
```

Sympy [F]

$$\int \left(d+ex^2\right)^{5/2}\sqrt{d^2-e^2x^4}\,dx=\int \sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}\left(d+ex^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)*(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))*(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \left(d+ex^2\right)^{5/2}\sqrt{d^2-e^2x^4}\,dx=\int \sqrt{-e^2x^4+d^2}\left(ex^2+d\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.42

$$\int \left(d+ex^2\right)^{5/2}\sqrt{d^2-e^2x^4}\,dx=-\frac{141\,d^4\log\left(\left|-\sqrt{-e}x+\sqrt{-ex^2+d}\right|\right)}{128\,\sqrt{-e}}-\frac{1}{384}\left(39\,d^3-2\left(115\,d^2e+4\left(6\,e^3x^2+23\,de^2\right)x^2\right)x^2\right)\sqrt{-ex^2+d}x$$

input
```
integrate((e*x^2+d)^(5/2)*(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
-141/128*d^4*log(abs(-sqrt(-e)*x + sqrt(-e*x^2 + d)))/sqrt(-e) - 1/384*(39
*d^3 - 2*(115*d^2*e + 4*(6*e^3*x^2 + 23*d*e^2)*x^2)*x^2)*sqrt(-e*x^2 + d)*
x
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{5/2}\sqrt{d^2-e^2x^4}\,dx=\int \sqrt{d^2-e^2\,x^4}\,\left(e\,x^2+d\right)^{5/2}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(5/2),x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.47

$$\int (d +ex^2)^{5/2}\sqrt{d^2-e^2x^4}\,dx=\frac{423\sqrt{e}\,asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)d^4-39\sqrt{-e\,x^2+d}\,d^3ex+230\sqrt{-e\,x^2+d}\,d^2e^2x^3+184\sqrt{-e}}{384e}$$

input
```
int((e*x^2+d)^(5/2)*(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(423*sqrt(e)*asin((sqrt(e)*x)/sqrt(d))*d**4 - 39*sqrt(d - e*x**2)*d**3*e*x
 + 230*sqrt(d - e*x**2)*d**2*e**2*x**3 + 184*sqrt(d - e*x**2)*d*e**3*x**5
+ 48*sqrt(d - e*x**2)*e**4*x**7)/(384*e)
```

3.131 $\int (d+ex^2)^{3/2}\sqrt{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int (d+ex^2)^{3/2}\sqrt{d^2-e^2x^4}\,dx = \frac{3d^2x\sqrt{d^2-e^2x^4}}{16\sqrt{d+ex^2}} + \frac{11dex^3\sqrt{d^2-e^2x^4}}{24\sqrt{d+ex^2}} + \frac{e^2x^5\sqrt{d^2-e^2x^4}}{6\sqrt{d+ex^2}} + \frac{13d^3\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{16\sqrt{e}}$$

output
```
3/16*d^2*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+11/24*d*e*x^3*(-e^2*x^4+d^
2)^(1/2)/(e*x^2+d)^(1/2)+1/6*e^2*x^5*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+
13/16*d^3*arctan(e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.89 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.69

$$\int (d+ex^2)^{3/2}\sqrt{d^2-e^2x^4}\,dx = \frac{1}{48}\left(\frac{x\sqrt{d^2-e^2x^4}(9d^2+22dex^2+8e^2x^4)}{\sqrt{d+ex^2}} + \frac{39id^3\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}\right)$$

input

```
Integrate[(d + e*x^2)^(3/2)*Sqrt[d^2 - e^2*x^4],x]
```

output

```
((x*Sqrt[d^2 - e^2*x^4]*(9*d^2 + 22*d*e*x^2 + 8*e^2*x^4))/Sqrt[d + e*x^2]
+ ((39*I)*d^3*Log[(-2*I)*Sqrt[e]*x + (2*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^
2]])/Sqrt[e])/48
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1396, 318, 25, 27, 299, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2}\sqrt{d^2-e^2x^4}\,dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \sqrt{d-ex^2}\left(ex^2+d\right)^2dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{\int -de\sqrt{d-ex^2}\left(11ex^2+7d\right)dx}{6e}-\frac{1}{6}x\left(d+ex^2\right)\left(d-ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int de\sqrt{d-ex^2}\left(11ex^2+7d\right)dx}{6e}-\frac{1}{6}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{6}d\int \sqrt{d-ex^2}\left(11ex^2+7d\right)dx-\frac{1}{6}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{6}d\left(\frac{39}{4}d\int\sqrt{d-ex^2}dx-\frac{11}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{1}{6}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{211}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{6}d\left(\frac{39}{4}d\left(\frac{1}{2}d\int\frac{1}{\sqrt{d-ex^2}}dx+\frac{1}{2}x\sqrt{d-ex^2}\right)-\frac{11}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{1}{6}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{6}d\left(\frac{39}{4}d\left(\frac{1}{2}d\int\frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}+\frac{1}{2}x\sqrt{d-ex^2}\right)-\frac{11}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{1}{6}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{6}d\left(\frac{39}{4}d\left(\frac{d\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d-ex^2}\right)-\frac{11}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{1}{6}x\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[(d + e*x^2)^(3/2)*Sqrt[d^2 - e^2*x^4],x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(-1/6*(x*(d - e*x^2)^(3/2)*(d + e*x^2)) + (d*((-11*x*
(d - e*x^2)^(3/2))/4 + (39*d*((x*Sqrt[d - e*x^2])/2 + (d*ArcTan[(Sqrt[e]*x
)/Sqrt[d - e*x^2]])/(2*Sqrt[e])))/4))/6))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]
)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(8 e^{\frac{5}{2}} x^{5} \sqrt{-e\,x^{2}+d}+22 d\,e^{\frac{3}{2}} x^{3} \sqrt{-e\,x^{2}+d}+9 \sqrt{-e\,x^{2}+d}\, \sqrt{e}\, d^{2} x+39 \arctan \left(\frac{\sqrt{e}\, x}{\sqrt{-e\,x^{2}+d}}\right) d^{3}\right)}{48 \sqrt{e\,x^{2}+d}\, \sqrt{-e\,x^{2}+d}\, \sqrt{e}}$	117
risch	$\frac{x \left(8 e^{2} x^{4}+22 d e\,x^{2}+9 d^{2}\right) \sqrt{-e\,x^{2}+d}\, \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e\,x^{2}+d}}\, \sqrt{e\,x^{2}+d}}{48 \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{13 d^{3} \arctan \left(\frac{\sqrt{e}\, x}{\sqrt{-e\,x^{2}+d}}\right) \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e\,x^{2}+d}}\, \sqrt{e\,x^{2}+d}}{16 \sqrt{e}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	154

input
```
int((e*x^2+d)^(3/2)*(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(-e^2*x^4+d^2)^(1/2)*(8*e^(5/2)*x^5*(-e*x^2+d)^(1/2)+22*d*e^(3/2)*x^3
*(-e*x^2+d)^(1/2)+9*(-e*x^2+d)^(1/2)*e^(1/2)*d^2*x+39*arctan(e^(1/2)*x/(-e
*x^2+d)^(1/2))*d^3)/(e*x^2+d)^(1/2)/(-e*x^2+d)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.77

$$\int (d+ex^{2})^{3/2} \sqrt{d^{2}-e^{2}x^{4}}\, dx = \left[-\frac{39\,(d^{3}ex^{2}+d^{4})\sqrt{-e}\log\left(-\frac{2\,e^{2}x^{4}+dex^{2}-2\,\sqrt{-e^{2}x^{4}+d^{2}}\sqrt{ex^{2}+d}\sqrt{-e}x-d^{2}}{ex^{2}+d}\right)-2\,(8\,e^{3}x^{5}}{96\,(e^{2}x^{2}+de)}\right.$$

$$\left.-\frac{39\,(d^{3}ex^{2}+d^{4})\sqrt{e}\arctan\left(\frac{\sqrt{-e^{2}x^{4}+d^{2}}\sqrt{ex^{2}+d}\sqrt{e}x}{e^{2}x^{4}-d^{2}}\right)-(8\,e^{3}x^{5}+22\,de^{2}x^{3}+9\,d^{2}ex)\sqrt{-e^{2}x^{4}+d^{2}}\sqrt{ex^{2}+d}}{48\,(e^{2}x^{2}+de)}\right]$$

input
```
integrate((e*x^2+d)^(3/2)*(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(39*(d^3*e*x^2 + d^4)*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt(-
e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)) - 2*(8*e^3*x
^5 + 22*d*e^2*x^3 + 9*d^2*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d))/(e^2*
x^2 + d*e), -1/48*(39*(d^3*e*x^2 + d^4)*sqrt(e)*arctan(sqrt(-e^2*x^4 + d^2
)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)) - (8*e^3*x^5 + 22*d*e^2*x^3 +
 9*d^2*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d))/(e^2*x^2 + d*e)]
```

Sympy [F]

$$\int \left(d+ex^2\right)^{3/2}\sqrt{d^2-e^2x^4}\,dx=\int \sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))*(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \left(d+ex^2\right)^{3/2}\sqrt{d^2-e^2x^4}\,dx=\int \sqrt{-e^2x^4+d^2}\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.44

$$\int \left(d+ex^2\right)^{3/2}\sqrt{d^2-e^2x^4}\,dx = -\frac{13\,d^3\log\left(\left|-\sqrt{-e}x+\sqrt{-ex^2+d}\right|\right)}{16\sqrt{-e}} + \frac{1}{48}\left(2\left(4\,e^2x^2+11\,de\right)x^2+9\,d^2\right)\sqrt{-ex^2+d}x$$

input
```
integrate((e*x^2+d)^(3/2)*(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
-13/16*d^3*log(abs(-sqrt(-e)*x + sqrt(-e*x^2 + d)))/sqrt(-e) + 1/48*(2*(4*
e^2*x^2 + 11*d*e)*x^2 + 9*d^2)*sqrt(-e*x^2 + d)*x
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2}\sqrt{d^2-e^2x^4}\,dx = \int \sqrt{d^2-e^2\,x^4}\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(3/2),x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int (d +ex^2)^{3/2}\sqrt{d^2-e^2x^4}\,dx = \frac{39\sqrt{e}\,asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)d^3+9\sqrt{-e\,x^2+d}\,d^2ex+22\sqrt{-e\,x^2+d}\,d\,e^2x^3+8\sqrt{-e\,x^2+d}}{48e}$$

input
```
int((e*x^2+d)^(3/2)*(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(39*sqrt(e)*asin((sqrt(e)*x)/sqrt(d))*d**3 + 9*sqrt(d - e*x**2)*d**2*e*x +
 22*sqrt(d - e*x**2)*d*e**2*x**3 + 8*sqrt(d - e*x**2)*e**3*x**5)/(48*e)
```

3.132 $\int \sqrt{d+ex^2}\sqrt{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \sqrt{d+ex^2}\sqrt{d^2-e^2x^4}\,dx=\frac{3dx\sqrt{d^2-e^2x^4}}{8\sqrt{d+ex^2}}+\frac{ex^3\sqrt{d^2-e^2x^4}}{4\sqrt{d+ex^2}}+\frac{5d^2\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{8\sqrt{e}}$$

output

```
3/8*d*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+1/4*e*x^3*(-e^2*x^4+d^2)^(1/2
)/(e*x^2+d)^(1/2)+5/8*d^2*arctan(e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^
(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int \sqrt{d+ex^2}\sqrt{d^2-e^2x^4}\,dx=\frac{1}{8}\left(\frac{x(3d+2ex^2)\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}+\frac{5id^2\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}\right)$$

input
```
Integrate[Sqrt[d + e*x^2]*Sqrt[d^2 - e^2*x^4],x]
```

output
```
((x*(3*d + 2*e*x^2)*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2] + ((5*I)*d^2*Log[
(-2*I)*Sqrt[e]*x + (2*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2]])/Sqrt[e])/8
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1396, 299, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}\sqrt{d^2-e^2x^4}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \sqrt{d-ex^2}\left(ex^2+d\right)dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{5}{4}d\int \sqrt{d-ex^2}dx-\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{211}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{5}{4}d\left(\frac{1}{2}d\int \frac{1}{\sqrt{d-ex^2}}dx+\frac{1}{2}x\sqrt{d-ex^2}\right)-\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{5}{4}d\left(\frac{1}{2}d\int \frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}+\frac{1}{2}x\sqrt{d-ex^2}\right)-\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{5}{4}d\left(\frac{d\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d-ex^2}\right)-\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[Sqrt[d + e*x^2]*Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/4*(x*(d - e*x^2)^(3/2)) + (5*d*((x*Sqrt[d - e*x^2
])/2 + (d*ArcTan[(Sqrt[e]*x)/Sqrt[d - e*x^2]])/(2*Sqrt[e])))/4))/(Sqrt[d -
 e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(2 e^{\frac{3}{2}} x^{3} \sqrt{-e\,x^{2}+d}+3 \sqrt{e}\, \sqrt{-e\,x^{2}+d}\, d x +5 \arctan\left(\frac{\sqrt{e}\, x}{\sqrt{-e\,x^{2}+d}}\right) d^{2}\right)}{8 \sqrt{e\,x^{2}+d}\, \sqrt{-e\,x^{2}+d}\, \sqrt{e}}$	96
risch	$\frac{x \left(2 e\,x^{2}+3 d \right) \sqrt{-e\,x^{2}+d}\, \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e\,x^{2}+d}}\, \sqrt{e\,x^{2}+d}}{8 \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{5 d^{2} \arctan\left(\frac{\sqrt{e}\, x}{\sqrt{-e\,x^{2}+d}}\right) \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e\,x^{2}+d}}\, \sqrt{e\,x^{2}+d}}{8 \sqrt{e}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	143

input

```
int((e*x^2+d)^(1/2)*(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/8*(-e^2*x^4+d^2)^(1/2)*(2*e^(3/2)*x^3*(-e*x^2+d)^(1/2)+3*e^(1/2)*(-e*x^2
+d)^(1/2)*d*x+5*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*d^2)/(e*x^2+d)^(1/2)/(-
e*x^2+d)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.19

$$\int \sqrt{d+ex^2}\sqrt{d^2-e^2x^4}\,dx$$

$$= \left[-\frac{5\,(d^2ex^2+d^3)\sqrt{-e}\log\left(-\frac{2\,e^2x^4+dex^2-2\,\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{-e}x-d^2}{ex^2+d}\right)-2\,\sqrt{-e^2x^4+d^2}(2\,e^2x^3+3\,dex)}{16\,(e^2x^2+de)}\right.$$

$$\left.-\frac{5\,(d^2ex^2+d^3)\sqrt{e}\arctan\left(\frac{\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{e}x}{e^2x^4-d^2}\right)-\sqrt{-e^2x^4+d^2}(2\,e^2x^3+3\,dex)\sqrt{ex^2+d}}{8\,(e^2x^2+de)}\right]$$

input
```
integrate((e*x^2+d)^(1/2)*(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(5*(d^2*e*x^2 + d^3)*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt(-e
^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)) - 2*sqrt(-e^2
*x^4 + d^2)*(2*e^2*x^3 + 3*d*e*x)*sqrt(e*x^2 + d))/(e^2*x^2 + d*e), -1/8*(
5*(d^2*e*x^2 + d^3)*sqrt(e)*arctan(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sq
rt(e)*x/(e^2*x^4 - d^2)) - sqrt(-e^2*x^4 + d^2)*(2*e^2*x^3 + 3*d*e*x)*sqrt
(e*x^2 + d))/(e^2*x^2 + d*e)]
```

Sympy [F]

$$\int \sqrt{d+ex^2}\sqrt{d^2-e^2x^4}\,dx=\int \sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}\sqrt{d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \sqrt{d+ex^2}\sqrt{d^2-e^2x^4}\,dx=\int \sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

$$\int \sqrt{d+ex^2}\sqrt{d^2-e^2x^4}\,dx = \frac{1}{8}\left(2\,ex^2+3\,d\right)\sqrt{-ex^2+d}x - \frac{5\,d^2\log\left(\left|-\sqrt{-e}x+\sqrt{-ex^2+d}\right|\right)}{8\,\sqrt{-e}}$$

input
```
integrate((e*x^2+d)^(1/2)*(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(2*e*x^2 + 3*d)*sqrt(-e*x^2 + d)*x - 5/8*d^2*log(abs(-sqrt(-e)*x + sqr
t(-e*x^2 + d)))/sqrt(-e)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex^2}\sqrt{d^2-e^2x^4}\,dx = \int \sqrt{d^2-e^2\,x^4}\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(1/2),x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46

$$\int \sqrt{d+ex^2}\sqrt{d^2-e^2x^4}\,dx = \frac{5\sqrt{e}\,asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)d^2+3\sqrt{-e\,x^2+d}\,dex+2\sqrt{-e\,x^2+d}\,e^2x^3}{8e}$$

input
```
int((e*x^2+d)^(1/2)*(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(5*sqrt(e)*asin((sqrt(e)*x)/sqrt(d))*d**2 + 3*sqrt(d - e*x**2)*d*e*x + 2*s
qrt(d - e*x**2)*e**2*x**3)/(8*e)
```

3.133 $\int \frac{\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 78

$$\int \frac{\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\,dx=\frac{x\sqrt{d^2-e^2x^4}}{2\sqrt{d+ex^2}}+\frac{d\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{2\sqrt{e}}$$

output
```
1/2*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+1/2*d*arctan(e^(1/2)*x*(e*x^2+d
)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\,dx=\frac{x\sqrt{d^2-e^2x^4}}{2\sqrt{d+ex^2}}+\frac{id\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/Sqrt[d + e*x^2],x]
```

output
```
(x*Sqrt[d^2 - e^2*x^4])/(2*Sqrt[d + e*x^2]) + ((I/2)*d*Log[(-2*I)*Sqrt[e]*
x + (2*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2]])/Sqrt[e]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1396, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \sqrt{d-ex^2}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{211}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{2}d\int \frac{1}{\sqrt{d-ex^2}}dx+\frac{1}{2}x\sqrt{d-ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{2}d\int \frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}+\frac{1}{2}x\sqrt{d-ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{d\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d-ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[Sqrt[d^2 - e^2*x^4]/Sqrt[d + e*x^2],x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d - e*x^2])/2 + (d*ArcTan[(Sqrt[e]*x)/Sqrt[d
 - e*x^2]])/(2*Sqrt[e])))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(x \sqrt{-e\, x^{2}+d}\, \sqrt{e}+d \arctan\left(\frac{\sqrt{e}\, x}{\sqrt{-e\, x^{2}+d}}\right)\right)}{2 \sqrt{e\, x^{2}+d}\, \sqrt{-e\, x^{2}+d}\, \sqrt{e}}$	73
risch	$\frac{x \sqrt{-e\, x^{2}+d}\, \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e\, x^{2}+d}}\, \sqrt{e\, x^{2}+d}}{2 \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{d \arctan\left(\frac{\sqrt{e}\, x}{\sqrt{-e\, x^{2}+d}}\right) \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e\, x^{2}+d}}\, \sqrt{e\, x^{2}+d}}{2 \sqrt{e}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	131

input

```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)*(x*(-e*x^2+d)^(1/2)*e^(1/2)+d*arc
tan(e^(1/2)*x/(-e*x^2+d)^(1/2)))/(-e*x^2+d)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.88

$$\int \frac{\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{2\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\,dex-(dex^2+d^2)\sqrt{-e}\log\left(-\frac{2\,e^2x^4+dex^2-2\,\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{-e}x-d^2}{ex^2+d}\right)}{4\,(e^2x^2+de)}, \sqrt{-e^2x^4+\right.$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*e*x - (d*e*x^2 + d^2)*sqrt(-e
)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(
-e)*x - d^2)/(e*x^2 + d)))/(e^2*x^2 + d*e), 1/2*(sqrt(-e^2*x^4 + d^2)*sqrt
(e*x^2 + d)*e*x - (d*e*x^2 + d^2)*sqrt(e)*arctan(sqrt(-e^2*x^4 + d^2)*sqrt
(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)))/(e^2*x^2 + d*e)]
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(e*x**2+d)**(1/2),x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/sqrt(e*x^2 + d), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\,dx=\frac{1}{2}\sqrt{-ex^2+d}x-\frac{d\log\left(\left|-\sqrt{-e}x+\sqrt{-ex^2+d}\right|\right)}{2\sqrt{-e}}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-e*x^2 + d)*x - 1/2*d*log(abs(-sqrt(-e)*x + sqrt(-e*x^2 + d)))/sq
rt(-e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{d^2-e^2\,x^4}}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^(1/2),x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}}{e\,x^2+d}dx$$

input

```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2),x)
```

output

```
int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4))/(d + e*x**2),x)
```

3.134 $\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{3/2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{\sqrt{e}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{\sqrt{e}}$$

output
```
-arctan(e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)+2^(1/2)*ar
ctan(2^(1/2)*e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{3/2}}\,dx=\frac{\frac{\sqrt{2}\sqrt{d^2-e^2x^4}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}-i\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^(3/2),x]
```

output
```
((Sqrt[2]*Sqrt[d^2 - e^2*x^4]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]])
/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) - I*Log[(-2*I)*Sqrt[e]*x + (2*Sqrt[d^2
- e^2*x^4])/Sqrt[d + e*x^2]])/Sqrt[e]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1396, 301, 224, 216, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{d-ex^2}}{ex^2+d}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 301}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(2d\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)}dx-\int \frac{1}{\sqrt{d-ex^2}}dx\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(2d\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)}dx-\int \frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(2d\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)}dx-\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{e}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(2d\int \frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}-\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{e}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{e}}-\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{e}}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^(3/2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(-(ArcTan[(Sqrt[e]*x)/Sqrt[d - e*x^2]]/Sqrt[e]) + (Sq
rt[2]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]])/Sqrt[e]))/(Sqrt[d - e*x
^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(74) = 148$.

Time = 0.43 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.91

method	result
default	$-\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(\sqrt{d}\, \sqrt{2}\, \ln \left(\frac{2 e \left(\sqrt{2}\, \sqrt{d}\, \sqrt{-e\, x^{2}+d}-\sqrt{-d e}\, x+d \right)}{e x -\sqrt{-d e}}\right) \sqrt{e}-\sqrt{d}\, \sqrt{2}\, \ln \left(\frac{2 e \left(\sqrt{2}\, \sqrt{d}\, \sqrt{-e\, x^{2}+d}+\sqrt{-d e}\, x+d \right)}{e x +\sqrt{-d e}}\right) \sqrt{e}+2 \sqrt{-d e}\, \mathrm{arcta} \right.}{2 \sqrt{e\, x^{2}+d}\, \sqrt{-e\, x^{2}+d}\, \sqrt{-d e}\, \sqrt{e}}$

input

```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)*(d^(1/2)*2^(1/2)*ln(2*e*(2^(1/2)
*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*e^(1/2)-d^
(1/2)*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(
e*x+(-d*e)^(1/2)))*e^(1/2)+2*(-d*e)^(1/2)*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2
)))/(-e*x^2+d)^(1/2)/(-d*e)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.97

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^{3/2}}\,dx = \left[\frac{\sqrt{2}e\sqrt{-\frac{1}{e}}\log\left(-\frac{3\,e^2x^4+2\,dex^2+2\,\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}ex\sqrt{-\frac{1}{e}}-d^2}{e^2x^4+2\,dex^2+d^2}\right)-\sqrt{-e}\log\left(-\frac{2\,e^2x^4+dex}{}\right.}{2\,e}\right.$$

$$\left.-\frac{\sqrt{2}\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{e}x}{e^2x^4-d^2}\right)-\sqrt{e}\arctan\left(\frac{\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{e}x}{e^2x^4-d^2}\right)}{e}\right]$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*e*sqrt(-1/e)*log(-(3*e^2*x^4 + 2*d*e*x^2 + 2*sqrt(2)*sqrt(-e
^2*x^4 + d^2)*sqrt(e*x^2 + d)*e*x*sqrt(-1/e) - d^2)/(e^2*x^4 + 2*d*e*x^2 +
 d^2)) - sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 + 2*sqrt(-e^2*x^4 + d^2)*sqrt(
e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)))/e, -(sqrt(2)*sqrt(e)*arctan(sqr
t(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)) - sqr
t(e)*arctan(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)
))/e]
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}}{\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(e*x**2+d)**(3/2),x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{3/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{3/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{3/2}}\,dx=\int \frac{\sqrt{d^2-e^2\,x^4}}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^(3/2),x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}}{e^2x^4+2de\,x^2+d^2}dx$$

input
```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(3/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4))/(d**2 + 2*d*e*x**2 + e**2*x*
*4),x)
```

3.135 $\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 93

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^{5/2}}\,dx=\frac{x\sqrt{d^2-e^2x^4}}{2d\left(d+ex^2\right)^{3/2}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{2\sqrt{2}d\sqrt{e}}$$

output
```
1/2*x*(-e^2*x^4+d^2)^(1/2)/d/(e*x^2+d)^(3/2)+1/4*arctan(2^(1/2)*e^(1/2)*x*
(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))*2^(1/2)/d/e^(1/2)
```

Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{d^2-e^2x^4}}{\left(d+ex^2\right)^{5/2}}\,dx=\frac{\sqrt{d^2-e^2x^4}\left(2\sqrt{e}x\sqrt{d-ex^2}+\sqrt{2}(d+ex^2)\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)\right)}{4d\sqrt{e}\sqrt{d-ex^2}\left(d+ex^2\right)^{3/2}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^(5/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(2*Sqrt[e]*x*Sqrt[d - e*x^2] + Sqrt[2]*(d + e*x^2)*Ar
cTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]]))/(4*d*Sqrt[e]*Sqrt[d - e*x^2]*(
d + e*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1396, 292, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{d-ex^2}}{(ex^2+d)^2}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{292}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{2}\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)}dx+\frac{x\sqrt{d-ex^2}}{2d(d+ex^2)}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{2}\int \frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}+\frac{x\sqrt{d-ex^2}}{2d(d+ex^2)}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{2}d\sqrt{e}}+\frac{x\sqrt{d-ex^2}}{2d(d+ex^2)}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^(5/2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d - e*x^2])/(2*d*(d + e*x^2)) + ArcTan[(Sqrt
[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]]/(2*Sqrt[2]*d*Sqrt[e])))/(Sqrt[d - e*x^2]*S
qrt[d + e*x^2])
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] && !IntegerQ[p] && !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(73) = 146.

Time = 0.46 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.31

method	result
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, e \bigg(-\sqrt{2}\, \ln \bigg(\frac{2 e \left(\sqrt{2}\, \sqrt{d}\, \sqrt{-e\, x^{2}+d}-\sqrt{-d e}\, x+d\right)}{e x-\sqrt{-d e}}\bigg) e\, x^{2} \sqrt{d}+\sqrt{2}\, \ln \bigg(\frac{2 e \left(\sqrt{2}\, \sqrt{d}\, \sqrt{-e\, x^{2}+d}+\sqrt{-d e}\, x+d\right)}{e x+\sqrt{-d e}}\bigg) e\, x^{2} \sqrt{d}-\sqrt{2}\, \ln \bigg(}{8 d \sqrt{e\, x^{2}+d}\, \sqrt{-e\, x^{2}+d}\, \left(e x-\sqrt{-d e}\right) (e x+}$

input
```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-e^2*x^4+d^2)^(1/2)*e*(-2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1
/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*e*x^2*d^(1/2)+2^(1/2)*ln(2*e*(2^
(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*e*x^2
*d^(1/2)-2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d
)/(e*x-(-d*e)^(1/2)))*d^(3/2)+2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(
1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*d^(3/2)+4*(-d*e)^(1/2)*(-e*x^2+
d)^(1/2)*x)/d/(e*x^2+d)^(1/2)/(-e*x^2+d)^(1/2)/(e*x-(-d*e)^(1/2))/(e*x+(-d
*e)^(1/2))/(-d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.16

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{5/2}}\,dx = \left[\frac{4\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}ex-\sqrt{2}(e^2x^4+2\,dex^2+d^2)\sqrt{-e}\log\left(-\frac{3\,e^2x^4+2\,dex^2-2\,\sqrt{2}\sqrt{-}}{e^2x^4+}\right.}{8\,(de^3x^4+2\,d^2e^2x^2+d^3e)}\right.$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*e*x - sqrt(2)*(e^2*x^4 + 2*d*
e*x^2 + d^2)*sqrt(-e)*log(-(3*e^2*x^4 + 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x^
4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)))/(
d*e^3*x^4 + 2*d^2*e^2*x^2 + d^3*e), 1/4*(2*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2
 + d)*e*x - sqrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e)*arctan(sqrt(2)*sqr
t(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)))/(d*e^3*x^4 +
 2*d^2*e^2*x^2 + d^3*e)]
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{5/2}}\,dx=\int \frac{\sqrt{-(-d+ex^2)(d+ex^2)}}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(e*x**2+d)**(5/2),x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{5/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{5/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{5/2}}\,dx=\int \frac{\sqrt{d^2-e^2\,x^4}}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^(5/2),x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{5/2}}\,dx=\frac{\sqrt{e\,x^{2}+d}\,\sqrt{-e^{2}x^{4}+d^{2}}\,x+2\left(\int \frac{\sqrt{e\,x^{2}+d}\,\sqrt{-e^{2}x^{4}+d^{2}}}{-e^{4}x^{8}-2d\,e^{3}x^{6}+2d^{3}e\,x^{2}+d^{4}}d x\right) d^{4}+4\left(\int \frac{\sqrt{e\,x^{2}+d}\,\sqrt{-e^{2}x^{4}}}{-e^{4}x^{8}-2d\,e^{3}x^{6}+2d^{3}}\right.}{3d\left(e^{2}x^{4}+2de\,x^{2}+d^{2}\right)}$$

input
```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(5/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x + 2*int((sqrt(d + e*x**2)*sqrt(
d**2 - e**2*x**4))/(d**4 + 2*d**3*e*x**2 - 2*d*e**3*x**6 - e**4*x**8),x)*d
**4 + 4*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4))/(d**4 + 2*d**3*e*x**
2 - 2*d*e**3*x**6 - e**4*x**8),x)*d**3*e*x**2 + 2*int((sqrt(d + e*x**2)*sq
rt(d**2 - e**2*x**4))/(d**4 + 2*d**3*e*x**2 - 2*d*e**3*x**6 - e**4*x**8),x
)*d**2*e**2*x**4)/(3*d*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.136 $\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 128

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{7/2}}\,dx = \frac{x\sqrt{d^2-e^2x^4}}{4d\,(d+ex^2)^{5/2}} + \frac{5x\sqrt{d^2-e^2x^4}}{16d^2\,(d+ex^2)^{3/2}} + \frac{7\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{16\sqrt{2}d^2\sqrt{e}}$$

output
```
1/4*x*(-e^2*x^4+d^2)^(1/2)/d/(e*x^2+d)^(5/2)+5/16*x*(-e^2*x^4+d^2)^(1/2)/d
^2/(e*x^2+d)^(3/2)+7/32*arctan(2^(1/2)*e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4
+d^2)^(1/2))*2^(1/2)/d^2/e^(1/2)
```

Mathematica [A] (verified)

Time = 3.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^4}\left(2\sqrt{e}x\sqrt{d-ex^2}(9d+5ex^2)+7\sqrt{2}(d+ex^2)^2\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)\right)}{32d^2\sqrt{e}\sqrt{d-ex^2}\,(d+ex^2)^{5/2}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(2*Sqrt[e]*x*Sqrt[d - e*x^2]*(9*d + 5*e*x^2) + 7*Sqrt
[2]*(d + e*x^2)^2*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]]))/(32*d^2*Sq
rt[e]*Sqrt[d - e*x^2]*(d + e*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1396, 296, 292, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{7/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\sqrt{d-ex^2}}{(ex^2+d)^3}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{296}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{7\int \frac{\sqrt{d-ex^2}}{(ex^2+d)^2}dx}{8d}+\frac{x(d-ex^2)^{3/2}}{8d^2(d+ex^2)^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{292}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{7\left(\frac{1}{2}\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)}dx+\frac{x\sqrt{d-ex^2}}{2d(d+ex^2)}\right)}{8d}+\frac{x(d-ex^2)^{3/2}}{8d^2(d+ex^2)^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{7\left(\frac{1}{2}\int \frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}+\frac{x\sqrt{d-ex^2}}{2d(d+ex^2)}\right)}{8d}+\frac{x(d-ex^2)^{3/2}}{8d^2(d+ex^2)^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{7\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{2}d\sqrt{e}}+\frac{x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)}\right)}{8d}+\frac{x\left(d-ex^2\right)^{3/2}}{8d^2\left(d+ex^2\right)^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[Sqrt[d^2 - e^2*x^4]/(d + e*x^2)^(7/2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((x*(d - e*x^2)^(3/2))/(8*d^2*(d + e*x^2)^2) + (7*((x
*Sqrt[d - e*x^2])/(2*d*(d + e*x^2)) + ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d -
e*x^2]]/(2*Sqrt[2]*d*Sqrt[e])))/(8*d)))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(102) = 204$.

Time = 0.46 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.53

method	result
default	$\frac{\sqrt{-e^{2}x^{4}+d^{2}}\, e^{2}\left(-7\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^{2}+d}-\sqrt{-de}\,x+d\right)}{ex-\sqrt{-de}}\right)\sqrt{2}\,e^{2}x^{4}\sqrt{d}+7\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^{2}+d}+\sqrt{-de}\,x+d\right)}{ex+\sqrt{-de}}\right)\sqrt{2}\,e^{2}x^{4}\sqrt{d}-14\ln\left(\right.\right.}{}$

input
```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(-e^2*x^4+d^2)^(1/2)*e^2*(-7*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)
-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*2^(1/2)*e^2*x^4*d^(1/2)+7*ln(2*e*(2
^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*2^(1
/2)*e^2*x^4*d^(1/2)-14*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/
2)*x+d)/(e*x-(-d*e)^(1/2)))*2^(1/2)*d^(3/2)*e*x^2+14*ln(2*e*(2^(1/2)*d^(1/
2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*2^(1/2)*d^(3/2)*
e*x^2+20*(-d*e)^(1/2)*(-e*x^2+d)^(1/2)*e*x^3-7*ln(2*e*(2^(1/2)*d^(1/2)*(-e
*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*2^(1/2)*d^(5/2)+7*ln(2
*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))
*2^(1/2)*d^(5/2)+36*d*(-d*e)^(1/2)*(-e*x^2+d)^(1/2)*x)/d^2/(e*x^2+d)^(1/2)
/(-e*x^2+d)^(1/2)/(e*x-(-d*e)^(1/2))^2/(e*x+(-d*e)^(1/2))^2/(-d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{7/2}}\,dx = \left[-\frac{7\sqrt{2}(e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3)\sqrt{-e}\log\left(-\frac{3\,e^2x^4+2\,dex^2-2\,\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{-}}{e^2x^4+2\,dex^2+d^2}\right.}{64\,(d^2e^4x^6+3\,d^3e^3x^4+3\,d^4e^2x^2+}\right.$$

$$-\frac{7\sqrt{2}(e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3)\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{e}x}{e^2x^4-d^2}\right)-2\sqrt{-e^2x^4+d^2}(5\,e^2x^3+9\,dex}{32\,(d^2e^4x^6+3\,d^3e^3x^4+3\,d^4e^2x^2+d^5e)}$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
[-1/64*(7*sqrt(2)*(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)*sqrt(-e)*log
(-(3*e^2*x^4 + 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*
sqrt(-e)*x - d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 4*sqrt(-e^2*x^4 + d^2)*(5
*e^2*x^3 + 9*d*e*x)*sqrt(e*x^2 + d))/(d^2*e^4*x^6 + 3*d^3*e^3*x^4 + 3*d^4*
e^2*x^2 + d^5*e), -1/32*(7*sqrt(2)*(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
d^3)*sqrt(e)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(e)*x
/(e^2*x^4 - d^2)) - 2*sqrt(-e^2*x^4 + d^2)*(5*e^2*x^3 + 9*d*e*x)*sqrt(e*x^
2 + d))/(d^2*e^4*x^6 + 3*d^3*e^3*x^4 + 3*d^4*e^2*x^2 + d^5*e)]
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{7/2}}\,dx = \int \frac{\sqrt{-(-d+ex^2)(d+ex^2)}}{(d+ex^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(1/2)/(e*x**2+d)**(7/2),x)
```

output
```
Integral(sqrt(-(-d + e*x**2)*(d + e*x**2))/(d + e*x**2)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{7/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{7/2}}\,dx=\int \frac{\sqrt{-e^2x^4+d^2}}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-e^2*x^4 + d^2)/(e*x^2 + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{7/2}}\,dx=\int \frac{\sqrt{d^2-e^2\,x^4}}{(e\,x^2+d)^{7/2}}\,dx$$

input
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^(7/2),x)
```

output
```
int((d^2 - e^2*x^4)^(1/2)/(d + e*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d^2-e^2x^4}}{(d+ex^2)^{7/2}}\,dx = \frac{5\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,dx+2\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,e\,x^3+14\bigg(\int \frac{\sqrt{e\,x^2+d}}{-e^5x^{10}-3d\,e^4x^8-2d^2}}{}$$

input
```
int((-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(7/2),x)
```

output
```
(5*sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*d*x + 2*sqrt(d + e*x**2)*sqrt(d
**2 - e**2*x**4)*e*x**3 + 14*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*
x**2)/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2*d**2*e**3*x**6 - 3*d*e*
*4*x**8 - e**5*x**10),x)*d**5*e + 42*int((sqrt(d + e*x**2)*sqrt(d**2 - e**
2*x**4)*x**2)/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2*d**2*e**3*x**6
 - 3*d*e**4*x**8 - e**5*x**10),x)*d**4*e**2*x**2 + 42*int((sqrt(d + e*x**2)
*sqrt(d**2 - e**2*x**4)*x**2)/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2
*d**2*e**3*x**6 - 3*d*e**4*x**8 - e**5*x**10),x)*d**3*e**3*x**4 + 14*int((
sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**2)/(d**5 + 3*d**4*e*x**2 + 2*d*
*3*e**2*x**4 - 2*d**2*e**3*x**6 - 3*d*e**4*x**8 - e**5*x**10),x)*d**2*e**4
*x**6)/(5*d**2*(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6))
```

3.137 $\int \left(d+ex^2\right)^{5/2} \left(d^2-e^2x^4\right)^{3/2} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 276

$$\int \left(d+ex^2\right)^{5/2} \left(d^2-e^2x^4\right)^{3/2} \, dx = \frac{185d^5x\sqrt{d^2-e^2x^4}}{1024\sqrt{d+ex^2}} + \frac{403d^4ex^3\sqrt{d^2-e^2x^4}}{512\sqrt{d+ex^2}} + \frac{55d^3e^2x^5\sqrt{d^2-e^2x^4}}{128\sqrt{d+ex^2}} - \frac{13d^2e^3x^7\sqrt{d^2-e^2x^4}}{64\sqrt{d+ex^2}} - \frac{7de^4x^9\sqrt{d^2-e^2x^4}}{24\sqrt{d+ex^2}} - \frac{e^5x^{11}\sqrt{d^2-e^2x^4}}{12\sqrt{d+ex^2}} + \frac{839d^6\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{1024\sqrt{e}}$$

output

```
185/1024*d^5*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+403/512*d^4*e*x^3*(-e^
2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+55/128*d^3*e^2*x^5*(-e^2*x^4+d^2)^(1/2)/(
e*x^2+d)^(1/2)-13/64*d^2*e^3*x^7*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)-7/24
*d*e^4*x^9*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)-1/12*e^5*x^11*(-e^2*x^4+d^
2)^(1/2)/(e*x^2+d)^(1/2)+839/1024*d^6*arctan(e^(1/2)*x*(e*x^2+d)^(1/2)/(-e
^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.51

$$\int \left(d+ex^2\right)^{5/2}\left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{\frac{x\sqrt{d^2-e^2x^4}\left(555d^5+2418d^4ex^2+1320d^3e^2x^4-624d^2e^3x^6-896de^4x^8-256e^5x^{10}\right)}{\sqrt{d+ex^2}}+\frac{2517id^6\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{3072}$$

input

```
Integrate[(d + e*x^2)^(5/2)*(d^2 - e^2*x^4)^(3/2),x]
```

output

```
((x*Sqrt[d^2 - e^2*x^4]*(555*d^5 + 2418*d^4*e*x^2 + 1320*d^3*e^2*x^4 - 624
*d^2*e^3*x^6 - 896*d*e^4*x^8 - 256*e^5*x^10))/Sqrt[d + e*x^2] + ((2517*I)*
d^6*Log[(-2*I)*Sqrt[e]*x + (2*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2]])/Sqrt[
e])/3072
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.83, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1396, 318, 25, 27, 403, 27, 403, 25, 27, 299, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{5/2}\left(d^2-e^2x^4\right)^{3/2}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^4dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{\int -de\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^2\left(25ex^2+13d\right)dx}{12e}-\frac{1}{12}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^3\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int de\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^2\left(25ex^2+13d\right)dx}{12e}-\frac{1}{12}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^3\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{12}d\int\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^2\left(25ex^2+13d\right)dx-\frac{1}{12}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^3\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{1}{12}d\left(-\frac{\int -5de\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)\left(71ex^2+31d\right)dx}{10e}-\frac{5}{2}x\left(d+ex^2\right)^2\left(d-ex^2\right)^{5/2}\right)-\frac{1}{12}x\left(d-ex^2\right)^{5/2}\big(d+e}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{1}{12}d\left(\frac{1}{2}d\int\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)\left(71ex^2+31d\right)dx-\frac{5}{2}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^2\right)-\frac{1}{12}x\big(d-ex^2\big)^{5/}}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{1}{12}d\Bigg(\frac{1}{2}d\left(-\frac{\int -de\left(d-ex^2\right)^{3/2}\left(603ex^2+319d\right)dx}{8e}-\frac{71}{8}x\left(d+ex^2\right)\left(d-ex^2\right)^{5/2}\right)-\frac{5}{2}x\left(d-ex^2\right)^{5/2}\big(d+ex^2}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{1}{12}d\Bigg(\frac{1}{2}d\left(\frac{\int de\left(d-ex^2\right)^{3/2}\left(603ex^2+319d\right)dx}{8e}-\frac{71}{8}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)\right)-\frac{5}{2}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^2\Bigg)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{1}{12}d\Bigg(\frac{1}{2}d\left(\frac{1}{8}d\int\left(d-ex^2\right)^{3/2}\left(603ex^2+319d\right)dx-\frac{71}{8}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)\right)-\frac{5}{2}x\left(d-ex^2\right)^{5/2}\big(d}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{1}{12}d\Bigg(\frac{1}{2}d\Bigg(\frac{1}{8}d\left(\frac{839}{2}d\int\left(d-ex^2\right)^{3/2}dx-\frac{201}{2}x\left(d-ex^2\right)^{5/2}\right)-\frac{71}{8}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)\Bigg)-\frac{5}{2}x\big(d-e}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 211$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{12}d\left(\frac{1}{2}d\left(\frac{1}{8}d\left(\frac{839}{2}d\left(\frac{3}{4}d\int\sqrt{d-ex^2}dx+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{201}{2}x\left(d-ex^2\right)^{5/2}\right)-\frac{71}{8}x\left(d-ex^2\right)^{5/2}\left(d\right.\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 211$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{12}d\left(\frac{1}{2}d\left(\frac{1}{8}d\left(\frac{839}{2}d\left(\frac{3}{4}d\left(\frac{1}{2}d\int\frac{1}{\sqrt{d-ex^2}}dx+\frac{1}{2}x\sqrt{d-ex^2}\right)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{201}{2}x\left(d-ex^2\right)^{5/2}\right)-\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 224$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{12}d\left(\frac{1}{2}d\left(\frac{1}{8}d\left(\frac{839}{2}d\left(\frac{3}{4}d\left(\frac{1}{2}d\int\frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}+\frac{1}{2}x\sqrt{d-ex^2}\right)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{201}{2}x\left(d-ex\right.\right.\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+}}$$

$$\Big\downarrow \ 216$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{12}d\left(\frac{1}{2}d\left(\frac{1}{8}d\left(\frac{839}{2}d\left(\frac{3}{4}d\left(\frac{d\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d-ex^2}\right)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{201}{2}x\left(d-ex^2\right)^{5}\right.\right.\right.\right.}{\sqrt{d-ex^2}\sqrt{d+}}$$

input

```
Int[(d + e*x^2)^(5/2)*(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(-1/12*(x*(d - e*x^2)^(5/2)*(d + e*x^2)^3) + (d*((-5*
x*(d - e*x^2)^(5/2)*(d + e*x^2)^2)/2 + (d*((-71*x*(d - e*x^2)^(5/2)*(d + e
*x^2))/8 + (d*((-201*x*(d - e*x^2)^(5/2))/2 + (839*d*((x*(d - e*x^2)^(3/2)
)/4 + (3*d*((x*Sqrt[d - e*x^2])/2 + (d*ArcTan[(Sqrt[e]*x)/Sqrt[d - e*x^2]]
)/(2*Sqrt[e])))/4))/2))/8))/2))/12))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.65

method	result
default	$\frac{\sqrt{-e^2x^4+d^2}\left(-256e^{\frac{11}{2}}x^{11}\sqrt{-e\,x^2+d}-896d\,e^{\frac{9}{2}}x^9\sqrt{-e\,x^2+d}-624d^2e^{\frac{7}{2}}x^7\sqrt{-e\,x^2+d}+1320d^3e^{\frac{5}{2}}x^5\sqrt{-e\,x^2+d}+2418d^4e^{\frac{3}{2}}x^3\sqrt{-e\right.}{3072\sqrt{e\,x^2+d}\,\sqrt{-e\,x^2+d}\,\sqrt{e}}$
risch	$\frac{x\left(-256e^5x^{10}-896d\,e^4x^8-624d^2e^3x^6+1320d^3e^2x^4+2418d^4e\,x^2+555d^5\right)\sqrt{-e\,x^2+d}\,\sqrt{\frac{-e^2x^4+d^2}{e\,x^2+d}}\,\sqrt{e\,x^2+d}}{3072\sqrt{-e^2x^4+d^2}}+\frac{839d^6\arctan\left(\frac{\sqrt{}}{\sqrt{-e}}\right.}{1024}$

input
```
int((e*x^2+d)^(5/2)*(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3072*(-e^2*x^4+d^2)^(1/2)*(-256*e^(11/2)*x^11*(-e*x^2+d)^(1/2)-896*d*e^(
9/2)*x^9*(-e*x^2+d)^(1/2)-624*d^2*e^(7/2)*x^7*(-e*x^2+d)^(1/2)+1320*d^3*e^
(5/2)*x^5*(-e*x^2+d)^(1/2)+2418*d^4*e^(3/2)*x^3*(-e*x^2+d)^(1/2)+555*(-e*x
^2+d)^(1/2)*e^(1/2)*d^5*x+2517*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*d^6)/(e*
x^2+d)^(1/2)/(-e*x^2+d)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.24

$$\int \left(d+ex^2\right)^{5/2}\left(d^2-e^2x^4\right)^{3/2}\,dx = \left[-\frac{2517\left(d^6ex^2+d^7\right)\sqrt{-e}\log\left(-\frac{2\,e^2x^4+dex^2-2\,\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{-e}x-d^2}{ex^2+d}\right)+2\left(256\,e^6x^{11}+89\right.}{6144\left(e^2x\right.}\right.$$

input
```
integrate((e*x^2+d)^(5/2)*(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6144*(2517*(d^6*e*x^2 + d^7)*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sq
rt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)) + 2*(256
*e^6*x^11 + 896*d*e^5*x^9 + 624*d^2*e^4*x^7 - 1320*d^3*e^3*x^5 - 2418*d^4*
e^2*x^3 - 555*d^5*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d))/(e^2*x^2 + d*
e), -1/3072*(2517*(d^6*e*x^2 + d^7)*sqrt(e)*arctan(sqrt(-e^2*x^4 + d^2)*sq
rt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)) + (256*e^6*x^11 + 896*d*e^5*x^9 +
 624*d^2*e^4*x^7 - 1320*d^3*e^3*x^5 - 2418*d^4*e^2*x^3 - 555*d^5*e*x)*sqrt
(-e^2*x^4 + d^2)*sqrt(e*x^2 + d))/(e^2*x^2 + d*e)]
```

Sympy [F]

$$\int \left(d+ex^2\right)^{5/2}\left(d^2-e^2x^4\right)^{3/2}\,dx = \int \left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}\left(d+ex^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)*(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)*(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \left(d+ex^2\right)^{5/2}\left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(-e^2x^4+d^2\right)^{\frac{3}{2}}\left(ex^2+d\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)*(e*x^2 + d)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.40

$$\int \left(d+ex^2\right)^{5/2}\left(d^2-e^2x^4\right)^{3/2}\,dx=-\frac{839\,d^6\log\left(\left|-\sqrt{-e}x+\sqrt{-ex^2+d}\right|\right)}{1024\sqrt{-e}}+\frac{1}{3072}\left(555\,d^5+2\left(1209\,d^4e+4\left(165\,d^3e^2-2\left(39\,d^2e^3+8\left(2\,e^5x^2+7\,de^4\right)x^2\right)x^2\right)x^2\right)x^2\right)\sqrt{-ex^2+d}x$$

input
```
integrate((e*x^2+d)^(5/2)*(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
-839/1024*d^6*log(abs(-sqrt(-e)*x + sqrt(-e*x^2 + d)))/sqrt(-e) + 1/3072*(
555*d^5 + 2*(1209*d^4*e + 4*(165*d^3*e^2 - 2*(39*d^2*e^3 + 8*(2*e^5*x^2 +
7*d*e^4)*x^2)*x^2)*x^2)*x^2)*sqrt(-e*x^2 + d)*x
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{5/2}\left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(d^2-e^2\,x^4\right)^{3/2}\left(e\,x^2+d\right)^{5/2}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^(5/2),x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.48

$$\int \left(d+ex^2\right)^{5/2} \left(d^2-e^2x^4\right)^{3/2} \, dx = \frac{2517\sqrt{e}\, asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right) d^6 + 555\sqrt{-e\,x^2+d}\, d^5 e x + 2418\sqrt{-e\,x^2+d}\, d^4 e^2 x^3 + 1320\sqrt{-e\,x^2+}}{3072e}$$

input
```
int((e*x^2+d)^(5/2)*(-e^2*x^4+d^2)^(3/2),x)
```

output
```
(2517*sqrt(e)*asin((sqrt(e)*x)/sqrt(d))*d**6 + 555*sqrt(d - e*x**2)*d**5*e
*x + 2418*sqrt(d - e*x**2)*d**4*e**2*x**3 + 1320*sqrt(d - e*x**2)*d**3*e**
3*x**5 - 624*sqrt(d - e*x**2)*d**2*e**4*x**7 - 896*sqrt(d - e*x**2)*d*e**5
*x**9 - 256*sqrt(d - e*x**2)*e**6*x**11)/(3072*e)
```

3.138 $\int (d+ex^2)^{3/2} (d^2-e^2x^4)^{3/2} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 236

$$\begin{aligned}\int (d+ex^2)^{3/2} (d^2-e^2x^4)^{3/2} \, dx &= \frac{91d^4x\sqrt{d^2-e^2x^4}}{256\sqrt{d+ex^2}} \\ &+ \frac{73d^3ex^3\sqrt{d^2-e^2x^4}}{128\sqrt{d+ex^2}} + \frac{9d^2e^2x^5\sqrt{d^2-e^2x^4}}{160\sqrt{d+ex^2}} - \frac{19de^3x^7\sqrt{d^2-e^2x^4}}{80\sqrt{d+ex^2}} \\ &- \frac{e^4x^9\sqrt{d^2-e^2x^4}}{10\sqrt{d+ex^2}} + \frac{165d^5\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{256\sqrt{e}}\end{aligned}$$

output
```
91/256*d^4*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+73/128*d^3*e*x^3*(-e^2*x
^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+9/160*d^2*e^2*x^5*(-e^2*x^4+d^2)^(1/2)/(e*x^
2+d)^(1/2)-19/80*d*e^3*x^7*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)-1/10*e^4*x
^9*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+165/256*d^5*arctan(e^(1/2)*x*(e*x^
2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.60 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.55

$$\int \left(d+ex^2\right)^{3/2} \left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{\frac{x\sqrt{d^2-e^2x^4}\left(455d^4+730d^3ex^2+72d^2e^2x^4-304de^3x^6-128e^4x^8\right)}{\sqrt{d+ex^2}}+\frac{825id^5\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{1280}$$

input
```
Integrate[(d + e*x^2)^(3/2)*(d^2 - e^2*x^4)^(3/2),x]
```

output
```
((x*Sqrt[d^2 - e^2*x^4]*(455*d^4 + 730*d^3*e*x^2 + 72*d^2*e^2*x^4 - 304*d*
e^3*x^6 - 128*e^4*x^8))/Sqrt[d + e*x^2] + ((825*I)*d^5*Log[(-2*I)*Sqrt[e]*
x + (2*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2]])/Sqrt[e])/1280
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.83, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1396, 318, 25, 27, 403, 25, 27, 299, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2} \left(d^2-e^2x^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^3dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{\int -de\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)\left(19ex^2+11d\right)dx}{10e}-\frac{1}{10}x\left(d+ex^2\right)^2\left(d-ex^2\right)^{5/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int de\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)\left(19ex^2+11d\right)dx}{10e}-\frac{1}{10}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^2\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{10}d\int\left(d-ex^2\right)^{3/2}\left(ex^2+d\right)\left(19ex^2+11d\right)dx-\frac{1}{10}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^2\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 403$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{10}d\left(-\frac{\int -de\left(d-ex^2\right)^{3/2}\left(183ex^2+107d\right)dx}{8e}-\frac{19}{8}x\left(d+ex^2\right)\left(d-ex^2\right)^{5/2}\right)-\frac{1}{10}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^2\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{10}d\left(\frac{\int de\left(d-ex^2\right)^{3/2}\left(183ex^2+107d\right)dx}{8e}-\frac{19}{8}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)\right)-\frac{1}{10}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^2\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{10}d\left(\frac{1}{8}d\int\left(d-ex^2\right)^{3/2}\left(183ex^2+107d\right)dx-\frac{19}{8}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)\right)-\frac{1}{10}x\left(d-ex^2\right)^{5/2}(d+e\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 299$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{10}d\left(\frac{1}{8}d\left(\frac{275}{2}d\int\left(d-ex^2\right)^{3/2}dx-\frac{61}{2}x\left(d-ex^2\right)^{5/2}\right)-\frac{19}{8}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)\right)-\frac{1}{10}x\left(d-ex^2\right)\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 211$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{10}d\left(\frac{1}{8}d\left(\frac{275}{2}d\left(\frac{3}{4}d\int\sqrt{d-ex^2}dx+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{61}{2}x\left(d-ex^2\right)^{5/2}\right)-\frac{19}{8}x\left(d-ex^2\right)^{5/2}(d+ex\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 211$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{10}d\left(\frac{1}{8}d\left(\frac{275}{2}d\left(\frac{3}{4}d\left(\frac{1}{2}d\int\frac{1}{\sqrt{d-ex^2}}dx+\frac{1}{2}x\sqrt{d-ex^2}\right)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{61}{2}x\left(d-ex^2\right)^{5/2}\right)-\frac{19}{8}x(d\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{10}d\left(\frac{1}{8}d\left(\frac{275}{2}d\left(\frac{3}{4}d\left(\frac{1}{2}d\int\frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}+\frac{1}{2}x\sqrt{d-ex^2}\right)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{61}{2}x\left(d-ex^2\right)^{5/2}\right)\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{10}d\left(\frac{1}{8}d\left(\frac{275}{2}d\left(\frac{3}{4}d\left(\frac{d\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d-ex^2}\right)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{61}{2}x\left(d-ex^2\right)^{5/2}\right)-\right.\right.}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d + e*x^2)^(3/2)*(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/10*(x*(d - e*x^2)^(5/2)*(d + e*x^2)^2) + (d*((-19
*x*(d - e*x^2)^(5/2)*(d + e*x^2))/8 + (d*((-61*x*(d - e*x^2)^(5/2))/2 + (2
75*d*((x*(d - e*x^2)^(3/2))/4 + (3*d*((x*Sqrt[d - e*x^2])/2 + (d*ArcTan[(S
qrt[e]*x)/Sqrt[d - e*x^2]])/(2*Sqrt[e])))/4))/2))/8))/10))/(Sqrt[d - e*x^2
]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.67

method	result
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(-128 e^{\frac{9}{2}} x^{9} \sqrt{-e\,x^{2}+d}-304 d\,e^{\frac{7}{2}} x^{7} \sqrt{-e\,x^{2}+d}+72 d^{2} e^{\frac{5}{2}} x^{5} \sqrt{-e\,x^{2}+d}+730 d^{3} e^{\frac{3}{2}} x^{3} \sqrt{-e\,x^{2}+d}+455 \sqrt{e}\, \sqrt{-e\,x^{2}+d}\, d^{4} x\right.}{1280 \sqrt{e\,x^{2}+d}\, \sqrt{-e\,x^{2}+d}\, \sqrt{e}}$
risch	$\frac{x \left(-128 e^{4} x^{8}-304 d\,e^{3} x^{6}+72 d^{2} e^{2} x^{4}+730 d^{3} e\,x^{2}+455 d^{4}\right) \sqrt{-e\,x^{2}+d}\, \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e\,x^{2}+d}}\, \sqrt{e\,x^{2}+d}}{1280 \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{165 d^{5} \arctan\left(\frac{\sqrt{e}\, x}{\sqrt{-e\,x^{2}+d}}\right) \sqrt{\frac{-e^{2} x^{4}}{e\,x^{2}+}}}{256 \sqrt{e}\, \sqrt{-e^{2} x^{4}+d^{2}}}$

input
```
int((e*x^2+d)^(3/2)*(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1280*(-e^2*x^4+d^2)^(1/2)*(-128*e^(9/2)*x^9*(-e*x^2+d)^(1/2)-304*d*e^(7/
2)*x^7*(-e*x^2+d)^(1/2)+72*d^2*e^(5/2)*x^5*(-e*x^2+d)^(1/2)+730*d^3*e^(3/2
)*x^3*(-e*x^2+d)^(1/2)+455*e^(1/2)*(-e*x^2+d)^(1/2)*d^4*x+825*arctan(e^(1/
2)*x/(-e*x^2+d)^(1/2))*d^5)/(e*x^2+d)^(1/2)/(-e*x^2+d)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.35

$$\int \left(d+ex^{2}\right)^{3/2} \left(d^{2}-e^{2}x^{4}\right)^{3/2}\,dx = \left[-\frac{825\left(d^{5}ex^{2}+d^{6}\right)\sqrt{-e}\log\left(-\frac{2\,e^{2}x^{4}+dex^{2}-2\,\sqrt{-e^{2}x^{4}+d^{2}}\sqrt{ex^{2}+d}\sqrt{-e}x-d^{2}}{ex^{2}+d}\right)+2\left(128\,e^{5}x^{9}+304\right.}{2560\left(e^{2}x^{2}+de\right)}\right.$$

input
```
integrate((e*x^2+d)^(3/2)*(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2560*(825*(d^5*e*x^2 + d^6)*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqr
t(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)) + 2*(128*
e^5*x^9 + 304*d*e^4*x^7 - 72*d^2*e^3*x^5 - 730*d^3*e^2*x^3 - 455*d^4*e*x)*
sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d))/(e^2*x^2 + d*e), -1/1280*(825*(d^5*e
*x^2 + d^6)*sqrt(e)*arctan(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/
(e^2*x^4 - d^2)) + (128*e^5*x^9 + 304*d*e^4*x^7 - 72*d^2*e^3*x^5 - 730*d^3
*e^2*x^3 - 455*d^4*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d))/(e^2*x^2 + d
*e)]
```

Sympy [F]

$$\int \left(d+ex^2\right)^{3/2} \left(d^2-e^2x^4\right)^{3/2} dx = \int \left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}} \left(d+ex^2\right)^{\frac{3}{2}} dx$$

input
```
integrate((e*x**2+d)**(3/2)*(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)*(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \left(d+ex^2\right)^{3/2} \left(d^2-e^2x^4\right)^{3/2} dx = \int \left(-e^2x^4+d^2\right)^{\frac{3}{2}} \left(ex^2+d\right)^{\frac{3}{2}} dx$$

input
```
integrate((e*x^2+d)^(3/2)*(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)*(e*x^2 + d)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.41

$$\int \left(d+ex^2\right)^{3/2}\left(d^2-e^2x^4\right)^{3/2}\,dx = -\frac{165\,d^5\log\left(\left|-\sqrt{-e}x+\sqrt{-ex^2+d}\right|\right)}{256\,\sqrt{-e}} + \frac{1}{1280}\left(455\,d^4+2\left(365\,d^3e+4\left(9\,d^2e^2-2\left(8\,e^4x^2+19\,de^3\right)x^2\right)x^2\right)x^2\right)\sqrt{-ex^2+d}x$$

input

```
integrate((e*x^2+d)^(3/2)*(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output

```
-165/256*d^5*log(abs(-sqrt(-e)*x + sqrt(-e*x^2 + d)))/sqrt(-e) + 1/1280*(4
55*d^4 + 2*(365*d^3*e + 4*(9*d^2*e^2 - 2*(8*e^4*x^2 + 19*d*e^3)*x^2)*x^2)*
x^2)*sqrt(-e*x^2 + d)*x
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2}\left(d^2-e^2x^4\right)^{3/2}\,dx = \int \left(d^2-e^2\,x^4\right)^{3/2}\left(e\,x^2+d\right)^{3/2}\,dx$$

input

```
int((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^(3/2),x)
```

output

```
int((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.48

$$\int \left(d+ex^2\right)^{3/2}\left(d^2 -e^2x^4\right)^{3/2}\,dx = \frac{825\sqrt{e}\,asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)d^5+455\sqrt{-e\,x^2+d}\,d^4ex+730\sqrt{-e\,x^2+d}\,d^3e^2x^3+72\sqrt{-e\,x^2+d}\,d^2e}{1280e}$$

input

```
int((e*x^2+d)^(3/2)*(-e^2*x^4+d^2)^(3/2),x)
```

output

```
(825*sqrt(e)*asin((sqrt(e)*x)/sqrt(d))*d**5 + 455*sqrt(d - e*x**2)*d**4*e*
x + 730*sqrt(d - e*x**2)*d**3*e**2*x**3 + 72*sqrt(d - e*x**2)*d**2*e**3*x*
*5 - 304*sqrt(d - e*x**2)*d*e**4*x**7 - 128*sqrt(d - e*x**2)*e**5*x**9)/(1
280*e)
```

3.139 $\int \sqrt{d+ex^2}\left(d^2-e^2x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 196

$$\begin{aligned}\int \sqrt{d+ex^2}\left(d^2-e^2x^4\right)^{3/2}\,dx &= \frac{61d^3x\sqrt{d^2-e^2x^4}}{128\sqrt{d+ex^2}}+\frac{61d^2ex^3\sqrt{d^2-e^2x^4}}{192\sqrt{d+ex^2}}\\ &\quad-\frac{7de^2x^5\sqrt{d^2-e^2x^4}}{48\sqrt{d+ex^2}}-\frac{e^3x^7\sqrt{d^2-e^2x^4}}{8\sqrt{d+ex^2}}+\frac{67d^4\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{128\sqrt{e}}\end{aligned}$$

output

```
61/128*d^3*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+61/192*d^2*e*x^3*(-e^2*x
^4+d^2)^(1/2)/(e*x^2+d)^(1/2)-7/48*d*e^2*x^5*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d
)^(1/2)-1/8*e^3*x^7*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+67/128*d^4*arctan
(e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.61

$$\int \sqrt{d+ex^2}\left(d^2-e^2x^4\right)^{3/2}\,dx=\frac{1}{384}\left(\frac{x\sqrt{d^2-e^2x^4}\left(183d^3+122d^2ex^2-56de^2x^4-48e^3x^6\right)}{\sqrt{d+ex^2}}+\frac{201id^4\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}\right)$$

input
```
Integrate[Sqrt[d + e*x^2]*(d^2 - e^2*x^4)^(3/2),x]
```

output
```
((x*Sqrt[d^2 - e^2*x^4]*(183*d^3 + 122*d^2*e*x^2 - 56*d*e^2*x^4 - 48*e^3*x
^6))/Sqrt[d + e*x^2] + ((201*I)*d^4*Log[(-2*I)*Sqrt[e]*x + (2*Sqrt[d^2 - e
^2*x^4])/Sqrt[d + e*x^2]])/Sqrt[e])/384
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1396, 318, 25, 27, 299, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}\left(d^2-e^2x^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \left(d-ex^2\right)^{3/2}\left(ex^2+d\right)^2dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(-\frac{\int -de\left(d-ex^2\right)^{3/2}\left(13ex^2+9d\right)dx}{8e}-\frac{1}{8}x\left(d+ex^2\right)\left(d-ex^2\right)^{5/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int de\left(d-ex^2\right)^{3/2}\left(13ex^2+9d\right)dx}{8e}-\frac{1}{8}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{8}d\int \left(d-ex^2\right)^{3/2}\left(13ex^2+9d\right)dx-\frac{1}{8}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 299$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{8}d\left(\frac{67}{6}d\int \left(d-ex^2\right)^{3/2}dx-\frac{13}{6}x\left(d-ex^2\right)^{5/2}\right)-\frac{1}{8}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 211$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{8}d\left(\frac{67}{6}d\left(\frac{3}{4}d\int \sqrt{d-ex^2}dx+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{13}{6}x\left(d-ex^2\right)^{5/2}\right)-\frac{1}{8}x\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 211$$

$$\frac{\sqrt{d^2-e^2x^4}\bigg(\frac{1}{8}d\bigg(\frac{67}{6}d\bigg(\frac{3}{4}d\bigg(\frac{1}{2}d\int \frac{1}{\sqrt{d-ex^2}}dx+\frac{1}{2}x\sqrt{d-ex^2}\bigg)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\bigg)-\frac{13}{6}x\left(d-ex^2\right)^{5/2}\bigg)-\frac{1}{8}x\big(d-ex^2}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 224$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{1}{8}d\Bigg(\frac{67}{6}d\Bigg(\frac{3}{4}d\Bigg(\frac{1}{2}d\int \frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}+\frac{1}{2}x\sqrt{d-ex^2}\Bigg)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\Bigg)-\frac{13}{6}x\left(d-ex^2\right)^{5/2}\Bigg)-\frac{1}{8}x}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 216$$

$$\frac{\sqrt{d^2-e^2x^4}\Bigg(\frac{1}{8}d\Bigg(\frac{67}{6}d\Bigg(\frac{3}{4}d\Bigg(\frac{d\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d-ex^2}\Bigg)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\Bigg)-\frac{13}{6}x\left(d-ex^2\right)^{5/2}\Bigg)-\frac{1}{8}x\big(d}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[Sqrt[d + e*x^2]*(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/8*(x*(d - e*x^2)^(5/2)*(d + e*x^2)) + (d*((-13*x*
(d - e*x^2)^(5/2))/6 + (67*d*((x*(d - e*x^2)^(3/2))/4 + (3*d*((x*Sqrt[d -
e*x^2])/2 + (d*ArcTan[(Sqrt[e]*x)/Sqrt[d - e*x^2]])/(2*Sqrt[e])))/4))/6))/
8))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.70

method	result
default	$\frac{\sqrt{-e^2x^4+d^2}\left(-48e^{\frac{7}{2}}x^7\sqrt{-e\,x^2+d}-56d\,e^{\frac{5}{2}}x^5\sqrt{-e\,x^2+d}+122d^2e^{\frac{3}{2}}x^3\sqrt{-e\,x^2+d}+183\sqrt{-e\,x^2+d}\,\sqrt{e}\,d^3x+201\arctan\left(\frac{\sqrt{e}\,x}{\sqrt{-e\,x^2+d}}\right)\right.}{384\sqrt{e\,x^2+d}\,\sqrt{-e\,x^2+d}\,\sqrt{e}}$
risch	$\frac{x\left(-48e^3x^6-56d\,e^2x^4+122d^2e\,x^2+183d^3\right)\sqrt{-e\,x^2+d}\,\sqrt{\frac{-e^2x^4+d^2}{e\,x^2+d}}\,\sqrt{e\,x^2+d}}{384\sqrt{-e^2x^4+d^2}}+\frac{67d^4\arctan\left(\frac{\sqrt{e}\,x}{\sqrt{-e\,x^2+d}}\right)\sqrt{\frac{-e^2x^4+d^2}{e\,x^2+d}}\,\sqrt{e\,x^2+d}}{128\sqrt{e}\,\sqrt{-e^2x^4+d^2}}$

input
```
int((e*x^2+d)^(1/2)*(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*(-e^2*x^4+d^2)^(1/2)*(-48*e^(7/2)*x^7*(-e*x^2+d)^(1/2)-56*d*e^(5/2)*
x^5*(-e*x^2+d)^(1/2)+122*d^2*e^(3/2)*x^3*(-e*x^2+d)^(1/2)+183*(-e*x^2+d)^(
1/2)*e^(1/2)*d^3*x+201*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*d^4)/(e*x^2+d)^(
1/2)/(-e*x^2+d)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.52

$$\int \sqrt{d+ex^2}\,(d^2 -e^2x^4)^{3/2}\,dx = \left[-\frac{201\,(d^4ex^2+d^5)\sqrt{-e}\log\left(-\frac{2\,e^2x^4+dex^2-2\,\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{-e}x-d^2}{ex^2+d}\right)+2\,(48\,e^4x^7+56\,de}{768\,(e^2x^2+de)}\right.$$
$$-\frac{201\,(d^4ex^2+d^5)\sqrt{e}\arctan\left(\frac{\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{e}x}{e^2x^4-d^2}\right)+(48\,e^4x^7+56\,de^3x^5-122\,d^2e^2x^3-183\,d^3ex)\sqrt{-e^2x}}{384\,(e^2x^2+de)}$$

input
```
integrate((e*x^2+d)^(1/2)*(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/768*(201*(d^4*e*x^2 + d^5)*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt
(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)) + 2*(48*e^
4*x^7 + 56*d*e^3*x^5 - 122*d^2*e^2*x^3 - 183*d^3*e*x)*sqrt(-e^2*x^4 + d^2)
*sqrt(e*x^2 + d))/(e^2*x^2 + d*e), -1/384*(201*(d^4*e*x^2 + d^5)*sqrt(e)*a
rctan(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)) + (4
8*e^4*x^7 + 56*d*e^3*x^5 - 122*d^2*e^2*x^3 - 183*d^3*e*x)*sqrt(-e^2*x^4 +
d^2)*sqrt(e*x^2 + d))/(e^2*x^2 + d*e)]
```

Sympy [F]

$$\int \sqrt{d+ex^2}\,(d^2-e^2x^4)^{3/2}\,dx = \int \left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}\sqrt{d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \sqrt{d+ex^2}\left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(-e^2x^4+d^2\right)^{\frac{3}{2}}\sqrt{ex^2+d}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)*sqrt(e*x^2 + d), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.42

$$\int \sqrt{d+ex^2}\left(d^2-e^2x^4\right)^{3/2}\,dx=-\frac{67\,d^4\log\left(\left|-\sqrt{-e}x+\sqrt{-ex^2+d}\right|\right)}{128\,\sqrt{-e}}+\frac{1}{384}\left(183\,d^3+2\left(61\,d^2e-4\left(6\,e^3x^2+7\,de^2\right)x^2\right)x^2\right)\sqrt{-ex^2+d}x$$

input
```
integrate((e*x^2+d)^(1/2)*(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
-67/128*d^4*log(abs(-sqrt(-e)*x + sqrt(-e*x^2 + d)))/sqrt(-e) + 1/384*(183
*d^3 + 2*(61*d^2*e - 4*(6*e^3*x^2 + 7*d*e^2)*x^2)*x^2)*sqrt(-e*x^2 + d)*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex^2}\left(d^2-e^2x^4\right)^{3/2}\,dx=\int \left(d^2-e^2\,x^4\right)^{3/2}\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^(1/2),x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)*(d + e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.47

$$\int \sqrt{d+ex^2}\left(d^2 -e^2x^4\right)^{3/2}\,dx = \frac{201\sqrt{e}\,asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)d^4 + 183\sqrt{-e\,x^2+d}\,d^3ex + 122\sqrt{-e\,x^2+d}\,d^2e^2x^3 - 56\sqrt{-e\,x^2+d}\,d\,e}{384e}$$

input

```
int((e*x^2+d)^(1/2)*(-e^2*x^4+d^2)^(3/2),x)
```

output

```
(201*sqrt(e)*asin((sqrt(e)*x)/sqrt(d))*d**4 + 183*sqrt(d - e*x**2)*d**3*e*
x + 122*sqrt(d - e*x**2)*d**2*e**2*x**3 - 56*sqrt(d - e*x**2)*d*e**3*x**5
- 48*sqrt(d - e*x**2)*e**4*x**7)/(384*e)
```

3.140 $\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\sqrt{d+ex^2}}\,dx=\frac{9d^2x\sqrt{d^2-e^2x^4}}{16\sqrt{d+ex^2}}+\frac{dex^3\sqrt{d^2-e^2x^4}}{24\sqrt{d+ex^2}}-\frac{e^2x^5\sqrt{d^2-e^2x^4}}{6\sqrt{d+ex^2}}+\frac{7d^3\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{16\sqrt{e}}$$

output

```
9/16*d^2*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+1/24*d*e*x^3*(-e^2*x^4+d^2
)^(1/2)/(e*x^2+d)^(1/2)-1/6*e^2*x^5*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+7
/16*d^3*arctan(e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.69

$$\int \frac{(d^2-e^2x^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \frac{1}{48}\left(\frac{x(27d^2+2dex^2-8e^2x^4)\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}} + \frac{21id^3\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}\right)$$

input

```
Integrate[(d^2 - e^2*x^4)^(3/2)/Sqrt[d + e*x^2],x]
```

output

```
((x*(27*d^2 + 2*d*e*x^2 - 8*e^2*x^4)*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2]
+ ((21*I)*d^3*Log[(-2*I)*Sqrt[e]*x + (2*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^
2]])/Sqrt[e])/48
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1396, 299, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int (d-ex^2)^{3/2}(ex^2+d)\,dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{7}{6}d\int (d-ex^2)^{3/2}dx-\frac{1}{6}x(d-ex^2)^{5/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{211}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{7}{6}d\left(\frac{3}{4}d\int\sqrt{d-ex^2}dx+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{1}{6}x\left(d-ex^2\right)^{5/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{211}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{7}{6}d\left(\frac{3}{4}d\left(\frac{1}{2}d\int\frac{1}{\sqrt{d-ex^2}}dx+\frac{1}{2}x\sqrt{d-ex^2}\right)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{1}{6}x\left(d-ex^2\right)^{5/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{7}{6}d\left(\frac{3}{4}d\left(\frac{1}{2}d\int\frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}+\frac{1}{2}x\sqrt{d-ex^2}\right)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{1}{6}x\left(d-ex^2\right)^{5/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{7}{6}d\left(\frac{3}{4}d\left(\frac{d\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d-ex^2}\right)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)-\frac{1}{6}x\left(d-ex^2\right)^{5/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/6*(x*(d - e*x^2)^(5/2)) + (7*d*((x*(d - e*x^2)^(3
/2))/4 + (3*d*((x*Sqrt[d - e*x^2])/2 + (d*ArcTan[(Sqrt[e]*x)/Sqrt[d - e*x^
2]])/(2*Sqrt[e])))/4))/6))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(-8 e^{\frac{5}{2}} x^{5} \sqrt{-e\,x^{2}+d}+2 d\,e^{\frac{3}{2}} x^{3} \sqrt{-e\,x^{2}+d}+27 \sqrt{-e\,x^{2}+d}\,\sqrt{e}\,d^{2} x+21 \arctan\left(\frac{\sqrt{e}\,x}{\sqrt{-e\,x^{2}+d}}\right) d^{3}\right)}{48 \sqrt{e\,x^{2}+d}\,\sqrt{-e\,x^{2}+d}\,\sqrt{e}}$	117
risch	$\frac{x \left(-8 e^{2} x^{4}+2 d e\,x^{2}+27 d^{2}\right) \sqrt{-e\,x^{2}+d}\,\sqrt{\frac{-e^{2} x^{4}+d^{2}}{e\,x^{2}+d}}\,\sqrt{e\,x^{2}+d}}{48 \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{7 d^{3} \arctan\left(\frac{\sqrt{e}\,x}{\sqrt{-e\,x^{2}+d}}\right) \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e\,x^{2}+d}}\,\sqrt{e\,x^{2}+d}}{16 \sqrt{e}\,\sqrt{-e^{2} x^{4}+d^{2}}}$	154

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(-e^2*x^4+d^2)^(1/2)*(-8*e^(5/2)*x^5*(-e*x^2+d)^(1/2)+2*d*e^(3/2)*x^3
*(-e*x^2+d)^(1/2)+27*(-e*x^2+d)^(1/2)*e^(1/2)*d^2*x+21*arctan(e^(1/2)*x/(-
e*x^2+d)^(1/2))*d^3)/(e*x^2+d)^(1/2)/(-e*x^2+d)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.76

$$\int \frac{(d^2-e^2x^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \left[-\frac{21\,(d^3ex^2+d^4)\sqrt{-e}\log\left(-\frac{2\,e^2x^4+dex^2-2\,\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{-e}x-d^2}{ex^2+d}\right)+2\,(8\,e^3x^5-2}{96\,(e^2x^2+de)}\right.$$

$$\left.-\frac{21\,(d^3ex^2+d^4)\sqrt{e}\arctan\left(\frac{\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{e}x}{e^2x^4-d^2}\right)+(8\,e^3x^5-2\,de^2x^3-27\,d^2ex)\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}}{48\,(e^2x^2+de)}\right]$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(21*(d^3*e*x^2 + d^4)*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt(-
e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)) + 2*(8*e^3*x
^5 - 2*d*e^2*x^3 - 27*d^2*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d))/(e^2*
x^2 + d*e), -1/48*(21*(d^3*e*x^2 + d^4)*sqrt(e)*arctan(sqrt(-e^2*x^4 + d^2
)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)) + (8*e^3*x^5 - 2*d*e^2*x^3 -
27*d^2*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d))/(e^2*x^2 + d*e)]
```

Sympy [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \int \frac{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**(1/2),x)
```

output
```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/sqrt(e*x^2 + d), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.44

$$\int \frac{(d^2-e^2x^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = -\frac{7\,d^3\log\left(\left|-\sqrt{-e}x+\sqrt{-ex^2+d}\right|\right)}{16\,\sqrt{-e}} - \frac{1}{48}\left(2\left(4\,e^2x^2-de\right)x^2-27\,d^2\right)\sqrt{-ex^2+d}x$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
-7/16*d^3*log(abs(-sqrt(-e)*x + sqrt(-e*x^2 + d)))/sqrt(-e) - 1/48*(2*(4*e
^2*x^2 - d*e)*x^2 - 27*d^2)*sqrt(-e*x^2 + d)*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \int \frac{(d^2-e^2\,x^4)^{3/2}}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(1/2),x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \frac{5\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,dx}{24} - \frac{\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,e\,x^3}{6} - \frac{17\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)d^2e}{24} + \frac{19\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^3}{24}$$

input

```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(1/2),x)
```

output

```
(5*sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*d*x - 4*sqrt(d + e*x**2)*sqrt(d
**2 - e**2*x**4)*e*x**3 - 17*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*
x**2)/(d**2 - e**2*x**4),x)*d**2*e + 19*int((sqrt(d + e*x**2)*sqrt(d**2 -
e**2*x**4))/(d**2 - e**2*x**4),x)*d**3)/24
```

3.141 $\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 113

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx=\frac{3dx\sqrt{d^2-e^2x^4}}{8\sqrt{d+ex^2}}+\frac{x(d^2-e^2x^4)^{3/2}}{4(d+ex^2)^{3/2}}+\frac{3d^2\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{8\sqrt{e}}$$

output
```
3/8*d*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+1/4*x*(-e^2*x^4+d^2)^(3/2)/(e
*x^2+d)^(3/2)+3/8*d^2*arctan(e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2
))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx=\frac{1}{8}\left(\frac{x(5d-2ex^2)\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}+\frac{3id^2\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}\right)$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(3/2),x]
```

output
```
((x*(5*d - 2*e*x^2)*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2] + ((3*I)*d^2*Log[
(-2*I)*Sqrt[e]*x + (2*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2]])/Sqrt[e])/8
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1396, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \left(d-ex^2\right)^{3/2}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{211}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{3}{4}d\int \sqrt{d-ex^2}dx+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{211}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{1}{\sqrt{d-ex^2}}dx+\frac{1}{2}x\sqrt{d-ex^2}\right)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}+\frac{1}{2}x\sqrt{d-ex^2}\right)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{3}{4}d\left(\frac{d\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d-ex^2}\right)+\frac{1}{4}x\left(d-ex^2\right)^{3/2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(3/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((x*(d - e*x^2)^(3/2))/4 + (3*d*((x*Sqrt[d - e*x^2])/
2 + (d*ArcTan[(Sqrt[e]*x)/Sqrt[d - e*x^2]])/(2*Sqrt[e])))/4))/(Sqrt[d - e*
x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\sqrt{-e^2x^4+d^2}\left(-2e^{\frac{3}{2}}x^3\sqrt{-e\,x^2+d}+5\sqrt{e}\,\sqrt{-e\,x^2+d}\,dx+3\arctan\left(\frac{\sqrt{e}\,x}{\sqrt{-e\,x^2+d}}\right)d^2\right)}{8\sqrt{e\,x^2+d}\,\sqrt{-e\,x^2+d}\,\sqrt{e}}$	96
risch	$\frac{x\left(-2e\,x^2+5d\right)\sqrt{-e\,x^2+d}\,\sqrt{\frac{-e^2x^4+d^2}{e\,x^2+d}}\,\sqrt{e\,x^2+d}}{8\sqrt{-e^2x^4+d^2}}+\frac{3d^2\arctan\left(\frac{\sqrt{e}\,x}{\sqrt{-e\,x^2+d}}\right)\sqrt{\frac{-e^2x^4+d^2}{e\,x^2+d}}\,\sqrt{e\,x^2+d}}{8\sqrt{e}\,\sqrt{-e^2x^4+d^2}}$	143

input

```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/8*(-e^2*x^4+d^2)^(1/2)*(-2*e^(3/2)*x^3*(-e*x^2+d)^(1/2)+5*e^(1/2)*(-e*x^
2+d)^(1/2)*d*x+3*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*d^2)/(e*x^2+d)^(1/2)/(
-e*x^2+d)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.24

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{3/2}}\,dx=\Bigg[-\frac{3\left(d^2ex^2+d^3\right)\sqrt{-e}\log\left(-\frac{2\,e^2x^4+dex^2-2\,\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{-e}x-d^2}{ex^2+d}\right)+2\,\sqrt{-e^2x^4+}}{16\left(e^2x^2+de\right)}$$

input

```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output

```
[-1/16*(3*(d^2*e*x^2 + d^3)*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt(-e
^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)) + 2*sqrt(-e^2
*x^4 + d^2)*(2*e^2*x^3 - 5*d*e*x)*sqrt(e*x^2 + d))/(e^2*x^2 + d*e), -1/8*(
3*(d^2*e*x^2 + d^3)*sqrt(e)*arctan(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sq
rt(e)*x/(e^2*x^4 - d^2)) + sqrt(-e^2*x^4 + d^2)*(2*e^2*x^3 - 5*d*e*x)*sqrt
(e*x^2 + d))/(e^2*x^2 + d*e)]
```

Sympy [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}}{\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**(3/2),x)
```

output
```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{\left(d^2-e^2\,x^4\right)^{3/2}}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

input

```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(3/2),x)
```

output

```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{3/2}}\,dx=-\frac{\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,x}{4}-\frac{7\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,x^2}{-e^2x^4+d^2}dx\right)de}{4}+\frac{5\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}}{-e^2x^4+d^2}dx\right)d^2}{4}$$

input

```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(3/2),x)
```

output

```
( - sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x - 7*int((sqrt(d + e*x**2)*sq
rt(d**2 - e**2*x**4)*x**2)/(d**2 - e**2*x**4),x)*d*e + 5*int((sqrt(d + e*x
**2)*sqrt(d**2 - e**2*x**4))/(d**2 - e**2*x**4),x)*d**2)/4
```

3.142 $\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 131

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = -\frac{x\sqrt{d^2-e^2x^4}}{2\sqrt{d+ex^2}} - \frac{5d\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{2\sqrt{e}} + \frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{\sqrt{e}}$$

output
```
-1/2*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)-5/2*d*arctan(e^(1/2)*x*(e*x^2+
d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)+2*2^(1/2)*d*arctan(2^(1/2)*e^(1/2)*
x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.77 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.24

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = -\frac{x\sqrt{d^2-e^2x^4}}{2\sqrt{d+ex^2}} + \frac{2\sqrt{2}d\sqrt{d^2-e^2x^4}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}} - \frac{5id\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(5/2),x]
```

output
```
-1/2*(x*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2] + (2*Sqrt[2]*d*Sqrt[d^2 - e^2
*x^4]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]])/(Sqrt[e]*Sqrt[d - e*x^2
]*Sqrt[d + e*x^2]) - (((5*I)/2)*d*Log[(-2*I)*Sqrt[e]*x + (2*Sqrt[d^2 - e^2
*x^4])/Sqrt[d + e*x^2]])/Sqrt[e]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = $\{1396, 318, 27, 398, 224, 216, 291, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(d-ex^2\right)^{3/2}}{ex^2+d}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 318}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{de\left(3d-5ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx}{2e}-\frac{1}{2}x\sqrt{d-ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{2}d\int \frac{3d-5ex^2}{\sqrt{d-ex^2}(ex^2+d)}dx-\frac{1}{2}x\sqrt{d-ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{2}d\left(8d\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)}dx-5\int \frac{1}{\sqrt{d-ex^2}}dx\right)-\frac{1}{2}x\sqrt{d-ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{2}d\left(8d\int\frac{1}{\sqrt{d-ex^2}(ex^2+d)}dx-5\int\frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}\right)-\frac{1}{2}x\sqrt{d-ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{2}d\left(8d\int\frac{1}{\sqrt{d-ex^2}(ex^2+d)}dx-\frac{5\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{e}}\right)-\frac{1}{2}x\sqrt{d-ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{2}d\left(8d\int\frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}-\frac{5\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{e}}\right)-\frac{1}{2}x\sqrt{d-ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{2}d\left(\frac{4\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{e}}-\frac{5\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{e}}\right)-\frac{1}{2}x\sqrt{d-ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(5/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-1/2*(x*Sqrt[d - e*x^2]) + (d*((-5*ArcTan[(Sqrt[e]*x
)/Sqrt[d - e*x^2]])/Sqrt[e] + (4*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d
 - e*x^2]])/Sqrt[e]))/2))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398

```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.56

method	result
default	$\frac{\sqrt{-e^{2}x^{4}+d^{2}}\left(2\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^{2}+d}+\sqrt{-de}\,x+d\right)}{ex+\sqrt{-de}}\right)\sqrt{e}\,d^{\frac{3}{2}}\sqrt{2}-2\sqrt{e}\,d^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^{2}+d}-\sqrt{-de}\,x+d\right)}{ex-\sqrt{-de}}\right)-\sqrt{-de}\,\sqrt{-e\,x\right.}{2\sqrt{e\,x^{2}+d}\,\sqrt{-e\,x^{2}+d}\,\sqrt{-de}\,\sqrt{e}}$
risch	$-\frac{x\sqrt{-e\,x^{2}+d}\,\sqrt{\frac{-e^{2}x^{4}+d^{2}}{e\,x^{2}+d}}\,\sqrt{e\,x^{2}+d}}{2\sqrt{-e^{2}x^{4}+d^{2}}}+\frac{\left(-\frac{5d\arctan\left(\frac{\sqrt{e}\,x}{\sqrt{-e\,x^{2}+d}}\right)}{2\sqrt{e}}-\frac{d^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{4d-2\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)+2\sqrt{2}\,\sqrt{d}\,\sqrt{-\left(x-\frac{\sqrt{-de}}{e}\right)^{2}e-2}}{x-\frac{\sqrt{-de}}{e}}\right.}{\sqrt{-de}}\right.}{}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)*(2*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^
2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*e^(1/2)*d^(3/2)*2^(1/2)-2
*e^(1/2)*d^(3/2)*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(
1/2)*x+d)/(e*x-(-d*e)^(1/2)))-(-d*e)^(1/2)*(-e*x^2+d)^(1/2)*e^(1/2)*x-5*(-
d*e)^(1/2)*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*d)/(-e*x^2+d)^(1/2)/(-d*e)^(
1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 401, normalized size of antiderivative = 3.06

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{2\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\,ex-4\sqrt{2}(de^2x^2+d^2e)\sqrt{-\frac{1}{e}}\log\left(-\frac{3e^2x^4+2dex^2+2\sqrt{2}\sqrt{-}}{e^2x^4+}\right.}{4}\right.$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*e*x - 4*sqrt(2)*(d*e^2*x^2 +
 d^2*e)*sqrt(-1/e)*log(-(3*e^2*x^4 + 2*d*e*x^2 + 2*sqrt(2)*sqrt(-e^2*x^4 +
 d^2)*sqrt(e*x^2 + d)*e*x*sqrt(-1/e) - d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)) +
 5*(d*e*x^2 + d^2)*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 + 2*sqrt(-e^2*x^4 +
d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)))/(e^2*x^2 + d*e), -1/2
*(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*e*x - 5*(d*e*x^2 + d^2)*sqrt(e)*arc
tan(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)) + 4*sq
rt(2)*(d*e^2*x^2 + d^2*e)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 +
 d)*sqrt(e)*x/(e^2*x^4 - d^2))/sqrt(e))/(e^2*x^2 + d*e)]
```

Sympy [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \int \frac{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}}{\left(d+ex^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**(5/2),x)
```

output
```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)/(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input

```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output

```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.26

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \frac{\sqrt{2}d^2\log\left(\frac{\left|2\left(\sqrt{-e}x-\sqrt{-ex^2+d}\right)^2-4\sqrt{2}|d|-6d\right|}{\left|2\left(\sqrt{-e}x-\sqrt{-ex^2+d}\right)^2+4\sqrt{2}|d|-6d\right|}\right)}{\sqrt{-e}|d|} - \frac{1}{2}\sqrt{-ex^2+d}x + \frac{d\log\left(\left(\sqrt{-e}x-\sqrt{-ex^2+d}\right)^2\right)}{\sqrt{-e}} + \frac{d\log\left(\left|-\sqrt{-e}x+\sqrt{-ex^2+d}\right|\right)}{2\sqrt{-e}}$$

input

```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output

```
sqrt(2)*d^2*log(abs(2*(sqrt(-e)*x - sqrt(-e*x^2 + d))^2 - 4*sqrt(2)*abs(d)
 - 6*d)/abs(2*(sqrt(-e)*x - sqrt(-e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))
/(sqrt(-e)*abs(d)) - 1/2*sqrt(-e*x^2 + d)*x + d*log((sqrt(-e)*x - sqrt(-e*
x^2 + d))^2)/sqrt(-e) + 1/2*d*log(abs(-sqrt(-e)*x + sqrt(-e*x^2 + d)))/sqr
t(-e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx=\int \frac{{\left(d^2-e^2\,x^4\right)}^{3/2}}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(5/2),x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx=\frac{2\sqrt{e\,x^{2}+d}\,\sqrt{-e^{2}x^{4}+d^{2}}\,x+5\left(\int \frac{\sqrt{e\,x^{2}+d}\,\sqrt{-e^{2}x^{4}+d^{2}}\,x^{4}}{-e^{3}x^{6}-d\,e^{2}x^{4}+d^{2}e\,x^{2}+d^{3}}d x \right) d\,e^{2}+5\left(\int \frac{\sqrt{e\,x^{2}+d}\,\sqrt{-e^{2}x}}{-e^{3}x^{6}-d\,e^{2}x^{4}+d}\right.}{e\,x}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(5/2),x)
```

output
```
(2*sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x + 5*int((sqrt(d + e*x**2)*sqr
t(d**2 - e**2*x**4)*x**4)/(d**3 + d**2*e*x**2 - d*e**2*x**4 - e**3*x**6),x
)*d*e**2 + 5*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**4)/(d**3 + d*
*2*e*x**2 - d*e**2*x**4 - e**3*x**6),x)*e**3*x**2 - int((sqrt(d + e*x**2)*
sqrt(d**2 - e**2*x**4))/(d**3 + d**2*e*x**2 - d*e**2*x**4 - e**3*x**6),x)*
d**3 - int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4))/(d**3 + d**2*e*x**2 -
 d*e**2*x**4 - e**3*x**6),x)*d**2*e*x**2)/(d + e*x**2)
```

3.143 $\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 123

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\frac{x\sqrt{d^2-e^2x^4}}{(d+ex^2)^{3/2}}+\frac{\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{\sqrt{e}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{\sqrt{2}\sqrt{e}}$$

output
```
x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(3/2)+arctan(e^(1/2)*x*(e*x^2+d)^(1/2)/(-
e^2*x^4+d^2)^(1/2))/e^(1/2)-1/2*2^(1/2)*arctan(2^(1/2)*e^(1/2)*x*(e*x^2+d)
^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.86 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.27

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\frac{x\sqrt{d^2-e^2x^4}}{(d+ex^2)^{3/2}}-\frac{\sqrt{d^2-e^2x^4}\arctan\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d-ex^2}}\right)}{\sqrt{2}\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}+\frac{i\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(7/2),x]
```

output
```
(x*Sqrt[d^2 - e^2*x^4])/(d + e*x^2)^(3/2) - (Sqrt[d^2 - e^2*x^4]*ArcTan[(S
qrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]])/(Sqrt[2]*Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[
d + e*x^2]) + (I*Log[(-2*I)*Sqrt[e]*x + (2*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e
*x^2]])/Sqrt[e]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1396, 315, 27, 385, 224, 216, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(d-ex^2\right)^{3/2}}{\left(ex^2+d\right)^2}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{2de^2x^2}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx}{2de}+\frac{x\sqrt{d-ex^2}}{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(e\int \frac{x^2}{\sqrt{d-ex^2}(ex^2+d)}dx+\frac{x\sqrt{d-ex^2}}{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{385}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(e\left(\frac{\int \frac{1}{\sqrt{d-ex^2}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx}{e}\right)+\frac{x\sqrt{d-ex^2}}{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 224$$

$$\frac{\sqrt{d^2-e^2x^4}\left(e\left(\frac{\int \frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}}{e}-\frac{d\int \frac{1}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx}{e}\right)+\frac{x\sqrt{d-ex^2}}{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 216$$

$$\frac{\sqrt{d^2-e^2x^4}\left(e\left(\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{e^{3/2}}-\frac{d\int \frac{1}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx}{e}\right)+\frac{x\sqrt{d-ex^2}}{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 291$$

$$\frac{\sqrt{d^2-e^2x^4}\left(e\left(\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{e^{3/2}}-\frac{d\int \frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}}{e}\right)+\frac{x\sqrt{d-ex^2}}{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 218$$

$$\frac{\sqrt{d^2-e^2x^4}\left(e\left(\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{e^{3/2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{2}e^{3/2}}\right)+\frac{x\sqrt{d-ex^2}}{d+ex^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(7/2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d - e*x^2])/(d + e*x^2) + e*(ArcTan[(Sqrt[e]
*x)/Sqrt[d - e*x^2]]/e^(3/2) - ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]]
/(Sqrt[2]*e^(3/2)))))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(99) = 198$.

Time = 0.46 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.03

method	result
default	$-\frac{\sqrt{-e^2x^4+d^2}\,\sqrt{e}\left(-\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^2+d}-\sqrt{-de}\,x+d\right)}{ex-\sqrt{-de}}\right)\sqrt{2}\,e^{\frac{3}{2}}x^2\sqrt{d}+\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^2+d}+\sqrt{-de}\,x+d\right)}{ex+\sqrt{-de}}\right)\sqrt{2}\,e^{\frac{3}{2}}x^2\sqrt{d}-\sqrt{e}\right.}{}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-e^2*x^4+d^2)^(1/2)*e^(1/2)*(-ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/
2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*2^(1/2)*e^(3/2)*x^2*d^(1/2)+ln(2*
e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*
2^(1/2)*e^(3/2)*x^2*d^(1/2)-e^(1/2)*d^(3/2)*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2
)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))+ln(2*e*(2^(1/2)*d
^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*e^(1/2)*d^(3
/2)*2^(1/2)-4*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*e*x^2*(-d*e)^(1/2)-4*(-d*
e)^(1/2)*(-e*x^2+d)^(1/2)*e^(1/2)*x-4*(-d*e)^(1/2)*arctan(e^(1/2)*x/(-e*x^
2+d)^(1/2))*d)/(e*x^2+d)^(1/2)/(-e*x^2+d)^(1/2)/(e*x-(-d*e)^(1/2))/(e*x+(-
d*e)^(1/2))/(-d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.59

$$\int\frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}\,dx=\left[\frac{4\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}ex-\sqrt{2}(e^2x^4+2\,dex^2+d^2)\sqrt{-e}\log\left(-\frac{3\,e^2x^4+2\,dex^2+2\,\sqrt{2}}{e^2x}\right.}{4\,(e}\right.$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*e*x - sqrt(2)*(e^2*x^4 + 2*d*
e*x^2 + d^2)*sqrt(-e)*log(-(3*e^2*x^4 + 2*d*e*x^2 + 2*sqrt(2)*sqrt(-e^2*x^
4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)) -
2*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt(
-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)))/(e^3*x^4 +
 2*d*e^2*x^2 + d^2*e), 1/2*(2*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*e*x + s
qrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e)*arctan(sqrt(2)*sqrt(-e^2*x^4 +
d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)) - 2*(e^2*x^4 + 2*d*e*x^2 +
 d^2)*sqrt(e)*arctan(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x
^4 - d^2)))/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)]
```

Sympy [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\int \frac{(-(-d+ex^2)\,(d+ex^2))^{\frac{3}{2}}}{(d+ex^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**(7/2),x)
```

output
```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)/(d + e*x**2)**(7/2), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^(7/2), x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\int \frac{(d^2-e^2\,x^4)^{3/2}}{(e\,x^2+d)^{7/2}}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(7/2),x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\frac{2\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,x+3\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,x^4}{-e^4x^8-2d\,e^3x^6+2d^3e\,x^2+d^4}dx\right)d^2e^2+6\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-}}{-e^4x^8-2d\,e^3x}\right.}{}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(7/2),x)
```

output

```
(2*sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x + 3*int((sqrt(d + e*x**2)*sqr
t(d**2 - e**2*x**4)*x**4)/(d**4 + 2*d**3*e*x**2 - 2*d*e**3*x**6 - e**4*x**
8),x)*d**2*e**2 + 6*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**4)/(d*
*4 + 2*d**3*e*x**2 - 2*d*e**3*x**6 - e**4*x**8),x)*d*e**3*x**2 + 3*int((sq
rt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**4)/(d**4 + 2*d**3*e*x**2 - 2*d*e*
*3*x**6 - e**4*x**8),x)*e**4*x**4 + int((sqrt(d + e*x**2)*sqrt(d**2 - e**2
*x**4))/(d**4 + 2*d**3*e*x**2 - 2*d*e**3*x**6 - e**4*x**8),x)*d**4 + 2*int
((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4))/(d**4 + 2*d**3*e*x**2 - 2*d*e**
3*x**6 - e**4*x**8),x)*d**3*e*x**2 + int((sqrt(d + e*x**2)*sqrt(d**2 - e**
2*x**4))/(d**4 + 2*d**3*e*x**2 - 2*d*e**3*x**6 - e**4*x**8),x)*d**2*e**2*x
**4)/(3*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.144 $\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{9/2}} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{9/2}} \, dx = \frac{x\sqrt{d^2-e^2x^4}}{2(d+ex^2)^{5/2}} + \frac{x\sqrt{d^2-e^2x^4}}{8d(d+ex^2)^{3/2}} + \frac{3\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{8\sqrt{2}d\sqrt{e}}$$

output
```
1/2*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(5/2)+1/8*x*(-e^2*x^4+d^2)^(1/2)/d/(e
*x^2+d)^(3/2)+3/16*arctan(2^(1/2)*e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)
^(1/2))*2^(1/2)/d/e^(1/2)
```

Mathematica [A] (verified)

Time = 5.01 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{9/2}} \, dx = \frac{\sqrt{d^2-e^2x^4}\left(2\sqrt{e}x\sqrt{d-ex^2}(5d+ex^2)+3\sqrt{2}(d+ex^2)^2\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)\right)}{16d\sqrt{e}\sqrt{d-ex^2}\,(d+ex^2)^{5/2}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(9/2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(2*Sqrt[e]*x*Sqrt[d - e*x^2]*(5*d + e*x^2) + 3*Sqrt[2
]*(d + e*x^2)^2*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]]))/(16*d*Sqrt[e
]*Sqrt[d - e*x^2]*(d + e*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1396, 292, 292, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(d-ex^2\right)^{3/2}}{\left(ex^2+d\right)^3}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{292}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{3}{4}\int \frac{\sqrt{d-ex^2}}{\left(ex^2+d\right)^2}dx+\frac{x\left(d-ex^2\right)^{3/2}}{4d\left(d+ex^2\right)^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{292}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx+\frac{x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)}\right)+\frac{x\left(d-ex^2\right)^{3/2}}{4d\left(d+ex^2\right)^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}+\frac{x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)}\right)+\frac{x\left(d-ex^2\right)^{3/2}}{4d\left(d+ex^2\right)^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{3}{4}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{2}d\sqrt{e}}+\frac{x\sqrt{d-ex^2}}{2d(d+ex^2)}\right)+\frac{x(d-ex^2)^{3/2}}{4d(d+ex^2)^2}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(9/2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((x*(d - e*x^2)^(3/2))/(4*d*(d + e*x^2)^2) + (3*((x*S
qrt[d - e*x^2])/(2*d*(d + e*x^2)) + ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*
x^2]]/(2*Sqrt[2]*d*Sqrt[e])))/4))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(99) = 198$.

Time = 0.47 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.62

method	result
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, e^{2} \left(3 \ln \left(\frac{2 e \left(\sqrt{2}\, \sqrt{d}\, \sqrt{-e\, x^{2}+d}+\sqrt{-d e}\, x+d\right)}{e x+\sqrt{-d e}}\right) \sqrt{2}\, e^{2} x^{4} \sqrt{d}-3 \ln \left(\frac{2 e \left(\sqrt{2}\, \sqrt{d}\, \sqrt{-e\, x^{2}+d}-\sqrt{-d e}\, x+d\right)}{e x-\sqrt{-d e}}\right) \sqrt{2}\, e^{2} x^{4} \sqrt{d}+6 \ln \left(\frac{2 e}{}\right.\right.}{}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*(-e^2*x^4+d^2)^(1/2)*e^2/d*(3*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2
)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*2^(1/2)*e^2*x^4*d^(1/2)-3*ln(2*e*(
2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*2^(
1/2)*e^2*x^4*d^(1/2)+6*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/
2)*x+d)/(e*x+(-d*e)^(1/2)))*2^(1/2)*d^(3/2)*e*x^2-6*ln(2*e*(2^(1/2)*d^(1/2
)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*2^(1/2)*d^(3/2)*e
*x^2+4*(-d*e)^(1/2)*(-e*x^2+d)^(1/2)*e*x^3+3*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x
^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*2^(1/2)*d^(5/2)-3*ln(2*e
*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*2
^(1/2)*d^(5/2)+20*d*(-d*e)^(1/2)*(-e*x^2+d)^(1/2)*x)/(e*x^2+d)^(1/2)/(-e*x
^2+d)^(1/2)/(e*x-(-d*e)^(1/2))^2/(e*x+(-d*e)^(1/2))^2/(-d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.88

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{9/2}}\,dx = \left[-\frac{3\sqrt{2}(e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3)\sqrt{-e}\log\left(-\frac{3\,e^2x^4+2\,dex^2-2\,\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}}{e^2x^4+2\,dex^2+d^2}\right.}{32\left(de^4x^6+3\,d^2e^3x^4+3\,d^3e^2x^2+\right.}\right.$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(9/2),x, algorithm="fricas")
```

output
```
[-1/32*(3*sqrt(2)*(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)*sqrt(-e)*log
(-(3*e^2*x^4 + 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*
sqrt(-e)*x - d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 4*sqrt(-e^2*x^4 + d^2)*(e
^2*x^3 + 5*d*e*x)*sqrt(e*x^2 + d))/(d*e^4*x^6 + 3*d^2*e^3*x^4 + 3*d^3*e^2*
x^2 + d^4*e), -1/16*(3*sqrt(2)*(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)
*sqrt(e)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^
2*x^4 - d^2)) - 2*sqrt(-e^2*x^4 + d^2)*(e^2*x^3 + 5*d*e*x)*sqrt(e*x^2 + d)
)/(d*e^4*x^6 + 3*d^2*e^3*x^4 + 3*d^3*e^2*x^2 + d^4*e)]
```

Sympy [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\int \frac{(-(-d+ex^2)\,(d+ex^2))^{\frac{3}{2}}}{(d+ex^2)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**(9/2),x)
```

output
```
Integral((-(-d + e*x**2)*(d + e*x**2))**(3/2)/(d + e*x**2)**(9/2), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(9/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^(9/2), x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx = \int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(9/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx = \int \frac{(d^2-e^2\,x^4)^{3/2}}{(e\,x^2+d)^{9/2}}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(9/2),x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx = \frac{5\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,dx+\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,e\,x^3+12\left(\int \frac{\sqrt{e\,x^2+}}{-e^5x^{10}-3d\,e^4x^8-2d}\right.}{}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(9/2),x)
```

output

```
(5*sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*d*x + sqrt(d + e*x**2)*sqrt(d**
2 - e**2*x**4)*e*x**3 + 12*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x*
*2)/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2*d**2*e**3*x**6 - 3*d*e**4
*x**8 - e**5*x**10),x)*d**5*e + 36*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*
x**4)*x**2)/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2*d**2*e**3*x**6 -
3*d*e**4*x**8 - e**5*x**10),x)*d**4*e**2*x**2 + 36*int((sqrt(d + e*x**2)*s
qrt(d**2 - e**2*x**4)*x**2)/(d**5 + 3*d**4*e*x**2 + 2*d**3*e**2*x**4 - 2*d
**2*e**3*x**6 - 3*d*e**4*x**8 - e**5*x**10),x)*d**3*e**3*x**4 + 12*int((sq
rt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**2)/(d**5 + 3*d**4*e*x**2 + 2*d**3
*e**2*x**4 - 2*d**2*e**3*x**6 - 3*d*e**4*x**8 - e**5*x**10),x)*d**2*e**4*x
**6)/(5*d*(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6))
```

3.145 $\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 160

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{11/2}}\,dx=\frac{x\sqrt{d^2-e^2x^4}}{3\left(d+ex^2\right)^{7/2}}+\frac{x\sqrt{d^2-e^2x^4}}{8d\left(d+ex^2\right)^{5/2}}+\frac{19x\sqrt{d^2-e^2x^4}}{96d^2\left(d+ex^2\right)^{3/2}}+\frac{11\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{32\sqrt{2}d^2\sqrt{e}}$$

output
```
1/3*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(7/2)+1/8*x*(-e^2*x^4+d^2)^(1/2)/d/(e
*x^2+d)^(5/2)+19/96*x*(-e^2*x^4+d^2)^(1/2)/d^2/(e*x^2+d)^(3/2)+11/64*arcta
n(2^(1/2)*e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))*2^(1/2)/d^2/e^(1
/2)
```

Mathematica [A] (verified)

Time = 4.92 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.84

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{11/2}}\,dx=\frac{\sqrt{d^2-e^2x^4}\left(2\sqrt{e}x\sqrt{d-ex^2}(63d^2+50dex^2+19e^2x^4)+33\sqrt{2}(d+ex^2)^3\arctan\left(\right.\right.}{192d^2\sqrt{e}\sqrt{d-ex^2}\left(d+ex^2\right)^{7/2}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(11/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(2*Sqrt[e]*x*Sqrt[d - e*x^2]*(63*d^2 + 50*d*e*x^2 + 1
9*e^2*x^4) + 33*Sqrt[2]*(d + e*x^2)^3*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d -
e*x^2]]))/(192*d^2*Sqrt[e]*Sqrt[d - e*x^2]*(d + e*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1396, 296, 292, 292, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{11/2}}dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(d-ex^2\right)^{3/2}}{\left(ex^2+d\right)^4}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 296}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{11\int \frac{\left(d-ex^2\right)^{3/2}}{\left(ex^2+d\right)^3}dx}{12d}+\frac{x\left(d-ex^2\right)^{5/2}}{12d^2\left(d+ex^2\right)^3}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 292}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{11\left(\frac{3}{4}\int \frac{\sqrt{d-ex^2}}{\left(ex^2+d\right)^2}dx+\frac{x\left(d-ex^2\right)^{3/2}}{4d\left(d+ex^2\right)^2}\right)}{12d}+\frac{x(d-ex^2)^{5/2}}{12d^2(d+ex^2)^3}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{292}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{11\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx+\frac{x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)}\right)+\frac{x\left(d-ex^2\right)^{3/2}}{4d\left(d+ex^2\right)^2}\right)}{12d}+\frac{x(d-ex^2)^{5/2}}{12d^2(d+ex^2)^3}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{11\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}+\frac{x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)}\right)+\frac{x\left(d-ex^2\right)^{3/2}}{4d\left(d+ex^2\right)^2}\right)}{12d}+\frac{x(d-ex^2)^{5/2}}{12d^2(d+ex^2)^3}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{11\left(\frac{3}{4}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{2}d\sqrt{e}}+\frac{x\sqrt{d-ex^2}}{2d\left(d+ex^2\right)}\right)+\frac{x\left(d-ex^2\right)^{3/2}}{4d\left(d+ex^2\right)^2}\right)}{12d}+\frac{x(d-ex^2)^{5/2}}{12d^2(d+ex^2)^3}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(11/2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((x*(d - e*x^2)^(5/2))/(12*d^2*(d + e*x^2)^3) + (11*(
(x*(d - e*x^2)^(3/2))/(4*d*(d + e*x^2)^2) + (3*((x*Sqrt[d - e*x^2])/(2*d*(
d + e*x^2)) + ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]]/(2*Sqrt[2]*d*Sqr
t[e])))/4))/(12*d)))/(Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(128) = 256$.

Time = 0.50 (sec) , antiderivative size = 596, normalized size of antiderivative = 3.72

method	result
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, e^{3} \left(-33 \sqrt{2}\, \ln \left(\frac{2 e \left(\sqrt{2}\, \sqrt{d}\, \sqrt{-e\, x^{2}+d}-\sqrt{-d e}\, x+d\right)}{e x -\sqrt{-d e}}\right) e^{3} x^{6} \sqrt{d}+33 \sqrt{2}\, \ln \left(\frac{2 e \left(\sqrt{2}\, \sqrt{d}\, \sqrt{-e\, x^{2}+d}+\sqrt{-d e}\, x+d\right)}{e x +\sqrt{-d e}}\right) e^{3} x^{6} \sqrt{d}-99 \sqrt{}\right.}{}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*(-e^2*x^4+d^2)^(1/2)*e^3*(-33*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^
2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*e^3*x^6*d^(1/2)+33*2^(1/2
)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(
1/2)))*e^3*x^6*d^(1/2)-99*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)
-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*d^(3/2)*e^2*x^4+99*2^(1/2)*ln(2*e*(
2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*d^(
3/2)*e^2*x^4+76*e^2*(-e*x^2+d)^(1/2)*(-d*e)^(1/2)*x^5-99*2^(1/2)*ln(2*e*(2
^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*d^(5
/2)*e*x^2+99*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)
*x+d)/(e*x+(-d*e)^(1/2)))*d^(5/2)*e*x^2+200*e*(-e*x^2+d)^(1/2)*d*(-d*e)^(1
/2)*x^3-33*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x
+d)/(e*x-(-d*e)^(1/2)))*d^(7/2)+33*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2
+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*d^(7/2)+252*(-e*x^2+d)^(1/
2)*d^2*(-d*e)^(1/2)*x)/d^2/(e*x^2+d)^(1/2)/(-e*x^2+d)^(1/2)/(e*x-(-d*e)^(1
/2))^3/(e*x+(-d*e)^(1/2))^3/(-d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.70

$$\int \frac{\left(d^{2}-e^{2} x^{4}\right)^{3/2}}{\left(d+e x^{2}\right)^{11/2}}\, dx = \left[-\frac{33 \sqrt{2}\left(e^{4} x^{8}+4\, d e^{3} x^{6}+6\, d^{2} e^{2} x^{4}+4\, d^{3} e x^{2}+d^{4}\right)\sqrt{-e}\log\left(-\frac{3\, e^{2} x^{4}+2\, d e x^{2}-2\, \sqrt{2}\sqrt{-e}}{e^{2} x^{4}+2}\right.}{384\left(d^{2} e^{5} x^{8}+4\, d^{3} e^{4} x^{6}+6\right.}\right.$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(11/2),x, algorithm="fricas")
```

output
```
[-1/384*(33*sqrt(2)*(e^4*x^8 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4 + 4*d^3*e*x^2 +
 d^4)*sqrt(-e)*log(-(3*e^2*x^4 + 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x^4 + d^2
)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 4*(19*e
^3*x^5 + 50*d*e^2*x^3 + 63*d^2*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d))/
(d^2*e^5*x^8 + 4*d^3*e^4*x^6 + 6*d^4*e^3*x^4 + 4*d^5*e^2*x^2 + d^6*e), -1/
192*(33*sqrt(2)*(e^4*x^8 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4 + 4*d^3*e*x^2 + d^4
)*sqrt(e)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/(e
^2*x^4 - d^2)) - 2*(19*e^3*x^5 + 50*d*e^2*x^3 + 63*d^2*e*x)*sqrt(-e^2*x^4
+ d^2)*sqrt(e*x^2 + d))/(d^2*e^5*x^8 + 4*d^3*e^4*x^6 + 6*d^4*e^3*x^4 + 4*d
^5*e^2*x^2 + d^6*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx = \int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(11/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^(11/2), x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx=\int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(11/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx=\int \frac{(d^2-e^2\,x^4)^{3/2}}{(e\,x^2+d)^{11/2}}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(11/2),x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(11/2), x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx=\frac{13\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,d^2x+7\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,de\,x^3+2\sqrt{e\,x^2+d}\,\sqrt{-e^2}{}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(11/2),x)
```

output

```
(13*sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*d**2*x + 7*sqrt(d + e*x**2)*sq
rt(d**2 - e**2*x**4)*d*e*x**3 + 2*sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*
e**2*x**5 + 44*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**2)/(d**6 +
4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 - 4*d*e**5*x**10 - e**
6*x**12),x)*d**7*e + 176*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**2
)/(d**6 + 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 - 4*d*e**5*x
**10 - e**6*x**12),x)*d**6*e**2*x**2 + 264*int((sqrt(d + e*x**2)*sqrt(d**2
 - e**2*x**4)*x**2)/(d**6 + 4*d**5*e*x**2 + 5*d**4*e**2*x**4 - 5*d**2*e**4
*x**8 - 4*d*e**5*x**10 - e**6*x**12),x)*d**5*e**3*x**4 + 176*int((sqrt(d +
 e*x**2)*sqrt(d**2 - e**2*x**4)*x**2)/(d**6 + 4*d**5*e*x**2 + 5*d**4*e**2*
x**4 - 5*d**2*e**4*x**8 - 4*d*e**5*x**10 - e**6*x**12),x)*d**4*e**4*x**6 +
 44*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**2)/(d**6 + 4*d**5*e*x*
*2 + 5*d**4*e**2*x**4 - 5*d**2*e**4*x**8 - 4*d*e**5*x**10 - e**6*x**12),x)
*d**3*e**5*x**8)/(13*d**2*(d**4 + 4*d**3*e*x**2 + 6*d**2*e**2*x**4 + 4*d*e
**3*x**6 + e**4*x**8))
```

3.146 $\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{13/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 195

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{13/2}}\,dx=\frac{x\sqrt{d^2-e^2x^4}}{4\left(d+ex^2\right)^{9/2}}+\frac{5x\sqrt{d^2-e^2x^4}}{48d\left(d+ex^2\right)^{7/2}}+\frac{17x\sqrt{d^2-e^2x^4}}{128d^2\left(d+ex^2\right)^{5/2}}+\frac{299x\sqrt{d^2-e^2x^4}}{1536d^3\left(d+ex^2\right)^{3/2}}+\frac{163\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{512\sqrt{2}d^3\sqrt{e}}$$

output

```
1/4*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(9/2)+5/48*x*(-e^2*x^4+d^2)^(1/2)/d/(
e*x^2+d)^(7/2)+17/128*x*(-e^2*x^4+d^2)^(1/2)/d^2/(e*x^2+d)^(5/2)+299/1536*
x*(-e^2*x^4+d^2)^(1/2)/d^3/(e*x^2+d)^(3/2)+163/1024*arctan(2^(1/2)*e^(1/2)
*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))*2^(1/2)/d^3/e^(1/2)
```

Mathematica [A] (verified)

Time = 4.99 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.74

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{13/2}}\,dx=\frac{\sqrt{d^2-e^2x^4}\Big(2\sqrt{e}x\sqrt{d-ex^2}\left(1047d^3+1465d^2ex^2+1101de^2x^4+299e^3x^6\right)+489\sqrt{}}{3072d^3\sqrt{e}\sqrt{d-ex^2}\left(d+ex^2\right)^{9/2}}$$

input
```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(13/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(2*Sqrt[e]*x*Sqrt[d - e*x^2]*(1047*d^3 + 1465*d^2*e*x
^2 + 1101*d*e^2*x^4 + 299*e^3*x^6) + 489*Sqrt[2]*(d + e*x^2)^4*ArcTan[(Sqr
t[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]]))/(3072*d^3*Sqrt[e]*Sqrt[d - e*x^2]*(d +
e*x^2)^(9/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1396, 315, 27, 402, 25, 27, 402, 25, 27, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{13/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{\left(d-ex^2\right)^{3/2}}{\left(ex^2+d\right)^5}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{2de\left(3d-2ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^4}dx}{8de}+\frac{x\sqrt{d-ex^2}}{4\left(d+ex^2\right)^4}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{4}\int \frac{3d-2ex^2}{\sqrt{d-ex^2}(ex^2+d)^4}dx+\frac{x\sqrt{d-ex^2}}{4(d+ex^2)^4}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{4}\left(\frac{5x\sqrt{d-ex^2}}{12d(d+ex^2)^3}-\frac{\int -\frac{de\left(31d-20ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^3}dx}{12d^2e}\right)+\frac{x\sqrt{d-ex^2}}{4(d+ex^2)^4}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{4}\left(\frac{\int \frac{de\left(31d-20ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^3}dx}{12d^2e}+\frac{5x\sqrt{d-ex^2}}{12d(d+ex^2)^3}\right)+\frac{x\sqrt{d-ex^2}}{4(d+ex^2)^4}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{4}\left(\frac{\int \frac{31d-20ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^3}dx}{12d}+\frac{5x\sqrt{d-ex^2}}{12d(d+ex^2)^3}\right)+\frac{x\sqrt{d-ex^2}}{4(d+ex^2)^4}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{4}\left(\frac{\frac{51x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}-\frac{\int -\frac{de\left(197d-102ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^2}dx}{8d^2e}}{12d}+\frac{5x\sqrt{d-ex^2}}{12d(d+ex^2)^3}\right)+\frac{x\sqrt{d-ex^2}}{4(d+ex^2)^4}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{de\left(197d-102ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^2}dx}{8d^2e}+\frac{51x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}}{12d}+\frac{5x\sqrt{d-ex^2}}{12d(d+ex^2)^3}\right)+\frac{x\sqrt{d-ex^2}}{4(d+ex^2)^4}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{197d-102ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^2}dx}{8d}+\frac{51x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}}{12d}+\frac{5x\sqrt{d-ex^2}}{12d(d+ex^2)^3}\right)+\frac{x\sqrt{d-ex^2}}{4(d+ex^2)^4}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{4}\left(\frac{\frac{\frac{299x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)}-\frac{\int -\frac{489d^2e}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx}{4d^2e}}{8d}+\frac{51x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}}{12d}+\frac{5x\sqrt{d-ex^2}}{12d(d+ex^2)^3}\right)+\frac{x\sqrt{d-ex^2}}{4(d+ex^2)^4}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{4}\left(\frac{\frac{\frac{489}{4}\int \frac{1}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx+\frac{299x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)}}{8d}+\frac{51x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}}{12d}+\frac{5x\sqrt{d-ex^2}}{12d(d+ex^2)^3}\right)+\frac{x\sqrt{d-ex^2}}{4(d+ex^2)^4}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 291$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{4}\left(\frac{\frac{\frac{489}{4}\int \frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}+\frac{299x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)}}{8d}+\frac{51x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}}{12d}+\frac{5x\sqrt{d-ex^2}}{12d(d+ex^2)^3}\right)+\frac{x\sqrt{d-ex^2}}{4(d+ex^2)^4}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 218$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{4}\left(\frac{\frac{\frac{489\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{4\sqrt{2}d\sqrt{e}}+\frac{299x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)}}{8d}+\frac{51x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}}{12d}+\frac{5x\sqrt{d-ex^2}}{12d(d+ex^2)^3}\right)+\frac{x\sqrt{d-ex^2}}{4(d+ex^2)^4}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(13/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d - e*x^2])/(4*(d + e*x^2)^4) + ((5*x*Sqrt[d
 - e*x^2])/(12*d*(d + e*x^2)^3) + ((51*x*Sqrt[d - e*x^2])/(8*d*(d + e*x^2)
^2) + ((299*x*Sqrt[d - e*x^2])/(4*d*(d + e*x^2)) + (489*ArcTan[(Sqrt[2]*Sq
rt[e]*x)/Sqrt[d - e*x^2]])/(4*Sqrt[2]*d*Sqrt[e]))/(8*d))/(12*d))/4))/(Sqrt
[d - e*x^2]*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. 2(157) = 314.

Time = 0.52 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.79

method	result
default	$-\frac{\sqrt{-e^{2}x^{4}+d^{2}}\, e^{4} \left(489 \ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^{2}+d}-\sqrt{-de}\,x+d\right)}{ex-\sqrt{-de}}\right)\sqrt{2}\,e^{4}x^{8}\sqrt{d}-489\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^{2}+d}+\sqrt{-de}\,x+d\right)}{ex+\sqrt{-de}}\right)\sqrt{2}\,e^{4}x^{8}\sqrt{d}+1\right.}{}$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6144*(-e^2*x^4+d^2)^(1/2)/d^3*e^4*(489*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+
d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*2^(1/2)*e^4*x^8*d^(1/2)-489
*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1
/2)))*2^(1/2)*e^4*x^8*d^(1/2)+1956*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2
)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*2^(1/2)*d^(3/2)*e^3*x^6-1956*ln(2*
e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*
2^(1/2)*d^(3/2)*e^3*x^6-1196*(-d*e)^(1/2)*(-e*x^2+d)^(1/2)*e^3*x^7+2934*ln
(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)
))*2^(1/2)*d^(5/2)*e^2*x^4-2934*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(
-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*2^(1/2)*d^(5/2)*e^2*x^4-4404*d*e^2*(-
d*e)^(1/2)*(-e*x^2+d)^(1/2)*x^5+1956*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1
/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*2^(1/2)*d^(7/2)*e*x^2-1956*ln(2*
e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*
2^(1/2)*d^(7/2)*e*x^2-5860*d^2*e*(-d*e)^(1/2)*(-e*x^2+d)^(1/2)*x^3+489*ln(
2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2))
)*2^(1/2)*d^(9/2)-489*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2
)*x+d)/(e*x+(-d*e)^(1/2)))*2^(1/2)*d^(9/2)-4188*d^3*(-d*e)^(1/2)*(-e*x^2+d
)^(1/2)*x)/(e*x^2+d)^(1/2)/(-e*x^2+d)^(1/2)/(e*x-(-d*e)^(1/2))^4/(e*x+(-d*
e)^(1/2))^4/(-d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.55

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{13/2}}\,dx=\left[-\frac{489\,\sqrt{2}(e^5x^{10}+5\,de^4x^8+10\,d^2e^3x^6+10\,d^3e^2x^4+5\,d^4ex^2+d^5)\sqrt{-e}\log\left(-\right.}{6144\,(d^3e^6x^{10}+}\right.$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(13/2),x, algorithm="fricas")
```

output
```
[-1/6144*(489*sqrt(2)*(e^5*x^10 + 5*d*e^4*x^8 + 10*d^2*e^3*x^6 + 10*d^3*e^
2*x^4 + 5*d^4*e*x^2 + d^5)*sqrt(-e)*log(-(3*e^2*x^4 + 2*d*e*x^2 - 2*sqrt(2
)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e^2*x^4 + 2*d*e*
x^2 + d^2)) - 4*(299*e^4*x^7 + 1101*d*e^3*x^5 + 1465*d^2*e^2*x^3 + 1047*d^
3*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d))/(d^3*e^6*x^10 + 5*d^4*e^5*x^8
 + 10*d^5*e^4*x^6 + 10*d^6*e^3*x^4 + 5*d^7*e^2*x^2 + d^8*e), -1/3072*(489*
sqrt(2)*(e^5*x^10 + 5*d*e^4*x^8 + 10*d^2*e^3*x^6 + 10*d^3*e^2*x^4 + 5*d^4*
e*x^2 + d^5)*sqrt(e)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*s
qrt(e)*x/(e^2*x^4 - d^2)) - 2*(299*e^4*x^7 + 1101*d*e^3*x^5 + 1465*d^2*e^2
*x^3 + 1047*d^3*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d))/(d^3*e^6*x^10 +
 5*d^4*e^5*x^8 + 10*d^5*e^4*x^6 + 10*d^6*e^3*x^4 + 5*d^7*e^2*x^2 + d^8*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{13/2}}\,dx = \int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^{\frac{13}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(13/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^(13/2), x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{13/2}}\,dx=\int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^{\frac{13}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(13/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{13/2}}\,dx=\int \frac{(d^2-e^2\,x^4)^{3/2}}{(e\,x^2+d)^{13/2}}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(13/2),x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(13/2), x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{13/2}}\,dx=\frac{423\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,d^3x+335\sqrt{e\,x^2+d}\,\sqrt{-e^2x^4+d^2}\,d^2e\,x^3+180\sqrt{e\,x^2+}}{}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(13/2),x)
```

output

```
(423*sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*d**3*x + 335*sqrt(d + e*x**2)
*sqrt(d**2 - e**2*x**4)*d**2*e*x**3 + 180*sqrt(d + e*x**2)*sqrt(d**2 - e**
2*x**4)*d*e**2*x**5 + 40*sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*e**3*x**7
 + 1956*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**2)/(d**7 + 5*d**6*
e*x**2 + 9*d**5*e**2*x**4 + 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 - 9*d**2*e
**5*x**10 - 5*d*e**6*x**12 - e**7*x**14),x)*d**9*e + 9780*int((sqrt(d + e*
x**2)*sqrt(d**2 - e**2*x**4)*x**2)/(d**7 + 5*d**6*e*x**2 + 9*d**5*e**2*x**
4 + 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 - 9*d**2*e**5*x**10 - 5*d*e**6*x**
12 - e**7*x**14),x)*d**8*e**2*x**2 + 19560*int((sqrt(d + e*x**2)*sqrt(d**2
 - e**2*x**4)*x**2)/(d**7 + 5*d**6*e*x**2 + 9*d**5*e**2*x**4 + 5*d**4*e**3
*x**6 - 5*d**3*e**4*x**8 - 9*d**2*e**5*x**10 - 5*d*e**6*x**12 - e**7*x**14
),x)*d**7*e**3*x**4 + 19560*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x
**2)/(d**7 + 5*d**6*e*x**2 + 9*d**5*e**2*x**4 + 5*d**4*e**3*x**6 - 5*d**3*
e**4*x**8 - 9*d**2*e**5*x**10 - 5*d*e**6*x**12 - e**7*x**14),x)*d**6*e**4*
x**6 + 9780*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**2)/(d**7 + 5*d
**6*e*x**2 + 9*d**5*e**2*x**4 + 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 - 9*d*
*2*e**5*x**10 - 5*d*e**6*x**12 - e**7*x**14),x)*d**5*e**5*x**8 + 1956*int(
(sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**2)/(d**7 + 5*d**6*e*x**2 + 9*d
**5*e**2*x**4 + 5*d**4*e**3*x**6 - 5*d**3*e**4*x**8 - 9*d**2*e**5*x**10 -
5*d*e**6*x**12 - e**7*x**14),x)*d**4*e**6*x**10)/(423*d**3*(d**5 + 5*d*...
```

3.147 $\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{15/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 230

$$\int \frac{\left(d^2-e^2x^4\right)^{3/2}}{\left(d+ex^2\right)^{15/2}}\,dx=\frac{x\sqrt{d^2-e^2x^4}}{5\left(d+ex^2\right)^{11/2}}+\frac{7x\sqrt{d^2-e^2x^4}}{80d\left(d+ex^2\right)^{9/2}}+\frac{33x\sqrt{d^2-e^2x^4}}{320d^2\left(d+ex^2\right)^{7/2}}+\frac{327x\sqrt{d^2-e^2x^4}}{2560d^3\left(d+ex^2\right)^{5/2}}+\frac{1887x\sqrt{d^2-e^2x^4}}{10240d^4\left(d+ex^2\right)^{3/2}}+\frac{609\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{2048\sqrt{2}d^4\sqrt{e}}$$

output
```
1/5*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(11/2)+7/80*x*(-e^2*x^4+d^2)^(1/2)/d/
(e*x^2+d)^(9/2)+33/320*x*(-e^2*x^4+d^2)^(1/2)/d^2/(e*x^2+d)^(7/2)+327/2560
*x*(-e^2*x^4+d^2)^(1/2)/d^3/(e*x^2+d)^(5/2)+1887/10240*x*(-e^2*x^4+d^2)^(1
/2)/d^4/(e*x^2+d)^(3/2)+609/4096*arctan(2^(1/2)*e^(1/2)*x*(e*x^2+d)^(1/2)/
(-e^2*x^4+d^2)^(1/2))*2^(1/2)/d^4/e^(1/2)
```

Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.60

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{15/2}}\,dx = \frac{\sqrt{d^2-e^2x^4}\left(\frac{2(7195d^4x+14480d^3ex^3+16302d^2e^2x^5+8856de^3x^7+1887e^4x^9)}{(d+ex^2)^5}+\frac{3045\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{e}\sqrt{d-ex^2}}\right.}{20480d^4\sqrt{d+ex^2}}$$

input

```
Integrate[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(15/2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((2*(7195*d^4*x + 14480*d^3*e*x^3 + 16302*d^2*e^2*x^5
 + 8856*d*e^3*x^7 + 1887*e^4*x^9))/(d + e*x^2)^5 + (3045*Sqrt[2]*ArcTan[(S
qrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]])/(Sqrt[e]*Sqrt[d - e*x^2])))/(20480*d^4
*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1396, 315, 27, 402, 27, 402, 27, 402, 25, 27, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{15/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d^2-e^2x^4}\int \frac{(d-ex^2)^{3/2}}{(ex^2+d)^6}dx}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{\int \frac{2de(4d-3ex^2)}{\sqrt{d-ex^2}(ex^2+d)^5}dx}{10de}+\frac{x\sqrt{d-ex^2}}{5(d+ex^2)^5}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\int \frac{4d-3ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^5}dx+\frac{x\sqrt{d-ex^2}}{5\left(d+ex^2\right)^5}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{7x\sqrt{d-ex^2}}{16d\left(d+ex^2\right)^4}-\frac{\int -\frac{3de\left(19d-14ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^4}dx}{16d^2e}\right)+\frac{x\sqrt{d-ex^2}}{5\left(d+ex^2\right)^5}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{3\int \frac{19d-14ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^4}dx}{16d}+\frac{7x\sqrt{d-ex^2}}{16d\left(d+ex^2\right)^4}\right)+\frac{x\sqrt{d-ex^2}}{5\left(d+ex^2\right)^5}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{3\left(\frac{11x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)^3}-\frac{\int -\frac{3de\left(65d-44ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^3}dx}{12d^2e}\right)}{16d}+\frac{7x\sqrt{d-ex^2}}{16d\left(d+ex^2\right)^4}\right)+\frac{x\sqrt{d-ex^2}}{5\left(d+ex^2\right)^5}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{3\left(\frac{\int \frac{65d-44ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^3}dx}{4d}+\frac{11x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)^3}\right)}{16d}+\frac{7x\sqrt{d-ex^2}}{16d\left(d+ex^2\right)^4}\right)+\frac{x\sqrt{d-ex^2}}{5\left(d+ex^2\right)^5}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{109x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}-\frac{\int -\frac{de\left(411d-218ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^2}dx}{8d^2e}}{4d}+\frac{11x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)^3}\right)}{16d}+\frac{7x\sqrt{d-ex^2}}{16d(d+ex^2)^4}\right)+\frac{x\sqrt{d-ex^2}}{5(d+ex^2)^5}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\int \frac{de\left(411d-218ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^2}dx}{8d^2e}+\frac{109x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}}{4d}+\frac{11x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)^3}\right)}{16d}+\frac{7x\sqrt{d-ex^2}}{16d(d+ex^2)^4}\right)+\frac{x\sqrt{d-ex^2}}{5(d+ex^2)^5}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\int \frac{411d-218ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^2}dx}{8d}+\frac{109x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}}{4d}+\frac{11x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)^3}\right)}{16d}+\frac{7x\sqrt{d-ex^2}}{16d(d+ex^2)^4}\right)+\frac{x\sqrt{d-ex^2}}{5(d+ex^2)^5}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\frac{629x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)}-\frac{\int -\frac{1015d^2e}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx}{4d^2e}}{8d}+\frac{109x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}}{4d}+\frac{11x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)^3}\right)}{16d}+\frac{7x\sqrt{d-ex^2}}{16d(d+ex^2)^4}\right)+\frac{x\sqrt{d-ex^2}}{5(d+ex^2)^5}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\frac{1015}{4}\int \frac{1}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx+\frac{629x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)}}{8d}+\frac{109x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}}{4d}+\frac{11x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)^3}\right)}{16d}+\frac{7x\sqrt{d-ex^2}}{16d(d+ex^2)^4}\right)+\frac{x\sqrt{d-ex^2}}{5(d+ex^2)^5}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\frac{1015}{4}\int \frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}+\frac{629x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)}}{8d}+\frac{109x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}}{4d}+\frac{11x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)^3}\right)}{16d}+\frac{7x\sqrt{d-ex^2}}{16d(d+ex^2)^4}\right)+\frac{x\sqrt{d-ex^2}}{5(d+ex^2)^5}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d^2-e^2x^4}\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\frac{1015\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{4\sqrt{2}d\sqrt{e}}+\frac{629x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)}}{8d}+\frac{109x\sqrt{d-ex^2}}{8d\left(d+ex^2\right)^2}}{4d}+\frac{11x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)^3}\right)}{16d}+\frac{7x\sqrt{d-ex^2}}{16d(d+ex^2)^4}\right)+\frac{x\sqrt{d-ex^2}}{5(d+ex^2)^5}\right)}{\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input

```
Int[(d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(15/2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*((x*Sqrt[d - e*x^2])/(5*(d + e*x^2)^5) + ((7*x*Sqrt[d
 - e*x^2])/(16*d*(d + e*x^2)^4) + (3*((11*x*Sqrt[d - e*x^2])/(4*d*(d + e*x
^2)^3) + ((109*x*Sqrt[d - e*x^2])/(8*d*(d + e*x^2)^2) + ((629*x*Sqrt[d - e
*x^2])/(4*d*(d + e*x^2)) + (1015*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2
]])/(4*Sqrt[2]*d*Sqrt[e]))/(8*d))/(4*d)))/(16*d))/5))/(Sqrt[d - e*x^2]*Sqr
t[d + e*x^2])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 883 vs. 2(186) = 372.

Time = 0.55 (sec) , antiderivative size = 884, normalized size of antiderivative = 3.84

method	result
default	$\sqrt{-e^2x^4+d^2}\, e^5 \Bigg(3045\sqrt{2}\, \ln\Bigg(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^2+d}+\sqrt{-de}\,x+d\right)}{ex+\sqrt{-de}}\Bigg) e^5x^{10}\sqrt{d}-3045\sqrt{2}\, \ln\Bigg(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^2+d}-\sqrt{-de}\,x+d\right)}{ex-\sqrt{-de}}\Bigg) e^5x^{10}\sqrt{d}-$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(15/2),x,method=_RETURNVERBOSE)
```

output
```
1/40960*(-e^2*x^4+d^2)^(1/2)*e^5/d^4*(3045*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)
*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*e^5*x^10*d^(1/2)-3
045*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*
x-(-d*e)^(1/2)))*e^5*x^10*d^(1/2)+15225*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-
e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*d^(3/2)*e^4*x^8-15225
*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(
-d*e)^(1/2)))*d^(3/2)*e^4*x^8+7548*e^4*(-d*e)^(1/2)*(-e*x^2+d)^(1/2)*x^9+3
0450*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e
*x+(-d*e)^(1/2)))*d^(5/2)*e^3*x^6-30450*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-
e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*d^(5/2)*e^3*x^6+35424
*e^3*d*(-d*e)^(1/2)*(-e*x^2+d)^(1/2)*x^7+30450*2^(1/2)*ln(2*e*(2^(1/2)*d^(
1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*d^(7/2)*e^2*x^
4-30450*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)
/(e*x-(-d*e)^(1/2)))*d^(7/2)*e^2*x^4+65208*e^2*d^2*(-d*e)^(1/2)*(-e*x^2+d)
^(1/2)*x^5+15225*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(
1/2)*x+d)/(e*x+(-d*e)^(1/2)))*d^(9/2)*e*x^2-15225*2^(1/2)*ln(2*e*(2^(1/2)*
d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*d^(9/2)*e*x
^2+57920*e*d^3*(-d*e)^(1/2)*(-e*x^2+d)^(1/2)*x^3+3045*2^(1/2)*ln(2*e*(2^(1
/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*d^(11/2
)-3045*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.45

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{15/2}}\,dx = \left[-\frac{3045\sqrt{2}(e^6x^{12}+6\,de^5x^{10}+15\,d^2e^4x^8+20\,d^3e^3x^6+15\,d^4e^2x^4+6\,d^5ex^2+d^6)}{40960\,(}\right.$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(15/2),x, algorithm="fricas")
```

output

```
[-1/40960*(3045*sqrt(2)*(e^6*x^12 + 6*d*e^5*x^10 + 15*d^2*e^4*x^8 + 20*d^3
*e^3*x^6 + 15*d^4*e^2*x^4 + 6*d^5*e*x^2 + d^6)*sqrt(-e)*log(-(3*e^2*x^4 +
2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^
2)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 4*(1887*e^5*x^9 + 8856*d*e^4*x^7 + 16302
*d^2*e^3*x^5 + 14480*d^3*e^2*x^3 + 7195*d^4*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt
(e*x^2 + d))/(d^4*e^7*x^12 + 6*d^5*e^6*x^10 + 15*d^6*e^5*x^8 + 20*d^7*e^4*
x^6 + 15*d^8*e^3*x^4 + 6*d^9*e^2*x^2 + d^10*e), -1/20480*(3045*sqrt(2)*(e^
6*x^12 + 6*d*e^5*x^10 + 15*d^2*e^4*x^8 + 20*d^3*e^3*x^6 + 15*d^4*e^2*x^4 +
 6*d^5*e*x^2 + d^6)*sqrt(e)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2
 + d)*sqrt(e)*x/(e^2*x^4 - d^2)) - 2*(1887*e^5*x^9 + 8856*d*e^4*x^7 + 1630
2*d^2*e^3*x^5 + 14480*d^3*e^2*x^3 + 7195*d^4*e*x)*sqrt(-e^2*x^4 + d^2)*sqr
t(e*x^2 + d))/(d^4*e^7*x^12 + 6*d^5*e^6*x^10 + 15*d^6*e^5*x^8 + 20*d^7*e^4
*x^6 + 15*d^8*e^3*x^4 + 6*d^9*e^2*x^2 + d^10*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{15/2}}\,dx = \text{Timed out}$$

input

```
integrate((-e**2*x**4+d**2)**(3/2)/(e*x**2+d)**(15/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{15/2}}\,dx = \int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^{\frac{15}{2}}}\,dx$$

input

```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(15/2),x, algorithm="maxima")
```

output

```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^(15/2), x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{15/2}}\,dx=\int \frac{(-e^2x^4+d^2)^{\frac{3}{2}}}{(ex^2+d)^{\frac{15}{2}}}\,dx$$

input
```
integrate((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(15/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + d^2)^(3/2)/(e*x^2 + d)^(15/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{15/2}}\,dx=\int \frac{(d^2-e^2\,x^4)^{3/2}}{(e\,x^2+d)^{15/2}}\,dx$$

input
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(15/2),x)
```

output
```
int((d^2 - e^2*x^4)^(3/2)/(d + e*x^2)^(15/2), x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^4)^{3/2}}{(d+ex^2)^{15/2}}\,dx=\text{Too large to display}$$

input
```
int((-e^2*x^4+d^2)^(3/2)/(e*x^2+d)^(15/2),x)
```

output

```
(285*sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*d**4*x + 275*sqrt(d + e*x**2)
*sqrt(d**2 - e**2*x**4)*d**3*e*x**3 + 206*sqrt(d + e*x**2)*sqrt(d**2 - e**
2*x**4)*d**2*e**2*x**5 + 88*sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*d*e**3
*x**7 + 16*sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*e**4*x**9 + 1740*int((s
qrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**2)/(d**8 + 6*d**7*e*x**2 + 14*d*
*6*e**2*x**4 + 14*d**5*e**3*x**6 - 14*d**3*e**5*x**10 - 14*d**2*e**6*x**12
 - 6*d*e**7*x**14 - e**8*x**16),x)*d**11*e + 10440*int((sqrt(d + e*x**2)*s
qrt(d**2 - e**2*x**4)*x**2)/(d**8 + 6*d**7*e*x**2 + 14*d**6*e**2*x**4 + 14
*d**5*e**3*x**6 - 14*d**3*e**5*x**10 - 14*d**2*e**6*x**12 - 6*d*e**7*x**14
 - e**8*x**16),x)*d**10*e**2*x**2 + 26100*int((sqrt(d + e*x**2)*sqrt(d**2
- e**2*x**4)*x**2)/(d**8 + 6*d**7*e*x**2 + 14*d**6*e**2*x**4 + 14*d**5*e**
3*x**6 - 14*d**3*e**5*x**10 - 14*d**2*e**6*x**12 - 6*d*e**7*x**14 - e**8*x
**16),x)*d**9*e**3*x**4 + 34800*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**
4)*x**2)/(d**8 + 6*d**7*e*x**2 + 14*d**6*e**2*x**4 + 14*d**5*e**3*x**6 - 1
4*d**3*e**5*x**10 - 14*d**2*e**6*x**12 - 6*d*e**7*x**14 - e**8*x**16),x)*d
**8*e**4*x**6 + 26100*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**2)/(
d**8 + 6*d**7*e*x**2 + 14*d**6*e**2*x**4 + 14*d**5*e**3*x**6 - 14*d**3*e**
5*x**10 - 14*d**2*e**6*x**12 - 6*d*e**7*x**14 - e**8*x**16),x)*d**7*e**5*x
**8 + 10440*int((sqrt(d + e*x**2)*sqrt(d**2 - e**2*x**4)*x**2)/(d**8 + 6*d
**7*e*x**2 + 14*d**6*e**2*x**4 + 14*d**5*e**3*x**6 - 14*d**3*e**5*x**10...
```

3.148 $\int \sqrt{1+x^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 21

$$\int \sqrt{1+x^2}\,dx=\frac{1}{2}x\sqrt{1+x^2}+\frac{\operatorname{arcsinh}(x)}{2}$$

output

```
1/2*x*(x^2+1)^(1/2)+1/2*arcsinh(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \sqrt{1+x^2}\,dx=\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\log\left(-x+\sqrt{1+x^2}\right)$$

input

```
Integrate[Sqrt[1 + x^2],x]
```

output

```
(x*Sqrt[1 + x^2])/2 - Log[-x + Sqrt[1 + x^2]]/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2+1}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^2+1}}dx+\frac{1}{2}\sqrt{x^2+1}x$$

$$\Big\downarrow \text{222}$$

$$\frac{\text{arcsinh}(x)}{2}+\frac{1}{2}\sqrt{x^2+1}x$$

input

```
Int[Sqrt[1 + x^2],x]
```

output

```
(x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Defintions of rubi rules used

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{x\sqrt{x^2+1}}{2}+\frac{\operatorname{arcsinh}(x)}{2}$	16
risch	$\frac{x\sqrt{x^2+1}}{2}+\frac{\operatorname{arcsinh}(x)}{2}$	16
trager	$\frac{x\sqrt{x^2+1}}{2}+\frac{\ln\left(\sqrt{x^2+1}+x\right)}{2}$	24
meijerg	$-\frac{-2\sqrt{\pi}\,x\sqrt{x^2+1}-2\sqrt{\pi}\,\operatorname{arcsinh}(x)}{4\sqrt{\pi}}$	27
pseudoelliptic	$\frac{x\sqrt{x^2+1}}{2}-\frac{\ln\left(\frac{\sqrt{x^2+1}-x}{x}\right)}{4}+\frac{\ln\left(\frac{\sqrt{x^2+1}+x}{x}\right)}{4}$	46

input

```
int((x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/2*x*(x^2+1)^(1/2)+1/2*arcsinh(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

input

```
integrate((x^2+1)^(1/2),x, algorithm="fricas")
```

output

```
1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{1+x^2}\,dx=\frac{x\sqrt{x^2+1}}{2}+\frac{\operatorname{asinh}(x)}{2}$$

input
```
integrate((x**2+1)**(1/2),x)
```

output
```
x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{1+x^2}\,dx=\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\operatorname{arsinh}(x)$$

input
```
integrate((x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^2 + 1)*x + 1/2*arcsinh(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \sqrt{1+x^2}\,dx=\frac{1}{2}\sqrt{x^2+1}x-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate((x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{1+x^2}\,dx=\frac{\operatorname{asinh}(x)}{2}+\frac{x\,\sqrt{x^2+1}}{2}$$

input
```
int((x^2 + 1)^(1/2),x)
```

output
```
asinh(x)/2 + (x*(x^2 + 1)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sqrt{1+x^2}\,dx=\frac{\sqrt{x^2+1}\,x}{2}+\frac{\log\left(\sqrt{x^2+1}+x\right)}{2}$$

input
```
int((x^2+1)^(1/2),x)
```

output
```
(sqrt(x**2 + 1)*x + log(sqrt(x**2 + 1) + x))/2
```

3.149 $\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} \, dx = \frac{1}{2}x\sqrt{1+x^2} + \frac{\operatorname{arcsinh}(x)}{2}$$

output
```
1/2*x*(x^2+1)^(1/2)+1/2*arcsinh(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 70 vs. 2(21) = 42.

Time = 0.91 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.33

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} \, dx = \frac{1}{2}\left(\frac{x\sqrt{1-x^4}}{\sqrt{1-x^2}} + \log\left(1-x^2\right) - \log\left(-x+x^3+\sqrt{1-x^2}\sqrt{1-x^4}\right)\right)$$

input
```
Integrate[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
```

output
```
((x*Sqrt[1 - x^4])/Sqrt[1 - x^2] + Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 -
x^2]*Sqrt[1 - x^4]])/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1386, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 1386}$$

$$\int \sqrt{x^2+1}dx$$

$$\Big\downarrow \text{ 211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^2+1}}dx+\frac{1}{2}\sqrt{x^2+1}x$$

$$\Big\downarrow \text{ 222}$$

$$\frac{\text{arcsinh}(x)}{2}+\frac{1}{2}\sqrt{x^2+1}x$$

input
```
Int[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

method	result	size
default	$\frac{\sqrt{-x^4+1}\left(x\sqrt{x^2+1}+\operatorname{arcsinh}(x)\right)}{2\sqrt{-x^2+1}\,\sqrt{x^2+1}}$	40
risch	$-\frac{x\sqrt{x^2+1}\,\sqrt{\frac{\left(-x^4+1\right)\left(-x^2+1\right)}{\left(x^2-1\right)^2}}\,\left(x^2-1\right)}{2\sqrt{-x^4+1}\,\sqrt{-x^2+1}}-\frac{\operatorname{arcsinh}(x)\sqrt{\frac{\left(-x^4+1\right)\left(-x^2+1\right)}{\left(x^2-1\right)^2}}\,\left(x^2-1\right)}{2\sqrt{-x^4+1}\,\sqrt{-x^2+1}}$	110

input
```
int((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-x^4+1)^(1/2)*(x*(x^2+1)^(1/2)+arcsinh(x))/(-x^2+1)^(1/2)/(x^2+1)^(1/
2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.71

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = -\frac{2\sqrt{-x^4+1}\sqrt{-x^2+1}x+\left(x^2-1\right)\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)-\left(x^2-1\right)\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)}{4\left(x^2-1\right)}$$

input
```
integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(-x^4 + 1)*sqrt(-x^2 + 1)*x + (x^2 - 1)*log((x^3 + sqrt(-x^4 +
 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) - (x^2 - 1)*log(-(x^3 - sqrt(-x^4 + 1)*
sqrt(-x^2 + 1) - x)/(x^3 - x)))/(x^2 - 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate((-x**4+1)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(-(x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx=\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx$$

input
```
int((1 - x^4)^(1/2)/(1 - x^2)^(1/2),x)
```

output
```
int((1 - x^4)^(1/2)/(1 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx=\int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}}dx$$

input
```
int((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x)
```

output
```
int((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x)
```

3.150 $\int \frac{(d+ex^2)^{7/2}}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{(d+ex^2)^{7/2}}{\sqrt{d^2-e^2x^4}}\,dx = -\frac{47d^2x\sqrt{d^2-e^2x^4}}{16\sqrt{d+ex^2}}-\frac{23dex^3\sqrt{d^2-e^2x^4}}{24\sqrt{d+ex^2}} -\frac{e^2x^5\sqrt{d^2-e^2x^4}}{6\sqrt{d+ex^2}}+\frac{63d^3\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{16\sqrt{e}}$$

output
```
-47/16*d^2*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)-23/24*d*e*x^3*(-e^2*x^4+
d^2)^(1/2)/(e*x^2+d)^(1/2)-1/6*e^2*x^5*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2
)+63/16*d^3*arctan(e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex^2)^{7/2}}{\sqrt{d^2-e^2x^4}}\,dx = -\frac{\sqrt{d^2-e^2x^4}(141d^2x+46dex^3+8e^2x^5)}{48\sqrt{d+ex^2}} +\frac{63id^3\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{16\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)^(7/2)/Sqrt[d^2 - e^2*x^4],x]
```

output
```
-1/48*(Sqrt[d^2 - e^2*x^4]*(141*d^2*x + 46*d*e*x^3 + 8*e^2*x^5))/Sqrt[d +
e*x^2] + (((63*I)/16)*d^3*Log[(-2*I)*Sqrt[e]*x + (2*Sqrt[d^2 - e^2*x^4])/S
qrt[d + e*x^2]])/Sqrt[e]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1396, 318, 25, 27, 403, 25, 27, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\left(ex^2+d\right)^3}{\sqrt{d-ex^2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{\int -\frac{de\left(ex^2+d\right)\left(15ex^2+7d\right)}{\sqrt{d-ex^2}}dx}{6e}-\frac{1}{6}x\sqrt{d-ex^2}\left(d+ex^2\right)^2\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{de\left(ex^2+d\right)\left(15ex^2+7d\right)}{\sqrt{d-ex^2}}dx}{6e}-\frac{1}{6}x\sqrt{d-ex^2}\left(d+ex^2\right)^2\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{6}d\int \frac{\left(ex^2+d\right)\left(15ex^2+7d\right)}{\sqrt{d-ex^2}}dx-\frac{1}{6}x\sqrt{d-ex^2}\left(d+ex^2\right)^2\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{6}d\left(-\frac{\int -\frac{de\left(103ex^2+43d\right)}{\sqrt{d-ex^2}}dx}{4e}-\frac{15}{4}x\sqrt{d-ex^2}\left(d+ex^2\right)\right)-\frac{1}{6}x\sqrt{d-ex^2}\left(d+ex^2\right)^2\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{6}d\left(\frac{\int \frac{de\left(103ex^2+43d\right)}{\sqrt{d-ex^2}}dx}{4e}-\frac{15}{4}x\sqrt{d-ex^2}\left(d+ex^2\right)\right)-\frac{1}{6}x\sqrt{d-ex^2}\left(d+ex^2\right)^2\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{6}d\left(\frac{1}{4}d\int \frac{103ex^2+43d}{\sqrt{d-ex^2}}dx-\frac{15}{4}x\sqrt{d-ex^2}\left(d+ex^2\right)\right)-\frac{1}{6}x\sqrt{d-ex^2}\left(d+ex^2\right)^2\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{6}d\left(\frac{1}{4}d\left(\frac{189}{2}d\int \frac{1}{\sqrt{d-ex^2}}dx-\frac{103}{2}x\sqrt{d-ex^2}\right)-\frac{15}{4}x\sqrt{d-ex^2}\left(d+ex^2\right)\right)-\frac{1}{6}x\sqrt{d-ex^2}(d+e\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{6}d\left(\frac{1}{4}d\left(\frac{189}{2}d\int \frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}-\frac{103}{2}x\sqrt{d-ex^2}\right)-\frac{15}{4}x\sqrt{d-ex^2}\left(d+ex^2\right)\right)-\frac{1}{6}x\sqrt{d-e\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{6}d\left(\frac{1}{4}d\left(\frac{189d\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{e}}-\frac{103}{2}x\sqrt{d-ex^2}\right)-\frac{15}{4}x\sqrt{d-ex^2}\left(d+ex^2\right)\right)-\frac{1}{6}x\sqrt{d-ex^2}\right.}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d + e*x^2)^(7/2)/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(-1/6*(x*Sqrt[d - e*x^2]*(d + e*x^2)^2) +
 (d*((-15*x*Sqrt[d - e*x^2]*(d + e*x^2))/4 + (d*((-103*x*Sqrt[d - e*x^2])/
2 + (189*d*ArcTan[(Sqrt[e]*x)/Sqrt[d - e*x^2]])/(2*Sqrt[e])))/4))/6))/Sqrt
[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(-8 e^{\frac{5}{2}} x^{5} \sqrt{-e \,x^{2}+d}-46 d \,e^{\frac{3}{2}} x^{3} \sqrt{-e \,x^{2}+d}-141 \sqrt{-e \,x^{2}+d}\, \sqrt{e}\, d^{2} x +189 \arctan \left(\frac{\sqrt{e}\, x}{\sqrt{-e \,x^{2}+d}}\right) d^{3}\right)}{48 \sqrt{e \,x^{2}+d}\, \sqrt{-e \,x^{2}+d}\, \sqrt{e}}$	117
risch	$-\frac{x \left(8 e^{2} x^{4}+46 d e \,x^{2}+141 d^{2}\right) \sqrt{-e \,x^{2}+d}\, \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e \,x^{2}+d}}\, \sqrt{e \,x^{2}+d}}{48 \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{63 d^{3} \arctan \left(\frac{\sqrt{e}\, x}{\sqrt{-e \,x^{2}+d}}\right) \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e \,x^{2}+d}}\, \sqrt{e \,x^{2}+d}}{16 \sqrt{e}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	154

input

```
int((e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/48*(-e^2*x^4+d^2)^(1/2)*(-8*e^(5/2)*x^5*(-e*x^2+d)^(1/2)-46*d*e^(3/2)*x^
3*(-e*x^2+d)^(1/2)-141*(-e*x^2+d)^(1/2)*e^(1/2)*d^2*x+189*arctan(e^(1/2)*x
/(-e*x^2+d)^(1/2))*d^3)/(e*x^2+d)^(1/2)/(-e*x^2+d)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex^2)^{7/2}}{\sqrt{d^2-e^2x^4}}\,dx = \left[-\frac{189\,(d^3ex^2+d^4)\sqrt{-e}\log\left(-\frac{2\,e^2x^4+dex^2-2\,\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{-e}x-d^2}{ex^2+d}\right)+2\,(8\,e^3x^5+46}{96\,(e^2x^2+de)}\right.$$

$$-\frac{189\,(d^3ex^2+d^4)\sqrt{e}\arctan\left(\frac{\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{e}x}{e^2x^4-d^2}\right)+(8\,e^3x^5+46\,de^2x^3+141\,d^2ex)\sqrt{-e^2x^4+d^2}\sqrt{ex^2+}}{48\,(e^2x^2+de)}$$

input
```
integrate((e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(189*(d^3*e*x^2 + d^4)*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt(
-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)) + 2*(8*e^3*
x^5 + 46*d*e^2*x^3 + 141*d^2*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d))/(e
^2*x^2 + d*e), -1/48*(189*(d^3*e*x^2 + d^4)*sqrt(e)*arctan(sqrt(-e^2*x^4 +
 d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)) + (8*e^3*x^5 + 46*d*e^2*x
^3 + 141*d^2*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d))/(e^2*x^2 + d*e)]
```

Sympy [F]

$$\int \frac{(d+ex^2)^{7/2}}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(d+ex^2)^{\frac{7}{2}}}{\sqrt{-(-d+ex^2)(d+ex^2)}}\,dx$$

input
```
integrate((e*x**2+d)**(7/2)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral((d + e*x**2)**(7/2)/sqrt(-(-d + e*x**2)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{7/2}}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(ex^2+d)^{\frac{7}{2}}}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(7/2)/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^{7/2}}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(ex^2+d)^{\frac{7}{2}}}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(7/2)/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(e\,x^2+d)^{7/2}}{\sqrt{d^2-e^2\,x^4}}\,dx$$

input
```
int((d + e*x^2)^(7/2)/(d^2 - e^2*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^(7/2)/(d^2 - e^2*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int \frac{(d+ex^2)^{7/2}}{\sqrt{d^2-e^2x^4}}\,dx = \frac{189\sqrt{e}\,asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)d^3 - 141\sqrt{-e\,x^2+d}\,d^2ex - 46\sqrt{-e\,x^2+d}\,d\,e^2x^3 - 8\sqrt{-e\,x^2+d}\,e}{48e}$$

input
```
int((e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(189*sqrt(e)*asin((sqrt(e)*x)/sqrt(d))*d**3 - 141*sqrt(d - e*x**2)*d**2*e*
x - 46*sqrt(d - e*x**2)*d*e**2*x**3 - 8*sqrt(d - e*x**2)*e**3*x**5)/(48*e)
```

3.151 $\int \frac{(d+ex^2)^{5/2}}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{d^2-e^2x^4}}\,dx = -\frac{11dx\sqrt{d^2-e^2x^4}}{8\sqrt{d+ex^2}} - \frac{ex^3\sqrt{d^2-e^2x^4}}{4\sqrt{d+ex^2}} + \frac{19d^2\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{8\sqrt{e}}$$

output
```
-11/8*d*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)-1/4*e*x^3*(-e^2*x^4+d^2)^(1
/2)/(e*x^2+d)^(1/2)+19/8*d^2*arctan(e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^
2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{d^2-e^2x^4}}\,dx = -\frac{(11dx+2ex^3)\sqrt{d^2-e^2x^4}}{8\sqrt{d+ex^2}} + \frac{19id^2\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{8\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)^(5/2)/Sqrt[d^2 - e^2*x^4],x]
```

output
```
-1/8*((11*d*x + 2*e*x^3)*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2] + (((19*I)/8
)*d^2*Log[(-2*I)*Sqrt[e]*x + (2*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2]])/Sqr
t[e]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1396, 318, 25, 27, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\left(ex^2+d\right)^2}{\sqrt{d-ex^2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{\int -\frac{de\left(9ex^2+5d\right)}{\sqrt{d-ex^2}}dx}{4e}-\frac{1}{4}x\sqrt{d-ex^2}\left(d+ex^2\right)\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{de\left(9ex^2+5d\right)}{\sqrt{d-ex^2}}dx}{4e}-\frac{1}{4}x\sqrt{d-ex^2}\left(d+ex^2\right)\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{4}d\int \frac{9ex^2+5d}{\sqrt{d-ex^2}}dx-\frac{1}{4}x\sqrt{d-ex^2}\left(d+ex^2\right)\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{4}d\left(\frac{19}{2}d\int\frac{1}{\sqrt{d-ex^2}}dx-\frac{9}{2}x\sqrt{d-ex^2}\right)-\frac{1}{4}x\sqrt{d-ex^2}\left(d+ex^2\right)\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 224$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{4}d\left(\frac{19}{2}d\int\frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}-\frac{9}{2}x\sqrt{d-ex^2}\right)-\frac{1}{4}x\sqrt{d-ex^2}\left(d+ex^2\right)\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 216$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{4}d\left(\frac{19d\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{e}}-\frac{9}{2}x\sqrt{d-ex^2}\right)-\frac{1}{4}x\sqrt{d-ex^2}\left(d+ex^2\right)\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d + e*x^2)^(5/2)/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(-1/4*(x*Sqrt[d - e*x^2]*(d + e*x^2)) + (
d*((-9*x*Sqrt[d - e*x^2])/2 + (19*d*ArcTan[(Sqrt[e]*x)/Sqrt[d - e*x^2]])/(
2*Sqrt[e])))/4))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(-2 e^{\frac{3}{2}} x^{3} \sqrt{-e \,x^{2}+d}-11 \sqrt{e}\, \sqrt{-e \,x^{2}+d}\, d x +19 \arctan \left(\frac{\sqrt{e}\, x}{\sqrt{-e \,x^{2}+d}}\right) d^{2}\right)}{8 \sqrt{e \,x^{2}+d}\, \sqrt{-e \,x^{2}+d}\, \sqrt{e}}$	96
risch	$-\frac{x \left(2 e \,x^{2}+11 d \right) \sqrt{-e \,x^{2}+d}\, \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e \,x^{2}+d}}\, \sqrt{e \,x^{2}+d}}{8 \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{19 d^{2} \arctan \left(\frac{\sqrt{e}\, x}{\sqrt{-e \,x^{2}+d}}\right) \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e \,x^{2}+d}}\, \sqrt{e \,x^{2}+d}}{8 \sqrt{e}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	143

input

```
int((e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/8*(-e^2*x^4+d^2)^(1/2)*(-2*e^(3/2)*x^3*(-e*x^2+d)^(1/2)-11*e^(1/2)*(-e*x
^2+d)^(1/2)*d*x+19*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*d^2)/(e*x^2+d)^(1/2)
/(-e*x^2+d)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{d^2-e^2x^4}}\,dx = \left[-\frac{19\,(d^2ex^2+d^3)\sqrt{-e}\log\left(-\frac{2\,e^2x^4+dex^2-2\,\sqrt{-e^2x^4+d^2}\,\sqrt{ex^2+d}\,\sqrt{-e}x-d^2}{ex^2+d}\right)+2\,\sqrt{-e^2x^4+d}}{16\,(e^2x^2+de)}\right.$$

$$\left.-\frac{19\,(d^2ex^2+d^3)\sqrt{e}\arctan\left(\frac{\sqrt{-e^2x^4+d^2}\,\sqrt{ex^2+d}\,\sqrt{e}x}{e^2x^4-d^2}\right)+\sqrt{-e^2x^4+d^2}(2\,e^2x^3+11\,dex)\sqrt{ex^2+d}}{8\,(e^2x^2+de)}\right]$$

input
```
integrate((e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(19*(d^2*e*x^2 + d^3)*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt(-
e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)) + 2*sqrt(-e^
2*x^4 + d^2)*(2*e^2*x^3 + 11*d*e*x)*sqrt(e*x^2 + d))/(e^2*x^2 + d*e), -1/8
*(19*(d^2*e*x^2 + d^3)*sqrt(e)*arctan(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)
*sqrt(e)*x/(e^2*x^4 - d^2)) + sqrt(-e^2*x^4 + d^2)*(2*e^2*x^3 + 11*d*e*x)*
sqrt(e*x^2 + d))/(e^2*x^2 + d*e)]
```

Sympy [F]

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(d+ex^2)^{\frac{5}{2}}}{\sqrt{-(-d+ex^2)\,(d+ex^2)}}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral((d + e*x**2)**(5/2)/sqrt(-(-d + e*x**2)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(5/2)/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(5/2)/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(e\,x^2+d)^{5/2}}{\sqrt{d^2-e^2\,x^4}}\,dx$$

input
```
int((d + e*x^2)^(5/2)/(d^2 - e^2*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^(5/2)/(d^2 - e^2*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{d^2-e^2x^4}}\,dx = \frac{19\sqrt{e}\,asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)d^2-11\sqrt{-e\,x^2+d}\,dex-2\sqrt{-e\,x^2+d}\,e^2x^3}{8e}$$

input

```
int((e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(1/2),x)
```

output

```
(19*sqrt(e)*asin((sqrt(e)*x)/sqrt(d))*d**2 - 11*sqrt(d - e*x**2)*d*e*x - 2
*sqrt(d - e*x**2)*e**2*x**3)/(8*e)
```

3.152 $\int \frac{(d+ex^2)^{3/2}}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 78

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{d^2-e^2x^4}}\,dx = -\frac{x\sqrt{d^2-e^2x^4}}{2\sqrt{d+ex^2}}+\frac{3d\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{2\sqrt{e}}$$

output
```
-1/2*x*(-e^2*x^4+d^2)^(1/2)/(e*x^2+d)^(1/2)+3/2*d*arctan(e^(1/2)*x*(e*x^2+
d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.55 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{d^2-e^2x^4}}\,dx = -\frac{x\sqrt{d^2-e^2x^4}}{2\sqrt{d+ex^2}}+\frac{3id\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)^(3/2)/Sqrt[d^2 - e^2*x^4],x]
```

output
```
-1/2*(x*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2] + (((3*I)/2)*d*Log[(-2*I)*Sqr
t[e]*x + (2*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2]])/Sqrt[e]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1396, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{ex^2+d}{\sqrt{d-ex^2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3}{2}d\int \frac{1}{\sqrt{d-ex^2}}dx-\frac{1}{2}x\sqrt{d-ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3}{2}d\int \frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}-\frac{1}{2}x\sqrt{d-ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3d\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{2\sqrt{e}}-\frac{1}{2}x\sqrt{d-ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d + e*x^2)^(3/2)/Sqrt[d^2 - e^2*x^4],x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(-1/2*(x*Sqrt[d - e*x^2]) + (3*d*ArcTan[(
Sqrt[e]*x)/Sqrt[d - e*x^2]])/(2*Sqrt[e])))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(-x \sqrt{-e\,x^{2}+d}\, \sqrt{e}+3 d \arctan\left(\frac{\sqrt{e}\, x}{\sqrt{-e\,x^{2}+d}}\right)\right)}{2 \sqrt{e\,x^{2}+d}\, \sqrt{-e\,x^{2}+d}\, \sqrt{e}}$	75
risch	$-\frac{x \sqrt{-e\,x^{2}+d}\, \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e\,x^{2}+d}}\, \sqrt{e\,x^{2}+d}}{2 \sqrt{-e^{2} x^{4}+d^{2}}}+\frac{3 d \arctan\left(\frac{\sqrt{e}\, x}{\sqrt{-e\,x^{2}+d}}\right) \sqrt{\frac{-e^{2} x^{4}+d^{2}}{e\,x^{2}+d}}\, \sqrt{e\,x^{2}+d}}{2 \sqrt{e}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	131

input
```
int((e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e^2*x^4+d^2)^(1/2)*(-x*(-e*x^2+d)^(1/2)*e^(1/2)+3*d*arctan(e^(1/2)*x
/(-e*x^2+d)^(1/2)))/(e*x^2+d)^(1/2)/(-e*x^2+d)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.88

$$\int \frac{\left(d+e x^{2}\right)^{3/2}}{\sqrt{d^{2}-e^{2} x^{4}}}\,dx=\left[-\frac{2\sqrt{-e^{2}x^{4}+d^{2}}\sqrt{ex^{2}+d}ex+3\left(dex^{2}+d^{2}\right)\sqrt{-e}\log\left(-\frac{2\,e^{2}x^{4}+dex^{2}-2\,\sqrt{-e^{2}x^{4}+d^{2}}\sqrt{ex^{2}+}}{ex^{2}+d}\right.}{4\left(e^{2}x^{2}+de\right)}\right.$$

$$\left.-\frac{\sqrt{-e^{2}x^{4}+d^{2}}\sqrt{ex^{2}+d}ex+3\left(dex^{2}+d^{2}\right)\sqrt{e}\arctan\left(\frac{\sqrt{-e^{2}x^{4}+d^{2}}\sqrt{ex^{2}+d}\sqrt{e}x}{e^{2}x^{4}-d^{2}}\right)}{2\left(e^{2}x^{2}+de\right)}\right]$$

input
```
integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*e*x + 3*(d*e*x^2 + d^2)*sqrt
(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sq
rt(-e)*x - d^2)/(e*x^2 + d)))/(e^2*x^2 + d*e), -1/2*(sqrt(-e^2*x^4 + d^2)*
sqrt(e*x^2 + d)*e*x + 3*(d*e*x^2 + d^2)*sqrt(e)*arctan(sqrt(-e^2*x^4 + d^2
)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)))/(e^2*x^2 + d*e)]
```

Sympy [F]

$$\int \frac{\left(d+e x^{2}\right)^{3/2}}{\sqrt{d^{2}-e^{2} x^{4}}}\,dx=\int \frac{\left(d+e x^{2}\right)^{\frac{3}{2}}}{\sqrt{-\left(-d+e x^{2}\right)\left(d+e x^{2}\right)}}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral((d + e*x**2)**(3/2)/sqrt(-(-d + e*x**2)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{\sqrt{d^2-e^2\,x^4}}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(d^2 - e^2*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^(3/2)/(d^2 - e^2*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.42

$$\int \frac{\left(d+e x^2\right)^{3/2}}{\sqrt{d^2-e^2 x^4}}\,dx=\frac{3\sqrt{e}\,asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)d-\sqrt{-e\,x^2+d}\,ex}{2e}$$

input
```
int((e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(3*sqrt(e)*asin((sqrt(e)*x)/sqrt(d))*d - sqrt(d - e*x**2)*e*x)/(2*e)
```

3.153 $\int \frac{\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 41

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{\sqrt{e}}$$

output
```
arctan(e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\,dx=\frac{i\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

input
```
Integrate[Sqrt[d + e*x^2]/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(I*Log[(-2*I)*Sqrt[e]*x + (2*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2]])/Sqrt[e
]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1396, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \color{blue}{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\sqrt{d-ex^2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \color{blue}{224}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \color{blue}{216}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{e}\sqrt{d^2-e^2x^4}}$$

input
```
Int[Sqrt[d + e*x^2]/Sqrt[d^2 - e^2*x^4],x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*ArcTan[(Sqrt[e]*x)/Sqrt[d - e*x^2]])/(Sqr
t[e]*Sqrt[d^2 - e^2*x^4])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \arctan\left(\frac{\sqrt{e}\, x}{\sqrt{-e\, x^{2}+d}}\right)}{\sqrt{e\, x^{2}+d}\, \sqrt{-e\, x^{2}+d}\, \sqrt{e}}$	54

input
```
int((e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x^2+d)^(1/2)*(-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^(1/2)/e^(1/2)*arctan(e^(
1/2)*x/(-e*x^2+d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.00

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\,dx = \left[-\frac{\sqrt{-e}\log\left(-\frac{2\,e^2x^4+dex^2-2\,\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{-e}x-d^2}{ex^2+d}\right)}{2\,e},\ -\frac{\arctan\left(\frac{\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{e}x}{e^2x^4-d^2}\right)}{\sqrt{e}}\right]$$

input
```
integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt(-e^2*x^4 + d^2)*sqrt(e*x
^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d))/e, -arctan(sqrt(-e^2*x^4 + d^2)*sqr
t(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2))/sqrt(e)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\,dx = \int \frac{\sqrt{d+ex^2}}{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x**2)/sqrt(-(-d + e*x**2)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{\sqrt{ex^2+d}}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/sqrt(-e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{\sqrt{ex^2+d}}{\sqrt{-e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)/sqrt(-e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\,dx=\int \frac{\sqrt{e\,x^2+d}}{\sqrt{d^2-e^2\,x^4}}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(d^2 - e^2*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^(1/2)/(d^2 - e^2*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\,dx = \frac{\sqrt{e}\,asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e}$$

input
```
int((e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(sqrt(e)*asin((sqrt(e)*x)/sqrt(d)))/e
```

3.154 $\int \frac{1}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 54

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{\sqrt{2}d\sqrt{e}}$$

output
```
1/2*arctan(2^(1/2)*e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))*2^(1/2)
/d/e^(1/2)
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx=\frac{\sqrt{d^2-e^2x^4}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{2}d\sqrt{e}\sqrt{d-ex^2}\sqrt{d+ex^2}}$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]])/(Sqrt[2]
*d*Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1396, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{2}d\sqrt{e}\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/(Sqrt[d + e*x^2]*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2
]])/(Sqrt[2]*d*Sqrt[e]*Sqrt[d^2 - e^2*x^4])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(43) = 86$.

Time = 0.81 (sec) , antiderivative size = 249, normalized size of antiderivative = 4.61

method	result
default	$\frac{\sqrt{-e^{2}x^{4}+d^{2}}\,\sqrt{e}\left(\sqrt{d}\,\sqrt{2}\,\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^{2}+d}-\sqrt{-de}\,x+d\right)}{ex-\sqrt{-de}}\right)\sqrt{e}-\sqrt{d}\,\sqrt{2}\,\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^{2}+d}+\sqrt{-de}\,x+d\right)}{ex+\sqrt{-de}}\right)\sqrt{e}-2\sqrt{-de}\,\mathrm{arc}\right.}{2\sqrt{e\,x^{2}+d}\,\sqrt{-e\,x^{2}+d}\left(-\sqrt{de}+\sqrt{-de}\right)\left(\sqrt{de}+\sqrt{-de}\right)\sqrt{-de}}$

input
```
int(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e^2*x^4+d^2)^(1/2)*e^(1/2)*(d^(1/2)*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*
(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*e^(1/2)-d^(1/2)*2^(
1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e
)^(1/2)))*e^(1/2)-2*(-d*e)^(1/2)*arctan(e^(1/2)*x/((e*x+(d*e)^(1/2))/e*(-e
*x+(d*e)^(1/2)))^(1/2))+2*(-d*e)^(1/2)*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2)))
/(e*x^2+d)^(1/2)/(-e*x^2+d)^(1/2)/(-(d*e)^(1/2)+(-d*e)^(1/2))/((d*e)^(1/2)
+(-d*e)^(1/2))/(-d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.83

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = \left[-\frac{\sqrt{2}\sqrt{-e}\log\left(-\frac{3\,e^2x^4+2\,dex^2-2\,\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{-e}x-d^2}{e^2x^4+2\,dex^2+d^2}\right)}{4\,de},\ -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{e}x}{e^2x^4-d^2}\right)}{2\,d\sqrt{e}}\right]$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(2)*sqrt(-e)*log(-(3*e^2*x^4 + 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x
^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2))/(
d*e), -1/2*sqrt(2)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqr
t(e)*x/(e^2*x^4 - d^2))/(d*sqrt(e))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x**2)*(d + e*x**2))*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2-e^2\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = \frac{\sqrt{e}\,\sqrt{2}\left(2atan\left(\frac{\tan\left(\frac{asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2}\right)}{\sqrt{2}+1}\right)-\log\left(-\sqrt{2}\,i+\tan\left(\frac{asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2}\right)+i\right)i+\log\left(\sqrt{2}\,i+\tan\left(\frac{asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2}\right.\right.\right.}{4de}$$

input
```
int(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(sqrt(e)*sqrt(2)*(2*atan(tan(asin((sqrt(e)*x)/sqrt(d))/2)/(sqrt(2) + 1)) -
 log( - sqrt(2)*i + tan(asin((sqrt(e)*x)/sqrt(d))/2) + i)*i + log(sqrt(2)*
i + tan(asin((sqrt(e)*x)/sqrt(d))/2) - i)*i))/(4*d*e)
```

3.155 $\int \frac{1}{(d+ex^2)^{3/2}\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 93

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{d^2-e^2x^4}}\,dx = \frac{x\sqrt{d^2-e^2x^4}}{4d^2(d+ex^2)^{3/2}} + \frac{3\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{4\sqrt{2}d^2\sqrt{e}}$$

output

```
1/4*x*(-e^2*x^4+d^2)^(1/2)/d^2/(e*x^2+d)^(3/2)+3/8*arctan(2^(1/2)*e^(1/2)*
x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))*2^(1/2)/d^2/e^(1/2)
```

Mathematica [A] (verified)

Time = 2.67 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{d^2-e^2x^4}}\,dx = \frac{\sqrt{d^2-e^2x^4}\left(2\sqrt{e}x\sqrt{d-ex^2}+3\sqrt{2}(d+ex^2)\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)\right)}{8d^2\sqrt{e}\sqrt{d-ex^2}\,(d+ex^2)^{3/2}}$$

input

```
Integrate[1/((d + e*x^2)^(3/2)*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(2*Sqrt[e]*x*Sqrt[d - e*x^2] + 3*Sqrt[2]*(d + e*x^2)*
ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]]))/(8*d^2*Sqrt[e]*Sqrt[d - e*x^
2]*(d + e*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1396, 296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)^2}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{296}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)}dx}{4d}+\frac{x\sqrt{d-ex^2}}{4d^2(d+ex^2)}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\int \frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}}{4d}+\frac{x\sqrt{d-ex^2}}{4d^2(d+ex^2)}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{3\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{4\sqrt{2}d^2\sqrt{e}}+\frac{x\sqrt{d-ex^2}}{4d^2(d+ex^2)}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/((d + e*x^2)^(3/2)*Sqrt[d^2 - e^2*x^4]),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d - e*x^2])/(4*d^2*(d + e*x^2))
+ (3*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]])/(4*Sqrt[2]*d^2*Sqrt[e]))
)/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(73) = 146$.

Time = 0.82 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.25

method	result
default	$-\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, e^{\frac{5}{2}} \left(3 \ln\left(\frac{2 e \left(\sqrt{2}\, \sqrt{d}\, \sqrt{-e\, x^{2}+d}-\sqrt{-d e}\, x+d\right)}{e x-\sqrt{-d e}}\right) \sqrt{2}\, e^{\frac{3}{2}} x^{2} \sqrt{d}-3 \ln\left(\frac{2 e \left(\sqrt{2}\, \sqrt{d}\, \sqrt{-e\, x^{2}+d}+\sqrt{-d e}\, x+d\right)}{e x+\sqrt{-d e}}\right) \sqrt{2}\, e^{\frac{3}{2}} x^{2} \sqrt{d}+3\sqrt{}\right.}{}$

input
```
int(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-e^2*x^4+d^2)^(1/2)*e^(5/2)*(3*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1
/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*2^(1/2)*e^(3/2)*x^2*d^(1/2)-3*ln
(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)
))*2^(1/2)*e^(3/2)*x^2*d^(1/2)+3*e^(1/2)*d^(3/2)*2^(1/2)*ln(2*e*(2^(1/2)*d
^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))-3*ln(2*e*(2^
(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*e^(1/
2)*d^(3/2)*2^(1/2)+4*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*e*x^2*(-d*e)^(1/2)
-4*arctan(e^(1/2)*x/((e*x+(d*e)^(1/2))/e*(-e*x+(d*e)^(1/2)))^(1/2))*e*x^2*
(-d*e)^(1/2)-4*(-d*e)^(1/2)*(-e*x^2+d)^(1/2)*e^(1/2)*x+4*(-d*e)^(1/2)*arct
an(e^(1/2)*x/(-e*x^2+d)^(1/2))*d-4*arctan(e^(1/2)*x/((e*x+(d*e)^(1/2))/e*(
-e*x+(d*e)^(1/2)))^(1/2))*d*(-d*e)^(1/2))/(e*x^2+d)^(1/2)/(-e*x^2+d)^(1/2)
/(-(d*e)^(1/2)+(-d*e)^(1/2))^2/((d*e)^(1/2)+(-d*e)^(1/2))^2/(-d*e)^(1/2)/(
e*x-(-d*e)^(1/2))/(e*x+(-d*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.20

$$\int \frac{1}{\left(d+e x^{2}\right)^{3/2} \sqrt{d^{2}-e^{2} x^{4}}}\, dx = \left[\frac{4 \sqrt{-e^{2} x^{4}+d^{2}} \sqrt{e x^{2}+d}\, d e x-3 \sqrt{2} (e^{2} x^{4}+2 d e x^{2}+d^{2}) \sqrt{-e} \log\left(-\frac{3 e^{2} x}{}\right.}{16 \left(d^{2} e^{3} x^{4}+2 d^{3} e^{2} x^{2}+d^{4} e\right)}\right.$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(4*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*e*x - 3*sqrt(2)*(e^2*x^4 + 2
*d*e*x^2 + d^2)*sqrt(-e)*log(-(3*e^2*x^4 + 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2
*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2))
)/(d^2*e^3*x^4 + 2*d^3*e^2*x^2 + d^4*e), 1/8*(2*sqrt(-e^2*x^4 + d^2)*sqrt(
e*x^2 + d)*e*x - 3*sqrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e)*arctan(sqrt
(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)))/(d^2*
e^3*x^4 + 2*d^3*e^2*x^2 + d^4*e)]
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x**2)*(d + e*x**2))*(d + e*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2-e^2\,x^4}\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.24

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{d^2-e^2x^4}}\,dx = \frac{6\sqrt{e}\,\sqrt{2}\,atan\left(\frac{\tan\left(\frac{asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2}\right)}{\sqrt{2}+1}\right)d+6\sqrt{e}\,\sqrt{2}\,atan\left(\frac{\tan\left(\frac{asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2}\right)}{\sqrt{2}+1}\right)e\,x^2}{}$$

input
```
int(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(1/2),x)
```

output

```
(6*sqrt(e)*sqrt(2)*atan(tan(asin((sqrt(e)*x)/sqrt(d))/2)/(sqrt(2) + 1))*d
+ 6*sqrt(e)*sqrt(2)*atan(tan(asin((sqrt(e)*x)/sqrt(d))/2)/(sqrt(2) + 1))*e
*x**2 + 4*sqrt(d - e*x**2)*e*x - 3*sqrt(e)*sqrt(2)*log( - sqrt(2)*i + tan(
asin((sqrt(e)*x)/sqrt(d))/2) + i)*d*i - 3*sqrt(e)*sqrt(2)*log( - sqrt(2)*i
 + tan(asin((sqrt(e)*x)/sqrt(d))/2) + i)*e*i*x**2 + 3*sqrt(e)*sqrt(2)*log(
sqrt(2)*i + tan(asin((sqrt(e)*x)/sqrt(d))/2) - i)*d*i + 3*sqrt(e)*sqrt(2)*
log(sqrt(2)*i + tan(asin((sqrt(e)*x)/sqrt(d))/2) - i)*e*i*x**2)/(16*d**2*e
*(d + e*x**2))
```

3.156 $\int \frac{1}{(d+ex^2)^{5/2}\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 128

$$\int \frac{1}{(d+ex^2)^{5/2}\sqrt{d^2-e^2x^4}}\,dx=\frac{x\sqrt{d^2-e^2x^4}}{8d^2(d+ex^2)^{5/2}}+\frac{9x\sqrt{d^2-e^2x^4}}{32d^3(d+ex^2)^{3/2}}+\frac{19\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{32\sqrt{2}d^3\sqrt{e}}$$

output

```
1/8*x*(-e^2*x^4+d^2)^(1/2)/d^2/(e*x^2+d)^(5/2)+9/32*x*(-e^2*x^4+d^2)^(1/2)
/d^3/(e*x^2+d)^(3/2)+19/64*arctan(2^(1/2)*e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*
x^4+d^2)^(1/2))*2^(1/2)/d^3/e^(1/2)
```

Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96

$$\int \frac{1}{(d+ex^2)^{5/2}\sqrt{d^2-e^2x^4}}\,dx=\frac{\sqrt{d^2-e^2x^4}\left(2\sqrt{e}x\sqrt{d-ex^2}(13d+9ex^2)+19\sqrt{2}(d+ex^2)^2\arctan\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{d-e}}\right.\right.}{64d^3\sqrt{e}\sqrt{d-ex^2}\,(d+ex^2)^{5/2}}$$

input

```
Integrate[1/((d + e*x^2)^(5/2)*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(2*Sqrt[e]*x*Sqrt[d - e*x^2]*(13*d + 9*e*x^2) + 19*Sq
rt[2]*(d + e*x^2)^2*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]]))/(64*d^3*
Sqrt[e]*Sqrt[d - e*x^2]*(d + e*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1396, 316, 25, 27, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\sqrt{d-ex^2}\left(ex^2+d\right)^3}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x\sqrt{d-ex^2}}{8d^2\left(d+ex^2\right)^2}-\frac{\int -\frac{e\left(7d-2ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^2}dx}{8d^2e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(7d-2ex^2\right)}{\sqrt{d-ex^2}\left(ex^2+d\right)^2}dx}{8d^2e}+\frac{x\sqrt{d-ex^2}}{8d^2\left(d+ex^2\right)^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{7d-2ex^2}{\sqrt{d-ex^2}\left(ex^2+d\right)^2}dx}{8d^2}+\frac{x\sqrt{d-ex^2}}{8d^2\left(d+ex^2\right)^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{9x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)}-\frac{\int -\frac{19d^2e}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx}{4d^2e}}{8d^2}+\frac{x\sqrt{d-ex^2}}{8d^2(d+ex^2)^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{19}{4}\int \frac{1}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx+\frac{9x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)}}{8d^2}+\frac{x\sqrt{d-ex^2}}{8d^2(d+ex^2)^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 291$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{19}{4}\int \frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}+\frac{9x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)}}{8d^2}+\frac{x\sqrt{d-ex^2}}{8d^2(d+ex^2)^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 218$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{19\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{4\sqrt{2}d\sqrt{e}}+\frac{9x\sqrt{d-ex^2}}{4d\left(d+ex^2\right)}}{8d^2}+\frac{x\sqrt{d-ex^2}}{8d^2(d+ex^2)^2}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/((d + e*x^2)^(5/2)*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*Sqrt[d - e*x^2])/(8*d^2*(d + e*x^2)^2
) + ((9*x*Sqrt[d - e*x^2])/(4*d*(d + e*x^2)) + (19*ArcTan[(Sqrt[2]*Sqrt[e]
*x)/Sqrt[d - e*x^2]])/(4*Sqrt[2]*d*Sqrt[e]))/(8*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(102) = 204.

Time = 0.84 (sec) , antiderivative size = 711, normalized size of antiderivative = 5.55

method	result
default	$-\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, e^{\frac{9}{2}} \left(19 \sqrt{2}\, \ln\left(\frac{2 e \left(\sqrt{2}\, \sqrt{d}\, \sqrt{-e\, x^{2}+d}-\sqrt{-d e}\, x+d\right)}{e x-\sqrt{-d e}}\right) e^{\frac{5}{2}} x^{4} \sqrt{d}-19 \sqrt{2}\, \ln\left(\frac{2 e \left(\sqrt{2}\, \sqrt{d}\, \sqrt{-e\, x^{2}+d}+\sqrt{-d e}\, x+d\right)}{e x+\sqrt{-d e}}\right) e^{\frac{5}{2}} x^{4} \sqrt{d}+3\right.}{}$

input
```
int(1/(e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-e^2*x^4+d^2)^(1/2)*e^(9/2)*(19*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e
*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*e^(5/2)*x^4*d^(1/2)-19
*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(
-d*e)^(1/2)))*e^(5/2)*x^4*d^(1/2)+38*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x
^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*d^(3/2)*e^(3/2)*x^2-38*2
^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d
*e)^(1/2)))*d^(3/2)*e^(3/2)*x^2+16*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*e^2*
x^4*(-d*e)^(1/2)-16*arctan(e^(1/2)*x/((e*x+(d*e)^(1/2))/e*(-e*x+(d*e)^(1/2
)))^(1/2))*e^2*x^4*(-d*e)^(1/2)-36*(-e*x^2+d)^(1/2)*e^(3/2)*(-d*e)^(1/2)*x
^3+19*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(
e*x-(-d*e)^(1/2)))*d^(5/2)*e^(1/2)-19*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*
x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*d^(5/2)*e^(1/2)+32*arct
an(e^(1/2)*x/(-e*x^2+d)^(1/2))*d*e*x^2*(-d*e)^(1/2)-32*arctan(e^(1/2)*x/((
e*x+(d*e)^(1/2))/e*(-e*x+(d*e)^(1/2)))^(1/2))*d*e*x^2*(-d*e)^(1/2)-52*(-e*
x^2+d)^(1/2)*d*e^(1/2)*(-d*e)^(1/2)*x+16*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2)
)*d^2*(-d*e)^(1/2)-16*arctan(e^(1/2)*x/((e*x+(d*e)^(1/2))/e*(-e*x+(d*e)^(1
/2)))^(1/2))*d^2*(-d*e)^(1/2))/(e*x^2+d)^(1/2)/(-e*x^2+d)^(1/2)/((d*e)^(1/
2)-(-d*e)^(1/2))^3/((d*e)^(1/2)+(-d*e)^(1/2))^3/(e*x-(-d*e)^(1/2))^2/(e*x+
(-d*e)^(1/2))^2/(-d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.86

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\sqrt{d^2-e^2x^4}}\,dx = \left[-\frac{19\,\sqrt{2}(e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3)\sqrt{-e}\log\left(-\frac{3\,e^2x^4+2\,dex^2-2\,\sqrt{2}\sqrt{-e^2}}{e^2x^4+2\,d}\right.}{128\,(d^3e^4x^6+3\,d^4e^3x^4}\right.$$

$$-\frac{19\,\sqrt{2}(e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3)\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{e}x}{e^2x^4-d^2}\right)-2\,\sqrt{-e^2x^4+d^2}(9\,e^2x^3+13\,d}{64\,(d^3e^4x^6+3\,d^4e^3x^4+3\,d^5e^2x^2+d^6e)}$$

input

```
integrate(1/(e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output

```
[-1/128*(19*sqrt(2)*(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)*sqrt(-e)*l
og(-(3*e^2*x^4 + 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d
)*sqrt(-e)*x - d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 4*sqrt(-e^2*x^4 + d^2)*
(9*e^2*x^3 + 13*d*e*x)*sqrt(e*x^2 + d))/(d^3*e^4*x^6 + 3*d^4*e^3*x^4 + 3*d
^5*e^2*x^2 + d^6*e), -1/64*(19*sqrt(2)*(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^
2 + d^3)*sqrt(e)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(
e)*x/(e^2*x^4 - d^2)) - 2*sqrt(-e^2*x^4 + d^2)*(9*e^2*x^3 + 13*d*e*x)*sqrt
(e*x^2 + d))/(d^3*e^4*x^6 + 3*d^4*e^3*x^4 + 3*d^5*e^2*x^2 + d^6*e)]
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}\left(d+ex^2\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(1/(e*x**2+d)**(5/2)/(-e**2*x**4+d**2)**(1/2),x)
```

output

```
Integral(1/(sqrt(-(-d + e*x**2)*(d + e*x**2))*(d + e*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^{5/2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^{5/2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{-e^2x^4+d^2}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-e^2*x^4 + d^2)*(e*x^2 + d)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{5/2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2-e^2\,x^4}\,(e\,x^2+d)^{5/2}}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.70

$$\int \frac{1}{(d+ex^2)^{5/2}\sqrt{d^2-e^2x^4}}\,dx = \frac{38\sqrt{e}\,\sqrt{2}\,atan\left(\frac{\tan\left(\frac{asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2}\right)}{\sqrt{2}+1}\right)d^2+76\sqrt{e}\,\sqrt{2}\,atan\left(\frac{\tan\left(\frac{asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2}\right)}{\sqrt{2}+1}\right)d}{}$$

input
```
int(1/(e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(38*sqrt(e)*sqrt(2)*atan(tan(asin((sqrt(e)*x)/sqrt(d))/2)/(sqrt(2) + 1))*d
**2 + 76*sqrt(e)*sqrt(2)*atan(tan(asin((sqrt(e)*x)/sqrt(d))/2)/(sqrt(2) +
1))*d*e*x**2 + 38*sqrt(e)*sqrt(2)*atan(tan(asin((sqrt(e)*x)/sqrt(d))/2)/(s
qrt(2) + 1))*e**2*x**4 + 52*sqrt(d - e*x**2)*d*e*x + 36*sqrt(d - e*x**2)*e
**2*x**3 - 19*sqrt(e)*sqrt(2)*log( - sqrt(2)*i + tan(asin((sqrt(e)*x)/sqrt
(d))/2) + i)*d**2*i - 38*sqrt(e)*sqrt(2)*log( - sqrt(2)*i + tan(asin((sqrt
(e)*x)/sqrt(d))/2) + i)*d*e*i*x**2 - 19*sqrt(e)*sqrt(2)*log( - sqrt(2)*i +
 tan(asin((sqrt(e)*x)/sqrt(d))/2) + i)*e**2*i*x**4 + 19*sqrt(e)*sqrt(2)*lo
g(sqrt(2)*i + tan(asin((sqrt(e)*x)/sqrt(d))/2) - i)*d**2*i + 38*sqrt(e)*sq
rt(2)*log(sqrt(2)*i + tan(asin((sqrt(e)*x)/sqrt(d))/2) - i)*d*e*i*x**2 + 1
9*sqrt(e)*sqrt(2)*log(sqrt(2)*i + tan(asin((sqrt(e)*x)/sqrt(d))/2) - i)*e*
*2*i*x**4)/(128*d**3*e*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.157 $\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 153

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{8d^2x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}} + \frac{19dx\sqrt{d^2-e^2x^4}}{8\sqrt{d-ex^2}} - \frac{ex^3\sqrt{d^2-e^2x^4}}{4\sqrt{d-ex^2}} - \frac{75d^2\operatorname{arctanh}\left(\frac{\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{8\sqrt{e}}$$

output
```
8*d^2*x*(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2)+19/8*d*x*(-e^2*x^4+d^2)^(1/2
)/(-e*x^2+d)^(1/2)-1/4*e*x^3*(-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^(1/2)-75/8*d^
2*arctanh(e^(1/2)*x*(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [A] (verified)

Time = 5.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{1}{8}\left(\frac{x(83d^2+17dex^2-2e^2x^4)\sqrt{d^2-e^2x^4}}{\sqrt{d-ex^2}\,(d+ex^2)} + \frac{75d^2\log(-d+ex^2)}{\sqrt{e}} - \frac{75d^2\log\left(dex - e\right.}{}\right.$$

input
```
Integrate[(d - e*x^2)^(9/2)/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
((x*(83*d^2 + 17*d*e*x^2 - 2*e^2*x^4)*Sqrt[d^2 - e^2*x^4])/(Sqrt[d - e*x^2
]*(d + e*x^2)) + (75*d^2*Log[-d + e*x^2])/Sqrt[e] - (75*d^2*Log[d*e*x - e^
2*x^3 + Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d^2 - e^2*x^4]])/Sqrt[e])/8
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1396, 315, 25, 27, 403, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{(d-ex^2)^3}{(ex^2+d)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int -\frac{de\left(d-9ex^2\right)\left(d-ex^2\right)}{\sqrt{ex^2+d}}dx}{de}+\frac{2x(d-ex^2)^2}{\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2x(d-ex^2)^2}{\sqrt{d+ex^2}}-\frac{\int \frac{de\left(d-9ex^2\right)\left(d-ex^2\right)}{\sqrt{ex^2+d}}dx}{de}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2x(d-ex^2)^2}{\sqrt{d+ex^2}}-\int \frac{(d-9ex^2)(d-ex^2)}{\sqrt{ex^2+d}}dx\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{\int \frac{5de\left(d-13ex^2\right)}{\sqrt{ex^2+d}}dx}{4e}+\frac{2x\left(d-ex^2\right)^2}{\sqrt{d+ex^2}}+\frac{1}{4}x\left(d-9ex^2\right)\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{5}{4}d\int \frac{d-13ex^2}{\sqrt{ex^2+d}}dx+\frac{2x\left(d-ex^2\right)^2}{\sqrt{d+ex^2}}+\frac{1}{4}x\left(d-9ex^2\right)\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 299$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{5}{4}d\left(\frac{15}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{13}{2}x\sqrt{d+ex^2}\right)+\frac{2x\left(d-ex^2\right)^2}{\sqrt{d+ex^2}}+\frac{1}{4}x\left(d-9ex^2\right)\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 224$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{5}{4}d\left(\frac{15}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{13}{2}x\sqrt{d+ex^2}\right)+\frac{2x\left(d-ex^2\right)^2}{\sqrt{d+ex^2}}+\frac{1}{4}x\left(d-9ex^2\right)\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 219$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{5}{4}d\left(\frac{15d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}-\frac{13}{2}x\sqrt{d+ex^2}\right)+\frac{2x\left(d-ex^2\right)^2}{\sqrt{d+ex^2}}+\frac{1}{4}x\left(d-9ex^2\right)\sqrt{d+ex^2}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d - e*x^2)^(9/2)/(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((2*x*(d - e*x^2)^2)/Sqrt[d + e*x^2] + (x
*(d - 9*e*x^2)*Sqrt[d + e*x^2])/4 - (5*d*((-13*x*Sqrt[d + e*x^2])/2 + (15*
d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/4))/Sqrt[d^2 - e^2*x
^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.63

method	result
default	$-\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(2 e^{\frac{5}{2}} x^{5}-17 d\, e^{\frac{3}{2}} x^{3}+75 \ln \left(x \sqrt{e}+\sqrt{e\, x^{2}+d}\right) d^{2} \sqrt{e\, x^{2}+d}-83 d^{2} x \sqrt{e}\right)}{8 \sqrt{-e\, x^{2}+d}\, \left(e\, x^{2}+d \right) \sqrt{e}}$
risch	$-\frac{x \left(-2 e\, x^{2}+19 d \right) \sqrt{e\, x^{2}+d}\, \sqrt{\frac{\left(-e\, x^{2}+d \right) \left(-e^{2} x^{4}+d^{2}\right)}{\left(e\, x^{2}-d \right)^{2}}}\, \left(e\, x^{2}-d \right)}{8 \sqrt{-e\, x^{2}+d}\, \sqrt{-e^{2} x^{4}+d^{2}}}-\frac{\left(\frac{8 d^{2} x}{\sqrt{e\, x^{2}+d}}-\frac{75 d^{2} \ln \left(x \sqrt{e}+\sqrt{e\, x^{2}+d}\right)}{8 \sqrt{e}}\right) \sqrt{\frac{\left(-e\, x^{2}+d \right) \left(-e^{2} x^{4}+d^{2}\right)}{\left(e\, x^{2}-d \right)^{2}}}}{\sqrt{-e\, x^{2}+d}\, \sqrt{-e^{2} x^{4}+d^{2}}}$

input
```
int((-e*x^2+d)^(9/2)/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-e^2*x^4+d^2)^(1/2)*(2*e^(5/2)*x^5-17*d*e^(3/2)*x^3+75*ln(x*e^(1/2)+
(e*x^2+d)^(1/2))*d^2*(e*x^2+d)^(1/2)-83*d^2*x*e^(1/2))/(-e*x^2+d)^(1/2)/(e
*x^2+d)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.93

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \left[\frac{75\,(d^2e^2x^4-d^4)\sqrt{e}\log\left(\frac{2\,e^2x^4-dex^2+2\,\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}\sqrt{e}x-d^2}{ex^2-d}\right)+2\,(2\,e^3x^5-17\,de}{16\,(e^3x^4-d^2e)}\right.$$

input
```
integrate((-e*x^2+d)^(9/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(75*(d^2*e^2*x^4 - d^4)*sqrt(e)*log((2*e^2*x^4 - d*e*x^2 + 2*sqrt(-e
^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(e)*x - d^2)/(e*x^2 - d)) + 2*(2*e^3*x^
5 - 17*d*e^2*x^3 - 83*d^2*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d))/(e^3
*x^4 - d^2*e), -1/8*(75*(d^2*e^2*x^4 - d^4)*sqrt(-e)*arctan(sqrt(-e^2*x^4
+ d^2)*sqrt(-e*x^2 + d)*sqrt(-e)*x/(e^2*x^4 - d^2)) - (2*e^3*x^5 - 17*d*e^
2*x^3 - 83*d^2*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d))/(e^3*x^4 - d^2*
e)]
```

Sympy [F]

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(d-ex^2)^{\frac{9}{2}}}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x**2+d)**(9/2)/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral((d - e*x**2)**(9/2)/(-(-d + e*x**2)*(d + e*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(-ex^2+d)^{\frac{9}{2}}}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(9/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e*x^2 + d)^(9/2)/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.41

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{75\,d^2\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{8\,\sqrt{e}} - \frac{((2\,e^2x^2-17\,de)x^2-83\,d^2)x}{8\,\sqrt{ex^2+d}}$$

input

```
integrate((-e*x^2+d)^(9/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output

```
75/8*d^2*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/sqrt(e) - 1/8*((2*e^2*x^2
- 17*d*e)*x^2 - 83*d^2)*x/sqrt(e*x^2 + d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(d-e\,x^2)^{9/2}}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input

```
int((d - e*x^2)^(9/2)/(d^2 - e^2*x^4)^(3/2),x)
```

output

```
int((d - e*x^2)^(9/2)/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{83\sqrt{e\,x^2+d}\,d^2ex+17\sqrt{e\,x^2+d}\,d\,e^2x^3-2\sqrt{e\,x^2+d}\,e^3x^5-75\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right.}{8e\,(e\,x^2+d)}$$

input

```
int((-e*x^2+d)^(9/2)/(-e^2*x^4+d^2)^(3/2),x)
```

output

```
(83*sqrt(d + e*x**2)*d**2*e*x + 17*sqrt(d + e*x**2)*d*e**2*x**3 - 2*sqrt(d
 + e*x**2)*e**3*x**5 - 75*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(
d))*d**3 - 75*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*d**2*e*x
**2 + 69*sqrt(e)*d**3 + 69*sqrt(e)*d**2*e*x**2)/(8*e*(d + e*x**2))
```

3.158 $\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 112

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{4dx\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}} + \frac{x\sqrt{d^2-e^2x^4}}{2\sqrt{d-ex^2}} - \frac{7d\,\text{arctanh}\left(\frac{\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{2\sqrt{e}}$$

output
```
4*d*x*(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2)+1/2*x*(-e^2*x^4+d^2)^(1/2)/(-e
*x^2+d)^(1/2)-7/2*d*arctanh(e^(1/2)*x*(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2
))/e^(1/2)
```

Mathematica [A] (verified)

Time = 5.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.13

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{1}{2}\left(\frac{x(9d+ex^2)\sqrt{d^2-e^2x^4}}{\sqrt{d-ex^2}\,(d+ex^2)} + \frac{7d\log(-d+ex^2)}{\sqrt{e}} - \frac{7d\log\left(dex-e^2x^3+\sqrt{e}\sqrt{d-ex\right.}}{\sqrt{e}}\right.$$

input
```
Integrate[(d - e*x^2)^(7/2)/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
((x*(9*d + e*x^2)*Sqrt[d^2 - e^2*x^4])/(Sqrt[d - e*x^2]*(d + e*x^2)) + (7*
d*Log[-d + e*x^2])/Sqrt[e] - (7*d*Log[d*e*x - e^2*x^3 + Sqrt[e]*Sqrt[d - e
*x^2]*Sqrt[d^2 - e^2*x^4]])/Sqrt[e])/2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1396, 315, 25, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-e x^2\right)^{7/2}}{\left(d^2-e^2 x^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-e x^2}\sqrt{d+e x^2}\int \frac{\left(d-e x^2\right)^2}{\left(e x^2+d\right)^{3/2}}dx}{\sqrt{d^2-e^2 x^4}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d-e x^2}\sqrt{d+e x^2}\left(\frac{\int -\frac{d e\left(d-5 e x^2\right)}{\sqrt{e x^2+d}}dx}{d e}+\frac{2 x\left(d-e x^2\right)}{\sqrt{d+e x^2}}\right)}{\sqrt{d^2-e^2 x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-e x^2}\sqrt{d+e x^2}\left(\frac{2 x\left(d-e x^2\right)}{\sqrt{d+e x^2}}-\frac{\int \frac{d e\left(d-5 e x^2\right)}{\sqrt{e x^2+d}}dx}{d e}\right)}{\sqrt{d^2-e^2 x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-e x^2}\sqrt{d+e x^2}\left(\frac{2 x\left(d-e x^2\right)}{\sqrt{d+e x^2}}-\int \frac{d-5 e x^2}{\sqrt{e x^2+d}}dx\right)}{\sqrt{d^2-e^2 x^4}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{7}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{5}{2}x\sqrt{d+ex^2}+\frac{2x(d-ex^2)}{\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 224$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{7}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{5}{2}x\sqrt{d+ex^2}+\frac{2x(d-ex^2)}{\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 219$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(-\frac{7d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{5}{2}x\sqrt{d+ex^2}+\frac{2x(d-ex^2)}{\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d - e*x^2)^(7/2)/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((2*x*(d - e*x^2))/Sqrt[d + e*x^2] + (5*x
*Sqrt[d + e*x^2])/2 - (7*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e
])))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

method	result	siz
default	$-\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(-e^{\frac{3}{2}} x^{3}+7 \ln \left(x \sqrt{e}+\sqrt{e\,x^{2}+d}\right) d \sqrt{e\,x^{2}+d}-9 \sqrt{e}\, d x \right)}{2 \sqrt{-e\,x^{2}+d}\, \left(e\,x^{2}+d \right) \sqrt{e}}$	83
risch	$-\frac{x \sqrt{e\,x^{2}+d}\, \sqrt{\frac{\left(-e\,x^{2}+d \right) \left(-e^{2} x^{4}+d^{2}\right)}{\left(e\,x^{2}-d \right)^{2}}}\, \left(e\,x^{2}-d \right)}{2 \sqrt{-e\,x^{2}+d}\, \sqrt{-e^{2} x^{4}+d^{2}}}-\frac{\left(\frac{4 d x}{\sqrt{e\,x^{2}+d}}-\frac{7 d \ln \left(x \sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2 \sqrt{e}}\right) \sqrt{\frac{\left(-e\,x^{2}+d \right) \left(-e^{2} x^{4}+d^{2}\right)}{\left(e\,x^{2}-d \right)^{2}}}\, \left(e\,x^{2}-d \right)}{\sqrt{-e\,x^{2}+d}\, \sqrt{-e^{2} x^{4}+d^{2}}}$	18

input

```
int((-e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*(-e^2*x^4+d^2)^(1/2)*(-e^(3/2)*x^3+7*ln(x*e^(1/2)+(e*x^2+d)^(1/2))*d*
(e*x^2+d)^(1/2)-9*e^(1/2)*d*x)/(-e*x^2+d)^(1/2)/(e*x^2+d)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.38

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \left[\frac{7\,(de^2x^4-d^3)\sqrt{e}\log\left(\frac{2\,e^2x^4-dex^2+2\,\sqrt{-e^2x^4+d^2}\,\sqrt{-ex^2+d}\,\sqrt{e}x-d^2}{ex^2-d}\right)-2\,\sqrt{-e^2x^4+d^2}(e^2x}{4\,(e^3x^4-d^2e)}\right.$$

input
```
integrate((-e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(7*(d*e^2*x^4 - d^3)*sqrt(e)*log((2*e^2*x^4 - d*e*x^2 + 2*sqrt(-e^2*x
^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(e)*x - d^2)/(e*x^2 - d)) - 2*sqrt(-e^2*x^4
 + d^2)*(e^2*x^3 + 9*d*e*x)*sqrt(-e*x^2 + d))/(e^3*x^4 - d^2*e), -1/2*(7*(
d*e^2*x^4 - d^3)*sqrt(-e)*arctan(sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqr
t(-e)*x/(e^2*x^4 - d^2)) + sqrt(-e^2*x^4 + d^2)*(e^2*x^3 + 9*d*e*x)*sqrt(-
e*x^2 + d))/(e^3*x^4 - d^2*e)]
```

Sympy [F]

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(d-ex^2)^{\frac{7}{2}}}{(-(-d+ex^2)\,(d+ex^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x**2+d)**(7/2)/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral((d - e*x**2)**(7/2)/(-(-d + e*x**2)*(d + e*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{(-ex^2+d)^{\frac{7}{2}}}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e*x^2 + d)^(7/2)/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.41

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{3/2}}\,dx=\frac{(ex^2+9\,d)x}{2\,\sqrt{ex^2+d}}+\frac{7\,d\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{2\,\sqrt{e}}$$

input
```
integrate((-e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(e*x^2 + 9*d)*x/sqrt(e*x^2 + d) + 7/2*d*log(abs(-sqrt(e)*x + sqrt(e*x^
2 + d)))/sqrt(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{(d-e\,x^2)^{7/2}}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input
```
int((d - e*x^2)^(7/2)/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
int((d - e*x^2)^(7/2)/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{36\sqrt{e\,x^2+d}\,dex + 4\sqrt{e\,x^2+d}\,e^2x^3 - 28\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)d^2 - 28\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+}}{\sqrt{}}\right.}{8e\,(e\,x^2+d)}$$

input
```
int((-e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
(36*sqrt(d + e*x**2)*d*e*x + 4*sqrt(d + e*x**2)*e**2*x**3 - 28*sqrt(e)*log
((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*d**2 - 28*sqrt(e)*log((sqrt(d + e
*x**2) + sqrt(e)*x)/sqrt(d))*d*e*x**2 + 33*sqrt(e)*d**2 + 33*sqrt(e)*d*e*x
**2)/(8*e*(d + e*x**2))
```

3.159 $\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 75

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{3/2}}\,dx=\frac{2x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{\sqrt{e}}$$

output
```
2*x*(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2)-arctanh(e^(1/2)*x*(-e*x^2+d)^(1/
2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [A] (verified)

Time = 4.89 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{3/2}}\,dx&=\frac{2x\sqrt{d^2-e^2x^4}}{\sqrt{d-ex^2}\,(d+ex^2)}+\frac{\log(-d+ex^2)}{\sqrt{e}}\\&\quad-\frac{\log\left(dex-e^2x^3+\sqrt{e}\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}\right)}{\sqrt{e}}\end{aligned}$$

input
```
Integrate[(d - e*x^2)^(5/2)/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(2*x*Sqrt[d^2 - e^2*x^4])/(Sqrt[d - e*x^2]*(d + e*x^2)) + Log[-d + e*x^2]/
Sqrt[e] - Log[d*e*x - e^2*x^3 + Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d^2 - e^2*x^4
]]/Sqrt[e]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1396, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-ex^2\right)^{5/2}}{\left(d^2-e^2x^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{d-ex^2}{\left(ex^2+d\right)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{298}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2x}{\sqrt{d+ex^2}}-\int \frac{1}{\sqrt{ex^2+d}}dx\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2x}{\sqrt{d+ex^2}}-\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2x}{\sqrt{d+ex^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d - e*x^2)^(5/2)/(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((2*x)/Sqrt[d + e*x^2] - ArcTanh[(Sqrt[e]
*x)/Sqrt[d + e*x^2]]/Sqrt[e]))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\left(-\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)\sqrt{e\,x^{2}+d}+2x\sqrt{e}\right)\sqrt{-e^{2}x^{4}+d^{2}}}{\sqrt{-e\,x^{2}+d}\,(e\,x^{2}+d)\sqrt{e}}$	72

input

```
int((-e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-ln(x*e^(1/2)+(e*x^2+d)^(1/2))*(e*x^2+d)^(1/2)+2*x*e^(1/2))*(-e^2*x^4+d^2
)^(1/2)/(-e*x^2+d)^(1/2)/(e*x^2+d)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.24

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \left[-\frac{4\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}ex-(e^2x^4-d^2)\sqrt{e}\log\left(\frac{2e^2x^4-dex^2+2\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+}}{ex^2-d}\right.}{2(e^3x^4-d^2e)}\right.$$

input
```
integrate((-e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(4*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*e*x - (e^2*x^4 - d^2)*sqrt(
e)*log((2*e^2*x^4 - d*e*x^2 + 2*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt
(e)*x - d^2)/(e*x^2 - d)))/(e^3*x^4 - d^2*e), -(2*sqrt(-e^2*x^4 + d^2)*sqr
t(-e*x^2 + d)*e*x + (e^2*x^4 - d^2)*sqrt(-e)*arctan(sqrt(-e^2*x^4 + d^2)*s
qrt(-e*x^2 + d)*sqrt(-e)*x/(e^2*x^4 - d^2)))/(e^3*x^4 - d^2*e)]
```

Sympy [F]

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(d-ex^2)^{\frac{5}{2}}}{(-(-d+ex^2)(d+ex^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x**2+d)**(5/2)/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral((d - e*x**2)**(5/2)/(-(-d + e*x**2)*(d + e*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(-ex^2+d)^{\frac{5}{2}}}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e*x^2 + d)^(5/2)/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.47

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{2\,x}{\sqrt{ex^2+d}} + \frac{\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{\sqrt{e}}$$

input
```
integrate((-e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
2*x/sqrt(e*x^2 + d) + log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/sqrt(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(d-e\,x^2)^{5/2}}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input
```
int((d - e*x^2)^(5/2)/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
int((d - e*x^2)^(5/2)/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{2\sqrt{e\,x^2+d}\,ex-\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)d-\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)e\,x^2+2\sqrt{e}\,d+2\sqrt{}}{e\,(e\,x^2+d)}$$

input

```
int((-e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(3/2),x)
```

output

```
(2*sqrt(d + e*x**2)*e*x - sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(
d))*d - sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*e*x**2 + 2*sqr
t(e)*d + 2*sqrt(e)*e*x**2)/(e*(d + e*x**2))
```

3.160 $\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 33

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{x\sqrt{d-ex^2}}{d\sqrt{d^2-e^2x^4}}$$

output
```
x*(-e*x^2+d)^(1/2)/d/(-e^2*x^4+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 4.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{x\sqrt{d^2-e^2x^4}}{\sqrt{d-ex^2}\,(d^2+dex^2)}$$

input
```
Integrate[(d - e*x^2)^(3/2)/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(x*Sqrt[d^2 - e^2*x^4])/(Sqrt[d - e*x^2]*(d^2 + d*e*x^2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1396, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(ex^2+d)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{208}$$

$$\frac{x\sqrt{d-ex^2}}{d\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d - e*x^2)^(3/2)/(d^2 - e^2*x^4)^(3/2),x]
```

output
```
(x*Sqrt[d - e*x^2])/(d*Sqrt[d^2 - e^2*x^4])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

method	result	size
gosper	$\frac{(e\,x^2+d)x(-e\,x^2+d)^{\frac{3}{2}}}{d(-e^2x^4+d^2)^{\frac{3}{2}}}$	37
orering	$\frac{(e\,x^2+d)x(-e\,x^2+d)^{\frac{3}{2}}}{d(-e^2x^4+d^2)^{\frac{3}{2}}}$	37
default	$\frac{\sqrt{-e^2x^4+d^2}\,x}{\sqrt{-e\,x^2+d}\,(e\,x^2+d)d}$	39

input
```
int((-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x^2+d)/d*x*(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{3/2}}\,dx = -\frac{\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}x}{de^2x^4-d^3}$$

input
```
integrate((-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*x/(d*e^2*x^4 - d^3)
```

Sympy [F]

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(d-ex^2)^{\frac{3}{2}}}{(-(-d+ex^2)(d+ex^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x**2+d)**(3/2)/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral((d - e*x**2)**(3/2)/(-(-d + e*x**2)*(d + e*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{(-ex^2+d)^{\frac{3}{2}}}{(-e^2x^4+d^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-e*x^2 + d)^(3/2)/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.42

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{x}{\sqrt{ex^2+d}d}$$

input
```
integrate((-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
x/(sqrt(e*x^2 + d)*d)
```

Mupad [B] (verification not implemented)

Time = 17.97 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{x\sqrt{d-e\,x^2}}{d\sqrt{d^2-e^2\,x^4}}$$

input
```
int((d - e*x^2)^(3/2)/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
(x*(d - e*x^2)^(1/2))/(d*(d^2 - e^2*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,ex+\sqrt{e}\,d+\sqrt{e}\,e\,x^2}{de\,(e\,x^2+d)}$$

input
```
int((-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*e*x + sqrt(e)*d + sqrt(e)*e*x**2)/(d*e*(d + e*x**2))
```

3.161 $\int \frac{\sqrt{d-ex^2}}{\left(d^2-e^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 95

$$\int \frac{\sqrt{d-ex^2}}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{x\sqrt{d-ex^2}}{2d^2\sqrt{d^2-e^2x^4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

output

```
1/2*x*(-e*x^2+d)^(1/2)/d^2/(-e^2*x^4+d^2)^(1/2)+1/4*arctanh(2^(1/2)*e^(1/2
)*x*(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))*2^(1/2)/d^2/e^(1/2)
```

Mathematica [A] (verified)

Time = 3.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{d-ex^2}}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{\sqrt{d^2-e^2x^4}\left(2\sqrt{e}x\sqrt{d+ex^2}+\sqrt{2}(d+ex^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)}{4d^2\sqrt{e}\sqrt{d-ex^2}\,(d+ex^2)^{3/2}}$$

input

```
Integrate[Sqrt[d - e*x^2]/(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(2*Sqrt[e]*x*Sqrt[d + e*x^2] + Sqrt[2]*(d + e*x^2)*Ar
cTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]))/(4*d^2*Sqrt[e]*Sqrt[d - e*x^2
]*(d + e*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1396, 296, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-ex^2}}{\left(d^2-e^2x^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)(ex^2+d)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{296}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{1}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx}{2d}+\frac{x}{2d^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2d}+\frac{x}{2d^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}+\frac{x}{2d^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[Sqrt[d - e*x^2]/(d^2 - e^2*x^4)^(3/2),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(2*d^2*Sqrt[d + e*x^2]) + ArcTanh[(Sqr
t[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(2*Sqrt[2]*d^2*Sqrt[e])))/Sqrt[d^2 - e^2*
x^4]
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(75) = 150$.

Time = 0.80 (sec) , antiderivative size = 343, normalized size of antiderivative = 3.61

method	result
default	$-\frac{\sqrt{-e^2x^4+d^2}\,e^2\left(\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{e\,x^2+d}+\sqrt{de}\,x+d\right)}{ex-\sqrt{de}}\right)\sqrt{2}\,de\,x^2-\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{e\,x^2+d}-\sqrt{de}\,x+d\right)}{ex+\sqrt{de}}\right)\sqrt{2}\,de\,x^2+4\sqrt{de}\,\sqrt{-\frac{(ex+\sqrt{-de}}{}}\right.}{4\sqrt{-e\,x^2+d}\,\sqrt{e\,x^2+d}\left(-\sqrt{de}+\sqrt{-de}\right)\left(\sqrt{de}+\sqrt{-de}\right)}$

input
```
int((-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-e^2*x^4+d^2)^(1/2)*e^2*(ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)+(d*
e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*2^(1/2)*d*e*x^2-ln(2*e*(2^(1/2)*d^(1/2)*(
e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2)))*2^(1/2)*d*e*x^2+4*(d*e)
^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)*d^(1/2)*x+ln(2*e*
(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*2^(1/
2)*d^2-ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)
^(1/2)))*2^(1/2)*d^2)/(-e*x^2+d)^(1/2)/(e*x^2+d)^(1/2)/(-(d*e)^(1/2)+(-d*e
)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)/d^(3/2)/(e*x-(-d*e)^(1/2))
/(e*x+(-d*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{d-ex^2}}{\left(d^2-e^2x^4\right)^{3/2}}\,dx=\left[-\frac{4\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}ex-\sqrt{2}(e^2x^4-d^2)\sqrt{e}\log\left(-\frac{3\,e^2x^4-2\,dex^2-2\,\sqrt{2}\sqrt{-e^2x^4+}{e^2x^4-2\,dex^2-}\right.}{8\left(d^2e^3x^4-d^4e\right)}\right.$$
$$\left.-\frac{2\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}ex-\sqrt{2}(e^2x^4-d^2)\sqrt{-e}\arctan\left(\frac{\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}\sqrt{-e}x}{e^2x^4-d^2}\right)}{4\left(d^2e^3x^4-d^4e\right)}\right]$$

input
```
integrate((-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(4*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*e*x - sqrt(2)*(e^2*x^4 - d^
2)*sqrt(e)*log(-(3*e^2*x^4 - 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sq
rt(-e*x^2 + d)*sqrt(e)*x - d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2)))/(d^2*e^3*x^4
 - d^4*e), -1/4*(2*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*e*x - sqrt(2)*(e^
2*x^4 - d^2)*sqrt(-e)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)
*sqrt(-e)*x/(e^2*x^4 - d^2)))/(d^2*e^3*x^4 - d^4*e)]
```

Sympy [F]

$$\int \frac{\sqrt{d-ex^2}}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{d-ex^2}}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x**2+d)**(1/2)/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral(sqrt(d - e*x**2)/(-(-d + e*x**2)*(d + e*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d-ex^2}}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-ex^2+d}}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e*x^2 + d)/(-e^2*x^4 + d^2)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d-ex^2}}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-ex^2+d}}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-e*x^2 + d)/(-e^2*x^4 + d^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-ex^2}}{(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{d-e\,x^2}}{(d^2-e^2\,x^4)^{3/2}}\,dx$$

input
```
int((d - e*x^2)^(1/2)/(d^2 - e^2*x^4)^(3/2),x)
```

output
```
int((d - e*x^2)^(1/2)/(d^2 - e^2*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.44

$$\int \frac{\sqrt{d-ex^2}}{(d^2-e^2x^4)^{3/2}}\,dx = \frac{4\sqrt{e\,x^2+d}\,ex-\sqrt{e}\,\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)d-\sqrt{e}\,\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-}{\sqrt{d}}\right.}{}$$

input
```
int((-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
(4*sqrt(d + e*x**2)*e*x - sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*
sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d - sqrt(e)*sqrt(2)*log((sqrt(d +
e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e*x**2 + sqrt(e)
*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sq
rt(d))*d + sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(
d) + sqrt(e)*x)/sqrt(d))*e*x**2 + sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) +
sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d + sqrt(e)*sqrt(2)*log((s
qrt(d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e*x**2 -
 sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(
e)*x)/sqrt(d))*d - sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2)
 + sqrt(d) + sqrt(e)*x)/sqrt(d))*e*x**2 + 4*sqrt(e)*d + 4*sqrt(e)*e*x**2)/
(8*d**2*e*(d + e*x**2))
```

3.162 $\int \frac{1}{\sqrt{d-ex^2}\left(d^2-e^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 131

$$\int \frac{1}{\sqrt{d-ex^2}\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{x}{4d^2\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}}+\frac{x\sqrt{d-ex^2}}{8d^3\sqrt{d^2-e^2x^4}}+\frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{8\sqrt{2}d^3\sqrt{e}}$$

output
```
1/4*x/d^2/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2)+1/8*x*(-e*x^2+d)^(1/2)/d^3
/(-e^2*x^4+d^2)^(1/2)+5/16*arctanh(2^(1/2)*e^(1/2)*x*(-e*x^2+d)^(1/2)/(-e^
2*x^4+d^2)^(1/2))*2^(1/2)/d^3/e^(1/2)
```

Mathematica [A] (verified)

Time = 2.96 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{d-ex^2}\left(d^2-e^2x^4\right)^{3/2}}\,dx=\frac{\sqrt{d^2-e^2x^4}\Big(-2\sqrt{e}x(-3d+ex^2)\sqrt{d+ex^2}+5\sqrt{2}(d^2-e^2x^4)\,\text{arctanh}\Big(}{16d^3\sqrt{e}\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{3/2}}$$

input
```
Integrate[1/(Sqrt[d - e*x^2]*(d^2 - e^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-2*Sqrt[e]*x*(-3*d + e*x^2)*Sqrt[d + e*x^2] + 5*Sqrt
[2]*(d^2 - e^2*x^4)*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]))/(16*d^3
*Sqrt[e]*(d - e*x^2)^(3/2)*(d + e*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1396, 316, 27, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d-ex^2}\left(d^2-e^2x^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\left(d-ex^2\right)^2\left(ex^2+d\right)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(2ex^2+3d\right)}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{4d^2e}+\frac{x}{4d^2(d-ex^2)\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{2ex^2+3d}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{4d^2}+\frac{x}{4d^2(d-ex^2)\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{x}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{5d^2e}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx}{2d^2e}}{4d^2}+\frac{x}{4d^2(d-ex^2)\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5}{2}\int \frac{1}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx+\frac{x}{2d\sqrt{d+ex^2}}}{4d^2}+\frac{x}{4d^2(d-ex^2)\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 291$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5}{2}\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{x}{2d\sqrt{d+ex^2}}}{4d^2}+\frac{x}{4d^2(d-ex^2)\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 221$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d\sqrt{e}}+\frac{x}{2d\sqrt{d+ex^2}}}{4d^2}+\frac{x}{4d^2(d-ex^2)\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[1/(Sqrt[d - e*x^2]*(d^2 - e^2*x^4)^(3/2)),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(4*d^2*(d - e*x^2)*Sqrt[d + e*x^2]) +
(x/(2*d*Sqrt[d + e*x^2]) + (5*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/(2*Sqrt[2]*d*Sqrt[e]))/(4*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(105) = 210$.

Time = 0.86 (sec) , antiderivative size = 1068, normalized size of antiderivative = 8.15

method	result	size
default	Expression too large to display	1068

input
```
int(1/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-e^2*x^4+d^2)^(1/2)*e^(11/2)*(-5*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*
x^2+d)^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*e^(7/2)*x^6*d^(1/2)+5*2^(
1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(
1/2)))*e^(7/2)*x^6*d^(1/2)-5*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/
2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*d^(3/2)*e^(5/2)*x^4+5*2^(1/2)*ln(2*
e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2)))*d^(
3/2)*e^(5/2)*x^4-8*ln((e^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))
^(1/2)+e*x)/e^(1/2))*e^3*x^6*(d*e)^(1/2)+8*ln(((e*x^2+d)^(1/2)*e^(1/2)+e*x
)/e^(1/2))*e^3*x^6*(d*e)^(1/2)+4*e^(5/2)*x^5*(e*x^2+d)^(1/2)*(d*e)^(1/2)-8
*e^(5/2)*x^5*(d*e)^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)
+5*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(
d*e)^(1/2)))*d^(5/2)*e^(3/2)*x^2-5*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+
d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2)))*d^(5/2)*e^(3/2)*x^2-8*ln((e^(
1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)+e*x)/e^(1/2))*d*e^2
*x^4*(d*e)^(1/2)+8*ln(((e*x^2+d)^(1/2)*e^(1/2)+e*x)/e^(1/2))*d*e^2*x^4*(d*
e)^(1/2)+8*d*e^(3/2)*x^3*(e*x^2+d)^(1/2)*(d*e)^(1/2)+5*2^(1/2)*ln(2*e*(2^(
1/2)*d^(1/2)*(e*x^2+d)^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*d^(7/2)*e
^(1/2)-5*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/
(e*x+(d*e)^(1/2)))*d^(7/2)*e^(1/2)+8*ln((e^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-
e*x+(-d*e)^(1/2)))^(1/2)+e*x)/e^(1/2))*d^2*e*x^2*(d*e)^(1/2)-8*ln(((e*x...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.81

$$\int \frac{1}{\sqrt{d-ex^2}\,(d^2-e^2x^4)^{3/2}}\,dx = \left[\frac{5\sqrt{2}(e^3x^6-de^2x^4-d^2ex^2+d^3)\sqrt{e}\log\left(-\frac{3e^2x^4-2dex^2-2\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{-e}}{e^2x^4-2dex^2+d^2}\right.}{32\,(d^3e^4x^6-d^4e^3x^4-d^5e}\right.$$

input
```
integrate(1/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output

```
[1/32*(5*sqrt(2)*(e^3*x^6 - d*e^2*x^4 - d^2*e*x^2 + d^3)*sqrt(e)*log(-(3*e
^2*x^4 - 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(
e)*x - d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2)) - 4*sqrt(-e^2*x^4 + d^2)*(e^2*x^3
 - 3*d*e*x)*sqrt(-e*x^2 + d))/(d^3*e^4*x^6 - d^4*e^3*x^4 - d^5*e^2*x^2 + d
^6*e), 1/16*(5*sqrt(2)*(e^3*x^6 - d*e^2*x^4 - d^2*e*x^2 + d^3)*sqrt(-e)*ar
ctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(-e)*x/(e^2*x^4 - d
^2)) - 2*sqrt(-e^2*x^4 + d^2)*(e^2*x^3 - 3*d*e*x)*sqrt(-e*x^2 + d))/(d^3*e
^4*x^6 - d^4*e^3*x^4 - d^5*e^2*x^2 + d^6*e)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d-ex^2}\,(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}\sqrt{d-ex^2}}\,dx$$

input

```
integrate(1/(-e*x**2+d)**(1/2)/(-e**2*x**4+d**2)**(3/2),x)
```

output

```
Integral(1/((-(-d + e*x**2)*(d + e*x**2))**(3/2)*sqrt(d - e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d-ex^2}\,(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}\sqrt{-ex^2+d}}\,dx$$

input

```
integrate(1/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output

```
integrate(1/((-e^2*x^4 + d^2)^(3/2)*sqrt(-e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d-ex^2}\,(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{1}{(-e^2x^4+d^2)^{\frac{3}{2}}\sqrt{-ex^2+d}}\,dx$$

input
```
integrate(1/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(3/2)*sqrt(-e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d-ex^2}\,(d^2-e^2x^4)^{3/2}}\,dx=\int \frac{1}{(d^2-e^2\,x^4)^{3/2}\,\sqrt{d-e\,x^2}}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)^(1/2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.85

$$\int \frac{1}{\sqrt{d-ex^2}\,(d^2-e^2x^4)^{3/2}}\,dx=\frac{12\sqrt{e\,x^2+d}\,dex-4\sqrt{e\,x^2+d}\,e^2x^3-5\sqrt{e}\,\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)}{}$$

input
```
int(1/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(3/2),x)
```

output

```
(12*sqrt(d + e*x**2)*d*e*x - 4*sqrt(d + e*x**2)*e**2*x**3 - 5*sqrt(e)*sqrt
(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d)
)*d**2 + 5*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(
d) + sqrt(e)*x)/sqrt(d))*e**2*x**4 + 5*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**
2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*d**2 - 5*sqrt(e)*sqrt
(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d)
)*e**2*x**4 + 5*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) -
sqrt(d) + sqrt(e)*x)/sqrt(d))*d**2 - 5*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**
2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e**2*x**4 - 5*sqrt(e)
*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sq
rt(d))*d**2 + 5*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) +
sqrt(d) + sqrt(e)*x)/sqrt(d))*e**2*x**4 - 12*sqrt(e)*d**2 + 12*sqrt(e)*e**
2*x**4)/(32*d**3*e*(d**2 - e**2*x**4))
```

3.163 $\int \frac{1}{(d-ex^2)^{3/2}(d^2-e^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 167

$$\int \frac{1}{(d-ex^2)^{3/2}(d^2-e^2x^4)^{3/2}}\,dx = \frac{x}{8d^2(d-ex^2)^{3/2}\sqrt{d^2-e^2x^4}} + \frac{11x}{32d^3\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}} - \frac{5x\sqrt{d-ex^2}}{64d^4\sqrt{d^2-e^2x^4}} + \frac{39\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{64\sqrt{2}d^4\sqrt{e}}$$

output
```
1/8*x/d^2/(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(1/2)+11/32*x/d^3/(-e*x^2+d)^(1/
2)/(-e^2*x^4+d^2)^(1/2)-5/64*x*(-e*x^2+d)^(1/2)/d^4/(-e^2*x^4+d^2)^(1/2)+3
9/128*arctanh(2^(1/2)*e^(1/2)*x*(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))*2^(
1/2)/d^4/e^(1/2)
```

Mathematica [A] (verified)

Time = 4.81 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{1}{(d-ex^2)^{3/2}(d^2-e^2x^4)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^4}\Bigl(-2\sqrt{e}x\sqrt{d+ex^2}(-25d^2+12dex^2+5e^2x^4)+39\sqrt{2}(d-}{128d^4\sqrt{e}(d-ex^2)^{5/2}(d+ex^2)^{3/2}}$$

input
```
Integrate[1/((d - e*x^2)^(3/2)*(d^2 - e^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(-2*Sqrt[e]*x*Sqrt[d + e*x^2]*(-25*d^2 + 12*d*e*x^2 +
 5*e^2*x^4) + 39*Sqrt[2]*(d - e*x^2)^2*(d + e*x^2)*ArcTanh[(Sqrt[2]*Sqrt[e
]*x)/Sqrt[d + e*x^2]]))/(128*d^4*Sqrt[e]*(d - e*x^2)^(5/2)*(d + e*x^2)^(3/
2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1396, 316, 27, 402, 27, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(d-ex^2\right)^{3/2}\left(d^2-e^2x^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\left(d-ex^2\right)^3\left(ex^2+d\right)^{3/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(4ex^2+7d\right)}{\left(d-ex^2\right)^2\left(ex^2+d\right)^{3/2}}dx}{8d^2e}+\frac{x}{8d^2\left(d-ex^2\right)^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{4ex^2+7d}{\left(d-ex^2\right)^2\left(ex^2+d\right)^{3/2}}dx}{8d^2}+\frac{x}{8d^2\left(d-ex^2\right)^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{de\left(22ex^2+17d\right)}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{4d^2e}+\frac{11x}{4d\left(d-ex^2\right)\sqrt{d+ex^2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{22ex^2+17d}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{4d}+\frac{11x}{4d\left(d-ex^2\right)\sqrt{d+ex^2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{-\frac{\int -\frac{39d^2e}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx}{2d^2e}-\frac{5x}{2d\sqrt{d+ex^2}}}{4d}+\frac{11x}{4d\left(d-ex^2\right)\sqrt{d+ex^2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{39}{2}\int \frac{1}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx-\frac{5x}{2d\sqrt{d+ex^2}}}{4d}+\frac{11x}{4d\left(d-ex^2\right)\sqrt{d+ex^2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 291$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{39}{2}\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{5x}{2d\sqrt{d+ex^2}}}{4d}+\frac{11x}{4d\left(d-ex^2\right)\sqrt{d+ex^2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{39\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d\sqrt{e}}-\frac{5x}{2d\sqrt{d+ex^2}}}{4d}+\frac{11x}{4d\left(d-ex^2\right)\sqrt{d+ex^2}}}{8d^2}+\frac{x}{8d^2\left(d-ex^2\right)^2\sqrt{d+ex^2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[1/((d - e*x^2)^(3/2)*(d^2 - e^2*x^4)^(3/2)),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(8*d^2*(d - e*x^2)^2*Sqrt[d + e*x^2])
+ ((11*x)/(4*d*(d - e*x^2)*Sqrt[d + e*x^2]) + ((-5*x)/(2*d*Sqrt[d + e*x^2]
) + (39*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[2]*d*Sqrt[e]
))/(4*d))/(8*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(135) = 270$.

Time = 0.88 (sec) , antiderivative size = 981, normalized size of antiderivative = 5.87

method	result
default	$\sqrt{-e^2x^4+d^2}\, e^{\frac{15}{2}} \left(39\sqrt{2}\, \ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{e\,x^2+d}+\sqrt{de}\,x+d\right)}{ex-\sqrt{de}}\right) e^{\frac{9}{2}} x^8 \sqrt{d} - 39\sqrt{2}\, \ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{e\,x^2+d}-\sqrt{de}\,x+d\right)}{ex+\sqrt{de}}\right) e^{\frac{9}{2}} x^8 \sqrt{d} - 48 \ln\left(\sqrt{\right.\right.$

input
```
int(1/(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(-e^2*x^4+d^2)^(1/2)*e^(15/2)*(39*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*
x^2+d)^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*e^(9/2)*x^8*d^(1/2)-39*2^
(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^
(1/2)))*e^(9/2)*x^8*d^(1/2)-48*ln(((e*x^2+d)^(1/2)*e^(1/2)+e*x)/e^(1/2))*e
^4*x^8*(d*e)^(1/2)+48*ln((e^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2
)))^(1/2)+e*x)/e^(1/2))*e^4*x^8*(d*e)^(1/2)-52*e^(7/2)*x^7*(d*e)^(1/2)*(e*
x^2+d)^(1/2)+32*e^(7/2)*x^7*(d*e)^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e
)^(1/2)))^(1/2)-78*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)+(d*e)^(
1/2)*x+d)/(e*x-(d*e)^(1/2)))*d^(5/2)*e^(5/2)*x^4+78*2^(1/2)*ln(2*e*(2^(1/2
)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2)))*d^(5/2)*e^(5
/2)*x^4-36*d*e^(5/2)*x^5*(d*e)^(1/2)*(e*x^2+d)^(1/2)-32*d*e^(5/2)*(d*e)^(1
/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)*x^5+96*ln(((e*x^2+d)
^(1/2)*e^(1/2)+e*x)/e^(1/2))*d^2*e^2*x^4*(d*e)^(1/2)-96*ln((e^(1/2)*(-(e*x
+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)+e*x)/e^(1/2))*d^2*e^2*x^4*(d*e
)^(1/2)+84*d^2*e^(3/2)*x^3*(d*e)^(1/2)*(e*x^2+d)^(1/2)-32*d^2*e^(3/2)*(d*e
)^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)*x^3+39*2^(1/2)*l
n(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))
*d^(9/2)*e^(1/2)-39*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^
(1/2)*x+d)/(e*x+(d*e)^(1/2)))*d^(9/2)*e^(1/2)+68*d^3*x*e^(1/2)*(d*e)^(1/2)
*(e*x^2+d)^(1/2)+32*d^3*(d*e)^(1/2)*e^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.38

$$\int \frac{1}{\left(d-ex^2\right)^{3/2}\left(d^2-e^2x^4\right)^{3/2}}\,dx = \left[\frac{39\,\sqrt{2}(e^4x^8-2\,de^3x^6+2\,d^3ex^2-d^4)\sqrt{e}\log\left(-\frac{3\,e^2x^4-2\,dex^2-2\,\sqrt{2}\sqrt{-e^2x^4}}{e^2x^4-2\,de}\right.}{256\,(d^4e^5x^8-2}\right.$$

input
```
integrate(1/(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/256*(39*sqrt(2)*(e^4*x^8 - 2*d*e^3*x^6 + 2*d^3*e*x^2 - d^4)*sqrt(e)*log
(-(3*e^2*x^4 - 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)
*sqrt(e)*x - d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2)) + 4*(5*e^3*x^5 + 12*d*e^2*x
^3 - 25*d^2*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d))/(d^4*e^5*x^8 - 2*d
^5*e^4*x^6 + 2*d^7*e^2*x^2 - d^8*e), 1/128*(39*sqrt(2)*(e^4*x^8 - 2*d*e^3*
x^6 + 2*d^3*e*x^2 - d^4)*sqrt(-e)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt
(-e*x^2 + d)*sqrt(-e)*x/(e^2*x^4 - d^2)) + 2*(5*e^3*x^5 + 12*d*e^2*x^3 - 2
5*d^2*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d))/(d^4*e^5*x^8 - 2*d^5*e^4
*x^6 + 2*d^7*e^2*x^2 - d^8*e)]
```

Sympy [F]

$$\int \frac{1}{(d-ex^2)^{3/2}\,(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{3}{2}}\left(d-ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e*x**2+d)**(3/2)/(-e**2*x**4+d**2)**(3/2),x)
```

output
```
Integral(1/((-(-d + e*x**2)*(d + e*x**2))**(3/2)*(d - e*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d-ex^2)^{3/2}\,(d^2-e^2x^4)^{3/2}}\,dx = \int \frac{1}{(-e^2x^4+d^2)^{\frac{3}{2}}(-ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(3/2)*(-e*x^2 + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d-ex^2)^{3/2}\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-e^2x^4+d^2\right)^{\frac{3}{2}}\left(-ex^2+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(3/2)*(-e*x^2 + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d-ex^2)^{3/2}\left(d^2-e^2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(d^2-e^2\,x^4\right)^{3/2}\left(d-e\,x^2\right)^{3/2}}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)^(3/2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(3/2)*(d - e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 755, normalized size of antiderivative = 4.52

$$\int \frac{1}{(d-ex^2)^{3/2}\left(d^2-e^2x^4\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(1/(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2),x)
```

output
```
(1100*sqrt(d + e*x**2)*d**2*e*x - 528*sqrt(d + e*x**2)*d*e**2*x**3 - 220*s
qrt(d + e*x**2)*e**3*x**5 - 429*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sq
rt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d**3 + 429*sqrt(e)*sqrt(2)*l
og((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d**
2*e*x**2 + 429*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - s
qrt(d) + sqrt(e)*x)/sqrt(d))*d*e**2*x**4 - 429*sqrt(e)*sqrt(2)*log((sqrt(d
 + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e**3*x**6 + 4
29*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqr
t(e)*x)/sqrt(d))*d**3 - 429*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d
)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*d**2*e*x**2 - 429*sqrt(e)*sqrt(2
)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*
d*e**2*x**4 + 429*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2)
+ sqrt(d) + sqrt(e)*x)/sqrt(d))*e**3*x**6 + 429*sqrt(e)*sqrt(2)*log((sqrt(
d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d**3 - 429*s
qrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)
*x)/sqrt(d))*d**2*e*x**2 - 429*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqr
t(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d*e**2*x**4 + 429*sqrt(e)*sqr
t(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d
))*e**3*x**6 - 429*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2)
 + sqrt(d) + sqrt(e)*x)/sqrt(d))*d**3 + 429*sqrt(e)*sqrt(2)*log((sqrt(d...
```

3.164 $\int \frac{(d-ex^2)^{13/2}}{(d^2-e^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 191

$$\int \frac{(d-ex^2)^{13/2}}{(d^2-e^2x^4)^{5/2}}\,dx=\frac{16d^3x(d-ex^2)^{3/2}}{3\left(d^2-e^2x^4\right)^{3/2}}-\frac{64d^2x\sqrt{d-ex^2}}{3\sqrt{d^2-e^2x^4}}-\frac{27dx\sqrt{d^2-e^2x^4}}{8\sqrt{d-ex^2}}+\frac{ex^3\sqrt{d^2-e^2x^4}}{4\sqrt{d-ex^2}}+\frac{163d^2\operatorname{arctanh}\left(\frac{\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{8\sqrt{e}}$$

output
```
16/3*d^3*x*(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2)-64/3*d^2*x*(-e*x^2+d)^(1/
2)/(-e^2*x^4+d^2)^(1/2)-27/8*d*x*(-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^(1/2)+1/4
*e*x^3*(-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^(1/2)+163/8*d^2*arctanh(e^(1/2)*x*(
-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [A] (verified)

Time = 5.76 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.81

$$\int \frac{(d-ex^2)^{13/2}}{(d^2-e^2x^4)^{5/2}}\,dx=\frac{1}{24}\left(-\frac{x\sqrt{d^2-e^2x^4}(465d^3+668d^2ex^2+69de^2x^4-6e^3x^6)}{\sqrt{d-ex^2}\,(d+ex^2)^2}-\frac{489d^2\log\left(-d+ex^2\right)}{\sqrt{e}}-\right.$$

input
```
Integrate[(d - e*x^2)^(13/2)/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(-((x*Sqrt[d^2 - e^2*x^4]*(465*d^3 + 668*d^2*e*x^2 + 69*d*e^2*x^4 - 6*e^3*
x^6))/(Sqrt[d - e*x^2]*(d + e*x^2)^2)) - (489*d^2*Log[-d + e*x^2])/Sqrt[e]
 + (489*d^2*Log[d*e*x - e^2*x^3 + Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d^2 - e^2*x
^4]])/Sqrt[e])/24
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {1396, 315, 27, 401, 25, 27, 403, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-ex^2\right)^{13/2}}{\left(d^2-e^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\left(d-ex^2\right)^4}{\left(ex^2+d\right)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{de\left(d-ex^2\right)^2\left(11ex^2+d\right)}{\left(ex^2+d\right)^{3/2}}dx}{3de}+\frac{2x\left(d-ex^2\right)^3}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\int \frac{\left(d-ex^2\right)^2\left(11ex^2+d\right)}{\left(ex^2+d\right)^{3/2}}dx+\frac{2x\left(d-ex^2\right)^3}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{401}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(-\frac{\int -\frac{de\left(11d-51ex^2\right)\left(d-ex^2\right)}{\sqrt{ex^2+d}}dx}{de}-\frac{10x\left(d-ex^2\right)^2}{\sqrt{d+ex^2}}\right)+\frac{2x\left(d-ex^2\right)^3}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(\frac{\int \frac{de\left(11d-51ex^2\right)\left(d-ex^2\right)}{\sqrt{ex^2+d}}dx}{de}-\frac{10x\left(d-ex^2\right)^2}{\sqrt{d+ex^2}}\right)+\frac{2x\left(d-ex^2\right)^3}{3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(\int \frac{(11d-51ex^2)\left(d-ex^2\right)}{\sqrt{ex^2+d}}dx-\frac{10x\left(d-ex^2\right)^2}{\sqrt{d+ex^2}}\right)+\frac{2x\left(d-ex^2\right)^3}{3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(\frac{\int \frac{de\left(55d-379ex^2\right)}{\sqrt{ex^2+d}}dx}{4e}-\frac{10x\left(d-ex^2\right)^2}{\sqrt{d+ex^2}}-\frac{1}{4}x\left(11d-51ex^2\right)\sqrt{d+ex^2}\right)+\frac{2x\left(d-ex^2\right)^3}{3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(\frac{1}{4}d\int \frac{55d-379ex^2}{\sqrt{ex^2+d}}dx-\frac{10x\left(d-ex^2\right)^2}{\sqrt{d+ex^2}}-\frac{1}{4}x\left(11d-51ex^2\right)\sqrt{d+ex^2}\right)+\frac{2x\left(d-ex^2\right)^3}{3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{1}{3}\left(\frac{1}{4}d\left(\frac{489}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{379}{2}x\sqrt{d+ex^2}\right)-\frac{10x\left(d-ex^2\right)^2}{\sqrt{d+ex^2}}-\frac{1}{4}x\left(11d-51ex^2\right)\sqrt{d+ex^2}\right)+}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{1}{3}\Bigg(\frac{1}{4}d\left(\frac{489}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{379}{2}x\sqrt{d+ex^2}\right)-\frac{10x\left(d-ex^2\right)^2}{\sqrt{d+ex^2}}-\frac{1}{4}x\left(11d-51ex^2\right)\sqrt{d+e}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\Bigg(\frac{1}{3}\Bigg(\frac{1}{4}d\left(\frac{489d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}-\frac{379}{2}x\sqrt{d+ex^2}\right)-\frac{10x\left(d-ex^2\right)^2}{\sqrt{d+ex^2}}-\frac{1}{4}x\left(11d-51ex^2\right)\sqrt{d+e}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d - e*x^2)^(13/2)/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((2*x*(d - e*x^2)^3)/(3*(d + e*x^2)^(3/2)
) + ((-10*x*(d - e*x^2)^2)/Sqrt[d + e*x^2] - (x*(11*d - 51*e*x^2)*Sqrt[d +
 e*x^2])/4 + (d*((-379*x*Sqrt[d + e*x^2])/2 + (489*d*ArcTanh[(Sqrt[e]*x)/S
qrt[d + e*x^2]])/(2*Sqrt[e])))/4)/3))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.74

method	result
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(6 x^{7} e^{\frac{7}{2}}-69 e^{\frac{5}{2}} d\, x^{5}+489 \ln \left(x \sqrt{e}+\sqrt{e\, x^{2}+d}\right) d^{2} e\, x^{2} \sqrt{e\, x^{2}+d}-668 e^{\frac{3}{2}} d^{2} x^{3}+489 \ln \left(x \sqrt{e}+\sqrt{e\, x^{2}+d}\right) d^{3} \sqrt{e\, x^{2}+d}-465\right.}{24 \sqrt{-e\, x^{2}+d}\, \left(e\, x^{2}+d\right)^{2} \sqrt{e}}$
risch	$\frac{x \left(-2 e\, x^{2}+27 d\right) \sqrt{e\, x^{2}+d}\, \sqrt{\frac{\left(-e\, x^{2}+d\right) \left(-e^{2} x^{4}+d^{2}\right)}{\left(e\, x^{2}-d\right)^{2}}}\, \left(e\, x^{2}-d\right)}{8 \sqrt{-e\, x^{2}+d}\, \sqrt{-e^{2} x^{4}+d^{2}}}-\frac{\left(\frac{163 d^{2} \ln \left(x \sqrt{e}+\sqrt{e\, x^{2}+d}\right)}{8 \sqrt{e}}+\frac{4 d^{3} \sqrt{\left(x-\frac{\sqrt{-d e}}{e}\right)^{2} e+2 \sqrt{-d e}\, \left(x-\frac{\sqrt{-d e}}{e}\right.}}{3 e \sqrt{-d e}\, \left(x-\frac{\sqrt{-d e}}{e}\right)^{2}}\right.}{}$

input
```
int((-e*x^2+d)^(13/2)/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-e^2*x^4+d^2)^(1/2)*(6*x^7*e^(7/2)-69*e^(5/2)*d*x^5+489*ln(x*e^(1/2)
+(e*x^2+d)^(1/2))*d^2*e*x^2*(e*x^2+d)^(1/2)-668*e^(3/2)*d^2*x^3+489*ln(x*e
^(1/2)+(e*x^2+d)^(1/2))*d^3*(e*x^2+d)^(1/2)-465*e^(1/2)*d^3*x)/(-e*x^2+d)^
(1/2)/(e*x^2+d)^2/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.06

$$\int \frac{(d-ex^2)^{13/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \left[\frac{489\left(d^2e^3x^6+d^3e^2x^4-d^4ex^2-d^5\right)\sqrt{e}\log\left(\frac{2\,e^2x^4-dex^2-2\,\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}\sqrt{e}x-d^2}{ex^2-d}\right)}{48\left(e^4x^6+de^3x^4-d\right.}\right.$$

input
```
integrate((-e*x^2+d)^(13/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(489*(d^2*e^3*x^6 + d^3*e^2*x^4 - d^4*e*x^2 - d^5)*sqrt(e)*log((2*e^
2*x^4 - d*e*x^2 - 2*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(e)*x - d^2)
/(e*x^2 - d)) - 2*(6*e^4*x^7 - 69*d*e^3*x^5 - 668*d^2*e^2*x^3 - 465*d^3*e*
x)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d))/(e^4*x^6 + d*e^3*x^4 - d^2*e^2*x
^2 - d^3*e), 1/24*(489*(d^2*e^3*x^6 + d^3*e^2*x^4 - d^4*e*x^2 - d^5)*sqrt(
-e)*arctan(sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(-e)*x/(e^2*x^4 - d^2
)) - (6*e^4*x^7 - 69*d*e^3*x^5 - 668*d^2*e^2*x^3 - 465*d^3*e*x)*sqrt(-e^2*
x^4 + d^2)*sqrt(-e*x^2 + d))/(e^4*x^6 + d*e^3*x^4 - d^2*e^2*x^2 - d^3*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^{13/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((-e*x**2+d)**(13/2)/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d-ex^2)^{13/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(-ex^2+d)^{\frac{13}{2}}}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(13/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-e*x^2 + d)^(13/2)/(-e^2*x^4 + d^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.40

$$\int \frac{(d-ex^2)^{13/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{1}{8}\left(2\,ex^2-27\,d\right)\sqrt{ex^2+d}x - \frac{163\,d^2\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{8\,\sqrt{e}} - \frac{16\,(4\,d^2ex^2+3\,d^3)x}{3\,(ex^2+d)^{\frac{3}{2}}}$$

input
```
integrate((-e*x^2+d)^(13/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
1/8*(2*e*x^2 - 27*d)*sqrt(e*x^2 + d)*x - 163/8*d^2*log(abs(-sqrt(e)*x + sq
rt(e*x^2 + d)))/sqrt(e) - 16/3*(4*d^2*e*x^2 + 3*d^3)*x/(e*x^2 + d)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^{13/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(d-e\,x^2)^{13/2}}{(d^2-e^2\,x^4)^{5/2}}\,dx$$

input
```
int((d - e*x^2)^(13/2)/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
int((d - e*x^2)^(13/2)/(d^2 - e^2*x^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.11

$$\int \frac{(d-ex^2)^{13/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{-3720\sqrt{e\,x^2+d}\,d^3ex-5344\sqrt{e\,x^2+d}\,d^2e^2x^3-552\sqrt{e\,x^2+d}\,d\,e^3x^5+48\sqrt{e\,x^2+}}{}$$

input
```
int((-e*x^2+d)^(13/2)/(-e^2*x^4+d^2)^(5/2),x)
```

output
```
( - 3720*sqrt(d + e*x**2)*d**3*e*x - 5344*sqrt(d + e*x**2)*d**2*e**2*x**3
- 552*sqrt(d + e*x**2)*d*e**3*x**5 + 48*sqrt(d + e*x**2)*e**4*x**7 + 3912*
sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*d**4 + 7824*sqrt(e)*lo
g((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*d**3*e*x**2 + 3912*sqrt(e)*log((
sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*d**2*e**2*x**4 + 111*sqrt(e)*d**4 +
 222*sqrt(e)*d**3*e*x**2 + 111*sqrt(e)*d**2*e**2*x**4)/(192*e*(d**2 + 2*d*
e*x**2 + e**2*x**4))
```

3.165 $\int \frac{(d-ex^2)^{11/2}}{(d^2-e^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 150

$$\int \frac{(d-ex^2)^{11/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{8d^2x(d-ex^2)^{3/2}}{3\,(d^2-e^2x^4)^{3/2}} - \frac{20dx\sqrt{d-ex^2}}{3\sqrt{d^2-e^2x^4}} - \frac{x\sqrt{d^2-e^2x^4}}{2\sqrt{d-ex^2}} + \frac{11d\,\mathrm{arctanh}\left(\frac{\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{2\sqrt{e}}$$

output
```
8/3*d^2*x*(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2)-20/3*d*x*(-e*x^2+d)^(1/2)/
(-e^2*x^4+d^2)^(1/2)-1/2*x*(-e^2*x^4+d^2)^(1/2)/(-e*x^2+d)^(1/2)+11/2*d*ar
ctanh(e^(1/2)*x*(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(1/2)
```

Mathematica [A] (verified)

Time = 5.63 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int \frac{(d-ex^2)^{11/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{1}{6}\left(-\frac{x\sqrt{d^2-e^2x^4}(27d^2+46dex^2+3e^2x^4)}{\sqrt{d-ex^2}\,(d+ex^2)^2} - \frac{33d\log(-d+ex^2)}{\sqrt{e}} + \frac{33d\log\left(dex - e\right.}{}\right.$$

input
```
Integrate[(d - e*x^2)^(11/2)/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(-((x*Sqrt[d^2 - e^2*x^4]*(27*d^2 + 46*d*e*x^2 + 3*e^2*x^4))/(Sqrt[d - e*x
^2]*(d + e*x^2)^2)) - (33*d*Log[-d + e*x^2])/Sqrt[e] + (33*d*Log[d*e*x - e
^2*x^3 + Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d^2 - e^2*x^4]])/Sqrt[e])/6
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1396, 315, 27, 401, 25, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-ex^2\right)^{11/2}}{\left(d^2-e^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{\left(d-ex^2\right)^3}{\left(ex^2+d\right)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{de\left(d-ex^2\right)\left(7ex^2+d\right)}{\left(ex^2+d\right)^{3/2}}dx}{3de}+\frac{2x\left(d-ex^2\right)^2}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\int \frac{\left(d-ex^2\right)\left(7ex^2+d\right)}{\left(ex^2+d\right)^{3/2}}dx+\frac{2x\left(d-ex^2\right)^2}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{401}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(-\frac{\int -\frac{de\left(7d-19ex^2\right)}{\sqrt{ex^2+d}}dx}{de}-\frac{6x\left(d-ex^2\right)}{\sqrt{d+ex^2}}\right)+\frac{2x\left(d-ex^2\right)^2}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(\frac{\int \frac{de\left(7d-19ex^2\right)}{\sqrt{ex^2+d}}dx}{de}-\frac{6x\left(d-ex^2\right)}{\sqrt{d+ex^2}}\right)+\frac{2x\left(d-ex^2\right)^2}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(\int \frac{7d-19ex^2}{\sqrt{ex^2+d}}dx-\frac{6x\left(d-ex^2\right)}{\sqrt{d+ex^2}}\right)+\frac{2x\left(d-ex^2\right)^2}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 299$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(\frac{33}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{19}{2}x\sqrt{d+ex^2}-\frac{6x\left(d-ex^2\right)}{\sqrt{d+ex^2}}\right)+\frac{2x\left(d-ex^2\right)^2}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 224$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(\frac{33}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{19}{2}x\sqrt{d+ex^2}-\frac{6x\left(d-ex^2\right)}{\sqrt{d+ex^2}}\right)+\frac{2x\left(d-ex^2\right)^2}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 219$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(\frac{33d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}-\frac{19}{2}x\sqrt{d+ex^2}-\frac{6x\left(d-ex^2\right)}{\sqrt{d+ex^2}}\right)+\frac{2x\left(d-ex^2\right)^2}{3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d - e*x^2)^(11/2)/(d^2 - e^2*x^4)^(5/2),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((2*x*(d - e*x^2)^2)/(3*(d + e*x^2)^(3/2)
) + ((-6*x*(d - e*x^2))/Sqrt[d + e*x^2] - (19*x*Sqrt[d + e*x^2])/2 + (33*d
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e]))/3))/Sqrt[d^2 - e^2*x^4
]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

method	result
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(-3 e^{\frac{5}{2}} x^{5}+33 \ln \left(x \sqrt{e}+\sqrt{e\, x^{2}+d}\right) d e\, x^{2} \sqrt{e\, x^{2}+d}-46 d\, e^{\frac{3}{2}} x^{3}+33 \ln \left(x \sqrt{e}+\sqrt{e\, x^{2}+d}\right) d^{2} \sqrt{e\, x^{2}+d}-27 d^{2} x \sqrt{e}\right)}{6 \sqrt{-e\, x^{2}+d}\, \left(e\, x^{2}+d\right)^{2} \sqrt{e}}$
risch	$\frac{x \sqrt{e\, x^{2}+d}\, \sqrt{\frac{\left(-e\, x^{2}+d\right) \left(-e^{2} x^{4}+d^{2}\right)}{\left(e\, x^{2}-d\right)^{2}}}\, \left(e\, x^{2}-d\right)}{2 \sqrt{-e\, x^{2}+d}\, \sqrt{-e^{2} x^{4}+d^{2}}}-\Bigg(\frac{11 d \ln \left(x \sqrt{e}+\sqrt{e\, x^{2}+d}\right)}{2 \sqrt{e}}+\frac{2 d^{2} \sqrt{\left(x-\frac{\sqrt{-d e}}{e}\right)^{2} e+2 \sqrt{-d e}\, \left(x-\frac{\sqrt{-d e}}{e}\right)}}{3 e \sqrt{-d e}\, \left(x-\frac{\sqrt{-d e}}{e}\right)^{2}}-\frac{10 d \sqrt{\left(x-\frac{\sqrt{-}}{e}\right.}}{}$

input
```
int((-e*x^2+d)^(11/2)/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-e^2*x^4+d^2)^(1/2)*(-3*e^(5/2)*x^5+33*ln(x*e^(1/2)+(e*x^2+d)^(1/2))*
d*e*x^2*(e*x^2+d)^(1/2)-46*d*e^(3/2)*x^3+33*ln(x*e^(1/2)+(e*x^2+d)^(1/2))*
d^2*(e*x^2+d)^(1/2)-27*d^2*x*e^(1/2))/(-e*x^2+d)^(1/2)/(e*x^2+d)^2/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.45

$$\int \frac{\left(d-ex^2\right)^{11/2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\left[\frac{33\,(de^3x^6+d^2e^2x^4-d^3ex^2-d^4)\sqrt{e}\log\left(\frac{2\,e^2x^4-dex^2-2\,\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}\sqrt{e}x-d^2}{ex^2-d}\right)+}{12\,(e^4x^6+de^3x^4-d^2e^2x^2-}\right.$$

input
```
integrate((-e*x^2+d)^(11/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(33*(d*e^3*x^6 + d^2*e^2*x^4 - d^3*e*x^2 - d^4)*sqrt(e)*log((2*e^2*x
^4 - d*e*x^2 - 2*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(e)*x - d^2)/(e
*x^2 - d)) + 2*(3*e^3*x^5 + 46*d*e^2*x^3 + 27*d^2*e*x)*sqrt(-e^2*x^4 + d^2
)*sqrt(-e*x^2 + d))/(e^4*x^6 + d*e^3*x^4 - d^2*e^2*x^2 - d^3*e), 1/6*(33*(
d*e^3*x^6 + d^2*e^2*x^4 - d^3*e*x^2 - d^4)*sqrt(-e)*arctan(sqrt(-e^2*x^4 +
 d^2)*sqrt(-e*x^2 + d)*sqrt(-e)*x/(e^2*x^4 - d^2)) + (3*e^3*x^5 + 46*d*e^2
*x^3 + 27*d^2*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d))/(e^4*x^6 + d*e^3
*x^4 - d^2*e^2*x^2 - d^3*e)]
```

Sympy [F]

$$\int \frac{(d-ex^2)^{11/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(d-ex^2)^{\frac{11}{2}}}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x**2+d)**(11/2)/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
Integral((d - e*x**2)**(11/2)/(-(-d + e*x**2)*(d + e*x**2))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^{11/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(-ex^2+d)^{\frac{11}{2}}}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(11/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-e*x^2 + d)^(11/2)/(-e^2*x^4 + d^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.41

$$\int \frac{(d-ex^2)^{11/2}}{(d^2-e^2x^4)^{5/2}}\,dx = -\frac{1}{2}\sqrt{ex^2+d}\,x - \frac{11\,d\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{2\sqrt{e}} - \frac{4\,(5\,dex^2+3\,d^2)x}{3\,(ex^2+d)^{\frac{3}{2}}}$$

input
```
integrate((-e*x^2+d)^(11/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(e*x^2 + d)*x - 11/2*d*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/sqr
t(e) - 4/3*(5*d*e*x^2 + 3*d^2)*x/(e*x^2 + d)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^{11/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(d-e\,x^2)^{11/2}}{(d^2-e^2\,x^4)^{5/2}}\,dx$$

input
```
int((d - e*x^2)^(11/2)/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
int((d - e*x^2)^(11/2)/(d^2 - e^2*x^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.26

$$\int \frac{(d-ex^2)^{11/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{-54\sqrt{e\,x^2+d}\,d^2ex-92\sqrt{e\,x^2+d}\,d\,e^2x^3-6\sqrt{e\,x^2+d}\,e^3x^5+66\sqrt{e}\log\Bigg(\frac{\sqrt{e\,x^2+d}+\sqrt{e}}{\sqrt{d}}}{}$$

input
```
int((-e*x^2+d)^(11/2)/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
( - 54*sqrt(d + e*x**2)*d**2*e*x - 92*sqrt(d + e*x**2)*d*e**2*x**3 - 6*sqr
t(d + e*x**2)*e**3*x**5 + 66*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sq
rt(d))*d**3 + 132*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*d**2
*e*x**2 + 66*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*d*e**2*x*
*4 - 15*sqrt(e)*d**3 - 30*sqrt(e)*d**2*e*x**2 - 15*sqrt(e)*d*e**2*x**4)/(1
2*e*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.166 $\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 110

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{5/2}}\,dx=\frac{4dx(d-ex^2)^{3/2}}{3\,(d^2-e^2x^4)^{3/2}}-\frac{4x\sqrt{d-ex^2}}{3\sqrt{d^2-e^2x^4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{\sqrt{e}}$$

output
```
4/3*d*x*(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2)-4/3*x*(-e*x^2+d)^(1/2)/(-e^2
*x^4+d^2)^(1/2)+arctanh(e^(1/2)*x*(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e
^(1/2)
```

Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{5/2}}\,dx=&-\frac{4ex^3\sqrt{d^2-e^2x^4}}{3\sqrt{d-ex^2}\,(d+ex^2)^2}-\frac{\log(-d+ex^2)}{\sqrt{e}}\\&+\frac{\log\left(dex-e^2x^3+\sqrt{e}\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}\right)}{\sqrt{e}}\end{aligned}$$

input
```
Integrate[(d - e*x^2)^(9/2)/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(-4*e*x^3*Sqrt[d^2 - e^2*x^4])/(3*Sqrt[d - e*x^2]*(d + e*x^2)^2) - Log[-d
+ e*x^2]/Sqrt[e] + Log[d*e*x - e^2*x^3 + Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d^2
- e^2*x^4]]/Sqrt[e]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1396, 315, 27, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{(d-ex^2)^2}{(ex^2+d)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{de(3ex^2+d)}{(ex^2+d)^{3/2}}dx}{3de}+\frac{2x(d-ex^2)}{3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\int \frac{3ex^2+d}{(ex^2+d)^{3/2}}dx+\frac{2x(d-ex^2)}{3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{298}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(3\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{2x}{\sqrt{d+ex^2}}\right)+\frac{2x(d-ex^2)}{3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(3\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{2x}{\sqrt{d+ex^2}}\right)+\frac{2x(d-ex^2)}{3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow 219$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{1}{3}\left(\frac{3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}-\frac{2x}{\sqrt{d+ex^2}}\right)+\frac{2x(d-ex^2)}{3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d - e*x^2)^(9/2)/(d^2 - e^2*x^4)^(5/2),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((2*x*(d - e*x^2))/(3*(d + e*x^2)^(3/2))
+ ((-2*x)/Sqrt[d + e*x^2] + (3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[
e])/3))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\, \left(3 \ln \left(x \sqrt{e}+\sqrt{e \,x^{2}+d}\right) e \,x^{2} \sqrt{e \,x^{2}+d}-4 e^{\frac{3}{2}} x^{3}+3 \ln \left(x \sqrt{e}+\sqrt{e \,x^{2}+d}\right) d \sqrt{e \,x^{2}+d}\right)}{3 \sqrt{-e \,x^{2}+d}\, \left(e \,x^{2}+d \right)^{2} \sqrt{e}}$	107

input
```
int((-e*x^2+d)^(9/2)/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-e^2*x^4+d^2)^(1/2)*(3*ln(x*e^(1/2)+(e*x^2+d)^(1/2))*e*x^2*(e*x^2+d)^
(1/2)-4*e^(3/2)*x^3+3*ln(x*e^(1/2)+(e*x^2+d)^(1/2))*d*(e*x^2+d)^(1/2))/(-e
*x^2+d)^(1/2)/(e*x^2+d)^2/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.95

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \left[\frac{8\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}e^2x^3+3\left(e^3x^6+de^2x^4-d^2ex^2-d^3\right)\sqrt{e}\log\left(\frac{2e^2x^4-dex^2}{}\right.}{6\left(e^4x^6+de^3x^4-d^2e^2x^2-d^3e\right)}\right.$$

input
```
integrate((-e*x^2+d)^(9/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(8*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*e^2*x^3 + 3*(e^3*x^6 + d*e^2
*x^4 - d^2*e*x^2 - d^3)*sqrt(e)*log((2*e^2*x^4 - d*e*x^2 - 2*sqrt(-e^2*x^4
 + d^2)*sqrt(-e*x^2 + d)*sqrt(e)*x - d^2)/(e*x^2 - d)))/(e^4*x^6 + d*e^3*x
^4 - d^2*e^2*x^2 - d^3*e), 1/3*(4*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*e^
2*x^3 + 3*(e^3*x^6 + d*e^2*x^4 - d^2*e*x^2 - d^3)*sqrt(-e)*arctan(sqrt(-e^
2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(-e)*x/(e^2*x^4 - d^2)))/(e^4*x^6 + d*e^
3*x^4 - d^2*e^2*x^2 - d^3*e)]
```

Sympy [F]

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(d-ex^2)^{\frac{9}{2}}}{\left(-(-d+ex^2)\,(d+ex^2)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x**2+d)**(9/2)/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
Integral((d - e*x**2)**(9/2)/(-(-d + e*x**2)*(d + e*x**2))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(-ex^2+d)^{\frac{9}{2}}}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(9/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-e*x^2 + d)^(9/2)/(-e^2*x^4 + d^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.35

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{5/2}}\,dx = -\frac{4\,ex^3}{3\,(ex^2+d)^{\frac{3}{2}}} - \frac{\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{\sqrt{e}}$$

input
```
integrate((-e*x^2+d)^(9/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
-4/3*e*x^3/(e*x^2 + d)^(3/2) - log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/sqrt
(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(d-e\,x^2)^{9/2}}{(d^2-e^2\,x^4)^{5/2}}\,dx$$

input
```
int((d - e*x^2)^(9/2)/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
int((d - e*x^2)^(9/2)/(d^2 - e^2*x^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.37

$$\int \frac{(d-ex^2)^{9/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{-4\sqrt{e\,x^2+d}\,e^2x^3+3\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)d^2+6\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)de\,x^2+3\sqrt{\vphantom{e}}}{3e\left(e^2x^4+2de\,x^2+d^2\right)}$$

input
```
int((-e*x^2+d)^(9/2)/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
( - 4*sqrt(d + e*x**2)*e**2*x**3 + 3*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(
e)*x)/sqrt(d))*d**2 + 6*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d)
)*d*e*x**2 + 3*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*e**2*x*
*4 - 4*sqrt(e)*d**2 - 8*sqrt(e)*d*e*x**2 - 4*sqrt(e)*e**2*x**4)/(3*e*(d**2
 + 2*d*e*x**2 + e**2*x**4))
```

3.167 $\int \frac{\left(d-ex^2\right)^{7/2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 70

$$\int \frac{\left(d-ex^2\right)^{7/2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{2x\left(d-ex^2\right)^{3/2}}{3\left(d^2-e^2x^4\right)^{3/2}}+\frac{x\sqrt{d-ex^2}}{3d\sqrt{d^2-e^2x^4}}$$

output
```
2/3*x*(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2)+1/3*x*(-e*x^2+d)^(1/2)/d/(-e^2
*x^4+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 5.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{\left(d-ex^2\right)^{7/2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{x\left(3d+ex^2\right)\sqrt{d^2-e^2x^4}}{3d\sqrt{d-ex^2}\left(d+ex^2\right)^2}$$

input
```
Integrate[(d - e*x^2)^(7/2)/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(x*(3*d + e*x^2)*Sqrt[d^2 - e^2*x^4])/(3*d*Sqrt[d - e*x^2]*(d + e*x^2)^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1396, 292, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{d-ex^2}{(ex^2+d)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{292}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2}{3}\int \frac{1}{(ex^2+d)^{3/2}}dx+\frac{x(d-ex^2)}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2x}{3d\sqrt{d+ex^2}}+\frac{x(d-ex^2)}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d - e*x^2)^(7/2)/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((x*(d - e*x^2))/(3*d*(d + e*x^2)^(3/2))
+ (2*x)/(3*d*Sqrt[d + e*x^2])))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{(e\,x^2+d)x(e\,x^2+3d)\left(-e\,x^2+d\right)^{\frac{5}{2}}}{3d\left(-e^2x^4+d^2\right)^{\frac{5}{2}}}$	47
orering	$\frac{(e\,x^2+d)x(e\,x^2+3d)\left(-e\,x^2+d\right)^{\frac{5}{2}}}{3d\left(-e^2x^4+d^2\right)^{\frac{5}{2}}}$	47
default	$\frac{\sqrt{-e^2x^4+d^2}\,x\left(e\,x^2+3d\right)}{3\sqrt{-e\,x^2+d}\,d\left(e\,x^2+d\right)^2}$	49

input
```
int((-e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(e*x^2+d)*x*(e*x^2+3*d)*(-e*x^2+d)^(5/2)/d/(-e^2*x^4+d^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{5/2}}\,dx = -\frac{\sqrt{-e^2x^4+d^2}(ex^3+3\,dx)\sqrt{-ex^2+d}}{3\,(de^3x^6+d^2e^2x^4-d^3ex^2-d^4)}$$

input
```
integrate((-e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(-e^2*x^4 + d^2)*(e*x^3 + 3*d*x)*sqrt(-e*x^2 + d)/(d*e^3*x^6 + d^
2*e^2*x^4 - d^3*e*x^2 - d^4)
```

Sympy [F]

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(d-ex^2)^{\frac{7}{2}}}{(-(-d+ex^2)\,(d+ex^2))^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x**2+d)**(7/2)/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
Integral((d - e*x**2)**(7/2)/(-(-d + e*x**2)*(d + e*x**2))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(-ex^2+d)^{\frac{7}{2}}}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-e*x^2 + d)^(7/2)/(-e^2*x^4 + d^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(-ex^2+d)^{\frac{7}{2}}}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((-e*x^2 + d)^(7/2)/(-e^2*x^4 + d^2)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 17.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.23

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{5/2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^4}\,\left(\frac{x\,\sqrt{d-e\,x^2}}{e^3}+\frac{x^3\,\sqrt{d-e\,x^2}}{3\,d\,e^2}\right)}{x^6-\frac{d^3}{e^3}+\frac{d\,x^4}{e}-\frac{d^2\,x^2}{e^2}}$$

input
```
int((d - e*x^2)^(7/2)/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
-((d^2 - e^2*x^4)^(1/2)*((x*(d - e*x^2)^(1/2))/e^3 + (x^3*(d - e*x^2)^(1/2
))/(3*d*e^2)))/(x^6 - d^3/e^3 + (d*x^4)/e - (d^2*x^2)/e^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

$$\int \frac{(d-ex^2)^{7/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{3\sqrt{e\,x^2+d}\,dex+\sqrt{e\,x^2+d}\,e^2x^3-3\sqrt{e}\,d^2-6\sqrt{e}\,de\,x^2-3\sqrt{e}\,e^2x^4}{3de\left(e^2x^4+2de\,x^2+d^2\right)}$$

input
```
int((-e*x^2+d)^(7/2)/(-e^2*x^4+d^2)^(5/2),x)
```

output
```
(3*sqrt(d + e*x**2)*d*e*x + sqrt(d + e*x**2)*e**2*x**3 - 3*sqrt(e)*d**2 -
6*sqrt(e)*d*e*x**2 - 3*sqrt(e)*e**2*x**4)/(3*d*e*(d**2 + 2*d*e*x**2 + e**2
*x**4))
```

3.168 $\int \frac{\left(d-ex^2\right)^{5/2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{\left(d-ex^2\right)^{5/2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{x\left(d-ex^2\right)^{3/2}}{3d\left(d^2-e^2x^4\right)^{3/2}}+\frac{2x\sqrt{d-ex^2}}{3d^2\sqrt{d^2-e^2x^4}}$$

output
```
1/3*x*(-e*x^2+d)^(3/2)/d/(-e^2*x^4+d^2)^(3/2)+2/3*x*(-e*x^2+d)^(1/2)/d^2/(
-e^2*x^4+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 5.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \frac{\left(d-ex^2\right)^{5/2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{x\left(3d+2ex^2\right)\sqrt{d^2-e^2x^4}}{3d^2\sqrt{d-ex^2}\left(d+ex^2\right)^2}$$

input
```
Integrate[(d - e*x^2)^(5/2)/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(x*(3*d + 2*e*x^2)*Sqrt[d^2 - e^2*x^4])/(3*d^2*Sqrt[d - e*x^2]*(d + e*x^2)
^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1396, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(ex^2+d)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2\int \frac{1}{(ex^2+d)^{3/2}}dx}{3d}+\frac{x}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{2x}{3d^2\sqrt{d+ex^2}}+\frac{x}{3d(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(d - e*x^2)^(5/2)/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(3*d*(d + e*x^2)^(3/2)) + (2*x)/(3*d^2
*Sqrt[d + e*x^2])))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{(e\,x^2+d)x(2e\,x^2+3d)(-e\,x^2+d)^{\frac{5}{2}}}{3d^2(-e^2x^4+d^2)^{\frac{5}{2}}}$	48
orering	$\frac{(e\,x^2+d)x(2e\,x^2+3d)(-e\,x^2+d)^{\frac{5}{2}}}{3d^2(-e^2x^4+d^2)^{\frac{5}{2}}}$	48
default	$\frac{\sqrt{-e^2x^4+d^2}\,x(2e\,x^2+3d)}{3\sqrt{-e\,x^2+d}\,(e\,x^2+d)^2d^2}$	50

input
```
int((-e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(e*x^2+d)*x*(2*e*x^2+3*d)*(-e*x^2+d)^(5/2)/d^2/(-e^2*x^4+d^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{5/2}}\,dx = -\frac{\sqrt{-e^2x^4+d^2}(2\,ex^3+3\,dx)\sqrt{-ex^2+d}}{3\,(d^2e^3x^6+d^3e^2x^4-d^4ex^2-d^5)}$$

input
```
integrate((-e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(-e^2*x^4 + d^2)*(2*e*x^3 + 3*d*x)*sqrt(-e*x^2 + d)/(d^2*e^3*x^6
+ d^3*e^2*x^4 - d^4*e*x^2 - d^5)
```

Sympy [F]

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(d-ex^2)^{\frac{5}{2}}}{(-(-d+ex^2)\,(d+ex^2))^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x**2+d)**(5/2)/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
Integral((d - e*x**2)**(5/2)/(-(-d + e*x**2)*(d + e*x**2))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(-ex^2+d)^{\frac{5}{2}}}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-e*x^2 + d)^(5/2)/(-e^2*x^4 + d^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(-ex^2+d)^{\frac{5}{2}}}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((-e*x^2 + d)^(5/2)/(-e^2*x^4 + d^2)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 17.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{5/2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^4}\,\left(\frac{x\,\sqrt{d-e\,x^2}}{d\,e^3}+\frac{2\,x^3\,\sqrt{d-e\,x^2}}{3\,d^2\,e^2}\right)}{x^6-\frac{d^3}{e^3}+\frac{d\,x^4}{e}-\frac{d^2\,x^2}{e^2}}$$

input
```
int((d - e*x^2)^(5/2)/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
-((d^2 - e^2*x^4)^(1/2)*((x*(d - e*x^2)^(1/2))/(d*e^3) + (2*x^3*(d - e*x^2
)^(1/2))/(3*d^2*e^2)))/(x^6 - d^3/e^3 + (d*x^4)/e - (d^2*x^2)/e^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{(d-ex^2)^{5/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{3\sqrt{e\,x^2+d}\,dex+2\sqrt{e\,x^2+d}\,e^2x^3-2\sqrt{e}\,d^2-4\sqrt{e}\,de\,x^2-2\sqrt{e}\,e^2x^4}{3d^2e\,(e^2x^4+2de\,x^2+d^2)}$$

input
```
int((-e*x^2+d)^(5/2)/(-e^2*x^4+d^2)^(5/2),x)
```

output
```
(3*sqrt(d + e*x**2)*d*e*x + 2*sqrt(d + e*x**2)*e**2*x**3 - 2*sqrt(e)*d**2
- 4*sqrt(e)*d*e*x**2 - 2*sqrt(e)*e**2*x**4)/(3*d**2*e*(d**2 + 2*d*e*x**2 +
 e**2*x**4))
```

3.169 $\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 131

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{x(d-ex^2)^{3/2}}{6d^2\,(d^2-e^2x^4)^{3/2}} + \frac{7x\sqrt{d-ex^2}}{12d^3\sqrt{d^2-e^2x^4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{4\sqrt{2}d^3\sqrt{e}}$$

output

```
1/6*x*(-e*x^2+d)^(3/2)/d^2/(-e^2*x^4+d^2)^(3/2)+7/12*x*(-e*x^2+d)^(1/2)/d^
3/(-e^2*x^4+d^2)^(1/2)+1/8*arctanh(2^(1/2)*e^(1/2)*x*(-e*x^2+d)^(1/2)/(-e^
2*x^4+d^2)^(1/2))*2^(1/2)/d^3/e^(1/2)
```

Mathematica [A] (verified)

Time = 5.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2x^4}\left(2\sqrt{e}x\sqrt{d+ex^2}(9d+7ex^2)+3\sqrt{2}(d+ex^2)^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)}{24d^3\sqrt{e}\sqrt{d-ex^2}\,(d+ex^2)^{5/2}}$$

input

```
Integrate[(d - e*x^2)^(3/2)/(d^2 - e^2*x^4)^(5/2),x]
```

output

```
(Sqrt[d^2 - e^2*x^4]*(2*Sqrt[e]*x*Sqrt[d + e*x^2]*(9*d + 7*e*x^2) + 3*Sqrt
[2]*(d + e*x^2)^2*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]))/(24*d^3*S
qrt[e]*Sqrt[d - e*x^2]*(d + e*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1396, 316, 25, 27, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-ex^2\right)^{3/2}}{\left(d^2-e^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \; 1396$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)(ex^2+d)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 316$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{x}{6d^2(d+ex^2)^{3/2}}-\frac{\int -\frac{e\left(5d-2ex^2\right)}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{6d^2e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(5d-2ex^2\right)}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{6d^2e}+\frac{x}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{5d-2ex^2}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{6d^2}+\frac{x}{6d^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{7x}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{3d^2e}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx}{2d^2e}}{6d^2}+\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3}{2}\int \frac{1}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx+\frac{7x}{2d\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3}{2}\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{7x}{2d\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d\sqrt{e}}+\frac{7x}{2d\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[(d - e*x^2)^(3/2)/(d^2 - e^2*x^4)^(5/2),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(6*d^2*(d + e*x^2)^(3/2)) + ((7*x)/(2*
d*Sqrt[d + e*x^2]) + (3*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*S
qrt[2]*d*Sqrt[e]))/(6*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(105) = 210.

Time = 0.57 (sec) , antiderivative size = 606, normalized size of antiderivative = 4.63

method	result
default	$\sqrt{-e^2x^4+d^2}\,e^2\left(3\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{e\,x^2+d}+\sqrt{de}\,x+d\right)}{ex-\sqrt{de}}\right)\sqrt{2}\,de\,x^2\sqrt{e\,x^2+d}\,\sqrt{-\frac{(ex+\sqrt{-de})\,(-ex+\sqrt{-de})}{e}}-3\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{e\,x^2+d}-\sqrt{de}\right.}{ex+\sqrt{de}}\right.\right.$

input
```
int((-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-e^2*x^4+d^2)^(1/2)*e^2/d^(5/2)*(3*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)
^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*2^(1/2)*d*e*x^2*(e*x^2+d)^(1/2)
*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)-3*ln(2*e*(2^(1/2)*d^(1/
2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2)))*2^(1/2)*d*e*x^2*(e*
x^2+d)^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)+16*e*x^3*(d
*e)^(1/2)*(e*x^2+d)^(1/2)*d^(1/2)+12*e*x^3*(d*e)^(1/2)*d^(1/2)*(-(e*x+(-d*
e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)+3*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)
^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*2^(1/2)*d^2*(e*x^2+d)^(1/2)*(-(
e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)-3*ln(2*e*(2^(1/2)*d^(1/2)*(
e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2)))*2^(1/2)*d^2*(e*x^2+d)^(
1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)+24*d^(3/2)*x*(d*e)^
(1/2)*(e*x^2+d)^(1/2)+12*d^(3/2)*x*(d*e)^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*
x+(-d*e)^(1/2)))^(1/2))/(-e*x^2+d)^(1/2)/(e*x^2+d)/((d*e)^(1/2)-(-d*e)^(1/
2))/((d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)/(e*x-(-d*e)^(1/2))/(-(e*x+(-d*e
)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)/(e*x+(-d*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.84

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \left[\frac{3\sqrt{2}(e^3x^6+de^2x^4-d^2ex^2-d^3)\sqrt{e}\log\left(-\frac{3e^2x^4-2\,dex^2-2\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}\sqrt{e}x-d}{e^2x^4-2\,dex^2+d^2}\right.}{48\,(d^3e^4x^6+d^4e^3x^4-d^5e^2x^2-d^6e}\right.$$

input
```
integrate((-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(2)*(e^3*x^6 + d*e^2*x^4 - d^2*e*x^2 - d^3)*sqrt(e)*log(-(3*e
^2*x^4 - 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(
e)*x - d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2)) - 4*sqrt(-e^2*x^4 + d^2)*(7*e^2*x
^3 + 9*d*e*x)*sqrt(-e*x^2 + d))/(d^3*e^4*x^6 + d^4*e^3*x^4 - d^5*e^2*x^2 -
 d^6*e), 1/24*(3*sqrt(2)*(e^3*x^6 + d*e^2*x^4 - d^2*e*x^2 - d^3)*sqrt(-e)*
arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(-e)*x/(e^2*x^4 -
 d^2)) - 2*sqrt(-e^2*x^4 + d^2)*(7*e^2*x^3 + 9*d*e*x)*sqrt(-e*x^2 + d))/(d
^3*e^4*x^6 + d^4*e^3*x^4 - d^5*e^2*x^2 - d^6*e)]
```

Sympy [F]

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(d-ex^2)^{\frac{3}{2}}}{(-(-d+ex^2)\,(d+ex^2))^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x**2+d)**(3/2)/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
Integral((d - e*x**2)**(3/2)/(-(-d + e*x**2)*(d + e*x**2))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{(-ex^2+d)^{\frac{3}{2}}}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-e*x^2 + d)^(3/2)/(-e^2*x^4 + d^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{5/2}}\,dx=\int \frac{(-ex^2+d)^{\frac{3}{2}}}{(-e^2x^4+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((-e*x^2 + d)^(3/2)/(-e^2*x^4 + d^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{5/2}}\,dx=\int \frac{(d-e\,x^2)^{3/2}}{(d^2-e^2\,x^4)^{5/2}}\,dx$$

input
```
int((d - e*x^2)^(3/2)/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
int((d - e*x^2)^(3/2)/(d^2 - e^2*x^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 542, normalized size of antiderivative = 4.14

$$\int \frac{(d-ex^2)^{3/2}}{(d^2-e^2x^4)^{5/2}}\,dx=\frac{36\sqrt{e\,x^2+d}\,dex+28\sqrt{e\,x^2+d}\,e^2x^3-3\sqrt{e}\,\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)d^2-6}{}$$

input
```
int((-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
(36*sqrt(d + e*x**2)*d*e*x + 28*sqrt(d + e*x**2)*e**2*x**3 - 3*sqrt(e)*sqr
t(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d
))*d**2 - 6*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt
(d) + sqrt(e)*x)/sqrt(d))*d*e*x**2 - 3*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**
2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e**2*x**4 + 3*sqrt(e)
*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sq
rt(d))*d**2 + 6*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) +
sqrt(d) + sqrt(e)*x)/sqrt(d))*d*e*x**2 + 3*sqrt(e)*sqrt(2)*log((sqrt(d + e
*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*e**2*x**4 + 3*sqr
t(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x
)/sqrt(d))*d**2 + 6*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2
) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d*e*x**2 + 3*sqrt(e)*sqrt(2)*log((sqrt(d
 + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e**2*x**4 - 3
*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(
e)*x)/sqrt(d))*d**2 - 6*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sq
rt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*d*e*x**2 - 3*sqrt(e)*sqrt(2)*log((sq
rt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*e**2*x**4
 - 20*sqrt(e)*d**2 - 40*sqrt(e)*d*e*x**2 - 20*sqrt(e)*e**2*x**4)/(48*d**3*
e*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.170 $\int \frac{\sqrt{d-ex^2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 167

$$\int \frac{\sqrt{d-ex^2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{x\sqrt{d-ex^2}}{4d^2\left(d^2-e^2x^4\right)^{3/2}}-\frac{x\left(d-ex^2\right)^{3/2}}{24d^3\left(d^2-e^2x^4\right)^{3/2}}+\frac{17x\sqrt{d-ex^2}}{48d^4\sqrt{d^2-e^2x^4}}+\frac{7\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{16\sqrt{2}d^4\sqrt{e}}$$

output

```
1/4*x*(-e*x^2+d)^(1/2)/d^2/(-e^2*x^4+d^2)^(3/2)-1/24*x*(-e*x^2+d)^(3/2)/d^
3/(-e^2*x^4+d^2)^(3/2)+17/48*x*(-e*x^2+d)^(1/2)/d^4/(-e^2*x^4+d^2)^(1/2)+7
/32*arctanh(2^(1/2)*e^(1/2)*x*(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))*2^(1/
2)/d^4/e^(1/2)
```

Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{d-ex^2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{\sqrt{d^2-e^2x^4}\left(2\sqrt{e}x\sqrt{d+ex^2}\left(27d^2+2dex^2-17e^2x^4\right)+21\sqrt{2}\left(d-ex^2\right)\left(d+ex^2\right)^2 \mathrm{a}\right.}{96d^4\sqrt{e}\left(d-ex^2\right)^{3/2}\left(d+ex^2\right)^{5/2}}$$

input

```
Integrate[Sqrt[d - e*x^2]/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(2*Sqrt[e]*x*Sqrt[d + e*x^2]*(27*d^2 + 2*d*e*x^2 - 17
*e^2*x^4) + 21*Sqrt[2]*(d - e*x^2)*(d + e*x^2)^2*ArcTanh[(Sqrt[2]*Sqrt[e]*
x)/Sqrt[d + e*x^2]]))/(96*d^4*Sqrt[e]*(d - e*x^2)^(3/2)*(d + e*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {1396, 316, 27, 402, 25, 27, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-ex^2}}{\left(d^2-e^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^2(ex^2+d)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(4ex^2+3d\right)}{\left(d-ex^2\right)\left(ex^2+d\right)^{5/2}}dx}{4d^2e}+\frac{x}{4d^2(d-ex^2)(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{4ex^2+3d}{\left(d-ex^2\right)\left(ex^2+d\right)^{5/2}}dx}{4d^2}+\frac{x}{4d^2(d-ex^2)(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{-\frac{\int -\frac{de\left(2ex^2+19d\right)}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{6d^2e}-\frac{x}{6d\left(d+ex^2\right)^{3/2}}}{4d^2}+\frac{x}{4d^2(d-ex^2)(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{de\left(2ex^2+19d\right)}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{6d^2e}-\frac{x}{6d\left(d+ex^2\right)^{3/2}}}{4d^2}+\frac{x}{4d^2(d-ex^2)(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{2ex^2+19d}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{6d}-\frac{x}{6d\left(d+ex^2\right)^{3/2}}}{4d^2}+\frac{x}{4d^2(d-ex^2)(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{17x}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{21d^2e}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx}{2d^2e}}{6d}-\frac{x}{6d\left(d+ex^2\right)^{3/2}}}{4d^2}+\frac{x}{4d^2(d-ex^2)(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{21}{2}\int \frac{1}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx+\frac{17x}{2d\sqrt{d+ex^2}}}{6d}-\frac{x}{6d\left(d+ex^2\right)^{3/2}}}{4d^2}+\frac{x}{4d^2(d-ex^2)(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 291$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{21}{2}\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{17x}{2d\sqrt{d+ex^2}}}{6d}-\frac{x}{6d\left(d+ex^2\right)^{3/2}}}{4d^2}+\frac{x}{4d^2(d-ex^2)(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{21\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d\sqrt{e}}+\frac{17x}{2d\sqrt{d+ex^2}}}{6d}-\frac{x}{6d\left(d+ex^2\right)^{3/2}}}{4d^2}+\frac{x}{4d^2(d-ex^2)(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[Sqrt[d - e*x^2]/(d^2 - e^2*x^4)^(5/2),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(4*d^2*(d - e*x^2)*(d + e*x^2)^(3/2))
+ (-1/6*x/(d*(d + e*x^2)^(3/2)) + ((17*x)/(2*d*Sqrt[d + e*x^2]) + (21*ArcT
anh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[2]*d*Sqrt[e]))/(6*d))/(4
*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(135) = 270$.

Time = 0.84 (sec) , antiderivative size = 745, normalized size of antiderivative = 4.46

method	result
default	$\sqrt{-e^2x^4+d^2}\, e^6 \Bigg(-21\sqrt{2}\, \ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{e\,x^2+d}+\sqrt{de}\,x+d\right)}{ex-\sqrt{de}}\right) d\,e^3x^6+21\sqrt{2}\, \ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{e\,x^2+d}-\sqrt{de}\,x+d\right)}{ex+\sqrt{de}}\right) d\,e^3x^6-21\sqrt{2}\, \ln\Bigg(\frac{2e(}{}$

input
```
int((-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-e^2*x^4+d^2)^(1/2)*e^6/d^(3/2)*(-21*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)
*(e*x^2+d)^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*d*e^3*x^6+21*2^(1/2)*
ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2))
)*d*e^3*x^6-21*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)+(d*e)^(1/2)
*x+d)/(e*x-(d*e)^(1/2)))*d^2*e^2*x^4+21*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e
*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2)))*d^2*e^2*x^4-80*e^2*x^5*d
^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)*(d*e)^(1/2)+12*e^
2*x^5*d^(1/2)*(d*e)^(1/2)*(e*x^2+d)^(1/2)+21*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/
2)*(e*x^2+d)^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*d^3*e*x^2-21*2^(1/2
)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2
)))*d^3*e*x^2-16*d^(3/2)*e*x^3*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))
^(1/2)*(d*e)^(1/2)+24*d^(3/2)*e*x^3*(d*e)^(1/2)*(e*x^2+d)^(1/2)+21*2^(1/2)
*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)
))*d^4-21*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)
/(e*x+(d*e)^(1/2)))*d^4+96*d^(5/2)*x*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(
1/2)))^(1/2)*(d*e)^(1/2)+12*d^(5/2)*x*(d*e)^(1/2)*(e*x^2+d)^(1/2))/(-e*x^2
+d)^(1/2)/(e*x^2+d)^(1/2)/(-(d*e)^(1/2)+(-d*e)^(1/2))^3/((d*e)^(1/2)+(-d*e
)^(1/2))^3/(e*x-(d*e)^(1/2))/(e*x+(d*e)^(1/2))/(e*x-(-d*e)^(1/2))^2/(e*x+(
-d*e)^(1/2))^2/(d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{d-ex^2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx = \left[\frac{21\,\sqrt{2}(e^4x^8-2\,d^2e^2x^4+d^4)\sqrt{e}\log\left(-\frac{3\,e^2x^4-2\,dex^2-2\,\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}\sqrt{e}x-d^2}{e^2x^4-2\,dex^2+d^2}\right)-}{192\,(d^4e^5x^8-2\,d^6e^3x^4+d^8}\right.$$

input
```
integrate((-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output

```
[1/192*(21*sqrt(2)*(e^4*x^8 - 2*d^2*e^2*x^4 + d^4)*sqrt(e)*log(-(3*e^2*x^4
 - 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(e)*x -
 d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2)) - 4*(17*e^3*x^5 - 2*d*e^2*x^3 - 27*d^2*
e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d))/(d^4*e^5*x^8 - 2*d^6*e^3*x^4 +
 d^8*e), 1/96*(21*sqrt(2)*(e^4*x^8 - 2*d^2*e^2*x^4 + d^4)*sqrt(-e)*arctan(
sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(-e)*x/(e^2*x^4 - d^2))
- 2*(17*e^3*x^5 - 2*d*e^2*x^3 - 27*d^2*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x
^2 + d))/(d^4*e^5*x^8 - 2*d^6*e^3*x^4 + d^8*e)]
```

Sympy [F]

$$\int \frac{\sqrt{d-ex^2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{\sqrt{d-ex^2}}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate((-e*x**2+d)**(1/2)/(-e**2*x**4+d**2)**(5/2),x)
```

output

```
Integral(sqrt(d - e*x**2)/(-(-d + e*x**2)*(d + e*x**2))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d-ex^2}}{(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{\sqrt{-ex^2+d}}{\left(-e^2x^4+d^2\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate((-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output

```
integrate(sqrt(-e*x^2 + d)/(-e^2*x^4 + d^2)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d-ex^2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\int \frac{\sqrt{-ex^2+d}}{\left(-e^2x^4+d^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-e*x^2 + d)/(-e^2*x^4 + d^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-ex^2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\int \frac{\sqrt{d-e\,x^2}}{\left(d^2-e^2\,x^4\right)^{5/2}}\,dx$$

input
```
int((d - e*x^2)^(1/2)/(d^2 - e^2*x^4)^(5/2),x)
```

output
```
int((d - e*x^2)^(1/2)/(d^2 - e^2*x^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 755, normalized size of antiderivative = 4.52

$$\int \frac{\sqrt{d-ex^2}}{\left(d^2-e^2x^4\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
(108*sqrt(d + e*x**2)*d**2*e*x + 8*sqrt(d + e*x**2)*d*e**2*x**3 - 68*sqrt(
d + e*x**2)*e**3*x**5 - 21*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)
*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d**3 - 21*sqrt(e)*sqrt(2)*log((sq
rt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d**2*e*x*
*2 + 21*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d)
+ sqrt(e)*x)/sqrt(d))*d*e**2*x**4 + 21*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**
2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e**3*x**6 + 21*sqrt(e
)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/s
qrt(d))*d**3 + 21*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2)
+ sqrt(d) + sqrt(e)*x)/sqrt(d))*d**2*e*x**2 - 21*sqrt(e)*sqrt(2)*log((sqrt
(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*d*e**2*x**4
 - 21*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) +
sqrt(e)*x)/sqrt(d))*e**3*x**6 + 21*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) +
 sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d**3 + 21*sqrt(e)*sqrt(2)
*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d
**2*e*x**2 - 21*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) -
sqrt(d) + sqrt(e)*x)/sqrt(d))*d*e**2*x**4 - 21*sqrt(e)*sqrt(2)*log((sqrt(d
 + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e**3*x**6 - 2
1*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt
(e)*x)/sqrt(d))*d**3 - 21*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(...
```

3.171 $\int \frac{1}{\sqrt{d-ex^2}\left(d^2-e^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 203

$$\int \frac{1}{\sqrt{d-ex^2}\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{x}{8d^2\sqrt{d-ex^2}\left(d^2-e^2x^4\right)^{3/2}}+\frac{13x\sqrt{d-ex^2}}{32d^3\left(d^2-e^2x^4\right)^{3/2}}-\frac{37x\left(d-ex^2\right)^{3/2}}{192d^4\left(d^2-e^2x^4\right)^{3/2}}+\frac{53x\sqrt{d-ex^2}}{384d^5\sqrt{d^2-e^2x^4}}+\frac{67\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{128\sqrt{2}d^5\sqrt{e}}$$

output

```
1/8*x/d^2/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(3/2)+13/32*x*(-e*x^2+d)^(1/2)/d
^3/(-e^2*x^4+d^2)^(3/2)-37/192*x*(-e*x^2+d)^(3/2)/d^4/(-e^2*x^4+d^2)^(3/2)
+53/384*x*(-e*x^2+d)^(1/2)/d^5/(-e^2*x^4+d^2)^(1/2)+67/256*arctanh(2^(1/2)
*e^(1/2)*x*(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))*2^(1/2)/d^5/e^(1/2)
```

Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{d-ex^2}\left(d^2-e^2x^4\right)^{5/2}}\,dx=\frac{\sqrt{d^2-e^2x^4}\Big(2\sqrt{e}x\sqrt{d+ex^2}\left(183d^3-61d^2ex^2-127de^2x^4+53e^3x^6\right)+20}{768d^5\sqrt{e}\left(d-ex^2\right)^{5/2}\left(d+ex^2\right)^{5/2}}$$

input

```
Integrate[1/(Sqrt[d - e*x^2]*(d^2 - e^2*x^4)^(5/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(2*Sqrt[e]*x*Sqrt[d + e*x^2]*(183*d^3 - 61*d^2*e*x^2
- 127*d*e^2*x^4 + 53*e^3*x^6) + 201*Sqrt[2]*(d^2 - e^2*x^4)^2*ArcTanh[(Sqr
t[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]))/(768*d^5*Sqrt[e]*(d - e*x^2)^(5/2)*(d +
 e*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {1396, 316, 27, 402, 27, 402, 25, 27, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d-ex^2}\left(d^2-e^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{(d-ex^2)^3(ex^2+d)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(6ex^2+7d\right)}{\left(d-ex^2\right)^2\left(ex^2+d\right)^{5/2}}dx}{8d^2e}+\frac{x}{8d^2(d-ex^2)^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{6ex^2+7d}{\left(d-ex^2\right)^2\left(ex^2+d\right)^{5/2}}dx}{8d^2}+\frac{x}{8d^2(d-ex^2)^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{de\left(52ex^2+15d\right)}{\left(d-ex^2\right)\left(ex^2+d\right)^{5/2}}dx}{4d^2e}+\frac{13x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{52ex^2+15d}{\left(d-ex^2\right)\left(ex^2+d\right)^{5/2}}dx}{4d}+\frac{13x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{-\frac{\int -\frac{de\left(74ex^2+127d\right)}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{6d^2e}-\frac{37x}{6d\left(d+ex^2\right)^{3/2}}}{4d}+\frac{13x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\int \frac{de\left(74ex^2+127d\right)}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{6d^2e}-\frac{37x}{6d\left(d+ex^2\right)^{3/2}}}{4d}+\frac{13x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\int \frac{74ex^2+127d}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{6d}-\frac{37x}{6d\left(d+ex^2\right)^{3/2}}}{4d}+\frac{13x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \color{blue}{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{53x}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{201d^2e}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx}{2d^2e}}{6d}-\frac{37x}{6d\left(d+ex^2\right)^{3/2}}}{4d}+\frac{13x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{201}{2}\int \frac{1}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx+\frac{53x}{2d\sqrt{d+ex^2}}}{6d}-\frac{37x}{6d\left(d+ex^2\right)^{3/2}}}{4d}+\frac{13x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \color{blue}{291}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{201}{2}\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{53x}{2d\sqrt{d+ex^2}}}{6d}-\frac{37x}{6d\left(d+ex^2\right)^{3/2}}}{4d}+\frac{13x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \color{blue}{221}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\frac{\frac{201\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d\sqrt{e}}+\frac{53x}{2d\sqrt{d+ex^2}}}{6d}-\frac{37x}{6d\left(d+ex^2\right)^{3/2}}}{4d}+\frac{13x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}}{8d^2}+\frac{x}{8d^2(d-ex^2)^2(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input

```
Int[1/(Sqrt[d - e*x^2]*(d^2 - e^2*x^4)^(5/2)),x]
```

output

```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(8*d^2*(d - e*x^2)^2*(d + e*x^2)^(3/2)
) + ((13*x)/(4*d*(d - e*x^2)*(d + e*x^2)^(3/2)) + ((-37*x)/(6*d*(d + e*x^2
)^(3/2)) + ((53*x)/(2*d*Sqrt[d + e*x^2]) + (201*ArcTanh[(Sqrt[2]*Sqrt[e]*x
)/Sqrt[d + e*x^2]])/(2*Sqrt[2]*d*Sqrt[e]))/(6*d))/(4*d))/(8*d^2)))/Sqrt[d^
2 - e^2*x^4]
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1659 vs. $2(165) = 330$.

Time = 0.89 (sec) , antiderivative size = 1660, normalized size of antiderivative = 8.18

method	result	size
default	Expression too large to display	1660

input
```
int(1/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(-e^2*x^4+d^2)^(1/2)*e^(19/2)*(252*d^4*x*(e*x^2+d)^(1/2)*(d*e)^(1/2)*
e^(1/2)-288*ln(((e*x^2+d)^(1/2)*e^(1/2)+e*x)/e^(1/2))*d*e^4*x^8*(d*e)^(1/2
)+288*ln((e^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)+e*x)/e
^(1/2))*d*e^4*x^8*(d*e)^(1/2)+576*ln(((e*x^2+d)^(1/2)*e^(1/2)+e*x)/e^(1/2)
)*d^2*e^3*x^6*(d*e)^(1/2)-576*ln((e^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d
*e)^(1/2)))^(1/2)+e*x)/e^(1/2))*d^2*e^3*x^6*(d*e)^(1/2)+576*ln(((e*x^2+d)^
(1/2)*e^(1/2)+e*x)/e^(1/2))*d^3*e^2*x^4*(d*e)^(1/2)-576*ln((e^(1/2)*(-(e*x
+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)+e*x)/e^(1/2))*d^3*e^2*x^4*(d*e
)^(1/2)-288*ln(((e*x^2+d)^(1/2)*e^(1/2)+e*x)/e^(1/2))*d^4*e*x^2*(d*e)^(1/2
)+288*ln((e^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)+e*x)/e
^(1/2))*d^4*e*x^2*(d*e)^(1/2)+201*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d
)^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*d^(3/2)*e^(9/2)*x^8-201*2^(1/2
)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2
)))*d^(3/2)*e^(9/2)*x^8-402*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2
)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*d^(5/2)*e^(7/2)*x^6+64*(d*e)^(1/2)*d
*e^(7/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)*x^7-896*d^2*e^(
5/2)*(d*e)^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)*x^5-64*
d^3*e^(3/2)*(d*e)^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*e)^(1/2)))^(1/2)*
x^3+402*2^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(
e*x+(d*e)^(1/2)))*d^(5/2)*e^(7/2)*x^6-204*e^(9/2)*x^9*(e*x^2+d)^(1/2)*(...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.48

$$\int \frac{1}{\sqrt{d-ex^2}\,(d^2-e^2x^4)^{5/2}}\,dx = \left[\frac{201\sqrt{2}(e^5x^{10}-de^4x^8-2\,d^2e^3x^6+2\,d^3e^2x^4+d^4ex^2-d^5)\sqrt{e}\log\left(-\frac{3\,e^2}{}\right.}{1536\,(d^5e^6x}\right.$$

input
```
integrate(1/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/1536*(201*sqrt(2)*(e^5*x^10 - d*e^4*x^8 - 2*d^2*e^3*x^6 + 2*d^3*e^2*x^4
 + d^4*e*x^2 - d^5)*sqrt(e)*log(-(3*e^2*x^4 - 2*d*e*x^2 - 2*sqrt(2)*sqrt(-
e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(e)*x - d^2)/(e^2*x^4 - 2*d*e*x^2 + d^
2)) - 4*(53*e^4*x^7 - 127*d*e^3*x^5 - 61*d^2*e^2*x^3 + 183*d^3*e*x)*sqrt(-
e^2*x^4 + d^2)*sqrt(-e*x^2 + d))/(d^5*e^6*x^10 - d^6*e^5*x^8 - 2*d^7*e^4*x
^6 + 2*d^8*e^3*x^4 + d^9*e^2*x^2 - d^10*e), 1/768*(201*sqrt(2)*(e^5*x^10 -
 d*e^4*x^8 - 2*d^2*e^3*x^6 + 2*d^3*e^2*x^4 + d^4*e*x^2 - d^5)*sqrt(-e)*arc
tan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(-e)*x/(e^2*x^4 - d^
2)) - 2*(53*e^4*x^7 - 127*d*e^3*x^5 - 61*d^2*e^2*x^3 + 183*d^3*e*x)*sqrt(-
e^2*x^4 + d^2)*sqrt(-e*x^2 + d))/(d^5*e^6*x^10 - d^6*e^5*x^8 - 2*d^7*e^4*x
^6 + 2*d^8*e^3*x^4 + d^9*e^2*x^2 - d^10*e)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d-ex^2}\,(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{5}{2}}\sqrt{d-ex^2}}\,dx$$

input
```
integrate(1/(-e*x**2+d)**(1/2)/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
Integral(1/((-(-d + e*x**2)*(d + e*x**2))**(5/2)*sqrt(d - e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d-ex^2}\,(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{(-e^2x^4+d^2)^{\frac{5}{2}}\sqrt{-ex^2+d}}\,dx$$

input
```
integrate(1/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(5/2)*sqrt(-e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d-ex^2}\left(d^2-e^2x^4\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^4+d^2\right)^{\frac{5}{2}}\sqrt{-ex^2+d}}\,dx$$

input
```
integrate(1/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(5/2)*sqrt(-e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d-ex^2}\left(d^2-e^2x^4\right)^{5/2}}\,dx = \int \frac{1}{\left(d^2-e^2\,x^4\right)^{5/2}\,\sqrt{d-e\,x^2}}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d - e*x^2)^(1/2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d - e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.98

$$\int \frac{1}{\sqrt{d-ex^2}\left(d^2-e^2x^4\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
(2196*sqrt(d + e*x**2)*d**3*e*x - 732*sqrt(d + e*x**2)*d**2*e**2*x**3 - 15
24*sqrt(d + e*x**2)*d*e**3*x**5 + 636*sqrt(d + e*x**2)*e**4*x**7 - 603*sqr
t(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x
)/sqrt(d))*d**4 + 1206*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqr
t(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d**2*e**2*x**4 - 603*sqrt(e)*sqrt(2)*
log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e*
*4*x**8 + 603*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sq
rt(d) + sqrt(e)*x)/sqrt(d))*d**4 - 1206*sqrt(e)*sqrt(2)*log((sqrt(d + e*x*
*2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*d**2*e**2*x**4 + 603
*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(
e)*x)/sqrt(d))*e**4*x**8 + 603*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqr
t(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d**4 - 1206*sqrt(e)*sqrt(2)*l
og((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d**
2*e**2*x**4 + 603*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2)
- sqrt(d) + sqrt(e)*x)/sqrt(d))*e**4*x**8 - 603*sqrt(e)*sqrt(2)*log((sqrt(
d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*d**4 + 1206*
sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e
)*x)/sqrt(d))*d**2*e**2*x**4 - 603*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) +
 sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*e**4*x**8 - 1172*sqrt(e)*
d**4 + 2344*sqrt(e)*d**2*e**2*x**4 - 1172*sqrt(e)*e**4*x**8)/(4608*d**5...
```

3.172 $\int \frac{1}{(d-ex^2)^{3/2}(d^2-e^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 239

$$\begin{aligned}\int \frac{1}{(d-ex^2)^{3/2}(d^2-e^2x^4)^{5/2}}\,dx &= \frac{x}{12d^2(d-ex^2)^{3/2}(d^2-e^2x^4)^{3/2}} \\ &+ \frac{19x}{96d^3\sqrt{d-ex^2}\,(d^2-e^2x^4)^{3/2}} + \frac{61x\sqrt{d-ex^2}}{128d^4(d^2-e^2x^4)^{3/2}} \\ &- \frac{71x(d-ex^2)^{3/2}}{256d^5(d^2-e^2x^4)^{3/2}} - \frac{9x\sqrt{d-ex^2}}{512d^6\sqrt{d^2-e^2x^4}} + \frac{275\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{512\sqrt{2}d^6\sqrt{e}}\end{aligned}$$

output
```
1/12*x/d^2/(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(3/2)+19/96*x/d^3/(-e*x^2+d)^(1
/2)/(-e^2*x^4+d^2)^(3/2)+61/128*x*(-e*x^2+d)^(1/2)/d^4/(-e^2*x^4+d^2)^(3/2
)-71/256*x*(-e*x^2+d)^(3/2)/d^5/(-e^2*x^4+d^2)^(3/2)-9/512*x*(-e*x^2+d)^(1
/2)/d^6/(-e^2*x^4+d^2)^(1/2)+275/1024*arctanh(2^(1/2)*e^(1/2)*x*(-e*x^2+d)
^(1/2)/(-e^2*x^4+d^2)^(1/2))*2^(1/2)/d^6/e^(1/2)
```

Mathematica [A] (verified)

Time = 5.43 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(d-ex^2\right)^{3/2}\left(d^2-e^2x^4\right)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2x^4}\Big(2\sqrt{e}x\sqrt{d+ex^2}(711d^4-436d^3ex^2-546d^2e^2x^4+372de^3x}{3072d^6\sqrt{e}\left(d-ex^2\right)^{7/}}$$

input
```
Integrate[1/((d - e*x^2)^(3/2)*(d^2 - e^2*x^4)^(5/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^4]*(2*Sqrt[e]*x*Sqrt[d + e*x^2]*(711*d^4 - 436*d^3*e*x^2
 - 546*d^2*e^2*x^4 + 372*d*e^3*x^6 + 27*e^4*x^8) + 825*Sqrt[2]*(d - e*x^2)
^3*(d + e*x^2)^2*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]))/(3072*d^6*
Sqrt[e]*(d - e*x^2)^(7/2)*(d + e*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {1396, 316, 27, 402, 27, 402, 27, 402, 27, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(d-ex^2\right)^{3/2}\left(d^2-e^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \textbf{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{1}{\left(d-ex^2\right)^4\left(ex^2+d\right)^{5/2}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \textbf{316}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{e\left(8ex^2+11d\right)}{\left(d-ex^2\right)^3\left(ex^2+d\right)^{5/2}}dx}{12d^2e}+\frac{x}{12d^2\left(d-ex^2\right)^3\left(d+ex^2\right)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\int \frac{8ex^2+11d}{\left(d-ex^2\right)^3\left(ex^2+d\right)^{5/2}}dx}{12d^2}+\frac{x}{12d^2(d-ex^2)^3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{\int \frac{3de\left(38ex^2+23d\right)}{\left(d-ex^2\right)^2\left(ex^2+d\right)^{5/2}}dx}{8d^2e}+\frac{19x}{8d\left(d-ex^2\right)^2\left(d+ex^2\right)^{3/2}}}{12d^2}+\frac{x}{12d^2(d-ex^2)^3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\int \frac{38ex^2+23d}{\left(d-ex^2\right)^2\left(ex^2+d\right)^{5/2}}dx}{8d}+\frac{19x}{8d\left(d-ex^2\right)^2\left(d+ex^2\right)^{3/2}}}{12d^2}+\frac{x}{12d^2(d-ex^2)^3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\int \frac{de\left(244ex^2+31d\right)}{\left(d-ex^2\right)\left(ex^2+d\right)^{5/2}}dx}{4d^2e}+\frac{61x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}\right)}{8d}+\frac{19x}{8d\left(d-ex^2\right)^2\left(d+ex^2\right)^{3/2}}}{12d^2}+\frac{x}{12d^2(d-ex^2)^3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\int \frac{244ex^2+31d}{\left(d-ex^2\right)\left(ex^2+d\right)^{5/2}}dx}{4d}+\frac{61x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}\right)}{8d}+\frac{19x}{8d\left(d-ex^2\right)^2\left(d+ex^2\right)^{3/2}}}{12d^2}+\frac{x}{12d^2(d-ex^2)^3(d+ex^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{-\frac{\int -\frac{3de\left(142ex^2+133d\right)}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{6d^2e}-\frac{71x}{2d\left(d+ex^2\right)^{3/2}}}{4d}+\frac{61x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}\right)}{8d}+\frac{19x}{8d\left(d-ex^2\right)^2\left(d+ex^2\right)^{3/2}}}{12d^2}+\frac{x}{12d^2(d-ex^2)}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{\int \frac{142ex^2+133d}{\left(d-ex^2\right)\left(ex^2+d\right)^{3/2}}dx}{2d}-\frac{71x}{2d\left(d+ex^2\right)^{3/2}}}{4d}+\frac{61x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}\right)}{8d}+\frac{19x}{8d\left(d-ex^2\right)^2\left(d+ex^2\right)^{3/2}}}{12d^2}+\frac{x}{12d^2(d-ex^2)^3(d}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{-\frac{\int -\frac{275d^2e}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx}{2d^2e}-\frac{9x}{2d\sqrt{d+ex^2}}}{2d}-\frac{71x}{2d\left(d+ex^2\right)^{3/2}}}{4d}+\frac{61x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}\right)}{8d}+\frac{19x}{8d\left(d-ex^2\right)^2\left(d+ex^2\right)^{3/2}}}{12d^2}+\frac{}{12d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{\frac{275}{2}\int \frac{1}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx-\frac{9x}{2d\sqrt{d+ex^2}}}{2d}-\frac{71x}{2d\left(d+ex^2\right)^{3/2}}}{4d}+\frac{61x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}\right)}{8d}+\frac{19x}{8d\left(d-ex^2\right)^2\left(d+ex^2\right)^{3/2}}}{12d^2}+\frac{}{12d^2}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \; 291$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{\frac{275}{2}\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{9x}{2d\sqrt{d+ex^2}}}{2d}-\frac{71x}{2d\left(d+ex^2\right)^{3/2}}}{4d}+\frac{61x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}\right)}{8d}+\frac{19x}{8d\left(d-ex^2\right)^2\left(d+ex^2\right)^{3/2}}}{12d^2}+\frac{}{12d}\right.}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{\frac{3\left(\frac{\frac{\frac{275\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d\sqrt{e}}-\frac{9x}{2d\sqrt{d+ex^2}}}{2d}-\frac{71x}{2d\left(d+ex^2\right)^{3/2}}}{4d}+\frac{61x}{4d\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}\right)}{8d}+\frac{19x}{8d\left(d-ex^2\right)^2\left(d+ex^2\right)^{3/2}}}{12d^2}+\frac{}{12d}\right.}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[1/((d - e*x^2)^(3/2)*(d^2 - e^2*x^4)^(5/2)),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(x/(12*d^2*(d - e*x^2)^3*(d + e*x^2)^(3/2
)) + ((19*x)/(8*d*(d - e*x^2)^2*(d + e*x^2)^(3/2)) + (3*((61*x)/(4*d*(d -
e*x^2)*(d + e*x^2)^(3/2)) + ((-71*x)/(2*d*(d + e*x^2)^(3/2)) + ((-9*x)/(2*
d*Sqrt[d + e*x^2]) + (275*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*Sqrt[2]*d*Sqrt[e]))/(2*d))/(4*d)))/(8*d))/(12*d^2)))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. $2(195) = 390$.

Time = 0.52 (sec) , antiderivative size = 1325, normalized size of antiderivative = 5.54

method	result	size
default	Expression too large to display	1325

input
```
int(1/(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96*(-e^2*x^4+d^2)^(1/2)*e^(23/2)*(960*ln(((e*x^2+d)^(1/2)*e^(1/2)+e*x)/
e^(1/2))*e^6*x^12*(d*e)^(1/2)-960*ln((e^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x
+(-d*e)^(1/2)))^(1/2)+e*x)/e^(1/2))*e^6*x^12*(d*e)^(1/2)+825*ln(2*e*(2^(1/
2)*d^(1/2)*(e*x^2+d)^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*2^(1/2)*d^(
13/2)*e^(1/2)-825*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)
/(e*x+(d*e)^(1/2)))*2^(1/2)*d^(13/2)*e^(1/2)+1132*e^(11/2)*x^11*(d*e)^(1/2
)*(e*x^2+d)^(1/2)-1024*e^(11/2)*x^11*(d*e)^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-
e*x+(-d*e)^(1/2)))^(1/2)+1152*d^5*(d*e)^(1/2)*e^(1/2)*(-(e*x+(-d*e)^(1/2))
/e*(-e*x+(-d*e)^(1/2)))^(1/2)*x+2475*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/
2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*2^(1/2)*d^(5/2)*e^(9/2)*x^8-2475*ln
(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2)))*
2^(1/2)*d^(5/2)*e^(9/2)*x^8-2475*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)+(
d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*2^(1/2)*d^(9/2)*e^(5/2)*x^4+2475*ln(2*e
*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2)))*2^(1
/2)*d^(9/2)*e^(5/2)*x^4-2880*ln((e^(1/2)*(-(e*x+(-d*e)^(1/2))/e*(-e*x+(-d*
e)^(1/2)))^(1/2)+e*x)/e^(1/2))*d^4*e^2*x^4*(d*e)^(1/2)+700*d*e^(9/2)*x^9*(
d*e)^(1/2)*(e*x^2+d)^(1/2)-3000*d^2*e^(7/2)*x^7*(d*e)^(1/2)*(e*x^2+d)^(1/2
)-1880*d^3*e^(5/2)*x^5*(d*e)^(1/2)*(e*x^2+d)^(1/2)+2380*d^4*e^(3/2)*x^3*(d
*e)^(1/2)*(e*x^2+d)^(1/2)-825*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)+(d*e
)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*2^(1/2)*e^(13/2)*x^12*d^(1/2)+825*ln(2*...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.38

$$\int \frac{1}{(d-ex^2)^{3/2}(d^2-e^2x^4)^{5/2}}\,dx = \left[\frac{825\sqrt{2}(e^6x^{12}-2\,de^5x^{10}-d^2e^4x^8+4\,d^3e^3x^6-d^4e^2x^4-2\,d^5ex^2+d}{61}\right.$$

input
```
integrate(1/(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/6144*(825*sqrt(2)*(e^6*x^12 - 2*d*e^5*x^10 - d^2*e^4*x^8 + 4*d^3*e^3*x^
6 - d^4*e^2*x^4 - 2*d^5*e*x^2 + d^6)*sqrt(e)*log(-(3*e^2*x^4 - 2*d*e*x^2 -
 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(e)*x - d^2)/(e^2*x^4
 - 2*d*e*x^2 + d^2)) + 4*(27*e^5*x^9 + 372*d*e^4*x^7 - 546*d^2*e^3*x^5 - 4
36*d^3*e^2*x^3 + 711*d^4*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d))/(d^6*
e^7*x^12 - 2*d^7*e^6*x^10 - d^8*e^5*x^8 + 4*d^9*e^4*x^6 - d^10*e^3*x^4 - 2
*d^11*e^2*x^2 + d^12*e), 1/3072*(825*sqrt(2)*(e^6*x^12 - 2*d*e^5*x^10 - d^
2*e^4*x^8 + 4*d^3*e^3*x^6 - d^4*e^2*x^4 - 2*d^5*e*x^2 + d^6)*sqrt(-e)*arct
an(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(-e)*x/(e^2*x^4 - d^2
)) + 2*(27*e^5*x^9 + 372*d*e^4*x^7 - 546*d^2*e^3*x^5 - 436*d^3*e^2*x^3 + 7
11*d^4*e*x)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d))/(d^6*e^7*x^12 - 2*d^7*e
^6*x^10 - d^8*e^5*x^8 + 4*d^9*e^4*x^6 - d^10*e^3*x^4 - 2*d^11*e^2*x^2 + d^
12*e)]
```

Sympy [F]

$$\int \frac{1}{(d-ex^2)^{3/2}(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex^2\right)\left(d+ex^2\right)\right)^{\frac{5}{2}}\left(d-ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e*x**2+d)**(3/2)/(-e**2*x**4+d**2)**(5/2),x)
```

output
```
Integral(1/((-(-d + e*x**2)*(d + e*x**2))**(5/2)*(d - e*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d-ex^2)^{3/2}(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{(-e^2x^4+d^2)^{\frac{5}{2}}(-ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(5/2)*(-e*x^2 + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d-ex^2)^{3/2}(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{(-e^2x^4+d^2)^{\frac{5}{2}}(-ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^4 + d^2)^(5/2)*(-e*x^2 + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d-ex^2)^{3/2}(d^2-e^2x^4)^{5/2}}\,dx = \int \frac{1}{(d^2-e^2\,x^4)^{5/2}\,(d-e\,x^2)^{3/2}}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d - e*x^2)^(3/2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)^(5/2)*(d - e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1181, normalized size of antiderivative = 4.94

$$\int \frac{1}{(d-ex^2)^{3/2}\,(d^2-e^2x^4)^{5/2}}\,dx = \text{Too large to display}$$

input

```
int(1/(-e*x^2+d)^(3/2)/(-e^2*x^4+d^2)^(5/2),x)
```

output

```
(2844*sqrt(d + e*x**2)*d**4*e*x - 1744*sqrt(d + e*x**2)*d**3*e**2*x**3 - 2
184*sqrt(d + e*x**2)*d**2*e**3*x**5 + 1488*sqrt(d + e*x**2)*d*e**4*x**7 +
108*sqrt(d + e*x**2)*e**5*x**9 - 825*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2)
 - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d**5 + 825*sqrt(e)*sqrt
(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d)
)*d**4*e*x**2 + 1650*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(
2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d**3*e**2*x**4 - 1650*sqrt(e)*sqrt(2)*l
og((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*d**
2*e**3*x**6 - 825*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2)
- sqrt(d) + sqrt(e)*x)/sqrt(d))*d*e**4*x**8 + 825*sqrt(e)*sqrt(2)*log((sqr
t(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*e**5*x**10
 + 825*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) +
 sqrt(e)*x)/sqrt(d))*d**5 - 825*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sq
rt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*d**4*e*x**2 - 1650*sqrt(e)*s
qrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt
(d))*d**3*e**2*x**4 + 1650*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)
*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*d**2*e**3*x**6 + 825*sqrt(e)*sqrt
(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d)
)*d*e**4*x**8 - 825*sqrt(e)*sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2
) + sqrt(d) + sqrt(e)*x)/sqrt(d))*e**5*x**10 + 825*sqrt(e)*sqrt(2)*log(...
```

3.173 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 2

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} \, dx = \arcsin(x)$$

output
```
arcsin(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 32 vs. $2(2) = 4$.

Time = 0.78 (sec) , antiderivative size = 32, normalized size of antiderivative = 16.00

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} \, dx = -\arctan\left(\frac{x\sqrt{1+x^2}\sqrt{1-x^4}}{-1+x^4}\right)$$

input
```
Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
```

output
```
-ArcTan[(x*Sqrt[1 + x^2]*Sqrt[1 - x^4])/(-1 + x^4)]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1386, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{1386}$$

$$\int \frac{1}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{223}$$

$$\arcsin(x)$$

input

```
Int[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
```

output

```
ArcSin[x]
```

Defintions of rubi rules used

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1386

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(2) = 4$.

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 14.50

method	result	size
default	$\frac{\sqrt{-x^4+1}\,\arcsin(x)}{\sqrt{x^2+1}\,\sqrt{-x^2+1}}$	29

input
```
int((x^2+1)^(1/2)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(x^2+1)^(1/2)*(-x^4+1)^(1/2)/(-x^2+1)^(1/2)*arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(2) = 4$.

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 14.00

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = -\arctan\left(\frac{\sqrt{-x^4+1}\sqrt{x^2+1}x}{x^4-1}\right)$$

input
```
integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt(-x^4 + 1)*sqrt(x^2 + 1)*x/(x^4 - 1))
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-(x-1)(x+1)(x^2+1)}}\,dx$$

input
```
integrate((x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
```

output
```
Integral(sqrt(x**2 + 1)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{1-x^4}}\,dx$$

input
```
int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2),x)
```

output
```
int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = asin(x)$$

input

```
int((x^2+1)^(1/2)/(-x^4+1)^(1/2),x)
```

output

```
asin(x)
```

3.174 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 2

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \operatorname{arcsinh}(x)$$

output
```
arcsinh(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(2) = 4.

Time = 0.77 (sec) , antiderivative size = 42, normalized size of antiderivative = 21.00

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \log\left(1-x^2\right) - \log\left(-x+x^3+\sqrt{1-x^2}\sqrt{1-x^4}\right)$$

input
```
Integrate[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
```

output
```
Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x^2]*Sqrt[1 - x^4]]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1386, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \; 1386$$

$$\int \frac{1}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \; 222$$

$$\text{arcsinh}(x)$$

input
```
Int[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
```

output
```
ArcSinh[x]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(2) = 4$.

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 14.50

method	result	size
default	$\frac{\sqrt{-x^4+1}\,\operatorname{arcsinh}(x)}{\sqrt{-x^2+1}\,\sqrt{x^2+1}}$	29

input

```
int((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/(-x^2+1)^(1/2)*(-x^4+1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(2) = 4$.

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 40.50

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right) + \frac{1}{2}\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)$$

input

```
integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output

```
-1/2*log((x^3 + sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) + 1/2*log(-(
x^3 - sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x))
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx=\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{-(x-1)(x+1)(x^2+1)}}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx=\int \frac{\sqrt{-x^2+1}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx=\int \frac{\sqrt{-x^2+1}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$$

input
```
int((1 - x^2)^(1/2)/(1 - x^4)^(1/2),x)
```

output
```
int((1 - x^2)^(1/2)/(1 - x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 4.50

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \log\left(\sqrt{x^2+1}+x\right)$$

input
```
int((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x)
```

output
```
log(sqrt(x**2 + 1) + x)
```

3.175 $\int \frac{\sqrt{-1+x^2}}{\sqrt{1-x^4}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{1-x^4}} \, dx = \arctan\left(\frac{x\sqrt{-1+x^2}}{\sqrt{1-x^4}}\right)$$

output
```
arctan(x*(x^2-1)^(1/2)/(-x^4+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{1-x^4}} \, dx = -\arctan\left(\frac{x\sqrt{-1+x^2}\sqrt{1-x^4}}{-1+x^4}\right)$$

input
```
Integrate[Sqrt[-1 + x^2]/Sqrt[1 - x^4],x]
```

output
```
-ArcTan[(x*Sqrt[-1 + x^2]*Sqrt[1 - x^4])/(-1 + x^4)]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1396, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{-x^2-1}\sqrt{x^2-1}\int \frac{1}{\sqrt{-x^2-1}}dx}{\sqrt{1-x^4}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{-x^2-1}\sqrt{x^2-1}\int \frac{1}{\frac{x^2}{-x^2-1}+1}d\frac{x}{\sqrt{-x^2-1}}}{\sqrt{1-x^4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{-x^2-1}\sqrt{x^2-1}\arctan\left(\frac{x}{\sqrt{-x^2-1}}\right)}{\sqrt{1-x^4}}$$

input
```
Int[Sqrt[-1 + x^2]/Sqrt[1 - x^4],x]
```

output
```
(Sqrt[-1 - x^2]*Sqrt[-1 + x^2]*ArcTan[x/Sqrt[-1 - x^2]])/Sqrt[1 - x^4]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

method	result	size
default	$\frac{\sqrt{-x^{4}+1}\, \arctan\left(\frac{x}{\sqrt{-x^{2}-1}}\right)}{\sqrt{x^{2}-1}\, \sqrt{-x^{2}-1}}$	39

input
```
int((x^2-1)^(1/2)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(x^2-1)^(1/2)*(-x^4+1)^(1/2)/(-x^2-1)^(1/2)*arctan(x/(-x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{1-x^4}}\,dx = -\arctan\left(\frac{\sqrt{-x^4+1}\sqrt{x^2-1}x}{x^4-1}\right)$$

input
```
integrate((x^2-1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt(-x^4 + 1)*sqrt(x^2 - 1)*x/(x^4 - 1))
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{(x-1)\,(x+1)}}{\sqrt{-(x-1)\,(x+1)\,(x^2+1)}}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(-x**4+1)**(1/2),x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2-1}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((x^2-1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - 1)/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2-1}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((x^2-1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - 1)/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{1-x^4}}\,dx=\int \frac{\sqrt{x^2-1}}{\sqrt{1-x^4}}\,dx$$

input

```
int((x^2 - 1)^(1/2)/(1 - x^4)^(1/2),x)
```

output

```
int((x^2 - 1)^(1/2)/(1 - x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{1-x^4}}\,dx=-asinh(x)\,i$$

input

```
int((x^2-1)^(1/2)/(-x^4+1)^(1/2),x)
```

output

```
 - asinh(x)*i
```

3.176 $\int \frac{\sqrt{-1-x^2}}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{1-x^4}}\,dx = \operatorname{arctanh}\left(\frac{x\sqrt{-1-x^2}}{\sqrt{1-x^4}}\right)$$

output
```
arctanh(x*(-x^2-1)^(1/2)/(-x^4+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{1-x^4}}\,dx = \log\left(1+x^2\right) - \log\left(x+x^3+\sqrt{-1-x^2}\sqrt{1-x^4}\right)$$

input
```
Integrate[Sqrt[-1 - x^2]/Sqrt[1 - x^4],x]
```

output
```
Log[1 + x^2] - Log[x + x^3 + Sqrt[-1 - x^2]*Sqrt[1 - x^4]]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1396, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-x^2-1}}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{-x^2-1}\sqrt{x^2-1}\int \frac{1}{\sqrt{x^2-1}}dx}{\sqrt{1-x^4}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{-x^2-1}\sqrt{x^2-1}\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}}{\sqrt{1-x^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{-x^2-1}\sqrt{x^2-1}\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{1-x^4}}$$

input
```
Int[Sqrt[-1 - x^2]/Sqrt[1 - x^4],x]
```

output
```
(Sqrt[-1 - x^2]*Sqrt[-1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[1 - x^4]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

method	result	size
default	$\frac{\sqrt{-x^4+1}\,\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{-x^2-1}\,\sqrt{x^2-1}}$	37

input
```
int((-x^2-1)^(1/2)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-x^2-1)^(1/2)*(-x^4+1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(21) = 42.

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.92

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2-1}+x}{x^3+x}\right) + \frac{1}{2}\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2-1}+x}{x^3+x}\right)$$

input
```
integrate((-x^2-1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log((x^3 + sqrt(-x^4 + 1)*sqrt(-x^2 - 1) + x)/(x^3 + x)) + 1/2*log(-(
x^3 - sqrt(-x^4 + 1)*sqrt(-x^2 - 1) + x)/(x^3 + x))
```

Sympy [F]

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-x^2-1}}{\sqrt{-(x-1)(x+1)(x^2+1)}}\,dx$$

input
```
integrate((-x**2-1)**(1/2)/(-x**4+1)**(1/2),x)
```

output
```
Integral(sqrt(-x**2 - 1)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-x^2-1}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((-x^2-1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 - 1)/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-x^2-1}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((-x^2-1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 - 1)/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-x^2-1}}{\sqrt{1-x^4}}\,dx$$

input
```
int((- x^2 - 1)^(1/2)/(1 - x^4)^(1/2),x)
```

output
```
int((- x^2 - 1)^(1/2)/(1 - x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{1-x^4}}\,dx = asin(x)\,i$$

input
```
int((-x^2-1)^(1/2)/(-x^4+1)^(1/2),x)
```

output
```
asin(x)*i
```

3.177 $\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\operatorname{arctanh}\left(\frac{x\sqrt{1+x^2}}{\sqrt{-1+x^4}}\right)$$

output
```
arctanh(x*(x^2+1)^(1/2)/(x^4-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=-\log\left(1+x^2\right)+\log\left(x+x^3+\sqrt{1+x^2}\sqrt{-1+x^4}\right)$$

input
```
Integrate[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]
```

output
```
-Log[1 + x^2] + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1396, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \textcolor{blue}{1396}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\int \frac{1}{\sqrt{x^2-1}}dx}{\sqrt{x^4-1}}$$

$$\Big\downarrow \textcolor{blue}{224}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}}{\sqrt{x^4-1}}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}$$

input
```
Int[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]
```

output
```
(Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

method	result	size
default	$\frac{\sqrt{x^4-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}\,\sqrt{x^2-1}}$	33

input

```
int((x^2+1)^(1/2)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/(x^2+1)^(1/2)*(x^4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.10

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right) - \frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)$$

input

```
integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3
 - sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x))
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}}\,dx$$

input
```
integrate((x**2+1)**(1/2)/(x**4-1)**(1/2),x)
```

output
```
Integral(sqrt(x**2 + 1)/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

input
```
int((x^2 + 1)^(1/2)/(x^4 - 1)^(1/2),x)
```

output
```
int((x^2 + 1)^(1/2)/(x^4 - 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \log\left(\sqrt{x^2-1}+x\right)$$

input
```
int((x^2+1)^(1/2)/(x^4-1)^(1/2),x)
```

output
```
log(sqrt(x**2 - 1) + x)
```

3.178 $\int \frac{\sqrt{1-x^2}}{\sqrt{-1+x^4}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+x^4}} \, dx = \arctan\left(\frac{x\sqrt{1-x^2}}{\sqrt{-1+x^4}}\right)$$

output
```
arctan(x*(-x^2+1)^(1/2)/(x^4-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+x^4}} \, dx = \arctan\left(\frac{x\sqrt{1-x^2}}{\sqrt{-1+x^4}}\right)$$

input
```
Integrate[Sqrt[1 - x^2]/Sqrt[-1 + x^4],x]
```

output
```
ArcTan[(x*Sqrt[1 - x^2])/Sqrt[-1 + x^4]]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1396, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{-x^2-1}\sqrt{1-x^2}\int \frac{1}{\sqrt{-x^2-1}}dx}{\sqrt{x^4-1}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\sqrt{-x^2-1}\sqrt{1-x^2}\int \frac{1}{\frac{x^2}{-x^2-1}+1}d\frac{x}{\sqrt{-x^2-1}}}{\sqrt{x^4-1}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt{-x^2-1}\sqrt{1-x^2}\arctan\left(\frac{x}{\sqrt{-x^2-1}}\right)}{\sqrt{x^4-1}}$$

input
```
Int[Sqrt[1 - x^2]/Sqrt[-1 + x^4],x]
```

output
```
(Sqrt[-1 - x^2]*Sqrt[1 - x^2]*ArcTan[x/Sqrt[-1 - x^2]])/Sqrt[-1 + x^4]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

method	result	size
default	$\frac{\sqrt{x^{4}-1}\, \arctan\left(\frac{x}{\sqrt{-x^{2}-1}}\right)}{\sqrt{-x^{2}+1}\, \sqrt{-x^{2}-1}}$	39

input
```
int((-x^2+1)^(1/2)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-x^2+1)^(1/2)*(x^4-1)^(1/2)/(-x^2-1)^(1/2)*arctan(x/(-x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+x^4}}\,dx = \arctan\left(\frac{\sqrt{-x^2+1}x}{\sqrt{x^4-1}}\right)$$

input
```
integrate((-x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
arctan(sqrt(-x^2 + 1)*x/sqrt(x^4 - 1))
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+x^4}}\,dx=\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{\sqrt{\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(x**4-1)**(1/2),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+x^4}}\,dx=\int \frac{\sqrt{-x^2+1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+x^4}}\,dx=\int \frac{\sqrt{-x^2+1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+x^4}}\,dx=\int \frac{\sqrt{1-x^2}}{\sqrt{x^4-1}}\,dx$$

input
```
int((1 - x^2)^(1/2)/(x^4 - 1)^(1/2),x)
```

output
```
int((1 - x^2)^(1/2)/(x^4 - 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+x^4}}\,dx=-asinh(x)\,i$$

input
```
int((-x^2+1)^(1/2)/(x^4-1)^(1/2),x)
```

output
```
- asinh(x)*i
```

3.179 $\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size $=19$, antiderivative size $=21$

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx=\operatorname{arctanh}\left(\frac{x\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\right)$$

output
```
arctanh(x*(x^2-1)^(1/2)/(x^4-1)^(1/2))
```

Mathematica [A] (verified)

Time $=0.69$ (sec) , antiderivative size $=38$, normalized size of antiderivative $=1.81$

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx=-\log\left(1-x^2\right)+\log\left(-x+x^3+\sqrt{-1+x^2}\sqrt{-1+x^4}\right)$$

input
```
Integrate[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]
```

output
```
-Log[1 - x^2] + Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1396, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\int \frac{1}{\sqrt{x^2+1}}dx}{\sqrt{x^4-1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\text{arcsinh}(x)}{\sqrt{x^4-1}}$$

input

```
Int[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]
```

output

```
(Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]
```

Defintions of rubi rules used

rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1396

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{\sqrt{x^4-1}\,\operatorname{arcsinh}(x)}{\sqrt{x^2-1}\,\sqrt{x^2+1}}$	25

input
```
int((x^2-1)^(1/2)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.48

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right) - \frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)$$

input
```
integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x)) - 1/2*log(-(x^3
 - sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x))
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{(x-1)\,(x+1)}}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(x**4-1)**(1/2),x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx=\int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

input
```
int((x^2 - 1)^(1/2)/(x^4 - 1)^(1/2),x)
```

output
```
int((x^2 - 1)^(1/2)/(x^4 - 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx=\log\left(\sqrt{x^2+1}+x\right)$$

input
```
int((x^2-1)^(1/2)/(x^4-1)^(1/2),x)
```

output
```
log(sqrt(x**2 + 1) + x)
```

3.180 $\int \frac{\sqrt{-1-x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{-1+x^4}}\,dx=\arctan\left(\frac{x\sqrt{-1-x^2}}{\sqrt{-1+x^4}}\right)$$

output

```
arctan(x*(-x^2-1)^(1/2)/(x^4-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{-1+x^4}}\,dx=\arctan\left(\frac{x\sqrt{-1-x^2}}{\sqrt{-1+x^4}}\right)$$

input

```
Integrate[Sqrt[-1 - x^2]/Sqrt[-1 + x^4],x]
```

output

```
ArcTan[(x*Sqrt[-1 - x^2])/Sqrt[-1 + x^4]]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1396, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-x^2-1}}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{-x^2-1}\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}}dx}{\sqrt{x^4-1}}$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{-x^2-1}\sqrt{1-x^2}\arcsin(x)}{\sqrt{x^4-1}}$$

input
```
Int[Sqrt[-1 - x^2]/Sqrt[-1 + x^4],x]
```

output
```
(Sqrt[-1 - x^2]*Sqrt[1 - x^2]*ArcSin[x])/Sqrt[-1 + x^4]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\sqrt{x^4-1}\,\arcsin(x)}{\sqrt{-x^2-1}\,\sqrt{-x^2+1}}$	29

input
```
int((-x^2-1)^(1/2)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-x^2-1)^(1/2)*(x^4-1)^(1/2)/(-x^2+1)^(1/2)*arcsin(x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.26

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}i\log\left(\frac{i\,x^3+\sqrt{x^4-1}\sqrt{-x^2-1}+i\,x}{x^3+x}\right) - \frac{1}{2}i\log\left(\frac{-i\,x^3+\sqrt{x^4-1}\sqrt{-x^2-1}-i\,x}{x^3+x}\right)$$

input
```
integrate((-x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*I*log((I*x^3 + sqrt(x^4 - 1)*sqrt(-x^2 - 1) + I*x)/(x^3 + x)) - 1/2*I*
log((-I*x^3 + sqrt(x^4 - 1)*sqrt(-x^2 - 1) - I*x)/(x^3 + x))
```

Sympy [F]

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{-x^2-1}}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}}\,dx$$

input
```
integrate((-x**2-1)**(1/2)/(x**4-1)**(1/2),x)
```

output
```
Integral(sqrt(-x**2 - 1)/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{-x^2-1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((-x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 - 1)/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{-x^2-1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((-x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 - 1)/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{-1+x^4}}\,dx=\int \frac{\sqrt{-x^2-1}}{\sqrt{x^4-1}}\,dx$$

input
```
int((- x^2 - 1)^(1/2)/(x^4 - 1)^(1/2),x)
```

output
```
int((- x^2 - 1)^(1/2)/(x^4 - 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{-1-x^2}}{\sqrt{-1+x^4}}\,dx=\log\left(\sqrt{x^2-1}+x\right)i$$

input
```
int((-x^2-1)^(1/2)/(x^4-1)^(1/2),x)
```

output
```
log(sqrt(x**2 - 1) + x)*i
```

3.181 $\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 45

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}} \, dx = -\operatorname{arctanh}\left(\frac{x\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\right)+\operatorname{arctanh}\left(\frac{x\sqrt{1+x^2}}{\sqrt{-1+x^4}}\right)$$

output
```
-arctanh(x*(x^2-1)^(1/2)/(x^4-1)^(1/2))+arctanh(x*(x^2+1)^(1/2)/(x^4-1)^(1
/2))
```

Mathematica [A] (verified)

Time = 4.67 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\begin{aligned}\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}} \, dx &= \log(1-x^2)-\log(1+x^2) \\ &\quad -\log\left(-x+x^3+\sqrt{-1+x^2}\sqrt{-1+x^4}\right) \\ &\quad +\log\left(x+x^3+\sqrt{1+x^2}\sqrt{-1+x^4}\right)\end{aligned}$$

input
```
Integrate[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

output
```
Log[1 - x^2] - Log[1 + x^2] - Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]
] + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\sqrt{x^2+1}\text{arcsinh}(x)}{\sqrt{x^4-1}}$$

input
```
Int[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

output
```
-((Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]) + (Sqrt[-1 + x
^2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

method	result	size
default	$-\frac{\sqrt{x^4-1}\,\operatorname{arcsinh}(x)}{\sqrt{x^2-1}\,\sqrt{x^2+1}}+\frac{\sqrt{x^4-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}\,\sqrt{x^2-1}}$	59

input
```
int((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)+1/(x^2+1)^(1/2)*(x
^4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(37) = 74$.

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.04

$$\begin{aligned}\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx &= \frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)\\ &\quad-\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)\\ &\quad-\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)\\ &\quad+\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)\end{aligned}$$

input
```
integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="frica
s")
```

output
```
1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3
 - sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log((x^3 + sqrt(x^4 -
 1)*sqrt(x^2 - 1) - x)/(x^3 - x)) + 1/2*log(-(x^3 - sqrt(x^4 - 1)*sqrt(x^2
 - 1) - x)/(x^3 - x))
```

Sympy [F]

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\int \frac{-\sqrt{x^2-1}+\sqrt{x^2+1}}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}}\,dx$$

input
```
integrate((-(x**2-1)**(1/2)+(x**2+1)**(1/2))/(x**4-1)**(1/2),x)
```

output
```
Integral((-sqrt(x**2 - 1) + sqrt(x**2 + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1
)), x)
```

Maxima [F]

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\int \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\int \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\int -\frac{\sqrt{x^2-1}-\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

input
```
int(-((x^2 - 1)^(1/2) - (x^2 + 1)^(1/2))/(x^4 - 1)^(1/2),x)
```

output
```
int(-((x^2 - 1)^(1/2) - (x^2 + 1)^(1/2))/(x^4 - 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.47

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\log\left(\sqrt{x^2-1}+x\right)-\log\left(\sqrt{x^2+1}+x\right)$$

input
```
int((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x)
```

output
```
log(sqrt(x**2 - 1) + x) - log(sqrt(x**2 + 1) + x)
```

3.182 $\int \left(\frac{3}{2}-3x^2\right)^p\,dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \left(\frac{3}{2}-3x^2\right)^p\,dx=\left(\frac{3}{2}\right)^p x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},2x^2\right)$$

output
```
(3/2)^p*x*hypergeom([1/2, -p],[3/2],2*x^2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(\frac{3}{2}-3x^2\right)^p\,dx=\left(\frac{3}{2}\right)^p x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},2x^2\right)$$

input
```
Integrate[(3/2 - 3*x^2)^p,x]
```

output
```
(3/2)^p*x*Hypergeometric2F1[1/2, -p, 3/2, 2*x^2]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{237\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{3}{2}-3x^2\right)^p dx$$

$$\Big\downarrow \text{ 237}$$

$$\left(\frac{3}{2}\right)^p x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},2x^2\right)$$

input
```
Int[(3/2 - 3*x^2)^p,x]
```

output
```
(3/2)^p*x*Hypergeometric2F1[1/2, -p, 3/2, 2*x^2]
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

method	result	size
meijerg	$3^p 2^{-p} x \,\text{hypergeom}\left(\left[\frac{1}{2},-p\right],\left[\frac{3}{2}\right],2x^2\right)$	24

input
```
int((3/2-3*x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
3^p*2^(-p)*x*hypergeom([1/2,-p],[3/2],2*x^2)
```

Fricas [F]

$$\int \left(\frac{3}{2}-3x^2\right)^p\,dx=\int \left(-3\,x^2+\frac{3}{2}\right)^p\,dx$$

input
```
integrate((3/2-3*x^2)^p,x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 3/2)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \left(\frac{3}{2}-3x^2\right)^p\,dx=2^{-p}3^px\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|2x^2e^{2i\pi}\right)$$

input
```
integrate((3/2-3*x**2)**p,x)
```

output
```
3**p*x*hyper((1/2, -p), (3/2,), 2*x**2*exp_polar(2*I*pi))/2**p
```

Maxima [F]

$$\int \left(\frac{3}{2}-3x^2\right)^p\,dx=\int \left(-3\,x^2+\frac{3}{2}\right)^p\,dx$$

input
```
integrate((3/2-3*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 3/2)^p, x)
```

Giac [F]

$$\int \left(\frac{3}{2}-3x^2\right)^p\,dx=\int \left(-3\,x^2+\frac{3}{2}\right)^p\,dx$$

input
```
integrate((3/2-3*x^2)^p,x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 3/2)^p, x)
```

Mupad [B] (verification not implemented)

Time = 17.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \left(\frac{3}{2}-3x^2\right)^p\,dx=\frac{x\,\left(\frac{3}{2}-3\,x^2\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ 2\,x^2\right)}{\left(1-2\,x^2\right)^p}$$

input
```
int((3/2 - 3*x^2)^p,x)
```

output
```
(x*(3/2 - 3*x^2)^p*hypergeom([1/2, -p], 3/2, 2*x^2))/(1 - 2*x^2)^p
```

Reduce [F]

$$\int \left(\frac{3}{2}-3x^2\right)^p dx=\frac{(-6x^2+3)^p x-4\left(\int \frac{(-6x^2+3)^p}{4p\,x^2+2x^2-2p-1}dx\right)p^2-2\left(\int \frac{(-6x^2+3)^p}{4p\,x^2+2x^2-2p-1}dx\right)p}{2^p\,(2p+1)}$$

input

```
int((3/2-3*x^2)^p,x)
```

output

```
(( - 6*x**2 + 3)**p*x - 4*int(( - 6*x**2 + 3)**p/(4*p*x**2 - 2*p + 2*x**2
- 1),x)*p**2 - 2*int(( - 6*x**2 + 3)**p/(4*p*x**2 - 2*p + 2*x**2 - 1),x)*p
)/(2**p*(2*p + 1))
```

3.183 $\int (2+4x^2)^{-p} (3-12x^4)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int (2+4x^2)^{-p} (3-12x^4)^p \, dx = \left(\frac{3}{2}\right)^p x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},2x^2\right)$$

output
```
(3/2)^p*x*hypergeom([1/2, -p],[3/2],2*x^2)
```

Mathematica [F]

$$\int (2+4x^2)^{-p} (3-12x^4)^p \, dx = \int (2+4x^2)^{-p} (3-12x^4)^p \, dx$$

input
```
Integrate[(3 - 12*x^4)^p/(2 + 4*x^2)^p,x]
```

output
```
Integrate[(3 - 12*x^4)^p/(2 + 4*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1386, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^2+2\right)^{-p} \left(3-12x^4\right)^p \, dx$$

$$\Big\downarrow \ 1386$$

$$\left(\frac{4}{3}\right)^{-p} \int \left(2-4x^2\right)^p dx$$

$$\Big\downarrow \ 237$$

$$\left(\frac{3}{2}\right)^p x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},2x^2\right)$$

input
```
Int[(3 - 12*x^4)^p/(2 + 4*x^2)^p,x]
```

output
```
(3/2)^p*x*Hypergeometric2F1[1/2, -p, 3/2, 2*x^2]
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

Maple [F]

$$\int \left(-12x^4+3\right)^p \left(4x^2+2\right)^{-p} dx$$

input
```
int((-12*x^4+3)^p/((4*x^2+2)^p),x)
```

output
```
int((-12*x^4+3)^p/((4*x^2+2)^p),x)
```

Fricas [F]

$$\int \left(2+4x^2\right)^{-p} \left(3-12x^4\right)^p \, dx = \int \frac{\left(-12\,x^4+3\right)^p}{\left(4\,x^2+2\right)^p} \, dx$$

input
```
integrate((-12*x^4+3)^p/((4*x^2+2)^p),x, algorithm="fricas")
```

output
```
integral((-12*x^4 + 3)^p/(4*x^2 + 2)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(2+4x^2\right)^{-p} \left(3-12x^4\right)^p \, dx = \text{Timed out}$$

input
```
integrate((-12*x**4+3)**p/((4*x**2+2)**p),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(2+4x^2\right)^{-p} \left(3-12x^4\right)^p \, dx = \int \frac{(-12\,x^4+3)^p}{(4\,x^2+2)^p} \, dx$$

input
```
integrate((-12*x^4+3)^p/((4*x^2+2)^p),x, algorithm="maxima")
```

output
```
integrate((-12*x^4 + 3)^p/(4*x^2 + 2)^p, x)
```

Giac [F]

$$\int \left(2+4x^2\right)^{-p} \left(3-12x^4\right)^p \, dx = \int \frac{(-12\,x^4+3)^p}{(4\,x^2+2)^p} \, dx$$

input
```
integrate((-12*x^4+3)^p/((4*x^2+2)^p),x, algorithm="giac")
```

output
```
integrate((-12*x^4 + 3)^p/(4*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(2+4x^2\right)^{-p} \left(3-12x^4\right)^p \, dx = \int \frac{(3-12\,x^4)^p}{(4\,x^2+2)^p} \, dx$$

input
```
int((3 - 12*x^4)^p/(4*x^2 + 2)^p,x)
```

output
```
int((3 - 12*x^4)^p/(4*x^2 + 2)^p, x)
```

Reduce [F]

$$\int \left(2+4x^2\right)^{-p}\left(3-12x^4\right)^p\,dx=\int \frac{(-12x^4+3)^p}{(4x^2+2)^p}dx$$

input
```
int((-12*x^4+3)^p/((4*x^2+2)^p),x)
```

output
```
int(( - 12*x**4 + 3)**p/(4*x**2 + 2)**p,x)
```

3.184 $\int (2-ex^2)^p (2+ex^2)^{p+q}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 42

$$\int (2-ex^2)^p (2+ex^2)^{p+q}\,dx = 2^{2p+q}x\,\mathrm{AppellF1}\left(\frac{1}{2},-p,-p-q,\frac{3}{2},\frac{ex^2}{2},-\frac{ex^2}{2}\right)$$

output
```
2^(2*p+q)*x*AppellF1(1/2,-p-q,-p,3/2,-1/2*e*x^2,1/2*e*x^2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(42) = 84.

Time = 0.24 (sec) , antiderivative size = 177, normalized size of antiderivative = 4.21

$$\int (2-ex^2)^p (2+ex^2)^{p+q}\,dx$$
$$= \frac{3x(2-ex^2)^p (2+ex^2)^{p+q}\,\mathrm{AppellF1}\left(\frac{1}{2},-p,-p-q,\frac{3}{2},\frac{ex^2}{2},-\frac{ex^2}{2}\right)}{3\,\mathrm{AppellF1}\left(\frac{1}{2},-p,-p-q,\frac{3}{2},\frac{ex^2}{2},-\frac{ex^2}{2}\right)+ex^2\left(-p\,\mathrm{AppellF1}\left(\frac{3}{2},1-p,-p-q,\frac{5}{2},\frac{ex^2}{2},-\frac{ex^2}{2}\right)+(p+q)\,\mathrm{A}\right.}$$

input
```
Integrate[(2 - e*x^2)^p*(2 + e*x^2)^(p + q),x]
```

output
```
(3*x*(2 - e*x^2)^p*(2 + e*x^2)^(p + q)*AppellF1[1/2, -p, -p - q, 3/2, (e*x
^2)/2, -1/2*(e*x^2)])/(3*AppellF1[1/2, -p, -p - q, 3/2, (e*x^2)/2, -1/2*(e
*x^2)] + e*x^2*(-(p*AppellF1[3/2, 1 - p, -p - q, 5/2, (e*x^2)/2, -1/2*(e*x
^2)]) + (p + q)*AppellF1[3/2, -p, 1 - p - q, 5/2, (e*x^2)/2, -1/2*(e*x^2)]
))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2-ex^2\right)^p \left(ex^2+2\right)^{p+q} \, dx$$

$$\Big\downarrow \text{333}$$

$$x 2^{2p+q} \text{AppellF1}\left(\frac{1}{2},-p,-p-q,\frac{3}{2},\frac{ex^2}{2},-\frac{ex^2}{2}\right)$$

input
```
Int[(2 - e*x^2)^p*(2 + e*x^2)^(p + q),x]
```

output
```
2^(2*p + q)*x*AppellF1[1/2, -p, -p - q, 3/2, (e*x^2)/2, -1/2*(e*x^2)]
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

Maple [F]

$$\int \left(-e\,x^{2}+2\right)^{p} \left(e\,x^{2}+2\right)^{p+q}dx$$

input
```
int((-e*x^2+2)^p*(e*x^2+2)^(p+q),x)
```

output
```
int((-e*x^2+2)^p*(e*x^2+2)^(p+q),x)
```

Fricas [F]

$$\int \left(2-ex^{2}\right)^{p} \left(2+ex^{2}\right)^{p+q}\,dx=\int \left(ex^{2}+2\right)^{p+q}\left(-ex^{2}+2\right)^{p}dx$$

input
```
integrate((-e*x^2+2)^p*(e*x^2+2)^(p+q),x, algorithm="fricas")
```

output
```
integral((e*x^2 + 2)^(p + q)*(-e*x^2 + 2)^p, x)
```

Sympy [F]

$$\int \left(2-ex^{2}\right)^{p} \left(2+ex^{2}\right)^{p+q}\,dx=\int \left(-ex^{2}+2\right)^{p} \left(ex^{2}+2\right)^{p+q}\,dx$$

input
```
integrate((-e*x**2+2)**p*(e*x**2+2)**(p+q),x)
```

output
```
Integral((-e*x**2 + 2)**p*(e*x**2 + 2)**(p + q), x)
```

Maxima [F]

$$\int \left(2-ex^2\right)^p \left(2+ex^2\right)^{p+q}\,dx=\int \left(ex^2+2\right)^{p+q}\left(-ex^2+2\right)^p\,dx$$

input
```
integrate((-e*x^2+2)^p*(e*x^2+2)^(p+q),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + 2)^(p + q)*(-e*x^2 + 2)^p, x)
```

Giac [F]

$$\int \left(2-ex^2\right)^p \left(2+ex^2\right)^{p+q}\,dx=\int \left(ex^2+2\right)^{p+q}\left(-ex^2+2\right)^p\,dx$$

input
```
integrate((-e*x^2+2)^p*(e*x^2+2)^(p+q),x, algorithm="giac")
```

output
```
integrate((e*x^2 + 2)^(p + q)*(-e*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(2-ex^2\right)^p \left(2+ex^2\right)^{p+q}\,dx=\int \left(2-e\,x^2\right)^p\,\left(e\,x^2+2\right)^{p+q}\,dx$$

input
```
int((2 - e*x^2)^p*(e*x^2 + 2)^(p + q),x)
```

output
```
int((2 - e*x^2)^p*(e*x^2 + 2)^(p + q), x)
```

Reduce [F]

$$\int \left(2-ex^2\right)^p \left(2+ex^2\right)^{p+q}\,dx$$

$$= \frac{\left(e\,x^2+2\right)^{p+q}\left(-e\,x^2+2\right)^p x+16\left(\int \frac{\left(e\,x^2+2\right)^{p+q}\left(-e\,x^2+2\right)^p x^2}{4e^2p\,x^4+2e^2q\,x^4+e^2x^4-16p-8q-4}dx\right)epq+8\left(\int \frac{\left(e\,x^2+2\right)^{p+q}\left(-e\,x^2+2\right)^p x^2}{4e^2p\,x^4+2e^2q\,x^4+e^2x^4-16p-8q-}\right.}{}$$

input

```
int((-e*x^2+2)^p*(e*x^2+2)^(p+q),x)
```

output

```
((e*x**2 + 2)**(p + q)*( - e*x**2 + 2)**p*x + 16*int(((e*x**2 + 2)**(p + q
)*( - e*x**2 + 2)**p*x**2)/(4*e**2*p*x**4 + 2*e**2*q*x**4 + e**2*x**4 - 16
*p - 8*q - 4),x)*e*p*q + 8*int(((e*x**2 + 2)**(p + q)*( - e*x**2 + 2)**p*x
**2)/(4*e**2*p*x**4 + 2*e**2*q*x**4 + e**2*x**4 - 16*p - 8*q - 4),x)*e*q**
2 + 4*int(((e*x**2 + 2)**(p + q)*( - e*x**2 + 2)**p*x**2)/(4*e**2*p*x**4 +
 2*e**2*q*x**4 + e**2*x**4 - 16*p - 8*q - 4),x)*e*q - 64*int(((e*x**2 + 2)
**(p + q)*( - e*x**2 + 2)**p)/(4*e**2*p*x**4 + 2*e**2*q*x**4 + e**2*x**4 -
 16*p - 8*q - 4),x)*p**2 - 64*int(((e*x**2 + 2)**(p + q)*( - e*x**2 + 2)**
p)/(4*e**2*p*x**4 + 2*e**2*q*x**4 + e**2*x**4 - 16*p - 8*q - 4),x)*p*q - 1
6*int(((e*x**2 + 2)**(p + q)*( - e*x**2 + 2)**p)/(4*e**2*p*x**4 + 2*e**2*q
*x**4 + e**2*x**4 - 16*p - 8*q - 4),x)*p - 16*int(((e*x**2 + 2)**(p + q)*(
 - e*x**2 + 2)**p)/(4*e**2*p*x**4 + 2*e**2*q*x**4 + e**2*x**4 - 16*p - 8*q
 - 4),x)*q**2 - 8*int(((e*x**2 + 2)**(p + q)*( - e*x**2 + 2)**p)/(4*e**2*p
*x**4 + 2*e**2*q*x**4 + e**2*x**4 - 16*p - 8*q - 4),x)*q)/(4*p + 2*q + 1)
```

3.185 $\int (2+ex^2)^q (4-e^2x^4)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 42

$$\int (2+ex^2)^q (4-e^2x^4)^p \, dx = 2^{2p+q} x \operatorname{AppellF1}\left(\frac{1}{2},-p,-p-q,\frac{3}{2},\frac{ex^2}{2},-\frac{ex^2}{2}\right)$$

output
```
2^(2*p+q)*x*AppellF1(1/2,-p-q,-p,3/2,-1/2*e*x^2,1/2*e*x^2)
```

Mathematica [F]

$$\int (2+ex^2)^q (4-e^2x^4)^p \, dx = \int (2+ex^2)^q (4-e^2x^4)^p \, dx$$

input
```
Integrate[(2 + e*x^2)^q*(4 - e^2*x^4)^p,x]
```

output
```
Integrate[(2 + e*x^2)^q*(4 - e^2*x^4)^p, x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1388, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-e^2x^4\right)^p \left(ex^2+2\right)^q\,dx$$

$$\Big\downarrow \text{1388}$$

$$\int \left(2-ex^2\right)^p \left(ex^2+2\right)^{p+q}dx$$

$$\Big\downarrow \text{333}$$

$$x2^{2p+q}\,\text{AppellF1}\left(\frac{1}{2},-p,-p-q,\frac{3}{2},\frac{ex^2}{2},-\frac{ex^2}{2}\right)$$

input
```
Int[(2 + e*x^2)^q*(4 - e^2*x^4)^p,x]
```

output
```
2^(2*p + q)*x*AppellF1[1/2, -p, -p - q, 3/2, (e*x^2)/2, -1/2*(e*x^2)]
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [F]

$$\int \left(e\,x^{2}+2\right)^{q} \left(-e^{2} x^{4}+4\right)^{p}d x$$

input
```
int((e*x^2+2)^q*(-e^2*x^4+4)^p,x)
```

output
```
int((e*x^2+2)^q*(-e^2*x^4+4)^p,x)
```

Fricas [F]

$$\int \left(2+ex^2\right)^q \left(4-e^2x^4\right)^p \, dx = \int \left(-e^{2} x^{4} + 4\right)^{p} {\left(e x^{2} + 2\right)}^{q} \,d x$$

input
```
integrate((e*x^2+2)^q*(-e^2*x^4+4)^p,x, algorithm="fricas")
```

output
```
integral((-e^2*x^4 + 4)^p*(e*x^2 + 2)^q, x)
```

Sympy [F]

$$\int \left(2+ex^2\right)^q \left(4-e^2x^4\right)^p \, dx = \int \left(-\left(ex^{2} - 2\right) \left(ex^{2} + 2\right)\right)^{p} \left(ex^{2} + 2\right)^{q}\, dx$$

input
```
integrate((e*x**2+2)**q*(-e**2*x**4+4)**p,x)
```

output
```
Integral((-(e*x**2 - 2)*(e*x**2 + 2))**p*(e*x**2 + 2)**q, x)
```

Maxima [F]

$$\int \left(2+ex^2\right)^q \left(4-e^2x^4\right)^p \, dx = \int \left(-e^2x^4+4\right)^p \left(ex^2+2\right)^q \, dx$$

input
```
integrate((e*x^2+2)^q*(-e^2*x^4+4)^p,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^4 + 4)^p*(e*x^2 + 2)^q, x)
```

Giac [F]

$$\int \left(2+ex^2\right)^q \left(4-e^2x^4\right)^p \, dx = \int \left(-e^2x^4+4\right)^p \left(ex^2+2\right)^q \, dx$$

input
```
integrate((e*x^2+2)^q*(-e^2*x^4+4)^p,x, algorithm="giac")
```

output
```
integrate((-e^2*x^4 + 4)^p*(e*x^2 + 2)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(2+ex^2\right)^q \left(4-e^2x^4\right)^p \, dx = \int \left(4-e^2\,x^4\right)^p \left(e\,x^2+2\right)^q \, dx$$

input
```
int((4 - e^2*x^4)^p*(e*x^2 + 2)^q,x)
```

output
```
int((4 - e^2*x^4)^p*(e*x^2 + 2)^q, x)
```

Reduce [F]

$$\int \left(2+ex^2\right)^q \left(4-e^2x^4\right)^p\,dx$$
$$= \frac{\left(e\,x^2+2\right)^q\left(-e^2x^4+4\right)^p x+16\left(\int \frac{\left(e\,x^2+2\right)^q\left(-e^2x^4+4\right)^p x^2}{4e^2p\,x^4+2e^2q\,x^4+e^2x^4-16p-8q-4}dx\right)epq+8\left(\int \frac{\left(e\,x^2+2\right)^q\left(-e^2x^4+4\right)^p x^2}{4e^2p\,x^4+2e^2q\,x^4+e^2x^4-16p-8q-4}\right.}{}$$

input

```
int((e*x^2+2)^q*(-e^2*x^4+4)^p,x)
```

output

```
((e*x**2 + 2)**q*( - e**2*x**4 + 4)**p*x + 16*int(((e*x**2 + 2)**q*( - e**
2*x**4 + 4)**p*x**2)/(4*e**2*p*x**4 + 2*e**2*q*x**4 + e**2*x**4 - 16*p - 8
*q - 4),x)*e*p*q + 8*int(((e*x**2 + 2)**q*( - e**2*x**4 + 4)**p*x**2)/(4*e
**2*p*x**4 + 2*e**2*q*x**4 + e**2*x**4 - 16*p - 8*q - 4),x)*e*q**2 + 4*int
(((e*x**2 + 2)**q*( - e**2*x**4 + 4)**p*x**2)/(4*e**2*p*x**4 + 2*e**2*q*x*
*4 + e**2*x**4 - 16*p - 8*q - 4),x)*e*q - 64*int(((e*x**2 + 2)**q*( - e**2
*x**4 + 4)**p)/(4*e**2*p*x**4 + 2*e**2*q*x**4 + e**2*x**4 - 16*p - 8*q - 4
),x)*p**2 - 64*int(((e*x**2 + 2)**q*( - e**2*x**4 + 4)**p)/(4*e**2*p*x**4
+ 2*e**2*q*x**4 + e**2*x**4 - 16*p - 8*q - 4),x)*p*q - 16*int(((e*x**2 + 2
)**q*( - e**2*x**4 + 4)**p)/(4*e**2*p*x**4 + 2*e**2*q*x**4 + e**2*x**4 - 1
6*p - 8*q - 4),x)*p - 16*int(((e*x**2 + 2)**q*( - e**2*x**4 + 4)**p)/(4*e*
*2*p*x**4 + 2*e**2*q*x**4 + e**2*x**4 - 16*p - 8*q - 4),x)*q**2 - 8*int(((
e*x**2 + 2)**q*( - e**2*x**4 + 4)**p)/(4*e**2*p*x**4 + 2*e**2*q*x**4 + e**
2*x**4 - 16*p - 8*q - 4),x)*q)/(4*p + 2*q + 1)
```

3.186 $\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx=-\frac{x\sqrt{1+x^4}}{1+x^2}+\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{1+x^4}}$$

output

```
-x*(x^4+1)^(1/2)/(x^2+1)+(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(2
*arctan(x)),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\begin{aligned}\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx&=x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^4\right)\\&\quad-\frac{1}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-x^4\right)\end{aligned}$$

input

```
Integrate[(1 - x^2)/Sqrt[1 + x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4] - (x^3*Hypergeometric2F1[1/2, 3/4
, 7/4, -x^4])/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{1510\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{\sqrt{x^4+1}}\,dx$$

$$\Big\downarrow \; 1510$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+1}}-\frac{x\sqrt{x^4+1}}{x^2+1}$$

input
```
Int[(1 - x^2)/Sqrt[1 + x^4],x]
```

output
```
-((x*Sqrt[1 + x^4])/(1 + x^2)) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*El
lipticE[2*ArcTan[x], 1/2])/Sqrt[1 + x^4]
```

Defintions of rubi rules used

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.52

method	result
meijerg	$-\frac{x^{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-x^{4}\right)}{3}+x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-x^{4}\right)$
default	$-\frac{i\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\left(\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\mathrm{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}+\frac{\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right.}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$
elliptic	$-\frac{i\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\left(\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\mathrm{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}+\frac{\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right.}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$

input
```
int((-x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3*hypergeom([1/2,3/4],[7/4],-x^4)+x*hypergeom([1/4,1/2],[5/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx=\frac{-i\sqrt{i}xE\left(\arcsin\left(\frac{\sqrt{i}}{x}\right)\mid -1\right)+2i\sqrt{i}xF\left(\arcsin\left(\frac{\sqrt{i}}{x}\right)\mid -1\right)-\sqrt{x^4+1}}{x}$$

input
```
integrate((-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(I)*x*elliptic_e(arcsin(sqrt(I)/x), -1) + 2*I*sqrt(I)*x*elliptic_f
(arcsin(sqrt(I)/x), -1) - sqrt(x^4 + 1))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx = -\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-x**2+1)/(x**4+1)**(1/2),x)
```

output
```
-x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi))/(4*gamma(
7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi))/(4*ga
mma(5/4))
```

Maxima [F]

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx = -\int \frac{x^2-1}{\sqrt{x^4+1}}\,dx$$

input
```
int(-(x^2 - 1)/(x^4 + 1)^(1/2),x)
```

output
```
-int((x^2 - 1)/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^4+1}}{x^4+1}dx - \left(\int \frac{\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)$$

input
```
int((-x^2+1)/(x^4+1)^(1/2),x)
```

output
```
int(sqrt(x**4 + 1)/(x**4 + 1),x) - int((sqrt(x**4 + 1)*x**2)/(x**4 + 1),x)
```

3.187 $\int \frac{1-e^2x^2}{\sqrt{1+e^4x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{1-e^2x^2}{\sqrt{1+e^4x^4}}\,dx=-\frac{x\sqrt{1+e^4x^4}}{1+e^2x^2}+\frac{(1+e^2x^2)\sqrt{\frac{1+e^4x^4}{(1+e^2x^2)^2}}E\left(2\arctan(ex)\,\middle|\,\frac{1}{2}\right)}{e\sqrt{1+e^4x^4}}$$

output
```
-x*(e^4*x^4+1)^(1/2)/(e^2*x^2+1)+(e^2*x^2+1)*((e^4*x^4+1)/(e^2*x^2+1)^2)^(
1/2)*EllipticE(sin(2*arctan(e*x)),1/2*2^(1/2))/e/(e^4*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.55

$$\begin{aligned}\int \frac{1-e^2x^2}{\sqrt{1+e^4x^4}}\,dx&=x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^4x^4\right)\\&\quad-\frac{1}{3}e^2x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^4x^4\right)\end{aligned}$$

input
```
Integrate[(1 - e^2*x^2)/Sqrt[1 + e^4*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, -(e^4*x^4)] - (e^2*x^3*Hypergeometric2F
1[1/2, 3/4, 7/4, -(e^4*x^4)])/3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{1510\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-e^2x^2}{\sqrt{e^4x^4+1}}dx$$

$$\Big\downarrow \text{1510}$$

$$\frac{(e^2x^2+1)\sqrt{\frac{e^4x^4+1}{(e^2x^2+1)^2}}E\left(2\arctan(ex)\left|\frac{1}{2}\right.\right)}{e\sqrt{e^4x^4+1}}-\frac{x\sqrt{e^4x^4+1}}{e^2x^2+1}$$

input
```
Int[(1 - e^2*x^2)/Sqrt[1 + e^4*x^4],x]
```

output
```
-((x*Sqrt[1 + e^4*x^4])/(1 + e^2*x^2)) + ((1 + e^2*x^2)*Sqrt[(1 + e^4*x^4)
/(1 + e^2*x^2)^2]*EllipticE[2*ArcTan[e*x], 1/2])/(e*Sqrt[1 + e^4*x^4])
```

Defintions of rubi rules used

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

method	result	size
meijerg	$-\frac{e^{2} x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right],\left[\frac{7}{4}\right],-e^{4} x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{5}{4}\right],-e^{4} x^{4}\right)$	40
default	$-\frac{i \sqrt{-i e^{2} x^{2}+1}\, \sqrt{i e^{2} x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{i e^{2}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{i e^{2}}, i\right)\right)}{\sqrt{i e^{2}}\, \sqrt{e^{4} x^{4}+1}}+\frac{\sqrt{-i e^{2} x^{2}+1}\, \sqrt{i e^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i e^{2}}, i\right)}{\sqrt{i e^{2}}\, \sqrt{e^{4} x^{4}+1}}$	138
elliptic	$-\frac{i \sqrt{-i e^{2} x^{2}+1}\, \sqrt{i e^{2} x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{i e^{2}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{i e^{2}}, i\right)\right)}{\sqrt{i e^{2}}\, \sqrt{e^{4} x^{4}+1}}+\frac{\sqrt{-i e^{2} x^{2}+1}\, \sqrt{i e^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i e^{2}}, i\right)}{\sqrt{i e^{2}}\, \sqrt{e^{4} x^{4}+1}}$	138

input
```
int((-e^2*x^2+1)/(e^4*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*e^2*x^3*hypergeom([1/2,3/4],[7/4],-e^4*x^4)+x*hypergeom([1/4,1/2],[5/
4],-e^4*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87

$$\int \frac{1-e^2x^2}{\sqrt{1+e^4x^4}}\,dx = -\frac{e^{2} x \left(-\frac{1}{e^{4}}\right)^{\frac{3}{4}} E(\arcsin\left(\frac{\left(-\frac{1}{e^{4}}\right)^{\frac{1}{4}}}{x}\right) \mid -1) - (e^{4} + e^{2}) x \left(-\frac{1}{e^{4}}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{1}{e^{4}}\right)^{\frac{1}{4}}}{x}\right) \mid -1) + \sqrt{e^{4} x^{4} + 1}}{e^{2} x}$$

input
```
integrate((-e^2*x^2+1)/(e^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-(e^2*x*(-1/e^4)^(3/4)*elliptic_e(arcsin((-1/e^4)^(1/4)/x), -1) - (e^4 + e
^2)*x*(-1/e^4)^(3/4)*elliptic_f(arcsin((-1/e^4)^(1/4)/x), -1) + sqrt(e^4*x
^4 + 1))/(e^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{1-e^2x^2}{\sqrt{1+e^4x^4}}\,dx = -\frac{e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|e^4x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|e^4x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-e**2*x**2+1)/(e**4*x**4+1)**(1/2),x)
```

output
```
-e**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**4*x**4*exp_polar(I*pi))
/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**4*x**4*exp_pol
ar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-e^2x^2}{\sqrt{1+e^4x^4}}\,dx = \int -\frac{e^2x^2-1}{\sqrt{e^4x^4+1}}\,dx$$

input
```
integrate((-e^2*x^2+1)/(e^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((e^2*x^2 - 1)/sqrt(e^4*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-e^2x^2}{\sqrt{1+e^4x^4}}\,dx = \int -\frac{e^2x^2-1}{\sqrt{e^4x^4+1}}\,dx$$

input
```
integrate((-e^2*x^2+1)/(e^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(e^2*x^2 - 1)/sqrt(e^4*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-e^2x^2}{\sqrt{1+e^4x^4}}\,dx = -\int \frac{e^2\,x^2-1}{\sqrt{e^4\,x^4+1}}\,dx$$

input
```
int(-(e^2*x^2 - 1)/(e^4*x^4 + 1)^(1/2),x)
```

output
```
-int((e^2*x^2 - 1)/(e^4*x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-e^2x^2}{\sqrt{1+e^4x^4}}\,dx = \int \frac{\sqrt{e^4x^4+1}}{e^4x^4+1}dx - \left(\int \frac{\sqrt{e^4x^4+1}\,x^2}{e^4x^4+1}dx\right)e^2$$

input
```
int((-e^2*x^2+1)/(e^4*x^4+1)^(1/2),x)
```

output
```
int(sqrt(e**4*x**4 + 1)/(e**4*x**4 + 1),x) - int((sqrt(e**4*x**4 + 1)*x**2
)/(e**4*x**4 + 1),x)*e**2
```

3.188 $\int \frac{d-ex^2}{\sqrt{d^2+e^2x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{d-ex^2}{\sqrt{d^2+e^2x^4}}\,dx = -\frac{x\sqrt{d^2+e^2x^4}}{d+ex^2}+\frac{\sqrt{d}(d+ex^2)\sqrt{\frac{d^2+e^2x^4}{(d+ex^2)^2}}E\left(2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|\frac{1}{2}\right)}{\sqrt{e}\sqrt{d^2+e^2x^4}}$$

output

```
-x*(e^2*x^4+d^2)^(1/2)/(e*x^2+d)+d^(1/2)*(e*x^2+d)*((e^2*x^4+d^2)/(e*x^2+d
)^2)^(1/2)*EllipticE(sin(2*arctan(e^(1/2)*x/d^(1/2))),1/2*2^(1/2))/e^(1/2)
/(e^2*x^4+d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int \frac{d-ex^2}{\sqrt{d^2+e^2x^4}}\,dx$$
$$=\frac{\sqrt{1+\frac{e^2x^4}{d^2}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{e^2x^4}{d^2}\right)-ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{e^2x^4}{d^2}\right)\right)}{3\sqrt{d^2+e^2x^4}}$$

input

```
Integrate[(d - e*x^2)/Sqrt[d^2 + e^2*x^4],x]
```

output
```
(Sqrt[1 + (e^2*x^4)/d^2]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((e^2*x^
4)/d^2)] - e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((e^2*x^4)/d^2)]))/(3*S
qrt[d^2 + e^2*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{1510\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{\sqrt{d^2+e^2x^4}}\,dx$$
$$\Big\downarrow \, 1510$$
$$\frac{\sqrt{d}\left(d+ex^2\right)\sqrt{\frac{d^2+e^2x^4}{\left(d+ex^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|\frac{1}{2}\right)}{\sqrt{e}\sqrt{d^2+e^2x^4}}-\frac{x\sqrt{d^2+e^2x^4}}{d+ex^2}$$

input
```
Int[(d - e*x^2)/Sqrt[d^2 + e^2*x^4],x]
```

output
```
-((x*Sqrt[d^2 + e^2*x^4])/(d + e*x^2)) + (Sqrt[d]*(d + e*x^2)*Sqrt[(d^2 +
e^2*x^4)/(d + e*x^2)^2]*EllipticE[2*ArcTan[(Sqrt[e]*x)/Sqrt[d]], 1/2])/(Sq
rt[e]*Sqrt[d^2 + e^2*x^4])
```

Defintions of rubi rules used

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.46

method	result	size
default	$\frac{d\sqrt{1-\frac{i e \,x^{2}}{d}}\, \sqrt{1+\frac{i e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i e}{d}}, i\right)}{\sqrt{\frac{i e}{d}}\, \sqrt{e^{2} x^{4}+d^{2}}}-\frac{i d \sqrt{1-\frac{i e \,x^{2}}{d}}\, \sqrt{1+\frac{i e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i e}{d}}, i\right)\right)}{\sqrt{\frac{i e}{d}}\, \sqrt{e^{2} x^{4}+d^{2}}}$	153
elliptic	$\frac{d\sqrt{1-\frac{i e \,x^{2}}{d}}\, \sqrt{1+\frac{i e \,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i e}{d}}, i\right)}{\sqrt{\frac{i e}{d}}\, \sqrt{e^{2} x^{4}+d^{2}}}-\frac{i d \sqrt{1-\frac{i e \,x^{2}}{d}}\, \sqrt{1+\frac{i e \,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i e}{d}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i e}{d}}, i\right)\right)}{\sqrt{\frac{i e}{d}}\, \sqrt{e^{2} x^{4}+d^{2}}}$	153

input
```
int((-e*x^2+d)/(e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(I*e/d)^(1/2)*(1-I*e*x^2/d)^(1/2)*(1+I*e*x^2/d)^(1/2)/(e^2*x^4+d^2)^(1/2
)*EllipticF(x*(I*e/d)^(1/2),I)-I*d/(I*e/d)^(1/2)*(1-I*e*x^2/d)^(1/2)*(1+I*
e*x^2/d)^(1/2)/(e^2*x^4+d^2)^(1/2)*(EllipticF(x*(I*e/d)^(1/2),I)-EllipticE
(x*(I*e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \frac{d-ex^2}{\sqrt{d^2+e^2x^4}}\,dx = -\frac{dex\left(-\frac{d^2}{e^2}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{d^2}{e^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-(de+e^2)x\left(-\frac{d^2}{e^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{d^2}{e^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+\sqrt{e^2x^4+d^2}}{dex}$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-(d*e*x*(-d^2/e^2)^(3/4)*elliptic_e(arcsin((-d^2/e^2)^(1/4)/x), -1) - (d*e
 + e^2)*x*(-d^2/e^2)^(3/4)*elliptic_f(arcsin((-d^2/e^2)^(1/4)/x), -1) + sq
rt(e^2*x^4 + d^2)*d)/(d*e*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

$$\int \frac{d-ex^2}{\sqrt{d^2+e^2x^4}}\,dx=\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}-\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{i\pi}}{d^2}\right)}{4d\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-e*x**2+d)/(e**2*x**4+d**2)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**2*x**4*exp_polar(I*pi)/d**2)/(4*
gamma(5/4)) - e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**2*x**4*exp_po
lar(I*pi)/d**2)/(4*d*gamma(7/4))
```

Maxima [F]

$$\int \frac{d-ex^2}{\sqrt{d^2+e^2x^4}}\,dx=\int -\frac{ex^2-d}{\sqrt{e^2x^4+d^2}}\,dx$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 - d)/sqrt(e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{d-ex^2}{\sqrt{d^2+e^2x^4}}\,dx=\int -\frac{ex^2-d}{\sqrt{e^2x^4+d^2}}\,dx$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(e*x^2 - d)/sqrt(e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d-ex^2}{\sqrt{d^2+e^2x^4}}\,dx=\int \frac{d-e\,x^2}{\sqrt{d^2+e^2\,x^4}}\,dx$$

input
```
int((d - e*x^2)/(d^2 + e^2*x^4)^(1/2),x)
```

output
```
int((d - e*x^2)/(d^2 + e^2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d-ex^2}{\sqrt{d^2+e^2x^4}}\,dx=\left(\int \frac{\sqrt{e^2x^4+d^2}}{e^2x^4+d^2}dx\right)d-\left(\int \frac{\sqrt{e^2x^4+d^2}\,x^2}{e^2x^4+d^2}dx\right)e$$

input
```
int((-e*x^2+d)/(e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 + e**2*x**4)/(d**2 + e**2*x**4),x)*d - int((sqrt(d**2 + e**2
*x**4)*x**2)/(d**2 + e**2*x**4),x)*e
```

3.189 $\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx=-\frac{x\sqrt{1+x^4}}{1+x^2}+\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{1+x^4}}$$

output
```
-x*(x^4+1)^(1/2)/(x^2+1)+(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(2
*arctan(x)),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx=x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^4\right)-\frac{1}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-x^4\right)$$

input
```
Integrate[(1 - x^2)/Sqrt[1 + x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4] - (x^3*Hypergeometric2F1[1/2, 3/4
, 7/4, -x^4])/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{1510\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{\sqrt{x^4+1}}dx$$
$$\Big\downarrow \text{ 1510}$$
$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+1}}-\frac{x\sqrt{x^4+1}}{x^2+1}$$

input
```
Int[(1 - x^2)/Sqrt[1 + x^4],x]
```

output
```
-((x*Sqrt[1 + x^4])/(1 + x^2)) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*El
lipticE[2*ArcTan[x], 1/2])/Sqrt[1 + x^4]
```

Defintions of rubi rules used

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.52

method	result
meijerg	$-\frac{x^3 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-x^4\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-x^4\right)$
default	$-\frac{i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\mathrm{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right.}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$
elliptic	$-\frac{i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\mathrm{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right.}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$

input
```
int((-x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3*hypergeom([1/2,3/4],[7/4],-x^4)+x*hypergeom([1/4,1/2],[5/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx=\frac{-i\sqrt{i}xE\left(\arcsin\left(\frac{\sqrt{i}}{x}\right)\mid -1\right)+2i\sqrt{i}xF\left(\arcsin\left(\frac{\sqrt{i}}{x}\right)\mid -1\right)-\sqrt{x^4+1}}{x}$$

input
```
integrate((-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(I)*x*elliptic_e(arcsin(sqrt(I)/x), -1) + 2*I*sqrt(I)*x*elliptic_f
(arcsin(sqrt(I)/x), -1) - sqrt(x^4 + 1))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx = -\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-x**2+1)/(x**4+1)**(1/2),x)
```

output
```
-x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi))/(4*gamma(
7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi))/(4*ga
mma(5/4))
```

Maxima [F]

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx=-\int \frac{x^2-1}{\sqrt{x^4+1}}\,dx$$

input
```
int(-(x^2 - 1)/(x^4 + 1)^(1/2),x)
```

output
```
-int((x^2 - 1)/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^4+1}}{x^4+1}dx-\left(\int \frac{\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)$$

input
```
int((-x^2+1)/(x^4+1)^(1/2),x)
```

output
```
int(sqrt(x**4 + 1)/(x**4 + 1),x) - int((sqrt(x**4 + 1)*x**2)/(x**4 + 1),x)
```

3.190 $\int \frac{1}{(1-x^2)\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{1}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

output

```
1/4*arctanh(2^(1/2)*x/(x^4+1)^(1/2))*2^(1/2)+1/4*(x^2+1)*((x^4+1)/(x^2+1)^
2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.34

$$\int \frac{1}{(1-x^2)\sqrt{1+x^4}}\,dx = -\sqrt[4]{-1}\,\text{EllipticPi}\left(i, i\operatorname{arcsinh}\left(\sqrt[4]{-1}x\right), -1\right)$$

input

```
Integrate[1/((1 - x^2)*Sqrt[1 + x^4]),x]
```

output

```
-((-1)^(1/4)*EllipticPi[I, I*ArcSinh[(-1)^(1/4)*x], -1])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1535, 761, 2213, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x^2)\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{1535}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+1}}dx+\frac{1}{2}\int \frac{x^2+1}{(1-x^2)\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\int \frac{x^2+1}{(1-x^2)\sqrt{x^4+1}}dx+\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

$$\Big\downarrow \text{2213}$$

$$\frac{1}{2}\int \frac{1}{1-\frac{2x^2}{x^4+1}}d\frac{x}{\sqrt{x^4+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}+\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

input
```
Int[1/((1 - x^2)*Sqrt[1 + x^4]),x]
```

output
```
ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ((1 + x^2)*Sqrt[(1 + x^4)
/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1535
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
1/(2*d)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[1/(2*d)   Int[(d - e*x^2)/((
d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
+ a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{(-1)^{\frac{3}{4}}\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}} x,-i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^{4}+1}}$	51
elliptic	$-\frac{(-1)^{\frac{3}{4}}\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}} x,-i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^{4}+1}}$	51

input
```
int(1/(-x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^
(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{1}{(1-x^2)\sqrt{1+x^4}}\,dx = -\frac{1}{2}i\sqrt{i}F\left(\arcsin\left(\sqrt{i}x\right)\,|-1\right) + \frac{1}{8}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)$$

input
```
integrate(1/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*I*sqrt(I)*elliptic_f(arcsin(sqrt(I)*x), -1) + 1/8*sqrt(2)*log((x^4 +
2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

Sympy [F]

$$\int \frac{1}{(1-x^2)\sqrt{1+x^4}}\,dx = -\int \frac{1}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

input
```
integrate(1/(-x**2+1)/(x**4+1)**(1/2),x)
```

output
```
-Integral(1/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{1}{\sqrt{x^4+1}(x^2-1)}\,dx$$

input
```
integrate(1/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{1}{\sqrt{x^4+1}(x^2-1)}\,dx$$

input
```
integrate(1/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-x^2)\sqrt{1+x^4}}\,dx = -\int \frac{1}{(x^2-1)\,\sqrt{x^4+1}}\,dx$$

input
```
int(-1/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
```

output
```
-int(1/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-x^2)\sqrt{1+x^4}}\,dx$$
$$= -\frac{\sqrt{2}\log(x^2-1)}{4} + \frac{\sqrt{2}\log\left(-\sqrt{x^4+1}\,\sqrt{2}-2x\right)}{4} + \frac{\left(\int \frac{\sqrt{x^4+1}}{x^4+1}dx\right)}{2}$$

input
```
int(1/(-x^2+1)/(x^4+1)^(1/2),x)
```

output
```
( - sqrt(2)*log(x**2 - 1) + sqrt(2)*log( - sqrt(x**4 + 1)*sqrt(2) - 2*x) +
 2*int(sqrt(x**4 + 1)/(x**4 + 1),x))/4
```

3.191 $\int \frac{1-x^2}{\sqrt{-1-x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{1-x^2}{\sqrt{-1-x^4}}\,dx=\frac{x\sqrt{-1-x^4}}{1+x^2}+\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{-1-x^4}}$$

output
```
x*(-x^4-1)^(1/2)/(x^2+1)+(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(2
*arctan(x)),1/2*2^(1/2))/(-x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{1-x^2}{\sqrt{-1-x^4}}\,dx=-\frac{\sqrt{1+x^4}\left(-3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^4\right)+x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-x^4\right)\right)}{3\sqrt{-1-x^4}}$$

input
```
Integrate[(1 - x^2)/Sqrt[-1 - x^4],x]
```

output
```
-1/3*(Sqrt[1 + x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4] + x^3*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -x^4]))/Sqrt[-1 - x^4]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{\sqrt{-x^4-1}}dx$$

$$\Big\downarrow \color{blue}{1510}$$

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{-x^4-1}}+\frac{\sqrt{-x^4-1}x}{x^2+1}$$

input
```
Int[(1 - x^2)/Sqrt[-1 - x^4],x]
```

output
```
(x*Sqrt[-1 - x^4])/(1 + x^2) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Elli
pticE[2*ArcTan[x], 1/2])/Sqrt[-1 - x^4]
```

Defintions of rubi rules used

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.54

method	result
meijerg	$-ix\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-x^{4}\right)+\frac{i x^{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-x^{4}\right)}{3}$
default	$\frac{i\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\left(\mathrm{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)-\mathrm{EllipticE}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^{4}-1}}+\frac{\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\mathrm{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^{4}-1}}$
elliptic	$\frac{i\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\left(\mathrm{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)-\mathrm{EllipticE}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^{4}-1}}+\frac{\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\mathrm{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^{4}-1}}$

input
```
int((-x^2+1)/(-x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*x*hypergeom([1/4,1/2],[5/4],-x^4)+1/3*I*x^3*hypergeom([1/2,3/4],[7/4],-
x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{1-x^2}{\sqrt{-1-x^4}}\,dx=-\frac{\sqrt{i}xE\left(\arcsin\left(\frac{\sqrt{i}}{x}\right)\middle|-1\right)-2\sqrt{i}xF\left(\arcsin\left(\frac{\sqrt{i}}{x}\right)\middle|-1\right)-\sqrt{-x^4-1}}{x}$$

input
```
integrate((-x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(I)*x*elliptic_e(arcsin(sqrt(I)/x), -1) - 2*sqrt(I)*x*elliptic_f(arc
sin(sqrt(I)/x), -1) - sqrt(-x^4 - 1))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{1-x^2}{\sqrt{-1-x^4}}\,dx = \frac{ix^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-x**2+1)/(-x**4-1)**(1/2),x)
```

output
```
I*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi))/(4*gamma
(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi))/(4
*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-x^2}{\sqrt{-1-x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{-x^4-1}}\,dx$$

input
```
integrate((-x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/sqrt(-x^4 - 1), x)
```

Giac [F]

$$\int \frac{1-x^2}{\sqrt{-1-x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{-x^4-1}}\,dx$$

input
```
integrate((-x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)/sqrt(-x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{\sqrt{-1-x^4}}\,dx=-\int \frac{x^2-1}{\sqrt{-x^4-1}}\,dx$$

input
```
int(-(x^2 - 1)/(- x^4 - 1)^(1/2),x)
```

output
```
-int((x^2 - 1)/(- x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-x^2}{\sqrt{-1-x^4}}\,dx=i\left(-\left(\int \frac{\sqrt{x^4+1}}{x^4+1}dx\right)+\int \frac{\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)$$

input
```
int((-x^2+1)/(-x^4-1)^(1/2),x)
```

output
```
i*( - int(sqrt(x**4 + 1)/(x**4 + 1),x) + int((sqrt(x**4 + 1)*x**2)/(x**4 +
 1),x))
```

3.192 $\int \frac{1}{(1-x^2)\sqrt{-1-x^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\begin{aligned}\int \frac{1}{(1-x^2)\sqrt{-1-x^4}}\,dx\\ &= \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-1-x^4}}\end{aligned}$$

output
```
1/4*arctan(2^(1/2)*x/(-x^4-1)^(1/2))*2^(1/2)+1/4*(x^2+1)*((x^4+1)/(x^2+1)^
2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(-x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

$$\int \frac{1}{(1-x^2)\sqrt{-1-x^4}}\,dx = -\frac{\sqrt[4]{-1}\sqrt{1+x^4}\,\text{EllipticPi}\left(i, i\text{arcsinh}\left(\sqrt[4]{-1}x\right),-1\right)}{\sqrt{-1-x^4}}$$

input
```
Integrate[1/((1 - x^2)*Sqrt[-1 - x^4]),x]
```

output
```
-(((-1)^(1/4)*Sqrt[1 + x^4]*EllipticPi[I, I*ArcSinh[(-1)^(1/4)*x], -1])/Sq
rt[-1 - x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1535, 761, 2213, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(1-x^2\right) \sqrt{-x^4-1}}dx$$

$$\Big\downarrow \text{1535}$$

$$\frac{1}{2} \int \frac{1}{\sqrt{-x^4-1}}dx+\frac{1}{2} \int \frac{x^2+1}{\left(1-x^2\right) \sqrt{-x^4-1}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2} \int \frac{x^2+1}{\left(1-x^2\right) \sqrt{-x^4-1}}dx+\frac{\left(x^2+1\right) \sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}} \text{EllipticF}\left(2 \arctan (x),\frac{1}{2}\right)}{4 \sqrt{-x^4-1}}$$

$$\Big\downarrow \text{2213}$$

$$\frac{1}{2} \int \frac{1}{\frac{2 x^2}{-x^4-1}+1}d\frac{x}{\sqrt{-x^4-1}}+\frac{\left(x^2+1\right) \sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}} \text{EllipticF}\left(2 \arctan (x),\frac{1}{2}\right)}{4 \sqrt{-x^4-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan \left(\frac{\sqrt{2} x}{\sqrt{-x^4-1}}\right)}{2 \sqrt{2}}+\frac{\left(x^2+1\right) \sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}} \text{EllipticF}\left(2 \arctan (x),\frac{1}{2}\right)}{4 \sqrt{-x^4-1}}$$

input
```
Int[1/((1 - x^2)*Sqrt[-1 - x^4]),x]
```

output
```
ArcTan[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/(2*Sqrt[2]) + ((1 + x^2)*Sqrt[(1 + x^4)
/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1535
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
1/(2*d)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[1/(2*d)   Int[(d - e*x^2)/((
d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
+ a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\operatorname{EllipticPi}\left(\sqrt{-i}\,x,i,\frac{\left(-1\right)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{\sqrt{-i}\,\sqrt{-x^{4}-1}}$	52
elliptic	$\frac{\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\operatorname{EllipticPi}\left(\sqrt{-i}\,x,i,\frac{\left(-1\right)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{\sqrt{-i}\,\sqrt{-x^{4}-1}}$	52

input
```
int(1/(-x^2+1)/(-x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-I)^(1/2)*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*EllipticPi((-I
)^(1/2)*x,I,(-1)^(1/4)/(-I)^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \frac{1}{(1-x^2)\sqrt{-1-x^4}}\,dx = -\frac{1}{2}\sqrt{i}F\left(\arcsin\left(\sqrt{i}x\right)\,|-1\right) - \frac{1}{4}\sqrt{-\frac{1}{2}}\log\left(\frac{2\sqrt{-\frac{1}{2}}x+\sqrt{-x^4-1}}{x^2-1}\right) + \frac{1}{4}\sqrt{-\frac{1}{2}}\log\left(-\frac{2\sqrt{-\frac{1}{2}}x-\sqrt{-x^4-1}}{x^2-1}\right)$$

input
```
integrate(1/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(I)*elliptic_f(arcsin(sqrt(I)*x), -1) - 1/4*sqrt(-1/2)*log((2*sqr
t(-1/2)*x + sqrt(-x^4 - 1))/(x^2 - 1)) + 1/4*sqrt(-1/2)*log(-(2*sqrt(-1/2)
*x - sqrt(-x^4 - 1))/(x^2 - 1))
```

Sympy [F]

$$\int \frac{1}{(1-x^2)\sqrt{-1-x^4}}\,dx = -\int \frac{1}{x^2\sqrt{-x^4-1}-\sqrt{-x^4-1}}\,dx$$

input
```
integrate(1/(-x**2+1)/(-x**4-1)**(1/2),x)
```

output
```
-Integral(1/(x**2*sqrt(-x**4 - 1) - sqrt(-x**4 - 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1-x^2)\sqrt{-1-x^4}}\,dx = \int -\frac{1}{\sqrt{-x^4-1}(x^2-1)}\,dx$$

input

```
integrate(1/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
```

output

```
-integrate(1/(sqrt(-x^4 - 1)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{(1-x^2)\sqrt{-1-x^4}}\,dx = \int -\frac{1}{\sqrt{-x^4-1}(x^2-1)}\,dx$$

input

```
integrate(1/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
```

output

```
integrate(-1/(sqrt(-x^4 - 1)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-x^2)\sqrt{-1-x^4}}\,dx = -\int \frac{1}{(x^2-1)\sqrt{-x^4-1}}\,dx$$

input

```
int(-1/((x^2 - 1)*(- x^4 - 1)^(1/2)),x)
```

output

```
-int(1/((x^2 - 1)*(- x^4 - 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-x^2)\sqrt{-1-x^4}}\,dx = \frac{i\left(-\sqrt{2}\log(x^2-1)+\sqrt{2}\log\left(\sqrt{x^4+1}\,\sqrt{2}-2x\right)-2\left(\int \frac{\sqrt{x^4+1}}{x^4+1}dx\right)\right)}{4}$$

input

```
int(1/(-x^2+1)/(-x^4-1)^(1/2),x)
```

output

```
(i*( - sqrt(2)*log(x**2 - 1) + sqrt(2)*log(sqrt(x**4 + 1)*sqrt(2) - 2*x) -
 2*int(sqrt(x**4 + 1)/(x**4 + 1),x)))/4
```

3.193 $\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 100

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx=\frac{x\sqrt{1+x^4}}{1+x^2}-\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{1+x^4}}+\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{\sqrt{1+x^4}}$$

output
```
x*(x^4+1)^(1/2)/(x^2+1)-(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(2*
arctan(x)),1/2*2^(1/2))/(x^4+1)^(1/2)+(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*In
verseJacobiAM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.40

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx=x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^4\right)+\frac{1}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-x^4\right)$$

input

```
Integrate[(1 + x^2)/Sqrt[1 + x^4],x]
```

output

```
x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4] + (x^3*Hypergeometric2F1[1/2, 3/4
, 7/4, -x^4])/3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{1512}$$

$$2\int \frac{1}{\sqrt{x^4+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+1}}-\int \frac{1-x^2}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{1510}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+1}}+\frac{\sqrt{x^4+1}x}{x^2+1}$$

input

```
Int[(1 + x^2)/Sqrt[1 + x^4],x]
```

output
```
(x*Sqrt[1 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Ellip
ticE[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2
)^2]*EllipticF[2*ArcTan[x], 1/2])/Sqrt[1 + x^4]
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.31

method	result
meijerg	$\frac{x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-x^{4}\right)$
default	$\frac{\sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}+\frac{i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$
elliptic	$\frac{\sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}+\frac{i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$

input
```
int((x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*hypergeom([1/2,3/4],[7/4],-x^4)+x*hypergeom([1/4,1/2],[5/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.28

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx = \frac{i\sqrt{i}xE\left(\arcsin\left(\frac{\sqrt{i}}{x}\right)|-1\right)+\sqrt{x^4+1}}{x}$$

input
```
integrate((x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(I)*x*elliptic_e(arcsin(sqrt(I)/x), -1) + sqrt(x^4 + 1))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.58

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((x**2+1)/(x**4+1)**(1/2),x)
```

output
```
x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi))/(4*gamma(7
/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi))/(4*gam
ma(5/4))
```

Maxima [F]

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + 1)/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{x^4+1}}\,dx$$

input
```
int((x^2 + 1)/(x^4 + 1)^(1/2),x)
```

output
```
int((x^2 + 1)/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^4+1}}{x^4+1}dx + \int \frac{\sqrt{x^4+1}\,x^2}{x^4+1}dx$$

input

```
int((x^2+1)/(x^4+1)^(1/2),x)
```

output

```
int(sqrt(x**4 + 1)/(x**4 + 1),x) + int((sqrt(x**4 + 1)*x**2)/(x**4 + 1),x)
```

3.194 $\int \frac{1+e^2x^2}{\sqrt{1+e^4x^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 150

$$\int \frac{1+e^2x^2}{\sqrt{1+e^4x^4}}\,dx = \frac{x\sqrt{1+e^4x^4}}{1+e^2x^2} - \frac{(1+e^2x^2)\sqrt{\frac{1+e^4x^4}{(1+e^2x^2)^2}}\,E\left(2\arctan(ex)\,\middle|\,\frac{1}{2}\right)}{e\sqrt{1+e^4x^4}} + \frac{(1+e^2x^2)\sqrt{\frac{1+e^4x^4}{(1+e^2x^2)^2}}\,\text{EllipticF}\left(2\arctan(ex),\frac{1}{2}\right)}{e\sqrt{1+e^4x^4}}$$

output

```
x*(e^4*x^4+1)^(1/2)/(e^2*x^2+1)-(e^2*x^2+1)*((e^4*x^4+1)/(e^2*x^2+1)^2)^(1
/2)*EllipticE(sin(2*arctan(e*x)),1/2*2^(1/2))/e/(e^4*x^4+1)^(1/2)+(e^2*x^2
+1)*((e^4*x^4+1)/(e^2*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(e*x),1/2*2^
(1/2))/e/(e^4*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.33

$$\int \frac{1+e^2x^2}{\sqrt{1+e^4x^4}}\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^4x^4\right) + \frac{1}{3}e^2x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^4x^4\right)$$

input
```
Integrate[(1 + e^2*x^2)/Sqrt[1 + e^4*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, -(e^4*x^4)] + (e^2*x^3*Hypergeometric2F
1[1/2, 3/4, 7/4, -(e^4*x^4)])/3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^2x^2+1}{\sqrt{e^4x^4+1}}dx$$

$$\Big\downarrow \text{ 1512}$$

$$2\int \frac{1}{\sqrt{e^4x^4+1}}dx-\int \frac{1-e^2x^2}{\sqrt{e^4x^4+1}}dx$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\left(e^2x^2+1\right)\sqrt{\frac{e^4x^4+1}{\left(e^2x^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan(ex),\frac{1}{2}\right)}{e\sqrt{e^4x^4+1}}-\int \frac{1-e^2x^2}{\sqrt{e^4x^4+1}}dx$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{\left(e^2x^2+1\right)\sqrt{\frac{e^4x^4+1}{\left(e^2x^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan(ex),\frac{1}{2}\right)}{e\sqrt{e^4x^4+1}}-\frac{\left(e^2x^2+1\right)\sqrt{\frac{e^4x^4+1}{\left(e^2x^2+1\right)^2}}E\left(2\arctan(ex)\left|\frac{1}{2}\right.\right)}{e\sqrt{e^4x^4+1}}+\frac{x\sqrt{e^4x^4+1}}{e^2x^2+1}$$

input
```
Int[(1 + e^2*x^2)/Sqrt[1 + e^4*x^4],x]
```

output
```
(x*Sqrt[1 + e^4*x^4])/(1 + e^2*x^2) - ((1 + e^2*x^2)*Sqrt[(1 + e^4*x^4)/(1
 + e^2*x^2)^2]*EllipticE[2*ArcTan[e*x], 1/2])/(e*Sqrt[1 + e^4*x^4]) + ((1
+ e^2*x^2)*Sqrt[(1 + e^4*x^4)/(1 + e^2*x^2)^2]*EllipticF[2*ArcTan[e*x], 1/
2])/(e*Sqrt[1 + e^4*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.27

method	result	size
meijerg	$\frac{e^{2} x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -e^{4} x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], -e^{4} x^{4}\right)$	40
default	$\frac{\sqrt{-i e^{2} x^{2}+1}\, \sqrt{i e^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i e^{2}}, i\right)}{\sqrt{i e^{2}}\, \sqrt{e^{4} x^{4}+1}}+\frac{i \sqrt{-i e^{2} x^{2}+1}\, \sqrt{i e^{2} x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{i e^{2}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{i e^{2}}, i\right)\right)}{\sqrt{i e^{2}}\, \sqrt{e^{4} x^{4}+1}}$	138
elliptic	$\frac{\sqrt{-i e^{2} x^{2}+1}\, \sqrt{i e^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i e^{2}}, i\right)}{\sqrt{i e^{2}}\, \sqrt{e^{4} x^{4}+1}}+\frac{i \sqrt{-i e^{2} x^{2}+1}\, \sqrt{i e^{2} x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{i e^{2}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{i e^{2}}, i\right)\right)}{\sqrt{i e^{2}}\, \sqrt{e^{4} x^{4}+1}}$	138

input
```
int((e^2*x^2+1)/(e^4*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/3*e^2*x^3*hypergeom([1/2,3/4],[7/4],-e^4*x^4)+x*hypergeom([1/4,1/2],[5/4
],-e^4*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.51

$$\int \frac{1+e^2x^2}{\sqrt{1+e^4x^4}}\,dx = \frac{e^2x\left(-\frac{1}{e^4}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{1}{e^4}\right)^{\frac{1}{4}}}{x}\right)|-1)+(e^4-e^2)x\left(-\frac{1}{e^4}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{e^4}\right)^{\frac{1}{4}}}{x}\right)|-1)+\sqrt{e^4x^4+1}}{e^2x}$$

input

```
integrate((e^2*x^2+1)/(e^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output

```
(e^2*x*(-1/e^4)^(3/4)*elliptic_e(arcsin((-1/e^4)^(1/4)/x), -1) + (e^4 - e^
2)*x*(-1/e^4)^(3/4)*elliptic_f(arcsin((-1/e^4)^(1/4)/x), -1) + sqrt(e^4*x^
4 + 1))/(e^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.45

$$\int \frac{1+e^2x^2}{\sqrt{1+e^4x^4}}\,dx = \frac{e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|e^4x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|e^4x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((e**2*x**2+1)/(e**4*x**4+1)**(1/2),x)
```

output

```
e**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**4*x**4*exp_polar(I*pi))/
(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**4*x**4*exp_pola
r(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+e^2x^2}{\sqrt{1+e^4x^4}}\,dx=\int \frac{e^2x^2+1}{\sqrt{e^4x^4+1}}\,dx$$

input
```
integrate((e^2*x^2+1)/(e^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e^2*x^2 + 1)/sqrt(e^4*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+e^2x^2}{\sqrt{1+e^4x^4}}\,dx=\int \frac{e^2x^2+1}{\sqrt{e^4x^4+1}}\,dx$$

input
```
integrate((e^2*x^2+1)/(e^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((e^2*x^2 + 1)/sqrt(e^4*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+e^2x^2}{\sqrt{1+e^4x^4}}\,dx=\int \frac{e^2\,x^2+1}{\sqrt{e^4\,x^4+1}}\,dx$$

input
```
int((e^2*x^2 + 1)/(e^4*x^4 + 1)^(1/2),x)
```

output
```
int((e^2*x^2 + 1)/(e^4*x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+e^2x^2}{\sqrt{1+e^4x^4}}\,dx=\int \frac{\sqrt{e^4x^4+1}}{e^4x^4+1}dx+\left(\int \frac{\sqrt{e^4x^4+1}\,x^2}{e^4x^4+1}dx\right)e^2$$

input

```
int((e^2*x^2+1)/(e^4*x^4+1)^(1/2),x)
```

output

```
int(sqrt(e**4*x**4 + 1)/(e**4*x**4 + 1),x) + int((sqrt(e**4*x**4 + 1)*x**2
)/(e**4*x**4 + 1),x)*e**2
```

3.195 $\int \frac{d+ex^2}{\sqrt{d^2+e^2x^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \frac{d+ex^2}{\sqrt{d^2+e^2x^4}}\,dx = \frac{x\sqrt{d^2+e^2x^4}}{d+ex^2} - \frac{\sqrt{d}(d+ex^2)\sqrt{\frac{d^2+e^2x^4}{(d+ex^2)^2}}\,E\left(2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|\frac{1}{2}\right)}{\sqrt{e}\sqrt{d^2+e^2x^4}} + \frac{\sqrt{d}(d+ex^2)\sqrt{\frac{d^2+e^2x^4}{(d+ex^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),\frac{1}{2}\right)}{\sqrt{e}\sqrt{d^2+e^2x^4}}$$

output
```
x*(e^2*x^4+d^2)^(1/2)/(e*x^2+d)-d^(1/2)*(e*x^2+d)*((e^2*x^4+d^2)/(e*x^2+d)
^2)^(1/2)*EllipticE(sin(2*arctan(e^(1/2)*x/d^(1/2))),1/2*2^(1/2))/e^(1/2)/
(e^2*x^4+d^2)^(1/2)+d^(1/2)*(e*x^2+d)*((e^2*x^4+d^2)/(e*x^2+d)^2)^(1/2)*In
verseJacobiAM(2*arctan(e^(1/2)*x/d^(1/2)),1/2*2^(1/2))/e^(1/2)/(e^2*x^4+d^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.48

$$\int \frac{d+ex^2}{\sqrt{d^2+e^2x^4}}\,dx = \frac{\sqrt{1+\frac{e^2x^4}{d^2}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{e^2x^4}{d^2}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{e^2x^4}{d^2}\right)\right)}{3\sqrt{d^2+e^2x^4}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[d^2 + e^2*x^4],x]
```

output
```
(Sqrt[1 + (e^2*x^4)/d^2]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((e^2*x^
4)/d^2)] + e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((e^2*x^4)/d^2)]))/(3*S
qrt[d^2 + e^2*x^4])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{d^2+e^2x^4}}dx$$

$$\Big\downarrow \text{1512}$$

$$2d\int \frac{1}{\sqrt{e^2x^4+d^2}}dx-d\int \frac{d-ex^2}{d\sqrt{e^2x^4+d^2}}dx$$

$$\Big\downarrow \text{27}$$

$$2d\int \frac{1}{\sqrt{e^2x^4+d^2}}dx-\int \frac{d-ex^2}{\sqrt{e^2x^4+d^2}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt{d}(d+ex^2)\sqrt{\frac{d^2+e^2x^4}{(d+ex^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),\frac{1}{2}\right)}{\sqrt{e}\sqrt{d^2+e^2x^4}}-\int\frac{d-ex^2}{\sqrt{e^2x^4+d^2}}dx$$

$$\Big\downarrow \text{1510}$$

$$\frac{\sqrt{d}(d+ex^2)\sqrt{\frac{d^2+e^2x^4}{(d+ex^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),\frac{1}{2}\right)}{\sqrt{e}\sqrt{d^2+e^2x^4}}-$$
$$\frac{\sqrt{d}(d+ex^2)\sqrt{\frac{d^2+e^2x^4}{(d+ex^2)^2}}E\left(2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|\frac{1}{2}\right)}{\sqrt{e}\sqrt{d^2+e^2x^4}}+\frac{x\sqrt{d^2+e^2x^4}}{d+ex^2}$$

input
```
Int[(d + e*x^2)/Sqrt[d^2 + e^2*x^4],x]
```

output
```
(x*Sqrt[d^2 + e^2*x^4])/(d + e*x^2) - (Sqrt[d]*(d + e*x^2)*Sqrt[(d^2 + e^2
*x^4)/(d + e*x^2)^2]*EllipticE[2*ArcTan[(Sqrt[e]*x)/Sqrt[d]], 1/2])/(Sqrt[
e]*Sqrt[d^2 + e^2*x^4]) + (Sqrt[d]*(d + e*x^2)*Sqrt[(d^2 + e^2*x^4)/(d + e
*x^2)^2]*EllipticF[2*ArcTan[(Sqrt[e]*x)/Sqrt[d]], 1/2])/(Sqrt[e]*Sqrt[d^2
+ e^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{d\sqrt{1-\frac{iex^{2}}{d}}\,\sqrt{1+\frac{iex^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{ie}{d}},i\right)}{\sqrt{\frac{ie}{d}}\,\sqrt{e^{2}x^{4}+d^{2}}}+\frac{id\sqrt{1-\frac{iex^{2}}{d}}\,\sqrt{1+\frac{iex^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{ie}{d}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{ie}{d}},i\right)\right)}{\sqrt{\frac{ie}{d}}\,\sqrt{e^{2}x^{4}+d^{2}}}$	153
elliptic	$\frac{d\sqrt{1-\frac{iex^{2}}{d}}\,\sqrt{1+\frac{iex^{2}}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{ie}{d}},i\right)}{\sqrt{\frac{ie}{d}}\,\sqrt{e^{2}x^{4}+d^{2}}}+\frac{id\sqrt{1-\frac{iex^{2}}{d}}\,\sqrt{1+\frac{iex^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{ie}{d}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{ie}{d}},i\right)\right)}{\sqrt{\frac{ie}{d}}\,\sqrt{e^{2}x^{4}+d^{2}}}$	153

input

```
int((e*x^2+d)/(e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
d/(I*e/d)^(1/2)*(1-I*e*x^2/d)^(1/2)*(1+I*e*x^2/d)^(1/2)/(e^2*x^4+d^2)^(1/2
)*EllipticF(x*(I*e/d)^(1/2),I)+I*d/(I*e/d)^(1/2)*(1-I*e*x^2/d)^(1/2)*(1+I*
e*x^2/d)^(1/2)/(e^2*x^4+d^2)^(1/2)*(EllipticF(x*(I*e/d)^(1/2),I)-EllipticE
(x*(I*e/d)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.53

$$\int \frac{d+ex^2}{\sqrt{d^2+e^2x^4}}\,dx = \frac{dex\left(-\frac{d^2}{e^2}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{d^2}{e^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-(de-e^2)x\left(-\frac{d^2}{e^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{d^2}{e^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+\sqrt{e^2x^4+d^2}d}{dex}$$

input

```
integrate((e*x^2+d)/(e^2*x^4+d^2)^(1/2),x, algorithm="fricas")
```

output
```
(d*e*x*(-d^2/e^2)^(3/4)*elliptic_e(arcsin((-d^2/e^2)^(1/4)/x), -1) - (d*e
- e^2)*x*(-d^2/e^2)^(3/4)*elliptic_f(arcsin((-d^2/e^2)^(1/4)/x), -1) + sqr
t(e^2*x^4 + d^2)*d)/(d*e*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.41

$$\int \frac{d+ex^2}{\sqrt{d^2+e^2x^4}}\,dx=\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^2x^4e^{i\pi}}{d^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^2x^4e^{i\pi}}{d^2}\right)}{4d\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(e**2*x**4+d**2)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), e**2*x**4*exp_polar(I*pi)/d**2)/(4*
gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), e**2*x**4*exp_po
lar(I*pi)/d**2)/(4*d*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{d^2+e^2x^4}}\,dx=\int \frac{ex^2+d}{\sqrt{e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)/(e^2*x^4+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(e^2*x^4 + d^2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{d^2+e^2x^4}}\,dx = \int \frac{ex^2+d}{\sqrt{e^2x^4+d^2}}\,dx$$

input
```
integrate((e*x^2+d)/(e^2*x^4+d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(e^2*x^4 + d^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{d^2+e^2x^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{d^2+e^2\,x^4}}\,dx$$

input
```
int((d + e*x^2)/(d^2 + e^2*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(d^2 + e^2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{\sqrt{d^2+e^2x^4}}\,dx = \left(\int \frac{\sqrt{e^2x^4+d^2}}{e^2x^4+d^2}dx\right)d+\left(\int \frac{\sqrt{e^2x^4+d^2}\,x^2}{e^2x^4+d^2}dx\right)e$$

input
```
int((e*x^2+d)/(e^2*x^4+d^2)^(1/2),x)
```

output
```
int(sqrt(d**2 + e**2*x**4)/(d**2 + e**2*x**4),x)*d + int((sqrt(d**2 + e**2
*x**4)*x**2)/(d**2 + e**2*x**4),x)*e
```

3.196 $\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 100

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx = \frac{x\sqrt{1+x^4}}{1+x^2} - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{1+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{\sqrt{1+x^4}}$$

output

```
x*(x^4+1)^(1/2)/(x^2+1)-(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(2*
arctan(x)),1/2*2^(1/2))/(x^4+1)^(1/2)+(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*In
verseJacobiAM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.40

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^4\right) + \frac{1}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-x^4\right)$$

input
```
Integrate[(1 + x^2)/Sqrt[1 + x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4] + (x^3*Hypergeometric2F1[1/2, 3/4
, 7/4, -x^4])/3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{1512}$$

$$2\int \frac{1}{\sqrt{x^4+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+1}}-\int \frac{1-x^2}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{1510}$$

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+1}}+\frac{\sqrt{x^4+1}x}{x^2+1}$$

input
```
Int[(1 + x^2)/Sqrt[1 + x^4],x]
```

output
```
(x*Sqrt[1 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Ellip
ticE[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2
)^2]*EllipticF[2*ArcTan[x], 1/2])/Sqrt[1 + x^4]
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.31

method	result
meijerg	$\frac{x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right],\left[\frac{7}{4}\right],-x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{5}{4}\right],-x^{4}\right)$
default	$\frac{\sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}+\frac{i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$
elliptic	$\frac{\sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}+\frac{i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$

input
```
int((x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*hypergeom([1/2,3/4],[7/4],-x^4)+x*hypergeom([1/4,1/2],[5/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.28

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx = \frac{i\,\sqrt{i}xE\left(\arcsin\left(\frac{\sqrt{i}}{x}\right)\,|-1\right)+\sqrt{x^4+1}}{x}$$

input
```
integrate((x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(I)*x*elliptic_e(arcsin(sqrt(I)/x), -1) + sqrt(x^4 + 1))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.58

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\,x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((x**2+1)/(x**4+1)**(1/2),x)
```

output
```
x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi))/(4*gamma(7
/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi))/(4*gam
ma(5/4))
```

Maxima [F]

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx=\int \frac{x^2+1}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx=\int \frac{x^2+1}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + 1)/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx=\int \frac{x^2+1}{\sqrt{x^4+1}}\,dx$$

input
```
int((x^2 + 1)/(x^4 + 1)^(1/2),x)
```

output
```
int((x^2 + 1)/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^4+1}}{x^4+1}dx + \int \frac{\sqrt{x^4+1}\,x^2}{x^4+1}dx$$

input

```
int((x^2+1)/(x^4+1)^(1/2),x)
```

output

```
int(sqrt(x**4 + 1)/(x**4 + 1),x) + int((sqrt(x**4 + 1)*x**2)/(x**4 + 1),x)
```

3.197 $\int \frac{1}{(1+x^2)\sqrt{1+x^4}}\, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{1}{(1+x^2)\sqrt{1+x^4}}\, dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x), \frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

output
```
1/4*arctan(2^(1/2)*x/(x^4+1)^(1/2))*2^(1/2)+1/4*(x^2+1)*((x^4+1)/(x^2+1)^2
)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.34

$$\int \frac{1}{(1+x^2)\sqrt{1+x^4}}\, dx = -\sqrt[4]{-1}\operatorname{EllipticPi}\left(-i, i\operatorname{arcsinh}\left(\sqrt[4]{-1}x\right), -1\right)$$

input
```
Integrate[1/((1 + x^2)*Sqrt[1 + x^4]),x]
```

output
```
-((-1)^(1/4)*EllipticPi[-I, I*ArcSinh[(-1)^(1/4)*x], -1])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1535, 761, 2213, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{1535}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+1}}dx+\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+1}}dx+\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

$$\Big\downarrow \text{2213}$$

$$\frac{1}{2}\int \frac{1}{\frac{2x^2}{x^4+1}+1}d\frac{x}{\sqrt{x^4+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}+\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

input
```
Int[1/((1 + x^2)*Sqrt[1 + x^4]),x]
```

output
```
ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1535
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
1/(2*d)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[1/(2*d)   Int[(d - e*x^2)/((
d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
+ a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{(-1)^{\frac{3}{4}} \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticPi}\left((-1)^{\frac{1}{4}} x , i, -\sqrt{-i}\, (-1)^{\frac{3}{4}}\right)}{\sqrt{x^{4}+1}}$	51
elliptic	$-\frac{(-1)^{\frac{3}{4}} \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticPi}\left((-1)^{\frac{1}{4}} x , i, -\sqrt{-i}\, (-1)^{\frac{3}{4}}\right)}{\sqrt{x^{4}+1}}$	51

input
```
int(1/(x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^
(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{1}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right) - \frac{1}{2}i\sqrt{i}F\left(\arcsin\left(\sqrt{i}x\right)\,|-1\right)$$

input
```
integrate(1/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) - 1/2*I*sqrt(I)*elliptic_f(arc
sin(sqrt(I)*x), -1)
```

Sympy [F]

$$\int \frac{1}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{1}{(x^2+1)\sqrt{x^4+1}}\,dx$$

input
```
integrate(1/(x**2+1)/(x**4+1)**(1/2),x)
```

output
```
Integral(1/((x**2 + 1)*sqrt(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+1}(x^2+1)}\,dx$$

input
```
integrate(1/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+1}(x^2+1)}\,dx$$

input
```
integrate(1/(x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{1}{(x^2+1)\sqrt{x^4+1}}\,dx$$

input
```
int(1/((x^2 + 1)*(x^4 + 1)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^4+1}}{x^6+x^4+x^2+1}dx$$

input

```
int(1/(x^2+1)/(x^4+1)^(1/2),x)
```

output

```
int(sqrt(x**4 + 1)/(x**6 + x**4 + x**2 + 1),x)
```

3.198 $\int \frac{1+x^2}{\sqrt{-1-x^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 107

$$\int \frac{1+x^2}{\sqrt{-1-x^4}}\,dx=-\frac{x\sqrt{-1-x^4}}{1+x^2}-\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{-1-x^4}}+\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{\sqrt{-1-x^4}}$$

output

```
-x*(-x^4-1)^(1/2)/(x^2+1)-(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(
2*arctan(x)),1/2*2^(1/2))/(-x^4-1)^(1/2)+(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)
*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(-x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{1+x^2}{\sqrt{-1-x^4}}\,dx=\frac{\sqrt{1+x^4}\left(3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^4\right)+x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-x^4\right)\right)}{3\sqrt{-1-x^4}}$$

input
```
Integrate[(1 + x^2)/Sqrt[-1 - x^4],x]
```

output
```
(Sqrt[1 + x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4] + x^3*Hypergeom
etric2F1[1/2, 3/4, 7/4, -x^4]))/(3*Sqrt[-1 - x^4])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{\sqrt{-x^4-1}}dx$$

$$\Big\downarrow \text{1512}$$

$$2\int \frac{1}{\sqrt{-x^4-1}}dx-\int \frac{1-x^2}{\sqrt{-x^4-1}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{\sqrt{-x^4-1}}-\int \frac{1-x^2}{\sqrt{-x^4-1}}dx$$

$$\Big\downarrow \text{1510}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{\sqrt{-x^4-1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{-x^4-1}}-\frac{\sqrt{-x^4-1}x}{x^2+1}$$

input
```
Int[(1 + x^2)/Sqrt[-1 - x^4],x]
```

output

```
-((x*Sqrt[-1 - x^4])/(1 + x^2)) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*E
llipticE[2*ArcTan[x], 1/2])/Sqrt[-1 - x^4] + ((1 + x^2)*Sqrt[(1 + x^4)/(1
+ x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/Sqrt[-1 - x^4]
```

Defintions of rubi rules used

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.32

method	result
meijerg	$-ix\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-x^{4}\right)-\frac{i x^{3}\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-x^{4}\right)}{3}$
default	$\frac{\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^{4}-1}}-\frac{i\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\left(\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)-\text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^{4}-1}}$
elliptic	$\frac{\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^{4}-1}}-\frac{i\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\left(\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)-\text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^{4}-1}}$

input
```
int((x^2+1)/(-x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*x*hypergeom([1/4,1/2],[5/4],-x^4)-1/3*I*x^3*hypergeom([1/2,3/4],[7/4],-
x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.29

$$\int \frac{1+x^2}{\sqrt{-1-x^4}}\,dx = \frac{\sqrt{i}xE\left(\arcsin\left(\frac{\sqrt{i}}{x}\right)|-1\right)-\sqrt{-x^4-1}}{x}$$

input
```
integrate((x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(I)*x*elliptic_e(arcsin(sqrt(I)/x), -1) - sqrt(-x^4 - 1))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.59

$$\int \frac{1+x^2}{\sqrt{-1-x^4}}\,dx = -\frac{ix^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((x**2+1)/(-x**4-1)**(1/2),x)
```

output
```
-I*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi))/(4*gamm
a(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi))/(
4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+x^2}{\sqrt{-1-x^4}}\,dx=\int \frac{x^2+1}{\sqrt{-x^4-1}}\,dx$$

input
```
integrate((x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/sqrt(-x^4 - 1), x)
```

Giac [F]

$$\int \frac{1+x^2}{\sqrt{-1-x^4}}\,dx=\int \frac{x^2+1}{\sqrt{-x^4-1}}\,dx$$

input
```
integrate((x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + 1)/sqrt(-x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{\sqrt{-1-x^4}}\,dx=\int \frac{x^2+1}{\sqrt{-x^4-1}}\,dx$$

input
```
int((x^2 + 1)/(- x^4 - 1)^(1/2),x)
```

output
```
int((x^2 + 1)/(- x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+x^2}{\sqrt{-1-x^4}}\,dx = -i\left(\int \frac{\sqrt{x^4+1}}{x^4+1}dx + \int \frac{\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)$$

input

```
int((x^2+1)/(-x^4-1)^(1/2),x)
```

output

```
- i*(int(sqrt(x**4 + 1)/(x**4 + 1),x) + int((sqrt(x**4 + 1)*x**2)/(x**4 +
1),x))
```

3.199 $\int \frac{1}{(1+x^2)\sqrt{-1-x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{1}{(1+x^2)\sqrt{-1-x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-1-x^4}}$$

output
```
1/4*arctanh(2^(1/2)*x/(-x^4-1)^(1/2))*2^(1/2)+1/4*(x^2+1)*((x^4+1)/(x^2+1)
^2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(-x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

$$\int \frac{1}{(1+x^2)\sqrt{-1-x^4}}\,dx = -\frac{\sqrt[4]{-1}\sqrt{1+x^4}\,\text{EllipticPi}\left(-i,i\operatorname{arcsinh}\left(\sqrt[4]{-1}x\right),-1\right)}{\sqrt{-1-x^4}}$$

input
```
Integrate[1/((1 + x^2)*Sqrt[-1 - x^4]),x]
```

output
```
-(((-1)^(1/4)*Sqrt[1 + x^4]*EllipticPi[-I, I*ArcSinh[(-1)^(1/4)*x], -1])/S
qrt[-1 - x^4])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1535, 761, 2213, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)\sqrt{-x^4-1}}dx$$

$$\Big\downarrow \text{1535}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{-x^4-1}}dx+\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{-x^4-1}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{-x^4-1}}dx+\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-x^4-1}}$$

$$\Big\downarrow \text{2213}$$

$$\frac{1}{2}\int \frac{1}{1-\frac{2x^2}{-x^4-1}}d\frac{x}{\sqrt{-x^4-1}}+\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-x^4-1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-x^4-1}}+\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}$$

input
```
Int[1/((1 + x^2)*Sqrt[-1 - x^4]),x]
```

output
```
ArcTanh[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/(2*Sqrt[2]) + ((1 + x^2)*Sqrt[(1 + x^4
)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1535
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
1/(2*d)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[1/(2*d)   Int[(d - e*x^2)/((
d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
+ a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{i\sqrt{-i}\,\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\operatorname{EllipticPi}\left(\sqrt{-i}\,x, -i, \frac{\left(-1\right)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{2\sqrt{-x^{4}-1}}+\frac{\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\operatorname{EllipticPi}\left(\sqrt{-i}\,x, -i, \frac{\left(-1\right)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{2\sqrt{-i}\,\sqrt{-x^{4}-1}}$	107
elliptic	$\frac{i\sqrt{-i}\,\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\operatorname{EllipticPi}\left(\sqrt{-i}\,x, -i, \frac{\left(-1\right)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{2\sqrt{-x^{4}-1}}+\frac{\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\operatorname{EllipticPi}\left(\sqrt{-i}\,x, -i, \frac{\left(-1\right)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{2\sqrt{-i}\,\sqrt{-x^{4}-1}}$	107

input
```
int(1/(x^2+1)/(-x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(-I)^(1/2)*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*EllipticPi
((-I)^(1/2)*x,-I,(-1)^(1/4)/(-I)^(1/2))+1/2/(-I)^(1/2)*(1+I*x^2)^(1/2)*(1-
I*x^2)^(1/2)/(-x^4-1)^(1/2)*EllipticPi((-I)^(1/2)*x,-I,(-1)^(1/4)/(-I)^(1/
2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{1}{(1+x^2)\sqrt{-1-x^4}}\,dx = -\frac{1}{2}\sqrt{i}F\left(\arcsin\left(\sqrt{i}x\right)\,|-1\right) + \frac{1}{8}\sqrt{2}\log\left(\frac{\sqrt{2}x+\sqrt{-x^4-1}}{x^2+1}\right) - \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}x-\sqrt{-x^4-1}}{x^2+1}\right)$$

input
```
integrate(1/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(I)*elliptic_f(arcsin(sqrt(I)*x), -1) + 1/8*sqrt(2)*log((sqrt(2)*
x + sqrt(-x^4 - 1))/(x^2 + 1)) - 1/8*sqrt(2)*log(-(sqrt(2)*x - sqrt(-x^4 -
 1))/(x^2 + 1))
```

Sympy [F]

$$\int \frac{1}{(1+x^2)\sqrt{-1-x^4}}\,dx = \int \frac{1}{(x^2+1)\sqrt{-x^4-1}}\,dx$$

input
```
integrate(1/(x**2+1)/(-x**4-1)**(1/2),x)
```

output
```
Integral(1/((x**2 + 1)*sqrt(-x**4 - 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^2)\sqrt{-1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4-1}(x^2+1)}\,dx$$

input
```
integrate(1/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^4 - 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x^2)\sqrt{-1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4-1}(x^2+1)}\,dx$$

input
```
integrate(1/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^4 - 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)\sqrt{-1-x^4}}\,dx = \int \frac{1}{(x^2+1)\sqrt{-x^4-1}}\,dx$$

input
```
int(1/((x^2 + 1)*(- x^4 - 1)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)*(- x^4 - 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1+x^2)\sqrt{-1-x^4}}\,dx = -\left(\int \frac{\sqrt{x^4+1}}{x^6+x^4+x^2+1}dx\right)i$$

input
```
int(1/(x^2+1)/(-x^4-1)^(1/2),x)
```

output
```
 - int(sqrt(x**4 + 1)/(x**6 + x**4 + x**2 + 1),x)*i
```

3.200 $\int \frac{A+Cx^4}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{A+Cx^4}{\left(a+bx^2\right)^{5/4}}\,dx=-\frac{14aCx}{5b^2\sqrt[4]{a+bx^2}}+\frac{2Cx\left(a+bx^2\right)^{3/4}}{5b^2}+\frac{2\left(5Ab^2+12a^2C\right)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}b^{5/2}\sqrt[4]{a+bx^2}}$$

output
```
-14/5*a*C*x/b^2/(b*x^2+a)^(1/4)+2/5*C*x*(b*x^2+a)^(3/4)/b^2+2/5*(5*A*b^2+1
2*C*a^2)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^
(1/2))/a^(1/2)/b^(5/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int \frac{A+Cx^4}{\left(a+bx^2\right)^{5/4}}\,dx=\frac{x\left(10Ab^2+2aC\left(6a+bx^2\right)-\left(5Ab^2+12a^2C\right)\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\right.\right.}{5ab^2\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(A + C*x^4)/(a + b*x^2)^(5/4),x]
```

output
```
(x*(10*A*b^2 + 2*a*C*(6*a + b*x^2) - (5*A*b^2 + 12*a^2*C)*(1 + (b*x^2)/a)^
(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(5*a*b^2*(a + b*x^2
)^(1/4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1472, 27, 299, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^4}{\left(a+bx^2\right)^{5/4}}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{2x\left(\frac{a^2C}{b^2}+A\right)}{a\sqrt[4]{a+bx^2}}-\frac{2\int \frac{b\left(\frac{2Ca^2}{b^2}+A\right)-aCx^2}{2b\sqrt[4]{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x\left(\frac{a^2C}{b^2}+A\right)}{a\sqrt[4]{a+bx^2}}-\frac{\int \frac{b\left(\frac{2Ca^2}{b^2}+A\right)-aCx^2}{\sqrt[4]{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{299}$$

$$\frac{2x\left(\frac{a^2C}{b^2}+A\right)}{a\sqrt[4]{a+bx^2}}-\frac{\frac{\left(12a^2C+5Ab^2\right)\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{5b}-\frac{2aCx\left(a+bx^2\right)^{3/4}}{5b}}{ab}$$

$$\Big\downarrow \text{227}$$

$$\frac{2x\left(\frac{a^2C}{b^2}+A\right)}{a\sqrt[4]{a+bx^2}}-\frac{\frac{\sqrt[4]{\frac{bx^2}{a}+1}\,(12a^2C+5Ab^2)\int\frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5b\sqrt[4]{a+bx^2}}-\frac{2aCx(a+bx^2)^{3/4}}{5b}}{ab}$$

$$\Big\downarrow\ 225$$

$$\frac{2x\left(\frac{a^2C}{b^2}+A\right)}{a\sqrt[4]{a+bx^2}}-\frac{\frac{\sqrt[4]{\frac{bx^2}{a}+1}\,(12a^2C+5Ab^2)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5b\sqrt[4]{a+bx^2}}-\frac{2aCx(a+bx^2)^{3/4}}{5b}}{ab}$$

$$\Big\downarrow\ 212$$

$$\frac{2x\left(\frac{a^2C}{b^2}+A\right)}{a\sqrt[4]{a+bx^2}}-\frac{\frac{\sqrt[4]{\frac{bx^2}{a}+1}\,(12a^2C+5Ab^2)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5b\sqrt[4]{a+bx^2}}-\frac{2aCx(a+bx^2)^{3/4}}{5b}}{ab}$$

input

```
Int[(A + C*x^4)/(a + b*x^2)^(5/4),x]
```

output

```
(2*(A + (a^2*C)/b^2)*x)/(a*(a + b*x^2)^(1/4)) - ((-2*a*C*x*(a + b*x^2)^(3/
4))/(5*b) + ((5*A*b^2 + 12*a^2*C)*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2
)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[
b]))/(5*b*(a + b*x^2)^(1/4)))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [F]

$$\int \frac{C\,x^{4}+A}{\left(b\,x^{2}+a\right)^{\frac{5}{4}}}dx$$

input
```
int((C*x^4+A)/(b*x^2+a)^(5/4),x)
```

output
```
int((C*x^4+A)/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{A+Cx^4}{(a+bx^2)^{5/4}}\,dx = \int \frac{Cx^4+A}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((C*x^4+A)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((C*x^4 + A)*(b*x^2 + a)^(3/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int \frac{A+Cx^4}{(a+bx^2)^{5/4}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{4}}} + \frac{Cx^5\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}}$$

input
```
integrate((C*x**4+A)/(b*x**2+a)**(5/4),x)
```

output
```
A*x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/4) + C*x**5*
hyper((5/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/4))
```

Maxima [F]

$$\int \frac{A+Cx^4}{(a+bx^2)^{5/4}}\,dx = \int \frac{Cx^4+A}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((C*x^4+A)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + A)/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{A+Cx^4}{(a+bx^2)^{5/4}}\,dx = \int \frac{Cx^4+A}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((C*x^4+A)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((C*x^4 + A)/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^4}{(a+bx^2)^{5/4}}\,dx = \int \frac{C\,x^4+A}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int((A + C*x^4)/(a + b*x^2)^(5/4),x)
```

output
```
int((A + C*x^4)/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{A+Cx^4}{(a+bx^2)^{5/4}}\,dx = \left(\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)c + \left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)a$$

input

```
int((C*x^4+A)/(b*x^2+a)^(5/4),x)
```

output

```
int(x**4/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*c + int(1
/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*a
```

3.201 $\int \frac{c+dx^2}{a+bx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 180

$$\int \frac{c+dx^2}{a+bx^4}\,dx=-\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

output

```
1/4*(b^(1/2)*c+a^(1/2)*d)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(
3/4)/b^(3/4)+1/4*(b^(1/2)*c+a^(1/2)*d)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))
*2^(1/2)/a^(3/4)/b^(3/4)+1/4*(b^(1/2)*c-a^(1/2)*d)*arctanh(2^(1/2)*a^(1/4)
*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(3/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^2}{a+bx^4}\,dx$$

$$= \frac{-2\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c-\sqrt{a}d\right)\bigl(\log\bigl(}{4\sqrt{2}a^{3/4}b^{3/4}}$$

input
```
Integrate[(c + d*x^2)/(a + b*x^4),x]
```

output
```
(-2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(S
qrt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c
 - Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*
b^(3/4))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{a+bx^4}\,dx$$

$$\Big\downarrow 1482$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{b}\left(\sqrt{a}-\sqrt{b}x^2\right)}{bx^4+a}dx}{2b}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{b}\left(\sqrt{b}x^2+\sqrt{a}\right)}{bx^4+a}dx}{2b}$$

$$\Big\downarrow 27$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt{b}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt{b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt{b}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt{b}}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}$$

input

```
Int[(c + d*x^2)/(a + b*x^4),x]
```

output
```
(((Sqrt[b]*c)/Sqrt[a] + d)*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqr
t[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*
a^(1/4)*b^(1/4))))/(2*Sqrt[b]) + (((Sqrt[b]*c)/Sqrt[a] - d)*(-1/2*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)
*b^(1/4))))/(2*Sqrt[b])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.19

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{\left(\_R^{2}d+c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\sqrt{2}\Bigg(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{ar}}{8}$

input

```
int((d*x^2+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output

```
1/4/b*sum((_R^2*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(121) = 242$.

Time = 0.08 (sec) , antiderivative size = 767, normalized size of antiderivative = 4.26

$$\begin{aligned}
\int \frac{c+dx^2}{a+bx^4}\,dx = {}& -\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
&+\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\Bigg) \\
&+\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
&-\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\Bigg) \\
&+\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
&+\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\Bigg) \\
&-\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
&-\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\Bigg)
\end{aligned}$$

input

```
integrate((d*x^2+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c
*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*
c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-
(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt
(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b)
)*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 +
 a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4
- 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqr
t(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^
2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/
(a^3*b^3)) - a*b^2*c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt(-(b^2*c^4
 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2
*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
- a*b^2*c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*
d^4)/(a^3*b^3)) - 2*c*d)/(a*b)))
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.61

$$\int \frac{c+dx^2}{a+bx^4}\,dx$$
$$= \operatorname{RootSum}\left(256t^4a^3b^3+64t^2a^2b^2cd+a^2d^4+2abc^2d^2+b^2c^4, \left(t\mapsto t\log\left(x+\frac{64t^3a^3b^2d+12ta^2bcd^2-4}{a^2d^4-b^2c^4}\right.\right.\right.$$

input
```
integrate((d*x**2+c)/(b*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*c
**2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d + 12*_t*
a**2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.23

$$\int \frac{c+dx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\sqrt{2}\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

input
```
integrate((d*x^2+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt
(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))
*sqrt(b)) + 1/4*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt
(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(b))*sqrt(b)) + 1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*log(sqrt(b)*x^
2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(
sqrt(b)*c - sqrt(a)*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(
a))/(a^(3/4)*b^(3/4))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.34

$$\int \frac{c+dx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

input
```
integrate((d*x^2+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)
*b^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*
log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3
)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 17.22 (sec) , antiderivative size = 599, normalized size of antiderivative = 3.33

$$\int \frac{c+dx^2}{a+bx^4}\,dx = -2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}} - \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{-\frac{b\,c^2\,\sqrt{-a^3\,b^3}-a\,d^2\,\sqrt{-a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}} - 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}-\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}} - \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}-\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{-\frac{a\,d^2\,\sqrt{-a^3\,b^3}-b\,c^2\,\sqrt{-a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}$$

input
```
int((c + d*x^2)/(a + b*x^4),x)
```

output
```
- 2*atanh((8*b^3*c^2*x*((d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3) - (c^2*(-a^3*b
^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 +
 (2*b*c^3*(-a^3*b^3)^(1/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2
*d^2*x*((d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3) - (c^2*(-a^3*b^3)^(1/2))/(16*a
^3*b^2) - (c*d)/(8*a*b))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^3*
b^3)^(1/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a))*(-(b*c^2*(-a^3*b^3)^(1/2)
 - a*d^2*(-a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) - 2*atanh((
8*b^3*c^2*x*((c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) - (d^2*(-
a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(
-a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^2*x*((c^
2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) - (d^2*(-a^3*b^3)^(1/2))/
(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(-a^3*b^3)^(1/2))
/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a))*(-(a*d^2*(-a^3*b^3)^(1/2) - b*c^2*(-
a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.33

$$\int \frac{c+dx^2}{a+bx^4}\,dx$$
$$= \frac{\sqrt{2}\left(-2\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d-2\sqrt{b}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c+2\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d+2\sqrt{b}\,ata}{}$$

input
```
int((d*x^2+c)/(b*x^4+a),x)
```

output
```
(b**(1/4)*a**(1/4)*sqrt(2)*( - 2*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*d - 2*sqrt(b)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c + 2*sqrt(a)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))
*d + 2*sqrt(b)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a*
*(1/4)*sqrt(2)))*c + sqrt(a)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
+ sqrt(b)*x**2)*d - sqrt(a)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sq
rt(b)*x**2)*d - sqrt(b)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqr
t(b)*x**2)*c + sqrt(b)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)
*x**2)*c))/(8*a*b)
```

3.202 $\int \frac{c-dx^2}{a+bx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 181

$$\int \frac{c-dx^2}{a+bx^4}\,dx = -\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

output
```
1/4*(b^(1/2)*c-a^(1/2)*d)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(
3/4)/b^(3/4)+1/4*(b^(1/2)*c-a^(1/2)*d)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))
*2^(1/2)/a^(3/4)/b^(3/4)+1/4*(b^(1/2)*c+a^(1/2)*d)*arctanh(2^(1/2)*a^(1/4)
*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(3/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.02

$$\int \frac{c-dx^2}{a+bx^4}\,dx = \frac{\left(-2\sqrt{b}c+2\sqrt{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c+\sqrt{a}d\right)\left(\log\right.}{4\sqrt{2}a^{3/4}b^{3/4}}$$

input

```
Integrate[(c - d*x^2)/(a + b*x^4),x]
```

output

```
((-2*Sqrt[b]*c + 2*Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*
(Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]
*c + Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] -
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4
)*b^(3/4))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-dx^2}{a+bx^4}\,dx$$

$$\Big\downarrow \text{1482}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{b}\left(\sqrt{a}-\sqrt{b}x^2\right)}{bx^4+a}dx}{2b}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{b}\left(\sqrt{b}x^2+\sqrt{a}\right)}{bx^4+a}dx}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt{b}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt{b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt{b}}$$

$$\Big\downarrow 27$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt{b}}$$

$$\Big\downarrow 1103$$

$$\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}$$

input
```
Int[(c - d*x^2)/(a + b*x^4),x]
```

output
```
(((Sqrt[b]*c)/Sqrt[a] - d)*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqr
t[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*
a^(1/4)*b^(1/4))))/(2*Sqrt[b]) + (((Sqrt[b]*c)/Sqrt[a] + d)*(-1/2*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)
*b^(1/4))))/(2*Sqrt[b])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.19

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4} b+a\right)}\frac{\left(-\_R^{2} d+c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4 b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8 a}-\frac{d\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{ar}\right.}{8}$

input
```
int((-d*x^2+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*sum((-_R^2*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(122) = 244$.

Time = 0.09 (sec) , antiderivative size = 767, normalized size of antiderivative = 4.24

$$
\begin{aligned}
\int \frac{c-dx^2}{a+bx^4}\,dx = {} & -\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
& +\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\Bigg) \\
& +\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
& -\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\Bigg) \\
& +\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
& +\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\Bigg) \\
& -\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
& -\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\Bigg)
\end{aligned}
$$

input

```
integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*
d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c
^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b
^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((
a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*l
og(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^
2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*
a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt(-
(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c
^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^
3*b^3)) - a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d
^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4
- 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*
d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) -
 a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*
d^4)/(a^3*b^3)) - 2*c*d)/(a*b)))
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.61

$$\int \frac{c-dx^2}{a+bx^4}\,dx =$$
$$-\operatorname{RootSum}\left(256t^4a^3b^3-64t^2a^2b^2cd+a^2d^4+2abc^2d^2+b^2c^4,\left(t\mapsto t\log\left(x+\frac{64t^3a^3b^2d-12ta^2bcd^2}{a^2d^4-b^2c^4}\right.\right.\right.$$

input
```
integrate((-d*x**2+c)/(b*x**4+a),x)
```

output
```
-RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*
c**2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d - 12*_t
*a**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.22

$$\int \frac{c-dx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\sqrt{2}\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

input

```
integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="maxima")
```

output

```
1/4*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt
(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))
*sqrt(b)) + 1/4*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt
(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(b))*sqrt(b)) + 1/8*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*log(sqrt(b)*x^
2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(
sqrt(b)*c + sqrt(a)*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(
a))/(a^(3/4)*b^(3/4))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.33

$$\int \frac{c-dx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

input

```
integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="giac")
```

output

```
1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)
*b^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*
log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3
)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 17.35 (sec) , antiderivative size = 603, normalized size of antiderivative = 3.33

$$\int \frac{c-dx^2}{a+bx^4}\,dx = 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}} - \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{\frac{a\,d^2\,\sqrt{-a^3\,b^3}-b\,c^2\,\sqrt{-a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}} + 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}+\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}} - \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}+\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{\frac{b\,c^2\,\sqrt{-a^3\,b^3}-a\,d^2\,\sqrt{-a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}$$

input

```
int((c - d*x^2)/(a + b*x^4),x)
```

output

```
2*atanh((8*b^3*c^2*x*((c*d)/(8*a*b) - (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2)
+ (d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (
2*b*c^3*(-a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d
^2*x*((c*d)/(8*a*b) - (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) + (d^2*(-a^3*b^3
)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(-a^3*b^
3)^(1/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a))*((a*d^2*(-a^3*b^3)^(1/2) -
b*c^2*(-a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) + 2*atanh((8*b
^3*c^2*x*((c*d)/(8*a*b) + (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(-a^3
*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^
3*b^3)^(1/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^2*x*((c*d)/
(8*a*b) + (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(-a^3*b^3)^(1/2))/(16
*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^3*b^3)^(1/2))/a^
2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a))*((b*c^2*(-a^3*b^3)^(1/2) - a*d^2*(-a^3*
b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.32

$$\int \frac{c-dx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\left(2\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d-2\sqrt{b}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c-2\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d+2\sqrt{b}\,atan\left(\right.\right.}{}$$

input
```
int((-d*x^2+c)/(b*x^4+a),x)
```

output
```
(b**(1/4)*a**(1/4)*sqrt(2)*(2*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*
sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*d - 2*sqrt(b)*atan((b**(1/4)*a**(1
/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c - 2*sqrt(a)*atan
((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*d
+ 2*sqrt(b)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1
/4)*sqrt(2)))*c - sqrt(a)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + s
qrt(b)*x**2)*d + sqrt(a)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(
b)*x**2)*d - sqrt(b)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b
)*x**2)*c + sqrt(b)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x*
*2)*c))/(8*a*b)
```

3.203 $\int \frac{c+dx^2}{a-bx^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \frac{c+dx^2}{a-bx^4} \, dx = \frac{\left(c-\frac{\sqrt{a}d}{\sqrt{b}}\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

output
```
1/2*(c-a^(1/2)*d/b^(1/2))*arctan(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(1/4)+1/2*(b
^(1/2)*c+a^(1/2)*d)*arctanh(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \frac{c+dx^2}{a-bx^4} \, dx$$
$$= \frac{2\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c+\sqrt{a}d\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\right)}{4a^{3/4}b^{3/4}}$$

input
```
Integrate[(c + d*x^2)/(a - b*x^4),x]
```

output
```
(2*(Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + Sqrt
[a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b
^(3/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1481, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{a-bx^4}\,dx$$

$$\Big\downarrow \text{1481}$$

$$\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}dx-\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{-bx^2-\sqrt{a}\sqrt{b}}dx$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}dx+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt[4]{a}b^{3/4}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt[4]{a}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt[4]{a}b^{3/4}}$$

input
```
Int[(c + d*x^2)/(a - b*x^4),x]
```

output
```
(((Sqrt[b]*c)/Sqrt[a] - d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(3/4)
) + (((Sqrt[b]*c)/Sqrt[a] + d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^
(3/4))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1481
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Simp[(e/2 + c*(d/(2*q)))   Int[1/(-q + c*x^2), x], x] + Simp[(
e/2 - c*(d/(2*q)))   Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] &
& NeQ[c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.42

method	result	size
risch	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4} b-a\right)}\frac{\left(\_R^{2} d+c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b}$	36
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}-\frac{d\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	104

input
```
int((d*x^2+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*sum((_R^2*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(57) = 114$.

Time = 0.08 (sec) , antiderivative size = 755, normalized size of antiderivative = 8.88

$$
\begin{aligned}
\int \frac{c+dx^2}{a-bx^4}\,dx = {} & \frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x\right. \\
& \left.+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right) \\
& -\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x\right. \\
& \left.-\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right) \\
& -\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x\right. \\
& \left.+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right) \\
& +\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x\right. \\
& \left.-\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)
\end{aligned}
$$

input

```
integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)
/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*
d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c
^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*
sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(
b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)
/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt((b^2*c^4
 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2
*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) +
 a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d
^4)/(a^3*b^3)) - 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a
^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 +
 a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3
)) - 2*c*d)/(a*b)))
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int \frac{c+dx^2}{a-bx^4}\,dx = -\operatorname{RootSum}\left(256t^4a^3b^3 - 64t^2a^2b^2cd - a^2d^4 + 2abc^2d^2 - b^2c^4, \left(t \mapsto t\log\left(x + \frac{-64t^3a^3b^2d + 12ta^2bcd}{a^2d^4 - b^2c^4}\right.\right.\right.$$

input
```
integrate((d*x**2+c)/(-b*x**4+a),x)
```

output
```
-RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*
c**2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d + 12*_
t*a**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.28

$$\int \frac{c+dx^2}{a-bx^4}\,dx = \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*(sqrt(b)*c - sqrt(a)*d)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(b)*c + sqrt(a)*d)*log((sqrt(
b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(57) = 114.

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.71

$$\begin{aligned}\int \frac{c+dx^2}{a-bx^4}\,dx = & -\frac{\sqrt{2}\left(b^2c+\sqrt{-ab}bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,\left(-ab^3\right)^{\frac{3}{4}}} \\ & - \frac{\sqrt{2}\left(b^2c-\sqrt{-ab}bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,\left(-ab^3\right)^{\frac{3}{4}}} \\ & - \frac{\sqrt{2}\left(b^2c-\sqrt{-ab}bd\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,\left(-ab^3\right)^{\frac{3}{4}}} \\ & + \frac{\sqrt{2}\left(b^2c-\sqrt{-ab}bd\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,\left(-ab^3\right)^{\frac{3}{4}}}\end{aligned}$$

input
```
integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c - sqrt(-a*b)
*b*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^
3)^(3/4) - 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)*log(x^2 + sqrt(2)*x*(-a/b)
^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)
*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

Mupad [B] (verification not implemented)

Time = 17.40 (sec) , antiderivative size = 579, normalized size of antiderivative = 6.81

$$\int \frac{c+dx^2}{a-bx^4}\,dx = 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}} + \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{-\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}-2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}} + 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}+\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}} + \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}+\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}$$

input
```
int((c + d*x^2)/(a - b*x^4),x)
```

output
```
2*atanh((8*b^3*c^2*x*((c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) -
 (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*
b*c^3*(a^3*b^3)^(1/2))/a^2 - (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x
*((c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(a^3*b^3)^(1/2
))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^(1/2
))/a^2 - (2*c*d^2*(a^3*b^3)^(1/2))/a))*(-(a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a
^3*b^3)^(1/2) - 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) + 2*atanh((8*b^3*c^2*x*
((c*d)/(8*a*b) + (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) + (d^2*(a^3*b^3)^(1/2)
)/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^(1/2)
)/a^2 + (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x*((c*d)/(8*a*b) + (c^
2*(a^3*b^3)^(1/2))/(16*a^3*b^2) + (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2
))/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(a^
3*b^3)^(1/2))/a))*((a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a^3*b^3)^(1/2) + 2*a^2*
b^2*c*d)/(16*a^3*b^3))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

$$\int \frac{c+dx^2}{a-bx^4}\,dx = \frac{-2\sqrt{a}\,atan\Bigl(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\Bigr)\,d+2\sqrt{b}\,atan\Bigl(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\Bigr)\,c-\sqrt{a}\log\Bigl(a^{\frac{1}{4}}-b^{\frac{1}{4}}x\Bigr)\,d+\sqrt{a}\log\Bigl(a^{\frac{1}{4}}+b^{\frac{1}{4}}x\Bigr)\,d-\sqrt{b}\log\Bigl(a^{\frac{1}{4}}-}{4b^{\frac{3}{4}}a^{\frac{3}{4}}}$$

input
```
int((d*x^2+c)/(-b*x^4+a),x)
```

output
```
(b**(1/4)*a**(1/4)*( - 2*sqrt(a)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*d +
 2*sqrt(b)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*c - sqrt(a)*log(a**(1/4)
- b**(1/4)*x)*d + sqrt(a)*log(a**(1/4) + b**(1/4)*x)*d - sqrt(b)*log(a**(1
/4) - b**(1/4)*x)*c + sqrt(b)*log(a**(1/4) + b**(1/4)*x)*c))/(4*a*b)
```

3.204 $\int \frac{c-dx^2}{a-bx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \frac{c-dx^2}{a-bx^4}\,dx=\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(c-\frac{\sqrt{a}d}{\sqrt{b}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}$$

output
```
1/2*(b^(1/2)*c+a^(1/2)*d)*arctan(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(3/4)+1/2*(c
-a^(1/2)*d/b^(1/2))*arctanh(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(1/4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \frac{c-dx^2}{a-bx^4}\,dx$$
$$=\frac{2\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c-\sqrt{a}d\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\right)}{4a^{3/4}b^{3/4}}$$

input
```
Integrate[(c - d*x^2)/(a - b*x^4),x]
```

output
```
(2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c - Sqrt
[a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b
^(3/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1481, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-dx^2}{a-bx^4}dx$$

$$\Big\downarrow \text{1481}$$

$$\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}dx-\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{-bx^2-\sqrt{a}\sqrt{b}}dx$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}dx+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt[4]{a}b^{3/4}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt[4]{a}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt[4]{a}b^{3/4}}$$

input
```
Int[(c - d*x^2)/(a - b*x^4),x]
```

output
```
(((Sqrt[b]*c)/Sqrt[a] + d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(3/4)
) + (((Sqrt[b]*c)/Sqrt[a] - d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^
(3/4))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1481
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Simp[(e/2 + c*(d/(2*q)))   Int[1/(-q + c*x^2), x], x] + Simp[(
e/2 - c*(d/(2*q)))   Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] &
& NeQ[c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

method	result	size
risch	$-\frac{\sum_{\_R=\text{RootOf}\left(\_Z^{4} b-a\right)}\frac{\left(-\_R^{2} d+c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b}$	37
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	104

input
```
int((-d*x^2+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*sum((-_R^2*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(57) = 114$.

Time = 0.09 (sec) , antiderivative size = 755, normalized size of antiderivative = 8.88

$$
\begin{aligned}
\int \frac{c-dx^2}{a-bx^4}\,dx = {} & \frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x\right. \\
& \left.+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right) \\
& -\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x\right. \\
& \left.-\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right) \\
& -\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x\right. \\
& \left.+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right) \\
& +\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x\right. \\
& \left.-\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)
\end{aligned}
$$

input
```
integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d
)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2
*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a
*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log
(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d
^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b
*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt((b^2*
c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 -
a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)
) + a*b^2*c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2
*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^
2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (
a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3
+ a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3
)) - 2*c*d)/(a*b)))
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int \frac{c-dx^2}{a-bx^4}\,dx$$
$$=\operatorname{RootSum}\left(256t^4a^3b^3+64t^2a^2b^2cd-a^2d^4+2abc^2d^2-b^2c^4, \left(t\mapsto t\log\left(x+\frac{-64t^3a^3b^2d-12ta^2bcd^2-}{a^2d^4-b^2c^4}\right.\right.\right.$$

input
```
integrate((-d*x**2+c)/(-b*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c
**2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d - 12*_t
*a**2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.28

$$\int \frac{c-dx^2}{a-bx^4}\,dx = \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*(sqrt(b)*c + sqrt(a)*d)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(b)*c - sqrt(a)*d)*log((sqrt(
b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(57) = 114$.

Time = 0.11 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.68

$$\begin{aligned}\int \frac{c-dx^2}{a-bx^4}\,dx = &-\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} \\ &-\frac{\sqrt{2}\left(b^2c+\sqrt{-ab}bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} \\ &-\frac{\sqrt{2}\left(b^2c+\sqrt{-ab}bd\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}} \\ &+\frac{\sqrt{2}\left(b^2c+\sqrt{-ab}bd\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}\end{aligned}$$

input
```
integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c + sqrt(-a*b)
*b*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^
3)^(3/4) - 1/8*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)*log(x^2 + sqrt(2)*x*(-a/b)
^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)
*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 579, normalized size of antiderivative = 6.81

$$\int \frac{c-dx^2}{a-bx^4}\,dx = -2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{-\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}} + \frac{8\,a\,b^2\,d^2\,x\,\sqrt{-\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{-\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}} - 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}} + \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}-2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}$$

input
```
int((c - d*x^2)/(a - b*x^4),x)
```

output
```
- 2*atanh((8*b^3*c^2*x*(- (c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^
2) - (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 +
 (2*b*c^3*(a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d
^2*x*(- (c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(a^3*b^3
)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3
)^(1/2))/a^2 + (2*c*d^2*(a^3*b^3)^(1/2))/a))*(-(a*d^2*(a^3*b^3)^(1/2) + b*
c^2*(a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) - 2*atanh((8*b^3*
c^2*x*((c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) + (d^2*(a^3*b^3)
^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)
^(1/2))/a^2 - (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x*((c^2*(a^3*b^3
)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) + (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3)
)^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^(1/2))/a^2 - (2*c*d
^2*(a^3*b^3)^(1/2))/a))*((a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a^3*b^3)^(1/2) -
2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

$$\int \frac{c-dx^2}{a-bx^4}\,dx$$
$$=\frac{2\sqrt{a}\,atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)d+2\sqrt{b}\,atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)c+\sqrt{a}\log\left(a^{\frac{1}{4}}-b^{\frac{1}{4}}x\right)d-\sqrt{a}\log\left(a^{\frac{1}{4}}+b^{\frac{1}{4}}x\right)d-\sqrt{b}\log\Big(a^{\frac{1}{4}}-b}{4b^{\frac{3}{4}}a^{\frac{3}{4}}}$$

input
```
int((-d*x^2+c)/(-b*x^4+a),x)
```

output
```
(b**(1/4)*a**(1/4)*(2*sqrt(a)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*d + 2*
sqrt(b)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*c + sqrt(a)*log(a**(1/4) - b
**(1/4)*x)*d - sqrt(a)*log(a**(1/4) + b**(1/4)*x)*d - sqrt(b)*log(a**(1/4)
 - b**(1/4)*x)*c + sqrt(b)*log(a**(1/4) + b**(1/4)*x)*c))/(4*a*b)
```

3.205 $\int \frac{2+3x^2}{4+9x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{2+3x^2}{4+9x^4}\,dx = -\frac{\arctan\left(1-\sqrt{3}x\right)}{2\sqrt{3}} + \frac{\arctan\left(1+\sqrt{3}x\right)}{2\sqrt{3}}$$

output
```
1/6*arctan(-1+x*3^(1/2))*3^(1/2)+1/6*arctan(1+x*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{4+9x^4}\,dx = \frac{-\arctan\left(1-\sqrt{3}x\right)+\arctan\left(1+\sqrt{3}x\right)}{2\sqrt{3}}$$

input
```
Integrate[(2 + 3*x^2)/(4 + 9*x^4),x]
```

output
```
(-ArcTan[1 - Sqrt[3]*x] + ArcTan[1 + Sqrt[3]*x])/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1476, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{9x^4+4}dx$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{6}\int \frac{1}{x^2-\frac{2x}{\sqrt{3}}+\frac{2}{3}}dx+\frac{1}{6}\int \frac{1}{x^2+\frac{2x}{\sqrt{3}}+\frac{2}{3}}dx$$

$$\Big\downarrow \text{1082}$$

$$\frac{\int \frac{1}{-\left(1-\sqrt{3}x\right)^2-1}d\left(1-\sqrt{3}x\right)}{2\sqrt{3}}-\frac{\int \frac{1}{-\left(\sqrt{3}x+1\right)^2-1}d\left(\sqrt{3}x+1\right)}{2\sqrt{3}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\arctan\left(1-\sqrt{3}x\right)}{2\sqrt{3}}$$

input
```
Int[(2 + 3*x^2)/(4 + 9*x^4),x]
```

output
```
-1/2*ArcTan[1 - Sqrt[3]*x]/Sqrt[3] + ArcTan[1 + Sqrt[3]*x]/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{2}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{3x^{3}\sqrt{3}}{4}+\frac{\sqrt{3}\,x}{2}\right)}{6}$
default	$\frac{\sqrt{6}\,\sqrt{2}\,\left(\ln\left(\frac{x^{2}+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^{2}-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}+1\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}-1\right)\right)}{48}+\frac{\sqrt{6}\,\sqrt{2}\,\left(\ln\left(\frac{x^{2}-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^{2}+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x}{2}\right.\right.}{48}$
meijerg	$\frac{\sqrt{6}\left(-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{3}\,\left(x^{4}\right)^{\frac{1}{4}}+\frac{3\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{4}\right)^{\frac{1}{4}}}{2-\sqrt{3}\,\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{3}\,\left(x^{4}\right)^{\frac{1}{4}}+\frac{3\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{4}\right)^{\frac{1}{4}}}{2+\sqrt{3}\,\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{1}{4}}}\right)}{24}-$

input
```
int((3*x^2+2)/(9*x^4+4),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctan(1/2*3^(1/2)*x)+1/6*3^(1/2)*arctan(3/4*x^3*3^(1/2)+1/2*3
^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{4+9x^4}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{4}\,\sqrt{3}\big(3\,x^3+2\,x\big)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{2}\,\sqrt{3}x\right)$$

input
```
integrate((3*x^2+2)/(9*x^4+4),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/4*sqrt(3)*(3*x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/2*sqr
t(3)*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{2+3x^2}{4+9x^4}\,dx = \frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{2}\right)+2\,\mathrm{atan}\left(\frac{3\sqrt{3}x^3}{4}+\frac{\sqrt{3}x}{2}\right)\right)}{12}$$

input
```
integrate((3*x**2+2)/(9*x**4+4),x)
```

output
```
sqrt(3)*(2*atan(sqrt(3)*x/2) + 2*atan(3*sqrt(3)*x**3/4 + sqrt(3)*x/2))/12
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{2+3x^2}{4+9x^4}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\Big(3\,x+\sqrt{3}\Big)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\Big(3\,x-\sqrt{3}\Big)\right)$$

input
```
integrate((3*x^2+2)/(9*x^4+4),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(3*x + sqrt(3))) + 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(3*x - sqrt(3)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int \frac{2+3x^2}{4+9x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{9}{8}\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{9}{8}\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)$$

input
```
integrate((3*x^2+2)/(9*x^4+4),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x + sqrt(2)*(4/9)^(1/4))) +
1/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x - sqrt(2)*(4/9)^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{2+3x^2}{4+9x^4}\,dx = \frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{3\sqrt{3}\,x^3}{4}+\frac{\sqrt{3}\,x}{2}\right)+\operatorname{atan}\left(\frac{\sqrt{3}\,x}{2}\right)\right)}{6}$$

input
```
int((3*x^2 + 2)/(9*x^4 + 4),x)
```

output
```
(3^(1/2)*(atan((3^(1/2)*x)/2 + (3*3^(1/2)*x^3)/4) + atan((3^(1/2)*x)/2)))/
6
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{2+3x^2}{4+9x^4}\,dx = \frac{\sqrt{3}\left(-atan\left(\frac{\sqrt{3}-3x}{\sqrt{3}}\right)+atan\left(\frac{\sqrt{3}+3x}{\sqrt{3}}\right)\right)}{6}$$

input

```
int((3*x^2+2)/(9*x^4+4),x)
```

output

```
(sqrt(3)*( - atan((sqrt(3) - 3*x)/sqrt(3)) + atan((sqrt(3) + 3*x)/sqrt(3))
))/6
```

3.206 $\int \frac{2-3x^2}{4+9x^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{2-3x^2}{4+9x^4} \, dx = \frac{\operatorname{arctanh}\left(\frac{2\sqrt{3}x}{2+3x^2}\right)}{2\sqrt{3}}$$

output
```
1/6*arctanh(2*3^(1/2)*x/(3*x^2+2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \frac{2-3x^2}{4+9x^4} \, dx = \frac{-\log\left(-2+2\sqrt{3}x-3x^2\right)+\log\left(2+2\sqrt{3}x+3x^2\right)}{4\sqrt{3}}$$

input
```
Integrate[(2 - 3*x^2)/(4 + 9*x^4),x]
```

output
```
(-Log[-2 + 2*Sqrt[3]*x - 3*x^2] + Log[2 + 2*Sqrt[3]*x + 3*x^2])/(4*Sqrt[3]
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1479, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-3x^2}{9x^4+4}dx$$

$$\Big\downarrow \text{1479}$$

$$-\frac{\int -\frac{2\left(\sqrt{3}-3x\right)}{3x^2-2\sqrt{3}x+2}dx}{4\sqrt{3}}-\frac{\int -\frac{2\sqrt{3}\left(\sqrt{3}x+1\right)}{3x^2+2\sqrt{3}x+2}dx}{4\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{3}-3x}{3x^2-2\sqrt{3}x+2}dx}{2\sqrt{3}}+\frac{1}{2}\int \frac{\sqrt{3}x+1}{3x^2+2\sqrt{3}x+2}dx$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log\left(3x^2+2\sqrt{3}x+2\right)}{4\sqrt{3}}-\frac{\log\left(3x^2-2\sqrt{3}x+2\right)}{4\sqrt{3}}$$

input

```
Int[(2 - 3*x^2)/(4 + 9*x^4),x]
```

output

```
-1/4*Log[2 - 2*Sqrt[3]*x + 3*x^2]/Sqrt[3] + Log[2 + 2*Sqrt[3]*x + 3*x^2]/(
4*Sqrt[3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

method	result
risch	$\frac{\sqrt{3}\,\ln\left(2\sqrt{3}\,x+3x^{2}+2\right)}{12}-\frac{\sqrt{3}\,\ln\left(-2\sqrt{3}\,x+3x^{2}+2\right)}{12}$
default	$\frac{\sqrt{6}\,\sqrt{2}\left(\ln\left(\frac{x^{2}+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^{2}-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}+1\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}-1\right)\right)}{48}-\frac{\sqrt{6}\,\sqrt{2}\left(\ln\left(\frac{x^{2}-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^{2}+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x}{2}\right.\right.}{48}$
meijerg	$\frac{\sqrt{6}\left(-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{3}\left(x^{4}\right)^{\frac{1}{4}}+\frac{3\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\left(x^{4}\right)^{\frac{1}{4}}}{2-\sqrt{3}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{3}\left(x^{4}\right)^{\frac{1}{4}}+\frac{3\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\left(x^{4}\right)^{\frac{1}{4}}}{2+\sqrt{3}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{1}{4}}}\right)}{24}-$

input
```
int((-3*x^2+2)/(9*x^4+4),x,method=_RETURNVERBOSE)
```

output
```
1/12*3^(1/2)*ln(2*3^(1/2)*x+3*x^2+2)-1/12*3^(1/2)*ln(-2*3^(1/2)*x+3*x^2+2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{2-3x^2}{4+9x^4}\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{9\,x^4+24\,x^2+4\,\sqrt{3}(3\,x^3+2\,x)+4}{9\,x^4+4}\right)$$

input
```
integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log((9*x^4 + 24*x^2 + 4*sqrt(3)*(3*x^3 + 2*x) + 4)/(9*x^4 + 4
))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(22) = 44$.

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{2-3x^2}{4+9x^4}\,dx = -\frac{\sqrt{3}\log\left(x^2-\frac{2\sqrt{3}x}{3}+\frac{2}{3}\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\frac{2\sqrt{3}x}{3}+\frac{2}{3}\right)}{12}$$

input
```
integrate((-3*x**2+2)/(9*x**4+4),x)
```

output
```
-sqrt(3)*log(x**2 - 2*sqrt(3)*x/3 + 2/3)/12 + sqrt(3)*log(x**2 + 2*sqrt(3)
*x/3 + 2/3)/12
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{2-3x^2}{4+9x^4}\,dx = \frac{1}{12}\sqrt{3}\log\left(3\,x^2+2\,\sqrt{3}x+2\right)-\frac{1}{12}\sqrt{3}\log\left(3\,x^2-2\,\sqrt{3}x+2\right)$$

input
```
integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*log(3*x^2 + 2*sqrt(3)*x + 2) - 1/12*sqrt(3)*log(3*x^2 - 2*sqr
t(3)*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{2-3x^2}{4+9x^4}\,dx = \frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}x+\frac{2}{3}\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}x+\frac{2}{3}\right)$$

input
```
integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(x^2 + sqrt(2)*(4/9)^(1/4)*x + 2/3) - 1/12*sqrt(3)*log(x^2
 - sqrt(2)*(4/9)^(1/4)*x + 2/3)
```

Mupad [B] (verification not implemented)

Time = 17.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{2-3x^2}{4+9x^4}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\,x^2+2}\right)}{6}$$

input
```
int(-(3*x^2 - 2)/(9*x^4 + 4),x)
```

output
```
(3^(1/2)*atanh((2*3^(1/2)*x)/(3*x^2 + 2)))/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{2-3x^2}{4+9x^4}\,dx=\frac{\sqrt{3}\,\left(-\log\left(-2\sqrt{3}\,x+3x^2+2\right)+\log\left(2\sqrt{3}\,x+3x^2+2\right)\right)}{12}$$

input

```
int((-3*x^2+2)/(9*x^4+4),x)
```

output

```
(sqrt(3)*( - log( - 2*sqrt(3)*x + 3*x**2 + 2) + log(2*sqrt(3)*x + 3*x**2 +
 2)))/12
```

3.207 $\int \frac{2+3x^2}{4-9x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{2+3x^2}{4-9x^4}\,dx=\frac{\operatorname{arctanh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

output
```
1/6*arctanh(1/2*x*6^(1/2))*6^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{2+3x^2}{4-9x^4}\,dx=\frac{-\log\left(\sqrt{6}-3x\right)+\log\left(\sqrt{6}+3x\right)}{2\sqrt{6}}$$

input
```
Integrate[(2 + 3*x^2)/(4 - 9*x^4),x]
```

output
```
(-Log[Sqrt[6] - 3*x] + Log[Sqrt[6] + 3*x])/(2*Sqrt[6])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1386, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{4-9x^4}dx$$

$$\Big\downarrow \text{1386}$$

$$\int \frac{1}{2-3x^2}dx$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

input

```
Int[(2 + 3*x^2)/(4 - 9*x^4),x]
```

output

```
ArcTanh[Sqrt[3/2]*x]/Sqrt[6]
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1386

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result
default	$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$
risch	$\frac{\sqrt{6}\,\ln\left(3x+\sqrt{6}\right)}{12}-\frac{\sqrt{6}\,\ln\left(3x-\sqrt{6}\right)}{12}$
meijerg	$-\frac{\sqrt{6}\,x\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-2\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)\right)}{24\left(x^{4}\right)^{\frac{1}{4}}}-\frac{\sqrt{6}\,x^{3}\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-\ln(1+\right.}{24}$

input
```
int((3*x^2+2)/(-9*x^4+4),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctanh(1/2*x*6^(1/2))*6^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{2+3x^2}{4-9x^4}\,dx=\frac{1}{12}\,\sqrt{6}\log\left(\frac{3\,x^2+2\,\sqrt{6}x+2}{3\,x^2-2}\right)$$

input
```
integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")
```

output
```
1/12*sqrt(6)*log((3*x^2 + 2*sqrt(6)*x + 2)/(3*x^2 - 2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{2+3x^2}{4-9x^4}\,dx = -\frac{\sqrt{6}\log\left(x-\frac{\sqrt{6}}{3}\right)}{12} + \frac{\sqrt{6}\log\left(x+\frac{\sqrt{6}}{3}\right)}{12}$$

input
```
integrate((3*x**2+2)/(-9*x**4+4),x)
```

output
```
-sqrt(6)*log(x - sqrt(6)/3)/12 + sqrt(6)*log(x + sqrt(6)/3)/12
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{2+3x^2}{4-9x^4}\,dx = -\frac{1}{12}\sqrt{6}\log\left(\frac{3x-\sqrt{6}}{3x+\sqrt{6}}\right)$$

input
```
integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")
```

output
```
-1/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{2+3x^2}{4-9x^4}\,dx = \frac{1}{12}\sqrt{6}\log\left(\left|x+\frac{1}{3}\sqrt{6}\right|\right) - \frac{1}{12}\sqrt{6}\log\left(\left|x-\frac{1}{3}\sqrt{6}\right|\right)$$

input
```
integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
```

output
```
1/12*sqrt(6)*log(abs(x + 1/3*sqrt(6))) - 1/12*sqrt(6)*log(abs(x - 1/3*sqrt
(6)))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2+3x^2}{4-9x^4}\,dx=\frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

input
```
int(-(3*x^2 + 2)/(9*x^4 - 4),x)
```

output
```
(6^(1/2)*atanh((6^(1/2)*x)/2))/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{2+3x^2}{4-9x^4}\,dx=\frac{\sqrt{6}\,\left(-\log\left(-\sqrt{6}+3x\right)+\log\left(\sqrt{6}+3x\right)\right)}{12}$$

input
```
int((3*x^2+2)/(-9*x^4+4),x)
```

output
```
(sqrt(6)*( - log( - sqrt(6) + 3*x) + log(sqrt(6) + 3*x)))/12
```

3.208 $\int \frac{2-3x^2}{4-9x^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{2-3x^2}{4-9x^4} \, dx = \frac{\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

output
```
1/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{2-3x^2}{4-9x^4} \, dx = \frac{\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

input
```
Integrate[(2 - 3*x^2)/(4 - 9*x^4),x]
```

output
```
ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1386, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-3x^2}{4-9x^4} \, dx$$

$$\Big\downarrow \text{1386}$$

$$\int \frac{1}{3x^2+2}dx$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

input
```
Int[(2 - 3*x^2)/(4 - 9*x^4),x]
```

output
```
ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result
default	$\frac{\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$
risch	$\frac{\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$
meijerg	$-\frac{\sqrt{6}\,x\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-2\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)\right)}{24\left(x^{4}\right)^{\frac{1}{4}}}+\frac{\sqrt{6}\,x^{3}\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\right.\right.}{24}$

input
```
int((-3*x^2+2)/(-9*x^4+4),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right)$$

input
```
integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")
```

output
```
1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

input
```
integrate((-3*x**2+2)/(-9*x**4+4),x)
```

output
```
sqrt(6)*atan(sqrt(6)*x/2)/6
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right)$$

input
```
integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")
```

output
```
1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right)$$

input
```
integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
```

output
```
1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

Mupad [B] (verification not implemented)

Time = 17.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2-3x^2}{4-9x^4}\,dx=\frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

input
```
int((3*x^2 - 2)/(9*x^4 - 4),x)
```

output
```
(6^(1/2)*atan((6^(1/2)*x)/2))/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2-3x^2}{4-9x^4}\,dx=\frac{\sqrt{6}\,atan\left(\frac{3x}{\sqrt{6}}\right)}{6}$$

input
```
int((-3*x^2+2)/(-9*x^4+4),x)
```

output
```
(sqrt(6)*atan((3*x)/sqrt(6)))/6
```

3.209 $\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 75

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx = -\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

output

```
1/2*b^(1/4)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(1/4)+1/2*b^(1/
4)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(1/4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx = \frac{\sqrt[4]{b}\left(-\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right)}{\sqrt{2}\sqrt[4]{a}}$$

input

```
Integrate[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

output

```
(b^(1/4)*(-ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + ArcTan[1 + (Sqrt[2]*b
^(1/4)*x)/a^(1/4)]))/(Sqrt[2]*a^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1476, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4} \, dx$$

$$\Big\downarrow \ 1476$$

$$\frac{1}{2}\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx+\frac{1}{2}\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx$$

$$\Big\downarrow \ 1082$$

$$\frac{\sqrt[4]{b}\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}}$$

$$\Big\downarrow \ 217$$

$$\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

input
```
Int[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

output
```
-((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))) + (
b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(51) = 102$.

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.72

method	result
default	$\frac{\sqrt{b}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8\sqrt{a}}+\sqrt{2}\,\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,a\right.$

input
```
int((a^(1/2)*b^(1/2)+b*x^2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/
b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^
(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8/(a/b)^(1/4)*2^(1/2)*(ln(
(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx$$
$$=\left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^4-4\sqrt{a}\sqrt{b}x^2+4\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3-ax\right)\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}+a}{bx^4+a}\right),\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\sqrt{\frac{1}{2}}x\right.\right.$$
$$\left.+\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\frac{\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3+ax\right)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{a}\right)\right]$$

input
```
integrate((a^(1/2)*b^(1/2)+b*x^2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*log((b*x^4 - 4*sqrt(a)*sqrt(b)*x^2 +
 4*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a)) + a)/(b*x^
4 + a)), sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(sqrt(b)/s
qrt(a))) + sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqrt(
b)*x^3 + a*x)*sqrt(sqrt(b)/sqrt(a))/a)]
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx=-\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$
$$+\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

input
```
integrate((a**(1/2)*b**(1/2)+b*x**2)/(b*x**4+a),x)
```

output
```
-sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(
a))/sqrt(b) - sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*l
og(sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(a))/sqrt(b) - sqrt(a)/sqrt(b) + x*
*2)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}$$

input
```
integrate((a^(1/2)*b^(1/2)+b*x^2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 1/2*sqrt(2)*sqrt(b)*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/sqrt(sqrt(a)*sqrt(b))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^(1/2)*b^(1/2)+b*x^2)/(b*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 17.67 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx=\frac{\sqrt{2}\,b^{1/4}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{2\,a^{1/4}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,b^{3/4}\,x^3}{2\,a^{3/4}}+\frac{\sqrt{2}\,b^{1/4}\,x}{2\,a^{1/4}}\right)\right)}{4\,a^{1/4}}$$

input
```
int((b*x^2 + a^(1/2)*b^(1/2))/(a + b*x^4),x)
```

output
```
(2^(1/2)*b^(1/4)*(2*atan((2^(1/2)*b^(1/4)*x)/(2*a^(1/4))) + 2*atan((2^(1/2
)*b^(3/4)*x^3)/(2*a^(3/4)) + (2^(1/2)*b^(1/4)*x)/(2*a^(1/4)))))/(4*a^(1/4)
)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx=\frac{b^{\frac{1}{4}}\sqrt{2}\left(-atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)\right)}{2a^{\frac{1}{4}}}$$

input
```
int((a^(1/2)*b^(1/2)+b*x^2)/(b*x^4+a),x)
```

output
```
(b**(1/4)*a**(3/4)*sqrt(2)*( - atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)
*x)/(b**(1/4)*a**(1/4)*sqrt(2))) + atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqr
t(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))))/(2*a)
```

3.210 $\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 51

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx=\frac{\sqrt[4]{b}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{\sqrt{2}\sqrt[4]{a}}$$

output

```
1/2*b^(1/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/
2)/a^(1/4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx \\ =\frac{\sqrt[4]{b}\left(-\log\left(-\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x-\sqrt{b}x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)\right)}{2\sqrt{2}\sqrt[4]{a}}$$

input

```
Integrate[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
```

output

```
(b^(1/4)*(-Log[-Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x - Sqrt[b]*x^2] + Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(2*Sqrt[2]*a^(1/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 106 vs. $2(51) = 102$.

Time $= 0.40$ (sec) , antiderivative size $= 106$, normalized size of antiderivative $= 2.08$, number of steps used $= 4$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used $= \{1479, 25, 27, 1103\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx$$

$$\Big\downarrow \text{ 1479}$$

$$-\frac{\sqrt[4]{b}\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt[4]{b}\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{b}\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

input
```
Int[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
```

output
```
-1/2*(b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(Sqr
t[2]*a^(1/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(2*Sqrt[2]*a^(1/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(36) = 72$.

Time = 0.21 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.00

method	result
default	$\frac{\sqrt{b}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8\sqrt{a}}-\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,a$

input
```
int((a^(1/2)*b^(1/2)-b*x^2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/
b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^
(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/8/(a/b)^(1/4)*2^(1/2)*(ln(
(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
)
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = \left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^4+4\sqrt{a}\sqrt{b}x^2+4\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3+ax\right)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}+a}{bx^4+a}\right), -\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\sqrt{\frac{1}{2}}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\right) + \sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\frac{\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3-ax\right)\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{a}\right)\right]$$

input
```
integrate((a^(1/2)*b^(1/2)-b*x^2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*log((b*x^4 + 4*sqrt(a)*sqrt(b)*x^2 +
4*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 + a*x)*sqrt(sqrt(b)/sqrt(a)) + a)/(b*x^4
+ a)), -sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(-sqrt(b)/
sqrt(a))) + sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqr
t(b)*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a))/a)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(48) = 96$.

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = -\frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{a}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4} + \frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{a}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

input
```
integrate((a**(1/2)*b**(1/2)-b*x**2)/(b*x**4+a),x)
```

output
```
-sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a)
)/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(
sqrt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a))/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/
4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = \frac{\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{4\,a^{\frac{1}{4}}} - \frac{\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{4\,a^{\frac{1}{4}}}$$

input
```
integrate((a^(1/2)*b^(1/2)-b*x^2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*b^(1/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))
/a^(1/4) - 1/4*sqrt(2)*b^(1/4)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/a^(1/4)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate((a^(1/2)*b^(1/2)-b*x^2)/(b*x^4+a),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 17.67 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\,b^{1/4}\,\mathrm{atanh}\left(\frac{2\,\sqrt{2}\,a^{1/4}\,b^{11/4}\,x}{2\,\sqrt{a}\,b^{5/2}+2\,b^3\,x^2}\right)}{2\,a^{1/4}}$$

input

```
int(-(b*x^2 - a^(1/2)*b^(1/2))/(a + b*x^4),x)
```

output

```
(2^(1/2)*b^(1/4)*atanh((2*2^(1/2)*a^(1/4)*b^(11/4)*x)/(2*a^(1/2)*b^(5/2) +
 2*b^3*x^2)))/(2*a^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = \frac{b^{\frac{1}{4}}\sqrt{2}\left(-\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)+\log\left(b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\right)}{4a^{\frac{1}{4}}}$$

input
```
int((a^(1/2)*b^(1/2)-b*x^2)/(b*x^4+a),x)
```

output
```
(b**(1/4)*a**(3/4)*sqrt(2)*( - log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a
) + sqrt(b)*x**2) + log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x*
*2)))/(4*a)
```

3.211 $\int \frac{d+ex^2}{d^2+e^2x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

output
```
1/2*arctan(-1+2^(1/2)*e^(1/2)*x/d^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)+1/2*arcta
n(1+2^(1/2)*e^(1/2)*x/d^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx=\frac{-\arctan\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)+\arctan\left(1+\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)/(d^2 + e^2*x^4),x]
```

output
```
(-ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]] + ArcTan[1 + (Sqrt[2]*Sqrt[e]*x)
/Sqrt[d]])/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1476, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{d^2+e^2x^4}dx$$

$$\Big\downarrow \text{1476}$$

$$\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}}dx}{2e}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}}dx}{2e}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

input

```
Int[(d + e*x^2)/(d^2 + e^2*x^4),x]
```

output

```
-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])) + Arc
Tan[1 + (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{\sqrt{2}\,\ln\left(e\,x^{2}\sqrt{-de}-dex\sqrt{2}-d\sqrt{-de}\right)}{4\sqrt{-de}}+\frac{\sqrt{2}\,\ln\left(e\,x^{2}\sqrt{-de}+dex\sqrt{2}-d\sqrt{-de}\right)}{4\sqrt{-de}}$
default	$\frac{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}{x^{2}-\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8d}+\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}{x^{2}+\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}\right.\right.}{}$

input
```
int((e*x^2+d)/(e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)/(-d*e)^(1/2)*ln(e*x^2*(-d*e)^(1/2)-d*e*x*2^(1/2)-d*(-d*e)^(1/
2))+1/4*2^(1/2)/(-d*e)^(1/2)*ln(e*x^2*(-d*e)^(1/2)+d*e*x*2^(1/2)-d*(-d*e)^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.83

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = \left[-\frac{\sqrt{2}\sqrt{-de}\log\left(\frac{e^2x^4-4\,dex^2-2\,\sqrt{2}(ex^3-dx)\sqrt{-de}+d^2}{e^2x^4+d^2}\right)}{4\,de}, \frac{\sqrt{2}\sqrt{de}\arctan\left(\frac{\sqrt{2}\sqrt{de}x}{2\,d}\right)+\sqrt{2}\sqrt{de}\arctan\left(\frac{\sqrt{2}(ex^3+dx}{2\,d^2}\right.}{2\,de}\right.$$

input
```
integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(2)*sqrt(-d*e)*log((e^2*x^4 - 4*d*e*x^2 - 2*sqrt(2)*(e*x^3 - d*x
)*sqrt(-d*e) + d^2)/(e^2*x^4 + d^2))/(d*e), 1/2*(sqrt(2)*sqrt(d*e)*arctan(
1/2*sqrt(2)*sqrt(d*e)*x/d) + sqrt(2)*sqrt(d*e)*arctan(1/2*sqrt(2)*(e*x^3 +
 d*x)*sqrt(d*e)/d^2))/(d*e)]
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = -\frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(-\sqrt{2}dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4} + \frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(\sqrt{2}dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4}$$

input
```
integrate((e*x**2+d)/(e**2*x**4+d**2),x)
```

output
```
-sqrt(2)*sqrt(-1/(d*e))*log(-sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4 +
sqrt(2)*sqrt(-1/(d*e))*log(sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(51) = 102$.

Time $= 0.11$ (sec) , antiderivative size $= 302$, normalized size of antiderivative $= 4.03$

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\left(e+\sqrt{e^2}\right)\log\left(\frac{2\sqrt{e^2}x+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}-\sqrt{2}\sqrt{-d\sqrt{e^2}}}{2\sqrt{e^2}x+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}+\sqrt{2}\sqrt{-d\sqrt{e^2}}}\right)}{8\sqrt{e^2}\sqrt{-d\sqrt{e^2}}} + \frac{\sqrt{2}\left(e+\sqrt{e^2}\right)\log\left(\frac{2\sqrt{e^2}x-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}-\sqrt{2}\sqrt{-d\sqrt{e^2}}}{2\sqrt{e^2}x-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}+\sqrt{2}\sqrt{-d\sqrt{e^2}}}\right)}{8\sqrt{e^2}\sqrt{-d\sqrt{e^2}}} - \frac{\sqrt{2}\left(e-\sqrt{e^2}\right)\log\left(\sqrt{e^2}x^2+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}x+d\right)}{8\sqrt{d}(e^2)^{\frac{3}{4}}} + \frac{\sqrt{2}\left(e-\sqrt{e^2}\right)\log\left(\sqrt{e^2}x^2-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}x+d\right)}{8\sqrt{d}(e^2)^{\frac{3}{4}}}$$

input
```
integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*(e + sqrt(e^2))*log((2*sqrt(e^2)*x + sqrt(2)*sqrt(d)*(e^2)^(1/
4) - sqrt(2)*sqrt(-d*sqrt(e^2)))/(2*sqrt(e^2)*x + sqrt(2)*sqrt(d)*(e^2)^(1
/4) + sqrt(2)*sqrt(-d*sqrt(e^2))))/(sqrt(e^2)*sqrt(-d*sqrt(e^2))) + 1/8*sq
rt(2)*(e + sqrt(e^2))*log((2*sqrt(e^2)*x - sqrt(2)*sqrt(d)*(e^2)^(1/4) - s
qrt(2)*sqrt(-d*sqrt(e^2)))/(2*sqrt(e^2)*x - sqrt(2)*sqrt(d)*(e^2)^(1/4) +
sqrt(2)*sqrt(-d*sqrt(e^2))))/(sqrt(e^2)*sqrt(-d*sqrt(e^2))) - 1/8*sqrt(2)*
(e - sqrt(e^2))*log(sqrt(e^2)*x^2 + sqrt(2)*sqrt(d)*(e^2)^(1/4)*x + d)/(sq
rt(d)*(e^2)^(3/4)) + 1/8*sqrt(2)*(e - sqrt(e^2))*log(sqrt(e^2)*x^2 - sqrt(
2)*sqrt(d)*(e^2)^(1/4)*x + d)/(sqrt(d)*(e^2)^(3/4))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\sqrt{-de}\log\left(x^2+\sqrt{2}x\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}+\sqrt{\frac{d^2}{e^2}}\right)}{4\,de} - \frac{\sqrt{2}\sqrt{-de}\log\left(x^2-\sqrt{2}x\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}+\sqrt{\frac{d^2}{e^2}}\right)}{4\,de}$$

input
```
integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*sqrt(-d*e)*log(x^2 + sqrt(2)*x*(d^2/e^2)^(1/4) + sqrt(d^2/e^2)
)/(d*e) - 1/4*sqrt(2)*sqrt(-d*e)*log(x^2 - sqrt(2)*x*(d^2/e^2)^(1/4) + sqr
t(d^2/e^2))/(d*e)
```

Mupad [B] (verification not implemented)

Time = 17.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e}\,x}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{3/2}\,x^3}{2\,d^{3/2}}+\frac{\sqrt{2}\,\sqrt{e}\,x}{2\,\sqrt{d}}\right)\right)}{4\,\sqrt{d}\,\sqrt{e}}$$

input
```
int((d + e*x^2)/(d^2 + e^2*x^4),x)
```

output
```
(2^(1/2)*(2*atan((2^(1/2)*e^(1/2)*x)/(2*d^(1/2))) + 2*atan((2^(1/2)*e^(3/2
)*x^3)/(2*d^(3/2)) + (2^(1/2)*e^(1/2)*x)/(2*d^(1/2)))))/(4*d^(1/2)*e^(1/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,\sqrt{2}\left(-atan\left(\frac{\sqrt{e}\,\sqrt{d}\,\sqrt{2}-2ex}{\sqrt{e}\,\sqrt{d}\,\sqrt{2}}\right)+atan\left(\frac{\sqrt{e}\,\sqrt{d}\,\sqrt{2}+2ex}{\sqrt{e}\,\sqrt{d}\,\sqrt{2}}\right)\right)}{2de}$$

input
```
int((e*x^2+d)/(e^2*x^4+d^2),x)
```

output
```
(sqrt(e)*sqrt(d)*sqrt(2)*( - atan((sqrt(e)*sqrt(d)*sqrt(2) - 2*e*x)/(sqrt(
e)*sqrt(d)*sqrt(2))) + atan((sqrt(e)*sqrt(d)*sqrt(2) + 2*e*x)/(sqrt(e)*sqr
t(d)*sqrt(2)))))/(2*d*e)
```

3.212 $\int \frac{d-ex^2}{d^2+e^2x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{e}x}{d+ex^2}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

output
```
1/2*arctanh(2^(1/2)*d^(1/2)*e^(1/2)*x/(e*x^2+d))*2^(1/2)/d^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.74

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx=\frac{-\log\left(-d+\sqrt{2}\sqrt{d}\sqrt{e}x-ex^2\right)+\log\left(d+\sqrt{2}\sqrt{d}\sqrt{e}x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

input
```
Integrate[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

output
```
(-Log[-d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x - e*x^2] + Log[d + Sqrt[2]*Sqrt[d]*Sq
rt[e]*x + e*x^2])/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 90 vs. $2(43) = 86$.

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{d^2+e^2x^4}dx$$

$$\Big\downarrow 1479$$

$$-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}\right)}dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}x+\sqrt{d}\right)}{\sqrt{e}\left(x^2+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}\right)}dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}\right)}dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}x+\sqrt{d}\right)}{\sqrt{e}\left(x^2+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}\right)}dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{e}x}{x^2-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}}dx}{2\sqrt{2}\sqrt{de}}+\frac{\int \frac{\sqrt{2}\sqrt{e}x+\sqrt{d}}{x^2+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}}dx}{2\sqrt{de}}$$

$$\Big\downarrow 1103$$

$$\frac{\log\left(\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\log\left(-\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

input
```
Int[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

output
```
-1/2*Log[d - Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[e])
+ Log[d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(32) = 64.

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.74

method	result
risch	$\frac{\sqrt{2}\,\ln\left(dex\sqrt{2}+e\,x^{2}\sqrt{de}+d\sqrt{de}\right)}{4\sqrt{de}}-\frac{\sqrt{2}\,\ln\left(-dex\sqrt{2}+e\,x^{2}\sqrt{de}+d\sqrt{de}\right)}{4\sqrt{de}}$
default	$\frac{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}{x^{2}-\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8d}-\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}{x^{2}+\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}\right)\right.$

input
```
int((-e*x^2+d)/(e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)/(d*e)^(1/2)*ln(d*e*x*2^(1/2)+e*x^2*(d*e)^(1/2)+d*(d*e)^(1/2))-
1/4*2^(1/2)/(d*e)^(1/2)*ln(-d*e*x*2^(1/2)+e*x^2*(d*e)^(1/2)+d*(d*e)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(32) = 64$.

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.26

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = \left[\frac{\sqrt{2}\sqrt{de}\log\left(\frac{e^2x^4+4\,dex^2+2\,\sqrt{2}\left(ex^3+dx\right)\sqrt{de}+d^2}{e^2x^4+d^2}\right)}{4\,de},\right.$$
$$\left.-\frac{\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\sqrt{-de}x}{2\,d}\right)-\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\left(ex^3-dx\right)\sqrt{-de}}{2\,d^2}\right)}{2\,de}\right]$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(2)*sqrt(d*e)*log((e^2*x^4 + 4*d*e*x^2 + 2*sqrt(2)*(e*x^3 + d*x)*
sqrt(d*e) + d^2)/(e^2*x^4 + d^2))/(d*e), -1/2*(sqrt(2)*sqrt(-d*e)*arctan(1
/2*sqrt(2)*sqrt(-d*e)*x/d) - sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(e*x^3
- d*x)*sqrt(-d*e)/d^2))/(d*e)]
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.86

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = -\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(-\sqrt{2}dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}$$
$$+\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(\sqrt{2}dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}$$

input
```
integrate((-e*x**2+d)/(e**2*x**4+d**2),x)
```

output
```
-sqrt(2)*sqrt(1/(d*e))*log(-sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4 + sq
rt(2)*sqrt(1/(d*e))*log(sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(32) = 64$.

Time = 0.11 (sec) , antiderivative size = 302, normalized size of antiderivative = 7.02

$$\begin{aligned}\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = &-\frac{\sqrt{2}\left(e-\sqrt{e^2}\right)\log\left(\frac{2\sqrt{e^2}x+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}-\sqrt{2}\sqrt{-d\sqrt{e^2}}}{2\sqrt{e^2}x+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}+\sqrt{2}\sqrt{-d\sqrt{e^2}}}\right)}{8\sqrt{e^2}\sqrt{-d\sqrt{e^2}}}\\ &-\frac{\sqrt{2}\left(e-\sqrt{e^2}\right)\log\left(\frac{2\sqrt{e^2}x-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}-\sqrt{2}\sqrt{-d\sqrt{e^2}}}{2\sqrt{e^2}x-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}+\sqrt{2}\sqrt{-d\sqrt{e^2}}}\right)}{8\sqrt{e^2}\sqrt{-d\sqrt{e^2}}}\\ &+\frac{\sqrt{2}\left(e+\sqrt{e^2}\right)\log\left(\sqrt{e^2}x^2+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}x+d\right)}{8\sqrt{d}(e^2)^{\frac{3}{4}}}\\ &-\frac{\sqrt{2}\left(e+\sqrt{e^2}\right)\log\left(\sqrt{e^2}x^2-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}x+d\right)}{8\sqrt{d}(e^2)^{\frac{3}{4}}}\end{aligned}$$

input

```
integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="maxima")
```

output

```
-1/8*sqrt(2)*(e - sqrt(e^2))*log((2*sqrt(e^2)*x + sqrt(2)*sqrt(d)*(e^2)^(1
/4) - sqrt(2)*sqrt(-d*sqrt(e^2)))/(2*sqrt(e^2)*x + sqrt(2)*sqrt(d)*(e^2)^(
1/4) + sqrt(2)*sqrt(-d*sqrt(e^2))))/(sqrt(e^2)*sqrt(-d*sqrt(e^2))) - 1/8*s
qrt(2)*(e - sqrt(e^2))*log((2*sqrt(e^2)*x - sqrt(2)*sqrt(d)*(e^2)^(1/4) -
sqrt(2)*sqrt(-d*sqrt(e^2)))/(2*sqrt(e^2)*x - sqrt(2)*sqrt(d)*(e^2)^(1/4) +
 sqrt(2)*sqrt(-d*sqrt(e^2))))/(sqrt(e^2)*sqrt(-d*sqrt(e^2))) + 1/8*sqrt(2)
*(e + sqrt(e^2))*log(sqrt(e^2)*x^2 + sqrt(2)*sqrt(d)*(e^2)^(1/4)*x + d)/(s
qrt(d)*(e^2)^(3/4)) - 1/8*sqrt(2)*(e + sqrt(e^2))*log(sqrt(e^2)*x^2 - sqrt
(2)*sqrt(d)*(e^2)^(1/4)*x + d)/(sqrt(d)*(e^2)^(3/4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(32) = 64$.

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.33

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\right)}{2\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}\right)}{2\,de} + \frac{\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\right)}{2\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}\right)}{2\,de}$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(d^2/e^2)^(1/4))/
(d^2/e^2)^(1/4))/(d*e) + 1/2*sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(d^2/e^2)^(1/4))/(d^2/e^2)^(1/4))/(d*e)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{2\sqrt{2}\sqrt{d}\,e^{7/2}\,x}{2\,e^4\,x^2+2\,d\,e^3}\right)}{2\sqrt{d}\sqrt{e}}$$

input
```
int((d - e*x^2)/(d^2 + e^2*x^4),x)
```

output
```
(2^(1/2)*atanh((2*2^(1/2)*d^(1/2)*e^(7/2)*x)/(2*d*e^3 + 2*e^4*x^2)))/(2*d^
(1/2)*e^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,\sqrt{2}\left(-\log\left(-\sqrt{e}\,\sqrt{d}\,\sqrt{2}\,x+d+e\,x^2\right)+\log\left(\sqrt{e}\,\sqrt{d}\,\sqrt{2}\,x+d+e\,x^2\right)\right)}{4de}$$

input

```
int((-e*x^2+d)/(e^2*x^4+d^2),x)
```

output

```
(sqrt(e)*sqrt(d)*sqrt(2)*( - log( - sqrt(e)*sqrt(d)*sqrt(2)*x + d + e*x**2
) + log(sqrt(e)*sqrt(d)*sqrt(2)*x + d + e*x**2)))/(4*d*e)
```

3.213 $\int \frac{5+2x^2}{-1+x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{5+2x^2}{-1+x^4}\,dx=-\frac{3\arctan(x)}{2}-\frac{7\text{arctanh}(x)}{2}$$

output
```
-3/2*arctan(x)-7/2*arctanh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{5+2x^2}{-1+x^4}\,dx=-\frac{3\arctan(x)}{2}+\frac{7}{4}\log(1-x)-\frac{7}{4}\log(1+x)$$

input
```
Integrate[(5 + 2*x^2)/(-1 + x^4),x]
```

output
```
(-3*ArcTan[x])/2 + (7*Log[1 - x])/4 - (7*Log[1 + x])/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1481, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+5}{x^4-1}dx$$

$$\Big\downarrow \text{1481}$$

$$\frac{7}{2}\int \frac{1}{x^2-1}dx-\frac{3}{2}\int \frac{1}{x^2+1}dx$$

$$\Big\downarrow \text{216}$$

$$\frac{7}{2}\int \frac{1}{x^2-1}dx-\frac{3\arctan(x)}{2}$$

$$\Big\downarrow \text{220}$$

$$-\frac{3\arctan(x)}{2}-\frac{7\text{arctanh}(x)}{2}$$

input
```
Int[(5 + 2*x^2)/(-1 + x^4),x]
```

output
```
(-3*ArcTan[x])/2 - (7*ArcTanh[x])/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1481
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Simp[(e/2 + c*(d/(2*q)))   Int[1/(-q + c*x^2), x], x] + Simp[(
e/2 - c*(d/(2*q)))   Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] &
& NeQ[c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

method	result
default	$\frac{7\ln(x-1)}{4}-\frac{7\ln(1+x)}{4}-\frac{3\arctan(x)}{2}$
risch	$\frac{7\ln(x-1)}{4}-\frac{7\ln(1+x)}{4}-\frac{3\arctan(x)}{2}$
parallelrisch	$\frac{7\ln(x-1)}{4}+\frac{3i\ln(x-i)}{4}-\frac{3i\ln(x+i)}{4}-\frac{7\ln(1+x)}{4}$
meijerg	$\frac{5x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{x^3\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{2(x^4)^{\frac{3}{4}}}$

input
```
int((2*x^2+5)/(x^4-1),x,method=_RETURNVERBOSE)
```

output
```
7/4*ln(x-1)-7/4*ln(1+x)-3/2*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{5+2x^2}{-1+x^4}\,dx=-\frac{3}{2}\arctan{(x)}-\frac{7}{4}\log{(x+1)}+\frac{7}{4}\log{(x-1)}$$

input
```
integrate((2*x^2+5)/(x^4-1),x, algorithm="fricas")
```

output
```
-3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{5+2x^2}{-1+x^4}\,dx = \frac{7\log{(x-1)}}{4} - \frac{7\log{(x+1)}}{4} - \frac{3\operatorname{atan}{(x)}}{2}$$

input
```
integrate((2*x**2+5)/(x**4-1),x)
```

output
```
7*log(x - 1)/4 - 7*log(x + 1)/4 - 3*atan(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{5+2x^2}{-1+x^4}\,dx = -\frac{3}{2}\arctan\left(x\right) - \frac{7}{4}\log\left(x+1\right) + \frac{7}{4}\log\left(x-1\right)$$

input
```
integrate((2*x^2+5)/(x^4-1),x, algorithm="maxima")
```

output
```
-3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{5+2x^2}{-1+x^4}\,dx = -\frac{3}{2}\arctan\left(x\right) - \frac{7}{4}\log\left(|x+1|\right) + \frac{7}{4}\log\left(|x-1|\right)$$

input
```
integrate((2*x^2+5)/(x^4-1),x, algorithm="giac")
```

output
```
-3/2*arctan(x) - 7/4*log(abs(x + 1)) + 7/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{5+2x^2}{-1+x^4}\,dx=-\frac{3\,\mathrm{atan}(x)}{2}-\frac{7\,\mathrm{atanh}(x)}{2}$$

input
```
int((2*x^2 + 5)/(x^4 - 1),x)
```

output
```
- (3*atan(x))/2 - (7*atanh(x))/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{5+2x^2}{-1+x^4}\,dx=-\frac{3atan(x)}{2}+\frac{7\log(x-1)}{4}-\frac{7\log(x+1)}{4}$$

input
```
int((2*x^2+5)/(x^4-1),x)
```

output
```
( - 6*atan(x) + 7*log(x - 1) - 7*log(x + 1))/4
```

3.214 $\int (d+ex^2) (a-cx^4)^{5/2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 216

$$\int (d+ex^2) (a-cx^4)^{5/2} \, dx = \frac{4a^2x(195d+77ex^2)\sqrt{a-cx^4}}{3003} + \frac{10ax(117d+77ex^2)(a-cx^4)^{3/2}}{9009} + \frac{1}{143}x(13d+11ex^2)(a-cx^4)^{5/2} + \frac{8a^{15/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{39c^{3/4}\sqrt{a-cx^4}} + \frac{8a^{13/4}\left(195d-\frac{77\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3003\sqrt[4]{c}\sqrt{a-cx^4}}$$

output
```
4/3003*a^2*x*(77*e*x^2+195*d)*(-c*x^4+a)^(1/2)+10/9009*a*x*(77*e*x^2+117*d
)*(-c*x^4+a)^(3/2)+1/143*x*(11*e*x^2+13*d)*(-c*x^4+a)^(5/2)+8/39*a^(15/4)*
e*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(3/4)/(-c*x^4+a)^(1/2
)+8/3003*a^(13/4)*(195*d-77*a^(1/2)*e/c^(1/2))*(1-c*x^4/a)^(1/2)*EllipticF
(c^(1/4)*x/a^(1/4),I)/c^(1/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.37

$$\int \left(d+ex^2\right) \left(a-cx^4\right)^{5/2} dx = \frac{a^2\sqrt{a-cx^4}\Big(3dx \operatorname{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},\frac{5}{4},\frac{cx^4}{a}\right)+ex^3 \operatorname{Hypergeometric2F1}\big(-\frac{5}{2},\frac{3}{4},\frac{7}{4},}{3\sqrt{1-\frac{cx^4}{a}}}$$

input

```
Integrate[(d + e*x^2)*(a - c*x^4)^(5/2),x]
```

output

```
(a^2*Sqrt[a - c*x^4]*(3*d*x*Hypergeometric2F1[-5/2, 1/4, 5/4, (c*x^4)/a] +
 e*x^3*Hypergeometric2F1[-5/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[1 - (c*x^4)/
a])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {1491, 27, 1491, 27, 1491, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)^{5/2} \left(d+ex^2\right) dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{5}{143}\int 2a\left(11ex^2+13d\right)\left(a-cx^4\right)^{3/2}dx+\frac{1}{143}x\left(a-cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{10}{143}a\int \left(11ex^2+13d\right)\left(a-cx^4\right)^{3/2}dx+\frac{1}{143}x\left(a-cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \text{1491}$$

$$\frac{10}{143}a\Biggl(\frac{1}{21}\int 2a\left(77ex^2+117d\right)\sqrt{a-cx^4}dx+\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(117d+77ex^2\right)\Biggr)+$$
$$\frac{1}{143}x\left(a-cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow\ 27$$

$$\frac{10}{143}a\Biggl(\frac{2}{21}a\int\left(77ex^2+117d\right)\sqrt{a-cx^4}dx+\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(117d+77ex^2\right)\Biggr)+$$
$$\frac{1}{143}x\left(a-cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow\ 1491$$

$$\frac{10}{143}a\Biggl(\frac{2}{21}a\Biggl(\frac{1}{15}\int\frac{6a\left(77ex^2+195d\right)}{\sqrt{a-cx^4}}dx+\frac{1}{5}x\sqrt{a-cx^4}\left(195d+77ex^2\right)\Biggr)+\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(117d+77ex^2\right)\Biggr)+$$
$$\frac{1}{143}x\left(a-cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow\ 27$$

$$\frac{10}{143}a\Biggl(\frac{2}{21}a\Biggl(\frac{2}{5}a\int\frac{77ex^2+195d}{\sqrt{a-cx^4}}dx+\frac{1}{5}x\sqrt{a-cx^4}\left(195d+77ex^2\right)\Biggr)+\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(117d+77ex^2\right)\Biggr)+$$
$$\frac{1}{143}x\left(a-cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow\ 1513$$

$$\frac{10}{143}a\Biggl(\frac{2}{21}a\Biggl(\frac{2}{5}a\Biggl(\left(195d-\frac{77\sqrt{a}e}{\sqrt{c}}\right)\int\frac{1}{\sqrt{a-cx^4}}dx+\frac{77\sqrt{a}e\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}\Biggr)+\frac{1}{5}x\sqrt{a-cx^4}\left(195d+77ex^2\right)\Biggr)$$
$$\frac{1}{143}x\left(a-cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow\ 27$$

$$\frac{10}{143}a\Biggl(\frac{2}{21}a\Biggl(\frac{2}{5}a\Biggl(\left(195d-\frac{77\sqrt{a}e}{\sqrt{c}}\right)\int\frac{1}{\sqrt{a-cx^4}}dx+\frac{77e\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\Biggr)+\frac{1}{5}x\sqrt{a-cx^4}\left(195d+77ex^2\right)\Biggr)+$$
$$\frac{1}{143}x\left(a-cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow\ 765$$

$$\frac{10}{143}a\Bigg(\frac{2}{21}a\Bigg(\frac{2}{5}a\Bigg(\frac{\sqrt{1-\frac{cx^4}{a}}\left(195d-\frac{77\sqrt{a}e}{\sqrt{c}}\right)\int\frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+\frac{77e\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\Bigg)+\frac{1}{5}x\sqrt{a-cx^4}\left(195d+77ex^2\right)$$

$$\frac{1}{143}x\left(a-cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow 762$$

$$\frac{10}{143}a\Bigg(\frac{2}{21}a\Bigg(\frac{2}{5}a\Bigg(\frac{77e\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(195d-\frac{77\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\Bigg)+\frac{1}{5}x\sqrt{a}$$

$$\frac{1}{143}x\left(a-cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow 1390$$

$$\frac{10}{143}a\Bigg(\frac{2}{21}a\Bigg(\frac{2}{5}a\Bigg(\frac{77e\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(195d-\frac{77\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\Bigg)$$

$$\frac{1}{143}x\left(a-cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow 1389$$

$$\frac{10}{143}a\Bigg(\frac{2}{21}a\Bigg(\frac{2}{5}a\Bigg(\frac{77\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(195d-\frac{77\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

$$\frac{1}{143}x\left(a-cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow 327$$

$$\frac{10}{143}a\Bigg(\frac{2}{21}a\Bigg(\frac{2}{5}a\Bigg(\frac{77a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(195d-\frac{77\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\Bigg(\arcsin}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

$$\frac{1}{143}x\left(a-cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

input
```
Int[(d + e*x^2)*(a - c*x^4)^(5/2),x]
```

output
```
(x*(13*d + 11*e*x^2)*(a - c*x^4)^(5/2))/143 + (10*a*((x*(117*d + 77*e*x^2)
*(a - c*x^4)^(3/2))/63 + (2*a*((x*(195*d + 77*e*x^2)*Sqrt[a - c*x^4])/5 +
(2*a*((77*a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/
4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(195*d - (77*Sqrt[a]*e)/Sqr
t[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(
1/4)*Sqrt[a - c*x^4])))/5))/21))/143
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

method	result
risch	$\frac{x\left(693c^2e\,x^{10}+819c^2d\,x^8-2156ace\,x^6-2808acd\,x^4+2387a^2e\,x^2+4329a^2d\right)\sqrt{-c\,x^4+a}}{9009}+\frac{8a^3\left(\frac{195d\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right.}{}$
default	$d\left(\frac{c^2x^9\sqrt{-c\,x^4+a}}{11}-\frac{24ac\,x^5\sqrt{-c\,x^4+a}}{77}+\frac{37a^2x\sqrt{-c\,x^4+a}}{77}+\frac{40a^3\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{77\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)+e\left(\right.$
elliptic	$\frac{e\,c^2x^{11}\sqrt{-c\,x^4+a}}{13}+\frac{c^2d\,x^9\sqrt{-c\,x^4+a}}{11}-\frac{28ace\,x^7\sqrt{-c\,x^4+a}}{117}-\frac{24acd\,x^5\sqrt{-c\,x^4+a}}{77}+\frac{31a^2e\,x^3\sqrt{-c\,x^4+a}}{117}+\frac{37a^2dx\sqrt{-c\,x}}{77}$

input
```
int((e*x^2+d)*(-c*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/9009*x*(693*c^2*e*x^10+819*c^2*d*x^8-2156*a*c*e*x^6-2808*a*c*d*x^4+2387*
a^2*e*x^2+4329*a^2*d)*(-c*x^4+a)^(1/2)+8/3003*a^3*(195*d/(1/a^(1/2)*c^(1/2
))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(
-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-77*e*a^(1/2)/(1/a
^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)
*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2
),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.75

$$\int \left(d+ex^2\right)\left(a-cx^4\right)^{5/2}\,dx =$$

$$-\frac{1848\,a^3\sqrt{-c}ex\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)-24\left(195\,a^2cd+77\,a^3e\right)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)}{9}$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/9009*(1848*a^3*sqrt(-c)*e*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x
), -1) - 24*(195*a^2*c*d + 77*a^3*e)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arc
sin((a/c)^(1/4)/x), -1) - (693*c^3*e*x^12 + 819*c^3*d*x^10 - 2156*a*c^2*e*
x^8 - 2808*a*c^2*d*x^6 + 2387*a^2*c*e*x^4 + 4329*a^2*c*d*x^2 - 1848*a^3*e)
*sqrt(-c*x^4 + a))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.26

$$\int \left(d+ex^2\right)\left(a-cx^4\right)^{5/2}\,dx = \frac{a^{\frac{5}{2}}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{5}{2}}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{a^{\frac{3}{2}}cdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)} - \frac{a^{\frac{3}{2}}cex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{a}c^2dx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)} + \frac{\sqrt{a}c^2ex^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

input

```
integrate((e*x**2+d)*(-c*x**4+a)**(5/2),x)
```

output

```
a**(5/2)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(2*I*pi
)/a)/(4*gamma(5/4)) + a**(5/2)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,)
, c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4)) - a**(3/2)*c*d*x**5*gamma(5/4
)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(2*gamma(9/4)) -
a**(3/2)*c*e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), c*x**4*exp_polar(
2*I*pi)/a)/(2*gamma(11/4)) + sqrt(a)*c**2*d*x**9*gamma(9/4)*hyper((-1/2, 9
/4), (13/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(13/4)) + sqrt(a)*c**2*e
*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), c*x**4*exp_polar(2*I*pi)/a
)/(4*gamma(15/4))
```

Maxima [F]

$$\int \left(d+ex^2\right)\left(a-cx^4\right)^{5/2}\,dx=\int \left(-cx^4+a\right)^{\frac{5}{2}}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 + a)^(5/2)*(e*x^2 + d), x)
```

Giac [F]

$$\int \left(d+ex^2\right)\left(a-cx^4\right)^{5/2}\,dx=\int \left(-cx^4+a\right)^{\frac{5}{2}}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(5/2)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)\left(a-cx^4\right)^{5/2}\,dx=\int \left(a-c\,x^4\right)^{5/2}\left(e\,x^2+d\right)\,dx$$

input
```
int((a - c*x^4)^(5/2)*(d + e*x^2),x)
```

output
```
int((a - c*x^4)^(5/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)\left(a-cx^4\right)^{5/2}\,dx = \frac{37\sqrt{-c\,x^4+a}\,a^2dx}{77}+\frac{31\sqrt{-c\,x^4+a}\,a^2e\,x^3}{117} - \frac{24\sqrt{-c\,x^4+a}\,acd\,x^5}{77}-\frac{28\sqrt{-c\,x^4+a}\,ace\,x^7}{117}+\frac{\sqrt{-c\,x^4+a}\,c^2d\,x^9}{11} + \frac{\sqrt{-c\,x^4+a}\,c^2e\,x^{11}}{13}+\frac{40\left(\int\frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)a^3d}{77}+\frac{8\left(\int\frac{\sqrt{-c\,x^4+a}\,x^2}{-c\,x^4+a}dx\right)a^3e}{39}$$

input

```
int((e*x^2+d)*(-c*x^4+a)^(5/2),x)
```

output

```
(4329*sqrt(a - c*x**4)*a**2*d*x + 2387*sqrt(a - c*x**4)*a**2*e*x**3 - 2808
*sqrt(a - c*x**4)*a*c*d*x**5 - 2156*sqrt(a - c*x**4)*a*c*e*x**7 + 819*sqrt
(a - c*x**4)*c**2*d*x**9 + 693*sqrt(a - c*x**4)*c**2*e*x**11 + 4680*int(sq
rt(a - c*x**4)/(a - c*x**4),x)*a**3*d + 1848*int((sqrt(a - c*x**4)*x**2)/(
a - c*x**4),x)*a**3*e)/9009
```

3.215 $\int (d+ex^2)\left(a-cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 186

$$\int (d+ex^2)\left(a-cx^4\right)^{3/2}\,dx = \frac{2}{105}ax\left(15d+7ex^2\right)\sqrt{a-cx^4} + \frac{1}{63}x\left(9d+7ex^2\right)\left(a-cx^4\right)^{3/2} + \frac{4a^{11/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15c^{3/4}\sqrt{a-cx^4}} + \frac{4a^{9/4}\left(15d-\frac{7\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{105\sqrt[4]{c}\sqrt{a-cx^4}}$$

output
```
2/105*a*x*(7*e*x^2+15*d)*(-c*x^4+a)^(1/2)+1/63*x*(7*e*x^2+9*d)*(-c*x^4+a)^
(3/2)+4/15*a^(11/4)*e*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(
3/4)/(-c*x^4+a)^(1/2)+4/105*a^(9/4)*(15*d-7*a^(1/2)*e/c^(1/2))*(1-c*x^4/a)
^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(1/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.42

$$\int (d+ex^2)(a-cx^4)^{3/2}\,dx=\frac{a\sqrt{a-cx^4}\left(3dx\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},c\right.\right.}{3\sqrt{1-\frac{cx^4}{a}}}$$

input
```
Integrate[(d + e*x^2)*(a - c*x^4)^(3/2),x]
```

output
```
(a*Sqrt[a - c*x^4]*(3*d*x*Hypergeometric2F1[-3/2, 1/4, 5/4, (c*x^4)/a] + e
*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[1 - (c*x^4)/a]
)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {1491, 27, 1491, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-cx^4)^{3/2}(d+ex^2)\,dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{21}\int 2a(7ex^2+9d)\sqrt{a-cx^4}dx+\frac{1}{63}x(a-cx^4)^{3/2}(9d+7ex^2)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{21}a\int (7ex^2+9d)\sqrt{a-cx^4}dx+\frac{1}{63}x(a-cx^4)^{3/2}(9d+7ex^2)$$

$$\Big\downarrow \text{1491}$$

$$\frac{2}{21}a\left(\frac{1}{15}\int\frac{6a\left(7ex^2+15d\right)}{\sqrt{a-cx^4}}dx+\frac{1}{5}x\sqrt{a-cx^4}\left(15d+7ex^2\right)\right)+\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{21}a\left(\frac{2}{5}a\int\frac{7ex^2+15d}{\sqrt{a-cx^4}}dx+\frac{1}{5}x\sqrt{a-cx^4}\left(15d+7ex^2\right)\right)+\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{1513}$$

$$\frac{2}{21}a\left(\frac{2}{5}a\left(\left(15d-\frac{7\sqrt{a}e}{\sqrt{c}}\right)\int\frac{1}{\sqrt{a-cx^4}}dx+\frac{7\sqrt{a}e\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)+\frac{1}{5}x\sqrt{a-cx^4}\left(15d+7ex^2\right)\right)+$$
$$\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{21}a\left(\frac{2}{5}a\left(\left(15d-\frac{7\sqrt{a}e}{\sqrt{c}}\right)\int\frac{1}{\sqrt{a-cx^4}}dx+\frac{7e\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)+\frac{1}{5}x\sqrt{a-cx^4}\left(15d+7ex^2\right)\right)+$$
$$\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{765}$$

$$\frac{2}{21}a\left(\frac{2}{5}a\left(\frac{\sqrt{1-\frac{cx^4}{a}}\left(15d-\frac{7\sqrt{a}e}{\sqrt{c}}\right)\int\frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+\frac{7e\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)+\frac{1}{5}x\sqrt{a-cx^4}\left(15d+7ex^2\right)\right)+$$
$$\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{762}$$

$$\frac{2}{21}a\left(\frac{2}{5}a\left(\frac{7e\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(15d-\frac{7\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)+\frac{1}{5}x\sqrt{a-cx^4}(15\right.$$
$$\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{1390}$$

$$\frac{2}{21}a\left(\frac{2}{5}a\left(\frac{7e\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(15d-\frac{7\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)+\frac{1}{5}x\sqrt{a$$

$$\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{1389}$$

$$\frac{2}{21}a\left(\frac{2}{5}a\left(\frac{7\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(15d-\frac{7\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)+\frac{1}{5}x$$

$$\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{327}$$

$$\frac{2}{21}a\left(\frac{2}{5}a\left(\frac{7a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(15d-\frac{7\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-\right.}{\sqrt[4]{c}\sqrt{a-cx^4}}\right.\right.$$

$$\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

input
```
Int[(d + e*x^2)*(a - c*x^4)^(3/2),x]
```

output
```
(x*(9*d + 7*e*x^2)*(a - c*x^4)^(3/2))/63 + (2*a*((x*(15*d + 7*e*x^2)*Sqrt[
a - c*x^4])/5 + (2*a*((7*a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c
^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(15*d - (7*S
qrt[a]*e)/Sqrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4
)], -1])/(c^(1/4)*Sqrt[a - c*x^4])))/5))/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08

method	result
risch	$\frac{x(-35ce\,x^6-45cd\,x^4+77ae\,x^2+135ad)\sqrt{-c\,x^4+a}}{315}+\frac{4a^2\Bigg(\frac{15d\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{7e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\sqrt{c}}}{}}{105}$
default	$d\left(-\frac{c\,x^5\sqrt{-c\,x^4+a}}{7}+\frac{3ax\sqrt{-c\,x^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)+e\Bigg(-\frac{c\,x^7\sqrt{-c\,x^4+a}}{9}+11$
elliptic	$-\frac{ce\,x^7\sqrt{-c\,x^4+a}}{9}-\frac{cd\,x^5\sqrt{-c\,x^4+a}}{7}+\frac{11ae\,x^3\sqrt{-c\,x^4+a}}{45}+\frac{3adx\sqrt{-c\,x^4+a}}{7}+\frac{4a^2d\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\Big(x\sqrt{}}{7\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}$

input
```
int((e*x^2+d)*(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*x*(-35*c*e*x^6-45*c*d*x^4+77*a*e*x^2+135*a*d)*(-c*x^4+a)^(1/2)+4/105
*a^2*(15*d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2)
)^(1/2),I)-7*e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)
^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF
(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.73

$$\int \left(d+ex^2\right)\left(a-cx^4\right)^{3/2}\,dx =$$

$$-\frac{84\,a^2\sqrt{-c}ex\left(\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1\right)-12\left(15\,acd+7\,a^2e\right)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1\right)+(35}{315\,cx}$$

input

```
integrate((e*x^2+d)*(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output

```
-1/315*(84*a^2*sqrt(-c)*e*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x),
-1) - 12*(15*a*c*d + 7*a^2*e)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/
c)^(1/4)/x), -1) + (35*c^2*e*x^8 + 45*c^2*d*x^6 - 77*a*c*e*x^4 - 135*a*c*d
*x^2 + 84*a^2*e)*sqrt(-c*x^4 + a))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)\left(a-cx^4\right)^{3/2}\,dx = \frac{a^{\frac{3}{2}}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{\sqrt{a}cdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{\sqrt{a}cex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input

```
integrate((e*x**2+d)*(-c*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(2*I*pi
)/a)/(4*gamma(5/4)) + a**(3/2)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,)
, c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4)) - sqrt(a)*c*d*x**5*gamma(5/4)
*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(9/4)) - s
qrt(a)*c*e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), c*x**4*exp_polar(2*
I*pi)/a)/(4*gamma(11/4))
```

Maxima [F]

$$\int \left(d+ex^2\right)\left(a-cx^4\right)^{3/2}\,dx=\int \left(-cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 + a)^(3/2)*(e*x^2 + d), x)
```

Giac [F]

$$\int \left(d+ex^2\right)\left(a-cx^4\right)^{3/2}\,dx=\int \left(-cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)\left(a-cx^4\right)^{3/2}\,dx=\int \left(a-c\,x^4\right)^{3/2}\left(e\,x^2+d\right)\,dx$$

input
```
int((a - c*x^4)^(3/2)*(d + e*x^2),x)
```

output
```
int((a - c*x^4)^(3/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)\left(a-cx^4\right)^{3/2}\,dx=\frac{3\sqrt{-c\,x^4+a}\,adx}{7}+\frac{11\sqrt{-c\,x^4+a}\,ae\,x^3}{45}-\frac{\sqrt{-c\,x^4+a}\,cd\,x^5}{7}-\frac{\sqrt{-c\,x^4+a}\,ce\,x^7}{9}+\frac{4\left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)a^2d}{7}+\frac{4\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-c\,x^4+a}dx\right)a^2e}{15}$$

input
```
int((e*x^2+d)*(-c*x^4+a)^(3/2),x)
```

output
```
(135*sqrt(a - c*x**4)*a*d*x + 77*sqrt(a - c*x**4)*a*e*x**3 - 45*sqrt(a - c
*x**4)*c*d*x**5 - 35*sqrt(a - c*x**4)*c*e*x**7 + 180*int(sqrt(a - c*x**4)/
(a - c*x**4),x)*a**2*d + 84*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a*
*2*e)/315
```

3.216 $\int (d+ex^2)\sqrt{a-cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 158

$$
\begin{aligned}
\int (d+ex^2)\sqrt{a-cx^4}\,dx
&= \frac{1}{15}x\left(5d+3ex^2\right)\sqrt{a-cx^4}+\frac{2a^{7/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{3/4}\sqrt{a-cx^4}} \\
&\quad+\frac{2a^{5/4}\left(5d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{15\sqrt[4]{c}\sqrt{a-cx^4}}
\end{aligned}
$$

output

```
1/15*x*(3*e*x^2+5*d)*(-c*x^4+a)^(1/2)+2/5*a^(7/4)*e*(1-c*x^4/a)^(1/2)*Elli
pticE(c^(1/4)*x/a^(1/4),I)/c^(3/4)/(-c*x^4+a)^(1/2)+2/15*a^(5/4)*(5*d-3*a^
(1/2)*e/c^(1/2))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(1/4)/
(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.92 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.49

$$\int \left(d+ex^2\right)\sqrt{a-cx^4}\,dx = \frac{\sqrt{a-cx^4}\left(3dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{1-\frac{cx^4}{a}}}$$

input

```
Integrate[(d + e*x^2)*Sqrt[a - c*x^4],x]
```

output

```
(Sqrt[a - c*x^4]*(3*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, (c*x^4)/a] + e*x
^3*Hypergeometric2F1[-1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[1 - (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1491, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-cx^4}\left(d+ex^2\right)\,dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{15}\int \frac{2a\left(3ex^2+5d\right)}{\sqrt{a-cx^4}}dx+\frac{1}{15}x\sqrt{a-cx^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{15}a\int \frac{3ex^2+5d}{\sqrt{a-cx^4}}dx+\frac{1}{15}x\sqrt{a-cx^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{1513}$$

$$\frac{2}{15}a\left(\left(5d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{3\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)+\frac{1}{15}x\sqrt{a-cx^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{15}a\left(\left(5d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{3e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)+\frac{1}{15}x\sqrt{a-cx^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{765}$$

$$\frac{2}{15}a\left(\frac{\sqrt{1-\frac{cx^4}{a}}\left(5d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+\frac{3e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)+\frac{1}{15}x\sqrt{a-cx^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{762}$$

$$\frac{2}{15}a\left(\frac{3e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)+$$
$$\frac{1}{15}x\sqrt{a-cx^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{1390}$$

$$\frac{2}{15}a\left(\frac{3e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)+$$
$$\frac{1}{15}x\sqrt{a-cx^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{1389}$$

$$\frac{2}{15}a\left(\frac{3\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)+$$
$$\frac{1}{15}x\sqrt{a-cx^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{327}$$

$$\frac{2}{15}a\left(\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)$$

$$\frac{1}{15}x\sqrt{a-cx^4}(5d+3ex^2)$$

input

```
Int[(d + e*x^2)*Sqrt[a - c*x^4],x]
```

output

```
(x*(5*d + 3*e*x^2)*Sqrt[a - c*x^4])/15 + (2*a*((3*a^(3/4)*e*Sqrt[1 - (c*x^
4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]
) + (a^(1/4)*(5*d - (3*Sqrt[a]*e)/Sqrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[A
rcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4])))/15
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.15

method	result
risch	$\frac{x(3e x^2+5d)\sqrt{-c x^4+a}}{15}+\frac{2a\left(\frac{5d\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}-\frac{3e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{Ellip}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}\,\sqrt{c}}\right.}{15}$
elliptic	$\frac{e x^3\sqrt{-c x^4+a}}{5}+\frac{dx\sqrt{-c x^4+a}}{3}+\frac{2ad\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}-\frac{2a^{\frac{3}{2}}e\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,(\mathrm{EllipticF}}{5\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c}}$
default	$d\left(\frac{x\sqrt{-c x^4+a}}{3}+\frac{2a\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}\right)+e\left(\frac{x^3\sqrt{-c x^4+a}}{5}-\frac{2a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,(\mathrm{El}}{5\sqrt{}}\right.$

input
```
int((e*x^2+d)*(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*e*x^2+5*d)*(-c*x^4+a)^(1/2)+2/15*a*(5*d/(1/a^(1/2)*c^(1/2))^(1/2
)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+
a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-3*e*a^(1/2)/(1/a^(1/2)*c
^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-Ell
ipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69

$$\int \left(d+ex^2\right)\sqrt{a-cx^4}\,dx = -\frac{6\,a\sqrt{-c}ex\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)-2\left(5\,cd+3\,ae\right)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)-(3\,cex^4}{15\,cx}$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(6*a*sqrt(-c)*e*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1)
- 2*(5*c*d + 3*a*e)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x
), -1) - (3*c*e*x^4 + 5*c*d*x^2 - 6*a*e)*sqrt(-c*x^4 + a))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.54

$$\int \left(d+ex^2\right)\sqrt{a-cx^4}\,dx = \frac{\sqrt{a}dx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}ex^3\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)*(-c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(2*I*pi)
/a)/(4*gamma(5/4)) + sqrt(a)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,),
c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4))
```

Maxima [F]

$$\int (d+ex^2)\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}(ex^2+d)\,dx$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(e*x^2 + d), x)
```

Giac [F]

$$\int (d+ex^2)\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}(ex^2+d)\,dx$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)\sqrt{a-cx^4}\,dx=\int \sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)\,dx$$

input
```
int((a - c*x^4)^(1/2)*(d + e*x^2),x)
```

output
```
int((a - c*x^4)^(1/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)\sqrt{a-cx^4}\,dx=\frac{\sqrt{-c\,x^4+a}\,dx}{3}+\frac{\sqrt{-c\,x^4+a}\,e\,x^3}{5}+\frac{2\left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)ad}{3}+\frac{2\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-c\,x^4+a}dx\right)ae}{5}$$

input
```
int((e*x^2+d)*(-c*x^4+a)^(1/2),x)
```

output
```
(5*sqrt(a - c*x**4)*d*x + 3*sqrt(a - c*x**4)*e*x**3 + 10*int(sqrt(a - c*x*
*4)/(a - c*x**4),x)*a*d + 6*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a*
e)/15
```

3.217 $\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 123

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

output
```
a^(3/4)*e*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(3/4)/(-c*x^4
+a)^(1/2)+a^(1/4)*(d-a^(1/2)*e/c^(1/2))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4
)*x/a^(1/4),I)/c^(1/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.63

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{a-cx^4}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

output
```
(Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] +
e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}dx$$

$$\Big\downarrow \text{1513}$$

$$\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{762}$$

$$\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

input

```
Int[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

output

```
(a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(d - (Sqrt[a]*e)/Sqrt[c])*Sqrt[1 - (
c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*
x^4])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.25

method	result
default	$\frac{d\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}\,\sqrt{c}}$
elliptic	$\frac{d\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}\,\sqrt{c}}$

input
```
int((e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I
)-e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1
/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/
2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.74

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = -\frac{a\sqrt{-c}ex\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)-(cd+ae)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)+\sqrt{-cx^4+a}ae}{acx}$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-(a*sqrt(-c)*e*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - (c*d
+ a*e)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + sqrt
(-c*x^4 + a)*a*e)/(a*c*x)
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp
_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int((d + e*x^2)/(a - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx=\left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)d+\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-c\,x^4+a}dx\right)e$$

input

```
int((e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

output

```
int(sqrt(a - c*x**4)/(a - c*x**4),x)*d + int((sqrt(a - c*x**4)*x**2)/(a -
c*x**4),x)*e
```

3.218 $\int \frac{d+ex^2}{\left(a-cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \frac{d+ex^2}{\left(a-cx^4\right)^{3/2}}\,dx = \frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}} - \frac{e\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a-cx^4}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}c^{3/4}\sqrt{a-cx^4}}$$

output

```
1/2*x*(e*x^2+d)/a/(-c*x^4+a)^(1/2)-1/2*e*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/
4)*x/a^(1/4),I)/a^(1/4)/c^(3/4)/(-c*x^4+a)^(1/2)+1/2*(c^(1/2)*d+a^(1/2)*e)
*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(3/4)/c^(3/4)/(-c*x^4+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.64

$$\int \frac{d+ex^2}{(a-cx^4)^{3/2}}\,dx = \frac{3dx+3dx\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+2ex^3\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeomet}}{6a\sqrt{a-cx^4}}$$

input
```
Integrate[(d + e*x^2)/(a - c*x^4)^(3/2),x]
```

output
```
(3*d*x + 3*d*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4
)/a] + 2*e*x^3*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, (c*x^4
)/a])/(6*a*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1493, 25, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a-cx^4)^{3/2}}dx$$

$$\Big\downarrow \ 1493$$

$$\frac{x(d+ex^2)}{2a\sqrt{a-cx^4}}-\frac{\int -\frac{d-ex^2}{\sqrt{a-cx^4}}dx}{2a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{d-ex^2}{\sqrt{a-cx^4}}dx}{2a}+\frac{x(d+ex^2)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \ 1513$$

$$\frac{\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{a-cx^4}}dx-\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{a-cx^4}}dx-\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\frac{\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

input
```
Int[(d + e*x^2)/(a - c*x^4)^(3/2),x]
```

output
```
(x*(d + e*x^2))/(2*a*Sqrt[a - c*x^4]) + (-((a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*
EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4])) + (
a^(1/4)*(d + (Sqrt[a]*e)/Sqrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^
(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4]))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.28

method	result
elliptic	$\frac{2c\left(\frac{e\,x^{3}}{4ac}+\frac{dx}{4ac}\right)}{\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}+\frac{d\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+\frac{e\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(\right.\right.}{2\sqrt{a}\,\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}\,\sqrt{c}}$
default	$d\left(\frac{x}{2a\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}\right)+e\left(\frac{x^{3}}{2a\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{El}\right.}{2\sqrt{a}}\right.$

input
```
int((e*x^2+d)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(1/4/a*e/c*x^3+1/4*d/a/c*x)/(-(x^4-1/c*a)*c)^(1/2)+1/2*d/a/(1/a^(1/2)*
c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(
1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+1/2/a^(1/2)
*e/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2
))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.77

$$\int \frac{d+ex^2}{\left(a-cx^4\right)^{3/2}}\,dx =$$
$$-\frac{(cex^4-ae)\sqrt{a}\left(\frac{c}{a}\right)^{\frac{3}{4}} E\left(\arcsin\left(x\left(\frac{c}{a}\right)^{\frac{1}{4}}\right) \mid -1\right) - ((cd+ce)x^4-ad-ae)\sqrt{a}\left(\frac{c}{a}\right)^{\frac{3}{4}} F\left(\arcsin\left(x\left(\frac{c}{a}\right)^{\frac{1}{4}}\right) \mid -1\right)}{2\left(ac^2x^4-a^2c\right)}$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((c*e*x^4 - a*e)*sqrt(a)*(c/a)^(3/4)*elliptic_e(arcsin(x*(c/a)^(1/4))
, -1) - ((c*d + c*e)*x^4 - a*d - a*e)*sqrt(a)*(c/a)^(3/4)*elliptic_f(arcsi
n(x*(c/a)^(1/4)), -1) + (c*e*x^3 + c*d*x)*sqrt(-c*x^4 + a))/(a*c^2*x^4 - a
^2*c)
```

Sympy [A] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

$$\int \frac{d+ex^2}{\left(a-cx^4\right)^{3/2}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(-c*x**4+a)**(3/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*a*
*(3/2)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), c*x**4*ex
p_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{(a-cx^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(a-cx^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a-cx^4)^{3/2}}\,dx = \int \frac{e\,x^2+d}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int((d + e*x^2)/(a - c*x^4)^(3/2),x)
```

output
```
int((d + e*x^2)/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(a-cx^4)^{3/2}}\,dx = \left(\int \frac{\sqrt{-c\,x^4+a}}{c^2x^8-2ac\,x^4+a^2}dx\right)d+\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{c^2x^8-2ac\,x^4+a^2}dx\right)e$$

input

```
int((e*x^2+d)/(-c*x^4+a)^(3/2),x)
```

output

```
int(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*d + int((sqrt(a -
c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*e
```

3.219 $\int \frac{d+ex^2}{(a-cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 188

$$\int \frac{d+ex^2}{(a-cx^4)^{5/2}}\,dx = \frac{x(d+ex^2)}{6a(a-cx^4)^{3/2}} + \frac{x(5d+3ex^2)}{12a^2\sqrt{a-cx^4}} - \frac{e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{5/4}c^{3/4}\sqrt{a-cx^4}} + \frac{(5\sqrt{c}d+3\sqrt{a}e)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{7/4}c^{3/4}\sqrt{a-cx^4}}$$

output

```
1/6*x*(e*x^2+d)/a/(-c*x^4+a)^(3/2)+1/12*x*(3*e*x^2+5*d)/a^2/(-c*x^4+a)^(1/
2)-1/4*e*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(5/4)/c^(3/4)/
(-c*x^4+a)^(1/2)+1/12*(5*c^(1/2)*d+3*a^(1/2)*e)*(1-c*x^4/a)^(1/2)*Elliptic
F(c^(1/4)*x/a^(1/4),I)/a^(7/4)/c^(3/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.66

$$\int \frac{d+ex^2}{(a-cx^4)^{5/2}}\,dx = \frac{dx(7a-5cx^4)+5dx(a-cx^4)\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+4ex^3(a}{12a^2\left(a-cx^4\right)^{3/2}}$$

input

```
Integrate[(d + e*x^2)/(a - c*x^4)^(5/2),x]
```

output

```
(d*x*(7*a - 5*c*x^4) + 5*d*x*(a - c*x^4)*Sqrt[1 - (c*x^4)/a]*Hypergeometri
c2F1[1/4, 1/2, 5/4, (c*x^4)/a] + 4*e*x^3*(a - c*x^4)*Sqrt[1 - (c*x^4)/a]*H
ypergeometric2F1[3/4, 5/2, 7/4, (c*x^4)/a])/(12*a^2*(a - c*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {1493, 25, 1493, 25, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a-cx^4)^{5/2}}dx$$

$$\Big\downarrow \text{1493}$$

$$\frac{x\left(d+ex^2\right)}{6a\left(a-cx^4\right)^{3/2}}-\frac{\int -\frac{3ex^2+5d}{(a-cx^4)^{3/2}}dx}{6a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3ex^2+5d}{(a-cx^4)^{3/2}}dx}{6a}+\frac{x\left(d+ex^2\right)}{6a\left(a-cx^4\right)^{3/2}}$$

$$\Big\downarrow \text{1493}$$

$$\frac{\frac{x(5d+3ex^2)}{2a\sqrt{a-cx^4}}-\frac{\int -\frac{5d-3ex^2}{\sqrt{a-cx^4}}dx}{2a}}{6a}+\frac{x(d+ex^2)}{6a(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{5d-3ex^2}{\sqrt{a-cx^4}}dx}{2a}+\frac{x(5d+3ex^2)}{2a\sqrt{a-cx^4}}}{6a}+\frac{x(d+ex^2)}{6a(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 1513$$

$$\frac{\frac{\left(\frac{3\sqrt{a}e}{\sqrt{c}}+5d\right)\int \frac{1}{\sqrt{a-cx^4}}dx-\frac{3\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x(5d+3ex^2)}{2a\sqrt{a-cx^4}}}{6a}+\frac{x(d+ex^2)}{6a(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\left(\frac{3\sqrt{a}e}{\sqrt{c}}+5d\right)\int \frac{1}{\sqrt{a-cx^4}}dx-\frac{3e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x(5d+3ex^2)}{2a\sqrt{a-cx^4}}}{6a}+\frac{x(d+ex^2)}{6a(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 765$$

$$\frac{\frac{\frac{\sqrt{1-\frac{cx^4}{a}}\left(\frac{3\sqrt{a}e}{\sqrt{c}}+5d\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-\frac{3e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x(5d+3ex^2)}{2a\sqrt{a-cx^4}}}{6a}+\frac{x(d+ex^2)}{6a(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 762$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\frac{3\sqrt{a}e}{\sqrt{c}}+5d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x(5d+3ex^2)}{2a\sqrt{a-cx^4}}}{6a}+\frac{x(d+ex^2)}{6a(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 1390$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\frac{3\sqrt{a}e}{\sqrt{c}}+5d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}}{2a}+\frac{x(5d+3ex^2)}{2a\sqrt{a-cx^4}}}{6a}+\frac{x(d+ex^2)}{6a(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 1389$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\frac{3\sqrt{a}e}{\sqrt{c}}+5d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}}{2a}+\frac{x\left(5d+3ex^2\right)}{2a\sqrt{a-cx^4}}}{6a}+\frac{x\left(d+ex^2\right)}{6a\left(a-cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\frac{3\sqrt{a}e}{\sqrt{c}}+5d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}}{2a}+\frac{x\left(5d+3ex^2\right)}{2a\sqrt{a-cx^4}}}{6a}+\frac{x\left(d+ex^2\right)}{6a\left(a-cx^4\right)^{3/2}}$$

input

```
Int[(d + e*x^2)/(a - c*x^4)^(5/2),x]
```

output

```
(x*(d + e*x^2))/(6*a*(a - c*x^4)^(3/2)) + ((x*(5*d + 3*e*x^2))/(2*a*Sqrt[a
 - c*x^4]) + ((-3*a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*
x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(5*d + (3*Sqrt[a]*e
)/Sqrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(c^(1/4)*Sqrt[a - c*x^4]))/(2*a))/(6*a)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.30

method	result
elliptic	$\frac{\left(\frac{e\,x^3}{6a\,c^2}+\frac{dx}{6a\,c^2}\right)\sqrt{-c\,x^4+a}}{\left(x^4-\frac{a}{c}\right)^2}+\frac{2c\left(\frac{e\,x^3}{8a^2c}+\frac{5dx}{24a^2c}\right)}{\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{5d\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{12a^2\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}+\frac{e\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,(}{4}$
default	$d\left(\frac{x\sqrt{-c\,x^4+a}}{6a\,c^2\left(x^4-\frac{a}{c}\right)^2}+\frac{5x}{12a^2\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{5\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{12a^2\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)+e\left(\frac{x^3\sqrt{-c\,x^4+a}}{6a\,c^2\left(x^4-\frac{a}{c}\right)^2}+\frac{}{4a^2\sqrt{}}\right.$

input
```
int((e*x^2+d)/(-c*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/6/a*e/c^2*x^3+1/6*d/a/c^2*x)*(-c*x^4+a)^(1/2)/(x^4-1/c*a)^2+2*c*(1/8/a^
2/c*e*x^3+5/24/a^2/c*d*x)/(-(x^4-1/c*a)*c)^(1/2)+5/12*d/a^2/(1/a^(1/2)*c^(
1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+1/4/a^(3/2)*e/
(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^
(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.02

$$\int \frac{d+ex^2}{\left(a-cx^4\right)^{5/2}}\,dx =$$

$$-\frac{3\,(c^2ex^8-2\,acex^4+a^2e)\sqrt{a}\left(\frac{c}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(\frac{c}{a}\right)^{\frac{1}{4}}\right)\,|-1)-((5\,c^2d+3\,c^2e)x^8-2\,(5\,acd+3\,ace)x^4+5}{12\,(a^2c^3x^8-2\,a^3}$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*(c^2*e*x^8 - 2*a*c*e*x^4 + a^2*e)*sqrt(a)*(c/a)^(3/4)*elliptic_e(
arcsin(x*(c/a)^(1/4)), -1) - ((5*c^2*d + 3*c^2*e)*x^8 - 2*(5*a*c*d + 3*a*c
*e)*x^4 + 5*a^2*d + 3*a^2*e)*sqrt(a)*(c/a)^(3/4)*elliptic_f(arcsin(x*(c/a)
^(1/4)), -1) + (3*c^2*e*x^7 + 5*c^2*d*x^5 - 5*a*c*e*x^3 - 7*a*c*d*x)*sqrt(
-c*x^4 + a))/(a^2*c^3*x^8 - 2*a^3*c^2*x^4 + a^4*c)
```

Sympy [A] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.44

$$\int \frac{d+ex^2}{(a-cx^4)^{5/2}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(-c*x**4+a)**(5/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*a*
*(5/2)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), c*x**4*ex
p_polar(2*I*pi)/a)/(4*a**(5/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{(a-cx^4)^{5/2}}\,dx = \int \frac{ex^2+d}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(-c*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(a-cx^4)^{5/2}}\,dx = \int \frac{ex^2+d}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(-c*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a-cx^4)^{5/2}}\,dx = \int \frac{e\,x^2+d}{(a-c\,x^4)^{5/2}}\,dx$$

input
```
int((d + e*x^2)/(a - c*x^4)^(5/2),x)
```

output
```
int((d + e*x^2)/(a - c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(a-cx^4)^{5/2}}\,dx = \left(\int \frac{\sqrt{-c\,x^4+a}}{-c^3x^{12}+3a\,c^2x^8-3a^2c\,x^4+a^3}dx\right)d + \left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-c^3x^{12}+3a\,c^2x^8-3a^2c\,x^4+a^3}dx\right)e$$

input
```
int((e*x^2+d)/(-c*x^4+a)^(5/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x
)*d + int((sqrt(a - c*x**4)*x**2)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 -
c**3*x**12),x)*e
```

3.220 $\int \left(d+ex^2\right)\left(9-x^4\right)^{5/2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 125

$$\int \left(d+ex^2\right)\left(9-x^4\right)^{5/2} \, dx = \frac{108x(195d+77ex^2)\sqrt{9-x^4}}{1001} + \frac{10x(117d+77ex^2)\left(9-x^4\right)^{3/2}}{1001} + \frac{1}{143}x\left(13d+11ex^2\right)\left(9-x^4\right)^{5/2} + \frac{1944}{13}\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right) + \frac{1944\sqrt{3}(65d-77e)\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-1\right)}{1001}$$

output

```
108/1001*x*(77*e*x^2+195*d)*(-x^4+9)^(1/2)+10/1001*x*(77*e*x^2+117*d)*(-x^
4+9)^(3/2)+1/143*x*(11*e*x^2+13*d)*(-x^4+9)^(5/2)+1944/13*3^(1/2)*e*Ellipt
icE(1/3*x*3^(1/2),I)+1944/1001*3^(1/2)*(65*d-77*e)*EllipticF(1/3*x*3^(1/2)
,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.36

$$\int (d+ex^2)(9-x^4)^{5/2}\,dx = 243dx\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},\frac{5}{4},\frac{x^4}{9}\right) + 81ex^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},\frac{7}{4},\frac{x^4}{9}\right)$$

input
```
Integrate[(d + e*x^2)*(9 - x^4)^(5/2),x]
```

output
```
243*d*x*Hypergeometric2F1[-5/2, 1/4, 5/4, x^4/9] + 81*e*x^3*Hypergeometric
2F1[-5/2, 3/4, 7/4, x^4/9]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {1491, 27, 1491, 27, 1491, 27, 1495, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (9-x^4)^{5/2}(d+ex^2)\,dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{5}{143}\int 18(11ex^2+13d)(9-x^4)^{3/2}dx + \frac{1}{143}x(9-x^4)^{5/2}(13d+11ex^2)$$

$$\Big\downarrow \text{27}$$

$$\frac{90}{143}\int (11ex^2+13d)(9-x^4)^{3/2}dx + \frac{1}{143}x(9-x^4)^{5/2}(13d+11ex^2)$$

$$\Big\downarrow \text{1491}$$

$$\frac{90}{143}\left(\frac{1}{21}\int 18\left(77ex^2+117d\right)\sqrt{9-x^4}dx+\frac{1}{63}x\left(9-x^4\right)^{3/2}\left(117d+77ex^2\right)\right)+$$
$$\frac{1}{143}x\left(9-x^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow 27$$

$$\frac{90}{143}\left(\frac{6}{7}\int \left(77ex^2+117d\right)\sqrt{9-x^4}dx+\frac{1}{63}x\left(9-x^4\right)^{3/2}\left(117d+77ex^2\right)\right)+$$
$$\frac{1}{143}x\left(9-x^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow 1491$$

$$\frac{90}{143}\left(\frac{6}{7}\left(\frac{1}{15}\int \frac{54\left(77ex^2+195d\right)}{\sqrt{9-x^4}}dx+\frac{1}{5}x\sqrt{9-x^4}\left(195d+77ex^2\right)\right)+\frac{1}{63}x\left(9-x^4\right)^{3/2}\left(117d+77ex^2\right)\right)+$$
$$\frac{1}{143}x\left(9-x^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow 27$$

$$\frac{90}{143}\left(\frac{6}{7}\left(\frac{18}{5}\int \frac{77ex^2+195d}{\sqrt{9-x^4}}dx+\frac{1}{5}x\sqrt{9-x^4}\left(195d+77ex^2\right)\right)+\frac{1}{63}x\left(9-x^4\right)^{3/2}\left(117d+77ex^2\right)\right)+$$
$$\frac{1}{143}x\left(9-x^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow 1495$$

$$\frac{90}{143}\left(\frac{6}{7}\left(\frac{18}{5}\int \frac{77ex^2+195d}{\sqrt{3-x^2}\sqrt{x^2+3}}dx+\frac{1}{5}x\sqrt{9-x^4}\left(195d+77ex^2\right)\right)+\frac{1}{63}x\left(9-x^4\right)^{3/2}\left(117d+77ex^2\right)\right)+$$
$$\frac{1}{143}x\left(9-x^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow 399$$

$$\frac{90}{143}\left(\frac{6}{7}\left(\frac{18}{5}\left(3(65d-77e)\int \frac{1}{\sqrt{3-x^2}\sqrt{x^2+3}}dx+77e\int \frac{\sqrt{x^2+3}}{\sqrt{3-x^2}}dx\right)+\frac{1}{5}x\sqrt{9-x^4}\left(195d+77ex^2\right)\right)+\frac{1}{63}x$$
$$\frac{1}{143}x\left(9-x^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow 284$$

$$\frac{90}{143}\left(\frac{6}{7}\left(\frac{18}{5}\left(3(65d-77e)\int \frac{1}{\sqrt{9-x^4}}dx+77e\int \frac{\sqrt{x^2+3}}{\sqrt{3-x^2}}dx\right)+\frac{1}{5}x\sqrt{9-x^4}\left(195d+77ex^2\right)\right)+\frac{1}{63}x\left(9-x^4\right)$$
$$\frac{1}{143}x\left(9-x^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \text{327}$$

$$\frac{90}{143}\left(\frac{6}{7}\left(\frac{18}{5}\left(3(65d-77e)\int \frac{1}{\sqrt{9-x^4}}dx+77\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)\right)+\frac{1}{5}x\sqrt{9-x^4}\left(195d+77ex^2\right)\right)+$$
$$\frac{1}{143}x\left(9-x^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \text{762}$$

$$\frac{90}{143}\left(\frac{6}{7}\left(\frac{18}{5}\left(\sqrt{3}(65d-77e)\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-1\right)+77\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)\right)+\frac{1}{5}x\sqrt{9-x^4}(1$$
$$\frac{1}{143}x\left(9-x^4\right)^{5/2}\left(13d+11ex^2\right)$$

input
```
Int[(d + e*x^2)*(9 - x^4)^(5/2),x]
```

output
```
(x*(13*d + 11*e*x^2)*(9 - x^4)^(5/2))/143 + (90*((x*(117*d + 77*e*x^2)*(9
- x^4)^(3/2))/63 + (6*((x*(195*d + 77*e*x^2)*Sqrt[9 - x^4])/5 + (18*(77*Sq
rt[3]*e*EllipticE[ArcSin[x/Sqrt[3]], -1] + Sqrt[3]*(65*d - 77*e)*EllipticF
[ArcSin[x/Sqrt[3]], -1]))/5))/7))/143
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1495
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q -
c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

method	result
meijerg	$81e\,x^{3} \operatorname{hypergeom}\left(\left[-\frac{5}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], \frac{x^{4}}{9}\right)+243dx \operatorname{hypergeom}\left(\left[-\frac{5}{2}, \frac{1}{4}\right], \left[\frac{5}{4}\right], \frac{x^{4}}{9}\right)$
risch	$-\frac{x\left(77e\,x^{10}+91d\,x^{8}-2156e\,x^{6}-2808d\,x^{4}+21483e\,x^{2}+38961d\right)\left(x^{4}-9\right)}{1001\sqrt{-x^{4}+9}}+\frac{3240d\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{3x^{2}+9}\ \operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3}, i\right)}{77\sqrt{-x^{4}+9}}-$
default	$d\left(\frac{x^{9}\sqrt{-x^{4}+9}}{11}-\frac{216x^{5}\sqrt{-x^{4}+9}}{77}+\frac{2997x\sqrt{-x^{4}+9}}{77}+\frac{3240\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{3x^{2}+9}\ \operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3}, i\right)}{77\sqrt{-x^{4}+9}}\right)+e\Bigg($
elliptic	$\frac{e\,x^{11}\sqrt{-x^{4}+9}}{13}+\frac{d\,x^{9}\sqrt{-x^{4}+9}}{11}-\frac{28e\,x^{7}\sqrt{-x^{4}+9}}{13}-\frac{216d\,x^{5}\sqrt{-x^{4}+9}}{77}+\frac{279e\,x^{3}\sqrt{-x^{4}+9}}{13}+\frac{2997dx\sqrt{-x^{4}+9}}{77}+$

input
```
int((e*x^2+d)*(-x^4+9)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
81*e*x^3*hypergeom([-5/2,3/4],[7/4],1/9*x^4)+243*d*x*hypergeom([-5/2,1/4],
[5/4],1/9*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int (d+ex^2)\,(9-x^4)^{5/2}\,dx = \frac{-449064i\,\sqrt{3}exE\left(\arcsin\left(\frac{\sqrt{3}}{x}\right)\,|-1\right)+1944i\,\sqrt{3}(65\,d+231\,e)xF\left(\arcsin\left(\frac{\sqrt{3}}{x}\right)\,|-1\right)+(77\,e}{1001\,x}$$

input
```
integrate((e*x^2+d)*(-x^4+9)^(5/2),x, algorithm="fricas")
```

output
```
1/1001*(-449064*I*sqrt(3)*e*x*elliptic_e(arcsin(sqrt(3)/x), -1) + 1944*I*s
qrt(3)*(65*d + 231*e)*x*elliptic_f(arcsin(sqrt(3)/x), -1) + (77*e*x^12 + 9
1*d*x^10 - 2156*e*x^8 - 2808*d*x^6 + 21483*e*x^4 + 38961*d*x^2 - 149688*e)
*sqrt(-x^4 + 9))/x
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(114) = 228$.

Time = 2.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.85

$$\int (d+ex^2)\left(9-x^4\right)^{5/2}\,dx = \frac{3dx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{x^4e^{2i\pi}}{9}\right)}{4\Gamma\left(\frac{13}{4}\right)} - \frac{27dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{x^4e^{2i\pi}}{9}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{243dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{x^4e^{2i\pi}}{9}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{3ex^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{x^4e^{2i\pi}}{9}\right)}{4\Gamma\left(\frac{15}{4}\right)} - \frac{27ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{x^4e^{2i\pi}}{9}\right)}{2\Gamma\left(\frac{11}{4}\right)} + \frac{243ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{x^4e^{2i\pi}}{9}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)*(-x**4+9)**(5/2),x)
```

output
```
3*d*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), x**4*exp_polar(2*I*pi)/9)/
(4*gamma(13/4)) - 27*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp
_polar(2*I*pi)/9)/(2*gamma(9/4)) + 243*d*x*gamma(1/4)*hyper((-1/2, 1/4), (
5/4,), x**4*exp_polar(2*I*pi)/9)/(4*gamma(5/4)) + 3*e*x**11*gamma(11/4)*hy
per((-1/2, 11/4), (15/4,), x**4*exp_polar(2*I*pi)/9)/(4*gamma(15/4)) - 27*
e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_polar(2*I*pi)/9)/(2
*gamma(11/4)) + 243*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_
polar(2*I*pi)/9)/(4*gamma(7/4))
```

Maxima [F]

$$\int \left(d+ex^2\right) \left(9-x^4\right)^{5/2} \, dx = \int \left(-x^4+9\right)^{\frac{5}{2}} \left(ex^2+d\right) dx$$

input
```
integrate((e*x^2+d)*(-x^4+9)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 9)^(5/2)*(e*x^2 + d), x)
```

Giac [F]

$$\int \left(d+ex^2\right) \left(9-x^4\right)^{5/2} \, dx = \int \left(-x^4+9\right)^{\frac{5}{2}} \left(ex^2+d\right) dx$$

input
```
integrate((e*x^2+d)*(-x^4+9)^(5/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 9)^(5/2)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right) \left(9-x^4\right)^{5/2} \, dx = \int \left(9-x^4\right)^{5/2} \left(e\,x^2+d\right) \, dx$$

input
```
int((9 - x^4)^(5/2)*(d + e*x^2),x)
```

output
```
int((9 - x^4)^(5/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right) \left(9-x^4\right)^{5/2} \, dx = \frac{\sqrt{-x^4+9}\, d\, x^9}{11} - \frac{216\sqrt{-x^4+9}\, d\, x^5}{77} + \frac{2997\sqrt{-x^4+9}\, dx}{77} + \frac{\sqrt{-x^4+9}\, e\, x^{11}}{13} - \frac{28\sqrt{-x^4+9}\, e\, x^7}{13} + \frac{279\sqrt{-x^4+9}\, e\, x^3}{13} - \frac{29160\left(\int \frac{\sqrt{-x^4+9}}{x^4-9}dx\right) d}{77} - \frac{1944\left(\int \frac{\sqrt{-x^4+9}\, x^2}{x^4-9}dx\right) e}{13}$$

input

```
int((e*x^2+d)*(-x^4+9)^(5/2),x)
```

output

```
(91*sqrt( - x**4 + 9)*d*x**9 - 2808*sqrt( - x**4 + 9)*d*x**5 + 38961*sqrt(
 - x**4 + 9)*d*x + 77*sqrt( - x**4 + 9)*e*x**11 - 2156*sqrt( - x**4 + 9)*e
*x**7 + 21483*sqrt( - x**4 + 9)*e*x**3 - 379080*int(sqrt( - x**4 + 9)/(x**
4 - 9),x)*d - 149688*int((sqrt( - x**4 + 9)*x**2)/(x**4 - 9),x)*e)/1001
```

3.221 $\int (d+ex^2)(9-x^4)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int (d+ex^2)(9-x^4)^{3/2}\,dx=\frac{6}{35}x(15d+7ex^2)\sqrt{9-x^4}+\frac{1}{63}x(9d+7ex^2)(9-x^4)^{3/2}+\frac{108}{5}\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)+\frac{108}{35}\sqrt{3}(5d-7e)\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-1\right)$$

output
```
6/35*x*(7*e*x^2+15*d)*(-x^4+9)^(1/2)+1/63*x*(7*e*x^2+9*d)*(-x^4+9)^(3/2)+1
08/5*3^(1/2)*e*EllipticE(1/3*x*3^(1/2),I)+108/35*3^(1/2)*(5*d-7*e)*Ellipti
cF(1/3*x*3^(1/2),I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

$$\int (d+ex^2)(9-x^4)^{3/2}\,dx=27dx\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{x^4}{9}\right)+9ex^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},\frac{x^4}{9}\right)$$

input
```
Integrate[(d + e*x^2)*(9 - x^4)^(3/2),x]
```

output
```
27*d*x*Hypergeometric2F1[-3/2, 1/4, 5/4, x^4/9] + 9*e*x^3*Hypergeometric2F
1[-3/2, 3/4, 7/4, x^4/9]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1491, 27, 1491, 27, 1495, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(9-x^4\right)^{3/2} \left(d+ex^2\right) \, dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{21} \int 18\left(7ex^2+9d\right) \sqrt{9-x^4}dx+\frac{1}{63} x\left(9-x^4\right)^{3/2} \left(9d+7ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7} \int \left(7ex^2+9d\right) \sqrt{9-x^4}dx+\frac{1}{63} x\left(9-x^4\right)^{3/2} \left(9d+7ex^2\right)$$

$$\Big\downarrow \text{1491}$$

$$\frac{6}{7} \left(\frac{1}{15} \int \frac{54\left(7ex^2+15d\right)}{\sqrt{9-x^4}}dx+\frac{1}{5} x\sqrt{9-x^4}\left(15d+7ex^2\right)\right)+\frac{1}{63} x\left(9-x^4\right)^{3/2} \left(9d+7ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7} \left(\frac{18}{5} \int \frac{7ex^2+15d}{\sqrt{9-x^4}}dx+\frac{1}{5} x\sqrt{9-x^4}\left(15d+7ex^2\right)\right)+\frac{1}{63} x\left(9-x^4\right)^{3/2} \left(9d+7ex^2\right)$$

$$\Big\downarrow \text{1495}$$

$$\frac{6}{7} \left(\frac{18}{5} \int \frac{7ex^2+15d}{\sqrt{3-x^2}\sqrt{x^2+3}}dx+\frac{1}{5} x\sqrt{9-x^4}\left(15d+7ex^2\right)\right)+\frac{1}{63} x\left(9-x^4\right)^{3/2} \left(9d+7ex^2\right)$$

$$\Big\downarrow \text{399}$$

$$\frac{6}{7}\left(\frac{18}{5}\left(3(5d-7e)\int \frac{1}{\sqrt{3-x^2}\sqrt{x^2+3}}dx+7e\int \frac{\sqrt{x^2+3}}{\sqrt{3-x^2}}dx\right)+\frac{1}{5}x\sqrt{9-x^4}\left(15d+7ex^2\right)\right)+\frac{1}{63}x\left(9-x^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{ 284}$$

$$\frac{6}{7}\left(\frac{18}{5}\left(3(5d-7e)\int \frac{1}{\sqrt{9-x^4}}dx+7e\int \frac{\sqrt{x^2+3}}{\sqrt{3-x^2}}dx\right)+\frac{1}{5}x\sqrt{9-x^4}\left(15d+7ex^2\right)\right)+\frac{1}{63}x\left(9-x^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{ 327}$$

$$\frac{6}{7}\left(\frac{18}{5}\left(3(5d-7e)\int \frac{1}{\sqrt{9-x^4}}dx+7\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)\right)+\frac{1}{5}x\sqrt{9-x^4}\left(15d+7ex^2\right)\right)+\frac{1}{63}x\left(9-x^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{ 762}$$

$$\frac{6}{7}\Bigg(\frac{18}{5}\left(\sqrt{3}(5d-7e)\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-1\right)+7\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)\right)+\frac{1}{5}x\sqrt{9-x^4}\Big(15d+7ex$$
$$\frac{1}{63}x\left(9-x^4\right)^{3/2}\left(9d+7ex^2\right)$$

input
```
Int[(d + e*x^2)*(9 - x^4)^(3/2),x]
```

output
```
(x*(9*d + 7*e*x^2)*(9 - x^4)^(3/2))/63 + (6*((x*(15*d + 7*e*x^2)*Sqrt[9 -
x^4])/5 + (18*(7*Sqrt[3]*e*EllipticE[ArcSin[x/Sqrt[3]], -1] + Sqrt[3]*(5*d
 - 7*e)*EllipticF[ArcSin[x/Sqrt[3]], -1]))/5))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1495
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q -
c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.34

method	result
meijerg	$9e\,x^3\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],\frac{x^4}{9}\right)+27dx\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{4}\right],\left[\frac{5}{4}\right],\frac{x^4}{9}\right)$
risch	$\frac{x\left(35e\,x^6+45d\,x^4-693e\,x^2-1215d\right)\left(x^4-9\right)}{315\sqrt{-x^4+9}}+\frac{36d\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\right)}{7\sqrt{-x^4+9}}-\frac{36e\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{3x^2+9}\,\left(\mathrm{Elli}\right.}{5\sqrt{}}$
default	$d\left(-\frac{x^5\sqrt{-x^4+9}}{7}+\frac{27x\sqrt{-x^4+9}}{7}+\frac{36\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\right)}{7\sqrt{-x^4+9}}\right)+e\left(-\frac{x^7\sqrt{-x^4+9}}{9}+\frac{11x^3\sqrt{-x^4+9}}{5}\right.$
elliptic	$-\frac{e\,x^7\sqrt{-x^4+9}}{9}-\frac{d\,x^5\sqrt{-x^4+9}}{7}+\frac{11e\,x^3\sqrt{-x^4+9}}{5}+\frac{27dx\sqrt{-x^4+9}}{7}+\frac{36d\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\right)}{7\sqrt{-x^4+9}}-\frac{36}{}$

input
```
int((e*x^2+d)*(-x^4+9)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
9*e*x^3*hypergeom([-3/2,3/4],[7/4],1/9*x^4)+27*d*x*hypergeom([-3/2,1/4],[5
/4],1/9*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int \left(d+ex^2\right)\left(9-x^4\right)^{3/2}\,dx=\frac{-20412i\,\sqrt{3}exE(\arcsin\left(\frac{\sqrt{3}}{x}\right)\mid -1)+972i\,\sqrt{3}(5\,d+21\,e)xF(\arcsin\left(\frac{\sqrt{3}}{x}\right)\mid -1)-(35\,ex^8+}{315\,x}$$

input
```
integrate((e*x^2+d)*(-x^4+9)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(-20412*I*sqrt(3)*e*x*elliptic_e(arcsin(sqrt(3)/x), -1) + 972*I*sqrt
(3)*(5*d + 21*e)*x*elliptic_f(arcsin(sqrt(3)/x), -1) - (35*e*x^8 + 45*d*x^
6 - 693*e*x^4 - 1215*d*x^2 + 6804*e)*sqrt(-x^4 + 9))/x
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.55

$$\int \left(d+ex^2\right)\left(9-x^4\right)^{3/2}\,dx = -\frac{3dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{2i\pi}}{9}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{27dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{2i\pi}}{9}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{3ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{2i\pi}}{9}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{27ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{2i\pi}}{9}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)*(-x**4+9)**(3/2),x)
```

output
```
-3*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(2*I*pi)/9)/
(4*gamma(9/4)) + 27*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_pol
ar(2*I*pi)/9)/(4*gamma(5/4)) - 3*e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/
4,), x**4*exp_polar(2*I*pi)/9)/(4*gamma(11/4)) + 27*e*x**3*gamma(3/4)*hype
r((-1/2, 3/4), (7/4,), x**4*exp_polar(2*I*pi)/9)/(4*gamma(7/4))
```

Maxima [F]

$$\int \left(d+ex^2\right)\left(9-x^4\right)^{3/2}\,dx = \int \left(-x^4+9\right)^{\frac{3}{2}}\left(ex^2+d\right)\,dx$$

input
```
integrate((e*x^2+d)*(-x^4+9)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 9)^(3/2)*(e*x^2 + d), x)
```

Giac [F]

$$\int \left(d+ex^2\right)\left(9-x^4\right)^{3/2}\,dx=\int \left(-x^4+9\right)^{\frac{3}{2}}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(-x^4+9)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 9)^(3/2)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)\left(9-x^4\right)^{3/2}\,dx=\int \left(9-x^4\right)^{3/2}\left(e\,x^2+d\right)\,dx$$

input
```
int((9 - x^4)^(3/2)*(d + e*x^2),x)
```

output
```
int((9 - x^4)^(3/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)\left(9-x^4\right)^{3/2}\,dx=-\frac{\sqrt{-x^4+9}\,d\,x^5}{7}+\frac{27\sqrt{-x^4+9}\,dx}{7}-\frac{\sqrt{-x^4+9}\,e\,x^7}{9}+\frac{11\sqrt{-x^4+9}\,e\,x^3}{5}-\frac{324\left(\int \frac{\sqrt{-x^4+9}}{x^4-9}dx\right)d}{7}-\frac{108\left(\int \frac{\sqrt{-x^4+9}\,x^2}{x^4-9}dx\right)e}{5}$$

input
```
int((e*x^2+d)*(-x^4+9)^(3/2),x)
```

output
```
( - 45*sqrt( - x**4 + 9)*d*x**5 + 1215*sqrt( - x**4 + 9)*d*x - 35*sqrt( -
x**4 + 9)*e*x**7 + 693*sqrt( - x**4 + 9)*e*x**3 - 14580*int(sqrt( - x**4 +
 9)/(x**4 - 9),x)*d - 6804*int((sqrt( - x**4 + 9)*x**2)/(x**4 - 9),x)*e)/3
15
```

3.222 $\int (d+ex^2)\sqrt{9-x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (d+ex^2)\sqrt{9-x^4}\,dx=\frac{1}{15}x\left(5d+3ex^2\right)\sqrt{9-x^4}+\frac{18}{5}\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)+\frac{2}{5}\sqrt{3}(5d-9e)\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-1\right)$$

output
```
1/15*x*(3*e*x^2+5*d)*(-x^4+9)^(1/2)+18/5*3^(1/2)*e*EllipticE(1/3*x*3^(1/2)
,I)+2/5*3^(1/2)*(5*d-9*e)*EllipticF(1/3*x*3^(1/2),I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int (d+ex^2)\sqrt{9-x^4}\,dx=3dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{x^4}{9}\right)+ex^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{x^4}{9}\right)$$

input
```
Integrate[(d + e*x^2)*Sqrt[9 - x^4],x]
```

output
```
3*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, x^4/9] + e*x^3*Hypergeometric2F1[-
1/2, 3/4, 7/4, x^4/9]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1491, 27, 1495, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{9-x^4}\left(d+ex^2\right)\,dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{15}\int \frac{18\left(3ex^2+5d\right)}{\sqrt{9-x^4}}dx+\frac{1}{15}x\sqrt{9-x^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{5}\int \frac{3ex^2+5d}{\sqrt{9-x^4}}dx+\frac{1}{15}x\sqrt{9-x^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{1495}$$

$$\frac{6}{5}\int \frac{3ex^2+5d}{\sqrt{3-x^2}\sqrt{x^2+3}}dx+\frac{1}{15}x\sqrt{9-x^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{399}$$

$$\frac{6}{5}\left((5d-9e)\int \frac{1}{\sqrt{3-x^2}\sqrt{x^2+3}}dx+3e\int \frac{\sqrt{x^2+3}}{\sqrt{3-x^2}}dx\right)+\frac{1}{15}x\sqrt{9-x^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{284}$$

$$\frac{6}{5}\left((5d-9e)\int \frac{1}{\sqrt{9-x^4}}dx+3e\int \frac{\sqrt{x^2+3}}{\sqrt{3-x^2}}dx\right)+\frac{1}{15}x\sqrt{9-x^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{327}$$

$$\frac{6}{5}\left((5d-9e)\int \frac{1}{\sqrt{9-x^4}}dx+3\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)\right)+\frac{1}{15}x\sqrt{9-x^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{762}$$

$$\frac{6}{5}\left(\frac{(5d-9e)\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-1\right)}{\sqrt{3}}+3\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)\right)+\frac{1}{15}x\sqrt{9-x^4}\left(5d+3ex^2\right)$$

input
```
Int[(d + e*x^2)*Sqrt[9 - x^4],x]
```

output
```
(x*(5*d + 3*e*x^2)*Sqrt[9 - x^4])/15 + (6*(3*Sqrt[3]*e*EllipticE[ArcSin[x/
Sqrt[3]], -1] + ((5*d - 9*e)*EllipticF[ArcSin[x/Sqrt[3]], -1])/Sqrt[3]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1495
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q -
c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.45

method	result
meijerg	$e\,x^{3} \operatorname{hypergeom}\left(\left[-\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], \frac{x^{4}}{9}\right)+3 d x \operatorname{hypergeom}\left(\left[-\frac{1}{2}, \frac{1}{4}\right], \left[\frac{5}{4}\right], \frac{x^{4}}{9}\right)$
risch	$-\frac{x\left(3 e\,x^{2}+5 d\right)\left(x^{4}-9\right)}{15 \sqrt{-x^{4}+9}}+\frac{2 d \sqrt{3}\, \sqrt{-3 x^{2}+9}\, \sqrt{3 x^{2}+9}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, x}{3}, i\right)}{3 \sqrt{-x^{4}+9}}-\frac{6 e \sqrt{3}\, \sqrt{-3 x^{2}+9}\, \sqrt{3 x^{2}+9}\, \left(\operatorname{EllipticF}\left(\frac{\sqrt{3}\, x}{3}, i\right)-\text{Ellipt}\right.}{5 \sqrt{-x^{4}+9}}$
elliptic	$\frac{e\,x^{3} \sqrt{-x^{4}+9}}{5}+\frac{d x \sqrt{-x^{4}+9}}{3}+\frac{2 d \sqrt{3}\, \sqrt{-3 x^{2}+9}\, \sqrt{3 x^{2}+9}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, x}{3}, i\right)}{3 \sqrt{-x^{4}+9}}-\frac{6 e \sqrt{3}\, \sqrt{-3 x^{2}+9}\, \sqrt{3 x^{2}+9}\, \left(\operatorname{EllipticF}\left(\frac{\sqrt{3}\, x}{3}, i\right)-\right.}{5 \sqrt{-x^{4}+9}}$
default	$d \left(\frac{x \sqrt{-x^{4}+9}}{3}+\frac{2 \sqrt{3}\, \sqrt{-3 x^{2}+9}\, \sqrt{3 x^{2}+9}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, x}{3}, i\right)}{3 \sqrt{-x^{4}+9}}\right)+e \left(\frac{x^{3} \sqrt{-x^{4}+9}}{5}-\frac{6 \sqrt{3}\, \sqrt{-3 x^{2}+9}\, \sqrt{3 x^{2}+9}\, \left(\operatorname{EllipticF}\left(\frac{\sqrt{3}}{3}\right.\right.}{5 \sqrt{-x^{4}+9}}\right.$

input
```
int((e*x^2+d)*(-x^4+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
e*x^3*hypergeom([-1/2,3/4],[7/4],1/9*x^4)+3*d*x*hypergeom([-1/2,1/4],[5/4]
,1/9*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \left(d+ex^2\right)\sqrt{9-x^4}\,dx$$
$$=\frac{-162i\sqrt{3}exE\left(\arcsin\left(\frac{\sqrt{3}}{x}\right)\mid -1\right)+6i\sqrt{3}(5\,d+27\,e)xF\left(\arcsin\left(\frac{\sqrt{3}}{x}\right)\mid -1\right)+(3\,ex^4+5\,dx^2-54\,e)\sqrt{-}}{15\,x}$$

input
```
integrate((e*x^2+d)*(-x^4+9)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(-162*I*sqrt(3)*e*x*elliptic_e(arcsin(sqrt(3)/x), -1) + 6*I*sqrt(3)*(
5*d + 27*e)*x*elliptic_f(arcsin(sqrt(3)/x), -1) + (3*e*x^4 + 5*d*x^2 - 54*
e)*sqrt(-x^4 + 9))/x
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \left(d+ex^2\right)\sqrt{9-x^4}\,dx=\frac{3dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{2i\pi}}{9}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{3ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{2i\pi}}{9}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)*(-x**4+9)**(1/2),x)
```

output
```
3*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(2*I*pi)/9)/(4*g
amma(5/4)) + 3*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar
(2*I*pi)/9)/(4*gamma(7/4))
```

Maxima [F]

$$\int (d+ex^2)\sqrt{9-x^4}\,dx=\int \sqrt{-x^4+9}(ex^2+d)\,dx$$

input
```
integrate((e*x^2+d)*(-x^4+9)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 9)*(e*x^2 + d), x)
```

Giac [F]

$$\int (d+ex^2)\sqrt{9-x^4}\,dx=\int \sqrt{-x^4+9}(ex^2+d)\,dx$$

input
```
integrate((e*x^2+d)*(-x^4+9)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + 9)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\sqrt{9-x^4}\,dx=\int \sqrt{9-x^4}\,\left(e\,x^2+d\right)\,dx$$

input
```
int((9 - x^4)^(1/2)*(d + e*x^2),x)
```

output
```
int((9 - x^4)^(1/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2)\sqrt{9-x^4}\,dx = \frac{\sqrt{-x^4+9}\,dx}{3}+\frac{\sqrt{-x^4+9}\,e\,x^3}{5} - 6\left(\int \frac{\sqrt{-x^4+9}}{x^4-9}dx\right)d - \frac{18\left(\int \frac{\sqrt{-x^4+9}\,x^2}{x^4-9}dx\right)e}{5}$$

input
```
int((e*x^2+d)*(-x^4+9)^(1/2),x)
```

output
```
(5*sqrt( - x**4 + 9)*d*x + 3*sqrt( - x**4 + 9)*e*x**3 - 90*int(sqrt( - x**
4 + 9)/(x**4 - 9),x)*d - 54*int((sqrt( - x**4 + 9)*x**2)/(x**4 - 9),x)*e)/
15
```

3.223 $\int \frac{d+ex^2}{\sqrt{9-x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \frac{d+ex^2}{\sqrt{9-x^4}}\,dx=\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)+\frac{(d-3e)\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-1\right)}{\sqrt{3}}$$

output
```
3^(1/2)*e*EllipticE(1/3*x*3^(1/2),I)+1/3*(d-3*e)*EllipticF(1/3*x*3^(1/2),I
)*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{d+ex^2}{\sqrt{9-x^4}}\,dx=\frac{1}{3}dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{x^4}{9}\right)+\frac{1}{9}ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{x^4}{9}\right)$$

input
```
Integrate[(d + e*x^2)/Sqrt[9 - x^4],x]
```

output
```
(d*x*Hypergeometric2F1[1/4, 1/2, 5/4, x^4/9])/3 + (e*x^3*Hypergeometric2F1
[1/2, 3/4, 7/4, x^4/9])/9
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1495, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{9-x^4}}dx$$

$$\Big\downarrow \text{1495}$$

$$\int \frac{d+ex^2}{\sqrt{3-x^2}\sqrt{x^2+3}}dx$$

$$\Big\downarrow \text{399}$$

$$(d-3e)\int \frac{1}{\sqrt{3-x^2}\sqrt{x^2+3}}dx+e\int \frac{\sqrt{x^2+3}}{\sqrt{3-x^2}}dx$$

$$\Big\downarrow \text{284}$$

$$(d-3e)\int \frac{1}{\sqrt{9-x^4}}dx+e\int \frac{\sqrt{x^2+3}}{\sqrt{3-x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$(d-3e)\int \frac{1}{\sqrt{9-x^4}}dx+\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)$$

$$\Big\downarrow \text{762}$$

$$\frac{(d-3e)\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-1\right)}{\sqrt{3}}+\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)$$

input
```
Int[(d + e*x^2)/Sqrt[9 - x^4],x]
```

output

```
Sqrt[3]*e*EllipticE[ArcSin[x/Sqrt[3]], -1] + ((d - 3*e)*EllipticF[ArcSin[x
/Sqrt[3]], -1])/Sqrt[3]
```

Defintions of rubi rules used

rule 284

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1495

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q -
c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
meijerg	$\frac{e\,x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], \frac{x^{4}}{9}\right)}{9}+\frac{d x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], \frac{x^{4}}{9}\right)}{3}$	34
default	$\frac{d\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{3x^{2}+9}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3}, i\right)}{9\sqrt{-x^{4}+9}}-\frac{e\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{3x^{2}+9}\,\left(\operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3}, i\right)-\operatorname{EllipticE}\left(\frac{\sqrt{3}\,x}{3}, i\right)\right)}{3\sqrt{-x^{4}+9}}$	98
elliptic	$\frac{d\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{3x^{2}+9}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3}, i\right)}{9\sqrt{-x^{4}+9}}-\frac{e\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{3x^{2}+9}\,\left(\operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3}, i\right)-\operatorname{EllipticE}\left(\frac{\sqrt{3}\,x}{3}, i\right)\right)}{3\sqrt{-x^{4}+9}}$	98

input
```
int((e*x^2+d)/(-x^4+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*e*x^3*hypergeom([1/2,3/4],[7/4],1/9*x^4)+1/3*d*x*hypergeom([1/4,1/2],[
5/4],1/9*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \frac{d+ex^{2}}{\sqrt{9-x^{4}}}\,dx = \frac{-9i\,\sqrt{3}exE(\arcsin\left(\frac{\sqrt{3}}{x}\right)\,|-1)+i\,\sqrt{3}(d+9\,e)xF(\arcsin\left(\frac{\sqrt{3}}{x}\right)\,|-1)-3\,\sqrt{-x^{4}+9}e}{3\,x}$$

input
```
integrate((e*x^2+d)/(-x^4+9)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-9*I*sqrt(3)*e*x*elliptic_e(arcsin(sqrt(3)/x), -1) + I*sqrt(3)*(d + 9
*e)*x*elliptic_f(arcsin(sqrt(3)/x), -1) - 3*sqrt(-x^4 + 9)*e)/x
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.74

$$\int \frac{d+ex^2}{\sqrt{9-x^4}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{x^4e^{2i\pi}}{9}\right)}{12\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{x^4e^{2i\pi}}{9}\right)}{12\Gamma\left(\frac{7}{4}\right)}$$

input

```
integrate((e*x**2+d)/(-x**4+9)**(1/2),x)
```

output

```
d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(2*I*pi)/9)/(12*gam
ma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(2*I*
pi)/9)/(12*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{9-x^4}}\,dx = \int \frac{ex^2+d}{\sqrt{-x^4+9}}\,dx$$

input

```
integrate((e*x^2+d)/(-x^4+9)^(1/2),x, algorithm="maxima")
```

output

```
integrate((e*x^2 + d)/sqrt(-x^4 + 9), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{9-x^4}}\,dx = \int \frac{ex^2+d}{\sqrt{-x^4+9}}\,dx$$

input

```
integrate((e*x^2+d)/(-x^4+9)^(1/2),x, algorithm="giac")
```

output

```
integrate((e*x^2 + d)/sqrt(-x^4 + 9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{9-x^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{9-x^4}}\,dx$$

input
```
int((d + e*x^2)/(9 - x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(9 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{\sqrt{9-x^4}}\,dx=-\left(\int \frac{\sqrt{-x^4+9}}{x^4-9}dx\right)d-\left(\int \frac{\sqrt{-x^4+9}\,x^2}{x^4-9}dx\right)e$$

input
```
int((e*x^2+d)/(-x^4+9)^(1/2),x)
```

output
```
 - (int(sqrt( - x**4 + 9)/(x**4 - 9),x)*d + int((sqrt( - x**4 + 9)*x**2)/(
x**4 - 9),x)*e)
```

3.224 $\int \frac{d+ex^2}{\left(9-x^4\right)^{3/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \frac{d+ex^2}{\left(9-x^4\right)^{3/2}} \, dx = \frac{x\left(d+ex^2\right)}{18\sqrt{9-x^4}} - \frac{eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)}{6\sqrt{3}} + \frac{(d+3e)\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-1\right)}{18\sqrt{3}}$$

output

```
1/18*x*(e*x^2+d)/(-x^4+9)^(1/2)-1/18*3^(1/2)*e*EllipticE(1/3*x*3^(1/2),I)+
1/54*(d+3*e)*EllipticF(1/3*x*3^(1/2),I)*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{d+ex^2}{\left(9-x^4\right)^{3/2}} \, dx = \frac{1}{162}x\left(\frac{9d}{\sqrt{9-x^4}} + 3d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{x^4}{9}\right) + 2ex^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},\frac{x^4}{9}\right)\right)$$

input
```
Integrate[(d + e*x^2)/(9 - x^4)^(3/2),x]
```

output
```
(x*((9*d)/Sqrt[9 - x^4] + 3*d*Hypergeometric2F1[1/4, 1/2, 5/4, x^4/9] + 2*
e*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, x^4/9]))/162
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1493, 25, 1495, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(9-x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1493}$$

$$\frac{x(d+ex^2)}{18\sqrt{9-x^4}}-\frac{1}{18}\int -\frac{d-ex^2}{\sqrt{9-x^4}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{18}\int \frac{d-ex^2}{\sqrt{9-x^4}}dx+\frac{x(d+ex^2)}{18\sqrt{9-x^4}}$$

$$\Big\downarrow \text{1495}$$

$$\frac{1}{18}\int \frac{d-ex^2}{\sqrt{3-x^2}\sqrt{x^2+3}}dx+\frac{x(d+ex^2)}{18\sqrt{9-x^4}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{18}\left((d+3e)\int \frac{1}{\sqrt{3-x^2}\sqrt{x^2+3}}dx-e\int \frac{\sqrt{x^2+3}}{\sqrt{3-x^2}}dx\right)+\frac{x(d+ex^2)}{18\sqrt{9-x^4}}$$

$$\Big\downarrow \text{284}$$

$$\frac{1}{18}\left((d+3e)\int \frac{1}{\sqrt{9-x^4}}dx-e\int \frac{\sqrt{x^2+3}}{\sqrt{3-x^2}}dx\right)+\frac{x(d+ex^2)}{18\sqrt{9-x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{18}\left((d+3e)\int \frac{1}{\sqrt{9-x^4}}dx-\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)\right)+\frac{x(d+ex^2)}{18\sqrt{9-x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{18}\left(\frac{(d+3e)\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-1\right)}{\sqrt{3}}-\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)\right)+\frac{x(d+ex^2)}{18\sqrt{9-x^4}}$$

input
```
Int[(d + e*x^2)/(9 - x^4)^(3/2),x]
```

output
```
(x*(d + e*x^2))/(18*Sqrt[9 - x^4]) + (-(Sqrt[3]*e*EllipticE[ArcSin[x/Sqrt[
3]], -1]) + ((d + 3*e)*EllipticF[ArcSin[x/Sqrt[3]], -1])/Sqrt[3])/18
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1495
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q -
c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

method	result
meijerg	$\frac{e\,x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{7}{4}\right],\frac{x^{4}}{9}\right)}{81}+\frac{d x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],\frac{x^{4}}{9}\right)}{27}$
risch	$\frac{x\left(e\,x^{2}+d\right)}{18\sqrt{-x^{4}+9}}+\frac{e\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{3x^{2}+9}\,\left(\operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\right)-\operatorname{EllipticE}\left(\frac{\sqrt{3}\,x}{3},i\right)\right)}{54\sqrt{-x^{4}+9}}+\frac{d\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{3x^{2}+9}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\right)}{162\sqrt{-x^{4}+9}}$
elliptic	$\frac{\frac{1}{18}e\,x^{3}+\frac{1}{18}d x}{\sqrt{-x^{4}+9}}+\frac{e\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{3x^{2}+9}\,\left(\operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\right)-\operatorname{EllipticE}\left(\frac{\sqrt{3}\,x}{3},i\right)\right)}{54\sqrt{-x^{4}+9}}+\frac{d\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{3x^{2}+9}\,\operatorname{EllipticF}(\frac{\sqrt{3}\,x}{3},}{162\sqrt{-x^{4}+9}}$
default	$d\left(\frac{x}{18\sqrt{-x^{4}+9}}+\frac{\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{3x^{2}+9}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\right)}{162\sqrt{-x^{4}+9}}\right)+e(\frac{x^{3}}{18\sqrt{-x^{4}+9}}+\frac{\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{3x^{2}+9}\,(\operatorname{EllipticF}(\frac{\sqrt{3}\,x}{3}}{54\sqrt{-x^{4}+9}}$

input
```
int((e*x^2+d)/(-x^4+9)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/81*e*x^3*hypergeom([3/4,3/2],[7/4],1/9*x^4)+1/27*d*x*hypergeom([1/4,3/2]
,[5/4],1/9*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \frac{d+ex^2}{(9-x^4)^{3/2}}\,dx =$$
$$-\frac{\sqrt{3}(ex^4-9\,e)E\left(\arcsin\left(\frac{1}{3}\sqrt{3}x\right)\,|-1\right)-\sqrt{3}((d+e)x^4-9\,d-9\,e)F\left(\arcsin\left(\frac{1}{3}\sqrt{3}x\right)\,|-1\right)+3\,(ex^3+dx}{54\,(x^4-9)}$$

input
```
integrate((e*x^2+d)/(-x^4+9)^(3/2),x, algorithm="fricas")
```

output
```
-1/54*(sqrt(3)*(e*x^4 - 9*e)*elliptic_e(arcsin(1/3*sqrt(3)*x), -1) - sqrt(
3)*((d + e)*x^4 - 9*d - 9*e)*elliptic_f(arcsin(1/3*sqrt(3)*x), -1) + 3*(e*
x^3 + d*x)*sqrt(-x^4 + 9))/(x^4 - 9)
```

Sympy [A] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{d+ex^2}{(9-x^4)^{3/2}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{2i\pi}}{9}\right)}{108\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{2i\pi}}{9}\right)}{108\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(-x**4+9)**(3/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(2*I*pi)/9)/(108*ga
mma(5/4)) + e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*exp_polar(2*I
*pi)/9)/(108*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{(9-x^4)^{3/2}}\,dx=\int \frac{ex^2+d}{(-x^4+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-x^4+9)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(-x^4 + 9)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(9-x^4)^{3/2}}\,dx=\int \frac{ex^2+d}{(-x^4+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-x^4+9)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(-x^4 + 9)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(9-x^4)^{3/2}}\,dx=\int \frac{e\,x^2+d}{(9-x^4)^{3/2}}\,dx$$

input
```
int((d + e*x^2)/(9 - x^4)^(3/2),x)
```

output
```
int((d + e*x^2)/(9 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{d+e x^2}{(9-x^4)^{3/2}} \, dx = \left(\int \frac{\sqrt{-x^4+9}}{x^8-18x^4+81}dx\right) d + \left(\int \frac{\sqrt{-x^4+9}\, x^2}{x^8-18x^4+81}dx\right) e$$

input

```
int((e*x^2+d)/(-x^4+9)^(3/2),x)
```

output

```
int(sqrt( - x**4 + 9)/(x**8 - 18*x**4 + 81),x)*d + int((sqrt( - x**4 + 9)*
x**2)/(x**8 - 18*x**4 + 81),x)*e
```

3.225 $\int \frac{d+ex^2}{\left(9-x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 96

$$\int \frac{d+ex^2}{\left(9-x^4\right)^{5/2}}\,dx=\frac{x\left(d+ex^2\right)}{54\left(9-x^4\right)^{3/2}}+\frac{x\left(5d+3ex^2\right)}{972\sqrt{9-x^4}}-\frac{eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)}{108\sqrt{3}}+\frac{(5d+9e)\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-1\right)}{972\sqrt{3}}$$

output
```
1/54*x*(e*x^2+d)/(-x^4+9)^(3/2)+1/972*x*(3*e*x^2+5*d)/(-x^4+9)^(1/2)-1/324
*3^(1/2)*e*EllipticE(1/3*x*3^(1/2),I)+1/2916*(5*d+9*e)*EllipticF(1/3*x*3^(
1/2),I)*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.73

$$\int \frac{d+ex^2}{\left(9-x^4\right)^{5/2}}\,dx=\frac{x\Bigl(\frac{3d\left(63-5x^4\right)}{\left(9-x^4\right)^{3/2}}+5d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{x^4}{9}\right)+4ex^2\,\text{Hypergeometric2F1}\Bigl(\frac{3}{4},\frac{5}{2},}{2916}$$

input
```
Integrate[(d + e*x^2)/(9 - x^4)^(5/2),x]
```

output
```
(x*((3*d*(63 - 5*x^4))/(9 - x^4)^(3/2) + 5*d*Hypergeometric2F1[1/4, 1/2, 5
/4, x^4/9] + 4*e*x^2*Hypergeometric2F1[3/4, 5/2, 7/4, x^4/9]))/2916
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1493, 25, 1493, 25, 1495, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(9-x^4)^{5/2}}dx$$

$$\Big\downarrow \text{ 1493}$$

$$\frac{x(d+ex^2)}{54(9-x^4)^{3/2}}-\frac{1}{54}\int -\frac{3ex^2+5d}{(9-x^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{54}\int \frac{3ex^2+5d}{(9-x^4)^{3/2}}dx+\frac{x(d+ex^2)}{54(9-x^4)^{3/2}}$$

$$\Big\downarrow \text{ 1493}$$

$$\frac{1}{54}\left(\frac{x(5d+3ex^2)}{18\sqrt{9-x^4}}-\frac{1}{18}\int -\frac{5d-3ex^2}{\sqrt{9-x^4}}dx\right)+\frac{x(d+ex^2)}{54(9-x^4)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{54}\left(\frac{1}{18}\int \frac{5d-3ex^2}{\sqrt{9-x^4}}dx+\frac{x(5d+3ex^2)}{18\sqrt{9-x^4}}\right)+\frac{x(d+ex^2)}{54(9-x^4)^{3/2}}$$

$$\Big\downarrow \text{ 1495}$$

$$\frac{1}{54}\left(\frac{1}{18}\int \frac{5d-3ex^2}{\sqrt{3-x^2}\sqrt{x^2+3}}dx+\frac{x(5d+3ex^2)}{18\sqrt{9-x^4}}\right)+\frac{x(d+ex^2)}{54(9-x^4)^{3/2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{54}\left(\frac{1}{18}\left((5d+9e)\int \frac{1}{\sqrt{3-x^2}\sqrt{x^2+3}}dx-3e\int \frac{\sqrt{x^2+3}}{\sqrt{3-x^2}}dx\right)+\frac{x\left(5d+3ex^2\right)}{18\sqrt{9-x^4}}\right)+\frac{x\left(d+ex^2\right)}{54\left(9-x^4\right)^{3/2}}$$

$$\Big\downarrow \text{284}$$

$$\frac{1}{54}\left(\frac{1}{18}\left((5d+9e)\int \frac{1}{\sqrt{9-x^4}}dx-3e\int \frac{\sqrt{x^2+3}}{\sqrt{3-x^2}}dx\right)+\frac{x\left(5d+3ex^2\right)}{18\sqrt{9-x^4}}\right)+\frac{x\left(d+ex^2\right)}{54\left(9-x^4\right)^{3/2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{54}\left(\frac{1}{18}\left((5d+9e)\int \frac{1}{\sqrt{9-x^4}}dx-3\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)\right)+\frac{x\left(5d+3ex^2\right)}{18\sqrt{9-x^4}}\right)+\frac{x\left(d+ex^2\right)}{54\left(9-x^4\right)^{3/2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{54}\left(\frac{1}{18}\left(\frac{(5d+9e)\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-1\right)}{\sqrt{3}}-3\sqrt{3}eE\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-1\right)\right)+\frac{x\left(5d+3ex^2\right)}{18\sqrt{9-x^4}}\right)+\frac{x\left(d+ex^2\right)}{54\left(9-x^4\right)^{3/2}}$$

input
```
Int[(d + e*x^2)/(9 - x^4)^(5/2),x]
```

output
```
(x*(d + e*x^2))/(54*(9 - x^4)^(3/2)) + ((x*(5*d + 3*e*x^2))/(18*Sqrt[9 - x
^4]) + (-3*Sqrt[3]*e*EllipticE[ArcSin[x/Sqrt[3]], -1] + ((5*d + 9*e)*Ellip
ticF[ArcSin[x/Sqrt[3]], -1])/Sqrt[3])/18)/54
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1495
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q -
c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.35

method	result
meijerg	$\frac{e\,x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{5}{2}\right],\left[\frac{7}{4}\right],\frac{x^{4}}{9}\right)}{729}+\frac{d x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{5}{2}\right],\left[\frac{5}{4}\right],\frac{x^{4}}{9}\right)}{243}$
risch	$\frac{x\left(3 e\,x^{6}+5 d\,x^{4}-45 e\,x^{2}-63 d\right)}{972\left(x^{4}-9\right)\sqrt{-x^{4}+9}}+\frac{5 d\sqrt{3}\,\sqrt{-3 x^{2}+9}\,\sqrt{3 x^{2}+9}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\right)}{8748\sqrt{-x^{4}+9}}+\frac{e\sqrt{3}\,\sqrt{-3 x^{2}+9}\,\sqrt{3 x^{2}+9}\,\Big(\operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\right)-\mathrm{E}}{972\sqrt{-x^{4}+9}}$
elliptic	$\frac{\left(\frac{1}{54} e\,x^{3}+\frac{1}{54} d x\right)\sqrt{-x^{4}+9}}{\left(x^{4}-9\right)^{2}}+\frac{\frac{1}{324} e\,x^{3}+\frac{5}{972} d x}{\sqrt{-x^{4}+9}}+\frac{5 d\sqrt{3}\,\sqrt{-3 x^{2}+9}\,\sqrt{3 x^{2}+9}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\right)}{8748\sqrt{-x^{4}+9}}+\frac{e\sqrt{3}\,\sqrt{-3 x^{2}+9}\,\sqrt{3 x^{2}+9}\,\Big(\mathrm{Ellip}}{972}$
default	$d\left(\frac{x\sqrt{-x^{4}+9}}{54\left(x^{4}-9\right)^{2}}+\frac{5 x}{972\sqrt{-x^{4}+9}}+\frac{5\sqrt{3}\,\sqrt{-3 x^{2}+9}\,\sqrt{3 x^{2}+9}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\right)}{8748\sqrt{-x^{4}+9}}\right)+e\Bigg(\frac{x^{3}\sqrt{-x^{4}+9}}{54\left(x^{4}-9\right)^{2}}+\frac{x^{3}}{324\sqrt{-x^{4}+9}}+\frac{\sqrt{3}}{}$

input
```
int((e*x^2+d)/(-x^4+9)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/729*e*x^3*hypergeom([3/4,5/2],[7/4],1/9*x^4)+1/243*d*x*hypergeom([1/4,5/
2],[5/4],1/9*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int \frac{d+e x^{2}}{\left(9-x^{4}\right)^{5/2}}\,dx =$$
$$-\frac{3\,\sqrt{3}(e x^{8}-18\,e x^{4}+81\,e)E\left(\arcsin\left(\frac{1}{3}\,\sqrt{3}x\right)\,|\,-1\right)-\sqrt{3}((5\,d+3\,e)x^{8}-18\,(5\,d+3\,e)x^{4}+405\,d+243\,e)}{2916\,(x^{8}-18\,x^{4}+81)}$$

input
```
integrate((e*x^2+d)/(-x^4+9)^(5/2),x, algorithm="fricas")
```

output
```
-1/2916*(3*sqrt(3)*(e*x^8 - 18*e*x^4 + 81*e)*elliptic_e(arcsin(1/3*sqrt(3)
*x), -1) - sqrt(3)*((5*d + 3*e)*x^8 - 18*(5*d + 3*e)*x^4 + 405*d + 243*e)*
elliptic_f(arcsin(1/3*sqrt(3)*x), -1) + 3*(3*e*x^7 + 5*d*x^5 - 45*e*x^3 -
63*d*x)*sqrt(-x^4 + 9))/(x^8 - 18*x^4 + 81)
```

Sympy [A] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.71

$$\int \frac{d+ex^2}{(9-x^4)^{5/2}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{2i\pi}}{9}\right)}{972\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{2i\pi}}{9}\right)}{972\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(-x**4+9)**(5/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), x**4*exp_polar(2*I*pi)/9)/(972*ga
mma(5/4)) + e*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), x**4*exp_polar(2*I
*pi)/9)/(972*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{(9-x^4)^{5/2}}\,dx = \int \frac{ex^2+d}{(-x^4+9)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-x^4+9)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(-x^4 + 9)^(5/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(9-x^4)^{5/2}}\,dx = \int \frac{ex^2+d}{(-x^4+9)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-x^4+9)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(-x^4 + 9)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(9-x^4)^{5/2}}\,dx=\int \frac{e\,x^2+d}{(9-x^4)^{5/2}}\,dx$$

input
```
int((d + e*x^2)/(9 - x^4)^(5/2),x)
```

output
```
int((d + e*x^2)/(9 - x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(9-x^4)^{5/2}}\,dx = -\left(\int \frac{\sqrt{-x^4+9}}{x^{12}-27x^8+243x^4-729}dx\right)d-\left(\int \frac{\sqrt{-x^4+9}\,x^2}{x^{12}-27x^8+243x^4-729}dx\right)e$$

input
```
int((e*x^2+d)/(-x^4+9)^(5/2),x)
```

output
```
 - (int(sqrt( - x**4 + 9)/(x**12 - 27*x**8 + 243*x**4 - 729),x)*d + int((s
qrt( - x**4 + 9)*x**2)/(x**12 - 27*x**8 + 243*x**4 - 729),x)*e)
```

3.226 $\int (1+bx^2)(1-b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int (1+bx^2)(1-b^2x^4)^{5/2}\,dx = \frac{4x(195+77bx^2)\sqrt{1-b^2x^4}}{3003} + \frac{10x(117+77bx^2)(1-b^2x^4)^{3/2}}{9009} + \frac{1}{143}x(13+11bx^2)(1-b^2x^4)^{5/2} + \frac{8E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{39\sqrt{b}} + \frac{944\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{3003\sqrt{b}}$$

output
```
4/3003*x*(77*b*x^2+195)*(-b^2*x^4+1)^(1/2)+10/9009*x*(77*b*x^2+117)*(-b^2*
x^4+1)^(3/2)+1/143*x*(11*b*x^2+13)*(-b^2*x^4+1)^(5/2)+8/39*EllipticE(b^(1/
2)*x,I)/b^(1/2)+944/3003*EllipticF(b^(1/2)*x,I)/b^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.38

$$\int (1+bx^2)(1-b^2x^4)^{5/2}\,dx = x\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},\frac{5}{4},b^2x^4\right) + \frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)$$

input
```
Integrate[(1 + b*x^2)*(1 - b^2*x^4)^(5/2),x]
```

output
```
x*Hypergeometric2F1[-5/2, 1/4, 5/4, b^2*x^4] + (b*x^3*Hypergeometric2F1[-5
/2, 3/4, 7/4, b^2*x^4])/3
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.88, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {1388, 318, 27, 403, 25, 27, 403, 27, 403, 27, 403, 27, 403, 27, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2+1\right)\left(1-b^2x^4\right)^{5/2}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}dx$$

$$\Big\downarrow \text{318}$$

$$-\frac{\int -2b\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}\left(12bx^2+7\right)dx}{13b}-\frac{1}{13}x\left(bx^2+1\right)^{5/2}\left(1-bx^2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{13}\int \left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}\left(12bx^2+7\right)dx-\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\Big\downarrow \text{403}$$

$$\frac{2}{13}\left(-\frac{\int -b\left(1-bx^2\right)^{5/2}\sqrt{bx^2+1}\left(161bx^2+89\right)dx}{11b}-\frac{12}{11}x\left(bx^2+1\right)^{3/2}\left(1-bx^2\right)^{7/2}\right)-\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{13}\left(\frac{\int b\left(1-bx^2\right)^{5/2}\sqrt{bx^2+1}\left(161bx^2+89\right)dx}{11b}-\frac{12}{11}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{3/2}\right)-\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\big\downarrow \; 27$$

$$\frac{2}{13}\left(\frac{1}{11}\int\left(1-bx^2\right)^{5/2}\sqrt{bx^2+1}\left(161bx^2+89\right)dx-\frac{12}{11}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{3/2}\right)-\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\big\downarrow \; 403$$

$$\frac{2}{13}\left(\frac{1}{11}\left(-\frac{\int -\frac{2b\left(1-bx^2\right)^{5/2}\left(642bx^2+481\right)}{\sqrt{bx^2+1}}dx}{9b}-\frac{161}{9}x\sqrt{bx^2+1}\left(1-bx^2\right)^{7/2}\right)-\frac{12}{11}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{3/2}\right)-\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\big\downarrow \; 27$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\int\frac{\left(1-bx^2\right)^{5/2}\left(642bx^2+481\right)}{\sqrt{bx^2+1}}dx-\frac{161}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}\Bigg)-\frac{12}{11}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{3/2}\Bigg)$$

$$\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\big\downarrow \; 403$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{\int\frac{5b\left(1-bx^2\right)^{3/2}\left(739bx^2+545\right)}{\sqrt{bx^2+1}}dx}{7b}+\frac{642}{7}x\sqrt{bx^2+1}\left(1-bx^2\right)^{5/2}\Bigg)-\frac{161}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}\Bigg)-\frac{12}{11}$$

$$\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\big\downarrow \; 27$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\int\frac{\left(1-bx^2\right)^{3/2}\left(739bx^2+545\right)}{\sqrt{bx^2+1}}dx+\frac{642}{7}x\sqrt{bx^2+1}\left(1-bx^2\right)^{5/2}\Bigg)-\frac{161}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}$$

$$\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\big\downarrow \; 403$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{\int \frac{6b\sqrt{1-bx^2}(408bx^2+331)}{\sqrt{bx^2+1}}dx}{5b}+\frac{739}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\Bigg)+\frac{642}{7}x\sqrt{bx^2+1}\left(1-bx^2\right)^{5/2}\Bigg)-$$

$$\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\Big\downarrow\ 27$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{6}{5}\int\frac{\sqrt{1-bx^2}(408bx^2+331)}{\sqrt{bx^2+1}}dx+\frac{739}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\Bigg)+\frac{642}{7}x\sqrt{bx^2+1}\left(1-bx^2\right)^{5/2}$$

$$\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\Big\downarrow\ 403$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{6}{5}\Bigg(\frac{\int\frac{3b(77bx^2+195)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{3b}+136x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{739}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\Bigg)+\frac{642}{7}x$$

$$\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\Big\downarrow\ 27$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{6}{5}\Bigg(\int\frac{77bx^2+195}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+136x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{739}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\Bigg)+\frac{642}{7}$$

$$\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\Big\downarrow\ 399$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{6}{5}\Bigg(118\int\frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+77\int\frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx+136x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{739}{5}x$$

$$\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\Big\downarrow\ 284$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{6}{5}\Bigg(118\int\frac{1}{\sqrt{1-b^2x^4}}dx+77\int\frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx+136x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{739}{5}x\sqrt{bx^2+1}(1$$

$$\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\Big\downarrow\ 327$$

$$\frac{2}{13}\left(\frac{1}{11}\left(\frac{2}{9}\left(\frac{5}{7}\left(\frac{6}{5}\left(118\int\frac{1}{\sqrt{1-b^2x^4}}dx+\frac{77E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}+136x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)+\frac{739}{5}x\sqrt{b}\right.\right.\right.\right.\right.$$

$$\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

$$\Big\downarrow \text{762}$$

$$\frac{2}{13}\left(\frac{1}{11}\left(\frac{2}{9}\left(\frac{5}{7}\left(\frac{6}{5}\left(\frac{118\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}+\frac{77E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}+136x\sqrt{1-bx^2}\sqrt{bx^2+1}\right.\right.\right.\right.\right.\right.$$

$$\frac{1}{13}x\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}$$

input
```
Int[(1 + b*x^2)*(1 - b^2*x^4)^(5/2),x]
```

output
```
-1/13*(x*(1 - b*x^2)^(7/2)*(1 + b*x^2)^(5/2)) + (2*((-12*x*(1 - b*x^2)^(7/
2)*(1 + b*x^2)^(3/2))/11 + ((-161*x*(1 - b*x^2)^(7/2)*Sqrt[1 + b*x^2])/9 +
 (2*((642*x*(1 - b*x^2)^(5/2)*Sqrt[1 + b*x^2])/7 + (5*((739*x*(1 - b*x^2)^
(3/2)*Sqrt[1 + b*x^2])/5 + (6*(136*x*Sqrt[1 - b*x^2]*Sqrt[1 + b*x^2] + (77
*EllipticE[ArcSin[Sqrt[b]*x], -1])/Sqrt[b] + (118*EllipticF[ArcSin[Sqrt[b]
*x], -1])/Sqrt[b]))/5))/7))/9)/11))/13
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.68 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.30

method	result
meijerg	$x \operatorname{hypergeom}\left(\left[-\frac{5}{2}, \frac{1}{4}\right], \left[\frac{5}{4}\right], b^2x^4\right) + \frac{b\,x^3 \operatorname{hypergeom}\left(\left[-\frac{5}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3}$
risch	$-\frac{x\left(693x^{10}b^5+819b^4x^8-2156b^3x^6-2808b^2x^4+2387b\,x^2+4329\right)\left(b^2x^4-1\right)}{9009\sqrt{-b^2x^4+1}} + \frac{40\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)}{77\sqrt{b}\,\sqrt{-b^2x^4+1}} - 8\sqrt{-}$
elliptic	$\frac{b^5x^{11}\sqrt{-b^2x^4+1}}{13} + \frac{b^4x^9\sqrt{-b^2x^4+1}}{11} - \frac{28b^3x^7\sqrt{-b^2x^4+1}}{117} - \frac{24b^2x^5\sqrt{-b^2x^4+1}}{77} + \frac{31b\,x^3\sqrt{-b^2x^4+1}}{117} + \frac{37x\sqrt{-b^2x^4+1}}{77} +$
default	$\frac{b^4x^9\sqrt{-b^2x^4+1}}{11} - \frac{24b^2x^5\sqrt{-b^2x^4+1}}{77} + \frac{37x\sqrt{-b^2x^4+1}}{77} + \frac{40\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)}{77\sqrt{b}\,\sqrt{-b^2x^4+1}} + b\left(\frac{b^4x^{11}\sqrt{-b^2x^4+1}}{13}\right.$

input
```
int((b*x^2+1)*(-b^2*x^4+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([-5/2,1/4],[5/4],b^2*x^4)+1/3*b*x^3*hypergeom([-5/2,3/4],[7/4]
,b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04

$$\int \left(1+bx^2\right)\left(1-b^2x^4\right)^{5/2}\,dx = \frac{\frac{24\sqrt{-b^2}(195\,b+77)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1\right)}{\sqrt{b}} - \frac{1848\sqrt{-b^2}xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1\right)}{\sqrt{b}} + (693\,b^7x^{12} + 819\,b^6x^{10} - 215}{9009\,b^2x}$$

input
```
integrate((b*x^2+1)*(-b^2*x^4+1)^(5/2),x, algorithm="fricas")
```

output
```
1/9009*(24*sqrt(-b^2)*(195*b + 77)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)
/sqrt(b) - 1848*sqrt(-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b)
 + (693*b^7*x^12 + 819*b^6*x^10 - 2156*b^5*x^8 - 2808*b^4*x^6 + 2387*b^3*x
^4 + 4329*b^2*x^2 - 1848*b)*sqrt(-b^2*x^4 + 1))/(b^2*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(107) = 214$.

Time = 2.51 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.97

$$\int (1+bx^2)\left(1-b^2x^4\right)^{5/2}\,dx = \frac{b^5x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{15}{4}\right)} + \frac{b^4x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{13}{4}\right)} - \frac{b^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{2\Gamma\left(\frac{11}{4}\right)} - \frac{b^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((b*x**2+1)*(-b**2*x**4+1)**(5/2),x)
```

output

```
b**5*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b**2*x**4*exp_polar(2*
I*pi))/(4*gamma(15/4)) + b**4*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,),
b**2*x**4*exp_polar(2*I*pi))/(4*gamma(13/4)) - b**3*x**7*gamma(7/4)*hyper(
(-1/2, 7/4), (11/4,), b**2*x**4*exp_polar(2*I*pi))/(2*gamma(11/4)) - b**2*
x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b**2*x**4*exp_polar(2*I*pi))/(2
*gamma(9/4)) + b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4*exp_
polar(2*I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*
*2*x**4*exp_polar(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1+bx^2\right)\left(1-b^2x^4\right)^{5/2}\,dx=\int \left(-b^2x^4+1\right)^{\frac{5}{2}}\left(bx^2+1\right)dx$$

input
```
integrate((b*x^2+1)*(-b^2*x^4+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-b^2*x^4 + 1)^(5/2)*(b*x^2 + 1), x)
```

Giac [F]

$$\int \left(1+bx^2\right)\left(1-b^2x^4\right)^{5/2}\,dx=\int \left(-b^2x^4+1\right)^{\frac{5}{2}}\left(bx^2+1\right)dx$$

input
```
integrate((b*x^2+1)*(-b^2*x^4+1)^(5/2),x, algorithm="giac")
```

output
```
integrate((-b^2*x^4 + 1)^(5/2)*(b*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+bx^2\right)\left(1-b^2x^4\right)^{5/2}\,dx=\int \left(1-b^2\,x^4\right)^{5/2}\,\left(b\,x^2+1\right)\,dx$$

input
```
int((1 - b^2*x^4)^(5/2)*(b*x^2 + 1),x)
```

output
```
int((1 - b^2*x^4)^(5/2)*(b*x^2 + 1), x)
```

Reduce [F]

$$\int \left(1+b x^2\right) \left(1-b^2 x^4\right)^{5/2} \, dx = \frac{\sqrt{-b^2x^4+1}\,b^5x^{11}}{13} + \frac{\sqrt{-b^2x^4+1}\,b^4x^9}{11} - \frac{28\sqrt{-b^2x^4+1}\,b^3x^7}{117} - \frac{24\sqrt{-b^2x^4+1}\,b^2x^5}{77} + \frac{31\sqrt{-b^2x^4+1}\,b\,x^3}{117} + \frac{37\sqrt{-b^2x^4+1}\,x}{77} - \frac{40\left(\int \frac{\sqrt{-b^2x^4+1}}{b^2x^4-1}dx\right)}{77} - \frac{8\left(\int \frac{\sqrt{-b^2x^4+1}\,x^2}{b^2x^4-1}dx\right)b}{39}$$

input
```
int((b*x^2+1)*(-b^2*x^4+1)^(5/2),x)
```

output
```
(693*sqrt( - b**2*x**4 + 1)*b**5*x**11 + 819*sqrt( - b**2*x**4 + 1)*b**4*x
**9 - 2156*sqrt( - b**2*x**4 + 1)*b**3*x**7 - 2808*sqrt( - b**2*x**4 + 1)*
b**2*x**5 + 2387*sqrt( - b**2*x**4 + 1)*b*x**3 + 4329*sqrt( - b**2*x**4 +
1)*x - 4680*int(sqrt( - b**2*x**4 + 1)/(b**2*x**4 - 1),x) - 1848*int((sqrt
( - b**2*x**4 + 1)*x**2)/(b**2*x**4 - 1),x)*b)/9009
```

3.227 $\int \left(1+bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 93

$$\begin{aligned}\int \left(1+bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx &= \frac{2}{105}x\left(15+7bx^2\right)\sqrt{1-b^2x^4}\\ &+ \frac{1}{63}x\left(9+7bx^2\right)\left(1-b^2x^4\right)^{3/2} + \frac{4E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{15\sqrt{b}}\\ &+ \frac{32\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{105\sqrt{b}}\end{aligned}$$

output

```
2/105*x*(7*b*x^2+15)*(-b^2*x^4+1)^(1/2)+1/63*x*(7*b*x^2+9)*(-b^2*x^4+1)^(3
/2)+4/15*EllipticE(b^(1/2)*x,I)/b^(1/2)+32/105*EllipticF(b^(1/2)*x,I)/b^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

$$\int (1+bx^2)\left(1-b^2x^4\right)^{3/2}\,dx = x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},b^2x^4\right) + \frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)$$

input
```
Integrate[(1 + b*x^2)*(1 - b^2*x^4)^(3/2),x]
```

output
```
x*Hypergeometric2F1[-3/2, 1/4, 5/4, b^2*x^4] + (b*x^3*Hypergeometric2F1[-3
/2, 3/4, 7/4, b^2*x^4])/3
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.72, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {1388, 318, 27, 403, 25, 27, 403, 27, 403, 27, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^2+1)\left(1-b^2x^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1388}$$

$$\int \left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{5/2}dx$$

$$\Big\downarrow \text{318}$$

$$-\frac{\int -2b\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}(8bx^2+5)\,dx}{9b}-\frac{1}{9}x\left(bx^2+1\right)^{3/2}\left(1-bx^2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\int \left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}(8bx^2+5)\,dx-\frac{1}{9}x\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \text{403}$$

$$\frac{2}{9}\left(-\frac{\int -\frac{b\left(1-bx^2\right)^{3/2}\left(59bx^2+43\right)}{\sqrt{bx^2+1}}dx}{7b}-\frac{8}{7}x\sqrt{bx^2+1}\left(1-bx^2\right)^{5/2}\right)-\frac{1}{9}x\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{9}\left(\frac{\int \frac{b\left(1-bx^2\right)^{3/2}\left(59bx^2+43\right)}{\sqrt{bx^2+1}}dx}{7b}-\frac{8}{7}x\left(1-bx^2\right)^{5/2}\sqrt{bx^2+1}\right)-\frac{1}{9}x\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\left(\frac{1}{7}\int \frac{\left(1-bx^2\right)^{3/2}\left(59bx^2+43\right)}{\sqrt{bx^2+1}}dx-\frac{8}{7}x\left(1-bx^2\right)^{5/2}\sqrt{bx^2+1}\right)-\frac{1}{9}x\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \text{403}$$

$$\frac{2}{9}\left(\frac{1}{7}\left(\frac{\int \frac{6b\sqrt{1-bx^2}\left(33bx^2+26\right)}{\sqrt{bx^2+1}}dx}{5b}+\frac{59}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\right)-\frac{8}{7}x\left(1-bx^2\right)^{5/2}\sqrt{bx^2+1}\right)-\frac{1}{9}x\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\left(\frac{1}{7}\left(\frac{6}{5}\int \frac{\sqrt{1-bx^2}\left(33bx^2+26\right)}{\sqrt{bx^2+1}}dx+\frac{59}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\right)-\frac{8}{7}x\left(1-bx^2\right)^{5/2}\sqrt{bx^2+1}\right)-\frac{1}{9}x\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \text{403}$$

$$\frac{2}{9}\left(\frac{1}{7}\left(\frac{6}{5}\left(\frac{\int \frac{3b\left(7bx^2+15\right)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{3b}+11x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)+\frac{59}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\right)-\frac{8}{7}x\left(1-bx^2\right)^{5/2}\sqrt{b}\right.$$

$$\frac{1}{9}x\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\Bigg(\frac{1}{7}\Bigg(\frac{6}{5}\Bigg(\int \frac{7bx^2+15}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+11x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{59}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\Bigg)-\frac{8}{7}x\left(1-bx^2\right)^{5/2}$$
$$\frac{1}{9}x\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \; 399$$

$$\frac{2}{9}\Bigg(\frac{1}{7}\Bigg(\frac{6}{5}\Bigg(8\int \frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+7\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx+11x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{59}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)$$
$$\frac{1}{9}x\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \; 284$$

$$\frac{2}{9}\Bigg(\frac{1}{7}\Bigg(\frac{6}{5}\Bigg(8\int \frac{1}{\sqrt{1-b^2x^4}}dx+7\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx+11x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{59}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\Bigg)-\frac{8}{7}$$
$$\frac{1}{9}x\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \; 327$$

$$\frac{2}{9}\Bigg(\frac{1}{7}\Bigg(\frac{6}{5}\Bigg(8\int \frac{1}{\sqrt{1-b^2x^4}}dx+\frac{7E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}+11x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{59}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)$$
$$\frac{1}{9}x\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \; 762$$

$$\frac{2}{9}\Bigg(\frac{1}{7}\Bigg(\frac{6}{5}\Bigg(\frac{8\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}+\frac{7E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}+11x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{59}{5}x$$
$$\frac{1}{9}x\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}$$

input

```
Int[(1 + b*x^2)*(1 - b^2*x^4)^(3/2),x]
```

output

```
-1/9*(x*(1 - b*x^2)^(5/2)*(1 + b*x^2)^(3/2)) + (2*((-8*x*(1 - b*x^2)^(5/2)
*Sqrt[1 + b*x^2])/7 + ((59*x*(1 - b*x^2)^(3/2)*Sqrt[1 + b*x^2])/5 + (6*(11
*x*Sqrt[1 - b*x^2]*Sqrt[1 + b*x^2] + (7*EllipticE[ArcSin[Sqrt[b]*x], -1])/
Sqrt[b] + (8*EllipticF[ArcSin[Sqrt[b]*x], -1])/Sqrt[b]))/5)/7))/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.65 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.39

method	result
meijerg	$x \,\text{hypergeom}\left(\left[-\frac{3}{2}, \frac{1}{4}\right], \left[\frac{5}{4}\right], b^{2} x^{4}\right)+\frac{b\,x^{3} \,\text{hypergeom}\left(\left[-\frac{3}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], b^{2} x^{4}\right)}{3}$
risch	$\frac{x \left(35 b^{3} x^{6}+45 b^{2} x^{4}-77 b\,x^{2}-135\right) \left(b^{2} x^{4}-1\right)}{315 \sqrt{-b^{2} x^{4}+1}}+\frac{4 \sqrt{-b\,x^{2}+1}\, \sqrt{b\,x^{2}+1}\, \text{EllipticF}\left(\sqrt{b}\, x , i\right)}{7 \sqrt{b}\, \sqrt{-b^{2} x^{4}+1}}-\frac{4 \sqrt{-b\,x^{2}+1}\, \sqrt{b\,x^{2}+1}\, \left(\text{EllipticF}\left(\sqrt{b}\, x\right.\right.}{15 \sqrt{b}\, \sqrt{-b^{2} x^{4}+}}$
elliptic	$-\frac{b^{3} x^{7} \sqrt{-b^{2} x^{4}+1}}{9}-\frac{b^{2} x^{5} \sqrt{-b^{2} x^{4}+1}}{7}+\frac{11 b\,x^{3} \sqrt{-b^{2} x^{4}+1}}{45}+\frac{3 x \sqrt{-b^{2} x^{4}+1}}{7}+\frac{4 \sqrt{-b\,x^{2}+1}\, \sqrt{b\,x^{2}+1}\, \text{EllipticF}\left(\sqrt{b}\, x , i\right)}{7 \sqrt{b}\, \sqrt{-b^{2} x^{4}+1}}-$
default	$-\frac{b^{2} x^{5} \sqrt{-b^{2} x^{4}+1}}{7}+\frac{3 x \sqrt{-b^{2} x^{4}+1}}{7}+\frac{4 \sqrt{-b\,x^{2}+1}\, \sqrt{b\,x^{2}+1}\, \text{EllipticF}\left(\sqrt{b}\, x , i\right)}{7 \sqrt{b}\, \sqrt{-b^{2} x^{4}+1}}+b \left(-\frac{b^{2} x^{7} \sqrt{-b^{2} x^{4}+1}}{9}+\frac{11 x^{3} \sqrt{-b^{2} x^{4}+1}}{45}\right.$

input
```
int((b*x^2+1)*(-b^2*x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([-3/2,1/4],[5/4],b^2*x^4)+1/3*b*x^3*hypergeom([-3/2,3/4],[7/4]
,b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.18

$$\int \left(1+bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx = \frac{\frac{12\sqrt{-b^2}(15\,b+7)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}} - \frac{84\sqrt{-b^2}xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}} - \left(35\,b^5x^8+45\,b^4x^6-77\,b^3x^4-\right.}{315\,b^2x}$$

input
```
integrate((b*x^2+1)*(-b^2*x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(12*sqrt(-b^2)*(15*b + 7)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sq
rt(b) - 84*sqrt(-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - (3
5*b^5*x^8 + 45*b^4*x^6 - 77*b^3*x^4 - 135*b^2*x^2 + 84*b)*sqrt(-b^2*x^4 +
1))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.67

$$\int \left(1+bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx = -\frac{b^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)} - \frac{b^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)*(-b**2*x**4+1)**(3/2),x)
```

output
```
-b**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b**2*x**4*exp_polar(2*I*
pi))/(4*gamma(11/4)) - b**2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b**
2*x**4*exp_polar(2*I*pi))/(4*gamma(9/4)) + b*x**3*gamma(3/4)*hyper((-1/2,
3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*h
yper((-1/2, 1/4), (5/4,), b**2*x**4*exp_polar(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1+bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx=\int \left(-b^2x^4+1\right)^{\frac{3}{2}}\left(bx^2+1\right)\,dx$$

input
```
integrate((b*x^2+1)*(-b^2*x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b^2*x^4 + 1)^(3/2)*(b*x^2 + 1), x)
```

Giac [F]

$$\int \left(1+bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx=\int \left(-b^2x^4+1\right)^{\frac{3}{2}}\left(bx^2+1\right)\,dx$$

input
```
integrate((b*x^2+1)*(-b^2*x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b^2*x^4 + 1)^(3/2)*(b*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+bx^2\right) \left(1-b^2x^4\right)^{3/2} \, dx=\int {\left(1-b^2\,x^4\right)}^{3/2}\,\left(b\,x^2+1\right) \,d x$$

input
```
int((1 - b^2*x^4)^(3/2)*(b*x^2 + 1),x)
```

output
```
int((1 - b^2*x^4)^(3/2)*(b*x^2 + 1), x)
```

Reduce [F]

$$\int \left(1+bx^2\right) \left(1-b^2x^4\right)^{3/2} \, dx=-\frac{\sqrt{-b^2x^4+1}\,b^3x^7}{9}-\frac{\sqrt{-b^2x^4+1}\,b^2x^5}{7}+\frac{11\sqrt{-b^2x^4+1}\,b\,x^3}{45}+\frac{3\sqrt{-b^2x^4+1}\,x}{7}-\frac{4\left(\int \frac{\sqrt{-b^2x^4+1}}{b^2x^4-1}dx\right)}{7}-\frac{4\left(\int \frac{\sqrt{-b^2x^4+1}\,x^2}{b^2x^4-1}dx\right) b}{15}$$

input
```
int((b*x^2+1)*(-b^2*x^4+1)^(3/2),x)
```

output
```
( - 35*sqrt( - b**2*x**4 + 1)*b**3*x**7 - 45*sqrt( - b**2*x**4 + 1)*b**2*x
**5 + 77*sqrt( - b**2*x**4 + 1)*b*x**3 + 135*sqrt( - b**2*x**4 + 1)*x - 18
0*int(sqrt( - b**2*x**4 + 1)/(b**2*x**4 - 1),x) - 84*int((sqrt( - b**2*x**
4 + 1)*x**2)/(b**2*x**4 - 1),x)*b)/315
```

3.228 $\int \left(1+bx^2\right) \sqrt{1-b^2 x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \left(1+bx^2\right) \sqrt{1-b^2x^4} \, dx = \frac{1}{15} x \left(5+3bx^2\right) \sqrt{1-b^2x^4} + \frac{2E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{5\sqrt{b}} + \frac{4 \operatorname{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{15\sqrt{b}}$$

output
```
1/15*x*(3*b*x^2+5)*(-b^2*x^4+1)^(1/2)+2/5*EllipticE(b^(1/2)*x,I)/b^(1/2)+4
/15*EllipticF(b^(1/2)*x,I)/b^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.62 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.68

$$\int \left(1+bx^2\right) \sqrt{1-b^2x^4} \, dx = x \operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},b^2x^4\right) + \frac{1}{3} b x^3 \operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)$$

input
```
Integrate[(1 + b*x^2)*Sqrt[1 - b^2*x^4],x]
```

output
```
x*Hypergeometric2F1[-1/2, 1/4, 5/4, b^2*x^4] + (b*x^3*Hypergeometric2F1[-1
/2, 3/4, 7/4, b^2*x^4])/3
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1388, 318, 27, 403, 27, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2+1\right) \sqrt{1-b^2x^4} \, dx$$

$$\Big\downarrow \text{1388}$$

$$\int \sqrt{1-bx^2} \left(bx^2+1\right)^{3/2} \, dx$$

$$\Big\downarrow \text{318}$$

$$-\frac{\int -\frac{2b\sqrt{1-bx^2}\left(4bx^2+3\right)}{\sqrt{bx^2+1}}dx}{5b}-\frac{1}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}\int \frac{\sqrt{1-bx^2}\left(4bx^2+3\right)}{\sqrt{bx^2+1}}dx-\frac{1}{5}x\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}$$

$$\Big\downarrow \text{403}$$

$$\frac{2}{5}\left(\frac{\int \frac{b\left(3bx^2+5\right)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{3b}+\frac{4}{3}x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)-\frac{1}{5}x\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{3bx^2+5}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+\frac{4}{3}x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)-\frac{1}{5}x\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}$$

$$\Big\downarrow \text{399}$$

$$\frac{2}{5}\left(\frac{1}{3}\left(2\int \frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+3\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)+\frac{4}{3}x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)-\frac{1}{5}x\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}$$

$$\Big\downarrow \; 284$$

$$\frac{2}{5}\left(\frac{1}{3}\left(2\int \frac{1}{\sqrt{1-b^2x^4}}dx+3\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)+\frac{4}{3}x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)-\frac{1}{5}x\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}$$

$$\Big\downarrow \; 327$$

$$\frac{2}{5}\left(\frac{1}{3}\left(2\int \frac{1}{\sqrt{1-b^2x^4}}dx+\frac{3E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)+\frac{4}{3}x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)-\frac{1}{5}x\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}$$

$$\Big\downarrow \; 762$$

$$\frac{2}{5}\left(\frac{1}{3}\left(\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}+\frac{3E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)+\frac{4}{3}x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)-\frac{1}{5}x\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}$$

input

```
Int[(1 + b*x^2)*Sqrt[1 - b^2*x^4],x]
```

output

```
-1/5*(x*(1 - b*x^2)^(3/2)*Sqrt[1 + b*x^2]) + (2*((4*x*Sqrt[1 - b*x^2]*Sqrt
[1 + b*x^2])/3 + ((3*EllipticE[ArcSin[Sqrt[b]*x], -1])/Sqrt[b] + (2*Ellipt
icF[ArcSin[Sqrt[b]*x], -1])/Sqrt[b])/3))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

method	result
meijerg	$x \operatorname{hypergeom}\left(\left[-\frac{1}{2}, \frac{1}{4}\right], \left[\frac{5}{4}\right], b^{2} x^{4}\right)+\frac{b\,x^{3} \operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^{2} x^{4}\right)}{3}$
risch	$-\frac{x \left(3 b\,x^{2}+5\right) \left(b^{2} x^{4}-1\right)}{15 \sqrt{-b^{2} x^{4}+1}}+\frac{2 \sqrt{-b\,x^{2}+1}\, \sqrt{b\,x^{2}+1}\, \operatorname{EllipticF}\left(\sqrt{b}\, x ,i\right)}{3 \sqrt{b}\, \sqrt{-b^{2} x^{4}+1}}-\frac{2 \sqrt{-b\,x^{2}+1}\, \sqrt{b\,x^{2}+1}\, (\operatorname{EllipticF}\left(\sqrt{b}\, x ,i\right)-\operatorname{EllipticE}(\sqrt{b}}{5 \sqrt{b}\, \sqrt{-b^{2} x^{4}+1}}$
elliptic	$\frac{b\,x^{3} \sqrt{-b^{2} x^{4}+1}}{5}+\frac{x \sqrt{-b^{2} x^{4}+1}}{3}+\frac{2 \sqrt{-b\,x^{2}+1}\, \sqrt{b\,x^{2}+1}\, \operatorname{EllipticF}\left(\sqrt{b}\, x ,i\right)}{3 \sqrt{b}\, \sqrt{-b^{2} x^{4}+1}}-\frac{2 \sqrt{-b\,x^{2}+1}\, \sqrt{b\,x^{2}+1}\, (\operatorname{EllipticF}\left(\sqrt{b}\, x ,i\right)-\operatorname{Ellip}}{5 \sqrt{b}\, \sqrt{-b^{2} x^{4}+1}}$
default	$\frac{x \sqrt{-b^{2} x^{4}+1}}{3}+\frac{2 \sqrt{-b\,x^{2}+1}\, \sqrt{b\,x^{2}+1}\, \operatorname{EllipticF}\left(\sqrt{b}\, x ,i\right)}{3 \sqrt{b}\, \sqrt{-b^{2} x^{4}+1}}+b \left(\frac{x^{3} \sqrt{-b^{2} x^{4}+1}}{5}-\frac{2 \sqrt{-b\,x^{2}+1}\, \sqrt{b\,x^{2}+1}\, (\operatorname{EllipticF}\left(\sqrt{b}\, x ,i\right)-\operatorname{Ell}}{5 b^{\frac{3}{2}} \sqrt{-b^{2} x^{4}+1}}\right.$

input
```
int((b*x^2+1)*(-b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([-1/2,1/4],[5/4],b^2*x^4)+1/3*b*x^3*hypergeom([-1/2,3/4],[7/4]
,b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \left(1+bx^2\right)\sqrt{1-b^2x^4}\,dx = \frac{\frac{2\sqrt{-b^2}(5\,b+3)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}} - \frac{6\sqrt{-b^2}xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}} + (3\,b^3x^4+5\,b^2x^2-6\,b)\sqrt{-b^2x^4+1}}{15\,b^2x}$$

input
```
integrate((b*x^2+1)*(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(2*sqrt(-b^2)*(5*b + 3)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(
b) - 6*sqrt(-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + (3*b^3
*x^4 + 5*b^2*x^2 - 6*b)*sqrt(-b^2*x^4 + 1))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \left(1+bx^2\right)\sqrt{1-b^2x^4}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)*(-b**2*x**4+1)**(1/2),x)
```

output
```
b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/
(4*gamma(7/4)) + x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b**2*x**4*exp_pol
ar(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int (1+bx^2)\sqrt{1-b^2x^4}\,dx=\int \sqrt{-b^2x^4+1}\,(bx^2+1)\,dx$$

input
```
integrate((b*x^2+1)*(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b^2*x^4 + 1)*(b*x^2 + 1), x)
```

Giac [F]

$$\int (1+bx^2)\sqrt{1-b^2x^4}\,dx=\int \sqrt{-b^2x^4+1}\,(bx^2+1)\,dx$$

input
```
integrate((b*x^2+1)*(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b^2*x^4 + 1)*(b*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+bx^2)\sqrt{1-b^2x^4}\,dx=\int \sqrt{1-b^2\,x^4}\,\left(b\,x^2+1\right)\,dx$$

input
```
int((1 - b^2*x^4)^(1/2)*(b*x^2 + 1),x)
```

output
```
int((1 - b^2*x^4)^(1/2)*(b*x^2 + 1), x)
```

Reduce [F]

$$\int (1+bx^2)\sqrt{1-b^2x^4}\,dx = \frac{\sqrt{-b^2x^4+1}\,b\,x^3}{5} + \frac{\sqrt{-b^2x^4+1}\,x}{3} - \frac{2\left(\int \frac{\sqrt{-b^2x^4+1}}{b^2x^4-1}dx\right)}{3} - \frac{2\left(\int \frac{\sqrt{-b^2x^4+1}\,x^2}{b^2x^4-1}dx\right)b}{5}$$

input

```
int((b*x^2+1)*(-b^2*x^4+1)^(1/2),x)
```

output

```
(3*sqrt( - b**2*x**4 + 1)*b*x**3 + 5*sqrt( - b**2*x**4 + 1)*x - 10*int(sqr
t( - b**2*x**4 + 1)/(b**2*x**4 - 1),x) - 6*int((sqrt( - b**2*x**4 + 1)*x**
2)/(b**2*x**4 - 1),x)*b)/15
```

3.229 $\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx=\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

output
```
EllipticE(b^(1/2)*x,I)/b^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

$$\begin{aligned}\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx &= x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)\\ &\quad+\frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)\end{aligned}$$

input
```
Integrate[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + (b*x^3*Hypergeometric2F1[1/2
, 3/4, 7/4, b^2*x^4])/3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{1388, 327\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\sqrt{1-b^2x^4}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx$$

$$\Big\downarrow \text{327}$$

$$\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

input
```
Int[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]
```

output
```
EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.95 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

method	result	size
meijerg	$\frac{b\,x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^{2} x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], b^{2} x^{4}\right)$	36
default	$\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2} x^{4}+1}}-\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)-\operatorname{EllipticE}\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^{2} x^{4}+1}}$	100
elliptic	$\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2} x^{4}+1}}-\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)-\operatorname{EllipticE}\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^{2} x^{4}+1}}$	100

input
```
int((b*x^2+1)/(-b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b*x^3*hypergeom([1/2,3/4],[7/4],b^2*x^4)+x*hypergeom([1/4,1/2],[5/4],b
^2*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(11) = 22.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.44

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx=\frac{\frac{\sqrt{-b^2}(b+1)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\mid -1\right)}{\sqrt{b}}-\frac{\sqrt{-b^2}xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\mid -1\right)}{\sqrt{b}}-\sqrt{-b^2x^4+1}b}{b^2x}$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b^2)*(b + 1)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - sqrt
(-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - sqrt(-b^2*x^4 + 1
)*b)/(b^2*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(12) = 24$.

Time = 0.81 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.38

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

output
```
b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(
4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar
(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{-b^2x^4+1}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{-b^2x^4+1}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \int \frac{b\,x^2+1}{\sqrt{1-b^2\,x^4}}\,dx$$

input
```
int((b*x^2 + 1)/(1 - b^2*x^4)^(1/2),x)
```

output
```
int((b*x^2 + 1)/(1 - b^2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = -\left(\int \frac{\sqrt{-b^2x^4+1}}{b\,x^2-1}dx\right)$$

input
```
int((b*x^2+1)/(-b^2*x^4+1)^(1/2),x)
```

output
```
- int(sqrt( - b**2*x**4 + 1)/(b*x**2 - 1),x)
```

3.230 $\int \frac{1+bx^2}{\left(1-b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{1+bx^2}{\left(1-b^2x^4\right)^{3/2}}\,dx=\frac{x(1+bx^2)}{2\sqrt{1-b^2x^4}}-\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{2\sqrt{b}}+\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}$$

output
```
1/2*x*(b*x^2+1)/(-b^2*x^4+1)^(1/2)-1/2*EllipticE(b^(1/2)*x,I)/b^(1/2)+Elli
pticF(b^(1/2)*x,I)/b^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{1+bx^2}{\left(1-b^2x^4\right)^{3/2}}\,dx=\frac{1}{6}x\left(\frac{3}{\sqrt{1-b^2x^4}}+3\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)+2bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},b^2x^4\right)\right)$$

input
```
Integrate[(1 + b*x^2)/(1 - b^2*x^4)^(3/2),x]
```

output
```
(x*(3/Sqrt[1 - b^2*x^4] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + 2*
b*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, b^2*x^4]))/6
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1388, 316, 27, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{(1-b^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{(1-bx^2)^{3/2}\sqrt{bx^2+1}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{b\sqrt{1-bx^2}}{\sqrt{bx^2+1}}dx}{2b}+\frac{x\sqrt{bx^2+1}}{2\sqrt{1-bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sqrt{1-bx^2}}{\sqrt{bx^2+1}}dx+\frac{x\sqrt{bx^2+1}}{2\sqrt{1-bx^2}}$$

$$\Big\downarrow \text{326}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx-\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)+\frac{x\sqrt{bx^2+1}}{2\sqrt{1-bx^2}}$$

$$\Big\downarrow \text{284}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-b^2x^4}}dx-\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)+\frac{x\sqrt{bx^2+1}}{2\sqrt{1-bx^2}}$$

$$\Big\downarrow 327$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-b^2x^4}}dx-\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)+\frac{x\sqrt{bx^2+1}}{2\sqrt{1-bx^2}}$$

$$\Big\downarrow 762$$

$$\frac{1}{2}\left(\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}-\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)+\frac{x\sqrt{bx^2+1}}{2\sqrt{1-bx^2}}$$

input
```
Int[(1 + b*x^2)/(1 - b^2*x^4)^(3/2),x]
```

output
```
(x*Sqrt[1 + b*x^2])/(2*Sqrt[1 - b*x^2]) + (-(EllipticE[ArcSin[Sqrt[b]*x],
-1]/Sqrt[b]) + (2*EllipticF[ArcSin[Sqrt[b]*x], -1])/Sqrt[b])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.58

method	result
meijerg	$x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right], \left[\frac{5}{4}\right], b^2 x^4\right)+\frac{b\,x^3 \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{7}{4}\right],b^2 x^4\right)}{3}$
elliptic	$-\frac{\left(-b^2 x^2-b\right)x}{2b\sqrt{\left(x^2-\frac{1}{b}\right)\left(-b^2 x^2-b\right)}}+\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)}{2\sqrt{b}\,\sqrt{-b^2 x^4+1}}+\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\Big(\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)-\operatorname{EllipticE}\Big(\sqrt{b}}{2\sqrt{b}\,\sqrt{-b^2 x^4+1}}$
default	$\frac{x}{2\sqrt{-\left(x^4-\frac{1}{b^2}\right)b^2}}+\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)}{2\sqrt{b}\,\sqrt{-b^2 x^4+1}}+b\Bigg(\frac{x^3}{2\sqrt{-\left(x^4-\frac{1}{b^2}\right)b^2}}+\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\Big(\operatorname{EllipticF}\Big(\sqrt{b}\,x,i}{2b^{\frac{3}{2}}\sqrt{-b^2 x^4+1}}$

input
```
int((b*x^2+1)/(-b^2*x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,3/2],[5/4],b^2*x^4)+1/3*b*x^3*hypergeom([3/4,3/2],[7/4],b
^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31

$$\int \frac{1+bx^2}{\left(1-b^2x^4\right)^{3/2}}\,dx = -\frac{\sqrt{-b^2x^4+1}bx + (b^2x^2-b)\sqrt{b}E(\arcsin\left(\sqrt{b}x\right)\,|-1) - ((b^2+b)x^2-b-1)\sqrt{b}F(\arcsin\left(\sqrt{b}x\right)\,|-1)}{2\,(b^2x^2-b)}$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-b^2*x^4 + 1)*b*x + (b^2*x^2 - b)*sqrt(b)*elliptic_e(arcsin(sqr
t(b)*x), -1) - ((b^2 + b)*x^2 - b - 1)*sqrt(b)*elliptic_f(arcsin(sqrt(b)*x
), -1))/(b^2*x^2 - b)
```

Sympy [A] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{1+bx^2}{\left(1-b^2x^4\right)^{3/2}}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(-b**2*x**4+1)**(3/2),x)
```

output
```
b*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(
4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b**2*x**4*exp_polar
(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{(1-b^2x^4)^{3/2}}\,dx = \int \frac{bx^2+1}{(-b^2x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{1+bx^2}{(1-b^2x^4)^{3/2}}\,dx = \int \frac{bx^2+1}{(-b^2x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{(1-b^2x^4)^{3/2}}\,dx = \int \frac{b\,x^2+1}{(1-b^2\,x^4)^{3/2}}\,dx$$

input
```
int((b*x^2 + 1)/(1 - b^2*x^4)^(3/2),x)
```

output
```
int((b*x^2 + 1)/(1 - b^2*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{(1-b^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{-b^2x^4+1}}{b^3x^6-b^2x^4-b\,x^2+1}dx$$

input
```
int((b*x^2+1)/(-b^2*x^4+1)^(3/2),x)
```

output
```
int(sqrt( - b**2*x**4 + 1)/(b**3*x**6 - b**2*x**4 - b*x**2 + 1),x)
```

3.231 $\int \frac{1+bx^2}{(1-b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{1+bx^2}{(1-b^2x^4)^{5/2}}\,dx = \frac{x(1+bx^2)}{6\,(1-b^2x^4)^{3/2}} + \frac{x(5+3bx^2)}{12\sqrt{1-b^2x^4}} - \frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{4\sqrt{b}} + \frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{3\sqrt{b}}$$

output
```
1/6*x*(b*x^2+1)/(-b^2*x^4+1)^(3/2)+1/12*x*(3*b*x^2+5)/(-b^2*x^4+1)^(1/2)-1
/4*EllipticE(b^(1/2)*x,I)/b^(1/2)+2/3*EllipticF(b^(1/2)*x,I)/b^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

$$\int \frac{1+bx^2}{(1-b^2x^4)^{5/2}}\,dx = \frac{1}{12}x\left(\frac{7-5b^2x^4}{(1-b^2x^4)^{3/2}} + 5\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right) + 4bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},b^2x^4\right)\right)$$

input
```
Integrate[(1 + b*x^2)/(1 - b^2*x^4)^(5/2),x]
```

output
```
(x*((7 - 5*b^2*x^4)/(1 - b^2*x^4)^(3/2) + 5*Hypergeometric2F1[1/4, 1/2, 5/
4, b^2*x^4] + 4*b*x^2*Hypergeometric2F1[3/4, 5/2, 7/4, b^2*x^4]))/12
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.38, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {1388, 316, 27, 402, 27, 402, 25, 27, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\left(1-b^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{b\left(3bx^2+5\right)}{\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{3/2}}dx}{6b}+\frac{x}{6\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \frac{3bx^2+5}{\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{3/2}}dx+\frac{x}{6\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}}$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{6}\left(\frac{\int \frac{2b\left(4bx^2+1\right)}{\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}dx}{2b}+\frac{4x}{\sqrt{1-bx^2}\sqrt{bx^2+1}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\int \frac{4bx^2+1}{\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}dx+\frac{4x}{\sqrt{1-bx^2}\sqrt{bx^2+1}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}}$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{6}\left(-\frac{\int -\frac{b(5-3bx^2)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{2b}-\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}+\frac{4x}{\sqrt{1-bx^2}\sqrt{bx^2+1}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}\left(\frac{\int \frac{b(5-3bx^2)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{2b}-\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}+\frac{4x}{\sqrt{1-bx^2}\sqrt{bx^2+1}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{5-3bx^2}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx-\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}+\frac{4x}{\sqrt{1-bx^2}\sqrt{bx^2+1}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(8\int \frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx-3\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)-\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}+\frac{4x}{\sqrt{1-bx^2}\sqrt{bx^2+1}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}}$$

$$\Big\downarrow \text{284}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(8\int \frac{1}{\sqrt{1-b^2x^4}}dx-3\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)-\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}+\frac{4x}{\sqrt{1-bx^2}\sqrt{bx^2+1}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(8\int \frac{1}{\sqrt{1-b^2x^4}}dx-\frac{3E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)-\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}+\frac{4x}{\sqrt{1-bx^2}\sqrt{bx^2+1}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{8\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}-\frac{3E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)-\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}+\frac{4x}{\sqrt{1-bx^2}\sqrt{bx^2+1}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}}$$

input
```
Int[(1 + b*x^2)/(1 - b^2*x^4)^(5/2),x]
```

output
```
x/(6*(1 - b*x^2)^(3/2)*Sqrt[1 + b*x^2]) + ((4*x)/(Sqrt[1 - b*x^2]*Sqrt[1 +
 b*x^2]) - (3*x*Sqrt[1 - b*x^2])/(2*Sqrt[1 + b*x^2]) + ((-3*EllipticE[ArcS
in[Sqrt[b]*x], -1])/Sqrt[b] + (8*EllipticF[ArcSin[Sqrt[b]*x], -1])/Sqrt[b]
)/2)/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.95 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.39

method	result
meijerg	$x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{5}{2}\right], \left[\frac{5}{4}\right], b^{2} x^{4}\right)+\frac{b\,x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{2}\right],\left[\frac{7}{4}\right], b^{2} x^{4}\right)}{3}$
default	$\frac{x\sqrt{-b^{2} x^{4}+1}}{6 b^{4}\left(x^{4}-\frac{1}{b^{2}}\right)^{2}}+\frac{5 x}{12 \sqrt{-\left(x^{4}-\frac{1}{b^{2}}\right) b^{2}}}+\frac{5 \sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x, i\right)}{12 \sqrt{b}\,\sqrt{-b^{2} x^{4}+1}}+b\left(\frac{x^{3} \sqrt{-b^{2} x^{4}+1}}{6 b^{4}\left(x^{4}-\frac{1}{b^{2}}\right)^{2}}+\frac{x^{3}}{4 \sqrt{-\left(x^{4}-\frac{1}{b^{2}}\right) b^{2}}}+\right.$
elliptic	$\frac{x\sqrt{-b^{2} x^{4}+1}}{12 b^{2}\left(x^{2}-\frac{1}{b}\right)^{2}}-\frac{3\left(-b^{2} x^{2}-b\right) x}{8 b \sqrt{\left(x^{2}-\frac{1}{b}\right)\left(-b^{2} x^{2}-b\right)}}+\frac{\left(-b^{2} x^{2}+b\right) x}{8 b \sqrt{\left(x^{2}+\frac{1}{b}\right)\left(-b^{2} x^{2}+b\right)}}+\frac{5 \sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x, i\right)}{12 \sqrt{b}\,\sqrt{-b^{2} x^{4}+1}}+$

input
```
int((b*x^2+1)/(-b^2*x^4+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,5/2],[5/4],b^2*x^4)+1/3*b*x^3*hypergeom([3/4,5/2],[7/4],b
^2*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(70) = 140$.

Time = 0.07 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.79

$$\int \frac{1+bx^2}{\left(1-b^2x^4\right)^{5/2}}\,dx =$$

$$-\frac{3\left(b^4x^6-b^3x^4-b^2x^2+b\right)\sqrt{b}E\left(\arcsin\left(\sqrt{b}x\right)\,|\,-1\right)-\left(\left(3\,b^4+5\,b^3\right)x^6-\left(3\,b^3+5\,b^2\right)x^4-\left(3\,b^2+5\,b\right)x^2-\right.}{12\left(b^4x^6-b^3x^4-b^2x^2+b\right)}$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*(b^4*x^6 - b^3*x^4 - b^2*x^2 + b)*sqrt(b)*elliptic_e(arcsin(sqrt(
b)*x), -1) - ((3*b^4 + 5*b^3)*x^6 - (3*b^3 + 5*b^2)*x^4 - (3*b^2 + 5*b)*x^
2 + 3*b + 5)*sqrt(b)*elliptic_f(arcsin(sqrt(b)*x), -1) + (3*b^3*x^5 + 2*b^
2*x^3 - 7*b*x)*sqrt(-b^2*x^4 + 1))/(b^4*x^6 - b^3*x^4 - b^2*x^2 + b)
```

Sympy [A] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.76

$$\int \frac{1+bx^2}{(1-b^2x^4)^{5/2}}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(-b**2*x**4+1)**(5/2),x)
```

output
```
b*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(
4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b**2*x**4*exp_polar
(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{(1-b^2x^4)^{5/2}}\,dx = \int \frac{bx^2+1}{(-b^2x^4+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 + 1)^(5/2), x)
```

Giac [F]

$$\int \frac{1+bx^2}{(1-b^2x^4)^{5/2}}\,dx = \int \frac{bx^2+1}{(-b^2x^4+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 + 1)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\left(1-b^2x^4\right)^{5/2}}\,dx=\int \frac{b\,x^2+1}{\left(1-b^2\,x^4\right)^{5/2}}\,dx$$

input
```
int((b*x^2 + 1)/(1 - b^2*x^4)^(5/2),x)
```

output
```
int((b*x^2 + 1)/(1 - b^2*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{\left(1-b^2x^4\right)^{5/2}}\,dx=-\left(\int \frac{\sqrt{-b^2x^4+1}}{b^5x^{10}-b^4x^8-2b^3x^6+2b^2x^4+b\,x^2-1}dx\right)$$

input
```
int((b*x^2+1)/(-b^2*x^4+1)^(5/2),x)
```

output
```
 - int(sqrt( - b**2*x**4 + 1)/(b**5*x**10 - b**4*x**8 - 2*b**3*x**6 + 2*b*
*2*x**4 + b*x**2 - 1),x)
```

3.232 $\int \frac{1+bx^2}{\left(1-b^2x^4\right)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{1+bx^2}{\left(1-b^2x^4\right)^{7/2}}\,dx=\frac{x(1+bx^2)}{10\left(1-b^2x^4\right)^{5/2}}+\frac{x(9+7bx^2)}{60\left(1-b^2x^4\right)^{3/2}}+\frac{x(15+7bx^2)}{40\sqrt{1-b^2x^4}}-\frac{7E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{40\sqrt{b}}+\frac{11\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{20\sqrt{b}}$$

output

```
1/10*x*(b*x^2+1)/(-b^2*x^4+1)^(5/2)+1/60*x*(7*b*x^2+9)/(-b^2*x^4+1)^(3/2)+
1/40*x*(7*b*x^2+15)/(-b^2*x^4+1)^(1/2)-7/40*EllipticE(b^(1/2)*x,I)/b^(1/2)
+11/20*EllipticF(b^(1/2)*x,I)/b^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68

$$\int \frac{1+bx^2}{\left(1-b^2x^4\right)^{7/2}}\,dx=\frac{1}{120}x\left(\frac{75-108b^2x^4+45b^4x^8}{\left(1-b^2x^4\right)^{5/2}}+45\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)+40bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{2},\frac{7}{4},b^2x^4\right)\right)$$

input
```
Integrate[(1 + b*x^2)/(1 - b^2*x^4)^(7/2),x]
```

output
```
(x*((75 - 108*b^2*x^4 + 45*b^4*x^8)/(1 - b^2*x^4)^(5/2) + 45*Hypergeometri
c2F1[1/4, 1/2, 5/4, b^2*x^4] + 40*b*x^2*Hypergeometric2F1[3/4, 7/2, 7/4, b
^2*x^4]))/120
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.62, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {1388, 316, 27, 402, 27, 402, 27, 402, 27, 402, 25, 27, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\left(1-b^2x^4\right)^{7/2}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{b\left(7bx^2+9\right)}{\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{5/2}}dx}{10b}+\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\int \frac{7bx^2+9}{\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{5/2}}dx+\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{10}\left(\frac{\int \frac{2b\left(40bx^2+19\right)}{\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{5/2}}dx}{6b}+\frac{8x}{3\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{3/2}}\right)+\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\frac{1}{3}\int \frac{40bx^2+19}{\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{5/2}}dx+\frac{8x}{3\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{3/2}}\right)+$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{\int -\frac{3b\left(7-59bx^2\right)}{\sqrt{1-bx^2}\left(bx^2+1\right)^{5/2}}dx}{2b}+\frac{59x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}\right)+\frac{8x}{3\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{3/2}}\right)+$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{59x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}-\frac{3}{2}\int \frac{7-59bx^2}{\sqrt{1-bx^2}\left(bx^2+1\right)^{5/2}}dx\right)+\frac{8x}{3\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{3/2}}\right)+$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{59x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}-\frac{3}{2}\left(\frac{11x\sqrt{1-bx^2}}{\left(bx^2+1\right)^{3/2}}-\frac{\int \frac{6b\left(11bx^2+4\right)}{\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}dx}{6b}\right)\right)+\frac{8x}{3\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{3/2}}\right)$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{59x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}-\frac{3}{2}\left(\frac{11x\sqrt{1-bx^2}}{\left(bx^2+1\right)^{3/2}}-\int \frac{11bx^2+4}{\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}dx\right)\right)+\frac{8x}{3\left(1-bx^2\right)^{3/2}\left(bx^2+1\right.}\right.$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{59x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}-\frac{3}{2}\left(\frac{\int -\frac{b\left(15-7bx^2\right)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{2b}+\frac{7x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}+\frac{11x\sqrt{1-bx^2}}{\left(bx^2+1\right)^{3/2}}\right)\right)+\frac{8x}{3\left(1-bx^2\right)^{3/2}}\right.$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{59x}{2\sqrt{1-bx^2}\,(bx^2+1)^{3/2}}-\frac{3}{2}\left(-\frac{\int \frac{b(15-7bx^2)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{2b}+\frac{7x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}+\frac{11x\sqrt{1-bx^2}}{(bx^2+1)^{3/2}}\right)\right)+\frac{8x}{3\,(1-bx^2)^{3/2}}\right.$$
$$\frac{x}{10\,(1-bx^2)^{5/2}\,(bx^2+1)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{59x}{2\sqrt{1-bx^2}\,(bx^2+1)^{3/2}}-\frac{3}{2}\left(-\frac{1}{2}\int\frac{15-7bx^2}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+\frac{7x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}+\frac{11x\sqrt{1-bx^2}}{(bx^2+1)^{3/2}}\right)\right)+\frac{}{3\,(1-\right.$$
$$\frac{x}{10\,(1-bx^2)^{5/2}\,(bx^2+1)^{3/2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{59x}{2\sqrt{1-bx^2}\,(bx^2+1)^{3/2}}-\frac{3}{2}\left(\frac{1}{2}\left(7\int\frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx-22\int\frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx\right)+\frac{7x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}+\frac{11x}{(b}\right.\right.\right.$$
$$\frac{x}{10\,(1-bx^2)^{5/2}\,(bx^2+1)^{3/2}}$$

$$\Big\downarrow \text{284}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{59x}{2\sqrt{1-bx^2}\,(bx^2+1)^{3/2}}-\frac{3}{2}\left(\frac{1}{2}\left(7\int\frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx-22\int\frac{1}{\sqrt{1-b^2x^4}}dx\right)+\frac{7x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}+\frac{11x\sqrt{1-b}}{(bx^2+1)^{3/}}\right.\right.\right.$$
$$\frac{x}{10\,(1-bx^2)^{5/2}\,(bx^2+1)^{3/2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{59x}{2\sqrt{1-bx^2}\,(bx^2+1)^{3/2}}-\frac{3}{2}\left(\frac{1}{2}\left(\frac{7E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}-22\int\frac{1}{\sqrt{1-b^2x^4}}dx\right)+\frac{7x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}+\frac{11}{(b}\right.\right.\right.$$
$$\frac{x}{10\,(1-bx^2)^{5/2}\,(bx^2+1)^{3/2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{59x}{2\sqrt{1-bx^2}\,(bx^2+1)^{3/2}}-\frac{3}{2}\left(\frac{1}{2}\left(\frac{7E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}-\frac{22\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}\right)+\frac{7x}{2}\right.\right.\right.$$
$$\frac{x}{10\,(1-bx^2)^{5/2}\,(bx^2+1)^{3/2}}$$

input
```
Int[(1 + b*x^2)/(1 - b^2*x^4)^(7/2),x]
```

output
```
x/(10*(1 - b*x^2)^(5/2)*(1 + b*x^2)^(3/2)) + ((8*x)/(3*(1 - b*x^2)^(3/2)*(
1 + b*x^2)^(3/2)) + ((59*x)/(2*Sqrt[1 - b*x^2]*(1 + b*x^2)^(3/2)) - (3*((1
1*x*Sqrt[1 - b*x^2])/(1 + b*x^2)^(3/2) + (7*x*Sqrt[1 - b*x^2])/(2*Sqrt[1 +
 b*x^2]) + ((7*EllipticE[ArcSin[Sqrt[b]*x], -1])/Sqrt[b] - (22*EllipticF[A
rcSin[Sqrt[b]*x], -1])/Sqrt[b])/2))/2)/3)/10
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.30

method	result
meijerg	$x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{7}{2}\right],\left[\frac{5}{4}\right],b^{2}x^{4}\right)+\frac{b\,x^{3}\,\text{hypergeom}\left(\left[\frac{3}{4},\frac{7}{2}\right],\left[\frac{7}{4}\right],b^{2}x^{4}\right)}{3}$
default	$-\frac{x\sqrt{-b^{2}x^{4}+1}}{10b^{6}\left(x^{4}-\frac{1}{b^{2}}\right)^{3}}+\frac{3x\sqrt{-b^{2}x^{4}+1}}{20b^{4}\left(x^{4}-\frac{1}{b^{2}}\right)^{2}}+\frac{3x}{8\sqrt{-\left(x^{4}-\frac{1}{b^{2}}\right)b^{2}}}+\frac{3\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\text{EllipticF}\left(\sqrt{b}\,x,i\right)}{8\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}+b\Bigg(-$
elliptic	$-\frac{x\sqrt{-b^{2}x^{4}+1}}{40b^{3}\left(x^{2}-\frac{1}{b}\right)^{3}}+\frac{11x\sqrt{-b^{2}x^{4}+1}}{120b^{2}\left(x^{2}-\frac{1}{b}\right)^{2}}-\frac{53\left(-b^{2}x^{2}-b\right)x}{160b\sqrt{\left(x^{2}-\frac{1}{b}\right)\left(-b^{2}x^{2}-b\right)}}+\frac{x\sqrt{-b^{2}x^{4}+1}}{48b^{2}\left(x^{2}+\frac{1}{b}\right)^{2}}+\frac{5\left(-b^{2}x^{2}+b\right)x}{32b\sqrt{\left(x^{2}+\frac{1}{b}\right)\left(-b^{2}x^{2}+b\right)}}+$

input

```
int((b*x^2+1)/(-b^2*x^4+1)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
x*hypergeom([1/4,7/2],[5/4],b^2*x^4)+1/3*b*x^3*hypergeom([3/4,7/2],[7/4],b
^2*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(93) = 186.

Time = 0.08 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.07

$$\int \frac{1+bx^2}{(1-b^2x^4)^{7/2}}\,dx =$$

$$-\frac{21\,(b^6x^{10}-b^5x^8-2\,b^4x^6+2\,b^3x^4+b^2x^2-b)\sqrt{b}E(\arcsin\left(\sqrt{b}x\right)\,|-1)-3\,((7\,b^6+15\,b^5)x^{10}-(7\,b^5+1}{}$$

input

```
integrate((b*x^2+1)/(-b^2*x^4+1)^(7/2),x, algorithm="fricas")
```

output

```
-1/120*(21*(b^6*x^10 - b^5*x^8 - 2*b^4*x^6 + 2*b^3*x^4 + b^2*x^2 - b)*sqrt
(b)*elliptic_e(arcsin(sqrt(b)*x), -1) - 3*((7*b^6 + 15*b^5)*x^10 - (7*b^5
+ 15*b^4)*x^8 - 2*(7*b^4 + 15*b^3)*x^6 + 2*(7*b^3 + 15*b^2)*x^4 + (7*b^2 +
 15*b)*x^2 - 7*b - 15)*sqrt(b)*elliptic_f(arcsin(sqrt(b)*x), -1) + (21*b^5
*x^9 + 24*b^4*x^7 - 80*b^3*x^5 - 28*b^2*x^3 + 75*b*x)*sqrt(-b^2*x^4 + 1))/
(b^6*x^10 - b^5*x^8 - 2*b^4*x^6 + 2*b^3*x^4 + b^2*x^2 - b)
```

Sympy [A] (verification not implemented)

Time = 12.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int \frac{1+bx^2}{(1-b^2x^4)^{7/2}}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{7}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{5}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((b*x**2+1)/(-b**2*x**4+1)**(7/2),x)
```

output
```
b*x**3*gamma(3/4)*hyper((3/4, 7/2), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(
4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 7/2), (5/4,), b**2*x**4*exp_polar
(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{(1-b^2x^4)^{7/2}}\,dx = \int \frac{bx^2+1}{(-b^2x^4+1)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 + 1)^(7/2), x)
```

Giac [F]

$$\int \frac{1+bx^2}{(1-b^2x^4)^{7/2}}\,dx = \int \frac{bx^2+1}{(-b^2x^4+1)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 + 1)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{(1-b^2x^4)^{7/2}}\,dx = \int \frac{b\,x^2+1}{(1-b^2\,x^4)^{7/2}}\,dx$$

input
```
int((b*x^2 + 1)/(1 - b^2*x^4)^(7/2),x)
```

output
```
int((b*x^2 + 1)/(1 - b^2*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{(1-b^2x^4)^{7/2}}\,dx = \int \frac{\sqrt{-b^2x^4+1}}{b^7x^{14}-b^6x^{12}-3b^5x^{10}+3b^4x^8+3b^3x^6-3b^2x^4-b\,x^2+1}dx$$

input
```
int((b*x^2+1)/(-b^2*x^4+1)^(7/2),x)
```

output
```
int(sqrt( - b**2*x**4 + 1)/(b**7*x**14 - b**6*x**12 - 3*b**5*x**10 + 3*b**
4*x**8 + 3*b**3*x**6 - 3*b**2*x**4 - b*x**2 + 1),x)
```

3.233 $\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{5/2}\,dx=\frac{4x\left(195-77bx^2\right)\sqrt{1-b^2x^4}}{3003}+\frac{10x\left(117-77bx^2\right)\left(1-b^2x^4\right)^{3/2}}{9009}+\frac{1}{143}x\left(13-11bx^2\right)\left(1-b^2x^4\right)^{5/2}-\frac{8E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{39\sqrt{b}}+\frac{2176\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{3003\sqrt{b}}$$

output
```
4/3003*x*(-77*b*x^2+195)*(-b^2*x^4+1)^(1/2)+10/9009*x*(-77*b*x^2+117)*(-b^
2*x^4+1)^(3/2)+1/143*x*(-11*b*x^2+13)*(-b^2*x^4+1)^(5/2)-8/39*EllipticE(b^
(1/2)*x,I)/b^(1/2)+2176/3003*EllipticF(b^(1/2)*x,I)/b^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.38

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{5/2}\,dx=x\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},\frac{5}{4},b^2x^4\right)-\frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)$$

input
```
Integrate[(1 - b*x^2)*(1 - b^2*x^4)^(5/2),x]
```

output
```
x*Hypergeometric2F1[-5/2, 1/4, 5/4, b^2*x^4] - (b*x^3*Hypergeometric2F1[-5
/2, 3/4, 7/4, b^2*x^4])/3
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.88, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {1388, 318, 27, 403, 25, 27, 403, 27, 403, 27, 403, 27, 403, 27, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{5/2}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{5/2}dx$$

$$\Big\downarrow \text{318}$$

$$-\frac{\int -2b\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}\left(10bx^2+7\right)dx}{13b}-\frac{1}{13}x\left(bx^2+1\right)^{3/2}\left(1-bx^2\right)^{9/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{13}\int \left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}\left(10bx^2+7\right)dx-\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \text{403}$$

$$\frac{2}{13}\left(-\frac{\int -\frac{b\left(1-bx^2\right)^{7/2}\left(107bx^2+87\right)}{\sqrt{bx^2+1}}dx}{11b}-\frac{10}{11}x\sqrt{bx^2+1}\left(1-bx^2\right)^{9/2}\right)-\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{13}\left(\frac{\int \frac{b\left(1-bx^2\right)^{7/2}\left(107bx^2+87\right)}{\sqrt{bx^2+1}}dx}{11b}-\frac{10}{11}x\left(1-bx^2\right)^{9/2}\sqrt{bx^2+1}\right)-\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow\ 27$$

$$\frac{2}{13}\left(\frac{1}{11}\int \frac{\left(1-bx^2\right)^{7/2}\left(107bx^2+87\right)}{\sqrt{bx^2+1}}dx-\frac{10}{11}x\left(1-bx^2\right)^{9/2}\sqrt{bx^2+1}\right)-$$
$$\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow\ 403$$

$$\frac{2}{13}\left(\frac{1}{11}\left(\frac{\int \frac{2b\left(1-bx^2\right)^{5/2}\left(411bx^2+338\right)}{\sqrt{bx^2+1}}dx}{9b}+\frac{107}{9}x\sqrt{bx^2+1}\left(1-bx^2\right)^{7/2}\right)-\frac{10}{11}x\left(1-bx^2\right)^{9/2}\sqrt{bx^2+1}\right)-$$
$$\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow\ 27$$

$$\frac{2}{13}\left(\frac{1}{11}\left(\frac{2}{9}\int \frac{\left(1-bx^2\right)^{5/2}\left(411bx^2+338\right)}{\sqrt{bx^2+1}}dx+\frac{107}{9}x\sqrt{bx^2+1}\left(1-bx^2\right)^{7/2}\right)-\frac{10}{11}x\left(1-bx^2\right)^{9/2}\sqrt{bx^2+1}\right)-$$
$$\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow\ 403$$

$$\frac{2}{13}\left(\frac{1}{11}\left(\frac{2}{9}\left(\frac{\int \frac{5b\left(1-bx^2\right)^{3/2}\left(431bx^2+391\right)}{\sqrt{bx^2+1}}dx}{7b}+\frac{411}{7}x\sqrt{bx^2+1}\left(1-bx^2\right)^{5/2}\right)+\frac{107}{9}x\sqrt{bx^2+1}\left(1-bx^2\right)^{7/2}\right)-\frac{10}{11}\right.$$
$$\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow\ 27$$

$$\frac{2}{13}\left(\frac{1}{11}\left(\frac{2}{9}\left(\frac{5}{7}\int \frac{\left(1-bx^2\right)^{3/2}\left(431bx^2+391\right)}{\sqrt{bx^2+1}}dx+\frac{411}{7}x\sqrt{bx^2+1}\left(1-bx^2\right)^{5/2}\right)+\frac{107}{9}x\sqrt{bx^2+1}\left(1-bx^2\right)^{7/2}\right.\right.$$
$$\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow\ 403$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{\int \frac{6b\sqrt{1-bx^2}(177bx^2+254)}{\sqrt{bx^2+1}}dx}{5b}+\frac{431}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\Bigg)+\frac{411}{7}x\sqrt{bx^2+1}\left(1-bx^2\right)^{5/2}\Bigg)+$$

$$\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow\ 27$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{6}{5}\int\frac{\sqrt{1-bx^2}(177bx^2+254)}{\sqrt{bx^2+1}}dx+\frac{431}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\Bigg)+\frac{411}{7}x\sqrt{bx^2+1}\left(1-bx^2\right)^{5/2}$$

$$\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow\ 403$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{6}{5}\Bigg(\frac{\int\frac{3b(195-77bx^2)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{3b}+59x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{431}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\Bigg)+\frac{411}{7}x\sqrt{b$$

$$\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow\ 27$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{6}{5}\Bigg(\int\frac{195-77bx^2}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+59x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{431}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\Bigg)+\frac{411}{7}x$$

$$\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow\ 399$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{6}{5}\Bigg(272\int\frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx-77\int\frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx+59x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{431}{5}x\sqrt{bx^2$$

$$\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow\ 284$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{6}{5}\Bigg(272\int\frac{1}{\sqrt{1-b^2x^4}}dx-77\int\frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx+59x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{431}{5}x\sqrt{bx^2+1}(1-$$

$$\frac{1}{13}x\left(1-bx^2\right)^{9/2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow\ 327$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{6}{5}\Bigg(272\int\frac{1}{\sqrt{1-b^{2}x^{4}}}dx-\frac{77E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}+59x\sqrt{1-bx^{2}}\sqrt{bx^{2}+1}\Bigg)+\frac{431}{5}x\sqrt{bx^{2}}$$

$$\frac{1}{13}x\left(1-bx^{2}\right)^{9/2}\left(bx^{2}+1\right)^{3/2}$$

$$\Big\downarrow \text{762}$$

$$\frac{2}{13}\Bigg(\frac{1}{11}\Bigg(\frac{2}{9}\Bigg(\frac{5}{7}\Bigg(\frac{6}{5}\Bigg(\frac{272\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}-\frac{77E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}+59x\sqrt{1-bx^{2}}\sqrt{bx^{2}+1}$$

$$\frac{1}{13}x\left(1-bx^{2}\right)^{9/2}\left(bx^{2}+1\right)^{3/2}$$

input

```
Int[(1 - b*x^2)*(1 - b^2*x^4)^(5/2),x]
```

output

```
-1/13*(x*(1 - b*x^2)^(9/2)*(1 + b*x^2)^(3/2)) + (2*((-10*x*(1 - b*x^2)^(9/
2)*Sqrt[1 + b*x^2])/11 + ((107*x*(1 - b*x^2)^(7/2)*Sqrt[1 + b*x^2])/9 + (2
*((411*x*(1 - b*x^2)^(5/2)*Sqrt[1 + b*x^2])/7 + (5*((431*x*(1 - b*x^2)^(3/
2)*Sqrt[1 + b*x^2])/5 + (6*(59*x*Sqrt[1 - b*x^2]*Sqrt[1 + b*x^2] - (77*Ell
ipticE[ArcSin[Sqrt[b]*x], -1])/Sqrt[b] + (272*EllipticF[ArcSin[Sqrt[b]*x],
 -1])/Sqrt[b]))/5))/7))/9)/11))/13
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.86 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.30

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[-\frac{5}{2},\frac{1}{4}\right],\left[\frac{5}{4}\right],b^2x^4\right)-\frac{b\,x^3\,\mathrm{hypergeom}\left(\left[-\frac{5}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3}$
risch	$\frac{x\left(693x^{10}b^5-819b^4x^8-2156b^3x^6+2808b^2x^4+2387b\,x^2-4329\right)\left(b^2x^4-1\right)}{9009\sqrt{-b^2x^4+1}}+\frac{40\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\mathrm{EllipticF}\left(\sqrt{b}\,x,i\right)}{77\sqrt{b}\,\sqrt{-b^2x^4+1}}+$ $8\sqrt{-b\,x$
elliptic	$-\frac{b^5x^{11}\sqrt{-b^2x^4+1}}{13}+\frac{b^4x^9\sqrt{-b^2x^4+1}}{11}+\frac{28b^3x^7\sqrt{-b^2x^4+1}}{117}-\frac{24b^2x^5\sqrt{-b^2x^4+1}}{77}-\frac{31b\,x^3\sqrt{-b^2x^4+1}}{117}+\frac{37x\sqrt{-b^2x^4+1}}{77}$
default	$-b\Bigg(\frac{b^4x^{11}\sqrt{-b^2x^4+1}}{13}-\frac{28b^2x^7\sqrt{-b^2x^4+1}}{117}+\frac{31x^3\sqrt{-b^2x^4+1}}{117}-$ $\frac{8\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\Big(\mathrm{EllipticF}\left(\sqrt{b}\,x,i\right)-\mathrm{EllipticE}\Big(\sqrt{b}\,x}{39b^{\frac{3}{2}}\sqrt{-b^2x^4+1}}$

input
```
int((-b*x^2+1)*(-b^2*x^4+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([-5/2,1/4],[5/4],b^2*x^4)-1/3*b*x^3*hypergeom([-5/2,3/4],[7/4]
,b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{5/2}\,dx=\frac{\frac{24\sqrt{-b^2}(195\,b-77)xF(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|-1)}{\sqrt{b}}+\frac{1848\sqrt{-b^2}xE(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|-1)}{\sqrt{b}}-(693\,b^7x^{12}-819\,b^6x^{10}-21}{9009\,b^2x}$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4+1)^(5/2),x, algorithm="fricas")
```

output
```
1/9009*(24*sqrt(-b^2)*(195*b - 77)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)
/sqrt(b) + 1848*sqrt(-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b)
 - (693*b^7*x^12 - 819*b^6*x^10 - 2156*b^5*x^8 + 2808*b^4*x^6 + 2387*b^3*x
^4 - 4329*b^2*x^2 - 1848*b)*sqrt(-b^2*x^4 + 1))/(b^2*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(107) = 214.

Time = 2.47 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.97

$$\begin{aligned}\int (1-bx^2)\left(1-b^2x^4\right)^{5/2}\,dx = &-\frac{b^5x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{15}{4}\right)}\\ &+\frac{b^4x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{b^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{2\Gamma\left(\frac{11}{4}\right)}\\ &-\frac{b^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{2\Gamma\left(\frac{9}{4}\right)}\\ &-\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}\end{aligned}$$

input

```
integrate((-b*x**2+1)*(-b**2*x**4+1)**(5/2),x)
```

output

```
-b**5*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b**2*x**4*exp_polar(2
*I*pi))/(4*gamma(15/4)) + b**4*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,),
 b**2*x**4*exp_polar(2*I*pi))/(4*gamma(13/4)) + b**3*x**7*gamma(7/4)*hyper
((-1/2, 7/4), (11/4,), b**2*x**4*exp_polar(2*I*pi))/(2*gamma(11/4)) - b**2
*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b**2*x**4*exp_polar(2*I*pi))/(
2*gamma(9/4)) - b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4*exp
_polar(2*I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b
**2*x**4*exp_polar(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{5/2}\,dx=\int -\left(-b^2x^4+1\right)^{\frac{5}{2}}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4+1)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((-b^2*x^4 + 1)^(5/2)*(b*x^2 - 1), x)
```

Giac [F]

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{5/2}\,dx=\int -\left(-b^2x^4+1\right)^{\frac{5}{2}}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(-b^2*x^4 + 1)^(5/2)*(b*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{5/2}\,dx=-\int \left(1-b^2\,x^4\right)^{5/2}\,\left(b\,x^2-1\right)\,dx$$

input
```
int(-(1 - b^2*x^4)^(5/2)*(b*x^2 - 1),x)
```

output
```
-int((1 - b^2*x^4)^(5/2)*(b*x^2 - 1), x)
```

Reduce [F]

$$\int \left(1-b x^2\right) \left(1-b^2 x^4\right)^{5/2}\,dx = -\frac{\sqrt{-b^2x^4+1}\,b^5x^{11}}{13}+\frac{\sqrt{-b^2x^4+1}\,b^4x^9}{11} +\frac{28\sqrt{-b^2x^4+1}\,b^3x^7}{117}-\frac{24\sqrt{-b^2x^4+1}\,b^2x^5}{77}-\frac{31\sqrt{-b^2x^4+1}\,b\,x^3}{117} +\frac{37\sqrt{-b^2x^4+1}\,x}{77}-\frac{40\left(\int \frac{\sqrt{-b^2x^4+1}}{b^2x^4-1}dx\right)}{77}+\frac{8\left(\int \frac{\sqrt{-b^2x^4+1}\,x^2}{b^2x^4-1}dx\right)b}{39}$$

input

```
int((-b*x^2+1)*(-b^2*x^4+1)^(5/2),x)
```

output

```
( - 693*sqrt( - b**2*x**4 + 1)*b**5*x**11 + 819*sqrt( - b**2*x**4 + 1)*b**
4*x**9 + 2156*sqrt( - b**2*x**4 + 1)*b**3*x**7 - 2808*sqrt( - b**2*x**4 +
1)*b**2*x**5 - 2387*sqrt( - b**2*x**4 + 1)*b*x**3 + 4329*sqrt( - b**2*x**4
 + 1)*x - 4680*int(sqrt( - b**2*x**4 + 1)/(b**2*x**4 - 1),x) + 1848*int((s
qrt( - b**2*x**4 + 1)*x**2)/(b**2*x**4 - 1),x)*b)/9009
```

3.234 $\int (1-bx^2)\left(1-b^2x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\begin{aligned}\int (1-bx^2)\left(1-b^2x^4\right)^{3/2}dx &= \frac{2}{105}x\left(15-7bx^2\right)\sqrt{1-b^2x^4}\\ &+\frac{1}{63}x\left(9-7bx^2\right)\left(1-b^2x^4\right)^{3/2}-\frac{4E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{15\sqrt{b}}\\ &+\frac{88\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{105\sqrt{b}}\end{aligned}$$

output

```
2/105*x*(-7*b*x^2+15)*(-b^2*x^4+1)^(1/2)+1/63*x*(-7*b*x^2+9)*(-b^2*x^4+1)^
(3/2)-4/15*EllipticE(b^(1/2)*x,I)/b^(1/2)+88/105*EllipticF(b^(1/2)*x,I)/b^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.75 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx=x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},b^2x^4\right)$$
$$-\frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)$$

input

```
Integrate[(1 - b*x^2)*(1 - b^2*x^4)^(3/2),x]
```

output

```
x*Hypergeometric2F1[-3/2, 1/4, 5/4, b^2*x^4] - (b*x^3*Hypergeometric2F1[-3
/2, 3/4, 7/4, b^2*x^4])/3
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.72, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.565$, Rules used = {1388, 318, 27, 403, 27, 403, 27, 403, 27, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1388}$$

$$\int \left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{3/2}dx$$

$$\Big\downarrow \text{318}$$

$$-\frac{\int -\frac{2b\left(1-bx^2\right)^{5/2}\left(6bx^2+5\right)}{\sqrt{bx^2+1}}dx}{9b}-\frac{1}{9}x\sqrt{bx^2+1}\left(1-bx^2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\int \frac{\left(1-bx^2\right)^{5/2}\left(6bx^2+5\right)}{\sqrt{bx^2+1}}dx-\frac{1}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{2}{9}\left(\frac{\int \frac{b\left(1-bx^2\right)^{3/2}\left(31bx^2+29\right)}{\sqrt{bx^2+1}}dx}{7b}+\frac{6}{7}x\sqrt{bx^2+1}\left(1-bx^2\right)^{5/2}\right)-\frac{1}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{9}\left(\frac{1}{7}\int \frac{\left(1-bx^2\right)^{3/2}\left(31bx^2+29\right)}{\sqrt{bx^2+1}}dx+\frac{6}{7}x\sqrt{bx^2+1}\left(1-bx^2\right)^{5/2}\right)-$$
$$\frac{1}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{2}{9}\left(\frac{1}{7}\left(\frac{\int \frac{6b\sqrt{1-bx^2}\left(12bx^2+19\right)}{\sqrt{bx^2+1}}dx}{5b}+\frac{31}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\right)+\frac{6}{7}x\sqrt{bx^2+1}\left(1-bx^2\right)^{5/2}\right)-$$
$$\frac{1}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{9}\left(\frac{1}{7}\left(\frac{6}{5}\int \frac{\sqrt{1-bx^2}\left(12bx^2+19\right)}{\sqrt{bx^2+1}}dx+\frac{31}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\right)+\frac{6}{7}x\sqrt{bx^2+1}\left(1-bx^2\right)^{5/2}\right)-$$
$$\frac{1}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{2}{9}\left(\frac{1}{7}\left(\frac{6}{5}\left(\frac{\int \frac{3b\left(15-7bx^2\right)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{3b}+4x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)+\frac{31}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\right)+\frac{6}{7}x\sqrt{bx^2+1}(1-bx$$
$$\frac{1}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{9}\left(\frac{1}{7}\left(\frac{6}{5}\left(\int \frac{15-7bx^2}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+4x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)+\frac{31}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\right)+\frac{6}{7}x\sqrt{bx^2+1}(1-$$
$$\frac{1}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{2}{9}\Bigg(\frac{1}{7}\Bigg(\frac{6}{5}\Bigg(22\int\frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx-7\int\frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx+4x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{31}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/}$$
$$\frac{1}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}$$

$$\Big\downarrow 284$$

$$\frac{2}{9}\Bigg(\frac{1}{7}\Bigg(\frac{6}{5}\Bigg(22\int\frac{1}{\sqrt{1-b^2x^4}}dx-7\int\frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx+4x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{31}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3/2}\Bigg)+\frac{6}{7}$$
$$\frac{1}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}$$

$$\Big\downarrow 327$$

$$\frac{2}{9}\Bigg(\frac{1}{7}\Bigg(\frac{6}{5}\Bigg(22\int\frac{1}{\sqrt{1-b^2x^4}}dx-\frac{7E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}+4x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{31}{5}x\sqrt{bx^2+1}\left(1-bx^2\right)^{3}$$
$$\frac{1}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}$$

$$\Big\downarrow 762$$

$$\frac{2}{9}\Bigg(\frac{1}{7}\Bigg(\frac{6}{5}\Bigg(\frac{22\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}-\frac{7E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}+4x\sqrt{1-bx^2}\sqrt{bx^2+1}\Bigg)+\frac{31}{5}x\sqrt{bx^2}$$
$$\frac{1}{9}x\left(1-bx^2\right)^{7/2}\sqrt{bx^2+1}$$

input

```
Int[(1 - b*x^2)*(1 - b^2*x^4)^(3/2),x]
```

output

```
-1/9*(x*(1 - b*x^2)^(7/2)*Sqrt[1 + b*x^2]) + (2*((6*x*(1 - b*x^2)^(5/2)*Sq
rt[1 + b*x^2])/7 + ((31*x*(1 - b*x^2)^(3/2)*Sqrt[1 + b*x^2])/5 + (6*(4*x*S
qrt[1 - b*x^2]*Sqrt[1 + b*x^2] - (7*EllipticE[ArcSin[Sqrt[b]*x], -1])/Sqrt
[b] + (22*EllipticF[ArcSin[Sqrt[b]*x], -1])/Sqrt[b]))/5)/7))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.88 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.39

method	result
meijerg	$x\,\text{hypergeom}\left(\left[-\frac{3}{2}, \frac{1}{4}\right], \left[\frac{5}{4}\right], b^2x^4\right) - \frac{b\,x^3\,\text{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3}$
risch	$-\frac{x\left(35b^3x^6-45b^2x^4-77b\,x^2+135\right)\left(b^2x^4-1\right)}{315\sqrt{-b^2x^4+1}} + \frac{4\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\text{EllipticF}\left(\sqrt{b}\,x,i\right)}{7\sqrt{b}\,\sqrt{-b^2x^4+1}} +$
elliptic	$\frac{b^3x^7\sqrt{-b^2x^4+1}}{9} - \frac{b^2x^5\sqrt{-b^2x^4+1}}{7} - \frac{11b\,x^3\sqrt{-b^2x^4+1}}{45} + \frac{3x\sqrt{-b^2x^4+1}}{7} + \frac{4\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\text{EllipticF}\left(\sqrt{b}\,x,i\right)}{7\sqrt{b}\,\sqrt{-b^2x^4+1}} +$
default	$-b\left(-\frac{b^2x^7\sqrt{-b^2x^4+1}}{9} + \frac{11x^3\sqrt{-b^2x^4+1}}{45} - \frac{4\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\left(\text{EllipticF}\left(\sqrt{b}\,x,i\right)-\text{EllipticE}\left(\sqrt{b}\,x,i\right)\right)}{15b^{\frac{3}{2}}\,\sqrt{-b^2x^4+1}}\right) -$

input
```
int((-b*x^2+1)*(-b^2*x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([-3/2,1/4],[5/4],b^2*x^4)-1/3*b*x^3*hypergeom([-3/2,3/4],[7/4]
,b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.17

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx=\frac{\frac{12\,\sqrt{-b^2}(15\,b-7)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}}+\frac{84\,\sqrt{-b^2}xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}}+(35\,b^5x^8-45\,b^4x^6-77\,b^3x^4+}{315\,b^2x}$$

input

```
integrate((-b*x^2+1)*(-b^2*x^4+1)^(3/2),x, algorithm="fricas")
```

output

```
1/315*(12*sqrt(-b^2)*(15*b - 7)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sq
rt(b) + 84*sqrt(-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + (3
5*b^5*x^8 - 45*b^4*x^6 - 77*b^3*x^4 + 135*b^2*x^2 + 84*b)*sqrt(-b^2*x^4 +
1))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.67

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx=\frac{b^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}-\frac{b^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((-b*x**2+1)*(-b**2*x**4+1)**(3/2),x)
```

output
```
b**3*x***7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b**2*x***4*exp_polar(2*I*p
i))/(4*gamma(11/4)) - b**2*x***5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b**2
*x***4*exp_polar(2*I*pi))/(4*gamma(9/4)) - b*x***3*gamma(3/4)*hyper((-1/2, 3
/4), (7/4,), b**2*x***4*exp_polar(2*I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hy
per((-1/2, 1/4), (5/4,), b**2*x***4*exp_polar(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx=\int -\left(-b^2x^4+1\right)^{\frac{3}{2}}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((-b^2*x^4 + 1)^(3/2)*(b*x^2 - 1), x)
```

Giac [F]

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx=\int -\left(-b^2x^4+1\right)^{\frac{3}{2}}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(-b^2*x^4 + 1)^(3/2)*(b*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx=-\int \left(1-b^2\,x^4\right)^{3/2}\,\left(b\,x^2-1\right)\,dx$$

input
```
int(-(1 - b^2*x^4)^(3/2)*(b*x^2 - 1),x)
```

output
```
-int((1 - b^2*x^4)^(3/2)*(b*x^2 - 1), x)
```

Reduce [F]

$$\int \left(1-bx^2\right)\left(1-b^2x^4\right)^{3/2}\,dx=\frac{\sqrt{-b^2x^4+1}\,b^3x^7}{9}-\frac{\sqrt{-b^2x^4+1}\,b^2x^5}{7}-\frac{11\sqrt{-b^2x^4+1}\,b\,x^3}{45}+\frac{3\sqrt{-b^2x^4+1}\,x}{7}-\frac{4\left(\int \frac{\sqrt{-b^2x^4+1}}{b^2x^4-1}dx\right)}{7}+\frac{4\left(\int \frac{\sqrt{-b^2x^4+1}\,x^2}{b^2x^4-1}dx\right)b}{15}$$

input
```
int((-b*x^2+1)*(-b^2*x^4+1)^(3/2),x)
```

output
```
(35*sqrt( - b**2*x**4 + 1)*b**3*x**7 - 45*sqrt( - b**2*x**4 + 1)*b**2*x**5
 - 77*sqrt( - b**2*x**4 + 1)*b*x**3 + 135*sqrt( - b**2*x**4 + 1)*x - 180*i
nt(sqrt( - b**2*x**4 + 1)/(b**2*x**4 - 1),x) + 84*int((sqrt( - b**2*x**4 +
 1)*x**2)/(b**2*x**4 - 1),x)*b)/315
```

3.235 $\int (1-bx^2)\sqrt{1-b^2x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int (1-bx^2)\sqrt{1-b^2x^4}\,dx=\frac{1}{15}x(5-3bx^2)\sqrt{1-b^2x^4}-\frac{2E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{5\sqrt{b}}+\frac{16\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{15\sqrt{b}}$$

output
```
1/15*x*(-3*b*x^2+5)*(-b^2*x^4+1)^(1/2)-2/5*EllipticE(b^(1/2)*x,I)/b^(1/2)+
16/15*EllipticF(b^(1/2)*x,I)/b^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.68

$$\int (1-bx^2)\sqrt{1-b^2x^4}\,dx=x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},b^2x^4\right)-\frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)$$

input

```
Integrate[(1 - b*x^2)*Sqrt[1 - b^2*x^4],x]
```

output

```
x*Hypergeometric2F1[-1/2, 1/4, 5/4, b^2*x^4] - (b*x^3*Hypergeometric2F1[-1
/2, 3/4, 7/4, b^2*x^4])/3
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1388, 318, 27, 403, 25, 27, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-bx^2\right)\sqrt{1-b^2x^4}\,dx$$

$$\Big\downarrow \text{1388}$$

$$\int \left(1-bx^2\right)^{3/2}\sqrt{bx^2+1}dx$$

$$\Big\downarrow \text{318}$$

$$\frac{\int \frac{2b\left(3-4bx^2\right)\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx}{5b}-\frac{1}{5}x\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}\int \frac{\left(3-4bx^2\right)\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx-\frac{1}{5}x\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \text{403}$$

$$\frac{2}{5}\left(\frac{4}{3}x\sqrt{1-bx^2}\sqrt{bx^2+1}-\frac{\int -\frac{b\left(5-3bx^2\right)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{3b}\right)-\frac{1}{5}x\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{5}\left(\frac{\int \frac{b\left(5-3bx^2\right)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{3b}+\frac{4}{3}x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)-\frac{1}{5}x\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow 27$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{5-3bx^2}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+\frac{4}{3}x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)-\frac{1}{5}x\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow 399$$

$$\frac{2}{5}\left(\frac{1}{3}\left(8\int \frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx-3\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)+\frac{4}{3}x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)-\frac{1}{5}x\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow 284$$

$$\frac{2}{5}\left(\frac{1}{3}\left(8\int \frac{1}{\sqrt{1-b^2x^4}}dx-3\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)+\frac{4}{3}x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)-\frac{1}{5}x\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow 327$$

$$\frac{2}{5}\left(\frac{1}{3}\left(8\int \frac{1}{\sqrt{1-b^2x^4}}dx-\frac{3E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)+\frac{4}{3}x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)-\frac{1}{5}x\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}$$

$$\Big\downarrow 762$$

$$\frac{2}{5}\left(\frac{1}{3}\left(\frac{8\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}-\frac{3E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)+\frac{4}{3}x\sqrt{1-bx^2}\sqrt{bx^2+1}\right)-\frac{1}{5}x\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}$$

input
```
Int[(1 - b*x^2)*Sqrt[1 - b^2*x^4],x]
```

output
```
-1/5*(x*Sqrt[1 - b*x^2]*(1 + b*x^2)^(3/2)) + (2*((4*x*Sqrt[1 - b*x^2]*Sqrt
[1 + b*x^2])/3 + ((-3*EllipticE[ArcSin[Sqrt[b]*x], -1])/Sqrt[b] + (8*Ellip
ticF[ArcSin[Sqrt[b]*x], -1])/Sqrt[b])/3))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.82 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

method	result
meijerg	$x\,\text{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{5}{4}\right],b^2x^4\right)-\frac{b\,x^3\,\text{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3}$
risch	$\frac{x\left(3b\,x^2-5\right)\left(b^2x^4-1\right)}{15\sqrt{-b^2x^4+1}}+\frac{2\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\text{EllipticF}\left(\sqrt{b}\,x,i\right)}{3\sqrt{b}\,\sqrt{-b^2x^4+1}}+\frac{2\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\left(\text{EllipticF}\left(\sqrt{b}\,x,i\right)-\text{EllipticE}\left(\sqrt{b}\,x,i\right.\right.}{5\sqrt{b}\,\sqrt{-b^2x^4+1}}$
elliptic	$-\frac{b\,x^3\sqrt{-b^2x^4+1}}{5}+\frac{x\sqrt{-b^2x^4+1}}{3}+\frac{2\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\text{EllipticF}\left(\sqrt{b}\,x,i\right)}{3\sqrt{b}\,\sqrt{-b^2x^4+1}}+\frac{2\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\left(\text{EllipticF}\left(\sqrt{b}\,x,i\right)-\text{Ell}\right.}{5\sqrt{b}\,\sqrt{-b^2x^4+1}}$
default	$-b\left(\frac{x^3\sqrt{-b^2x^4+1}}{5}-\frac{2\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\left(\text{EllipticF}\left(\sqrt{b}\,x,i\right)-\text{EllipticE}\left(\sqrt{b}\,x,i\right)\right)}{5b^{\frac{3}{2}}\sqrt{-b^2x^4+1}}\right)+\frac{x\sqrt{-b^2x^4+1}}{3}+\frac{2\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2}}{3\sqrt{b}}$

input
```
int((-b*x^2+1)*(-b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([-1/2,1/4],[5/4],b^2*x^4)-1/3*b*x^3*hypergeom([-1/2,3/4],[7/4]
,b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int \left(1-bx^2\right)\sqrt{1-b^2x^4}\,dx$$
$$=\frac{\frac{2\sqrt{-b^2}(5\,b-3)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\mid-1\right)}{\sqrt{b}}+\frac{6\sqrt{-b^2}xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\mid-1\right)}{\sqrt{b}}-\left(3\,b^3x^4-5\,b^2x^2-6\,b\right)\sqrt{-b^2x^4+1}}{15\,b^2x}$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(2*sqrt(-b^2)*(5*b - 3)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(
b) + 6*sqrt(-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - (3*b^3
*x^4 - 5*b^2*x^2 - 6*b)*sqrt(-b^2*x^4 + 1))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \left(1-bx^2\right)\sqrt{1-b^2x^4}\,dx=-\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)*(-b**2*x**4+1)**(1/2),x)
```

output
```
-b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))
/(4*gamma(7/4)) + x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b**2*x**4*exp_po
lar(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int (1-bx^2)\sqrt{1-b^2x^4}\,dx = \int -\sqrt{-b^2x^4+1}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-b^2*x^4 + 1)*(b*x^2 - 1), x)
```

Giac [F]

$$\int (1-bx^2)\sqrt{1-b^2x^4}\,dx = \int -\sqrt{-b^2x^4+1}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(-b^2*x^4 + 1)*(b*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-bx^2)\sqrt{1-b^2x^4}\,dx = -\int \sqrt{1-b^2\,x^4}\,\left(b\,x^2-1\right)\,dx$$

input
```
int(-(1 - b^2*x^4)^(1/2)*(b*x^2 - 1),x)
```

output
```
-int((1 - b^2*x^4)^(1/2)*(b*x^2 - 1), x)
```

Reduce [F]

$$\int (1-bx^2)\sqrt{1-b^2x^4}\,dx = -\frac{\sqrt{-b^2x^4+1}\,b\,x^3}{5}+\frac{\sqrt{-b^2x^4+1}\,x}{3} - \frac{2\left(\int \frac{\sqrt{-b^2x^4+1}}{b^2x^4-1}dx\right)}{3}+\frac{2\left(\int \frac{\sqrt{-b^2x^4+1}\,x^2}{b^2x^4-1}dx\right)b}{5}$$

input
```
int((-b*x^2+1)*(-b^2*x^4+1)^(1/2),x)
```

output
```
( - 3*sqrt( - b**2*x**4 + 1)*b*x**3 + 5*sqrt( - b**2*x**4 + 1)*x - 10*int(
sqrt( - b**2*x**4 + 1)/(b**2*x**4 - 1),x) + 6*int((sqrt( - b**2*x**4 + 1)*
x**2)/(b**2*x**4 - 1),x)*b)/15
```

3.236 $\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx=-\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}+\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}$$

output
```
-EllipticE(b^(1/2)*x,I)/b^(1/2)+2*EllipticF(b^(1/2)*x,I)/b^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\begin{aligned}\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx&=x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)\\&\quad-\frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)\end{aligned}$$

input
```
Integrate[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] - (b*x^3*Hypergeometric2F1[1/2
, 3/4, 7/4, b^2*x^4])/3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1388, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}dx$$

$$\Big\downarrow \; 1388$$

$$\int \frac{\sqrt{1-bx^2}}{\sqrt{bx^2+1}}dx$$

$$\Big\downarrow \; 326$$

$$2\int \frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx-\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx$$

$$\Big\downarrow \; 284$$

$$2\int \frac{1}{\sqrt{1-b^2x^4}}dx-\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx$$

$$\Big\downarrow \; 327$$

$$2\int \frac{1}{\sqrt{1-b^2x^4}}dx-\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

$$\Big\downarrow \; 762$$

$$\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}-\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

input
```
Int[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

output
```
-(EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]) + (2*EllipticF[ArcSin[Sqrt[b]*
x], -1])/Sqrt[b]
```

Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
meijerg	$-\frac{b\,x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], b^{2} x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], b^{2} x^{4}\right)$	36
default	$\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(\sqrt{b}\,x , i\right)-\operatorname{EllipticE}\left(\sqrt{b}\,x , i\right)\right)}{\sqrt{b}\,\sqrt{-b^{2} x^{4}+1}}+\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x , i\right)}{\sqrt{b}\,\sqrt{-b^{2} x^{4}+1}}$	99
elliptic	$\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(\sqrt{b}\,x , i\right)-\operatorname{EllipticE}\left(\sqrt{b}\,x , i\right)\right)}{\sqrt{b}\,\sqrt{-b^{2} x^{4}+1}}+\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x , i\right)}{\sqrt{b}\,\sqrt{-b^{2} x^{4}+1}}$	99

input
```
int((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b*x^3*hypergeom([1/2,3/4],[7/4],b^2*x^4)+x*hypergeom([1/4,1/2],[5/4],
b^2*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(25) = 50.

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx=\frac{\frac{\sqrt{-b^2}(b-1)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\middle|-1\right)}{\sqrt{b}}+\frac{\sqrt{-b^2}xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\middle|-1\right)}{\sqrt{b}}+\sqrt{-b^2x^4+1}b}{b^2x}$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b^2)*(b - 1)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + sqrt
(-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + sqrt(-b^2*x^4 + 1
)*b)/(b^2*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(27) = 54$.

Time = 0.83 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.00

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx = -\frac{bx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

output
```
-b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/
(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_pola
r(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{-b^2x^4+1}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx=\int -\frac{bx^2-1}{\sqrt{-b^2x^4+1}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx=-\int \frac{b\,x^2-1}{\sqrt{1-b^2\,x^4}}\,dx$$

input
```
int(-(b*x^2 - 1)/(1 - b^2*x^4)^(1/2),x)
```

output
```
-int((b*x^2 - 1)/(1 - b^2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx=\int \frac{\sqrt{-b^2x^4+1}}{b\,x^2+1}dx$$

input
```
int((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x)
```

output
```
int(sqrt( - b**2*x**4 + 1)/(b*x**2 + 1),x)
```

3.237 $\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{3/2}}\,dx=\frac{x\left(1-bx^2\right)}{2\sqrt{1-b^2x^4}}+\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{2\sqrt{b}}$$

output
```
1/2*x*(-b*x^2+1)/(-b^2*x^4+1)^(1/2)+1/2*EllipticE(b^(1/2)*x,I)/b^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{3/2}}\,dx=\frac{1}{6}x\left(\frac{3}{\sqrt{1-b^2x^4}}+3\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)\right.$$
$$\left.-2bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},b^2x^4\right)\right)$$

input
```
Integrate[(1 - b*x^2)/(1 - b^2*x^4)^(3/2),x]
```

output
```
(x*(3/Sqrt[1 - b^2*x^4] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] - 2*
b*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, b^2*x^4]))/6
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1388, 316, 25, 27, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}-\frac{\int -\frac{b\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx}{2b}+\frac{x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx+\frac{x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{2\sqrt{b}}+\frac{x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}$$

input
```
Int[(1 - b*x^2)/(1 - b^2*x^4)^(3/2),x]
```

output
```
(x*Sqrt[1 - b*x^2])/(2*Sqrt[1 + b*x^2]) + EllipticE[ArcSin[Sqrt[b]*x], -1]
/(2*Sqrt[b])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],b^2x^4\right)-\frac{b\,x^3\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3}$
elliptic	$\frac{\left(-b^2x^2+b\right)x}{2b\sqrt{\left(x^2+\frac{1}{b}\right)\left(-b^2x^2+b\right)}}+\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)}{2\sqrt{b}\,\sqrt{-b^2x^4+1}}-\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\left(\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)-\operatorname{EllipticE}\left(\sqrt{b}\,x,i\right.\right.}{2\sqrt{b}\,\sqrt{-b^2x^4+1}}$
default	$-b\left(\frac{x^3}{2\sqrt{-\left(x^4-\frac{1}{b^2}\right)b^2}}+\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\left(\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)-\operatorname{EllipticE}\left(\sqrt{b}\,x,i\right)\right)}{2b^{\frac{3}{2}}\sqrt{-b^2x^4+1}}\right)+\frac{x}{2\sqrt{-\left(x^4-\frac{1}{b^2}\right)b^2}}+\frac{\sqrt{-b\,x^2+1}}{}$

input
```
int((-b*x^2+1)/(-b^2*x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,3/2],[5/4],b^2*x^4)-1/3*b*x^3*hypergeom([3/4,3/2],[7/4],b
^2*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(35) = 70.

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

$$\int\frac{1-bx^2}{\left(1-b^2x^4\right)^{3/2}}\,dx=\frac{\sqrt{-b^2x^4+1}bx+\left(b^2x^2+b\right)\sqrt{b}E\left(\arcsin\left(\sqrt{b}x\right)\mid-1\right)-\left(\left(b^2-b\right)x^2+b-1\right)\sqrt{b}F(\mathrm{arc}}{2\left(b^2x^2+b\right)}$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-b^2*x^4 + 1)*b*x + (b^2*x^2 + b)*sqrt(b)*elliptic_e(arcsin(sqrt
(b)*x), -1) - ((b^2 - b)*x^2 + b - 1)*sqrt(b)*elliptic_f(arcsin(sqrt(b)*x)
, -1))/(b^2*x^2 + b)
```

Sympy [A] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{1-bx^2}{(1-b^2x^4)^{3/2}}\,dx = -\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(-b**2*x**4+1)**(3/2),x)
```

output
```
-b*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b**2*x**4*exp_polar(2*I*pi))/
(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b**2*x**4*exp_pola
r(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{(1-b^2x^4)^{3/2}}\,dx = \int -\frac{bx^2-1}{(-b^2x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/(-b^2*x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{(1-b^2x^4)^{3/2}}\,dx = \int -\frac{bx^2-1}{(-b^2x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/(-b^2*x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{(1-b^2x^4)^{3/2}}\,dx = -\int \frac{b\,x^2-1}{(1-b^2\,x^4)^{3/2}}\,dx$$

input
```
int(-(b*x^2 - 1)/(1 - b^2*x^4)^(3/2),x)
```

output
```
-int((b*x^2 - 1)/(1 - b^2*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{(1-b^2x^4)^{3/2}}\,dx = -\left(\int \frac{\sqrt{-b^2x^4+1}}{b^3x^6+b^2x^4-b\,x^2-1}dx\right)$$

input
```
int((-b*x^2+1)/(-b^2*x^4+1)^(3/2),x)
```

output
```
- int(sqrt( - b**2*x**4 + 1)/(b**3*x**6 + b**2*x**4 - b*x**2 - 1),x)
```

3.238 $\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{5/2}}\,dx=\frac{x\left(1-bx^2\right)}{6\left(1-b^2x^4\right)^{3/2}}+\frac{x\left(5-3bx^2\right)}{12\sqrt{1-b^2x^4}}+\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{4\sqrt{b}}+\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{6\sqrt{b}}$$

output
```
1/6*x*(-b*x^2+1)/(-b^2*x^4+1)^(3/2)+1/12*x*(-3*b*x^2+5)/(-b^2*x^4+1)^(1/2)
+1/4*EllipticE(b^(1/2)*x,I)/b^(1/2)+1/6*EllipticF(b^(1/2)*x,I)/b^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{5/2}}\,dx=\frac{1}{12}x\left(\frac{7-5b^2x^4}{\left(1-b^2x^4\right)^{3/2}}+5\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)-4bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},b^2x^4\right)\right)$$

input
```
Integrate[(1 - b*x^2)/(1 - b^2*x^4)^(5/2),x]
```

output
```
(x*((7 - 5*b^2*x^4)/(1 - b^2*x^4)^(3/2) + 5*Hypergeometric2F1[1/4, 1/2, 5/
4, b^2*x^4] - 4*b*x^2*Hypergeometric2F1[3/4, 5/2, 7/4, b^2*x^4]))/12
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.44, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {1388, 316, 27, 402, 27, 402, 25, 27, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{(1-b^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{(1-bx^2)^{3/2}(bx^2+1)^{5/2}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{b(3bx^2+1)}{\sqrt{1-bx^2}(bx^2+1)^{5/2}}dx}{2b}+\frac{x}{2\sqrt{1-bx^2}(bx^2+1)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{3bx^2+1}{\sqrt{1-bx^2}(bx^2+1)^{5/2}}dx+\frac{x}{2\sqrt{1-bx^2}(bx^2+1)^{3/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{2b(bx^2+4)}{\sqrt{1-bx^2}(bx^2+1)^{3/2}}dx}{6b}-\frac{x\sqrt{1-bx^2}}{3(bx^2+1)^{3/2}}\right)+\frac{x}{2\sqrt{1-bx^2}(bx^2+1)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{bx^2+4}{\sqrt{1-bx^2}(bx^2+1)^{3/2}}dx-\frac{x\sqrt{1-bx^2}}{3(bx^2+1)^{3/2}}\right)+\frac{x}{2\sqrt{1-bx^2}(bx^2+1)^{3/2}}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}-\frac{\int -\frac{b\left(3bx^2+5\right)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{2b}\right)-\frac{x\sqrt{1-bx^2}}{3\left(bx^2+1\right)^{3/2}}\right)+\frac{x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{\int \frac{b\left(3bx^2+5\right)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{2b}+\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}\right)-\frac{x\sqrt{1-bx^2}}{3\left(bx^2+1\right)^{3/2}}\right)+\frac{x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{3bx^2+5}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}\right)-\frac{x\sqrt{1-bx^2}}{3\left(bx^2+1\right)^{3/2}}\right)+\frac{x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+3\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)+\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}\right)-\frac{x\sqrt{1-bx^2}}{3\left(bx^2+1\right)^{3/2}}\right)+\frac{x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 284}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-b^2x^4}}dx+3\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)+\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}\right)-\frac{x\sqrt{1-bx^2}}{3\left(bx^2+1\right)^{3/2}}\right)+\frac{x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-b^2x^4}}dx+\frac{3E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)+\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}\right)-\frac{x\sqrt{1-bx^2}}{3\left(bx^2+1\right)^{3/2}}\right)+\frac{x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{2\,\mathrm{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}+\frac{3E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)+\frac{3x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}\right)-\frac{x\sqrt{1-bx^2}}{3\left(bx^2+1\right)^{3/2}}\right)+\frac{x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}$$

input

```
Int[(1 - b*x^2)/(1 - b^2*x^4)^(5/2),x]
```

output

```
x/(2*Sqrt[1 - b*x^2]*(1 + b*x^2)^(3/2)) + (-1/3*(x*Sqrt[1 - b*x^2])/(1 + b
*x^2)^(3/2) + ((3*x*Sqrt[1 - b*x^2])/(2*Sqrt[1 + b*x^2]) + ((3*EllipticE[A
rcSin[Sqrt[b]*x], -1])/Sqrt[b] + (2*EllipticF[ArcSin[Sqrt[b]*x], -1])/Sqrt
[b])/2)/3)/2
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.39

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{5}{2}\right],\left[\frac{5}{4}\right],b^2x^4\right)-\frac{b\,x^3\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{5}{2}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3}$
default	$-b\left(\frac{x^3\sqrt{-b^2x^4+1}}{6b^4\left(x^4-\frac{1}{b^2}\right)^2}+\frac{x^3}{4\sqrt{-\left(x^4-\frac{1}{b^2}\right)b^2}}+\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\left(\mathrm{EllipticF}\left(\sqrt{b}\,x,i\right)-\mathrm{EllipticE}\left(\sqrt{b}\,x,i\right)\right)}{4b^{\frac{3}{2}}\sqrt{-b^2x^4+1}}\right)+\frac{x\sqrt{-b^2x^4+1}}{6b^4\left(x^4-\frac{1}{b^2}\right)}$
elliptic	$\frac{x\sqrt{-b^2x^4+1}}{12b^2\left(x^2+\frac{1}{b}\right)^2}+\frac{3\left(-b^2x^2+b\right)x}{8b\sqrt{\left(x^2+\frac{1}{b}\right)\left(-b^2x^2+b\right)}}-\frac{\left(-b^2x^2-b\right)x}{8b\sqrt{\left(x^2-\frac{1}{b}\right)\left(-b^2x^2-b\right)}}+\frac{5\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\mathrm{EllipticF}\left(\sqrt{b}\,x,i\right)}{12\sqrt{b}\,\sqrt{-b^2x^4+1}}-\sqrt{-b\,x^2+}$

input
```
int((-b*x^2+1)/(-b^2*x^4+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,5/2],[5/4],b^2*x^4)-1/3*b*x^3*hypergeom([3/4,5/2],[7/4],b
^2*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(70) = 140.

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.78

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{5/2}}\,dx=\frac{3\,(b^4x^6+b^3x^4-b^2x^2-b)\sqrt{b}E(\arcsin\left(\sqrt{b}x\right)\,|-1)-((3\,b^4-5\,b^3)x^6+(3\,b^3-5\,b^2)}{12\,(b^4x^6 -}$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(3*(b^4*x^6 + b^3*x^4 - b^2*x^2 - b)*sqrt(b)*elliptic_e(arcsin(sqrt(b
)*x), -1) - ((3*b^4 - 5*b^3)*x^6 + (3*b^3 - 5*b^2)*x^4 - (3*b^2 - 5*b)*x^2
 - 3*b + 5)*sqrt(b)*elliptic_f(arcsin(sqrt(b)*x), -1) + (3*b^3*x^5 - 2*b^2
*x^3 - 7*b*x)*sqrt(-b^2*x^4 + 1))/(b^4*x^6 + b^3*x^4 - b^2*x^2 - b)
```

Sympy [A] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{5/2}}\,dx = -\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(-b**2*x**4+1)**(5/2),x)
```

output
```
-b*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b**2*x**4*exp_polar(2*I*pi))/
(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b**2*x**4*exp_pola
r(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{5/2}}\,dx = \int -\frac{bx^2-1}{\left(-b^2x^4+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/(-b^2*x^4 + 1)^(5/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{5/2}}\,dx = \int -\frac{bx^2-1}{\left(-b^2x^4+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/(-b^2*x^4 + 1)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{(1-b^2x^4)^{5/2}}\,dx=-\int \frac{b\,x^2-1}{(1-b^2\,x^4)^{5/2}}\,dx$$

input
```
int(-(b*x^2 - 1)/(1 - b^2*x^4)^(5/2),x)
```

output
```
-int((b*x^2 - 1)/(1 - b^2*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{(1-b^2x^4)^{5/2}}\,dx=\int \frac{\sqrt{-b^2x^4+1}}{b^5x^{10}+b^4x^8-2b^3x^6-2b^2x^4+b\,x^2+1}dx$$

input
```
int((-b*x^2+1)/(-b^2*x^4+1)^(5/2),x)
```

output
```
int(sqrt( - b**2*x**4 + 1)/(b**5*x**10 + b**4*x**8 - 2*b**3*x**6 - 2*b**2*
x**4 + b*x**2 + 1),x)
```

3.239 $\int \frac{1-bx^2}{(1-b^2x^4)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{1-bx^2}{(1-b^2x^4)^{7/2}}\,dx=\frac{x(1-bx^2)}{10(1-b^2x^4)^{5/2}}+\frac{x(9-7bx^2)}{60(1-b^2x^4)^{3/2}}+\frac{x(15-7bx^2)}{40\sqrt{1-b^2x^4}}+\frac{7E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{40\sqrt{b}}+\frac{\operatorname{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{5\sqrt{b}}$$

output
```
1/10*x*(-b*x^2+1)/(-b^2*x^4+1)^(5/2)+1/60*x*(-7*b*x^2+9)/(-b^2*x^4+1)^(3/2
)+1/40*x*(-7*b*x^2+15)/(-b^2*x^4+1)^(1/2)+7/40*EllipticE(b^(1/2)*x,I)/b^(1
/2)+1/5*EllipticF(b^(1/2)*x,I)/b^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68

$$\int \frac{1-bx^2}{(1-b^2x^4)^{7/2}}\,dx=\frac{1}{120}x\left(\frac{75-108b^2x^4+45b^4x^8}{(1-b^2x^4)^{5/2}}+45\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)-40bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{2},\frac{7}{4},b^2x^4\right)\right)$$

input
```
Integrate[(1 - b*x^2)/(1 - b^2*x^4)^(7/2),x]
```

output
```
(x*((75 - 108*b^2*x^4 + 45*b^4*x^8)/(1 - b^2*x^4)^(5/2) + 45*Hypergeometri
c2F1[1/4, 1/2, 5/4, b^2*x^4] - 40*b*x^2*Hypergeometric2F1[3/4, 7/2, 7/4, b
^2*x^4]))/120
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.55, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {1388, 316, 27, 402, 27, 402, 27, 402, 27, 402, 25, 27, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{7/2}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{b\left(7bx^2+5\right)}{\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{7/2}}dx}{6b}+\frac{x}{6\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \frac{7bx^2+5}{\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{7/2}}dx+\frac{x}{6\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{5/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{6}\left(\frac{\int -\frac{2b\left(1-30bx^2\right)}{\sqrt{1-bx^2}\left(bx^2+1\right)^{7/2}}dx}{2b}+\frac{6x}{\sqrt{1-bx^2}\left(bx^2+1\right)^{5/2}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{6x}{\sqrt{1-bx^2}\left(bx^2+1\right)^{5/2}}-\int \frac{1-30bx^2}{\sqrt{1-bx^2}\left(bx^2+1\right)^{7/2}}dx\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{5/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{6}\left(\frac{\int \frac{3b\left(31bx^2+7\right)}{\sqrt{1-bx^2}\left(bx^2+1\right)^{5/2}}dx}{10b}-\frac{31x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}+\frac{6x}{\sqrt{1-bx^2}\left(bx^2+1\right)^{5/2}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{3}{10}\int \frac{31bx^2+7}{\sqrt{1-bx^2}\left(bx^2+1\right)^{5/2}}dx-\frac{31x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}+\frac{6x}{\sqrt{1-bx^2}\left(bx^2+1\right)^{5/2}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{5/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{6}\left(\frac{3}{10}\left(-\frac{\int -\frac{6b\left(4bx^2+11\right)}{\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}dx}{6b}-\frac{4x\sqrt{1-bx^2}}{\left(bx^2+1\right)^{3/2}}\right)-\frac{31x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}+\frac{6x}{\sqrt{1-bx^2}\left(bx^2+1\right)^{5/2}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{3}{10}\left(\int \frac{4bx^2+11}{\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}dx-\frac{4x\sqrt{1-bx^2}}{\left(bx^2+1\right)^{3/2}}\right)-\frac{31x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}+\frac{6x}{\sqrt{1-bx^2}\left(bx^2+1\right)^{5/2}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{5/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{6}\left(\frac{3}{10}\left(-\frac{\int -\frac{b\left(7bx^2+15\right)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{2b}+\frac{7x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}-\frac{4x\sqrt{1-bx^2}}{\left(bx^2+1\right)^{3/2}}\right)-\frac{31x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}+\frac{6x}{\sqrt{1-bx^2}\left(bx^2+1\right)^{5/2}}\right)+\frac{x}{6\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}\left(\frac{3}{10}\left(\frac{\int \frac{b(7bx^2+15)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{2b}+\frac{7x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}-\frac{4x\sqrt{1-bx^2}}{(bx^2+1)^{3/2}}\right)-\frac{31x\sqrt{1-bx^2}}{10(bx^2+1)^{5/2}}+\frac{6x}{\sqrt{1-bx^2}(bx^2+1)^{5/2}}\right)+$$

$$\frac{x}{6(1-bx^2)^{3/2}(bx^2+1)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{6}\left(\frac{3}{10}\left(\frac{1}{2}\int \frac{7bx^2+15}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+\frac{7x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}-\frac{4x\sqrt{1-bx^2}}{(bx^2+1)^{3/2}}\right)-\frac{31x\sqrt{1-bx^2}}{10(bx^2+1)^{5/2}}+\frac{6x}{\sqrt{1-bx^2}(bx^2+1)^{5/2}}\right.$$

$$\frac{x}{6(1-bx^2)^{3/2}(bx^2+1)^{5/2}}$$

$$\Big\downarrow \ 399$$

$$\frac{1}{6}\left(\frac{3}{10}\left(\frac{1}{2}\left(8\int \frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+7\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)+\frac{7x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}-\frac{4x\sqrt{1-bx^2}}{(bx^2+1)^{3/2}}\right)-\frac{31x\sqrt{1-bx^2}}{10(bx^2+1)^{5/2}}\right.$$

$$\frac{x}{6(1-bx^2)^{3/2}(bx^2+1)^{5/2}}$$

$$\Big\downarrow \ 284$$

$$\frac{1}{6}\left(\frac{3}{10}\left(\frac{1}{2}\left(8\int \frac{1}{\sqrt{1-b^2x^4}}dx+7\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)+\frac{7x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}-\frac{4x\sqrt{1-bx^2}}{(bx^2+1)^{3/2}}\right)-\frac{31x\sqrt{1-bx^2}}{10(bx^2+1)^{5/2}}+\right.$$

$$\frac{x}{6(1-bx^2)^{3/2}(bx^2+1)^{5/2}}$$

$$\Big\downarrow \ 327$$

$$\frac{1}{6}\left(\frac{3}{10}\left(\frac{1}{2}\left(8\int \frac{1}{\sqrt{1-b^2x^4}}dx+\frac{7E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)+\frac{7x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}-\frac{4x\sqrt{1-bx^2}}{(bx^2+1)^{3/2}}\right)-\frac{31x\sqrt{1-bx^2}}{10(bx^2+1)^{5/2}}\right.$$

$$\frac{x}{6(1-bx^2)^{3/2}(bx^2+1)^{5/2}}$$

$$\Big\downarrow \ 762$$

$$\frac{1}{6}\left(\frac{3}{10}\left(\frac{1}{2}\left(\frac{8\,\mathrm{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}+\frac{7E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)+\frac{7x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}-\frac{4x\sqrt{1-bx^2}}{(bx^2+1)^{3/2}}\right)-\right.$$

$$\frac{x}{6(1-bx^2)^{3/2}(bx^2+1)^{5/2}}$$

input

```
Int[(1 - b*x^2)/(1 - b^2*x^4)^(7/2),x]
```

output

```
x/(6*(1 - b*x^2)^(3/2)*(1 + b*x^2)^(5/2)) + ((6*x)/(Sqrt[1 - b*x^2]*(1 + b
*x^2)^(5/2)) - (31*x*Sqrt[1 - b*x^2])/(10*(1 + b*x^2)^(5/2)) + (3*((-4*x*S
qrt[1 - b*x^2])/(1 + b*x^2)^(3/2) + (7*x*Sqrt[1 - b*x^2])/(2*Sqrt[1 + b*x^
2]) + ((7*EllipticE[ArcSin[Sqrt[b]*x], -1])/Sqrt[b] + (8*EllipticF[ArcSin[
Sqrt[b]*x], -1])/Sqrt[b])/2))/10)/6
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.30

method	result
meijerg	$x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{7}{2}\right], \left[\frac{5}{4}\right], b^{2} x^{4}\right)-\frac{b\,x^{3} \mathrm{hypergeom}\left(\left[\frac{3}{4}, \frac{7}{2}\right], \left[\frac{7}{4}\right], b^{2} x^{4}\right)}{3}$
elliptic	$\frac{x \sqrt{-b^{2} x^{4}+1}}{48 b^{2} \left(x^{2}-\frac{1}{b}\right)^{2}}-\frac{5 \left(-b^{2} x^{2}-b \right) x}{32 b \sqrt{\left(x^{2}-\frac{1}{b}\right) \left(-b^{2} x^{2}-b \right)}}+\frac{x \sqrt{-b^{2} x^{4}+1}}{40 b^{3} \left(x^{2}+\frac{1}{b}\right)^{3}}+\frac{11 x \sqrt{-b^{2} x^{4}+1}}{120 b^{2} \left(x^{2}+\frac{1}{b}\right)^{2}}+\frac{53 \left(-b^{2} x^{2}+b \right) x}{160 b \sqrt{\left(x^{2}+\frac{1}{b}\right) \left(-b^{2} x^{2}+b \right)}}+\frac{3 \sqrt{-b\,x^{2}+1}}{8}$
default	$-b \left(-\frac{x^{3} \sqrt{-b^{2} x^{4}+1}}{10 b^{6} \left(x^{4}-\frac{1}{b^{2}}\right)^{3}}+\frac{7 x^{3} \sqrt{-b^{2} x^{4}+1}}{60 b^{4} \left(x^{4}-\frac{1}{b^{2}}\right)^{2}}+\frac{7 x^{3}}{40 \sqrt{-\left(x^{4}-\frac{1}{b^{2}}\right) b^{2}}}+\frac{7 \sqrt{-b\,x^{2}+1}\, \sqrt{b\,x^{2}+1}\, \left(\mathrm{EllipticF}\left(\sqrt{b}\, x , i\right)-\mathrm{EllipticE}\left(\sqrt{b}\right.\right.}{40 b^{\frac{3}{2}} \sqrt{-b^{2} x^{4}+1}}\right.$

input
```
int((-b*x^2+1)/(-b^2*x^4+1)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,7/2],[5/4],b^2*x^4)-1/3*b*x^3*hypergeom([3/4,7/2],[7/4],b
^2*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(93) = 186$.

Time = 0.08 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.99

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{7/2}}\,dx = \frac{21\left(b^6x^{10}+b^5x^8-2\,b^4x^6-2\,b^3x^4+b^2x^2+b\right)\sqrt{b}E\left(\arcsin\left(\sqrt{b}x\right)\,|-1\right)-3\left(\left(7\,b^6-\right.\right.}{}$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(7/2),x, algorithm="fricas")
```

output
```
1/120*(21*(b^6*x^10 + b^5*x^8 - 2*b^4*x^6 - 2*b^3*x^4 + b^2*x^2 + b)*sqrt(
b)*elliptic_e(arcsin(sqrt(b)*x), -1) - 3*((7*b^6 - 15*b^5)*x^10 + (7*b^5 -
 15*b^4)*x^8 - 2*(7*b^4 - 15*b^3)*x^6 - 2*(7*b^3 - 15*b^2)*x^4 + (7*b^2 -
15*b)*x^2 + 7*b - 15)*sqrt(b)*elliptic_f(arcsin(sqrt(b)*x), -1) + (21*b^5*
x^9 - 24*b^4*x^7 - 80*b^3*x^5 + 28*b^2*x^3 + 75*b*x)*sqrt(-b^2*x^4 + 1))/(
b^6*x^10 + b^5*x^8 - 2*b^4*x^6 - 2*b^3*x^4 + b^2*x^2 + b)
```

Sympy [A] (verification not implemented)

Time = 14.56 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{7/2}}\,dx = -\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\\frac{7}{4}\end{matrix}\middle|\,b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\\frac{5}{4}\end{matrix}\middle|\,b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(-b**2*x**4+1)**(7/2),x)
```

output
```
-b*x**3*gamma(3/4)*hyper((3/4, 7/2), (7/4,), b**2*x**4*exp_polar(2*I*pi))/
(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 7/2), (5/4,), b**2*x**4*exp_pola
r(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{(1-b^2x^4)^{7/2}}\,dx = \int -\frac{bx^2-1}{(-b^2x^4+1)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(7/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/(-b^2*x^4 + 1)^(7/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{(1-b^2x^4)^{7/2}}\,dx = \int -\frac{bx^2-1}{(-b^2x^4+1)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(7/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/(-b^2*x^4 + 1)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{(1-b^2x^4)^{7/2}}\,dx = -\int \frac{b\,x^2-1}{(1-b^2\,x^4)^{7/2}}\,dx$$

input
```
int(-(b*x^2 - 1)/(1 - b^2*x^4)^(7/2),x)
```

output
```
-int((b*x^2 - 1)/(1 - b^2*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{7/2}}\,dx = -\left(\int \frac{\sqrt{-b^2x^4+1}}{b^7x^{14}+b^6x^{12}-3b^5x^{10}-3b^4x^8+3b^3x^6+3b^2x^4-b\,x^2-1}dx\right)$$

input
```
int((-b*x^2+1)/(-b^2*x^4+1)^(7/2),x)
```

output
```
 - int(sqrt( - b**2*x**4 + 1)/(b**7*x**14 + b**6*x**12 - 3*b**5*x**10 - 3*
b**4*x**8 + 3*b**3*x**6 + 3*b**2*x**4 - b*x**2 - 1),x)
```

3.240 $\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{9/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{9/2}}\,dx=\frac{x(1-bx^2)}{14\left(1-b^2x^4\right)^{7/2}}+\frac{x(13-11bx^2)}{140\left(1-b^2x^4\right)^{5/2}}+\frac{x(117-77bx^2)}{840\left(1-b^2x^4\right)^{3/2}}+\frac{x(195-77bx^2)}{560\sqrt{1-b^2x^4}}+\frac{11E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{80\sqrt{b}}+\frac{59\,\mathrm{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{280\sqrt{b}}$$

output
```
1/14*x*(-b*x^2+1)/(-b^2*x^4+1)^(7/2)+1/140*x*(-11*b*x^2+13)/(-b^2*x^4+1)^(
5/2)+1/840*x*(-77*b*x^2+117)/(-b^2*x^4+1)^(3/2)+1/560*x*(-77*b*x^2+195)/(-
b^2*x^4+1)^(1/2)+11/80*EllipticE(b^(1/2)*x,I)/b^(1/2)+59/280*EllipticF(b^(
1/2)*x,I)/b^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{9/2}}\,dx=\frac{x\left(-\frac{3\left(-365+793b^2x^4-663b^4x^8+195b^6x^{12}\right)}{\left(1-b^2x^4\right)^{7/2}}+585\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)-560bx}{1680}$$

input
```
Integrate[(1 - b*x^2)/(1 - b^2*x^4)^(9/2),x]
```

output
```
(x*((-3*(-365 + 793*b^2*x^4 - 663*b^4*x^8 + 195*b^6*x^12))/(1 - b^2*x^4)^(
7/2) + 585*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] - 560*b*x^2*Hypergeom
etric2F1[3/4, 9/2, 7/4, b^2*x^4]))/1680
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.76, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {1388, 316, 27, 402, 27, 402, 25, 27, 402, 27, 402, 27, 402, 27, 402, 25, 27, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\left(1-b^2x^4\right)^{9/2}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{\left(1-bx^2\right)^{7/2}\left(bx^2+1\right)^{9/2}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{b\left(11bx^2+9\right)}{\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{9/2}}dx}{10b}+\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\int \frac{11bx^2+9}{\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{9/2}}dx+\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{10}\left(\frac{\int \frac{2b\left(90bx^2+17\right)}{\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{9/2}}dx}{6b}+\frac{10x}{3\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{7/2}}\right)+\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\frac{1}{3}\int \frac{90bx^2+17}{\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{9/2}}dx+\frac{10x}{3\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{7/2}}\right)+\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow\ 402$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{\int -\frac{b\left(73-749bx^2\right)}{\sqrt{1-bx^2}\left(bx^2+1\right)^{9/2}}dx}{2b}+\frac{107x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{7/2}}\right)+\frac{10x}{3\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{7/2}}\right)+\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow\ 25$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{107x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{7/2}}-\frac{\int \frac{b\left(73-749bx^2\right)}{\sqrt{1-bx^2}\left(bx^2+1\right)^{9/2}}dx}{2b}\right)+\frac{10x}{3\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{7/2}}\right)+\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow\ 27$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{107x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{7/2}}-\frac{1}{2}\int \frac{73-749bx^2}{\sqrt{1-bx^2}\left(bx^2+1\right)^{9/2}}dx\right)+\frac{10x}{3\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{7/2}}\right)+\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow\ 402$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{\int -\frac{10b\left(20-411bx^2\right)}{\sqrt{1-bx^2}\left(bx^2+1\right)^{7/2}}dx}{14b}-\frac{411x\sqrt{1-bx^2}}{7\left(bx^2+1\right)^{7/2}}\right)+\frac{107x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{7/2}}\right)+\frac{10x}{3\left(1-bx^2\right)^{3/2}\left(bx^2+1\right)^{7/}}\right.$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow\ 27$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{1}{2}\left(-\frac{5}{7}\int \frac{20-411bx^2}{\sqrt{1-bx^2}\left(bx^2+1\right)^{7/2}}dx-\frac{411x\sqrt{1-bx^2}}{7\left(bx^2+1\right)^{7/2}}\right)+\frac{107x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{7/2}}\right)+\frac{10x}{3\left(1-bx^2\right)^{3/2}\left(bx\right.}\right.$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow\ 402$$

$$\frac{1}{10}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(-\frac{5}{7}\Bigg(\frac{431x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}-\frac{\int\frac{3b\left(431bx^2+77\right)}{\sqrt{1-bx^2}\left(bx^2+1\right)^{5/2}}dx}{10b}\Bigg)-\frac{411x\sqrt{1-bx^2}}{7\left(bx^2+1\right)^{7/2}}\Bigg)+\frac{107x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{7/2}}\Bigg)+$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow\ 27$$

$$\frac{1}{10}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(-\frac{5}{7}\Bigg(\frac{431x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}-\frac{3}{10}\int\frac{431bx^2+77}{\sqrt{1-bx^2}\left(bx^2+1\right)^{5/2}}dx\Bigg)-\frac{411x\sqrt{1-bx^2}}{7\left(bx^2+1\right)^{7/2}}\Bigg)+\frac{107x}{2\sqrt{1-bx^2}\left(bx^2+1\right)^{7}}$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow\ 402$$

$$\frac{1}{10}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(-\frac{5}{7}\Bigg(\frac{431x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}-\frac{3}{10}\Bigg(-\frac{\int-\frac{6b\left(59bx^2+136\right)}{\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}dx}{6b}-\frac{59x\sqrt{1-bx^2}}{\left(bx^2+1\right)^{3/2}}\Bigg)\Bigg)-\frac{411x\sqrt{1-bx^2}}{7\left(bx^2+1\right)^{7/2}}\Bigg)+$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow\ 27$$

$$\frac{1}{10}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(-\frac{5}{7}\Bigg(\frac{431x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}-\frac{3}{10}\Bigg(\int\frac{59bx^2+136}{\sqrt{1-bx^2}\left(bx^2+1\right)^{3/2}}dx-\frac{59x\sqrt{1-bx^2}}{\left(bx^2+1\right)^{3/2}}\Bigg)\Bigg)-\frac{411x\sqrt{1-bx^2}}{7\left(bx^2+1\right)^{7/2}}\Bigg)+$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow\ 402$$

$$\frac{1}{10}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(-\frac{5}{7}\Bigg(\frac{431x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}-\frac{3}{10}\Bigg(-\frac{\int-\frac{b\left(77bx^2+195\right)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{2b}+\frac{77x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}-\frac{59x\sqrt{1-bx^2}}{\left(bx^2+1\right)^{3/2}}\Bigg)\Bigg)-\frac{411x\sqrt{1}}{7\left(bx^2+\right.}$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow\ 25$$

$$\frac{1}{10}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(-\frac{5}{7}\Bigg(\frac{431x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}-\frac{3}{10}\Bigg(\frac{\int\frac{b\left(77bx^2+195\right)}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{2b}+\frac{77x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}-\frac{59x\sqrt{1-bx^2}}{\left(bx^2+1\right)^{3/2}}\Bigg)\Bigg)-\frac{411x\sqrt{1-b}}{7\left(bx^2+1\right)^{7}}$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{1}{2}\left(-\frac{5}{7}\left(\frac{431x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}-\frac{3}{10}\left(\frac{1}{2}\int\frac{77bx^2+195}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+\frac{77x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}-\frac{59x\sqrt{1-bx^2}}{\left(bx^2+1\right)^{3/2}}\right)\right)-\frac{411x}{7\left(bx\right.}\right.\right.\right.$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow \; 399$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{1}{2}\left(-\frac{5}{7}\left(\frac{431x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}-\frac{3}{10}\left(\frac{1}{2}\left(118\int\frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx+77\int\frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)+\frac{77x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}\right.\right.\right.\right.\right.\right.$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow \; 284$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{1}{2}\left(-\frac{5}{7}\left(\frac{431x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}-\frac{3}{10}\left(\frac{1}{2}\left(118\int\frac{1}{\sqrt{1-b^2x^4}}dx+77\int\frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)+\frac{77x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}-\frac{59x\sqrt{}}{\left(bx^2\right.}\right.\right.\right.\right.\right.$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{1}{2}\left(-\frac{5}{7}\left(\frac{431x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}-\frac{3}{10}\left(\frac{1}{2}\left(118\int\frac{1}{\sqrt{1-b^2x^4}}dx+\frac{77E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)+\frac{77x\sqrt{1-bx^2}}{2\sqrt{bx^2+1}}\right.\right.\right.\right.\right.$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

$$\Big\downarrow \; 762$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{1}{2}\left(-\frac{5}{7}\left(\frac{431x\sqrt{1-bx^2}}{10\left(bx^2+1\right)^{5/2}}-\frac{3}{10}\left(\frac{1}{2}\left(\frac{118\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}+\frac{77E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)\right.\right.\right.\right.\right.$$

$$\frac{x}{10\left(1-bx^2\right)^{5/2}\left(bx^2+1\right)^{7/2}}$$

input

```
Int[(1 - b*x^2)/(1 - b^2*x^4)^(9/2),x]
```

output
```
x/(10*(1 - b*x^2)^(5/2)*(1 + b*x^2)^(7/2)) + ((10*x)/(3*(1 - b*x^2)^(3/2)*
(1 + b*x^2)^(7/2)) + ((107*x)/(2*Sqrt[1 - b*x^2]*(1 + b*x^2)^(7/2)) + ((-4
11*x*Sqrt[1 - b*x^2])/(7*(1 + b*x^2)^(7/2)) - (5*((431*x*Sqrt[1 - b*x^2])/
(10*(1 + b*x^2)^(5/2)) - (3*((-59*x*Sqrt[1 - b*x^2])/(1 + b*x^2)^(3/2) + (
77*x*Sqrt[1 - b*x^2])/(2*Sqrt[1 + b*x^2]) + ((77*EllipticE[ArcSin[Sqrt[b]*
x], -1])/Sqrt[b] + (118*EllipticF[ArcSin[Sqrt[b]*x], -1])/Sqrt[b])/2))/10)
)/7)/2)/3)/10
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.24

method	result
meijerg	$x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{9}{2}\right], \left[\frac{5}{4}\right], b^{2} x^{4}\right)-\frac{b \,x^{3} \mathrm{hypergeom}\left(\left[\frac{3}{4}, \frac{9}{2}\right], \left[\frac{7}{4}\right], b^{2} x^{4}\right)}{3}$
elliptic	$-\frac{x \sqrt{-b^{2} x^{4}+1}}{160 b^{3} \left(x^{2}-\frac{1}{b}\right)^{3}}+\frac{x \sqrt{-b^{2} x^{4}+1}}{30 b^{2} \left(x^{2}-\frac{1}{b}\right)^{2}}-\frac{27 \left(-b^{2} x^{2}-b \right) x}{160 b \sqrt{\left(x^{2}-\frac{1}{b}\right) \left(-b^{2} x^{2}-b \right)}}+\frac{x \sqrt{-b^{2} x^{4}+1}}{112 b^{4} \left(x^{2}+\frac{1}{b}\right)^{4}}+\frac{39 x \sqrt{-b^{2} x^{4}+1}}{1120 b^{3} \left(x^{2}+\frac{1}{b}\right)^{3}}+\frac{157 x \sqrt{-b^{2} x^{4}+1}}{1680 b^{2} \left(x^{2}+\frac{1}{b}\right)^{2}}$
default	$-b \left(\frac{x^{3} \sqrt{-b^{2} x^{4}+1}}{14 b^{8} \left(x^{4}-\frac{1}{b^{2}}\right)^{4}}-\frac{11 x^{3} \sqrt{-b^{2} x^{4}+1}}{140 b^{6} \left(x^{4}-\frac{1}{b^{2}}\right)^{3}}+\frac{11 x^{3} \sqrt{-b^{2} x^{4}+1}}{120 b^{4} \left(x^{4}-\frac{1}{b^{2}}\right)^{2}}+\frac{11 x^{3}}{80 \sqrt{-\left(x^{4}-\frac{1}{b^{2}}\right) b^{2}}}+\frac{11 \sqrt{-b \,x^{2}+1}\, \sqrt{b \,x^{2}+1}\, \left(\mathrm{EllipticF}\left(\sqrt{}\right.\right.}{80 b^{\frac{3}{2}} \sqrt{-b^{2} x^{4}}}\right.$

input
```
int((-b*x^2+1)/(-b^2*x^4+1)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,9/2],[5/4],b^2*x^4)-1/3*b*x^3*hypergeom([3/4,9/2],[7/4],b
^2*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(116) = 232$.

Time = 0.08 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.22

$$\int \frac{1-bx^2}{(1-b^2x^4)^{9/2}}\,dx = \frac{231\,(b^8x^{14}+b^7x^{12}-3\,b^6x^{10}-3\,b^5x^8+3\,b^4x^6+3\,b^3x^4-b^2x^2-b)\sqrt{b}E(\arcsin\left(\sqrt{b}x\right.}{}$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(9/2),x, algorithm="fricas")
```

output
```
1/1680*(231*(b^8*x^14 + b^7*x^12 - 3*b^6*x^10 - 3*b^5*x^8 + 3*b^4*x^6 + 3*
b^3*x^4 - b^2*x^2 - b)*sqrt(b)*elliptic_e(arcsin(sqrt(b)*x), -1) - 3*((77*
b^8 - 195*b^7)*x^14 + (77*b^7 - 195*b^6)*x^12 - 3*(77*b^6 - 195*b^5)*x^10
- 3*(77*b^5 - 195*b^4)*x^8 + 3*(77*b^4 - 195*b^3)*x^6 + 3*(77*b^3 - 195*b^
2)*x^4 - (77*b^2 - 195*b)*x^2 - 77*b + 195)*sqrt(b)*elliptic_f(arcsin(sqrt
(b)*x), -1) + (231*b^7*x^13 - 354*b^6*x^11 - 1201*b^5*x^9 + 788*b^4*x^7 +
1921*b^3*x^5 - 458*b^2*x^3 - 1095*b*x)*sqrt(-b^2*x^4 + 1))/(b^8*x^14 + b^7
*x^12 - 3*b^6*x^10 - 3*b^5*x^8 + 3*b^4*x^6 + 3*b^3*x^4 - b^2*x^2 - b)
```

Sympy [A] (verification not implemented)

Time = 35.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.48

$$\int \frac{1-bx^2}{(1-b^2x^4)^{9/2}}\,dx = -\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{9}{2}\\ \frac{7}{4}\end{matrix}\middle|\,b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{9}{2}\\ \frac{5}{4}\end{matrix}\middle|\,b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(-b**2*x**4+1)**(9/2),x)
```

output
```
-b*x**3*gamma(3/4)*hyper((3/4, 9/2), (7/4,), b**2*x**4*exp_polar(2*I*pi))/
(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 9/2), (5/4,), b**2*x**4*exp_pola
r(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{(1-b^2x^4)^{9/2}}\,dx = \int -\frac{bx^2-1}{(-b^2x^4+1)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(9/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/(-b^2*x^4 + 1)^(9/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{(1-b^2x^4)^{9/2}}\,dx = \int -\frac{bx^2-1}{(-b^2x^4+1)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(9/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/(-b^2*x^4 + 1)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{(1-b^2x^4)^{9/2}}\,dx = -\int \frac{b\,x^2-1}{(1-b^2\,x^4)^{9/2}}\,dx$$

input
```
int(-(b*x^2 - 1)/(1 - b^2*x^4)^(9/2),x)
```

output
```
-int((b*x^2 - 1)/(1 - b^2*x^4)^(9/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{(1-b^2x^4)^{9/2}}\,dx = \int \frac{\sqrt{-b^2x^4+1}}{b^9x^{18}+b^8x^{16}-4b^7x^{14}-4b^6x^{12}+6b^5x^{10}+6b^4x^8-4b^3x^6-4b^2x^4+b\,x^2+1}dx$$

input
```
int((-b*x^2+1)/(-b^2*x^4+1)^(9/2),x)
```

output
```
int(sqrt( - b**2*x**4 + 1)/(b**9*x**18 + b**8*x**16 - 4*b**7*x**14 - 4*b**
6*x**12 + 6*b**5*x**10 + 6*b**4*x**8 - 4*b**3*x**6 - 4*b**2*x**4 + b*x**2
+ 1),x)
```

3.241 $\int \left(d+ex^2\right) \left(a+cx^4\right)^{5/2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 319

$$\begin{aligned}\int \left(d+ex^2\right) \left(a+cx^4\right)^{5/2} \, dx &= \frac{8a^3ex\sqrt{a+cx^4}}{39\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{4a^2x(195d+77ex^2)\sqrt{a+cx^4}}{3003} \\ &+\frac{10ax(117d+77ex^2)\left(a+cx^4\right)^{3/2}}{9009}+\frac{1}{143}x\left(13d+11ex^2\right)\left(a+cx^4\right)^{5/2} \\ &-\frac{8a^{13/4}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{39c^{3/4}\sqrt{a+cx^4}} \\ &+\frac{4a^{11/4}\left(195\sqrt{c}d+77\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3003c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

output
```
8/39*a^3*e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)+4/3003*a^2*x*(7
7*e*x^2+195*d)*(c*x^4+a)^(1/2)+10/9009*a*x*(77*e*x^2+117*d)*(c*x^4+a)^(3/2
)+1/143*x*(11*e*x^2+13*d)*(c*x^4+a)^(5/2)-8/39*a^(13/4)*e*(a^(1/2)+c^(1/2)
*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(
1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)+4/3003*a^(11/4)*(195
*c^(1/2)*d+77*a^(1/2)*e)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)
*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c^
(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.25

$$\int \left(d+ex^2\right) \left(a+cx^4\right)^{5/2} dx = \frac{a^2\sqrt{a+cx^4}\left(3dx \operatorname{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)+ex^3 \operatorname{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},\frac{7}{4}\right.\right.}{3\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(d + e*x^2)*(a + c*x^4)^(5/2),x]
```

output
```
(a^2*Sqrt[a + c*x^4]*(3*d*x*Hypergeometric2F1[-5/2, 1/4, 5/4, -((c*x^4)/a)
] + e*x^3*Hypergeometric2F1[-5/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[1 + (c
*x^4)/a])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1491, 27, 1491, 27, 1491, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^{5/2} \left(d+ex^2\right) dx$$

$$\Big\downarrow \text{ 1491}$$

$$\frac{5}{143} \int 2a\left(11ex^2+13d\right) \left(cx^4+a\right)^{3/2} dx+\frac{1}{143}x\left(a+cx^4\right)^{5/2} \left(13d+11ex^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{10}{143}a \int \left(11ex^2+13d\right) \left(cx^4+a\right)^{3/2} dx+\frac{1}{143}x\left(a+cx^4\right)^{5/2} \left(13d+11ex^2\right)$$

$$\Big\downarrow \text{ 1491}$$

$$\frac{10}{143}a\left(\frac{1}{21}\int 2a\left(77ex^2+117d\right)\sqrt{cx^4+a}dx+\frac{1}{63}x\left(a+cx^4\right)^{3/2}\left(117d+77ex^2\right)\right)+$$
$$\frac{1}{143}x\left(a+cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \ 27$$

$$\frac{10}{143}a\left(\frac{2}{21}a\int \left(77ex^2+117d\right)\sqrt{cx^4+a}dx+\frac{1}{63}x\left(a+cx^4\right)^{3/2}\left(117d+77ex^2\right)\right)+$$
$$\frac{1}{143}x\left(a+cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \ 1491$$

$$\frac{10}{143}a\left(\frac{2}{21}a\left(\frac{1}{15}\int \frac{6a\left(77ex^2+195d\right)}{\sqrt{cx^4+a}}dx+\frac{1}{5}x\sqrt{a+cx^4}\left(195d+77ex^2\right)\right)+\frac{1}{63}x\left(a+cx^4\right)^{3/2}\left(117d+77ex^2\right)\right)+$$
$$\frac{1}{143}x\left(a+cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \ 27$$

$$\frac{10}{143}a\left(\frac{2}{21}a\left(\frac{2}{5}a\int \frac{77ex^2+195d}{\sqrt{cx^4+a}}dx+\frac{1}{5}x\sqrt{a+cx^4}\left(195d+77ex^2\right)\right)+\frac{1}{63}x\left(a+cx^4\right)^{3/2}\left(117d+77ex^2\right)\right)+$$
$$\frac{1}{143}x\left(a+cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \ 1512$$

$$\frac{10}{143}a\left(\frac{2}{21}a\left(\frac{2}{5}a\left(\left(\frac{77\sqrt{a}e}{\sqrt{c}}+195d\right)\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{77\sqrt{a}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{5}x\sqrt{a+cx^4}\left(195d+77ex^2\right)\right)$$
$$\frac{1}{143}x\left(a+cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \ 27$$

$$\frac{10}{143}a\left(\frac{2}{21}a\left(\frac{2}{5}a\left(\left(\frac{77\sqrt{a}e}{\sqrt{c}}+195d\right)\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{77e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{5}x\sqrt{a+cx^4}\left(195d+77ex^2\right)\right)+$$
$$\frac{1}{143}x\left(a+cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \ 761$$

$$\frac{10}{143}a\Bigg(\frac{2}{21}a\Bigg(\frac{2}{5}a\Bigg(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{77\sqrt{a}e}{\sqrt{c}}+195d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{77e\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^4+a}}}{\sqrt{c}}$$

$$\frac{1}{143}x\left(a+cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \text{1510}$$

$$\frac{10}{143}a\Bigg(\frac{2}{21}a\Bigg(\frac{2}{5}a\Bigg(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{77\sqrt{a}e}{\sqrt{c}}+195d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{77e\Bigg(\frac{\sqrt[4]{a}(\sqrt{a}+}{}}{}$$

$$\frac{1}{143}x\left(a+cx^4\right)^{5/2}\left(13d+11ex^2\right)$$

input
```
Int[(d + e*x^2)*(a + c*x^4)^(5/2),x]
```

output
```
(x*(13*d + 11*e*x^2)*(a + c*x^4)^(5/2))/143 + (10*a*((x*(117*d + 77*e*x^2)
*(a + c*x^4)^(3/2))/63 + (2*a*((x*(195*d + 77*e*x^2)*Sqrt[a + c*x^4])/5 +
(2*a*((-77*e*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/Sqrt[c]
 + ((195*d + (77*Sqrt[a]*e)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/
2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))/5))/21))/143
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.75

method	result
risch	$\frac{x\left(693c^{2}e\,x^{10}+819c^{2}d\,x^{8}+2156ace\,x^{6}+2808acd\,x^{4}+2387a^{2}e\,x^{2}+4329a^{2}d\right)\sqrt{c\,x^{4}+a}}{9009}+\frac{8a^{3}\Bigg(\frac{195d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\ \mathrm{EllipticF}\big(}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}}{}$
default	$d\left(\frac{c^{2}x^{9}\sqrt{c\,x^{4}+a}}{11}+\frac{24ac\,x^{5}\sqrt{c\,x^{4}+a}}{77}+\frac{37a^{2}x\sqrt{c\,x^{4}+a}}{77}+\frac{40a^{3}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{77\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+e\Bigg(\frac{c^{2}}{}$
elliptic	$\frac{e\,c^{2}x^{11}\sqrt{c\,x^{4}+a}}{13}+\frac{c^{2}d\,x^{9}\sqrt{c\,x^{4}+a}}{11}+\frac{28ace\,x^{7}\sqrt{c\,x^{4}+a}}{117}+\frac{24acd\,x^{5}\sqrt{c\,x^{4}+a}}{77}+\frac{31a^{2}e\,x^{3}\sqrt{c\,x^{4}+a}}{117}+\frac{37a^{2}dx\sqrt{c\,x^{4}+a}}{77}+\frac{40}{}$

input

```
int((e*x^2+d)*(c*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/9009*x*(693*c^2*e*x^10+819*c^2*d*x^8+2156*a*c*e*x^6+2808*a*c*d*x^4+2387*
a^2*e*x^2+4329*a^2*d)*(c*x^4+a)^(1/2)+8/3003*a^3*(195*d/(I/a^(1/2)*c^(1/2)
)^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c
*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+77*I*e*a^(1/2)/(I/a
^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)
*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2)
,I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.50

$$\int \left(d+ex^{2}\right)\left(a+cx^{4}\right)^{5/2}dx=\frac{1848\,a^{3}\sqrt{c}ex\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,\middle|\,-1\right)+24\left(195\,a^{2}cd-77\,a^{3}e\right)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\Bigg(\arcsin\Bigg(}{}$$

input

```
integrate((e*x^2+d)*(c*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
1/9009*(1848*a^3*sqrt(c)*e*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x
), -1) + 24*(195*a^2*c*d - 77*a^3*e)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arc
sin((-a/c)^(1/4)/x), -1) + (693*c^3*e*x^12 + 819*c^3*d*x^10 + 2156*a*c^2*e
*x^8 + 2808*a*c^2*d*x^6 + 2387*a^2*c*e*x^4 + 4329*a^2*c*d*x^2 + 1848*a^3*e
)*sqrt(c*x^4 + a))/(c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.46 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.82

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^{5/2}dx=\frac{a^{\frac{5}{2}}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{\frac{5}{2}}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}cdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}+\frac{a^{\frac{3}{2}}cex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}c^2dx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{\sqrt{a}c^2ex^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((e*x**2+d)*(c*x**4+a)**(5/2),x)
```

output
```
a**(5/2)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/
a)/(4*gamma(5/4)) + a**(5/2)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,),
c*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**(3/2)*c*d*x**5*gamma(5/4)*hy
per((-1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(2*gamma(9/4)) + a**(3/
2)*c*e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/
a)/(2*gamma(11/4)) + sqrt(a)*c**2*d*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13
/4,), c*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4)) + sqrt(a)*c**2*e*x**11*gam
ma(11/4)*hyper((-1/2, 11/4), (15/4,), c*x**4*exp_polar(I*pi)/a)/(4*gamma(1
5/4))
```

Maxima [F]

$$\int (d+ex^2)\left(a+cx^4\right)^{5/2}\,dx=\int \left(cx^4+a\right)^{\frac{5}{2}}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(5/2)*(e*x^2 + d), x)
```

Giac [F]

$$\int (d+ex^2)\left(a+cx^4\right)^{5/2}\,dx=\int \left(cx^4+a\right)^{\frac{5}{2}}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(5/2)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right) \left(a+cx^4\right)^{5/2} \, dx=\int \left(c\,x^4+a\right)^{5/2}\,\left(e\,x^2+d\right) \,dx$$

input
```
int((a + c*x^4)^(5/2)*(d + e*x^2),x)
```

output
```
int((a + c*x^4)^(5/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right) \left(a+cx^4\right)^{5/2} \, dx=\frac{37\sqrt{c\,x^4+a}\,a^2dx}{77}+\frac{31\sqrt{c\,x^4+a}\,a^2e\,x^3}{117}+\frac{24\sqrt{c\,x^4+a}\,acd\,x^5}{77}+\frac{28\sqrt{c\,x^4+a}\,ace\,x^7}{117}+\frac{\sqrt{c\,x^4+a}\,c^2d\,x^9}{11}+\frac{\sqrt{c\,x^4+a}\,c^2e\,x^{11}}{13}+\frac{40\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)a^3d}{77}+\frac{8\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}dx\right)a^3e}{39}$$

input
```
int((e*x^2+d)*(c*x^4+a)^(5/2),x)
```

output
```
(4329*sqrt(a + c*x**4)*a**2*d*x + 2387*sqrt(a + c*x**4)*a**2*e*x**3 + 2808
*sqrt(a + c*x**4)*a*c*d*x**5 + 2156*sqrt(a + c*x**4)*a*c*e*x**7 + 819*sqrt
(a + c*x**4)*c**2*d*x**9 + 693*sqrt(a + c*x**4)*c**2*e*x**11 + 4680*int(sq
rt(a + c*x**4)/(a + c*x**4),x)*a**3*d + 1848*int((sqrt(a + c*x**4)*x**2)/(
a + c*x**4),x)*a**3*e)/9009
```

3.242 $\int (d+ex^2)(a+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 290

$$\int (d+ex^2)\left(a+cx^4\right)^{3/2}dx=\frac{4a^2ex\sqrt{a+cx^4}}{15\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{2}{105}ax\left(15d+7ex^2\right)\sqrt{a+cx^4}$$
$$+\frac{1}{63}x\left(9d+7ex^2\right)\left(a+cx^4\right)^{3/2}-\frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}$$
$$+\frac{2a^{7/4}\left(15\sqrt{c}d+7\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105c^{3/4}\sqrt{a+cx^4}}$$

output

```
4/15*a^2*e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)+2/105*a*x*(7*e*
x^2+15*d)*(c*x^4+a)^(1/2)+1/63*x*(7*e*x^2+9*d)*(c*x^4+a)^(3/2)-4/15*a^(9/4
)*e*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Ellipt
icE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)+
2/105*a^(7/4)*(15*c^(1/2)*d+7*a^(1/2)*e)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/
(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4))
,1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.27

$$\int (d+ex^2)\,(a+cx^4)^{3/2}\,dx = \frac{a\sqrt{a+cx^4}\left(3dx\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},\right.\right.}{3\sqrt{1+\frac{cx^4}{a}}}$$

input

```
Integrate[(d + e*x^2)*(a + c*x^4)^(3/2),x]
```

output

```
(a*Sqrt[a + c*x^4]*(3*d*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^4)/a)]
+ e*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[1 + (c*x
^4)/a])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1491, 27, 1491, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4)^{3/2}\,(d+ex^2)\,dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{21}\int 2a(7ex^2+9d)\sqrt{cx^4+a}dx+\frac{1}{63}x(a+cx^4)^{3/2}\,(9d+7ex^2)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{21}a\int (7ex^2+9d)\sqrt{cx^4+a}dx+\frac{1}{63}x(a+cx^4)^{3/2}\,(9d+7ex^2)$$

$$\Big\downarrow \text{1491}$$

$$\frac{2}{21}a\left(\frac{1}{15}\int \frac{6a\left(7ex^2+15d\right)}{\sqrt{cx^4+a}}dx+\frac{1}{5}x\sqrt{a+cx^4}\left(15d+7ex^2\right)\right)+\frac{1}{63}x\left(a+cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \; 27$$

$$\frac{2}{21}a\left(\frac{2}{5}a\int \frac{7ex^2+15d}{\sqrt{cx^4+a}}dx+\frac{1}{5}x\sqrt{a+cx^4}\left(15d+7ex^2\right)\right)+\frac{1}{63}x\left(a+cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \; 1512$$

$$\frac{2}{21}a\left(\frac{2}{5}a\left(\left(\frac{7\sqrt{a}e}{\sqrt{c}}+15d\right)\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{7\sqrt{a}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{5}x\sqrt{a+cx^4}\left(15d+7ex^2\right)\right)+\frac{1}{63}x\left(a+cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \; 27$$

$$\frac{2}{21}a\left(\frac{2}{5}a\left(\left(\frac{7\sqrt{a}e}{\sqrt{c}}+15d\right)\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{7e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{5}x\sqrt{a+cx^4}\left(15d+7ex^2\right)\right)+\frac{1}{63}x\left(a+cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \; 761$$

$$\frac{2}{21}a\left(\frac{2}{5}a\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{7\sqrt{a}e}{\sqrt{c}}+15d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{7e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{5}\right.$$
$$\frac{1}{63}x\left(a+cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \; 1510$$

$$\frac{2}{21}a\left(\frac{2}{5}a\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{7\sqrt{a}e}{\sqrt{c}}+15d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{7e\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{}}{}\right.}{}\right.\right.$$
$$\frac{1}{63}x\left(a+cx^4\right)^{3/2}\left(9d+7ex^2\right)$$

input
```
Int[(d + e*x^2)*(a + c*x^4)^(3/2),x]
```

output
```
(x*(9*d + 7*e*x^2)*(a + c*x^4)^(3/2))/63 + (2*a*((x*(15*d + 7*e*x^2)*Sqrt[
a + c*x^4])/5 + (2*a*((-7*e*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)
) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*
x^4])))/Sqrt[c] + ((15*d + (7*Sqrt[a]*e)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))/5))/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.74

method	result
risch	$\frac{x\left(35ce\,x^{6}+45cd\,x^{4}+77ae\,x^{2}+135ad\right)\sqrt{c\,x^{4}+a}}{315}+\frac{4a^{2}\left(\frac{15d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{7ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+}}{}\right.}{105}$
default	$d\left(\frac{c\,x^{5}\sqrt{c\,x^{4}+a}}{7}+\frac{3ax\sqrt{c\,x^{4}+a}}{7}+\frac{4a^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+e\left(\frac{c\,x^{7}\sqrt{c\,x^{4}+a}}{9}+\frac{11a\,x^{3}\sqrt{c}}{45}\right.$
elliptic	$\frac{ce\,x^{7}\sqrt{c\,x^{4}+a}}{9}+\frac{cd\,x^{5}\sqrt{c\,x^{4}+a}}{7}+\frac{11ae\,x^{3}\sqrt{c\,x^{4}+a}}{45}+\frac{3adx\sqrt{c\,x^{4}+a}}{7}+\frac{4a^{2}d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$

input

```
int((e*x^2+d)*(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/315*x*(35*c*e*x^6+45*c*d*x^4+77*a*e*x^2+135*a*d)*(c*x^4+a)^(1/2)+4/105*a
^2*(15*d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(
1/2),I)+7*I*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^
(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x
*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.46

$$\int \left(d+ex^2\right) \left(a +cx^4\right)^{3/2}\,dx = \frac{84\,a^2\sqrt{c}ex\left(-\frac{a}{c}\right)^{\frac{3}{4}} E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right) \mid -1) + 12\,(15\,acd-7\,a^2e)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)}{x}\right.}{315\,cx}$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(84*a^2*sqrt(c)*e*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x),
-1) + 12*(15*a*c*d - 7*a^2*e)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a
/c)^(1/4)/x), -1) + (35*c^2*e*x^8 + 45*c^2*d*x^6 + 77*a*c*e*x^4 + 135*a*c*
d*x^2 + 84*a^2*e)*sqrt(c*x^4 + a))/(c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.59

$$\int \left(d+ex^2\right) \left(a+cx^4\right)^{3/2}\,dx = \frac{a^{\frac{3}{2}}dx\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}ex^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}cdx^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{\sqrt{a}cex^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x**2+d)*(c*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/
a)/(4*gamma(5/4)) + a**(3/2)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,),
c*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*c*d*x**5*gamma(5/4)*hyp
er((-1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + sqrt(a)
*c*e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)
/(4*gamma(11/4))
```

Maxima [F]

$$\int (d+ex^2)\left(a+cx^4\right)^{3/2}\,dx=\int \left(cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2)*(e*x^2 + d), x)
```

Giac [F]

$$\int (d+ex^2)\left(a+cx^4\right)^{3/2}\,dx=\int \left(cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2) \left(a+cx^4\right)^{3/2} \, dx=\int \left(c\,x^4+a\right)^{3/2}\,\left(e\,x^2+d\right) \,dx$$

input
```
int((a + c*x^4)^(3/2)*(d + e*x^2),x)
```

output
```
int((a + c*x^4)^(3/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2) \left(a+cx^4\right)^{3/2} \, dx=\frac{3\sqrt{c\,x^4+a}\,a d x}{7}+\frac{11\sqrt{c\,x^4+a}\,a e\,x^3}{45}+\frac{\sqrt{c\,x^4+a}\,c d\,x^5}{7}+\frac{\sqrt{c\,x^4+a}\,c e\,x^7}{9}+\frac{4\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}d x\right) a^2 d}{7}+\frac{4\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}d x\right) a^2 e}{15}$$

input
```
int((e*x^2+d)*(c*x^4+a)^(3/2),x)
```

output
```
(135*sqrt(a + c*x**4)*a*d*x + 77*sqrt(a + c*x**4)*a*e*x**3 + 45*sqrt(a + c
*x**4)*c*d*x**5 + 35*sqrt(a + c*x**4)*c*e*x**7 + 180*int(sqrt(a + c*x**4)/
(a + c*x**4),x)*a**2*d + 84*int((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)*a*
*2*e)/315
```

3.243 $\int (d+ex^2)\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 261

$$\begin{aligned}\int (d+ex^2)\sqrt{a+cx^4}\,dx &= \frac{2aex\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{1}{15}x\left(5d+3ex^2\right)\sqrt{a+cx^4}\\ &\quad-\frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}\\ &\quad+\frac{a^{3/4}\left(5\sqrt{c}d+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

output
```
2/5*a*e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)+1/15*x*(3*e*x^2+5*
d)*(c*x^4+a)^(1/2)-2/5*a^(5/4)*e*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)
+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1
/2))/c^(3/4)/(c*x^4+a)^(1/2)+1/15*a^(3/4)*(5*c^(1/2)*d+3*a^(1/2)*e)*(a^(1/
2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(
2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.95 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.30

$$\int \left(d+ex^2\right)\sqrt{a+cx^4}\,dx = \frac{\sqrt{a+cx^4}\left(3dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)\right)}{3\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(d + e*x^2)*Sqrt[a + c*x^4],x]
```

output
```
(Sqrt[a + c*x^4]*(3*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^4)/a)] +
e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[1 + (c*x^4
)/a])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1491, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^4}\left(d+ex^2\right)\,dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{15}\int \frac{2a\left(3ex^2+5d\right)}{\sqrt{cx^4+a}}dx+\frac{1}{15}x\sqrt{a+cx^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{15}a\int \frac{3ex^2+5d}{\sqrt{cx^4+a}}dx+\frac{1}{15}x\sqrt{a+cx^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{1512}$$

$$\frac{2}{15}a\left(\left(\frac{3\sqrt{a}e}{\sqrt{c}}+5d\right)\int\frac{1}{\sqrt{cx^4+a}}dx-\frac{3\sqrt{a}e\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{15}x\sqrt{a+cx^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{15}a\left(\left(\frac{3\sqrt{a}e}{\sqrt{c}}+5d\right)\int\frac{1}{\sqrt{cx^4+a}}dx-\frac{3e\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{15}x\sqrt{a+cx^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{15}a\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{3\sqrt{a}e}{\sqrt{c}}+5d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{3e\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+$$
$$\frac{1}{15}x\sqrt{a+cx^4}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{1510}$$

$$\frac{2}{15}a\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{3\sqrt{a}e}{\sqrt{c}}+5d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{3e\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)}}}{\sqrt[4]{c}\sqrt{a+}}\right.}{}\right.$$
$$\frac{1}{15}x\sqrt{a+cx^4}\left(5d+3ex^2\right)$$

input
```
Int[(d + e*x^2)*Sqrt[a + c*x^4],x]
```

output
```
(x*(5*d + 3*e*x^2)*Sqrt[a + c*x^4])/15 + (2*a*((-3*e*(-((x*Sqrt[a + c*x^4]
)/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(c^(1/4)*Sqrt[a + c*x^4])))/Sqrt[c] + ((5*d + (3*Sqrt[a]*e)/Sqrt[c])*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4
])))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.75

method	result
risch	$\frac{x(3e\,x^2+5d)\sqrt{c\,x^4+a}}{15}+\frac{2a\left(\frac{5d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{3ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{E}\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}\right.}{15}$
elliptic	$\frac{e\,x^3\sqrt{c\,x^4+a}}{5}+\frac{dx\sqrt{c\,x^4+a}}{3}+\frac{2ad\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{2ia^{\frac{3}{2}}e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\right.}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c}}$
default	$d\left(\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)+e\left(\frac{x^3\sqrt{c\,x^4+a}}{5}+\frac{2ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{E}\right.}{5\sqrt{}}\right.$

input
```
int((e*x^2+d)*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*e*x^2+5*d)*(c*x^4+a)^(1/2)+2/15*a*(5*d/(I/a^(1/2)*c^(1/2))^(1/2)
*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)
^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3*I*e*a^(1/2)/(I/a^(1/2)*c
^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-Elli
pticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.41

$$\int \left(d+ex^2\right)\sqrt{a+cx^4}\,dx$$
$$=\frac{6\,a\sqrt{c}ex\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid-1)+2\,(5\,cd-3\,ae)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid-1)+(3\,cex^4}{15\,cx}$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(6*a*sqrt(c)*e*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1)
+ 2*(5*c*d - 3*a*e)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/
x), -1) + (3*c*e*x^4 + 5*c*d*x^2 + 6*a*e)*sqrt(c*x^4 + a))/(c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.31

$$\int \left(d+ex^2\right)\sqrt{a+cx^4}\,dx=\frac{\sqrt{a}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)*(c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a
)/(4*gamma(5/4)) + sqrt(a)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*
x**4*exp_polar(I*pi)/a)/(4*gamma(7/4))
```

Maxima [F]

$$\int \left(d+ex^2\right)\sqrt{a+cx^4}\,dx=\int \sqrt{cx^4+a}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x^2 + d), x)
```

Giac [F]

$$\int (d+ex^2)\sqrt{a+cx^4}\,dx=\int \sqrt{cx^4+a}\,(ex^2+d)\,dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\sqrt{a+cx^4}\,dx=\int \sqrt{c\,x^4+a}\,\left(e\,x^2+d\right)\,dx$$

input
```
int((a + c*x^4)^(1/2)*(d + e*x^2),x)
```

output
```
int((a + c*x^4)^(1/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2)\sqrt{a+cx^4}\,dx=\frac{\sqrt{c\,x^4+a}\,dx}{3}+\frac{\sqrt{c\,x^4+a}\,e\,x^3}{5}+\frac{2\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)ad}{3}+\frac{2\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}dx\right)ae}{5}$$

input
```
int((e*x^2+d)*(c*x^4+a)^(1/2),x)
```

output
```
(5*sqrt(a + c*x**4)*d*x + 3*sqrt(a + c*x**4)*e*x**3 + 10*int(sqrt(a + c*x*
*4)/(a + c*x**4),x)*a*d + 6*int((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)*a*
e)/15
```

3.244 $\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 227

$$\begin{aligned}\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx &= \frac{ex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} \\ &\quad + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

output

```
e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)-a^(1/4)*e*(a^(1/2)+c^(1/
2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c
^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)+1/2*(c^(1/2)*d+a^(
1/2)*e)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*In
verseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(3/4)/(c*
x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.34

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx$$
$$=\frac{\sqrt{1+\frac{cx^4}{a}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)\right)}{3\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

output
```
(Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
 + e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[a + c*x^
4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx$$
$$\Big\downarrow \text{1512}$$
$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{\sqrt{a}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}$$
$$\Big\downarrow \text{27}$$
$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \textcolor{blue}{761}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{e\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \textcolor{blue}{1510}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{e\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

input

```
Int[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

output

```
-((e*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/Sqrt[c]) + ((d
+ (Sqrt[a]*e)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)
*c^(1/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.74

method	result
default	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$

input

```
int((e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
+I*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/
2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.39

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \frac{a\sqrt{c}ex\left(-\frac{a}{c}\right)^{\frac{3}{4}} E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right) | -1) + (cd-ae)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right) | -1) + \sqrt{cx^4+a}ae}{acx}$$

input

```
integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output

```
(a*sqrt(c)*e*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) + (c*d
- a*e)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + sqr
t(c*x^4 + a)*a*e)/(a*c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.34

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input

```
integrate((e*x**2+d)/(c*x**4+a)**(1/2),x)
```

output

```
d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_p
olar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x^2)/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)d+\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}dx\right)e$$

input

```
int((e*x^2+d)/(c*x^4+a)^(1/2),x)
```

output

```
int(sqrt(a + c*x**4)/(a + c*x**4),x)*d + int((sqrt(a + c*x**4)*x**2)/(a +
c*x**4),x)*e
```

3.245 $\int \frac{d+ex^2}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 262

$$\begin{aligned}\int \frac{d+ex^2}{(a+cx^4)^{3/2}}\,dx &= \frac{x(d+ex^2)}{2a\sqrt{a+cx^4}} - \frac{ex\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\ &+ \frac{e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} \\ &+ \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

output

```
1/2*x*(e*x^2+d)/a/(c*x^4+a)^(1/2)-1/2*e*x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(1/
2)+c^(1/2)*x^2)+1/2*e*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^
2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4
)/c^(3/4)/(c*x^4+a)^(1/2)+1/4*(c^(1/2)*d-a^(1/2)*e)*(a^(1/2)+c^(1/2)*x^2)*
((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)
*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.38

$$\int \frac{d+ex^2}{\left(a+cx^4\right)^{3/2}}\,dx = \frac{3dx+3dx\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+2ex^3\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeom}}{6a\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x^2)/(a + c*x^4)^(3/2),x]
```

output
```
(3*d*x + 3*d*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x
^4)/a)] + 2*e*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((
c*x^4)/a)])/(6*a*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1493, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\left(a+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1493}$$

$$\frac{x\left(d+ex^2\right)}{2a\sqrt{a+cx^4}}-\frac{\int -\frac{d-ex^2}{\sqrt{cx^4+a}}dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d-ex^2}{\sqrt{cx^4+a}}dx}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{1512}$$

$$\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{\sqrt{a}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{e\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+c}}{\sqrt{a}+\sqrt{}}\right.}{\sqrt{c}}}{2a}$$

$$\frac{x\left(d+ex^2\right)}{2a\sqrt{a+cx^4}}$$

input
```
Int[(d + e*x^2)/(a + c*x^4)^(3/2),x]
```

output
```
(x*(d + e*x^2))/(2*a*Sqrt[a + c*x^4]) + ((e*(-((x*Sqrt[a + c*x^4])/(Sqrt[a
] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(
1/4)*Sqrt[a + c*x^4])))/Sqrt[c] + ((d - (Sqrt[a]*e)/Sqrt[c])*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.81

method	result
elliptic	$-\frac{2c\left(-\frac{e\,x^{3}}{4ac}-\frac{dx}{4ac}\right)}{\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}-\frac{ie\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{E}\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$
default	$d\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+e\left(\frac{x^{3}}{2a\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}-\frac{i\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{E}\right.}{2\sqrt{c}}\right.$

input
```
int((e*x^2+d)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/4/a*e/c*x^3-1/4*d/a/c*x)/((x^4+1/c*a)*c)^(1/2)+1/2*d/a/(I/a^(1/2)
*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^
(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/2*I/a^(1/
2)*e/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/
2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.45

$$\int \frac{d+ex^{2}}{\left(a+cx^{4}\right)^{3/2}}\,dx=\frac{\left(cex^{4}+ae\right)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\mid-1\right)-\left(\left(cd+ce\right)x^{4}+ad+ae\right)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}}{2\left(ac^{2}x^{4}+a^{2}c\right)}$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((c*e*x^4 + a*e)*sqrt(a)*(-c/a)^(3/4)*elliptic_e(arcsin(x*(-c/a)^(1/4)
), -1) - ((c*d + c*e)*x^4 + a*d + a*e)*sqrt(a)*(-c/a)^(3/4)*elliptic_f(arc
sin(x*(-c/a)^(1/4)), -1) + (c*e*x^3 + c*d*x)*sqrt(c*x^4 + a))/(a*c^2*x^4 +
 a^2*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.30

$$\int \frac{d+ex^2}{\left(a+cx^4\right)^{3/2}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(c*x**4+a)**(3/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(
3/2)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), c*x**4*exp_
polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\left(a+cx^4\right)^{3/2}}\,dx = \int \frac{ex^2+d}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\left(a+cx^4\right)^{3/2}}\,dx = \int \frac{ex^2+d}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a+cx^4)^{3/2}}\,dx=\int \frac{e\,x^2+d}{{\left(c\,x^4+a\right)}^{3/2}} \,dx$$

input
```
int((d + e*x^2)/(a + c*x^4)^(3/2),x)
```

output
```
int((d + e*x^2)/(a + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(a+cx^4)^{3/2}}\,dx=\left(\int \frac{\sqrt{c\,x^{4}+a}}{c^{2} x^{8}+2 a c\,x^{4}+a^{2}}d x \right) d +\left(\int \frac{\sqrt{c\,x^{4}+a}\,x^{2}}{c^{2} x^{8}+2 a c\,x^{4}+a^{2}}d x \right) e$$

input
```
int((e*x^2+d)/(c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*d + int((sqrt(a +
c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*e
```

3.246 $\int \frac{d+ex^2}{\left(a+cx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 292

$$\begin{aligned}\int \frac{d+ex^2}{(a+cx^4)^{5/2}}\,dx &= \frac{x(d+ex^2)}{6a\,(a+cx^4)^{3/2}} + \frac{x(5d+3ex^2)}{12a^2\sqrt{a+cx^4}} - \frac{ex\sqrt{a+cx^4}}{4a^2\sqrt{c}\,(\sqrt{a}+\sqrt{c}x^2)} \\ &+ \frac{e(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}c^{3/4}\sqrt{a+cx^4}} \\ &+ \frac{(5\sqrt{c}d-3\sqrt{a}e)\,(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{24a^{9/4}c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

output

```
1/6*x*(e*x^2+d)/a/(c*x^4+a)^(3/2)+1/12*x*(3*e*x^2+5*d)/a^2/(c*x^4+a)^(1/2)
-1/4*e*x*(c*x^4+a)^(1/2)/a^2/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)+1/4*e*(a^(1/2)+
c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(7/4)/c^(3/4)/(c*x^4+a)^(1/2)+1/24*
(5*c^(1/2)*d-3*a^(1/2)*e)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2
)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a
^(9/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.42

$$\int \frac{d+ex^2}{(a+cx^4)^{5/2}}\,dx = \frac{dx(7a+5cx^4)+5dx(a+cx^4)\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+4ex^3(}{12a^2\,(a+cx^4)^{3/2}}$$

input
```
Integrate[(d + e*x^2)/(a + c*x^4)^(5/2),x]
```

output
```
(d*x*(7*a + 5*c*x^4) + 5*d*x*(a + c*x^4)*Sqrt[1 + (c*x^4)/a]*Hypergeometri
c2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + 4*e*x^3*(a + c*x^4)*Sqrt[1 + (c*x^4)/a
]*Hypergeometric2F1[3/4, 5/2, 7/4, -((c*x^4)/a)])/(12*a^2*(a + c*x^4)^(3/2
))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1493, 25, 1493, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a+cx^4)^{5/2}}dx$$

$$\Big\downarrow \text{ 1493}$$

$$\frac{x(d+ex^2)}{6a\,(a+cx^4)^{3/2}}-\frac{\int -\frac{3ex^2+5d}{(cx^4+a)^{3/2}}dx}{6a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3ex^2+5d}{(cx^4+a)^{3/2}}dx}{6a}+\frac{x(d+ex^2)}{6a\,(a+cx^4)^{3/2}}$$

$$\Big\downarrow \text{ 1493}$$

$$\frac{\frac{x\left(5d+3ex^2\right)}{2a\sqrt{a+cx^4}}-\frac{\int -\frac{5d-3ex^2}{\sqrt{cx^4+a}}dx}{2a}}{6a}+\frac{x\left(d+ex^2\right)}{6a\left(a+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{5d-3ex^2}{\sqrt{cx^4+a}}dx}{2a}+\frac{x\left(5d+3ex^2\right)}{2a\sqrt{a+cx^4}}}{6a}+\frac{x\left(d+ex^2\right)}{6a\left(a+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 1512$$

$$\frac{\frac{\left(5d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{3\sqrt{a}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}}{2a}+\frac{x\left(5d+3ex^2\right)}{2a\sqrt{a+cx^4}}}{6a}+\frac{x\left(d+ex^2\right)}{6a\left(a+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(5d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{3e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}}{2a}+\frac{x\left(5d+3ex^2\right)}{2a\sqrt{a+cx^4}}}{6a}+\frac{x\left(d+ex^2\right)}{6a\left(a+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{\frac{3e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(5d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}}{2a}+\frac{x\left(5d+3ex^2\right)}{2a\sqrt{a+cx^4}}}{6a}+$$
$$\frac{x\left(d+ex^2\right)}{6a\left(a+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 1510$$

$$\frac{\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(5d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{3e\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}}{2a}}{6a}$$
$$\frac{x\left(d+ex^2\right)}{6a\left(a+cx^4\right)^{3/2}}$$

input
```
Int[(d + e*x^2)/(a + c*x^4)^(5/2),x]
```

output
```
(x*(d + e*x^2))/(6*a*(a + c*x^4)^(3/2)) + ((x*(5*d + 3*e*x^2))/(2*a*Sqrt[a
 + c*x^4]) + ((3*e*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1
/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/S
qrt[c] + ((5*d - (3*Sqrt[a]*e)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]))/(2*a))/(6*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.88

method	result
elliptic	$\frac{\left(\frac{e\,x^3}{6a\,c^2}+\frac{dx}{6a\,c^2}\right)\sqrt{c\,x^4+a}}{\left(x^4+\frac{a}{c}\right)^2}-\frac{2c\left(-\frac{e\,x^3}{8a^2c}-\frac{5dx}{24a^2c}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{5d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{12a^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-$
default	$d\left(\frac{x\sqrt{c\,x^4+a}}{6a\,c^2\left(x^4+\frac{a}{c}\right)^2}+\frac{5x}{12a^2\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{5\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{12a^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)+e\left(\frac{x^3\sqrt{c\,x^4+a}}{6a\,c^2\left(x^4+\frac{a}{c}\right)^2}+\right.$

input
```
int((e*x^2+d)/(c*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/6/a*e/c^2*x^3+1/6*d/a/c^2*x)*(c*x^4+a)^(1/2)/(x^4+1/c*a)^2-2*c*(-1/8/a^
2/c*e*x^3-5/24/a^2/c*d*x)/((x^4+1/c*a)*c)^(1/2)+5/12*d/a^2/(I/a^(1/2)*c^(1
/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)
/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/4*I/a^(3/2)*e/
(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(
1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.67

$$\int \frac{d+ex^2}{(a+cx^4)^{5/2}}\,dx = \frac{3\left(c^2ex^8+2\,acex^4+a^2e\right)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}} E\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)|-1\right)-\left(\left(5\,c^2d+3\,c^2e\right)x^8+\right.}{}$$

input

```
integrate((e*x^2+d)/(c*x^4+a)^(5/2),x, algorithm="fricas")
```

output

```
1/12*(3*(c^2*e*x^8 + 2*a*c*e*x^4 + a^2*e)*sqrt(a)*(-c/a)^(3/4)*elliptic_e(
arcsin(x*(-c/a)^(1/4)), -1) - ((5*c^2*d + 3*c^2*e)*x^8 + 2*(5*a*c*d + 3*a*
c*e)*x^4 + 5*a^2*d + 3*a^2*e)*sqrt(a)*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c
/a)^(1/4)), -1) + (3*c^2*e*x^7 + 5*c^2*d*x^5 + 5*a*c*e*x^3 + 7*a*c*d*x)*sq
rt(c*x^4 + a))/(a^2*c^3*x^8 + 2*a^3*c^2*x^4 + a^4*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.67 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.27

$$\int \frac{d+ex^2}{(a+cx^4)^{5/2}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input

```
integrate((e*x**2+d)/(c*x**4+a)**(5/2),x)
```

output

```
d*x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(
5/2)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), c*x**4*exp_
polar(I*pi)/a)/(4*a**(5/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{(a+cx^4)^{5/2}}\,dx = \int \frac{ex^2+d}{(cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(c*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(a+cx^4)^{5/2}}\,dx = \int \frac{ex^2+d}{(cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(c*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a+cx^4)^{5/2}}\,dx = \int \frac{e\,x^2+d}{(c\,x^4+a)^{5/2}}\,dx$$

input
```
int((d + e*x^2)/(a + c*x^4)^(5/2),x)
```

output
```
int((d + e*x^2)/(a + c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(a+cx^4)^{5/2}}\,dx = \left(\int \frac{\sqrt{c\,x^4+a}}{c^3x^{12}+3a\,c^2x^8+3a^2c\,x^4+a^3}dx\right)d + \left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c^3x^{12}+3a\,c^2x^8+3a^2c\,x^4+a^3}dx\right)e$$

input
```
int((e*x^2+d)/(c*x^4+a)^(5/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a**3 + 3*a**2*c*x**4 + 3*a*c**2*x**8 + c**3*x**12),x
)*d + int((sqrt(a + c*x**4)*x**2)/(a**3 + 3*a**2*c*x**4 + 3*a*c**2*x**8 +
c**3*x**12),x)*e
```

3.247 $\int (d+ex^2)(9+x^4)^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 211

$$\begin{aligned}\int (d+ex^2)(9+x^4)^{5/2}\,dx &= \frac{1944ex\sqrt{9+x^4}}{13(3+x^2)}+\frac{108x(195d+77ex^2)\sqrt{9+x^4}}{1001}\\
&+\frac{10x(117d+77ex^2)(9+x^4)^{3/2}}{1001}+\frac{1}{143}x(13d+11ex^2)(9+x^4)^{5/2}\\
&-\frac{1944\sqrt{3}e(3+x^2)\sqrt{\frac{9+x^4}{(3+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{3}}\right)|\frac{1}{2}\right)}{13\sqrt{9+x^4}}\\
&+\frac{972\sqrt{3}(65d+77e)(3+x^2)\sqrt{\frac{9+x^4}{(3+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{1001\sqrt{9+x^4}}\end{aligned}$$

output
```
1944*e*x*(x^4+9)^(1/2)/(13*x^2+39)+108/1001*x*(77*e*x^2+195*d)*(x^4+9)^(1/
2)+10/1001*x*(77*e*x^2+117*d)*(x^4+9)^(3/2)+1/143*x*(11*e*x^2+13*d)*(x^4+9
)^(5/2)-1944/13*3^(1/2)*e*(x^2+3)*((x^4+9)/(x^2+3)^2)^(1/2)*EllipticE(sin(
2*arctan(1/3*x*3^(1/2))),1/2*2^(1/2))/(x^4+9)^(1/2)+972/1001*3^(1/2)*(65*d
+77*e)*(x^2+3)*((x^4+9)/(x^2+3)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*x*3^
(1/2)),1/2*2^(1/2))/(x^4+9)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.64 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.21

$$\int (d+ex^2)\left(9+x^4\right)^{5/2}\,dx = 243dx\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{9}\right) + 81ex^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{9}\right)$$

input
```
Integrate[(d + e*x^2)*(9 + x^4)^(5/2),x]
```

output
```
243*d*x*Hypergeometric2F1[-5/2, 1/4, 5/4, -1/9*x^4] + 81*e*x^3*Hypergeomet
ric2F1[-5/2, 3/4, 7/4, -1/9*x^4]
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1491, 27, 1491, 27, 1491, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^4+9\right)^{5/2}\left(d+ex^2\right)dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{5}{143}\int 18\left(11ex^2+13d\right)\left(x^4+9\right)^{3/2}dx+\frac{1}{143}x\left(x^4+9\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{90}{143}\int \left(11ex^2+13d\right)\left(x^4+9\right)^{3/2}dx+\frac{1}{143}x\left(x^4+9\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \text{1491}$$

$$\frac{90}{143}\left(\frac{1}{21}\int 18\left(77ex^2+117d\right)\sqrt{x^4+9}dx+\frac{1}{63}x\left(x^4+9\right)^{3/2}\left(117d+77ex^2\right)\right)+\frac{1}{143}x\left(x^4+9\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{90}{143}\left(\frac{6}{7}\int\left(77ex^2+117d\right)\sqrt{x^4+9}dx+\frac{1}{63}x\left(x^4+9\right)^{3/2}\left(117d+77ex^2\right)\right)+\frac{1}{143}x\left(x^4+9\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \text{ 1491}$$

$$\frac{90}{143}\left(\frac{6}{7}\left(\frac{1}{15}\int\frac{54\left(77ex^2+195d\right)}{\sqrt{x^4+9}}dx+\frac{1}{5}x\sqrt{x^4+9}\left(195d+77ex^2\right)\right)+\frac{1}{63}x\left(x^4+9\right)^{3/2}\left(117d+77ex^2\right)\right)+\frac{1}{143}x\left(x^4+9\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{90}{143}\left(\frac{6}{7}\left(\frac{18}{5}\int\frac{77ex^2+195d}{\sqrt{x^4+9}}dx+\frac{1}{5}x\sqrt{x^4+9}\left(195d+77ex^2\right)\right)+\frac{1}{63}x\left(x^4+9\right)^{3/2}\left(117d+77ex^2\right)\right)+\frac{1}{143}x\left(x^4+9\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \text{ 1512}$$

$$\frac{90}{143}\Bigg(\frac{6}{7}\Bigg(\frac{18}{5}\left(3(65d+77e)\int\frac{1}{\sqrt{x^4+9}}dx-231e\int\frac{3-x^2}{3\sqrt{x^4+9}}dx\right)+\frac{1}{5}x\sqrt{x^4+9}\left(195d+77ex^2\right)\Bigg)+\frac{1}{63}x\big(x^4+ \quad \frac{1}{143}x\left(x^4+9\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{90}{143}\Bigg(\frac{6}{7}\Bigg(\frac{18}{5}\left(3(65d+77e)\int\frac{1}{\sqrt{x^4+9}}dx-77e\int\frac{3-x^2}{\sqrt{x^4+9}}dx\right)+\frac{1}{5}x\sqrt{x^4+9}\left(195d+77ex^2\right)\Bigg)+\frac{1}{63}x\left(x^4+9\right)^{3} \quad \frac{1}{143}x\left(x^4+9\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow \text{ 761}$$

$$\frac{90}{143}\Bigg(\frac{6}{7}\Bigg(\frac{18}{5}\left(\frac{\sqrt{3}\left(x^2+3\right)\sqrt{\frac{x^4+9}{\left(x^2+3\right)^2}}(65d+77e)\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{2\sqrt{x^4+9}}-77e\int\frac{3-x^2}{\sqrt{x^4+9}}dx\right)+\frac{1}{5}x\sqrt{x^4}$$

$$\frac{1}{143}x\left(x^4+9\right)^{5/2}\left(13d+11ex^2\right)$$

$$\Big\downarrow 1510$$

$$\frac{90}{143}\left(\frac{6}{7}\left(\frac{18}{5}\left(\frac{\sqrt{3}(x^2+3)\sqrt{\frac{x^4+9}{(x^2+3)^2}}(65d+77e)\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{2\sqrt{x^4+9}}-77e\left(\frac{\sqrt{3}(x^2+3)\sqrt{\frac{x^4+9}{(x^2+3)^2}}E\left(\right.}{\sqrt{x^4+9}}\right.\right.\right.\right.$$

$$\frac{1}{143}x\left(x^4+9\right)^{5/2}\left(13d+11ex^2\right)$$

input

```
Int[(d + e*x^2)*(9 + x^4)^(5/2),x]
```

output

```
(x*(13*d + 11*e*x^2)*(9 + x^4)^(5/2))/143 + (90*((x*(117*d + 77*e*x^2)*(9
+ x^4)^(3/2))/63 + (6*((x*(195*d + 77*e*x^2)*Sqrt[9 + x^4])/5 + (18*(-77*e
*(-((x*Sqrt[9 + x^4])/(3 + x^2)) + (Sqrt[3]*(3 + x^2)*Sqrt[(9 + x^4)/(3 +
x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[3]], 1/2])/Sqrt[9 + x^4]) + (Sqrt[3]*(65
*d + 77*e)*(3 + x^2)*Sqrt[(9 + x^4)/(3 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt
[3]], 1/2])/(2*Sqrt[9 + x^4])))/5))/7))/143
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.16

method	result
meijerg	$81e\,x^3 \operatorname{hypergeom}\left(\left[-\frac{5}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -\frac{x^4}{9}\right) + 243dx \operatorname{hypergeom}\left(\left[-\frac{5}{2}, \frac{1}{4}\right], \left[\frac{5}{4}\right], -\frac{x^4}{9}\right)$
risch	$\frac{x\left(77e\,x^{10}+91d\,x^8+2156e\,x^6+2808d\,x^4+21483e\,x^2+38961d\right)\sqrt{x^4+9}}{1001} + \frac{3240d\sqrt{-3ix^2+9}\,\sqrt{3ix^2+9}\ \operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right), i\right)}{77\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^4+9}} +$
default	$d\left(\frac{x^9\sqrt{x^4+9}}{11} + \frac{216x^5\sqrt{x^4+9}}{77} + \frac{2997x\sqrt{x^4+9}}{77} + \frac{3240\sqrt{-3ix^2+9}\,\sqrt{3ix^2+9}\ \operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right), i\right)}{77\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^4+9}}\right) + e\left(\frac{x^{11}\sqrt{x^4+9}}{13}\right.$
elliptic	$\frac{e\,x^{11}\sqrt{x^4+9}}{13} + \frac{d\,x^9\sqrt{x^4+9}}{11} + \frac{28e\,x^7\sqrt{x^4+9}}{13} + \frac{216d\,x^5\sqrt{x^4+9}}{77} + \frac{279e\,x^3\sqrt{x^4+9}}{13} + \frac{2997dx\sqrt{x^4+9}}{77} + \frac{3240d\sqrt{-3ix^2+9}}{77}$

input
```
int((e*x^2+d)*(x^4+9)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
81*e*x^3*hypergeom([-5/2,3/4],[7/4],-1/9*x^4)+243*d*x*hypergeom([-5/2,1/4]
,[5/4],-1/9*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.45

$$\int (d+ex^2)\,(9+x^4)^{5/2}\,dx = \frac{449064i\sqrt{3i}exE\left(\arcsin\left(\frac{\sqrt{3i}}{x}\right)\middle|-1\right) + 1944i\sqrt{3i}(65\,d-231\,e)xF\left(\arcsin\left(\frac{\sqrt{3i}}{x}\right)\middle|-1\right) + (77}{1001\,x}$$

input

```
integrate((e*x^2+d)*(x^4+9)^(5/2),x, algorithm="fricas")
```

output

```
1/1001*(449064*I*sqrt(3*I)*e*x*elliptic_e(arcsin(sqrt(3*I)/x), -1) + 1944*
I*sqrt(3*I)*(65*d - 231*e)*x*elliptic_f(arcsin(sqrt(3*I)/x), -1) + (77*e*x
^12 + 91*d*x^10 + 2156*e*x^8 + 2808*d*x^6 + 21483*e*x^4 + 38961*d*x^2 + 14
9688*e)*sqrt(x^4 + 9))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.05

$$\begin{aligned}\int (d+ex^2)\,(9+x^4)^{5/2}\,dx = {} & \frac{3dx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{4\Gamma\left(\frac{13}{4}\right)} \\ & + \frac{27dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{243dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{4\Gamma\left(\frac{5}{4}\right)} \\ & + \frac{3ex^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{4\Gamma\left(\frac{15}{4}\right)} + \frac{27ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{2\Gamma\left(\frac{11}{4}\right)} \\ & + \frac{243ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{4\Gamma\left(\frac{7}{4}\right)}\end{aligned}$$

input
```
integrate((e*x**2+d)*(x**4+9)**(5/2),x)
```

output
```
3*d*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), x**4*exp_polar(I*pi)/9)/(4
*gamma(13/4)) + 27*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_p
olar(I*pi)/9)/(2*gamma(9/4)) + 243*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,
), x**4*exp_polar(I*pi)/9)/(4*gamma(5/4)) + 3*e*x**11*gamma(11/4)*hyper((-
1/2, 11/4), (15/4,), x**4*exp_polar(I*pi)/9)/(4*gamma(15/4)) + 27*e*x**7*g
amma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/9)/(2*gamma(11/
4)) + 243*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi
)/9)/(4*gamma(7/4))
```

Maxima [F]

$$\int \left(d+ex^2\right)\left(9+x^4\right)^{5/2}\,dx=\int \left(x^4+9\right)^{\frac{5}{2}}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(x^4+9)^(5/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 9)^(5/2)*(e*x^2 + d), x)
```

Giac [F]

$$\int \left(d+ex^2\right)\left(9+x^4\right)^{5/2}\,dx=\int \left(x^4+9\right)^{\frac{5}{2}}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(x^4+9)^(5/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 9)^(5/2)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)\left(9+x^4\right)^{5/2}\,dx=\int \left(x^4+9\right)^{5/2}\left(e\,x^2+d\right)\,dx$$

input
```
int((x^4 + 9)^(5/2)*(d + e*x^2),x)
```

output
```
int((x^4 + 9)^(5/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)\left(9+x^4\right)^{5/2}\,dx=\frac{\sqrt{x^4+9}\,d\,x^9}{11}+\frac{216\sqrt{x^4+9}\,d\,x^5}{77}+\frac{2997\sqrt{x^4+9}\,dx}{77}+\frac{\sqrt{x^4+9}\,e\,x^{11}}{13}+\frac{28\sqrt{x^4+9}\,e\,x^7}{13}+\frac{279\sqrt{x^4+9}\,e\,x^3}{13}+\frac{29160\left(\int \frac{\sqrt{x^4+9}}{x^4+9}dx\right)d}{77}+\frac{1944\left(\int \frac{\sqrt{x^4+9}\,x^2}{x^4+9}dx\right)e}{13}$$

input
```
int((e*x^2+d)*(x^4+9)^(5/2),x)
```

output
```
(91*sqrt(x**4 + 9)*d*x**9 + 2808*sqrt(x**4 + 9)*d*x**5 + 38961*sqrt(x**4 +
 9)*d*x + 77*sqrt(x**4 + 9)*e*x**11 + 2156*sqrt(x**4 + 9)*e*x**7 + 21483*s
qrt(x**4 + 9)*e*x**3 + 379080*int(sqrt(x**4 + 9)/(x**4 + 9),x)*d + 149688*
int((sqrt(x**4 + 9)*x**2)/(x**4 + 9),x)*e)/1001
```

3.248 $\int \left(d+ex^2\right)\left(9+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 187

$$\begin{aligned}\int \left(d+ex^2\right)\left(9+x^4\right)^{3/2}\,dx &= \frac{108ex\sqrt{9+x^4}}{5\left(3+x^2\right)}+\frac{6}{35}x\left(15d+7ex^2\right)\sqrt{9+x^4} \\ &+\frac{1}{63}x\left(9d+7ex^2\right)\left(9+x^4\right)^{3/2}-\frac{108\sqrt{3}e(3+x^2)\sqrt{\frac{9+x^4}{(3+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{3}}\right)|\frac{1}{2}\right)}{5\sqrt{9+x^4}} \\ &+\frac{54\sqrt{3}(5d+7e)\left(3+x^2\right)\sqrt{\frac{9+x^4}{(3+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{35\sqrt{9+x^4}}\end{aligned}$$

output

```
108*e*x*(x^4+9)^(1/2)/(5*x^2+15)+6/35*x*(7*e*x^2+15*d)*(x^4+9)^(1/2)+1/63*
x*(7*e*x^2+9*d)*(x^4+9)^(3/2)-108/5*3^(1/2)*e*(x^2+3)*((x^4+9)/(x^2+3)^2)^
(1/2)*EllipticE(sin(2*arctan(1/3*x*3^(1/2))),1/2*2^(1/2))/(x^4+9)^(1/2)+54
/35*3^(1/2)*(5*d+7*e)*(x^2+3)*((x^4+9)/(x^2+3)^2)^(1/2)*InverseJacobiAM(2*
arctan(1/3*x*3^(1/2)),1/2*2^(1/2))/(x^4+9)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.55 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.24

$$\int (d+ex^2)\left(9+x^4\right)^{3/2}\,dx = 27dx\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{9}\right) + 9ex^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{9}\right)$$

input

```
Integrate[(d + e*x^2)*(9 + x^4)^(3/2),x]
```

output

```
27*d*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -1/9*x^4] + 9*e*x^3*Hypergeometri
c2F1[-3/2, 3/4, 7/4, -1/9*x^4]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1491, 27, 1491, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^4+9\right)^{3/2}\left(d+ex^2\right)dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{21}\int 18\left(7ex^2+9d\right)\sqrt{x^4+9}dx+\frac{1}{63}x\left(x^4+9\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\int \left(7ex^2+9d\right)\sqrt{x^4+9}dx+\frac{1}{63}x\left(x^4+9\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{1491}$$

$$\frac{6}{7}\left(\frac{1}{15}\int \frac{54\left(7ex^2+15d\right)}{\sqrt{x^4+9}}dx+\frac{1}{5}x\sqrt{x^4+9}\left(15d+7ex^2\right)\right)+\frac{1}{63}x\left(x^4+9\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{18}{5}\int\frac{7ex^2+15d}{\sqrt{x^4+9}}dx+\frac{1}{5}x\sqrt{x^4+9}\left(15d+7ex^2\right)\right)+\frac{1}{63}x\left(x^4+9\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{1512}$$

$$\frac{6}{7}\left(\frac{18}{5}\left(3(5d+7e)\int\frac{1}{\sqrt{x^4+9}}dx-21e\int\frac{3-x^2}{3\sqrt{x^4+9}}dx\right)+\frac{1}{5}x\sqrt{x^4+9}\left(15d+7ex^2\right)\right)+$$
$$\frac{1}{63}x\left(x^4+9\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{18}{5}\left(3(5d+7e)\int\frac{1}{\sqrt{x^4+9}}dx-7e\int\frac{3-x^2}{\sqrt{x^4+9}}dx\right)+\frac{1}{5}x\sqrt{x^4+9}\left(15d+7ex^2\right)\right)+$$
$$\frac{1}{63}x\left(x^4+9\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{7}\left(\frac{18}{5}\left(\frac{\sqrt{3}\left(x^2+3\right)\sqrt{\frac{x^4+9}{\left(x^2+3\right)^2}}(5d+7e)\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{2\sqrt{x^4+9}}-7e\int\frac{3-x^2}{\sqrt{x^4+9}}dx\right)+\frac{1}{5}x\sqrt{x^4+9}(15d\right.$$
$$\frac{1}{63}x\left(x^4+9\right)^{3/2}\left(9d+7ex^2\right)$$

$$\Big\downarrow \text{1510}$$

$$\frac{6}{7}\left(\frac{18}{5}\left(\frac{\sqrt{3}\left(x^2+3\right)\sqrt{\frac{x^4+9}{\left(x^2+3\right)^2}}(5d+7e)\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{2\sqrt{x^4+9}}-7e\left(\frac{\sqrt{3}\left(x^2+3\right)\sqrt{\frac{x^4+9}{\left(x^2+3\right)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt{x^4+9}}\right.\right.\right.$$
$$\frac{1}{63}x\left(x^4+9\right)^{3/2}\left(9d+7ex^2\right)$$

input
```
Int[(d + e*x^2)*(9 + x^4)^(3/2),x]
```

output
```
(x*(9*d + 7*e*x^2)*(9 + x^4)^(3/2))/63 + (6*((x*(15*d + 7*e*x^2)*Sqrt[9 +
x^4])/5 + (18*(-7*e*(-((x*Sqrt[9 + x^4])/(3 + x^2)) + (Sqrt[3]*(3 + x^2)*S
qrt[(9 + x^4)/(3 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[3]], 1/2])/Sqrt[9 + x
^4]) + (Sqrt[3]*(5*d + 7*e)*(3 + x^2)*Sqrt[(9 + x^4)/(3 + x^2)^2]*Elliptic
F[2*ArcTan[x/Sqrt[3]], 1/2])/(2*Sqrt[9 + x^4])))/5))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.18

method	result
meijerg	$9e\,x^3 \operatorname{hypergeom}\left(\left[-\frac{3}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -\frac{x^4}{9}\right) + 27dx \operatorname{hypergeom}\left(\left[-\frac{3}{2}, \frac{1}{4}\right], \left[\frac{5}{4}\right], -\frac{x^4}{9}\right)$
risch	$\frac{x(35e\,x^6+45d\,x^4+693e\,x^2+1215d)\sqrt{x^4+9}}{315} + \frac{36d\sqrt{-3ix^2+9}\,\sqrt{3ix^2+9}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right),i\right)}{7\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^4+9}} +$
default	$d\left(\frac{x^5\sqrt{x^4+9}}{7} + \frac{27x\sqrt{x^4+9}}{7} + \frac{36\sqrt{-3ix^2+9}\,\sqrt{3ix^2+9}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right),i\right)}{7\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^4+9}}\right) + e\left(\frac{x^7\sqrt{x^4+9}}{9} + \frac{11x^3\sqrt{x^4+9}}{5} +\right.$
elliptic	$\frac{e\,x^7\sqrt{x^4+9}}{9} + \frac{d\,x^5\sqrt{x^4+9}}{7} + \frac{11e\,x^3\sqrt{x^4+9}}{5} + \frac{27dx\sqrt{x^4+9}}{7} + \frac{36d\sqrt{-3ix^2+9}\,\sqrt{3ix^2+9}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right),i\right)}{7\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^4+9}} +$

input
```
int((e*x^2+d)*(x^4+9)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
9*e*x^3*hypergeom([-3/2,3/4],[7/4],-1/9*x^4)+27*d*x*hypergeom([-3/2,1/4],[
5/4],-1/9*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.44

$$\int \left(d+ex^2\right)\left(9+x^4\right)^{3/2}\,dx = \frac{20412i\,\sqrt{3i}exE(\arcsin\left(\frac{\sqrt{3i}}{x}\right)\,|-1) + 972i\,\sqrt{3i}(5\,d-21\,e)xF(\arcsin\left(\frac{\sqrt{3i}}{x}\right)\,|-1) + (35\,ex^8 +}{315\,x}$$

input
```
integrate((e*x^2+d)*(x^4+9)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(20412*I*sqrt(3*I)*e*x*elliptic_e(arcsin(sqrt(3*I)/x), -1) + 972*I*s
qrt(3*I)*(5*d - 21*e)*x*elliptic_f(arcsin(sqrt(3*I)/x), -1) + (35*e*x^8 +
45*d*x^6 + 693*e*x^4 + 1215*d*x^2 + 6804*e)*sqrt(x^4 + 9))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.78

$$\int (d+ex^2)\left(9+x^4\right)^{3/2}\,dx=\frac{3dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{27dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{3ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{27ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)*(x**4+9)**(3/2),x)
```

output
```
3*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/9)/(4*
gamma(9/4)) + 27*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(
I*pi)/9)/(4*gamma(5/4)) + 3*e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,),
x**4*exp_polar(I*pi)/9)/(4*gamma(11/4)) + 27*e*x**3*gamma(3/4)*hyper((-1/2
, 3/4), (7/4,), x**4*exp_polar(I*pi)/9)/(4*gamma(7/4))
```

Maxima [F]

$$\int (d+ex^2)\left(9+x^4\right)^{3/2}\,dx=\int (x^4+9)^{\frac{3}{2}}(ex^2+d)\,dx$$

input
```
integrate((e*x^2+d)*(x^4+9)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 9)^(3/2)*(e*x^2 + d), x)
```

Giac [F]

$$\int \left(d+ex^2\right) \left(9+x^4\right)^{3/2} \, dx = \int \left(x^4+9\right)^{\frac{3}{2}} \left(ex^2+d\right) \, dx$$

input
```
integrate((e*x^2+d)*(x^4+9)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 9)^(3/2)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right) \left(9+x^4\right)^{3/2} \, dx = \int \left(x^4+9\right)^{3/2} \left(e\,x^2+d\right) \, dx$$

input
```
int((x^4 + 9)^(3/2)*(d + e*x^2),x)
```

output
```
int((x^4 + 9)^(3/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right) \left(9+x^4\right)^{3/2} \, dx = \frac{\sqrt{x^4+9}\,d\,x^5}{7} + \frac{27\sqrt{x^4+9}\,dx}{7} + \frac{\sqrt{x^4+9}\,e\,x^7}{9} + \frac{11\sqrt{x^4+9}\,e\,x^3}{5} + \frac{324\left(\int \frac{\sqrt{x^4+9}}{x^4+9}dx\right) d}{7} + \frac{108\left(\int \frac{\sqrt{x^4+9}\,x^2}{x^4+9}dx\right) e}{5}$$

input
```
int((e*x^2+d)*(x^4+9)^(3/2),x)
```

output
```
(45*sqrt(x**4 + 9)*d*x**5 + 1215*sqrt(x**4 + 9)*d*x + 35*sqrt(x**4 + 9)*e*
x**7 + 693*sqrt(x**4 + 9)*e*x**3 + 14580*int(sqrt(x**4 + 9)/(x**4 + 9),x)*
d + 6804*int((sqrt(x**4 + 9)*x**2)/(x**4 + 9),x)*e)/315
```

3.249 $\int (d+ex^2)\sqrt{9+x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 163

$$
\begin{aligned}
\int (d+ex^2)\sqrt{9+x^4}\,dx &= \frac{18ex\sqrt{9+x^4}}{5(3+x^2)}+\frac{1}{15}x(5d+3ex^2)\sqrt{9+x^4} \\
&\quad -\frac{18\sqrt{3}e(3+x^2)\sqrt{\frac{9+x^4}{(3+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{3}}\right)|\frac{1}{2}\right)}{5\sqrt{9+x^4}} \\
&\quad +\frac{\sqrt{3}(5d+9e)(3+x^2)\sqrt{\frac{9+x^4}{(3+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{5\sqrt{9+x^4}}
\end{aligned}
$$

output
```
18*e*x*(x^4+9)^(1/2)/(5*x^2+15)+1/15*x*(3*e*x^2+5*d)*(x^4+9)^(1/2)-18/5*3^
(1/2)*e*(x^2+3)*((x^4+9)/(x^2+3)^2)^(1/2)*EllipticE(sin(2*arctan(1/3*x*3^(
1/2))),1/2*2^(1/2))/(x^4+9)^(1/2)+1/5*3^(1/2)*(5*d+9*e)*(x^2+3)*((x^4+9)/(
x^2+3)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*x*3^(1/2)),1/2*2^(1/2))/(x^4+
9)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.27

$$\int (d+ex^2)\sqrt{9+x^4}\,dx = 3dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{9}\right) + ex^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{9}\right)$$

input
```
Integrate[(d + e*x^2)*Sqrt[9 + x^4],x]
```

output
```
3*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -1/9*x^4] + e*x^3*Hypergeometric2F
1[-1/2, 3/4, 7/4, -1/9*x^4]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1491, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^4+9}\left(d+ex^2\right)\,dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{15}\int \frac{18\left(3ex^2+5d\right)}{\sqrt{x^4+9}}dx+\frac{1}{15}x\sqrt{x^4+9}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{5}\int \frac{3ex^2+5d}{\sqrt{x^4+9}}dx+\frac{1}{15}x\sqrt{x^4+9}\left(5d+3ex^2\right)$$

$$\Big\downarrow \text{1512}$$

$$\frac{6}{5}\left((5d+9e)\int \frac{1}{\sqrt{x^4+9}}dx-9e\int \frac{3-x^2}{3\sqrt{x^4+9}}dx\right)+\frac{1}{15}x\sqrt{x^4+9}\left(5d+3ex^2\right)$$

$$\Big\downarrow \ \color{blue}{27}$$

$$\frac{6}{5}\left((5d+9e)\int \frac{1}{\sqrt{x^4+9}}dx-3e\int \frac{3-x^2}{\sqrt{x^4+9}}dx\right)+\frac{1}{15}x\sqrt{x^4+9}\left(5d+3ex^2\right)$$

$$\Big\downarrow \ \color{blue}{761}$$

$$\frac{6}{5}\left(\frac{(x^2+3)\sqrt{\frac{x^4+9}{(x^2+3)^2}}(5d+9e)\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{2\sqrt{3}\sqrt{x^4+9}}-3e\int \frac{3-x^2}{\sqrt{x^4+9}}dx\right)+$$
$$\frac{1}{15}x\sqrt{x^4+9}\left(5d+3ex^2\right)$$

$$\Big\downarrow \ \color{blue}{1510}$$

$$\frac{6}{5}\Bigg(\frac{(x^2+3)\sqrt{\frac{x^4+9}{(x^2+3)^2}}(5d+9e)\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{2\sqrt{3}\sqrt{x^4+9}}-3e\Bigg(\frac{\sqrt{3}(x^2+3)\sqrt{\frac{x^4+9}{(x^2+3)^2}}E\Big(2\arctan\left(\frac{x}{\sqrt{3}}\right)|\frac{1}{2}\Big)}{\sqrt{x^4+9}}$$
$$\frac{1}{15}x\sqrt{x^4+9}\left(5d+3ex^2\right)$$

input
```
Int[(d + e*x^2)*Sqrt[9 + x^4],x]
```

output
```
(x*(5*d + 3*e*x^2)*Sqrt[9 + x^4])/15 + (6*(-3*e*(-((x*Sqrt[9 + x^4])/(3 +
x^2)) + (Sqrt[3]*(3 + x^2)*Sqrt[(9 + x^4)/(3 + x^2)^2]*EllipticE[2*ArcTan[
x/Sqrt[3]], 1/2])/Sqrt[9 + x^4]) + ((5*d + 9*e)*(3 + x^2)*Sqrt[(9 + x^4)/(
3 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[3]], 1/2])/(2*Sqrt[3]*Sqrt[9 + x^4])
))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.20

method	result
meijerg	$e\,x^3 \operatorname{hypergeom}\left(\left[-\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -\frac{x^4}{9}\right) + 3dx \operatorname{hypergeom}\left(\left[-\frac{1}{2}, \frac{1}{4}\right], \left[\frac{5}{4}\right], -\frac{x^4}{9}\right)$
risch	$\frac{x\left(3e\,x^2+5d\right)\sqrt{x^4+9}}{15} + \frac{2d\sqrt{-3ix^2+9}\,\sqrt{3ix^2+9}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right), i\right)}{3\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^4+9}} + \frac{6ie\sqrt{-3ix^2+9}\,\sqrt{3ix^2+9}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\right.\right.}{5\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^4}}$
elliptic	$\frac{e\,x^3\sqrt{x^4+9}}{5} + \frac{dx\sqrt{x^4+9}}{3} + \frac{2d\sqrt{-3ix^2+9}\,\sqrt{3ix^2+9}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right), i\right)}{3\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^4+9}} + \frac{6ie\sqrt{-3ix^2+9}\,\sqrt{3ix^2+9}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\right.\right.\right.}{5\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)}$
default	$d\left(\frac{x\sqrt{x^4+9}}{3} + \frac{2\sqrt{-3ix^2+9}\,\sqrt{3ix^2+9}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right), i\right)}{3\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^4+9}}\right) + e\left(\frac{x^3\sqrt{x^4+9}}{5} + \frac{6i\sqrt{-3ix^2+9}\,\sqrt{3ix^2+9}\,\left(\operatorname{EllipticF}\left(x\right.\right.}{5\left(\frac{\sqrt{6}}{6}\right.}\right.$

input
```
int((e*x^2+d)*(x^4+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
e*x^3*hypergeom([-1/2,3/4],[7/4],-1/9*x^4)+3*d*x*hypergeom([-1/2,1/4],[5/4
],-1/9*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.43

$$\int \left(d+ex^2\right)\sqrt{9+x^4}\,dx$$
$$=\frac{162i\sqrt{3i}exE\left(\arcsin\left(\frac{\sqrt{3i}}{x}\right)|-1\right)+6i\sqrt{3i}(5d-27e)xF\left(\arcsin\left(\frac{\sqrt{3i}}{x}\right)|-1\right)+(3ex^4+5dx^2+54e)\sqrt{x}}{15x}$$

input
```
integrate((e*x^2+d)*(x^4+9)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(162*I*sqrt(3*I)*e*x*elliptic_e(arcsin(sqrt(3*I)/x), -1) + 6*I*sqrt(3
*I)*(5*d - 27*e)*x*elliptic_f(arcsin(sqrt(3*I)/x), -1) + (3*e*x^4 + 5*d*x^
2 + 54*e)*sqrt(x^4 + 9))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.44

$$\int \left(d+ex^2\right)\sqrt{9+x^4}\,dx=\frac{3dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{3ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)*(x**4+9)**(1/2),x)
```

output
```
3*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi)/9)/(4*gam
ma(5/4)) + 3*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I
*pi)/9)/(4*gamma(7/4))
```

Maxima [F]

$$\int (d+ex^2)\sqrt{9+x^4}\,dx=\int \sqrt{x^4+9}(ex^2+d)\,dx$$

input
```
integrate((e*x^2+d)*(x^4+9)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 9)*(e*x^2 + d), x)
```

Giac [F]

$$\int (d+ex^2)\sqrt{9+x^4}\,dx=\int \sqrt{x^4+9}(ex^2+d)\,dx$$

input
```
integrate((e*x^2+d)*(x^4+9)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 9)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\sqrt{9+x^4}\,dx=\int \sqrt{x^4+9}\,(e\,x^2+d)\,dx$$

input
```
int((x^4 + 9)^(1/2)*(d + e*x^2),x)
```

output
```
int((x^4 + 9)^(1/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2)\sqrt{9+x^4}\,dx = \frac{\sqrt{x^4+9}\,dx}{3} + \frac{\sqrt{x^4+9}\,e\,x^3}{5} + 6\left(\int \frac{\sqrt{x^4+9}}{x^4+9}dx\right)d + \frac{18\left(\int \frac{\sqrt{x^4+9}\,x^2}{x^4+9}dx\right)e}{5}$$

input

```
int((e*x^2+d)*(x^4+9)^(1/2),x)
```

output

```
(5*sqrt(x**4 + 9)*d*x + 3*sqrt(x**4 + 9)*e*x**3 + 90*int(sqrt(x**4 + 9)/(x
**4 + 9),x)*d + 54*int((sqrt(x**4 + 9)*x**2)/(x**4 + 9),x)*e)/15
```

3.250 $\int \frac{d+ex^2}{\sqrt{9+x^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 132

$$\int \frac{d+ex^2}{\sqrt{9+x^4}}\,dx = \frac{ex\sqrt{9+x^4}}{3+x^2} - \frac{\sqrt{3}e(3+x^2)\sqrt{\frac{9+x^4}{(3+x^2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{3}}\right)|\frac{1}{2}\right)}{\sqrt{9+x^4}} + \frac{(d+3e)\,(3+x^2)\sqrt{\frac{9+x^4}{(3+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{2\sqrt{3}\sqrt{9+x^4}}$$

output
```
e*x*(x^4+9)^(1/2)/(x^2+3)-3^(1/2)*e*(x^2+3)*((x^4+9)/(x^2+3)^2)^(1/2)*Elli
pticE(sin(2*arctan(1/3*x*3^(1/2))),1/2*2^(1/2))/(x^4+9)^(1/2)+1/6*(d+3*e)*
(x^2+3)*((x^4+9)/(x^2+3)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*x*3^(1/2)),
1/2*2^(1/2))*3^(1/2)/(x^4+9)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.37

$$\int \frac{d+ex^2}{\sqrt{9+x^4}}\,dx = \frac{1}{3}dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{9}\right) + \frac{1}{9}ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{9}\right)$$

input
```
Integrate[(d + e*x^2)/Sqrt[9 + x^4],x]
```

output
```
(d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -1/9*x^4])/3 + (e*x^3*Hypergeometric
2F1[1/2, 3/4, 7/4, -1/9*x^4])/9
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{x^4+9}}dx$$

$$\Big\downarrow \text{1512}$$

$$(d+3e)\int \frac{1}{\sqrt{x^4+9}}dx-3e\int \frac{3-x^2}{3\sqrt{x^4+9}}dx$$

$$\Big\downarrow \text{27}$$

$$(d+3e)\int \frac{1}{\sqrt{x^4+9}}dx-e\int \frac{3-x^2}{\sqrt{x^4+9}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{(x^2+3)\sqrt{\frac{x^4+9}{(x^2+3)^2}}(d+3e)\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{2\sqrt{3}\sqrt{x^4+9}}-e\int \frac{3-x^2}{\sqrt{x^4+9}}dx$$

$$\Big\downarrow \text{1510}$$

$$\frac{(x^2+3)\sqrt{\frac{x^4+9}{(x^2+3)^2}}(d+3e)\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{2\sqrt{3}\sqrt{x^4+9}}-e\left(\frac{\sqrt{3}(x^2+3)\sqrt{\frac{x^4+9}{(x^2+3)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{3}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+9}}-\frac{x\sqrt{x^4+9}}{x^2+3}\right)$$

input

```
Int[(d + e*x^2)/Sqrt[9 + x^4],x]
```

output

```
-(e*(-((x*Sqrt[9 + x^4])/(3 + x^2)) + (Sqrt[3]*(3 + x^2)*Sqrt[(9 + x^4)/(3
 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[3]], 1/2])/Sqrt[9 + x^4])) + ((d + 3*
e)*(3 + x^2)*Sqrt[(9 + x^4)/(3 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[3]], 1/
2])/(2*Sqrt[3]*Sqrt[9 + x^4])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.26

method	result
meijerg	$\frac{e\,x^{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{x^{4}}{9}\right)}{9}+\frac{dx\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-\frac{x^{4}}{9}\right)}{3}$
default	$\frac{d\sqrt{-3ix^{2}+9}\,\sqrt{3ix^{2}+9}\ \mathrm{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right),i\right)}{9\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^{4}+9}}+\frac{ie\sqrt{-3ix^{2}+9}\,\sqrt{3ix^{2}+9}\ \Bigl(\mathrm{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right),i\right)-\mathrm{EllipticE}\Bigl(x\Bigl(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}}{3\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^{4}+9}}$
elliptic	$\frac{d\sqrt{-3ix^{2}+9}\,\sqrt{3ix^{2}+9}\ \mathrm{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right),i\right)}{9\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^{4}+9}}+\frac{ie\sqrt{-3ix^{2}+9}\,\sqrt{3ix^{2}+9}\ \Bigl(\mathrm{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right),i\right)-\mathrm{EllipticE}\Bigl(x\Bigl(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}}{3\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^{4}+9}}$

input
```
int((e*x^2+d)/(x^4+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*e*x^3*hypergeom([1/2,3/4],[7/4],-1/9*x^4)+1/3*d*x*hypergeom([1/4,1/2],
[5/4],-1/9*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.41

$$\int \frac{d+ex^2}{\sqrt{9+x^4}}\,dx = \frac{9i\,\sqrt{3i}exE(\arcsin\left(\frac{\sqrt{3i}}{x}\right)\mid-1)+i\,\sqrt{3i}(d-9\,e)xF(\arcsin\left(\frac{\sqrt{3i}}{x}\right)\mid-1)+3\,\sqrt{x^4+9}e}{3\,x}$$

input
```
integrate((e*x^2+d)/(x^4+9)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(9*I*sqrt(3*I)*e*x*elliptic_e(arcsin(sqrt(3*I)/x), -1) + I*sqrt(3*I)*(
d - 9*e)*x*elliptic_f(arcsin(sqrt(3*I)/x), -1) + 3*sqrt(x^4 + 9)*e)/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.49

$$\int \frac{d+ex^2}{\sqrt{9+x^4}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{12\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{12\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(x**4+9)**(1/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi)/9)/(12*gamma
(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/
9)/(12*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{9+x^4}}\,dx = \int \frac{ex^2+d}{\sqrt{x^4+9}}\,dx$$

input
```
integrate((e*x^2+d)/(x^4+9)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(x^4 + 9), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{9+x^4}}\,dx = \int \frac{ex^2+d}{\sqrt{x^4+9}}\,dx$$

input
```
integrate((e*x^2+d)/(x^4+9)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(x^4 + 9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{9+x^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{x^4+9}}\,dx$$

input
```
int((d + e*x^2)/(x^4 + 9)^(1/2),x)
```

output
```
int((d + e*x^2)/(x^4 + 9)^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{\sqrt{9+x^4}}\,dx=\left(\int \frac{\sqrt{x^4+9}}{x^4+9}dx\right)d+\left(\int \frac{\sqrt{x^4+9}\,x^2}{x^4+9}dx\right)e$$

input
```
int((e*x^2+d)/(x^4+9)^(1/2),x)
```

output
```
int(sqrt(x**4 + 9)/(x**4 + 9),x)*d + int((sqrt(x**4 + 9)*x**2)/(x**4 + 9),
x)*e
```

3.251 $\int \frac{d+ex^2}{\left(9+x^4\right)^{3/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 158

$$\begin{aligned}\int \frac{d+ex^2}{\left(9+x^4\right)^{3/2}} \, dx = {} & \frac{x(d+ex^2)}{18\sqrt{9+x^4}} - \frac{ex\sqrt{9+x^4}}{18\left(3+x^2\right)} \\ & + \frac{e(3+x^2)\sqrt{\frac{9+x^4}{(3+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{3}}\right)|\frac{1}{2}\right)}{6\sqrt{3}\sqrt{9+x^4}} \\ & + \frac{(d-3e)\left(3+x^2\right)\sqrt{\frac{9+x^4}{(3+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{36\sqrt{3}\sqrt{9+x^4}}\end{aligned}$$

output

```
1/18*x*(e*x^2+d)/(x^4+9)^(1/2)-e*x*(x^4+9)^(1/2)/(18*x^2+54)+1/18*3^(1/2)*
e*(x^2+3)*((x^4+9)/(x^2+3)^2)^(1/2)*EllipticE(sin(2*arctan(1/3*x*3^(1/2)))
,1/2*2^(1/2))/(x^4+9)^(1/2)+1/108*(d-3*e)*(x^2+3)*((x^4+9)/(x^2+3)^2)^(1/2
)*InverseJacobiAM(2*arctan(1/3*x*3^(1/2)),1/2*2^(1/2))*3^(1/2)/(x^4+9)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.39

$$\int \frac{d+ex^2}{(9+x^4)^{3/2}}\,dx = \frac{1}{162}x\left(\frac{9d}{\sqrt{9+x^4}} + 3d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{9}\right)\right.$$
$$\left.+ 2ex^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{x^4}{9}\right)\right)$$

input
```
Integrate[(d + e*x^2)/(9 + x^4)^(3/2),x]
```

output
```
(x*((9*d)/Sqrt[9 + x^4] + 3*d*Hypergeometric2F1[1/4, 1/2, 5/4, -1/9*x^4] +
 2*e*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, -1/9*x^4]))/162
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1493, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(x^4+9)^{3/2}}\,dx$$

$$\Big\downarrow \text{1493}$$

$$\frac{x(d+ex^2)}{18\sqrt{x^4+9}} - \frac{1}{18}\int -\frac{d-ex^2}{\sqrt{x^4+9}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{18}\int \frac{d-ex^2}{\sqrt{x^4+9}}dx + \frac{x(d+ex^2)}{18\sqrt{x^4+9}}$$

$$\Big\downarrow \text{1512}$$

$$\frac{1}{18}\left((d-3e)\int \frac{1}{\sqrt{x^4+9}}dx+3e\int \frac{3-x^2}{3\sqrt{x^4+9}}dx\right)+\frac{x\left(d+ex^2\right)}{18\sqrt{x^4+9}}$$

$$\Big\downarrow 27$$

$$\frac{1}{18}\left((d-3e)\int \frac{1}{\sqrt{x^4+9}}dx+e\int \frac{3-x^2}{\sqrt{x^4+9}}dx\right)+\frac{x\left(d+ex^2\right)}{18\sqrt{x^4+9}}$$

$$\Big\downarrow 761$$

$$\frac{1}{18}\left(e\int \frac{3-x^2}{\sqrt{x^4+9}}dx+\frac{\left(x^2+3\right)\sqrt{\frac{x^4+9}{\left(x^2+3\right)^2}}(d-3e)\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{2\sqrt{3}\sqrt{x^4+9}}\right)+\frac{x\left(d+ex^2\right)}{18\sqrt{x^4+9}}$$

$$\Big\downarrow 1510$$

$$\frac{1}{18}\left(\frac{\left(x^2+3\right)\sqrt{\frac{x^4+9}{\left(x^2+3\right)^2}}(d-3e)\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{2\sqrt{3}\sqrt{x^4+9}}+e\left(\frac{\sqrt{3}\left(x^2+3\right)\sqrt{\frac{x^4+9}{\left(x^2+3\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{3}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+9}}\right.\right.$$

$$\frac{x\left(d+ex^2\right)}{18\sqrt{x^4+9}}$$

input

```
Int[(d + e*x^2)/(9 + x^4)^(3/2),x]
```

output

```
(x*(d + e*x^2))/(18*Sqrt[9 + x^4]) + (e*(-((x*Sqrt[9 + x^4])/(3 + x^2)) +
(Sqrt[3]*(3 + x^2)*Sqrt[(9 + x^4)/(3 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[3
]], 1/2])/Sqrt[9 + x^4]) + ((d - 3*e)*(3 + x^2)*Sqrt[(9 + x^4)/(3 + x^2)^2
]*EllipticF[2*ArcTan[x/Sqrt[3]], 1/2])/(2*Sqrt[3]*Sqrt[9 + x^4]))/18
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.22

method	result
meijerg	$\frac{e\,x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{7}{4}\right],-\frac{x^{4}}{9}\right)}{81}+\frac{d x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],-\frac{x^{4}}{9}\right)}{27}$
risch	$\frac{x\left(e\,x^{2}+d\right)}{18\sqrt{x^{4}+9}}-\frac{i e\sqrt{-3 i x^{2}+9}\,\sqrt{3 i x^{2}+9}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right),i\right)\right)}{54\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^{4}+9}}+\frac{d\sqrt{-3 i x^{2}+9}\,\sqrt{3 i x^{2}+9}\,\text{Ellipt}}{162\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right.}$
elliptic	$-\frac{2\left(-\frac{1}{36}e\,x^{3}-\frac{1}{36}d x\right)}{\sqrt{x^{4}+9}}-\frac{i e\sqrt{-3 i x^{2}+9}\,\sqrt{3 i x^{2}+9}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right),i\right)\right)}{54\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^{4}+9}}+\frac{d\sqrt{-3 i x^{2}+9}\,\sqrt{3i}}{162(}$
default	$d\left(\frac{x}{18\sqrt{x^{4}+9}}+\frac{\sqrt{-3 i x^{2}+9}\,\sqrt{3 i x^{2}+9}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right),i\right)}{162\left(\frac{\sqrt{6}}{6}+\frac{i\sqrt{6}}{6}\right)\sqrt{x^{4}+9}}\right)+e\left(\frac{x^{3}}{18\sqrt{x^{4}+9}}-\frac{i\sqrt{-3 i x^{2}+9}\,\sqrt{3 i x^{2}+9}\,\left(\operatorname{EllipticF}\left(x\left(\right.\right.\right.}{54\left(\frac{\sqrt{6}}{6}\right.}\right.$

input
```
int((e*x^2+d)/(x^4+9)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/81*e*x^3*hypergeom([3/4,3/2],[7/4],-1/9*x^4)+1/27*d*x*hypergeom([1/4,3/2
],[5/4],-1/9*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{d+ex^2}{(9+x^4)^{3/2}}\,dx = \frac{\sqrt{\frac{1}{3}i}(i\,ex^4+9i\,e)E(\arcsin\left(\sqrt{\frac{1}{3}}ix\right)\,|-1)+\sqrt{\frac{1}{3}}i(-i\,(d+e)x^4-9i\,d-9i\,e)F(\arcsin (}{18\,(x^4+9)}$$

input
```
integrate((e*x^2+d)/(x^4+9)^(3/2),x, algorithm="fricas")
```

output
```
1/18*(sqrt(1/3*I)*(I*e*x^4 + 9*I*e)*elliptic_e(arcsin(sqrt(1/3*I)*x), -1)
+ sqrt(1/3*I)*(-I*(d + e)*x^4 - 9*I*d - 9*I*e)*elliptic_f(arcsin(sqrt(1/3*
I)*x), -1) + (e*x^3 + d*x)*sqrt(x^4 + 9))/(x^4 + 9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.41

$$\int \frac{d+ex^2}{(9+x^4)^{3/2}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{x^4e^{i\pi}}{9}\right)}{108\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\,\frac{x^4e^{i\pi}}{9}\right)}{108\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(x**4+9)**(3/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi)/9)/(108*gamm
a(5/4)) + e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*exp_polar(I*pi)
/9)/(108*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{(9+x^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(x^4+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(x^4+9)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(x^4 + 9)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(9+x^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(x^4+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(x^4+9)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(x^4 + 9)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(9+x^4)^{3/2}}\,dx = \int \frac{e\,x^2+d}{(x^4+9)^{3/2}}\,dx$$

input
```
int((d + e*x^2)/(x^4 + 9)^(3/2),x)
```

output
```
int((d + e*x^2)/(x^4 + 9)^(3/2), x)
```

Reduce [F]

$$\int \frac{d+e x^2}{\left(9+x^4\right)^{3/2}}\,dx=\left(\int \frac{\sqrt{x^4+9}}{x^8+18x^4+81}dx\right)d+\left(\int \frac{\sqrt{x^4+9}\,x^2}{x^8+18x^4+81}dx\right)e$$

input
```
int((e*x^2+d)/(x^4+9)^(3/2),x)
```

output
```
int(sqrt(x**4 + 9)/(x**8 + 18*x**4 + 81),x)*d + int((sqrt(x**4 + 9)*x**2)/
(x**8 + 18*x**4 + 81),x)*e
```

3.252 $\int \frac{d+ex^2}{\left(9+x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 184

$$
\begin{aligned}
\int \frac{d+ex^2}{\left(9+x^4\right)^{5/2}}\,dx &= \frac{x(d+ex^2)}{54\left(9+x^4\right)^{3/2}}+\frac{x(5d+3ex^2)}{972\sqrt{9+x^4}} \\
&\quad -\frac{ex\sqrt{9+x^4}}{324\left(3+x^2\right)}+\frac{e(3+x^2)\sqrt{\frac{9+x^4}{(3+x^2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{3}}\right)|\frac{1}{2}\right)}{108\sqrt{3}\sqrt{9+x^4}} \\
&\quad +\frac{(5d-9e)\left(3+x^2\right)\sqrt{\frac{9+x^4}{(3+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{1944\sqrt{3}\sqrt{9+x^4}}
\end{aligned}
$$

output

```
1/54*x*(e*x^2+d)/(x^4+9)^(3/2)+1/972*x*(3*e*x^2+5*d)/(x^4+9)^(1/2)-e*x*(x^
4+9)^(1/2)/(324*x^2+972)+1/324*3^(1/2)*e*(x^2+3)*((x^4+9)/(x^2+3)^2)^(1/2)
*EllipticE(sin(2*arctan(1/3*x*3^(1/2))),1/2*2^(1/2))/(x^4+9)^(1/2)+1/5832*
(5*d-9*e)*(x^2+3)*((x^4+9)/(x^2+3)^2)^(1/2)*InverseJacobiAM(2*arctan(1/3*x
*3^(1/2)),1/2*2^(1/2))*3^(1/2)/(x^4+9)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.37

$$\int \frac{d+ex^2}{(9+x^4)^{5/2}}\,dx = \frac{x\left(\frac{3d(63+5x^4)}{(9+x^4)^{3/2}} + 5d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{9}\right) + 4ex^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\right.\right.}{2916}$$

input
```
Integrate[(d + e*x^2)/(9 + x^4)^(5/2),x]
```

output
```
(x*((3*d*(63 + 5*x^4))/(9 + x^4)^(3/2) + 5*d*Hypergeometric2F1[1/4, 1/2, 5
/4, -1/9*x^4] + 4*e*x^2*Hypergeometric2F1[3/4, 5/2, 7/4, -1/9*x^4]))/2916
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1493, 25, 1493, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(x^4+9)^{5/2}}dx$$

$$\Big\downarrow \text{1493}$$

$$\frac{x(d+ex^2)}{54(x^4+9)^{3/2}} - \frac{1}{54}\int -\frac{3ex^2+5d}{(x^4+9)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{54}\int \frac{3ex^2+5d}{(x^4+9)^{3/2}}dx + \frac{x(d+ex^2)}{54(x^4+9)^{3/2}}$$

$$\Big\downarrow \text{1493}$$

$$\frac{1}{54}\left(\frac{x(5d+3ex^2)}{18\sqrt{x^4+9}} - \frac{1}{18}\int -\frac{5d-3ex^2}{\sqrt{x^4+9}}dx\right) + \frac{x(d+ex^2)}{54(x^4+9)^{3/2}}$$

$$\Big\downarrow 25$$

$$\frac{1}{54}\left(\frac{1}{18}\int \frac{5d-3ex^2}{\sqrt{x^4+9}}dx+\frac{x\left(5d+3ex^2\right)}{18\sqrt{x^4+9}}\right)+\frac{x\left(d+ex^2\right)}{54\left(x^4+9\right)^{3/2}}$$

$$\Big\downarrow 1512$$

$$\frac{1}{54}\left(\frac{1}{18}\left((5d-9e)\int \frac{1}{\sqrt{x^4+9}}dx+9e\int \frac{3-x^2}{3\sqrt{x^4+9}}dx\right)+\frac{x\left(5d+3ex^2\right)}{18\sqrt{x^4+9}}\right)+\frac{x\left(d+ex^2\right)}{54\left(x^4+9\right)^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{1}{54}\left(\frac{1}{18}\left((5d-9e)\int \frac{1}{\sqrt{x^4+9}}dx+3e\int \frac{3-x^2}{\sqrt{x^4+9}}dx\right)+\frac{x\left(5d+3ex^2\right)}{18\sqrt{x^4+9}}\right)+\frac{x\left(d+ex^2\right)}{54\left(x^4+9\right)^{3/2}}$$

$$\Big\downarrow 761$$

$$\frac{1}{54}\left(\frac{1}{18}\left(3e\int \frac{3-x^2}{\sqrt{x^4+9}}dx+\frac{\left(x^2+3\right)\sqrt{\frac{x^4+9}{\left(x^2+3\right)^2}}(5d-9e)\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{2\sqrt{3}\sqrt{x^4+9}}\right)+\frac{x\left(5d+3ex^2\right)}{18\sqrt{x^4+9}}\right)+$$
$$\frac{x\left(d+ex^2\right)}{54\left(x^4+9\right)^{3/2}}$$

$$\Big\downarrow 1510$$

$$\frac{1}{54}\left(\frac{1}{18}\left(\frac{\left(x^2+3\right)\sqrt{\frac{x^4+9}{\left(x^2+3\right)^2}}(5d-9e)\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{3}}\right),\frac{1}{2}\right)}{2\sqrt{3}\sqrt{x^4+9}}+3e\left(\frac{\sqrt{3}\left(x^2+3\right)\sqrt{\frac{x^4+9}{\left(x^2+3\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{}}\right.\right.}{\sqrt{x^4+9}}\right.\right.\right.$$
$$\frac{x\left(d+ex^2\right)}{54\left(x^4+9\right)^{3/2}}$$

input
```
Int[(d + e*x^2)/(9 + x^4)^(5/2),x]
```

output
```
(x*(d + e*x^2))/(54*(9 + x^4)^(3/2)) + ((x*(5*d + 3*e*x^2))/(18*Sqrt[9 + x
^4]) + (3*e*(-((x*Sqrt[9 + x^4])/(3 + x^2)) + (Sqrt[3]*(3 + x^2)*Sqrt[(9 +
 x^4)/(3 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[3]], 1/2])/Sqrt[9 + x^4]) + (
(5*d - 9*e)*(3 + x^2)*Sqrt[(9 + x^4)/(3 + x^2)^2]*EllipticF[2*ArcTan[x/Sqr
t[3]], 1/2])/(2*Sqrt[3]*Sqrt[9 + x^4]))/18)/54
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.18

method	result
meijerg	$\frac{e\,x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{5}{2}\right],\left[\frac{7}{4}\right],-\frac{x^{4}}{9}\right)}{729}+\frac{d x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{5}{2}\right],\left[\frac{5}{4}\right],-\frac{x^{4}}{9}\right)}{243}$
risch	$\frac{x\left(3 e\,x^{6}+5 d\,x^{4}+45 e\,x^{2}+63 d\right)}{972\left(x^{4}+9\right)^{\frac{3}{2}}}+\frac{5 d \sqrt{-3 i x^{2}+9}\,\sqrt{3 i x^{2}+9}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i \sqrt{6}}{6}\right),i\right)}{8748\left(\frac{\sqrt{6}}{6}+\frac{i \sqrt{6}}{6}\right)\sqrt{x^{4}+9}}-$
elliptic	$\frac{\frac{1}{54} e\,x^{3}+\frac{1}{54} d x}{\left(x^{4}+9\right)^{\frac{3}{2}}}-\frac{2\left(-\frac{1}{648} e\,x^{3}-\frac{5}{1944} d x\right)}{\sqrt{x^{4}+9}}+\frac{5 d \sqrt{-3 i x^{2}+9}\,\sqrt{3 i x^{2}+9}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i \sqrt{6}}{6}\right),i\right)}{8748\left(\frac{\sqrt{6}}{6}+\frac{i \sqrt{6}}{6}\right)\sqrt{x^{4}+9}}-$
default	$d\left(\frac{x}{54\left(x^{4}+9\right)^{\frac{3}{2}}}+\frac{5 x}{972\sqrt{x^{4}+9}}+\frac{5\sqrt{-3 i x^{2}+9}\,\sqrt{3 i x^{2}+9}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{6}}{6}+\frac{i \sqrt{6}}{6}\right),i\right)}{8748\left(\frac{\sqrt{6}}{6}+\frac{i \sqrt{6}}{6}\right)\sqrt{x^{4}+9}}\right)+e\left(\frac{x^{3}}{54\left(x^{4}+9\right)^{\frac{3}{2}}}+\frac{x^{3}}{324\sqrt{x^{4}+9}}-\right.$

input
```
int((e*x^2+d)/(x^4+9)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/729*e*x^3*hypergeom([3/4,5/2],[7/4],-1/9*x^4)+1/243*d*x*hypergeom([1/4,5
/2],[5/4],-1/9*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.65

$$\int \frac{d+ex^2}{\left(9+x^4\right)^{5/2}}\,dx =$$

$$-\frac{3\sqrt{\frac{1}{3}i}(-i\,ex^8-18i\,ex^4-81i\,e)E(\arcsin\left(\sqrt{\frac{1}{3}i}x\right)\,|-1)-\sqrt{\frac{1}{3}i}(-i\,(5\,d+3\,e)x^8-18i\,(5\,d+3\,e)x^4-}{972\,(x^8+18\,x^4+81)}$$

input
```
integrate((e*x^2+d)/(x^4+9)^(5/2),x, algorithm="fricas")
```

output
```
-1/972*(3*sqrt(1/3*I)*(-I*e*x^8 - 18*I*e*x^4 - 81*I*e)*elliptic_e(arcsin(s
qrt(1/3*I)*x), -1) - sqrt(1/3*I)*(-I*(5*d + 3*e)*x^8 - 18*I*(5*d + 3*e)*x^
4 - 405*I*d - 243*I*e)*elliptic_f(arcsin(sqrt(1/3*I)*x), -1) - (3*e*x^7 +
5*d*x^5 + 45*e*x^3 + 63*d*x)*sqrt(x^4 + 9))/(x^8 + 18*x^4 + 81)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.35

$$\int \frac{d+ex^2}{(9+x^4)^{5/2}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{972\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{9}\right)}{972\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(x**4+9)**(5/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), x**4*exp_polar(I*pi)/9)/(972*gamm
a(5/4)) + e*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), x**4*exp_polar(I*pi)
/9)/(972*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{(9+x^4)^{5/2}}\,dx = \int \frac{ex^2+d}{(x^4+9)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(x^4+9)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(x^4 + 9)^(5/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(9+x^4)^{5/2}}\,dx = \int \frac{ex^2+d}{(x^4+9)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(x^4+9)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(x^4 + 9)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(9+x^4)^{5/2}}\,dx = \int \frac{e\,x^2+d}{(x^4+9)^{5/2}}\,dx$$

input
```
int((d + e*x^2)/(x^4 + 9)^(5/2),x)
```

output
```
int((d + e*x^2)/(x^4 + 9)^(5/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(9+x^4)^{5/2}}\,dx = \left(\int \frac{\sqrt{x^4+9}}{x^{12}+27x^8+243x^4+729}dx\right)d + \left(\int \frac{\sqrt{x^4+9}\,x^2}{x^{12}+27x^8+243x^4+729}dx\right)e$$

input
```
int((e*x^2+d)/(x^4+9)^(5/2),x)
```

output
```
int(sqrt(x**4 + 9)/(x**12 + 27*x**8 + 243*x**4 + 729),x)*d + int((sqrt(x**
4 + 9)*x**2)/(x**12 + 27*x**8 + 243*x**4 + 729),x)*e
```

3.253 $\int \left(1-bx^2\right) \left(1+b^2x^4\right)^{5/2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 238

$$\begin{aligned}\int \left(1-bx^2\right) \left(1+b^2x^4\right)^{5/2} \, dx &= \frac{4x(195-77bx^2)\sqrt{1+b^2x^4}}{3003} \\ &- \frac{8x\sqrt{1+b^2x^4}}{39\left(1+bx^2\right)} + \frac{10x(117-77bx^2)\left(1+b^2x^4\right)^{3/2}}{9009} \\ &+ \frac{1}{143}x\left(13-11bx^2\right)\left(1+b^2x^4\right)^{5/2} + \frac{8(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)\middle|\frac{1}{2}\right)}{39\sqrt{b}\sqrt{1+b^2x^4}} \\ &+ \frac{472(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{3003\sqrt{b}\sqrt{1+b^2x^4}}\end{aligned}$$

output
```
4/3003*x*(-77*b*x^2+195)*(b^2*x^4+1)^(1/2)-8*x*(b^2*x^4+1)^(1/2)/(39*b*x^2
+39)+10/9009*x*(-77*b*x^2+117)*(b^2*x^4+1)^(3/2)+1/143*x*(-11*b*x^2+13)*(b
^2*x^4+1)^(5/2)+8/39*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(s
in(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)+472/3003*(b
*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/2)*x
),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.79 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.20

$$\int (1-bx^2)\left(1+b^2x^4\right)^{5/2}\,dx = x\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},\frac{5}{4},-b^2x^4\right) - \frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)$$

input
```
Integrate[(1 - b*x^2)*(1 + b^2*x^4)^(5/2),x]
```

output
```
x*Hypergeometric2F1[-5/2, 1/4, 5/4, -(b^2*x^4)] - (b*x^3*Hypergeometric2F1
[-5/2, 3/4, 7/4, -(b^2*x^4)])/3
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1491, 27, 1491, 27, 1491, 27, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-bx^2)\left(b^2x^4+1\right)^{5/2}dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{5}{143}\int 2(13-11bx^2)\left(b^2x^4+1\right)^{3/2}dx+\frac{1}{143}x(13-11bx^2)\left(b^2x^4+1\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{10}{143}\int (13-11bx^2)\left(b^2x^4+1\right)^{3/2}dx+\frac{1}{143}x(13-11bx^2)\left(b^2x^4+1\right)^{5/2}$$

$$\Big\downarrow \text{1491}$$

$$\frac{10}{143}\left(\frac{1}{21}\int 2\left(117-77bx^2\right)\sqrt{b^2x^4+1}dx+\frac{1}{63}x\left(117-77bx^2\right)\left(b^2x^4+1\right)^{3/2}\right)+\frac{1}{143}x\left(13-11bx^2\right)\left(b^2x^4+1\right)^{5/2}$$

$$\Big\downarrow \ 27$$

$$\frac{10}{143}\left(\frac{2}{21}\int \left(117-77bx^2\right)\sqrt{b^2x^4+1}dx+\frac{1}{63}x\left(117-77bx^2\right)\left(b^2x^4+1\right)^{3/2}\right)+\frac{1}{143}x\left(13-11bx^2\right)\left(b^2x^4+1\right)^{5/2}$$

$$\Big\downarrow \ 1491$$

$$\frac{10}{143}\left(\frac{2}{21}\left(\frac{1}{15}\int \frac{6\left(195-77bx^2\right)}{\sqrt{b^2x^4+1}}dx+\frac{1}{5}x\sqrt{b^2x^4+1}\left(195-77bx^2\right)\right)+\frac{1}{63}x\left(117-77bx^2\right)\left(b^2x^4+1\right)^{3/2}\right)+\frac{1}{143}x\left(13-11bx^2\right)\left(b^2x^4+1\right)^{5/2}$$

$$\Big\downarrow \ 27$$

$$\frac{10}{143}\left(\frac{2}{21}\left(\frac{2}{5}\int \frac{195-77bx^2}{\sqrt{b^2x^4+1}}dx+\frac{1}{5}x\sqrt{b^2x^4+1}\left(195-77bx^2\right)\right)+\frac{1}{63}x\left(117-77bx^2\right)\left(b^2x^4+1\right)^{3/2}\right)+\frac{1}{143}x\left(13-11bx^2\right)\left(b^2x^4+1\right)^{5/2}$$

$$\Big\downarrow \ 1512$$

$$\frac{10}{143}\left(\frac{2}{21}\left(\frac{2}{5}\left(118\int \frac{1}{\sqrt{b^2x^4+1}}dx+77\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{1}{5}x\sqrt{b^2x^4+1}\left(195-77bx^2\right)\right)+\frac{1}{63}x\left(117-77bx^2\right)\right.$$
$$\frac{1}{143}x\left(13-11bx^2\right)\left(b^2x^4+1\right)^{5/2}$$

$$\Big\downarrow \ 761$$

$$\frac{10}{143}\left(\frac{2}{21}\left(\frac{2}{5}\left(77\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx+\frac{59\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}\right)+\frac{1}{5}x\sqrt{b^2x^4+1}(1\right.\right.$$
$$\frac{1}{143}x\left(13-11bx^2\right)\left(b^2x^4+1\right)^{5/2}$$

$$\Big\downarrow \ 1510$$

$$\frac{10}{143}\left(\frac{2}{21}\left(\frac{2}{5}\left(\frac{59\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}+77\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,E\left(2\arctan\left(\sqrt{}\right.\right.}{\sqrt{b}\sqrt{b^2x^4+1}}\right.\right.\right.\right.$$

$$\frac{1}{143}x\left(13-11bx^2\right)\left(b^2x^4+1\right)^{5/2}$$

input

```
Int[(1 - b*x^2)*(1 + b^2*x^4)^(5/2),x]
```

output

```
(x*(13 - 11*b*x^2)*(1 + b^2*x^4)^(5/2))/143 + (10*((x*(117 - 77*b*x^2)*(1
+ b^2*x^4)^(3/2))/63 + (2*((x*(195 - 77*b*x^2)*Sqrt[1 + b^2*x^4])/5 + (2*(
77*(-((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)
/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2
*x^4])) + (59*(1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*Ar
cTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])))/5))/21))/143
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.16

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[-\frac{5}{2},\frac{1}{4}\right],\left[\frac{5}{4}\right],-b^2x^4\right)-\frac{b\,x^3\,\mathrm{hypergeom}\left(\left[-\frac{5}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^2x^4\right)}{3}$
risch	$-\frac{x\left(693x^{10}b^5-819b^4x^8+2156b^3x^6-2808b^2x^4+2387b\,x^2-4329\right)\sqrt{b^2x^4+1}}{9009}+\frac{40\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\ \mathrm{EllipticF}\left(x\sqrt{ib},i\right)}{77\sqrt{ib}\,\sqrt{b^2x^4+1}}-$
elliptic	$-\frac{b^5x^{11}\sqrt{b^2x^4+1}}{13}+\frac{b^4x^9\sqrt{b^2x^4+1}}{11}-\frac{28b^3x^7\sqrt{b^2x^4+1}}{117}+\frac{24b^2x^5\sqrt{b^2x^4+1}}{77}-\frac{31b\,x^3\sqrt{b^2x^4+1}}{117}+\frac{37x\sqrt{b^2x^4+1}}{77}+$
default	$-b\left(\frac{b^4x^{11}\sqrt{b^2x^4+1}}{13}+\frac{28b^2x^7\sqrt{b^2x^4+1}}{117}+\frac{31x^3\sqrt{b^2x^4+1}}{117}+\right.$

input
```
int((-b*x^2+1)*(b^2*x^4+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([-5/2,1/4],[5/4],-b^2*x^4)-1/3*b*x^3*hypergeom([-5/2,3/4],[7/4
],-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.53

$$\int \left(1-bx^2\right)\left(1+b^2x^4\right)^{5/2}\,dx =$$

$$-\frac{1848\,bx\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)|-1) - 24\,(195\,b^2+77\,b)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)|-1) + (693}{9009\,bx}$$

input

```
integrate((-b*x^2+1)*(b^2*x^4+1)^(5/2),x, algorithm="fricas")
```

output

```
-1/9009*(1848*b*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1)
 - 24*(195*b^2 + 77*b)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x)
, -1) + (693*b^6*x^12 - 819*b^5*x^10 + 2156*b^4*x^8 - 2808*b^3*x^6 + 2387*
b^2*x^4 - 4329*b*x^2 + 1848)*sqrt(b^2*x^4 + 1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.32 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \left(1-bx^2\right)\left(1+b^2x^4\right)^{5/2}\,dx = &-\frac{b^5x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{15}{4}\right)}\\ &+\frac{b^4x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{13}{4}\right)}\\ &-\frac{b^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{2\Gamma\left(\frac{11}{4}\right)}+\frac{b^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{2\Gamma\left(\frac{9}{4}\right)}\\ &-\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}\end{aligned}$$

input
```
integrate((-b*x**2+1)*(b**2*x**4+1)**(5/2),x)
```

output
```
-b**5*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b**2*x**4*exp_polar(I
*pi))/(4*gamma(15/4)) + b**4*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b
**2*x**4*exp_polar(I*pi))/(4*gamma(13/4)) - b**3*x**7*gamma(7/4)*hyper((-1
/2, 7/4), (11/4,), b**2*x**4*exp_polar(I*pi))/(2*gamma(11/4)) + b**2*x**5*
gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b**2*x**4*exp_polar(I*pi))/(2*gamma(
9/4)) - b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I
*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b**2*x**4*e
xp_polar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int (1-bx^2)\left(1+b^2x^4\right)^{5/2}\,dx=\int -\left(b^2x^4+1\right)^{\frac{5}{2}}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(b^2*x^4+1)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((b^2*x^4 + 1)^(5/2)*(b*x^2 - 1), x)
```

Giac [F]

$$\int (1-bx^2)\left(1+b^2x^4\right)^{5/2}\,dx=\int -\left(b^2x^4+1\right)^{\frac{5}{2}}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(b^2*x^4+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(b^2*x^4 + 1)^(5/2)*(b*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-bx^2\right)\left(1+b^2x^4\right)^{5/2}\,dx=-\int \left(b^2\,x^4+1\right)^{5/2}\,\left(b\,x^2-1\right)\,dx$$

input
```
int(-(b^2*x^4 + 1)^(5/2)*(b*x^2 - 1),x)
```

output
```
-int((b^2*x^4 + 1)^(5/2)*(b*x^2 - 1), x)
```

Reduce [F]

$$\int \left(1-bx^2\right)\left(1+b^2x^4\right)^{5/2}\,dx=-\frac{\sqrt{b^2x^4+1}\,b^5x^{11}}{13}+\frac{\sqrt{b^2x^4+1}\,b^4x^9}{11}-\frac{28\sqrt{b^2x^4+1}\,b^3x^7}{117}+\frac{24\sqrt{b^2x^4+1}\,b^2x^5}{77}-\frac{31\sqrt{b^2x^4+1}\,b\,x^3}{117}+\frac{37\sqrt{b^2x^4+1}\,x}{77}+\frac{40\left(\int \frac{\sqrt{b^2x^4+1}}{b^2x^4+1}dx\right)}{77}-\frac{8\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^2x^4+1}dx\right)b}{39}$$

input
```
int((-b*x^2+1)*(b^2*x^4+1)^(5/2),x)
```

output
```
( - 693*sqrt(b**2*x**4 + 1)*b**5*x**11 + 819*sqrt(b**2*x**4 + 1)*b**4*x**9
 - 2156*sqrt(b**2*x**4 + 1)*b**3*x**7 + 2808*sqrt(b**2*x**4 + 1)*b**2*x**5
 - 2387*sqrt(b**2*x**4 + 1)*b*x**3 + 4329*sqrt(b**2*x**4 + 1)*x + 4680*int
(sqrt(b**2*x**4 + 1)/(b**2*x**4 + 1),x) - 1848*int((sqrt(b**2*x**4 + 1)*x*
*2)/(b**2*x**4 + 1),x)*b)/9009
```

3.254 $\int \left(1-bx^2\right)\left(1+b^2x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 212

$$\int \left(1-bx^2\right)\left(1+b^2x^4\right)^{3/2}\,dx = \frac{2}{105}x\left(15-7bx^2\right)\sqrt{1+b^2x^4}-\frac{4x\sqrt{1+b^2x^4}}{15\left(1+bx^2\right)} + \frac{1}{63}x\left(9-7bx^2\right)\left(1+b^2x^4\right)^{3/2} + \frac{4(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)\middle|\frac{1}{2}\right)}{15\sqrt{b}\sqrt{1+b^2x^4}} + \frac{16(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{105\sqrt{b}\sqrt{1+b^2x^4}}$$

output

```
2/105*x*(-7*b*x^2+15)*(b^2*x^4+1)^(1/2)-4*x*(b^2*x^4+1)^(1/2)/(15*b*x^2+15
)+1/63*x*(-7*b*x^2+9)*(b^2*x^4+1)^(3/2)+4/15*(b*x^2+1)*((b^2*x^4+1)/(b*x^2
+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(b^2*
x^4+1)^(1/2)+16/105*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*InverseJacob
iAM(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.49 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.22

$$\int (1-bx^2)\left(1+b^2x^4\right)^{3/2}\,dx = x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-b^2x^4\right) - \frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)$$

input
```
Integrate[(1 - b*x^2)*(1 + b^2*x^4)^(3/2),x]
```

output
```
x*Hypergeometric2F1[-3/2, 1/4, 5/4, -(b^2*x^4)] - (b*x^3*Hypergeometric2F1
[-3/2, 3/4, 7/4, -(b^2*x^4)])/3
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1491, 27, 1491, 27, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-bx^2)\left(b^2x^4+1\right)^{3/2}dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{21}\int 2(9-7bx^2)\sqrt{b^2x^4+1}dx+\frac{1}{63}x(9-7bx^2)\left(b^2x^4+1\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{21}\int (9-7bx^2)\sqrt{b^2x^4+1}dx+\frac{1}{63}x(9-7bx^2)\left(b^2x^4+1\right)^{3/2}$$

$$\Big\downarrow \text{1491}$$

$$\frac{2}{21}\left(\frac{1}{15}\int \frac{6(15-7bx^2)}{\sqrt{b^2x^4+1}}dx+\frac{1}{5}x\sqrt{b^2x^4+1}(15-7bx^2)\right)+\frac{1}{63}x(9-7bx^2)\left(b^2x^4+1\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{21}\left(\frac{2}{5}\int\frac{15-7bx^2}{\sqrt{b^2x^4+1}}dx+\frac{1}{5}x\sqrt{b^2x^4+1}\left(15-7bx^2\right)\right)+\frac{1}{63}x\left(9-7bx^2\right)\left(b^2x^4+1\right)^{3/2}$$

$$\Big\downarrow \text{1512}$$

$$\frac{2}{21}\left(\frac{2}{5}\left(8\int\frac{1}{\sqrt{b^2x^4+1}}dx+7\int\frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{1}{5}x\sqrt{b^2x^4+1}\left(15-7bx^2\right)\right)+$$
$$\frac{1}{63}x\left(9-7bx^2\right)\left(b^2x^4+1\right)^{3/2}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{21}\left(\frac{2}{5}\left(7\int\frac{1-bx^2}{\sqrt{b^2x^4+1}}dx+\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}\right)+\frac{1}{5}x\sqrt{b^2x^4+1}(15-7bx^2\right.$$
$$\frac{1}{63}x\left(9-7bx^2\right)\left(b^2x^4+1\right)^{3/2}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2}{21}\Bigg(\frac{2}{5}\Bigg(\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}+7\Bigg(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}$$
$$\frac{1}{63}x\left(9-7bx^2\right)\left(b^2x^4+1\right)^{3/2}$$

input

```
Int[(1 - b*x^2)*(1 + b^2*x^4)^(3/2),x]
```

output

```
(x*(9 - 7*b*x^2)*(1 + b^2*x^4)^(3/2))/63 + (2*((x*(15 - 7*b*x^2)*Sqrt[1 +
b^2*x^4])/5 + (2*(7*(-((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*S
qrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqr
t[b]*Sqrt[1 + b^2*x^4])) + (4*(1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2
]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])))/5))/2
1
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.18

method	result
meijerg	$x \,\text{hypergeom}\left(\left[-\frac{3}{2}, \frac{1}{4}\right], \left[\frac{5}{4}\right], -b^2x^4\right) - \frac{b\,x^3 \,\text{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^2x^4\right)}{3}$
risch	$-\frac{x\left(35b^3x^6-45b^2x^4+77b\,x^2-135\right)\sqrt{b^2x^4+1}}{315} + \frac{4\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\text{EllipticF}\left(x\sqrt{ib},i\right)}{7\sqrt{ib}\,\sqrt{b^2x^4+1}} - \frac{4i\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\left(\text{EllipticF}\right.}{15\sqrt{ib}\,\sqrt{}}$
elliptic	$-\frac{b^3x^7\sqrt{b^2x^4+1}}{9} + \frac{b^2x^5\sqrt{b^2x^4+1}}{7} - \frac{11b\,x^3\sqrt{b^2x^4+1}}{45} + \frac{3x\sqrt{b^2x^4+1}}{7} + \frac{4\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\text{EllipticF}\left(x\sqrt{ib},i\right)}{7\sqrt{ib}\,\sqrt{b^2x^4+1}} -$
default	$-b\left(\frac{b^2x^7\sqrt{b^2x^4+1}}{9} + \frac{11x^3\sqrt{b^2x^4+1}}{45} + \frac{4i\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\left(\text{EllipticF}\left(x\sqrt{ib},i\right)-\text{EllipticE}\left(x\sqrt{ib},i\right)\right)}{15\sqrt{ib}\,\sqrt{b^2x^4+1}\,b}\right) + \frac{b^2x^5\sqrt{b^2x^4+}}{7}$

input
```
int((-b*x^2+1)*(b^2*x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([-3/2,1/4],[5/4],-b^2*x^4)-1/3*b*x^3*hypergeom([-3/2,3/4],[7/4
],-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.52

$$\int \left(1-bx^2\right)\left(1+b^2x^4\right)^{3/2}\,dx =$$

$$-\frac{84\,bx\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1) - 12\left(15\,b^2+7\,b\right)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1) + (35\,b^4x^8}{315\,bx}$$

input
```
integrate((-b*x^2+1)*(b^2*x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/315*(84*b*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1) - 1
2*(15*b^2 + 7*b)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x), -1)
 + (35*b^4*x^8 - 45*b^3*x^6 + 77*b^2*x^4 - 135*b*x^2 + 84)*sqrt(b^2*x^4 +
1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.70

$$\int \left(1-bx^2\right)\left(1+b^2x^4\right)^{3/2}\,dx = -\frac{b^3x^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{b^2x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{bx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)*(b**2*x**4+1)**(3/2),x)
```

output
```
-b**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b**2*x**4*exp_polar(I*pi
))/(4*gamma(11/4)) + b**2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b**2*
x**4*exp_polar(I*pi))/(4*gamma(9/4)) - b*x**3*gamma(3/4)*hyper((-1/2, 3/4)
, (7/4,), b**2*x**4*exp_polar(I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((
-1/2, 1/4), (5/4,), b**2*x**4*exp_polar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1-bx^2\right)\left(1+b^2x^4\right)^{3/2}\,dx = \int -\left(b^2x^4+1\right)^{\frac{3}{2}}\left(bx^2-1\right)\,dx$$

input
```
integrate((-b*x^2+1)*(b^2*x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((b^2*x^4 + 1)^(3/2)*(b*x^2 - 1), x)
```

Giac [F]

$$\int \left(1-bx^2\right)\left(1+b^2x^4\right)^{3/2}\,dx=\int -\left(b^2x^4+1\right)^{\frac{3}{2}}\left(bx^2-1\right)\,dx$$

input
```
integrate((-b*x^2+1)*(b^2*x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(b^2*x^4 + 1)^(3/2)*(b*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-bx^2\right)\left(1+b^2x^4\right)^{3/2}\,dx=-\int \left(b^2\,x^4+1\right)^{3/2}\,\left(b\,x^2-1\right)\,dx$$

input
```
int(-(b^2*x^4 + 1)^(3/2)*(b*x^2 - 1),x)
```

output
```
-int((b^2*x^4 + 1)^(3/2)*(b*x^2 - 1), x)
```

Reduce [F]

$$\int \left(1-bx^2\right)\left(1+b^2x^4\right)^{3/2}\,dx=-\frac{\sqrt{b^2x^4+1}\,b^3x^7}{9}+\frac{\sqrt{b^2x^4+1}\,b^2x^5}{7}-\frac{11\sqrt{b^2x^4+1}\,b\,x^3}{45}+\frac{3\sqrt{b^2x^4+1}\,x}{7}+\frac{4\left(\int \frac{\sqrt{b^2x^4+1}}{b^2x^4+1}dx\right)}{7}-\frac{4\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^2x^4+1}dx\right)b}{15}$$

input
```
int((-b*x^2+1)*(b^2*x^4+1)^(3/2),x)
```

output
```
( - 35*sqrt(b**2*x**4 + 1)*b**3*x**7 + 45*sqrt(b**2*x**4 + 1)*b**2*x**5 -
77*sqrt(b**2*x**4 + 1)*b*x**3 + 135*sqrt(b**2*x**4 + 1)*x + 180*int(sqrt(b
**2*x**4 + 1)/(b**2*x**4 + 1),x) - 84*int((sqrt(b**2*x**4 + 1)*x**2)/(b**2
*x**4 + 1),x)*b)/315
```

3.255 $\int \left(1-bx^2\right) \sqrt{1+b^2x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 186

$$\begin{aligned}\int \left(1-bx^2\right) \sqrt{1+b^2x^4} \, dx &= \frac{1}{15}x\left(5-3bx^2\right)\sqrt{1+b^2x^4}-\frac{2x\sqrt{1+b^2x^4}}{5\left(1+bx^2\right)} \\ &\quad+\frac{2(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)\middle|\frac{1}{2}\right)}{5\sqrt{b}\sqrt{1+b^2x^4}} \\ &\quad+\frac{2(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{15\sqrt{b}\sqrt{1+b^2x^4}}\end{aligned}$$

output

```
1/15*x*(-3*b*x^2+5)*(b^2*x^4+1)^(1/2)-2*x*(b^2*x^4+1)^(1/2)/(5*b*x^2+5)+2/
5*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/2)
*x)),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)+2/15*(b*x^2+1)*((b^2*x^4+1)/(b
*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b^(1/2)/
(b^2*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.92 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.25

$$\int (1-bx^2)\sqrt{1+b^2x^4}\,dx = x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-b^2x^4\right) - \frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)$$

input
```
Integrate[(1 - b*x^2)*Sqrt[1 + b^2*x^4],x]
```

output
```
x*Hypergeometric2F1[-1/2, 1/4, 5/4, -(b^2*x^4)] - (b*x^3*Hypergeometric2F1
[-1/2, 3/4, 7/4, -(b^2*x^4)])/3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1491, 27, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-bx^2)\sqrt{b^2x^4+1}\,dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{15}\int \frac{2(5-3bx^2)}{\sqrt{b^2x^4+1}}dx+\frac{1}{15}x\sqrt{b^2x^4+1}(5-3bx^2)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{15}\int \frac{5-3bx^2}{\sqrt{b^2x^4+1}}dx+\frac{1}{15}x\sqrt{b^2x^4+1}(5-3bx^2)$$

$$\Big\downarrow \text{1512}$$

$$\frac{2}{15}\left(2\int \frac{1}{\sqrt{b^2x^4+1}}dx+3\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{1}{15}x\sqrt{b^2x^4+1}(5-3bx^2)$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{15}\left(3\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx+\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}\right)+\frac{1}{15}x\sqrt{b^2x^4+1}\left(5-3bx^2\right)$$

$$\Big\downarrow \text{1510}$$

$$\frac{2}{15}\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}+3\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\right.\right.$$

$$\frac{1}{15}x\sqrt{b^2x^4+1}\left(5-3bx^2\right)$$

input
```
Int[(1 - b*x^2)*Sqrt[1 + b^2*x^4],x]
```

output
```
(x*(5 - 3*b*x^2)*Sqrt[1 + b^2*x^4])/15 + (2*(3*(-((x*Sqrt[1 + b^2*x^4])/(1
 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*Ar
cTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])) + ((1 + b*x^2)*Sqrt[(1
 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*S
qrt[1 + b^2*x^4])))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.20

method	result
meijerg	$x\,\operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{5}{4}\right],-b^2x^4\right)-\frac{b\,x^3\operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^2x^4\right)}{3}$
risch	$-\frac{x\left(3b\,x^2-5\right)\sqrt{b^2x^4+1}}{15}+\frac{2\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{ib},i\right)}{3\sqrt{ib}\,\sqrt{b^2x^4+1}}-\frac{2i\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\bigl(\operatorname{EllipticF}\left(x\sqrt{ib},i\right)-\operatorname{EllipticE}}{5\sqrt{ib}\,\sqrt{b^2x^4+1}}$
elliptic	$-\frac{b\,x^3\sqrt{b^2x^4+1}}{5}+\frac{x\sqrt{b^2x^4+1}}{3}+\frac{2\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{ib},i\right)}{3\sqrt{ib}\,\sqrt{b^2x^4+1}}-\frac{2i\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\bigl(\operatorname{EllipticF}\left(x\sqrt{ib},i\right)-\mathrm{E}}{5\sqrt{ib}\,\sqrt{b^2x^4+1}}$
default	$-b\left(\frac{x^3\sqrt{b^2x^4+1}}{5}+\frac{2i\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{ib},i\right)-\operatorname{EllipticE}\left(x\sqrt{ib},i\right)\right)}{5\sqrt{ib}\,\sqrt{b^2x^4+1}\,b}\right)+\frac{x\sqrt{b^2x^4+1}}{3}+\frac{2\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x}}{3\sqrt{ib}}$

input
```
int((-b*x^2+1)*(b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([-1/2,1/4],[5/4],-b^2*x^4)-1/3*b*x^3*hypergeom([-1/2,3/4],[7/4
],-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.51

$$\int \left(1-bx^2\right)\sqrt{1+b^2x^4}\,dx =$$

$$-\frac{6\,bx\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-2\,(5\,b^2+3\,b)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+(3\,b^2x^4-}{15\,bx}$$

input
```
integrate((-b*x^2+1)*(b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(6*b*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1) - 2*(
5*b^2 + 3*b)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x), -1) + (
3*b^2*x^4 - 5*b*x^2 + 6)*sqrt(b^2*x^4 + 1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int \left(1-bx^2\right)\sqrt{1+b^2x^4}\,dx$$

$$=-\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)*(b**2*x**4+1)**(1/2),x)
```

output
```
-b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(
4*gamma(7/4)) + x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b**2*x**4*exp_pola
r(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1-bx^2\right)\sqrt{1+b^2x^4}\,dx=\int -\sqrt{b^2x^4+1}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(b^2*x^4 + 1)*(b*x^2 - 1), x)
```

Giac [F]

$$\int \left(1-bx^2\right)\sqrt{1+b^2x^4}\,dx=\int -\sqrt{b^2x^4+1}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(b^2*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(b^2*x^4 + 1)*(b*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-bx^2\right)\sqrt{1+b^2x^4}\,dx=-\int \sqrt{b^2\,x^4+1}\,\left(b\,x^2-1\right)\,dx$$

input
```
int(-(b^2*x^4 + 1)^(1/2)*(b*x^2 - 1),x)
```

output
```
-int((b^2*x^4 + 1)^(1/2)*(b*x^2 - 1), x)
```

Reduce [F]

$$\int (1-bx^2)\sqrt{1+b^2x^4}\,dx = -\frac{\sqrt{b^2x^4+1}\,b\,x^3}{5}+\frac{\sqrt{b^2x^4+1}\,x}{3}+\frac{2\left(\int \frac{\sqrt{b^2x^4+1}}{b^2x^4+1}dx\right)}{3}-\frac{2\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^2x^4+1}dx\right)b}{5}$$

input

```
int((-b*x^2+1)*(b^2*x^4+1)^(1/2),x)
```

output

```
( - 3*sqrt(b**2*x**4 + 1)*b*x**3 + 5*sqrt(b**2*x**4 + 1)*x + 10*int(sqrt(b
**2*x**4 + 1)/(b**2*x**4 + 1),x) - 6*int((sqrt(b**2*x**4 + 1)*x**2)/(b**2*
x**4 + 1),x)*b)/15
```

3.256 $\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = -\frac{x\sqrt{1+b^2x^4}}{1+bx^2} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}}$$

output
```
-x*(b^2*x^4+1)^(1/2)/(b*x^2+1)+(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*E
llipticE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\begin{aligned}\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = {} & x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right) \\ & -\frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)\end{aligned}$$

input
```
Integrate[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] - (b*x^3*Hypergeometric2F1[
1/2, 3/4, 7/4, -(b^2*x^4)])/3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{1510\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx$$

$$\Big\downarrow \color{blue}{1510}$$

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

input
```
Int[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

output
```
-((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1
+ b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4
])
```

Defintions of rubi rules used

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.86 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.43

method	result	size
meijerg	$-\frac{b\,x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^{2} x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], -b^{2} x^{4}\right)$	38
default	$-\frac{i \sqrt{-i b\,x^{2}+1}\, \sqrt{i b\,x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{i b}, i\right)-\operatorname{EllipticE}\left(x \sqrt{i b}, i\right)\right)}{\sqrt{i b}\, \sqrt{b^{2} x^{4}+1}}+\frac{\sqrt{-i b\,x^{2}+1}\, \sqrt{i b\,x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i b}, i\right)}{\sqrt{i b}\, \sqrt{b^{2} x^{4}+1}}$	120
elliptic	$-\frac{i \sqrt{-i b\,x^{2}+1}\, \sqrt{i b\,x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{i b}, i\right)-\operatorname{EllipticE}\left(x \sqrt{i b}, i\right)\right)}{\sqrt{i b}\, \sqrt{b^{2} x^{4}+1}}+\frac{\sqrt{-i b\,x^{2}+1}\, \sqrt{i b\,x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i b}, i\right)}{\sqrt{i b}\, \sqrt{b^{2} x^{4}+1}}$	120

input
```
int((-b*x^2+1)/(b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b*x^3*hypergeom([1/2,3/4],[7/4],-b^2*x^4)+x*hypergeom([1/4,1/2],[5/4]
,-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = -\frac{bx\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} E(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right) \mid -1) - (b^2+b)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right) \mid -1) + \sqrt{b^2x^4+1}}{bx}$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-(b*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1) - (b^2 + b)*
x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x), -1) + sqrt(b^2*x^4 +
 1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = -\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

output
```
-b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4
*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(
I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{b^2x^4+1}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{b^2x^4+1}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")
```

output

```
integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx=-\int \frac{b\,x^2-1}{\sqrt{b^2\,x^4+1}}\,dx$$

input

```
int(-(b*x^2 - 1)/(b^2*x^4 + 1)^(1/2),x)
```

output

```
-int((b*x^2 - 1)/(b^2*x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx=\int \frac{\sqrt{b^2x^4+1}}{b^2x^4+1}dx-\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^2x^4+1}dx\right)b$$

input

```
int((-b*x^2+1)/(b^2*x^4+1)^(1/2),x)
```

output

```
int(sqrt(b**2*x**4 + 1)/(b**2*x**4 + 1),x) - int((sqrt(b**2*x**4 + 1)*x**2
)/(b**2*x**4 + 1),x)*b
```

3.257 $\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 186

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{3/2}}\,dx = \frac{x(1-bx^2)}{2\sqrt{1+b^2x^4}}+\frac{x\sqrt{1+b^2x^4}}{2\left(1+bx^2\right)} - \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{2\sqrt{b}\sqrt{1+b^2x^4}} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{2\sqrt{b}\sqrt{1+b^2x^4}}$$

output

```
1/2*x*(-b*x^2+1)/(b^2*x^4+1)^(1/2)+x*(b^2*x^4+1)^(1/2)/(2*b*x^2+2)-1/2*(b*
x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/2)*x)),
1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)+1/2*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1
)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b^(1/2)/(b^2*x
^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.35

$$\int \frac{1-bx^2}{(1+b^2x^4)^{3/2}}\,dx = \frac{1}{6}x\left(\frac{3}{\sqrt{1+b^2x^4}}+3\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)\right.$$
$$\left.-\,2bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-b^2x^4\right)\right)$$

input
```
Integrate[(1 - b*x^2)/(1 + b^2*x^4)^(3/2),x]
```

output
```
(x*(3/Sqrt[1 + b^2*x^4] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] -
 2*b*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, -(b^2*x^4)]))/6
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1493, 25, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{(b^2x^4+1)^{3/2}}\,dx$$
$$\Big\downarrow \text{1493}$$
$$\frac{x(1-bx^2)}{2\sqrt{b^2x^4+1}}-\frac{1}{2}\int -\frac{bx^2+1}{\sqrt{b^2x^4+1}}dx$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{2}\int \frac{bx^2+1}{\sqrt{b^2x^4+1}}dx+\frac{x(1-bx^2)}{2\sqrt{b^2x^4+1}}$$
$$\Big\downarrow \text{1512}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{b^2x^4+1}}dx-\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{x\left(1-bx^2\right)}{2\sqrt{b^2x^4+1}}$$

$$\Big\downarrow \; 761$$

$$\frac{1}{2}\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{x\left(1-bx^2\right)}{2\sqrt{b^2x^4+1}}$$

$$\Big\downarrow \; 1510$$

$$\frac{1}{2}\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}+\frac{x\sqrt{b^2x}}{bx^2+}\right.$$

$$\frac{x\left(1-bx^2\right)}{2\sqrt{b^2x^4+1}}$$

input
```
Int[(1 - b*x^2)/(1 + b^2*x^4)^(3/2),x]
```

output
```
(x*(1 - b*x^2))/(2*Sqrt[1 + b^2*x^4]) + ((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)
 - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[
b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)
/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2
*x^4]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.20

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right], \left[\frac{5}{4}\right], -b^{2} x^{4}\right)-\frac{b\,x^{3}\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{7}{4}\right],-b^{2} x^{4}\right)}{3}$
elliptic	$-\frac{2 b^{2}\left(\frac{x^{3}}{4 b}-\frac{x}{4 b^{2}}\right)}{\sqrt{\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}+\frac{\sqrt{-i b\,x^{2}+1}\,\sqrt{i b\,x^{2}+1}\;\mathrm{EllipticF}\left(x\sqrt{i b}, i\right)}{2\sqrt{i b}\,\sqrt{b^{2} x^{4}+1}}+\frac{i\sqrt{-i b\,x^{2}+1}\,\sqrt{i b\,x^{2}+1}\left(\mathrm{EllipticF}\left(x\sqrt{i b}, i\right)-\mathrm{EllipticE}\left(x\sqrt{i b}, i\right)\right.}{2\sqrt{i b}\,\sqrt{b^{2} x^{4}+1}}$
default	$-b\left(\frac{x^{3}}{2\sqrt{\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}-\frac{i\sqrt{-i b\,x^{2}+1}\,\sqrt{i b\,x^{2}+1}\left(\mathrm{EllipticF}\left(x\sqrt{i b}, i\right)-\mathrm{EllipticE}\left(x\sqrt{i b}, i\right)\right)}{2\sqrt{i b}\,\sqrt{b^{2} x^{4}+1}\,b}\right)+\frac{x}{2\sqrt{\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}+\sqrt{-i b\,x^{2}+}$

input
```
int((-b*x^2+1)/(b^2*x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,3/2],[5/4],-b^2*x^4)-1/3*b*x^3*hypergeom([3/4,3/2],[7/4],
-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.58

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{3/2}}\,dx =$$

$$-\frac{(b^3x^4+b)(-b^2)^{\frac{3}{4}}\,E(\arcsin\left((-b^2)^{\frac{1}{4}}\,x\right)\,|-1) - ((b^3-b^2)x^4+b-1)(-b^2)^{\frac{3}{4}}\,F(\arcsin\left((-b^2)^{\frac{1}{4}}\,x\right)\,|-1)+}{2\left(b^4x^4+b^2\right)}$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((b^3*x^4 + b)*(-b^2)^(3/4)*elliptic_e(arcsin((-b^2)^(1/4)*x), -1) -
((b^3 - b^2)*x^4 + b - 1)*(-b^2)^(3/4)*elliptic_f(arcsin((-b^2)^(1/4)*x),
-1) + (b^3*x^3 - b^2*x)*sqrt(b^2*x^4 + 1))/(b^4*x^4 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.35

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{3/2}}\,dx = -\frac{bx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(b**2*x**4+1)**(3/2),x)
```

output
```
-b*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b**2*x**4*exp_polar(I*pi))/(4
*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b**2*x**4*exp_polar(
I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{(1+b^2x^4)^{3/2}}\,dx = \int -\frac{bx^2-1}{(b^2x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/(b^2*x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{(1+b^2x^4)^{3/2}}\,dx = \int -\frac{bx^2-1}{(b^2x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/(b^2*x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{(1+b^2x^4)^{3/2}}\,dx = -\int \frac{b\,x^2-1}{(b^2\,x^4+1)^{3/2}}\,dx$$

input
```
int(-(b*x^2 - 1)/(b^2*x^4 + 1)^(3/2),x)
```

output
```
-int((b*x^2 - 1)/(b^2*x^4 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{3/2}}\,dx=\int \frac{\sqrt{b^2x^4+1}}{b^4x^8+2b^2x^4+1}dx-\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^4x^8+2b^2x^4+1}dx\right)b$$

input
```
int((-b*x^2+1)/(b^2*x^4+1)^(3/2),x)
```

output
```
int(sqrt(b**2*x**4 + 1)/(b**4*x**8 + 2*b**2*x**4 + 1),x) - int((sqrt(b**2*
x**4 + 1)*x**2)/(b**4*x**8 + 2*b**2*x**4 + 1),x)*b
```

3.258 $\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 212

$$\begin{aligned}\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{5/2}}\,dx &= \frac{x(1-bx^2)}{6\left(1+b^2x^4\right)^{3/2}}+\frac{x(5-3bx^2)}{12\sqrt{1+b^2x^4}}\\ &+\frac{x\sqrt{1+b^2x^4}}{4\left(1+bx^2\right)}-\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{4\sqrt{b}\sqrt{1+b^2x^4}}\\ &+\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{3\sqrt{b}\sqrt{1+b^2x^4}}\end{aligned}$$

output

```
1/6*x*(-b*x^2+1)/(b^2*x^4+1)^(3/2)+1/12*x*(-3*b*x^2+5)/(b^2*x^4+1)^(1/2)+x
*(b^2*x^4+1)^(1/2)/(4*b*x^2+4)-1/4*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/
2)*EllipticE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/
2)+1/3*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(
b^(1/2)*x),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

$$\int \frac{1-bx^2}{(1+b^2x^4)^{5/2}}\,dx = \frac{1}{12}x\left(\frac{7+5b^2x^4}{(1+b^2x^4)^{3/2}}\right.$$
$$\left.+5\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)-4bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},-b^2x^4\right)\right)$$

input
```
Integrate[(1 - b*x^2)/(1 + b^2*x^4)^(5/2),x]
```

output
```
(x*((7 + 5*b^2*x^4)/(1 + b^2*x^4)^(3/2) + 5*Hypergeometric2F1[1/4, 1/2, 5/
4, -(b^2*x^4)] - 4*b*x^2*Hypergeometric2F1[3/4, 5/2, 7/4, -(b^2*x^4)]))/12
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1493, 25, 1493, 25, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{(b^2x^4+1)^{5/2}}dx$$
$$\Big\downarrow \text{1493}$$
$$\frac{x(1-bx^2)}{6(b^2x^4+1)^{3/2}}-\frac{1}{6}\int -\frac{5-3bx^2}{(b^2x^4+1)^{3/2}}dx$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{6}\int \frac{5-3bx^2}{(b^2x^4+1)^{3/2}}dx+\frac{x(1-bx^2)}{6(b^2x^4+1)^{3/2}}$$
$$\Big\downarrow \text{1493}$$

$$\frac{1}{6}\left(\frac{x\left(5-3bx^2\right)}{2\sqrt{b^2x^4+1}}-\frac{1}{2}\int -\frac{3bx^2+5}{\sqrt{b^2x^4+1}}dx\right)+\frac{x\left(1-bx^2\right)}{6\left(b^2x^4+1\right)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{3bx^2+5}{\sqrt{b^2x^4+1}}dx+\frac{x\left(5-3bx^2\right)}{2\sqrt{b^2x^4+1}}\right)+\frac{x\left(1-bx^2\right)}{6\left(b^2x^4+1\right)^{3/2}}$$

$$\Big\downarrow \ 1512$$

$$\frac{1}{6}\left(\frac{1}{2}\left(8\int \frac{1}{\sqrt{b^2x^4+1}}dx-3\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{x\left(5-3bx^2\right)}{2\sqrt{b^2x^4+1}}\right)+\frac{x\left(1-bx^2\right)}{6\left(b^2x^4+1\right)^{3/2}}$$

$$\Big\downarrow \ 761$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-3\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{x\left(5-3bx^2\right)}{2\sqrt{b^2x^4+1}}\right)+\frac{x\left(1-bx^2\right)}{6\left(b^2x^4+1\right)^{3/2}}$$

$$\Big\downarrow \ 1510$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-3\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\right.\right.\right.$$

$$\frac{x\left(1-bx^2\right)}{6\left(b^2x^4+1\right)^{3/2}}$$

input
```
Int[(1 - b*x^2)/(1 + b^2*x^4)^(5/2),x]
```

output
```
(x*(1 - b*x^2))/(6*(1 + b^2*x^4)^(3/2)) + ((x*(5 - 3*b*x^2))/(2*Sqrt[1 + b
^2*x^4]) + (-3*(-((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(
1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*
Sqrt[1 + b^2*x^4])) + (4*(1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*Ell
ipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]))/2)/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.18

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{5}{2}\right],\left[\frac{5}{4}\right],-b^2x^4\right)-\frac{b\,x^3\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{5}{2}\right],\left[\frac{7}{4}\right],-b^2x^4\right)}{3}$
elliptic	$\frac{\left(-\frac{x^3}{6b^3}+\frac{x}{6b^4}\right)\sqrt{b^2x^4+1}}{\left(x^4+\frac{1}{b^2}\right)^2}-\frac{2b^2\left(\frac{x^3}{8b}-\frac{5x}{24b^2}\right)}{\sqrt{\left(x^4+\frac{1}{b^2}\right)b^2}}+\frac{5\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\ \mathrm{EllipticF}\left(x\sqrt{ib},i\right)}{12\sqrt{ib}\,\sqrt{b^2x^4+1}}+\frac{i\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\left(\mathrm{EllipticF}\left(\right.\right.}{4\sqrt{ib}\,\sqrt{b^2}}$
default	$-b\left(\frac{x^3\sqrt{b^2x^4+1}}{6b^4\left(x^4+\frac{1}{b^2}\right)^2}+\frac{x^3}{4\sqrt{\left(x^4+\frac{1}{b^2}\right)b^2}}-\frac{i\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{ib},i\right)-\mathrm{EllipticE}\left(x\sqrt{ib},i\right)\right)}{4\sqrt{ib}\,\sqrt{b^2x^4+1}\,b}\right)+\frac{x\sqrt{b^2x^4+}}{6b^4\left(x^4+\frac{1}{b^2}\right.}$

input
```
int((-b*x^2+1)/(b^2*x^4+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,5/2],[5/4],-b^2*x^4)-1/3*b*x^3*hypergeom([3/4,5/2],[7/4],
-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.76

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{5/2}}\,dx =$$

$$-\frac{3\left(b^5x^8+2\,b^3x^4+b\right)\left(-b^2\right)^{\frac{3}{4}}\,E\left(\arcsin\left(\left(-b^2\right)^{\frac{1}{4}}\,x\right)\,|\,-1\right)-\left(\left(3\,b^5-5\,b^4\right)x^8+2\left(3\,b^3-5\,b^2\right)x^4+3\,b-5\right)(-}{12\left(b^6x^8+2\,b^4x^4+b^2\right)}$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*(b^5*x^8 + 2*b^3*x^4 + b)*(-b^2)^(3/4)*elliptic_e(arcsin((-b^2)^(
1/4)*x), -1) - ((3*b^5 - 5*b^4)*x^8 + 2*(3*b^3 - 5*b^2)*x^4 + 3*b - 5)*(-b
^2)^(3/4)*elliptic_f(arcsin((-b^2)^(1/4)*x), -1) + (3*b^5*x^7 - 5*b^4*x^5
+ 5*b^3*x^3 - 7*b^2*x)*sqrt(b^2*x^4 + 1))/(b^6*x^8 + 2*b^4*x^4 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.31

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{5/2}}\,dx = -\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\,b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|\,b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(b**2*x**4+1)**(5/2),x)
```

output
```
-b*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b**2*x**4*exp_polar(I*pi))/(4
*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b**2*x**4*exp_polar(
I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{5/2}}\,dx = \int -\frac{bx^2-1}{\left(b^2x^4+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/(b^2*x^4 + 1)^(5/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{5/2}}\,dx = \int -\frac{bx^2-1}{\left(b^2x^4+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/(b^2*x^4 + 1)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{(1+b^2x^4)^{5/2}}\,dx = -\int \frac{b\,x^2-1}{(b^2\,x^4+1)^{5/2}}\,dx$$

input
```
int(-(b*x^2 - 1)/(b^2*x^4 + 1)^(5/2),x)
```

output
```
-int((b*x^2 - 1)/(b^2*x^4 + 1)^(5/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{(1+b^2x^4)^{5/2}}\,dx = \int \frac{\sqrt{b^2x^4+1}}{b^6x^{12}+3b^4x^8+3b^2x^4+1}dx - \left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^6x^{12}+3b^4x^8+3b^2x^4+1}dx\right) b$$

input
```
int((-b*x^2+1)/(b^2*x^4+1)^(5/2),x)
```

output
```
int(sqrt(b**2*x**4 + 1)/(b**6*x**12 + 3*b**4*x**8 + 3*b**2*x**4 + 1),x) -
int((sqrt(b**2*x**4 + 1)*x**2)/(b**6*x**12 + 3*b**4*x**8 + 3*b**2*x**4 + 1
),x)*b
```

3.259 $\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 238

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{7/2}}\,dx = \frac{x(1-bx^2)}{10\left(1+b^2x^4\right)^{5/2}} + \frac{x(9-7bx^2)}{60\left(1+b^2x^4\right)^{3/2}} + \frac{x(15-7bx^2)}{40\sqrt{1+b^2x^4}} + \frac{7x\sqrt{1+b^2x^4}}{40\left(1+bx^2\right)} - \frac{7(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{40\sqrt{b}\sqrt{1+b^2x^4}} + \frac{11(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{40\sqrt{b}\sqrt{1+b^2x^4}}$$

output

```
1/10*x*(-b*x^2+1)/(b^2*x^4+1)^(5/2)+1/60*x*(-7*b*x^2+9)/(b^2*x^4+1)^(3/2)+
1/40*x*(-7*b*x^2+15)/(b^2*x^4+1)^(1/2)+7*x*(b^2*x^4+1)^(1/2)/(40*b*x^2+40)
-7/40*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(
1/2)*x)),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)+11/40*(b*x^2+1)*((b^2*x^4+
1)/(b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b^(
1/2)/(b^2*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.35

$$\int \frac{1-bx^2}{(1+b^2x^4)^{7/2}}\,dx = \frac{1}{120}x\left(\frac{3(25+36b^2x^4+15b^4x^8)}{(1+b^2x^4)^{5/2}}\right.$$
$$\left.+45\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)-40bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{2},\frac{7}{4},-b^2x^4\right)\right)$$

input

```
Integrate[(1 - b*x^2)/(1 + b^2*x^4)^(7/2),x]
```

output

```
(x*((3*(25 + 36*b^2*x^4 + 15*b^4*x^8))/(1 + b^2*x^4)^(5/2) + 45*Hypergeome
tric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] - 40*b*x^2*Hypergeometric2F1[3/4, 7/2,
7/4, -(b^2*x^4)]))/120
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1493, 25, 1493, 27, 1493, 25, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{(b^2x^4+1)^{7/2}}dx$$
$$\Big\downarrow \text{1493}$$
$$\frac{x(1-bx^2)}{10\,(b^2x^4+1)^{5/2}}-\frac{1}{10}\int -\frac{9-7bx^2}{(b^2x^4+1)^{5/2}}dx$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{10}\int \frac{9-7bx^2}{(b^2x^4+1)^{5/2}}dx+\frac{x(1-bx^2)}{10\,(b^2x^4+1)^{5/2}}$$

$$\Big\downarrow \; 1493$$

$$\frac{1}{10}\left(\frac{x\left(9-7bx^2\right)}{6\left(b^2x^4+1\right)^{3/2}}-\frac{1}{6}\int -\frac{3\left(15-7bx^2\right)}{\left(b^2x^4+1\right)^{3/2}}dx\right)+\frac{x\left(1-bx^2\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{10}\left(\frac{1}{2}\int \frac{15-7bx^2}{\left(b^2x^4+1\right)^{3/2}}dx+\frac{x\left(9-7bx^2\right)}{6\left(b^2x^4+1\right)^{3/2}}\right)+\frac{x\left(1-bx^2\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

$$\Big\downarrow \; 1493$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{x\left(15-7bx^2\right)}{2\sqrt{b^2x^4+1}}-\frac{1}{2}\int -\frac{7bx^2+15}{\sqrt{b^2x^4+1}}dx\right)+\frac{x\left(9-7bx^2\right)}{6\left(b^2x^4+1\right)^{3/2}}\right)+\frac{x\left(1-bx^2\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{7bx^2+15}{\sqrt{b^2x^4+1}}dx+\frac{x\left(15-7bx^2\right)}{2\sqrt{b^2x^4+1}}\right)+\frac{x\left(9-7bx^2\right)}{6\left(b^2x^4+1\right)^{3/2}}\right)+\frac{x\left(1-bx^2\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

$$\Big\downarrow \; 1512$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\left(22\int \frac{1}{\sqrt{b^2x^4+1}}dx-7\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{x\left(15-7bx^2\right)}{2\sqrt{b^2x^4+1}}\right)+\frac{x\left(9-7bx^2\right)}{6\left(b^2x^4+1\right)^{3/2}}\right)+$$
$$\frac{x\left(1-bx^2\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{11\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-7\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{x\left(15-7bx^2\right)}{2\sqrt{b^2x^4+1}}\right)+\right.\right.$$
$$\frac{x\left(1-bx^2\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

$$\Big\downarrow \; 1510$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{11\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-7\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)\right.}{\sqrt{b}\sqrt{b^2x^4+1}}\right.\right.\right.\right.$$
$$\frac{x\left(1-bx^2\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

input

```
Int[(1 - b*x^2)/(1 + b^2*x^4)^(7/2),x]
```

output

```
(x*(1 - b*x^2))/(10*(1 + b^2*x^4)^(5/2)) + ((x*(9 - 7*b*x^2))/(6*(1 + b^2*
x^4)^(3/2)) + ((x*(15 - 7*b*x^2))/(2*Sqrt[1 + b^2*x^4]) + (-7*(-((x*Sqrt[1
 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]
*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])) + (11*(
1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x]
, 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]))/2)/2)/10
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.16

method	result
meijerg	$x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{7}{2}\right], \left[\frac{5}{4}\right], -b^{2} x^{4}\right)-\frac{b\,x^{3} \,\mathrm{hypergeom}\left(\left[\frac{3}{4}, \frac{7}{2}\right], \left[\frac{7}{4}\right], -b^{2} x^{4}\right)}{3}$
elliptic	$\frac{\left(-\frac{x^{3}}{10 b^{5}}+\frac{x}{10 b^{6}}\right) \sqrt{b^{2} x^{4}+1}}{\left(x^{4}+\frac{1}{b^{2}}\right)^{3}}+\frac{\left(-\frac{7 x^{3}}{60 b^{3}}+\frac{3 x}{20 b^{4}}\right) \sqrt{b^{2} x^{4}+1}}{\left(x^{4}+\frac{1}{b^{2}}\right)^{2}}-\frac{2 b^{2} \left(\frac{7 x^{3}}{80 b}-\frac{3 x}{16 b^{2}}\right)}{\sqrt{\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}+\frac{3 \sqrt{-i b\,x^{2}+1}\, \sqrt{i b\,x^{2}+1}\, \mathrm{EllipticF}\left(x \sqrt{i b}, i\right)}{8 \sqrt{i b}\, \sqrt{b^{2} x^{4}+1}}+$ 7
default	$-b (\frac{x^{3} \sqrt{b^{2} x^{4}+1}}{10 b^{6} \left(x^{4}+\frac{1}{b^{2}}\right)^{3}}+\frac{7 x^{3} \sqrt{b^{2} x^{4}+1}}{60 b^{4} \left(x^{4}+\frac{1}{b^{2}}\right)^{2}}+\frac{7 x^{3}}{40 \sqrt{\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}-\frac{7 i \sqrt{-i b\,x^{2}+1}\, \sqrt{i b\,x^{2}+1}\, (\mathrm{EllipticF}\left(x \sqrt{i b}, i\right)-\mathrm{EllipticE}(x\sqrt{ib}}{40 \sqrt{i b}\, \sqrt{b^{2} x^{4}+1}\, b}$

input

```
int((-b*x^2+1)/(b^2*x^4+1)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
x*hypergeom([1/4,7/2],[5/4],-b^2*x^4)-1/3*b*x^3*hypergeom([3/4,7/2],[7/4],
-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.88

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{7/2}}\, dx =$$

$$-\frac{21\,(b^7x^{12}+3\,b^5x^8+3\,b^3x^4+b)(-b^2)^{\frac{3}{4}}\,E(\arcsin\left(\left(-b^2\right)^{\frac{1}{4}}\,x\right)\,|\,-1)-3\,((7\,b^7-15\,b^6)x^{12}+3\,(7\,b^5-15\,b^4}{}$$

input

```
integrate((-b*x^2+1)/(b^2*x^4+1)^(7/2),x, algorithm="fricas")
```

output
```
-1/120*(21*(b^7*x^12 + 3*b^5*x^8 + 3*b^3*x^4 + b)*(-b^2)^(3/4)*elliptic_e(
arcsin((-b^2)^(1/4)*x), -1) - 3*((7*b^7 - 15*b^6)*x^12 + 3*(7*b^5 - 15*b^4
)*x^8 + 3*(7*b^3 - 15*b^2)*x^4 + 7*b - 15)*(-b^2)^(3/4)*elliptic_f(arcsin(
(-b^2)^(1/4)*x), -1) + (21*b^7*x^11 - 45*b^6*x^9 + 56*b^5*x^7 - 108*b^4*x^
5 + 47*b^3*x^3 - 75*b^2*x)*sqrt(b^2*x^4 + 1))/(b^8*x^12 + 3*b^6*x^8 + 3*b^
4*x^4 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 31.68 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.28

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{7/2}}\,dx = -\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(b**2*x**4+1)**(7/2),x)
```

output
```
-b*x**3*gamma(3/4)*hyper((3/4, 7/2), (7/4,), b**2*x**4*exp_polar(I*pi))/(4
*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 7/2), (5/4,), b**2*x**4*exp_polar(
I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{7/2}}\,dx = \int -\frac{bx^2-1}{\left(b^2x^4+1\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(7/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/(b^2*x^4 + 1)^(7/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{7/2}}\,dx=\int -\frac{bx^2-1}{\left(b^2x^4+1\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(7/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/(b^2*x^4 + 1)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{7/2}}\,dx=-\int \frac{b\,x^2-1}{\left(b^2\,x^4+1\right)^{7/2}}\,dx$$

input
```
int(-(b*x^2 - 1)/(b^2*x^4 + 1)^(7/2),x)
```

output
```
-int((b*x^2 - 1)/(b^2*x^4 + 1)^(7/2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{1-bx^2}{\left(1+b^2x^4\right)^{7/2}}\,dx&=\int \frac{\sqrt{b^2x^4+1}}{b^8x^{16}+4b^6x^{12}+6b^4x^8+4b^2x^4+1}dx\\&\quad-\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^8x^{16}+4b^6x^{12}+6b^4x^8+4b^2x^4+1}dx\right)b\end{aligned}$$

input
```
int((-b*x^2+1)/(b^2*x^4+1)^(7/2),x)
```

output
```
int(sqrt(b**2*x**4 + 1)/(b**8*x**16 + 4*b**6*x**12 + 6*b**4*x**8 + 4*b**2*
x**4 + 1),x) - int((sqrt(b**2*x**4 + 1)*x**2)/(b**8*x**16 + 4*b**6*x**12 +
 6*b**4*x**8 + 4*b**2*x**4 + 1),x)*b
```

3.260 $\int (1+bx^2)\left(1+b^2x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 212

$$\int (1+bx^2)\left(1+b^2x^4\right)^{3/2}\,dx = \frac{4x\sqrt{1+b^2x^4}}{15\left(1+bx^2\right)} + \frac{2}{105}x\left(15+7bx^2\right)\sqrt{1+b^2x^4} + \frac{1}{63}x\left(9+7bx^2\right)\left(1+b^2x^4\right)^{3/2} - \frac{4(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{15\sqrt{b}\sqrt{1+b^2x^4}} + \frac{44(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{105\sqrt{b}\sqrt{1+b^2x^4}}$$

output

```
4*x*(b^2*x^4+1)^(1/2)/(15*b*x^2+15)+2/105*x*(7*b*x^2+15)*(b^2*x^4+1)^(1/2)
+1/63*x*(7*b*x^2+9)*(b^2*x^4+1)^(3/2)-4/15*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1
)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(b^2*x^
4+1)^(1/2)+44/105*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*InverseJacobiA
M(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.22

$$\int (1+bx^2)\left(1+b^2x^4\right)^{3/2}\,dx = x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-b^2x^4\right) + \frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)$$

input
```
Integrate[(1 + b*x^2)*(1 + b^2*x^4)^(3/2),x]
```

output
```
x*Hypergeometric2F1[-3/2, 1/4, 5/4, -(b^2*x^4)] + (b*x^3*Hypergeometric2F1
[-3/2, 3/4, 7/4, -(b^2*x^4)])/3
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1491, 27, 1491, 27, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^2+1)\left(b^2x^4+1\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{21}\int 2(7bx^2+9)\sqrt{b^2x^4+1}dx+\frac{1}{63}x(7bx^2+9)\left(b^2x^4+1\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{21}\int (7bx^2+9)\sqrt{b^2x^4+1}dx+\frac{1}{63}x(7bx^2+9)\left(b^2x^4+1\right)^{3/2}$$

$$\Big\downarrow \text{1491}$$

$$\frac{2}{21}\left(\frac{1}{15}\int \frac{6(7bx^2+15)}{\sqrt{b^2x^4+1}}dx+\frac{1}{5}x\sqrt{b^2x^4+1}(7bx^2+15)\right)+\frac{1}{63}x(7bx^2+9)\left(b^2x^4+1\right)^{3/2}$$

$$\Big\downarrow 27$$

$$\frac{2}{21}\left(\frac{2}{5}\int \frac{7bx^2+15}{\sqrt{b^2x^4+1}}dx+\frac{1}{5}x\sqrt{b^2x^4+1}\left(7bx^2+15\right)\right)+\frac{1}{63}x\left(7bx^2+9\right)\left(b^2x^4+1\right)^{3/2}$$

$$\Big\downarrow 1512$$

$$\frac{2}{21}\left(\frac{2}{5}\left(22\int \frac{1}{\sqrt{b^2x^4+1}}dx-7\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{1}{5}x\sqrt{b^2x^4+1}\left(7bx^2+15\right)\right)+$$
$$\frac{1}{63}x\left(7bx^2+9\right)\left(b^2x^4+1\right)^{3/2}$$

$$\Big\downarrow 761$$

$$\frac{2}{21}\left(\frac{2}{5}\left(\frac{11\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-7\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{1}{5}x\sqrt{b^2x^4+1}\left(7bx^2+1\right.\right.$$
$$\frac{1}{63}x\left(7bx^2+9\right)\left(b^2x^4+1\right)^{3/2}$$

$$\Big\downarrow 1510$$

$$\frac{2}{21}\left(\frac{2}{5}\left(\frac{11\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-7\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}\right.\right.\right.$$
$$\frac{1}{63}x\left(7bx^2+9\right)\left(b^2x^4+1\right)^{3/2}$$

input
```
Int[(1 + b*x^2)*(1 + b^2*x^4)^(3/2),x]
```

output
```
(x*(9 + 7*b*x^2)*(1 + b^2*x^4)^(3/2))/63 + (2*((x*(15 + 7*b*x^2)*Sqrt[1 +
b^2*x^4])/5 + (2*(-7*(-((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*
Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sq
rt[b]*Sqrt[1 + b^2*x^4])) + (11*(1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)
^2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])))/5))
/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.18

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{4}\right],\left[\frac{5}{4}\right],-b^2x^4\right)+\frac{b\,x^3\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^2x^4\right)}{3}$
risch	$\frac{x\left(35b^3x^6+45b^2x^4+77b\,x^2+135\right)\sqrt{b^2x^4+1}}{315}+\frac{4\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\ \mathrm{EllipticF}\left(x\sqrt{ib},i\right)}{7\sqrt{ib}\,\sqrt{b^2x^4+1}}+$
elliptic	$\frac{b^3x^7\sqrt{b^2x^4+1}}{9}+\frac{b^2x^5\sqrt{b^2x^4+1}}{7}+\frac{11b\,x^3\sqrt{b^2x^4+1}}{45}+\frac{3x\sqrt{b^2x^4+1}}{7}+\frac{4\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\ \mathrm{EllipticF}\left(x\sqrt{ib},i\right)}{7\sqrt{ib}\,\sqrt{b^2x^4+1}}+$
default	$\frac{b^2x^5\sqrt{b^2x^4+1}}{7}+\frac{3x\sqrt{b^2x^4+1}}{7}+\frac{4\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\ \mathrm{EllipticF}\left(x\sqrt{ib},i\right)}{7\sqrt{ib}\,\sqrt{b^2x^4+1}}+b\left(\frac{b^2x^7\sqrt{b^2x^4+1}}{9}+\frac{11x^3\sqrt{b^2x^4+1}}{45}+\right.$

input
```
int((b*x^2+1)*(b^2*x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([-3/2,1/4],[5/4],-b^2*x^4)+1/3*b*x^3*hypergeom([-3/2,3/4],[7/4
],-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.52

$$\int \left(1+bx^2\right)\left(1+b^2x^4\right)^{3/2}\,dx=\frac{84\,bx\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)+12\left(15\,b^2-7\,b\right)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\middle|-\right.}{315\,bx}$$

input
```
integrate((b*x^2+1)*(b^2*x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(84*b*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1) + 12
*(15*b^2 - 7*b)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x), -1)
+ (35*b^4*x^8 + 45*b^3*x^6 + 77*b^2*x^4 + 135*b*x^2 + 84)*sqrt(b^2*x^4 + 1
))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.70

$$\int (1+bx^2)\,(1+b^2x^4)^{3/2}\,dx = \frac{b^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{b^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)*(b**2*x**4+1)**(3/2),x)
```

output
```
b**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b**2*x**4*exp_polar(I*pi)
)/(4*gamma(11/4)) + b**2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b**2*x
**4*exp_polar(I*pi))/(4*gamma(9/4)) + b*x**3*gamma(3/4)*hyper((-1/2, 3/4),
 (7/4,), b**2*x**4*exp_polar(I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((-
1/2, 1/4), (5/4,), b**2*x**4*exp_polar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int (1+bx^2)\,(1+b^2x^4)^{3/2}\,dx = \int (b^2x^4+1)^{\frac{3}{2}}\,(bx^2+1)\,dx$$

input
```
integrate((b*x^2+1)*(b^2*x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 1)^(3/2)*(b*x^2 + 1), x)
```

Giac [F]

$$\int (1+bx^2)\left(1+b^2x^4\right)^{3/2}\,dx=\int \left(b^2x^4+1\right)^{\frac{3}{2}}\left(bx^2+1\right)dx$$

input
```
integrate((b*x^2+1)*(b^2*x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 1)^(3/2)*(b*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+bx^2)\left(1+b^2x^4\right)^{3/2}\,dx=\int \left(b^2\,x^4+1\right)^{3/2}\left(b\,x^2+1\right)\,dx$$

input
```
int((b^2*x^4 + 1)^(3/2)*(b*x^2 + 1),x)
```

output
```
int((b^2*x^4 + 1)^(3/2)*(b*x^2 + 1), x)
```

Reduce [F]

$$\int (1+bx^2)\left(1+b^2x^4\right)^{3/2}\,dx=\frac{\sqrt{b^2x^4+1}\,b^3x^7}{9}+\frac{\sqrt{b^2x^4+1}\,b^2x^5}{7}+\frac{11\sqrt{b^2x^4+1}\,b\,x^3}{45}+\frac{3\sqrt{b^2x^4+1}\,x}{7}+\frac{4\left(\int \frac{\sqrt{b^2x^4+1}}{b^2x^4+1}dx\right)}{7}+\frac{4\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^2x^4+1}dx\right)b}{15}$$

input
```
int((b*x^2+1)*(b^2*x^4+1)^(3/2),x)
```

output
```
(35*sqrt(b**2*x**4 + 1)*b**3*x**7 + 45*sqrt(b**2*x**4 + 1)*b**2*x**5 + 77*
sqrt(b**2*x**4 + 1)*b*x**3 + 135*sqrt(b**2*x**4 + 1)*x + 180*int(sqrt(b**2
*x**4 + 1)/(b**2*x**4 + 1),x) + 84*int((sqrt(b**2*x**4 + 1)*x**2)/(b**2*x*
*4 + 1),x)*b)/315
```

3.261 $\int \left(1+b x^2\right) \sqrt{1+b^2 x^4} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 186

$$\begin{aligned}\int \left(1+b x^2\right) \sqrt{1+b^2 x^4} \, dx = {} & \frac{2 x \sqrt{1+b^2 x^4}}{5 \left(1+b x^2\right)}+\frac{1}{15} x \left(5+3 b x^2\right) \sqrt{1+b^2 x^4} \\ & -\frac{2(1+b x^2) \sqrt{\frac{1+b^2 x^4}{(1+b x^2)^2}}\, E\left(2 \arctan \left(\sqrt{b} x\right) |\frac{1}{2}\right)}{5 \sqrt{b} \sqrt{1+b^2 x^4}} \\ & +\frac{8(1+b x^2) \sqrt{\frac{1+b^2 x^4}{(1+b x^2)^2}}\, \text{EllipticF}\left(2 \arctan \left(\sqrt{b} x\right),\frac{1}{2}\right)}{15 \sqrt{b} \sqrt{1+b^2 x^4}}\end{aligned}$$

output

```
2*x*(b^2*x^4+1)^(1/2)/(5*b*x^2+5)+1/15*x*(3*b*x^2+5)*(b^2*x^4+1)^(1/2)-2/5
*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/2)*
x)),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)+8/15*(b*x^2+1)*((b^2*x^4+1)/(b*
x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b^(1/2)/(
b^2*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.79 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.25

$$\int (1+bx^2)\sqrt{1+b^2x^4}\,dx = x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-b^2x^4\right) + \frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)$$

input
```
Integrate[(1 + b*x^2)*Sqrt[1 + b^2*x^4],x]
```

output
```
x*Hypergeometric2F1[-1/2, 1/4, 5/4, -(b^2*x^4)] + (b*x^3*Hypergeometric2F1
[-1/2, 3/4, 7/4, -(b^2*x^4)])/3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1491, 27, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^2+1)\sqrt{b^2x^4+1}\,dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{15}\int \frac{2(3bx^2+5)}{\sqrt{b^2x^4+1}}dx+\frac{1}{15}x\sqrt{b^2x^4+1}(3bx^2+5)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{15}\int \frac{3bx^2+5}{\sqrt{b^2x^4+1}}dx+\frac{1}{15}x\sqrt{b^2x^4+1}(3bx^2+5)$$

$$\Big\downarrow \text{1512}$$

$$\frac{2}{15}\left(8\int \frac{1}{\sqrt{b^2x^4+1}}dx-3\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{1}{15}x\sqrt{b^2x^4+1}(3bx^2+5)$$

$$\Big\downarrow 761$$

$$\frac{2}{15}\left(\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-3\int\frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{1}{15}x\sqrt{b^2x^4+1}\left(3bx^2+5\right)$$

$$\Big\downarrow 1510$$

$$\frac{2}{15}\left(\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-3\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\right.\right.$$

$$\frac{1}{15}x\sqrt{b^2x^4+1}\left(3bx^2+5\right)$$

input

```
Int[(1 + b*x^2)*Sqrt[1 + b^2*x^4],x]
```

output

```
(x*(5 + 3*b*x^2)*Sqrt[1 + b^2*x^4])/15 + (2*(-3*(-((x*Sqrt[1 + b^2*x^4])/(
1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*A
rcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])) + (4*(1 + b*x^2)*Sqrt
[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b
]*Sqrt[1 + b^2*x^4])))/15
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.20

method	result
meijerg	$x \operatorname{hypergeom}\left(\left[-\frac{1}{2}, \frac{1}{4}\right], \left[\frac{5}{4}\right], -b^{2} x^{4}\right)+\frac{b\,x^{3} \operatorname{hypergeom}\left(\left[-\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -b^{2} x^{4}\right)}{3}$
risch	$\frac{x \left(3 b\,x^{2}+5\right) \sqrt{b^{2} x^{4}+1}}{15}+\frac{2 \sqrt{-i b\,x^{2}+1}\, \sqrt{i b\,x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i b}, i\right)}{3 \sqrt{i b}\, \sqrt{b^{2} x^{4}+1}}+\frac{2 i \sqrt{-i b\,x^{2}+1}\, \sqrt{i b\,x^{2}+1}\, (\operatorname{EllipticF}\left(x \sqrt{i b}, i\right)-\operatorname{EllipticE}(x}{5 \sqrt{i b}\, \sqrt{b^{2} x^{4}+1}}$
elliptic	$\frac{b\,x^{3} \sqrt{b^{2} x^{4}+1}}{5}+\frac{x \sqrt{b^{2} x^{4}+1}}{3}+\frac{2 \sqrt{-i b\,x^{2}+1}\, \sqrt{i b\,x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i b}, i\right)}{3 \sqrt{i b}\, \sqrt{b^{2} x^{4}+1}}+\frac{2 i \sqrt{-i b\,x^{2}+1}\, \sqrt{i b\,x^{2}+1}\, (\operatorname{EllipticF}\left(x \sqrt{i b}, i\right)-\mathrm{Ellip}}{5 \sqrt{i b}\, \sqrt{b^{2} x^{4}+1}}$
default	$\frac{x \sqrt{b^{2} x^{4}+1}}{3}+\frac{2 \sqrt{-i b\,x^{2}+1}\, \sqrt{i b\,x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i b}, i\right)}{3 \sqrt{i b}\, \sqrt{b^{2} x^{4}+1}}+b \Bigg(\frac{x^{3} \sqrt{b^{2} x^{4}+1}}{5}+\frac{2 i \sqrt{-i b\,x^{2}+1}\, \sqrt{i b\,x^{2}+1}\, (\operatorname{EllipticF}\left(x \sqrt{i b}, i\right)-\mathrm{E}}{5 \sqrt{i b}\, \sqrt{b^{2} x^{4}+1}\, b}$

input
```
int((b*x^2+1)*(b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([-1/2,1/4],[5/4],-b^2*x^4)+1/3*b*x^3*hypergeom([-1/2,3/4],[7/4
],-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.51

$$\int \left(1+bx^2\right)\sqrt{1+b^2x^4}\,dx$$

$$= \frac{6\,bx\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} E(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right) | -1) + 2\,(5\,b^2-3\,b)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right) | -1) + (3\,b^2x^4+5\,b}{15\,bx}$$

input
```
integrate((b*x^2+1)*(b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(6*b*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1) + 2*(5
*b^2 - 3*b)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x), -1) + (3
*b^2*x^4 + 5*b*x^2 + 6)*sqrt(b^2*x^4 + 1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int \left(1+bx^2\right)\sqrt{1+b^2x^4}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)*(b**2*x**4+1)**(1/2),x)
```

output
```
b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4
*gamma(7/4)) + x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b**2*x**4*exp_polar
(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1+bx^2\right)\sqrt{1+b^2x^4}\,dx=\int \sqrt{b^2x^4+1}\left(bx^2+1\right)dx$$

input
```
integrate((b*x^2+1)*(b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b^2*x^4 + 1)*(b*x^2 + 1), x)
```

Giac [F]

$$\int \left(1+bx^2\right)\sqrt{1+b^2x^4}\,dx=\int \sqrt{b^2x^4+1}\left(bx^2+1\right)dx$$

input
```
integrate((b*x^2+1)*(b^2*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b^2*x^4 + 1)*(b*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+bx^2\right)\sqrt{1+b^2x^4}\,dx=\int \sqrt{b^2\,x^4+1}\,\left(b\,x^2+1\right)\,dx$$

input
```
int((b^2*x^4 + 1)^(1/2)*(b*x^2 + 1),x)
```

output
```
int((b^2*x^4 + 1)^(1/2)*(b*x^2 + 1), x)
```

Reduce [F]

$$\int \left(1+b x^2\right) \sqrt{1+b^2 x^4}\, dx = \frac{\sqrt{b^2 x^4+1}\, b\, x^3}{5} + \frac{\sqrt{b^2 x^4+1}\, x}{3} + \frac{2\left(\int \frac{\sqrt{b^2 x^4+1}}{b^2 x^4+1} dx\right)}{3} + \frac{2\left(\int \frac{\sqrt{b^2 x^4+1}\, x^2}{b^2 x^4+1} dx\right) b}{5}$$

input
```
int((b*x^2+1)*(b^2*x^4+1)^(1/2),x)
```

output
```
(3*sqrt(b**2*x**4 + 1)*b*x**3 + 5*sqrt(b**2*x**4 + 1)*x + 10*int(sqrt(b**2
*x**4 + 1)/(b**2*x**4 + 1),x) + 6*int((sqrt(b**2*x**4 + 1)*x**2)/(b**2*x**
4 + 1),x)*b)/15
```

3.262 $\int \frac{1+bx^2}{\sqrt{1+b^2x^4}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}} \, dx = \frac{x\sqrt{1+b^2x^4}}{1+bx^2} - \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\, E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\, \text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}}$$

output
```
x*(b^2*x^4+1)^(1/2)/(b*x^2+1)-(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*El
lipticE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)+(b
*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/2)*x
),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.31

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}} \, dx = x \,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right) + \frac{1}{3}bx^3 \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)$$

input
```
Integrate[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + (b*x^3*Hypergeometric2F1[
1/2, 3/4, 7/4, -(b^2*x^4)])/3
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\sqrt{b^2x^4+1}}dx$$

$$\Big\downarrow \text{1512}$$

$$2\int \frac{1}{\sqrt{b^2x^4+1}}dx-\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx$$

$$\Big\downarrow \text{1510}$$

$$\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}+\frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

input
```
Int[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

output
```
(x*Sqrt[1 + b^2*x^4])/(1 + b*x^2) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b
*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
+ ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b
]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.25

method	result	size
meijerg	$\frac{b\,x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^{2} x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-b^{2} x^{4}\right)$	38
default	$\frac{\sqrt{-i b\,x^{2}+1}\,\sqrt{i b\,x^{2}+1}\,\operatorname{EllipticF}\left(x\sqrt{i b},i\right)}{\sqrt{i b}\,\sqrt{b^{2} x^{4}+1}}+\frac{i\sqrt{-i b\,x^{2}+1}\,\sqrt{i b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{i b},i\right)-\operatorname{EllipticE}\left(x\sqrt{i b},i\right)\right)}{\sqrt{i b}\,\sqrt{b^{2} x^{4}+1}}$	120
elliptic	$\frac{\sqrt{-i b\,x^{2}+1}\,\sqrt{i b\,x^{2}+1}\,\operatorname{EllipticF}\left(x\sqrt{i b},i\right)}{\sqrt{i b}\,\sqrt{b^{2} x^{4}+1}}+\frac{i\sqrt{-i b\,x^{2}+1}\,\sqrt{i b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{i b},i\right)-\operatorname{EllipticE}\left(x\sqrt{i b},i\right)\right)}{\sqrt{i b}\,\sqrt{b^{2} x^{4}+1}}$	120

input
```
int((b*x^2+1)/(b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b*x^3*hypergeom([1/2,3/4],[7/4],-b^2*x^4)+x*hypergeom([1/4,1/2],[5/4],
-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.48

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \frac{bx\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+(b^2-b)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+\sqrt{b^2x^4+1}}{bx}$$

input
```
integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(b*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1) + (b^2 - b)*x
*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x), -1) + sqrt(b^2*x^4 +
1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

output
```
b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(I
*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{b^2x^4+1}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{b^2x^4+1}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \int \frac{b\,x^2+1}{\sqrt{b^2\,x^4+1}}\,dx$$

input
```
int((b*x^2 + 1)/(b^2*x^4 + 1)^(1/2),x)
```

output
```
int((b*x^2 + 1)/(b^2*x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx=\int \frac{\sqrt{b^2x^4+1}}{b^2x^4+1}dx+\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^2x^4+1}dx\right)b$$

input

```
int((b*x^2+1)/(b^2*x^4+1)^(1/2),x)
```

output

```
int(sqrt(b**2*x**4 + 1)/(b**2*x**4 + 1),x) + int((sqrt(b**2*x**4 + 1)*x**2
)/(b**2*x**4 + 1),x)*b
```

3.263 $\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{3/2}}\,dx=\frac{x(1+bx^2)}{2\sqrt{1+b^2x^4}}-\frac{x\sqrt{1+b^2x^4}}{2\left(1+bx^2\right)}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{2\sqrt{b}\sqrt{1+b^2x^4}}$$

output

```
1/2*x*(b*x^2+1)/(b^2*x^4+1)^(1/2)-x*(b^2*x^4+1)^(1/2)/(2*b*x^2+2)+1/2*(b*x
^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/2)*x)),1
/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.55

$$\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{3/2}}\,dx=\frac{1}{6}x\left(\frac{3}{\sqrt{1+b^2x^4}}+3\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+2bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-b^2x^4\right)\right)$$

input
```
Integrate[(1 + b*x^2)/(1 + b^2*x^4)^(3/2),x]
```

output
```
(x*(3/Sqrt[1 + b^2*x^4] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] +
 2*b*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, -(b^2*x^4)]))/6
```

Rubi [A] (verified)

Time $=0.36$ (sec) , antiderivative size $=119$, normalized size of antiderivative $=1.00$, number of steps used $=3$, number of rules used $=3$, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used $=\{1493, 25, 1510\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{(b^2x^4+1)^{3/2}}dx$$

$$\Big\downarrow \text{1493}$$

$$\frac{x(bx^2+1)}{2\sqrt{b^2x^4+1}}-\frac{1}{2}\int -\frac{1-bx^2}{\sqrt{b^2x^4+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx+\frac{x(bx^2+1)}{2\sqrt{b^2x^4+1}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{1}{2}\left(\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{x\sqrt{b^2x^4+1}}{bx^2+1}\right)+\frac{x(bx^2+1)}{2\sqrt{b^2x^4+1}}$$

input
```
Int[(1 + b*x^2)/(1 + b^2*x^4)^(3/2),x]
```

output
```
(x*(1 + b*x^2))/(2*Sqrt[1 + b^2*x^4]) + (-((x*Sqrt[1 + b^2*x^4])/(1 + b*x^
2)) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sq
rt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.71 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.32

method	result
meijerg	$x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right], \left[\frac{5}{4}\right], -b^{2} x^{4}\right)+\frac{b \,x^{3} \mathrm{hypergeom}\left(\left[\frac{3}{4}, \frac{3}{2}\right], \left[\frac{7}{4}\right], -b^{2} x^{4}\right)}{3}$
elliptic	$-\frac{2 b^{2} \left(-\frac{x^{3}}{4 b}-\frac{x}{4 b^{2}}\right)}{\sqrt{\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}+\frac{\sqrt{-i b \,x^{2}+1}\, \sqrt{i b \,x^{2}+1}\, \mathrm{EllipticF}\left(x \sqrt{i b}, i\right)}{2 \sqrt{i b}\, \sqrt{b^{2} x^{4}+1}}-\frac{i \sqrt{-i b \,x^{2}+1}\, \sqrt{i b \,x^{2}+1}\, (\mathrm{EllipticF}\left(x \sqrt{i b}, i\right)-\mathrm{EllipticE}(x \sqrt{i b},}{2 \sqrt{i b}\, \sqrt{b^{2} x^{4}+1}}$
default	$\frac{x}{2 \sqrt{\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}+\frac{\sqrt{-i b \,x^{2}+1}\, \sqrt{i b \,x^{2}+1}\, \mathrm{EllipticF}\left(x \sqrt{i b}, i\right)}{2 \sqrt{i b}\, \sqrt{b^{2} x^{4}+1}}+b \left(\frac{x^{3}}{2 \sqrt{\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}-\frac{i \sqrt{-i b \,x^{2}+1}\, \sqrt{i b \,x^{2}+1}\, (\mathrm{EllipticF}(x \sqrt{ib}}{2 \sqrt{i b}\, \sqrt{b^{2} x^{4}+1}\, b}\right.$

input
```
int((b*x^2+1)/(b^2*x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,3/2],[5/4],-b^2*x^4)+1/3*b*x^3*hypergeom([3/4,3/2],[7/4],
-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{3/2}}\,dx = \frac{(b^3x^4+b)(-b^2)^{\frac{3}{4}}\,E(\arcsin\left((-b^2)^{\frac{1}{4}}\,x\right)\,|-1) - ((b^3+b^2)x^4+b+1)(-b^2)^{\frac{3}{4}}\,F(\arcsin}{2\left(b^4x^4+b^2\right)}$$

input
```
integrate((b*x^2+1)/(b^2*x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((b^3*x^4 + b)*(-b^2)^(3/4)*elliptic_e(arcsin((-b^2)^(1/4)*x), -1) - (
(b^3 + b^2)*x^4 + b + 1)*(-b^2)^(3/4)*elliptic_f(arcsin((-b^2)^(1/4)*x), -
1) + (b^3*x^3 + b^2*x)*sqrt(b^2*x^4 + 1))/(b^4*x^4 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.55

$$\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{3/2}}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\right| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\right| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(b**2*x**4+1)**(3/2),x)
```

output
```
b*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b**2*x**4*exp_polar(I
*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{(1+b^2x^4)^{3/2}}\,dx=\int \frac{bx^2+1}{(b^2x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/(b^2*x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{1+bx^2}{(1+b^2x^4)^{3/2}}\,dx=\int \frac{bx^2+1}{(b^2x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/(b^2*x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{(1+b^2x^4)^{3/2}}\,dx=\int \frac{b\,x^2+1}{(b^2\,x^4+1)^{3/2}}\,dx$$

input
```
int((b*x^2 + 1)/(b^2*x^4 + 1)^(3/2),x)
```

output
```
int((b*x^2 + 1)/(b^2*x^4 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{3/2}}\,dx = \int \frac{\sqrt{b^2x^4+1}}{b^4x^8+2b^2x^4+1}dx + \left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^4x^8+2b^2x^4+1}dx\right)b$$

input

```
int((b*x^2+1)/(b^2*x^4+1)^(3/2),x)
```

output

```
int(sqrt(b**2*x**4 + 1)/(b**4*x**8 + 2*b**2*x**4 + 1),x) + int((sqrt(b**2*
x**4 + 1)*x**2)/(b**4*x**8 + 2*b**2*x**4 + 1),x)*b
```

3.264 $\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 211

$$\begin{aligned}\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{5/2}}\,dx &= \frac{x(1+bx^2)}{6\left(1+b^2x^4\right)^{3/2}}+\frac{x(5+3bx^2)}{12\sqrt{1+b^2x^4}} \\ &\quad -\frac{x\sqrt{1+b^2x^4}}{4\left(1+bx^2\right)}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{4\sqrt{b}\sqrt{1+b^2x^4}} \\ &\quad +\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{12\sqrt{b}\sqrt{1+b^2x^4}}\end{aligned}$$

output

```
1/6*x*(b*x^2+1)/(b^2*x^4+1)^(3/2)+1/12*x*(3*b*x^2+5)/(b^2*x^4+1)^(1/2)-x*(
b^2*x^4+1)^(1/2)/(4*b*x^2+4)+1/4*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)
*EllipticE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)
+1/12*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b
^(1/2)*x),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

$$\int \frac{1+bx^2}{(1+b^2x^4)^{5/2}}\,dx = \frac{1}{12}x\left(\frac{7+5b^2x^4}{(1+b^2x^4)^{3/2}}\right.$$
$$\left.+5\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+4bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},-b^2x^4\right)\right)$$

input
```
Integrate[(1 + b*x^2)/(1 + b^2*x^4)^(5/2),x]
```

output
```
(x*((7 + 5*b^2*x^4)/(1 + b^2*x^4)^(3/2) + 5*Hypergeometric2F1[1/4, 1/2, 5/
4, -(b^2*x^4)] + 4*b*x^2*Hypergeometric2F1[3/4, 5/2, 7/4, -(b^2*x^4)]))/12
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1493, 25, 1493, 25, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{(b^2x^4+1)^{5/2}}dx$$
$$\Big\downarrow \text{1493}$$
$$\frac{x(bx^2+1)}{6(b^2x^4+1)^{3/2}}-\frac{1}{6}\int -\frac{3bx^2+5}{(b^2x^4+1)^{3/2}}dx$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{6}\int \frac{3bx^2+5}{(b^2x^4+1)^{3/2}}dx+\frac{x(bx^2+1)}{6(b^2x^4+1)^{3/2}}$$
$$\Big\downarrow \text{1493}$$

$$\frac{1}{6}\left(\frac{x\left(3bx^2+5\right)}{2\sqrt{b^2x^4+1}}-\frac{1}{2}\int -\frac{5-3bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{x\left(bx^2+1\right)}{6\left(b^2x^4+1\right)^{3/2}}$$

$$\Big\downarrow 25$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{5-3bx^2}{\sqrt{b^2x^4+1}}dx+\frac{x\left(3bx^2+5\right)}{2\sqrt{b^2x^4+1}}\right)+\frac{x\left(bx^2+1\right)}{6\left(b^2x^4+1\right)^{3/2}}$$

$$\Big\downarrow 1512$$

$$\frac{1}{6}\left(\frac{1}{2}\left(2\int \frac{1}{\sqrt{b^2x^4+1}}dx+3\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{x\left(3bx^2+5\right)}{2\sqrt{b^2x^4+1}}\right)+\frac{x\left(bx^2+1\right)}{6\left(b^2x^4+1\right)^{3/2}}$$

$$\Big\downarrow 761$$

$$\frac{1}{6}\left(\frac{1}{2}\left(3\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx+\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}\right)+\frac{x\left(3bx^2+5\right)}{2\sqrt{b^2x^4+1}}\right)+\frac{x\left(bx^2+1\right)}{6\left(b^2x^4+1\right)^{3/2}}$$

$$\Big\downarrow 1510$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}+3\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\right.\right.\right.$$

$$\frac{x\left(bx^2+1\right)}{6\left(b^2x^4+1\right)^{3/2}}$$

input
```
Int[(1 + b*x^2)/(1 + b^2*x^4)^(5/2),x]
```

output
```
(x*(1 + b*x^2))/(6*(1 + b^2*x^4)^(3/2)) + ((x*(5 + 3*b*x^2))/(2*Sqrt[1 + b
^2*x^4]) + (3*(-((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1
 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*S
qrt[1 + b^2*x^4])) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*Ellipt
icF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]))/2)/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.18

method	result
meijerg	$x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{5}{2}\right],\left[\frac{5}{4}\right],-b^2x^4\right)+\frac{b\,x^3\,\text{hypergeom}\left(\left[\frac{3}{4},\frac{5}{2}\right],\left[\frac{7}{4}\right],-b^2x^4\right)}{3}$
elliptic	$\frac{\left(\frac{x^3}{6b^3}+\frac{x}{6b^4}\right)\sqrt{b^2x^4+1}}{\left(x^4+\frac{1}{b^2}\right)^2}-\frac{2b^2\left(-\frac{x^3}{8b}-\frac{5x}{24b^2}\right)}{\sqrt{\left(x^4+\frac{1}{b^2}\right)b^2}}+\frac{5\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\text{EllipticF}\left(x\sqrt{ib},i\right)}{12\sqrt{ib}\,\sqrt{b^2x^4+1}}-\frac{i\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\left(\text{EllipticF}\left(\right.\right.}{4\sqrt{ib}\,\sqrt{b^2}}$
default	$\frac{x\sqrt{b^2x^4+1}}{6b^4\left(x^4+\frac{1}{b^2}\right)^2}+\frac{5x}{12\sqrt{\left(x^4+\frac{1}{b^2}\right)b^2}}+\frac{5\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\text{EllipticF}\left(x\sqrt{ib},i\right)}{12\sqrt{ib}\,\sqrt{b^2x^4+1}}+b\left(\frac{x^3\sqrt{b^2x^4+1}}{6b^4\left(x^4+\frac{1}{b^2}\right)^2}+\frac{x^3}{4\sqrt{\left(x^4+\frac{1}{b^2}\right)b^2}}-\right.$

input
```
int((b*x^2+1)/(b^2*x^4+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,5/2],[5/4],-b^2*x^4)+1/3*b*x^3*hypergeom([3/4,5/2],[7/4],
-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.77

$$\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{5/2}}\,dx=\frac{3\left(b^5x^8+2\,b^3x^4+b\right)\left(-b^2\right)^{\frac{3}{4}}\,E\left(\arcsin\left(\left(-b^2\right)^{\frac{1}{4}}\,x\right)\,|-1\right)-\left(\left(3\,b^5+5\,b^4\right)x^8+2\left(3\,b^3+\right.\right.}{12\,(b}$$

input
```
integrate((b*x^2+1)/(b^2*x^4+1)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(3*(b^5*x^8 + 2*b^3*x^4 + b)*(-b^2)^(3/4)*elliptic_e(arcsin((-b^2)^(1
/4)*x), -1) - ((3*b^5 + 5*b^4)*x^8 + 2*(3*b^3 + 5*b^2)*x^4 + 3*b + 5)*(-b^
2)^(3/4)*elliptic_f(arcsin((-b^2)^(1/4)*x), -1) + (3*b^5*x^7 + 5*b^4*x^5 +
 5*b^3*x^3 + 7*b^2*x)*sqrt(b^2*x^4 + 1))/(b^6*x^8 + 2*b^4*x^4 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.55 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.31

$$\int \frac{1+bx^2}{(1+b^2x^4)^{5/2}}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{5}{2}\\ \frac{7}{4}\end{matrix}\right| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{5}{2}\\ \frac{5}{4}\end{matrix}\right| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((b*x**2+1)/(b**2*x**4+1)**(5/2),x)
```

output

```
b*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b**2*x**4*exp_polar(I
*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{(1+b^2x^4)^{5/2}}\,dx = \int \frac{bx^2+1}{(b^2x^4+1)^{\frac{5}{2}}}\,dx$$

input

```
integrate((b*x^2+1)/(b^2*x^4+1)^(5/2),x, algorithm="maxima")
```

output

```
integrate((b*x^2 + 1)/(b^2*x^4 + 1)^(5/2), x)
```

Giac [F]

$$\int \frac{1+bx^2}{(1+b^2x^4)^{5/2}}\,dx = \int \frac{bx^2+1}{(b^2x^4+1)^{\frac{5}{2}}}\,dx$$

input

```
integrate((b*x^2+1)/(b^2*x^4+1)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/(b^2*x^4 + 1)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{5/2}}\,dx=\int \frac{b\,x^2+1}{{\left(b^2\,x^4+1\right)}^{5/2}}\,dx$$

input
```
int((b*x^2 + 1)/(b^2*x^4 + 1)^(5/2),x)
```

output
```
int((b*x^2 + 1)/(b^2*x^4 + 1)^(5/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{5/2}}\,dx=\int \frac{\sqrt{b^2x^4+1}}{b^6x^{12}+3b^4x^8+3b^2x^4+1}dx+\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^6x^{12}+3b^4x^8+3b^2x^4+1}dx\right)b$$

input
```
int((b*x^2+1)/(b^2*x^4+1)^(5/2),x)
```

output
```
int(sqrt(b**2*x**4 + 1)/(b**6*x**12 + 3*b**4*x**8 + 3*b**2*x**4 + 1),x) +
int((sqrt(b**2*x**4 + 1)*x**2)/(b**6*x**12 + 3*b**4*x**8 + 3*b**2*x**4 + 1
),x)*b
```

3.265 $\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 237

$$\begin{aligned}\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{7/2}}\,dx &= \frac{x(1+bx^2)}{10\left(1+b^2x^4\right)^{5/2}}+\frac{x(9+7bx^2)}{60\left(1+b^2x^4\right)^{3/2}}+\frac{x(15+7bx^2)}{40\sqrt{1+b^2x^4}}\\ &\quad-\frac{7x\sqrt{1+b^2x^4}}{40\left(1+bx^2\right)}+\frac{7(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{40\sqrt{b}\sqrt{1+b^2x^4}}\\ &\quad+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{10\sqrt{b}\sqrt{1+b^2x^4}}\end{aligned}$$

output

```
1/10*x*(b*x^2+1)/(b^2*x^4+1)^(5/2)+1/60*x*(7*b*x^2+9)/(b^2*x^4+1)^(3/2)+1/
40*x*(7*b*x^2+15)/(b^2*x^4+1)^(1/2)-7*x*(b^2*x^4+1)^(1/2)/(40*b*x^2+40)+7/
40*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/2
)*x)),1/2*2^(1/2))/b^(1/2)/(b^2*x^4+1)^(1/2)+1/10*(b*x^2+1)*((b^2*x^4+1)/(
b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b^(1/2)
/(b^2*x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.35

$$\int \frac{1+bx^2}{(1+b^2x^4)^{7/2}}\,dx = \frac{1}{120}x\left(\frac{3(25+36b^2x^4+15b^4x^8)}{(1+b^2x^4)^{5/2}}\right.$$
$$\left.+45\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+40bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{2},\frac{7}{4},-b^2x^4\right)\right)$$

input
```
Integrate[(1 + b*x^2)/(1 + b^2*x^4)^(7/2),x]
```

output
```
(x*((3*(25 + 36*b^2*x^4 + 15*b^4*x^8))/(1 + b^2*x^4)^(5/2) + 45*Hypergeome
tric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + 40*b*x^2*Hypergeometric2F1[3/4, 7/2,
7/4, -(b^2*x^4)]))/120
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1493, 25, 1493, 27, 1493, 25, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{(b^2x^4+1)^{7/2}}\,dx$$

$$\Big\downarrow \text{1493}$$

$$\frac{x(bx^2+1)}{10(b^2x^4+1)^{5/2}}-\frac{1}{10}\int -\frac{7bx^2+9}{(b^2x^4+1)^{5/2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{10}\int \frac{7bx^2+9}{(b^2x^4+1)^{5/2}}dx+\frac{x(bx^2+1)}{10(b^2x^4+1)^{5/2}}$$

$$\Big\downarrow \text{1493}$$

$$\frac{1}{10}\left(\frac{x\left(7bx^2+9\right)}{6\left(b^2x^4+1\right)^{3/2}}-\frac{1}{6}\int -\frac{3\left(7bx^2+15\right)}{\left(b^2x^4+1\right)^{3/2}}dx\right)+\frac{x\left(bx^2+1\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\frac{1}{2}\int \frac{7bx^2+15}{\left(b^2x^4+1\right)^{3/2}}dx+\frac{x\left(7bx^2+9\right)}{6\left(b^2x^4+1\right)^{3/2}}\right)+\frac{x\left(bx^2+1\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

$$\Big\downarrow \text{1493}$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{x\left(7bx^2+15\right)}{2\sqrt{b^2x^4+1}}-\frac{1}{2}\int -\frac{15-7bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{x\left(7bx^2+9\right)}{6\left(b^2x^4+1\right)^{3/2}}\right)+\frac{x\left(bx^2+1\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{15-7bx^2}{\sqrt{b^2x^4+1}}dx+\frac{x\left(7bx^2+15\right)}{2\sqrt{b^2x^4+1}}\right)+\frac{x\left(7bx^2+9\right)}{6\left(b^2x^4+1\right)^{3/2}}\right)+\frac{x\left(bx^2+1\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

$$\Big\downarrow \text{1512}$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\left(8\int \frac{1}{\sqrt{b^2x^4+1}}dx+7\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx\right)+\frac{x\left(7bx^2+15\right)}{2\sqrt{b^2x^4+1}}\right)+\frac{x\left(7bx^2+9\right)}{6\left(b^2x^4+1\right)^{3/2}}\right)+ \frac{x\left(bx^2+1\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\left(7\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx+\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}\right)+\frac{x\left(7bx^2+15\right)}{2\sqrt{b^2x^4+1}}\right)+\frac{x}{6\,(}\right.$$

$$\frac{x\left(bx^2+1\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}+7\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\right.}{\sqrt{b}\sqrt{b^2x^4+1}}\right.\right.\right.\right.$$

$$\frac{x\left(bx^2+1\right)}{10\left(b^2x^4+1\right)^{5/2}}$$

input
```
Int[(1 + b*x^2)/(1 + b^2*x^4)^(7/2),x]
```

output
```
(x*(1 + b*x^2))/(10*(1 + b^2*x^4)^(5/2)) + ((x*(9 + 7*b*x^2))/(6*(1 + b^2*
x^4)^(3/2)) + ((x*(15 + 7*b*x^2))/(2*Sqrt[1 + b^2*x^4]) + (7*(-((x*Sqrt[1
+ b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*
EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])) + (4*(1
+ b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x],
1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]))/2)/2)/10
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.16

method	result
meijerg	$x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{7}{2}\right], \left[\frac{5}{4}\right], -b^2x^4\right) + \frac{b\,x^3 \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{7}{2}\right],\left[\frac{7}{4}\right],-b^2x^4\right)}{3}$
elliptic	$\frac{\left(\frac{x^3}{10b^5}+\frac{x}{10b^6}\right)\sqrt{b^2x^4+1}}{\left(x^4+\frac{1}{b^2}\right)^3} + \frac{\left(\frac{7x^3}{60b^3}+\frac{3x}{20b^4}\right)\sqrt{b^2x^4+1}}{\left(x^4+\frac{1}{b^2}\right)^2} - \frac{2b^2\left(-\frac{7x^3}{80b}-\frac{3x}{16b^2}\right)}{\sqrt{\left(x^4+\frac{1}{b^2}\right)b^2}} + \frac{3\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\ \operatorname{EllipticF}\left(x\sqrt{ib},i\right)}{8\sqrt{ib}\,\sqrt{b^2x^4+1}} - 7i$
default	$\frac{x\sqrt{b^2x^4+1}}{10b^6\left(x^4+\frac{1}{b^2}\right)^3} + \frac{3x\sqrt{b^2x^4+1}}{20b^4\left(x^4+\frac{1}{b^2}\right)^2} + \frac{3x}{8\sqrt{\left(x^4+\frac{1}{b^2}\right)b^2}} + \frac{3\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\ \operatorname{EllipticF}\left(x\sqrt{ib},i\right)}{8\sqrt{ib}\,\sqrt{b^2x^4+1}} + b\left(\frac{x^3\sqrt{b^2x^4+1}}{10b^6\left(x^4+\frac{1}{b^2}\right)^3} +\right.$

input
```
int((b*x^2+1)/(b^2*x^4+1)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,7/2],[5/4],-b^2*x^4)+1/3*b*x^3*hypergeom([3/4,7/2],[7/4],
-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.89

$$\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{7/2}}\,dx = \frac{21\left(b^7x^{12}+3\,b^5x^8+3\,b^3x^4+b\right)\left(-b^2\right)^{\frac{3}{4}}\,E\left(\arcsin\left(\left(-b^2\right)^{\frac{1}{4}}\,x\right)\,|\,-1\right) - 3\left(\left(7\,b^7+15\,b^6\right)\right.}{}$$

input
```
integrate((b*x^2+1)/(b^2*x^4+1)^(7/2),x, algorithm="fricas")
```

output
```
1/120*(21*(b^7*x^12 + 3*b^5*x^8 + 3*b^3*x^4 + b)*(-b^2)^(3/4)*elliptic_e(a
rcsin((-b^2)^(1/4)*x), -1) - 3*((7*b^7 + 15*b^6)*x^12 + 3*(7*b^5 + 15*b^4)
*x^8 + 3*(7*b^3 + 15*b^2)*x^4 + 7*b + 15)*(-b^2)^(3/4)*elliptic_f(arcsin((
-b^2)^(1/4)*x), -1) + (21*b^7*x^11 + 45*b^6*x^9 + 56*b^5*x^7 + 108*b^4*x^5
 + 47*b^3*x^3 + 75*b^2*x)*sqrt(b^2*x^4 + 1))/(b^8*x^12 + 3*b^6*x^8 + 3*b^4
*x^4 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 31.86 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.28

$$\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{7/2}}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(b**2*x**4+1)**(7/2),x)
```

output
```
b*x**3*gamma(3/4)*hyper((3/4, 7/2), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 7/2), (5/4,), b**2*x**4*exp_polar(I
*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{\left(1+b^2x^4\right)^{7/2}}\,dx = \int \frac{bx^2+1}{\left(b^2x^4+1\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4+1)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/(b^2*x^4 + 1)^(7/2), x)
```

Giac [F]

$$\int \frac{1+bx^2}{(1+b^2x^4)^{7/2}}\,dx=\int \frac{bx^2+1}{(b^2x^4+1)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4+1)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/(b^2*x^4 + 1)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{(1+b^2x^4)^{7/2}}\,dx=\int \frac{b\,x^2+1}{(b^2\,x^4+1)^{7/2}}\,dx$$

input
```
int((b*x^2 + 1)/(b^2*x^4 + 1)^(7/2),x)
```

output
```
int((b*x^2 + 1)/(b^2*x^4 + 1)^(7/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{(1+b^2x^4)^{7/2}}\,dx=\int \frac{\sqrt{b^2x^4+1}}{b^8x^{16}+4b^6x^{12}+6b^4x^8+4b^2x^4+1}dx+\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^8x^{16}+4b^6x^{12}+6b^4x^8+4b^2x^4+1}dx\right)b$$

input
```
int((b*x^2+1)/(b^2*x^4+1)^(7/2),x)
```

output
```
int(sqrt(b**2*x**4 + 1)/(b**8*x**16 + 4*b**6*x**12 + 6*b**4*x**8 + 4*b**2*
x**4 + 1),x) + int((sqrt(b**2*x**4 + 1)*x**2)/(b**8*x**16 + 4*b**6*x**12 +
 6*b**4*x**8 + 4*b**2*x**4 + 1),x)*b
```

3.266 $\int \left(1+bx^2\right)\left(-1+b^2x^4\right)^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 171

$$\begin{aligned}\int \left(1+bx^2\right)\left(-1+b^2x^4\right)^{5/2}\,dx &= \frac{4x(195+77bx^2)\sqrt{-1+b^2x^4}}{3003}\\ &\quad-\frac{10x(117+77bx^2)\left(-1+b^2x^4\right)^{3/2}}{9009}\\ &\quad+\frac{1}{143}x\left(13+11bx^2\right)\left(-1+b^2x^4\right)^{5/2}-\frac{8\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{39\sqrt{b}\sqrt{-1+b^2x^4}}\\ &\quad-\frac{944\sqrt{1-b^2x^4}\,\mathrm{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{3003\sqrt{b}\sqrt{-1+b^2x^4}}\end{aligned}$$

output

```
4/3003*x*(77*b*x^2+195)*(b^2*x^4-1)^(1/2)-10/9009*x*(77*b*x^2+117)*(b^2*x^
4-1)^(3/2)+1/143*x*(11*b*x^2+13)*(b^2*x^4-1)^(5/2)-8/39*(-b^2*x^4+1)^(1/2)
*EllipticE(b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)-944/3003*(-b^2*x^4+1)^(1
/2)*EllipticF(b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.43

$$\int (1+bx^2)\,(-1+b^2x^4)^{5/2}\,dx=\frac{\sqrt{-1+b^2x^4}(3x\,\text{Hypergeometric2F1}\,(-\frac{5}{2},\frac{1}{4},\frac{5}{4},b^2x^4)+bx^3\,\text{Hypergeometric2F1}\,(-\frac{5}{2},\frac{3}{4},\frac{7}{4},b}{3\sqrt{1-b^2x^4}}$$

input
```
Integrate[(1 + b*x^2)*(-1 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[-1 + b^2*x^4]*(3*x*Hypergeometric2F1[-5/2, 1/4, 5/4, b^2*x^4] + b*x^
3*Hypergeometric2F1[-5/2, 3/4, 7/4, b^2*x^4]))/(3*Sqrt[1 - b^2*x^4])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^2+1)\,(b^2x^4-1)^{5/2}\,dx$$
$$\Big\downarrow \text{ 1571}$$
$$\int (bx^2+1)\,(b^2x^4-1)^{5/2}dx$$

input
```
Int[(1 + b*x^2)*(-1 + b^2*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.00 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.51

method	result
meijerg	$\frac{\mathrm{signum}\left(b^{2} x^{4}-1\right)^{\frac{5}{2}} x\,\mathrm{hypergeom}\left(\left[-\frac{5}{2},\frac{1}{4}\right],\left[\frac{5}{4}\right],b^{2} x^{4}\right)}{\left(-\mathrm{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{5}{2}}}+\frac{b\,\mathrm{signum}\left(b^{2} x^{4}-1\right)^{\frac{5}{2}} x^{3}\,\mathrm{hypergeom}\left(\left[-\frac{5}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^{2} x^{4}\right)}{3\left(-\mathrm{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{5}{2}}}$
risch	$\frac{x\left(693 x^{10} b^{5}+819 b^{4} x^{8}-2156 b^{3} x^{6}-2808 b^{2} x^{4}+2387 b\,x^{2}+4329\right)\sqrt{b^{2} x^{4}-1}}{9009}-\frac{40\sqrt{b\,x^{2}+1}\,\sqrt{-b\,x^{2}+1}\,\mathrm{EllipticF}\left(x\sqrt{-b},i\right)}{77\sqrt{-b}\,\sqrt{b^{2} x^{4}-1}}-8\sqrt{b\,x^{2}}$
elliptic	$\frac{b^{5} x^{11}\sqrt{b^{2} x^{4}-1}}{13}+\frac{b^{4} x^{9}\sqrt{b^{2} x^{4}-1}}{11}-\frac{28 b^{3} x^{7}\sqrt{b^{2} x^{4}-1}}{117}-\frac{24 b^{2} x^{5}\sqrt{b^{2} x^{4}-1}}{77}+\frac{31 b\,x^{3}\sqrt{b^{2} x^{4}-1}}{117}+\frac{37 x\sqrt{b^{2} x^{4}-1}}{77}-40\sqrt{b\,x^{2}+}$
default	$\frac{b^{4} x^{9}\sqrt{b^{2} x^{4}-1}}{11}-\frac{24 b^{2} x^{5}\sqrt{b^{2} x^{4}-1}}{77}+\frac{37 x\sqrt{b^{2} x^{4}-1}}{77}-\frac{40\sqrt{b\,x^{2}+1}\,\sqrt{-b\,x^{2}+1}\,\mathrm{EllipticF}\left(x\sqrt{-b},i\right)}{77\sqrt{-b}\,\sqrt{b^{2} x^{4}-1}}+b\left(\frac{b^{4} x^{11}\sqrt{b^{2} x^{4}-1}}{13}-2\right.$

input
```
int((b*x^2+1)*(b^2*x^4-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
signum(b^2*x^4-1)^(5/2)/(-signum(b^2*x^4-1))^(5/2)*x*hypergeom([-5/2,1/4],
[5/4],b^2*x^4)+1/3*b*signum(b^2*x^4-1)^(5/2)/(-signum(b^2*x^4-1))^(5/2)*x^
3*hypergeom([-5/2,3/4],[7/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62

$$\int \left(1+bx^{2}\right)\left(-1+b^{2}x^{4}\right)^{5/2}\,dx=\frac{\frac{24\,(195\,b+77)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\mid -1\right)}{\sqrt{b}}+(693\,b^{6}x^{12}+819\,b^{5}x^{10}-2156\,b^{4}x^{8}-2808\,b^{3}x^{6}+2387\,b^{2}x^{4}+}{9009\,bx}$$

input
```
integrate((b*x^2+1)*(b^2*x^4-1)^(5/2),x, algorithm="fricas")
```

output
```
1/9009*(24*(195*b + 77)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) +
(693*b^6*x^12 + 819*b^5*x^10 - 2156*b^4*x^8 - 2808*b^3*x^6 + 2387*b^2*x^4
+ 4329*b*x^2 - 1848)*sqrt(b^2*x^4 - 1) - 1848*x*elliptic_e(arcsin(1/(sqrt(
b)*x)), -1)/sqrt(b))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.20

$$
\begin{aligned}
\int \left(1+bx^2\right)\left(-1+b^2x^4\right)^{5/2}\,dx &= \frac{ib^5x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\,b^2x^4\right)}{4\Gamma\left(\frac{15}{4}\right)} \\
&+ \frac{ib^4x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\,b^2x^4\right)}{4\Gamma\left(\frac{13}{4}\right)} - \frac{ib^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\,b^2x^4\right)}{2\Gamma\left(\frac{11}{4}\right)} \\
&- \frac{ib^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\,b^2x^4\right)}{2\Gamma\left(\frac{9}{4}\right)} \\
&+ \frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\,b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}
\end{aligned}
$$

input
```
integrate((b*x**2+1)*(b**2*x**4-1)**(5/2),x)
```

output
```
I*b**5*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b**2*x**4)/(4*gamma(
15/4)) + I*b**4*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b**2*x**4)/(4*
gamma(13/4)) - I*b**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b**2*x**
4)/(2*gamma(11/4)) - I*b**2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b**
2*x**4)/(2*gamma(9/4)) + I*b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*
*2*x**4)/(4*gamma(7/4)) + I*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b**2*x
**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1+bx^2\right) \left(-1+b^2x^4\right)^{5/2} \, dx = \int \left(b^2x^4-1\right)^{\frac{5}{2}} \left(bx^2+1\right) dx$$

input
```
integrate((b*x^2+1)*(b^2*x^4-1)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 - 1)^(5/2)*(b*x^2 + 1), x)
```

Giac [F]

$$\int \left(1+bx^2\right) \left(-1+b^2x^4\right)^{5/2} \, dx = \int \left(b^2x^4-1\right)^{\frac{5}{2}} \left(bx^2+1\right) dx$$

input
```
integrate((b*x^2+1)*(b^2*x^4-1)^(5/2),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 - 1)^(5/2)*(b*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+bx^2\right) \left(-1+b^2x^4\right)^{5/2} \, dx = \int \left(b^2\,x^4-1\right)^{5/2}\,\left(b\,x^2+1\right) \,d x$$

input
```
int((b^2*x^4 - 1)^(5/2)*(b*x^2 + 1),x)
```

output
```
int((b^2*x^4 - 1)^(5/2)*(b*x^2 + 1), x)
```

Reduce [F]

$$\int \left(1+bx^2\right)\left(-1+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{b^2x^4-1}\,b^5x^{11}}{13}+\frac{\sqrt{b^2x^4-1}\,b^4x^9}{11}-\frac{28\sqrt{b^2x^4-1}\,b^3x^7}{117}-\frac{24\sqrt{b^2x^4-1}\,b^2x^5}{77}+\frac{31\sqrt{b^2x^4-1}\,b\,x^3}{117}+\frac{37\sqrt{b^2x^4-1}\,x}{77}-\frac{40\left(\int\frac{\sqrt{b^2x^4-1}}{b^2x^4-1}dx\right)}{77}-\frac{8\left(\int\frac{\sqrt{b^2x^4-1}\,x^2}{b^2x^4-1}dx\right)b}{39}$$

input
```
int((b*x^2+1)*(b^2*x^4-1)^(5/2),x)
```

output
```
(693*sqrt(b**2*x**4 - 1)*b**5*x**11 + 819*sqrt(b**2*x**4 - 1)*b**4*x**9 -
2156*sqrt(b**2*x**4 - 1)*b**3*x**7 - 2808*sqrt(b**2*x**4 - 1)*b**2*x**5 +
2387*sqrt(b**2*x**4 - 1)*b*x**3 + 4329*sqrt(b**2*x**4 - 1)*x - 4680*int(sq
rt(b**2*x**4 - 1)/(b**2*x**4 - 1),x) - 1848*int((sqrt(b**2*x**4 - 1)*x**2)
/(b**2*x**4 - 1),x)*b)/9009
```

3.267 $\int (1+bx^2)(-1+b^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 145

$$\int (1+bx^2)\left(-1+b^2x^4\right)^{3/2}\,dx=-\frac{2}{105}x\left(15+7bx^2\right)\sqrt{-1+b^2x^4}+\frac{1}{63}x\left(9+7bx^2\right)\left(-1+b^2x^4\right)^{3/2}+\frac{4\sqrt{1-b^2x^4}E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{15\sqrt{b}\sqrt{-1+b^2x^4}}+\frac{32\sqrt{1-b^2x^4}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{105\sqrt{b}\sqrt{-1+b^2x^4}}$$

output
```
-2/105*x*(7*b*x^2+15)*(b^2*x^4-1)^(1/2)+1/63*x*(7*b*x^2+9)*(b^2*x^4-1)^(3/
2)+4/15*(-b^2*x^4+1)^(1/2)*EllipticE(b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2
)+32/105*(-b^2*x^4+1)^(1/2)*EllipticF(b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.67 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.51

$$\int (1+bx^2)\left(-1+b^2x^4\right)^{3/2}\,dx = -\frac{\sqrt{-1+b^2x^4}\left(3x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)\right)}{3\sqrt{1-b^2x^4}}$$

input
```
Integrate[(1 + b*x^2)*(-1 + b^2*x^4)^(3/2),x]
```

output
```
-1/3*(Sqrt[-1 + b^2*x^4]*(3*x*Hypergeometric2F1[-3/2, 1/4, 5/4, b^2*x^4] +
 b*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, b^2*x^4]))/Sqrt[1 - b^2*x^4]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^2+1)\left(b^2x^4-1\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int (bx^2+1)\left(b^2x^4-1\right)^{3/2}dx$$

input
```
Int[(1 + b*x^2)*(-1 + b^2*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.73 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

method	result
meijerg	$\frac{\mathrm{signum}(b^2x^4-1)^{\frac{3}{2}}x\,\mathrm{hypergeom}([-\frac{3}{2},\frac{1}{4}],[\frac{5}{4}],b^2x^4)}{(-\mathrm{signum}(b^2x^4-1))^{\frac{3}{2}}}+\frac{b\,\mathrm{signum}(b^2x^4-1)^{\frac{3}{2}}x^3\,\mathrm{hypergeom}([-\frac{3}{2},\frac{3}{4}],[\frac{7}{4}],b^2x^4)}{3(-\mathrm{signum}(b^2x^4-1))^{\frac{3}{2}}}$
risch	$\frac{x(35b^3x^6+45b^2x^4-77b\,x^2-135)\sqrt{b^2x^4-1}}{315}+\frac{4\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\mathrm{EllipticF}(x\sqrt{-b},i)}{7\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{4\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,(\mathrm{EllipticF}(x\sqrt{-}}{15\sqrt{-b}\,\sqrt{b^2x^4}}$
elliptic	$\frac{b^3x^7\sqrt{b^2x^4-1}}{9}+\frac{b^2x^5\sqrt{b^2x^4-1}}{7}-\frac{11b\,x^3\sqrt{b^2x^4-1}}{45}-\frac{3x\sqrt{b^2x^4-1}}{7}+\frac{4\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\mathrm{EllipticF}(x\sqrt{-b},i)}{7\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{4\sqrt{b\,x^2+}}{}$
default	$\frac{b^2x^5\sqrt{b^2x^4-1}}{7}-\frac{3x\sqrt{b^2x^4-1}}{7}+\frac{4\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\mathrm{EllipticF}(x\sqrt{-b},i)}{7\sqrt{-b}\,\sqrt{b^2x^4-1}}+b\left(\frac{b^2x^7\sqrt{b^2x^4-1}}{9}-\frac{11x^3\sqrt{b^2x^4-1}}{45}+\frac{4\sqrt{b\,x^2}}{}\right.$

input
```
int((b*x^2+1)*(b^2*x^4-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
signum(b^2*x^4-1)^(3/2)/(-signum(b^2*x^4-1))^(3/2)*x*hypergeom([-3/2,1/4],
[5/4],b^2*x^4)+1/3*b*signum(b^2*x^4-1)^(3/2)/(-signum(b^2*x^4-1))^(3/2)*x^
3*hypergeom([-3/2,3/4],[7/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

$$\int \left(1+bx^2\right)\left(-1+b^2x^4\right)^{3/2}\,dx =$$

$$-\frac{\frac{12\,(15\,b+7)xF(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1)}{\sqrt{b}}-(35\,b^4x^8+45\,b^3x^6-77\,b^2x^4-135\,bx^2+84)\sqrt{b^2x^4-1}-\frac{84\,xE(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1)}{\sqrt{b}}}{315\,bx}$$

input
```
integrate((b*x^2+1)*(b^2*x^4-1)^(3/2),x, algorithm="fricas")
```

output
```
-1/315*(12*(15*b + 7)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - (3
5*b^4*x^8 + 45*b^3*x^6 - 77*b^2*x^4 - 135*b*x^2 + 84)*sqrt(b^2*x^4 - 1) -
84*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int (1+bx^2)(-1+b^2x^4)^{3/2}\,dx = \frac{ib^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|b^2x^4\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{ib^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|b^2x^4\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)*(b**2*x**4-1)**(3/2),x)
```

output
```
I*b**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b**2*x**4)/(4*gamma(11/
4)) + I*b**2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b**2*x**4)/(4*gamm
a(9/4)) - I*b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4)/(4*gam
ma(7/4)) - I*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b**2*x**4)/(4*gamma(5
/4))
```

Maxima [F]

$$\int (1+bx^2) \left(-1+b^2x^4\right)^{3/2} \, dx = \int \left(b^2x^4-1\right)^{\frac{3}{2}} (bx^2+1) \, dx$$

input
```
integrate((b*x^2+1)*(b^2*x^4-1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 - 1)^(3/2)*(b*x^2 + 1), x)
```

Giac [F]

$$\int (1+bx^2) \left(-1+b^2x^4\right)^{3/2} \, dx = \int \left(b^2x^4-1\right)^{\frac{3}{2}} (bx^2+1) \, dx$$

input
```
integrate((b*x^2+1)*(b^2*x^4-1)^(3/2),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 - 1)^(3/2)*(b*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+bx^2) \left(-1+b^2x^4\right)^{3/2} \, dx = \int \left(b^2\,x^4-1\right)^{3/2}\,\left(b\,x^2+1\right) \, dx$$

input
```
int((b^2*x^4 - 1)^(3/2)*(b*x^2 + 1),x)
```

output
```
int((b^2*x^4 - 1)^(3/2)*(b*x^2 + 1), x)
```

Reduce [F]

$$\int \left(1+bx^2\right)\left(-1+b^2x^4\right)^{3/2}\,dx=\frac{\sqrt{b^2x^4-1}\,b^3x^7}{9}+\frac{\sqrt{b^2x^4-1}\,b^2x^5}{7}-\frac{11\sqrt{b^2x^4-1}\,b\,x^3}{45}-\frac{3\sqrt{b^2x^4-1}\,x}{7}+\frac{4\left(\int \frac{\sqrt{b^2x^4-1}}{b^2x^4-1}dx\right)}{7}+\frac{4\left(\int \frac{\sqrt{b^2x^4-1}\,x^2}{b^2x^4-1}dx\right)b}{15}$$

input
```
int((b*x^2+1)*(b^2*x^4-1)^(3/2),x)
```

output
```
(35*sqrt(b**2*x**4 - 1)*b**3*x**7 + 45*sqrt(b**2*x**4 - 1)*b**2*x**5 - 77*
sqrt(b**2*x**4 - 1)*b*x**3 - 135*sqrt(b**2*x**4 - 1)*x + 180*int(sqrt(b**2
*x**4 - 1)/(b**2*x**4 - 1),x) + 84*int((sqrt(b**2*x**4 - 1)*x**2)/(b**2*x*
*4 - 1),x)*b)/315
```

3.268 $\int (1+bx^2) \sqrt{-1+b^2x^4} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\begin{aligned}\int (1+bx^2) \sqrt{-1+b^2x^4} \, dx = {} & \frac{1}{15}x(5+3bx^2) \sqrt{-1+b^2x^4} \\ & - \frac{2\sqrt{1-b^2x^4}E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{5\sqrt{b}\sqrt{-1+b^2x^4}} \\ & - \frac{4\sqrt{1-b^2x^4}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{15\sqrt{b}\sqrt{-1+b^2x^4}}\end{aligned}$$

output
```
1/15*x*(3*b*x^2+5)*(b^2*x^4-1)^(1/2)-2/5*(-b^2*x^4+1)^(1/2)*EllipticE(b^(1
/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)-4/15*(-b^2*x^4+1)^(1/2)*EllipticF(b^(1/
2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.95 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

$$\int \left(1+bx^2\right)\sqrt{-1+b^2x^4}\,dx$$
$$=\frac{\sqrt{-1+b^2x^4}\left(3x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)\right)}{3\sqrt{1-b^2x^4}}$$

input

```
Integrate[(1 + b*x^2)*Sqrt[-1 + b^2*x^4],x]
```

output

```
(Sqrt[-1 + b^2*x^4]*(3*x*Hypergeometric2F1[-1/2, 1/4, 5/4, b^2*x^4] + b*x^
3*Hypergeometric2F1[-1/2, 3/4, 7/4, b^2*x^4]))/(3*Sqrt[1 - b^2*x^4])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2+1\right)\sqrt{b^2x^4-1}\,dx$$
$$\Big\downarrow \ 1571$$
$$\int \left(bx^2+1\right)\sqrt{b^2x^4-1}dx$$

input

```
Int[(1 + b*x^2)*Sqrt[-1 + b^2*x^4],x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.99 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.74

method	result
meijerg	$\frac{\sqrt{\operatorname{signum}(b^2x^4-1)}\,x\operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{5}{4}\right],b^2x^4\right)}{\sqrt{-\operatorname{signum}(b^2x^4-1)}}+\frac{b\sqrt{\operatorname{signum}(b^2x^4-1)}\,x^3\operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3\sqrt{-\operatorname{signum}(b^2x^4-1)}}$
risch	$\frac{x(3b\,x^2+5)\sqrt{b^2x^4-1}}{15}-\frac{2\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-b},i\right)}{3\sqrt{-b}\,\sqrt{b^2x^4-1}}-\frac{2\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,(\operatorname{EllipticF}(x\sqrt{-b},i)-\operatorname{EllipticE}(x\sqrt{-}}{5\sqrt{-b}\,\sqrt{b^2x^4-1}}$
elliptic	$\frac{b\,x^3\sqrt{b^2x^4-1}}{5}+\frac{x\sqrt{b^2x^4-1}}{3}-\frac{2\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-b},i\right)}{3\sqrt{-b}\,\sqrt{b^2x^4-1}}-\frac{2\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,(\operatorname{EllipticF}(x\sqrt{-b},i)-\text{Elliptic}}{5\sqrt{-b}\,\sqrt{b^2x^4-1}}$
default	$\frac{x\sqrt{b^2x^4-1}}{3}-\frac{2\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-b},i\right)}{3\sqrt{-b}\,\sqrt{b^2x^4-1}}+b\Bigg(\frac{x^3\sqrt{b^2x^4-1}}{5}-\frac{2\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,(\operatorname{EllipticF}(x\sqrt{-b},i)-\text{Ellipt}}{5\sqrt{-b}\,\sqrt{b^2x^4-1}\,b}$

input
```
int((b*x^2+1)*(b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
signum(b^2*x^4-1)^(1/2)/(-signum(b^2*x^4-1))^(1/2)*x*hypergeom([-1/2,1/4],
[5/4],b^2*x^4)+1/3*b*signum(b^2*x^4-1)^(1/2)/(-signum(b^2*x^4-1))^(1/2)*x^
3*hypergeom([-1/2,3/4],[7/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

$$\int \left(1+bx^2\right)\sqrt{-1+b^2x^4}\,dx$$
$$=\frac{\frac{2\,(5\,b+3)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1\right)}{\sqrt{b}}+\left(3\,b^2x^4+5\,bx^2-6\right)\sqrt{b^2x^4-1}-\frac{6\,xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1\right)}{\sqrt{b}}}{15\,bx}$$

input
```
integrate((b*x^2+1)*(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(2*(5*b + 3)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + (3*b^2
*x^4 + 5*b*x^2 - 6)*sqrt(b^2*x^4 - 1) - 6*x*elliptic_e(arcsin(1/(sqrt(b)*x
)), -1)/sqrt(b))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.53

$$\int \left(1+bx^2\right)\sqrt{-1+b^2x^4}\,dx=\frac{ibx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ix\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\,b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)*(b**2*x**4-1)**(1/2),x)
```

output
```
I*b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) +
 I*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1+bx^2\right)\sqrt{-1+b^2x^4}\,dx=\int \sqrt{b^2x^4-1}\left(bx^2+1\right)dx$$

input
```
integrate((b*x^2+1)*(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b^2*x^4 - 1)*(b*x^2 + 1), x)
```

Giac [F]

$$\int \left(1+bx^2\right) \sqrt{-1+b^2x^4}\, dx = \int \sqrt{b^2x^4-1}\left(bx^2+1\right) dx$$

input
```
integrate((b*x^2+1)*(b^2*x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b^2*x^4 - 1)*(b*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+bx^2\right) \sqrt{-1+b^2x^4}\, dx = \int \sqrt{b^2\,x^4-1}\,\left(b\,x^2+1\right)\,dx$$

input
```
int((b^2*x^4 - 1)^(1/2)*(b*x^2 + 1),x)
```

output
```
int((b^2*x^4 - 1)^(1/2)*(b*x^2 + 1), x)
```

Reduce [F]

$$\int \left(1+bx^2\right) \sqrt{-1+b^2x^4}\, dx = \frac{\sqrt{b^2x^4-1}\,b\,x^3}{5} + \frac{\sqrt{b^2x^4-1}\,x}{3} - \frac{2\left(\int \frac{\sqrt{b^2x^4-1}}{b^2x^4-1}dx\right)}{3} - \frac{2\left(\int \frac{\sqrt{b^2x^4-1}\,x^2}{b^2x^4-1}dx\right)b}{5}$$

input
```
int((b*x^2+1)*(b^2*x^4-1)^(1/2),x)
```

output
```
(3*sqrt(b**2*x**4 - 1)*b*x**3 + 5*sqrt(b**2*x**4 - 1)*x - 10*int(sqrt(b**2
*x**4 - 1)/(b**2*x**4 - 1),x) - 6*int((sqrt(b**2*x**4 - 1)*x**2)/(b**2*x**
4 - 1),x)*b)/15
```

3.269 $\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = \frac{\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}$$

output

```
(-b^2*x^4+1)^(1/2)*EllipticE(b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx$$
$$= \frac{\sqrt{1-b^2x^4}\left(3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)\right)}{3\sqrt{-1+b^2x^4}}$$

input

```
Integrate[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

output
```
(Sqrt[1 - b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + b*x^3*
Hypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/(3*Sqrt[-1 + b^2*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1390, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\sqrt{b^2x^4-1}}dx$$

$$\Big\downarrow \text{1390}$$

$$\frac{\sqrt{1-b^2x^4}\int \frac{bx^2+1}{\sqrt{1-b^2x^4}}dx}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{1388}$$

$$\frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{1-b^2x^4}E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

input
```
Int[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

output
```
(Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^
2*x^4])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.98 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.05

method	result	size
meijerg	$\frac{b\sqrt{-\operatorname{signum}(b^2x^4-1)}\,x^3\operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3\sqrt{\operatorname{signum}(b^2x^4-1)}}+\frac{\sqrt{-\operatorname{signum}(b^2x^4-1)}\,x\operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],b^2x^4\right)}{\sqrt{\operatorname{signum}(b^2x^4-1)}}$	88
default	$\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-b},i\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-b},i\right)-\operatorname{EllipticE}\left(x\sqrt{-b},i\right)\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$	107
elliptic	$\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-b},i\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-b},i\right)-\operatorname{EllipticE}\left(x\sqrt{-b},i\right)\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$	107

input
```
int((b*x^2+1)/(b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b/signum(b^2*x^4-1)^(1/2)*(-signum(b^2*x^4-1))^(1/2)*x^3*hypergeom([1/
2,3/4],[7/4],b^2*x^4)+1/signum(b^2*x^4-1)^(1/2)*(-signum(b^2*x^4-1))^(1/2)
*x*hypergeom([1/4,1/2],[5/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx=-\frac{\frac{(b+1)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}}-\frac{xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}}-\sqrt{b^2x^4-1}}{bx}$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-((b + 1)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - x*elliptic_e(a
rcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - sqrt(b^2*x^4 - 1))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx=-\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

output
```
-I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) -
 I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx=\int \frac{bx^2+1}{\sqrt{b^2x^4-1}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

Giac [F]

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx=\int \frac{bx^2+1}{\sqrt{b^2x^4-1}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx=\int \frac{b\,x^2+1}{\sqrt{b^2\,x^4-1}}\,dx$$

input
```
int((b*x^2 + 1)/(b^2*x^4 - 1)^(1/2),x)
```

output
```
int((b*x^2 + 1)/(b^2*x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx=\int \frac{\sqrt{b^2x^4-1}}{b\,x^2-1}dx$$

input
```
int((b*x^2+1)/(b^2*x^4-1)^(1/2),x)
```

output
```
int(sqrt(b**2*x**4 - 1)/(b*x**2 - 1),x)
```

3.270 $\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx=-\frac{x\left(1+bx^2\right)}{2\sqrt{-1+b^2x^4}}+\frac{\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{2\sqrt{b}\sqrt{-1+b^2x^4}}-\frac{\sqrt{1-b^2x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}$$

output

```
-1/2*x*(b*x^2+1)/(b^2*x^4-1)^(1/2)+1/2*(-b^2*x^4+1)^(1/2)*EllipticE(b^(1/2
)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)-(-b^2*x^4+1)^(1/2)*EllipticF(b^(1/2)*x,I)
/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx=$$

$$-\frac{x\Big(3+3\sqrt{1-b^2x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)+2bx^2\sqrt{1-b^2x^4}\,\text{Hypergeometric2F1}\Big(\frac{3}{4},\frac{3}{2},\frac{7}{4},b^2x^4}{6\sqrt{-1+b^2x^4}}$$

input
```
Integrate[(1 + b*x^2)/(-1 + b^2*x^4)^(3/2),x]
```

output
```
-1/6*(x*(3 + 3*Sqrt[1 - b^2*x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4]
 + 2*b*x^2*Sqrt[1 - b^2*x^4]*Hypergeometric2F1[3/4, 3/2, 7/4, b^2*x^4]))/S
qrt[-1 + b^2*x^4]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\left(b^2x^4-1\right)^{3/2}}dx$$

$$\Big\downarrow \ 1571$$

$$\int \frac{bx^2+1}{\left(b^2x^4-1\right)^{3/2}}dx$$

input
```
Int[(1 + b*x^2)/(-1 + b^2*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.00 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

method	result
meijerg	$\frac{\left(-\operatorname{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{3}{2}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right],\left[\frac{5}{4}\right], b^{2} x^{4}\right)}{\operatorname{signum}\left(b^{2} x^{4}-1\right)^{\frac{3}{2}}}+\frac{b\left(-\operatorname{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{3}{2}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{3}{2}\right],\left[\frac{7}{4}\right], b^{2} x^{4}\right)}{3 \operatorname{signum}\left(b^{2} x^{4}-1\right)^{\frac{3}{2}}}$
elliptic	$-\frac{\left(b^{2} x^{2}+b\right) x}{2 b \sqrt{\left(x^{2}-\frac{1}{b}\right)\left(b^{2} x^{2}+b\right)}}-\frac{\sqrt{b x^{2}+1} \sqrt{-b x^{2}+1} \operatorname{EllipticF}(x \sqrt{-b}, i)}{2 \sqrt{-b} \sqrt{b^{2} x^{4}-1}}+\frac{\sqrt{b x^{2}+1} \sqrt{-b x^{2}+1}\left(\operatorname{EllipticF}(x \sqrt{-b}, i)-\operatorname{EllipticE}(x \sqrt{-}\right.}{2 \sqrt{-b} \sqrt{b^{2} x^{4}-1}}$
default	$-\frac{x}{2 \sqrt{\left(x^{4}-\frac{1}{b^{2}}\right) b^{2}}}-\frac{\sqrt{b x^{2}+1} \sqrt{-b x^{2}+1} \operatorname{EllipticF}(x \sqrt{-b}, i)}{2 \sqrt{-b} \sqrt{b^{2} x^{4}-1}}+b\left(-\frac{x^{3}}{2 \sqrt{\left(x^{4}-\frac{1}{b^{2}}\right) b^{2}}}+\frac{\sqrt{b x^{2}+1} \sqrt{-b x^{2}+1}\left(\operatorname{EllipticF}(x \sqrt{-}\right.}{2 \sqrt{-b} \sqrt{b^{2} x^{4}-}}\right.$

input
```
int((b*x^2+1)/(b^2*x^4-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/signum(b^2*x^4-1)^(3/2)*(-signum(b^2*x^4-1))^(3/2)*x*hypergeom([1/4,3/2]
,[5/4],b^2*x^4)+1/3*b/signum(b^2*x^4-1)^(3/2)*(-signum(b^2*x^4-1))^(3/2)*x
^3*hypergeom([3/4,3/2],[7/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx =$$
$$-\frac{\sqrt{b^2x^4-1}bx-(-i\,b^2x^2+i\,b)\sqrt{b}E(\arcsin\left(\sqrt{b}x\right)\,|-1)-(i\,(b^2+b)x^2-i\,b-i)\sqrt{b}F(\arcsin\left(\sqrt{b}x\right)\,|-}{2\,(b^2x^2-b)}$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(b^2*x^4 - 1)*b*x - (-I*b^2*x^2 + I*b)*sqrt(b)*elliptic_e(arcsin
(sqrt(b)*x), -1) - (I*(b^2 + b)*x^2 - I*b - I)*sqrt(b)*elliptic_f(arcsin(s
qrt(b)*x), -1))/(b^2*x^2 - b)
```

Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \frac{1+bx^2}{(-1+b^2x^4)^{3/2}}\,dx = \frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(b**2*x**4-1)**(3/2),x)
```

output
```
I*b*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b**2*x**4)/(4*gamma(7/4)) +
I*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{(-1+b^2x^4)^{3/2}}\,dx = \int \frac{bx^2+1}{(b^2x^4-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/(b^2*x^4 - 1)^(3/2), x)
```

Giac [F]

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx = \int \frac{bx^2+1}{\left(b^2x^4-1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/(b^2*x^4 - 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx = \int \frac{b\,x^2+1}{\left(b^2\,x^4-1\right)^{3/2}}\,dx$$

input
```
int((b*x^2 + 1)/(b^2*x^4 - 1)^(3/2),x)
```

output
```
int((b*x^2 + 1)/(b^2*x^4 - 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx = \int \frac{\sqrt{b^2x^4-1}}{b^3x^6-b^2x^4-b\,x^2+1}dx$$

input
```
int((b*x^2+1)/(b^2*x^4-1)^(3/2),x)
```

output
```
int(sqrt(b**2*x**4 - 1)/(b**3*x**6 - b**2*x**4 - b*x**2 + 1),x)
```

3.271 $\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{5/2}}\,dx = -\frac{x(1+bx^2)}{6\left(-1+b^2x^4\right)^{3/2}}+\frac{x(5+3bx^2)}{12\sqrt{-1+b^2x^4}} - \frac{\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{4\sqrt{b}\sqrt{-1+b^2x^4}}+\frac{2\sqrt{1-b^2x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{3\sqrt{b}\sqrt{-1+b^2x^4}}$$

output
```
-1/6*x*(b*x^2+1)/(b^2*x^4-1)^(3/2)+1/12*x*(3*b*x^2+5)/(b^2*x^4-1)^(1/2)-1/
4*(-b^2*x^4+1)^(1/2)*EllipticE(b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)+2/3*
(-b^2*x^4+1)^(1/2)*EllipticF(b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.68

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{5/2}}\,dx = \frac{x\left(-7+5b^2x^4-5(1-b^2x^4)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)-4bx^2(1-b^2x^4}{12\left(-1+b^2x^4\right)^{3/2}}$$

input
```
Integrate[(1 + b*x^2)/(-1 + b^2*x^4)^(5/2),x]
```

output
```
(x*(-7 + 5*b^2*x^4 - 5*(1 - b^2*x^4)^(3/2)*Hypergeometric2F1[1/4, 1/2, 5/4
, b^2*x^4] - 4*b*x^2*(1 - b^2*x^4)^(3/2)*Hypergeometric2F1[3/4, 5/2, 7/4,
b^2*x^4]))/(12*(-1 + b^2*x^4)^(3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{(b^2x^4-1)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1571}$$

$$\int \frac{bx^2+1}{(b^2x^4-1)^{5/2}}dx$$

input
```
Int[(1 + b*x^2)/(-1 + b^2*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

method	result
meijerg	$\frac{\left(-\operatorname{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{5}{2}} x \,\operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{5}{2}\right], \left[\frac{5}{4}\right], b^{2} x^{4}\right)}{\operatorname{signum}\left(b^{2} x^{4}-1\right)^{\frac{5}{2}}}+\frac{b \left(-\operatorname{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{5}{2}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{2}\right], \left[\frac{7}{4}\right], b^{2} x^{4}\right)}{3 \operatorname{signum}\left(b^{2} x^{4}-1\right)^{\frac{5}{2}}}$
default	$-\frac{x \sqrt{b^{2} x^{4}-1}}{6 b^{4} \left(x^{4}-\frac{1}{b^{2}}\right)^{2}}+\frac{5 x}{12 \sqrt{\left(x^{4}-\frac{1}{b^{2}}\right) b^{2}}}+\frac{5 \sqrt{b \,x^{2}+1}\, \sqrt{-b \,x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{-b}, i\right)}{12 \sqrt{-b}\, \sqrt{b^{2} x^{4}-1}}+b \Bigg(-\frac{x^{3} \sqrt{b^{2} x^{4}-1}}{6 b^{4} \left(x^{4}-\frac{1}{b^{2}}\right)^{2}}+\frac{x^{3}}{4 \sqrt{\left(x^{4}-\frac{1}{b^{2}}\right) b^{2}}}$
elliptic	$-\frac{x \sqrt{b^{2} x^{4}-1}}{12 b^{2} \left(x^{2}-\frac{1}{b}\right)^{2}}+\frac{3 \left(b^{2} x^{2}+b \right) x}{8 b \sqrt{\left(x^{2}-\frac{1}{b}\right) \left(b^{2} x^{2}+b \right)}}-\frac{\left(b^{2} x^{2}-b \right) x}{8 b \sqrt{\left(x^{2}+\frac{1}{b}\right) \left(b^{2} x^{2}-b \right)}}+\frac{5 \sqrt{b \,x^{2}+1}\, \sqrt{-b \,x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{-b}, i\right)}{12 \sqrt{-b}\, \sqrt{b^{2} x^{4}-1}}-\sqrt{b \,x^{2}+1}$

input
```
int((b*x^2+1)/(b^2*x^4-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/signum(b^2*x^4-1)^(5/2)*(-signum(b^2*x^4-1))^(5/2)*x*hypergeom([1/4,5/2]
,[5/4],b^2*x^4)+1/3*b/signum(b^2*x^4-1)^(5/2)*(-signum(b^2*x^4-1))^(5/2)*x
^3*hypergeom([3/4,5/2],[7/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{5/2}}\,dx =$$

$$-\frac{3\,(-i\,b^4x^6+i\,b^3x^4+i\,b^2x^2-i\,b)\sqrt{b}E(\arcsin\left(\sqrt{b}x\right)\,|-1)-(-i\,(3\,b^4+5\,b^3)x^6+i\,(3\,b^3+5\,b^2)x^4+i\,(3}{12\,(b^4x^6-b^3x^4-b^2x^2}$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*(-I*b^4*x^6 + I*b^3*x^4 + I*b^2*x^2 - I*b)*sqrt(b)*elliptic_e(arc
sin(sqrt(b)*x), -1) - (-I*(3*b^4 + 5*b^3)*x^6 + I*(3*b^3 + 5*b^2)*x^4 + I*
(3*b^2 + 5*b)*x^2 - 3*I*b - 5*I)*sqrt(b)*elliptic_f(arcsin(sqrt(b)*x), -1)
 - (3*b^3*x^5 + 2*b^2*x^3 - 7*b*x)*sqrt(b^2*x^4 - 1))/(b^4*x^6 - b^3*x^4 -
 b^2*x^2 + b)
```

Sympy [A] (verification not implemented)

Time = 5.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.42

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{5/2}}\,dx=-\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(b**2*x**4-1)**(5/2),x)
```

output
```
-I*b*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b**2*x**4)/(4*gamma(7/4)) -
 I*x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{5/2}}\,dx=\int \frac{bx^2+1}{\left(b^2x^4-1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/(b^2*x^4 - 1)^(5/2), x)
```

Giac [F]

$$\int \frac{1+bx^2}{(-1+b^2x^4)^{5/2}}\,dx=\int \frac{bx^2+1}{(b^2x^4-1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/(b^2*x^4 - 1)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{(-1+b^2x^4)^{5/2}}\,dx=\int \frac{b\,x^2+1}{(b^2\,x^4-1)^{5/2}}\,dx$$

input
```
int((b*x^2 + 1)/(b^2*x^4 - 1)^(5/2),x)
```

output
```
int((b*x^2 + 1)/(b^2*x^4 - 1)^(5/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{(-1+b^2x^4)^{5/2}}\,dx=\int \frac{\sqrt{b^2x^4-1}}{b^5x^{10}-b^4x^8-2b^3x^6+2b^2x^4+b\,x^2-1}dx$$

input
```
int((b*x^2+1)/(b^2*x^4-1)^(5/2),x)
```

output
```
int(sqrt(b**2*x**4 - 1)/(b**5*x**10 - b**4*x**8 - 2*b**3*x**6 + 2*b**2*x**
4 + b*x**2 - 1),x)
```

3.272 $\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{7/2}}\,dx = -\frac{x(1+bx^2)}{10\left(-1+b^2x^4\right)^{5/2}}+\frac{x(9+7bx^2)}{60\left(-1+b^2x^4\right)^{3/2}}-\frac{x(15+7bx^2)}{40\sqrt{-1+b^2x^4}}+\frac{7\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{40\sqrt{b}\sqrt{-1+b^2x^4}}-\frac{11\sqrt{1-b^2x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{20\sqrt{b}\sqrt{-1+b^2x^4}}$$

output
```
-1/10*x*(b*x^2+1)/(b^2*x^4-1)^(5/2)+1/60*x*(7*b*x^2+9)/(b^2*x^4-1)^(3/2)-1
/40*x*(7*b*x^2+15)/(b^2*x^4-1)^(1/2)+7/40*(-b^2*x^4+1)^(1/2)*EllipticE(b^(
1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)-11/20*(-b^2*x^4+1)^(1/2)*EllipticF(b^(
1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{7/2}}\,dx =$$

$$-\frac{x\left(75-108b^2x^4+45b^4x^8+45(1-b^2x^4)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)+40bx^2(1-b^2x^4)^{5/2}\,\text{Hyp}\right.}{120\left(-1+b^2x^4\right)^{5/2}}$$

input
```
Integrate[(1 + b*x^2)/(-1 + b^2*x^4)^(7/2),x]
```

output
```
-1/120*(x*(75 - 108*b^2*x^4 + 45*b^4*x^8 + 45*(1 - b^2*x^4)^(5/2)*Hypergeo
metric2F1[1/4, 1/2, 5/4, b^2*x^4] + 40*b*x^2*(1 - b^2*x^4)^(5/2)*Hypergeom
etric2F1[3/4, 7/2, 7/4, b^2*x^4]))/(-1 + b^2*x^4)^(5/2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{(b^2x^4-1)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 1571}$$

$$\int \frac{bx^2+1}{(b^2x^4-1)^{7/2}}dx$$

input
```
Int[(1 + b*x^2)/(-1 + b^2*x^4)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.00 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.52

method	result
meijerg	$\frac{\left(-\operatorname{signum}\left(b^2x^4-1\right)\right)^{\frac{7}{2}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{7}{2}\right],\left[\frac{5}{4}\right],b^2x^4\right)}{\operatorname{signum}\left(b^2x^4-1\right)^{\frac{7}{2}}}+\frac{b\left(-\operatorname{signum}\left(b^2x^4-1\right)\right)^{\frac{7}{2}} x^3 \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{7}{2}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3\operatorname{signum}\left(b^2x^4-1\right)^{\frac{7}{2}}}$
elliptic	$-\frac{x\sqrt{b^2x^4-1}}{40b^3\left(x^2-\frac{1}{b}\right)^3}+\frac{11x\sqrt{b^2x^4-1}}{120b^2\left(x^2-\frac{1}{b}\right)^2}-\frac{53\left(b^2x^2+b\right)x}{160b\sqrt{\left(x^2-\frac{1}{b}\right)\left(b^2x^2+b\right)}}+\frac{x\sqrt{b^2x^4-1}}{48b^2\left(x^2+\frac{1}{b}\right)^2}+\frac{5\left(b^2x^2-b\right)x}{32b\sqrt{\left(x^2+\frac{1}{b}\right)\left(b^2x^2-b\right)}}-\frac{3\sqrt{b\,x^2+1}\,\sqrt{-}}{8}$
default	$-\frac{x\sqrt{b^2x^4-1}}{10b^6\left(x^4-\frac{1}{b^2}\right)^3}+\frac{3x\sqrt{b^2x^4-1}}{20b^4\left(x^4-\frac{1}{b^2}\right)^2}-\frac{3x}{8\sqrt{\left(x^4-\frac{1}{b^2}\right)b^2}}-\frac{3\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\;\operatorname{EllipticF}\left(x\sqrt{-b},i\right)}{8\sqrt{-b}\,\sqrt{b^2x^4-1}}+b\Bigg(-\frac{x^3\sqrt{b^2x^4-1}}{10b^6\left(x^4-\frac{1}{b^2}\right)}$

input
```
int((b*x^2+1)/(b^2*x^4-1)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/signum(b^2*x^4-1)^(7/2)*(-signum(b^2*x^4-1))^(7/2)*x*hypergeom([1/4,7/2]
,[5/4],b^2*x^4)+1/3*b/signum(b^2*x^4-1)^(7/2)*(-signum(b^2*x^4-1))^(7/2)*x
^3*hypergeom([3/4,7/2],[7/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.46

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{7/2}}\,dx =$$

$$-\frac{21\left(i\,b^6x^{10}-i\,b^5x^8-2i\,b^4x^6+2i\,b^3x^4+i\,b^2x^2-i\,b\right)\sqrt{b}E\left(\arcsin\left(\sqrt{b}x\right)\,|-1\right)+3\left(-i\left(7\,b^6+15\,b^5\right)x^{10}+\right.}{}$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(7/2),x, algorithm="fricas")
```

output

```
-1/120*(21*(I*b^6*x^10 - I*b^5*x^8 - 2*I*b^4*x^6 + 2*I*b^3*x^4 + I*b^2*x^2
 - I*b)*sqrt(b)*elliptic_e(arcsin(sqrt(b)*x), -1) + 3*(-I*(7*b^6 + 15*b^5)
*x^10 + I*(7*b^5 + 15*b^4)*x^8 + 2*I*(7*b^4 + 15*b^3)*x^6 - 2*I*(7*b^3 + 1
5*b^2)*x^4 - I*(7*b^2 + 15*b)*x^2 + 7*I*b + 15*I)*sqrt(b)*elliptic_f(arcsi
n(sqrt(b)*x), -1) + (21*b^5*x^9 + 24*b^4*x^7 - 80*b^3*x^5 - 28*b^2*x^3 + 7
5*b*x)*sqrt(b^2*x^4 - 1))/(b^6*x^10 - b^5*x^8 - 2*b^4*x^6 + 2*b^3*x^4 + b^
2*x^2 - b)
```

Sympy [A] (verification not implemented)

Time = 13.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.35

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{7/2}}\,dx=\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((b*x**2+1)/(b**2*x**4-1)**(7/2),x)
```

output

```
I*b*x**3*gamma(3/4)*hyper((3/4, 7/2), (7/4,), b**2*x**4)/(4*gamma(7/4)) +
I*x*gamma(1/4)*hyper((1/4, 7/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{\left(-1+b^2x^4\right)^{7/2}}\,dx=\int \frac{bx^2+1}{\left(b^2x^4-1\right)^{\frac{7}{2}}}\,dx$$

input

```
integrate((b*x^2+1)/(b^2*x^4-1)^(7/2),x, algorithm="maxima")
```

output

```
integrate((b*x^2 + 1)/(b^2*x^4 - 1)^(7/2), x)
```

Giac [F]

$$\int \frac{1+bx^2}{(-1+b^2x^4)^{7/2}}\,dx=\int \frac{bx^2+1}{(b^2x^4-1)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/(b^2*x^4 - 1)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{(-1+b^2x^4)^{7/2}}\,dx=\int \frac{b\,x^2+1}{(b^2\,x^4-1)^{7/2}}\,dx$$

input
```
int((b*x^2 + 1)/(b^2*x^4 - 1)^(7/2),x)
```

output
```
int((b*x^2 + 1)/(b^2*x^4 - 1)^(7/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{(-1+b^2x^4)^{7/2}}\,dx=\int \frac{\sqrt{b^2x^4-1}}{b^7x^{14}-b^6x^{12}-3b^5x^{10}+3b^4x^8+3b^3x^6-3b^2x^4-b\,x^2+1}dx$$

input
```
int((b*x^2+1)/(b^2*x^4-1)^(7/2),x)
```

output
```
int(sqrt(b**2*x**4 - 1)/(b**7*x**14 - b**6*x**12 - 3*b**5*x**10 + 3*b**4*x
**8 + 3*b**3*x**6 - 3*b**2*x**4 - b*x**2 + 1),x)
```

3.273 $\int (1-bx^2)\left(-1+b^2x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int (1-bx^2)\left(-1+b^2x^4\right)^{3/2}dx = -\frac{2}{105}x\left(15-7bx^2\right)\sqrt{-1+b^2x^4} + \frac{1}{63}x\left(9-7bx^2\right)\left(-1+b^2x^4\right)^{3/2} - \frac{4\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{15\sqrt{b}\sqrt{-1+b^2x^4}} + \frac{88\sqrt{1-b^2x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{105\sqrt{b}\sqrt{-1+b^2x^4}}$$

output

```
-2/105*x*(-7*b*x^2+15)*(b^2*x^4-1)^(1/2)+1/63*x*(-7*b*x^2+9)*(b^2*x^4-1)^(
3/2)-4/15*(-b^2*x^4+1)^(1/2)*EllipticE(b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1
/2)+88/105*(-b^2*x^4+1)^(1/2)*EllipticF(b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(
1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.65 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.51

$$\int (1-bx^2)\,(-1+b^2x^4)^{3/2}\,dx = \frac{\sqrt{-1+b^2x^4}\left(-3x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4}\right.\right.}{3\sqrt{1-b^2x^4}}$$

input
```
Integrate[(1 - b*x^2)*(-1 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[-1 + b^2*x^4]*(-3*x*Hypergeometric2F1[-3/2, 1/4, 5/4, b^2*x^4] + b*x
^3*Hypergeometric2F1[-3/2, 3/4, 7/4, b^2*x^4]))/(3*Sqrt[1 - b^2*x^4])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-bx^2)\,(b^2x^4-1)^{3/2}\,dx$$
$$\Big\downarrow \text{1571}$$
$$\int (1-bx^2)\,(b^2x^4-1)^{3/2}dx$$

input
```
Int[(1 - b*x^2)*(-1 + b^2*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

method	result
meijerg	$\frac{\operatorname{signum}(b^2x^4-1)^{\frac{3}{2}} x \operatorname{hypergeom}([-\frac{3}{2},\frac{1}{4}],[\frac{5}{4}],b^2x^4)}{(-\operatorname{signum}(b^2x^4-1))^{\frac{3}{2}}}-\frac{b \operatorname{signum}(b^2x^4-1)^{\frac{3}{2}} x^3 \operatorname{hypergeom}([-\frac{3}{2},\frac{3}{4}],[\frac{7}{4}],b^2x^4)}{3(-\operatorname{signum}(b^2x^4-1))^{\frac{3}{2}}}$
risch	$-\frac{x(35b^3x^6-45b^2x^4-77b x^2+135)\sqrt{b^2x^4-1}}{315}+\frac{4\sqrt{b x^2+1}\,\sqrt{-b x^2+1}\ \text{EllipticF}(x\sqrt{-b},i)}{7\sqrt{-b}\,\sqrt{b^2x^4-1}}-\frac{4\sqrt{b x^2+1}\,\sqrt{-b x^2+1}\ (\text{EllipticF}(x\sqrt{}}{15\sqrt{-b}\,\sqrt{b^2}}$
elliptic	$-\frac{b^3x^7\sqrt{b^2x^4-1}}{9}+\frac{b^2x^5\sqrt{b^2x^4-1}}{7}+\frac{11b x^3\sqrt{b^2x^4-1}}{45}-\frac{3x\sqrt{b^2x^4-1}}{7}+\frac{4\sqrt{b x^2+1}\,\sqrt{-b x^2+1}\ \text{EllipticF}(x\sqrt{-b},i)}{7\sqrt{-b}\,\sqrt{b^2x^4-1}}-\frac{4\sqrt{b x^2}}{}$
default	$-b\left(\frac{b^2x^7\sqrt{b^2x^4-1}}{9}-\frac{11x^3\sqrt{b^2x^4-1}}{45}+\frac{4\sqrt{b x^2+1}\,\sqrt{-b x^2+1}\ (\text{EllipticF}(x\sqrt{-b},i)-\text{EllipticE}(x\sqrt{-b},i))}{15\sqrt{-b}\,\sqrt{b^2x^4-1}\,b}\right)+\frac{b^2x^5\sqrt{b^2x^4-1}}{7}$

input
```
int((-b*x^2+1)*(b^2*x^4-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
signum(b^2*x^4-1)^(3/2)/(-signum(b^2*x^4-1))^(3/2)*x*hypergeom([-3/2,1/4],
[5/4],b^2*x^4)-1/3*b*signum(b^2*x^4-1)^(3/2)/(-signum(b^2*x^4-1))^(3/2)*x^
3*hypergeom([-3/2,3/4],[7/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.62

$$\int \left(1-bx^2\right)\left(-1+b^2x^4\right)^{3/2}\,dx =$$

$$-\frac{\frac{12\,(15\,b-7)xF(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1)}{\sqrt{b}}+(35\,b^4x^8-45\,b^3x^6-77\,b^2x^4+135\,bx^2+84)\sqrt{b^2x^4-1}+\frac{84\,xE(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1}{\sqrt{b}}}{315\,bx}$$

input
```
integrate((-b*x^2+1)*(b^2*x^4-1)^(3/2),x, algorithm="fricas")
```

output
```
-1/315*(12*(15*b - 7)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + (3
5*b^4*x^8 - 45*b^3*x^6 - 77*b^2*x^4 + 135*b*x^2 + 84)*sqrt(b^2*x^4 - 1) +
84*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int (1-bx^2)\left(-1+b^2x^4\right)^{3/2}\,dx = -\frac{ib^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{ib^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)*(b**2*x**4-1)**(3/2),x)
```

output
```
-I*b**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b**2*x**4)/(4*gamma(11
/4)) + I*b**2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b**2*x**4)/(4*gam
ma(9/4)) + I*b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4)/(4*ga
mma(7/4)) - I*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b**2*x**4)/(4*gamma(
5/4))
```

Maxima [F]

$$\int (1-bx^2)\left(-1+b^2x^4\right)^{3/2}\,dx = \int -\left(b^2x^4-1\right)^{\frac{3}{2}}\left(bx^2-1\right)\,dx$$

input
```
integrate((-b*x^2+1)*(b^2*x^4-1)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((b^2*x^4 - 1)^(3/2)*(b*x^2 - 1), x)
```

Giac [F]

$$\int \left(1-bx^2\right) \left(-1+b^2x^4\right)^{3/2} \, dx = \int -\left(b^2x^4-1\right)^{\frac{3}{2}} \left(bx^2-1\right) dx$$

input
```
integrate((-b*x^2+1)*(b^2*x^4-1)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(b^2*x^4 - 1)^(3/2)*(b*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-bx^2\right) \left(-1+b^2x^4\right)^{3/2} \, dx = -\int \left(b^2\,x^4-1\right)^{3/2}\,\left(b\,x^2-1\right) \, dx$$

input
```
int(-(b^2*x^4 - 1)^(3/2)*(b*x^2 - 1),x)
```

output
```
-int((b^2*x^4 - 1)^(3/2)*(b*x^2 - 1), x)
```

Reduce [F]

$$\begin{aligned}\int \left(1-bx^2\right) \left(-1+b^2x^4\right)^{3/2} \, dx &= -\frac{\sqrt{b^2x^4-1}\,b^3x^7}{9} + \frac{\sqrt{b^2x^4-1}\,b^2x^5}{7} \\ &\quad + \frac{11\sqrt{b^2x^4-1}\,b\,x^3}{45} - \frac{3\sqrt{b^2x^4-1}\,x}{7} + \frac{4\left(\int \frac{\sqrt{b^2x^4-1}}{b^2x^4-1}dx\right)}{7} - \frac{4\left(\int \frac{\sqrt{b^2x^4-1}\,x^2}{b^2x^4-1}dx\right)b}{15}\end{aligned}$$

input
```
int((-b*x^2+1)*(b^2*x^4-1)^(3/2),x)
```

output

```
( - 35*sqrt(b**2*x**4 - 1)*b**3*x**7 + 45*sqrt(b**2*x**4 - 1)*b**2*x**5 +
77*sqrt(b**2*x**4 - 1)*b*x**3 - 135*sqrt(b**2*x**4 - 1)*x + 180*int(sqrt(b
**2*x**4 - 1)/(b**2*x**4 - 1),x) - 84*int((sqrt(b**2*x**4 - 1)*x**2)/(b**2
*x**4 - 1),x)*b)/315
```

3.274 $\int (1-bx^2)\sqrt{-1+b^2x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\begin{aligned}\int (1-bx^2)\sqrt{-1+b^2x^4}\,dx = {} & \frac{1}{15}x(5-3bx^2)\sqrt{-1+b^2x^4} \\ & + \frac{2\sqrt{1-b^2x^4}E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{5\sqrt{b}\sqrt{-1+b^2x^4}} \\ & - \frac{16\sqrt{1-b^2x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{15\sqrt{b}\sqrt{-1+b^2x^4}}\end{aligned}$$

output
```
1/15*x*(-3*b*x^2+5)*(b^2*x^4-1)^(1/2)+2/5*(-b^2*x^4+1)^(1/2)*EllipticE(b^(
1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)-16/15*(-b^2*x^4+1)^(1/2)*EllipticF(b^(
1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

$$\int (1-bx^2)\sqrt{-1+b^2x^4}\,dx = -\frac{\sqrt{-1+b^2x^4}\left(-3x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)\right)}{3\sqrt{1-b^2x^4}}$$

input
```
Integrate[(1 - b*x^2)*Sqrt[-1 + b^2*x^4],x]
```

output
```
-1/3*(Sqrt[-1 + b^2*x^4]*(-3*x*Hypergeometric2F1[-1/2, 1/4, 5/4, b^2*x^4]
+ b*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, b^2*x^4]))/Sqrt[1 - b^2*x^4]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-bx^2)\sqrt{b^2x^4-1}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int (1-bx^2)\sqrt{b^2x^4-1}dx$$

input
```
Int[(1 - b*x^2)*Sqrt[-1 + b^2*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.74

method	result
meijerg	$\frac{\sqrt{\text{signum}(b^2x^4-1)}\,x \text{ hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{5}{4}\right],b^2x^4\right)}{\sqrt{-\text{signum}(b^2x^4-1)}} - \frac{b\sqrt{\text{signum}(b^2x^4-1)}\,x^3 \text{ hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3\sqrt{-\text{signum}(b^2x^4-1)}}$
risch	$-\frac{x(3b x^2-5)\sqrt{b^2x^4-1}}{15} - \frac{2\sqrt{b x^2+1}\sqrt{-b x^2+1}\text{ EllipticF}(x\sqrt{-b},i)}{3\sqrt{-b}\sqrt{b^2x^4-1}} + \frac{2\sqrt{b x^2+1}\sqrt{-b x^2+1}\,(\text{EllipticF}(x\sqrt{-b},i)-\text{EllipticE}(x\sqrt{}}{5\sqrt{-b}\sqrt{b^2x^4-1}}$
elliptic	$-\frac{b x^3\sqrt{b^2x^4-1}}{5} + \frac{x\sqrt{b^2x^4-1}}{3} - \frac{2\sqrt{b x^2+1}\sqrt{-b x^2+1}\text{ EllipticF}(x\sqrt{-b},i)}{3\sqrt{-b}\sqrt{b^2x^4-1}} + \frac{2\sqrt{b x^2+1}\sqrt{-b x^2+1}\,(\text{EllipticF}(x\sqrt{-b},i)-\text{Ellip}}{5\sqrt{-b}\sqrt{b^2x^4-1}}$
default	$-b\left(\frac{x^3\sqrt{b^2x^4-1}}{5} - \frac{2\sqrt{b x^2+1}\sqrt{-b x^2+1}\,(\text{EllipticF}(x\sqrt{-b},i)-\text{EllipticE}(x\sqrt{-b},i))}{5\sqrt{-b}\sqrt{b^2x^4-1}\,b}\right) + \frac{x\sqrt{b^2x^4-1}}{3} - \frac{2\sqrt{b x^2+1}\sqrt{-b x^2+1}}{3\sqrt{-b}\sqrt{b}}$

input
```
int((-b*x^2+1)*(b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
signum(b^2*x^4-1)^(1/2)/(-signum(b^2*x^4-1))^(1/2)*x*hypergeom([-1/2,1/4],
[5/4],b^2*x^4)-1/3*b*signum(b^2*x^4-1)^(1/2)/(-signum(b^2*x^4-1))^(1/2)*x^
3*hypergeom([-1/2,3/4],[7/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int (1-bx^2)\sqrt{-1+b^2x^4}\,dx = \frac{\frac{2\,(5\,b-3)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1\right)}{\sqrt{b}} - (3\,b^2x^4-5\,bx^2-6)\sqrt{b^2x^4-1} + \frac{6\,xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1\right)}{\sqrt{b}}}{15\,bx}$$

input
```
integrate((-b*x^2+1)*(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(2*(5*b - 3)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - (3*b^2
*x^4 - 5*b*x^2 - 6)*sqrt(b^2*x^4 - 1) + 6*x*elliptic_e(arcsin(1/(sqrt(b)*x
)), -1)/sqrt(b))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.53

$$\int \left(1-bx^2\right)\sqrt{-1+b^2x^4}\,dx = -\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)*(b**2*x**4-1)**(1/2),x)
```

output
```
-I*b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4))
+ I*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1-bx^2\right)\sqrt{-1+b^2x^4}\,dx = \int -\sqrt{b^2x^4-1}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(b^2*x^4 - 1)*(b*x^2 - 1), x)
```

Giac [F]

$$\int \left(1-bx^2\right)\sqrt{-1+b^2x^4}\,dx=\int -\sqrt{b^2x^4-1}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(b^2*x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(b^2*x^4 - 1)*(b*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-bx^2\right)\sqrt{-1+b^2x^4}\,dx=-\int \sqrt{b^2\,x^4-1}\,\left(b\,x^2-1\right)\,dx$$

input
```
int(-(b^2*x^4 - 1)^(1/2)*(b*x^2 - 1),x)
```

output
```
-int((b^2*x^4 - 1)^(1/2)*(b*x^2 - 1), x)
```

Reduce [F]

$$\int \left(1-bx^2\right)\sqrt{-1+b^2x^4}\,dx=-\frac{\sqrt{b^2x^4-1}\,b\,x^3}{5}+\frac{\sqrt{b^2x^4-1}\,x}{3}-\frac{2\left(\int\frac{\sqrt{b^2x^4-1}}{b^2x^4-1}dx\right)}{3}+\frac{2\left(\int\frac{\sqrt{b^2x^4-1}\,x^2}{b^2x^4-1}dx\right)b}{5}$$

input
```
int((-b*x^2+1)*(b^2*x^4-1)^(1/2),x)
```

output
```
( - 3*sqrt(b**2*x**4 - 1)*b*x**3 + 5*sqrt(b**2*x**4 - 1)*x - 10*int(sqrt(b
**2*x**4 - 1)/(b**2*x**4 - 1),x) + 6*int((sqrt(b**2*x**4 - 1)*x**2)/(b**2*
x**4 - 1),x)*b)/15
```

3.275 $\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\frac{\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}} + \frac{2\sqrt{1-b^2x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}$$

output
```
-(-b^2*x^4+1)^(1/2)*EllipticE(b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)+2*(-b
^2*x^4+1)^(1/2)*EllipticF(b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\frac{\sqrt{1-b^2x^4}\left(-3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)\right)}{3\sqrt{-1+b^2x^4}}$$

input
```
Integrate[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

output
```
-1/3*(Sqrt[1 - b^2*x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] +
b*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/Sqrt[-1 + b^2*x^4]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1390, 1388, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\sqrt{b^2x^4-1}}dx$$

$$\Big\downarrow \text{1390}$$

$$\frac{\sqrt{1-b^2x^4}\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}dx}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{1388}$$

$$\frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{1-bx^2}}{\sqrt{bx^2+1}}dx}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{326}$$

$$\frac{\sqrt{1-b^2x^4}\left(2\int \frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx-\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{284}$$

$$\frac{\sqrt{1-b^2x^4}\left(2\int \frac{1}{\sqrt{1-b^2x^4}}dx-\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{1-b^2x^4}\left(2\int \frac{1}{\sqrt{1-b^2x^4}}dx-\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{1-b^2x^4}\left(\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}-\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\right)}{\sqrt{b^2x^4-1}}$$

input
```
Int[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

output
```
(Sqrt[1 - b^2*x^4]*(-(EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]) + (2*Ellip
ticF[ArcSin[Sqrt[b]*x], -1])/Sqrt[b]))/Sqrt[-1 + b^2*x^4]
```

Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

method	result	size
meijerg	$-\frac{b\sqrt{-\operatorname{signum}(b^2x^4-1)}\,x^3\operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3\sqrt{\operatorname{signum}(b^2x^4-1)}}+\frac{\sqrt{-\operatorname{signum}(b^2x^4-1)}\,x\operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],b^2x^4\right)}{\sqrt{\operatorname{signum}(b^2x^4-1)}}$	88
default	$-\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-b},i\right)-\operatorname{EllipticE}\left(x\sqrt{-b},i\right)\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-b},i\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$	108
elliptic	$-\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-b},i\right)-\operatorname{EllipticE}\left(x\sqrt{-b},i\right)\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-b},i\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$	108

input
```
int((-b*x^2+1)/(b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b/signum(b^2*x^4-1)^(1/2)*(-signum(b^2*x^4-1))^(1/2)*x^3*hypergeom([1
/2,3/4],[7/4],b^2*x^4)+1/signum(b^2*x^4-1)^(1/2)*(-signum(b^2*x^4-1))^(1/2
)*x*hypergeom([1/4,1/2],[5/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\frac{\frac{(b-1)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1\right)}{\sqrt{b}}+\frac{xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1\right)}{\sqrt{b}}+\sqrt{b^2x^4-1}}{bx}$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-((b - 1)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + x*elliptic_e(a
rcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + sqrt(b^2*x^4 - 1))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = \frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\,b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\,b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

output
```
I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) -
I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{b^2x^4-1}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{b^2x^4-1}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\int \frac{b\,x^2-1}{\sqrt{b^2\,x^4-1}}\,dx$$

input
```
int(-(b*x^2 - 1)/(b^2*x^4 - 1)^(1/2),x)
```

output
```
-int((b*x^2 - 1)/(b^2*x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\left(\int \frac{\sqrt{b^2x^4-1}}{b\,x^2+1}dx\right)$$

input
```
int((-b*x^2+1)/(b^2*x^4-1)^(1/2),x)
```

output
```
- int(sqrt(b**2*x**4 - 1)/(b*x**2 + 1),x)
```

3.276 $\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx=-\frac{x\left(1-bx^2\right)}{2\sqrt{-1+b^2x^4}}-\frac{\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{2\sqrt{b}\sqrt{-1+b^2x^4}}$$

output
```
-1/2*x*(-b*x^2+1)/(b^2*x^4-1)^(1/2)-1/2*(-b^2*x^4+1)^(1/2)*EllipticE(b^(1/
2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx=\frac{x\left(-3-3\sqrt{1-b^2x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)+2bx^2\sqrt{1-b^2x^4}\,\text{Hypergeo}\right.}{6\sqrt{-1+b^2x^4}}$$

input
```
Integrate[(1 - b*x^2)/(-1 + b^2*x^4)^(3/2),x]
```

output
```
(x*(-3 - 3*Sqrt[1 - b^2*x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + 2
*b*x^2*Sqrt[1 - b^2*x^4]*Hypergeometric2F1[3/4, 3/2, 7/4, b^2*x^4]))/(6*Sq
rt[-1 + b^2*x^4])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{(b^2x^4-1)^{3/2}}\,dx$$

$$\Big\downarrow \ 1571$$

$$\int \frac{1-bx^2}{(b^2x^4-1)^{3/2}}dx$$

input
```
Int[(1 - b*x^2)/(-1 + b^2*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

method	result
meijerg	$\frac{\left(-\operatorname{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{3}{2}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right],\left[\frac{5}{4}\right], b^{2} x^{4}\right)}{\operatorname{signum}\left(b^{2} x^{4}-1\right)^{\frac{3}{2}}}-\frac{b\left(-\operatorname{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{3}{2}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{3}{2}\right],\left[\frac{7}{4}\right], b^{2} x^{4}\right)}{3 \operatorname{signum}\left(b^{2} x^{4}-1\right)^{\frac{3}{2}}}$
elliptic	$\frac{\left(b^{2} x^{2}-b\right) x}{2 b \sqrt{\left(x^{2}+\frac{1}{b}\right)\left(b^{2} x^{2}-b\right)}}-\frac{\sqrt{b\,x^{2}+1}\,\sqrt{-b\,x^{2}+1}\ \operatorname{EllipticF}\left(x \sqrt{-b}, i\right)}{2 \sqrt{-b}\,\sqrt{b^{2} x^{4}-1}}-\frac{\sqrt{b\,x^{2}+1}\,\sqrt{-b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x \sqrt{-b}, i\right)-\operatorname{EllipticE}\left(x \sqrt{-b},\right.\right.}{2 \sqrt{-b}\,\sqrt{b^{2} x^{4}-1}}$
default	$-b\left(-\frac{x^{3}}{2 \sqrt{\left(x^{4}-\frac{1}{b^{2}}\right) b^{2}}}+\frac{\sqrt{b\,x^{2}+1}\,\sqrt{-b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x \sqrt{-b}, i\right)-\operatorname{EllipticE}\left(x \sqrt{-b}, i\right)\right)}{2 \sqrt{-b}\,\sqrt{b^{2} x^{4}-1}\, b}\right)-\frac{x}{2 \sqrt{\left(x^{4}-\frac{1}{b^{2}}\right) b^{2}}}-\frac{\sqrt{b\,x^{2}+1}\,\sqrt{}}{}$

input
```
int((-b*x^2+1)/(b^2*x^4-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/signum(b^2*x^4-1)^(3/2)*(-signum(b^2*x^4-1))^(3/2)*x*hypergeom([1/4,3/2]
,[5/4],b^2*x^4)-1/3*b/signum(b^2*x^4-1)^(3/2)*(-signum(b^2*x^4-1))^(3/2)*x
^3*hypergeom([3/4,3/2],[7/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx=\frac{\sqrt{b^2x^4-1}bx+\left(i\,b^2x^2+i\,b\right)\sqrt{b}E\left(\arcsin\left(\sqrt{b}x\right)\,|\,-1\right)+\left(-i\,\left(b^2-b\right)x^2-i\,b+i\right)\sqrt{}}{2\left(b^2x^2+b\right)}$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(b^2*x^4 - 1)*b*x + (I*b^2*x^2 + I*b)*sqrt(b)*elliptic_e(arcsin(s
qrt(b)*x), -1) + (-I*(b^2 - b)*x^2 - I*b + I)*sqrt(b)*elliptic_f(arcsin(sq
rt(b)*x), -1))/(b^2*x^2 + b)
```

Sympy [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx = -\frac{ibx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ix\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(b**2*x**4-1)**(3/2),x)
```

output
```
-I*b*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b**2*x**4)/(4*gamma(7/4)) +
 I*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx = \int -\frac{bx^2-1}{\left(b^2x^4-1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/(b^2*x^4 - 1)^(3/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx = \int -\frac{bx^2-1}{\left(b^2x^4-1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/(b^2*x^4 - 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx=-\int \frac{b\,x^2-1}{\left(b^2\,x^4-1\right)^{3/2}}\,dx$$

input
```
int(-(b*x^2 - 1)/(b^2*x^4 - 1)^(3/2),x)
```

output
```
-int((b*x^2 - 1)/(b^2*x^4 - 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{3/2}}\,dx=-\left(\int \frac{\sqrt{b^2x^4-1}}{b^3x^6+b^2x^4-b\,x^2-1}dx\right)$$

input
```
int((-b*x^2+1)/(b^2*x^4-1)^(3/2),x)
```

output
```
- int(sqrt(b**2*x**4 - 1)/(b**3*x**6 + b**2*x**4 - b*x**2 - 1),x)
```

3.277 $\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{5/2}}\,dx=-\frac{x(1-bx^2)}{6\left(-1+b^2x^4\right)^{3/2}}+\frac{x(5-3bx^2)}{12\sqrt{-1+b^2x^4}}+\frac{\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{4\sqrt{b}\sqrt{-1+b^2x^4}}+\frac{\sqrt{1-b^2x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{6\sqrt{b}\sqrt{-1+b^2x^4}}$$

output

```
-1/6*x*(-b*x^2+1)/(b^2*x^4-1)^(3/2)+1/12*x*(-3*b*x^2+5)/(b^2*x^4-1)^(1/2)+
1/4*(-b^2*x^4+1)^(1/2)*EllipticE(b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)+1/
6*(-b^2*x^4+1)^(1/2)*EllipticF(b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.68

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{5/2}}\,dx=\frac{x\Big(-7+5b^2x^4-5(1-b^2x^4)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)+4bx^2(1-b^2x}{12\left(-1+b^2x^4\right)^{3/2}}$$

input
```
Integrate[(1 - b*x^2)/(-1 + b^2*x^4)^(5/2),x]
```

output
```
(x*(-7 + 5*b^2*x^4 - 5*(1 - b^2*x^4)^(3/2)*Hypergeometric2F1[1/4, 1/2, 5/4
, b^2*x^4] + 4*b*x^2*(1 - b^2*x^4)^(3/2)*Hypergeometric2F1[3/4, 5/2, 7/4,
b^2*x^4]))/(12*(-1 + b^2*x^4)^(3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\left(b^2x^4-1\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{1-bx^2}{\left(b^2x^4-1\right)^{5/2}}dx$$

input
```
Int[(1 - b*x^2)/(-1 + b^2*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

method	result
meijerg	$\frac{\left(-\operatorname{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{5}{2}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{5}{2}\right],\left[\frac{5}{4}\right],b^{2} x^{4}\right)}{\operatorname{signum}\left(b^{2} x^{4}-1\right)^{\frac{5}{2}}}-\frac{b\left(-\operatorname{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{5}{2}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{5}{2}\right],\left[\frac{7}{4}\right],b^{2} x^{4}\right)}{3 \operatorname{signum}\left(b^{2} x^{4}-1\right)^{\frac{5}{2}}}$
elliptic	$-\frac{x\sqrt{b^{2} x^{4}-1}}{12 b^{2}\left(x^{2}+\frac{1}{b}\right)^{2}}-\frac{3\left(b^{2} x^{2}-b\right) x}{8 b\sqrt{\left(x^{2}+\frac{1}{b}\right)\left(b^{2} x^{2}-b\right)}}+\frac{\left(b^{2} x^{2}+b\right) x}{8 b\sqrt{\left(x^{2}-\frac{1}{b}\right)\left(b^{2} x^{2}+b\right)}}+\frac{5\sqrt{b\,x^{2}+1}\,\sqrt{-b\,x^{2}+1}\,\operatorname{EllipticF}\left(x\sqrt{-b},i\right)}{12\sqrt{-b}\,\sqrt{b^{2} x^{4}-1}}+\sqrt{b\,x^{2}+1}$
default	$-b\left(-\frac{x^{3}\sqrt{b^{2} x^{4}-1}}{6 b^{4}\left(x^{4}-\frac{1}{b^{2}}\right)^{2}}+\frac{x^{3}}{4\sqrt{\left(x^{4}-\frac{1}{b^{2}}\right) b^{2}}}-\frac{\sqrt{b\,x^{2}+1}\,\sqrt{-b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-b},i\right)-\operatorname{EllipticE}\left(x\sqrt{-b},i\right)\right)}{4\sqrt{-b}\,\sqrt{b^{2} x^{4}-1}\,b}\right)-\frac{x\sqrt{b^{2} x^{4}-}}{6 b^{4}\left(x^{4}-\frac{1}{b^{2}}\right.}$

input
```
int((-b*x^2+1)/(b^2*x^4-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/signum(b^2*x^4-1)^(5/2)*(-signum(b^2*x^4-1))^(5/2)*x*hypergeom([1/4,5/2]
,[5/4],b^2*x^4)-1/3*b/signum(b^2*x^4-1)^(5/2)*(-signum(b^2*x^4-1))^(5/2)*x
^3*hypergeom([3/4,5/2],[7/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{5/2}}\,dx =$$

$$-\frac{3\,(i\,b^4x^6+i\,b^3x^4-i\,b^2x^2-i\,b)\sqrt{b}E(\arcsin\left(\sqrt{b}x\right)\,|-1)-(i\,(3\,b^4-5\,b^3)x^6+i\,(3\,b^3-5\,b^2)x^4-i\,(3\,b^2-}{12\,(b^4x^6+b^3x^4-b^2x^2-}$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*(I*b^4*x^6 + I*b^3*x^4 - I*b^2*x^2 - I*b)*sqrt(b)*elliptic_e(arcs
in(sqrt(b)*x), -1) - (I*(3*b^4 - 5*b^3)*x^6 + I*(3*b^3 - 5*b^2)*x^4 - I*(3
*b^2 - 5*b)*x^2 - 3*I*b + 5*I)*sqrt(b)*elliptic_f(arcsin(sqrt(b)*x), -1) +
 (3*b^3*x^5 - 2*b^2*x^3 - 7*b*x)*sqrt(b^2*x^4 - 1))/(b^4*x^6 + b^3*x^4 - b
^2*x^2 - b)
```

Sympy [A] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.41

$$\int \frac{1-bx^2}{(-1+b^2x^4)^{5/2}}\,dx = \frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(b**2*x**4-1)**(5/2),x)
```

output
```
I*b*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b**2*x**4)/(4*gamma(7/4)) -
I*x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{(-1+b^2x^4)^{5/2}}\,dx = \int -\frac{bx^2-1}{(b^2x^4-1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/(b^2*x^4 - 1)^(5/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{(-1+b^2x^4)^{5/2}}\,dx = \int -\frac{bx^2-1}{(b^2x^4-1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/(b^2*x^4 - 1)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{(-1+b^2x^4)^{5/2}}\,dx = -\int \frac{b\,x^2-1}{(b^2\,x^4-1)^{5/2}}\,dx$$

input
```
int(-(b*x^2 - 1)/(b^2*x^4 - 1)^(5/2),x)
```

output
```
-int((b*x^2 - 1)/(b^2*x^4 - 1)^(5/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{(-1+b^2x^4)^{5/2}}\,dx = -\left(\int \frac{\sqrt{b^2x^4-1}}{b^5x^{10}+b^4x^8-2b^3x^6-2b^2x^4+b\,x^2+1}dx\right)$$

input
```
int((-b*x^2+1)/(b^2*x^4-1)^(5/2),x)
```

output
```
 - int(sqrt(b**2*x**4 - 1)/(b**5*x**10 + b**4*x**8 - 2*b**3*x**6 - 2*b**2*
x**4 + b*x**2 + 1),x)
```

3.278 $\int \frac{1-bx^2}{(-1+b^2x^4)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 171

$$\int \frac{1-bx^2}{(-1+b^2x^4)^{7/2}}\,dx = -\frac{x(1-bx^2)}{10\left(-1+b^2x^4\right)^{5/2}} + \frac{x(9-7bx^2)}{60\left(-1+b^2x^4\right)^{3/2}} - \frac{x(15-7bx^2)}{40\sqrt{-1+b^2x^4}} - \frac{7\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{40\sqrt{b}\sqrt{-1+b^2x^4}} - \frac{\sqrt{1-b^2x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{5\sqrt{b}\sqrt{-1+b^2x^4}}$$

output
```
-1/10*x*(-b*x^2+1)/(b^2*x^4-1)^(5/2)+1/60*x*(-7*b*x^2+9)/(b^2*x^4-1)^(3/2)
-1/40*x*(-7*b*x^2+15)/(b^2*x^4-1)^(1/2)-7/40*(-b^2*x^4+1)^(1/2)*EllipticE(
b^(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)-1/5*(-b^2*x^4+1)^(1/2)*EllipticF(b^
(1/2)*x,I)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62

$$\int \frac{1-bx^2}{(-1+b^2x^4)^{7/2}}\,dx = \frac{x\Big(-75+108b^2x^4-45b^4x^8-45(1-b^2x^4)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)+}{120\left(-1+b^2x^4\right)^{5/2}}$$

input
```
Integrate[(1 - b*x^2)/(-1 + b^2*x^4)^(7/2),x]
```

output
```
(x*(-75 + 108*b^2*x^4 - 45*b^4*x^8 - 45*(1 - b^2*x^4)^(5/2)*Hypergeometric
2F1[1/4, 1/2, 5/4, b^2*x^4] + 40*b*x^2*(1 - b^2*x^4)^(5/2)*Hypergeometric2
F1[3/4, 7/2, 7/4, b^2*x^4]))/(120*(-1 + b^2*x^4)^(5/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\left(b^2x^4-1\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 1571}$$

$$\int \frac{1-bx^2}{\left(b^2x^4-1\right)^{7/2}}dx$$

input
```
Int[(1 - b*x^2)/(-1 + b^2*x^4)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.51

method	result
meijerg	$\frac{\left(-\operatorname{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{7}{2}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{7}{2}\right],\left[\frac{5}{4}\right],b^{2} x^{4}\right)}{\operatorname{signum}\left(b^{2} x^{4}-1\right)^{\frac{7}{2}}}-\frac{b \left(-\operatorname{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{7}{2}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{7}{2}\right],\left[\frac{7}{4}\right],b^{2} x^{4}\right)}{3 \operatorname{signum}\left(b^{2} x^{4}-1\right)^{\frac{7}{2}}}$
elliptic	$\frac{x \sqrt{b^{2} x^{4}-1}}{48 b^{2} \left(x^{2}-\frac{1}{b}\right)^{2}}-\frac{5 \left(b^{2} x^{2}+b \right) x}{32 b \sqrt{\left(x^{2}-\frac{1}{b}\right) \left(b^{2} x^{2}+b \right)}}+\frac{x \sqrt{b^{2} x^{4}-1}}{40 b^{3} \left(x^{2}+\frac{1}{b}\right)^{3}}+\frac{11 x \sqrt{b^{2} x^{4}-1}}{120 b^{2} \left(x^{2}+\frac{1}{b}\right)^{2}}+\frac{53 \left(b^{2} x^{2}-b \right) x}{160 b \sqrt{\left(x^{2}+\frac{1}{b}\right) \left(b^{2} x^{2}-b \right)}}-\frac{3 \sqrt{b \,x^{2}+1}\, \sqrt{-b \,x}}{8 \sqrt{-}}$
default	$-b \left(-\frac{x^{3} \sqrt{b^{2} x^{4}-1}}{10 b^{6} \left(x^{4}-\frac{1}{b^{2}}\right)^{3}}+\frac{7 x^{3} \sqrt{b^{2} x^{4}-1}}{60 b^{4} \left(x^{4}-\frac{1}{b^{2}}\right)^{2}}-\frac{7 x^{3}}{40 \sqrt{\left(x^{4}-\frac{1}{b^{2}}\right) b^{2}}}+\frac{7 \sqrt{b \,x^{2}+1}\, \sqrt{-b \,x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{-b}, i\right)-\operatorname{EllipticE}\left(x \sqrt{-}\right.\right.}{40 \sqrt{-b}\, \sqrt{b^{2} x^{4}-1}\, b}\right.$

input
```
int((-b*x^2+1)/(b^2*x^4-1)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/signum(b^2*x^4-1)^(7/2)*(-signum(b^2*x^4-1))^(7/2)*x*hypergeom([1/4,7/2]
,[5/4],b^2*x^4)-1/3*b/signum(b^2*x^4-1)^(7/2)*(-signum(b^2*x^4-1))^(7/2)*x
^3*hypergeom([3/4,7/2],[7/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.44

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{7/2}}\,dx =$$

$$-\frac{21\left(-i\,b^6x^{10}-i\,b^5x^8+2i\,b^4x^6+2i\,b^3x^4-i\,b^2x^2-i\,b\right)\sqrt{b}E\left(\arcsin\left(\sqrt{b}x\right)\,|\,-1\right)+3\left(i\left(7\,b^6-15\,b^5\right)x^{10}+\right.}{}$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(7/2),x, algorithm="fricas")
```

output

```
-1/120*(21*(-I*b^6*x^10 - I*b^5*x^8 + 2*I*b^4*x^6 + 2*I*b^3*x^4 - I*b^2*x^
2 - I*b)*sqrt(b)*elliptic_e(arcsin(sqrt(b)*x), -1) + 3*(I*(7*b^6 - 15*b^5)
*x^10 + I*(7*b^5 - 15*b^4)*x^8 - 2*I*(7*b^4 - 15*b^3)*x^6 - 2*I*(7*b^3 - 1
5*b^2)*x^4 + I*(7*b^2 - 15*b)*x^2 + 7*I*b - 15*I)*sqrt(b)*elliptic_f(arcsi
n(sqrt(b)*x), -1) - (21*b^5*x^9 - 24*b^4*x^7 - 80*b^3*x^5 + 28*b^2*x^3 + 7
5*b*x)*sqrt(b^2*x^4 - 1))/(b^6*x^10 + b^5*x^8 - 2*b^4*x^6 - 2*b^3*x^4 + b^
2*x^2 + b)
```

Sympy [A] (verification not implemented)

Time = 11.94 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.35

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{7/2}}\,dx=-\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{7}{4}\end{matrix}\middle|\,b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{5}{4}\end{matrix}\middle|\,b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((-b*x**2+1)/(b**2*x**4-1)**(7/2),x)
```

output

```
-I*b*x**3*gamma(3/4)*hyper((3/4, 7/2), (7/4,), b**2*x**4)/(4*gamma(7/4)) +
 I*x*gamma(1/4)*hyper((1/4, 7/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{7/2}}\,dx=\int -\frac{bx^2-1}{\left(b^2x^4-1\right)^{\frac{7}{2}}}\,dx$$

input

```
integrate((-b*x^2+1)/(b^2*x^4-1)^(7/2),x, algorithm="maxima")
```

output

```
-integrate((b*x^2 - 1)/(b^2*x^4 - 1)^(7/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{7/2}}\,dx=\int -\frac{bx^2-1}{\left(b^2x^4-1\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(7/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/(b^2*x^4 - 1)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{7/2}}\,dx=-\int \frac{b\,x^2-1}{\left(b^2\,x^4-1\right)^{7/2}}\,dx$$

input
```
int(-(b*x^2 - 1)/(b^2*x^4 - 1)^(7/2),x)
```

output
```
-int((b*x^2 - 1)/(b^2*x^4 - 1)^(7/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{7/2}}\,dx=$$
$$-\left(\int \frac{\sqrt{b^2x^4-1}}{b^7x^{14}+b^6x^{12}-3b^5x^{10}-3b^4x^8+3b^3x^6+3b^2x^4-b\,x^2-1}dx\right)$$

input
```
int((-b*x^2+1)/(b^2*x^4-1)^(7/2),x)
```

output
```
 - int(sqrt(b**2*x**4 - 1)/(b**7*x**14 + b**6*x**12 - 3*b**5*x**10 - 3*b**
4*x**8 + 3*b**3*x**6 + 3*b**2*x**4 - b*x**2 - 1),x)
```

3.279 $\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{9/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 197

$$\begin{aligned}\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{9/2}}\,dx &= -\frac{x(1-bx^2)}{14\left(-1+b^2x^4\right)^{7/2}} + \frac{x(13-11bx^2)}{140\left(-1+b^2x^4\right)^{5/2}} \\ &\quad - \frac{x(117-77bx^2)}{840\left(-1+b^2x^4\right)^{3/2}} + \frac{x(195-77bx^2)}{560\sqrt{-1+b^2x^4}} + \frac{11\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{80\sqrt{b}\sqrt{-1+b^2x^4}} \\ &\quad + \frac{59\sqrt{1-b^2x^4}\,\operatorname{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{280\sqrt{b}\sqrt{-1+b^2x^4}}\end{aligned}$$

output

```
-1/14*x*(-b*x^2+1)/(b^2*x^4-1)^(7/2)+1/140*x*(-11*b*x^2+13)/(b^2*x^4-1)^(5
/2)-1/840*x*(-77*b*x^2+117)/(b^2*x^4-1)^(3/2)+1/560*x*(-77*b*x^2+195)/(b^2
*x^4-1)^(1/2)+11/80*(-b^2*x^4+1)^(1/2)*EllipticE(b^(1/2)*x,I)/b^(1/2)/(b^2
*x^4-1)^(1/2)+59/280*(-b^2*x^4+1)^(1/2)*EllipticF(b^(1/2)*x,I)/b^(1/2)/(b^
2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.58

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{9/2}}\,dx = \frac{x\bigl(-1095+2379b^2x^4-1989b^4x^8+585b^6x^{12}-585\left(1-b^2x^4\right)^{7/2}\,\text{Hypergeometric2}}{1680\,(-1+b}$$

input
```
Integrate[(1 - b*x^2)/(-1 + b^2*x^4)^(9/2),x]
```

output
```
(x*(-1095 + 2379*b^2*x^4 - 1989*b^4*x^8 + 585*b^6*x^12 - 585*(1 - b^2*x^4)
^(7/2)*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + 560*b*x^2*(1 - b^2*x^4)
^(7/2)*Hypergeometric2F1[3/4, 9/2, 7/4, b^2*x^4]))/(1680*(-1 + b^2*x^4)^(7
/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\left(b^2x^4-1\right)^{9/2}}dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{1-bx^2}{\left(b^2x^4-1\right)^{9/2}}dx$$

input
```
Int[(1 - b*x^2)/(-1 + b^2*x^4)^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.45

method	result
meijerg	$\frac{\left(-\operatorname{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{9}{2}} x \,\operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{9}{2}\right], \left[\frac{5}{4}\right], b^{2} x^{4}\right)}{\operatorname{signum}\left(b^{2} x^{4}-1\right)^{\frac{9}{2}}}-\frac{b \left(-\operatorname{signum}\left(b^{2} x^{4}-1\right)\right)^{\frac{9}{2}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{9}{2}\right], \left[\frac{7}{4}\right], b^{2} x^{4}\right)}{3 \,\operatorname{signum}\left(b^{2} x^{4}-1\right)^{\frac{9}{2}}}$
elliptic	$\frac{x \sqrt{b^{2} x^{4}-1}}{160 b^{3} \left(x^{2}-\frac{1}{b}\right)^{3}}-\frac{x \sqrt{b^{2} x^{4}-1}}{30 b^{2} \left(x^{2}-\frac{1}{b}\right)^{2}}+\frac{27 \left(b^{2} x^{2}+b \right) x}{160 b \sqrt{\left(x^{2}-\frac{1}{b}\right) \left(b^{2} x^{2}+b \right)}}-\frac{x \sqrt{b^{2} x^{4}-1}}{112 b^{4} \left(x^{2}+\frac{1}{b}\right)^{4}}-\frac{39 x \sqrt{b^{2} x^{4}-1}}{1120 b^{3} \left(x^{2}+\frac{1}{b}\right)^{3}}-\frac{157 x \sqrt{b^{2} x^{4}-1}}{1680 b^{2} \left(x^{2}+\frac{1}{b}\right)^{2}}-$
default	$-b \left(-\frac{x^{3} \sqrt{b^{2} x^{4}-1}}{14 b^{8} \left(x^{4}-\frac{1}{b^{2}}\right)^{4}}+\frac{11 x^{3} \sqrt{b^{2} x^{4}-1}}{140 b^{6} \left(x^{4}-\frac{1}{b^{2}}\right)^{3}}-\frac{11 x^{3} \sqrt{b^{2} x^{4}-1}}{120 b^{4} \left(x^{4}-\frac{1}{b^{2}}\right)^{2}}+\frac{11 x^{3}}{80 \sqrt{\left(x^{4}-\frac{1}{b^{2}}\right) b^{2}}}-\frac{11 \sqrt{b \,x^{2}+1}\, \sqrt{-b \,x^{2}+1}\, (\operatorname{EllipticF}(x}{80 \sqrt{-b}\, \sqrt{b^{2} x}}\right.$

input
```
int((-b*x^2+1)/(b^2*x^4-1)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/signum(b^2*x^4-1)^(9/2)*(-signum(b^2*x^4-1))^(9/2)*x*hypergeom([1/4,9/2]
,[5/4],b^2*x^4)-1/3*b/signum(b^2*x^4-1)^(9/2)*(-signum(b^2*x^4-1))^(9/2)*x
^3*hypergeom([3/4,9/2],[7/4],b^2*x^4)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(158) = 316.

Time = 0.08 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.67

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{9/2}}\,dx =$$

$$-\frac{231\left(i\,b^8x^{14}+i\,b^7x^{12}-3i\,b^6x^{10}-3i\,b^5x^8+3i\,b^4x^6+3i\,b^3x^4-i\,b^2x^2-i\,b\right)\sqrt{b}E\left(\arcsin\left(\sqrt{b}x\right)\,|-1\right)+3}{}$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(9/2),x, algorithm="fricas")
```

output
```
-1/1680*(231*(I*b^8*x^14 + I*b^7*x^12 - 3*I*b^6*x^10 - 3*I*b^5*x^8 + 3*I*b
^4*x^6 + 3*I*b^3*x^4 - I*b^2*x^2 - I*b)*sqrt(b)*elliptic_e(arcsin(sqrt(b)*
x), -1) + 3*(-I*(77*b^8 - 195*b^7)*x^14 - I*(77*b^7 - 195*b^6)*x^12 + 3*I*
(77*b^6 - 195*b^5)*x^10 + 3*I*(77*b^5 - 195*b^4)*x^8 - 3*I*(77*b^4 - 195*b
^3)*x^6 - 3*I*(77*b^3 - 195*b^2)*x^4 + I*(77*b^2 - 195*b)*x^2 + 77*I*b - 1
95*I)*sqrt(b)*elliptic_f(arcsin(sqrt(b)*x), -1) + (231*b^7*x^13 - 354*b^6*
x^11 - 1201*b^5*x^9 + 788*b^4*x^7 + 1921*b^3*x^5 - 458*b^2*x^3 - 1095*b*x)
*sqrt(b^2*x^4 - 1))/(b^8*x^14 + b^7*x^12 - 3*b^6*x^10 - 3*b^5*x^8 + 3*b^4*
x^6 + 3*b^3*x^4 - b^2*x^2 - b)
```

Sympy [A] (verification not implemented)

Time = 35.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.30

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{9/2}}\,dx = \frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{9}{2}\\\frac{7}{4}\end{matrix}\right| b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{9}{2}\\\frac{5}{4}\end{matrix}\right| b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(b**2*x**4-1)**(9/2),x)
```

output
```
I*b*x**3*gamma(3/4)*hyper((3/4, 9/2), (7/4,), b**2*x**4)/(4*gamma(7/4)) -
I*x*gamma(1/4)*hyper((1/4, 9/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{9/2}}\,dx = \int -\frac{bx^2-1}{\left(b^2x^4-1\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(9/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/(b^2*x^4 - 1)^(9/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{9/2}}\,dx=\int -\frac{bx^2-1}{\left(b^2x^4-1\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(9/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/(b^2*x^4 - 1)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{9/2}}\,dx=-\int \frac{b\,x^2-1}{{\left(b^2\,x^4-1\right)}^{9/2}}\,dx$$

input
```
int(-(b*x^2 - 1)/(b^2*x^4 - 1)^(9/2),x)
```

output
```
-int((b*x^2 - 1)/(b^2*x^4 - 1)^(9/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{\left(-1+b^2x^4\right)^{9/2}}\,dx=$$
$$-\left(\int \frac{\sqrt{b^2x^4-1}}{b^9x^{18}+b^8x^{16}-4b^7x^{14}-4b^6x^{12}+6b^5x^{10}+6b^4x^8-4b^3x^6-4b^2x^4+b\,x^2+1}dx\right)$$

input
```
int((-b*x^2+1)/(b^2*x^4-1)^(9/2),x)
```

output

```
 - int(sqrt(b**2*x**4 - 1)/(b**9*x**18 + b**8*x**16 - 4*b**7*x**14 - 4*b**
6*x**12 + 6*b**5*x**10 + 6*b**4*x**8 - 4*b**3*x**6 - 4*b**2*x**4 + b*x**2
+ 1),x)
```

3.280 $\int (1-bx^2)(-1-b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 244

$$\begin{aligned}\int (1-bx^2)\left(-1-b^2x^4\right)^{5/2}dx &= \frac{4x(195-77bx^2)\sqrt{-1-b^2x^4}}{3003}\\
&\quad-\frac{8x\sqrt{-1-b^2x^4}}{39\,(1+bx^2)}-\frac{10x(117-77bx^2)\left(-1-b^2x^4\right)^{3/2}}{9009}\\
&\quad+\frac{1}{143}x(13-11bx^2)\left(-1-b^2x^4\right)^{5/2}-\frac{8(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{39\sqrt{b}\sqrt{-1-b^2x^4}}\\
&\quad-\frac{472(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{3003\sqrt{b}\sqrt{-1-b^2x^4}}\end{aligned}$$

output

```
4/3003*x*(-77*b*x^2+195)*(-b^2*x^4-1)^(1/2)-8*x*(-b^2*x^4-1)^(1/2)/(39*b*x
^2+39)-10/9009*x*(-77*b*x^2+117)*(-b^2*x^4-1)^(3/2)+1/143*x*(-11*b*x^2+13)
*(-b^2*x^4-1)^(5/2)-8/39*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*Ellipti
cE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)-472/30
03*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1
/2)*x),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.89 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.31

$$\int \left(1-bx^2\right)\left(-1-b^2x^4\right)^{5/2}\,dx =$$
$$-\frac{\sqrt{-1-b^2x^4}\left(-3x\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},\frac{5}{4},-b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)\right)}{3\sqrt{1+b^2x^4}}$$

input
```
Integrate[(1 - b*x^2)*(-1 - b^2*x^4)^(5/2),x]
```

output
```
-1/3*(Sqrt[-1 - b^2*x^4]*(-3*x*Hypergeometric2F1[-5/2, 1/4, 5/4, -(b^2*x^4
)] + b*x^3*Hypergeometric2F1[-5/2, 3/4, 7/4, -(b^2*x^4)]))/Sqrt[1 + b^2*x^
4]
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1491, 27, 1491, 27, 1491, 27, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-bx^2\right)\left(-b^2x^4-1\right)^{5/2}dx$$
$$\Big\downarrow \text{1491}$$
$$\frac{5}{143}\int -2\left(13-11bx^2\right)\left(-b^2x^4-1\right)^{3/2}dx+\frac{1}{143}x\left(13-11bx^2\right)\left(-b^2x^4-1\right)^{5/2}$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{143}x\left(13-11bx^2\right)\left(-b^2x^4-1\right)^{5/2}-\frac{10}{143}\int \left(13-11bx^2\right)\left(-b^2x^4-1\right)^{3/2}dx$$
$$\Big\downarrow \text{1491}$$

$$\frac{1}{143}x\left(13-11bx^2\right)\left(-b^2x^4-1\right)^{5/2}-\frac{10}{143}\left(\frac{1}{21}\int -2\left(117-77bx^2\right)\sqrt{-b^2x^4-1}dx+\frac{1}{63}x\left(117-77bx^2\right)\left(-b^2x^4-1\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{143}x\left(13-11bx^2\right)\left(-b^2x^4-1\right)^{5/2}-\frac{10}{143}\left(\frac{1}{63}x\left(117-77bx^2\right)\left(-b^2x^4-1\right)^{3/2}-\frac{2}{21}\int \left(117-77bx^2\right)\sqrt{-b^2x^4-1}dx\right)$$

$$\Big\downarrow \text{ 1491}$$

$$\frac{1}{143}x\left(13-11bx^2\right)\left(-b^2x^4-1\right)^{5/2}-\frac{10}{143}\left(\frac{1}{63}x\left(117-77bx^2\right)\left(-b^2x^4-1\right)^{3/2}-\frac{2}{21}\left(\frac{1}{15}\int -\frac{6\left(195-77bx^2\right)}{\sqrt{-b^2x^4-1}}dx+\frac{1}{5}x\sqrt{-b^2x^4-1}\left(195-77bx^2\right)\right)\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{143}x\left(13-11bx^2\right)\left(-b^2x^4-1\right)^{5/2}-\frac{10}{143}\left(\frac{1}{63}x\left(117-77bx^2\right)\left(-b^2x^4-1\right)^{3/2}-\frac{2}{21}\left(\frac{1}{5}x\left(195-77bx^2\right)\sqrt{-b^2x^4-1}-\frac{2}{5}\int \frac{195-77bx^2}{\sqrt{-b^2x^4-1}}dx\right)\right)$$

$$\Big\downarrow \text{ 1512}$$

$$\frac{1}{143}x\left(13-11bx^2\right)\left(-b^2x^4-1\right)^{5/2}-\frac{10}{143}\left(\frac{1}{63}x\left(117-77bx^2\right)\left(-b^2x^4-1\right)^{3/2}-\frac{2}{21}\left(\frac{1}{5}x\left(195-77bx^2\right)\sqrt{-b^2x^4-1}-\frac{2}{5}\left(118\int \frac{1}{\sqrt{-b^2x^4-1}}dx+77\right.\right.\right.$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{143}x\left(13-11bx^2\right)\left(-b^2x^4-1\right)^{5/2}-\frac{10}{143}\left(\frac{1}{63}x\left(117-77bx^2\right)\left(-b^2x^4-1\right)^{3/2}-\frac{2}{21}\left(\frac{1}{5}x\left(195-77bx^2\right)\sqrt{-b^2x^4-1}-\frac{2}{5}\left(77\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx+\frac{59(}{}\right.\right.\right.$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{1}{143}x\left(13-11bx^2\right)\left(-b^2x^4-1\right)^{5/2}-\frac{10}{143}\left(\frac{1}{63}x\left(117-77bx^2\right)\left(-b^2x^4-1\right)^{3/2}-\frac{2}{21}\left(\frac{1}{5}x\left(195-77bx^2\right)\sqrt{-b^2x^4-1}-\frac{2}{5}\left(\frac{59\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\text{Elli}}{\sqrt{b}\sqrt{-b}}\right.\right.\right.$$

input
```
Int[(1 - b*x^2)*(-1 - b^2*x^4)^(5/2),x]
```

output
```
(x*(13 - 11*b*x^2)*(-1 - b^2*x^4)^(5/2))/143 - (10*((x*(117 - 77*b*x^2)*(-
1 - b^2*x^4)^(3/2))/63 - (2*((x*(195 - 77*b*x^2)*Sqrt[-1 - b^2*x^4])/5 - (
2*(77*((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4
)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b
^2*x^4])) + (59*(1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*
ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])))/5))/21))/143
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 2.43 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.37

method	result
meijerg	$\frac{\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{5}{2}} x \operatorname{hypergeom}\left(\left[-\frac{5}{2}, \frac{1}{4}\right],\left[\frac{5}{4}\right],-b^{2} x^{4}\right)}{\operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{5}{2}}}-\frac{b\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{5}{2}} x^{3} \operatorname{hypergeom}\left(\left[-\frac{5}{2}, \frac{3}{4}\right],\left[\frac{7}{4}\right],-b^{2} x^{4}\right)}{3 \operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{5}{2}}}$
risch	$\frac{x\left(693 x^{10} b^{5}-819 b^{4} x^{8}+2156 b^{3} x^{6}-2808 b^{2} x^{4}+2387 b x^{2}-4329\right)\left(b^{2} x^{4}+1\right)}{9009 \sqrt{-b^{2} x^{4}-1}}-\frac{40 \sqrt{i b x^{2}+1} \sqrt{-i b x^{2}+1} \operatorname{EllipticF}\left(x \sqrt{-i b}, i\right)}{77 \sqrt{-i b} \sqrt{-b^{2} x^{4}-1}}-$
elliptic	$-\frac{b^{5} x^{11} \sqrt{-b^{2} x^{4}-1}}{13}+\frac{b^{4} x^{9} \sqrt{-b^{2} x^{4}-1}}{11}-\frac{28 b^{3} x^{7} \sqrt{-b^{2} x^{4}-1}}{117}+\frac{24 b^{2} x^{5} \sqrt{-b^{2} x^{4}-1}}{77}-\frac{31 b x^{3} \sqrt{-b^{2} x^{4}-1}}{117}+\frac{37 x \sqrt{-b^{2} x^{4}-1}}{77}$
default	$-b\left(\frac{b^{4} x^{11} \sqrt{-b^{2} x^{4}-1}}{13}+\frac{28 b^{2} x^{7} \sqrt{-b^{2} x^{4}-1}}{117}+\frac{31 x^{3} \sqrt{-b^{2} x^{4}-1}}{117}+\frac{8 i \sqrt{i b x^{2}+1} \sqrt{-i b x^{2}+1}\left(\operatorname{EllipticF}\left(x \sqrt{-i b}, i\right)-\operatorname{EllipticE}\right.}{39 \sqrt{-i b} \sqrt{-b^{2} x^{4}-1} b}\right.$

input
```
int((-b*x^2+1)*(-b^2*x^4-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-signum(b^2*x^4+1))^(5/2)/signum(b^2*x^4+1)^(5/2)*x*hypergeom([-5/2,1/4],
[5/4],-b^2*x^4)-1/3*b*(-signum(b^2*x^4+1))^(5/2)/signum(b^2*x^4+1)^(5/2)*x
^3*hypergeom([-5/2,3/4],[7/4],-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.57

$$\int \left(1-bx^2\right) \left(-1-b^2x^4\right)^{5/2}\,dx = \frac{24\sqrt{-b^2}(195\,b+77)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-1848\sqrt{-b^2}x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}E(\arcsin}{}$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4-1)^(5/2),x, algorithm="fricas")
```

output
```
1/9009*(24*sqrt(-b^2)*(195*b + 77)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/
b^2)^(1/4)/x), -1) - 1848*sqrt(-b^2)*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-
1/b^2)^(1/4)/x), -1) - (693*b^6*x^12 - 819*b^5*x^10 + 2156*b^4*x^8 - 2808*
b^3*x^6 + 2387*b^2*x^4 - 4329*b*x^2 + 1848)*sqrt(-b^2*x^4 - 1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.56 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.97

$$\int \left(1-bx^2\right)\left(-1-b^2x^4\right)^{5/2}\,dx=-\frac{ib^5x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{15}{4}\right)}+\frac{ib^4x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{13}{4}\right)}-\frac{ib^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{2\Gamma\left(\frac{11}{4}\right)}+\frac{ib^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{2\Gamma\left(\frac{9}{4}\right)}-\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)*(-b**2*x**4-1)**(5/2),x)
```

output
```
-I*b**5*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b**2*x**4*exp_polar
(I*pi))/(4*gamma(15/4)) + I*b**4*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,
), b**2*x**4*exp_polar(I*pi))/(4*gamma(13/4)) - I*b**3*x**7*gamma(7/4)*hyp
er((-1/2, 7/4), (11/4,), b**2*x**4*exp_polar(I*pi))/(2*gamma(11/4)) + I*b*
*2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b**2*x**4*exp_polar(I*pi))/(
2*gamma(9/4)) - I*b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4*e
xp_polar(I*pi))/(4*gamma(7/4)) + I*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,),
 b**2*x**4*exp_polar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1-bx^2\right)\left(-1-b^2x^4\right)^{5/2}\,dx=\int -\left(-b^2x^4-1\right)^{\frac{5}{2}}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4-1)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((-b^2*x^4 - 1)^(5/2)*(b*x^2 - 1), x)
```

Giac [F]

$$\int \left(1-bx^2\right)\left(-1-b^2x^4\right)^{5/2}\,dx=\int -\left(-b^2x^4-1\right)^{\frac{5}{2}}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4-1)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(-b^2*x^4 - 1)^(5/2)*(b*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-bx^2\right)\left(-1-b^2x^4\right)^{5/2}\,dx=-\int \left(-b^2\,x^4-1\right)^{5/2}\,\left(b\,x^2-1\right)\,dx$$

input
```
int(-(- b^2*x^4 - 1)^(5/2)*(b*x^2 - 1),x)
```

output
```
-int((- b^2*x^4 - 1)^(5/2)*(b*x^2 - 1), x)
```

Reduce [F]

$$\int \left(1-bx^2\right)\left(-1-b^2x^4\right)^{5/2}\,dx=\frac{i\Bigl(-693\sqrt{b^2x^4+1}\,b^5x^{11}+819\sqrt{b^2x^4+1}\,b^4x^9-2156\sqrt{b^2x^4+1}\,b^3x^7+2808\sqrt{b^2x^4+1}\,b^2x}{9}$$

input
```
int((-b*x^2+1)*(-b^2*x^4-1)^(5/2),x)
```

output
```
(i*( - 693*sqrt(b**2*x**4 + 1)*b**5*x**11 + 819*sqrt(b**2*x**4 + 1)*b**4*x
**9 - 2156*sqrt(b**2*x**4 + 1)*b**3*x**7 + 2808*sqrt(b**2*x**4 + 1)*b**2*x
**5 - 2387*sqrt(b**2*x**4 + 1)*b*x**3 + 4329*sqrt(b**2*x**4 + 1)*x + 4680*
int(sqrt(b**2*x**4 + 1)/(b**2*x**4 + 1),x) - 1848*int((sqrt(b**2*x**4 + 1)
*x**2)/(b**2*x**4 + 1),x)*b))/9009
```

3.281 $\int (1-bx^2)\left(-1-b^2x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 217

$$\begin{aligned}\int (1-bx^2)\left(-1-b^2x^4\right)^{3/2}\,dx &= -\frac{2}{105}x\left(15-7bx^2\right)\sqrt{-1-b^2x^4}+\frac{4x\sqrt{-1-b^2x^4}}{15\left(1+bx^2\right)}\\ &+\frac{1}{63}x\left(9-7bx^2\right)\left(-1-b^2x^4\right)^{3/2}+\frac{4(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{15\sqrt{b}\sqrt{-1-b^2x^4}}\\ &+\frac{16(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{105\sqrt{b}\sqrt{-1-b^2x^4}}\end{aligned}$$

output

```
-2/105*x*(-7*b*x^2+15)*(-b^2*x^4-1)^(1/2)+4*x*(-b^2*x^4-1)^(1/2)/(15*b*x^2
+15)+1/63*x*(-7*b*x^2+9)*(-b^2*x^4-1)^(3/2)+4/15*(b*x^2+1)*((b^2*x^4+1)/(b
*x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(
-b^2*x^4-1)^(1/2)+16/105*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*Inverse
JacobiAM(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.57 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.35

$$\int (1-bx^2)\left(-1-b^2x^4\right)^{3/2}\,dx=\frac{\sqrt{-1-b^2x^4}\left(-3x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4}\right.\right.}{3\sqrt{1+b^2x^4}}$$

input
```
Integrate[(1 - b*x^2)*(-1 - b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[-1 - b^2*x^4]*(-3*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -(b^2*x^4)] +
b*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -(b^2*x^4)]))/(3*Sqrt[1 + b^2*x^4]
)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1491, 27, 1491, 27, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-bx^2)\left(-b^2x^4-1\right)^{3/2}dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{21}\int -2(9-7bx^2)\sqrt{-b^2x^4-1}dx+\frac{1}{63}x(9-7bx^2)\left(-b^2x^4-1\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{63}x(9-7bx^2)\left(-b^2x^4-1\right)^{3/2}-\frac{2}{21}\int (9-7bx^2)\sqrt{-b^2x^4-1}dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{63}x\left(9-7bx^2\right)\left(-b^2x^4-1\right)^{3/2}-\frac{2}{21}\left(\frac{1}{15}\int -\frac{6\left(15-7bx^2\right)}{\sqrt{-b^2x^4-1}}dx+\frac{1}{5}x\sqrt{-b^2x^4-1}\left(15-7bx^2\right)\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{63}x\left(9-7bx^2\right)\left(-b^2x^4-1\right)^{3/2}-\frac{2}{21}\left(\frac{1}{5}x\left(15-7bx^2\right)\sqrt{-b^2x^4-1}-\frac{2}{5}\int\frac{15-7bx^2}{\sqrt{-b^2x^4-1}}dx\right)$$

$$\Big\downarrow \ 1512$$

$$\frac{1}{63}x\left(9-7bx^2\right)\left(-b^2x^4-1\right)^{3/2}- \frac{2}{21}\left(\frac{1}{5}x\left(15-7bx^2\right)\sqrt{-b^2x^4-1}-\frac{2}{5}\left(8\int\frac{1}{\sqrt{-b^2x^4-1}}dx+7\int\frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx\right)\right)$$

$$\Big\downarrow \ 761$$

$$\frac{1}{63}x\left(9-7bx^2\right)\left(-b^2x^4-1\right)^{3/2}- \frac{2}{21}\left(\frac{1}{5}x\left(15-7bx^2\right)\sqrt{-b^2x^4-1}-\frac{2}{5}\left(7\int\frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx+\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\right.}{\sqrt{b}\sqrt{-b^2x^4-1}}\right.\right.$$

$$\Big\downarrow \ 1510$$

$$\frac{1}{63}x\left(9-7bx^2\right)\left(-b^2x^4-1\right)^{3/2}- \frac{2}{21}\left(\frac{1}{5}x\left(15-7bx^2\right)\sqrt{-b^2x^4-1}-\frac{2}{5}\left(\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+7\left(\frac{\left(bx^2+1\right)\sqrt{}}{}\right.\right.\right.$$

input

```
Int[(1 - b*x^2)*(-1 - b^2*x^4)^(3/2),x]
```

output

```
(x*(9 - 7*b*x^2)*(-1 - b^2*x^4)^(3/2))/63 - (2*((x*(15 - 7*b*x^2)*Sqrt[-1
 - b^2*x^4])/5 - (2*(7*((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*S
qrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqr
t[b]*Sqrt[-1 - b^2*x^4])) + (4*(1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^
2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])))/5))
/21
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 2.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.41

method	result
meijerg	$\frac{\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{3}{2}} x \operatorname{hypergeom}\left(\left[-\frac{3}{2}, \frac{1}{4}\right],\left[\frac{5}{4}\right],-b^{2} x^{4}\right)}{\operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{3}{2}}}-\frac{b\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{3}{2}} x^{3} \operatorname{hypergeom}\left(\left[-\frac{3}{2}, \frac{3}{4}\right],\left[\frac{7}{4}\right],-b^{2} x^{4}\right)}{3 \operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{3}{2}}}$
risch	$-\frac{x\left(35 b^{3} x^{6}-45 b^{2} x^{4}+77 b x^{2}-135\right)\left(b^{2} x^{4}+1\right)}{315 \sqrt{-b^{2} x^{4}-1}}+\frac{4 \sqrt{i b x^{2}+1} \sqrt{-i b x^{2}+1} \operatorname{EllipticF}(x \sqrt{-i b}, i)}{7 \sqrt{-i b} \sqrt{-b^{2} x^{4}-1}}+$
elliptic	$\frac{b^{3} x^{7} \sqrt{-b^{2} x^{4}-1}}{9}-\frac{b^{2} x^{5} \sqrt{-b^{2} x^{4}-1}}{7}+\frac{11 b x^{3} \sqrt{-b^{2} x^{4}-1}}{45}-\frac{3 x \sqrt{-b^{2} x^{4}-1}}{7}+\frac{4 \sqrt{i b x^{2}+1} \sqrt{-i b x^{2}+1} \operatorname{EllipticF}(x \sqrt{-i b}, i)}{7 \sqrt{-i b} \sqrt{-b^{2} x^{4}-1}}+$
default	$-b\left(-\frac{b^{2} x^{7} \sqrt{-b^{2} x^{4}-1}}{9}-\frac{11 x^{3} \sqrt{-b^{2} x^{4}-1}}{45}-\frac{4 i \sqrt{i b x^{2}+1} \sqrt{-i b x^{2}+1}(\operatorname{EllipticF}(x \sqrt{-i b}, i)-\operatorname{EllipticE}(x \sqrt{-i b}, i))}{15 \sqrt{-i b} \sqrt{-b^{2} x^{4}-1}\, b}\right)-$

input
```
int((-b*x^2+1)*(-b^2*x^4-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-signum(b^2*x^4+1))^(3/2)/signum(b^2*x^4+1)^(3/2)*x*hypergeom([-3/2,1/4],
[5/4],-b^2*x^4)-1/3*b*(-signum(b^2*x^4+1))^(3/2)/signum(b^2*x^4+1)^(3/2)*x
^3*hypergeom([-3/2,3/4],[7/4],-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.56

$$\int \left(1-bx^2\right)\left(-1-b^2x^4\right)^{3/2}\,dx =$$

$$-\frac{12\sqrt{-b^2}(15\,b+7)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} F\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right) - 84\sqrt{-b^2}x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} E\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right) -}{315\,bx}$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4-1)^(3/2),x, algorithm="fricas")
```

output
```
-1/315*(12*sqrt(-b^2)*(15*b + 7)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^
2)^(1/4)/x), -1) - 84*sqrt(-b^2)*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^
2)^(1/4)/x), -1) - (35*b^4*x^8 - 45*b^3*x^6 + 77*b^2*x^4 - 135*b*x^2 + 84)
*sqrt(-b^2*x^4 - 1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.71

$$\int \left(1-bx^2\right)\left(-1-b^2x^4\right)^{3/2}\,dx=\frac{ib^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}-\frac{ib^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)*(-b**2*x**4-1)**(3/2),x)
```

output
```
I*b**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b**2*x**4*exp_polar(I*p
i))/(4*gamma(11/4)) - I*b**2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*
*2*x**4*exp_polar(I*pi))/(4*gamma(9/4)) + I*b*x**3*gamma(3/4)*hyper((-1/2,
 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*gamma(7/4)) - I*x*gamma(1/4)*
hyper((-1/2, 1/4), (5/4,), b**2*x**4*exp_polar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1-bx^2\right)\left(-1-b^2x^4\right)^{3/2}\,dx=\int -\left(-b^2x^4-1\right)^{\frac{3}{2}}\left(bx^2-1\right)\,dx$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4-1)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((-b^2*x^4 - 1)^(3/2)*(b*x^2 - 1), x)
```

Giac [F]

$$\int \left(1-bx^2\right)\left(-1-b^2x^4\right)^{3/2}\,dx=\int -\left(-b^2x^4-1\right)^{\frac{3}{2}}\left(bx^2-1\right)\,dx$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4-1)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(-b^2*x^4 - 1)^(3/2)*(b*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-bx^2\right)\left(-1-b^2x^4\right)^{3/2}\,dx=-\int \left(-b^2\,x^4-1\right)^{3/2}\left(b\,x^2-1\right)\,dx$$

input
```
int(-(- b^2*x^4 - 1)^(3/2)*(b*x^2 - 1),x)
```

output
```
-int((- b^2*x^4 - 1)^(3/2)*(b*x^2 - 1), x)
```

Reduce [F]

$$\int \left(1-bx^2\right)\left(-1-b^2x^4\right)^{3/2}\,dx=\frac{i\left(35\sqrt{b^2x^4+1}\,b^3x^7-45\sqrt{b^2x^4+1}\,b^2x^5+77\sqrt{b^2x^4+1}\,b\,x^3-135\sqrt{b^2x^4+1}\,x-180\left(\int\right.\right.}{315}$$

input
```
int((-b*x^2+1)*(-b^2*x^4-1)^(3/2),x)
```

output
```
(i*(35*sqrt(b**2*x**4 + 1)*b**3*x**7 - 45*sqrt(b**2*x**4 + 1)*b**2*x**5 +
77*sqrt(b**2*x**4 + 1)*b*x**3 - 135*sqrt(b**2*x**4 + 1)*x - 180*int(sqrt(b
**2*x**4 + 1)/(b**2*x**4 + 1),x) + 84*int((sqrt(b**2*x**4 + 1)*x**2)/(b**2
*x**4 + 1),x)*b))/315
```

3.282 $\int (1-bx^2)\sqrt{-1-b^2x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 190

$$\begin{aligned}\int (1-bx^2)\sqrt{-1-b^2x^4}\,dx = {} & \frac{1}{15}x\left(5-3bx^2\right)\sqrt{-1-b^2x^4} - \frac{2x\sqrt{-1-b^2x^4}}{5\left(1+bx^2\right)} \\ & - \frac{2(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{5\sqrt{b}\sqrt{-1-b^2x^4}} \\ & - \frac{2(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{15\sqrt{b}\sqrt{-1-b^2x^4}}\end{aligned}$$

output
```
1/15*x*(-3*b*x^2+5)*(-b^2*x^4-1)^(1/2)-2*x*(-b^2*x^4-1)^(1/2)/(5*b*x^2+5)-
2/5*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/
2)*x)),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)-2/15*(b*x^2+1)*((b^2*x^4+1)
/(b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b^(1/
2)/(-b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.40

$$\int \left(1-bx^2\right)\sqrt{-1-b^2x^4}\,dx = \\ -\frac{\sqrt{-1-b^2x^4}\big(-3x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)}{3\sqrt{1+b^2x^4}}$$

input
```
Integrate[(1 - b*x^2)*Sqrt[-1 - b^2*x^4],x]
```

output
```
-1/3*(Sqrt[-1 - b^2*x^4]*(-3*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -(b^2*x^4
)] + b*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -(b^2*x^4)]))/Sqrt[1 + b^2*x^
4]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1491, 27, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-bx^2\right)\sqrt{-b^2x^4-1}\,dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{15}\int -\frac{2\left(5-3bx^2\right)}{\sqrt{-b^2x^4-1}}dx+\frac{1}{15}x\sqrt{-b^2x^4-1}\left(5-3bx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}x\left(5-3bx^2\right)\sqrt{-b^2x^4-1}-\frac{2}{15}\int \frac{5-3bx^2}{\sqrt{-b^2x^4-1}}dx$$

$$\Big\downarrow \text{1512}$$

$$\frac{1}{15}x\left(5-3bx^2\right)\sqrt{-b^2x^4-1}-\frac{2}{15}\left(2\int \frac{1}{\sqrt{-b^2x^4-1}}dx+3\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{15}x\left(5-3bx^2\right)\sqrt{-b^2x^4-1}-\frac{2}{15}\left(3\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx+\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}\right)$$

$$\Big\downarrow \text{1510}$$

$$\frac{1}{15}x\left(5-3bx^2\right)\sqrt{-b^2x^4-1}-\frac{2}{15}\Bigg(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+3\Bigg(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+x$$

input
```
Int[(1 - b*x^2)*Sqrt[-1 - b^2*x^4],x]
```

output
```
(x*(5 - 3*b*x^2)*Sqrt[-1 - b^2*x^4])/15 - (2*(3*((x*Sqrt[-1 - b^2*x^4])/(1
 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*Arc
Tan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])) + ((1 + b*x^2)*Sqrt[(1
 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*S
qrt[-1 - b^2*x^4])))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 2.43 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

method	result
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(b^{2} x^{4}+1\right)}\, x \operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{5}{4}\right],-b^{2} x^{4}\right)}{\sqrt{\operatorname{signum}\left(b^{2} x^{4}+1\right)}}-\frac{b \sqrt{-\operatorname{signum}\left(b^{2} x^{4}+1\right)}\, x^{3} \operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^{2} x^{4}\right)}{3 \sqrt{\operatorname{signum}\left(b^{2} x^{4}+1\right)}}$
risch	$\frac{x \left(3 b \,x^{2}-5\right) \left(b^{2} x^{4}+1\right)}{15 \sqrt{-b^{2} x^{4}-1}}-\frac{2 \sqrt{i b \,x^{2}+1}\, \sqrt{-i b \,x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{-i b}, i\right)}{3 \sqrt{-i b}\, \sqrt{-b^{2} x^{4}-1}}-\frac{2 i \sqrt{i b \,x^{2}+1}\, \sqrt{-i b \,x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{-i b}, i\right)-\operatorname{EllipticE}(\right)}{5 \sqrt{-i b}\, \sqrt{-b^{2} x^{4}-1}}$
elliptic	$-\frac{b \,x^{3} \sqrt{-b^{2} x^{4}-1}}{5}+\frac{x \sqrt{-b^{2} x^{4}-1}}{3}-\frac{2 \sqrt{i b \,x^{2}+1}\, \sqrt{-i b \,x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{-i b}, i\right)}{3 \sqrt{-i b}\, \sqrt{-b^{2} x^{4}-1}}-\frac{2 i \sqrt{i b \,x^{2}+1}\, \sqrt{-i b \,x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{-i b}\right)\right)}{5 \sqrt{-i b}\, \sqrt{-b^{2} x^{4}-}}$
default	$-b \left(\frac{x^{3} \sqrt{-b^{2} x^{4}-1}}{5}+\frac{2 i \sqrt{i b \,x^{2}+1}\, \sqrt{-i b \,x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{-i b}, i\right)-\operatorname{EllipticE}\left(x \sqrt{-i b}, i\right)\right)}{5 \sqrt{-i b}\, \sqrt{-b^{2} x^{4}-1}\, b}\right)+\frac{x \sqrt{-b^{2} x^{4}-1}}{3}-\frac{2 \sqrt{i b \,x^{2}+1}\, \sqrt{-}}{3}$

input

```
int((-b*x^2+1)*(-b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
(-signum(b^2*x^4+1))^(1/2)/signum(b^2*x^4+1)^(1/2)*x*hypergeom([-1/2,1/4],
[5/4],-b^2*x^4)-1/3*b*(-signum(b^2*x^4+1))^(1/2)/signum(b^2*x^4+1)^(1/2)*x
^3*hypergeom([-1/2,3/4],[7/4],-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.56

$$\int (1-bx^2)\sqrt{-1-b^2x^4}\,dx$$
$$= \frac{2\sqrt{-b^2}(5\,b+3)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1) - 6\sqrt{-b^2}x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} E(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1) - (3\,b}{15\,bx}$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(2*sqrt(-b^2)*(5*b + 3)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(
1/4)/x), -1) - 6*sqrt(-b^2)*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1
/4)/x), -1) - (3*b^2*x^4 - 5*b*x^2 + 6)*sqrt(-b^2*x^4 - 1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.38

$$\int (1-bx^2)\sqrt{-1-b^2x^4}\,dx = -\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)*(-b**2*x**4-1)**(1/2),x)
```

output
```
-I*b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))
/(4*gamma(7/4)) + I*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b**2*x**4*exp_
polar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int (1-bx^2)\sqrt{-1-b^2x^4}\,dx = \int -\sqrt{-b^2x^4-1}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-b^2*x^4 - 1)*(b*x^2 - 1), x)
```

Giac [F]

$$\int (1-bx^2)\sqrt{-1-b^2x^4}\,dx = \int -\sqrt{-b^2x^4-1}\left(bx^2-1\right)dx$$

input
```
integrate((-b*x^2+1)*(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(-b^2*x^4 - 1)*(b*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-bx^2)\sqrt{-1-b^2x^4}\,dx = -\int \sqrt{-b^2\,x^4-1}\,\left(b\,x^2-1\right)\,dx$$

input
```
int(-(- b^2*x^4 - 1)^(1/2)*(b*x^2 - 1),x)
```

output
```
-int((- b^2*x^4 - 1)^(1/2)*(b*x^2 - 1), x)
```

Reduce [F]

$$\int \left(1-bx^2\right)\sqrt{-1-b^2x^4}\,dx = \frac{i\left(-3\sqrt{b^2x^4+1}\,b\,x^3+5\sqrt{b^2x^4+1}\,x+10\left(\int \frac{\sqrt{b^2x^4+1}}{b^2x^4+1}dx\right)-6\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^2x^4+1}dx\right)b\right)}{15}$$

input

```
int((-b*x^2+1)*(-b^2*x^4-1)^(1/2),x)
```

output

```
(i*( - 3*sqrt(b**2*x**4 + 1)*b*x**3 + 5*sqrt(b**2*x**4 + 1)*x + 10*int(sqr
t(b**2*x**4 + 1)/(b**2*x**4 + 1),x) - 6*int((sqrt(b**2*x**4 + 1)*x**2)/(b*
*2*x**4 + 1),x)*b))/15
```

3.283 $\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx=\frac{x\sqrt{-1-b^2x^4}}{1+bx^2}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}}$$

output
```
x*(-b^2*x^4-1)^(1/2)/(b*x^2+1)+(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*E
llipticE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx=-\frac{\sqrt{1+b^2x^4}\left(-3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)\right)}{3\sqrt{-1-b^2x^4}}$$

input
```
Integrate[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

output

```
-1/3*(Sqrt[1 + b^2*x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)]
 + b*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)]))/Sqrt[-1 - b^2*x^4]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{1510\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}\,dx$$

$$\Big\downarrow \, {\color{blue}1510}$$

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}$$

input

```
Int[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

output

```
(x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 +
b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4]
)
```

Defintions of rubi rules used

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

method	result	size
meijerg	$-\frac{b\sqrt{\mathrm{signum}(b^{2}x^{4}+1)}\, x^{3}\, \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^{2}x^{4}\right)}{3\sqrt{-\mathrm{signum}(b^{2}x^{4}+1)}}+\frac{\sqrt{\mathrm{signum}(b^{2}x^{4}+1)}\, x\, \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-b^{2}x^{4}\right)}{\sqrt{-\mathrm{signum}(b^{2}x^{4}+1)}}$	90
default	$\frac{i\sqrt{ib\,x^{2}+1}\,\sqrt{-ib\,x^{2}+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{-ib},i\right)-\mathrm{EllipticE}\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^{2}x^{4}-1}}+\frac{\sqrt{ib\,x^{2}+1}\,\sqrt{-ib\,x^{2}+1}\ \mathrm{EllipticF}\left(x\sqrt{-ib},i\right)}{\sqrt{-ib}\,\sqrt{-b^{2}x^{4}-1}}$	122
elliptic	$\frac{i\sqrt{ib\,x^{2}+1}\,\sqrt{-ib\,x^{2}+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{-ib},i\right)-\mathrm{EllipticE}\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^{2}x^{4}-1}}+\frac{\sqrt{ib\,x^{2}+1}\,\sqrt{-ib\,x^{2}+1}\ \mathrm{EllipticF}\left(x\sqrt{-ib},i\right)}{\sqrt{-ib}\,\sqrt{-b^{2}x^{4}-1}}$	122

input
```
int((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b/(-signum(b^2*x^4+1))^(1/2)*signum(b^2*x^4+1)^(1/2)*x^3*hypergeom([1
/2,3/4],[7/4],-b^2*x^4)+1/(-signum(b^2*x^4+1))^(1/2)*signum(b^2*x^4+1)^(1/
2)*x*hypergeom([1/4,1/2],[5/4],-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx =$$

$$-\frac{\sqrt{-b^2}(b+1)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)|-1)-\sqrt{-b^2}x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)|-1)-\sqrt{-b^2}}{bx}$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-b^2)*(b + 1)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x),
 -1) - sqrt(-b^2)*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1
) - sqrt(-b^2*x^4 - 1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx = \frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

output
```
I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(
4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_pol
ar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{-b^2x^4-1}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{-b^2x^4-1}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx=-\int \frac{b\,x^2-1}{\sqrt{-b^2\,x^4-1}}\,dx$$

input
```
int(-(b*x^2 - 1)/(- b^2*x^4 - 1)^(1/2),x)
```

output
```
-int((b*x^2 - 1)/(- b^2*x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx=i\left(-\left(\int \frac{\sqrt{b^2x^4+1}}{b^2x^4+1}dx\right)+\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^2x^4+1}dx\right)b\right)$$

input
```
int((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x)
```

output
```
i*( - int(sqrt(b**2*x**4 + 1)/(b**2*x**4 + 1),x) + int((sqrt(b**2*x**4 + 1
)*x**2)/(b**2*x**4 + 1),x)*b)
```

3.284 $\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 190

$$\begin{aligned}\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{3/2}}\,dx &= -\frac{x(1-bx^2)}{2\sqrt{-1-b^2x^4}}+\frac{x\sqrt{-1-b^2x^4}}{2\left(1+bx^2\right)}\\ &+\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{2\sqrt{b}\sqrt{-1-b^2x^4}}\\ &-\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{2\sqrt{b}\sqrt{-1-b^2x^4}}\end{aligned}$$

output

```
-1/2*x*(-b*x^2+1)/(-b^2*x^4-1)^(1/2)+x*(-b^2*x^4-1)^(1/2)/(2*b*x^2+2)+1/2*
(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/2)*x
)),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)-1/2*(b*x^2+1)*((b^2*x^4+1)/(b*x
^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b^(1/2)/(-
b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.48

$$\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{3/2}}\,dx = \frac{x\left(-3-3\sqrt{1+b^2x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+2bx^2\sqrt{1+b^2x^4}\,\text{Hyperg}\right.}{6\sqrt{-1-b^2x^4}}$$

input

```
Integrate[(1 - b*x^2)/(-1 - b^2*x^4)^(3/2),x]
```

output

```
(x*(-3 - 3*Sqrt[1 + b^2*x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)]
+ 2*b*x^2*Sqrt[1 + b^2*x^4]*Hypergeometric2F1[3/4, 3/2, 7/4, -(b^2*x^4)]))
/(6*Sqrt[-1 - b^2*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1493, 25, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\left(-b^2x^4-1\right)^{3/2}}dx$$

$$\Big\downarrow \text{1493}$$

$$\frac{1}{2}\int -\frac{bx^2+1}{\sqrt{-b^2x^4-1}}dx-\frac{x\left(1-bx^2\right)}{2\sqrt{-b^2x^4-1}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}dx-\frac{x\left(1-bx^2\right)}{2\sqrt{-b^2x^4-1}}$$

$$\Big\downarrow \text{1512}$$

$$\frac{1}{2}\left(\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx-2\int \frac{1}{\sqrt{-b^2x^4-1}}dx\right)-\frac{x\left(1-bx^2\right)}{2\sqrt{-b^2x^4-1}}$$

$$\Big\downarrow \textcolor{blue}{761}$$

$$\frac{1}{2}\left(\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx-\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}\right)-\frac{x(1-bx^2)}{2\sqrt{-b^2x^4-1}}$$

$$\Big\downarrow \textcolor{blue}{1510}$$

$$\frac{1}{2}\left(-\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+\frac{x\sqrt{-}}{bx}\right.$$

$$\frac{x(1-bx^2)}{2\sqrt{-b^2x^4-1}}$$

input
```
Int[(1 - b*x^2)/(-1 - b^2*x^4)^(3/2),x]
```

output
```
-1/2*(x*(1 - b*x^2))/Sqrt[-1 - b^2*x^4] + ((x*Sqrt[-1 - b^2*x^4])/(1 + b*x
^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sq
rt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4]) - ((1 + b*x^2)*Sqrt[(1 + b^2*
x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1
- b^2*x^4]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 1.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

method	result
meijerg	$\frac{\operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{3}{2}} x \,\operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right], \left[\frac{5}{4}\right], -b^{2} x^{4}\right)}{\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{3}{2}}}-\frac{b \,\operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{3}{2}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{3}{2}\right], \left[\frac{7}{4}\right], -b^{2} x^{4}\right)}{3\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{3}{2}}}$
elliptic	$\frac{2 b^{2}\left(\frac{x^{3}}{4 b}-\frac{x}{4 b^{2}}\right)}{\sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}-\frac{\sqrt{i b \,x^{2}+1}\, \sqrt{-i b \,x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{-i b}, i\right)}{2 \sqrt{-i b}\, \sqrt{-b^{2} x^{4}-1}}+\frac{i \sqrt{i b \,x^{2}+1}\, \sqrt{-i b \,x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{-i b}, i\right)-\operatorname{EllipticE}\left(x \sqrt{-i b}\right.\right.}{2 \sqrt{-i b}\, \sqrt{-b^{2} x^{4}-1}}$
default	$-b \left(-\frac{x^{3}}{2 \sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}-\frac{i \sqrt{i b \,x^{2}+1}\, \sqrt{-i b \,x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{-i b}, i\right)-\operatorname{EllipticE}\left(x \sqrt{-i b}, i\right)\right)}{2 \sqrt{-i b}\, \sqrt{-b^{2} x^{4}-1}\, b}\right)-\frac{x}{2 \sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}-$

input
```
int((-b*x^2+1)/(-b^2*x^4-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-signum(b^2*x^4+1))^(3/2)*signum(b^2*x^4+1)^(3/2)*x*hypergeom([1/4,3/2]
,[5/4],-b^2*x^4)-1/3*b/(-signum(b^2*x^4+1))^(3/2)*signum(b^2*x^4+1)^(3/2)*
x^3*hypergeom([3/4,3/2],[7/4],-b^2*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.61

$$\int \frac{1-bx^2}{(-1-b^2x^4)^{3/2}}\,dx = \frac{(-i\,b^3x^4-i\,b)(-b^2)^{\frac{3}{4}}\,E(\arcsin\left((-b^2)^{\frac{1}{4}}\,x\right)\,|-1)+(i\,(b^3-b^2)x^4+i\,b-i)(-b^2)^{\frac{3}{4}}}{2\,(b^4x^4+b^2)}$$

input

```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(3/2),x, algorithm="fricas")
```

output

```
1/2*((-I*b^3*x^4 - I*b)*(-b^2)^(3/4)*elliptic_e(arcsin((-b^2)^(1/4)*x), -1
) + (I*(b^3 - b^2)*x^4 + I*b - I)*(-b^2)^(3/4)*elliptic_f(arcsin((-b^2)^(1
/4)*x), -1) - (b^3*x^3 - b^2*x)*sqrt(-b^2*x^4 - 1))/(b^4*x^4 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.89 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.37

$$\int \frac{1-bx^2}{(-1-b^2x^4)^{3/2}}\,dx = -\frac{ibx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\,b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ix\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\,b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((-b*x**2+1)/(-b**2*x**4-1)**(3/2),x)
```

output

```
-I*b*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b**2*x**4*exp_polar(I*pi))/
(4*gamma(7/4)) + I*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b**2*x**4*exp_po
lar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{(-1-b^2x^4)^{3/2}}\,dx = \int -\frac{bx^2-1}{(-b^2x^4-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/(-b^2*x^4 - 1)^(3/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{(-1-b^2x^4)^{3/2}}\,dx = \int -\frac{bx^2-1}{(-b^2x^4-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/(-b^2*x^4 - 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{(-1-b^2x^4)^{3/2}}\,dx = -\int \frac{b\,x^2-1}{(-b^2\,x^4-1)^{3/2}}\,dx$$

input
```
int(-(b*x^2 - 1)/(- b^2*x^4 - 1)^(3/2),x)
```

output
```
-int((b*x^2 - 1)/(- b^2*x^4 - 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{3/2}}\,dx = i\left(\int \frac{\sqrt{b^2x^4+1}}{b^4x^8+2b^2x^4+1}dx - \left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^4x^8+2b^2x^4+1}dx\right)b\right)$$

input

```
int((-b*x^2+1)/(-b^2*x^4-1)^(3/2),x)
```

output

```
i*(int(sqrt(b**2*x**4 + 1)/(b**4*x**8 + 2*b**2*x**4 + 1),x) - int((sqrt(b*
*2*x**4 + 1)*x**2)/(b**4*x**8 + 2*b**2*x**4 + 1),x)*b)
```

3.285 $\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 217

$$\begin{aligned}\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{5/2}}\,dx &= -\frac{x(1-bx^2)}{6\left(-1-b^2x^4\right)^{3/2}}+\frac{x(5-3bx^2)}{12\sqrt{-1-b^2x^4}} \\ &\quad -\frac{x\sqrt{-1-b^2x^4}}{4\left(1+bx^2\right)}-\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)\middle|\frac{1}{2}\right)}{4\sqrt{b}\sqrt{-1-b^2x^4}} \\ &\quad +\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}\,\operatorname{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{3\sqrt{b}\sqrt{-1-b^2x^4}}\end{aligned}$$

output
```
-1/6*x*(-b*x^2+1)/(-b^2*x^4-1)^(3/2)+1/12*x*(-3*b*x^2+5)/(-b^2*x^4-1)^(1/2
)-x*(-b^2*x^4-1)^(1/2)/(4*b*x^2+4)-1/4*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)
^(1/2)*EllipticE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1
)^(1/2)+1/3*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(b^(1/2)*x),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.46

$$\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{5/2}}\,dx =$$

$$-\frac{x\left(7+5b^2x^4+5(1+b^2x^4)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)-4bx^2(1+b^2x^4)^{3/2}\,\text{Hypergeometric2}\right.}{12\left(-1-b^2x^4\right)^{3/2}}$$

input
```
Integrate[(1 - b*x^2)/(-1 - b^2*x^4)^(5/2),x]
```

output
```
-1/12*(x*(7 + 5*b^2*x^4 + 5*(1 + b^2*x^4)^(3/2)*Hypergeometric2F1[1/4, 1/2
, 5/4, -(b^2*x^4)] - 4*b*x^2*(1 + b^2*x^4)^(3/2)*Hypergeometric2F1[3/4, 5/
2, 7/4, -(b^2*x^4)]))/(-1 - b^2*x^4)^(3/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1493, 25, 1493, 25, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\left(-b^2x^4-1\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1493}$$

$$\frac{1}{6}\int -\frac{5-3bx^2}{\left(-b^2x^4-1\right)^{3/2}}dx-\frac{x\left(1-bx^2\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{6}\int \frac{5-3bx^2}{\left(-b^2x^4-1\right)^{3/2}}dx-\frac{x\left(1-bx^2\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$

$$\Big\downarrow \; 1493$$

$$\frac{1}{6}\left(\frac{x\left(5-3bx^2\right)}{2\sqrt{-b^2x^4-1}}-\frac{1}{2}\int -\frac{3bx^2+5}{\sqrt{-b^2x^4-1}}dx\right)-\frac{x\left(1-bx^2\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{3bx^2+5}{\sqrt{-b^2x^4-1}}dx+\frac{x\left(5-3bx^2\right)}{2\sqrt{-b^2x^4-1}}\right)-\frac{x\left(1-bx^2\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$

$$\Big\downarrow \; 1512$$

$$\frac{1}{6}\left(\frac{1}{2}\left(8\int \frac{1}{\sqrt{-b^2x^4-1}}dx-3\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx\right)+\frac{x\left(5-3bx^2\right)}{2\sqrt{-b^2x^4-1}}\right)-\frac{x\left(1-bx^2\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-3\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx\right)+\frac{x\left(5-3bx^2\right)}{2\sqrt{-b^2x^4-1}}\right)-$$
$$\frac{x\left(1-bx^2\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$

$$\Big\downarrow \; 1510$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-3\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+\right.\right.\right.$$
$$\frac{x\left(1-bx^2\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$

input

```
Int[(1 - b*x^2)/(-1 - b^2*x^4)^(5/2),x]
```

output

```
-1/6*(x*(1 - b*x^2))/(-1 - b^2*x^4)^(3/2) + ((x*(5 - 3*b*x^2))/(2*Sqrt[-1
- b^2*x^4]) + (-3*((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[
(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]
*Sqrt[-1 - b^2*x^4])) + (4*(1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*E
llipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4]))/2)/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 1.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.41

method	result
meijerg	$\frac{\mathrm{signum}\left(b^{2} x^{4}+1\right)^{\frac{5}{2}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{5}{2}\right], \left[\frac{5}{4}\right], -b^{2} x^{4}\right)}{\left(-\mathrm{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{5}{2}}}-\frac{b \,\mathrm{signum}\left(b^{2} x^{4}+1\right)^{\frac{5}{2}} x^{3} \mathrm{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{2}\right], \left[\frac{7}{4}\right], -b^{2} x^{4}\right)}{3 \left(-\mathrm{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{5}{2}}}$
elliptic	$\frac{\left(\frac{x^{3}}{6 b^{3}}-\frac{x}{6 b^{4}}\right) \sqrt{-b^{2} x^{4}-1}}{\left(x^{4}+\frac{1}{b^{2}}\right)^{2}}+\frac{2 b^{2} \left(-\frac{x^{3}}{8 b}+\frac{5 x}{24 b^{2}}\right)}{\sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}+\frac{5 \sqrt{i b \,x^{2}+1}\, \sqrt{-i b \,x^{2}+1}\, \mathrm{EllipticF}\left(x \sqrt{-i b}, i\right)}{12 \sqrt{-i b}\, \sqrt{-b^{2} x^{4}-1}}-\frac{i \sqrt{i b \,x^{2}+1}\, \sqrt{-i b \,x^{2}+1}\, (\mathrm{Elliptic}}{4 \sqrt{-i b}}$
default	$-b \left(-\frac{x^{3} \sqrt{-b^{2} x^{4}-1}}{6 b^{4} \left(x^{4}+\frac{1}{b^{2}}\right)^{2}}+\frac{x^{3}}{4 \sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}+\frac{i \sqrt{i b \,x^{2}+1}\, \sqrt{-i b \,x^{2}+1}\, \left(\mathrm{EllipticF}\left(x \sqrt{-i b}, i\right)-\mathrm{EllipticE}\left(x \sqrt{-i b}, i\right)\right)}{4 \sqrt{-i b}\, \sqrt{-b^{2} x^{4}-1}\, b}\right)-\frac{x \sqrt{-}}{6 b^{4} \left(x\right.}$

input
```
int((-b*x^2+1)/(-b^2*x^4-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-signum(b^2*x^4+1))^(5/2)*signum(b^2*x^4+1)^(5/2)*x*hypergeom([1/4,5/2]
,[5/4],-b^2*x^4)-1/3*b/(-signum(b^2*x^4+1))^(5/2)*signum(b^2*x^4+1)^(5/2)*
x^3*hypergeom([3/4,5/2],[7/4],-b^2*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.77

$$\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{5/2}}\,dx =$$

$$-\frac{3\,(-i\,b^5x^8-2i\,b^3x^4-i\,b)(-b^2)^{\frac{3}{4}}\,E(\arcsin\left((-b^2)^{\frac{1}{4}}\,x\right)\,|\,-1)-(-i\,(3\,b^5-5\,b^4)x^8-2i\,(3\,b^3-5\,b^2)x^4-}{12\,(b^6x^8+2\,b^4x^4}$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*(-I*b^5*x^8 - 2*I*b^3*x^4 - I*b)*(-b^2)^(3/4)*elliptic_e(arcsin((
-b^2)^(1/4)*x), -1) - (-I*(3*b^5 - 5*b^4)*x^8 - 2*I*(3*b^3 - 5*b^2)*x^4 -
3*I*b + 5*I)*(-b^2)^(3/4)*elliptic_f(arcsin((-b^2)^(1/4)*x), -1) - (3*b^5*
x^7 - 5*b^4*x^5 + 5*b^3*x^3 - 7*b^2*x)*sqrt(-b^2*x^4 - 1))/(b^6*x^8 + 2*b^
4*x^4 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.32

$$\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{5/2}}\,dx = \frac{ibx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\,b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|\,b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(-b**2*x**4-1)**(5/2),x)
```

output
```
I*b*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b**2*x**4*exp_polar(I*pi))/(
4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b**2*x**4*exp_pol
ar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{5/2}}\,dx = \int -\frac{bx^2-1}{\left(-b^2x^4-1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/(-b^2*x^4 - 1)^(5/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{(-1-b^2x^4)^{5/2}}\,dx=\int -\frac{bx^2-1}{(-b^2x^4-1)^{\frac{5}{2}}}\,dx$$

input

```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(5/2),x, algorithm="giac")
```

output

```
integrate(-(b*x^2 - 1)/(-b^2*x^4 - 1)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{(-1-b^2x^4)^{5/2}}\,dx=-\int \frac{b\,x^2-1}{(-b^2\,x^4-1)^{5/2}}\,dx$$

input

```
int(-(b*x^2 - 1)/(- b^2*x^4 - 1)^(5/2),x)
```

output

```
-int((b*x^2 - 1)/(- b^2*x^4 - 1)^(5/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{(-1-b^2x^4)^{5/2}}\,dx=i\left(-\left(\int \frac{\sqrt{b^2x^4+1}}{b^6x^{12}+3b^4x^8+3b^2x^4+1}dx\right)+\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^6x^{12}+3b^4x^8+3b^2x^4+1}dx\right)b\right)$$

input

```
int((-b*x^2+1)/(-b^2*x^4-1)^(5/2),x)
```

output

```
i*( - int(sqrt(b**2*x**4 + 1)/(b**6*x**12 + 3*b**4*x**8 + 3*b**2*x**4 + 1)
,x) + int((sqrt(b**2*x**4 + 1)*x**2)/(b**6*x**12 + 3*b**4*x**8 + 3*b**2*x*
*4 + 1),x)*b)
```

3.286 $\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 244

$$\begin{aligned}\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{7/2}}\,dx &= -\frac{x(1-bx^2)}{10\left(-1-b^2x^4\right)^{5/2}}+\frac{x(9-7bx^2)}{60\left(-1-b^2x^4\right)^{3/2}}\\ &\quad-\frac{x(15-7bx^2)}{40\sqrt{-1-b^2x^4}}+\frac{7x\sqrt{-1-b^2x^4}}{40\left(1+bx^2\right)}+\frac{7(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{40\sqrt{b}\sqrt{-1-b^2x^4}}\\ &\quad-\frac{11(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{40\sqrt{b}\sqrt{-1-b^2x^4}}\end{aligned}$$

output
```
-1/10*x*(-b*x^2+1)/(-b^2*x^4-1)^(5/2)+1/60*x*(-7*b*x^2+9)/(-b^2*x^4-1)^(3/
2)-1/40*x*(-7*b*x^2+15)/(-b^2*x^4-1)^(1/2)+7*x*(-b^2*x^4-1)^(1/2)/(40*b*x^
2+40)+7/40*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(sin(2*arcta
n(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)-11/40*(b*x^2+1)*((b^
2*x^4+1)/(b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/2)*x),1/2*2^(1/2
))/b^(1/2)/(-b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.44

$$\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{7/2}}\,dx=\frac{x\left(-75-108b^2x^4-45b^4x^8-45\left(1+b^2x^4\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)\right.}{120\left(-1-b^2x^4\right)^{5/2}}$$

input
```
Integrate[(1 - b*x^2)/(-1 - b^2*x^4)^(7/2),x]
```

output
```
(x*(-75 - 108*b^2*x^4 - 45*b^4*x^8 - 45*(1 + b^2*x^4)^(5/2)*Hypergeometric
2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + 40*b*x^2*(1 + b^2*x^4)^(5/2)*Hypergeometr
ic2F1[3/4, 7/2, 7/4, -(b^2*x^4)]))/(120*(-1 - b^2*x^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1493, 25, 1493, 27, 1493, 25, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\left(-b^2x^4-1\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 1493}$$

$$\frac{1}{10}\int -\frac{9-7bx^2}{\left(-b^2x^4-1\right)^{5/2}}dx-\frac{x\left(1-bx^2\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{10}\int \frac{9-7bx^2}{\left(-b^2x^4-1\right)^{5/2}}dx-\frac{x\left(1-bx^2\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \text{ 1493}$$

$$\frac{1}{10}\left(\frac{x\left(9-7bx^2\right)}{6\left(-b^2x^4-1\right)^{3/2}}-\frac{1}{6}\int -\frac{3\left(15-7bx^2\right)}{\left(-b^2x^4-1\right)^{3/2}}dx\right)-\frac{x\left(1-bx^2\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{10}\left(\frac{1}{2}\int \frac{15-7bx^2}{\left(-b^2x^4-1\right)^{3/2}}dx+\frac{x\left(9-7bx^2\right)}{6\left(-b^2x^4-1\right)^{3/2}}\right)-\frac{x\left(1-bx^2\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \ 1493$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\int -\frac{7bx^2+15}{\sqrt{-b^2x^4-1}}dx-\frac{x\left(15-7bx^2\right)}{2\sqrt{-b^2x^4-1}}\right)+\frac{x\left(9-7bx^2\right)}{6\left(-b^2x^4-1\right)^{3/2}}\right)-\frac{x\left(1-bx^2\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{10}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{7bx^2+15}{\sqrt{-b^2x^4-1}}dx-\frac{x\left(15-7bx^2\right)}{2\sqrt{-b^2x^4-1}}\right)+\frac{x\left(9-7bx^2\right)}{6\left(-b^2x^4-1\right)^{3/2}}\right)-\frac{x\left(1-bx^2\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \ 1512$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\left(7\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx-22\int \frac{1}{\sqrt{-b^2x^4-1}}dx\right)-\frac{x\left(15-7bx^2\right)}{2\sqrt{-b^2x^4-1}}\right)+\frac{x\left(9-7bx^2\right)}{6\left(-b^2x^4-1\right)^{3/2}}\right)-\frac{x\left(1-bx^2\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \ 761$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\left(7\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx-\frac{11\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}\right)-\frac{x\left(15-7bx^2\right)}{2\sqrt{-b^2x^4-1}}\right)+\right.\right.$$

$$\frac{x\left(1-bx^2\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \ 1510$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\left(7\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}\right)-\frac{11\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{Elliptic}}{\sqrt{b}\sqrt{-b^2x^4}}\right.\right.\right.$$

$$\frac{x\left(1-bx^2\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

input
```
Int[(1 - b*x^2)/(-1 - b^2*x^4)^(7/2),x]
```

output
```
-1/10*(x*(1 - b*x^2))/(-1 - b^2*x^4)^(5/2) + ((x*(9 - 7*b*x^2))/(6*(-1 - b
^2*x^4)^(3/2)) + (-1/2*(x*(15 - 7*b*x^2))/Sqrt[-1 - b^2*x^4] + (7*((x*Sqrt
[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^
2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])) - (1
1*(1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]
*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4]))/2)/2)/10
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 1.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.37

method	result
meijerg	$\frac{\operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{7}{2}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{7}{2}\right],\left[\frac{5}{4}\right],-b^{2} x^{4}\right)}{\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{7}{2}}}-\frac{b \operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{7}{2}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{7}{2}\right],\left[\frac{7}{4}\right],-b^{2} x^{4}\right)}{3\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{7}{2}}}$
elliptic	$\frac{\left(-\frac{x^{3}}{10 b^{5}}+\frac{x}{10 b^{6}}\right) \sqrt{-b^{2} x^{4}-1}}{\left(x^{4}+\frac{1}{b^{2}}\right)^{3}}+\frac{\left(-\frac{7 x^{3}}{60 b^{3}}+\frac{3 x}{20 b^{4}}\right) \sqrt{-b^{2} x^{4}-1}}{\left(x^{4}+\frac{1}{b^{2}}\right)^{2}}+\frac{2 b^{2}\left(\frac{7 x^{3}}{80 b}-\frac{3 x}{16 b^{2}}\right)}{\sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}-\frac{3 \sqrt{i b\, x^{2}+1}\, \sqrt{-i b\, x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{-i b}, i\right)}{8 \sqrt{-i b}\, \sqrt{-b^{2} x^{4}-1}}$
default	$-b \left(\frac{x^{3} \sqrt{-b^{2} x^{4}-1}}{10 b^{6}\left(x^{4}+\frac{1}{b^{2}}\right)^{3}}+\frac{7 x^{3} \sqrt{-b^{2} x^{4}-1}}{60 b^{4}\left(x^{4}+\frac{1}{b^{2}}\right)^{2}}-\frac{7 x^{3}}{40 \sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}-\frac{7 i \sqrt{i b\, x^{2}+1}\, \sqrt{-i b\, x^{2}+1}\, (\operatorname{EllipticF}\left(x \sqrt{-i b}, i\right)-\operatorname{EllipticE}(x}{40 \sqrt{-i b}\, \sqrt{-b^{2} x^{4}-1}\, b}\right.$

input
```
int((-b*x^2+1)/(-b^2*x^4-1)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-signum(b^2*x^4+1))^(7/2)*signum(b^2*x^4+1)^(7/2)*x*hypergeom([1/4,7/2]
,[5/4],-b^2*x^4)-1/3*b/(-signum(b^2*x^4+1))^(7/2)*signum(b^2*x^4+1)^(7/2)*
x^3*hypergeom([3/4,7/2],[7/4],-b^2*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.88

$$\int \frac{1-bx^2}{\left(-1-b^2x^4\right)^{7/2}}\,dx =$$

$$-\frac{21\,(i\,b^7x^{12}+3i\,b^5x^8+3i\,b^3x^4+i\,b)(-b^2)^{\frac{3}{4}}\,E(\arcsin\left(\left(-b^2\right)^{\frac{1}{4}}\,x\right)\,|-1)+3\,(-i\,(7\,b^7-15\,b^6)x^{12}-3i\,(7\,b^5}{}$$

input

```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(7/2),x, algorithm="fricas")
```

output

```
-1/120*(21*(I*b^7*x^12 + 3*I*b^5*x^8 + 3*I*b^3*x^4 + I*b)*(-b^2)^(3/4)*ell
iptic_e(arcsin((-b^2)^(1/4)*x), -1) + 3*(-I*(7*b^7 - 15*b^6)*x^12 - 3*I*(7
*b^5 - 15*b^4)*x^8 - 3*I*(7*b^3 - 15*b^2)*x^4 - 7*I*b + 15*I)*(-b^2)^(3/4)
*elliptic_f(arcsin((-b^2)^(1/4)*x), -1) + (21*b^7*x^11 - 45*b^6*x^9 + 56*b
^5*x^7 - 108*b^4*x^5 + 47*b^3*x^3 - 75*b^2*x)*sqrt(-b^2*x^4 - 1))/(b^8*x^1
2 + 3*b^6*x^8 + 3*b^4*x^4 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 33.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.29

$$\int \frac{1-bx^2}{(-1-b^2x^4)^{7/2}}\,dx = -\frac{ibx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ix\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((-b*x**2+1)/(-b**2*x**4-1)**(7/2),x)
```

output

```
-I*b*x**3*gamma(3/4)*hyper((3/4, 7/2), (7/4,), b**2*x**4*exp_polar(I*pi))/
(4*gamma(7/4)) + I*x*gamma(1/4)*hyper((1/4, 7/2), (5/4,), b**2*x**4*exp_po
lar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{(-1-b^2x^4)^{7/2}}\,dx = \int -\frac{bx^2-1}{(-b^2x^4-1)^{\frac{7}{2}}}\,dx$$

input

```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(7/2),x, algorithm="maxima")
```

output

```
-integrate((b*x^2 - 1)/(-b^2*x^4 - 1)^(7/2), x)
```

Giac [F]

$$\int \frac{1-bx^2}{(-1-b^2x^4)^{7/2}}\,dx = \int -\frac{bx^2-1}{(-b^2x^4-1)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(7/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/(-b^2*x^4 - 1)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{(-1-b^2x^4)^{7/2}}\,dx = -\int \frac{b\,x^2-1}{(-b^2\,x^4-1)^{7/2}}\,dx$$

input
```
int(-(b*x^2 - 1)/(- b^2*x^4 - 1)^(7/2),x)
```

output
```
-int((b*x^2 - 1)/(- b^2*x^4 - 1)^(7/2), x)
```

Reduce [F]

$$\int \frac{1-bx^2}{(-1-b^2x^4)^{7/2}}\,dx = i\left(\int \frac{\sqrt{b^2x^4+1}}{b^8x^{16}+4b^6x^{12}+6b^4x^8+4b^2x^4+1}dx - \left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^8x^{16}+4b^6x^{12}+6b^4x^8+4b^2x^4+1}dx\right)b\right)$$

input
```
int((-b*x^2+1)/(-b^2*x^4-1)^(7/2),x)
```

output
```
i*(int(sqrt(b**2*x**4 + 1)/(b**8*x**16 + 4*b**6*x**12 + 6*b**4*x**8 + 4*b*
*2*x**4 + 1),x) - int((sqrt(b**2*x**4 + 1)*x**2)/(b**8*x**16 + 4*b**6*x**1
2 + 6*b**4*x**8 + 4*b**2*x**4 + 1),x)*b)
```

3.287 $\int (1+bx^2)(-1-b^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 217

$$\int (1+bx^2)\left(-1-b^2x^4\right)^{3/2}dx = -\frac{4x\sqrt{-1-b^2x^4}}{15\left(1+bx^2\right)}-\frac{2}{105}x\left(15+7bx^2\right)\sqrt{-1-b^2x^4}+\frac{1}{63}x\left(9+7bx^2\right)\left(-1-b^2x^4\right)^{3/2}-\frac{4(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{15\sqrt{b}\sqrt{-1-b^2x^4}}+\frac{44(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{105\sqrt{b}\sqrt{-1-b^2x^4}}$$

output

```
-4*x*(-b^2*x^4-1)^(1/2)/(15*b*x^2+15)-2/105*x*(7*b*x^2+15)*(-b^2*x^4-1)^(1
/2)+1/63*x*(7*b*x^2+9)*(-b^2*x^4-1)^(3/2)-4/15*(b*x^2+1)*((b^2*x^4+1)/(b*x
^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(-b
^2*x^4-1)^(1/2)+44/105*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*InverseJa
cobiAM(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.57 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.35

$$\int (1+bx^2)\left(-1-b^2x^4\right)^{3/2}\,dx = -\frac{\sqrt{-1-b^2x^4}\left(3x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)\right)}{3\sqrt{1+b^2x^4}}$$

input

```
Integrate[(1 + b*x^2)*(-1 - b^2*x^4)^(3/2),x]
```

output

```
-1/3*(Sqrt[-1 - b^2*x^4]*(3*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -(b^2*x^4)
] + b*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -(b^2*x^4)]))/Sqrt[1 + b^2*x^4
]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1491, 27, 1491, 27, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^2+1)\left(-b^2x^4-1\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{21}\int -2(7bx^2+9)\sqrt{-b^2x^4-1}dx+\frac{1}{63}x(7bx^2+9)\left(-b^2x^4-1\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{63}x(7bx^2+9)\left(-b^2x^4-1\right)^{3/2}-\frac{2}{21}\int (7bx^2+9)\sqrt{-b^2x^4-1}dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{63}x\left(7bx^2+9\right)\left(-b^2x^4-1\right)^{3/2}-\frac{2}{21}\left(\frac{1}{15}\int -\frac{6\left(7bx^2+15\right)}{\sqrt{-b^2x^4-1}}dx+\frac{1}{5}x\sqrt{-b^2x^4-1}\left(7bx^2+15\right)\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{63}x\left(7bx^2+9\right)\left(-b^2x^4-1\right)^{3/2}-\frac{2}{21}\left(\frac{1}{5}x\left(7bx^2+15\right)\sqrt{-b^2x^4-1}-\frac{2}{5}\int \frac{7bx^2+15}{\sqrt{-b^2x^4-1}}dx\right)$$

$$\Big\downarrow \text{ 1512}$$

$$\frac{1}{63}x\left(7bx^2+9\right)\left(-b^2x^4-1\right)^{3/2}-$$
$$\frac{2}{21}\left(\frac{1}{5}x\left(7bx^2+15\right)\sqrt{-b^2x^4-1}-\frac{2}{5}\left(22\int \frac{1}{\sqrt{-b^2x^4-1}}dx-7\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx\right)\right)$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{63}x\left(7bx^2+9\right)\left(-b^2x^4-1\right)^{3/2}-$$
$$\frac{2}{21}\left(\frac{1}{5}x\left(7bx^2+15\right)\sqrt{-b^2x^4-1}-\frac{2}{5}\left(\frac{11\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-7\int \frac{1-bx^2}{\sqrt{-b^2x^4-}}\right.\right.$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{1}{63}x\left(7bx^2+9\right)\left(-b^2x^4-1\right)^{3/2}-$$
$$\frac{2}{21}\left(\frac{1}{5}x\left(7bx^2+15\right)\sqrt{-b^2x^4-1}-\frac{2}{5}\left(\frac{11\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-7\left(\frac{\left(bx^2+1\right)\sqrt{}}{}\right.\right.\right.$$

input
```
Int[(1 + b*x^2)*(-1 - b^2*x^4)^(3/2),x]
```

output
```
(x*(9 + 7*b*x^2)*(-1 - b^2*x^4)^(3/2))/63 - (2*((x*(15 + 7*b*x^2)*Sqrt[-1
- b^2*x^4])/5 - (2*(-7*((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*
Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sq
rt[b]*Sqrt[-1 - b^2*x^4])) + (11*(1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2
)^2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])))/5
))/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 1.85 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.41

method	result
meijerg	$\frac{(-\operatorname{signum}(b^2x^4+1))^{\frac{3}{2}} x \operatorname{hypergeom}([-\frac{3}{2},\frac{1}{4}],[\frac{5}{4}],-b^2x^4)}{\operatorname{signum}(b^2x^4+1)^{\frac{3}{2}}} + \frac{b(-\operatorname{signum}(b^2x^4+1))^{\frac{3}{2}} x^3 \operatorname{hypergeom}([-\frac{3}{2},\frac{3}{4}],[\frac{7}{4}],-b^2x^4)}{3\operatorname{signum}(b^2x^4+1)^{\frac{3}{2}}}$
risch	$\frac{x(35b^3x^6+45b^2x^4+77b\,x^2+135)(b^2x^4+1)}{315\sqrt{-b^2x^4-1}} + \frac{4\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\operatorname{EllipticF}(x\sqrt{-ib},i)}{7\sqrt{-ib}\,\sqrt{-b^2x^4-1}} - \frac{4i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,(\operatorname{EllipticF}(}{15\sqrt{-ib}\,\sqrt{-}}$
elliptic	$-\frac{b^3x^7\sqrt{-b^2x^4-1}}{9} - \frac{b^2x^5\sqrt{-b^2x^4-1}}{7} - \frac{11b\,x^3\sqrt{-b^2x^4-1}}{45} - \frac{3x\sqrt{-b^2x^4-1}}{7} + \frac{4\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\operatorname{EllipticF}(x\sqrt{-ib},i)}{7\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$
default	$-\frac{b^2x^5\sqrt{-b^2x^4-1}}{7} - \frac{3x\sqrt{-b^2x^4-1}}{7} + \frac{4\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\operatorname{EllipticF}(x\sqrt{-ib},i)}{7\sqrt{-ib}\,\sqrt{-b^2x^4-1}} + b\Bigg(-\frac{b^2x^7\sqrt{-b^2x^4-1}}{9} - \frac{11x^3\sqrt{-b^2x^4}}{45}$

input
```
int((b*x^2+1)*(-b^2*x^4-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-signum(b^2*x^4+1))^(3/2)/signum(b^2*x^4+1)^(3/2)*x*hypergeom([-3/2,1/4],
[5/4],-b^2*x^4)+1/3*b*(-signum(b^2*x^4+1))^(3/2)/signum(b^2*x^4+1)^(3/2)*x
^3*hypergeom([-3/2,3/4],[7/4],-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.56

$$\int \left(1+bx^2\right)\left(-1-b^2x^4\right)^{3/2}\,dx =$$

$$-\frac{12\sqrt{-b^2}(15\,b-7)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+84\sqrt{-b^2}x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+(}{315\,bx}$$

input
```
integrate((b*x^2+1)*(-b^2*x^4-1)^(3/2),x, algorithm="fricas")
```

output
```
-1/315*(12*sqrt(-b^2)*(15*b - 7)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^
2)^(1/4)/x), -1) + 84*sqrt(-b^2)*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^
2)^(1/4)/x), -1) + (35*b^4*x^8 + 45*b^3*x^6 + 77*b^2*x^4 + 135*b*x^2 + 84)
*sqrt(-b^2*x^4 - 1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.72

$$\int (1+bx^2)\left(-1-b^2x^4\right)^{3/2}\,dx = -\frac{ib^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)} - \frac{ib^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((b*x**2+1)*(-b**2*x**4-1)**(3/2),x)
```

output

```
-I*b**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b**2*x**4*exp_polar(I*
pi))/(4*gamma(11/4)) - I*b**2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b
**2*x**4*exp_polar(I*pi))/(4*gamma(9/4)) - I*b*x**3*gamma(3/4)*hyper((-1/2
, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*gamma(7/4)) - I*x*gamma(1/4)
*hyper((-1/2, 1/4), (5/4,), b**2*x**4*exp_polar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int (1+bx^2)\left(-1-b^2x^4\right)^{3/2}\,dx = \int \left(-b^2x^4-1\right)^{\frac{3}{2}}\left(bx^2+1\right)dx$$

input

```
integrate((b*x^2+1)*(-b^2*x^4-1)^(3/2),x, algorithm="maxima")
```

output

```
integrate((-b^2*x^4 - 1)^(3/2)*(b*x^2 + 1), x)
```

Giac [F]

$$\int \left(1+bx^2\right) \left(-1-b^2x^4\right)^{3/2} \, dx = \int \left(-b^2x^4-1\right)^{\frac{3}{2}} \left(bx^2+1\right) dx$$

input
```
integrate((b*x^2+1)*(-b^2*x^4-1)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b^2*x^4 - 1)^(3/2)*(b*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+bx^2\right) \left(-1-b^2x^4\right)^{3/2} \, dx = \int \left(-b^2\,x^4-1\right)^{3/2} \left(b\,x^2+1\right) \, dx$$

input
```
int((- b^2*x^4 - 1)^(3/2)*(b*x^2 + 1),x)
```

output
```
int((- b^2*x^4 - 1)^(3/2)*(b*x^2 + 1), x)
```

Reduce [F]

$$\int \left(1+bx^2\right) \left(-1 -b^2x^4\right)^{3/2} \, dx = \frac{i\left(-35\sqrt{b^2x^4+1}\,b^3x^7-45\sqrt{b^2x^4+1}\,b^2x^5-77\sqrt{b^2x^4+1}\,b\,x^3-135\sqrt{b^2x^4+1}\,x-180\left(\int\right.\right.}{315}$$

input
```
int((b*x^2+1)*(-b^2*x^4-1)^(3/2),x)
```

output
```
(i*( - 35*sqrt(b**2*x**4 + 1)*b**3*x**7 - 45*sqrt(b**2*x**4 + 1)*b**2*x**5
 - 77*sqrt(b**2*x**4 + 1)*b*x**3 - 135*sqrt(b**2*x**4 + 1)*x - 180*int(sqr
t(b**2*x**4 + 1)/(b**2*x**4 + 1),x) - 84*int((sqrt(b**2*x**4 + 1)*x**2)/(b
**2*x**4 + 1),x)*b))/315
```

3.288 $\int \left(1+bx^2\right) \sqrt{-1-b^2x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 190

$$\begin{aligned}\int \left(1+bx^2\right) \sqrt{-1-b^2x^4}\,dx &= \frac{2x\sqrt{-1-b^2x^4}}{5\left(1+bx^2\right)}+\frac{1}{15}x\left(5+3bx^2\right)\sqrt{-1-b^2x^4} \\ &\quad+\frac{2(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)\middle|\frac{1}{2}\right)}{5\sqrt{b}\sqrt{-1-b^2x^4}} \\ &\quad-\frac{8(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{15\sqrt{b}\sqrt{-1-b^2x^4}}\end{aligned}$$

output

```
2*x*(-b^2*x^4-1)^(1/2)/(5*b*x^2+5)+1/15*x*(3*b*x^2+5)*(-b^2*x^4-1)^(1/2)+2
/5*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/2
)*x)),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)-8/15*(b*x^2+1)*((b^2*x^4+1)/
(b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b^(1/2
)/(-b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.40

$$\int (1+bx^2)\sqrt{-1-b^2x^4}\,dx$$
$$=\frac{\sqrt{-1-b^2x^4}\left(3x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)\right)}{3\sqrt{1+b^2x^4}}$$

input
```
Integrate[(1 + b*x^2)*Sqrt[-1 - b^2*x^4],x]
```

output
```
(Sqrt[-1 - b^2*x^4]*(3*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -(b^2*x^4)] + b
*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -(b^2*x^4)]))/(3*Sqrt[1 + b^2*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1491, 27, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^2+1)\sqrt{-b^2x^4-1}\,dx$$
$$\Big\downarrow \text{1491}$$
$$\frac{1}{15}\int -\frac{2(3bx^2+5)}{\sqrt{-b^2x^4-1}}dx+\frac{1}{15}x\sqrt{-b^2x^4-1}(3bx^2+5)$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{15}x(3bx^2+5)\sqrt{-b^2x^4-1}-\frac{2}{15}\int\frac{3bx^2+5}{\sqrt{-b^2x^4-1}}dx$$
$$\Big\downarrow \text{1512}$$
$$\frac{1}{15}x(3bx^2+5)\sqrt{-b^2x^4-1}-\frac{2}{15}\left(8\int\frac{1}{\sqrt{-b^2x^4-1}}dx-3\int\frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{15}x\left(3bx^2+5\right)\sqrt{-b^2x^4-1}-\frac{2}{15}\left(\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-3\int\frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx\right)$$

$$\Big\downarrow \text{1510}$$

$$\frac{1}{15}x\left(3bx^2+5\right)\sqrt{-b^2x^4-1}-\frac{2}{15}\left(\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-3\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+\right.\right.$$

input
```
Int[(1 + b*x^2)*Sqrt[-1 - b^2*x^4],x]
```

output
```
(x*(5 + 3*b*x^2)*Sqrt[-1 - b^2*x^4])/15 - (2*(-3*((x*Sqrt[-1 - b^2*x^4])/(
1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*Ar
cTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])) + (4*(1 + b*x^2)*Sqrt
[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b
]*Sqrt[-1 - b^2*x^4])))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 2.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

method	result
meijerg	$\frac{\sqrt{-\operatorname{signum}(b^2x^4+1)}\,x\operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{5}{4}\right],-b^2x^4\right)}{\sqrt{\operatorname{signum}(b^2x^4+1)}}+\frac{b\sqrt{-\operatorname{signum}(b^2x^4+1)}\,x^3\operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^2x^4\right)}{3\sqrt{\operatorname{signum}(b^2x^4+1)}}$
risch	$-\frac{x(3bx^2+5)(b^2x^4+1)}{15\sqrt{-b^2x^4-1}}-\frac{2\sqrt{ibx^2+1}\,\sqrt{-ibx^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-ib},i\right)}{3\sqrt{-ib}\,\sqrt{-b^2x^4-1}}+\frac{2i\sqrt{ibx^2+1}\,\sqrt{-ibx^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-ib},i\right)-\mathrm{Elliptic}\right.}{5\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$
elliptic	$\frac{bx^3\sqrt{-b^2x^4-1}}{5}+\frac{x\sqrt{-b^2x^4-1}}{3}-\frac{2\sqrt{ibx^2+1}\,\sqrt{-ibx^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-ib},i\right)}{3\sqrt{-ib}\,\sqrt{-b^2x^4-1}}+\frac{2i\sqrt{ibx^2+1}\,\sqrt{-ibx^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-ib},i\right)\right.}{5\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$
default	$\frac{x\sqrt{-b^2x^4-1}}{3}-\frac{2\sqrt{ibx^2+1}\,\sqrt{-ibx^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-ib},i\right)}{3\sqrt{-ib}\,\sqrt{-b^2x^4-1}}+b\left(\frac{x^3\sqrt{-b^2x^4-1}}{5}+\frac{2i\sqrt{ibx^2+1}\,\sqrt{-ibx^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-ib},\right.\right.}{5\sqrt{-ib}\,\sqrt{-b^2x^4-1}}\right.$

input
```
int((b*x^2+1)*(-b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-signum(b^2*x^4+1))^(1/2)/signum(b^2*x^4+1)^(1/2)*x*hypergeom([-1/2,1/4],
[5/4],-b^2*x^4)+1/3*b*(-signum(b^2*x^4+1))^(1/2)/signum(b^2*x^4+1)^(1/2)*x
^3*hypergeom([-1/2,3/4],[7/4],-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.55

$$\int \left(1+bx^2\right)\sqrt{-1-b^2x^4}\,dx$$

$$= \frac{2\sqrt{-b^2}(5\,b-3)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)|-1)+6\sqrt{-b^2}x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)|-1)+(3\,b}{15\,bx}$$

input
```
integrate((b*x^2+1)*(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(2*sqrt(-b^2)*(5*b - 3)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(
1/4)/x), -1) + 6*sqrt(-b^2)*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1
/4)/x), -1) + (3*b^2*x^4 + 5*b*x^2 + 6)*sqrt(-b^2*x^4 - 1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.38

$$\int \left(1+bx^2\right)\sqrt{-1-b^2x^4}\,dx = \frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)*(-b**2*x**4-1)**(1/2),x)
```

output
```
I*b*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/
(4*gamma(7/4)) + I*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b**2*x**4*exp_p
olar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int (1+bx^2)\sqrt{-1-b^2x^4}\,dx=\int \sqrt{-b^2x^4-1}\left(bx^2+1\right)dx$$

input
```
integrate((b*x^2+1)*(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b^2*x^4 - 1)*(b*x^2 + 1), x)
```

Giac [F]

$$\int (1+bx^2)\sqrt{-1-b^2x^4}\,dx=\int \sqrt{-b^2x^4-1}\left(bx^2+1\right)dx$$

input
```
integrate((b*x^2+1)*(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b^2*x^4 - 1)*(b*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+bx^2)\sqrt{-1-b^2x^4}\,dx=\int \sqrt{-b^2\,x^4-1}\,\left(b\,x^2+1\right)\,dx$$

input
```
int((- b^2*x^4 - 1)^(1/2)*(b*x^2 + 1),x)
```

output
```
int((- b^2*x^4 - 1)^(1/2)*(b*x^2 + 1), x)
```

Reduce [F]

$$\int (1+bx^2)\sqrt{-1-b^2x^4}\,dx = \frac{i\left(3\sqrt{b^2x^4+1}\,b\,x^3+5\sqrt{b^2x^4+1}\,x+10\left(\int \frac{\sqrt{b^2x^4+1}}{b^2x^4+1}dx\right)+6\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^2x^4+1}dx\right)b\right)}{15}$$

input

```
int((b*x^2+1)*(-b^2*x^4-1)^(1/2),x)
```

output

```
(i*(3*sqrt(b**2*x**4 + 1)*b*x**3 + 5*sqrt(b**2*x**4 + 1)*x + 10*int(sqrt(b
**2*x**4 + 1)/(b**2*x**4 + 1),x) + 6*int((sqrt(b**2*x**4 + 1)*x**2)/(b**2*
x**4 + 1),x)*b))/15
```

3.289 $\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx=-\frac{x\sqrt{-1-b^2x^4}}{1+bx^2}-\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}}$$

output
```
-x*(-b^2*x^4-1)^(1/2)/(b*x^2+1)-(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*
EllipticE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)
+(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/2
)*x),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.49

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx=\frac{\sqrt{1+b^2x^4}\left(3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)\right)}{3\sqrt{-1-b^2x^4}}$$

input
```
Integrate[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

output
```
(Sqrt[1 + b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + b*x
^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)]))/(3*Sqrt[-1 - b^2*x^4])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}dx$$

$$\Big\downarrow \text{1512}$$

$$2\int \frac{1}{\sqrt{-b^2x^4-1}}dx-\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx$$

$$\Big\downarrow \text{1510}$$

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}$$

input
```
Int[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

output

```
-((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2)) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1
 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x
^4]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[S
qrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])
```

Defintions of rubi rules used

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.58

method	result	size
meijerg	$\frac{b\sqrt{\operatorname{signum}(b^2x^4+1)}\,x^3\operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^2x^4\right)}{3\sqrt{-\operatorname{signum}(b^2x^4+1)}}+\frac{\sqrt{\operatorname{signum}(b^2x^4+1)}\,x\operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-b^2x^4\right)}{\sqrt{-\operatorname{signum}(b^2x^4+1)}}$	90
default	$\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-ib},i\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}-\frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-ib},i\right)-\operatorname{EllipticE}\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$	122
elliptic	$\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-ib},i\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}-\frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-ib},i\right)-\operatorname{EllipticE}\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$	122

input

```
int((b*x^2+1)/(-b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/3*b/(-signum(b^2*x^4+1))^(1/2)*signum(b^2*x^4+1)^(1/2)*x^3*hypergeom([1/
2,3/4],[7/4],-b^2*x^4)+1/(-signum(b^2*x^4+1))^(1/2)*signum(b^2*x^4+1)^(1/2
)*x*hypergeom([1/4,1/2],[5/4],-b^2*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.54

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx =$$

$$-\frac{\sqrt{-b^2}(b-1)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right) \mid -1) + \sqrt{-b^2}x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} E(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right) \mid -1) + \sqrt{-b^2}}{bx}$$

input

```
integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

output

```
-(sqrt(-b^2)*(b - 1)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x),
 -1) + sqrt(-b^2)*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1
) + sqrt(-b^2*x^4 - 1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = -\frac{ibx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\right| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{1}{2}\\ \frac{5}{4}\end{matrix}\right| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

output

```
-I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/
(4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_po
lar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx=\int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

Giac [F]

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx=\int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx=\int \frac{b\,x^2+1}{\sqrt{-b^2\,x^4-1}}\,dx$$

input
```
int((b*x^2 + 1)/(- b^2*x^4 - 1)^(1/2),x)
```

output
```
int((b*x^2 + 1)/(- b^2*x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = -i\left(\int \frac{\sqrt{b^2x^4+1}}{b^2x^4+1}dx + \left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^2x^4+1}dx\right)b\right)$$

input
```
int((b*x^2+1)/(-b^2*x^4-1)^(1/2),x)
```

output
```
 - i*(int(sqrt(b**2*x**4 + 1)/(b**2*x**4 + 1),x) + int((sqrt(b**2*x**4 + 1
)*x**2)/(b**2*x**4 + 1),x)*b)
```

3.290 $\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{3/2}}\,dx=-\frac{x\left(1+bx^2\right)}{2\sqrt{-1-b^2x^4}}-\frac{x\sqrt{-1-b^2x^4}}{2\left(1+bx^2\right)}-\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)\middle|\frac{1}{2}\right)}{2\sqrt{b}\sqrt{-1-b^2x^4}}$$

output
```
-1/2*x*(b*x^2+1)/(-b^2*x^4-1)^(1/2)-x*(-b^2*x^4-1)^(1/2)/(2*b*x^2+2)-1/2*(
b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/2)*x)
),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{3/2}}\,dx=$$

$$-\frac{x\left(3+3\sqrt{1+b^2x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+2bx^2\sqrt{1+b^2x^4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\right.\right.}{6\sqrt{-1-b^2x^4}}$$

input
```
Integrate[(1 + b*x^2)/(-1 - b^2*x^4)^(3/2),x]
```

output
```
-1/6*(x*(3 + 3*Sqrt[1 + b^2*x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^
4)] + 2*b*x^2*Sqrt[1 + b^2*x^4]*Hypergeometric2F1[3/4, 3/2, 7/4, -(b^2*x^4
)]))/Sqrt[-1 - b^2*x^4]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = $\{1493, 25, 1510\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\left(-b^2x^4-1\right)^{3/2}}dx$$

$$\Big\downarrow \text{1493}$$

$$\frac{1}{2}\int -\frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx-\frac{x\left(bx^2+1\right)}{2\sqrt{-b^2x^4-1}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx-\frac{x\left(bx^2+1\right)}{2\sqrt{-b^2x^4-1}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{1}{2}\left(-\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}\right)-\frac{x\left(bx^2+1\right)}{2\sqrt{-b^2x^4-1}}$$

input
```
Int[(1 + b*x^2)/(-1 - b^2*x^4)^(3/2),x]
```

output
```
-1/2*(x*(1 + b*x^2))/Sqrt[-1 - b^2*x^4] + (-((x*Sqrt[-1 - b^2*x^4])/(1 + b
*x^2)) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan
[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

method	result
meijerg	$\frac{\operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{3}{2}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],-b^{2} x^{4}\right)}{\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{3}{2}}}+\frac{b \operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{3}{2}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{7}{4}\right],-b^{2} x^{4}\right)}{3\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{3}{2}}}$
elliptic	$\frac{2 b^{2}\left(-\frac{x^{3}}{4 b}-\frac{x}{4 b^{2}}\right)}{\sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}-\frac{\sqrt{i b\,x^{2}+1}\,\sqrt{-i b\,x^{2}+1}\,\operatorname{EllipticF}\left(x\sqrt{-i b},i\right)}{2\sqrt{-i b}\,\sqrt{-b^{2} x^{4}-1}}-\frac{i\sqrt{i b\,x^{2}+1}\,\sqrt{-i b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-i b},i\right)-\operatorname{EllipticE}(x\sqrt{-ib}}{2\sqrt{-i b}\,\sqrt{-b^{2} x^{4}-1}}$
default	$-\frac{x}{2\sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}-\frac{\sqrt{i b\,x^{2}+1}\,\sqrt{-i b\,x^{2}+1}\,\operatorname{EllipticF}\left(x\sqrt{-i b},i\right)}{2\sqrt{-i b}\,\sqrt{-b^{2} x^{4}-1}}+b\left(-\frac{x^{3}}{2\sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}-\frac{i\sqrt{i b\,x^{2}+1}\,\sqrt{-i b\,x^{2}+1}\,(\mathrm{Ellipt}}{2\sqrt{-ib}}\right.$

input
```
int((b*x^2+1)/(-b^2*x^4-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-signum(b^2*x^4+1))^(3/2)*signum(b^2*x^4+1)^(3/2)*x*hypergeom([1/4,3/2]
,[5/4],-b^2*x^4)+1/3*b/(-signum(b^2*x^4+1))^(3/2)*signum(b^2*x^4+1)^(3/2)*
x^3*hypergeom([3/4,3/2],[7/4],-b^2*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{3/2}}\,dx = \frac{(i\,b^3x^4+i\,b)(-b^2)^{\frac{3}{4}}\,E\left(\arcsin\left((-b^2)^{\frac{1}{4}}\,x\right)\,|-1\right)+\left(-i\,(b^3+b^2)x^4-i\,b-i\right)(-b^2)^{\frac{3}{4}}}{2\,(b^4x^4+b^2)}$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((I*b^3*x^4 + I*b)*(-b^2)^(3/4)*elliptic_e(arcsin((-b^2)^(1/4)*x), -1)
 + (-I*(b^3 + b^2)*x^4 - I*b - I)*(-b^2)^(3/4)*elliptic_f(arcsin((-b^2)^(1
/4)*x), -1) + (b^3*x^3 + b^2*x)*sqrt(-b^2*x^4 - 1))/(b^4*x^4 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.95 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.57

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{3/2}}\,dx = \frac{ibx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ix\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(-b**2*x**4-1)**(3/2),x)
```

output
```
I*b*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b**2*x**4*exp_polar(I*pi))/(
4*gamma(7/4)) + I*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b**2*x**4*exp_pol
ar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{(-1-b^2x^4)^{3/2}}\,dx=\int \frac{bx^2+1}{(-b^2x^4-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 - 1)^(3/2), x)
```

Giac [F]

$$\int \frac{1+bx^2}{(-1-b^2x^4)^{3/2}}\,dx=\int \frac{bx^2+1}{(-b^2x^4-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 - 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{(-1-b^2x^4)^{3/2}}\,dx=\int \frac{b\,x^2+1}{(-b^2\,x^4-1)^{3/2}}\,dx$$

input
```
int((b*x^2 + 1)/(- b^2*x^4 - 1)^(3/2),x)
```

output
```
int((b*x^2 + 1)/(- b^2*x^4 - 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{3/2}}\,dx = i\left(\int \frac{\sqrt{b^2x^4+1}}{b^4x^8+2b^2x^4+1}dx + \left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^4x^8+2b^2x^4+1}dx\right)b\right)$$

input

```
int((b*x^2+1)/(-b^2*x^4-1)^(3/2),x)
```

output

```
i*(int(sqrt(b**2*x**4 + 1)/(b**4*x**8 + 2*b**2*x**4 + 1),x) + int((sqrt(b*
*2*x**4 + 1)*x**2)/(b**4*x**8 + 2*b**2*x**4 + 1),x)*b)
```

3.291 $\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 216

$$\begin{aligned}\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{5/2}}\,dx =& -\frac{x(1+bx^2)}{6\left(-1-b^2x^4\right)^{3/2}}+\frac{x(5+3bx^2)}{12\sqrt{-1-b^2x^4}} \\ &+\frac{x\sqrt{-1-b^2x^4}}{4\left(1+bx^2\right)}+\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)\middle|\frac{1}{2}\right)}{4\sqrt{b}\sqrt{-1-b^2x^4}} \\ &+\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{12\sqrt{b}\sqrt{-1-b^2x^4}}\end{aligned}$$

output

```
-1/6*x*(b*x^2+1)/(-b^2*x^4-1)^(3/2)+1/12*x*(3*b*x^2+5)/(-b^2*x^4-1)^(1/2)+
x*(-b^2*x^4-1)^(1/2)/(4*b*x^2+4)+1/4*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(
1/2)*EllipticE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^
(1/2)+1/12*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arc
tan(b^(1/2)*x),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.46

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{5/2}}\,dx =$$
$$-\frac{x\left(7+5b^2x^4+5\left(1+b^2x^4\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+4bx^2\left(1+b^2x^4\right)^{3/2}\text{Hypergeometric2}}{12\left(-1-b^2x^4\right)^{3/2}}$$

input
```
Integrate[(1 + b*x^2)/(-1 - b^2*x^4)^(5/2),x]
```

output
```
-1/12*(x*(7 + 5*b^2*x^4 + 5*(1 + b^2*x^4)^(3/2)*Hypergeometric2F1[1/4, 1/2
, 5/4, -(b^2*x^4)] + 4*b*x^2*(1 + b^2*x^4)^(3/2)*Hypergeometric2F1[3/4, 5/
2, 7/4, -(b^2*x^4)]))/(-1 - b^2*x^4)^(3/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1493, 25, 1493, 25, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\left(-b^2x^4-1\right)^{5/2}}dx$$
$$\Big\downarrow \text{1493}$$
$$\frac{1}{6}\int -\frac{3bx^2+5}{\left(-b^2x^4-1\right)^{3/2}}dx-\frac{x\left(bx^2+1\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$
$$\Big\downarrow \text{25}$$
$$-\frac{1}{6}\int \frac{3bx^2+5}{\left(-b^2x^4-1\right)^{3/2}}dx-\frac{x\left(bx^2+1\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$

$$\Big\downarrow \ 1493$$

$$\frac{1}{6}\left(\frac{x\left(3bx^2+5\right)}{2\sqrt{-b^2x^4-1}}-\frac{1}{2}\int -\frac{5-3bx^2}{\sqrt{-b^2x^4-1}}dx\right)-\frac{x\left(bx^2+1\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{5-3bx^2}{\sqrt{-b^2x^4-1}}dx+\frac{x\left(3bx^2+5\right)}{2\sqrt{-b^2x^4-1}}\right)-\frac{x\left(bx^2+1\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$

$$\Big\downarrow \ 1512$$

$$\frac{1}{6}\left(\frac{1}{2}\left(2\int \frac{1}{\sqrt{-b^2x^4-1}}dx+3\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx\right)+\frac{x\left(3bx^2+5\right)}{2\sqrt{-b^2x^4-1}}\right)-\frac{x\left(bx^2+1\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$

$$\Big\downarrow \ 761$$

$$\frac{1}{6}\left(\frac{1}{2}\left(3\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx+\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}\right)+\frac{x\left(3bx^2+5\right)}{2\sqrt{-b^2x^4-1}}\right)-$$
$$\frac{x\left(bx^2+1\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$

$$\Big\downarrow \ 1510$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+3\left(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+\right.\right.\right.$$
$$\frac{x\left(bx^2+1\right)}{6\left(-b^2x^4-1\right)^{3/2}}$$

input

```
Int[(1 + b*x^2)/(-1 - b^2*x^4)^(5/2),x]
```

output

```
-1/6*(x*(1 + b*x^2))/(-1 - b^2*x^4)^(3/2) + ((x*(5 + 3*b*x^2))/(2*Sqrt[-1
- b^2*x^4]) + (3*((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(
1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*
Sqrt[-1 - b^2*x^4])) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*Elli
pticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4]))/2)/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 1.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.42

method	result
meijerg	$\frac{\operatorname{signum}(b^2x^4+1)^{\frac{5}{2}}x\operatorname{hypergeom}([\frac{1}{4},\frac{5}{2}],[\frac{5}{4}],-b^2x^4)}{(-\operatorname{signum}(b^2x^4+1))^{\frac{5}{2}}}+\frac{b\operatorname{signum}(b^2x^4+1)^{\frac{5}{2}}x^3\operatorname{hypergeom}([\frac{3}{4},\frac{5}{2}],[\frac{7}{4}],-b^2x^4)}{3(-\operatorname{signum}(b^2x^4+1))^{\frac{5}{2}}}$
elliptic	$\frac{\left(-\frac{x^3}{6b^3}-\frac{x}{6b^4}\right)\sqrt{-b^2x^4-1}}{\left(x^4+\frac{1}{b^2}\right)^2}+\frac{2b^2\left(\frac{x^3}{8b}+\frac{5x}{24b^2}\right)}{\sqrt{-\left(x^4+\frac{1}{b^2}\right)b^2}}+\frac{5\sqrt{ibx^2+1}\sqrt{-ibx^2+1}\operatorname{EllipticF}(x\sqrt{-ib},i)}{12\sqrt{-ib}\sqrt{-b^2x^4-1}}+\frac{i\sqrt{ibx^2+1}\sqrt{-ibx^2+1}\,(\mathrm{Elliptic}}{4\sqrt{-ib}}$
default	$-\frac{x\sqrt{-b^2x^4-1}}{6b^4\left(x^4+\frac{1}{b^2}\right)^2}+\frac{5x}{12\sqrt{-\left(x^4+\frac{1}{b^2}\right)b^2}}+\frac{5\sqrt{ibx^2+1}\sqrt{-ibx^2+1}\operatorname{EllipticF}(x\sqrt{-ib},i)}{12\sqrt{-ib}\sqrt{-b^2x^4-1}}+b\left(-\frac{x^3\sqrt{-b^2x^4-1}}{6b^4\left(x^4+\frac{1}{b^2}\right)^2}+\frac{x^3}{4\sqrt{-(x^4+}}\right.$

input
```
int((b*x^2+1)/(-b^2*x^4-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-signum(b^2*x^4+1))^(5/2)*signum(b^2*x^4+1)^(5/2)*x*hypergeom([1/4,5/2]
,[5/4],-b^2*x^4)+1/3*b/(-signum(b^2*x^4+1))^(5/2)*signum(b^2*x^4+1)^(5/2)*
x^3*hypergeom([3/4,5/2],[7/4],-b^2*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int \frac{1+bx^2}{(-1-b^2x^4)^{5/2}}\,dx =$$

$$-\frac{3\,(i\,b^5x^8+2i\,b^3x^4+i\,b)(-b^2)^{\frac{3}{4}}\,E(\arcsin\left((-b^2)^{\frac{1}{4}}\,x\right)\,|-1)-(i\,(3\,b^5+5\,b^4)x^8+2i\,(3\,b^3+5\,b^2)x^4+3i\,b}{12\,(b^6x^8+2\,b^4x^4+}$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*(I*b^5*x^8 + 2*I*b^3*x^4 + I*b)*(-b^2)^(3/4)*elliptic_e(arcsin((-
b^2)^(1/4)*x), -1) - (I*(3*b^5 + 5*b^4)*x^8 + 2*I*(3*b^3 + 5*b^2)*x^4 + 3*
I*b + 5*I)*(-b^2)^(3/4)*elliptic_f(arcsin((-b^2)^(1/4)*x), -1) + (3*b^5*x^
7 + 5*b^4*x^5 + 5*b^3*x^3 + 7*b^2*x)*sqrt(-b^2*x^4 - 1))/(b^6*x^8 + 2*b^4*
x^4 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.33

$$\int \frac{1+bx^2}{(-1-b^2x^4)^{5/2}}\,dx = -\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(-b**2*x**4-1)**(5/2),x)
```

output
```
-I*b*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b**2*x**4*exp_polar(I*pi))/
(4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b**2*x**4*exp_po
lar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{(-1-b^2x^4)^{5/2}}\,dx = \int \frac{bx^2+1}{(-b^2x^4-1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 - 1)^(5/2), x)
```

Giac [F]

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{5/2}}\,dx=\int \frac{bx^2+1}{\left(-b^2x^4-1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 - 1)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{5/2}}\,dx=\int \frac{b\,x^2+1}{\left(-b^2\,x^4-1\right)^{5/2}}\,dx$$

input
```
int((b*x^2 + 1)/(- b^2*x^4 - 1)^(5/2),x)
```

output
```
int((b*x^2 + 1)/(- b^2*x^4 - 1)^(5/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{5/2}}\,dx=-i\left(\int \frac{\sqrt{b^2x^4+1}}{b^6x^{12}+3b^4x^8+3b^2x^4+1}dx+\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^6x^{12}+3b^4x^8+3b^2x^4+1}dx\right)b\right)$$

input
```
int((b*x^2+1)/(-b^2*x^4-1)^(5/2),x)
```

output
```
 - i*(int(sqrt(b**2*x**4 + 1)/(b**6*x**12 + 3*b**4*x**8 + 3*b**2*x**4 + 1)
,x) + int((sqrt(b**2*x**4 + 1)*x**2)/(b**6*x**12 + 3*b**4*x**8 + 3*b**2*x*
*4 + 1),x)*b)
```

3.292 $\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{7/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 243

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{7/2}} \, dx = -\frac{x(1+bx^2)}{10\left(-1-b^2x^4\right)^{5/2}}+\frac{x(9+7bx^2)}{60\left(-1-b^2x^4\right)^{3/2}} -\frac{x(15+7bx^2)}{40\sqrt{-1-b^2x^4}}-\frac{7x\sqrt{-1-b^2x^4}}{40\left(1+bx^2\right)}-\frac{7(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{40\sqrt{b}\sqrt{-1-b^2x^4}} -\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{10\sqrt{b}\sqrt{-1-b^2x^4}}$$

output
```
-1/10*x*(b*x^2+1)/(-b^2*x^4-1)^(5/2)+1/60*x*(7*b*x^2+9)/(-b^2*x^4-1)^(3/2)
-1/40*x*(7*b*x^2+15)/(-b^2*x^4-1)^(1/2)-7*x*(-b^2*x^4-1)^(1/2)/(40*b*x^2+4
0)-7/40*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b
^(1/2)*x)),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1/2)-1/10*(b*x^2+1)*((b^2*x^
4+1)/(b*x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b
^(1/2)/(-b^2*x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.44

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{7/2}}\,dx =$$

$$-\frac{x\Big(75+108b^2x^4+45b^4x^8+45(1+b^2x^4)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+40bx^2(1+b^2x^4)^{5/2}\,\text{Hy}}{120\left(-1-b^2x^4\right)^{5/2}}$$

input
```
Integrate[(1 + b*x^2)/(-1 - b^2*x^4)^(7/2),x]
```

output
```
-1/120*(x*(75 + 108*b^2*x^4 + 45*b^4*x^8 + 45*(1 + b^2*x^4)^(5/2)*Hypergeo
metric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + 40*b*x^2*(1 + b^2*x^4)^(5/2)*Hyperg
eometric2F1[3/4, 7/2, 7/4, -(b^2*x^4)]))/(-1 - b^2*x^4)^(5/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1493, 25, 1493, 27, 1493, 25, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\left(-b^2x^4-1\right)^{7/2}}dx$$

$$\Big\downarrow \text{1493}$$

$$\frac{1}{10}\int -\frac{7bx^2+9}{\left(-b^2x^4-1\right)^{5/2}}dx-\frac{x\left(bx^2+1\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{10}\int \frac{7bx^2+9}{\left(-b^2x^4-1\right)^{5/2}}dx-\frac{x\left(bx^2+1\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \ 1493$$

$$\frac{1}{10}\left(\frac{x\left(7bx^2+9\right)}{6\left(-b^2x^4-1\right)^{3/2}}-\frac{1}{6}\int -\frac{3\left(7bx^2+15\right)}{\left(-b^2x^4-1\right)^{3/2}}dx\right)-\frac{x\left(bx^2+1\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{10}\left(\frac{1}{2}\int \frac{7bx^2+15}{\left(-b^2x^4-1\right)^{3/2}}dx+\frac{x\left(7bx^2+9\right)}{6\left(-b^2x^4-1\right)^{3/2}}\right)-\frac{x\left(bx^2+1\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \ 1493$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\int -\frac{15-7bx^2}{\sqrt{-b^2x^4-1}}dx-\frac{x\left(7bx^2+15\right)}{2\sqrt{-b^2x^4-1}}\right)+\frac{x\left(7bx^2+9\right)}{6\left(-b^2x^4-1\right)^{3/2}}\right)-\frac{x\left(bx^2+1\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{10}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{15-7bx^2}{\sqrt{-b^2x^4-1}}dx-\frac{x\left(7bx^2+15\right)}{2\sqrt{-b^2x^4-1}}\right)+\frac{x\left(7bx^2+9\right)}{6\left(-b^2x^4-1\right)^{3/2}}\right)-\frac{x\left(bx^2+1\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \ 1512$$

$$\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\left(-8\int \frac{1}{\sqrt{-b^2x^4-1}}dx-7\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx\right)-\frac{x\left(7bx^2+15\right)}{2\sqrt{-b^2x^4-1}}\right)+\frac{x\left(7bx^2+9\right)}{6\left(-b^2x^4-1\right)^{3/2}}\right)-\frac{x\left(bx^2+1\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \ 761$$

$$\frac{1}{10}\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\left(-7\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx-\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}\right)-\frac{x\left(7bx^2+15\right)}{2\sqrt{-b^2x^4-1}}\Bigg) -$$

$$\frac{x\left(bx^2+1\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

$$\Big\downarrow \ 1510$$

$$\frac{1}{10}\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(-\frac{4\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-7\Bigg(\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

$$\frac{x\left(bx^2+1\right)}{10\left(-b^2x^4-1\right)^{5/2}}$$

input
```
Int[(1 + b*x^2)/(-1 - b^2*x^4)^(7/2),x]
```

output
```
-1/10*(x*(1 + b*x^2))/(-1 - b^2*x^4)^(5/2) + ((x*(9 + 7*b*x^2))/(6*(-1 - b
^2*x^4)^(3/2)) + (-1/2*(x*(15 + 7*b*x^2))/Sqrt[-1 - b^2*x^4] + (-7*((x*Sqr
t[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)
^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])) - (
4*(1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]
*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4]))/2)/2)/10
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 1.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.37

method	result
meijerg	$\frac{\operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{7}{2}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{7}{2}\right],\left[\frac{5}{4}\right],-b^{2} x^{4}\right)}{\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{7}{2}}}+\frac{b \operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{7}{2}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{7}{2}\right],\left[\frac{7}{4}\right],-b^{2} x^{4}\right)}{3\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{7}{2}}}$
elliptic	$\frac{\left(\frac{x^{3}}{10 b^{5}}+\frac{x}{10 b^{6}}\right) \sqrt{-b^{2} x^{4}-1}}{\left(x^{4}+\frac{1}{b^{2}}\right)^{3}}+\frac{\left(\frac{7 x^{3}}{60 b^{3}}+\frac{3 x}{20 b^{4}}\right) \sqrt{-b^{2} x^{4}-1}}{\left(x^{4}+\frac{1}{b^{2}}\right)^{2}}+\frac{2 b^{2}\left(-\frac{7 x^{3}}{80 b}-\frac{3 x}{16 b^{2}}\right)}{\sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}-\frac{3 \sqrt{i b x^{2}+1} \sqrt{-i b x^{2}+1} \operatorname{EllipticF}\left(x \sqrt{-i b}, i\right)}{8 \sqrt{-i b} \sqrt{-b^{2} x^{4}-1}}-$
default	$\frac{x \sqrt{-b^{2} x^{4}-1}}{10 b^{6}\left(x^{4}+\frac{1}{b^{2}}\right)^{3}}+\frac{3 x \sqrt{-b^{2} x^{4}-1}}{20 b^{4}\left(x^{4}+\frac{1}{b^{2}}\right)^{2}}-\frac{3 x}{8 \sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}-\frac{3 \sqrt{i b x^{2}+1} \sqrt{-i b x^{2}+1} \operatorname{EllipticF}\left(x \sqrt{-i b}, i\right)}{8 \sqrt{-i b} \sqrt{-b^{2} x^{4}-1}}+b\left(\frac{x^{3} \sqrt{-b^{2} x^{4}-1}}{10 b^{6}\left(x^{4}+\frac{1}{b^{2}}\right)^{3}}\right.$

input
```
int((b*x^2+1)/(-b^2*x^4-1)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-signum(b^2*x^4+1))^(7/2)*signum(b^2*x^4+1)^(7/2)*x*hypergeom([1/4,7/2]
,[5/4],-b^2*x^4)+1/3*b/(-signum(b^2*x^4+1))^(7/2)*signum(b^2*x^4+1)^(7/2)*
x^3*hypergeom([3/4,7/2],[7/4],-b^2*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.89

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{7/2}}\,dx =$$

$$-\frac{21\,(-i\,b^7x^{12}-3i\,b^5x^8-3i\,b^3x^4-i\,b)(-b^2)^{\frac{3}{4}}\,E(\arcsin\left((-b^2)^{\frac{1}{4}}\,x\right)\,|\,-1)+3\,(i\,(7\,b^7+15\,b^6)x^{12}+3i\,(7\,b^5}{}$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(7/2),x, algorithm="fricas")
```

output
```
-1/120*(21*(-I*b^7*x^12 - 3*I*b^5*x^8 - 3*I*b^3*x^4 - I*b)*(-b^2)^(3/4)*el
liptic_e(arcsin((-b^2)^(1/4)*x), -1) + 3*(I*(7*b^7 + 15*b^6)*x^12 + 3*I*(7
*b^5 + 15*b^4)*x^8 + 3*I*(7*b^3 + 15*b^2)*x^4 + 7*I*b + 15*I)*(-b^2)^(3/4)
*elliptic_f(arcsin((-b^2)^(1/4)*x), -1) - (21*b^7*x^11 + 45*b^6*x^9 + 56*b
^5*x^7 + 108*b^4*x^5 + 47*b^3*x^3 + 75*b^2*x)*sqrt(-b^2*x^4 - 1))/(b^8*x^1
2 + 3*b^6*x^8 + 3*b^4*x^4 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 33.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.29

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{7/2}}\,dx = \frac{ibx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\\frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ix\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(-b**2*x**4-1)**(7/2),x)
```

output
```
I*b*x**3*gamma(3/4)*hyper((3/4, 7/2), (7/4,), b**2*x**4*exp_polar(I*pi))/(
4*gamma(7/4)) + I*x*gamma(1/4)*hyper((1/4, 7/2), (5/4,), b**2*x**4*exp_pol
ar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{7/2}}\,dx = \int \frac{bx^2+1}{\left(-b^2x^4-1\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 - 1)^(7/2), x)
```

Giac [F]

$$\int \frac{1+bx^2}{(-1-b^2x^4)^{7/2}}\,dx = \int \frac{bx^2+1}{(-b^2x^4-1)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 - 1)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{(-1-b^2x^4)^{7/2}}\,dx = \int \frac{b\,x^2+1}{(-b^2\,x^4-1)^{7/2}}\,dx$$

input
```
int((b*x^2 + 1)/(- b^2*x^4 - 1)^(7/2),x)
```

output
```
int((b*x^2 + 1)/(- b^2*x^4 - 1)^(7/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{(-1-b^2x^4)^{7/2}}\,dx = i\left(\int \frac{\sqrt{b^2x^4+1}}{b^8x^{16}+4b^6x^{12}+6b^4x^8+4b^2x^4+1}dx + \left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^8x^{16}+4b^6x^{12}+6b^4x^8+4b^2x^4+1}dx\right)b\right)$$

input
```
int((b*x^2+1)/(-b^2*x^4-1)^(7/2),x)
```

output
```
i*(int(sqrt(b**2*x**4 + 1)/(b**8*x**16 + 4*b**6*x**12 + 6*b**4*x**8 + 4*b*
*2*x**4 + 1),x) + int((sqrt(b**2*x**4 + 1)*x**2)/(b**8*x**16 + 4*b**6*x**1
2 + 6*b**4*x**8 + 4*b**2*x**4 + 1),x)*b)
```

3.293 $\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{9/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 270

$$\begin{aligned}\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{9/2}}\,dx &= -\frac{x(1+bx^2)}{14\left(-1-b^2x^4\right)^{7/2}}+\frac{x(13+11bx^2)}{140\left(-1-b^2x^4\right)^{5/2}} \\ &\quad -\frac{x(117+77bx^2)}{840\left(-1-b^2x^4\right)^{3/2}}+\frac{x(195+77bx^2)}{560\sqrt{-1-b^2x^4}}+\frac{11x\sqrt{-1-b^2x^4}}{80\left(1+bx^2\right)} \\ &\quad +\frac{11(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{80\sqrt{b}\sqrt{-1-b^2x^4}} \\ &\quad +\frac{59(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{560\sqrt{b}\sqrt{-1-b^2x^4}}\end{aligned}$$

output
```
-1/14*x*(b*x^2+1)/(-b^2*x^4-1)^(7/2)+1/140*x*(11*b*x^2+13)/(-b^2*x^4-1)^(5
/2)-1/840*x*(77*b*x^2+117)/(-b^2*x^4-1)^(3/2)+1/560*x*(77*b*x^2+195)/(-b^2
*x^4-1)^(1/2)+11*x*(-b^2*x^4-1)^(1/2)/(80*b*x^2+80)+11/80*(b*x^2+1)*((b^2*
x^4+1)/(b*x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/2)*x)),1/2*2^(1/2))/
b^(1/2)/(-b^2*x^4-1)^(1/2)+59/560*(b*x^2+1)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2
)*InverseJacobiAM(2*arctan(b^(1/2)*x),1/2*2^(1/2))/b^(1/2)/(-b^2*x^4-1)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.43

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{9/2}}\,dx =$$

$$-\frac{x\Big(1095+2379b^2x^4+1989b^4x^8+585b^6x^{12}+585(1+b^2x^4)^{7/2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+56}{1680\left(-1-b^2x^4\right)^{7/2}}$$

input
```
Integrate[(1 + b*x^2)/(-1 - b^2*x^4)^(9/2),x]
```

output
```
-1/1680*(x*(1095 + 2379*b^2*x^4 + 1989*b^4*x^8 + 585*b^6*x^12 + 585*(1 + b
^2*x^4)^(7/2)*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + 560*b*x^2*(1
+ b^2*x^4)^(7/2)*Hypergeometric2F1[3/4, 9/2, 7/4, -(b^2*x^4)]))/(-1 - b^2*
x^4)^(7/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1493, 25, 1493, 25, 1493, 27, 1493, 25, 1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\left(-b^2x^4-1\right)^{9/2}}\,dx$$

$$\Big\downarrow \text{1493}$$

$$\frac{1}{14}\int -\frac{11bx^2+13}{\left(-b^2x^4-1\right)^{7/2}}dx-\frac{x\left(bx^2+1\right)}{14\left(-b^2x^4-1\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{14}\int \frac{11bx^2+13}{\left(-b^2x^4-1\right)^{7/2}}dx-\frac{x\left(bx^2+1\right)}{14\left(-b^2x^4-1\right)^{7/2}}$$

$$\Big\downarrow 1493$$

$$\frac{1}{14}\left(\frac{x\left(11bx^2+13\right)}{10\left(-b^2x^4-1\right)^{5/2}}-\frac{1}{10}\int -\frac{77bx^2+117}{\left(-b^2x^4-1\right)^{5/2}}dx\right)-\frac{x\left(bx^2+1\right)}{14\left(-b^2x^4-1\right)^{7/2}}$$

$$\Big\downarrow 25$$

$$\frac{1}{14}\left(\frac{1}{10}\int \frac{77bx^2+117}{\left(-b^2x^4-1\right)^{5/2}}dx+\frac{x\left(11bx^2+13\right)}{10\left(-b^2x^4-1\right)^{5/2}}\right)-\frac{x\left(bx^2+1\right)}{14\left(-b^2x^4-1\right)^{7/2}}$$

$$\Big\downarrow 1493$$

$$\frac{1}{14}\left(\frac{1}{10}\left(\frac{1}{6}\int -\frac{3\left(77bx^2+195\right)}{\left(-b^2x^4-1\right)^{3/2}}dx-\frac{x\left(77bx^2+117\right)}{6\left(-b^2x^4-1\right)^{3/2}}\right)+\frac{x\left(11bx^2+13\right)}{10\left(-b^2x^4-1\right)^{5/2}}\right)-\frac{x\left(bx^2+1\right)}{14\left(-b^2x^4-1\right)^{7/2}}$$

$$\Big\downarrow 27$$

$$\frac{1}{14}\left(\frac{1}{10}\left(-\frac{1}{2}\int \frac{77bx^2+195}{\left(-b^2x^4-1\right)^{3/2}}dx-\frac{x\left(77bx^2+117\right)}{6\left(-b^2x^4-1\right)^{3/2}}\right)+\frac{x\left(11bx^2+13\right)}{10\left(-b^2x^4-1\right)^{5/2}}\right)-\frac{x\left(bx^2+1\right)}{14\left(-b^2x^4-1\right)^{7/2}}$$

$$\Big\downarrow 1493$$

$$\frac{1}{14}\left(\frac{1}{10}\left(\frac{1}{2}\left(\frac{x\left(77bx^2+195\right)}{2\sqrt{-b^2x^4-1}}-\frac{1}{2}\int -\frac{195-77bx^2}{\sqrt{-b^2x^4-1}}dx\right)-\frac{x\left(77bx^2+117\right)}{6\left(-b^2x^4-1\right)^{3/2}}\right)+\frac{x\left(11bx^2+13\right)}{10\left(-b^2x^4-1\right)^{5/2}}\right)-\frac{x\left(bx^2+1\right)}{14\left(-b^2x^4-1\right)^{7/2}}$$

$$\Big\downarrow 25$$

$$\frac{1}{14}\left(\frac{1}{10}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{195-77bx^2}{\sqrt{-b^2x^4-1}}dx+\frac{x\left(77bx^2+195\right)}{2\sqrt{-b^2x^4-1}}\right)-\frac{x\left(77bx^2+117\right)}{6\left(-b^2x^4-1\right)^{3/2}}\right)+\frac{x\left(11bx^2+13\right)}{10\left(-b^2x^4-1\right)^{5/2}}\right)-\frac{x\left(bx^2+1\right)}{14\left(-b^2x^4-1\right)^{7/2}}$$

$$\Big\downarrow 1512$$

$$\frac{1}{14}\Bigg(\frac{1}{10}\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(118\int \frac{1}{\sqrt{-b^2x^4-1}}dx+77\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx\Bigg)+\frac{x(77bx^2+195)}{2\sqrt{-b^2x^4-1}}\Bigg)-\frac{x(77bx^2+117)}{6(-b^2x^4-1)^{3/2}}\Bigg)+\frac{x}{10}$$

$$\frac{x(bx^2+1)}{14(-b^2x^4-1)^{7/2}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{14}\Bigg(\frac{1}{10}\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(77\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx+\frac{59(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}\Bigg)+\frac{x(77bx^2+1}{2\sqrt{-b^2x^4}}$$

$$\frac{x(bx^2+1)}{14(-b^2x^4-1)^{7/2}}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{1}{14}\Bigg(\frac{1}{10}\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(\frac{59(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+77\Bigg(\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\Big(2\arctan\Big(}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

$$\frac{x(bx^2+1)}{14(-b^2x^4-1)^{7/2}}$$

input
```
Int[(1 + b*x^2)/(-1 - b^2*x^4)^(9/2),x]
```

output
```
-1/14*(x*(1 + b*x^2))/(-1 - b^2*x^4)^(7/2) + ((x*(13 + 11*b*x^2))/(10*(-1
- b^2*x^4)^(5/2)) + (-1/6*(x*(117 + 77*b*x^2))/(-1 - b^2*x^4)^(3/2) + ((x*
(195 + 77*b*x^2))/(2*Sqrt[-1 - b^2*x^4]) + (77*((x*Sqrt[-1 - b^2*x^4])/(1
+ b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticE[2*ArcT
an[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])) + (59*(1 + b*x^2)*Sqrt[
(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]
*Sqrt[-1 - b^2*x^4]))/2)/2)/10)/14
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 1.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.33

method	result
meijerg	$\frac{\operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{9}{2}} x \,\text{hypergeom}\left(\left[\frac{1}{4}, \frac{9}{2}\right], \left[\frac{5}{4}\right], -b^{2} x^{4}\right)}{\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{9}{2}}}+\frac{b \operatorname{signum}\left(b^{2} x^{4}+1\right)^{\frac{9}{2}} x^{3} \,\text{hypergeom}\left(\left[\frac{3}{4}, \frac{9}{2}\right], \left[\frac{7}{4}\right], -b^{2} x^{4}\right)}{3\left(-\operatorname{signum}\left(b^{2} x^{4}+1\right)\right)^{\frac{9}{2}}}$
elliptic	$\frac{\left(-\frac{x^{3}}{14 b^{7}}-\frac{x}{14 b^{8}}\right) \sqrt{-b^{2} x^{4}-1}}{\left(x^{4}+\frac{1}{b^{2}}\right)^{4}}+\frac{\left(-\frac{11 x^{3}}{140 b^{5}}-\frac{13 x}{140 b^{6}}\right) \sqrt{-b^{2} x^{4}-1}}{\left(x^{4}+\frac{1}{b^{2}}\right)^{3}}+\frac{\left(-\frac{11 x^{3}}{120 b^{3}}-\frac{39 x}{280 b^{4}}\right) \sqrt{-b^{2} x^{4}-1}}{\left(x^{4}+\frac{1}{b^{2}}\right)^{2}}+\frac{2 b^{2}\left(\frac{11 x^{3}}{160 b}+\frac{39 x}{224 b^{2}}\right)}{\sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}+39\sqrt{ib}$
default	$-\frac{x \sqrt{-b^{2} x^{4}-1}}{14 b^{8}\left(x^{4}+\frac{1}{b^{2}}\right)^{4}}-\frac{13 x \sqrt{-b^{2} x^{4}-1}}{140 b^{6}\left(x^{4}+\frac{1}{b^{2}}\right)^{3}}-\frac{39 x \sqrt{-b^{2} x^{4}-1}}{280 b^{4}\left(x^{4}+\frac{1}{b^{2}}\right)^{2}}+\frac{39 x}{112 \sqrt{-\left(x^{4}+\frac{1}{b^{2}}\right) b^{2}}}+\frac{39 \sqrt{i b\, x^{2}+1}\, \sqrt{-i b\, x^{2}+1}\ \text{EllipticF}\left(x \sqrt{-}\right.}{112 \sqrt{-i b}\, \sqrt{-b^{2} x^{4}-1}}$

input
```
int((b*x^2+1)/(-b^2*x^4-1)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-signum(b^2*x^4+1))^(9/2)*signum(b^2*x^4+1)^(9/2)*x*hypergeom([1/4,9/2]
,[5/4],-b^2*x^4)+1/3*b/(-signum(b^2*x^4+1))^(9/2)*signum(b^2*x^4+1)^(9/2)*
x^3*hypergeom([3/4,9/2],[7/4],-b^2*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.97

$$\int \frac{1+bx^2}{(-1-b^2x^4)^{9/2}}\,dx =$$

$$-\frac{231\,(i\,b^9x^{16}+4i\,b^7x^{12}+6i\,b^5x^8+4i\,b^3x^4+i\,b)(-b^2)^{\frac{3}{4}}\,E(\arcsin\left((-b^2)^{\frac{1}{4}}\,x\right)\,|-1)+3\,(-i\,(77\,b^9+195\,b^8}{}$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(9/2),x, algorithm="fricas")
```

output
```
-1/1680*(231*(I*b^9*x^16 + 4*I*b^7*x^12 + 6*I*b^5*x^8 + 4*I*b^3*x^4 + I*b)
*(-b^2)^(3/4)*elliptic_e(arcsin((-b^2)^(1/4)*x), -1) + 3*(-I*(77*b^9 + 195
*b^8)*x^16 - 4*I*(77*b^7 + 195*b^6)*x^12 - 6*I*(77*b^5 + 195*b^4)*x^8 - 4*
I*(77*b^3 + 195*b^2)*x^4 - 77*I*b - 195*I)*(-b^2)^(3/4)*elliptic_f(arcsin(
(-b^2)^(1/4)*x), -1) + (231*b^9*x^15 + 585*b^8*x^13 + 847*b^7*x^11 + 1989*
b^6*x^9 + 1133*b^5*x^7 + 2379*b^4*x^5 + 637*b^3*x^3 + 1095*b^2*x)*sqrt(-b^
2*x^4 - 1))/(b^10*x^16 + 4*b^8*x^12 + 6*b^6*x^8 + 4*b^4*x^4 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 89.76 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.26

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{9/2}}\,dx = -\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{9}{2}\\ \frac{7}{4}\end{matrix}\middle|\, b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{9}{2}\\ \frac{5}{4}\end{matrix}\middle|\, b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(-b**2*x**4-1)**(9/2),x)
```

output
```
-I*b*x**3*gamma(3/4)*hyper((3/4, 9/2), (7/4,), b**2*x**4*exp_polar(I*pi))/
(4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 9/2), (5/4,), b**2*x**4*exp_po
lar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{9/2}}\,dx = \int \frac{bx^2+1}{\left(-b^2x^4-1\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 - 1)^(9/2), x)
```

Giac [F]

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{9/2}}\,dx=\int \frac{bx^2+1}{\left(-b^2x^4-1\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/(-b^2*x^4 - 1)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{9/2}}\,dx=\int \frac{b\,x^2+1}{\left(-b^2\,x^4-1\right)^{9/2}}\,dx$$

input
```
int((b*x^2 + 1)/(- b^2*x^4 - 1)^(9/2),x)
```

output
```
int((b*x^2 + 1)/(- b^2*x^4 - 1)^(9/2), x)
```

Reduce [F]

$$\int \frac{1+bx^2}{\left(-1-b^2x^4\right)^{9/2}}\,dx=-i\left(\left(\int \frac{\sqrt{b^2x^4+1}}{b^{10}x^{20}+5b^8x^{16}+10b^6x^{12}+10b^4x^8+5b^2x^4+1}dx\right.\right.$$
$$\left.\left.+\left(\int \frac{\sqrt{b^2x^4+1}\,x^2}{b^{10}x^{20}+5b^8x^{16}+10b^6x^{12}+10b^4x^8+5b^2x^4+1}dx\right)b\right)\right)$$

input
```
int((b*x^2+1)/(-b^2*x^4-1)^(9/2),x)
```

output

```
 - i*(int(sqrt(b**2*x**4 + 1)/(b**10*x**20 + 5*b**8*x**16 + 10*b**6*x**12
+ 10*b**4*x**8 + 5*b**2*x**4 + 1),x) + int((sqrt(b**2*x**4 + 1)*x**2)/(b**
10*x**20 + 5*b**8*x**16 + 10*b**6*x**12 + 10*b**4*x**8 + 5*b**2*x**4 + 1),
x)*b)
```

3.294 $\int (d+ex^2)^4 (a+cx^4)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 106

$$\int (d+ex^2)^4 (a+cx^4)\, dx = ad^4x+\frac{4}{3}ad^3ex^3+\frac{1}{5}d^2(cd^2+6ae^2)\,x^5+\frac{4}{7}de(cd^2+ae^2)\,x^7 +\frac{1}{9}e^2(6cd^2+ae^2)\,x^9+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

output
```
a*d^4*x+4/3*a*d^3*e*x^3+1/5*d^2*(6*a*e^2+c*d^2)*x^5+4/7*d*e*(a*e^2+c*d^2)*
x^7+1/9*e^2*(a*e^2+6*c*d^2)*x^9+4/11*c*d*e^3*x^11+1/13*c*e^4*x^13
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^4 (a+cx^4)\, dx = ad^4x+\frac{4}{3}ad^3ex^3+\frac{1}{5}d^2(cd^2+6ae^2)\,x^5+\frac{4}{7}de(cd^2+ae^2)\,x^7 +\frac{1}{9}e^2(6cd^2+ae^2)\,x^9+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

input
```
Integrate[(d + e*x^2)^4*(a + c*x^4),x]
```

output
```
a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^
2 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 +
(c*e^4*x^13)/13
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)\left(d+ex^2\right)^4 dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(e^2x^8\left(ae^2+6cd^2\right)+4dex^6\left(ae^2+cd^2\right)+d^2x^4\left(6ae^2+cd^2\right)+ad^4+4ad^3ex^2+4cde^3x^{10}+ce^4x^{12}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}e^2x^9\left(ae^2+6cd^2\right)+\frac{4}{7}dex^7\left(ae^2+cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+cd^2\right)+ad^4x+\frac{4}{3}ad^3ex^3+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

input
```
Int[(d + e*x^2)^4*(a + c*x^4),x]
```

output
```
a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^
2 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 +
(c*e^4*x^13)/13
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

method	result
norman	$\frac{c\,e^4x^{13}}{13}+\frac{4cd\,e^3x^{11}}{11}+\left(\frac{1}{9}e^4a+\frac{2}{3}c\,d^2e^2\right)x^9+\left(\frac{4}{7}ad\,e^3+\frac{4}{7}c\,d^3e\right)x^7+\left(\frac{6}{5}a\,d^2e^2+\frac{1}{5}d^4c\right)x^5+$
default	$\frac{c\,e^4x^{13}}{13}+\frac{4cd\,e^3x^{11}}{11}+\frac{(e^4a+6c\,d^2e^2)x^9}{9}+\frac{(4ad\,e^3+4c\,d^3e)x^7}{7}+\frac{(6a\,d^2e^2+d^4c)x^5}{5}+\frac{4a\,d^3e\,x^3}{3}+a\,d^4x$
gosper	$\frac{1}{13}c\,e^4x^{13}+\frac{4}{11}cd\,e^3x^{11}+\frac{1}{9}x^9e^4a+\frac{2}{3}x^9c\,d^2e^2+\frac{4}{7}x^7ad\,e^3+\frac{4}{7}x^7c\,d^3e+\frac{6}{5}x^5a\,d^2e^2+\frac{1}{5}x^5d^4c$
risch	$\frac{1}{13}c\,e^4x^{13}+\frac{4}{11}cd\,e^3x^{11}+\frac{1}{9}x^9e^4a+\frac{2}{3}x^9c\,d^2e^2+\frac{4}{7}x^7ad\,e^3+\frac{4}{7}x^7c\,d^3e+\frac{6}{5}x^5a\,d^2e^2+\frac{1}{5}x^5d^4c$
parallelrisch	$\frac{1}{13}c\,e^4x^{13}+\frac{4}{11}cd\,e^3x^{11}+\frac{1}{9}x^9e^4a+\frac{2}{3}x^9c\,d^2e^2+\frac{4}{7}x^7ad\,e^3+\frac{4}{7}x^7c\,d^3e+\frac{6}{5}x^5a\,d^2e^2+\frac{1}{5}x^5d^4c$
orering	$\frac{x(3465e^4c\,x^{12}+16380d\,e^3c\,x^{10}+5005a\,e^4x^8+30030c\,d^2e^2x^8+25740ad\,e^3x^6+25740c\,d^3e\,x^6+54054a\,d^2e^2x^4+9009c\,d^4x^4+60}{45045}$

input
```
int((e*x^2+d)^4*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/13*c*e^4*x^13+4/11*c*d*e^3*x^11+(1/9*e^4*a+2/3*c*d^2*e^2)*x^9+(4/7*a*d*e
^3+4/7*c*d^3*e)*x^7+(6/5*a*d^2*e^2+1/5*d^4*c)*x^5+4/3*a*d^3*e*x^3+a*d^4*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \left(d+ex^2\right)^4\left(a+cx^4\right)\,dx=\frac{1}{13}\,ce^4x^{13}+\frac{4}{11}\,cde^3x^{11}+\frac{1}{9}\left(6\,cd^2e^2+ae^4\right)x^9+\frac{4}{3}\,ad^3ex^3+\frac{4}{7}\left(cd^3e+ade^3\right)x^7+ad^4x+\frac{1}{5}\left(cd^4+6\,ad^2e^2\right)x^5$$

input
```
integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="fricas")
```

output
```
1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 1/9*(6*c*d^2*e^2 + a*e^4)*x^9 + 4/3*
a*d^3*e*x^3 + 4/7*(c*d^3*e + a*d*e^3)*x^7 + a*d^4*x + 1/5*(c*d^4 + 6*a*d^2
*e^2)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04

$$\int (d+ex^2)^4 (a+cx^4)\, dx = ad^4x + \frac{4ad^3ex^3}{3} + \frac{4cde^3x^{11}}{11} + \frac{ce^4x^{13}}{13} + x^9\left(\frac{ae^4}{9} + \frac{2cd^2e^2}{3}\right) + x^7\cdot\left(\frac{4ade^3}{7} + \frac{4cd^3e}{7}\right) + x^5\cdot\left(\frac{6ad^2e^2}{5} + \frac{cd^4}{5}\right)$$

input
```
integrate((e*x**2+d)**4*(c*x**4+a),x)
```

output
```
a*d**4*x + 4*a*d**3*e*x**3/3 + 4*c*d*e**3*x**11/11 + c*e**4*x**13/13 + x**
9*(a*e**4/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 4*c*d**3*e/7) + x**5
*(6*a*d**2*e**2/5 + c*d**4/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int (d+ex^2)^4 (a+cx^4)\, dx = \frac{1}{13}\,ce^4x^{13} + \frac{4}{11}\,cde^3x^{11} + \frac{1}{9}\,(6\,cd^2e^2 + ae^4)x^9 + \frac{4}{3}\,ad^3ex^3 + \frac{4}{7}\,(cd^3e + ade^3)x^7 + ad^4x + \frac{1}{5}\,(cd^4 + 6\,ad^2e^2)x^5$$

input
```
integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="maxima")
```

output
```
1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 1/9*(6*c*d^2*e^2 + a*e^4)*x^9 + 4/3*
a*d^3*e*x^3 + 4/7*(c*d^3*e + a*d*e^3)*x^7 + a*d^4*x + 1/5*(c*d^4 + 6*a*d^2
*e^2)*x^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \left(d+ex^2\right)^4 \left(a+cx^4\right) \, dx = \frac{1}{13}ce^4x^{13}+\frac{4}{11}cde^3x^{11}+\frac{2}{3}cd^2e^2x^9+\frac{1}{9}ae^4x^9+\frac{4}{7}cd^3ex^7 +\frac{4}{7}ade^3x^7+\frac{1}{5}cd^4x^5+\frac{6}{5}ad^2e^2x^5+\frac{4}{3}ad^3ex^3+ad^4x$$

input
```
integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="giac")
```

output
```
1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 2/3*c*d^2*e^2*x^9 + 1/9*a*e^4*x^9 +
4/7*c*d^3*e*x^7 + 4/7*a*d*e^3*x^7 + 1/5*c*d^4*x^5 + 6/5*a*d^2*e^2*x^5 + 4/
3*a*d^3*e*x^3 + a*d^4*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \left(d+ex^2\right)^4 \left(a+cx^4\right) \, dx = x^5\left(\frac{c\,d^4}{5}+\frac{6\,a\,d^2\,e^2}{5}\right)+x^9\left(\frac{2\,c\,d^2\,e^2}{3}+\frac{a\,e^4}{9}\right) +x^7\left(\frac{4\,c\,d^3\,e}{7}+\frac{4\,a\,d\,e^3}{7}\right)+\frac{c\,e^4\,x^{13}}{13} +a\,d^4\,x+\frac{4\,a\,d^3\,e\,x^3}{3}+\frac{4\,c\,d\,e^3\,x^{11}}{11}$$

input
```
int((a + c*x^4)*(d + e*x^2)^4,x)
```

output
```
x^5*((c*d^4)/5 + (6*a*d^2*e^2)/5) + x^9*((a*e^4)/9 + (2*c*d^2*e^2)/3) + x^
7*((4*a*d*e^3)/7 + (4*c*d^3*e)/7) + (c*e^4*x^13)/13 + a*d^4*x + (4*a*d^3*e
*x^3)/3 + (4*c*d*e^3*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^4\left(a+cx^4\right)\,dx = \frac{x(3465c\,e^4x^{12}+16380cd\,e^3x^{10}+5005a\,e^4x^8+30030c\,d^2e^2x^8+25740ad\,e^3x^6+25740c\,d^3e\,x^6+54054a}{45045}$$

input

```
int((e*x^2+d)^4*(c*x^4+a),x)
```

output

```
(x*(45045*a*d**4 + 60060*a*d**3*e*x**2 + 54054*a*d**2*e**2*x**4 + 25740*a*
d*e**3*x**6 + 5005*a*e**4*x**8 + 9009*c*d**4*x**4 + 25740*c*d**3*e*x**6 +
30030*c*d**2*e**2*x**8 + 16380*c*d*e**3*x**10 + 3465*c*e**4*x**12))/45045
```

3.295 $\int (d+ex^2)^3 (a+cx^4) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 79

$$\int (d+ex^2)^3 (a+cx^4) \, dx = ad^3x + ad^2ex^3 + \frac{1}{5}d(cd^2+3ae^2)\,x^5 + \frac{1}{7}e(3cd^2+ae^2)\,x^7 + \frac{1}{3}cde^2x^9 + \frac{1}{11}ce^3x^{11}$$

output
```
a*d^3*x+a*d^2*e*x^3+1/5*d*(3*a*e^2+c*d^2)*x^5+1/7*e*(a*e^2+3*c*d^2)*x^7+1/
3*c*d*e^2*x^9+1/11*c*e^3*x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^3 (a+cx^4) \, dx = ad^3x + ad^2ex^3 + \frac{1}{5}d(cd^2+3ae^2)\,x^5 + \frac{1}{7}e(3cd^2+ae^2)\,x^7 + \frac{1}{3}cde^2x^9 + \frac{1}{11}ce^3x^{11}$$

input
```
Integrate[(d + e*x^2)^3*(a + c*x^4),x]
```

output
```
a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)
*x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)\left(d+ex^2\right)^3 dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(ex^6\left(ae^2+3cd^2\right)+dx^4\left(3ae^2+cd^2\right)+ad^3+3ad^2ex^2+3cde^2x^8+ce^3x^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}ex^7\left(ae^2+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+cd^2\right)+ad^3x+ad^2ex^3+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

input
```
Int[(d + e*x^2)^3*(a + c*x^4),x]
```

output
```
a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)
*x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{c\,e^{3} x^{11}}{11}+\frac{c d\,e^{2} x^{9}}{3}+\frac{\left(a\,e^{3}+3 c\,d^{2} e\right) x^{7}}{7}+\frac{\left(3 d\,e^{2} a+d^{3} c\right) x^{5}}{5}+a\,d^{2} e\,x^{3}+a\,d^{3} x$	72
norman	$\frac{c\,e^{3} x^{11}}{11}+\frac{c d\,e^{2} x^{9}}{3}+\left(\frac{1}{7} a\,e^{3}+\frac{3}{7} c\,d^{2} e\right) x^{7}+\left(\frac{3}{5} d\,e^{2} a+\frac{1}{5} d^{3} c\right) x^{5}+a\,d^{2} e\,x^{3}+a\,d^{3} x$	72
gosper	$\frac{1}{11} c\,e^{3} x^{11}+\frac{1}{3} c d\,e^{2} x^{9}+\frac{1}{7} x^{7} a\,e^{3}+\frac{3}{7} x^{7} c\,d^{2} e+\frac{3}{5} x^{5} d\,e^{2} a+\frac{1}{5} x^{5} d^{3} c+a\,d^{2} e\,x^{3}+a\,d^{3} x$	74
risch	$\frac{1}{11} c\,e^{3} x^{11}+\frac{1}{3} c d\,e^{2} x^{9}+\frac{1}{7} x^{7} a\,e^{3}+\frac{3}{7} x^{7} c\,d^{2} e+\frac{3}{5} x^{5} d\,e^{2} a+\frac{1}{5} x^{5} d^{3} c+a\,d^{2} e\,x^{3}+a\,d^{3} x$	74
parallelrisch	$\frac{1}{11} c\,e^{3} x^{11}+\frac{1}{3} c d\,e^{2} x^{9}+\frac{1}{7} x^{7} a\,e^{3}+\frac{3}{7} x^{7} c\,d^{2} e+\frac{3}{5} x^{5} d\,e^{2} a+\frac{1}{5} x^{5} d^{3} c+a\,d^{2} e\,x^{3}+a\,d^{3} x$	74
orering	$\frac{x \left(105 e^{3} c\,x^{10}+385 d\,e^{2} c\,x^{8}+165 a\,e^{3} x^{6}+495 c\,d^{2} e\,x^{6}+693 a d\,e^{2} x^{4}+231 c\,d^{3} x^{4}+1155 d^{2} e a\,x^{2}+1155 d^{3} a \right)}{1155}$	78

input
```
int((e*x^2+d)^3*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/11*c*e^3*x^11+1/3*c*d*e^2*x^9+1/7*(a*e^3+3*c*d^2*e)*x^7+1/5*(3*a*d*e^2+c
*d^3)*x^5+a*d^2*e*x^3+a*d^3*x
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right)\,dx = \frac{1}{11}\,ce^3x^{11}+\frac{1}{3}\,cde^2x^9+\frac{1}{7}\,\left(3\,cd^2e+ae^3\right)x^7 + ad^2ex^3+\frac{1}{5}\,\left(cd^3+3\,ade^2\right)x^5+ad^3x$$

input
```
integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="fricas")
```

output
```
1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 1/7*(3*c*d^2*e + a*e^3)*x^7 + a*d^2*e*
x^3 + 1/5*(c*d^3 + 3*a*d*e^2)*x^5 + a*d^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right) \, dx = ad^3x + ad^2ex^3 + \frac{cde^2x^9}{3} + \frac{ce^3x^{11}}{11} + x^7\left(\frac{ae^3}{7} + \frac{3cd^2e}{7}\right) + x^5 \cdot \left(\frac{3ade^2}{5} + \frac{cd^3}{5}\right)$$

input
```
integrate((e*x**2+d)**3*(c*x**4+a),x)
```

output
```
a*d**3*x + a*d**2*e*x**3 + c*d*e**2*x**9/3 + c*e**3*x**11/11 + x**7*(a*e**
3/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + c*d**3/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right) \, dx = \frac{1}{11}\,ce^3x^{11} + \frac{1}{3}\,cde^2x^9 + \frac{1}{7}\left(3\,cd^2e + ae^3\right)x^7 + ad^2ex^3 + \frac{1}{5}\left(cd^3 + 3\,ade^2\right)x^5 + ad^3x$$

input
```
integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="maxima")
```

output
```
1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 1/7*(3*c*d^2*e + a*e^3)*x^7 + a*d^2*e*
x^3 + 1/5*(c*d^3 + 3*a*d*e^2)*x^5 + a*d^3*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right) \, dx = \frac{1}{11}\,ce^3x^{11} + \frac{1}{3}\,cde^2x^9 + \frac{3}{7}\,cd^2ex^7 + \frac{1}{7}\,ae^3x^7 + \frac{1}{5}\,cd^3x^5 + \frac{3}{5}\,ade^2x^5 + ad^2ex^3 + ad^3x$$

input

```
integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="giac")
```

output

```
1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 3/7*c*d^2*e*x^7 + 1/7*a*e^3*x^7 + 1/5*
c*d^3*x^5 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + a*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right) \, dx = x^5 \left(\frac{c\,d^3}{5} + \frac{3\,a\,d\,e^2}{5}\right) + x^7 \left(\frac{3\,c\,d^2\,e}{7} + \frac{a\,e^3}{7}\right) + \frac{c\,e^3\,x^{11}}{11} + a\,d^3\,x + a\,d^2\,e\,x^3 + \frac{c\,d\,e^2\,x^9}{3}$$

input

```
int((a + c*x^4)*(d + e*x^2)^3,x)
```

output

```
x^5*((c*d^3)/5 + (3*a*d*e^2)/5) + x^7*((a*e^3)/7 + (3*c*d^2*e)/7) + (c*e^3
*x^11)/11 + a*d^3*x + a*d^2*e*x^3 + (c*d*e^2*x^9)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \left(d+ex^2\right)^3\left(a+cx^4\right)\,dx$$
$$=\frac{x(105c\,e^3x^{10}+385cd\,e^2x^8+165a\,e^3x^6+495c\,d^2e\,x^6+693ad\,e^2x^4+231c\,d^3x^4+1155a\,d^2e\,x^2+1155a\,d}{1155}$$

input
```
int((e*x^2+d)^3*(c*x^4+a),x)
```

output
```
(x*(1155*a*d**3 + 1155*a*d**2*e*x**2 + 693*a*d*e**2*x**4 + 165*a*e**3*x**6
 + 231*c*d**3*x**4 + 495*c*d**2*e*x**6 + 385*c*d*e**2*x**8 + 105*c*e**3*x*
*10))/1155
```

3.296 $\int (d+ex^2)^2 (a+cx^4) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int (d+ex^2)^2 (a+cx^4) \, dx = ad^2x + \frac{2}{3}adex^3 + \frac{1}{5}(cd^2+ae^2)x^5 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9$$

output
```
a*d^2*x+2/3*a*d*e*x^3+1/5*(a*e^2+c*d^2)*x^5+2/7*c*d*e*x^7+1/9*c*e^2*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^2 (a+cx^4) \, dx = ad^2x + \frac{2}{3}adex^3 + \frac{1}{5}(cd^2+ae^2)x^5 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9$$

input
```
Integrate[(d + e*x^2)^2*(a + c*x^4),x]
```

output
```
a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c
*e^2*x^9)/9
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)\left(d+ex^2\right)^2\,dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(x^4\left(ae^2+cd^2\right)+ad^2+2adex^2+2cdex^6+ce^2x^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5\left(ae^2+cd^2\right)+ad^2x+\frac{2}{3}adex^3+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

input
```
Int[(d + e*x^2)^2*(a + c*x^4),x]
```

output
```
a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c
*e^2*x^9)/9
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result	size
default	$a\,d^{2} x +\frac{2 a d e\,x^{3}}{3}+\frac{\left(a\,e^{2}+c\,d^{2}\right) x^{5}}{5}+\frac{2 c d e\,x^{7}}{7}+\frac{c\,e^{2} x^{9}}{9}$	49
norman	$\frac{c\,e^{2} x^{9}}{9}+\frac{2 c d e\,x^{7}}{7}+\left(\frac{a\,e^{2}}{5}+\frac{c\,d^{2}}{5}\right) x^{5}+\frac{2 a d e\,x^{3}}{3}+a\,d^{2} x$	50
gosper	$\frac{1}{9} c\,e^{2} x^{9}+\frac{2}{7} c d e\,x^{7}+\frac{1}{5} x^{5} a\,e^{2}+\frac{1}{5} x^{5} c\,d^{2}+\frac{2}{3} a d e\,x^{3}+a\,d^{2} x$	51
risch	$\frac{1}{9} c\,e^{2} x^{9}+\frac{2}{7} c d e\,x^{7}+\frac{1}{5} x^{5} a\,e^{2}+\frac{1}{5} x^{5} c\,d^{2}+\frac{2}{3} a d e\,x^{3}+a\,d^{2} x$	51
parallelrisch	$\frac{1}{9} c\,e^{2} x^{9}+\frac{2}{7} c d e\,x^{7}+\frac{1}{5} x^{5} a\,e^{2}+\frac{1}{5} x^{5} c\,d^{2}+\frac{2}{3} a d e\,x^{3}+a\,d^{2} x$	51
orering	$\frac{x \left(35 c\,e^{2} x^{8}+90 e c d\,x^{6}+63 a\,e^{2} x^{4}+63 c\,d^{2} x^{4}+210 a d e\,x^{2}+315 a\,d^{2}\right)}{315}$	54

input
```
int((e*x^2+d)^2*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
a*d^2*x+2/3*a*d*e*x^3+1/5*(a*e^2+c*d^2)*x^5+2/7*c*d*e*x^7+1/9*c*e^2*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)\,dx=\frac{1}{9}\,ce^{2}x^{9}+\frac{2}{7}\,cdex^{7}+\frac{2}{3}\,adex^{3}+\frac{1}{5}\,{\left(cd^{2}+ae^{2}\right)}x^{5}+ad^{2}x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="fricas")
```

output
```
1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 2/3*a*d*e*x^3 + 1/5*(c*d^2 + a*e^2)*x^5 +
a*d^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right) \, dx = ad^2x + \frac{2adex^3}{3} + \frac{2cdex^7}{7} + \frac{ce^2x^9}{9} + x^5\left(\frac{ae^2}{5} + \frac{cd^2}{5}\right)$$

input
```
integrate((e*x**2+d)**2*(c*x**4+a),x)
```

output
```
a*d**2*x + 2*a*d*e*x**3/3 + 2*c*d*e*x**7/7 + c*e**2*x**9/9 + x**5*(a*e**2/
5 + c*d**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right) \, dx = \frac{1}{9}\,ce^2x^9 + \frac{2}{7}\,cdex^7 + \frac{2}{3}\,adex^3 + \frac{1}{5}\,\left(cd^2+ae^2\right)x^5 + ad^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="maxima")
```

output
```
1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 2/3*a*d*e*x^3 + 1/5*(c*d^2 + a*e^2)*x^5 +
a*d^2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right) \, dx = \frac{1}{9}\,ce^2x^9 + \frac{2}{7}\,cdex^7 + \frac{1}{5}\,cd^2x^5 + \frac{1}{5}\,ae^2x^5 + \frac{2}{3}\,adex^3 + ad^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="giac")
```

output
```
1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 1/5*c*d^2*x^5 + 1/5*a*e^2*x^5 + 2/3*a*d*e*
x^3 + a*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)\,dx=x^5\left(\frac{c\,d^2}{5}+\frac{a\,e^2}{5}\right)+\frac{c\,e^2\,x^9}{9}+a\,d^2\,x+\frac{2\,a\,d\,e\,x^3}{3}+\frac{2\,c\,d\,e\,x^7}{7}$$

input
```
int((a + c*x^4)*(d + e*x^2)^2,x)
```

output
```
x^5*((a*e^2)/5 + (c*d^2)/5) + (c*e^2*x^9)/9 + a*d^2*x + (2*a*d*e*x^3)/3 +
(2*c*d*e*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)\,dx$$
$$=\frac{x(35c\,e^2x^8+90cde\,x^6+63a\,e^2x^4+63c\,d^2x^4+210ade\,x^2+315a\,d^2)}{315}$$

input
```
int((e*x^2+d)^2*(c*x^4+a),x)
```

output
```
(x*(315*a*d**2 + 210*a*d*e*x**2 + 63*a*e**2*x**4 + 63*c*d**2*x**4 + 90*c*d
*e*x**6 + 35*c*e**2*x**8))/315
```

3.297 $\int (d+ex^2)(a+cx^4)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int (d+ex^2)(a+cx^4)\,dx = adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

output
```
a*d*x+1/3*a*e*x^3+1/5*c*d*x^5+1/7*c*e*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (d+ex^2)(a+cx^4)\,dx = adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

input
```
Integrate[(d + e*x^2)*(a + c*x^4),x]
```

output
```
a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4)(d+ex^2)\,dx$$

$$\Big\downarrow \text{1468}$$

$$\int (ad+aex^2+cdx^4+cex^6)\,dx$$

$$\Big\downarrow \text{2009}$$

$$adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

input
```
Int[(d + e*x^2)*(a + c*x^4),x]
```

output
```
a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
gosper	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27
default	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27
norman	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27
risch	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27
parallelrisch	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27
orering	$\frac{x(15ce\,x^6+21cd\,x^4+35ae\,x^2+105ad)}{105}$	30

input
```
int((e*x^2+d)*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
a*d*x+1/3*a*e*x^3+1/5*c*d*x^5+1/7*c*e*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int (d+ex^2)\,(a+cx^4)\;dx=\frac{1}{7}\,cex^7+\frac{1}{5}\,cdx^5+\frac{1}{3}\,aex^3+adx$$

input
```
integrate((e*x^2+d)*(c*x^4+a),x, algorithm="fricas")
```

output
```
1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/3*a*e*x^3 + a*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \left(d+ex^2\right) \left(a+cx^4\right) \, dx = adx + \frac{aex^3}{3} + \frac{cdx^5}{5} + \frac{cex^7}{7}$$

input
```
integrate((e*x**2+d)*(c*x**4+a),x)
```

output
```
a*d*x + a*e*x**3/3 + c*d*x**5/5 + c*e*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \left(d+ex^2\right) \left(a+cx^4\right) \, dx = \frac{1}{7}\,cex^7 + \frac{1}{5}\,cdx^5 + \frac{1}{3}\,aex^3 + adx$$

input
```
integrate((e*x^2+d)*(c*x^4+a),x, algorithm="maxima")
```

output
```
1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/3*a*e*x^3 + a*d*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \left(d+ex^2\right) \left(a+cx^4\right) \, dx = \frac{1}{7}\,cex^7 + \frac{1}{5}\,cdx^5 + \frac{1}{3}\,aex^3 + adx$$

input
```
integrate((e*x^2+d)*(c*x^4+a),x, algorithm="giac")
```

output
```
1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/3*a*e*x^3 + a*d*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx=\frac{c\,e\,x^7}{7}+\frac{c\,d\,x^5}{5}+\frac{a\,e\,x^3}{3}+a\,d\,x$$

input
```
int((a + c*x^4)*(d + e*x^2),x)
```

output
```
a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx=\frac{x(15ce\,x^6+21cd\,x^4+35ae\,x^2+105ad)}{105}$$

input
```
int((e*x^2+d)*(c*x^4+a),x)
```

output
```
(x*(105*a*d + 35*a*e*x**2 + 21*c*d*x**4 + 15*c*e*x**6))/105
```

3.298 $\int \frac{a+cx^4}{d+ex^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{a+cx^4}{d+ex^2} \, dx = -\frac{cdx}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}$$

output
```
-c*d*x/e^2+1/3*c*x^3/e+(a*e^2+c*d^2)*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)/e^(
5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{a+cx^4}{d+ex^2} \, dx = -\frac{cdx}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}$$

input
```
Integrate[(a + c*x^4)/(d + e*x^2),x]
```

output
```
-((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(Sqrt[d]*e^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{d+ex^2}dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(\frac{ae^2+cd^2}{e^2\left(d+ex^2\right)}-\frac{cd}{e^2}+\frac{cx^2}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(ae^2+cd^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}-\frac{cdx}{e^2}+\frac{cx^3}{3e}$$

input
```
Int[(a + c*x^4)/(d + e*x^2),x]
```

output
```
-((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(Sqrt[d]*e^(5/2))
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{c\left(-\frac{1}{3}e\,x^{3}+dx\right)}{e^{2}}+\frac{\left(a\,e^{2}+c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e^{2}\sqrt{de}}$	47
risch	$\frac{c\,x^{3}}{3e}-\frac{cdx}{e^{2}}-\frac{\ln\left(ex+\sqrt{-de}\right)a}{2\sqrt{-de}}-\frac{\ln\left(ex+\sqrt{-de}\right)c\,d^{2}}{2e^{2}\sqrt{-de}}+\frac{\ln\left(-ex+\sqrt{-de}\right)a}{2\sqrt{-de}}+\frac{\ln\left(-ex+\sqrt{-de}\right)c\,d^{2}}{2e^{2}\sqrt{-de}}$	113

input
```
int((c*x^4+a)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-c/e^2*(-1/3*e*x^3+d*x)+(a*e^2+c*d^2)/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.38

$$\int \frac{a+cx^4}{d+ex^2}\,dx$$
$$=\left[\frac{2\,cde^2x^3-6\,cd^2ex-3\,(cd^2+ae^2)\sqrt{-de}\log\left(\frac{ex^2-2\,\sqrt{-de}x-d}{ex^2+d}\right)}{6\,de^3},\frac{cde^2x^3-3\,cd^2ex+3\,(cd^2+ae^2)\sqrt{de}\,\mathrm{arc}}{3\,de^3}\right.$$

input
```
integrate((c*x^4+a)/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/6*(2*c*d*e^2*x^3 - 6*c*d^2*e*x - 3*(c*d^2 + a*e^2)*sqrt(-d*e)*log((e*x^
2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d*e^3), 1/3*(c*d*e^2*x^3 - 3*c*d^2*
e*x + 3*(c*d^2 + a*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(49) = 98$.

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.89

$$\int \frac{a+cx^4}{d+ex^2}\,dx = -\frac{cdx}{e^2}+\frac{cx^3}{3e}-\frac{\sqrt{-\frac{1}{de^5}}\,(ae^2+cd^2)\log\left(-de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{de^5}}\,(ae^2+cd^2)\log\left(de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2}$$

input
```
integrate((c*x**4+a)/(e*x**2+d),x)
```

output
```
-c*d*x/e**2 + c*x**3/(3*e) - sqrt(-1/(d*e**5))*(a*e**2 + c*d**2)*log(-d*e*
*2*sqrt(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2 + c*d**2)*log(d*e*
*2*sqrt(-1/(d*e**5)) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{a+cx^4}{d+ex^2}\,dx = \frac{(cd^2+ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^2} + \frac{ce^2x^3-3\,cdex}{3\,e^3}$$

input
```
integrate((c*x^4+a)/(e*x^2+d),x, algorithm="giac")
```

output
```
(c*d^2 + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^2) + 1/3*(c*e^2*x^3 - 3
*c*d*e*x)/e^3
```

Mupad [B] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{a+cx^4}{d+ex^2}\,dx = \frac{c\,x^3}{3\,e} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2+a\,e^2\right)}{\sqrt{d}\,e^{5/2}} - \frac{c\,d\,x}{e^2}$$

input
```
int((a + c*x^4)/(d + e*x^2),x)
```

output
```
(c*x^3)/(3*e) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + c*d^2))/(d^(1/2)*e^(5/
2)) - (c*d*x)/e^2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int \frac{a+cx^4}{d+ex^2}\,dx = \frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^2 + 3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)c\,d^2 - 3c\,d^2ex + cd\,e^2x^3}{3d\,e^3}$$

input
```
int((c*x^4+a)/(e*x^2+d),x)
```

output

```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e**2 + 3*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*c*d**2 - 3*c*d**2*e*x + c*d*e**2*x**3)/(3*
d*e**3)
```

3.299 $\int \frac{a+cx^4}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{a+cx^4}{(d+ex^2)^2}\,dx=\frac{cx}{e^2}+\frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d\,(d+ex^2)}-\frac{(3cd^2-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

output
```
c*x/e^2+1/2*(a+c*d^2/e^2)*x/d/(e*x^2+d)-1/2*(-a*e^2+3*c*d^2)*arctan(e^(1/2
)*x/d^(1/2))/d^(3/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^4}{(d+ex^2)^2}\,dx=\frac{cx}{e^2}+\frac{(cd^2+ae^2)\,x}{2de^2\,(d+ex^2)}-\frac{(3cd^2-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

input
```
Integrate[(a + c*x^4)/(d + e*x^2)^2,x]
```

output
```
(c*x)/e^2 + ((c*d^2 + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 - a*e^2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1472, 25, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d(d+ex^2)}-\frac{\int -\frac{2cdx^2+\left(a-\frac{cd^2}{e^2}\right)e}{e(ex^2+d)}dx}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2cdx^2+\left(a-\frac{cd^2}{e^2}\right)e}{e(ex^2+d)}dx}{2d}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d(d+ex^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2cdx^2+\left(a-\frac{cd^2}{e^2}\right)e}{ex^2+d}dx}{2de}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d(d+ex^2)}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{2cdx}{e}-\frac{(3cd^2-ae^2)\int \frac{1}{ex^2+d}dx}{e}}{2de}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d(d+ex^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2cdx}{e}-\frac{(3cd^2-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}}{2de}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d(d+ex^2)}$$

input

```
Int[(a + c*x^4)/(d + e*x^2)^2,x]
```

output
```
((a + (c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) + ((2*c*d*x)/e - ((3*c*d^2 - a*e^2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(3/2)))/(2*d*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{cx}{e^2}+\frac{\frac{\left(a\,e^2+c\,d^2\right)x}{2d\left(e\,x^2+d\right)}+\frac{\left(a\,e^2-3c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}}{e^2}$	70
risch	$\frac{cx}{e^2}+\frac{(a\,e^2+c\,d^2)x}{2d\,e^2(e\,x^2+d)}-\frac{\ln(ex+\sqrt{-de})a}{4\sqrt{-de}\,d}+\frac{3d\ln(ex+\sqrt{-de})c}{4e^2\sqrt{-de}}+\frac{\ln(-ex+\sqrt{-de})a}{4\sqrt{-de}\,d}-\frac{3d\ln(-ex+\sqrt{-de})c}{4e^2\sqrt{-de}}$	133

input
```
int((c*x^4+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c*x/e^2+1/e^2*(1/2*(a*e^2+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2-3*c*d^2)/d/(d*e)
^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.00

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^2}\,dx = \left[\frac{4\,cd^2e^2x^3+(3\,cd^3-ade^2+(3\,cd^2e-ae^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)+2\,(3\,cd^3e+ade^3)x}{4\,(d^2e^4x^2+d^3e^3)}, \frac{2\,cd^2e^2x}{}\right.$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - a*d*e^2 + (3*c*d^2*e - a*e^3)*x^2)*sqrt
(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^3*e + a*d*
e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2*e^2*x^3 - (3*c*d^3 - a*d*e^2
 + (3*c*d^2*e - a*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e +
 a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(68) = 136$.

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.86

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{cx}{e^2}+\frac{x(ae^2+cd^2)}{2d^2e^2+2de^3x^2}-\frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2-3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2-3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

input
```
integrate((c*x**4+a)/(e*x**2+d)**2,x)
```

output
```
c*x/e**2 + x*(a*e**2 + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt(-1/(d*
*3*e**5))*(a*e**2 - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4 +
 sqrt(-1/(d**3*e**5))*(a*e**2 - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5
)) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{a+cx^4}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} - \frac{(3\,cd^2-ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^2} + \frac{cd^2x+ae^2x}{2\,(ex^2+d)de^2}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
c*x/e^2 - 1/2*(3*c*d^2 - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^2) +
1/2*(c*d^2*x + a*e^2*x)/((e*x^2 + d)*d*e^2)
```

Mupad [B] (verification not implemented)

Time = 17.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(a\,e^2-3\,c\,d^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2+a\,e^2\right)}{2\,d\,\left(e^3\,x^2+d\,e^2\right)}$$

input
```
int((a + c*x^4)/(d + e*x^2)^2,x)
```

output
```
(c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2))/(2*d^(3/2)*e^(5/
2)) + (x*(a*e^2 + c*d^2))/(2*d*(d*e^2 + e^3*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.88

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^2}\,dx$$

$$= \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) a d\,e^2 + \sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) a\,e^3x^2 - 3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) c\,d^3 - 3\sqrt{e}\,\sqrt{d}\,atan}{2d^2e^3\left(e\,x^2+d\right)}$$

input
```
int((c*x^4+a)/(e*x^2+d)^2,x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e**2 + sqrt(e)*sqrt(d)*
atan((e*x)/(sqrt(e)*sqrt(d)))*a*e**3*x**2 - 3*sqrt(e)*sqrt(d)*atan((e*x)/(
sqrt(e)*sqrt(d)))*c*d**3 - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))
*c*d**2*e*x**2 + a*d*e**3*x + 3*c*d**3*e*x + 2*c*d**2*e**2*x**3)/(2*d**2*e
**3*(d + e*x**2))
```

3.300 $\int \frac{a+cx^4}{\left(d+ex^2\right)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^3}\,dx = \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\left(d+ex^2\right)^2} + \frac{\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)x}{8\left(d+ex^2\right)} + \frac{3(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

output
```
1/4*(a+c*d^2/e^2)*x/d/(e*x^2+d)^2+(3*a/d^2-5*c/e^2)*x/(8*e*x^2+8*d)+3/8*(a
*e^2+c*d^2)*arctan(e^(1/2)*x/d^(1/2))/d^(5/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^3}\,dx = \frac{ae^2x(5d+3ex^2)-cd^2x(3d+5ex^2)}{8d^2e^2\left(d+ex^2\right)^2} + \frac{3(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

input
```
Integrate[(a + c*x^4)/(d + e*x^2)^3,x]
```

output
```
(a*e^2*x*(5*d + 3*e*x^2) - c*d^2*x*(3*d + 5*e*x^2))/(8*d^2*e^2*(d + e*x^2)
^2) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1472, 25, 25, 27, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{(d+ex^2)^3}dx$$

$$\Big\downarrow 1472$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2}-\frac{\int -\frac{4cdx^2+\left(3a-\frac{cd^2}{e^2}\right)e}{e(ex^2+d)^2}dx}{4d}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{\frac{cd^2}{e}-4cx^2d-3ae}{e(ex^2+d)^2}dx}{4d}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2}$$

$$\Big\downarrow 25$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2}-\frac{\int \frac{\frac{cd^2}{e}-4cx^2d-3ae}{e(ex^2+d)^2}dx}{4d}$$

$$\Big\downarrow 27$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2}-\frac{\int \frac{\frac{cd^2}{e}-4cx^2d-3ae}{(ex^2+d)^2}dx}{4de}$$

$$\Big\downarrow 298$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2}-\frac{\frac{x\left(\frac{5cd}{e}-\frac{3ae}{d}\right)}{2(d+ex^2)}-\frac{3(ae^2+cd^2)\int \frac{1}{ex^2+d}dx}{2de}}{4de}$$

$$\Big\downarrow 218$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2}-\frac{\frac{x\left(\frac{5cd}{e}-\frac{3ae}{d}\right)}{2(d+ex^2)}-\frac{3(ae^2+cd^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{3/2}}}{4de}$$

input
```
Int[(a + c*x^4)/(d + e*x^2)^3,x]
```

output
```
((a + (c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) - ((((5*c*d)/e - (3*a*e)/d)*x)/(
2*(d + e*x^2)) - (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2
)*e^(3/2)))/(4*d*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

method	result	si
default	$\frac{\frac{\left(3 a \,e^{2}-5 c \,d^{2}\right) x^{3}}{8 d^{2} e}+\frac{\left(5 a \,e^{2}-3 c \,d^{2}\right) x}{8 d \,e^{2}}}{\left(e \,x^{2}+d \right)^{2}}+\frac{3 \left(a \,e^{2}+c \,d^{2}\right) \arctan \left(\frac{e x}{\sqrt{d e}}\right)}{8 d^{2} e^{2} \sqrt{d e}}$	92
risch	$\frac{\frac{\left(3 a \,e^{2}-5 c \,d^{2}\right) x^{3}}{8 d^{2} e}+\frac{\left(5 a \,e^{2}-3 c \,d^{2}\right) x}{8 d \,e^{2}}}{\left(e \,x^{2}+d \right)^{2}}-\frac{3 \ln \left(e x +\sqrt{-d e}\right) a}{16 \sqrt{-d e}\, d^{2}}-\frac{3 \ln \left(e x +\sqrt{-d e}\right) c}{16 \sqrt{-d e}\, e^{2}}+\frac{3 \ln \left(-e x +\sqrt{-d e}\right) a}{16 \sqrt{-d e}\, d^{2}}+\frac{3 \ln \left(-e x +\sqrt{-d e}\right) c}{16 \sqrt{-d e}\, e^{2}}$	1

input
```
int((c*x^4+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*a*e^2-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-3*c*d^2)/d/e^2*x)/(e*x^2+d)^
2+3/8*(a*e^2+c*d^2)/d^2/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.29

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^3}\,dx$$

$$= \left[-\frac{2\,(5\,cd^3e^2-3\,ade^4)x^3+3\,(cd^4+ad^2e^2+(cd^2e^2+ae^4)x^4+2\,(cd^3e+ade^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}}{ex^2+d}\right.}{16\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}\right.$$

$$-\frac{(5\,cd^3e^2-3\,ade^4)x^3-3\,(cd^4+ad^2e^2+(cd^2e^2+ae^4)x^4+2\,(cd^3e+ade^3)x^2)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)+(3}{8\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(5*c*d^3*e^2 - 3*a*d*e^4)*x^3 + 3*(c*d^4 + a*d^2*e^2 + (c*d^2*e^
2 + a*e^4)*x^4 + 2*(c*d^3*e + a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt
(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^4*e - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 +
 2*d^4*e^4*x^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - 3*a*d*e^4)*x^3 - 3*(c*d^4
+ a*d^2*e^2 + (c*d^2*e^2 + a*e^4)*x^4 + 2*(c*d^3*e + a*d*e^3)*x^2)*sqrt(d*
e)*arctan(sqrt(d*e)*x/d) + (3*c*d^4*e - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d
^4*e^4*x^2 + d^5*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(90) = 180$.

Time = 0.36 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.35

$$\int \frac{a+cx^4}{(d+ex^2)^3}\,dx = -\frac{3\sqrt{-\frac{1}{d^5e^5}}(ae^2+cd^2)\log\left(-\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}(ae^2+cd^2)}{3ae^2+3cd^2}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{d^5e^5}}(ae^2+cd^2)\log\left(\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}(ae^2+cd^2)}{3ae^2+3cd^2}+x\right)}{16} + \frac{x^3\cdot(3ae^3-5cd^2e)+x(5ade^2-3cd^3)}{8d^4e^2+16d^3e^3x^2+8d^2e^4x^4}$$

input
```
integrate((c*x**4+a)/(e*x**2+d)**3,x)
```

output
```
-3*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)*log(-3*d**3*e**2*sqrt(-1/(d**5*e
**5))*(a*e**2 + c*d**2)/(3*a*e**2 + 3*c*d**2) + x)/16 + 3*sqrt(-1/(d**5*e*
*5))*(a*e**2 + c*d**2)*log(3*d**3*e**2*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d*
*2)/(3*a*e**2 + 3*c*d**2) + x)/16 + (x**3*(3*a*e**3 - 5*c*d**2*e) + x*(5*a
*d*e**2 - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e**3*x**2 + 8*d**2*e**4*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^3}\,dx = \frac{3\,(cd^2+ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^2e^2} - \frac{5\,cd^2ex^3-3\,ae^3x^3+3\,cd^3x-5\,ade^2x}{8\,\left(ex^2+d\right)^2d^2e^2}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
3/8*(c*d^2 + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^2) - 1/8*(5*c*d
^2*e*x^3 - 3*a*e^3*x^3 + 3*c*d^3*x - 5*a*d*e^2*x)/((e*x^2 + d)^2*d^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^3}\,dx = \frac{\frac{x^3\,\left(3\,a\,e^2-5\,c\,d^2\right)}{8\,d^2\,e}+\frac{x\,\left(5\,a\,e^2-3\,c\,d^2\right)}{8\,d\,e^2}}{d^2+2\,d\,e\,x^2+e^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2+a\,e^2\right)}{8\,d^{5/2}\,e^{5/2}}$$

input
```
int((a + c*x^4)/(d + e*x^2)^3,x)
```

output
```
((x^3*(3*a*e^2 - 5*c*d^2))/(8*d^2*e) + (x*(5*a*e^2 - 3*c*d^2))/(8*d*e^2))/
(d^2 + e^2*x^4 + 2*d*e*x^2) + (3*atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + c*d^2)
)/(8*d^(5/2)*e^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.38

$$\int \frac{a+cx^4}{(d+ex^2)^3}\,dx$$
$$= \frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,d^2e^2+6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad\,e^3x^2+3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^4x^4+3\sqrt{e}\,\sqrt{d}\,ata}{8d^3}$$

input
```
int((c*x^4+a)/(e*x^2+d)^3,x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2*e**2 + 6*sqrt(e)*s
qrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e**3*x**2 + 3*sqrt(e)*sqrt(d)*ata
n((e*x)/(sqrt(e)*sqrt(d)))*a*e**4*x**4 + 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqr
t(e)*sqrt(d)))*c*d**4 + 6*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c*
d**3*e*x**2 + 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c*d**2*e**2*
x**4 + 5*a*d**2*e**3*x + 3*a*d*e**4*x**3 - 3*c*d**4*e*x - 5*c*d**3*e**2*x*
*3)/(8*d**3*e**3*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.301 $\int \frac{a+cx^4}{\left(d+ex^2\right)^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 123

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^4} \, dx = \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\left(d+ex^2\right)^3}+\frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\left(d+ex^2\right)^2} +\frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)x}{16d\left(d+ex^2\right)}+\frac{\left(cd^2+5ae^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

output

```
1/6*(a+c*d^2/e^2)*x/d/(e*x^2+d)^3+1/24*(5*a/d^2-7*c/e^2)*x/(e*x^2+d)^2+1/1
6*(5*a/d^2+c/e^2)*x/d/(e*x^2+d)+1/16*(5*a*e^2+c*d^2)*arctan(e^(1/2)*x/d^(1
/2))/d^(7/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^4} \, dx = \frac{x\left(cd^2\left(-3d^2-8dex^2+3e^2x^4\right)+ae^2\left(33d^2+40dex^2+15e^2x^4\right)\right)}{48d^3e^2\left(d+ex^2\right)^3} +\frac{\left(cd^2+5ae^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

input
```
Integrate[(a + c*x^4)/(d + e*x^2)^4,x]
```

output
```
(x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + a*e^2*(33*d^2 + 40*d*e*x^2 +
15*e^2*x^4)))/(48*d^3*e^2*(d + e*x^2)^3) + ((c*d^2 + 5*a*e^2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = $\{1472, 25, 25, 27, 298, 215, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^4}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}-\frac{\int -\frac{6cdx^2+\left(5a-\frac{cd^2}{e^2}\right)e}{e\left(ex^2+d\right)^3}dx}{6d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{\frac{cd^2}{e}-6cx^2d-5ae}{e\left(ex^2+d\right)^3}dx}{6d}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}-\frac{\int \frac{\frac{cd^2}{e}-6cx^2d-5ae}{e\left(ex^2+d\right)^3}dx}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}-\frac{\int \frac{\frac{cd^2}{e}-6cx^2d-5ae}{\left(ex^2+d\right)^3}dx}{6de}$$

$$\Big\downarrow \text{298}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}-\frac{\frac{x\left(\frac{7cd}{e}-\frac{5ae}{d}\right)}{4(d+ex^2)^2}-\frac{3}{4}\left(\frac{5ae}{d}+\frac{cd}{e}\right)\int\frac{1}{(ex^2+d)^2}dx}{6de}$$

$$\Big\downarrow 215$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}-\frac{\frac{x\left(\frac{7cd}{e}-\frac{5ae}{d}\right)}{4(d+ex^2)^2}-\frac{3}{4}\left(\frac{5ae}{d}+\frac{cd}{e}\right)\left(\frac{\int\frac{1}{ex^2+d}dx}{2d}+\frac{x}{2d(d+ex^2)}\right)}{6de}$$

$$\Big\downarrow 218$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}-\frac{\frac{x\left(\frac{7cd}{e}-\frac{5ae}{d}\right)}{4(d+ex^2)^2}-\frac{3}{4}\left(\frac{5ae}{d}+\frac{cd}{e}\right)\left(\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}+\frac{x}{2d(d+ex^2)}\right)}{6de}$$

input

```
Int[(a + c*x^4)/(d + e*x^2)^4,x]
```

output

```
((a + (c*d^2)/e^2)*x)/(6*d*(d + e*x^2)^3) - ((((7*c*d)/e - (5*a*e)/d)*x)/(
4*(d + e*x^2)^2) - (3*((c*d)/e + (5*a*e)/d)*(x/(2*d*(d + e*x^2)) + ArcTan[
(Sqrt[e]*x)/Sqrt[d]]/(2*d^(3/2)*Sqrt[e])))/4)/(6*d*e)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

method	result
default	$\frac{\frac{\left(5a\,e^{2}+c\,d^{2}\right)x^{5}}{16d^{3}}+\frac{\left(5a\,e^{2}-c\,d^{2}\right)x^{3}}{6d^{2}e}+\frac{\left(11a\,e^{2}-c\,d^{2}\right)x}{16d\,e^{2}}}{\left(e\,x^{2}+d\right)^{3}}+\frac{\left(5a\,e^{2}+c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16d^{3}e^{2}\sqrt{de}}$
risch	$\frac{\frac{\left(5a\,e^{2}+c\,d^{2}\right)x^{5}}{16d^{3}}+\frac{\left(5a\,e^{2}-c\,d^{2}\right)x^{3}}{6d^{2}e}+\frac{\left(11a\,e^{2}-c\,d^{2}\right)x}{16d\,e^{2}}}{\left(e\,x^{2}+d\right)^{3}}-\frac{5\ln\left(ex+\sqrt{-de}\right)a}{32\sqrt{-de}\,d^{3}}-\frac{\ln\left(ex+\sqrt{-de}\right)c}{32\sqrt{-de}\,e^{2}d}+\frac{5\ln\left(-ex+\sqrt{-de}\right)a}{32\sqrt{-de}\,d^{3}}+$

input
```
int((c*x^4+a)/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(5*a*e^2+c*d^2)/d^3*x^5+1/6*(5*a*e^2-c*d^2)/d^2/e*x^3+1/16*(11*a*e^2
-c*d^2)/d/e^2*x)/(e*x^2+d)^3+1/16*(5*a*e^2+c*d^2)/d^3/e^2/(d*e)^(1/2)*arct
an(e*x/(d*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.45

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^4}\,dx$$

$$= \left[\frac{6\,(cd^3e^3+5\,ade^5)x^5-16\,(cd^4e^2-5\,ad^2e^4)x^3-3\,((cd^2e^3+5\,ae^5)x^6+cd^5+5\,ad^3e^2+3\,(cd^3e^2+5\,ade}{96\,(d^4e^6x^6+3\,d^5e^5x^4+3\,d^6e^4x^2}\right.$$

input

```
integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="fricas")
```

output

```
[1/96*(6*(c*d^3*e^3 + 5*a*d*e^5)*x^5 - 16*(c*d^4*e^2 - 5*a*d^2*e^4)*x^3 -
3*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d^5 + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + 5*a*d*
e^4)*x^4 + 3*(c*d^4*e + 5*a*d^2*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-
d*e)*x - d)/(e*x^2 + d)) - 6*(c*d^5*e - 11*a*d^3*e^3)*x)/(d^4*e^6*x^6 + 3*
d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3), 1/48*(3*(c*d^3*e^3 + 5*a*d*e^5)*x^
5 - 8*(c*d^4*e^2 - 5*a*d^2*e^4)*x^3 + 3*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d^5
 + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + 5*a*d^2*e^3)
*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(c*d^5*e - 11*a*d^3*e^3)*x)/(d^4
*e^6*x^6 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3)]
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.66

$$\begin{aligned}\int \frac{a+cx^4}{\left(d+ex^2\right)^4}\,dx &= -\frac{\sqrt{-\frac{1}{d^7e^5}}\cdot\left(5ae^2+cd^2\right)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}\\ &\quad+\frac{\sqrt{-\frac{1}{d^7e^5}}\cdot\left(5ae^2+cd^2\right)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}\\ &\quad+\frac{x^5\cdot\left(15ae^4+3cd^2e^2\right)+x^3\cdot\left(40ade^3-8cd^3e\right)+x\left(33ad^2e^2-3cd^4\right)}{48d^6e^2+144d^5e^3x^2+144d^4e^4x^4+48d^3e^5x^6}\end{aligned}$$

input

```
integrate((c*x**4+a)/(e*x**2+d)**4,x)
```

output

```
-sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(-d**4*e**2*sqrt(-1/(d**7*e**
5)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(d**4*e**2*sqrt(
-1/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*c*d**2*e**2) + x**3*(40*a*d
*e**3 - 8*c*d**3*e) + x*(33*a*d**2*e**2 - 3*c*d**4))/(48*d**6*e**2 + 144*d
**5*e**3*x**2 + 144*d**4*e**4*x**4 + 48*d**3*e**5*x**6)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{(d+ex^2)^4}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{a+cx^4}{(d+ex^2)^4}\,dx = \frac{(cd^2+5\,ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\,\sqrt{de}d^3e^2} + \frac{3\,cd^2e^2x^5+15\,ae^4x^5-8\,cd^3ex^3+40\,ade^3x^3-3\,cd^4x+33\,ad^2e^2x}{48\,(ex^2+d)^3d^3e^2}$$

input

```
integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="giac")
```

output

```
1/16*(c*d^2 + 5*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3*e^2) + 1/48*(3
*c*d^2*e^2*x^5 + 15*a*e^4*x^5 - 8*c*d^3*e*x^3 + 40*a*d*e^3*x^3 - 3*c*d^4*x
 + 33*a*d^2*e^2*x)/((e*x^2 + d)^3*d^3*e^2)
```

Mupad [B] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^4}{(d+ex^2)^4}\,dx = \frac{\frac{x^5\,(c\,d^2+5\,a\,e^2)}{16\,d^3}+\frac{x^3\,(5\,a\,e^2-c\,d^2)}{6\,d^2\,e}+\frac{x\,(11\,a\,e^2-c\,d^2)}{16\,d\,e^2}}{d^3+3\,d^2\,e\,x^2+3\,d\,e^2\,x^4+e^3\,x^6}+\frac{\operatorname{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(c\,d^2+5\,a\,e^2)}{16\,d^{7/2}\,e^{5/2}}$$

input
```
int((a + c*x^4)/(d + e*x^2)^4,x)
```

output
```
((x^5*(5*a*e^2 + c*d^2))/(16*d^3) + (x^3*(5*a*e^2 - c*d^2))/(6*d^2*e) + (x
*(11*a*e^2 - c*d^2))/(16*d*e^2))/(d^3 + e^3*x^6 + 3*d^2*e*x^2 + 3*d*e^2*x^
4) + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2))/(16*d^(7/2)*e^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.54

$$\int \frac{a+cx^4}{(d+ex^2)^4}\,dx = \frac{15\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,d^3e^2+45\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,d^2e^3x^2+45\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad\,e^4x^4+15\sqrt{e}}{}$$

input
```
int((c*x^4+a)/(e*x^2+d)^4,x)
```

output
```
(15*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**3*e**2 + 45*sqrt(e)
*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2*e**3*x**2 + 45*sqrt(e)*sqrt(
d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e**4*x**4 + 15*sqrt(e)*sqrt(d)*atan((
e*x)/(sqrt(e)*sqrt(d)))*a*e**5*x**6 + 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e
)*sqrt(d)))*c*d**5 + 9*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c*d**
4*e*x**2 + 9*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c*d**3*e**2*x**
4 + 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c*d**2*e**3*x**6 + 33*
a*d**3*e**3*x + 40*a*d**2*e**4*x**3 + 15*a*d*e**5*x**5 - 3*c*d**5*e*x - 8*
c*d**4*e**2*x**3 + 3*c*d**3*e**3*x**5)/(48*d**4*e**3*(d**3 + 3*d**2*e*x**2
 + 3*d*e**2*x**4 + e**3*x**6))
```

3.302 $\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \, dx = a^2d^3x+a^2d^2ex^3+\frac{1}{5}ad\left(2cd^2+3ae^2\right)x^5 +\frac{1}{7}ae\left(6cd^2+ae^2\right)x^7+\frac{1}{9}cd\left(cd^2+6ae^2\right)x^9 +\frac{1}{11}ce\left(3cd^2+2ae^2\right)x^{11}+\frac{3}{13}c^2de^2x^{13}+\frac{1}{15}c^2e^3x^{15}$$

output
```
a^2*d^3*x+a^2*d^2*e*x^3+1/5*a*d*(3*a*e^2+2*c*d^2)*x^5+1/7*a*e*(a*e^2+6*c*d
^2)*x^7+1/9*c*d*(6*a*e^2+c*d^2)*x^9+1/11*c*e*(2*a*e^2+3*c*d^2)*x^11+3/13*c
^2*d*e^2*x^13+1/15*c^2*e^3*x^15
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \, dx = a^2d^3x+a^2d^2ex^3+\frac{1}{5}ad\left(2cd^2+3ae^2\right)x^5 +\frac{1}{7}ae\left(6cd^2+ae^2\right)x^7+\frac{1}{9}cd\left(cd^2+6ae^2\right)x^9 +\frac{1}{11}ce\left(3cd^2+2ae^2\right)x^{11}+\frac{3}{13}c^2de^2x^{13}+\frac{1}{15}c^2e^3x^{15}$$

input
```
Integrate[(d + e*x^2)^3*(a + c*x^4)^2,x]
```

output
```
a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^
2 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^
2)*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*x^15)/15
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^2 \left(d+ex^2\right)^3 \, dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(a^2d^3+3a^2d^2ex^2+cex^{10}\left(2ae^2+3cd^2\right)+cdx^8\left(6ae^2+cd^2\right)+aex^6\left(ae^2+6cd^2\right)+adx^4\left(3ae^2+2cd^2\right)+3c^2$$

$$\Big\downarrow \text{2009}$$

$$a^2d^3x+a^2d^2ex^3+\frac{1}{11}cex^{11}\left(2ae^2+3cd^2\right)+\frac{1}{9}cdx^9\left(6ae^2+cd^2\right)+\frac{1}{7}aex^7\left(ae^2+6cd^2\right)+\frac{1}{5}adx^5\left(3ae^2+2cd^2\right)+\frac{3}{13}c^2de^2x^{13}+\frac{1}{15}c^2e^3x^{15}$$

input
```
Int[(d + e*x^2)^3*(a + c*x^4)^2,x]
```

output
```
a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^
2 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^
2)*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*x^15)/15
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

method	result
norman	$a^2d^3x+a^2d^2e\,x^3+\left(\frac{3}{5}d\,e^2a^2+\frac{2}{5}d^3ac\right)x^5+\left(\frac{1}{7}e^3a^2+\frac{6}{7}d^2eac\right)x^7+\left(\frac{2}{3}d\,e^2ac+\frac{1}{9}d^3c^2\right)x^9+$
default	$\frac{c^2e^3x^{15}}{15}+\frac{3c^2d\,e^2x^{13}}{13}+\frac{(2e^3ac+3d^2e\,c^2)x^{11}}{11}+\frac{(6d\,e^2ac+d^3c^2)x^9}{9}+\frac{(e^3a^2+6d^2eac)x^7}{7}+\frac{(3d\,e^2a^2+2d^3ac)x^5}{5}+$
gosper	$a^2d^3x+a^2d^2e\,x^3+\frac{3}{5}x^5d\,e^2a^2+\frac{2}{5}x^5d^3ac+\frac{1}{7}x^7e^3a^2+\frac{6}{7}x^7d^2eac+\frac{2}{3}x^9d\,e^2ac+\frac{1}{9}x^9d^3c^2+$
risch	$a^2d^3x+a^2d^2e\,x^3+\frac{3}{5}x^5d\,e^2a^2+\frac{2}{5}x^5d^3ac+\frac{1}{7}x^7e^3a^2+\frac{6}{7}x^7d^2eac+\frac{2}{3}x^9d\,e^2ac+\frac{1}{9}x^9d^3c^2+$
parallelrisch	$a^2d^3x+a^2d^2e\,x^3+\frac{3}{5}x^5d\,e^2a^2+\frac{2}{5}x^5d^3ac+\frac{1}{7}x^7e^3a^2+\frac{6}{7}x^7d^2eac+\frac{2}{3}x^9d\,e^2ac+\frac{1}{9}x^9d^3c^2+$
orering	$\frac{x(3003e^3c^2x^{14}+10395d\,e^2c^2x^{12}+8190ac\,e^3x^{10}+12285c^2d^2e\,x^{10}+30030acd\,e^2x^8+5005c^2d^3x^8+6435a^2e^3x^6+38610ac\,d^2e\,x}{45045}$

input
```
int((e*x^2+d)^3*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d^3*x+a^2*d^2*e*x^3+(3/5*d*e^2*a^2+2/5*d^3*a*c)*x^5+(1/7*e^3*a^2+6/7*d
^2*e*a*c)*x^7+(2/3*d*e^2*a*c+1/9*d^3*c^2)*x^9+(2/11*e^3*a*c+3/11*d^2*e*c^2
)*x^11+3/13*c^2*d*e^2*x^13+1/15*c^2*e^3*x^15
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \, dx = \frac{1}{15}\,c^2e^3x^{15} + \frac{3}{13}\,c^2de^2x^{13} + \frac{1}{11}\,\left(3\,c^2d^2e + 2\,ace^3\right)x^{11} + \frac{1}{9}\,\left(c^2d^3 + 6\,acde^2\right)x^9 + a^2d^2ex^3 + \frac{1}{7}\,\left(6\,acd^2e + a^2e^3\right)x^7 + a^2d^3x + \frac{1}{5}\,\left(2\,acd^3 + 3\,a^2de^2\right)x^5$$

input
```
integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 1/11*(3*c^2*d^2*e + 2*a*c*e^3)*x
^11 + 1/9*(c^2*d^3 + 6*a*c*d*e^2)*x^9 + a^2*d^2*e*x^3 + 1/7*(6*a*c*d^2*e +
 a^2*e^3)*x^7 + a^2*d^3*x + 1/5*(2*a*c*d^3 + 3*a^2*d*e^2)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \, dx = a^2d^3x + a^2d^2ex^3 + \frac{3c^2de^2x^{13}}{13} + \frac{c^2e^3x^{15}}{15} + x^{11} \cdot \left(\frac{2ace^3}{11} + \frac{3c^2d^2e}{11}\right) + x^9 \cdot \left(\frac{2acde^2}{3} + \frac{c^2d^3}{9}\right) + x^7\left(\frac{a^2e^3}{7} + \frac{6acd^2e}{7}\right) + x^5 \cdot \left(\frac{3a^2de^2}{5} + \frac{2acd^3}{5}\right)$$

input
```
integrate((e*x**2+d)**3*(c*x**4+a)**2,x)
```

output
```
a**2*d**3*x + a**2*d**2*e*x**3 + 3*c**2*d*e**2*x**13/13 + c**2*e**3*x**15/
15 + x**11*(2*a*c*e**3/11 + 3*c**2*d**2*e/11) + x**9*(2*a*c*d*e**2/3 + c**
2*d**3/9) + x**7*(a**2*e**3/7 + 6*a*c*d**2*e/7) + x**5*(3*a**2*d*e**2/5 +
2*a*c*d**3/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \left(d+ex^2\right)^3\left(a+cx^4\right)^2\,dx &= \frac{1}{15}c^2e^3x^{15}+\frac{3}{13}c^2de^2x^{13}+\frac{1}{11}\left(3\,c^2d^2e+2\,ace^3\right)x^{11}\\&\quad+\frac{1}{9}\left(c^2d^3+6\,acde^2\right)x^9+a^2d^2ex^3\\&\quad+\frac{1}{7}\left(6\,acd^2e+a^2e^3\right)x^7+a^2d^3x+\frac{1}{5}\left(2\,acd^3+3\,a^2de^2\right)x^5\end{aligned}$$

input

```
integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="maxima")
```

output

```
1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 1/11*(3*c^2*d^2*e + 2*a*c*e^3)*x
^11 + 1/9*(c^2*d^3 + 6*a*c*d*e^2)*x^9 + a^2*d^2*e*x^3 + 1/7*(6*a*c*d^2*e +
 a^2*e^3)*x^7 + a^2*d^3*x + 1/5*(2*a*c*d^3 + 3*a^2*d*e^2)*x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \left(d+ex^2\right)^3\left(a+cx^4\right)^2\,dx &= \frac{1}{15}c^2e^3x^{15}+\frac{3}{13}c^2de^2x^{13}+\frac{3}{11}c^2d^2ex^{11}+\frac{2}{11}ace^3x^{11}\\&\quad+\frac{1}{9}c^2d^3x^9+\frac{2}{3}acde^2x^9+\frac{6}{7}acd^2ex^7+\frac{1}{7}a^2e^3x^7\\&\quad+\frac{2}{5}acd^3x^5+\frac{3}{5}a^2de^2x^5+a^2d^2ex^3+a^2d^3x\end{aligned}$$

input

```
integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="giac")
```

output

```
1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 3/11*c^2*d^2*e*x^11 + 2/11*a*c*e
^3*x^11 + 1/9*c^2*d^3*x^9 + 2/3*a*c*d*e^2*x^9 + 6/7*a*c*d^2*e*x^7 + 1/7*a^
2*e^3*x^7 + 2/5*a*c*d^3*x^5 + 3/5*a^2*d*e^2*x^5 + a^2*d^2*e*x^3 + a^2*d^3*
x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \, dx = x^5 \left(\frac{3\,a^2\,d\,e^2}{5}+\frac{2\,c\,a\,d^3}{5}\right)+x^7 \left(\frac{a^2\,e^3}{7}+\frac{6\,c\,a\,d^2\,e}{7}\right) + x^9 \left(\frac{c^2\,d^3}{9}+\frac{2\,a\,c\,d\,e^2}{3}\right)+x^{11} \left(\frac{3\,c^2\,d^2\,e}{11}+\frac{2\,a\,c\,e^3}{11}\right) + a^2\,d^3\,x+\frac{c^2\,e^3\,x^{15}}{15}+a^2\,d^2\,e\,x^3+\frac{3\,c^2\,d\,e^2\,x^{13}}{13}$$

input
```
int((a + c*x^4)^2*(d + e*x^2)^3,x)
```

output
```
x^5*((3*a^2*d*e^2)/5 + (2*a*c*d^3)/5) + x^7*((a^2*e^3)/7 + (6*a*c*d^2*e)/7
) + x^9*((c^2*d^3)/9 + (2*a*c*d*e^2)/3) + x^11*((3*c^2*d^2*e)/11 + (2*a*c*
e^3)/11) + a^2*d^3*x + (c^2*e^3*x^15)/15 + a^2*d^2*e*x^3 + (3*c^2*d*e^2*x^
13)/13
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \, dx = \frac{x(3003c^2e^3x^{14}+10395c^2d\,e^2x^{12}+8190ac\,e^3x^{10}+12285c^2d^2e\,x^{10}+30030acd\,e^2x^8+5005c^2d^3x^8+6435}{45045}$$

input
```
int((e*x^2+d)^3*(c*x^4+a)^2,x)
```

output
```
(x*(45045*a**2*d**3 + 45045*a**2*d**2*e*x**2 + 27027*a**2*d*e**2*x**4 + 64
35*a**2*e**3*x**6 + 18018*a*c*d**3*x**4 + 38610*a*c*d**2*e*x**6 + 30030*a*
c*d*e**2*x**8 + 8190*a*c*e**3*x**10 + 5005*c**2*d**3*x**8 + 12285*c**2*d**
2*e*x**10 + 10395*c**2*d*e**2*x**12 + 3003*c**2*e**3*x**14))/45045
```

3.303 $\int \left(d+e x^2\right)^2 \left(a+c x^4\right)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \left(d+e x^2\right)^2 \left(a+c x^4\right)^2 \, dx = a^2 d^2 x+\frac{2}{3} a^2 d e x^3+\frac{1}{5} a \left(2 c d^2+a e^2\right) x^5+\frac{4}{7} a c d e x^7 +\frac{1}{9} c \left(c d^2+2 a e^2\right) x^9+\frac{2}{11} c^2 d e x^{11}+\frac{1}{13} c^2 e^2 x^{13}$$

output
```
a^2*d^2*x+2/3*a^2*d*e*x^3+1/5*a*(a*e^2+2*c*d^2)*x^5+4/7*a*c*d*e*x^7+1/9*c*
(2*a*e^2+c*d^2)*x^9+2/11*c^2*d*e*x^11+1/13*c^2*e^2*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \left(d+e x^2\right)^2 \left(a+c x^4\right)^2 \, dx = a^2 d^2 x+\frac{2}{3} a^2 d e x^3+\frac{1}{5} a \left(2 c d^2+a e^2\right) x^5+\frac{4}{7} a c d e x^7 +\frac{1}{9} c \left(c d^2+2 a e^2\right) x^9+\frac{2}{11} c^2 d e x^{11}+\frac{1}{13} c^2 e^2 x^{13}$$

input
```
Integrate[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

output
```
a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*a*c*d*e*x
^7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/11 + (c^2*e^2*x^13)
/13
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^2 \left(d+ex^2\right)^2 dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(a^2d^2+2a^2dex^2+cx^8\left(2ae^2+cd^2\right)+ax^4\left(ae^2+2cd^2\right)+4acdex^6+2c^2dex^{10}+c^2e^2x^{12}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{9}cx^9\left(2ae^2+cd^2\right)+\frac{1}{5}ax^5\left(ae^2+2cd^2\right)+\frac{4}{7}acdex^7+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

input
```
Int[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

output
```
a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*a*c*d*e*x
^7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/11 + (c^2*e^2*x^13)
/13
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
default	$\frac{c^2e^2x^{13}}{13}+\frac{2c^2de\,x^{11}}{11}+\frac{(2e^2ac+d^2c^2)x^9}{9}+\frac{4acde\,x^7}{7}+\frac{(a^2e^2+2d^2ac)x^5}{5}+\frac{2a^2de\,x^3}{3}+a^2d^2x$
norman	$\frac{c^2e^2x^{13}}{13}+\frac{2c^2de\,x^{11}}{11}+\left(\frac{2}{9}e^2ac+\frac{1}{9}d^2c^2\right)x^9+\frac{4acde\,x^7}{7}+\left(\frac{1}{5}a^2e^2+\frac{2}{5}d^2ac\right)x^5+\frac{2a^2de\,x^3}{3}+a^2d^2x$
gosper	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9d^2c^2+\frac{4}{7}acde\,x^7+\frac{1}{5}x^5a^2e^2+\frac{2}{5}x^5d^2ac+\frac{2}{3}a^2de\,x^3$
risch	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9d^2c^2+\frac{4}{7}acde\,x^7+\frac{1}{5}x^5a^2e^2+\frac{2}{5}x^5d^2ac+\frac{2}{3}a^2de\,x^3$
parallelrisch	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9d^2c^2+\frac{4}{7}acde\,x^7+\frac{1}{5}x^5a^2e^2+\frac{2}{5}x^5d^2ac+\frac{2}{3}a^2de\,x^3$
orering	$\frac{x(3465e^2c^2x^{12}+8190de\,c^2x^{10}+10010ac\,e^2x^8+5005c^2d^2x^8+25740deac\,x^6+9009a^2e^2x^4+18018ac\,d^2x^4+30030de\,a^2x^2+450}{45045}$

input
```
int((e*x^2+d)^2*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*c^2*e^2*x^13+2/11*c^2*d*e*x^11+1/9*(2*a*c*e^2+c^2*d^2)*x^9+4/7*a*c*d*
e*x^7+1/5*(a^2*e^2+2*a*c*d^2)*x^5+2/3*a^2*d*e*x^3+a^2*d^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \left(d+ex^2\right)^2\left(a+cx^4\right)^2\,dx=\frac{1}{13}\,c^2e^2x^{13}+\frac{2}{11}\,c^2dex^{11}+\frac{4}{7}\,acdex^7+\frac{1}{9}\,\left(c^2d^2+2\,ace^2\right)x^9+\frac{2}{3}\,a^2dex^3+\frac{1}{5}\,\left(2\,acd^2+a^2e^2\right)x^5+a^2d^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 4/7*a*c*d*e*x^7 + 1/9*(c^2*d^2 + 2
*a*c*e^2)*x^9 + 2/3*a^2*d*e*x^3 + 1/5*(2*a*c*d^2 + a^2*e^2)*x^5 + a^2*d^2*
x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = a^2d^2x + \frac{2a^2dex^3}{3} + \frac{4acdex^7}{7} + \frac{2c^2dex^{11}}{11} + \frac{c^2e^2x^{13}}{13} + x^9 \cdot \left(\frac{2ace^2}{9} + \frac{c^2d^2}{9}\right) + x^5\left(\frac{a^2e^2}{5} + \frac{2acd^2}{5}\right)$$

input
```
integrate((e*x**2+d)**2*(c*x**4+a)**2,x)
```

output
```
a**2*d**2*x + 2*a**2*d*e*x**3/3 + 4*a*c*d*e*x**7/7 + 2*c**2*d*e*x**11/11 +
 c**2*e**2*x**13/13 + x**9*(2*a*c*e**2/9 + c**2*d**2/9) + x**5*(a**2*e**2/
5 + 2*a*c*d**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = \frac{1}{13}\,c^2e^2x^{13} + \frac{2}{11}\,c^2dex^{11} + \frac{4}{7}\,acdex^7 + \frac{1}{9}\,\left(c^2d^2 + 2\,ace^2\right)x^9 + \frac{2}{3}\,a^2dex^3 + \frac{1}{5}\,\left(2\,acd^2 + a^2e^2\right)x^5 + a^2d^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 4/7*a*c*d*e*x^7 + 1/9*(c^2*d^2 + 2
*a*c*e^2)*x^9 + 2/3*a^2*d*e*x^3 + 1/5*(2*a*c*d^2 + a^2*e^2)*x^5 + a^2*d^2*
x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = \frac{1}{13}c^2e^2x^{13}+\frac{2}{11}c^2dex^{11}+\frac{1}{9}c^2d^2x^9+\frac{2}{9}ace^2x^9 +\frac{4}{7}acdex^7+\frac{2}{5}acd^2x^5+\frac{1}{5}a^2e^2x^5+\frac{2}{3}a^2dex^3+a^2d^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 1/9*c^2*d^2*x^9 + 2/9*a*c*e^2*x^9
+ 4/7*a*c*d*e*x^7 + 2/5*a*c*d^2*x^5 + 1/5*a^2*e^2*x^5 + 2/3*a^2*d*e*x^3 +
a^2*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = x^5\left(\frac{a^2\,e^2}{5}+\frac{2\,c\,a\,d^2}{5}\right)+x^9\left(\frac{c^2\,d^2}{9}+\frac{2\,a\,c\,e^2}{9}\right)+a^2\,d^2\,x +\frac{c^2\,e^2\,x^{13}}{13}+\frac{2\,a^2\,d\,e\,x^3}{3}+\frac{2\,c^2\,d\,e\,x^{11}}{11}+\frac{4\,a\,c\,d\,e\,x^7}{7}$$

input
```
int((a + c*x^4)^2*(d + e*x^2)^2,x)
```

output
```
x^5*((a^2*e^2)/5 + (2*a*c*d^2)/5) + x^9*((c^2*d^2)/9 + (2*a*c*e^2)/9) + a^
2*d^2*x + (c^2*e^2*x^13)/13 + (2*a^2*d*e*x^3)/3 + (2*c^2*d*e*x^11)/11 + (4
*a*c*d*e*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx$$
$$= \frac{x(3465c^2e^2x^{12}+8190c^2de\,x^{10}+10010ac\,e^2x^8+5005c^2d^2x^8+25740acde\,x^6+9009a^2e^2x^4+18018ac\,d^2}{45045}$$

input
```
int((e*x^2+d)^2*(c*x^4+a)^2,x)
```

output
```
(x*(45045*a**2*d**2 + 30030*a**2*d*e*x**2 + 9009*a**2*e**2*x**4 + 18018*a*
c*d**2*x**4 + 25740*a*c*d*e*x**6 + 10010*a*c*e**2*x**8 + 5005*c**2*d**2*x*
*8 + 8190*c**2*d*e*x**10 + 3465*c**2*e**2*x**12))/45045
```

3.304 $\int (d+ex^2)(a+cx^4)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int (d+ex^2)(a+cx^4)^2\,dx=a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

output
```
a^2*d*x+1/3*a^2*e*x^3+2/5*a*c*d*x^5+2/7*a*c*e*x^7+1/9*c^2*d*x^9+1/11*c^2*e
*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (d+ex^2)(a+cx^4)^2\,dx=a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

input
```
Integrate[(d + e*x^2)*(a + c*x^4)^2,x]
```

output
```
a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/
9 + (c^2*e*x^11)/11
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^2 \left(d+ex^2\right) dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(a^2d+a^2ex^2+2acdx^4+2acex^6+c^2dx^8+c^2ex^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

input
```
Int[(d + e*x^2)*(a + c*x^4)^2,x]
```

output
```
a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/
9 + (c^2*e*x^11)/11
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
gosper	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51
default	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51
norman	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51
risch	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51
parallelrisch	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51
orering	$\frac{x\left(315c^2e\,x^{10}+385d\,c^2x^8+990ace\,x^6+1386acd\,x^4+1155a^2e\,x^2+3465a^2d\right)}{3465}$	54

input
```
int((e*x^2+d)*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d*x+1/3*a^2*e*x^3+2/5*a*c*d*x^5+2/7*a*c*e*x^7+1/9*c^2*d*x^9+1/11*c^2*e
*x^11
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx=\frac{1}{11}\,c^2ex^{11}+\frac{1}{9}\,c^2dx^9+\frac{2}{7}\,acex^7+\frac{2}{5}\,acdx^5+\frac{1}{3}\,a^2ex^3+a^2dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 + 1/3*a^2*
e*x^3 + a^2*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx=a^2dx+\frac{a^2ex^3}{3}+\frac{2acdx^5}{5}+\frac{2acex^7}{7}+\frac{c^2dx^9}{9}+\frac{c^2ex^{11}}{11}$$

input
```
integrate((e*x**2+d)*(c*x**4+a)**2,x)
```

output
```
a**2*d*x + a**2*e*x**3/3 + 2*a*c*d*x**5/5 + 2*a*c*e*x**7/7 + c**2*d*x**9/9
 + c**2*e*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx=\frac{1}{11}\,c^2ex^{11}+\frac{1}{9}\,c^2dx^9+\frac{2}{7}\,acex^7+\frac{2}{5}\,acdx^5+\frac{1}{3}\,a^2ex^3+a^2dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 + 1/3*a^2*
e*x^3 + a^2*d*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx=\frac{1}{11}\,c^2ex^{11}+\frac{1}{9}\,c^2dx^9+\frac{2}{7}\,acex^7+\frac{2}{5}\,acdx^5+\frac{1}{3}\,a^2ex^3+a^2dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 + 1/3*a^2*
e*x^3 + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx=\frac{e\,a^2\,x^3}{3}+d\,a^2\,x+\frac{2\,e\,a\,c\,x^7}{7}+\frac{2\,d\,a\,c\,x^5}{5}+\frac{e\,c^2\,x^{11}}{11}+\frac{d\,c^2\,x^9}{9}$$

input
```
int((a + c*x^4)^2*(d + e*x^2),x)
```

output
```
(a^2*e*x^3)/3 + (c^2*d*x^9)/9 + (c^2*e*x^11)/11 + a^2*d*x + (2*a*c*d*x^5)/
5 + (2*a*c*e*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx$$
$$=\frac{x(315c^2e\,x^{10}+385c^2d\,x^8+990ace\,x^6+1386acd\,x^4+1155a^2e\,x^2+3465a^2d)}{3465}$$

input
```
int((e*x^2+d)*(c*x^4+a)^2,x)
```

output
```
(x*(3465*a**2*d + 1155*a**2*e*x**2 + 1386*a*c*d*x**4 + 990*a*c*e*x**6 + 38
5*c**2*d*x**8 + 315*c**2*e*x**10))/3465
```

3.305 $\int (a+cx^4)^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int (a+cx^4)^2 \, dx = a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

output
```
a^2*x+2/5*a*c*x^5+1/9*c^2*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+cx^4)^2 \, dx = a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

input
```
Integrate[(a + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^2 dx$$

$$\Big\downarrow \text{747}$$

$$\int \left(a^2+2acx^4+c^2x^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2x+\frac{2}{5}acx^5+\frac{c^2x^9}{9}$$

input
```
Int[(a + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$a^2x+\frac{2}{5}ac\,x^5+\frac{1}{9}c^2x^9$	22
default	$a^2x+\frac{2}{5}ac\,x^5+\frac{1}{9}c^2x^9$	22
norman	$a^2x+\frac{2}{5}ac\,x^5+\frac{1}{9}c^2x^9$	22
risch	$a^2x+\frac{2}{5}ac\,x^5+\frac{1}{9}c^2x^9$	22
parallelrisch	$a^2x+\frac{2}{5}ac\,x^5+\frac{1}{9}c^2x^9$	22
orering	$\frac{x\left(5c^2x^8+18ac\,x^4+45a^2\right)}{45}$	25

input
```
int((c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/5*a*c*x^5+1/9*c^2*x^9
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{5}\,acx^5+a^2x$$

input
```
integrate((c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \left(a+cx^4\right)^2\,dx=a^2x+\frac{2acx^5}{5}+\frac{c^2x^9}{9}$$

input
```
integrate((c*x**4+a)**2,x)
```

output
```
a**2*x + 2*a*c*x**5/5 + c**2*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{5}\,acx^5+a^2x$$

input
```
integrate((c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{5}\,acx^5+a^2x$$

input
```
integrate((c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^4\right)^2\,dx=a^2\,x+\frac{2\,a\,c\,x^5}{5}+\frac{c^2\,x^9}{9}$$

input
```
int((a + c*x^4)^2,x)
```

output
```
a^2*x + (c^2*x^9)/9 + (2*a*c*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \left(a+cx^4\right)^2\,dx=\frac{x(5c^2x^8+18ac\,x^4+45a^2)}{45}$$

input
```
int((c*x^4+a)^2,x)
```

output
```
(x*(45*a**2 + 18*a*c*x**4 + 5*c**2*x**8))/45
```

3.306 $\int \frac{\left(a+cx^4\right)^2}{d+ex^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{\left(a+cx^4\right)^2}{d+ex^2}\,dx=-\frac{cd\left(cd^2+2ae^2\right)x}{e^4}+\frac{c\left(cd^2+2ae^2\right)x^3}{3e^3}-\frac{c^2dx^5}{5e^2}+\frac{c^2x^7}{7e}+\frac{\left(cd^2+ae^2\right)^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}$$

output
```
-c*d*(2*a*e^2+c*d^2)*x/e^4+1/3*c*(2*a*e^2+c*d^2)*x^3/e^3-1/5*c^2*d*x^5/e^2
+1/7*c^2*x^7/e+(a*e^2+c*d^2)^2*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+cx^4\right)^2}{d+ex^2}\,dx=\frac{cx\left(70ae^2\left(-3d+ex^2\right)+c\left(-105d^3+35d^2ex^2-21de^2x^4+15e^3x^6\right)\right)}{105e^4}+\frac{\left(cd^2+ae^2\right)^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2),x]
```

output
```
(c*x*(70*a*e^2*(-3*d + e*x^2) + c*(-105*d^3 + 35*d^2*e*x^2 - 21*d*e^2*x^4
+ 15*e^3*x^6)))/(105*e^4) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*e^(9/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{d+ex^2} \, dx$$

$$\Big\downarrow \text{ 1468}$$

$$\int \left(\frac{a^2e^4+2acd^2e^2+c^2d^4}{e^4(d+ex^2)}-\frac{cd(2ae^2+cd^2)}{e^4}+\frac{cx^2(2ae^2+cd^2)}{e^3}-\frac{c^2dx^4}{e^2}+\frac{c^2x^6}{e}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(ae^2+cd^2)^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}-\frac{cdx(2ae^2+cd^2)}{e^4}+\frac{cx^3(2ae^2+cd^2)}{3e^3}-\frac{c^2dx^5}{5e^2}+\frac{c^2x^7}{7e}$$

input
```
Int[(a + c*x^4)^2/(d + e*x^2),x]
```

output
```
-((c*d*(c*d^2 + 2*a*e^2)*x)/e^4) + (c*(c*d^2 + 2*a*e^2)*x^3)/(3*e^3) - (c^
2*d*x^5)/(5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)
/Sqrt[d]])/(Sqrt[d]*e^(9/2))
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.96

method	result
default	$-\frac{c\left(-\frac{c\,x^{7} e^{3}}{7}+\frac{d c\,x^{5} e^{2}}{5}-\frac{e\left(2 a\,e^{2}+c\,d^{2}\right) x^{3}}{3}+x d\left(2 a\,e^{2}+c\,d^{2}\right)\right)}{e^{4}}+\frac{\left(a^{2} e^{4}+2 a c\,d^{2} e^{2}+c^{2} d^{4}\right) \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{e^{4} \sqrt{d e}}$
risch	$\frac{c^{2} x^{7}}{7 e}-\frac{c^{2} d\,x^{5}}{5 e^{2}}+\frac{2 c a\,x^{3}}{3 e}+\frac{c^{2} d^{2} x^{3}}{3 e^{3}}-\frac{2 c a d x}{e^{2}}-\frac{c^{2} d^{3} x}{e^{4}}-\frac{\ln\left(e x+\sqrt{-d e}\right) a^{2}}{2 \sqrt{-d e}}-\frac{\ln\left(e x+\sqrt{-d e}\right) a c\,d^{2}}{e^{2} \sqrt{-d e}}-\frac{\ln\left(e x+\sqrt{-d e}\right) c^{2} d^{4}}{2 e^{4} \sqrt{-d e}}$

input
```
int((c*x^4+a)^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-c/e^4*(-1/7*c*x^7*e^3+1/5*d*c*x^5*e^2-1/3*e*(2*a*e^2+c*d^2)*x^3+x*d*(2*a*
e^2+c*d^2))+(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/e^4/(d*e)^(1/2)*arctan(e*x/(d*
e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.48

$$\int \frac{\left(a+cx^4\right)^2}{d+ex^2}\,dx$$

$$= \left[\frac{30\,c^2de^4x^7 - 42\,c^2d^2e^3x^5 + 70\,(c^2d^3e^2 + 2\,acde^4)x^3 - 105\,(c^2d^4 + 2\,acd^2e^2 + a^2e^4)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-d}}{ex^2+d}\right.}{210\,de^5}\right.$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="fricas")
```

output

```
[1/210*(30*c^2*d*e^4*x^7 - 42*c^2*d^2*e^3*x^5 + 70*(c^2*d^3*e^2 + 2*a*c*d*
e^4)*x^3 - 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-d*e)*log((e*x^2 -
 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 210*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*
e^5), 1/105*(15*c^2*d*e^4*x^7 - 21*c^2*d^2*e^3*x^5 + 35*(c^2*d^3*e^2 + 2*a
*c*d*e^4)*x^3 + 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)*arctan(s
qrt(d*e)*x/d) - 105*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*e^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(100) = 200$.

Time = 0.24 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.19

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = -\frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + x^3\cdot\left(\frac{2ac}{3e} + \frac{c^2d^2}{3e^3}\right) + x\left(-\frac{2acd}{e^2} - \frac{c^2d^3}{e^4}\right) - \frac{\sqrt{-\frac{1}{de^9}}\,(ae^2+cd^2)^2\log\left(-\frac{de^4\sqrt{-\frac{1}{de^9}}\,(ae^2+cd^2)^2}{a^2e^4+2acd^2e^2+c^2d^4} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^9}}\,(ae^2+cd^2)^2\log\left(\frac{de^4\sqrt{-\frac{1}{de^9}}\,(ae^2+cd^2)^2}{a^2e^4+2acd^2e^2+c^2d^4} + x\right)}{2}$$

input

```
integrate((c*x**4+a)**2/(e*x**2+d),x)
```

output

```
-c**2*d*x**5/(5*e**2) + c**2*x**7/(7*e) + x**3*(2*a*c/(3*e) + c**2*d**2/(3
*e**3)) + x*(-2*a*c*d/e**2 - c**2*d**3/e**4) - sqrt(-1/(d*e**9))*(a*e**2 +
 c*d**2)**2*log(-d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4
+ 2*a*c*d**2*e**2 + c**2*d**4) + x)/2 + sqrt(-1/(d*e**9))*(a*e**2 + c*d**2
)**2*log(d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4 + 2*a*c*
d**2*e**2 + c**2*d**4) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = \frac{(c^2d^4+2\,acd^2e^2+a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^4} + \frac{15\,c^2e^6x^7-21\,c^2de^5x^5+35\,c^2d^2e^4x^3+70\,ace^6x^3-105\,c^2d^3e^3x-210\,acde^5x}{105\,e^7}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="giac")
```

output
```
(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4)
+ 1/105*(15*c^2*e^6*x^7 - 21*c^2*d*e^5*x^5 + 35*c^2*d^2*e^4*x^3 + 70*a*c*e
^6*x^3 - 105*c^2*d^3*e^3*x - 210*a*c*d*e^5*x)/e^7
```

Mupad [B] (verification not implemented)

Time = 16.95 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.31

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = x^3\left(\frac{c^2\,d^2}{3\,e^3}+\frac{2\,a\,c}{3\,e}\right)+\frac{c^2\,x^7}{7\,e}-\frac{c^2\,d\,x^5}{5\,e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,(c\,d^2+a\,e^2)^2}{\sqrt{d}\,(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4)}\right)\,(c\,d^2+a\,e^2)^2}{\sqrt{d}\,e^{9/2}} - \frac{d\,x\left(\frac{c^2\,d^2}{e^3}+\frac{2\,a\,c}{e}\right)}{e}$$

input
```
int((a + c*x^4)^2/(d + e*x^2),x)
```

output
```
x^3*((c^2*d^2)/(3*e^3) + (2*a*c)/(3*e)) + (c^2*x^7)/(7*e) - (c^2*d*x^5)/(5
*e^2) + (atan((e^(1/2)*x*(a*e^2 + c*d^2)^2)/(d^(1/2)*(a^2*e^4 + c^2*d^4 +
2*a*c*d^2*e^2)))*(a*e^2 + c*d^2)^2)/(d^(1/2)*e^(9/2)) - (d*x*((c^2*d^2)/e^
3 + (2*a*c)/e))/e
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.44

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = \frac{105\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2e^4+210\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ac\,d^2e^2+105\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)c^2d^4-210ac\,d}{105d\,e^5}$$

input
```
int((c*x^4+a)^2/(e*x^2+d),x)
```

output
```
(105*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*e**4 + 210*sqrt(e)
*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c*d**2*e**2 + 105*sqrt(e)*sqrt(d)
*atan((e*x)/(sqrt(e)*sqrt(d)))*c**2*d**4 - 210*a*c*d**2*e**3*x + 70*a*c*d*
e**4*x**3 - 105*c**2*d**4*e*x + 35*c**2*d**3*e**2*x**3 - 21*c**2*d**2*e**3
*x**5 + 15*c**2*d*e**4*x**7)/(105*d*e**5)
```

3.307 $\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx = \frac{c(3cd^2+2ae^2)\,x}{e^4} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+ae^2)^2\,x}{2de^4\,(d+ex^2)} - \frac{(7cd^2-ae^2)\,(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}$$

output
```
c*(2*a*e^2+3*c*d^2)*x/e^4-2/3*c^2*d*x^3/e^3+1/5*c^2*x^5/e^2+1/2*(a*e^2+c*d
^2)^2*x/d/e^4/(e*x^2+d)-1/2*(-a*e^2+7*c*d^2)*(a*e^2+c*d^2)*arctan(e^(1/2)*
x/d^(1/2))/d^(3/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx = \frac{c(3cd^2+2ae^2)\,x}{e^4} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+ae^2)^2\,x}{2de^4\,(d+ex^2)} - \frac{(7c^2d^4+6acd^2e^2-a^2e^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^2,x]
```

output
```
(c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2)
+ ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c^2*d^4 + 6*a*c*d^2*e^
2 - a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1472, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(ae^2+cd^2\right)^2}{2de^4\left(d+ex^2\right)}-\frac{\int -\frac{\frac{2c^2dx^6}{e}-\frac{2c^2d^2x^4}{e^2}+\frac{2cd\left(cd^2+2ae^2\right)x^2}{e^3}+a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{ex^2+d}dx}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{2c^2dx^6}{e}-\frac{2c^2d^2x^4}{e^2}+\frac{2cd\left(cd^2+2ae^2\right)x^2}{e^3}+a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{ex^2+d}dx}{2d}+\frac{x\left(ae^2+cd^2\right)^2}{2de^4\left(d+ex^2\right)}$$

$$\Big\downarrow \text{2341}$$

$$\frac{\int \left(\frac{2c^2dx^4}{e^2}-\frac{4c^2d^2x^2}{e^3}+\frac{2cd(3cd^2+2ae^2)}{e^4}+\frac{-7c^2d^4-6ace^2d^2+a^2e^4}{e^4(ex^2+d)}\right)dx}{2d}+\frac{x\left(ae^2+cd^2\right)^2}{2de^4\left(d+ex^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\left(7cd^2-ae^2\right)\left(ae^2+cd^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}+\frac{2cdx\left(2ae^2+3cd^2\right)}{e^4}-\frac{4c^2d^2x^3}{3e^3}+\frac{2c^2dx^5}{5e^2}}{2d}+\frac{x\left(ae^2+cd^2\right)^2}{2de^4\left(d+ex^2\right)}$$

input
```
Int[(a + c*x^4)^2/(d + e*x^2)^2,x]
```

output
```
((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) + ((2*c*d*(3*c*d^2 + 2*a*e^2)*
x)/e^4 - (4*c^2*d^2*x^3)/(3*e^3) + (2*c^2*d*x^5)/(5*e^2) - ((7*c*d^2 - a*e
^2)*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2)))/(2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

method	result
default	$\frac{c\left(\frac{1}{5}c\,x^5e^2-\frac{2}{3}cd\,x^3e+2xa\,e^2+3xc\,d^2\right)}{e^4}+\frac{\frac{\left(a^2e^4+2ac\,d^2e^2+c^2d^4\right)x}{2d\left(e\,x^2+d\right)}+\frac{\left(a^2e^4-6ac\,d^2e^2-7c^2d^4\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}}{e^4}$
risch	$\frac{c^2x^5}{5e^2}-\frac{2c^2d\,x^3}{3e^3}+\frac{2cxa}{e^2}+\frac{3c^2x\,d^2}{e^4}+\frac{\left(a^2e^4+2ac\,d^2e^2+c^2d^4\right)x}{2d\,e^4\left(e\,x^2+d\right)}-\frac{\ln\left(ex+\sqrt{-de}\right)a^2}{4\sqrt{-de}\,d}+\frac{3d\ln\left(ex+\sqrt{-de}\right)ac}{2e^2\sqrt{-de}}+\frac{7d^3\ln(ex+}{4e^4\sqrt{-}}$

input
```
int((c*x^4+a)^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c/e^4*(1/5*c*x^5*e^2-2/3*c*d*x^3*e+2*x*a*e^2+3*x*c*d^2)+1/e^4*(1/2*(a^2*e^
4+2*a*c*d^2*e^2+c^2*d^4)/d*x/(e*x^2+d)+1/2*(a^2*e^4-6*a*c*d^2*e^2-7*c^2*d^
4)/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.01

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx$$

$$= \left[\frac{12\,c^2d^2e^4x^7 - 28\,c^2d^3e^3x^5 + 20\,(7\,c^2d^4e^2 + 6\,acd^2e^4)x^3 + 15\,(7\,c^2d^5 + 6\,acd^3e^2 - a^2de^4 + (7\,c^2d^4e + 6\,a}{60\,(d^2e^6x^2 + d^3e^5)}\right.$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*c^2*d^2*e^4*x^7 - 28*c^2*d^3*e^3*x^5 + 20*(7*c^2*d^4*e^2 + 6*a*c
*d^2*e^4)*x^3 + 15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (7*c^2*d^4*e +
 6*a*c*d^2*e^3 - a^2*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)
/(e*x^2 + d)) + 30*(7*c^2*d^5*e + 6*a*c*d^3*e^3 + a^2*d*e^5)*x)/(d^2*e^6*x
^2 + d^3*e^5), 1/30*(6*c^2*d^2*e^4*x^7 - 14*c^2*d^3*e^3*x^5 + 10*(7*c^2*d^
4*e^2 + 6*a*c*d^2*e^4)*x^3 - 15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (
7*c^2*d^4*e + 6*a*c*d^2*e^3 - a^2*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d
) + 15*(7*c^2*d^5*e + 6*a*c*d^3*e^3 + a^2*d*e^5)*x)/(d^2*e^6*x^2 + d^3*e^5
)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(122) = 244$.

Time = 0.43 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.40

$$\begin{aligned}\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx &= -\frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + x\left(\frac{2ac}{e^2}+\frac{3c^2d^2}{e^4}\right) + \frac{x(a^2e^4+2acd^2e^2+c^2d^4)}{2d^2e^4+2de^5x^2} \\ &\quad - \frac{\sqrt{-\frac{1}{d^3e^9}}\,(ae^2-7cd^2)\,(ae^2+cd^2)\log\left(-\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}\,(ae^2-7cd^2)\,(ae^2+cd^2)}{a^2e^4-6acd^2e^2-7c^2d^4}+x\right)}{4} \\ &\quad + \frac{\sqrt{-\frac{1}{d^3e^9}}\,(ae^2-7cd^2)\,(ae^2+cd^2)\log\left(\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}\,(ae^2-7cd^2)\,(ae^2+cd^2)}{a^2e^4-6acd^2e^2-7c^2d^4}+x\right)}{4}\end{aligned}$$

input
```
integrate((c*x**4+a)**2/(e*x**2+d)**2,x)
```

output
```
-2*c**2*d*x**3/(3*e**3) + c**2*x**5/(5*e**2) + x*(2*a*c/e**2 + 3*c**2*d**2
/e**4) + x*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)/(2*d**2*e**4 + 2*d*e*
*5*x**2) - sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)*log(
-d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**
2*e**4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)/4 + sqrt(-1/(d**3*e**9))*(a*e
**2 - 7*c*d**2)*(a*e**2 + c*d**2)*log(d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e*
*2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**2*e**4 - 6*a*c*d**2*e**2 - 7*c**2*d**
4) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.10

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx = -\frac{(7\,c^2d^4+6\,acd^2e^2-a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^4} + \frac{c^2d^4x+2\,acd^2e^2x+a^2e^4x}{2\,(ex^2+d)de^4} + \frac{3\,c^2e^8x^5-10\,c^2de^7x^3+45\,c^2d^2e^6x+30\,ace^8x}{15\,e^{10}}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/2*(7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e
)*d*e^4) + 1/2*(c^2*d^4*x + 2*a*c*d^2*e^2*x + a^2*e^4*x)/((e*x^2 + d)*d*e^
4) + 1/15*(3*c^2*e^8*x^5 - 10*c^2*d*e^7*x^3 + 45*c^2*d^2*e^6*x + 30*a*c*e^
8*x)/e^10
```

Mupad [B] (verification not implemented)

Time = 16.94 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.40

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx = x\left(\frac{3\,c^2\,d^2}{e^4}+\frac{2\,a\,c}{e^2}\right)+\frac{c^2\,x^5}{5\,e^2}-\frac{2\,c^2\,d\,x^3}{3\,e^3}+\frac{x\,(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4)}{2\,d\,\left(e^5\,x^2+d\,e^4\right)} - \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,\left(c\,d^2+a\,e^2\right)\,\left(a\,e^2-7\,c\,d^2\right)}{\sqrt{d}\,\left(-a^2\,e^4+6\,a\,c\,d^2\,e^2+7\,c^2\,d^4\right)}\right)\,\left(c\,d^2+a\,e^2\right)\,\left(a\,e^2-7\,c\,d^2\right)}{2\,d^{3/2}\,e^{9/2}}$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^2,x)
```

output
```
x*((3*c^2*d^2)/e^4 + (2*a*c)/e^2) + (c^2*x^5)/(5*e^2) - (2*c^2*d*x^3)/(3*e
^3) + (x*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(2*d*(d*e^4 + e^5*x^2)) - (a
tan((e^(1/2)*x*(a*e^2 + c*d^2)*(a*e^2 - 7*c*d^2))/(d^(1/2)*(7*c^2*d^4 - a^
2*e^4 + 6*a*c*d^2*e^2)))*(a*e^2 + c*d^2)*(a*e^2 - 7*c*d^2))/(2*d^(3/2)*e^(
9/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.01

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx$$
$$= \frac{15\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2d\,e^4+15\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2e^5x^2-90\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ac\,d^3e^2-90\sqrt{e}\,\sqrt{}}{}$$

input
```
int((c*x^4+a)^2/(e*x^2+d)^2,x)
```

output
```
(15*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*d*e**4 + 15*sqrt(e)
*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*e**5*x**2 - 90*sqrt(e)*sqrt(d)
*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c*d**3*e**2 - 90*sqrt(e)*sqrt(d)*atan((e*
x)/(sqrt(e)*sqrt(d)))*a*c*d**2*e**3*x**2 - 105*sqrt(e)*sqrt(d)*atan((e*x)/
(sqrt(e)*sqrt(d)))*c**2*d**5 - 105*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqr
t(d)))*c**2*d**4*e*x**2 + 15*a**2*d*e**5*x + 90*a*c*d**3*e**3*x + 60*a*c*d
**2*e**4*x**3 + 105*c**2*d**5*e*x + 70*c**2*d**4*e**2*x**3 - 14*c**2*d**3*
e**3*x**5 + 6*c**2*d**2*e**4*x**7)/(30*d**2*e**5*(d + e*x**2))
```

3.308 $\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 155

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx=-\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}+\frac{\left(cd^2+ae^2\right)^2x}{4de^4\left(d+ex^2\right)^2}-\frac{\left(13cd^2-3ae^2\right)\left(cd^2+ae^2\right)x}{8d^2e^4\left(d+ex^2\right)}+\frac{\left(35c^2d^4+6acd^2e^2+3a^2e^4\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}$$

output
```
-3*c^2*d*x/e^4+1/3*c^2*x^3/e^3+1/4*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^2-1/8
*(-3*a*e^2+13*c*d^2)*(a*e^2+c*d^2)*x/d^2/e^4/(e*x^2+d)+1/8*(3*a^2*e^4+6*a*
c*d^2*e^2+35*c^2*d^4)*arctan(e^(1/2)*x/d^(1/2))/d^(5/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx=\frac{x(3a^2e^4(5d+3ex^2)-6acd^2e^2(3d+5ex^2)-c^2d^2(105d^3+175d^2ex^2+56de^2x^4-8e^3x^6))}{24d^2e^4\left(d+ex^2\right)^2}+\frac{\left(35c^2d^4+6acd^2e^2+3a^2e^4\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^3,x]
```

output
```
(x*(3*a^2*e^4*(5*d + 3*e*x^2) - 6*a*c*d^2*e^2*(3*d + 5*e*x^2) - c^2*d^2*(1
05*d^3 + 175*d^2*e*x^2 + 56*d*e^2*x^4 - 8*e^3*x^6)))/(24*d^2*e^4*(d + e*x^
2)^2) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(8*d^(5/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1472, 25, 2345, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}-\frac{\int -\frac{\frac{4c^2dx^6}{e}-\frac{4c^2d^2x^4}{e^2}+\frac{4cd\left(cd^2+2ae^2\right)x^2}{e^3}+3a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{\left(ex^2+d\right)^2}dx}{4d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{4c^2dx^6}{e}-\frac{4c^2d^2x^4}{e^2}+\frac{4cd\left(cd^2+2ae^2\right)x^2}{e^3}+3a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{\left(ex^2+d\right)^2}dx}{4d}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{2d(d+ex^2)}-\frac{\int -\frac{\frac{11c^2d^4}{e^4}-\frac{16c^2x^2d^3}{e^3}+\frac{8c^2x^4d^2}{e^2}+\frac{6acd^2}{e^2}+3a^2}{ex^2+d}dx}{2d}}{4d}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\frac{11c^2d^4}{e^4}-\frac{16c^2x^2d^3}{e^3}+\frac{8c^2x^4d^2}{e^2}+\frac{6acd^2}{e^2}+3a^2}{ex^2+d}dx}{2d}+\frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{2d(d+ex^2)}}{4d}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 1467$$

$$\frac{\frac{\int \left(-\frac{24c^2d^3}{e^4}+\frac{8c^2x^2d^2}{e^3}+\frac{35c^2d^4+6ace^2d^2+3a^2e^4}{e^4\left(ex^2+d\right)}\right)dx}{2d}+\frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{2d(d+ex^2)}}{4d}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{\left(3a^2e^4+6acd^2e^2+35c^2d^4\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}-\frac{24c^2d^3x}{e^4}+\frac{8c^2d^2x^3}{3e^3}}{2d}+\frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{2d(d+ex^2)}}{4d}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}$$

input

```
Int[(a + c*x^4)^2/(d + e*x^2)^3,x]
```

output

```
((c*d^2 + a*e^2)^2*x)/(4*d*e^4*(d + e*x^2)^2) + (((3*a^2 - (13*c^2*d^4)/e^
4 - (10*a*c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) + ((-24*c^2*d^3*x)/e^4 + (8*c^2
*d^2*x^3)/(3*e^3) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2)))/(2*d))/(4*d)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1467

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.98

method	result
default	$-\frac{c^{2}\left(-\frac{1}{3}e\,x^{3}+3dx\right)}{e^{4}}+\frac{\frac{\frac{e\left(3a^{2}e^{4}-10ac\,d^{2}e^{2}-13c^{2}d^{4}\right)x^{3}}{8d^{2}}+\frac{\left(5a^{2}e^{4}-6ac\,d^{2}e^{2}-11c^{2}d^{4}\right)x}{8d}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(3a^{2}e^{4}+6ac\,d^{2}e^{2}+35c^{2}d^{4}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8d^{2}\sqrt{de}}}{e^{4}}$
risch	$\frac{c^{2}x^{3}}{3e^{3}}-\frac{3c^{2}dx}{e^{4}}+\frac{\frac{e\left(3a^{2}e^{4}-10ac\,d^{2}e^{2}-13c^{2}d^{4}\right)x^{3}}{8d^{2}}+\frac{\left(5a^{2}e^{4}-6ac\,d^{2}e^{2}-11c^{2}d^{4}\right)x}{8d}}{e^{4}\left(e\,x^{2}+d\right)^{2}}-\frac{3\ln\left(ex+\sqrt{-de}\right)a^{2}}{16\sqrt{-de}\,d^{2}}-\frac{3\ln\left(ex+\sqrt{-de}\right)ac}{8e^{2}\sqrt{-de}}-$

input
```
int((c*x^4+a)^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-c^2/e^4*(-1/3*e*x^3+3*d*x)+1/e^4*((1/8*e*(3*a^2*e^4-10*a*c*d^2*e^2-13*c^2
*d^4)/d^2*x^3+1/8*(5*a^2*e^4-6*a*c*d^2*e^2-11*c^2*d^4)/d*x)/(e*x^2+d)^2+1/
8*(3*a^2*e^4+6*a*c*d^2*e^2+35*c^2*d^4)/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.33

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx$$

$$= \Bigg[\frac{16\,c^2d^3e^4x^7 - 112\,c^2d^4e^3x^5 - 2\,(175\,c^2d^5e^2 + 30\,acd^3e^4 - 9\,a^2de^6)x^3 - 3\,(35\,c^2d^6 + 6\,acd^4e^2 + 3\,a^2d^2e^4}{}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*c^2*d^3*e^4*x^7 - 112*c^2*d^4*e^3*x^5 - 2*(175*c^2*d^5*e^2 + 30*
a*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 - 3*(35*c^2*d^6 + 6*a*c*d^4*e^2 + 3*a^2*d^2
*e^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + 3*a^2*e^6)*x^4 + 2*(35*c^2*d^5*e
+ 6*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x
 - d)/(e*x^2 + d)) - 6*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x)/(
d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5), 1/24*(8*c^2*d^3*e^4*x^7 - 56*c^2*d
^4*e^3*x^5 - (175*c^2*d^5*e^2 + 30*a*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 + 3*(35*
c^2*d^6 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^4
+ 3*a^2*e^6)*x^4 + 2*(35*c^2*d^5*e + 6*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*sqr
t(d*e)*arctan(sqrt(d*e)*x/d) - 3*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2
*e^5)*x)/(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5)]
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.66

$$\begin{aligned}\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx = &-\frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} \\ &- \frac{\sqrt{-\frac{1}{d^5e^9}}\cdot(3a^2e^4 + 6acd^2e^2 + 35c^2d^4)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16} \\ &+ \frac{\sqrt{-\frac{1}{d^5e^9}}\cdot(3a^2e^4 + 6acd^2e^2 + 35c^2d^4)\log\left(d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16} \\ &+ \frac{x^3\cdot(3a^2e^5 - 10acd^2e^3 - 13c^2d^4e) + x(5a^2de^4 - 6acd^3e^2 - 11c^2d^5)}{8d^4e^4 + 16d^3e^5x^2 + 8d^2e^6x^4}\end{aligned}$$

input
```
integrate((c*x**4+a)**2/(e*x**2+d)**3,x)
```

output
```
-3*c**2*d*x/e**4 + c**2*x**3/(3*e**3) - sqrt(-1/(d**5*e**9))*(3*a**2*e**4
+ 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(-d**3*e**4*sqrt(-1/(d**5*e**9)) + x)
/16 + sqrt(-1/(d**5*e**9))*(3*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*
log(d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + (x**3*(3*a**2*e**5 - 10*a*c*d
**2*e**3 - 13*c**2*d**4*e) + x*(5*a**2*d*e**4 - 6*a*c*d**3*e**2 - 11*c**2*
d**5))/(8*d**4*e**4 + 16*d**3*e**5*x**2 + 8*d**2*e**6*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03

$$\begin{aligned}
\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx &= \frac{(35\,c^2d^4+6\,acd^2e^2+3\,a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^2e^4} \\
&\quad - \frac{13\,c^2d^4ex^3+10\,acd^2e^3x^3-3\,a^2e^5x^3+11\,c^2d^5x+6\,acd^3e^2x-5\,a^2de^4x}{8\,(ex^2+d)^2d^2e^4} \\
&\quad + \frac{c^2e^6x^3-9\,c^2de^5x}{3\,e^9}
\end{aligned}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
1/8*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d
*e)*d^2*e^4) - 1/8*(13*c^2*d^4*e*x^3 + 10*a*c*d^2*e^3*x^3 - 3*a^2*e^5*x^3
+ 11*c^2*d^5*x + 6*a*c*d^3*e^2*x - 5*a^2*d*e^4*x)/((e*x^2 + d)^2*d^2*e^4)
+ 1/3*(c^2*e^6*x^3 - 9*c^2*d*e^5*x)/e^9
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.06

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx = \frac{c^2\,x^3}{3\,e^3} - \frac{\frac{x^3\,\left(-3\,a^2\,e^5+10\,a\,c\,d^2\,e^3+13\,c^2\,d^4\,e\right)}{8\,d^2}+\frac{x\,\left(-5\,a^2\,e^4+6\,a\,c\,d^2\,e^2+11\,c^2\,d^4\right)}{8\,d}}{d^2\,e^4+2\,d\,e^5\,x^2+e^6\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,a^2\,e^4+6\,a\,c\,d^2\,e^2+35\,c^2\,d^4\right)}{8\,d^{5/2}\,e^{9/2}} - \frac{3\,c^2\,d\,x}{e^4}$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^3,x)
```

output
```
(c^2*x^3)/(3*e^3) - ((x^3*(13*c^2*d^4*e - 3*a^2*e^5 + 10*a*c*d^2*e^3))/(8*
d^2) + (x*(11*c^2*d^4 - 5*a^2*e^4 + 6*a*c*d^2*e^2))/(8*d))/(d^2*e^4 + e^6*
x^4 + 2*d*e^5*x^2) + (atan((e^(1/2)*x)/d^(1/2))*(3*a^2*e^4 + 35*c^2*d^4 +
6*a*c*d^2*e^2))/(8*d^(5/2)*e^(9/2)) - (3*c^2*d*x)/e^4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.43

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx$$
$$= \frac{9\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2d^2e^4+18\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2d\,e^5x^2+9\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2e^6x^4+18\sqrt{e}\,\sqrt{}}{}$$

input
```
int((c*x^4+a)^2/(e*x^2+d)^3,x)
```

output
```
(9*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*d**2*e**4 + 18*sqrt(
e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*d*e**5*x**2 + 9*sqrt(e)*sqrt
(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*e**6*x**4 + 18*sqrt(e)*sqrt(d)*atan
((e*x)/(sqrt(e)*sqrt(d)))*a*c*d**4*e**2 + 36*sqrt(e)*sqrt(d)*atan((e*x)/(s
qrt(e)*sqrt(d)))*a*c*d**3*e**3*x**2 + 18*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(
e)*sqrt(d)))*a*c*d**2*e**4*x**4 + 105*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*
sqrt(d)))*c**2*d**6 + 210*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c*
*2*d**5*e*x**2 + 105*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c**2*d*
*4*e**2*x**4 + 15*a**2*d**2*e**5*x + 9*a**2*d*e**6*x**3 - 18*a*c*d**4*e**3
*x - 30*a*c*d**3*e**4*x**3 - 105*c**2*d**6*e*x - 175*c**2*d**5*e**2*x**3 -
 56*c**2*d**4*e**3*x**5 + 8*c**2*d**3*e**4*x**7)/(24*d**3*e**5*(d**2 + 2*d
*e*x**2 + e**2*x**4))
```

3.309 $\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 187

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^4}\,dx &= \frac{c^2x}{e^4}+\frac{\left(cd^2+ae^2\right)^2x}{6de^4\left(d+ex^2\right)^3}-\frac{\left(19cd^2-5ae^2\right)\left(cd^2+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2}\\ &\quad+\frac{\left(29c^2d^4+2acd^2e^2+5a^2e^4\right)x}{16d^3e^4\left(d+ex^2\right)}\\ &\quad-\frac{\left(35c^2d^4-2acd^2e^2-5a^2e^4\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}\end{aligned}$$

output
```
c^2*x/e^4+1/6*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^3-1/24*(-5*a*e^2+19*c*d^2)
*(a*e^2+c*d^2)*x/d^2/e^4/(e*x^2+d)^2+1/16*(5*a^2*e^4+2*a*c*d^2*e^2+29*c^2*
d^4)*x/d^3/e^4/(e*x^2+d)-1/16*(-5*a^2*e^4-2*a*c*d^2*e^2+35*c^2*d^4)*arctan
(e^(1/2)*x/d^(1/2))/d^(7/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.93

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx$$
$$= \frac{x(-2acd^2e^2(3d^2+8dex^2-3e^2x^4)+a^2e^4(33d^2+40dex^2+15e^2x^4)+c^2d^3(105d^3+280d^2ex^2+231de^2x}{48d^3e^4\left(d+ex^2\right)^3}$$
$$-\frac{(35c^2d^4-2acd^2e^2-5a^2e^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

output
```
(x*(-2*a*c*d^2*e^2*(3*d^2 + 8*d*e*x^2 - 3*e^2*x^4) + a^2*e^4*(33*d^2 + 40*
d*e*x^2 + 15*e^2*x^4) + c^2*d^3*(105*d^3 + 280*d^2*e*x^2 + 231*d*e^2*x^4 +
 48*e^3*x^6)))/(48*d^3*e^4*(d + e*x^2)^3) - ((35*c^2*d^4 - 2*a*c*d^2*e^2 -
 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1472, 25, 2345, 27, 1471, 25, 25, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}-\frac{\int -\frac{\frac{6c^2dx^6}{e}-\frac{6c^2d^2x^4}{e^2}+\frac{6cd\left(cd^2+2ae^2\right)x^2}{e^3}+5a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{(ex^2+d)^3}dx}{6d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{6c^2dx^6}{e}-\frac{6c^2d^2x^4}{e^2}+\frac{6cd\left(cd^2+2ae^2\right)x^2}{e^3}+5a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{(ex^2+d)^3}dx}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

$$\Big\downarrow \ 2345$$

$$\frac{\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}-\frac{\int -\frac{3\left(\frac{8c^2d^2x^4}{e^2}-\frac{16c^2d^3x^2}{e^3}+\frac{5c^2d^4+2ace^2d^2+5a^2e^4}{e^4}\right)}{\left(ex^2+d\right)^2}dx}{4d}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3\int \frac{\frac{5c^2d^4}{e^4}-\frac{16c^2x^2d^3}{e^3}+\frac{8c^2x^4d^2}{e^2}+\frac{2acd^2}{e^2}+5a^2}{\left(ex^2+d\right)^2}dx}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

$$\Big\downarrow \ 1471$$

$$\frac{\frac{3\left(\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{2d\left(d+ex^2\right)}-\frac{\int -\frac{16c^2x^2d^3+\left(-\frac{19c^2d^4}{e^4}+\frac{2acd^2}{e^2}+5a^2\right)e^3}{e^3\left(ex^2+d\right)}dx}{2d}\right)}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3\left(\frac{\int -\frac{\frac{19c^2d^4}{e}-16c^2x^2d^3-2aced^2-5a^2e^3}{e^3\left(ex^2+d\right)}dx}{2d}+\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{2d\left(d+ex^2\right)}\right)}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3\left(\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{2d\left(d+ex^2\right)}-\frac{\int \frac{\frac{19c^2d^4}{e}-16c^2x^2d^3-2aced^2-5a^2e^3}{e^3\left(ex^2+d\right)}dx}{2d}\right)}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3\left(\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{2d\left(d+ex^2\right)}-\frac{\int \frac{\frac{19c^2d^4}{e}-16c^2x^2d^3-2aced^2-5a^2e^3}{ex^2+d}dx}{2de^3}\right)}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{3\left(\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{2d\left(d+ex^2\right)}-\frac{\frac{\left(-5a^2e^4-2acd^2e^2+35c^2d^4\right)\int \frac{1}{ex^2+d}dx}{e}-\frac{16c^2d^3x}{e}}{2de^3}\right)}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{3\left(\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{2d\left(d+ex^2\right)}-\frac{\frac{\left(-5a^2e^4-2acd^2e^2+35c^2d^4\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{16c^2d^3x}{e}}{2de^3}\right)}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

input

```
Int[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

output

```
((c*d^2 + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) + (((5*a^2 - (19*c^2*d^4)/e^
4 - (14*a*c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) + (3*(((5*a^2 + (29*c^2*d^4)/
e^4 + (2*a*c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) - ((-16*c^2*d^3*x)/e + ((35*c^
2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e
^(3/2)))/(2*d*e^3)))/(4*d))/(6*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.96

method	result
default	$\frac{c^2x}{e^4}+\frac{\frac{\frac{e^2\left(5a^2e^4+2ac\,d^2e^2+29c^2d^4\right)x^5}{16d^3}+\frac{e\left(5a^2e^4-2ac\,d^2e^2+17c^2d^4\right)x^3}{6d^2}+\frac{\left(11a^2e^4-2ac\,d^2e^2+19c^2d^4\right)x}{16d}}{\left(e\,x^2+d\right)^3}+\frac{\left(5a^2e^4+2ac\,d^2e^2-35c^2d^4\right)\mathrm{arct}}{16d^3\sqrt{de}}}{e^4}$
risch	$\frac{c^2x}{e^4}+\frac{\frac{e^2\left(5a^2e^4+2ac\,d^2e^2+29c^2d^4\right)x^5}{16d^3}+\frac{e\left(5a^2e^4-2ac\,d^2e^2+17c^2d^4\right)x^3}{6d^2}+\frac{\left(11a^2e^4-2ac\,d^2e^2+19c^2d^4\right)x}{16d}}{e^4(e\,x^2+d)^3}-\frac{5\ln\left(ex+\sqrt{-de}\right)a^2}{32\sqrt{-de}\,d^3}-\frac{\ln(}{1}$

input
```
int((c*x^4+a)^2/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

output
```
c^2*x/e^4+1/e^4*((1/16*e^2*(5*a^2*e^4+2*a*c*d^2*e^2+29*c^2*d^4)/d^3*x^5+1/
6*e*(5*a^2*e^4-2*a*c*d^2*e^2+17*c^2*d^4)/d^2*x^3+1/16*(11*a^2*e^4-2*a*c*d^
2*e^2+19*c^2*d^4)/d*x)/(e*x^2+d)^3+1/16*(5*a^2*e^4+2*a*c*d^2*e^2-35*c^2*d^
4)/d^3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.54

$$\int\frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^4}\,dx = \left[\frac{96\,c^2d^4e^4x^7+6\,(77\,c^2d^5e^3+2\,acd^3e^5+5\,a^2de^7)x^5+16\,(35\,c^2d^6e^2-2\,acd^4e^4+5\,a^2d^2e^6)x^3+3\,(35\,c^2d}{}\right.$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="fricas")
```

output

```
[1/96*(96*c^2*d^4*e^4*x^7 + 6*(77*c^2*d^5*e^3 + 2*a*c*d^3*e^5 + 5*a^2*d*e^
7)*x^5 + 16*(35*c^2*d^6*e^2 - 2*a*c*d^4*e^4 + 5*a^2*d^2*e^6)*x^3 + 3*(35*c
^2*d^7 - 2*a*c*d^5*e^2 - 5*a^2*d^3*e^4 + (35*c^2*d^4*e^3 - 2*a*c*d^2*e^5 -
 5*a^2*e^7)*x^6 + 3*(35*c^2*d^5*e^2 - 2*a*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3
*(35*c^2*d^6*e - 2*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x^2)*sqrt(-d*e)*log((e*x^2
 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^7*e - 2*a*c*d^5*e^3 + 11
*a^2*d^3*e^5)*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5),
1/48*(48*c^2*d^4*e^4*x^7 + 3*(77*c^2*d^5*e^3 + 2*a*c*d^3*e^5 + 5*a^2*d*e^7
)*x^5 + 8*(35*c^2*d^6*e^2 - 2*a*c*d^4*e^4 + 5*a^2*d^2*e^6)*x^3 - 3*(35*c^2
*d^7 - 2*a*c*d^5*e^2 - 5*a^2*d^3*e^4 + (35*c^2*d^4*e^3 - 2*a*c*d^2*e^5 - 5
*a^2*e^7)*x^6 + 3*(35*c^2*d^5*e^2 - 2*a*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3*(
35*c^2*d^6*e - 2*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d
*e)*x/d) + 3*(35*c^2*d^7*e - 2*a*c*d^5*e^3 + 11*a^2*d^3*e^5)*x)/(d^4*e^8*x
^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5)]
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.56

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{c^2x}{e^4} - \frac{\sqrt{-\frac{1}{d^7e^9}}\cdot(5a^2e^4+2acd^2e^2-35c^2d^4)\log\left(-d^4e^4\sqrt{-\frac{1}{d^7e^9}}+x\right)}{32} + \frac{\sqrt{-\frac{1}{d^7e^9}}\cdot(5a^2e^4+2acd^2e^2-35c^2d^4)\log\left(d^4e^4\sqrt{-\frac{1}{d^7e^9}}+x\right)}{32} + \frac{x^5\cdot(15a^2e^6+6acd^2e^4+87c^2d^4e^2)+x^3\cdot(40a^2de^5-16acd^3e^3+136c^2d^5e)+x(33a^2d^2e^4-6acd^4e^2+}{48d^6e^4+144d^5e^5x^2+144d^4e^6x^4+48d^3e^7x^6}$$

input

```
integrate((c*x**4+a)**2/(e*x**2+d)**4,x)
```

output

```
c**2*x/e**4 - sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**
2*d**4)*log(-d**4*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + sqrt(-1/(d**7*e**9))
*(5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2*d**4)*log(d**4*e**4*sqrt(-1/(d**
7*e**9)) + x)/32 + (x**5*(15*a**2*e**6 + 6*a*c*d**2*e**4 + 87*c**2*d**4*e*
*2) + x**3*(40*a**2*d*e**5 - 16*a*c*d**3*e**3 + 136*c**2*d**5*e) + x*(33*a
**2*d**2*e**4 - 6*a*c*d**4*e**2 + 57*c**2*d**6))/(48*d**6*e**4 + 144*d**5*
e**5*x**2 + 144*d**4*e**6*x**4 + 48*d**3*e**7*x**6)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{c^2x}{e^4} - \frac{(35\,c^2d^4 - 2\,acd^2e^2 - 5\,a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\,\sqrt{de}d^3e^4} + \frac{87\,c^2d^4e^2x^5 + 6\,acd^2e^4x^5 + 15\,a^2e^6x^5 + 136\,c^2d^5ex^3 - 16\,acd^3e^3x^3 + 40\,a^2de^5x^3 + 57\,c^2d^6x - 6\,acd^4e^2}{48\,(ex^2+d)^3d^3e^4}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="giac")
```

output
```
c^2*x/e^4 - 1/16*(35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*arctan(e*x/sqrt(
d*e))/(sqrt(d*e)*d^3*e^4) + 1/48*(87*c^2*d^4*e^2*x^5 + 6*a*c*d^2*e^4*x^5 +
 15*a^2*e^6*x^5 + 136*c^2*d^5*e*x^3 - 16*a*c*d^3*e^3*x^3 + 40*a^2*d*e^5*x^
3 + 57*c^2*d^6*x - 6*a*c*d^4*e^2*x + 33*a^2*d^2*e^4*x)/((e*x^2 + d)^3*d^3*
e^4)
```

Mupad [B] (verification not implemented)

Time = 17.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{\frac{x^3\,(5\,a^2\,e^5-2\,a\,c\,d^2\,e^3+17\,c^2\,d^4\,e)}{6\,d^2}+\frac{x\,(11\,a^2\,e^4-2\,a\,c\,d^2\,e^2+19\,c^2\,d^4)}{16\,d}+\frac{x^5\,(5\,a^2\,e^6+2\,a\,c\,d^2\,e^4+29\,c^2\,d^4\,e^2)}{16\,d^3}}{d^3\,e^4+3\,d^2\,e^5\,x^2+3\,d\,e^6\,x^4+e^7\,x^6} + \frac{c^2\,x}{e^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(5\,a^2\,e^4+2\,a\,c\,d^2\,e^2-35\,c^2\,d^4)}{16\,d^{7/2}\,e^{9/2}}$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^4,x)
```

output
```
((x^3*(5*a^2*e^5 + 17*c^2*d^4*e - 2*a*c*d^2*e^3))/(6*d^2) + (x*(11*a^2*e^4
 + 19*c^2*d^4 - 2*a*c*d^2*e^2))/(16*d) + (x^5*(5*a^2*e^6 + 29*c^2*d^4*e^2
+ 2*a*c*d^2*e^4))/(16*d^3))/(d^3*e^4 + e^7*x^6 + 3*d*e^6*x^4 + 3*d^2*e^5*x
^2) + (c^2*x)/e^4 + (atan((e^(1/2)*x)/d^(1/2))*(5*a^2*e^4 - 35*c^2*d^4 + 2
*a*c*d^2*e^2))/(16*d^(7/2)*e^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.70

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{15\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2d^3e^4+45\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2d^2e^5x^2+45\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2d\,e^6x^4+15}{}$$

input
```
int((c*x^4+a)^2/(e*x^2+d)^4,x)
```

output

```
(15*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*d**3*e**4 + 45*sqrt
(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*d**2*e**5*x**2 + 45*sqrt(e)
*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*d*e**6*x**4 + 15*sqrt(e)*sqrt(
d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*e**7*x**6 + 6*sqrt(e)*sqrt(d)*atan((
e*x)/(sqrt(e)*sqrt(d)))*a*c*d**5*e**2 + 18*sqrt(e)*sqrt(d)*atan((e*x)/(sqr
t(e)*sqrt(d)))*a*c*d**4*e**3*x**2 + 18*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)
*sqrt(d)))*a*c*d**3*e**4*x**4 + 6*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt
(d)))*a*c*d**2*e**5*x**6 - 105*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)
))*c**2*d**7 - 315*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c**2*d**6
*e*x**2 - 315*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c**2*d**5*e**2
*x**4 - 105*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c**2*d**4*e**3*x
**6 + 33*a**2*d**3*e**5*x + 40*a**2*d**2*e**6*x**3 + 15*a**2*d*e**7*x**5 -
 6*a*c*d**5*e**3*x - 16*a*c*d**4*e**4*x**3 + 6*a*c*d**3*e**5*x**5 + 105*c*
*2*d**7*e*x + 280*c**2*d**6*e**2*x**3 + 231*c**2*d**5*e**3*x**5 + 48*c**2*
d**4*e**4*x**7)/(48*d**4*e**5*(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3
*x**6))
```

3.310 $\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 226

$$
\begin{aligned}
\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = {} & \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} - \frac{(25cd^2-7ae^2)(cd^2+ae^2)x}{48d^2e^4(d+ex^2)^3} \\
& + \frac{(163c^2d^4+6acd^2e^2+35a^2e^4)x}{192d^3e^4(d+ex^2)^2} - \frac{(93c^2d^4-6acd^2e^2-35a^2e^4)x}{128d^4e^4(d+ex^2)} \\
& + \frac{(35c^2d^4+6acd^2e^2+35a^2e^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}}
\end{aligned}
$$

output

```
1/8*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^4-1/48*(-7*a*e^2+25*c*d^2)*(a*e^2+c*
d^2)*x/d^2/e^4/(e*x^2+d)^3+1/192*(35*a^2*e^4+6*a*c*d^2*e^2+163*c^2*d^4)*x/
d^3/e^4/(e*x^2+d)^2-1/128*(-35*a^2*e^4-6*a*c*d^2*e^2+93*c^2*d^4)*x/d^4/e^4
/(e*x^2+d)+1/128*(35*a^2*e^4+6*a*c*d^2*e^2+35*c^2*d^4)*arctan(e^(1/2)*x/d^
(1/2))/d^(9/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.88

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx$$
$$= \frac{\frac{\sqrt{d}\sqrt{e}x(-6acd^2e^2(3d^3+11d^2ex^2-11de^2x^4-3e^3x^6)+a^2e^4(279d^3+511d^2ex^2+385de^2x^4+105e^3x^6)-c^2d^4(105d^3+385d^2ex^2+511de^2x^4+279e}{(d+ex^2)^4}}{384d^{9/2}e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

output
```
((Sqrt[d]*Sqrt[e]*x*(-6*a*c*d^2*e^2*(3*d^3 + 11*d^2*e*x^2 - 11*d*e^2*x^4 -
 3*e^3*x^6) + a^2*e^4*(279*d^3 + 511*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x
^6) - c^2*d^4*(105*d^3 + 385*d^2*e*x^2 + 511*d*e^2*x^4 + 279*e^3*x^6)))/(d
 + e*x^2)^4 + 3*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*
x)/Sqrt[d]])/(384*d^(9/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1472, 25, 2345, 25, 1471, 27, 25, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx$$
$$\Big\downarrow \text{1472}$$
$$\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}-\frac{\int -\frac{\frac{8c^2dx^6}{e}-\frac{8c^2d^2x^4}{e^2}+\frac{8cd\left(cd^2+2ae^2\right)x^2}{e^3}+7a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{(ex^2+d)^4}dx}{8d}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{8c^2dx^6}{e}-\frac{8c^2d^2x^4}{e^2}+\frac{8cd\left(cd^2+2ae^2\right)x^2}{e^3}+7a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{(ex^2+d)^4}dx}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}-\frac{\int -\frac{\frac{19c^2d^4}{e^4}-\frac{96c^2x^2d^3}{e^3}+\frac{48c^2x^4d^2}{e^2}+\frac{6acd^2}{e^2}+35a^2}{\left(ex^2+d\right)^3}dx}{6d}}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\frac{19c^2d^4}{e^4}-\frac{96c^2x^2d^3}{e^3}+\frac{48c^2x^4d^2}{e^2}+\frac{6acd^2}{e^2}+35a^2}{\left(ex^2+d\right)^3}dx}{6d}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\frac{\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{4d\left(d+ex^2\right)^2}-\frac{\int -\frac{3\left(64c^2x^2d^3+\left(-\frac{29c^2d^4}{e^4}+\frac{6acd^2}{e^2}+35a^2\right)e^3\right)}{e^3\left(ex^2+d\right)^2}dx}{4d}}{6d}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int -\frac{\frac{29c^2d^4}{e}-64c^2x^2d^3-6aced^2-35a^2e^3}{\left(ex^2+d\right)^2}dx}{4de^3}+\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{4d\left(d+ex^2\right)^2}}{6d}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{4d\left(d+ex^2\right)^2}-\frac{3\int \frac{\frac{29c^2d^4}{e}-64c^2x^2d^3-6aced^2-35a^2e^3}{\left(ex^2+d\right)^2}dx}{4de^3}}{6d}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{4d\left(d+ex^2\right)^2}-\frac{3\left(\frac{x\left(-35a^2e^4-6acd^2e^2+93c^2d^4\right)}{2de\left(d+ex^2\right)}-\frac{\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\int\frac{1}{ex^2+d}dx}{2de}\right)}{4de^3}}{6d}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

$$\Big\downarrow 218$$

$$\frac{\frac{\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{4d\left(d+ex^2\right)^2}-\frac{3\left(\frac{x\left(-35a^2e^4-6acd^2e^2+93c^2d^4\right)}{2de\left(d+ex^2\right)}-\frac{\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{3/2}}\right)}{4de^3}}{6d}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

input
```
Int[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

output
```
((c*d^2 + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) + (((7*a^2 - (25*c^2*d^4)/e^
4 - (18*a*c*d^2)/e^2)*x)/(6*d*(d + e*x^2)^3) + (((35*a^2 + (163*c^2*d^4)/e
^4 + (6*a*c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) - (3*(((93*c^2*d^4 - 6*a*c*d^
2*e^2 - 35*a^2*e^4)*x)/(2*d*e*(d + e*x^2)) - ((35*c^2*d^4 + 6*a*c*d^2*e^2
+ 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(3/2))))/(4*d*e^3)
)/(6*d))/(8*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.94

method	result
default	$\frac{\frac{\left(35a^{2}e^{4}+6ac\,d^{2}e^{2}-93c^{2}d^{4}\right)x^{7}}{128d^{4}e}+\frac{\left(385a^{2}e^{4}+66ac\,d^{2}e^{2}-511c^{2}d^{4}\right)x^{5}}{384d^{3}e^{2}}+\frac{\left(511a^{2}e^{4}-66ac\,d^{2}e^{2}-385c^{2}d^{4}\right)x^{3}}{384d^{2}e^{3}}+\frac{\left(93a^{2}e^{4}-6ac\,d^{2}e^{2}-35c^{2}d^{4}\right)x}{128d\,e^{4}}}{\left(e\,x^{2}+d\right)^{4}}+$
risch	$\frac{\frac{\left(35a^{2}e^{4}+6ac\,d^{2}e^{2}-93c^{2}d^{4}\right)x^{7}}{128d^{4}e}+\frac{\left(385a^{2}e^{4}+66ac\,d^{2}e^{2}-511c^{2}d^{4}\right)x^{5}}{384d^{3}e^{2}}+\frac{\left(511a^{2}e^{4}-66ac\,d^{2}e^{2}-385c^{2}d^{4}\right)x^{3}}{384d^{2}e^{3}}+\frac{\left(93a^{2}e^{4}-6ac\,d^{2}e^{2}-35c^{2}d^{4}\right)x}{128d\,e^{4}}}{\left(e\,x^{2}+d\right)^{4}}-$

input

```
int((c*x^4+a)^2/(e*x^2+d)^5,x,method=_RETURNVERBOSE)
```

output

```
(1/128*(35*a^2*e^4+6*a*c*d^2*e^2-93*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+
66*a*c*d^2*e^2-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4-66*a*c*d^2*e^2-
385*c^2*d^4)/d^2/e^3*x^3+1/128*(93*a^2*e^4-6*a*c*d^2*e^2-35*c^2*d^4)/d/e^4
*x)/(e*x^2+d)^4+1/128*(35*a^2*e^4+6*a*c*d^2*e^2+35*c^2*d^4)/d^4/e^4/(d*e)^
(1/2)*arctan(e*x/(d*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 806, normalized size of antiderivative = 3.57

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \text{Too large to display}$$

input

```
integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="fricas")
```

output

```
[-1/768*(6*(93*c^2*d^5*e^4 - 6*a*c*d^3*e^6 - 35*a^2*d*e^8)*x^7 + 2*(511*c^
2*d^6*e^3 - 66*a*c*d^4*e^5 - 385*a^2*d^2*e^7)*x^5 + 2*(385*c^2*d^7*e^2 + 6
6*a*c*d^5*e^4 - 511*a^2*d^3*e^6)*x^3 + 3*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*
a^2*d^4*e^4 + (35*c^2*d^4*e^4 + 6*a*c*d^2*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^
2*d^5*e^3 + 6*a*c*d^3*e^5 + 35*a^2*d*e^7)*x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*
d^4*e^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d^7*e + 6*a*c*d^5*e^3 + 35*a^2*d
^3*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*
(35*c^2*d^8*e + 6*a*c*d^6*e^3 - 93*a^2*d^4*e^5)*x)/(d^5*e^9*x^8 + 4*d^6*e^
8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5), -1/384*(3*(93*c^2*d^5*e^
4 - 6*a*c*d^3*e^6 - 35*a^2*d*e^8)*x^7 + (511*c^2*d^6*e^3 - 66*a*c*d^4*e^5
- 385*a^2*d^2*e^7)*x^5 + (385*c^2*d^7*e^2 + 66*a*c*d^5*e^4 - 511*a^2*d^3*e
^6)*x^3 - 3*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2*d^4*e^4 + (35*c^2*d^4*e^4
 + 6*a*c*d^2*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^5*e^3 + 6*a*c*d^3*e^5 + 3
5*a^2*d*e^7)*x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4*e^4 + 35*a^2*d^2*e^6)*x^4
 + 4*(35*c^2*d^7*e + 6*a*c*d^5*e^3 + 35*a^2*d^3*e^5)*x^2)*sqrt(d*e)*arctan
(sqrt(d*e)*x/d) + 3*(35*c^2*d^8*e + 6*a*c*d^6*e^3 - 93*a^2*d^4*e^5)*x)/(d^
5*e^9*x^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5)]
```

Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.48

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = -\frac{\sqrt{-\frac{1}{d^9e^9}}\cdot(35a^2e^4+6acd^2e^2+35c^2d^4)\log\left(-d^5e^4\sqrt{-\frac{1}{d^9e^9}}+x\right)}{256} + \frac{\sqrt{-\frac{1}{d^9e^9}}\cdot(35a^2e^4+6acd^2e^2+35c^2d^4)\log\left(d^5e^4\sqrt{-\frac{1}{d^9e^9}}+x\right)}{256} + \frac{x^7\cdot(105a^2e^7+18acd^2e^5-279c^2d^4e^3)+x^5\cdot(385a^2de^6+66acd^3e^4-511c^2d^5e^2)+x^3\cdot(511a^2d^2e^5-}{384d^8e^4+1536d^7e^5x^2+2304d^6e^6x^4+1536d^5e^7x^6+}$$

input
```
integrate((c*x**4+a)**2/(e*x**2+d)**5,x)
```

output
```
-sqrt(-1/(d**9*e**9))*(35*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(
-d**5*e**4*sqrt(-1/(d**9*e**9)) + x)/256 + sqrt(-1/(d**9*e**9))*(35*a**2*e
**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(d**5*e**4*sqrt(-1/(d**9*e**9)) +
 x)/256 + (x**7*(105*a**2*e**7 + 18*a*c*d**2*e**5 - 279*c**2*d**4*e**3) +
x**5*(385*a**2*d*e**6 + 66*a*c*d**3*e**4 - 511*c**2*d**5*e**2) + x**3*(511
*a**2*d**2*e**5 - 66*a*c*d**4*e**3 - 385*c**2*d**6*e) + x*(279*a**2*d**3*e
**4 - 18*a*c*d**5*e**2 - 105*c**2*d**7))/(384*d**8*e**4 + 1536*d**7*e**5*x
**2 + 2304*d**6*e**6*x**4 + 1536*d**5*e**7*x**6 + 384*d**4*e**8*x**8)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \frac{(35\,c^2d^4+6\,acd^2e^2+35\,a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{128\sqrt{de}d^4e^4} - \frac{279\,c^2d^4e^3x^7-18\,acd^2e^5x^7-105\,a^2e^7x^7+511\,c^2d^5e^2x^5-66\,acd^3e^4x^5-385\,a^2de^6x^5+385\,c^2d^6ex^3+}{384\,(ex^2+d)^4d^4e^4}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="giac")
```

output
```
1/128*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqr
t(d*e)*d^4*e^4) - 1/384*(279*c^2*d^4*e^3*x^7 - 18*a*c*d^2*e^5*x^7 - 105*a^
2*e^7*x^7 + 511*c^2*d^5*e^2*x^5 - 66*a*c*d^3*e^4*x^5 - 385*a^2*d*e^6*x^5 +
 385*c^2*d^6*e*x^3 + 66*a*c*d^4*e^3*x^3 - 511*a^2*d^2*e^5*x^3 + 105*c^2*d^
7*x + 18*a*c*d^5*e^2*x - 279*a^2*d^3*e^4*x)/((e*x^2 + d)^4*d^4*e^4)
```

Mupad [B] (verification not implemented)

Time = 17.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.06

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(35\,a^2\,e^4+6\,a\,c\,d^2\,e^2+35\,c^2\,d^4)}{128\,d^{9/2}\,e^{9/2}} - \frac{\frac{x\,(-93\,a^2\,e^4+6\,a\,c\,d^2\,e^2+35\,c^2\,d^4)}{128\,d\,e^4}-\frac{x^7\,(35\,a^2\,e^4+6\,a\,c\,d^2\,e^2-93\,c^2\,d^4)}{128\,d^4\,e}+\frac{x^3\,(-511\,a^2\,e^4+66\,a\,c\,d^2\,e^2+385\,c^2\,d^4)}{384\,d^2\,e^3}-\frac{x^5\,(385\,a^2\,e^4+66}{384}}{d^4+4\,d^3\,e\,x^2+6\,d^2\,e^2\,x^4+4\,d\,e^3\,x^6+e^4\,x^8}$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^5,x)
```

output
```
(atan((e^(1/2)*x)/d^(1/2))*(35*a^2*e^4 + 35*c^2*d^4 + 6*a*c*d^2*e^2))/(128
*d^(9/2)*e^(9/2)) - ((x*(35*c^2*d^4 - 93*a^2*e^4 + 6*a*c*d^2*e^2))/(128*d*
e^4) - (x^7*(35*a^2*e^4 - 93*c^2*d^4 + 6*a*c*d^2*e^2))/(128*d^4*e) + (x^3*
(385*c^2*d^4 - 511*a^2*e^4 + 66*a*c*d^2*e^2))/(384*d^2*e^3) - (x^5*(385*a^
2*e^4 - 511*c^2*d^4 + 66*a*c*d^2*e^2))/(384*d^3*e^2))/(d^4 + e^4*x^8 + 4*d
^3*e*x^2 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.79

$$\int \frac{(a + cx^4)^2}{(d + ex^2)^5}\, dx = \text{Too large to display}$$

input
```
int((c*x^4+a)^2/(e*x^2+d)^5,x)
```

output
```
(105*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*d**4*e**4 + 420*sq
rt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*d**3*e**5*x**2 + 630*sqrt
(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*d**2*e**6*x**4 + 420*sqrt(e
)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*d*e**7*x**6 + 105*sqrt(e)*sqr
t(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*e**8*x**8 + 18*sqrt(e)*sqrt(d)*ata
n((e*x)/(sqrt(e)*sqrt(d)))*a*c*d**6*e**2 + 72*sqrt(e)*sqrt(d)*atan((e*x)/(
sqrt(e)*sqrt(d)))*a*c*d**5*e**3*x**2 + 108*sqrt(e)*sqrt(d)*atan((e*x)/(sqr
t(e)*sqrt(d)))*a*c*d**4*e**4*x**4 + 72*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)
*sqrt(d)))*a*c*d**3*e**5*x**6 + 18*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqr
t(d)))*a*c*d**2*e**6*x**8 + 105*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d
)))*c**2*d**8 + 420*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c**2*d**
7*e*x**2 + 630*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c**2*d**6*e**
2*x**4 + 420*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c**2*d**5*e**3*
x**6 + 105*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c**2*d**4*e**4*x*
*8 + 279*a**2*d**4*e**5*x + 511*a**2*d**3*e**6*x**3 + 385*a**2*d**2*e**7*x
**5 + 105*a**2*d*e**8*x**7 - 18*a*c*d**6*e**3*x - 66*a*c*d**5*e**4*x**3 +
66*a*c*d**4*e**5*x**5 + 18*a*c*d**3*e**6*x**7 - 105*c**2*d**8*e*x - 385*c*
*2*d**7*e**2*x**3 - 511*c**2*d**6*e**3*x**5 - 279*c**2*d**5*e**4*x**7)/(38
4*d**5*e**5*(d**4 + 4*d**3*e*x**2 + 6*d**2*e**2*x**4 + 4*d*e**3*x**6 + e**
4*x**8))
```

3.311 $\int \frac{(d+ex^2)^4}{a+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 353

$$
\begin{aligned}
\int \frac{(d+ex^2)^4}{a+cx^4}\,dx
&= \frac{e^2(6cd^2-ae^2)\,x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c} \\
&\quad - \frac{\left(c^2d^4+4\sqrt{a}c^{3/2}d^3e-6acd^2e^2-4a^{3/2}\sqrt{c}de^3+a^2e^4\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{9/4}} \\
&\quad + \frac{\left(c^2d^4+4\sqrt{a}c^{3/2}d^3e-6acd^2e^2-4a^{3/2}\sqrt{c}de^3+a^2e^4\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{9/4}} \\
&\quad + \frac{\left(c^2d^4-4\sqrt{a}c^{3/2}d^3e-6acd^2e^2+4a^{3/2}\sqrt{c}de^3+a^2e^4\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}c^{9/4}}
\end{aligned}
$$

output
```
e^2*(-a*e^2+6*c*d^2)*x/c^2+4/3*d*e^3*x^3/c+1/5*e^4*x^5/c+1/4*(c^2*d^4+4*a^
(1/2)*c^(3/2)*d^3*e-6*a*c*d^2*e^2-4*a^(3/2)*c^(1/2)*d*e^3+a^2*e^4)*arctan(
-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(9/4)+1/4*(c^2*d^4+4*a^(1/
2)*c^(3/2)*d^3*e-6*a*c*d^2*e^2-4*a^(3/2)*c^(1/2)*d*e^3+a^2*e^4)*arctan(1+2
^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(9/4)+1/4*(c^2*d^4-4*a^(1/2)*c
^(3/2)*d^3*e-6*a*c*d^2*e^2+4*a^(3/2)*c^(1/2)*d*e^3+a^2*e^4)*arctanh(2^(1/2
)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)/c^(9/4)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx$$

$$= \frac{-120a^{3/4}\sqrt[4]{c}e^2(-6cd^2+ae^2)\,x+160a^{3/4}c^{5/4}de^3x^3+24a^{3/4}c^{5/4}e^4x^5-30\sqrt{2}\big(c^2d^4+4\sqrt{a}c^{3/2}d^3e-6acd^2}{}$$

input
```
Integrate[(d + e*x^2)^4/(a + c*x^4),x]
```

output
```
(-120*a^(3/4)*c^(1/4)*e^2*(-6*c*d^2 + a*e^2)*x + 160*a^(3/4)*c^(5/4)*d*e^3
*x^3 + 24*a^(3/4)*c^(5/4)*e^4*x^5 - 30*Sqrt[2]*(c^2*d^4 + 4*Sqrt[a]*c^(3/2
)*d^3*e - 6*a*c*d^2*e^2 - 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*x)/a^(1/4)] + 30*Sqrt[2]*(c^2*d^4 + 4*Sqrt[a]*c^(3/2)*d^3*e
 - 6*a*c*d^2*e^2 - 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*x)/a^(1/4)] - 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/2)*d^3*e - 6*a*
c*d^2*e^2 + 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2] + 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/2)*d^3*
e - 6*a*c*d^2*e^2 + 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(120*a^(3/4)*c^(9/4))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^4}{a+cx^4}dx$$

$$\Big\downarrow \text{1485}$$

$$\int \left(\frac{a^2e^4+4cdex^2\left(cd^2-ae^2\right)-6acd^2e^2+c^2d^4}{c^2\left(a+cx^4\right)}+\frac{e^2\left(6cd^2-ae^2\right)}{c^2}+\frac{4de^3x^2}{c}+\frac{e^4x^4}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(a^2e^4-6acd^2e^2+4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)}{2\sqrt{2}a^{3/4}c^{9/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(a^2e^4-6acd^2e^2+4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)}{2\sqrt{2}a^{3/4}c^{9/4}}-\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}}+\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}}+\frac{e^2x\left(6cd^2-ae^2\right)}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}$$

input

```
Int[(d + e*x^2)^4/(a + c*x^4),x]
```

output
```
(e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (4*d*e^3*x^3)/(3*c) + (e^4*x^5)/(5*c) - ((
c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))
*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) + ((
c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) - ((
c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4
)*c^(9/4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(
c*d^2 - a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4
*Sqrt[2]*a^(3/4)*c^(9/4))
```

Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.33

method	result
risch	$\frac{e^{4} x^{5}}{5 c}+\frac{4 d \,e^{3} x^{3}}{3 c}-\frac{e^{4} x a}{c^{2}}+\frac{6 e^{2} x \,d^{2}}{c}+\frac{\sum_{\_R =\operatorname{RootOf}\left(c \,\_Z^{4}+a \right)}\frac{\left(4 e c d \left(-a \,e^{2}+c \,d^{2}\right) \_R^{2}+a^{2} e^{4}-6 a c \,d^{2} e^{2}+c^{2} d^{4}\right) \ln \left(x -\_R \right)}{\_R^{3}}}{4 c^{3}}$
default	$-\frac{e^{2} \left(-\frac{1}{5} c \,x^{5} e^{2}-\frac{4}{3} c d \,x^{3} e +x a \,e^{2}-6 x c \,d^{2}\right)}{c^{2}}+\frac{\frac{\left(a^{2} e^{4}-6 a c \,d^{2} e^{2}+c^{2} d^{4}\right) \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \Big(\ln \left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\, \mathrm{arc}}{8 a}}{}$

input
```
int((e*x^2+d)^4/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output

```
1/5*e^4*x^5/c+4/3*d*e^3*x^3/c-e^4/c^2*x*a+6*e^2/c*x*d^2+1/4/c^3*sum((4*e*c
*d*(-a*e^2+c*d^2)*_R^2+a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)/_R^3*ln(x-_R),_R=Roo
tOf(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2878 vs. $2(278) = 556$.

Time = 9.62 (sec) , antiderivative size = 2878, normalized size of antiderivative = 8.15

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx = \text{Too large to display}$$

input

```
integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="fricas")
```

output

```
1/60*(12*c*e^4*x^5 + 80*c*d*e^3*x^3 + 15*c^2*sqrt(-(8*c^3*d^7*e - 56*a*c^2
*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*d^16 - 56*a*c
^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*
d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14
 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 - 36*a
^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c^3*d
^6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x + (a*c^8*d
^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 + 23
9*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 + 4*(a^3*c^8*d^3*e
- a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^
4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 +
 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8
*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqr
t(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^1
0*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^1
2 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) - 15*c^2*sqrt(-(8*
c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt
(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10
*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12
 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 2...
```

Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx = x\left(-\frac{ae^4}{c^2}+\frac{6d^2e^2}{c}\right) + \operatorname{RootSum}\left(256t^4a^3c^9 + t^2\left(-256a^5c^5de^7 + 1792a^4c^6d^3e^5 - 1792a^3c^7d^5e^3 + 256a^2c^8d^7e\right) + a^8e^{16} + 8a^7\right. + \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c}$$

input
```
integrate((e*x**2+d)**4/(c*x**4+a),x)
```

output
```
x*(-a*e**4/c**2 + 6*d**2*e**2/c) + RootSum(256*_t**4*a**3*c**9 + _t**2*(-2
56*a**5*c**5*d*e**7 + 1792*a**4*c**6*d**3*e**5 - 1792*a**3*c**7*d**5*e**3
+ 256*a**2*c**8*d**7*e) + a**8*e**16 + 8*a**7*c*d**2*e**14 + 28*a**6*c**2*
d**4*e**12 + 56*a**5*c**3*d**6*e**10 + 70*a**4*c**4*d**8*e**8 + 56*a**3*c*
*5*d**10*e**6 + 28*a**2*c**6*d**12*e**4 + 8*a*c**7*d**14*e**2 + c**8*d**16
, Lambda(_t, _t*log(x + (256*_t**3*a**4*c**7*d*e**3 - 256*_t**3*a**3*c**8*
d**3*e + 4*_t*a**7*c**2*e**12 - 264*_t*a**6*c**3*d**2*e**10 + 1980*_t*a**5
*c**4*d**4*e**8 - 3696*_t*a**4*c**5*d**6*e**6 + 1980*_t*a**3*c**6*d**8*e**
4 - 264*_t*a**2*c**7*d**10*e**2 + 4*_t*a*c**8*d**12)/(a**8*e**16 - 24*a**7
*c*d**2*e**14 - 36*a**6*c**2*d**4*e**12 + 88*a**5*c**3*d**6*e**10 + 198*a*
*4*c**4*d**8*e**8 + 88*a**3*c**5*d**10*e**6 - 36*a**2*c**6*d**12*e**4 - 24
*a*c**7*d**14*e**2 + c**8*d**16))) + 4*d*e**3*x**3/(3*c) + e**4*x**5/(5*c
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx = \frac{3\,ce^4x^5 + 20\,cde^3x^3 + 15\,(6\,cd^2e^2 - ae^4)x}{15\,c^2} + \frac{\frac{2\sqrt{2}\left(c^{\frac{5}{2}}d^4 + 4\sqrt{a}c^2d^3e - 6\,ac^{\frac{3}{2}}d^2e^2 - 4\,a^{\frac{3}{2}}cde^3 + a^2\sqrt{c}e^4\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\left(c^{\frac{5}{2}}d^4 + 4\sqrt{a}c^2d^3e - 6\,ac^{\frac{3}{2}}d^2e^2 - 4\,a^{\frac{3}{2}}cd\right.}{\sqrt{a}\sqrt{\sqrt{}}}}{}$$

input
```
integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/15*(3*c*e^4*x^5 + 20*c*d*e^3*x^3 + 15*(6*c*d^2*e^2 - a*e^4)*x)/c^2 + 1/8
*(2*sqrt(2)*(c^(5/2)*d^4 + 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 - 4*a
^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2
)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*s
qrt(c)) + 2*sqrt(2)*(c^(5/2)*d^4 + 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e
^2 - 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
- sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(c))*sqrt(c)) + sqrt(2)*(c^(5/2)*d^4 - 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)
*d^2*e^2 + 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*log(sqrt(c)*x^2 + sqrt(2)*
a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c^(5/2)*d^4 - 4*
sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 + 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*
e^4)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/
4)))/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.44

$$
\begin{aligned}
\int \frac{(d+ex^2)^4}{a+cx^4}\,dx
&= \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4 - 6\,(ac^3)^{\frac{1}{4}}ac^2d^2e^2 + (ac^3)^{\frac{1}{4}}a^2ce^4 + 4\,(ac^3)^{\frac{3}{4}}cd^3e - 4\,(ac^3)^{\frac{3}{4}}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)\right.}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{4\,ac^4} \\
&\quad + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4 - 6\,(ac^3)^{\frac{1}{4}}ac^2d^2e^2 + (ac^3)^{\frac{1}{4}}a^2ce^4 + 4\,(ac^3)^{\frac{3}{4}}cd^3e - 4\,(ac^3)^{\frac{3}{4}}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\right.}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{4\,ac^4} \\
&\quad + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4 - 6\,(ac^3)^{\frac{1}{4}}ac^2d^2e^2 + (ac^3)^{\frac{1}{4}}a^2ce^4 - 4\,(ac^3)^{\frac{3}{4}}cd^3e + 4\,(ac^3)^{\frac{3}{4}}ade^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}\right.}{8\,ac^4} \\
&\quad - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4 - 6\,(ac^3)^{\frac{1}{4}}ac^2d^2e^2 + (ac^3)^{\frac{1}{4}}a^2ce^4 - 4\,(ac^3)^{\frac{3}{4}}cd^3e + 4\,(ac^3)^{\frac{3}{4}}ade^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}\right.}{8\,ac^4} \\
&\quad + \frac{3\,c^4e^4x^5 + 20\,c^4de^3x^3 + 90\,c^4d^2e^2x - 15\,ac^3e^4x}{15\,c^5}
\end{aligned}
$$

input
```
integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="giac")
```

output

```
1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + (a*c^
3)^(1/4)*a^2*c*e^4 + 4*(a*c^3)^(3/4)*c*d^3*e - 4*(a*c^3)^(3/4)*a*d*e^3)*ar
ctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/4*sq
rt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + (a*c^3)^(1/
4)*a^2*c*e^4 + 4*(a*c^3)^(3/4)*c*d^3*e - 4*(a*c^3)^(3/4)*a*d*e^3)*arctan(1
/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*
((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + (a*c^3)^(1/4)*a^2
*c*e^4 - 4*(a*c^3)^(3/4)*c*d^3*e + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 + sqrt
(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^
4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)^(3
/4)*c*d^3*e + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(a*c^4) + 1/15*(3*c^4*e^4*x^5 + 20*c^4*d*e^3*x^3 + 90*c^4*d^2*e^
2*x - 15*a*c^3*e^4*x)/c^5
```

Mupad [B] (verification not implemented)

Time = 17.78 (sec) , antiderivative size = 4022, normalized size of antiderivative = 11.39

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx = \text{Too large to display}$$

input

```
int((d + e*x^2)^4/(a + c*x^4),x)
```

output

```
atan((((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*
a^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2
)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e
+ 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6
*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4
*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9)^(1/2
) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*
c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*
a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*
a^3*c^9))^(1/2)*1i + ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*
c*d^2*e^6 + 70*a^2*c^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*
a^2*c^5*d^2*e^2)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8
*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5
 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) +
 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*
(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*
d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c
^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c
^9)^(1/2))/(16*a^3*c^9))^(1/2)*1i)/(((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^
6*e^2 - 28*a^3*c*d^2*e^6 + 70*a^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 812, normalized size of antiderivative = 2.30

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx = \text{Too large to display}$$

input

```
int((e*x^2+d)^4/(c*x^4+a),x)
```

output

```
(120*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)
*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d*e**3 - 120*c**(1/4)*a**(3/4)*sqrt(2
)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2
)))*c**2*d**3*e - 30*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*e**4 + 180*c**(3/4)*
a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*
a**(1/4)*sqrt(2)))*a*c*d**2*e**2 - 30*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(
1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*d**
4 - 120*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt
(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d*e**3 + 120*c**(1/4)*a**(3/4)*sqr
t(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqr
t(2)))*c**2*d**3*e + 30*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*
sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*e**4 - 180*c**(3/
4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/
4)*a**(1/4)*sqrt(2)))*a*c*d**2*e**2 + 30*c**(3/4)*a**(1/4)*sqrt(2)*atan((c
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*
d**4 - 60*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + s
qrt(a) + sqrt(c)*x**2)*a*c*d*e**3 + 60*c**(1/4)*a**(3/4)*sqrt(2)*log( - c*
*(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c**2*d**3*e + 60*c**(1
/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c...
```

3.312 $\int \frac{(d+ex^2)^3}{a+cx^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 278

$$\begin{aligned}\int \frac{(d+ex^2)^3}{a+cx^4} \, dx = {} & \frac{3de^2x}{c} + \frac{e^3x^3}{3c} \\ & - \frac{\left(\sqrt{c}d(cd^2-3ae^2)+\sqrt{a}e(3cd^2-ae^2)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{7/4}} \\ & + \frac{\left(\sqrt{c}d(cd^2-3ae^2)+\sqrt{a}e(3cd^2-ae^2)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{7/4}} \\ & + \frac{\left(\sqrt{c}d(cd^2-3ae^2)-\sqrt{a}e(3cd^2-ae^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}c^{7/4}}\end{aligned}$$

output
```
3*d*e^2*x/c+1/3*e^3*x^3/c+1/4*(c^(1/2)*d*(-3*a*e^2+c*d^2)+a^(1/2)*e*(-a*e^
2+3*c*d^2))*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(7/4)+1
/4*(c^(1/2)*d*(-3*a*e^2+c*d^2)+a^(1/2)*e*(-a*e^2+3*c*d^2))*arctan(1+2^(1/2
)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(7/4)+1/4*(c^(1/2)*d*(-3*a*e^2+c*d^
2)-a^(1/2)*e*(-a*e^2+3*c*d^2))*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+
c^(1/2)*x^2))*2^(1/2)/a^(3/4)/c^(7/4)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx$$

$$= \frac{72a^{3/4}c^{3/4}de^2x + 8a^{3/4}c^{3/4}e^3x^3 + 6\sqrt{2}\left(-c^{3/2}d^3 - 3\sqrt{a}cd^2e + 3a\sqrt{c}de^2 + a^{3/2}e^3\right)\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) +}{}$$

input
```
Integrate[(d + e*x^2)^3/(a + c*x^4),x]
```

output
```
(72*a^(3/4)*c^(3/4)*d*e^2*x + 8*a^(3/4)*c^(3/4)*e^3*x^3 + 6*Sqrt[2]*(-(c^(
3/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1
- (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*
e - 3*a*Sqrt[c]*d*e^2 - a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4
)] - 3*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3
/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2
]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^(3/4)*c^(7/4))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx$$

$$\Big\downarrow \text{1485}$$

$$\int \left(\frac{ex^2(3cd^2-ae^2)-3ade^2+cd^3}{c(a+cx^4)} + \frac{3de^2}{c} + \frac{e^3x^2}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d\left(cd^2-3ae^2\right)+\sqrt{a}e\left(3cd^2-ae^2\right)\right)}{2\sqrt{2}a^{3/4}c^{7/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d\left(cd^2-3ae^2\right)+\sqrt{a}e\left(3cd^2-ae^2\right)\right)}{2\sqrt{2}a^{3/4}c^{7/4}}-\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)-\sqrt{a}e\left(3cd^2-ae^2\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}+\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)-\sqrt{a}e\left(3cd^2-ae^2\right)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}+\frac{3de^2x}{c}+\frac{e^3x^3}{3c}$$

input
```
Int[(d + e*x^2)^3/(a + c*x^4),x]
```

output
```
(3*d*e^2*x)/c + (e^3*x^3)/(3*c) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*
e*(3*c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a
^(3/4)*c^(7/4)) + ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e*(3*c*d^2 - a*e
^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(7/4))
 - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4)) +
 ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4))
```

Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.29

method	result
risch	$\frac{e^{3} x^{3}}{3 c}+\frac{3 d\,e^{2} x}{c}+\frac{\sum_{\_R =\operatorname{RootOf}\left(c\,\_Z^{4}+a \right)}\frac{\left(e \left(-a\,e^{2}+3 c\,d^{2}\right) \_R^{2}-3 d\,e^{2} a +d^{3} c \right) \ln \left(x -\_R \right)}{\_R^{3}}}{4 c^{2}}$
default	$\frac{e^{2} \left(\frac{1}{3} e\,x^{3}+3 d x \right)}{c}+\frac{\frac{\left(-3 d\,e^{2} a +d^{3} c \right) \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8 a}+\frac{\left(-a\,e^{3}+3 c\,d^{2} e\right.}{}}{c}$

input
```
int((e*x^2+d)^3/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*e^3*x^3/c+3*d*e^2*x/c+1/4/c^2*sum((e*(-a*e^2+3*c*d^2)*_R^2-3*d*e^2*a+d
^3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2133 vs. 2(217) = 434.

Time = 2.04 (sec) , antiderivative size = 2133, normalized size of antiderivative = 7.67

$$\int \frac{\left(d+e x^2\right)^3}{a+c x^4}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/12*(4*e^3*x^3 + 36*d*e^2*x - 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6
*a^2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e
^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e
^12)/(a^3*c^7)))/(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*
d^8*e^4 + 27*a^4*c^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x + (a*c^6*d^
9 - 18*a^2*c^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c
^2*d*e^8 + (3*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10
*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 3
0*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e
^3 + 6*a^2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4
*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 +
 a^6*e^12)/(a^3*c^7)))/(a*c^3))) + 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3
 + 6*a^2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d
^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a
^6*e^12)/(a^3*c^7)))/(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*
c^4*d^8*e^4 + 27*a^4*c^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x - (a*c^
6*d^9 - 18*a^2*c^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a
^5*c^2*d*e^8 + (3*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*
d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8
 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*...
```

Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx$$

$$= \operatorname{RootSum}\Big(256t^4a^3c^7 + t^2\cdot\left(192a^4c^4de^5 - 640a^3c^5d^3e^3 + 192a^2c^6d^5e\right) + a^6e^{12} + 6a^5cd^2e^{10} + 15a^4c^2d^4e^8$$

$$+ \frac{3de^2x}{c} + \frac{e^3x^3}{3c}$$

input
```
integrate((e*x**2+d)**3/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c**7 + _t**2*(192*a**4*c**4*d*e**5 - 640*a**3*c**5*
d**3*e**3 + 192*a**2*c**6*d**5*e) + a**6*e**12 + 6*a**5*c*d**2*e**10 + 15*
a**4*c**2*d**4*e**8 + 20*a**3*c**3*d**6*e**6 + 15*a**2*c**4*d**8*e**4 + 6*
a*c**5*d**10*e**2 + c**6*d**12, Lambda(_t, _t*log(x + (-64*_t**3*a**4*c**5
*e**3 + 192*_t**3*a**3*c**6*d**2*e - 36*_t*a**5*c**2*d*e**8 + 336*_t*a**4*
c**3*d**3*e**6 - 504*_t*a**3*c**4*d**5*e**4 + 144*_t*a**2*c**5*d**7*e**2 -
 4*_t*a*c**6*d**9)/(a**6*e**12 - 12*a**5*c*d**2*e**10 - 27*a**4*c**2*d**4*
e**8 + 27*a**2*c**4*d**8*e**4 + 12*a*c**5*d**10*e**2 - c**6*d**12)))) + 3*
d*e**2*x/c + e**3*x**3/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.23

$$\int \frac{\left(d+ex^2\right)^3}{a+cx^4}\,dx=\frac{e^3x^3+9\,de^2x}{3\,c}$$
$$+\frac{\frac{2\,\sqrt{2}\left(c^{\frac{3}{2}}d^3+3\,\sqrt{a}cd^2e-3\,a\sqrt{c}de^2-a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\,\sqrt{2}\left(c^{\frac{3}{2}}d^3+3\,\sqrt{a}cd^2e-3\,a\sqrt{c}de^2-a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}\right)}{2\,\sqrt{}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}}{}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/3*(e^3*x^3 + 9*d*e^2*x)/c + 1/8*(2*sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^
2*e - 3*a*sqrt(c)*d*e^2 - a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + s
qrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
c))*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*
e^2 - a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/
4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(
2)*(c^(3/2)*d^3 - 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3)*log
(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sq
rt(2)*(c^(3/2)*d^3 - 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3)*
log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/
c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.48

$$
\begin{aligned}
\int \frac{(d+ex^2)^3}{a+cx^4}\,dx &= \frac{c^2e^3x^3+9\,c^2de^2x}{3\,c^3} \\
&+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3-3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2+3\,(ac^3)^{\frac{3}{4}}\,cd^2e-(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4} \\
&+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3-3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2+3\,(ac^3)^{\frac{3}{4}}\,cd^2e-(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4} \\
&+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3-3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2-3\,(ac^3)^{\frac{3}{4}}\,cd^2e+(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^4} \\
&- \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3-3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2-3\,(ac^3)^{\frac{3}{4}}\,cd^2e+(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^4}
\end{aligned}
$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="giac")
```

output
```
1/3*(c^2*e^3*x^3 + 9*c^2*d*e^2*x)/c^3 + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3
 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a
*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4)
+ 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*
c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 -
 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e
^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a
*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*
e + (a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c
^4)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 2712, normalized size of antiderivative = 9.76

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx = \text{Too large to display}$$

input

```
int((d + e*x^2)^3/(a + c*x^4),x)
```

output

```
(e^3*x^3)/(3*c) - atan((a^3*e^6*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (5*d^
3*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c^7)
^(1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^4*
e^2*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c
^2 + 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^3*
c^7)^(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*(
-a^3*c^7)^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-
a^3*c^7)^(1/2))/(a*c^2)) - (c^3*d^6*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (
5*d^3*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*
c^7)^(1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*
d^4*e^2*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^
9)/c^2 + 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-
a^3*c^7)^(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e
^6*(-a^3*c^7)^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^
2*(-a^3*c^7)^(1/2))/(a*c^2)) + (a*c^2*d^4*e^2*x*((e^6*(-a^3*c^7)^(1/2))/(1
6*c^7) + (5*d^3*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (
d^6*(-a^3*c^7)^(1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c
^6) + (15*d^4*e^2*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e
 + (2*a^4*e^9)/c^2 + 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3
 + (2*d^9*(-a^3*c^7)^(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 624, normalized size of antiderivative = 2.24

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx = \text{Too large to display}$$

input

```
int((e*x^2+d)^3/(c*x^4+a),x)
```

output

```
(6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x
)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*e**3 - 18*c**(1/4)*a**(3/4)*sqrt(2)*atan(
(c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*d
**2*e + 18*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*s
qrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d*e**2 - 6*c**(3/4)*a**(1/4)*sqrt
(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt
(2)))*c*d**3 - 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*e**3 + 18*c**(1/4)*a**(3/4)
*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)
*sqrt(2)))*c*d**2*e - 18*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)
*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d*e**2 + 6*c**(3/4)
*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)
*a**(1/4)*sqrt(2)))*c*d**3 - 3*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a
**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*e**3 + 9*c**(1/4)*a**(3/4)*s
qrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*d**2
*e + 3*c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
 + sqrt(c)*x**2)*a*e**3 - 9*c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4
)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*d**2*e + 9*c**(3/4)*a**(1/4)*sqrt(
2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*d*e**2 -
 3*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(...
```

3.313 $\int \frac{\left(d+ex^2\right)^2}{a+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 220

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^2}{a+cx^4}\,dx = {} & \frac{e^2x}{c}-\frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}}\\ & +\frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}}\\ & +\frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}c^{5/4}}\end{aligned}$$

output

```
e^2*x/c+1/4*(c*d^2+2*a^(1/2)*c^(1/2)*d*e-a*e^2)*arctan(-1+2^(1/2)*c^(1/4)*
x/a^(1/4))*2^(1/2)/a^(3/4)/c^(5/4)+1/4*(c*d^2+2*a^(1/2)*c^(1/2)*d*e-a*e^2)
*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(5/4)+1/4*(c*d^2-2*
a^(1/2)*c^(1/2)*d*e-a*e^2)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1
/2)*x^2))*2^(1/2)/a^(3/4)/c^(5/4)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$$
$$= \frac{8a^{3/4}\sqrt[4]{c}e^2x-2\sqrt{2}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt{2}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan}{}$$

input
```
Integrate[(d + e*x^2)^2/(a + c*x^4),x]
```

output
```
(8*a^(3/4)*c^(1/4)*e^2*x - 2*Sqrt[2]*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^
2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(c*d^2 + 2*Sqrt[a]*
Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-(
c*d^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2] + Sqrt[2]*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(8*a^(3/4)*c^(5/4))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.35, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$$
$$\Big\downarrow \text{1485}$$
$$\int \left(\frac{-ae^2+cd^2+2cdex^2}{c\,(a+cx^4)}+\frac{e^2}{c}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)}{2\sqrt{2}a^{3/4}c^{5/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)}{2\sqrt{2}a^{3/4}c^{5/4}}-\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}+\frac{e^2x}{c}$$

input
```
Int[(d + e*x^2)^2/(a + c*x^4),x]
```

output
```
(e^2*x)/c - ((c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 + 2*Sqrt[a]*Sqrt
[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*c^(5/4)) - ((c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2
 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
+ Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4))
```

Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.25

method	result
risch	$\frac{e^2 x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(2\_R^2 cde-a\,e^2+c\,d^2\right)\ln\left(x-\_R\right)}{\_R^3}}{4c^2}$
default	$\frac{e^2 x}{c}+\frac{\frac{\left(-a\,e^2+c\,d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{ed\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{}}\right.\right.}{}}{c}$

input
```
int((e*x^2+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
e^2*x/c+1/4/c^2*sum((2*_R^2*c*d*e-a*e^2+c*d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^
4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1480 vs. $2(161) = 322$.

Time = 0.46 (sec) , antiderivative size = 1480, normalized size of antiderivative = 6.73

$$\int \frac{\left(d+ex^2\right)^2}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(4*e^2*x + c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*
a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)
))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*
d^2*e^6 + a^4*e^8)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4
- a^4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2
*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*
d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*
a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) - c*sqrt(-(4*c*d^3*e - 4*a*
d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*
a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e
^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^
2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d
^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(
a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*
d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*
c^2))) + c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*
d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*
c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^
6 + a^4*e^8)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*
c*e^6 - 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^...
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$$

$$= \operatorname{RootSum}\left(256t^4a^3c^5 + t^2\left(-128a^3c^3de^3 + 128a^2c^4d^3e\right) + a^4e^8 + 4a^3cd^2e^6 + 6a^2c^2d^4e^4 + 4ac^3d^6e^2 + c^4d\right.$$

$$+ \frac{e^2x}{c}$$

input
```
integrate((e*x**2+d)**2/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c**5 + _t**2*(-128*a**3*c**3*d*e**3 + 128*a**2*c**4
*d**3*e) + a**4*e**8 + 4*a**3*c*d**2*e**6 + 6*a**2*c**2*d**4*e**4 + 4*a*c*
*3*d**6*e**2 + c**4*d**8, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c**4*d*e
- 4*_t*a**4*c*e**6 + 60*_t*a**3*c**2*d**2*e**4 - 60*_t*a**2*c**3*d**4*e**2
 + 4*_t*a*c**4*d**6)/(a**4*e**8 - 4*a**3*c*d**2*e**6 - 10*a**2*c**2*d**4*e
**4 - 4*a*c**3*d**6*e**2 + c**4*d**8)))) + e**2*x/c
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.31

$$\int \frac{\left(d+ex^2\right)^2}{a+cx^4}\,dx = \frac{e^2x}{c} + \frac{\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^2+2\sqrt{a}cde-a\sqrt{c}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^2+2\sqrt{a}cde-a\sqrt{c}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \sqrt{2}}{8c}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="maxima")
```

output
```
e^2*x/c + 1/8*(2*sqrt(2)*(c^(3/2)*d^2 + 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*a
rctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqr
t(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^2 +
2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)
*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sq
rt(c)) + sqrt(2)*(c^(3/2)*d^2 - 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*log(sqrt(
c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*
(c^(3/2)*d^2 - 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*
a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{(d+ex^2)^2}{a+cx^4}\,dx &= \frac{e^2x}{c} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^2 - (ac^3)^{\frac{1}{4}}ace^2 + 2\,(ac^3)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} \\ &+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^2 - (ac^3)^{\frac{1}{4}}ace^2 + 2\,(ac^3)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} \\ &+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^2 - (ac^3)^{\frac{1}{4}}ace^2 - 2\,(ac^3)^{\frac{3}{4}}de\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^3} \\ &- \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^2 - (ac^3)^{\frac{1}{4}}ace^2 - 2\,(ac^3)^{\frac{3}{4}}de\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^3}\end{aligned}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")
```

output
```
e^2*x/c + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(
a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/
4))/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 +
 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e
^2 - 2*(a*c^3)^(3/4)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*
c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c
^3)^(3/4)*d*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

Mupad [B] (verification not implemented)

Time = 17.37 (sec) , antiderivative size = 1479, normalized size of antiderivative = 6.72

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^2/(a + c*x^4),x)
```

output

```
(e^2*x)/c - 2*atanh((8*c^3*d^4*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4
*(-a^3*c^5)^(1/2))/(16*a^3*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d
^2*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c
^5)^(1/2))/a^2 + 4*c^2*d^5*e + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3
*e^3 - (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(
a*c)) + (8*a^2*c*e^4*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4*(-a^3*c^5
)^(1/2))/(16*a^3*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d^2*e^2*(-a
^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c^5)^(1/2))
/a^2 + 4*c^2*d^5*e + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3 - (14
*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c)) - (4
8*a*c^2*d^2*e^2*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4*(-a^3*c^5)^(1/
2))/(16*a^3*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d^2*e^2*(-a^3*c^
5)^(1/2))/(8*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c^5)^(1/2))/a^2
+ 4*c^2*d^5*e + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3 - (14*d^2*
e^4*(-a^3*c^5)^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c)))*((a^2*e^
4*(-a^3*c^5)^(1/2) + c^2*d^4*(-a^3*c^5)^(1/2) - 4*a^2*c^4*d^3*e + 4*a^3*c^
3*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^5)^(1/2))/(16*a^3*c^5))^(1/2) - 2*atanh((8
*c^3*d^4*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) - (d^4*(-a^3*c^5)^(1/2))/(16
*a^3*c^3) - (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) + (3*d^2*e^2*(-a^3*c^5)^(1/2
))/(8*a^2*c^4))^(1/2))/((2*d^6*(-a^3*c^5)^(1/2))/a^2 + 4*a^2*d*e^5 + 4*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.05

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx = \text{Too large to display}$$

input

```
int((e*x^2+d)^2/(c*x^4+a),x)
```

output

```
( - 4*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*d*e + 2*c**(3/4)*a**(1/4)*sqrt(2)*atan
((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*
e**2 - 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*d**2 + 4*c**(1/4)*a**(3/4)*sqrt(2)*
atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))
)*c*d*e - 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*e**2 + 2*c**(3/4)*a**(1/4)*sqrt(
2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(
2)))*c*d**2 + 2*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)
*x + sqrt(a) + sqrt(c)*x**2)*c*d*e - 2*c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1
/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*d*e + c**(3/4)*a**(1/4)
*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*e*
*2 - c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a
) + sqrt(c)*x**2)*c*d**2 - c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)
*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*e**2 + c**(3/4)*a**(1/4)*sqrt(2)*lo
g(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*d**2 + 8*a*c*e**
2*x)/(8*a*c**2)
```

3.314 $\int \frac{d+ex^2}{a+cx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 180

$$\begin{aligned}\int \frac{d+ex^2}{a+cx^4}\,dx = & -\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} \\ & +\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} \\ & +\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}\end{aligned}$$

output

```
1/4*(c^(1/2)*d+a^(1/2)*e)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(
3/4)/c^(3/4)+1/4*(c^(1/2)*d+a^(1/2)*e)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))
*2^(1/2)/a^(3/4)/c^(3/4)+1/4*(c^(1/2)*d-a^(1/2)*e)*arctanh(2^(1/2)*a^(1/4)
*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)/c^(3/4)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02

$$\int \frac{d+ex^2}{a+cx^4}\,dx$$
$$=\frac{-2\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-\left(\sqrt{c}d-\sqrt{a}e\right)\left(\log\left(\sqrt{}\right.\right.}{4\sqrt{2}a^{3/4}c^{3/4}}$$

input
```
Integrate[(d + e*x^2)/(a + c*x^4),x]
```

output
```
(-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(S
qrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d
 - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(4*Sqrt[2]*a^(3/4)*
c^(3/4))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{a+cx^4}\,dx$$
$$\Big\downarrow \text{1482}$$
$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}{cx^4+a}dx}{2c}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}{cx^4+a}dx}{2c}$$
$$\Big\downarrow \text{27}$$
$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}$$

input
```
Int[(d + e*x^2)/(a + c*x^4),x]
```

output

```
(((Sqrt[c]*d)/Sqrt[a] + e)*(-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqr
t[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*
a^(1/4)*c^(1/4))))/(2*Sqrt[c]) + (((Sqrt[c]*d)/Sqrt[a] - e)*(-1/2*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)
*c^(1/4))))/(2*Sqrt[c])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.19

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+a\right)}\frac{\left(\_R^{2}e+d\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4c}$
default	$\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\,\mathrm{ar}\right.}{8}$

input
```
int((e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum((_R^2*e+d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(121) = 242$.

Time = 0.09 (sec) , antiderivative size = 767, normalized size of antiderivative = 4.26

$$
\begin{aligned}
\int \frac{d+ex^2}{a+cx^4}\,dx &= -\frac{1}{4}\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\log\left(-\left(c^2d^4-a^2e^4\right)x\right.\\
&\quad\left.+\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+ac^2d^3-a^2cde^2\right)\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\right)\\
&\quad+\frac{1}{4}\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\log\left(-\left(c^2d^4-a^2e^4\right)x\right.\\
&\quad\left.-\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+ac^2d^3-a^2cde^2\right)\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\right)\\
&\quad+\frac{1}{4}\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\log\left(-\left(c^2d^4-a^2e^4\right)x\right.\\
&\quad\left.+\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-ac^2d^3+a^2cde^2\right)\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\right)\\
&\quad-\frac{1}{4}\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\log\left(-\left(c^2d^4-a^2e^4\right)x\right.\\
&\quad\left.-\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-ac^2d^3+a^2cde^2\right)\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\right)
\end{aligned}
$$

input

```
integrate((e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d
*e)/(a*c))*log(-(c^2*d^4 - a^2*e^4)*x + (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*
d^2*e^2 + a^2*e^4)/(a^3*c^3)) + a*c^2*d^3 - a^2*c*d*e^2)*sqrt(-(a*c*sqrt(-
(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt
(-(a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c)
)*log(-(c^2*d^4 - a^2*e^4)*x - (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 +
 a^2*e^4)/(a^3*c^3)) + a*c^2*d^3 - a^2*c*d*e^2)*sqrt(-(a*c*sqrt(-(c^2*d^4
- 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt((a*c*sqr
t(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))*log(-(c^
2*d^4 - a^2*e^4)*x + (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a^3*c^3)) - a*c^2*d^3 + a^2*c*d*e^2)*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))) - 1/4*sqrt((a*c*sqrt(-(c^2*d^4
 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))*log(-(c^2*d^4 - a^2
*e^4)*x - (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3))
- a*c^2*d^3 + a^2*c*d*e^2)*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*
e^4)/(a^3*c^3)) - 2*d*e)/(a*c)))
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.61

$$\int \frac{d+ex^2}{a+cx^4}\,dx$$

$$= \mathrm{RootSum}\Bigg(256t^4a^3c^3+64t^2a^2c^2de+a^2e^4+2acd^2e^2+c^2d^4, \Bigg(t\mapsto t\log\Bigg(x+\frac{64t^3a^3c^2e+12ta^2cde^2-}{a^2e^4-c^2d^4}$$

input
```
integrate((e*x**2+d)/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c**3 + 64*_t**2*a**2*c**2*d*e + a**2*e**4 + 2*a*c*d
**2*e**2 + c**2*d**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*c**2*e + 12*_t*
a**2*c*d*e**2 - 4*_t*a*c**2*d**3)/(a**2*e**4 - c**2*d**4))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.23

$$\int \frac{d+ex^2}{a+cx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{3}{4}}}$$

input

```
integrate((e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output

```
1/4*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))
*sqrt(c)) + 1/4*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt
(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(c))*sqrt(c)) + 1/8*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^
2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - 1/8*sqrt(2)*(
sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(
a))/(a^(3/4)*c^(3/4))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.34

$$\int \frac{d+ex^2}{a+cx^4}\,dx = \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3} - \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

input

```
integrate((e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output

```
1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)
*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*
log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3
)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 599, normalized size of antiderivative = 3.33

$$\int \frac{d+ex^2}{a+cx^4}\,dx = -2\,\mathrm{atanh}\left(\frac{8\,c^3\,d^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}-\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}-\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}} - \frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}-\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}-\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}\right)\sqrt{-\frac{c\,d^2\,\sqrt{-a^3\,c^3}-a\,e^2\,\sqrt{-a^3\,c^3}+2\,a^2\,c^2\,d\,e}{16\,a^3\,c^3}} - 2\,\mathrm{atanh}\left(\frac{8\,c^3\,d^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}-\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}+\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}} - \frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}-\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}+\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}\right)\sqrt{-\frac{a\,e^2\,\sqrt{-a^3\,c^3}-c\,d^2\,\sqrt{-a^3\,c^3}+2\,a^2\,c^2\,d\,e}{16\,a^3\,c^3}}$$

input

```
int((d + e*x^2)/(a + c*x^4),x)
```

output

```
- 2*atanh((8*c^3*d^2*x*((e^2*(-a^3*c^3)^(1/2))/(16*a^2*c^3) - (d^2*(-a^3*c
^3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 +
 (2*c*d^3*(-a^3*c^3)^(1/2))/a^2 - (2*d*e^2*(-a^3*c^3)^(1/2))/a) - (8*a*c^2
*e^2*x*((e^2*(-a^3*c^3)^(1/2))/(16*a^2*c^3) - (d^2*(-a^3*c^3)^(1/2))/(16*a
^3*c^2) - (d*e)/(8*a*c))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 + (2*c*d^3*(-a^3*
c^3)^(1/2))/a^2 - (2*d*e^2*(-a^3*c^3)^(1/2))/a))*(-(c*d^2*(-a^3*c^3)^(1/2)
 - a*e^2*(-a^3*c^3)^(1/2) + 2*a^2*c^2*d*e)/(16*a^3*c^3))^(1/2) - 2*atanh((
8*c^3*d^2*x*((d^2*(-a^3*c^3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c) - (e^2*(-
a^3*c^3)^(1/2))/(16*a^2*c^3))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 - (2*c*d^3*(
-a^3*c^3)^(1/2))/a^2 + (2*d*e^2*(-a^3*c^3)^(1/2))/a) - (8*a*c^2*e^2*x*((d^
2*(-a^3*c^3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c) - (e^2*(-a^3*c^3)^(1/2))/
(16*a^2*c^3))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 - (2*c*d^3*(-a^3*c^3)^(1/2))
/a^2 + (2*d*e^2*(-a^3*c^3)^(1/2))/a))*(-(a*e^2*(-a^3*c^3)^(1/2) - c*d^2*(-
a^3*c^3)^(1/2) + 2*a^2*c^2*d*e)/(16*a^3*c^3))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.33

$$\int \frac{d+ex^2}{a+cx^4}\,dx$$

$$= \frac{\sqrt{2}\,\Bigg(-2\sqrt{a}\,atan\bigg(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\bigg)\,e-2\sqrt{c}\,atan\bigg(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\bigg)\,d+2\sqrt{a}\,atan\bigg(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\bigg)\,e+2\sqrt{c}\,a}{}$$

input
```
int((e*x^2+d)/(c*x^4+a),x)
```

output
```
(c**(1/4)*a**(1/4)*sqrt(2)*( - 2*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*e - 2*sqrt(c)*atan((c**(1/4)*a*
*(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d + 2*sqrt(a)*a
tan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))
*e + 2*sqrt(c)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a*
*(1/4)*sqrt(2)))*d + sqrt(a)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
+ sqrt(c)*x**2)*e - sqrt(a)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sq
rt(c)*x**2)*e - sqrt(c)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqr
t(c)*x**2)*d + sqrt(c)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)
*x**2)*d))/(8*a*c)
```

3.315 $\int \frac{1}{a+cx^4}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 134

$$\int \frac{1}{a+cx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

output
```
1/4*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(1/4)+1/4*arcta
n(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(1/4)+1/4*arctanh(2^(1/2)
*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)/c^(1/4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+cx^4}\,dx$$
$$= \frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{}\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

input
```
Integrate[(a + c*x^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
c^(1/4))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.49, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+cx^4}dx$$

$$\Big\downarrow \text{ 755}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

input

```
Int[(a + c*x^4)^(-1),x]
```

output

```
(-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + Ar
cTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a
]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]
*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.20

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}}{4c}$	27
default	$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}$	102

input
```
int(1/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \frac{1}{a+cx^4}\,dx = {} & \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right) \\ & +\frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right) \\ & -\frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right) \\ & -\frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)\end{aligned}$$

input
```
integrate(1/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/(a^3*c))^(1/4) + x) + 1/4*I*(-1/(a^3*c))^
(1/4)*log(I*a*(-1/(a^3*c))^(1/4) + x) - 1/4*I*(-1/(a^3*c))^(1/4)*log(-I*a*
(-1/(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4)
 + x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.15

$$\int \frac{1}{a+cx^4}\,dx = \operatorname{RootSum}\left(256t^4a^3c+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

input
```
integrate(1/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.26

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}$$

input

```
integrate(1/(c*x^4+a),x, algorithm="maxima")
```

output

```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1
/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^
(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2
 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(87) = 174$.

Time = 0.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.34

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}$$

input
```
integrate(1/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
+ sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log
(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.25

$$\int \frac{1}{a+cx^4}\,dx=-\frac{\operatorname{atan}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)+\operatorname{atanh}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)}{2\,{\left(-a\right)}^{3/4}\,c^{1/4}}$$

input
```
int(1/(a + c*x^4),x)
```

output
```
-(atan((c^(1/4)*x)/(-a)^(1/4)) + atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3
/4)*c^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+cx^4}\,dx$$

$$=\frac{\sqrt{2}\,\Bigg(-2atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+\log\Big(c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{}}{8c^{\frac{1}{4}}a^{\frac{3}{4}}}$$

input
```
int(1/(c*x^4+a),x)
```

output

```
(c**(3/4)*a**(1/4)*sqrt(2)*( - 2*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))) + 2*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))) - log( - c**(1/4)*a**(1/4)*sqrt(2
)*x + sqrt(a) + sqrt(c)*x**2) + log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
+ sqrt(c)*x**2)))/(8*a*c)
```

3.316 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 257

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx &= \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)} \\ &\quad + \frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)} \\ &\quad + \frac{\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)}\end{aligned}$$

output

```
e^(3/2)*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)/(a*e^2+c*d^2)+1/4*c^(1/4)*(c^(1/
2)*d-a^(1/2)*e)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/(a*e^
2+c*d^2)+1/4*c^(1/4)*(c^(1/2)*d-a^(1/2)*e)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1
/4))*2^(1/2)/a^(3/4)/(a*e^2+c*d^2)+1/4*c^(1/4)*(c^(1/2)*d+a^(1/2)*e)*arcta
nh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)/(a*e^2
+c*d^2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)\,(a+cx^4)}\,dx$$
$$= \frac{8a^{3/4}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{2}\sqrt[4]{c}\sqrt{d}\Big(\left(-2\sqrt{c}d+2\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\Big(}{8a^{3/4}\sqrt{d}\,(cd}$$

input
```
Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
(8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*(
(-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(
Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*
d + Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(3/4)*Sqrt[d
]*(c*d^2 + a*e^2))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)\,(d+ex^2)}\,dx$$
$$\Big\downarrow \text{1485}$$
$$\int \left(\frac{e^2}{(d+ex^2)\,(ae^2+cd^2)}+\frac{c(d-ex^2)}{(a+cx^4)\,(ae^2+cd^2)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)}-\frac{\sqrt[4]{c}(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(ae^2+cd^2)}$$

input
```
Int[1/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
(e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)
*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/
4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[
a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a
^(3/4)*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.98

method	result
default	$\frac{e^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(a e^2+c d^2)\sqrt{de}} + \frac{c\Bigg(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a} - \frac{e\sqrt{2}\,\Big(\ln\Big(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}}}{}}{a e^2+c d^2}$
risch	$\frac{\sqrt{-de}\,e\ln\left(\left(-16a^2 d e^5-d^3 a c e^3-c^2 d^5 e\right)x+16(-de)^{\frac{3}{2}}a^2 e^3-4(-de)^{\frac{3}{2}}a c d^2 e-5 a c e^2\sqrt{-de}\,d^3-c^2 d^5\sqrt{-de}\right)}{2d(a e^2+c d^2)} - \frac{\sqrt{-de}\,e\ln\Big(\big(-16a^2}{}$

input
```
int(1/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
e^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+c/(a*e^2+c*d^2)*(1/8
*d*(1/c*a)^(1/4)/a*2^(1/2)*(ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))
/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+2*arctan(2^(1/2)/(1/c*a)^(1/
4)*x+1)+2*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1))-1/8*e/c/(1/c*a)^(1/4)*2^(1/2)
*(ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1
/2)+(1/c*a)^(1/2)))+2*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+2*arctan(2^(1/2)/(
1/c*a)^(1/4)*x-1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. $2(190) = 380$.

Time = 0.44 (sec) , antiderivative size = 4084, normalized size of antiderivative = 15.89

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.34

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \frac{e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2+ae^2)\sqrt{de}} + \frac{\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} + \frac{\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} + \frac{\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
e^2*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) + 1/2*((a*c^3)^(1/4)
*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/4)
*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)
*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sq
rt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3
)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2
 + sqrt(2)*a^2*c^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 18.47 (sec) , antiderivative size = 4802, normalized size of antiderivative = 18.68

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)*(d + e*x^2)),x)
```

output
```
atan((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a
^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*
(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^
2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/
2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^
4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*
d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 +
448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*
d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(
a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5
*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(
a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i - (((a*e^2*(-a^3*c)^(1
/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a
^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a
^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))
^(1/2)*(256*a^4*c^4*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2)
+ 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*
a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*
e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(
16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.28

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \frac{2c^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)de - 2c^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d^2 - 2c^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)}{}$$

input

```
int(1/(e*x^2+d)/(c*x^4+a),x)
```

output

```
(2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x
)/(c**(1/4)*a**(1/4)*sqrt(2)))*d*e - 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**
(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d**2 -
2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)
/(c**(1/4)*a**(1/4)*sqrt(2)))*d*e + 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(
1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d**2 + 8
*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e - c**(1/4)*a**(3/4)*sqr
t(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*d*e + c*
*(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c
)*x**2)*d*e - c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x
 + sqrt(a) + sqrt(c)*x**2)*d**2 + c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*a
**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*d**2)/(8*a*d*(a*e**2 + c*d**2)
)
```

3.317 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 336

$$
\begin{aligned}
\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)}\,dx &= \frac{e^2x}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{e^{3/2}\left(5cd^2+ae^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2+ae^2\right)^2} \\
&\quad-\frac{c^{3/4}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad+\frac{c^{3/4}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad+\frac{c^{3/4}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}
\end{aligned}
$$

output
```
1/2*e^2*x/d/(a*e^2+c*d^2)/(e*x^2+d)+1/2*e^(3/2)*(a*e^2+5*c*d^2)*arctan(e^(
1/2)*x/d^(1/2))/d^(3/2)/(a*e^2+c*d^2)^2+1/4*c^(3/4)*(c*d^2-2*a^(1/2)*c^(1/
2)*d*e-a*e^2)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/(a*e^2+
c*d^2)^2+1/4*c^(3/4)*(c*d^2-2*a^(1/2)*c^(1/2)*d*e-a*e^2)*arctan(1+2^(1/2)*
c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/(a*e^2+c*d^2)^2+1/4*c^(3/4)*(c*d^2+2*a^
(1/2)*c^(1/2)*d*e-a*e^2)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2
)*x^2))*2^(1/2)/a^(3/4)/(a*e^2+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex^2)^2(a+cx^4)}\,dx$$

$$= \frac{\frac{4e^2(cd^2+ae^2)x}{d(d+ex^2)}+\frac{4e^{3/2}(5cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}}+\frac{2\sqrt{2}c^{3/4}(-cd^2+2\sqrt{a}\sqrt{c}de+ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}}-\frac{2\sqrt{2}c^{3/4}(-cd^2+2\sqrt{a}\sqrt{c}d}{}}{}$$

input
```
Integrate[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

output
```
((4*e^2*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (4*e^(3/2)*(5*c*d^2 + a*e^2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-(c*d^2) + 2*Sq
rt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4
) - (2*Sqrt[2]*c^(3/4)*(-(c*d^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[2]*c^(3/4)*(-(c*d^2) - 2*
Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/a^(3/4) + (Sqrt[2]*c^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e
^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4))/(8*(c
*d^2 + a*e^2)^2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.35, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{1485}$$

$$\int \left(\frac{2cde^2}{(d+ex^2)(ae^2+cd^2)^2}+\frac{e^2}{(d+ex^2)^2(ae^2+cd^2)}+\frac{c(-ae^2+cd^2-2cdex^2)}{(a+cx^4)(ae^2+cd^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{2c\sqrt{d}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(ae^2+cd^2\right)}+\frac{e^2x}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

input

```
Int[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

output

```
(e^2*x)/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (2*c*Sqrt[d]*e^(3/2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(2*d^(3/2)*(c*d^2 + a*e^2)) - (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e
- a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^
2 + a*e^2)^2) + (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1
+ (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c
^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(3
/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
```

Defintions of rubi rules used

rule 1485

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.91

method	result
default	$\frac{e^{2}\left(\frac{\left(a\,e^{2}+c\,d^{2}\right)x}{2d\left(e\,x^{2}+d\right)}+\frac{\left(a\,e^{2}+5c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{2}}-\frac{c\left(\frac{\left(a\,e^{2}-c\,d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arcta}\right.}{8a}\right.}{}$
risch	Expression too large to display

input
```
int(1/(e*x^2+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
e^2/(a*e^2+c*d^2)^2*(1/2*(a*e^2+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+5*c*d^2)/d
/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-c/(a*e^2+c*d^2)^2*(1/8*(a*e^2-c*d^2)
*(1/c*a)^(1/4)/a*2^(1/2)*(ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(
x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+2*arctan(2^(1/2)/(1/c*a)^(1/4)
*x+1)+2*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1))+1/4*e*d/(1/c*a)^(1/4)*2^(1/2)*(
ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2
)+(1/c*a)^(1/2)))+2*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/
c*a)^(1/4)*x-1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4193 vs. $2(265) = 530$.

Time = 8.63 (sec) , antiderivative size = 8409, normalized size of antiderivative = 25.03

$$\int \frac{1}{\left(d+ex^{2}\right)^{2}\left(a+cx^{4}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2 (a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**2/(c*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2 (a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(265) = 530$.

Time = 0.12 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.58

$$
\begin{aligned}
&\int \frac{1}{(d+ex^2)^2\,(a+cx^4)}\,dx \\
&= \frac{e^2x}{2\,(cd^3+ade^2)(ex^2+d)} \\
&+ \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{1}{4}}\,ace^2-2\,(ac^3)^{\frac{3}{4}}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\,\left(\sqrt{2}ac^3d^4+2\,\sqrt{2}a^2c^2d^2e^2+\sqrt{2}a^3ce^4\right)} \\
&+ \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{1}{4}}\,ace^2-2\,(ac^3)^{\frac{3}{4}}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\,\left(\sqrt{2}ac^3d^4+2\,\sqrt{2}a^2c^2d^2e^2+\sqrt{2}a^3ce^4\right)} \\
&+ \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{1}{4}}\,ace^2+2\,(ac^3)^{\frac{3}{4}}\,de\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\,\left(\sqrt{2}ac^3d^4+2\,\sqrt{2}a^2c^2d^2e^2+\sqrt{2}a^3ce^4\right)} \\
&- \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{1}{4}}\,ace^2+2\,(ac^3)^{\frac{3}{4}}\,de\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\,\left(\sqrt{2}ac^3d^4+2\,\sqrt{2}a^2c^2d^2e^2+\sqrt{2}a^3ce^4\right)} \\
&+ \frac{(5\,cd^2e^2+ae^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,(c^2d^5+2\,acd^3e^2+a^2de^4)\sqrt{de}}
\end{aligned}
$$

input

```
integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")
```

output

```
1/2*e^2*x/((c*d^3 + a*d*e^2)*(e*x^2 + d)) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (
a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e
^2 + sqrt(2)*a^3*c*e^4) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e
^2 - 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c
*e^4) + 1/4*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/
4)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 +
2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^4) - 1/4*((a*c^3)^(1/4)*c^2*d^
2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*log(x^2 - sqrt(2)*x*(a/c)
^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(
2)*a^3*c*e^4) + 1/2*(5*c*d^2*e^2 + a*e^4)*arctan(e*x/sqrt(d*e))/((c^2*d^5
+ 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d*e))
```

Mupad [B] (verification not implemented)

Time = 19.32 (sec) , antiderivative size = 16369, normalized size of antiderivative = 48.72

$$\int \frac{1}{(d+ex^2)^2 (a+cx^4)} dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)*(d + e*x^2)^2),x)
```

output
```
(e^2*x)/(2*d*(d + e*x^2)*(a*e^2 + c*d^2)) - atan(((((((256*a^8*c^4*d*e^16
- 128*a*c^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 89
60*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176
*a^7*c^5*d^3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*
d^4*e^6 + 6*a^2*c^2*d^6*e^4)) + (x*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-
a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c
^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^
2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14
*e^5 + 4608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^
11 - 4608*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17)
)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*
d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^
3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 +
 a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^
(1/2) + (x*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 10
24*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^
5*c^6*d^3*e^12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e
^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(
1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))
/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a...
```

Reduce [B] (verification not implemented)

Time = 16.34 (sec) , antiderivative size = 1155, normalized size of antiderivative = 3.44

$$\int \frac{1}{(d+ex^2)^2 (a+cx^4)} dx = \text{Too large to display}$$

input
```
int(1/(e*x^2+d)^2/(c*x^4+a),x)
```

output

```
(4*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x
)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*d**4*e + 4*c**(1/4)*a**(3/4)*sqrt(2)*atan
((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*
d**3*e**2*x**2 + 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d**3*e**2 + 2*c**(3/4)*a*
*(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a*
*(1/4)*sqrt(2)))*a*d**2*e**3*x**2 - 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(
1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*d**5 -
 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x
)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*d**4*e*x**2 - 4*c**(1/4)*a**(3/4)*sqrt(2)
*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)
))*c*d**4*e - 4*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2)
+ 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*d**3*e**2*x**2 - 2*c**(3/4)*
a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*
a**(1/4)*sqrt(2)))*a*d**3*e**2 - 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4
)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d**2*e**3
*x**2 + 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sq
rt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*d**5 + 2*c**(3/4)*a**(1/4)*sqrt(2)
*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)
))*c*d**4*e*x**2 + 4*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2...
```

3.318 $\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 268

$$\begin{aligned}\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx &= \frac{x\left(cd\left(d^2-\frac{3ae^2}{c}\right)+e(3cd^2-ae^2)\,x^2\right)}{4ac\,(a+cx^4)}\\ &\quad-\frac{3\left(\sqrt{c}d+\sqrt{a}e\right)(cd^2+ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}}\\ &\quad+\frac{3\left(\sqrt{c}d+\sqrt{a}e\right)(cd^2+ae^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}}\\ &\quad+\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}c^{7/4}}\end{aligned}$$

output

```
1/4*x*(c*d*(d^2-3*a*e^2/c)+e*(-a*e^2+3*c*d^2)*x^2)/a/c/(c*x^4+a)+3/16*(c^(
1/2)*d+a^(1/2)*e)*(a*e^2+c*d^2)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/
2)/a^(7/4)/c^(7/4)+3/16*(c^(1/2)*d+a^(1/2)*e)*(a*e^2+c*d^2)*arctan(1+2^(1/
2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(7/4)+3/16*(c^(1/2)*d-a^(1/2)*e)*(
a*e^2+c*d^2)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1
/2)/a^(7/4)/c^(7/4)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$$
$$= \frac{-\frac{8a^{3/4}c^{3/4}(ae^2x(3d+ex^2)-cd^2x(d+3ex^2))}{a+cx^4} - 6\sqrt{2}\left(c^{3/2}d^3+\sqrt{a}cd^2e+a\sqrt{c}de^2+a^{3/2}e^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+}{}$$

input
```
Integrate[(d + e*x^2)^3/(a + c*x^4)^2,x]
```

output
```
((-8*a^(3/4)*c^(3/4)*(a*e^2*x*(3*d + e*x^2) - c*d^2*x*(d + 3*e*x^2)))/(a +
 c*x^4) - 6*Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(
3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*(c^(3/2)*d^3
 + Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*(-(c^(3/2)*d^3) + Sqrt[a]*c*d^2*e - a*Sqrt[c
]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2] + 3*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - a^(3/2)
*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^(7/4)*
c^(7/4))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {1519, 25, 2397, 27, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \text{1519}$$
$$-\frac{\int -\frac{3cde^2x^4+3e(cd^2+ae^2)x^2+cd^3}{(cx^4+a)^2}dx}{c}-\frac{e^3x^3}{c(a+cx^4)}$$

$$\Big\downarrow\ 25$$

$$\frac{\int \frac{3cde^2x^4+3e(cd^2+ae^2)x^2+cd^3}{(cx^4+a)^2}dx}{c}-\frac{e^3x^3}{c\left(a+cx^4\right)}$$

$$\Big\downarrow\ 2397$$

$$\frac{\frac{x(3ex^2(ae^2+cd^2)+d(cd^2-3ae^2))}{4a(a+cx^4)}-\frac{\int -\frac{3c\left(cd^2+ae^2\right)\left(ex^2+d\right)}{cx^4+a}dx}{4ac}}{c}-\frac{e^3x^3}{c\left(a+cx^4\right)}$$

$$\Big\downarrow\ 27$$

$$\frac{\frac{3(ae^2+cd^2)\int \frac{ex^2+d}{cx^4+a}dx}{4a}+\frac{x(3ex^2(ae^2+cd^2)+d(cd^2-3ae^2))}{4a(a+cx^4)}}{c}-\frac{e^3x^3}{c\left(a+cx^4\right)}$$

$$\Big\downarrow\ 1482$$

$$\frac{\frac{3(ae^2+cd^2)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}{cx^4+a}dx}{2c}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}{cx^4+a}dx}{2c}\right)}{4a}+\frac{x(3ex^2(ae^2+cd^2)+d(cd^2-3ae^2))}{4a(a+cx^4)}}{c}-\frac{e^3x^3}{c\left(a+cx^4\right)}$$

$$\Big\downarrow\ 27$$

$$\frac{\frac{3(ae^2+cd^2)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}\right)}{4a}+\frac{x(3ex^2(ae^2+cd^2)+d(cd^2-3ae^2))}{4a(a+cx^4)}}{c}-\frac{e^3x^3}{c\left(a+cx^4\right)}$$

$$\Big\downarrow\ 1476$$

$$\frac{\frac{3\left(ae^2+cd^2\right)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{4a}+\frac{x\left(3ex^2\left(ae^2+cd^2\right)+d\left(cd^2-3ae^2\right)\right)}{4a\left(a+cx^4\right)}}{c}-$$

$$\frac{e^3x^3}{c\left(a+cx^4\right)}$$

$$\Big\downarrow 1082$$

$$\frac{\frac{3\left(ae^2+cd^2\right)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}\right)}{4a}+x(3e}{c}$$

$$\frac{e^3x^3}{c\left(a+cx^4\right)}$$

$$\Big\downarrow 217$$

$$\frac{\frac{3\left(ae^2+cd^2\right)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}\right)}{4a}+\frac{x\left(3ex^2\left(ae^2+cd^2\right)+d\left(cd^2-3ae^2\right)\right)}{4a\left(a+cx^4\right)}}{c}-$$

$$\frac{e^3x^3}{c\left(a+cx^4\right)}$$

$$\Big\downarrow \ 1479$$

$$\frac{\frac{3\left(ae^2+cd^2\right)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}\right.}{4a}}{c}$$

$$\frac{e^3x^3}{c\left(a+cx^4\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3\left(ae^2+cd^2\right)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\right.}{2\sqrt{c}}\right.}{4a}}{c}$$

$$\frac{e^3x^3}{c\left(a+cx^4\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3\left(ae^2+cd^2\right)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}\right)}{4a}+x(}{c}$$

$$\frac{e^3x^3}{c\left(a+cx^4\right)}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{3\left(ae^2+cd^2\right)\left(\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right.}{2\sqrt{c}}\right.}{4a}}{c}$$

$$\frac{e^3x^3}{c\left(a+cx^4\right)}$$

input
```
Int[(d + e*x^2)^3/(a + c*x^4)^2,x]
```

output
```
-((e^3*x^3)/(c*(a + c*x^4))) + ((x*(d*(c*d^2 - 3*a*e^2) + 3*e*(c*d^2 + a*e
^2)*x^2))/(4*a*(a + c*x^4)) + (3*(c*d^2 + a*e^2)*((((Sqrt[c]*d)/Sqrt[a] +
e)*(-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) +
 ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sq
rt[c]) + (((Sqrt[c]*d)/Sqrt[a] - e)*(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[c]))
)/(4*a))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.44

method	result
risch	$\frac{-\frac{e\left(a\,e^{2}-3c\,d^{2}\right)x^{3}}{4ac}-\frac{d\left(3a\,e^{2}-c\,d^{2}\right)x}{4ac}}{c\,x^{4}+a}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}\left(c\,\_Z^{4}+a\right)}\frac{\left(e\left(a\,e^{2}+c\,d^{2}\right)\_R^{2}+d\left(a\,e^{2}+c\,d^{2}\right)\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{16a\,c^{2}}$
default	$\frac{-\frac{e\left(a\,e^{2}-3c\,d^{2}\right)x^{3}}{4ac}-\frac{d\left(3a\,e^{2}-c\,d^{2}\right)x}{4ac}}{c\,x^{4}+a}+\frac{3\left(a\,e^{2}+c\,d^{2}\right)\Bigg(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\Bigg(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\Bigg(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-}{8a}}{4ac}$

input

```
int((e*x^2+d)^3/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output

```
(-1/4*e*(a*e^2-3*c*d^2)/a/c*x^3-1/4*d*(3*a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+3/1
6/a/c^2*sum((e*(a*e^2+c*d^2)*_R^2+d*(a*e^2+c*d^2))/_R^3*ln(x-_R),_R=RootOf
(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. $2(207) = 414$.

Time = 0.73 (sec) , antiderivative size = 2116, normalized size of antiderivative = 7.90

$$\int \frac{\left(d+ex^2\right)^3}{\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

output

```
1/16*(4*(3*c*d^2*e - a*e^3)*x^3 - 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*c^2*d^5*e
 + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^
2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^
10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 + 3*a*c^4*d^8*e^2
+ 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x +
27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 + a^6*
c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6
*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*sqrt(-(2
*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c
^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^
5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) + 3*(a*c^2*x^4 + a^2*c)*s
qrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12
+ 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8
 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 +
 3*a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8
 - a^5*e^10)*x - 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5
*c^2*d*e^6 + a^6*c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^
4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^
7*c^7)))*sqrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-
(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^...
```

Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$$
$$=\operatorname{RootSum}\Big(65536t^4a^7c^7+t^2\cdot\left(9216a^6c^4de^5+18432a^5c^5d^3e^3+9216a^4c^6d^5e\right)+81a^6e^{12}+486a^5cd^2e^{10}+$$
$$+\frac{x^3(-ae^3+3cd^2e)+x(-3ade^2+cd^3)}{4a^2c+4ac^2x^4}$$

input
```
integrate((e*x**2+d)**3/(c*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*c**7 + _t**2*(9216*a**6*c**4*d*e**5 + 18432*a**5*
c**5*d**3*e**3 + 9216*a**4*c**6*d**5*e) + 81*a**6*e**12 + 486*a**5*c*d**2*
e**10 + 1215*a**4*c**2*d**4*e**8 + 1620*a**3*c**3*d**6*e**6 + 1215*a**2*c*
*4*d**8*e**4 + 486*a*c**5*d**10*e**2 + 81*c**6*d**12, Lambda(_t, _t*log(x
+ (4096*_t**3*a**6*c**5*e + 432*_t*a**5*c**2*d*e**6 + 720*_t*a**4*c**3*d**
3*e**4 + 144*_t*a**3*c**4*d**5*e**2 - 144*_t*a**2*c**5*d**7)/(27*a**5*e**1
0 + 81*a**4*c*d**2*e**8 + 54*a**3*c**2*d**4*e**6 - 54*a**2*c**3*d**6*e**4
- 81*a*c**4*d**8*e**2 - 27*c**5*d**10)))) + (x**3*(-a*e**3 + 3*c*d**2*e) +
 x*(-3*a*d*e**2 + c*d**3))/(4*a**2*c + 4*a*c**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx=\frac{(3\,cd^2e-ae^3)x^3+(cd^3-3\,ade^2)x}{4\,(ac^2x^4+a^2c)}$$
$$+\frac{3\,(cd^2+ae^2)\left(\frac{2\,\sqrt{2}(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\,\sqrt{2}(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}(\sqrt{c}d-}{}\right.}{32\,ac}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

output

```
1/4*((3*c*d^2*e - a*e^3)*x^3 + (c*d^3 - 3*a*d*e^2)*x)/(a*c^2*x^4 + a^2*c)
+ 3/32*(c*d^2 + a*e^2)*(2*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(
2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)
*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan
(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(sqrt(c)*d - sqrt(a)*e
)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4))
 - sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(207) = 414$.

Time = 0.14 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx = \frac{3\,cd^2ex^3 - ae^3x^3 + cd^3x - 3\,ade^2x}{4\,(cx^4+a)ac}$$
$$+ \frac{3\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 + (ac^3)^{\frac{3}{4}}\,cd^2e + (ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^4}$$
$$+ \frac{3\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 + (ac^3)^{\frac{3}{4}}\,cd^2e + (ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^4}$$
$$+ \frac{3\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 - (ac^3)^{\frac{3}{4}}\,cd^2e - (ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c^4}$$
$$- \frac{3\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 - (ac^3)^{\frac{3}{4}}\,cd^2e - (ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c^4}$$

input

```
integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

output

```
1/4*(3*c*d^2*e*x^3 - a*e^3*x^3 + c*d^3*x - 3*a*d*e^2*x)/((c*x^4 + a)*a*c)
+ 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3
)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 +
 (a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/
32*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3
/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(
a/c))/(a^2*c^4) - 3/32*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^
2*d*e^2 - (a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x
*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 17.64 (sec) , antiderivative size = 2560, normalized size of antiderivative = 9.55

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input

```
int((d + e*x^2)^3/(a + c*x^4)^2,x)
```

output

```
- ((d*x*(3*a*e^2 - c*d^2))/(4*a*c) + (e*x^3*(a*e^2 - 3*c*d^2))/(4*a*c))/(a
 + c*x^4) - 2*atanh((9*c^3*d^6*x*((9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) -
 (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d^6*(-a^7*c^7)^(1/2
))/(256*a^7*c^4) - (9*d*e^5)/(128*a*c^3) + (9*d^2*e^4*(-a^7*c^7)^(1/2))/(2
56*a^5*c^6) - (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*c
*d^6*e^3)/16 - (27*a^3*e^9)/(32*c^2) + (27*c^2*d^8*e)/(32*a) - (27*a^2*d^2
*e^7)/(16*c) + (27*d^9*(-a^7*c^7)^(1/2))/(32*a^5*c) - (27*d*e^8*(-a^7*c^7)
^(1/2))/(32*a*c^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^2*c^4) + (27*d^7*
e^2*(-a^7*c^7)^(1/2))/(16*a^4*c^2))) + (9*a*e^6*x*((9*e^6*(-a^7*c^7)^(1/2)
)/(256*a^4*c^7) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d^
6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d*e^5)/(128*a*c^3) + (9*d^2*e^4*(-a
^7*c^7)^(1/2))/(256*a^5*c^6) - (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))
^(1/2))/(2*((27*a*e^9)/(32*c^2) + (27*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16
*a^2) - (27*c^2*d^8*e)/(32*a^3) - (27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c) + (
27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^3*c^5) + (27*d^3*e^6*(-a^7*c^7)^(1/2))/(1
6*a^4*c^4) - (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^6*c^2))) + (9*c*d^2*e^4*x
*((9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (9*d^5*e)/(128*a^3*c) - (9*d^3*
e^3)/(64*a^2*c^2) - (9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d*e^5)/(12
8*a*c^3) + (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) - (9*d^4*e^2*(-a^7*c
^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*a*e^9)/(32*c^2) + (27*d^2*e^7)...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1304, normalized size of antiderivative = 4.87

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input

```
int((e*x^2+d)^3/(c*x^4+a)^2,x)
```

output

```
( - 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*e**3 - 6*c**(1/4)*a**(3/4)*sqrt(2)*
atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))
)*a*c*d**2*e - 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*e**3*x**4 - 6*c**(1/4)*a*
*(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a*
*(1/4)*sqrt(2)))*c**2*d**2*e*x**4 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(
1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*d*e
**2 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt
(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d**3 - 6*c**(3/4)*a**(1/4)*sqrt(2)
*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)
))*a*c*d*e**2*x**4 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*d**3*x**4 + 6*c**(
1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(
1/4)*a**(1/4)*sqrt(2)))*a**2*e**3 + 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(
1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d**2
*e + 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*e**3*x**4 + 6*c**(1/4)*a**(3/4)*sqr
t(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqr
t(2)))*c**2*d**2*e*x**4 + 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a*...
```

3.319 $\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 249

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx = \frac{x\left(d^2-\frac{ae^2}{c}+2dex^2\right)}{4a\,(a+cx^4)} - \frac{\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} + \frac{\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} + \frac{\left(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}c^{5/4}}$$

output
```
1/4*x*(d^2-a*e^2/c+2*d*e*x^2)/a/(c*x^4+a)+1/16*(3*c*d^2+2*a^(1/2)*c^(1/2)*
d*e+a*e^2)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(5/4)+1/
16*(3*c*d^2+2*a^(1/2)*c^(1/2)*d*e+a*e^2)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4
))*2^(1/2)/a^(7/4)/c^(5/4)+1/16*(3*c*d^2-2*a^(1/2)*c^(1/2)*d*e+a*e^2)*arct
anh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(7/4)/c^(5/
4)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx$$
$$= -\frac{8a^{3/4}\sqrt[4]{c}(ae^2x-cdx(d+2ex^2))}{a+cx^4} - 2\sqrt{2}\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\sqrt{2}\left(3cd^2+2\sqrt{a}\right.$$

input
```
Integrate[(d + e*x^2)^2/(a + c*x^4)^2,x]
```

output
```
((-8*a^(3/4)*c^(1/4)*(a*e^2*x - c*d*x*(d + 2*e*x^2)))/(a + c*x^4) - 2*Sqrt
[2]*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*
x)/a^(1/4)] + 2*Sqrt[2]*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e
 + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]
*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(32*a^(7/4)*c^(5/4))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.31, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {1519, 25, 1493, 27, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \text{1519}$$
$$-\frac{\int -\frac{3cd^2+6cex^2d+ae^2}{(cx^4+a)^2}dx}{3c} - \frac{e^2x}{3c\,(a+cx^4)}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3cd^2+6cex^2d+ae^2}{\left(cx^4+a\right)^2}dx}{3c}-\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1493$$

$$\frac{\frac{x\left(ae^2+3cd^2+6cdex^2\right)}{4a\left(a+cx^4\right)}-\frac{\int -\frac{3\left(3cd^2+2cex^2d+ae^2\right)}{cx^4+a}dx}{4a}}{3c}-\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{3cd^2+2cex^2d+ae^2}{cx^4+a}dx}{4a}+\frac{x\left(ae^2+3cd^2+6cdex^2\right)}{4a\left(a+cx^4\right)}}{3c}-\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1482$$

$$\frac{\frac{3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}{cx^4+a}dx}{2\sqrt{a}\sqrt{c}}+\frac{\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}{cx^4+a}dx}{2\sqrt{a}\sqrt{c}}\right)}{4a}+\frac{x\left(ae^2+3cd^2+6cdex^2\right)}{4a\left(a+cx^4\right)}}{3c}-$$

$$\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x\left(ae^2+3cd^2+6cdex^2\right)}{4a\left(a+cx^4\right)}}{3c}-$$

$$\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{3\left(\frac{\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{a}}+\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x\left(ae^2+3cd^2+6cdex^2\right.}{4a\left(a+cx^4\right)}}{3c}$$

$$\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\left(\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right.}{2\sqrt{a}}\right.}{4a}}{3c}$$

$$\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)}{2\sqrt{a}}\right)}{4a}+\frac{x\left(ae^2+3cd^2+6cdex^2\right)}{4a\left(a+cx^4\right)}}{3c}-$$

$$\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \ 1479$$

$$\frac{\frac{3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{a}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right.}{2\sqrt{a}}\right.}{4a}}{3c}$$

$$\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{a}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\Big(}{2\sqrt{a}}\right.}{4a}}{3c}$$

$$\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)}{2\sqrt{a}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(2\sqrt{a}\sqrt{c}de+ae^2+3\right.}{2\sqrt{a}}\right.}{4a}}{3c}$$

$$\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{3\left(\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)}{2\sqrt{a}}+\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}}\right.}{2\sqrt{a}}\right.}{4a}}{3c}$$

$$\frac{e^2x}{3c\left(a+cx^4\right)}$$

input

```
Int[(d + e*x^2)^2/(a + c*x^4)^2,x]
```

output

```
-1/3*(e^2*x)/(c*(a + c*x^4)) + ((x*(3*c*d^2 + a*e^2 + 6*c*d*e*x^2))/(4*a*(
a + c*x^4)) + (3*(((3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*(-(ArcTan[1 -
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqr
t[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[a]) + ((3*c*d
^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[a]))
)/(4*a))/(3*c)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

method	result
risch	$\frac{\frac{de\,x^{3}}{2a}-\frac{\left(a\,e^{2}-c\,d^{2}\right)x}{4ac}}{c\,x^{4}+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+a\right)}\frac{\left(2de\_R^{2}+\frac{a\,e^{2}+3c\,d^{2}}{c}\right)\ln\left(x-\_R\right)}{\_R^{3}}}{16ac}$
default	$\frac{\frac{de\,x^{3}}{2a}-\frac{\left(a\,e^{2}-c\,d^{2}\right)x}{4ac}}{c\,x^{4}+a}+\frac{\frac{\left(a\,e^{2}+3c\,d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+ed\sqrt{2}\Bigg(\ln\Bigg(}{4ac}$

input
```
int((e*x^2+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*d*e/a*x^3-1/4*(a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+1/16/a/c*sum((2*d*e*_R^2
+1/c*(a*e^2+3*c*d^2))/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1596 vs. $2(188) = 376$.

Time = 0.50 (sec) , antiderivative size = 1596, normalized size of antiderivative = 6.41

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(8*c*d*e*x^3 + (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 +
 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c
^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*
e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*
sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^
6 + a^4*e^8)/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^2*
d^2*e^4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 +
 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e +
 4*a*d*e^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(81*c^4
*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/
(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^
3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c^
4*d*e*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*
d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^
4*c^2*d^2*e^4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6
*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d
^3*e + 4*a*d*e^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt((a^3*c^2*sqrt(-(8
1*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*
e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108
*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (...
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx$$
$$= \operatorname{RootSum}\left(65536t^4a^7c^5 + t^2\cdot\left(2048a^5c^3de^3 + 6144a^4c^4d^3e\right) + a^4e^8 + 20a^3cd^2e^6 + 118a^2c^2d^4e^4 + 180ac^3\right.$$
$$+ \frac{2cdex^3 + x(-ae^2 + cd^2)}{4a^2c + 4ac^2x^4}$$

input
```
integrate((e*x**2+d)**2/(c*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*c**5 + _t**2*(2048*a**5*c**3*d*e**3 + 6144*a**4*c
**4*d**3*e) + a**4*e**8 + 20*a**3*c*d**2*e**6 + 118*a**2*c**2*d**4*e**4 +
180*a*c**3*d**6*e**2 + 81*c**4*d**8, Lambda(_t, _t*log(x + (-8192*_t**3*a*
*6*c**4*d*e + 16*_t*a**5*c*e**6 - 48*_t*a**4*c**2*d**2*e**4 - 144*_t*a**3*
c**3*d**4*e**2 + 432*_t*a**2*c**4*d**6)/(a**4*e**8 + 12*a**3*c*d**2*e**6 +
 38*a**2*c**2*d**4*e**4 + 108*a*c**3*d**6*e**2 + 81*c**4*d**8)))) + (2*c*d
*e*x**3 + x*(-a*e**2 + c*d**2))/(4*a**2*c + 4*a*c**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx = \frac{2\,cdex^3 + (cd^2 - ae^2)x}{4\,(ac^2x^4 + a^2c)}$$
$$+ \frac{\frac{2\sqrt{2}\left(3\,c^{\frac{3}{2}}d^2 + 2\,\sqrt{a}cde + a\sqrt{c}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\left(3\,c^{\frac{3}{2}}d^2 + 2\,\sqrt{a}cde + a\sqrt{c}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} +}{32\,ac}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(2*c*d*e*x^3 + (c*d^2 - a*e^2)*x)/(a*c^2*x^4 + a^2*c) + 1/32*(2*sqrt(2
)*(3*c^(3/2)*d^2 + 2*sqrt(a)*c*d*e + a*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*
sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3*c^(3/2)*d^2 + 2*sqrt(a)*c*d*e + a
*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/s
qrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3
*c^(3/2)*d^2 - 2*sqrt(a)*c*d*e + a*sqrt(c)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*
a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*c^(3/2)*d^2 -
2*sqrt(a)*c*d*e + a*sqrt(c)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)
*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.41

$$\begin{aligned}
&\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx \\
&= \frac{2\,cdex^3+cd^2x-ae^2x}{4\,(cx^4+a)ac} \\
&+ \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{1}{4}}\,ace^2+2\,(ac^3)^{\frac{3}{4}}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} \\
&+ \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{1}{4}}\,ace^2+2\,(ac^3)^{\frac{3}{4}}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} \\
&+ \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{1}{4}}\,ace^2-2\,(ac^3)^{\frac{3}{4}}\,de\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} \\
&- \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{1}{4}}\,ace^2-2\,(ac^3)^{\frac{3}{4}}\,de\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}
\end{aligned}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(2*c*d*e*x^3 + c*d^2*x - a*e^2*x)/((c*x^4 + a)*a*c) + 1/16*sqrt(2)*(3*
(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*arcta
n(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sq
rt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d
*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3)
+ 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3
)^(3/4)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32
*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4
)*d*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 1565, normalized size of antiderivative = 6.29

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^2/(a + c*x^4)^2,x)
```

output
```
2*atanh((9*c^3*d^4*x*((9*d^4*(-a^7*c^5)^(1/2))/(256*a^7*c^3) - (3*d^3*e)/(
64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5) +
(d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/(2*((27*d^6*(-a^7*c^5)^(1
/2))/(32*a^5) - (c*d^3*e^3)/8 - (a*d*e^5)/16 - (9*c^2*d^5*e)/(16*a) + (e^6
*(-a^7*c^5)^(1/2))/(32*a^2*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^3*c^2
) + (15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*a^4*c))) + (c*e^4*x*((9*d^4*(-a^7*c^
5)^(1/2))/(256*a^7*c^3) - (3*d^3*e)/(64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e
^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5) + (d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c
^4))^(1/2))/(2*((27*d^6*(-a^7*c^5)^(1/2))/(32*a^7) - (d*e^5)/(16*a) - (c*d
^3*e^3)/(8*a^2) - (9*c^2*d^5*e)/(16*a^3) + (e^6*(-a^7*c^5)^(1/2))/(32*a^4*
c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^5*c^2) + (15*d^4*e^2*(-a^7*c^5)^
(1/2))/(32*a^6*c))) + (c^2*d^2*e^2*x*((9*d^4*(-a^7*c^5)^(1/2))/(256*a^7*c^
3) - (3*d^3*e)/(64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^7*c^5)^(1/2))/
(256*a^5*c^5) + (d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/((27*d^6*
(-a^7*c^5)^(1/2))/(32*a^6) - (d*e^5)/16 - (c*d^3*e^3)/(8*a) - (9*c^2*d^5*e
)/(16*a^2) + (e^6*(-a^7*c^5)^(1/2))/(32*a^3*c^3) + (5*d^2*e^4*(-a^7*c^5)^(
1/2))/(32*a^4*c^2) + (15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*a^5*c)))*((a^2*e^4*
(-a^7*c^5)^(1/2) + 9*c^2*d^4*(-a^7*c^5)^(1/2) - 12*a^4*c^4*d^3*e - 4*a^5*c
^3*d*e^3 + 2*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(256*a^7*c^5))^(1/2) - 2*atanh(
(9*c^3*d^4*x*(- (d*e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c) - (9*d^4*(-...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 957, normalized size of antiderivative = 3.84

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x^2+d)^2/(c*x^4+a)^2,x)
```

output

```
( - 4*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d*e - 4*c**(1/4)*a**(3/4)*sqrt(2)*at
an((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*
c**2*d*e*x**4 - 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2
) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*e**2 - 6*c**(3/4)*a**(1
/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1
/4)*sqrt(2)))*a*c*d**2 - 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/
4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*e**2*x**4 - 6*c
**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c
**(1/4)*a**(1/4)*sqrt(2)))*c**2*d**2*x**4 + 4*c**(1/4)*a**(3/4)*sqrt(2)*at
an((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*
a*c*d*e + 4*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*d*e*x**4 + 2*c**(3/4)*a**(1/4
)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4
)*sqrt(2)))*a**2*e**2 + 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4
)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d**2 + 2*c**(3/4
)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4
)*a**(1/4)*sqrt(2)))*a*c*e**2*x**4 + 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*d*
*2*x**4 + 2*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*...
```

3.320 $\int \frac{d+ex^2}{\left(a+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 207

$$\begin{aligned}\int \frac{d+ex^2}{\left(a+cx^4\right)^2}\,dx={}&\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}-\frac{\left(3\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}\\&+\frac{\left(3\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}\\&+\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}\end{aligned}$$

output

```
1/4*x*(e*x^2+d)/a/(c*x^4+a)+1/16*(3*c^(1/2)*d+a^(1/2)*e)*arctan(-1+2^(1/2)
*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(3/4)+1/16*(3*c^(1/2)*d+a^(1/2)*e)*a
rctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(3/4)+1/16*(3*c^(1/2)
*d-a^(1/2)*e)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(
1/2)/a^(7/4)/c^(3/4)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.29

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx$$
$$=\frac{\frac{8ax(d+ex^2)}{a+cx^4}-\frac{2\sqrt{2}\sqrt[4]{a}(3\sqrt{c}d+\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}}+\frac{2\sqrt{2}\sqrt[4]{a}(3\sqrt{c}d+\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}}+\frac{\sqrt{2}\left(-3\sqrt[4]{a}\sqrt{c}d+a^{3/4}e\right.}{}}{32a^2}$$

input
```
Integrate[(d + e*x^2)/(a + c*x^4)^2,x]
```

output
```
((8*a*x*(d + e*x^2))/(a + c*x^4) - (2*Sqrt[2]*a^(1/4)*(3*Sqrt[c]*d + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (2*Sqrt[2]*a^(1/4
)*(3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/
4) + (Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d - a
^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))
/(32*a^2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.28, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {1493, 25, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \text{1493}$$
$$\frac{x(d+ex^2)}{4a(a+cx^4)}-\frac{\int -\frac{ex^2+3d}{cx^4+a}dx}{4a}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ex^2+3d}{cx^4+a}dx}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 1482}$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}{cx^4+a}dx}{2c}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}{cx^4+a}dx}{2c}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}+\right.}{2\sqrt{c}}}{4a}$$

$$\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}}{4a}+$$

$$\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}}{4a}+$$

$$\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}}{4a}+$$

$$\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

input

```
Int[(d + e*x^2)/(a + c*x^4)^2,x]
```

output

```
(x*(d + e*x^2))/(4*a*(a + c*x^4)) + ((((3*Sqrt[c]*d)/Sqrt[a] + e)*(-(ArcTa
n[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 +
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[c]) + ((
(3*Sqrt[c]*d)/Sqrt[a] - e)*(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[c]))/(4*a)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\frac{e\,x^{3}}{4a}+\frac{dx}{4a}}{c\,x^{4}+a}+\frac{\sum_{\_R =\mathrm{RootOf}\left(c\,\_Z^{4}+a \right)}\frac{\left(\_R^{2} e +3 d \right) \ln \left(x -\_R \right)}{\_R^{3}}}{16 a c}$
default	$d \left(\frac{x}{4 a \left(c\,x^{4}+a \right)}+\frac{3 \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32 a^{2}}\right)+e \left(\frac{x^{3}}{4 a \left(c\,x^{4}+a\right.}\right.$

input

```
int((e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output

```
(1/4/a*e*x^3+1/4*d/a*x)/(c*x^4+a)+1/16/a/c*sum((_R^2*e+3*d)/_R^3*ln(x-_R),
_R=RootOf(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 873 vs. 2(146) = 292.

Time = 0.09 (sec) , antiderivative size = 873, normalized size of antiderivative = 4.22

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output

```
1/16*(4*e*x^3 - (a*c*x^4 + a^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^
2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^2*e^4)*
x + (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) +
27*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^
2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))) + (a*c*x^4 + a^2)*sqrt(-(a^
3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3
*c))*log(-(81*c^2*d^4 - a^2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c
*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 27*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-(a^
3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3
*c))) + (a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a
^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^2*e^4)*x + (a^6*
c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 27*a^2*c^
2*d^3 + 3*a^3*c*d*e^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^
2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))) - (a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(
81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))*log(-(
81*c^2*d^4 - a^2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 +
a^2*e^4)/(a^7*c^3)) - 27*a^2*c^2*d^3 + 3*a^3*c*d*e^2)*sqrt((a^3*c*sqrt(-(8
1*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))) + 4*d*
x)/(a*c*x^4 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.66

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx$$
$$= \operatorname{RootSum}\Bigg(65536t^4a^7c^3+3072t^2a^4c^2de+a^2e^4+18acd^2e^2+81c^2d^4, \Bigg(t\mapsto t\log\Bigg(x+\frac{4096t^3a^6c^2e+14}{a^2e^4}$$
$$+\frac{dx+ex^3}{4a^2+4acx^4}$$

input
```
integrate((e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*c**3 + 3072*_t**2*a**4*c**2*d*e + a**2*e**4 + 18*
a*c*d**2*e**2 + 81*c**2*d**4, Lambda(_t, _t*log(x + (4096*_t**3*a**6*c**2*
e + 144*_t*a**3*c*d*e**2 - 432*_t*a**2*c**2*d**3)/(a**2*e**4 - 81*c**2*d**
4)))) + (d*x + e*x**3)/(4*a**2 + 4*a*c*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.22

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx = \frac{ex^3+dx}{4\,(acx^4+a^2)}$$
$$+\frac{\frac{2\sqrt{2}(3\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}(3\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}(3\sqrt{c}d-\sqrt{a}e)\log\Big(\sqrt{c}x^2}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{32\,a}$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output

```
1/4*(e*x^3 + d*x)/(a*c*x^4 + a^2) + 1/32*(2*sqrt(2)*(3*sqrt(c)*d + sqrt(a)
*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3*sqrt(c)
*d + sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))
/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*
(3*sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sq
rt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x
^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.29

$$\begin{aligned}\int \frac{d+ex^2}{(a+cx^4)^2}\,dx = {} & \frac{ex^3+dx}{4\,(cx^4+a)a} + \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d+(ac^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} \\ & + \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d+(ac^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} \\ & + \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d-(ac^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} \\ & - \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d-(ac^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}\end{aligned}$$

input

```
integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output

```
1/4*(e*x^3 + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d +
(a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4
))/(a^2*c^3) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arct
an(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/32*s
qrt(2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)
^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c
^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.08

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx = \frac{\frac{e\,x^3}{4\,a}+\frac{d\,x}{4\,a}}{c\,x^4+a} - 2\,\mathrm{atanh}\left(\frac{c^2\,e^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}-\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}}}{2\,\left(\frac{c\,e^3}{32\,a}-\frac{9\,c^2\,d^2\,e}{32\,a^2}-\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^6}+\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^5}\right)} - \frac{9\,c^3\,d^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}-\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}}}{2\,\left(\frac{c\,e^3}{32}-\frac{9\,c^2\,d^2\,e}{32\,a}-\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^5}+\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^4}\right)}\right)\sqrt{-\frac{9\,c\,d^2\,\sqrt{-a^7\,c^3}-a\,e^2\,\sqrt{-a^7\,c^3}+6\,a^4\,c^2\,d\,e}{256\,a^7\,c^3}} - 2\,\mathrm{atanh}\left(\frac{c^2\,e^2\,x\,\sqrt{\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}-\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}}}{2\,\left(\frac{c\,e^3}{32\,a}-\frac{9\,c^2\,d^2\,e}{32\,a^2}+\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^6}-\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^5}\right)} - \frac{9\,c^3\,d^2\,x\,\sqrt{\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}-\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}}}{2\,\left(\frac{c\,e^3}{32}-\frac{9\,c^2\,d^2\,e}{32\,a}+\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^5}-\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^4}\right)}\right)\sqrt{-\frac{a\,e^2\,\sqrt{-a^7\,c^3}-9\,c\,d^2\,\sqrt{-a^7\,c^3}+6\,a^4\,c^2\,d\,e}{256\,a^7\,c^3}}$$

input
```
int((d + e*x^2)/(a + c*x^4)^2,x)
```

output
```
((e*x^3)/(4*a) + (d*x)/(4*a))/(a + c*x^4) - 2*atanh((c^2*e^2*x*((e^2*(-a^7
*c^3)^(1/2))/(256*a^6*c^3) - (9*d^2*(-a^7*c^3)^(1/2))/(256*a^7*c^2) - (3*d
*e)/(128*a^3*c))^(1/2))/(2*((c*e^3)/(32*a) - (9*c^2*d^2*e)/(32*a^2) - (27*
c*d^3*(-a^7*c^3)^(1/2))/(32*a^6) + (3*d*e^2*(-a^7*c^3)^(1/2))/(32*a^5))) -
 (9*c^3*d^2*x*((e^2*(-a^7*c^3)^(1/2))/(256*a^6*c^3) - (9*d^2*(-a^7*c^3)^(1
/2))/(256*a^7*c^2) - (3*d*e)/(128*a^3*c))^(1/2))/(2*((c*e^3)/32 - (9*c^2*d
^2*e)/(32*a) - (27*c*d^3*(-a^7*c^3)^(1/2))/(32*a^5) + (3*d*e^2*(-a^7*c^3)^
(1/2))/(32*a^4))))*(-(9*c*d^2*(-a^7*c^3)^(1/2) - a*e^2*(-a^7*c^3)^(1/2) +
6*a^4*c^2*d*e)/(256*a^7*c^3))^(1/2) - 2*atanh((c^2*e^2*x*((9*d^2*(-a^7*c^3
)^(1/2))/(256*a^7*c^2) - (3*d*e)/(128*a^3*c) - (e^2*(-a^7*c^3)^(1/2))/(256
*a^6*c^3))^(1/2))/(2*((c*e^3)/(32*a) - (9*c^2*d^2*e)/(32*a^2) + (27*c*d^3*
(-a^7*c^3)^(1/2))/(32*a^6) - (3*d*e^2*(-a^7*c^3)^(1/2))/(32*a^5))) - (9*c^
3*d^2*x*((9*d^2*(-a^7*c^3)^(1/2))/(256*a^7*c^2) - (3*d*e)/(128*a^3*c) - (e
^2*(-a^7*c^3)^(1/2))/(256*a^6*c^3))^(1/2))/(2*((c*e^3)/32 - (9*c^2*d^2*e)/
(32*a) + (27*c*d^3*(-a^7*c^3)^(1/2))/(32*a^5) - (3*d*e^2*(-a^7*c^3)^(1/2))
/(32*a^4))))*(-(a*e^2*(-a^7*c^3)^(1/2) - 9*c*d^2*(-a^7*c^3)^(1/2) + 6*a^4*
c^2*d*e)/(256*a^7*c^3))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.95

$$\int \frac{d+ex^2}{\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

input

```
int((e*x^2+d)/(c*x^4+a)^2,x)
```

output

```
( - 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*e - 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((
c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*e*
x**4 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d - 6*c**(3/4)*a**(1/4)*sqrt(2)*ata
n((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c
*d*x**4 + 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*e + 2*c**(1/4)*a**(3/4)*sqrt(2)*
atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))
)*c*e*x**4 + 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) +
 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d + 6*c**(3/4)*a**(1/4)*sqrt(
2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(
2)))*c*d*x**4 + c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)
*x + sqrt(a) + sqrt(c)*x**2)*a*e + c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/
4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*e*x**4 - c**(1/4)*a**(3/
4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*e -
 c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqr
t(c)*x**2)*c*e*x**4 - 3*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)
*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*d - 3*c**(3/4)*a**(1/4)*sqrt(2)*log
( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*d*x**4 + 3*...
```

3.321 $\int \frac{1}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 151

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4a\,(a+cx^4)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

output

```
1/4*x/a/(c*x^4+a)+3/16*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4
)/c^(1/4)+3/16*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(1/4)
+3/16*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(
7/4)/c^(1/4)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx$$

$$=\frac{\frac{8a^{3/4}x}{a+cx^4}-\frac{6\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{6\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}-\frac{3\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}}+\frac{3\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{\sqrt[4]{c}}}{32a^{7/4}}$$

input
```
Integrate[(a + c*x^4)^(-2),x]
```

output
```
((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/
4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^
(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/c^(1/4))/(32*a^(7/4))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.49, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx$$

$$\Big\downarrow \text{749}$$

$$\frac{3\int \frac{1}{cx^4+a}dx}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1476$$

$$\frac{3\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+$$

$$\frac{x}{4a\left(a+cx^4\right)}$$

input
```
Int[(a + c*x^4)^(-2),x]
```

output
```
x/(4*a*(a + c*x^4)) + (3*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt
[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a
^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])))/(4*a
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.30

method	result	size
risch	$\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16ac}$	46
default	$\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}$	118

input
```
int(1/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a/(c*x^4+a)+3/16/a/c*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.21

$$\int\frac{1}{(a+cx^4)^2}\,dx$$
$$=\frac{3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\left(-i\,acx^4-i\,a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(i\,a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\,(i\,ac}{16\,(acx^4+}$$

input
```
integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x)
 - 3*(-I*a*c*x^4 - I*a^2)*(-1/(a^7*c))^(1/4)*log(I*a^2*(-1/(a^7*c))^(1/4)
+ x) - 3*(I*a*c*x^4 + I*a^2)*(-1/(a^7*c))^(1/4)*log(-I*a^2*(-1/(a^7*c))^(1
/4) + x) - 3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4
) + x) + 4*x)/(a*c*x^4 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.26

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4a^2+4acx^4} + \text{RootSum}\left(65536t^4a^7c+81, \left(t \mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

input
```
integrate(1/(c*x**4+a)**2,x)
```

output
```
x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*l
og(16*_t*a**2/3 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,(acx^4+a^2)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2\right.}{}\right.}{32\,a}$$

input
```
integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sq
rt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(
2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)
))/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,(cx^4+a)a} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c}$$

input

```
integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

output

```
1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32
*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c
) - 3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c)
)/(a^2*c)
```

Mupad [B] (verification not implemented)

Time = 17.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.38

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,a\,\left(c\,x^4+a\right)} + \frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{{(-a)}^{1/4}}\right)}{8\,{(-a)}^{7/4}\,c^{1/4}} + \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{{(-a)}^{1/4}}\right)}{8\,{(-a)}^{7/4}\,c^{1/4}}$$

input

```
int(1/(a + c*x^4)^2,x)
```

output
```
x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/
4)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.02

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx$$
$$= \frac{-6c^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-6c^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^4+6c^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+}{}$$

input
```
int(1/(c*x^4+a)^2,x)
```

output
```
( - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c*
*(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
 + 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)
*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a + 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
- 3*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
 + sqrt(c)*x**2)*a - 3*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*
sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 + 3*c**(3/4)*a**(1/4)*sqrt(2)*l
og(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a + 3*c**(3/4)*a*
*(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c
*x**4 + 8*a*c*x)/(32*a**2*c*(a + c*x**4))
```

3.322 $\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 371

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx &= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2} \\ &\quad + \frac{\sqrt[4]{c}\left(\sqrt{a}e(cd^2+5ae^2)-\sqrt{c}d(3cd^2+7ae^2)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^2+ae^2)^2} \\ &\quad - \frac{\sqrt[4]{c}\left(\sqrt{a}e(cd^2+5ae^2)-\sqrt{c}d(3cd^2+7ae^2)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^2+ae^2)^2} \\ &\quad + \frac{\sqrt[4]{c}\left(\sqrt{a}e(cd^2+5ae^2)+\sqrt{c}d(3cd^2+7ae^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}\,(cd^2+ae^2)^2}\end{aligned}$$

output
```
1/4*c*x*(-e*x^2+d)/a/(a*e^2+c*d^2)/(c*x^4+a)+e^(7/2)*arctan(e^(1/2)*x/d^(1
/2))/d^(1/2)/(a*e^2+c*d^2)^2-1/16*c^(1/4)*(a^(1/2)*e*(5*a*e^2+c*d^2)-c^(1/
2)*d*(7*a*e^2+3*c*d^2))*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/
4)/(a*e^2+c*d^2)^2-1/16*c^(1/4)*(a^(1/2)*e*(5*a*e^2+c*d^2)-c^(1/2)*d*(7*a*
e^2+3*c*d^2))*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/(a*e^2+c
*d^2)^2+1/16*c^(1/4)*(a^(1/2)*e*(5*a*e^2+c*d^2)+c^(1/2)*d*(7*a*e^2+3*c*d^2
))*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(7/4
)/(a*e^2+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.16

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$$

$$=\frac{\frac{8c(cd^2+ae^2)x(d-ex^2)}{a(a+cx^4)}+\frac{32e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\sqrt[4]{c}(-3c^{3/2}d^3+\sqrt{a}cd^2e-7a\sqrt{c}de^2+5a^{3/2}e^3)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}}-\frac{2\sqrt{2}\sqrt[4]{c}(-}{}}{}$$

input
```
Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2)*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[
a]*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^
2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/a^(7/4) - (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*
Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + S
qrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e +
 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 689, normalized size of antiderivative = 1.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1568, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2(d+ex^2)}dx$$

$$\Big\downarrow \ 1568$$

$$\int \left(-\frac{ce^2(ex^2-d)}{(a+cx^4)(ae^2+cd^2)^2}+\frac{c(d-ex^2)}{(a+cx^4)^2(ae^2+cd^2)}+\frac{e^4}{(d+ex^2)(ae^2+cd^2)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sqrt[4]{c}e^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2}+\frac{\sqrt[4]{c}e^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2}-\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(3\sqrt{c}d-\sqrt{a}e)}{8\sqrt{2}a^{7/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(3\sqrt{c}d-\sqrt{a}e)}{8\sqrt{2}a^{7/4}(ae^2+cd^2)}-\frac{\sqrt[4]{c}e^2(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2}+\frac{\sqrt[4]{c}e^2(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2}-\frac{\sqrt[4]{c}(\sqrt{a}e+3\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}(\sqrt{a}e+3\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(ae^2+cd^2)}+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(ae^2+cd^2)^2}+\frac{cx(d-ex^2)}{4a(a+cx^4)(ae^2+cd^2)}$$

input

```
Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d -
Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*
d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a
^(3/4)*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/
4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + S
qrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d
 + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*
Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(
c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 1568
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.90

method	result
default	$\frac{e^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(a e^2+c d^2)^2\sqrt{de}}+\frac{c\Bigg(\frac{-\frac{e(a e^2+c d^2)x^3}{4a}+\frac{d(a e^2+c d^2)x}{4a}}{c x^4+a}+\frac{\frac{(7d e^2 a+3d^3 c)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\Bigg(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{8a}}{}}{(a}$
risch	Expression too large to display

input

```
int(1/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output

```
e^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+c/(a*e^2+c*d^2)^2*
((-1/4*e*(a*e^2+c*d^2)/a*x^3+1/4*d*(a*e^2+c*d^2)/a*x)/(c*x^4+a)+1/4/a*(1/8
*(7*a*d*e^2+3*c*d^3)*(1/c*a)^(1/4)/a*2^(1/2)*(ln((x^2+(1/c*a)^(1/4)*x*2^(1
/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+2*arctan(2
^(1/2)/(1/c*a)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1))+1/8*(-5*a*e
^3-c*d^2*e)/c/(1/c*a)^(1/4)*2^(1/2)*(ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*
a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+2*arctan(2^(1/2)/(1
/c*a)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. $2(303) = 606$.

Time = 11.08 (sec) , antiderivative size = 9892, normalized size of antiderivative = 26.66

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input

```
integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output

```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

input

```
integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

output

```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(303) = 606$.

Time = 0.13 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.67

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
e^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e))
+ 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/
4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + s
qrt(2)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2
*d*e^2 - (a*c^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)
*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 +
7*(a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^
3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*s
qrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*(3*(a*c^3)^(1/4)*c^3*
d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4
)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4
 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/4*(c*e*x^3 - c*d*x
)/((c*x^4 + a)*(a*c*d^2 + a^2*e^2))
```

Mupad [B] (verification not implemented)

Time = 19.52 (sec) , antiderivative size = 17945, normalized size of antiderivative = 48.37

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
((c*d*x)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x
^4) - atan(((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5
*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*
a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(25
6*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2
*d^4*e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6
*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-
a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d
^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65
536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 5
89824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11
 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((
9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 4
4*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*
a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*
e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d
^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^
9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3
*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1438, normalized size of antiderivative = 3.88

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x^2+d)/(c*x^4+a)^2,x)
```

output

```
(10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*d*e**3 + 2*c**(1/4)*a**(3/4)*sqrt(2)*
atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))
)*a*c*d**3*e + 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2
) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d*e**3*x**4 + 2*c**(1/4)
*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)
*a**(1/4)*sqrt(2)))*c**2*d**3*e*x**4 - 14*c**(3/4)*a**(1/4)*sqrt(2)*atan((
c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2
*d**2*e**2 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d**4 - 14*c**(3/4)*a**(1/4)
*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)
*sqrt(2)))*a*c*d**2*e**2*x**4 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)
*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*d**4*x*
*4 - 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt
(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*d*e**3 - 2*c**(1/4)*a**(3/4)*sqrt
(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt
(2)))*a*c*d**3*e - 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sq
rt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d*e**3*x**4 - 2*c**(
1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(
1/4)*a**(1/4)*sqrt(2)))*c**2*d**3*e*x**4 + 14*c**(3/4)*a**(1/4)*sqrt(2)...
```

3.323 $\int \frac{1}{(d+ex^2)^2(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 492

$$
\begin{aligned}
\int \frac{1}{(d+ex^2)^2(a+cx^4)^2}\,dx = {} & -\frac{e^2(cd^2-ae^2)\,x}{2ad\,(cd^2+ae^2)^2\,(d+ex^2)} \\
& + \frac{cx(d-ex^2)}{4a\,(cd^2+ae^2)\,(d+ex^2)\,(a+cx^4)} + \frac{e^{7/2}(9cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\,(cd^2+ae^2)^3} \\
& - \frac{c^{3/4}\left(3c^2d^4-2\sqrt{a}c^{3/2}d^3e+12acd^2e^2-18a^{3/2}\sqrt{c}de^3-7a^2e^4\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^2+ae^2)^3} \\
& + \frac{c^{3/4}\left(3c^2d^4-2\sqrt{a}c^{3/2}d^3e+12acd^2e^2-18a^{3/2}\sqrt{c}de^3-7a^2e^4\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^2+ae^2)^3} \\
& + \frac{c^{3/4}\left(3c^2d^4+12acd^2e^2-7a^2e^4+2\sqrt{a}\sqrt{c}de(cd^2+9ae^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}\,(cd^2+ae^2)^3}
\end{aligned}
$$

output

```
-1/2*e^2*(-a*e^2+c*d^2)*x/a/d/(a*e^2+c*d^2)^2/(e*x^2+d)+1/4*c*x*(-e*x^2+d)
/a/(a*e^2+c*d^2)/(e*x^2+d)/(c*x^4+a)+1/2*e^(7/2)*(a*e^2+9*c*d^2)*arctan(e^
(1/2)*x/d^(1/2))/d^(3/2)/(a*e^2+c*d^2)^3+1/16*c^(3/4)*(3*c^2*d^4-2*a^(1/2)
*c^(3/2)*d^3*e+12*a*c*d^2*e^2-18*a^(3/2)*c^(1/2)*d*e^3-7*a^2*e^4)*arctan(-
1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/(a*e^2+c*d^2)^3+1/16*c^(3/4)*
(3*c^2*d^4-2*a^(1/2)*c^(3/2)*d^3*e+12*a*c*d^2*e^2-18*a^(3/2)*c^(1/2)*d*e^3
-7*a^2*e^4)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/(a*e^2+c*d
^2)^3+1/16*c^(3/4)*(3*c^2*d^4+12*a*c*d^2*e^2-7*a^2*e^4+2*a^(1/2)*c^(1/2)*d
*e*(9*a*e^2+c*d^2))*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2
))*2^(1/2)/a^(7/4)/(a*e^2+c*d^2)^3
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx$$

$$= \frac{\frac{16e^4(cd^2+ae^2)x}{d(d+ex^2)} + \frac{8c(cd^2+ae^2)x(-ae^2+cd(d-2ex^2))}{a(a+cx^4)} + \frac{16e^{7/2}(9cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2\sqrt{2}c^{3/4}\left(-3c^2d^4+2\sqrt{a}c^{3/2}d^3e-12acd^2e^2-\right.}{}}{}$$

input

```
Integrate[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

output

```
((16*e^4*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (8*c*(c*d^2 + a*e^2)*x*(-(a*
e^2) + c*d*(d - 2*e*x^2)))/(a*(a + c*x^4)) + (16*e^(7/2)*(9*c*d^2 + a*e^2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-3*c^2*d^4 + 2
*Sqrt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 + 7*a^2
*e^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(3/4
)*(-3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt
[c]*d*e^3 + 7*a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) -
(Sqrt[2]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e + 12*a*c*d^2*e^2 + 1
8*a^(3/2)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/
2)*d^3*e + 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^
2)^3)
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 864, normalized size of antiderivative = 1.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1568, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2 (d+ex^2)^2}dx$$

$$\Big\downarrow \text{1568}$$

$$\int \left(-\frac{ce^2 \left(ae^2-3cd^2+4cdex^2\right)}{(a+cx^4) \left(ae^2+cd^2\right)^3}+\frac{c\left(-ae^2+cd^2-2cdex^2\right)}{(a+cx^4)^2 \left(ae^2+cd^2\right)^2}+\frac{4cde^4}{(d+ex^2) \left(ae^2+cd^2\right)^3}+\frac{e^4}{(d+ex^2)^2 \left(ae^2+cd^2\right)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{xe^4}{2d \left(cd^2+ae^2\right)^2 (ex^2+d)}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) e^{7/2}}{2d^{3/2} \left(cd^2+ae^2\right)^2}+\frac{4c\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) e^{7/2}}{\left(cd^2+ae^2\right)^3}-$$

$$\frac{c^{3/4}\left(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) e^2}{2\sqrt{2}a^{3/4} \left(cd^2+ae^2\right)^3}+$$

$$\frac{c^{3/4}\left(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right) e^2}{2\sqrt{2}a^{3/4} \left(cd^2+ae^2\right)^3}-$$

$$\frac{c^{3/4}\left(3cd^2+4\sqrt{a}\sqrt{c}ed-ae^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right) e^2}{4\sqrt{2}a^{3/4} \left(cd^2+ae^2\right)^3}+$$

$$\frac{c^{3/4}\left(3cd^2+4\sqrt{a}\sqrt{c}ed-ae^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right) e^2}{4\sqrt{2}a^{3/4} \left(cd^2+ae^2\right)^3}-$$

$$\frac{c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4} \left(cd^2+ae^2\right)^2}+$$

$$\frac{c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4} \left(cd^2+ae^2\right)^2}-$$

$$\frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}ed-3ae^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4} \left(cd^2+ae^2\right)^2}+$$

$$\frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}ed-3ae^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4} \left(cd^2+ae^2\right)^2}+\frac{cx\left(cd^2-2cex^2d-ae^2\right)}{4a \left(cd^2+ae^2\right)^2 (cx^4+a)}$$

input
```
Int[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

output
```
(e^4*x)/(2*d*(c*d^2 + a*e^2)^2*(d + e*x^2)) + (c*x*(c*d^2 - a*e^2 - 2*c*d*
e*x^2))/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^4)) + (4*c*Sqrt[d]*e^(7/2)*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^3 + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(2*d^(3/2)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sq
rt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*
e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 +
a*e^2)^2) + (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[
1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) +
(c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(
3*c*d^2 + 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3
*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*
e^2*(3*c*d^2 + 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/
4)*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2)
```

Defintions of rubi rules used

rule 1568
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.82

method	result
default	$\frac{e^{4}\left(\frac{\left(a\,e^{2}+c\,d^{2}\right)x}{2d\left(e\,x^{2}+d\right)}+\frac{\left(a\,e^{2}+9c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{3}}-\frac{c\left(\frac{\frac{ecd\left(a\,e^{2}+c\,d^{2}\right)x^{3}}{2a}+\frac{\left(a^{2}e^{4}-c^{2}d^{4}\right)x}{4a}}{c\,x^{4}+a}+\frac{\left(7a^{2}e^{4}-12ac\,d^{2}e^{2}-3c^{2}d^{4}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\right.\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(1/(e*x^2+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
e^4/(a*e^2+c*d^2)^3*(1/2*(a*e^2+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+9*c*d^2)/d
/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-c/(a*e^2+c*d^2)^3*((1/2*e*c*d*(a*e^2
+c*d^2)/a*x^3+1/4*(a^2*e^4-c^2*d^4)/a*x)/(c*x^4+a)+1/4/a*(1/8*(7*a^2*e^4-1
2*a*c*d^2*e^2-3*c^2*d^4)*(1/c*a)^(1/4)/a*2^(1/2)*(ln((x^2+(1/c*a)^(1/4)*x*
2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+2*arct
an(2^(1/2)/(1/c*a)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1))+1/8*(18
*a*c*d*e^3+2*c^2*d^3*e)/c/(1/c*a)^(1/4)*2^(1/2)*(ln((x^2-(1/c*a)^(1/4)*x*2
^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+2*arcta
n(2^(1/2)/(1/c*a)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7634 vs. 2(412) = 824.

Time = 100.17 (sec) , antiderivative size = 15292, normalized size of antiderivative = 31.08

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**2/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. 2(412) = 824.

Time = 0.16 (sec) , antiderivative size = 879, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

input

```
integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

output

```
1/8*(3*(a*c^3)^(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 7*(a*c^3)^
(1/4)*a^2*c*e^4 - 2*(a*c^3)^(3/4)*c*d^3*e - 18*(a*c^3)^(3/4)*a*d*e^3)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d
^6 + 3*sqrt(2)*a^3*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c
*e^6) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 7*
(a*c^3)^(1/4)*a^2*c*e^4 - 2*(a*c^3)^(3/4)*c*d^3*e - 18*(a*c^3)^(3/4)*a*d*e
^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a
^2*c^4*d^6 + 3*sqrt(2)*a^3*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(
2)*a^5*c*e^6) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2
*e^2 - 7*(a*c^3)^(1/4)*a^2*c*e^4 + 2*(a*c^3)^(3/4)*c*d^3*e + 18*(a*c^3)^(3
/4)*a*d*e^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4
*d^6 + 3*sqrt(2)*a^3*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5
*c*e^6) - 1/16*(3*(a*c^3)^(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 -
 7*(a*c^3)^(1/4)*a^2*c*e^4 + 2*(a*c^3)^(3/4)*c*d^3*e + 18*(a*c^3)^(3/4)*a*
d*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^6 +
 3*sqrt(2)*a^3*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c*e^6
) + 1/2*(9*c*d^2*e^4 + a*e^6)*arctan(e*x/sqrt(d*e))/((c^3*d^7 + 3*a*c^2*d^
5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*sqrt(d*e)) - 1/4*(2*c^2*d^2*e^2*x^5 -
 2*a*c*e^4*x^5 + c^2*d^3*e*x^3 + a*c*d*e^3*x^3 - c^2*d^4*x + a*c*d^2*e^2*x
 - 2*a^2*e^4*x)/((a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4)*(c*e*x^6 + c...
```

Mupad [B] (verification not implemented)

Time = 21.29 (sec) , antiderivative size = 28923, normalized size of antiderivative = 58.79

$$\int \frac{1}{(d+ex^2)^2 (a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^2*(d + e*x^2)^2),x)
```

output
```
((x*(2*a^2*e^4 + c^2*d^4 - a*c*d^2*e^2))/(4*a*d*(a^2*e^4 + c^2*d^4 + 2*a*c
*d^2*e^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)) + (c*e^2*x^5*(a*e^2 - c*d^2))
/(2*a*d*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a*d + a*e*x^2 + c*d*x^4 + c
*e*x^6) + atan(((((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c
^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 128243
2*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15
- 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2*e^19)/(512*(a^4*c^8*d^18 +
a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e
^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*
a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e^24 - 24576*a^4*c^15*d^23*e^2 -
 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*d^19*e^6 + 1966080*a^7*c^12*d^
17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 25460736*a^9*c^10*d^13*e^12 + 34750
464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9*e^16 + 16588800*a^12*c^7*d^
7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^14*c^5*d^3*e^22)/(512*(a^4*c
^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*
c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*
e^10 + 28*a^10*c^2*d^6*e^12)) - (x*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*
d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*
d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^
3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(25...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3860, normalized size of antiderivative = 7.85

$$\int \frac{1}{(d+ex^2)^2 (a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(e*x^2+d)^2/(c*x^4+a)^2,x)
```

output

```
(36*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*c*d**4*e**3 + 36*c**(1/4)*a**(3/4)*sq
rt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sq
rt(2)))*a**2*c*d**3*e**4*x**2 + 4*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)
*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c**2*d**6*
e + 4*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c**2*d**5*e**2*x**2 + 36*c**(1/4)*a**(
3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(
1/4)*sqrt(2)))*a*c**2*d**4*e**3*x**4 + 36*c**(1/4)*a**(3/4)*sqrt(2)*atan((
c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*
*2*d**3*e**4*x**6 + 4*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**3*d**6*e*x**4 + 4*c**
(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**
(1/4)*a**(1/4)*sqrt(2)))*c**3*d**5*e**2*x**6 + 14*c**(3/4)*a**(1/4)*sqrt(2
)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2
)))*a**3*d**3*e**4 + 14*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**3*d**2*e**5*x**2 -
24*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x
)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*c*d**5*e**2 - 24*c**(3/4)*a**(1/4)*sqr
t(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*...
```

3.324 $\int (d+ex^2)^{3/2}(a-cx^4)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 146

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)\,dx = \frac{3}{128}d\left(16a-\frac{cd^2}{e^2}\right)x\sqrt{d+ex^2} + \frac{1}{64}\left(16a-\frac{cd^2}{e^2}\right)x\left(d+ex^2\right)^{3/2} + \frac{cdx(d+ex^2)^{5/2}}{16e^2} - \frac{cx^3(d+ex^2)^{5/2}}{8e} - \frac{3d^2(cd^2-16ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{128e^{5/2}}$$

output

```
3/128*d*(16*a-c*d^2/e^2)*x*(e*x^2+d)^(1/2)+1/64*(16*a-c*d^2/e^2)*x*(e*x^2+
d)^(3/2)+1/16*c*d*x*(e*x^2+d)^(5/2)/e^2-1/8*c*x^3*(e*x^2+d)^(5/2)/e-3/128*
d^2*(-16*a*e^2+c*d^2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.81

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)\,dx = -\frac{x\sqrt{d+ex^2}(-3cd^3-80ade^2+2cd^2ex^2-32ae^3x^2+24cde^2x^4+16ce^3x^6)}{128e^2} - \frac{3(-cd^4+16ad^2e^2)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{128e^{5/2}}$$

input
```
Integrate[(d + e*x^2)^(3/2)*(a - c*x^4),x]
```

output
```
-1/128*(x*Sqrt[d + e*x^2]*(-3*c*d^3 - 80*a*d*e^2 + 2*c*d^2*e*x^2 - 32*a*e^
3*x^2 + 24*c*d*e^2*x^4 + 16*c*e^3*x^6))/e^2 - (3*(-(c*d^4) + 16*a*d^2*e^2)
*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(128*e^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1474, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right) \left(d+ex^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1474}$$

$$\frac{\int \left(3cdx^2+8ae\right) \left(ex^2+d\right)^{3/2}dx}{8e}-\frac{cx^3 \left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{cdx\left(d+ex^2\right)^{5/2}}{2e}-\frac{\left(cd^2-16ae^2\right)\int \left(ex^2+d\right)^{3/2}dx}{2e}}{8e}-\frac{cx^3 \left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{cdx\left(d+ex^2\right)^{5/2}}{2e}-\frac{\left(cd^2-16ae^2\right)\left(\frac{3}{4}d\int \sqrt{ex^2+d}dx+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{2e}}{8e}-\frac{cx^3 \left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{cdx\left(d+ex^2\right)^{5/2}}{2e}-\frac{\left(cd^2-16ae^2\right)\left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{2e}}{8e}-\frac{cx^3 \left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{cdx(d+ex^2)^{5/2}}{2e}-\frac{(cd^2-16ae^2)\left(\frac{3}{4}d\left(\frac{1}{2}d\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x(d+ex^2)^{3/2}\right)}{2e}}{8e}-\frac{cx^3(d+ex^2)^{5/2}}{8e}$$

$$\Big\downarrow 219$$

$$\frac{\frac{cdx(d+ex^2)^{5/2}}{2e}-\frac{(cd^2-16ae^2)\left(\frac{3}{4}d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x(d+ex^2)^{3/2}\right)}{2e}}{8e}-\frac{cx^3(d+ex^2)^{5/2}}{8e}$$

input

```
Int[(d + e*x^2)^(3/2)*(a - c*x^4),x]
```

output

```
-1/8*(c*x^3*(d + e*x^2)^(5/2))/e + ((c*d*x*(d + e*x^2)^(5/2))/(2*e) - ((c*
d^2 - 16*a*e^2)*((x*(d + e*x^2)^(3/2))/4 + (3*d*((x*Sqrt[d + e*x^2])/2 + (
d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/4))/(2*e))/(8*e)
```

Defintions of rubi rules used

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{\frac{5\left(\frac{3}{5} a\, d^{2} e^{2}-\frac{3}{80} d^{4} c\right) \operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)}{8}+\frac{5 x\sqrt{e\,x^{2}+d}\left(d\left(-\frac{3 c\,x^{4}}{10}+a\right) e^{\frac{5}{2}}+\frac{2 x^{2}\left(-\frac{c\,x^{4}}{2}+a\right) e^{\frac{7}{2}}}{5}+\frac{3 d^{2} c\left(-\frac{2 e^{\frac{3}{2}} x^{2}}{3}+\sqrt{e}\, d\right)}{80}\right)}{8}}{e^{\frac{5}{2}}}$
risch	$\frac{x\left(-16 e^{3} c\,x^{6}-24 c d\,e^{2} x^{4}+32 a\,e^{3} x^{2}-2 c\,d^{2} e\,x^{2}+80 d\,e^{2} a+3 d^{3} c\right)\sqrt{e\,x^{2}+d}}{128 e^{2}}+\frac{3 d^{2}\left(16 a\,e^{2}-c\,d^{2}\right) \ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{128 e^{\frac{5}{2}}}$
default	$a\left(\frac{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{4}+\frac{3 d\left(\frac{x\sqrt{e\,x^{2}+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2\sqrt{e}}\right)}{4}\right)-c\left(\frac{x^{3}\left(e\,x^{2}+d\right)^{\frac{5}{2}}}{8 e}-\frac{3 d\left(\frac{x\left(e\,x^{2}+d\right)^{\frac{5}{2}}}{6 e}-\frac{d\left(\frac{x\left(e\,x^{2}+d\right)}{4}\right.}{}\right.}{}\right.$

input
```
int((e*x^2+d)^(3/2)*(-c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
5/8/e^(5/2)*((3/5*a*d^2*e^2-3/80*d^4*c)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))
+x*(e*x^2+d)^(1/2)*(d*(-3/10*c*x^4+a)*e^(5/2)+2/5*x^2*(-1/2*c*x^4+a)*e^(7/
2)+3/80*d^2*c*(-2/3*e^(3/2)*x^2+e^(1/2)*d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.62

$$\int (d+ex^2)^{3/2} (a -cx^4)\ dx = \left[-\frac{3\,(cd^4-16\,ad^2e^2)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\,(16\,ce^4x^7+24\,cde^3x^5+2\,(cd}{256\,e^3} \right.$$

input
```
integrate((e*x^2+d)^(3/2)*(-c*x^4+a),x, algorithm="fricas")
```

output
```
[-1/256*(3*(c*d^4 - 16*a*d^2*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)
*sqrt(e)*x - d) + 2*(16*c*e^4*x^7 + 24*c*d*e^3*x^5 + 2*(c*d^2*e^2 - 16*a*e
^4)*x^3 - (3*c*d^3*e + 80*a*d*e^3)*x)*sqrt(e*x^2 + d))/e^3, 1/128*(3*(c*d^
4 - 16*a*d^2*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (16*c*e^4*
x^7 + 24*c*d*e^3*x^5 + 2*(c*d^2*e^2 - 16*a*e^4)*x^3 - (3*c*d^3*e + 80*a*d*
e^3)*x)*sqrt(e*x^2 + d))/e^3]
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int (d+ex^2)^{3/2} (a -cx^4)\ dx = \begin{cases} \sqrt{d+ex^2}\left(-\frac{3cdx^5}{16}-\frac{cex^7}{8}+\frac{x^3\left(ae^2-\frac{cd^2}{16}\right)}{4e}+\frac{x\left(2ade-\frac{3d\left(ae^2-\frac{cd^2}{16}\right)}{4e}\right)}{2e}\right)+\left(ad^2-\frac{d\left(2ade-\frac{3d\left(ae^2-\frac{cd^2}{16}\right.}{4e}\right.}{2e}\right. \\ d^{\frac{3}{2}}\left(ax-\frac{cx^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**(3/2)*(-c*x**4+a),x)
```

output
```
Piecewise((sqrt(d + e*x**2)*(-3*c*d*x**5/16 - c*e*x**7/8 + x**3*(a*e**2 -
c*d**2/16)/(4*e) + x*(2*a*d*e - 3*d*(a*e**2 - c*d**2/16)/(4*e))/(2*e)) + (
a*d**2 - d*(2*a*d*e - 3*d*(a*e**2 - c*d**2/16)/(4*e))/(2*e))*Piecewise((lo
g(2*sqrt(e)*sqrt(d + e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqrt(e
*x**2), True)), Ne(e, 0)), (d**(3/2)*(a*x - c*x**5/5), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(-c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)\,dx=$$
$$-\frac{1}{128}\left(2\left(4\left(2\,cex^2+3\,cd\right)x^2+\frac{cd^2e^5-16\,ae^7}{e^6}\right)x^2-\frac{3\,cd^3e^4+80\,ade^6}{e^6}\right)\sqrt{ex^2+d}\,x$$
$$+\frac{3\left(cd^4-16\,ad^2e^2\right)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{128\,e^{\frac{5}{2}}}$$

input
```
integrate((e*x^2+d)^(3/2)*(-c*x^4+a),x, algorithm="giac")
```

output
```
-1/128*(2*(4*(2*c*e*x^2 + 3*c*d)*x^2 + (c*d^2*e^5 - 16*a*e^7)/e^6)*x^2 - (
3*c*d^3*e^4 + 80*a*d*e^6)/e^6)*sqrt(e*x^2 + d)*x + 3/128*(c*d^4 - 16*a*d^2
*e^2)*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)\,dx=\int \left(a-c\,x^4\right)\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int((a - c*x^4)*(d + e*x^2)^(3/2),x)
```

output
```
int((a - c*x^4)*(d + e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.14

$$\int \left(d+ex^2\right)^{3/2}(a$$

$$-cx^4)\,dx=\frac{80\sqrt{e\,x^2+d}\,a d\,e^3x+32\sqrt{e\,x^2+d}\,a\,e^4x^3+3\sqrt{e\,x^2+d}\,c\,d^3ex-2\sqrt{e\,x^2+d}\,c\,d^2e^2x^3-24\sqrt{e\,x}}{1}$$

input
```
int((e*x^2+d)^(3/2)*(-c*x^4+a),x)
```

output
```
(80*sqrt(d + e*x**2)*a*d*e**3*x + 32*sqrt(d + e*x**2)*a*e**4*x**3 + 3*sqrt
(d + e*x**2)*c*d**3*e*x - 2*sqrt(d + e*x**2)*c*d**2*e**2*x**3 - 24*sqrt(d
+ e*x**2)*c*d*e**3*x**5 - 16*sqrt(d + e*x**2)*c*e**4*x**7 + 48*sqrt(e)*log
((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d**2*e**2 - 3*sqrt(e)*log((sqrt
(d + e*x**2) + sqrt(e)*x)/sqrt(d))*c*d**4)/(128*e**3)
```

3.325 $\int \sqrt{d+ex^2}\left(a-cx^4\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)\,dx=\frac{1}{16}\left(8a-\frac{cd^2}{e^2}\right)x\sqrt{d+ex^2}+\frac{cdx\left(d+ex^2\right)^{3/2}}{8e^2}-\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}-\frac{d\left(cd^2-8ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16e^{5/2}}$$

output
```
1/16*(8*a-c*d^2/e^2)*x*(e*x^2+d)^(1/2)+1/8*c*d*x*(e*x^2+d)^(3/2)/e^2-1/6*c
*x^3*(e*x^2+d)^(3/2)/e-1/16*d*(-8*a*e^2+c*d^2)*arctanh(e^(1/2)*x/(e*x^2+d)
^(1/2))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)\,dx=-\frac{x\sqrt{d+ex^2}\left(-3cd^2-24ae^2+2cdex^2+8ce^2x^4\right)}{48e^2}+\frac{\left(cd^3-8ade^2\right)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{16e^{5/2}}$$

input
```
Integrate[Sqrt[d + e*x^2]*(a - c*x^4),x]
```

output
```
-1/48*(x*Sqrt[d + e*x^2]*(-3*c*d^2 - 24*a*e^2 + 2*c*d*e*x^2 + 8*c*e^2*x^4)
)/e^2 + ((c*d^3 - 8*a*d*e^2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(16*e^(5
/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1474, 27, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)\sqrt{d+ex^2}dx$$

$$\Big\downarrow \text{1474}$$

$$\frac{\int 3\left(cdx^2+2ae\right)\sqrt{ex^2+d}dx}{6e}-\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \left(cdx^2+2ae\right)\sqrt{ex^2+d}dx}{2e}-\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{cdx\left(d+ex^2\right)^{3/2}}{4e}-\frac{\left(cd^2-8ae^2\right)\int \sqrt{ex^2+d}dx}{4e}}{2e}-\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{cdx\left(d+ex^2\right)^{3/2}}{4e}-\frac{\left(cd^2-8ae^2\right)\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}}{2e}-\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{cdx\left(d+ex^2\right)^{3/2}}{4e}-\frac{\left(cd^2-8ae^2\right)\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}}{2e}-\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{cdx\left(d+ex^2\right)^{3/2}}{4e}-\frac{\left(cd^2-8ae^2\right)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}}{2e}-\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

input

```
Int[Sqrt[d + e*x^2]*(a - c*x^4),x]
```

output

```
-1/6*(c*x^3*(d + e*x^2)^(3/2))/e + ((c*d*x*(d + e*x^2)^(3/2))/(4*e) - ((c*
d^2 - 8*a*e^2)*((x*Sqrt[d + e*x^2])/2 + (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*
x^2]])/(2*Sqrt[e])))/(4*e))/(2*e)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{d\left(a e^2-\frac{c d^2}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{e x^2+d}}{x\sqrt{e}}\right)+x\sqrt{e x^2+d}\left(\left(-\frac{c x^4}{3}+a\right)e^{\frac{5}{2}}+\frac{dc\left(-\frac{2e^{\frac{3}{2}}x^2}{3}+\sqrt{e}\,d\right)}{8}\right)}{2e^{\frac{5}{2}}}$
risch	$\frac{x\left(-8c x^4 e^2-2cd x^2 e+24a e^2+3c d^2\right)\sqrt{e x^2+d}}{48e^2}+\frac{d\left(8a e^2-c d^2\right)\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{16e^{\frac{5}{2}}}$
default	$a\left(\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}\right)-c\left(\frac{x^3\left(e x^2+d\right)^{\frac{3}{2}}}{6e}-\frac{d\left(\frac{x\left(e x^2+d\right)^{\frac{3}{2}}}{4e}-\frac{d\left(\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}\right.}{4e}\right.}{2e}\right.$

input
```
int((e*x^2+d)^(1/2)*(-c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/e^(5/2)*(d*(a*e^2-1/8*c*d^2)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))+x*(e*x
^2+d)^(1/2)*((-1/3*c*x^4+a)*e^(5/2)+1/8*d*c*(-2/3*e^(3/2)*x^2+e^(1/2)*d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.65

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)\,dx$$

$$= \Bigg[-\frac{3\left(cd^3-8\,ade^2\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\left(8\,ce^3x^5+2\,cde^2x^3-3\left(cd^2e+8\,ae^3\right)x\right)}{96\,e^3}$$

input
```
integrate((e*x^2+d)^(1/2)*(-c*x^4+a),x, algorithm="fricas")
```

output
```
[-1/96*(3*(c*d^3 - 8*a*d*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqr
t(e)*x - d) + 2*(8*c*e^3*x^5 + 2*c*d*e^2*x^3 - 3*(c*d^2*e + 8*a*e^3)*x)*sq
rt(e*x^2 + d))/e^3, 1/48*(3*(c*d^3 - 8*a*d*e^2)*sqrt(-e)*arctan(sqrt(-e)*x
/sqrt(e*x^2 + d)) - (8*c*e^3*x^5 + 2*c*d*e^2*x^3 - 3*(c*d^2*e + 8*a*e^3)*x
)*sqrt(e*x^2 + d))/e^3]
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.06

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)\,dx$$

$$= \begin{cases} \sqrt{d+ex^2}\left(-\frac{cdx^3}{24e}-\frac{cx^5}{6}+\frac{x\left(ae+\frac{cd^2}{8e}\right)}{2e}\right)+\left(ad-\frac{d\left(ae+\frac{cd^2}{8e}\right)}{2e}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{e}\sqrt{d+ex^2}+2ex\right)}{\sqrt{e}} & \text{for}\ d\neq 0\\ \frac{x\log(x)}{\sqrt{ex^2}} & \text{otherwise}\end{cases}\right) & \text{for}\ e \\ \sqrt{d}\left(ax-\frac{cx^5}{5}\right) & \text{othe}\end{cases}$$

input
```
integrate((e*x**2+d)**(1/2)*(-c*x**4+a),x)
```

output
```
Piecewise((sqrt(d + e*x**2)*(-c*d*x**3/(24*e) - c*x**5/6 + x*(a*e + c*d**2
/(8*e))/(2*e)) + (a*d - d*(a*e + c*d**2/(8*e))/(2*e))*Piecewise((log(2*sqr
t(e)*sqrt(d + e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqrt(e*x**2),
 True)), Ne(e, 0)), (sqrt(d)*(a*x - c*x**5/5), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)\,dx=\text{Exception raised: ValueError}$$

input

```
integrate((e*x^2+d)^(1/2)*(-c*x^4+a),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)\,dx=-\frac{1}{48}\left(2\left(4\,cx^2+\frac{cd}{e}\right)x^2-\frac{3\left(cd^2e^2+8\,ae^4\right)}{e^4}\right)\sqrt{ex^2+d}x+\frac{\left(cd^3-8\,ade^2\right)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{16\,e^{\frac{5}{2}}}$$

input

```
integrate((e*x^2+d)^(1/2)*(-c*x^4+a),x, algorithm="giac")
```

output

```
-1/48*(2*(4*c*x^2 + c*d/e)*x^2 - 3*(c*d^2*e^2 + 8*a*e^4)/e^4)*sqrt(e*x^2 +
 d)*x + 1/16*(c*d^3 - 8*a*d*e^2)*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^
(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)\,dx=\int \left(a-c\,x^4\right)\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a - c*x^4)*(d + e*x^2)^(1/2),x)
```

output
```
int((a - c*x^4)*(d + e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.11

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)\,dx$$
$$=\frac{24\sqrt{e\,x^2+d}\,a\,e^3x+3\sqrt{e\,x^2+d}\,c\,d^2ex-2\sqrt{e\,x^2+d}\,cd\,e^2x^3-8\sqrt{e\,x^2+d}\,c\,e^3x^5+24\sqrt{e}\log\Bigl(\frac{\sqrt{e\,x^2+d}+\sqrt{e}}{\sqrt{d}}}{48e^3}$$

input
```
int((e*x^2+d)^(1/2)*(-c*x^4+a),x)
```

output
```
(24*sqrt(d + e*x**2)*a*e**3*x + 3*sqrt(d + e*x**2)*c*d**2*e*x - 2*sqrt(d +
 e*x**2)*c*d*e**2*x**3 - 8*sqrt(d + e*x**2)*c*e**3*x**5 + 24*sqrt(e)*log((
sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d*e**2 - 3*sqrt(e)*log((sqrt(d +
e*x**2) + sqrt(e)*x)/sqrt(d))*c*d**3)/(48*e**3)
```

3.326 $\int \frac{a-cx^4}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{a-cx^4}{\sqrt{d+ex^2}}\,dx=\frac{3cdx\sqrt{d+ex^2}}{8e^2}-\frac{cx^3\sqrt{d+ex^2}}{4e}-\frac{(3cd^2-8ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{5/2}}$$

output
```
3/8*c*d*x*(e*x^2+d)^(1/2)/e^2-1/4*c*x^3*(e*x^2+d)^(1/2)/e-1/8*(-8*a*e^2+3*
c*d^2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

$$\int \frac{a-cx^4}{\sqrt{d+ex^2}}\,dx=-\frac{c\sqrt{d+ex^2}(-3dx+2ex^3)}{8e^2}+\frac{(3cd^2-8ae^2)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{8e^{5/2}}$$

input
```
Integrate[(a - c*x^4)/Sqrt[d + e*x^2],x]
```

output
```
-1/8*(c*Sqrt[d + e*x^2]*(-3*d*x + 2*e*x^3))/e^2 + ((3*c*d^2 - 8*a*e^2)*Log
[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(8*e^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1474, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-cx^4}{\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{1474}$$

$$\frac{\int \frac{3cdx^2+4ae}{\sqrt{ex^2+d}}dx}{4e}-\frac{cx^3\sqrt{d+ex^2}}{4e}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{3cdx\sqrt{d+ex^2}}{2e}-\frac{\left(3cd^2-8ae^2\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}}{4e}-\frac{cx^3\sqrt{d+ex^2}}{4e}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3cdx\sqrt{d+ex^2}}{2e}-\frac{\left(3cd^2-8ae^2\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}}{4e}-\frac{cx^3\sqrt{d+ex^2}}{4e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3cdx\sqrt{d+ex^2}}{2e}-\frac{\left(3cd^2-8ae^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}}{4e}-\frac{cx^3\sqrt{d+ex^2}}{4e}$$

input

```
Int[(a - c*x^4)/Sqrt[d + e*x^2],x]
```

output

```
-1/4*(c*x^3*Sqrt[d + e*x^2])/e + ((3*c*d*x*Sqrt[d + e*x^2])/(2*e) - ((3*c*
d^2 - 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(3/2)))/(4*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{cx\left(-2e\,x^{2}+3d\right)\sqrt{e\,x^{2}+d}}{8e^{2}}+\frac{\left(8a\,e^{2}-3c\,d^{2}\right)\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{8e^{\frac{5}{2}}}$	62
default	$\frac{a\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{\sqrt{e}}-c\left(\frac{x^{3}\sqrt{e\,x^{2}+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)$	88
pseudoelliptic	$\frac{-2c\,e^{\frac{3}{2}}x^{3}\sqrt{e\,x^{2}+d}+3cdx\sqrt{e}\,\sqrt{e\,x^{2}+d}+8\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)a\,e^{2}-3\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)c\,d^{2}}{8e^{\frac{5}{2}}}$	88

input
```
int((-c*x^4+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*c*x*(-2*e*x^2+3*d)/e^2*(e*x^2+d)^(1/2)+1/8*(8*a*e^2-3*c*d^2)/e^(5/2)*l
n(x*e^(1/2)+(e*x^2+d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.72

$$\int \frac{a-cx^4}{\sqrt{d+ex^2}}\,dx = \left[-\frac{(3\,cd^2-8\,ae^2)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\,(2\,ce^2x^3-3\,cdex)\sqrt{ex^2+d}}{16\,e^3}, (3\,cd^2-8\,ae^2\right.$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((3*c*d^2 - 8*a*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(
e)*x - d) + 2*(2*c*e^2*x^3 - 3*c*d*e*x)*sqrt(e*x^2 + d))/e^3, 1/8*((3*c*d^
2 - 8*a*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*c*e^2*x^3 -
3*c*d*e*x)*sqrt(e*x^2 + d))/e^3]
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int \frac{a-cx^4}{\sqrt{d+ex^2}}\,dx = \begin{cases} \left(a-\frac{3cd^2}{8e^2}\right)\left(\begin{cases} \frac{\log\left(2\sqrt{e}\sqrt{d+ex^2}+2ex\right)}{\sqrt{e}} & \text{for}\: d \neq 0 \\ \frac{x\log\left(x\right)}{\sqrt{ex^2}} & \text{otherwise} \end{cases}\right)+\sqrt{d+ex^2}\cdot\left(\frac{3cdx}{8e^2}-\frac{cx^3}{4e}\right) & \text{for}\: e \neq 0 \\ \frac{ax-\frac{cx^5}{5}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

input
```
integrate((-c*x**4+a)/(e*x**2+d)**(1/2),x)
```

output
```
Piecewise(((a - 3*c*d**2/(8*e**2))*Piecewise((log(2*sqrt(e)*sqrt(d + e*x**
2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqrt(e*x**2), True)) + sqrt(d +
e*x**2)*(3*c*d*x/(8*e**2) - c*x**3/(4*e)), Ne(e, 0)), ((a*x - c*x**5/5)/sq
rt(d), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a-cx^4}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int \frac{a-cx^4}{\sqrt{d+ex^2}}\,dx = -\frac{1}{8}\sqrt{ex^2+d}\left(\frac{2\,cx^2}{e}-\frac{3\,cd}{e^2}\right)x + \frac{(3\,cd^2-8\,ae^2)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{8\,e^{\frac{5}{2}}}$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(e*x^2 + d)*(2*c*x^2/e - 3*c*d/e^2)*x + 1/8*(3*c*d^2 - 8*a*e^2)*l
og(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a-cx^4}{\sqrt{d+ex^2}}\,dx=\int \frac{a-c\,x^4}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a - c*x^4)/(d + e*x^2)^(1/2),x)
```

output
```
int((a - c*x^4)/(d + e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07

$$\int \frac{a-cx^4}{\sqrt{d+ex^2}}\,dx = \frac{3\sqrt{e\,x^2+d}\,cdex-2\sqrt{e\,x^2+d}\,c\,e^2x^3+8\sqrt{e}\,\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,e^2-3\sqrt{e}\,\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)c\,d^2}{8e^3}$$

input
```
int((-c*x^4+a)/(e*x^2+d)^(1/2),x)
```

output
```
(3*sqrt(d + e*x**2)*c*d*e*x - 2*sqrt(d + e*x**2)*c*e**2*x**3 + 8*sqrt(e)*l
og((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*e**2 - 3*sqrt(e)*log((sqrt(d
+ e*x**2) + sqrt(e)*x)/sqrt(d))*c*d**2)/(8*e**3)
```

3.327 $\int \frac{a-cx^4}{\left(d+ex^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{3/2}} \, dx=\frac{\left(a-\frac{cd^2}{e^2}\right) x}{d\sqrt{d+ex^2}}-\frac{cx\sqrt{d+ex^2}}{2e^2}+\frac{3cd\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}$$

output
```
(a-c*d^2/e^2)*x/d/(e*x^2+d)^(1/2)-1/2*c*x*(e*x^2+d)^(1/2)/e^2+3/2*c*d*arct
anh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{3/2}} \, dx=\frac{-3cd^2x+2ae^2x-cdex^3}{2de^2\sqrt{d+ex^2}}-\frac{3cd\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{2e^{5/2}}$$

input
```
Integrate[(a - c*x^4)/(d + e*x^2)^(3/2),x]
```

output
```
(-3*c*d^2*x + 2*a*e^2*x - c*d*e*x^3)/(2*d*e^2*Sqrt[d + e*x^2]) - (3*c*d*Lo
g[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(2*e^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1472, 25, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-cx^4}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1472}$$

$$\frac{x\left(a-\frac{cd^2}{e^2}\right)}{d\sqrt{d+ex^2}}-\frac{\int -\frac{cd(d-ex^2)}{e^2\sqrt{ex^2+d}}dx}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{cd(d-ex^2)}{e^2\sqrt{ex^2+d}}dx}{d}+\frac{x\left(a-\frac{cd^2}{e^2}\right)}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\int \frac{d-ex^2}{\sqrt{ex^2+d}}dx}{e^2}+\frac{x\left(a-\frac{cd^2}{e^2}\right)}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{c\left(\frac{3}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{1}{2}x\sqrt{d+ex^2}\right)}{e^2}+\frac{x\left(a-\frac{cd^2}{e^2}\right)}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{c\left(\frac{3}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{1}{2}x\sqrt{d+ex^2}\right)}{e^2}+\frac{x\left(a-\frac{cd^2}{e^2}\right)}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x\left(a-\frac{cd^2}{e^2}\right)}{d\sqrt{d+ex^2}}+\frac{c\left(\frac{3d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}-\frac{1}{2}x\sqrt{d+ex^2}\right)}{e^2}$$

input
```
Int[(a - c*x^4)/(d + e*x^2)^(3/2),x]
```

output
```
((a - (c*d^2)/e^2)*x)/(d*Sqrt[d + e*x^2]) + (c*(-1/2*(x*Sqrt[d + e*x^2]) +
 (3*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/e^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

method	result	size
risch	$-\frac{cx\sqrt{e\,x^{2}+d}}{2e^{2}}-\frac{cdx}{e^{2}\sqrt{e\,x^{2}+d}}+\frac{ax}{d\sqrt{e\,x^{2}+d}}+\frac{3cd\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2e^{\frac{5}{2}}}$	73
pseudoelliptic	$\frac{\frac{3\ \operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)\sqrt{e\,x^{2}+d}\,c\,d^{2}e^{2}}{2}+x\,e^{\frac{5}{2}}\left(-\frac{1}{2}cd\,x^{2}e+a\,e^{2}-\frac{3}{2}c\,d^{2}\right)}{\sqrt{e\,x^{2}+d}\,e^{\frac{9}{2}}d}$	78
default	$\frac{ax}{d\sqrt{e\,x^{2}+d}}-c\left(\frac{x^{3}}{2e\sqrt{e\,x^{2}+d}}-\frac{3d\left(-\frac{x}{e\sqrt{e\,x^{2}+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{e^{\frac{3}{2}}}\right)}{2e}\right)$	80

input

```
int((-c*x^4+a)/(e*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*c*x*(e*x^2+d)^(1/2)/e^2-1/e^2*c*d*x/(e*x^2+d)^(1/2)+a*x/d/(e*x^2+d)^(
1/2)+3/2/e^(5/2)*c*d*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.58

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{3/2}}\,dx = \left[\frac{3\left(cd^2ex^2+cd^3\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-2\left(cde^2x^3+\left(3\,cd^2e-2\,ae\right.\right.}{4\left(de^4x^2+d^2e^3\right)}\right.$$

$$\left.-\frac{3\left(cd^2ex^2+cd^3\right)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+\left(cde^2x^3+\left(3\,cd^2e-2\,ae^3\right)x\right)\sqrt{ex^2+d}}{2\left(de^4x^2+d^2e^3\right)}\right]$$

input

```
integrate((-c*x^4+a)/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output

```
[1/4*(3*(c*d^2*e*x^2 + c*d^3)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqr
t(e)*x - d) - 2*(c*d*e^2*x^3 + (3*c*d^2*e - 2*a*e^3)*x)*sqrt(e*x^2 + d))/(
d*e^4*x^2 + d^2*e^3), -1/2*(3*(c*d^2*e*x^2 + c*d^3)*sqrt(-e)*arctan(sqrt(-
e)*x/sqrt(e*x^2 + d)) + (c*d*e^2*x^3 + (3*c*d^2*e - 2*a*e^3)*x)*sqrt(e*x^2
 + d))/(d*e^4*x^2 + d^2*e^3)]
```

Sympy [A] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

$$\int \frac{a-cx^4}{(d+ex^2)^{3/2}}\,dx = \frac{ax}{d^{\frac{3}{2}}\sqrt{1+\frac{ex^2}{d}}} - c\left(\frac{3\sqrt{d}x}{2e^2\sqrt{1+\frac{ex^2}{d}}} - \frac{3d\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{d}e\sqrt{1+\frac{ex^2}{d}}}\right)$$

input

```
integrate((-c*x**4+a)/(e*x**2+d)**(3/2),x)
```

output

```
a*x/(d**(3/2)*sqrt(1 + e*x**2/d)) - c*(3*sqrt(d)*x/(2*e**2*sqrt(1 + e*x**2
/d)) - 3*d*asinh(sqrt(e)*x/sqrt(d))/(2*e**(5/2)) + x**3/(2*sqrt(d)*e*sqrt(
1 + e*x**2/d)))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a-cx^4}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((-c*x^4+a)/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{a-cx^4}{(d+ex^2)^{3/2}}\,dx = -\frac{\left(\frac{cx^2}{e}+\frac{3\,cd^2e-2\,ae^3}{de^3}\right)x}{2\,\sqrt{ex^2+d}} - \frac{3\,cd\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{2\,e^{\frac{5}{2}}}$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(c*x^2/e + (3*c*d^2*e - 2*a*e^3)/(d*e^3))*x/sqrt(e*x^2 + d) - 3/2*c*d
*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a-cx^4}{(d+ex^2)^{3/2}}\,dx = \int \frac{a-c\,x^4}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

input
```
int((a - c*x^4)/(d + e*x^2)^(3/2),x)
```

output
```
int((a - c*x^4)/(d + e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.12

$$\int \frac{a-cx^4}{(d+ex^2)^{3/2}}\,dx = \frac{8\sqrt{e\,x^2+d}\,a\,e^3x-12\sqrt{e\,x^2+d}\,c\,d^2ex-4\sqrt{e\,x^2+d}\,cd\,e^2x^3+12\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right.}{8d\,e}$$

input
```
int((-c*x^4+a)/(e*x^2+d)^(3/2),x)
```

output

```
(8*sqrt(d + e*x**2)*a*e**3*x - 12*sqrt(d + e*x**2)*c*d**2*e*x - 4*sqrt(d +
 e*x**2)*c*d*e**2*x**3 + 12*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqr
t(d))*c*d**3 + 12*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*c*d*
*2*e*x**2 + 8*sqrt(e)*a*d*e**2 + 8*sqrt(e)*a*e**3*x**2 - 9*sqrt(e)*c*d**3
- 9*sqrt(e)*c*d**2*e*x**2)/(8*d*e**3*(d + e*x**2))
```

3.328 $\int \frac{a-cx^4}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{\left(a-\frac{cd^2}{e^2}\right)x}{3d\left(d+ex^2\right)^{3/2}} + \frac{2\left(\frac{a}{d^2}+\frac{2c}{e^2}\right)x}{3\sqrt{d+ex^2}} - \frac{c\,\mathrm{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

output
```
1/3*(a-c*d^2/e^2)*x/d/(e*x^2+d)^(3/2)+2/3*(a/d^2+2*c/e^2)*x/(e*x^2+d)^(1/2
)-c*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x\left(3cd^3+3ade^2+4cd^2ex^2+2ae^3x^2\right)}{3d^2e^2\left(d+ex^2\right)^{3/2}} + \frac{c\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{e^{5/2}}$$

input
```
Integrate[(a - c*x^4)/(d + e*x^2)^(5/2),x]
```

output
```
(x*(3*c*d^3 + 3*a*d*e^2 + 4*c*d^2*e*x^2 + 2*a*e^3*x^2))/(3*d^2*e^2*(d + e*
x^2)^(3/2)) + (c*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/e^(5/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1472, 25, 27, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-cx^4}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \ 1472$$

$$\frac{x\left(a-\frac{cd^2}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{\left(\frac{cd^2}{e^2}+2a\right)e-3cdx^2}{e\left(ex^2+d\right)^{3/2}}dx}{3d}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\frac{cd^2}{e}-3cx^2d+2ae}{e\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{x\left(a-\frac{cd^2}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\frac{cd^2}{e}-3cx^2d+2ae}{\left(ex^2+d\right)^{3/2}}dx}{3de}+\frac{x\left(a-\frac{cd^2}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \ 298$$

$$\frac{\frac{2x\left(\frac{ae}{d}+\frac{2cd}{e}\right)}{\sqrt{d+ex^2}}-\frac{3cd\int \frac{1}{\sqrt{ex^2+d}}dx}{e}}{3de}+\frac{x\left(a-\frac{cd^2}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{2x\left(\frac{ae}{d}+\frac{2cd}{e}\right)}{\sqrt{d+ex^2}}-\frac{3cd\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}}{3de}+\frac{x\left(a-\frac{cd^2}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{2x\left(\frac{ae}{d}+\frac{2cd}{e}\right)}{\sqrt{d+ex^2}}-\frac{3cd\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}}{3de}+\frac{x\left(a-\frac{cd^2}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}$$

input
```
Int[(a - c*x^4)/(d + e*x^2)^(5/2),x]
```

output
```
((a - (c*d^2)/e^2)*x)/(3*d*(d + e*x^2)^(3/2)) + ((2*((2*c*d)/e + (a*e)/d)*
x)/Sqrt[d + e*x^2] - (3*c*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(3/2))
/(3*d*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

method	result	size
pseudoelliptic	$\frac{-(e\,x^2+d)^{\frac{3}{2}} c\,d^2 \operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}}{x\sqrt{e}}\right)+x\left(\frac{4c\,d^2 e^{\frac{3}{2}} x^2}{3}+c\,d^3\sqrt{e}+\frac{2a\,e^{\frac{7}{2}} x^2}{3}+a d\,e^{\frac{5}{2}}\right)}{e^{\frac{5}{2}} (e\,x^2+d)^{\frac{3}{2}} d^2}$	88
default	$a\left(\frac{x}{3d(e\,x^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{e\,x^2+d}}\right)-c\left(-\frac{x^3}{3e(e\,x^2+d)^{\frac{3}{2}}}+\frac{-\frac{x}{e\sqrt{e\,x^2+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{e^{\frac{3}{2}}}}{e}\right)$	96

input
```
int((-c*x^4+a)/(e*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^(5/2)/(e*x^2+d)^(3/2)*(-(e*x^2+d)^(3/2)*c*d^2*arctanh((e*x^2+d)^(1/2)/
x/e^(1/2))+x*(4/3*c*d^2*e^(3/2)*x^2+c*d^3*e^(1/2)+2/3*a*e^(7/2)*x^2+a*d*e^
(5/2)))/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.17

$$\int \frac{a-cx^4}{(d+ex^2)^{5/2}}\,dx = \left[\frac{3\,(cd^2e^2x^4+2\,cd^3ex^2+cd^4)\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\,(2\,(2\,cd^2e^2+}{6\,(d^2e^5x^4+2\,d^3e^4x^2+d^4e^3)}\right.$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(e)*log(-2*e*x^2 + 2*s
qrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(2*(2*c*d^2*e^2 + a*e^4)*x^3 + 3*(c*d^3*
e + a*d*e^3)*x)*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 + d^4*e^3),
1/3*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(-e)*arctan(sqrt(-e)*x/
sqrt(e*x^2 + d)) + (2*(2*c*d^2*e^2 + a*e^4)*x^3 + 3*(c*d^3*e + a*d*e^3)*x)
*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 + d^4*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(80) = 160$.

Time = 4.83 (sec) , antiderivative size = 403, normalized size of antiderivative = 4.69

$$\int \frac{a-cx^4}{(d+ex^2)^{5/2}}\,dx = a\left(\frac{3dx}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}} + \frac{2ex^3}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}\right) - c\left(\frac{3d^{\frac{39}{2}}e^{11}\sqrt{1+\frac{ex^2}{d}}\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}} + \frac{3d^{\frac{37}{2}}e^{12}x^2\sqrt{1+\frac{ex^2}{d}}\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}} - \frac{3d^{19}e^{\frac{23}{2}}x}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}} - \frac{4d^{18}e^{\frac{25}{2}}x^3}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}}\right)$$

input

```
integrate((-c*x**4+a)/(e*x**2+d)**(5/2),x)
```

output

```
a*(3*d*x/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 + e*x**
2/d)) + 2*e*x**3/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1
 + e*x**2/d))) - c*(3*d**(39/2)*e**11*sqrt(1 + e*x**2/d)*asinh(sqrt(e)*x/s
qrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*
x**2*sqrt(1 + e*x**2/d)) + 3*d**(37/2)*e**12*x**2*sqrt(1 + e*x**2/d)*asinh
(sqrt(e)*x/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2
)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) - 3*d**19*e**(23/2)*x/(3*d**(39/2)*e*
*(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)
) - 4*d**18*e**(25/2)*x**3/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d
**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x\left(\frac{2\left(2\,cd^2e^2+ae^4\right)x^2}{d^2e^3}+\frac{3\left(cd^3e+ade^3\right)}{d^2e^3}\right)}{3\left(ex^2+d\right)^{\frac{3}{2}}}+\frac{c\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{e^{\frac{5}{2}}}$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*(2*(2*c*d^2*e^2 + a*e^4)*x^2/(d^2*e^3) + 3*(c*d^3*e + a*d*e^3)/(d^2*
e^3))/(e*x^2 + d)^(3/2) + c*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a-c\,x^4}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a - c*x^4)/(d + e*x^2)^(5/2),x)
```

output
```
int((a - c*x^4)/(d + e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.59

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{3\sqrt{e\,x^2+d}\,ad\,e^3x+2\sqrt{e\,x^2+d}\,a\,e^4x^3+3\sqrt{e\,x^2+d}\,c\,d^3ex+4\sqrt{e\,x^2+d}\,c\,d^2e^2x^3-3}{}$$

input
```
int((-c*x^4+a)/(e*x^2+d)^(5/2),x)
```

output
```
(3*sqrt(d + e*x**2)*a*d*e**3*x + 2*sqrt(d + e*x**2)*a*e**4*x**3 + 3*sqrt(d
 + e*x**2)*c*d**3*e*x + 4*sqrt(d + e*x**2)*c*d**2*e**2*x**3 - 3*sqrt(e)*lo
g((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*c*d**4 - 6*sqrt(e)*log((sqrt(d +
 e*x**2) + sqrt(e)*x)/sqrt(d))*c*d**3*e*x**2 - 3*sqrt(e)*log((sqrt(d + e*x
**2) + sqrt(e)*x)/sqrt(d))*c*d**2*e**2*x**4 - 2*sqrt(e)*a*d**2*e**2 - 4*sq
rt(e)*a*d*e**3*x**2 - 2*sqrt(e)*a*e**4*x**4)/(3*d**2*e**3*(d**2 + 2*d*e*x*
*2 + e**2*x**4))
```

3.329 $\int \frac{a-cx^4}{\left(d+ex^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{7/2}}\,dx=\frac{\left(a-\frac{cd^2}{e^2}\right)x}{5d\left(d+ex^2\right)^{5/2}}+\frac{2\left(\frac{2a}{d^2}+\frac{3c}{e^2}\right)x}{15\left(d+ex^2\right)^{3/2}}+\frac{\left(\frac{8a}{d^2}-\frac{3c}{e^2}\right)x}{15d\sqrt{d+ex^2}}$$

output
```
1/5*(a-c*d^2/e^2)*x/d/(e*x^2+d)^(5/2)+2/15*(2*a/d^2+3*c/e^2)*x/(e*x^2+d)^(
3/2)+1/15*(8*a/d^2-3*c/e^2)*x/d/(e*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{7/2}}\,dx=\frac{15ad^2x+20adex^3-3cd^2x^5+8ae^2x^5}{15d^3\left(d+ex^2\right)^{5/2}}$$

input
```
Integrate[(a - c*x^4)/(d + e*x^2)^(7/2),x]
```

output
```
(15*a*d^2*x + 20*a*d*e*x^3 - 3*c*d^2*x^5 + 8*a*e^2*x^5)/(15*d^3*(d + e*x^2
)^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1470, 362, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-cx^4}{(d+ex^2)^{7/2}} \, dx$$

$$\Big\downarrow \; 1470$$

$$\frac{\int \frac{x^2(4ae-cdx^2)}{(ex^2+d)^{7/2}}dx}{d}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \; 362$$

$$\frac{\frac{x^3\left(\frac{4ae}{d}+\frac{cd}{e}\right)}{5(d+ex^2)^{5/2}}-\frac{1}{5}\left(\frac{3cd}{e}-\frac{8ae}{d}\right)\int \frac{x^2}{(ex^2+d)^{5/2}}dx}{d}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \; 242$$

$$\frac{\frac{x^3\left(\frac{4ae}{d}+\frac{cd}{e}\right)}{5(d+ex^2)^{5/2}}-\frac{x^3\left(\frac{3cd}{e}-\frac{8ae}{d}\right)}{15d(d+ex^2)^{3/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

input

```
Int[(a - c*x^4)/(d + e*x^2)^(7/2),x]
```

output

```
(a*x)/(d*(d + e*x^2)^(5/2)) + ((((c*d)/e + (4*a*e)/d)*x^3)/(5*(d + e*x^2)^
(5/2)) - (((3*c*d)/e - (8*a*e)/d)*x^3)/(15*d*(d + e*x^2)^(3/2)))/d
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1470
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d + e*x^2)^q*(d*
PolynomialQuotient[(a + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q
+ 1/2, 0] && LtQ[4*p + 2*q + 1, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

method	result
gosper	$\frac{x\left(8a\,e^{2}x^{4}-3c\,d^{2}x^{4}+20ade\,x^{2}+15a\,d^{2}\right)}{15\left(e\,x^{2}+d\right)^{\frac{5}{2}}d^{3}}$
trager	$\frac{x\left(8a\,e^{2}x^{4}-3c\,d^{2}x^{4}+20ade\,x^{2}+15a\,d^{2}\right)}{15\left(e\,x^{2}+d\right)^{\frac{5}{2}}d^{3}}$
pseudoelliptic	$\frac{x\left(8a\,e^{2}x^{4}-3c\,d^{2}x^{4}+20ade\,x^{2}+15a\,d^{2}\right)}{15\left(e\,x^{2}+d\right)^{\frac{5}{2}}d^{3}}$
orering	$\frac{x\left(8a\,e^{2}x^{4}-3c\,d^{2}x^{4}+20ade\,x^{2}+15a\,d^{2}\right)}{15\left(e\,x^{2}+d\right)^{\frac{5}{2}}d^{3}}$
default	$a\left(\frac{x}{5d\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^{2}\sqrt{e\,x^{2}+d}}}{d}\right)-c\left(-\frac{x^{3}}{2e\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{3d\left(-\frac{x}{4e\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{d\left(\frac{x}{5d\left(e\,x^{2}+d\right)^{\frac{5}{2}}}\right.}{}\right.}{2e}\right.$

input
```
int((-c*x^4+a)/(e*x^2+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(8*a*e^2*x^4-3*c*d^2*x^4+20*a*d*e*x^2+15*a*d^2)/(e*x^2+d)^(5/2)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{a-cx^{4}}{\left(d+ex^{2}\right)^{7/2}}\,dx=\frac{\left(20\,adex^{3}-\left(3\,cd^{2}-8\,ae^{2}\right)x^{5}+15\,ad^{2}x\right)\sqrt{ex^{2}+d}}{15\left(d^{3}e^{3}x^{6}+3\,d^{4}e^{2}x^{4}+3\,d^{5}ex^{2}+d^{6}\right)}$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(20*a*d*e*x^3 - (3*c*d^2 - 8*a*e^2)*x^5 + 15*a*d^2*x)*sqrt(e*x^2 + d)
/(d^3*e^3*x^6 + 3*d^4*e^2*x^4 + 3*d^5*e*x^2 + d^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(83) = 166$.

Time = 9.56 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.30

$$\int \frac{a-cx^4}{(d+ex^2)^{7/2}}\,dx = a\left(\frac{15d^5x}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}\right.$$
$$+\frac{35d^4ex^3}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}$$
$$+\frac{28d^3e^2x^5}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}$$
$$\left.+\frac{8d^2e^3x^7}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}\right)$$
$$-\frac{cx^5}{5d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+10d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+5d^{\frac{3}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}}$$

input

```
integrate((-c*x**4+a)/(e*x**2+d)**(7/2),x)
```

output

```
a*(15*d**5*x/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1
 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e*
*3*x**6*sqrt(1 + e*x**2/d)) + 35*d**4*e*x**3/(15*d**(17/2)*sqrt(1 + e*x**2
/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt
(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 28*d**3*e**2
*x**5/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x*
*2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6
*sqrt(1 + e*x**2/d)) + 8*d**2*e**3*x**7/(15*d**(17/2)*sqrt(1 + e*x**2/d) +
 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 +
e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d))) - c*x**5/(5*d**(7/
2)*sqrt(1 + e*x**2/d) + 10*d**(5/2)*e*x**2*sqrt(1 + e*x**2/d) + 5*d**(3/2)
*e**2*x**4*sqrt(1 + e*x**2/d))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.29

$$\int \frac{a-cx^4}{(d+ex^2)^{7/2}}\,dx = \frac{cx^3}{2\,(ex^2+d)^{\frac{5}{2}}e} + \frac{8\,ax}{15\,\sqrt{ex^2+d}d^3} + \frac{4\,ax}{15\,(ex^2+d)^{\frac{3}{2}}d^2} + \frac{ax}{5\,(ex^2+d)^{\frac{5}{2}}d} - \frac{cx}{10\,(ex^2+d)^{\frac{3}{2}}e^2} - \frac{cx}{5\,\sqrt{ex^2+d}de^2} + \frac{3\,cdx}{10\,(ex^2+d)^{\frac{5}{2}}e^2}$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
1/2*c*x^3/((e*x^2 + d)^(5/2)*e) + 8/15*a*x/(sqrt(e*x^2 + d)*d^3) + 4/15*a*
x/((e*x^2 + d)^(3/2)*d^2) + 1/5*a*x/((e*x^2 + d)^(5/2)*d) - 1/10*c*x/((e*x
^2 + d)^(3/2)*e^2) - 1/5*c*x/(sqrt(e*x^2 + d)*d*e^2) + 3/10*c*d*x/((e*x^2
+ d)^(5/2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.63

$$\int \frac{a-cx^4}{(d+ex^2)^{7/2}}\,dx = \frac{\left(x^2\left(\frac{20\,ae}{d^2} - \frac{(3\,cd^2e^2-8\,ae^4)x^2}{d^3e^2}\right) + \frac{15\,a}{d}\right)x}{15\,(ex^2+d)^{\frac{5}{2}}}$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
1/15*(x^2*(20*a*e/d^2 - (3*c*d^2*e^2 - 8*a*e^4)*x^2/(d^3*e^2)) + 15*a/d)*x
/(e*x^2 + d)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 17.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int \frac{a-cx^4}{(d+ex^2)^{7/2}}\,dx = \frac{6\,c\,d^3\,x\,(e\,x^2+d) - 3\,c\,d^4\,x + 8\,a\,e^2\,x\,(e\,x^2+d)^2 - 3\,c\,d^2\,x\,(e\,x^2+d)^2 + 3\,a\,d^2\,e^2\,x + 4}{15\,d^3\,e^2\,(e\,x^2+d)^{5/2}}$$

input
```
int((a - c*x^4)/(d + e*x^2)^(7/2),x)
```

output
```
(6*c*d^3*x*(d + e*x^2) - 3*c*d^4*x + 8*a*e^2*x*(d + e*x^2)^2 - 3*c*d^2*x*(
d + e*x^2)^2 + 3*a*d^2*e^2*x + 4*a*d*e^2*x*(d + e*x^2))/(15*d^3*e^2*(d + e
*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.27

$$\int \frac{a-cx^4}{(d+ex^2)^{7/2}}\,dx = \frac{15\sqrt{e\,x^2+d}\,a\,d^2e^3x + 20\sqrt{e\,x^2+d}\,ad\,e^4x^3 + 8\sqrt{e\,x^2+d}\,a\,e^5x^5 - 3\sqrt{e\,x^2+d}\,c\,d^2e^3x^5}{15}$$

input
```
int((-c*x^4+a)/(e*x^2+d)^(7/2),x)
```

output
```
(15*sqrt(d + e*x**2)*a*d**2*e**3*x + 20*sqrt(d + e*x**2)*a*d*e**4*x**3 + 8
*sqrt(d + e*x**2)*a*e**5*x**5 - 3*sqrt(d + e*x**2)*c*d**2*e**3*x**5 - 8*sq
rt(e)*a*d**3*e**2 - 24*sqrt(e)*a*d**2*e**3*x**2 - 24*sqrt(e)*a*d*e**4*x**4
 - 8*sqrt(e)*a*e**5*x**6 - 3*sqrt(e)*c*d**5 - 9*sqrt(e)*c*d**4*e*x**2 - 9*
sqrt(e)*c*d**3*e**2*x**4 - 3*sqrt(e)*c*d**2*e**3*x**6)/(15*d**3*e**3*(d**3
 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6))
```

3.330 $\int \frac{a-cx^4}{\left(d+ex^2\right)^{9/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 126

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{9/2}}\,dx=\frac{\left(a-\frac{cd^2}{e^2}\right)x}{7d\left(d+ex^2\right)^{7/2}}+\frac{2\left(\frac{3a}{d^2}+\frac{4c}{e^2}\right)x}{35\left(d+ex^2\right)^{5/2}}+\frac{\left(\frac{8a}{d^2}-\frac{c}{e^2}\right)x}{35d\left(d+ex^2\right)^{3/2}}-\frac{2\left(cd^2-8ae^2\right)x}{35d^4e^2\sqrt{d+ex^2}}$$

output
```
1/7*(a-c*d^2/e^2)*x/d/(e*x^2+d)^(7/2)+2/35*(3*a/d^2+4*c/e^2)*x/(e*x^2+d)^(
5/2)+1/35*(8*a/d^2-c/e^2)*x/d/(e*x^2+d)^(3/2)-2/35*(-8*a*e^2+c*d^2)*x/d^4/
e^2/(e*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.59

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{9/2}}\,dx=\frac{35ad^3x+70ad^2ex^3-7cd^3x^5+56ade^2x^5-2cd^2ex^7+16ae^3x^7}{35d^4\left(d+ex^2\right)^{7/2}}$$

input
```
Integrate[(a - c*x^4)/(d + e*x^2)^(9/2),x]
```

output
```
(35*a*d^3*x + 70*a*d^2*e*x^3 - 7*c*d^3*x^5 + 56*a*d*e^2*x^5 - 2*c*d^2*e*x^
7 + 16*a*e^3*x^7)/(35*d^4*(d + e*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1470, 362, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{1470}$$

$$\frac{\int \frac{x^2\left(6ae-cdx^2\right)}{\left(ex^2+d\right)^{9/2}}dx}{d}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{x^3\left(\frac{6ae}{d}+\frac{cd}{e}\right)}{7\left(d+ex^2\right)^{7/2}}-\frac{3}{7}\left(\frac{cd}{e}-\frac{8ae}{d}\right)\int \frac{x^2}{\left(ex^2+d\right)^{7/2}}dx}{d}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{x^3\left(\frac{6ae}{d}+\frac{cd}{e}\right)}{7\left(d+ex^2\right)^{7/2}}-\frac{3}{7}\left(\frac{cd}{e}-\frac{8ae}{d}\right)\left(\frac{2e\int \frac{x^4}{\left(ex^2+d\right)^{7/2}}dx}{3d}+\frac{x^3}{3d\left(d+ex^2\right)^{5/2}}\right)}{d}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{\frac{x^3\left(\frac{6ae}{d}+\frac{cd}{e}\right)}{7\left(d+ex^2\right)^{7/2}}-\frac{3}{7}\left(\frac{2ex^5}{15d^2\left(d+ex^2\right)^{5/2}}+\frac{x^3}{3d\left(d+ex^2\right)^{5/2}}\right)\left(\frac{cd}{e}-\frac{8ae}{d}\right)}{d}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

input
```
Int[(a - c*x^4)/(d + e*x^2)^(9/2),x]
```

output
```
(a*x)/(d*(d + e*x^2)^(7/2)) + ((((c*d)/e + (6*a*e)/d)*x^3)/(7*(d + e*x^2)^
(7/2)) - (3*((c*d)/e - (8*a*e)/d)*(x^3/(3*d*(d + e*x^2)^(5/2)) + (2*e*x^5)
/(15*d^2*(d + e*x^2)^(5/2))))/7)/d
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1470
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d + e*x^2)^q*(d*
PolynomialQuotient[(a + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q
+ 1/2, 0] && LtQ[4*p + 2*q + 1, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.51

method	result
pseudoelliptic	$\frac{x\left(\left(-\frac{c\,x^{4}}{5}+a\right)d^{3}+2x^{2}e\left(-\frac{c\,x^{4}}{35}+a\right)d^{2}+\frac{8ad\,e^{2}x^{4}}{5}+\frac{16a\,e^{3}x^{6}}{35}\right)}{\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$
gosper	$\frac{x\left(16a\,e^{3}x^{6}-2c\,d^{2}e\,x^{6}+56ad\,e^{2}x^{4}-7c\,d^{3}x^{4}+70a\,d^{2}e\,x^{2}+35a\,d^{3}\right)}{35\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$
trager	$\frac{x\left(16a\,e^{3}x^{6}-2c\,d^{2}e\,x^{6}+56ad\,e^{2}x^{4}-7c\,d^{3}x^{4}+70a\,d^{2}e\,x^{2}+35a\,d^{3}\right)}{35\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$
orering	$\frac{x\left(16a\,e^{3}x^{6}-2c\,d^{2}e\,x^{6}+56ad\,e^{2}x^{4}-7c\,d^{3}x^{4}+70a\,d^{2}e\,x^{2}+35a\,d^{3}\right)}{35\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$
default	$a\left(\frac{x}{7d(e\,x^{2}+d)^{\frac{7}{2}}}+\frac{\frac{6x}{35d\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^{2}\sqrt{e\,x^{2}+d}}\right)}{7d}}{d}\right)-c\left(-\frac{x^{3}}{4e(e\,x^{2}+d)^{\frac{7}{2}}}+\frac{3d\left(-\frac{x}{6e\left(e\,x^{2}+d\right)}\right.}{}\right.$

input
```
int((-c*x^4+a)/(e*x^2+d)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
x/(e*x^2+d)^(7/2)*((-1/5*c*x^4+a)*d^3+2*x^2*e*(-1/35*c*x^4+a)*d^2+8/5*a*d*
e^2*x^4+16/35*a*e^3*x^6)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

$$\int \frac{a-cx^4}{(d+ex^2)^{9/2}}\,dx = -\frac{(2\,(cd^2e-8\,ae^3)x^7-70\,ad^2ex^3+7\,(cd^3-8\,ade^2)x^5-35\,ad^3x)\sqrt{ex^2+d}}{35\,(d^4e^4x^8+4\,d^5e^3x^6+6\,d^6e^2x^4+4\,d^7ex^2+d^8)}$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*(2*(c*d^2*e - 8*a*e^3)*x^7 - 70*a*d^2*e*x^3 + 7*(c*d^3 - 8*a*d*e^2)*
x^5 - 35*a*d^3*x)*sqrt(e*x^2 + d)/(d^4*e^4*x^8 + 4*d^5*e^3*x^6 + 6*d^6*e^2
*x^4 + 4*d^7*e*x^2 + d^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. 2(119) = 238.

Time = 22.09 (sec) , antiderivative size = 1469, normalized size of antiderivative = 11.66

$$\int \frac{a-cx^4}{(d+ex^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-c*x**4+a)/(e*x**2+d)**(9/2),x)
```

output
```
a*(35*d**14*x/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt
(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2
)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d
) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*
sqrt(1 + e*x**2/d)) + 175*d**13*e*x**3/(35*d**(37/2)*sqrt(1 + e*x**2/d) +
210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 +
 e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e*
*4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) +
 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 371*d**12*e**2*x**5/(35*d**
(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*
d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 +
e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**
5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) +
 429*d**11*e**3*x**7/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x*
*2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d
**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e
*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6
*x**12*sqrt(1 + e*x**2/d)) + 286*d**10*e**4*x**9/(35*d**(37/2)*sqrt(1 + e*
x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**
4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.22

$$\int \frac{a-cx^4}{(d+ex^2)^{9/2}}\,dx = \frac{cx^3}{4\,(ex^2+d)^{\frac{7}{2}}e} + \frac{16\,ax}{35\,\sqrt{ex^2+d}d^4} + \frac{8\,ax}{35\,(ex^2+d)^{\frac{3}{2}}d^3} + \frac{6\,ax}{35\,(ex^2+d)^{\frac{5}{2}}d^2} + \frac{ax}{7\,(ex^2+d)^{\frac{7}{2}}d} - \frac{3\,cx}{140\,(ex^2+d)^{\frac{5}{2}}e^2} - \frac{2\,cx}{35\,\sqrt{ex^2+d}d^2e^2} - \frac{cx}{35\,(ex^2+d)^{\frac{3}{2}}de^2} + \frac{3\,cdx}{28\,(ex^2+d)^{\frac{7}{2}}e^2}$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(9/2),x, algorithm="maxima")
```

output
```
1/4*c*x^3/((e*x^2 + d)^(7/2)*e) + 16/35*a*x/(sqrt(e*x^2 + d)*d^4) + 8/35*a
*x/((e*x^2 + d)^(3/2)*d^3) + 6/35*a*x/((e*x^2 + d)^(5/2)*d^2) + 1/7*a*x/((
e*x^2 + d)^(7/2)*d) - 3/140*c*x/((e*x^2 + d)^(5/2)*e^2) - 2/35*c*x/(sqrt(e
*x^2 + d)*d^2*e^2) - 1/35*c*x/((e*x^2 + d)^(3/2)*d*e^2) + 3/28*c*d*x/((e*x
^2 + d)^(7/2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.68

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{9/2}}\,dx = -\frac{\left(\left(x^2\left(\frac{2\left(cd^2e^4-8\,ae^6\right)x^2}{d^4e^3}+\frac{7\left(cd^3e^3-8\,ade^5\right)}{d^4e^3}\right)-\frac{70\,ae}{d^2}\right)x^2-\frac{35\,a}{d}\right)x}{35\left(ex^2+d\right)^{\frac{7}{2}}}$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(9/2),x, algorithm="giac")
```

output
```
-1/35*((x^2*(2*(c*d^2*e^4 - 8*a*e^6)*x^2/(d^4*e^3) + 7*(c*d^3*e^3 - 8*a*d*
e^5)/(d^4*e^3)) - 70*a*e/d^2)*x^2 - 35*a/d)*x/(e*x^2 + d)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 17.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{9/2}}\,dx = \frac{x\left(\frac{a}{7\,d}-\frac{c\,d}{7\,e^2}\right)}{\left(e\,x^2+d\right)^{7/2}}+\frac{x\left(\frac{c}{5\,e^2}+\frac{c\,d^2+6\,a\,e^2}{35\,d^2\,e^2}\right)}{\left(e\,x^2+d\right)^{5/2}} + \frac{x\left(8\,a\,e^2-c\,d^2\right)}{35\,d^3\,e^2\left(e\,x^2+d\right)^{3/2}}+\frac{x\left(16\,a\,e^2-2\,c\,d^2\right)}{35\,d^4\,e^2\sqrt{e\,x^2+d}}$$

input
```
int((a - c*x^4)/(d + e*x^2)^(9/2),x)
```

output
```
(x*(a/(7*d) - (c*d)/(7*e^2)))/(d + e*x^2)^(7/2) + (x*(c/(5*e^2) + (6*a*e^2
 + c*d^2)/(35*d^2*e^2)))/(d + e*x^2)^(5/2) + (x*(8*a*e^2 - c*d^2))/(35*d^3
*e^2*(d + e*x^2)^(3/2)) + (x*(16*a*e^2 - 2*c*d^2))/(35*d^4*e^2*(d + e*x^2)
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.29

$$\int \frac{a-cx^4}{(d+ex^2)^{9/2}}\,dx = \frac{35\sqrt{e\,x^2+d}\,a\,d^3e^3x+70\sqrt{e\,x^2+d}\,a\,d^2e^4x^3+56\sqrt{e\,x^2+d}\,ad\,e^5x^5+16\sqrt{e\,x^2+d}\,a\,e}{}$$

input
```
int((-c*x^4+a)/(e*x^2+d)^(9/2),x)
```

output
```
(35*sqrt(d + e*x**2)*a*d**3*e**3*x + 70*sqrt(d + e*x**2)*a*d**2*e**4*x**3
+ 56*sqrt(d + e*x**2)*a*d*e**5*x**5 + 16*sqrt(d + e*x**2)*a*e**6*x**7 - 7*
sqrt(d + e*x**2)*c*d**3*e**3*x**5 - 2*sqrt(d + e*x**2)*c*d**2*e**4*x**7 -
16*sqrt(e)*a*d**4*e**2 - 64*sqrt(e)*a*d**3*e**3*x**2 - 96*sqrt(e)*a*d**2*e
**4*x**4 - 64*sqrt(e)*a*d*e**5*x**6 - 16*sqrt(e)*a*e**6*x**8 + 2*sqrt(e)*c
*d**6 + 8*sqrt(e)*c*d**5*e*x**2 + 12*sqrt(e)*c*d**4*e**2*x**4 + 8*sqrt(e)*
c*d**3*e**3*x**6 + 2*sqrt(e)*c*d**2*e**4*x**8)/(35*d**4*e**3*(d**4 + 4*d**
3*e*x**2 + 6*d**2*e**2*x**4 + 4*d*e**3*x**6 + e**4*x**8))
```

3.331 $\int \frac{a-cx^4}{\left(d+ex^2\right)^{11/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 160

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{11/2}} \, dx = \frac{\left(a-\frac{cd^2}{e^2}\right)x}{9d\left(d+ex^2\right)^{9/2}} + \frac{2\left(\frac{4a}{d^2}+\frac{5c}{e^2}\right)x}{63\left(d+ex^2\right)^{7/2}} + \frac{\left(\frac{16a}{d^2}-\frac{c}{e^2}\right)x}{105d\left(d+ex^2\right)^{5/2}} - \frac{4(cd^2-16ae^2)\,x}{315d^4e^2\left(d+ex^2\right)^{3/2}} - \frac{8(cd^2-16ae^2)\,x}{315d^5e^2\sqrt{d+ex^2}}$$

output

```
1/9*(a-c*d^2/e^2)*x/d/(e*x^2+d)^(9/2)+2/63*(4*a/d^2+5*c/e^2)*x/(e*x^2+d)^(
7/2)+1/105*(16*a/d^2-c/e^2)*x/d/(e*x^2+d)^(5/2)-4/315*(-16*a*e^2+c*d^2)*x/
d^4/e^2/(e*x^2+d)^(3/2)-8/315*(-16*a*e^2+c*d^2)*x/d^5/e^2/(e*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.59

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{11/2}} \, dx = \frac{-cd^2x^5(63d^2+36dex^2+8e^2x^4)+a(315d^4x+840d^3ex^3+1008d^2e^2x^5+576de^3x^7+}{315d^5\left(d+ex^2\right)^{9/2}}$$

input

```
Integrate[(a - c*x^4)/(d + e*x^2)^(11/2),x]
```

output
```
(-(c*d^2*x^5*(63*d^2 + 36*d*e*x^2 + 8*e^2*x^4)) + a*(315*d^4*x + 840*d^3*e
*x^3 + 1008*d^2*e^2*x^5 + 576*d*e^3*x^7 + 128*e^4*x^9))/(315*d^5*(d + e*x^
2)^(9/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1470, 362, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{11/2}}dx$$

$$\Big\downarrow \text{1470}$$

$$\frac{\int \frac{x^2\left(8ae-cdx^2\right)}{\left(ex^2+d\right)^{11/2}}dx}{d}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{x^3\left(\frac{8ae}{d}+\frac{cd}{e}\right)}{9\left(d+ex^2\right)^{9/2}}-\frac{1}{3}\left(\frac{cd}{e}-\frac{16ae}{d}\right)\int \frac{x^2}{\left(ex^2+d\right)^{9/2}}dx}{d}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{x^3\left(\frac{8ae}{d}+\frac{cd}{e}\right)}{9\left(d+ex^2\right)^{9/2}}-\frac{1}{3}\left(\frac{cd}{e}-\frac{16ae}{d}\right)\left(\frac{4e\int \frac{x^4}{\left(ex^2+d\right)^{9/2}}dx}{3d}+\frac{x^3}{3d\left(d+ex^2\right)^{7/2}}\right)}{d}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{x^3\left(\frac{8ae}{d}+\frac{cd}{e}\right)}{9(d+ex^2)^{9/2}}-\frac{1}{3}\left(\frac{cd}{e}-\frac{16ae}{d}\right)\left(\frac{4e\left(\frac{2e\int\frac{x^6}{\left(ex^2+d\right)^{9/2}}dx}{5d}+\frac{x^5}{5d\left(d+ex^2\right)^{7/2}}\right)}{3d}+\frac{x^3}{3d(d+ex^2)^{7/2}}\right)}{d}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{\frac{x^3\left(\frac{8ae}{d}+\frac{cd}{e}\right)}{9(d+ex^2)^{9/2}}-\frac{1}{3}\left(\frac{4e\left(\frac{2ex^7}{35d^2\left(d+ex^2\right)^{7/2}}+\frac{x^5}{5d\left(d+ex^2\right)^{7/2}}\right)}{3d}+\frac{x^3}{3d(d+ex^2)^{7/2}}\right)\left(\frac{cd}{e}-\frac{16ae}{d}\right)}{d}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

input
```
Int[(a - c*x^4)/(d + e*x^2)^(11/2),x]
```

output
```
(a*x)/(d*(d + e*x^2)^(9/2)) + (((((c*d)/e + (8*a*e)/d)*x^3)/(9*(d + e*x^2)^
(9/2)) - (((c*d)/e - (16*a*e)/d)*(x^3/(3*d*(d + e*x^2)^(7/2)) + (4*e*(x^5/
(5*d*(d + e*x^2)^(7/2)) + (2*e*x^7)/(35*d^2*(d + e*x^2)^(7/2))))/(3*d)))/3
)/d
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1470
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d + e*x^2)^q*(d*
PolynomialQuotient[(a + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q
+ 1/2, 0] && LtQ[4*p + 2*q + 1, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

method	result
pseudoelliptic	$\frac{\left(\left(-\frac{c\,x^{4}}{5}+a\right) d^{4}+\frac{8 x^{2} \left(-\frac{3 c\,x^{4}}{70}+a\right) e\,d^{3}}{3}+\frac{16 x^{4} \left(-\frac{c\,x^{4}}{126}+a\right) e^{2} d^{2}}{5}+\frac{64 a d\,e^{3} x^{6}}{35}+\frac{128 a\,e^{4} x^{8}}{315}\right) x}{\left(e\,x^{2}+d\right)^{\frac{9}{2}} d^{5}}$
gosper	$\frac{x \left(128 a\,e^{4} x^{8}-8 c\,d^{2} e^{2} x^{8}+576 a d\,e^{3} x^{6}-36 c\,d^{3} e\,x^{6}+1008 a\,d^{2} e^{2} x^{4}-63 c\,d^{4} x^{4}+840 a\,d^{3} e\,x^{2}+315 d^{4} a\right)}{315 \left(e\,x^{2}+d\right)^{\frac{9}{2}} d^{5}}$
trager	$\frac{x \left(128 a\,e^{4} x^{8}-8 c\,d^{2} e^{2} x^{8}+576 a d\,e^{3} x^{6}-36 c\,d^{3} e\,x^{6}+1008 a\,d^{2} e^{2} x^{4}-63 c\,d^{4} x^{4}+840 a\,d^{3} e\,x^{2}+315 d^{4} a\right)}{315 \left(e\,x^{2}+d\right)^{\frac{9}{2}} d^{5}}$
orering	$\frac{x \left(128 a\,e^{4} x^{8}-8 c\,d^{2} e^{2} x^{8}+576 a d\,e^{3} x^{6}-36 c\,d^{3} e\,x^{6}+1008 a\,d^{2} e^{2} x^{4}-63 c\,d^{4} x^{4}+840 a\,d^{3} e\,x^{2}+315 d^{4} a\right)}{315 \left(e\,x^{2}+d\right)^{\frac{9}{2}} d^{5}}$
default	$a \left(\frac{x}{9 d \left(e\,x^{2}+d\right)^{\frac{9}{2}}}+\frac{\frac{8 x}{63 d \left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{8 \left(\frac{6 x}{35 d \left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{6 \left(\frac{4 x}{15 d \left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8 x}{15 d^{2} \sqrt{e\,x^{2}+d}}\right)}{7 d}\right)}{9 d}}{d}\right)-c \left(-\frac{x^{3}}{6 e \left(e\,x^{2}+d\right)^{\frac{9}{2}}}\right.$

input
```
int((-c*x^4+a)/(e*x^2+d)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
((-1/5*c*x^4+a)*d^4+8/3*x^2*(-3/70*c*x^4+a)*e*d^3+16/5*x^4*(-1/126*c*x^4+a
)*e^2*d^2+64/35*a*d*e^3*x^6+128/315*a*e^4*x^8)*x/(e*x^2+d)^(9/2)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{11/2}}\,dx =$$
$$-\frac{\left(8\left(cd^2e^2-16\,ae^4\right)x^9-840\,ad^3ex^3+36\left(cd^3e-16\,ade^3\right)x^7-315\,ad^4x+63\left(cd^4-16\,ad^2e^2\right)x^5\right)\sqrt{ex^2+}}{315\left(d^5e^5x^{10}+5\,d^6e^4x^8+10\,d^7e^3x^6+10\,d^8e^2x^4+5\,d^9ex^2+d^{10}\right)}$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(11/2),x, algorithm="fricas")
```

output
```
-1/315*(8*(c*d^2*e^2 - 16*a*e^4)*x^9 - 840*a*d^3*e*x^3 + 36*(c*d^3*e - 16*
a*d*e^3)*x^7 - 315*a*d^4*x + 63*(c*d^4 - 16*a*d^2*e^2)*x^5)*sqrt(e*x^2 + d
)/(d^5*e^5*x^10 + 5*d^6*e^4*x^8 + 10*d^7*e^3*x^6 + 10*d^8*e^2*x^4 + 5*d^9*
e*x^2 + d^10)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3738 vs. 2(155) = 310.

Time = 47.78 (sec) , antiderivative size = 3738, normalized size of antiderivative = 23.36

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-c*x**4+a)/(e*x**2+d)**(11/2),x)
```

output

```
a*(315*d**30*x/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e*x**2*s
qrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 37800*d
**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*sqrt(1 +
 e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*d**(59/
2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(1 + e*x
**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(53/2)*e*
*9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*x**2/d)
) + 2730*d**29*e*x**3/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e
*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) +
37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*s
qrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*
d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(
1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(5
3/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*
x**2/d)) + 10773*d**28*e**2*x**5/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*
d**(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e
*x**2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*
e**4*x**8*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/
d) + 66150*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*
x**14*sqrt(1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.18

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{11/2}}\,dx = \frac{cx^3}{6\,(ex^2+d)^{\frac{9}{2}}e} + \frac{128\,ax}{315\,\sqrt{ex^2+d}d^5} + \frac{64\,ax}{315\,(ex^2+d)^{\frac{3}{2}}d^4} + \frac{16\,ax}{105\,(ex^2+d)^{\frac{5}{2}}d^3} + \frac{8\,ax}{63\,(ex^2+d)^{\frac{7}{2}}d^2} + \frac{ax}{9\,(ex^2+d)^{\frac{9}{2}}d} - \frac{cx}{126\,(ex^2+d)^{\frac{7}{2}}e^2} - \frac{8\,cx}{315\,\sqrt{ex^2+d}d^3e^2} - \frac{4\,cx}{315\,(ex^2+d)^{\frac{3}{2}}d^2e^2} - \frac{cx}{105\,(ex^2+d)^{\frac{5}{2}}de^2} + \frac{cdx}{18\,(ex^2+d)^{\frac{9}{2}}e^2}$$

input

```
integrate((-c*x^4+a)/(e*x^2+d)^(11/2),x, algorithm="maxima")
```

output
```
1/6*c*x^3/((e*x^2 + d)^(9/2)*e) + 128/315*a*x/(sqrt(e*x^2 + d)*d^5) + 64/3
15*a*x/((e*x^2 + d)^(3/2)*d^4) + 16/105*a*x/((e*x^2 + d)^(5/2)*d^3) + 8/63
*a*x/((e*x^2 + d)^(7/2)*d^2) + 1/9*a*x/((e*x^2 + d)^(9/2)*d) - 1/126*c*x/(
(e*x^2 + d)^(7/2)*e^2) - 8/315*c*x/(sqrt(e*x^2 + d)*d^3*e^2) - 4/315*c*x/(
(e*x^2 + d)^(3/2)*d^2*e^2) - 1/105*c*x/((e*x^2 + d)^(5/2)*d*e^2) + 1/18*c*
d*x/((e*x^2 + d)^(9/2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{11/2}}\,dx = -\frac{\left(\left(\left(4\,x^2\left(\frac{2\,(cd^2e^6-16\,ae^8)x^2}{d^5e^4}+\frac{9\,(cd^3e^5-16\,ade^7)}{d^5e^4}\right)+\frac{63\,(cd^4e^4-16\,ad^2e^6)}{d^5e^4}\right)x^2-\frac{840\,ae}{d^2}\right)x^2-\frac{315\,a}{d}\right)x}{315\,\left(ex^2+d\right)^{\frac{9}{2}}}$$

input
```
integrate((-c*x^4+a)/(e*x^2+d)^(11/2),x, algorithm="giac")
```

output
```
-1/315*(((4*x^2*(2*(c*d^2*e^6 - 16*a*e^8)*x^2/(d^5*e^4) + 9*(c*d^3*e^5 - 1
6*a*d*e^7)/(d^5*e^4)) + 63*(c*d^4*e^4 - 16*a*d^2*e^6)/(d^5*e^4))*x^2 - 840
*a*e/d^2)*x^2 - 315*a/d)*x/(e*x^2 + d)^(9/2)
```

Mupad [B] (verification not implemented)

Time = 17.85 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98

$$\int \frac{a-cx^4}{\left(d+ex^2\right)^{11/2}}\,dx = \frac{x\left(\frac{a}{9\,d}-\frac{c\,d}{9\,e^2}\right)}{\left(e\,x^2+d\right)^{9/2}}+\frac{x\left(\frac{c}{7\,e^2}+\frac{c\,d^2+8\,a\,e^2}{63\,d^2\,e^2}\right)}{\left(e\,x^2+d\right)^{7/2}} + \frac{x\,\left(16\,a\,e^2-c\,d^2\right)}{105\,d^3\,e^2\,\left(e\,x^2+d\right)^{5/2}}+\frac{x\,\left(64\,a\,e^2-4\,c\,d^2\right)}{315\,d^4\,e^2\,\left(e\,x^2+d\right)^{3/2}}+\frac{x\,\left(128\,a\,e^2-8\,c\,d^2\right)}{315\,d^5\,e^2\,\sqrt{e\,x^2+d}}$$

input
```
int((a - c*x^4)/(d + e*x^2)^(11/2),x)
```

output
```
(x*(a/(9*d) - (c*d)/(9*e^2)))/(d + e*x^2)^(9/2) + (x*(c/(7*e^2) + (8*a*e^2
 + c*d^2)/(63*d^2*e^2)))/(d + e*x^2)^(7/2) + (x*(16*a*e^2 - c*d^2))/(105*d
^3*e^2*(d + e*x^2)^(5/2)) + (x*(64*a*e^2 - 4*c*d^2))/(315*d^4*e^2*(d + e*x
^2)^(3/2)) + (x*(128*a*e^2 - 8*c*d^2))/(315*d^5*e^2*(d + e*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.29

$$\int \frac{a-cx^4}{(d+ex^2)^{11/2}}\,dx = \frac{315\sqrt{e\,x^2+d}\,a\,d^4e^3x+840\sqrt{e\,x^2+d}\,a\,d^3e^4x^3+1008\sqrt{e\,x^2+d}\,a\,d^2e^5x^5+576\sqrt{e\,x^2}}{}$$

input
```
int((-c*x^4+a)/(e*x^2+d)^(11/2),x)
```

output
```
(315*sqrt(d + e*x**2)*a*d**4*e**3*x + 840*sqrt(d + e*x**2)*a*d**3*e**4*x**
3 + 1008*sqrt(d + e*x**2)*a*d**2*e**5*x**5 + 576*sqrt(d + e*x**2)*a*d*e**6
*x**7 + 128*sqrt(d + e*x**2)*a*e**7*x**9 - 63*sqrt(d + e*x**2)*c*d**4*e**3
*x**5 - 36*sqrt(d + e*x**2)*c*d**3*e**4*x**7 - 8*sqrt(d + e*x**2)*c*d**2*e
**5*x**9 - 128*sqrt(e)*a*d**5*e**2 - 640*sqrt(e)*a*d**4*e**3*x**2 - 1280*s
qrt(e)*a*d**3*e**4*x**4 - 1280*sqrt(e)*a*d**2*e**5*x**6 - 640*sqrt(e)*a*d*
e**6*x**8 - 128*sqrt(e)*a*e**7*x**10 + 8*sqrt(e)*c*d**7 + 40*sqrt(e)*c*d**
6*e*x**2 + 80*sqrt(e)*c*d**5*e**2*x**4 + 80*sqrt(e)*c*d**4*e**3*x**6 + 40*
sqrt(e)*c*d**3*e**4*x**8 + 8*sqrt(e)*c*d**2*e**5*x**10)/(315*d**5*e**3*(d*
*5 + 5*d**4*e*x**2 + 10*d**3*e**2*x**4 + 10*d**2*e**3*x**6 + 5*d*e**4*x**8
 + e**5*x**10))
```

3.332 $\int \left(d+ex^2\right)^{3/2} \left(a-cx^4\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 270

$$\begin{aligned}\int \left(d+ex^2\right)^{3/2} \left(a-cx^4\right)^2 \, dx &= \frac{d(7c^2d^4-48acd^2e^2+384a^2e^4)\, x\sqrt{d+ex^2}}{1024e^4} \\ &+ \frac{(7c^2d^4-48acd^2e^2+384a^2e^4)\, x(d+ex^2)^{3/2}}{1536e^4} - \frac{cd(7cd^2-48ae^2)\, x(d+ex^2)^{5/2}}{384e^4} \\ &+ \frac{c(7cd^2-48ae^2)\, x^3(d+ex^2)^{5/2}}{192e^3} - \frac{7c^2dx^5(d+ex^2)^{5/2}}{120e^2} \\ &+ \frac{c^2x^7(d+ex^2)^{5/2}}{12e} + \frac{d^2(7c^2d^4-48acd^2e^2+384a^2e^4)\,\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{1024e^{9/2}}\end{aligned}$$

output

```
1/1024*d*(384*a^2*e^4-48*a*c*d^2*e^2+7*c^2*d^4)*x*(e*x^2+d)^(1/2)/e^4+1/15
36*(384*a^2*e^4-48*a*c*d^2*e^2+7*c^2*d^4)*x*(e*x^2+d)^(3/2)/e^4-1/384*c*d*
(-48*a*e^2+7*c*d^2)*x*(e*x^2+d)^(5/2)/e^4+1/192*c*(-48*a*e^2+7*c*d^2)*x^3*
(e*x^2+d)^(5/2)/e^3-7/120*c^2*d*x^5*(e*x^2+d)^(5/2)/e^2+1/12*c^2*x^7*(e*x^
2+d)^(5/2)/e+1/1024*d^2*(384*a^2*e^4-48*a*c*d^2*e^2+7*c^2*d^4)*arctanh(e^(
1/2)*x/(e*x^2+d)^(1/2))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.73

$$\int \left(d+ex^2\right)^{3/2} \left(a-cx^4\right)^2 \, dx = \frac{\sqrt{e}x\sqrt{d+ex^2}(1920a^2e^4(5d+2ex^2)-240ace^2(-3d^3+2d^2ex^2+24de^2x^4+16e^3x^6)+c^2(-1}{}$$

input
```
Integrate[(d + e*x^2)^(3/2)*(a - c*x^4)^2,x]
```

output
```
(Sqrt[e]*x*Sqrt[d + e*x^2]*(1920*a^2*e^4*(5*d + 2*e*x^2) - 240*a*c*e^2*(-3
*d^3 + 2*d^2*e*x^2 + 24*d*e^2*x^4 + 16*e^3*x^6) + c^2*(-105*d^5 + 70*d^4*e
*x^2 - 56*d^3*e^2*x^4 + 48*d^2*e^3*x^6 + 1664*d*e^4*x^8 + 1280*e^5*x^10))
- 15*(7*c^2*d^6 - 48*a*c*d^4*e^2 + 384*a^2*d^2*e^4)*Log[-(Sqrt[e]*x) + Sqr
t[d + e*x^2]])/(15360*e^(9/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1474, 2346, 27, 1474, 27, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)^2 \left(d+ex^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1474}$$

$$\frac{\int \left(ex^2+d\right)^{3/2} \left(-7c^2dx^6-24acex^4+12a^2e\right)dx}{12e}+\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{\int 5\left(ex^2+d\right)^{3/2}\left(c\left(7cd^2-48ae^2\right)x^4+24a^2e^2\right)dx}{10e}-\frac{7c^2dx^5\left(d+ex^2\right)^{5/2}}{10e}}{12e}+\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int (ex^2+d)^{3/2}(c(7cd^2-48ae^2)x^4+24a^2e^2)dx}{2e}-\frac{7c^2dx^5(d+ex^2)^{5/2}}{10e}}{12e}+\frac{c^2x^7(d+ex^2)^{5/2}}{12e}$$

$$\Big\downarrow \; 1474$$

$$\frac{\frac{\frac{\int 3\left(ex^2+d\right)^{3/2}\left(64a^2e^3-cd\left(7cd^2-48ae^2\right)x^2\right)dx}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}\left(7cd^2-48ae^2\right)}{8e}}{2e}-\frac{7c^2dx^5(d+ex^2)^{5/2}}{10e}}{12e}+$$

$$\frac{c^2x^7(d+ex^2)^{5/2}}{12e}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\int \left(ex^2+d\right)^{3/2}\left(64a^2e^3-cd\left(7cd^2-48ae^2\right)x^2\right)dx}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}\left(7cd^2-48ae^2\right)}{8e}}{2e}-\frac{7c^2dx^5(d+ex^2)^{5/2}}{10e}}{12e}+$$

$$\frac{c^2x^7(d+ex^2)^{5/2}}{12e}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{\frac{3\left(\frac{\left(384a^2e^4-48acd^2e^2+7c^2d^4\right)\int \left(ex^2+d\right)^{3/2}dx}{6e}-\frac{cdx\left(d+ex^2\right)^{5/2}\left(7cd^2-48ae^2\right)}{6e}\right)}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}\left(7cd^2-48ae^2\right)}{8e}}{2e}-\frac{7c^2dx^5(d+ex^2)^{5/2}}{10e}}{12e}+$$

$$\frac{c^2x^7(d+ex^2)^{5/2}}{12e}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{\frac{3\left(\frac{\left(384a^2e^4-48acd^2e^2+7c^2d^4\right)\left(\frac{3}{4}d\int \sqrt{ex^2+d}dx+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{6e}-\frac{cdx\left(d+ex^2\right)^{5/2}\left(7cd^2-48ae^2\right)}{6e}\right)}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}\left(7cd^2-48ae^2\right)}{8e}}{2e}-\frac{7c^2dx^5(d+ex}{10e}}{12e}$$

$$\frac{c^2x^7(d+ex^2)^{5/2}}{12e}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{\frac{3\left(\frac{\left(384a^2e^4-48acd^2e^2+7c^2d^4\right)\left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{6e}-\frac{cdx\left(d+ex^2\right)^{5/2}\left(7cd^2-48ae^2\right)}{6e}\right)}{8e}+\frac{cx^3(d+ex^2)^{5/2}(7cd^2-48ae^2}{8e}}{2e}}{12e}$$

$$\frac{c^2x^7(d+ex^2)^{5/2}}{12e}$$

$$\Big\downarrow \color{blue}{224}$$

$$\frac{\frac{\frac{3\left(\frac{\left(384a^2e^4-48acd^2e^2+7c^2d^4\right)\left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{6e}-\frac{cdx\left(d+ex^2\right)^{5/2}\left(7cd^2-48ae^2\right)}{6e}\right)}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}(7c}{8e}}{2e}}{12e}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

$$\Big\downarrow \color{blue}{219}$$

$$\frac{\frac{\frac{3\left(\frac{\left(384a^2e^4-48acd^2e^2+7c^2d^4\right)\left(\frac{3}{4}d\left(\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{6e}-\frac{cdx\left(d+ex^2\right)^{5/2}\left(7cd^2-48ae^2\right)}{6e}\right)}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}(7cd}{8e}}{2e}}{12e}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

input
```
Int[(d + e*x^2)^(3/2)*(a - c*x^4)^2,x]
```

output
```
(c^2*x^7*(d + e*x^2)^(5/2))/(12*e) + ((-7*c^2*d*x^5*(d + e*x^2)^(5/2))/(10
*e) + ((c*(7*c*d^2 - 48*a*e^2)*x^3*(d + e*x^2)^(5/2))/(8*e) + (3*(-1/6*(c*
d*(7*c*d^2 - 48*a*e^2)*x*(d + e*x^2)^(5/2))/e + ((7*c^2*d^4 - 48*a*c*d^2*e
^2 + 384*a^2*e^4)*((x*(d + e*x^2)^(3/2))/4 + (3*d*((x*Sqrt[d + e*x^2])/2 +
 (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/4))/(6*e)))/(8*e))
/(2*e))/(12*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{\frac{3d^2\left(a^2e^4-\frac{1}{8}ac\,d^2e^2+\frac{7}{384}c^2d^4\right)\operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}}{x\sqrt{e}}\right)}{8}+\frac{5x\sqrt{e\,x^2+d}\left(\frac{2x^2\left(\frac{1}{3}c^2x^8-ac\,x^4+a^2\right)e^{\frac{11}{2}}}{5}+\left(\left(\frac{13}{75}c^2x^8-\frac{3}{5}ac\,x^4+a^2\right)e^{\frac{9}{2}}+\frac{3d}{}\right.\right.}{8}}{e^{\frac{9}{2}}}$
risch	$\frac{x(1280e^5c^2x^{10}+1664d\,c^2e^4x^8-3840ac\,e^5x^6+48c^2d^2e^3x^6-5760acd\,e^4x^4-56c^2d^3e^2x^4+3840a^2e^5x^2-480ac\,d^2e^3x^2+70c^2}{15360e^4}$
default	$a^2\left(\frac{x\left(e\,x^2+d\right)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2\sqrt{e}}\right)}{4}\right)+c^2\left(\frac{x^7\left(e\,x^2+d\right)^{\frac{5}{2}}}{12e}-\frac{7d\left(\frac{x^5\left(e\,x^2+d\right)^{\frac{5}{2}}}{10e}-\frac{d\left(\frac{x^3\left(e\,x^{\right.}}{8}\right.}{}\right.}{}\right.$

input
```
int((e*x^2+d)^(3/2)*(-c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
5/8/e^(9/2)*(3/5*d^2*(a^2*e^4-1/8*a*c*d^2*e^2+7/384*c^2*d^4)*arctanh((e*x^
2+d)^(1/2)/x/e^(1/2))+x*(e*x^2+d)^(1/2)*(2/5*x^2*(1/3*c^2*x^8-a*c*x^4+a^2)
*e^(11/2)+((13/75*c^2*x^8-3/5*a*c*x^4+a^2)*e^(9/2)+3/40*d*(d*(-7/90*c*x^4+
a)*e^(5/2)-2/3*x^2*(-1/10*c*x^4+a)*e^(7/2)-7/48*d^2*c*(-2/3*e^(3/2)*x^2+e^
(1/2)*d))*c)*d))
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.59

$$\int \left(d+ex^2\right)^{3/2} (a$$

$$-cx^4)^2\,dx = \left[\frac{15\,(7\,c^2d^6 - 48\,acd^4e^2 + 384\,a^2d^2e^4)\sqrt{e}\log\left(-2\,ex^2 - 2\,\sqrt{ex^2+d}\sqrt{e}x - d\right) + 2\,(1280\,c^2e^6x^{1}}{}\right.$$

$$-\frac{15\,(7\,c^2d^6 - 48\,acd^4e^2 + 384\,a^2d^2e^4)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - (1280\,c^2e^6x^{11} + 1664\,c^2de^5x^9 + 48\,(c^2d^2e^4 -}{}$$

input
```
integrate((e*x^2+d)^(3/2)*(-c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/30720*(15*(7*c^2*d^6 - 48*a*c*d^4*e^2 + 384*a^2*d^2*e^4)*sqrt(e)*log(-2
*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(1280*c^2*e^6*x^11 + 1664*c^
2*d*e^5*x^9 + 48*(c^2*d^2*e^4 - 80*a*c*e^6)*x^7 - 8*(7*c^2*d^3*e^3 + 720*a
*c*d*e^5)*x^5 + 10*(7*c^2*d^4*e^2 - 48*a*c*d^2*e^4 + 384*a^2*e^6)*x^3 - 15
*(7*c^2*d^5*e - 48*a*c*d^3*e^3 - 640*a^2*d*e^5)*x)*sqrt(e*x^2 + d))/e^5, -
1/15360*(15*(7*c^2*d^6 - 48*a*c*d^4*e^2 + 384*a^2*d^2*e^4)*sqrt(-e)*arctan
(sqrt(-e)*x/sqrt(e*x^2 + d)) - (1280*c^2*e^6*x^11 + 1664*c^2*d*e^5*x^9 + 4
8*(c^2*d^2*e^4 - 80*a*c*e^6)*x^7 - 8*(7*c^2*d^3*e^3 + 720*a*c*d*e^5)*x^5 +
 10*(7*c^2*d^4*e^2 - 48*a*c*d^2*e^4 + 384*a^2*e^6)*x^3 - 15*(7*c^2*d^5*e -
 48*a*c*d^3*e^3 - 640*a^2*d*e^5)*x)*sqrt(e*x^2 + d))/e^5]
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.44

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^2\,dx = \begin{cases}\sqrt{d+ex^2}\cdot\left(\frac{13c^2dx^9}{120}+\frac{c^2ex^{11}}{12}+\frac{x^7\left(-2ace^2+\frac{c^2d^2}{40}\right)}{8e}+\frac{x^5\left(-4acde-\frac{7d\left(-2ace^2+\frac{c^2d^2}{40}\right)}{8e}\right)}{6e}+\frac{x^3\left(a^2e^2-2acd^2-\right.}{}\right. \\ d^{\frac{3}{2}}\left(a^2x-\frac{2acx^5}{5}+\frac{c^2x^9}{9}\right)\end{cases}$$

input
```
integrate((e*x**2+d)**(3/2)*(-c*x**4+a)**2,x)
```

output
```
Piecewise((sqrt(d + e*x**2)*(13*c**2*d*x**9/120 + c**2*e*x**11/12 + x**7*(
-2*a*c*e**2 + c**2*d**2/40)/(8*e) + x**5*(-4*a*c*d*e - 7*d*(-2*a*c*e**2 +
c**2*d**2/40)/(8*e))/(6*e) + x**3*(a**2*e**2 - 2*a*c*d**2 - 5*d*(-4*a*c*d*
e - 7*d*(-2*a*c*e**2 + c**2*d**2/40)/(8*e))/(6*e))/(4*e) + x*(2*a**2*d*e -
 3*d*(a**2*e**2 - 2*a*c*d**2 - 5*d*(-4*a*c*d*e - 7*d*(-2*a*c*e**2 + c**2*d
**2/40)/(8*e))/(6*e))/(4*e))/(2*e)) + (a**2*d**2 - d*(2*a**2*d*e - 3*d*(a*
*2*e**2 - 2*a*c*d**2 - 5*d*(-4*a*c*d*e - 7*d*(-2*a*c*e**2 + c**2*d**2/40)/
(8*e))/(6*e))/(4*e))/(2*e))*Piecewise((log(2*sqrt(e)*sqrt(d + e*x**2) + 2*
e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqrt(e*x**2), True)), Ne(e, 0)), (d**(3
/2)*(a**2*x - 2*a*c*x**5/5 + c**2*x**9/9), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(-c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.83

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^2\,dx=\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,c^2ex^2+13\,c^2d\right)x^2+\frac{3\left(c^2d^2e^9-80\,ace^{11}\right)}{e^{10}}\right)x^2-\frac{7\,c^2d^3e^8+720\,acde^1}{e^{10}}\right.\right.\right.\right.$$
$$-\frac{\left(7\,c^2d^6-48\,acd^4e^2+384\,a^2d^2e^4\right)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{1024\,e^{\frac{9}{2}}}$$

input
```
integrate((e*x^2+d)^(3/2)*(-c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/15360*(2*(4*(2*(8*(10*c^2*e*x^2 + 13*c^2*d)*x^2 + 3*(c^2*d^2*e^9 - 80*a*
c*e^11)/e^10)*x^2 - (7*c^2*d^3*e^8 + 720*a*c*d*e^10)/e^10)*x^2 + 5*(7*c^2*
d^4*e^7 - 48*a*c*d^2*e^9 + 384*a^2*e^11)/e^10)*x^2 - 15*(7*c^2*d^5*e^6 - 4
8*a*c*d^3*e^8 - 640*a^2*d*e^10)/e^10)*sqrt(e*x^2 + d)*x - 1/1024*(7*c^2*d^
6 - 48*a*c*d^4*e^2 + 384*a^2*d^2*e^4)*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)
))/e^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2} \left(a-cx^4\right)^2 \, dx = \int \left(a-c\,x^4\right)^2 \left(e\,x^2+d\right)^{3/2} \,dx$$

input
```
int((a - c*x^4)^2*(d + e*x^2)^(3/2),x)
```

output
```
int((a - c*x^4)^2*(d + e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.24

$$\int \left(d+ex^2\right)^{3/2} \left(a -cx^4\right)^2 \, dx = \frac{9600\sqrt{e\,x^2+d}\,a^2 d\,e^5 x+3840\sqrt{e\,x^2+d}\,a^2 e^6 x^3+720\sqrt{e\,x^2+d}\,ac\,d^3 e^3 x-480\sqrt{e\,x^2+d}\,ac\,d}{}$$

input
```
int((e*x^2+d)^(3/2)*(-c*x^4+a)^2,x)
```

output
```
(9600*sqrt(d + e*x**2)*a**2*d*e**5*x + 3840*sqrt(d + e*x**2)*a**2*e**6*x**
3 + 720*sqrt(d + e*x**2)*a*c*d**3*e**3*x - 480*sqrt(d + e*x**2)*a*c*d**2*e
**4*x**3 - 5760*sqrt(d + e*x**2)*a*c*d*e**5*x**5 - 3840*sqrt(d + e*x**2)*a
*c*e**6*x**7 - 105*sqrt(d + e*x**2)*c**2*d**5*e*x + 70*sqrt(d + e*x**2)*c*
*2*d**4*e**2*x**3 - 56*sqrt(d + e*x**2)*c**2*d**3*e**3*x**5 + 48*sqrt(d +
e*x**2)*c**2*d**2*e**4*x**7 + 1664*sqrt(d + e*x**2)*c**2*d*e**5*x**9 + 128
0*sqrt(d + e*x**2)*c**2*e**6*x**11 + 5760*sqrt(e)*log((sqrt(d + e*x**2) +
sqrt(e)*x)/sqrt(d))*a**2*d**2*e**4 - 720*sqrt(e)*log((sqrt(d + e*x**2) + s
qrt(e)*x)/sqrt(d))*a*c*d**4*e**2 + 105*sqrt(e)*log((sqrt(d + e*x**2) + sqr
t(e)*x)/sqrt(d))*c**2*d**6)/(15360*e**5)
```

3.333 $\int \sqrt{d+ex^2}\left(a-cx^4\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 221

$$\begin{aligned}\int \sqrt{d+ex^2}\left(a-cx^4\right)^2 \, dx = {} & \frac{\left(7c^2d^4-32acd^2e^2+128a^2e^4\right)x\sqrt{d+ex^2}}{256e^4} \\ & - \frac{cd\left(7cd^2-32ae^2\right)x\left(d+ex^2\right)^{3/2}}{128e^4} \\ & + \frac{c\left(7cd^2-32ae^2\right)x^3\left(d+ex^2\right)^{3/2}}{96e^3} \\ & - \frac{7c^2dx^5\left(d+ex^2\right)^{3/2}}{80e^2} + \frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e} \\ & + \frac{d\left(7c^2d^4-32acd^2e^2+128a^2e^4\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{256e^{9/2}}\end{aligned}$$

output

```
1/256*(128*a^2*e^4-32*a*c*d^2*e^2+7*c^2*d^4)*x*(e*x^2+d)^(1/2)/e^4-1/128*c
*d*(-32*a*e^2+7*c*d^2)*x*(e*x^2+d)^(3/2)/e^4+1/96*c*(-32*a*e^2+7*c*d^2)*x^
3*(e*x^2+d)^(3/2)/e^3-7/80*c^2*d*x^5*(e*x^2+d)^(3/2)/e^2+1/10*c^2*x^7*(e*x
^2+d)^(3/2)/e+1/256*d*(128*a^2*e^4-32*a*c*d^2*e^2+7*c^2*d^4)*arctanh(e^(1/
2)*x/(e*x^2+d)^(1/2))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.74

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)^2\,dx$$
$$=\frac{\sqrt{e}x\sqrt{d+ex^2}(1920a^2e^4-160ace^2(-3d^2+2dex^2+8e^2x^4)+c^2(-105d^4+70d^3ex^2-56d^2e^2x^4+48de^3}{3840e^{9/2}}$$

input
```
Integrate[Sqrt[d + e*x^2]*(a - c*x^4)^2,x]
```

output
```
(Sqrt[e]*x*Sqrt[d + e*x^2]*(1920*a^2*e^4 - 160*a*c*e^2*(-3*d^2 + 2*d*e*x^2
 + 8*e^2*x^4) + c^2*(-105*d^4 + 70*d^3*e*x^2 - 56*d^2*e^2*x^4 + 48*d*e^3*x
^6 + 384*e^4*x^8)) - 15*(7*c^2*d^5 - 32*a*c*d^3*e^2 + 128*a^2*d*e^4)*Log[-
(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(3840*e^(9/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1474, 2346, 27, 1474, 27, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)^2\sqrt{d+ex^2}\,dx$$
$$\Big\downarrow \text{1474}$$
$$\frac{\int \sqrt{ex^2+d}\left(-7c^2dx^6-20acex^4+10a^2e\right)dx}{10e}+\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$
$$\Big\downarrow \text{2346}$$
$$\frac{\frac{\int 5\sqrt{ex^2+d}\left(c\left(7cd^2-32ae^2\right)x^4+16a^2e^2\right)dx}{8e}-\frac{7c^2dx^5\left(d+ex^2\right)^{3/2}}{8e}}{10e}+\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$
$$\Big\downarrow \text{27}$$

$$\frac{\frac{5\int \sqrt{ex^2+d}\left(c\left(7cd^2-32ae^2\right)x^4+16a^2e^2\right)dx}{8e}-\frac{7c^2dx^5\left(d+ex^2\right)^{3/2}}{8e}}{10e}+\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

$\downarrow$ 1474

$$\frac{\frac{5\left(\frac{\int 3\sqrt{ex^2+d}\left(32a^2e^3-cd\left(7cd^2-32ae^2\right)x^2\right)dx}{6e}+\frac{cx^3\left(d+ex^2\right)^{3/2}\left(7cd^2-32ae^2\right)}{6e}\right)}{8e}-\frac{7c^2dx^5\left(d+ex^2\right)^{3/2}}{8e}}{10e}+\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

$\downarrow$ 27

$$\frac{\frac{5\left(\frac{\int \sqrt{ex^2+d}\left(32a^2e^3-cd\left(7cd^2-32ae^2\right)x^2\right)dx}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}\left(7cd^2-32ae^2\right)}{6e}\right)}{8e}-\frac{7c^2dx^5\left(d+ex^2\right)^{3/2}}{8e}}{10e}+\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

$\downarrow$ 299

$$\frac{\frac{5\left(\frac{\frac{\left(128a^2e^4-32acd^2e^2+7c^2d^4\right)\int \sqrt{ex^2+d}dx}{4e}-\frac{cdx\left(d+ex^2\right)^{3/2}\left(7cd^2-32ae^2\right)}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}\left(7cd^2-32ae^2\right)}{6e}\right)}{8e}-\frac{7c^2dx^5\left(d+ex^2\right)^{3/2}}{8e}}{10e}+\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

$\downarrow$ 211

$$\frac{\frac{5\left(\frac{\frac{\left(128a^2e^4-32acd^2e^2+7c^2d^4\right)\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}-\frac{cdx\left(d+ex^2\right)^{3/2}\left(7cd^2-32ae^2\right)}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}\left(7cd^2-32ae^2\right)}{6e}\right)}{8e}-\frac{7c^2dx^5\left(d+ex^2\right.}{8e}}{10e}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

$\downarrow$ 224

$$\frac{\frac{5\left(\frac{\frac{\left(128a^2e^4-32acd^2e^2+7c^2d^4\right)\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}-\frac{cdx\left(d+ex^2\right)^{3/2}\left(7cd^2-32ae^2\right)}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}\left(7cd^2-32ae^2\right)}{6e}\right)}{8e}-7c^2dx}{10e}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{5\left(\frac{\frac{\left(128a^2e^4-32acd^2e^2+7c^2d^4\right)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}-\frac{cdx\left(d+ex^2\right)^{3/2}\left(7cd^2-32ae^2\right)}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}\left(7cd^2-32ae^2\right)}{6e}\right)}{8e}-7c^2dx^5}{10e}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

input
```
Int[Sqrt[d + e*x^2]*(a - c*x^4)^2,x]
```

output
```
(c^2*x^7*(d + e*x^2)^(3/2))/(10*e) + ((-7*c^2*d*x^5*(d + e*x^2)^(3/2))/(8*
e) + (5*((c*(7*c*d^2 - 32*a*e^2)*x^3*(d + e*x^2)^(3/2))/(6*e) + (-1/4*(c*d
*(7*c*d^2 - 32*a*e^2)*x*(d + e*x^2)^(3/2))/e + ((7*c^2*d^4 - 32*a*c*d^2*e^
2 + 128*a^2*e^4)*((x*Sqrt[d + e*x^2])/2 + (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
e*x^2]])/(2*Sqrt[e])))/(4*e))/(2*e)))/(8*e))/(10*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

rule 2346

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{d\left(a^2e^4-\frac{1}{4}acd^2e^2+\frac{7}{128}c^2d^4\right)\operatorname{arctanh}\left(\frac{\sqrt{ex^2+d}}{x\sqrt{e}}\right)+x\left(\left(\frac{1}{5}c^2x^8-\frac{2}{3}acx^4+a^2\right)e^{\frac{9}{2}}+\frac{\left(d\left(-\frac{7cx^4}{60}+a\right)e^{\frac{5}{2}}-\frac{2x^2\left(-\frac{3cx^4}{20}+a\right)e^{\frac{7}{2}}}{3}-\right.}{4}\right.}{2e^{\frac{9}{2}}}$
risch	$\frac{x\left(384c^2e^4x^8+48dc^2e^3x^6-1280ace^4x^4-56c^2d^2e^2x^4-320acde^3x^2+70c^2d^3ex^2+1920a^2e^4+480acd^2e^2-105c^2d^4\right)\sqrt{ex^2+}}{3840e^4}$
default	$a^2\left(\frac{x\sqrt{ex^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)}{2\sqrt{e}}\right)+c^2\left(\frac{x^7\left(ex^2+d\right)^{\frac{3}{2}}}{10e}-\frac{7d\left(\frac{x^5\left(ex^2+d\right)^{\frac{3}{2}}}{8e}-\frac{5d\left(\frac{x^3\left(ex^2+d\right)^{\frac{3}{2}}}{6e}-\frac{d\left(\frac{x\left(ex^2\right.}{4e}\right.}{}\right.}{}\right.}{1}\right.$

input
```
int((e*x^2+d)^(1/2)*(-c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/e^(9/2)*(d*(a^2*e^4-1/4*a*c*d^2*e^2+7/128*c^2*d^4)*arctanh((e*x^2+d)^(
1/2)/x/e^(1/2))+x*((1/5*c^2*x^8-2/3*a*c*x^4+a^2)*e^(9/2)+1/4*(d*(-7/60*c*x
^4+a)*e^(5/2)-2/3*x^2*(-3/20*c*x^4+a)*e^(7/2)-7/32*d^2*c*(-2/3*e^(3/2)*x^2
+e^(1/2)*d))*d*c)*(e*x^2+d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.60

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)^2\,dx$$

$$= \left[\frac{15\left(7\,c^2d^5-32\,acd^3e^2+128\,a^2de^4\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\left(384\,c^2e^5x^9+48\,c^2de^4x^7\right.}{7}\right.$$

$$-\frac{15\left(7\,c^2d^5-32\,acd^3e^2+128\,a^2de^4\right)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\left(384\,c^2e^5x^9+48\,c^2de^4x^7-8\left(7\,c^2d^2e^3+1\right.\right.}{3840\,e^5}$$

input
```
integrate((e*x^2+d)^(1/2)*(-c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/7680*(15*(7*c^2*d^5 - 32*a*c*d^3*e^2 + 128*a^2*d*e^4)*sqrt(e)*log(-2*e*
x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(384*c^2*e^5*x^9 + 48*c^2*d*e^4
*x^7 - 8*(7*c^2*d^2*e^3 + 160*a*c*e^5)*x^5 + 10*(7*c^2*d^3*e^2 - 32*a*c*d*
e^4)*x^3 - 15*(7*c^2*d^4*e - 32*a*c*d^2*e^3 - 128*a^2*e^5)*x)*sqrt(e*x^2 +
 d))/e^5, -1/3840*(15*(7*c^2*d^5 - 32*a*c*d^3*e^2 + 128*a^2*d*e^4)*sqrt(-e
)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (384*c^2*e^5*x^9 + 48*c^2*d*e^4*x^7
 - 8*(7*c^2*d^2*e^3 + 160*a*c*e^5)*x^5 + 10*(7*c^2*d^3*e^2 - 32*a*c*d*e^4)
*x^3 - 15*(7*c^2*d^4*e - 32*a*c*d^2*e^3 - 128*a^2*e^5)*x)*sqrt(e*x^2 + d))
/e^5]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.27

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)^2\,dx$$
$$=\begin{cases}\sqrt{d+ex^2}\left(\frac{c^2dx^7}{80e}+\frac{c^2x^9}{10}+\frac{x^5\left(-2ace-\frac{7c^2d^2}{80e}\right)}{6e}+\frac{x^3\left(-2acd-\frac{5d\left(-2ace-\frac{7c^2d^2}{80e}\right)}{6e}\right)}{4e}+\frac{x\left(a^2e-\frac{3d\left(-2acd-\frac{5d\left(-2ace-\frac{7c^2d^2}{80e}\right)}{6e}\right)}{4e}\right.}{2e}\right. \\ \sqrt{d}\left(a^2x-\frac{2acx^5}{5}+\frac{c^2x^9}{9}\right)\end{cases}$$

input
```
integrate((e*x**2+d)**(1/2)*(-c*x**4+a)**2,x)
```

output
```
Piecewise((sqrt(d + e*x**2)*(c**2*d*x**7/(80*e) + c**2*x**9/10 + x**5*(-2*
a*c*e - 7*c**2*d**2/(80*e))/(6*e) + x**3*(-2*a*c*d - 5*d*(-2*a*c*e - 7*c**
2*d**2/(80*e))/(6*e))/(4*e) + x*(a**2*e - 3*d*(-2*a*c*d - 5*d*(-2*a*c*e -
7*c**2*d**2/(80*e))/(6*e))/(4*e))/(2*e)) + (a**2*d - d*(a**2*e - 3*d*(-2*a
*c*d - 5*d*(-2*a*c*e - 7*c**2*d**2/(80*e))/(6*e))/(4*e))/(2*e))*Piecewise(
(log(2*sqrt(e)*sqrt(d + e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqr
t(e*x**2), True)), Ne(e, 0)), (sqrt(d)*(a**2*x - 2*a*c*x**5/5 + c**2*x**9/
9), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(-c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.83

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)^2\,dx$$
$$=\frac{1}{3840}\left(2\left(4\left(6\left(8\,c^2x^2+\frac{c^2d}{e}\right)x^2-\frac{7\,c^2d^2e^6+160\,ace^8}{e^8}\right)x^2+\frac{5\,(7\,c^2d^3e^5-32\,acde^7)}{e^8}\right)x^2-\frac{15\,(7\,c^2d^4e$$
$$-\frac{(7\,c^2d^5-32\,acd^3e^2+128\,a^2de^4)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{256\,e^{\frac{9}{2}}}$$

input
```
integrate((e*x^2+d)^(1/2)*(-c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*c^2*x^2 + c^2*d/e)*x^2 - (7*c^2*d^2*e^6 + 160*a*c*e^8)/
e^8)*x^2 + 5*(7*c^2*d^3*e^5 - 32*a*c*d*e^7)/e^8)*x^2 - 15*(7*c^2*d^4*e^4 -
 32*a*c*d^2*e^6 - 128*a^2*e^8)/e^8)*sqrt(e*x^2 + d)*x - 1/256*(7*c^2*d^5 -
 32*a*c*d^3*e^2 + 128*a^2*d*e^4)*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^
(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)^2\,dx=\int \left(a-c\,x^4\right)^2\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a - c*x^4)^2*(d + e*x^2)^(1/2),x)
```

output
```
int((a - c*x^4)^2*(d + e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.22

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)^2 \, dx$$

$$= \frac{1920\sqrt{e\,x^2+d}\,a^2e^5x+480\sqrt{e\,x^2+d}\,ac\,d^2e^3x-320\sqrt{e\,x^2+d}\,acd\,e^4x^3-1280\sqrt{e\,x^2+d}\,ac\,e^5x^5-105}{}$$

input
```
int((e*x^2+d)^(1/2)*(-c*x^4+a)^2,x)
```

output
```
(1920*sqrt(d + e*x**2)*a**2*e**5*x + 480*sqrt(d + e*x**2)*a*c*d**2*e**3*x
- 320*sqrt(d + e*x**2)*a*c*d*e**4*x**3 - 1280*sqrt(d + e*x**2)*a*c*e**5*x*
*5 - 105*sqrt(d + e*x**2)*c**2*d**4*e*x + 70*sqrt(d + e*x**2)*c**2*d**3*e*
*2*x**3 - 56*sqrt(d + e*x**2)*c**2*d**2*e**3*x**5 + 48*sqrt(d + e*x**2)*c*
*2*d*e**4*x**7 + 384*sqrt(d + e*x**2)*c**2*e**5*x**9 + 1920*sqrt(e)*log((s
qrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a**2*d*e**4 - 480*sqrt(e)*log((sqrt(
d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*c*d**3*e**2 + 105*sqrt(e)*log((sqrt(d
+ e*x**2) + sqrt(e)*x)/sqrt(d))*c**2*d**5)/(3840*e**5)
```

3.334 $\int \frac{\left(a-cx^4\right)^2}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 174

$$\begin{aligned}\int \frac{\left(a-cx^4\right)^2}{\sqrt{d+ex^2}}\,dx = &-\frac{cd\left(35cd^2-96ae^2\right)x\sqrt{d+ex^2}}{128e^4}+\frac{c\left(35cd^2-96ae^2\right)x^3\sqrt{d+ex^2}}{192e^3}\\ &-\frac{7c^2dx^5\sqrt{d+ex^2}}{48e^2}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}\\ &+\frac{\left(35c^2d^4-96acd^2e^2+128a^2e^4\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{128e^{9/2}}\end{aligned}$$

output
```
-1/128*c*d*(-96*a*e^2+35*c*d^2)*x*(e*x^2+d)^(1/2)/e^4+1/192*c*(-96*a*e^2+3
5*c*d^2)*x^3*(e*x^2+d)^(1/2)/e^3-7/48*c^2*d*x^5*(e*x^2+d)^(1/2)/e^2+1/8*c^
2*x^7*(e*x^2+d)^(1/2)/e+1/128*(128*a^2*e^4-96*a*c*d^2*e^2+35*c^2*d^4)*arct
anh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.79

$$\int \frac{\left(a-cx^4\right)^2}{\sqrt{d+ex^2}}\,dx = \frac{cx\sqrt{d+ex^2}\left(-105cd^3+288ade^2+70cd^2ex^2-192ae^3x^2-56cde^2x^4+48ce^3x^6\right)}{384e^4} + \frac{\left(35c^2d^4-96acd^2e^2+128a^2e^4\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{-\sqrt{d}+\sqrt{d+ex^2}}\right)}{64e^{9/2}}$$

input

```
Integrate[(a - c*x^4)^2/Sqrt[d + e*x^2],x]
```

output

```
(c*x*Sqrt[d + e*x^2]*(-105*c*d^3 + 288*a*d*e^2 + 70*c*d^2*e*x^2 - 192*a*e^
3*x^2 - 56*c*d*e^2*x^4 + 48*c*e^3*x^6))/(384*e^4) + ((35*c^2*d^4 - 96*a*c*
d^2*e^2 + 128*a^2*e^4)*ArcTanh[(Sqrt[e]*x)/(-Sqrt[d] + Sqrt[d + e*x^2])])/
(64*e^(9/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1474, 2346, 1474, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-cx^4\right)^2}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{1474}$$

$$\frac{\int \frac{-7c^2dx^6-16acex^4+8a^2e}{\sqrt{ex^2+d}}dx}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{\int \frac{c\left(35cd^2-96ae^2\right)x^4+48a^2e^2}{\sqrt{ex^2+d}}dx}{6e}-\frac{7c^2dx^5\sqrt{d+ex^2}}{6e}}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

$$\Big\downarrow \ 1474$$

$$\frac{\frac{\frac{\int \frac{3\left(64a^2e^3-cd\left(35cd^2-96ae^2\right)x^2\right)}{\sqrt{ex^2+d}}dx}{4e}+\frac{cx^3\sqrt{d+ex^2}\left(35cd^2-96ae^2\right)}{4e}}{6e}-\frac{7c^2dx^5\sqrt{d+ex^2}}{6e}}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3\int \frac{64a^2e^3-cd\left(35cd^2-96ae^2\right)x^2}{\sqrt{ex^2+d}}dx}{4e}+\frac{cx^3\sqrt{d+ex^2}\left(35cd^2-96ae^2\right)}{4e}}{6e}-\frac{7c^2dx^5\sqrt{d+ex^2}}{6e}}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{\frac{3\left(\frac{\left(128a^2e^4-96acd^2e^2+35c^2d^4\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}-\frac{cdx\sqrt{d+ex^2}\left(35cd^2-96ae^2\right)}{2e}\right)}{4e}+\frac{cx^3\sqrt{d+ex^2}\left(35cd^2-96ae^2\right)}{4e}}{6e}-\frac{7c^2dx^5\sqrt{d+ex^2}}{6e}}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{\frac{3\left(\frac{\left(128a^2e^4-96acd^2e^2+35c^2d^4\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}-\frac{cdx\sqrt{d+ex^2}\left(35cd^2-96ae^2\right)}{2e}\right)}{4e}+\frac{cx^3\sqrt{d+ex^2}\left(35cd^2-96ae^2\right)}{4e}}{6e}-\frac{7c^2dx^5\sqrt{d+ex^2}}{6e}}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{3\left(\frac{\left(128a^2e^4-96acd^2e^2+35c^2d^4\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}-\frac{cdx\sqrt{d+ex^2}\left(35cd^2-96ae^2\right)}{2e}\right)}{4e}+\frac{cx^3\sqrt{d+ex^2}\left(35cd^2-96ae^2\right)}{4e}}{6e}-\frac{7c^2dx^5\sqrt{d+ex^2}}{6e}}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

input

```
Int[(a - c*x^4)^2/Sqrt[d + e*x^2],x]
```

output

```
(c^2*x^7*Sqrt[d + e*x^2])/(8*e) + ((-7*c^2*d*x^5*Sqrt[d + e*x^2])/(6*e) +
((c*(35*c*d^2 - 96*a*e^2)*x^3*Sqrt[d + e*x^2])/(4*e) + (3*(-1/2*(c*d*(35*c
*d^2 - 96*a*e^2)*x*Sqrt[d + e*x^2])/e + ((35*c^2*d^4 - 96*a*c*d^2*e^2 + 12
8*a^2*e^4)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(3/2))))/(4*e))/(6*e
))/(8*e)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{(a^2e^4-\frac{3}{4}acd^2e^2+\frac{35}{128}c^2d^4)\operatorname{arctanh}(\frac{\sqrt{ex^2+d}}{x\sqrt{e}})+\frac{3x\sqrt{ex^2+d}(d(-\frac{7cx^4}{36}+a)e^{\frac{5}{2}}-\frac{2x^2(-\frac{cx^4}{4}+a)e^{\frac{7}{2}}}{3}-\frac{35d^2c(-\frac{2e^{\frac{3}{2}}x^2}{3}+\sqrt{e}d}{96}}{4}}{e^{\frac{9}{2}}}$
risch	$\frac{cx(48e^3cx^6-56de^2cx^4-192ae^3x^2+70cd^2ex^2+288de^2a-105d^3c)\sqrt{ex^2+d}}{384e^4}+\frac{(128a^2e^4-96acd^2e^2+35c^2d^4)\ln(x\sqrt{e}+}{128e^{\frac{9}{2}}}$
default	$\frac{a^2\ln(x\sqrt{e}+\sqrt{ex^2+d})}{\sqrt{e}}+c^2(\frac{x^7\sqrt{ex^2+d}}{8e}-\frac{7d(\frac{x^5\sqrt{ex^2+d}}{6e}-\frac{5d(\frac{x^3\sqrt{ex^2+d}}{4e}-\frac{3d(\frac{x\sqrt{ex^2+d}}{2e}-\frac{d\ln(x\sqrt{e}+\sqrt{ex^2+d})}{2e^{\frac{3}{2}}})}{4e}}{6e}}{8e}$

input
```
int((-c*x^4+a)^2/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^(9/2)*((a^2*e^4-3/4*a*c*d^2*e^2+35/128*c^2*d^4)*arctanh((e*x^2+d)^(1/2
)/x/e^(1/2))+3/4*x*(e*x^2+d)^(1/2)*(d*(-7/36*c*x^4+a)*e^(5/2)-2/3*x^2*(-1/
4*c*x^4+a)*e^(7/2)-35/96*d^2*c*(-2/3*e^(3/2)*x^2+e^(1/2)*d))*c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.62

$$\int \frac{(a-cx^4)^2}{\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{3\,(35\,c^2d^4 - 96\,acd^2e^2 + 128\,a^2e^4)\sqrt{e}\log\left(-2\,ex^2 - 2\,\sqrt{ex^2+d}\sqrt{e}x - d\right) + 2\,(48\,c^2e^4x^7 - 56\,c^2de^3x^5 +}{768\,e^5}\right.$$
$$- \frac{3\,(35\,c^2d^4 - 96\,acd^2e^2 + 128\,a^2e^4)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - (48\,c^2e^4x^7 - 56\,c^2de^3x^5 + 2\,(35\,c^2d^2e^2 - 96}{384\,e^5}$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(35*c^2*d^4 - 96*a*c*d^2*e^2 + 128*a^2*e^4)*sqrt(e)*log(-2*e*x^2
 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(48*c^2*e^4*x^7 - 56*c^2*d*e^3*x^5
 + 2*(35*c^2*d^2*e^2 - 96*a*c*e^4)*x^3 - 3*(35*c^2*d^3*e - 96*a*c*d*e^3)*x
)*sqrt(e*x^2 + d))/e^5, -1/384*(3*(35*c^2*d^4 - 96*a*c*d^2*e^2 + 128*a^2*e
^4)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (48*c^2*e^4*x^7 - 56*c^2
*d*e^3*x^5 + 2*(35*c^2*d^2*e^2 - 96*a*c*e^4)*x^3 - 3*(35*c^2*d^3*e - 96*a*
c*d*e^3)*x)*sqrt(e*x^2 + d))/e^5]
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\int \frac{(a-cx^4)^2}{\sqrt{d+ex^2}}\,dx$$
$$= \begin{cases} \left(a^2 + \frac{3d^2\left(-2ac+\frac{35c^2d^2}{48e^2}\right)}{8e^2}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{e}\sqrt{d+ex^2}+2ex\right)}{\sqrt{e}} & \text{for}\ d \neq 0 \\ \frac{x\log(x)}{\sqrt{ex^2}} & \text{otherwise}\end{cases}\right) + \sqrt{d+ex^2}\left(-\frac{7c^2dx^5}{48e^2} + \frac{c^2x^7}{8e} - \frac{3dx\left(-2ac+\right.}{8e^2}\right. \\ \frac{a^2x - \frac{2acx^5}{5} + \frac{c^2x^9}{9}}{\sqrt{d}} \end{cases}$$

input
```
integrate((-c*x**4+a)**2/(e*x**2+d)**(1/2),x)
```

output

```
Piecewise(((a**2 + 3*d**2*(-2*a*c + 35*c**2*d**2/(48*e**2))/(8*e**2))*Piec
ewise((log(2*sqrt(e)*sqrt(d + e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(
x)/sqrt(e*x**2), True)) + sqrt(d + e*x**2)*(-7*c**2*d*x**5/(48*e**2) + c**
2*x**7/(8*e) - 3*d*x*(-2*a*c + 35*c**2*d**2/(48*e**2))/(8*e**2) + x**3*(-2
*a*c + 35*c**2*d**2/(48*e**2))/(4*e)), Ne(e, 0)), ((a**2*x - 2*a*c*x**5/5
+ c**2*x**9/9)/sqrt(d), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a-cx^4)^2}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((-c*x^4+a)^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.83

$$\begin{aligned}
&\int \frac{(a-cx^4)^2}{\sqrt{d+ex^2}}\,dx \\
&= \frac{1}{384}\left(2\left(4\left(\frac{6\,c^2x^2}{e}-\frac{7\,c^2d}{e^2}\right)x^2+\frac{35\,c^2d^2e^4-96\,ace^6}{e^7}\right)x^2-\frac{3\,(35\,c^2d^3e^3-96\,acde^5)}{e^7}\right)\sqrt{ex^2+d}x \\
&\quad -\frac{(35\,c^2d^4-96\,acd^2e^2+128\,a^2e^4)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{128\,e^{\frac{9}{2}}}
\end{aligned}$$

input

```
integrate((-c*x^4+a)^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*c^2*x^2/e - 7*c^2*d/e^2)*x^2 + (35*c^2*d^2*e^4 - 96*a*c*e^6
)/e^7)*x^2 - 3*(35*c^2*d^3*e^3 - 96*a*c*d*e^5)/e^7)*sqrt(e*x^2 + d)*x - 1/
128*(35*c^2*d^4 - 96*a*c*d^2*e^2 + 128*a^2*e^4)*log(abs(-sqrt(e)*x + sqrt(
e*x^2 + d)))/e^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^2}{\sqrt{d+ex^2}}\,dx = \int \frac{(a-c\,x^4)^2}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a - c*x^4)^2/(d + e*x^2)^(1/2),x)
```

output
```
int((a - c*x^4)^2/(d + e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.20

$$\int \frac{(a-cx^4)^2}{\sqrt{d+ex^2}}\,dx = \frac{288\sqrt{e\,x^2+d}\,acd\,e^3x-192\sqrt{e\,x^2+d}\,ac\,e^4x^3-105\sqrt{e\,x^2+d}\,c^2d^3ex+70\sqrt{e\,x^2+d}\,c^2d^2e^2x^3-56\sqrt{e\,x^2}}{}$$

input
```
int((-c*x^4+a)^2/(e*x^2+d)^(1/2),x)
```

output
```
(288*sqrt(d + e*x**2)*a*c*d*e**3*x - 192*sqrt(d + e*x**2)*a*c*e**4*x**3 -
105*sqrt(d + e*x**2)*c**2*d**3*e*x + 70*sqrt(d + e*x**2)*c**2*d**2*e**2*x*
*3 - 56*sqrt(d + e*x**2)*c**2*d*e**3*x**5 + 48*sqrt(d + e*x**2)*c**2*e**4*
x**7 + 384*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a**2*e**4 -
 288*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*c*d**2*e**2 + 1
05*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*c**2*d**4)/(384*e**
5)
```

3.335 $\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 159

$$\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{\left(cd^2-ae^2\right)^2 x}{de^4\sqrt{d+ex^2}} + \frac{c\left(19cd^2-16ae^2\right)x\sqrt{d+ex^2}}{16e^4} - \frac{11c^2dx^3\sqrt{d+ex^2}}{24e^3} + \frac{c^2x^5\sqrt{d+ex^2}}{6e^2} - \frac{cd\left(35cd^2-48ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16e^{9/2}}$$

output
```
(-a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^(1/2)+1/16*c*(-16*a*e^2+19*c*d^2)*x*(e*
x^2+d)^(1/2)/e^4-11/24*c^2*d*x^3*(e*x^2+d)^(1/2)/e^3+1/6*c^2*x^5*(e*x^2+d)
^(1/2)/e^2-1/16*c*d*(-48*a*e^2+35*c*d^2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2)
)/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.82

$$\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{x\left(48a^2e^4-48acde^2\left(3d+ex^2\right)+c^2d\left(105d^3+35d^2ex^2-14de^2x^4+8e^3x^6\right)\right)}{48de^4\sqrt{d+ex^2}} + \frac{cd\left(35cd^2-48ae^2\right)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{16e^{9/2}}$$

input
```
Integrate[(a - c*x^4)^2/(d + e*x^2)^(3/2),x]
```

output
```
(x*(48*a^2*e^4 - 48*a*c*d*e^2*(3*d + e*x^2) + c^2*d*(105*d^3 + 35*d^2*e*x^
2 - 14*d*e^2*x^4 + 8*e^3*x^6)))/(48*d*e^4*Sqrt[d + e*x^2]) + (c*d*(35*c*d^
2 - 48*a*e^2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(16*e^(9/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1472, 2346, 1473, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(cd^2-ae^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\int \frac{-\frac{c^2dx^6}{e}+\frac{c^2d^2x^4}{e^2}-\frac{cd\left(cd^2-2ae^2\right)x^2}{e^3}+\frac{cd^2\left(cd^2-2ae^2\right)}{e^4}}{\sqrt{ex^2+d}}dx}{d}$$

$$\Big\downarrow \text{2346}$$

$$\frac{x\left(cd^2-ae^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\frac{\int \frac{\frac{11c^2d^2x^4}{e}+6cd\left(2a-\frac{cd^2}{e^2}\right)x^2+\frac{6cd^2\left(cd^2-2ae^2\right)}{e^3}}{\sqrt{ex^2+d}}dx}{6e}-\frac{c^2dx^5\sqrt{d+ex^2}}{6e^2}}{d}$$

$$\Big\downarrow \text{1473}$$

$$\frac{x\left(cd^2-ae^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\frac{\frac{\int -\frac{3cd\left(\left(\frac{19cd^2}{e}-16ae\right)x^2+8d\left(2a-\frac{cd^2}{e^2}\right)\right)}{\sqrt{ex^2+d}}dx}{4e}+\frac{11c^2d^2x^3\sqrt{d+ex^2}}{4e^2}}{6e}-\frac{c^2dx^5\sqrt{d+ex^2}}{6e^2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\left(cd^2-ae^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\frac{\frac{11c^2d^2x^3\sqrt{d+ex^2}}{4e^2}-\frac{3cd\int\frac{\left(\frac{19cd^2}{e}-16ae\right)x^2+8d\left(2a-\frac{cd^2}{e^2}\right)}{\sqrt{ex^2+d}}dx}{4e}}{6e}-\frac{c^2dx^5\sqrt{d+ex^2}}{6e^2}}{d}$$

$$\Big\downarrow \textcolor{blue}{299}$$

$$\frac{x\left(cd^2-ae^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\frac{\frac{11c^2d^2x^3\sqrt{d+ex^2}}{4e^2}-\frac{3cd\left(\frac{1}{2}d\left(48a-\frac{35cd^2}{e^2}\right)\int\frac{1}{\sqrt{ex^2+d}}dx-\frac{1}{2}x\sqrt{d+ex^2}\left(16a-\frac{19cd^2}{e^2}\right)\right)}{4e}}{6e}-\frac{c^2dx^5\sqrt{d+ex^2}}{6e^2}}{d}$$

$$\Big\downarrow \textcolor{blue}{224}$$

$$\frac{x\left(cd^2-ae^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\frac{\frac{11c^2d^2x^3\sqrt{d+ex^2}}{4e^2}-\frac{3cd\left(\frac{1}{2}d\left(48a-\frac{35cd^2}{e^2}\right)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{1}{2}x\sqrt{d+ex^2}\left(16a-\frac{19cd^2}{e^2}\right)\right)}{4e}}{6e}-\frac{c^2dx^5\sqrt{d+ex^2}}{6e^2}}{d}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{x\left(cd^2-ae^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\frac{\frac{11c^2d^2x^3\sqrt{d+ex^2}}{4e^2}-\frac{3cd\left(\frac{d\left(48a-\frac{35cd^2}{e^2}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}-\frac{1}{2}x\sqrt{d+ex^2}\left(16a-\frac{19cd^2}{e^2}\right)\right)}{4e}}{6e}-\frac{c^2dx^5\sqrt{d+ex^2}}{6e^2}}{d}$$

input

```
Int[(a - c*x^4)^2/(d + e*x^2)^(3/2),x]
```

output

```
((c*d^2 - a*e^2)^2*x)/(d*e^4*Sqrt[d + e*x^2]) - (-1/6*(c^2*d*x^5*Sqrt[d +
e*x^2])/e^2 + ((11*c^2*d^2*x^3*Sqrt[d + e*x^2])/(4*e^2) - (3*c*d*(-1/2*((1
6*a - (19*c*d^2)/e^2)*x*Sqrt[d + e*x^2]) + (d*(48*a - (35*c*d^2)/e^2)*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/(4*e))/(6*e))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{3d^2\left(a\,e^2-\frac{35c\,d^2}{48}\right)\sqrt{e\,x^2+d}\,c\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}}{x\sqrt{e}}\right)+\left(-3d^2\left(\frac{7c\,x^4}{72}+a\right)c\,e^{\frac{5}{2}}-\left(-\frac{c\,x^4}{6}+a\right)x^2dc\,e^{\frac{7}{2}}+\frac{35e^{\frac{3}{2}}c^2d^3x^2}{48}+\frac{35\sqrt{e}\,c^2d^4}{16}}{e^{\frac{9}{2}}\sqrt{e\,x^2+d}\,d}$
risch	$-\frac{cx\left(-8c\,x^4e^2+22cd\,x^2e+48a\,e^2-57c\,d^2\right)\sqrt{e\,x^2+d}}{48e^4}-\frac{2cdxa}{e^2\sqrt{e\,x^2+d}}+\frac{c^2d^3x}{e^4\sqrt{e\,x^2+d}}+\frac{3cd\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)a}{e^{\frac{5}{2}}}-35c$
default	$\frac{a^2x}{d\sqrt{e\,x^2+d}}+c^2\left(\frac{x^7}{6e\sqrt{e\,x^2+d}}-\frac{7d\left(\frac{x^5}{4e\sqrt{e\,x^2+d}}-\frac{5d\left(\frac{x^3}{2e\sqrt{e\,x^2+d}}-\frac{3d\left(-\frac{x}{e\sqrt{e\,x^2+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{e^{\frac{3}{2}}}\right)}{2e}\right)}{4e}\right)}{6e}\right)-2c$

input
```
int((-c*x^4+a)^2/(e*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^(9/2)*(3*d^2*(a*e^2-35/48*c*d^2)*(e*x^2+d)^(1/2)*c*arctanh((e*x^2+d)^(
1/2)/x/e^(1/2))+(-3*d^2*(7/72*c*x^4+a)*c*e^(5/2)-(-1/6*c*x^4+a)*x^2*d*c*e^
(7/2)+35/48*e^(3/2)*c^2*d^3*x^2+35/16*e^(1/2)*c^2*d^4+e^(9/2)*a^2)*x)/(e*x
^2+d)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.32

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \left[-\frac{3\,(35\,c^2d^5-48\,acd^3e^2+(35\,c^2d^4e-48\,acd^2e^3)x^2)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}\right.}{}\right.$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(35*c^2*d^5 - 48*a*c*d^3*e^2 + (35*c^2*d^4*e - 48*a*c*d^2*e^3)*x
^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*(8*c^2*d*e
^4*x^7 - 14*c^2*d^2*e^3*x^5 + (35*c^2*d^3*e^2 - 48*a*c*d*e^4)*x^3 + 3*(35*
c^2*d^4*e - 48*a*c*d^2*e^3 + 16*a^2*e^5)*x)*sqrt(e*x^2 + d))/(d*e^6*x^2 +
d^2*e^5), 1/48*(3*(35*c^2*d^5 - 48*a*c*d^3*e^2 + (35*c^2*d^4*e - 48*a*c*d^
2*e^3)*x^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (8*c^2*d*e^4*x^7
 - 14*c^2*d^2*e^3*x^5 + (35*c^2*d^3*e^2 - 48*a*c*d*e^4)*x^3 + 3*(35*c^2*d^
4*e - 48*a*c*d^2*e^3 + 16*a^2*e^5)*x)*sqrt(e*x^2 + d))/(d*e^6*x^2 + d^2*e^
5)]
```

Sympy [F]

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(-a+cx^4)^2}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*x**4+a)**2/(e*x**2+d)**(3/2),x)
```

output
```
Integral((-a + c*x**4)**2/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((-c*x^4+a)^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \frac{\left(\left(2\left(\frac{4c^2x^2}{e}-\frac{7c^2d}{e^2}\right)x^2+\frac{35c^2d^3e^4-48acde^6}{de^7}\right)x^2+\frac{3(35c^2d^4e^3-48acd^2e^5+16a^2e^7)}{de^7}\right)x}{48\sqrt{ex^2+d}} + \frac{(35c^2d^3-48acde^2)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{16e^{\frac{9}{2}}}$$

input

```
integrate((-c*x^4+a)^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output

```
1/48*((2*(4*c^2*x^2/e - 7*c^2*d/e^2)*x^2 + (35*c^2*d^3*e^4 - 48*a*c*d*e^6)
/(d*e^7))*x^2 + 3*(35*c^2*d^4*e^3 - 48*a*c*d^2*e^5 + 16*a^2*e^7)/(d*e^7))*
x/sqrt(e*x^2 + d) + 1/16*(35*c^2*d^3 - 48*a*c*d*e^2)*log(abs(-sqrt(e)*x +
sqrt(e*x^2 + d)))/e^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(a-c\,x^4)^2}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a - c*x^4)^2/(d + e*x^2)^(3/2),x)
```

output
```
int((a - c*x^4)^2/(d + e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.25

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \frac{384\sqrt{e\,x^2+d}\,a^2e^5x - 1152\sqrt{e\,x^2+d}\,acd^2e^3x - 384\sqrt{e\,x^2+d}\,acd\,e^4x^3 + 840\sqrt{e\,x^2+}}{}$$

input
```
int((-c*x^4+a)^2/(e*x^2+d)^(3/2),x)
```

output
```
(384*sqrt(d + e*x**2)*a**2*e**5*x - 1152*sqrt(d + e*x**2)*a*c*d**2*e**3*x
- 384*sqrt(d + e*x**2)*a*c*d*e**4*x**3 + 840*sqrt(d + e*x**2)*c**2*d**4*e*
x + 280*sqrt(d + e*x**2)*c**2*d**3*e**2*x**3 - 112*sqrt(d + e*x**2)*c**2*d
**2*e**3*x**5 + 64*sqrt(d + e*x**2)*c**2*d*e**4*x**7 + 1152*sqrt(e)*log((s
qrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*c*d**3*e**2 + 1152*sqrt(e)*log((sq
rt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*c*d**2*e**3*x**2 - 840*sqrt(e)*log(
(sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*c**2*d**5 - 840*sqrt(e)*log((sqrt(
d + e*x**2) + sqrt(e)*x)/sqrt(d))*c**2*d**4*e*x**2 + 384*sqrt(e)*a**2*d*e*
*4 + 384*sqrt(e)*a**2*e**5*x**2 - 864*sqrt(e)*a*c*d**3*e**2 - 864*sqrt(e)*
a*c*d**2*e**3*x**2 + 525*sqrt(e)*c**2*d**5 + 525*sqrt(e)*c**2*d**4*e*x**2)
/(384*d*e**5*(d + e*x**2))
```

3.336 $\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 172

$$\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{\left(cd^2-ae^2\right)^2 x}{3de^4\left(d+ex^2\right)^{3/2}} - \frac{2\left(cd^2-ae^2\right)\left(5cd^2+ae^2\right)x}{3d^2e^4\sqrt{d+ex^2}} - \frac{11c^2dx\sqrt{d+ex^2}}{8e^4} + \frac{c^2x^3\sqrt{d+ex^2}}{4e^3} + \frac{c\left(35cd^2-16ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{9/2}}$$

output

```
1/3*(-a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^(3/2)-2/3*(-a*e^2+c*d^2)*(a*e^2+5*c
*d^2)*x/d^2/e^4/(e*x^2+d)^(1/2)-11/8*c^2*d*x*(e*x^2+d)^(1/2)/e^4+1/4*c^2*x
^3*(e*x^2+d)^(1/2)/e^3+1/8*c*(-16*a*e^2+35*c*d^2)*arctanh(e^(1/2)*x/(e*x^2
+d)^(1/2))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

$$\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x(8a^2e^4(3d+2ex^2)+16acd^2e^2(3d+4ex^2)-c^2d^2(105d^3+140d^2ex^2+21de^2x^4-6e}{24d^2e^4\left(d+ex^2\right)^{3/2}} - \frac{c(35cd^2-16ae^2)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{8e^{9/2}}$$

input
```
Integrate[(a - c*x^4)^2/(d + e*x^2)^(5/2),x]
```

output
```
(x*(8*a^2*e^4*(3*d + 2*e*x^2) + 16*a*c*d^2*e^2*(3*d + 4*e*x^2) - c^2*d^2*(
105*d^3 + 140*d^2*e*x^2 + 21*d*e^2*x^4 - 6*e^3*x^6)))/(24*d^2*e^4*(d + e*x
^2)^(3/2)) - (c*(35*c*d^2 - 16*a*e^2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])
/(8*e^(9/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1472, 25, 2345, 27, 1473, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(cd^2-ae^2\right)^2}{3de^4\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{\frac{3c^2dx^6}{e}-\frac{3c^2d^2x^4}{e^2}+\frac{3cd\left(cd^2-2ae^2\right)x^2}{e^3}+2a^2+\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{\left(ex^2+d\right)^{3/2}}dx}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{3c^2dx^6}{e}-\frac{3c^2d^2x^4}{e^2}+\frac{3cd\left(cd^2-2ae^2\right)x^2}{e^3}+2a^2+\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{x\left(cd^2-ae^2\right)^2}{3de^4\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{2x\left(a^2+\frac{4acd^2}{e^2}-\frac{5c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}-\frac{\int -\frac{3\left(\frac{c^2d^2x^4}{e^2}-\frac{2c^2d^3x^2}{e^3}+\frac{cd^2\left(3cd^2-2ae^2\right)}{e^4}\right)}{\sqrt{ex^2+d}}dx}{d}}{3d}+\frac{x\left(cd^2-ae^2\right)^2}{3de^4\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{\frac{c^2 d^2 x^4}{e^2}-\frac{2 c^2 d^3 x^2}{e^3}+\frac{c d^2 \left(3 c d^2-2 a e^2\right)}{e^4}}{\sqrt{e x^2+d}}dx}{d}+\frac{2 x \left(a^2+\frac{4 a c d^2}{e^2}-\frac{5 c^2 d^4}{e^4}\right)}{d \sqrt{d+e x^2}}}{3 d}+\frac{x \left(c d^2-a e^2\right)^2}{3 d e^4 \left(d+e x^2\right)^{3/2}}$$

$$\Big\downarrow \; 1473$$

$$\frac{\frac{3 \left(\frac{\int \frac{c d^2 \left(4 \left(3 c d^2-2 a e^2\right)-11 c d e x^2\right)}{e^3 \sqrt{e x^2+d}}dx}{4 e}+\frac{c^2 d^2 x^3 \sqrt{d+e x^2}}{4 e^3}\right)}{d}+\frac{2 x \left(a^2+\frac{4 a c d^2}{e^2}-\frac{5 c^2 d^4}{e^4}\right)}{d \sqrt{d+e x^2}}}{3 d}+\frac{x \left(c d^2-a e^2\right)^2}{3 d e^4 \left(d+e x^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3 \left(\frac{c d^2 \int \frac{4 \left(3 c d^2-2 a e^2\right)-11 c d e x^2}{\sqrt{e x^2+d}}dx}{4 e^4}+\frac{c^2 d^2 x^3 \sqrt{d+e x^2}}{4 e^3}\right)}{d}+\frac{2 x \left(a^2+\frac{4 a c d^2}{e^2}-\frac{5 c^2 d^4}{e^4}\right)}{d \sqrt{d+e x^2}}}{3 d}+\frac{x \left(c d^2-a e^2\right)^2}{3 d e^4 \left(d+e x^2\right)^{3/2}}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{3 \left(\frac{c d^2 \left(\frac{1}{2} \left(35 c d^2-16 a e^2\right) \int \frac{1}{\sqrt{e x^2+d}}dx-\frac{11}{2} c d x \sqrt{d+e x^2}\right)}{4 e^4}+\frac{c^2 d^2 x^3 \sqrt{d+e x^2}}{4 e^3}\right)}{d}+\frac{2 x \left(a^2+\frac{4 a c d^2}{e^2}-\frac{5 c^2 d^4}{e^4}\right)}{d \sqrt{d+e x^2}}}{3 d}+ \\ \frac{x \left(c d^2-a e^2\right)^2}{3 d e^4 \left(d+e x^2\right)^{3/2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{3 \left(\frac{c d^2 \left(\frac{1}{2} \left(35 c d^2-16 a e^2\right) \int \frac{1}{1-\frac{e x^2}{e x^2+d}}d\frac{x}{\sqrt{e x^2+d}}-\frac{11}{2} c d x \sqrt{d+e x^2}\right)}{4 e^4}+\frac{c^2 d^2 x^3 \sqrt{d+e x^2}}{4 e^3}\right)}{d}+\frac{2 x \left(a^2+\frac{4 a c d^2}{e^2}-\frac{5 c^2 d^4}{e^4}\right)}{d \sqrt{d+e x^2}}}{3 d}+ \\ \frac{x \left(c d^2-a e^2\right)^2}{3 d e^4 \left(d+e x^2\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{2x\left(a^2+\frac{4acd^2}{e^2}-\frac{5c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}+\frac{3\left(\frac{cd^2\left(\frac{(35cd^2-16ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}-\frac{11}{2}cdx\sqrt{d+ex^2}\right)}{4e^4}+\frac{c^2d^2x^3\sqrt{d+ex^2}}{4e^3}\right)}{d}}{3d}+\frac{x\left(cd^2-ae^2\right)^2}{3de^4\left(d+ex^2\right)^{3/2}}$$

input

```
Int[(a - c*x^4)^2/(d + e*x^2)^(5/2),x]
```

output

```
((c*d^2 - a*e^2)^2*x)/(3*d*e^4*(d + e*x^2)^(3/2)) + ((2*(a^2 - (5*c^2*d^4)
/e^4 + (4*a*c*d^2)/e^2)*x)/(d*Sqrt[d + e*x^2]) + (3*((c^2*d^2*x^3*Sqrt[d +
 e*x^2])/(4*e^3) + (c*d^2*((-11*c*d*x*Sqrt[d + e*x^2])/2 + ((35*c*d^2 - 16
*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/(4*e^4)))/d)/(
3*d)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{-2d^2\left(a e^2-\frac{35c d^2}{16}\right)(e x^2+d)^{\frac{3}{2}} c \operatorname{arctanh}\left(\frac{\sqrt{e x^2+d}}{x\sqrt{e}}\right)+\frac{2x\Big(-\frac{105c^2 d^5\sqrt{e}}{16}+e^{\frac{3}{2}}\Big(-\frac{35c^2d^4x^2}{4}+3ce\left(-\frac{7c x^4}{16}+a\right)d^3+4x^2\Big(\frac{3c x^4}{32}+a}{3}}{e^{\frac{9}{2}}(e x^2+d)^{\frac{3}{2}}d^2}$
default	$a^2\left(\frac{x}{3d(e x^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{e x^2+d}}\right)+c^2\Bigg(\frac{x^7}{4e(e x^2+d)^{\frac{3}{2}}}-\frac{7d\Bigg(\frac{x^5}{2e\left(e x^2+d\right)^{\frac{3}{2}}}-\frac{5d\Big(-\frac{x^3}{3e\left(e x^2+d\right)^{\frac{3}{2}}}+\frac{-\frac{x}{e\sqrt{e x^2+d}}+\ln}{}}{2e}}{4e}$
risch	$-\frac{c^2x(-2e x^2+11d)\sqrt{e x^2+d}}{8e^4}-\frac{\frac{c\left(16a e^2-35c d^2\right)\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{\sqrt{e}}+\frac{2\left(a^2e^4-2ac d^2e^2+c^2d^4\right)\Bigg(-\frac{\sqrt{\left(x-\frac{\sqrt{-de}}{e}\right)^2 e+2\sqrt{-de}}}{3\sqrt{-de}\left(x-\frac{\sqrt{-d}}{e}\right.}}{e}}{}$

input

```
int((-c*x^4+a)^2/(e*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
2/3/e^(9/2)*(-3*d^2*(a*e^2-35/16*c*d^2)*(e*x^2+d)^(3/2)*c*arctanh((e*x^2+d
)^(1/2)/x/e^(1/2))+x*(-105/16*c^2*d^5*e^(1/2)+e^(3/2)*(-35/4*c^2*d^4*x^2+3
*c*e*(-7/16*c*x^4+a)*d^3+4*x^2*(3/32*c*x^4+a)*c*e^2*d^2+3/2*e^3*a^2*d+e^4*
x^2*a^2)))/(e*x^2+d)^(3/2)/d^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.78

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{5/2}}\,dx = \Bigg[-\frac{3\,(35\,c^2d^6-16\,acd^4e^2+(35\,c^2d^4e^2-16\,acd^2e^4)x^4+2\,(35\,c^2d^5e-16\,acd^3e^3)x^2)\sqrt{}}{}$$

$$-\frac{3\,(35\,c^2d^6-16\,acd^4e^2+(35\,c^2d^4e^2-16\,acd^2e^4)x^4+2\,(35\,c^2d^5e-16\,acd^3e^3)x^2)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-}{24\,(d^2e^7x^4}$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(35*c^2*d^6 - 16*a*c*d^4*e^2 + (35*c^2*d^4*e^2 - 16*a*c*d^2*e^4)
*x^4 + 2*(35*c^2*d^5*e - 16*a*c*d^3*e^3)*x^2)*sqrt(e)*log(-2*e*x^2 + 2*sqr
t(e*x^2 + d)*sqrt(e)*x - d) - 2*(6*c^2*d^2*e^4*x^7 - 21*c^2*d^3*e^3*x^5 -
4*(35*c^2*d^4*e^2 - 16*a*c*d^2*e^4 - 4*a^2*e^6)*x^3 - 3*(35*c^2*d^5*e - 16
*a*c*d^3*e^3 - 8*a^2*d*e^5)*x)*sqrt(e*x^2 + d))/(d^2*e^7*x^4 + 2*d^3*e^6*x
^2 + d^4*e^5), -1/24*(3*(35*c^2*d^6 - 16*a*c*d^4*e^2 + (35*c^2*d^4*e^2 - 1
6*a*c*d^2*e^4)*x^4 + 2*(35*c^2*d^5*e - 16*a*c*d^3*e^3)*x^2)*sqrt(-e)*arcta
n(sqrt(-e)*x/sqrt(e*x^2 + d)) - (6*c^2*d^2*e^4*x^7 - 21*c^2*d^3*e^3*x^5 -
4*(35*c^2*d^4*e^2 - 16*a*c*d^2*e^4 - 4*a^2*e^6)*x^3 - 3*(35*c^2*d^5*e - 16
*a*c*d^3*e^3 - 8*a^2*d*e^5)*x)*sqrt(e*x^2 + d))/(d^2*e^7*x^4 + 2*d^3*e^6*x
^2 + d^4*e^5)]
```

Sympy [F]

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(-a+cx^4)^2}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*x**4+a)**2/(e*x**2+d)**(5/2),x)
```

output
```
Integral((-a + c*x**4)**2/(d + e*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{5/2}}\,dx = \frac{\left(\left(3\left(\frac{2\,c^2x^2}{e}-\frac{7\,c^2d}{e^2}\right)x^2-\frac{4\,(35\,c^2d^4e^4-16\,acd^2e^6-4\,a^2e^8)}{d^2e^7}\right)x^2-\frac{3\,(35\,c^2d^5e^3-16\,acd^3e^5-8\,a^2de^7)}{d^2e^7}\right)x}{24\,(ex^2+d)^{\frac{3}{2}}} - \frac{(35\,c^2d^2-16\,ace^2)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{8\,e^{\frac{9}{2}}}$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
1/24*((3*(2*c^2*x^2/e - 7*c^2*d/e^2)*x^2 - 4*(35*c^2*d^4*e^4 - 16*a*c*d^2*
e^6 - 4*a^2*e^8)/(d^2*e^7))*x^2 - 3*(35*c^2*d^5*e^3 - 16*a*c*d^3*e^5 - 8*a
^2*d*e^7)/(d^2*e^7))*x/(e*x^2 + d)^(3/2) - 1/8*(35*c^2*d^2 - 16*a*c*e^2)*l
og(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(a-c\,x^4)^2}{(e\,x^2+d)^{5/2}}\,dx$$

input

```
int((a - c*x^4)^2/(d + e*x^2)^(5/2),x)
```

output

```
int((a - c*x^4)^2/(d + e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.71

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{5/2}}\,dx = \frac{192\sqrt{e\,x^2+d}\,a^2d\,e^5x+128\sqrt{e\,x^2+d}\,a^2e^6x^3+384\sqrt{e\,x^2+d}\,ac\,d^3e^3x+512\sqrt{e\,x^2+}}{}$$

input

```
int((-c*x^4+a)^2/(e*x^2+d)^(5/2),x)
```

output

```
(192*sqrt(d + e*x**2)*a**2*d*e**5*x + 128*sqrt(d + e*x**2)*a**2*e**6*x**3
+ 384*sqrt(d + e*x**2)*a*c*d**3*e**3*x + 512*sqrt(d + e*x**2)*a*c*d**2*e**
4*x**3 - 840*sqrt(d + e*x**2)*c**2*d**5*e*x - 1120*sqrt(d + e*x**2)*c**2*d
**4*e**2*x**3 - 168*sqrt(d + e*x**2)*c**2*d**3*e**3*x**5 + 48*sqrt(d + e*x
**2)*c**2*d**2*e**4*x**7 - 384*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/
sqrt(d))*a*c*d**4*e**2 - 768*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sq
rt(d))*a*c*d**3*e**3*x**2 - 384*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)
/sqrt(d))*a*c*d**2*e**4*x**4 + 840*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)
*x)/sqrt(d))*c**2*d**6 + 1680*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/s
qrt(d))*c**2*d**5*e*x**2 + 840*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/
sqrt(d))*c**2*d**4*e**2*x**4 - 128*sqrt(e)*a**2*d**2*e**4 - 256*sqrt(e)*a*
*2*d*e**5*x**2 - 128*sqrt(e)*a**2*e**6*x**4 + 175*sqrt(e)*c**2*d**6 + 350*
sqrt(e)*c**2*d**5*e*x**2 + 175*sqrt(e)*c**2*d**4*e**2*x**4)/(192*d**2*e**5
*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.337 $\int \frac{(a-cx^4)^2}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 186

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{7/2}}\,dx=\frac{(cd^2-ae^2)^2x}{5de^4(d+ex^2)^{5/2}}-\frac{4(cd^2-ae^2)(4cd^2+ae^2)x}{15d^2e^4(d+ex^2)^{3/2}}+\frac{2(29c^2d^4-3acd^2e^2+4a^2e^4)x}{15d^3e^4\sqrt{d+ex^2}}+\frac{c^2x\sqrt{d+ex^2}}{2e^4}-\frac{7c^2d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{9/2}}$$

output
```
1/5*(-a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^(5/2)-4/15*(-a*e^2+c*d^2)*(a*e^2+4*
c*d^2)*x/d^2/e^4/(e*x^2+d)^(3/2)+2/15*(4*a^2*e^4-3*a*c*d^2*e^2+29*c^2*d^4)
*x/d^3/e^4/(e*x^2+d)^(1/2)+1/2*c^2*x*(e*x^2+d)^(1/2)/e^4-7/2*c^2*d*arctanh
(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.75

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{7/2}}\,dx=\frac{x(-12acd^2e^4x^4+2a^2e^4(15d^2+20dex^2+8e^2x^4)+c^2d^3(105d^3+245d^2ex^2+161de^2x}{30d^3e^4(d+ex^2)^{5/2}}+\frac{7c^2d\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{2e^{9/2}}$$

input
```
Integrate[(a - c*x^4)^2/(d + e*x^2)^(7/2),x]
```

output
```
(x*(-12*a*c*d^2*e^4*x^4 + 2*a^2*e^4*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4) + c^
2*d^3*(105*d^3 + 245*d^2*e*x^2 + 161*d*e^2*x^4 + 15*e^3*x^6)))/(30*d^3*e^4
*(d + e*x^2)^(5/2)) + (7*c^2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(2*e^(
9/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1472, 25, 2345, 25, 1471, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(cd^2-ae^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}-\frac{\int -\frac{\frac{5c^2dx^6}{e}-\frac{5c^2d^2x^4}{e^2}+\frac{5cd\left(cd^2-2ae^2\right)x^2}{e^3}+4a^2+\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{\left(ex^2+d\right)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{5c^2dx^6}{e}-\frac{5c^2d^2x^4}{e^2}+\frac{5cd\left(cd^2-2ae^2\right)x^2}{e^3}+4a^2+\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{\left(ex^2+d\right)^{5/2}}dx}{5d}+\frac{x\left(cd^2-ae^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{4x\left(a^2+\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}-\frac{\int -\frac{\frac{13c^2d^4}{e^4}-\frac{30c^2x^2d^3}{e^3}+\frac{15c^2x^4d^2}{e^2}-\frac{6acd^2}{e^2}+8a^2}{\left(ex^2+d\right)^{3/2}}dx}{3d}}{5d}+\frac{x\left(cd^2-ae^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\frac{13c^2d^4}{e^4}-\frac{30c^2x^2d^3}{e^3}+\frac{15c^2x^4d^2}{e^2}-\frac{6acd^2}{e^2}+8a^2}{\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{4x\left(a^2+\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}}{5d}+\frac{x\left(cd^2-ae^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow 1471$$

$$\frac{\frac{\frac{2x\left(4a^2-\frac{3acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}-\frac{\int \frac{15c^2d^3\left(3d-ex^2\right)}{e^4\sqrt{ex^2+d}}dx}{d}}{3d}+\frac{4x\left(a^2+\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}}{5d}+\frac{x\left(cd^2-ae^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{2x\left(4a^2-\frac{3acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}-\frac{15c^2d^2\int \frac{3d-ex^2}{\sqrt{ex^2+d}}dx}{e^4}}{3d}+\frac{4x\left(a^2+\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}}{5d}+\frac{x\left(cd^2-ae^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow 299$$

$$\frac{\frac{\frac{2x\left(4a^2-\frac{3acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}-\frac{15c^2d^2\left(\frac{7}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{1}{2}x\sqrt{d+ex^2}\right)}{e^4}}{3d}+\frac{4x\left(a^2+\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}}{5d}+\frac{x\left(cd^2-ae^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow 224$$

$$\frac{\frac{\frac{2x\left(4a^2-\frac{3acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}-\frac{15c^2d^2\left(\frac{7}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{1}{2}x\sqrt{d+ex^2}\right)}{e^4}}{3d}+\frac{4x\left(a^2+\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}}{5d}+\frac{x\left(cd^2-ae^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow 219$$

$$\frac{\frac{\frac{2x\left(4a^2-\frac{3acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}-\frac{15c^2d^2\left(\frac{7d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}-\frac{1}{2}x\sqrt{d+ex^2}\right)}{e^4}}{3d}+\frac{4x\left(a^2+\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}}{5d}+\frac{x\left(cd^2-ae^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

input

```
Int[(a - c*x^4)^2/(d + e*x^2)^(7/2),x]
```

output
```
((c*d^2 - a*e^2)^2*x)/(5*d*e^4*(d + e*x^2)^(5/2)) + ((4*(a^2 - (4*c^2*d^4)
/e^4 + (3*a*c*d^2)/e^2)*x)/(3*d*(d + e*x^2)^(3/2)) + ((2*(4*a^2 + (29*c^2*
d^4)/e^4 - (3*a*c*d^2)/e^2)*x)/(d*Sqrt[d + e*x^2]) - (15*c^2*d^2*(-1/2*(x*
Sqrt[d + e*x^2]) + (7*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e]))
)/e^4)/(3*d))/(5*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{-\frac{7\left(e\,x^{2}+d\right)^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right) c^{2} d^{4}}{2}+x\left(a\,d^{2}\left(-\frac{2c\,x^{4}}{5}+a\right)e^{\frac{9}{2}}+\frac{4e^{\frac{11}{2}}a^{2}d\,x^{2}}{3}+\frac{8e^{\frac{13}{2}}a^{2}x^{4}}{15}+\frac{161d^{3}\left(\frac{15x^{6}e^{\frac{7}{2}}}{161}+e^{\frac{5}{2}}d\,x^{4}+\frac{35e^{\frac{3}{2}}d^{2}}{23}\right.}{30}\right.}{e^{\frac{9}{2}}\left(e\,x^{2}+d\right)^{\frac{5}{2}}d^{3}}$
default	$a^{2}\left(\frac{x}{5d(e\,x^{2}+d)^{\frac{5}{2}}}+\frac{\frac{4x}{15d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^{2}\sqrt{e\,x^{2}+d}}}{d}\right)+c^{2}\left(\frac{x^{7}}{2e(e\,x^{2}+d)^{\frac{5}{2}}}-\frac{7d\left(-\frac{x^{5}}{5e\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{-\frac{x^{3}}{3e\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+}{}\right.}{2e}\right.$
risch	Expression too large to display

input
```
int((-c*x^4+a)^2/(e*x^2+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^(9/2)/(e*x^2+d)^(5/2)*(-7/2*(e*x^2+d)^(5/2)*arctanh((e*x^2+d)^(1/2)/x/
e^(1/2))*c^2*d^4+x*(a*d^2*(-2/5*c*x^4+a)*e^(9/2)+4/3*e^(11/2)*a^2*d*x^2+8/
15*e^(13/2)*a^2*x^4+161/30*d^3*(15/161*x^6*e^(7/2)+e^(5/2)*d*x^4+35/23*e^(
3/2)*d^2*x^2+15/23*e^(1/2)*d^3)*c^2))/d^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.47

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{7/2}}\,dx = \Bigg[\frac{105\left(c^2d^4e^3x^6+3\,c^2d^5e^2x^4+3\,c^2d^6ex^2+c^2d^7\right)\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-\right.}{6}$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
[1/60*(105*(c^2*d^4*e^3*x^6 + 3*c^2*d^5*e^2*x^4 + 3*c^2*d^6*e*x^2 + c^2*d^
7)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(15*c^2*d^3
*e^4*x^7 + (161*c^2*d^4*e^3 - 12*a*c*d^2*e^5 + 16*a^2*e^7)*x^5 + 5*(49*c^2
*d^5*e^2 + 8*a^2*d*e^6)*x^3 + 15*(7*c^2*d^6*e + 2*a^2*d^2*e^5)*x)*sqrt(e*x
^2 + d))/(d^3*e^8*x^6 + 3*d^4*e^7*x^4 + 3*d^5*e^6*x^2 + d^6*e^5), 1/30*(10
5*(c^2*d^4*e^3*x^6 + 3*c^2*d^5*e^2*x^4 + 3*c^2*d^6*e*x^2 + c^2*d^7)*sqrt(-
e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (15*c^2*d^3*e^4*x^7 + (161*c^2*d^4
*e^3 - 12*a*c*d^2*e^5 + 16*a^2*e^7)*x^5 + 5*(49*c^2*d^5*e^2 + 8*a^2*d*e^6)
*x^3 + 15*(7*c^2*d^6*e + 2*a^2*d^2*e^5)*x)*sqrt(e*x^2 + d))/(d^3*e^8*x^6 +
 3*d^4*e^7*x^4 + 3*d^5*e^6*x^2 + d^6*e^5)]
```

Sympy [F]

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{7/2}}\,dx = \int \frac{(-a+cx^4)^2}{(d+ex^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-c*x**4+a)**2/(e*x**2+d)**(7/2),x)
```

output
```
Integral((-a + c*x**4)**2/(d + e*x**2)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((-c*x^4+a)^2/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{7/2}}\,dx = \frac{\left(\left(\left(\frac{15\,c^2x^2}{e} + \frac{161\,c^2d^4e^5-12\,acd^2e^7+16\,a^2e^9}{d^3e^7}\right)x^2 + \frac{5\,(49\,c^2d^5e^4+8\,a^2de^8)}{d^3e^7}\right)x^2 + \frac{15\,(7\,c^2d^6e^3+2\,a^2d^2e^7)}{d^3e^7}\right.}{30\,(ex^2+d)^{\frac{5}{2}}} + \frac{7\,c^2d\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{2\,e^{\frac{9}{2}}}$$

input

```
integrate((-c*x^4+a)^2/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output

```
1/30*(((15*c^2*x^2/e + (161*c^2*d^4*e^5 - 12*a*c*d^2*e^7 + 16*a^2*e^9)/(d^
3*e^7))*x^2 + 5*(49*c^2*d^5*e^4 + 8*a^2*d*e^8)/(d^3*e^7))*x^2 + 15*(7*c^2*
d^6*e^3 + 2*a^2*d^2*e^7)/(d^3*e^7))*x/(e*x^2 + d)^(5/2) + 7/2*c^2*d*log(ab
s(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{7/2}}\,dx=\int \frac{{\left(a-c\,x^4\right)}^2}{{\left(e\,x^2+d\right)}^{7/2}} \,d x$$

input
```
int((a - c*x^4)^2/(d + e*x^2)^(7/2),x)
```

output
```
int((a - c*x^4)^2/(d + e*x^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.72

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{7/2}}\,dx=\frac{120\sqrt{e\,x^2+d}\,a^2d^2e^5x+160\sqrt{e\,x^2+d}\,a^2d\,e^6x^3+64\sqrt{e\,x^2+d}\,a^2e^7x^5-48\sqrt{e\,x^2+d}}{}$$

input
```
int((-c*x^4+a)^2/(e*x^2+d)^(7/2),x)
```

output
```
(120*sqrt(d + e*x**2)*a**2*d**2*e**5*x + 160*sqrt(d + e*x**2)*a**2*d*e**6*
x**3 + 64*sqrt(d + e*x**2)*a**2*e**7*x**5 - 48*sqrt(d + e*x**2)*a*c*d**2*e
**5*x**5 + 420*sqrt(d + e*x**2)*c**2*d**6*e*x + 980*sqrt(d + e*x**2)*c**2*
d**5*e**2*x**3 + 644*sqrt(d + e*x**2)*c**2*d**4*e**3*x**5 + 60*sqrt(d + e*
x**2)*c**2*d**3*e**4*x**7 - 420*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)
/sqrt(d))*c**2*d**7 - 1260*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt
(d))*c**2*d**6*e*x**2 - 1260*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sq
rt(d))*c**2*d**5*e**2*x**4 - 420*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x
)/sqrt(d))*c**2*d**4*e**3*x**6 - 64*sqrt(e)*a**2*d**3*e**4 - 192*sqrt(e)*a
**2*d**2*e**5*x**2 - 192*sqrt(e)*a**2*d*e**6*x**4 - 64*sqrt(e)*a**2*e**7*x
**6 - 48*sqrt(e)*a*c*d**5*e**2 - 144*sqrt(e)*a*c*d**4*e**3*x**2 - 144*sqrt
(e)*a*c*d**3*e**4*x**4 - 48*sqrt(e)*a*c*d**2*e**5*x**6 - 203*sqrt(e)*c**2*
d**7 - 609*sqrt(e)*c**2*d**6*e*x**2 - 609*sqrt(e)*c**2*d**5*e**2*x**4 - 20
3*sqrt(e)*c**2*d**4*e**3*x**6)/(120*d**3*e**5*(d**3 + 3*d**2*e*x**2 + 3*d*
e**2*x**4 + e**3*x**6))
```

3.338 $\int \frac{(a-cx^4)^2}{(d+ex^2)^{9/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 210

$$\begin{aligned}\int \frac{(a-cx^4)^2}{(d+ex^2)^{9/2}}\,dx &= \frac{(cd^2-ae^2)^2 x}{7de^4(d+ex^2)^{7/2}}\\ &-\frac{2(cd^2-ae^2)(11cd^2+3ae^2)x}{35d^2e^4(d+ex^2)^{5/2}}+\frac{2(61c^2d^4-3acd^2e^2+12a^2e^4)x}{105d^3e^4(d+ex^2)^{3/2}}\\ &-\frac{4(44c^2d^4+3acd^2e^2-12a^2e^4)x}{105d^4e^4\sqrt{d+ex^2}}+\frac{c^2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{9/2}}\end{aligned}$$

output

```
1/7*(-a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^(7/2)-2/35*(-a*e^2+c*d^2)*(3*a*e^2+
11*c*d^2)*x/d^2/e^4/(e*x^2+d)^(5/2)+2/105*(12*a^2*e^4-3*a*c*d^2*e^2+61*c^2
*d^4)*x/d^3/e^4/(e*x^2+d)^(3/2)-4/105*(-12*a^2*e^4+3*a*c*d^2*e^2+44*c^2*d^
4)*x/d^4/e^4/(e*x^2+d)^(1/2)+c^2*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(9/2
)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.75

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{9/2}}\,dx =$$
$$-\frac{x(6acd^2e^4x^4(7d+2ex^2)-3a^2e^4(35d^3+70d^2ex^2+56de^2x^4+16e^3x^6)+c^2d^4(105d^3+350d^2ex^2+406de}{105d^4e^4\left(d+ex^2\right)^{7/2}}$$
$$-\frac{c^2\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{e^{9/2}}$$

input
```
Integrate[(a - c*x^4)^2/(d + e*x^2)^(9/2),x]
```

output
```
-1/105*(x*(6*a*c*d^2*e^4*x^4*(7*d + 2*e*x^2) - 3*a^2*e^4*(35*d^3 + 70*d^2*
e*x^2 + 56*d*e^2*x^4 + 16*e^3*x^6) + c^2*d^4*(105*d^3 + 350*d^2*e*x^2 + 40
6*d*e^2*x^4 + 176*e^3*x^6)))/(d^4*e^4*(d + e*x^2)^(7/2)) - (c^2*Log[-(Sqrt
[e]*x) + Sqrt[d + e*x^2]])/e^(9/2)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1472, 25, 2345, 25, 1471, 25, 25, 27, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{9/2}}dx$$

$$\Big\downarrow \text{ 1472}$$

$$\frac{x\left(cd^2-ae^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}-\frac{\int -\frac{\frac{7c^2dx^6}{e}-\frac{7c^2d^2x^4}{e^2}+\frac{7cd\left(cd^2-2ae^2\right)x^2}{e^3}+6a^2+\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{(ex^2+d)^{7/2}}dx}{7d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\frac{7c^2dx^6}{e}-\frac{7c^2d^2x^4}{e^2}+\frac{7cd\left(cd^2-2ae^2\right)x^2}{e^3}+6a^2+\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{(ex^2+d)^{7/2}}dx}{7d}+\frac{x\left(cd^2-ae^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \ 2345$$

$$\frac{\frac{2x\left(3a^2+\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}-\frac{\int -\frac{\frac{17c^2d^4}{e^4}-\frac{70c^2x^2d^3}{e^3}+\frac{35c^2x^4d^2}{e^2}-\frac{6acd^2}{e^2}+24a^2}{\left(ex^2+d\right)^{5/2}}dx}{5d}}{7d}+\frac{x\left(cd^2-ae^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{\frac{17c^2d^4}{e^4}-\frac{70c^2x^2d^3}{e^3}+\frac{35c^2x^4d^2}{e^2}-\frac{6acd^2}{e^2}+24a^2}{\left(ex^2+d\right)^{5/2}}dx}{5d}+\frac{2x\left(3a^2+\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(cd^2-ae^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \ 1471$$

$$\frac{\frac{\frac{2x\left(12a^2-\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{105c^2x^2d^3+\left(-\frac{71c^2d^4}{e^4}-\frac{12acd^2}{e^2}+48a^2\right)e^3}{e^3\left(ex^2+d\right)^{3/2}}dx}{3d}}{5d}+\frac{2x\left(3a^2+\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(cd^2-ae^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{\int -\frac{\frac{71c^2d^4}{e}-105c^2x^2d^3+12aced^2-48a^2e^3}{e^3\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{2x\left(12a^2-\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}}{5d}+\frac{2x\left(3a^2+\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(cd^2-ae^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{2x\left(12a^2-\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\int \frac{\frac{71c^2d^4}{e}-105c^2x^2d^3+12aced^2-48a^2e^3}{e^3\left(ex^2+d\right)^{3/2}}dx}{3d}}{5d}+\frac{2x\left(3a^2+\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(cd^2-ae^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{2x\left(12a^2-\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\int \frac{\frac{71c^2d^4}{e}-105c^2x^2d^3+12aced^2-48a^2e^3}{\left(ex^2+d\right)^{3/2}}dx}{3de^3}}{5d}+\frac{2x\left(3a^2+\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(cd^2-ae^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{\frac{2x\left(12a^2-\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\frac{4x\left(-12a^2e^4+3acd^2e^2+44c^2d^4\right)}{de\sqrt{d+ex^2}}-\frac{105c^2d^3\int \frac{1}{\sqrt{ex^2+d}}dx}{e}}{3de^3}}{5d}+\frac{2x\left(3a^2+\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(cd^2-ae^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{2x\left(12a^2-\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\frac{4x\left(-12a^2e^4+3acd^2e^2+44c^2d^4\right)}{de\sqrt{d+ex^2}}-\frac{105c^2d^3\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}}{3de^3}}{5d}+\frac{2x\left(3a^2+\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(cd^2-ae^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{2x\left(12a^2-\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\frac{4x\left(-12a^2e^4+3acd^2e^2+44c^2d^4\right)}{de\sqrt{d+ex^2}}-\frac{105c^2d^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}}{3de^3}}{5d}+\frac{2x\left(3a^2+\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(cd^2-ae^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

input

```
Int[(a - c*x^4)^2/(d + e*x^2)^(9/2),x]
```

output

```
((c*d^2 - a*e^2)^2*x)/(7*d*e^4*(d + e*x^2)^(7/2)) + ((2*(3*a^2 - (11*c^2*d
^4)/e^4 + (8*a*c*d^2)/e^2)*x)/(5*d*(d + e*x^2)^(5/2)) + ((2*(12*a^2 + (61*
c^2*d^4)/e^4 - (3*a*c*d^2)/e^2)*x)/(3*d*(d + e*x^2)^(3/2)) - ((4*(44*c^2*d
^4 + 3*a*c*d^2*e^2 - 12*a^2*e^4)*x)/(d*e*Sqrt[d + e*x^2]) - (105*c^2*d^3*A
rcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(3/2))/(3*d*e^3))/(5*d))/(7*d)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{c^{2} d^{4} \operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right) e^{4} \left(e\,x^{2}+d\right)^{\frac{7}{2}}+\frac{x\,e^{\frac{9}{2}} \left(48 a^{2} e^{7} x^{6}-12 a c\,d^{2} e^{5} x^{6}-176 c^{2} d^{4} e^{3} x^{6}+168 a^{2} d\,e^{6} x^{4}-42 a c\,d^{3} e^{4} x^{4}-406 c^{2} d^{5} e^{2} x^{4}+\right.}{105}}{\left(e\,x^{2}+d\right)^{\frac{7}{2}} e^{\frac{17}{2}} d^{4}}$
default	$a^{2} \left(\frac{x}{7 d \left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{\frac{6 x}{35 d \left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{6 \left(\frac{4 x}{15 d \left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8 x}{15 d^{2} \sqrt{e\,x^{2}+d}}\right)}{7 d}}{d}\right)+c^{2} \left(-\frac{x^{7}}{7 e \left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5 e \left(e\,x^{2}+d\right)^{\frac{5}{2}}}}{}\right.$

input
```
int((-c*x^4+a)^2/(e*x^2+d)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
(c^2*d^4*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))*e^4*(e*x^2+d)^(7/2)+1/105*x*e^
(9/2)*(48*a^2*e^7*x^6-12*a*c*d^2*e^5*x^6-176*c^2*d^4*e^3*x^6+168*a^2*d*e^6
*x^4-42*a*c*d^3*e^4*x^4-406*c^2*d^5*e^2*x^4+210*a^2*d^2*e^5*x^2-350*c^2*d^
6*e*x^2+105*a^2*d^3*e^4-105*c^2*d^7))/(e*x^2+d)^(7/2)/e^(17/2)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.65

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{9/2}}\,dx = \left[\frac{105\,(c^2d^4e^4x^8+4\,c^2d^5e^3x^6+6\,c^2d^6e^2x^4+4\,c^2d^7ex^2+c^2d^8)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex}\right.}{}\right.$$

$$-\frac{105\,(c^2d^4e^4x^8+4\,c^2d^5e^3x^6+6\,c^2d^6e^2x^4+4\,c^2d^7ex^2+c^2d^8)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+(4\,(44\,c^2d^4e^4+3\,ac}{105\,(d^4e^9x^8+4\,d}$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(c^2*d^4*e^4*x^8 + 4*c^2*d^5*e^3*x^6 + 6*c^2*d^6*e^2*x^4 + 4*c
^2*d^7*e*x^2 + c^2*d^8)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x
 - d) - 2*(4*(44*c^2*d^4*e^4 + 3*a*c*d^2*e^6 - 12*a^2*e^8)*x^7 + 14*(29*c^
2*d^5*e^3 + 3*a*c*d^3*e^5 - 12*a^2*d*e^7)*x^5 + 70*(5*c^2*d^6*e^2 - 3*a^2*
d^2*e^6)*x^3 + 105*(c^2*d^7*e - a^2*d^3*e^5)*x)*sqrt(e*x^2 + d))/(d^4*e^9*
x^8 + 4*d^5*e^8*x^6 + 6*d^6*e^7*x^4 + 4*d^7*e^6*x^2 + d^8*e^5), -1/105*(10
5*(c^2*d^4*e^4*x^8 + 4*c^2*d^5*e^3*x^6 + 6*c^2*d^6*e^2*x^4 + 4*c^2*d^7*e*x
^2 + c^2*d^8)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (4*(44*c^2*d^4
*e^4 + 3*a*c*d^2*e^6 - 12*a^2*e^8)*x^7 + 14*(29*c^2*d^5*e^3 + 3*a*c*d^3*e^
5 - 12*a^2*d*e^7)*x^5 + 70*(5*c^2*d^6*e^2 - 3*a^2*d^2*e^6)*x^3 + 105*(c^2*
d^7*e - a^2*d^3*e^5)*x)*sqrt(e*x^2 + d))/(d^4*e^9*x^8 + 4*d^5*e^8*x^6 + 6*
d^6*e^7*x^4 + 4*d^7*e^6*x^2 + d^8*e^5)]
```

Sympy [F]

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{9/2}}\,dx = \int \frac{(-a+cx^4)^2}{(d+ex^2)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-c*x**4+a)**2/(e*x**2+d)**(9/2),x)
```

output
```
Integral((-a + c*x**4)**2/(d + e*x**2)**(9/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(9/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.91

$$\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{9/2}}\,dx = -\frac{\left(2\left(x^2\left(\frac{2\left(44\,c^2d^4e^6+3\,acd^2e^8-12\,a^2e^{10}\right)x^2}{d^4e^7}+\frac{7\left(29\,c^2d^5e^5+3\,acd^3e^7-12\,a^2de^9\right)}{d^4e^7}\right)+\frac{35\left(5\,c^2d^6e^4-3\,a^2d^2e^8\right)}{d^4e^7}\right)x^2+\frac{105\,(c^2d^7e^3-a^2}{d^4e^7}}{105\left(ex^2+d\right)^{\frac{7}{2}}} - \frac{c^2\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{e^{\frac{9}{2}}}$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(9/2),x, algorithm="giac")
```

output
```
-1/105*(2*(x^2*(2*(44*c^2*d^4*e^6 + 3*a*c*d^2*e^8 - 12*a^2*e^10)*x^2/(d^4*
e^7) + 7*(29*c^2*d^5*e^5 + 3*a*c*d^3*e^7 - 12*a^2*d*e^9)/(d^4*e^7)) + 35*(
5*c^2*d^6*e^4 - 3*a^2*d^2*e^8)/(d^4*e^7))*x^2 + 105*(c^2*d^7*e^3 - a^2*d^3
*e^7)/(d^4*e^7))*x/(e*x^2 + d)^(7/2) - c^2*log(abs(-sqrt(e)*x + sqrt(e*x^2
 + d)))/e^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{9/2}}\,dx = \int \frac{(a-c\,x^4)^2}{(e\,x^2+d)^{9/2}}\,dx$$

input
```
int((a - c*x^4)^2/(d + e*x^2)^(9/2),x)
```

output
```
int((a - c*x^4)^2/(d + e*x^2)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 641, normalized size of antiderivative = 3.05

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{9/2}}\,dx = \frac{56\sqrt{e}\,c^2d^8 - 42\sqrt{e\,x^2+d}\,ac\,d^3e^5x^5 - 12\sqrt{e\,x^2+d}\,ac\,d^2e^6x^7 + 420\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}}{\sqrt{d}}\right.}{}$$

input
```
int((-c*x^4+a)^2/(e*x^2+d)^(9/2),x)
```

output

```
(105*sqrt(d + e*x**2)*a**2*d**3*e**5*x + 210*sqrt(d + e*x**2)*a**2*d**2*e*
*6*x**3 + 168*sqrt(d + e*x**2)*a**2*d*e**7*x**5 + 48*sqrt(d + e*x**2)*a**2
*e**8*x**7 - 42*sqrt(d + e*x**2)*a*c*d**3*e**5*x**5 - 12*sqrt(d + e*x**2)*
a*c*d**2*e**6*x**7 - 105*sqrt(d + e*x**2)*c**2*d**7*e*x - 350*sqrt(d + e*x
**2)*c**2*d**6*e**2*x**3 - 406*sqrt(d + e*x**2)*c**2*d**5*e**3*x**5 - 176*
sqrt(d + e*x**2)*c**2*d**4*e**4*x**7 + 105*sqrt(e)*log((sqrt(d + e*x**2) +
 sqrt(e)*x)/sqrt(d))*c**2*d**8 + 420*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(
e)*x)/sqrt(d))*c**2*d**7*e*x**2 + 630*sqrt(e)*log((sqrt(d + e*x**2) + sqrt
(e)*x)/sqrt(d))*c**2*d**6*e**2*x**4 + 420*sqrt(e)*log((sqrt(d + e*x**2) +
sqrt(e)*x)/sqrt(d))*c**2*d**5*e**3*x**6 + 105*sqrt(e)*log((sqrt(d + e*x**2
) + sqrt(e)*x)/sqrt(d))*c**2*d**4*e**4*x**8 - 48*sqrt(e)*a**2*d**4*e**4 -
192*sqrt(e)*a**2*d**3*e**5*x**2 - 288*sqrt(e)*a**2*d**2*e**6*x**4 - 192*sq
rt(e)*a**2*d*e**7*x**6 - 48*sqrt(e)*a**2*e**8*x**8 + 12*sqrt(e)*a*c*d**6*e
**2 + 48*sqrt(e)*a*c*d**5*e**3*x**2 + 72*sqrt(e)*a*c*d**4*e**4*x**4 + 48*s
qrt(e)*a*c*d**3*e**5*x**6 + 12*sqrt(e)*a*c*d**2*e**6*x**8 + 56*sqrt(e)*c**
2*d**8 + 224*sqrt(e)*c**2*d**7*e*x**2 + 336*sqrt(e)*c**2*d**6*e**2*x**4 +
224*sqrt(e)*c**2*d**5*e**3*x**6 + 56*sqrt(e)*c**2*d**4*e**4*x**8)/(105*d**
4*e**5*(d**4 + 4*d**3*e*x**2 + 6*d**2*e**2*x**4 + 4*d*e**3*x**6 + e**4*x**
8))
```

3.339 $\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{11/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 231

$$\begin{aligned}\int \frac{(a-cx^4)^2}{(d+ex^2)^{11/2}}\,dx &= \frac{(cd^2-ae^2)^2\,x}{9de^4\,(d+ex^2)^{9/2}} \\ &\quad-\frac{4(cd^2-ae^2)\,(7cd^2+2ae^2)\,x}{63d^2e^4\,(d+ex^2)^{7/2}}+\frac{2(35c^2d^4-acd^2e^2+8a^2e^4)\,x}{105d^3e^4\,(d+ex^2)^{5/2}} \\ &\quad-\frac{4(35c^2d^4+2acd^2e^2-16a^2e^4)\,x}{315d^4e^4\,(d+ex^2)^{3/2}}+\frac{(35c^2d^4-16acd^2e^2+128a^2e^4)\,x}{315d^5e^4\sqrt{d+ex^2}}\end{aligned}$$

output

```
1/9*(-a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^(9/2)-4/63*(-a*e^2+c*d^2)*(2*a*e^2+
7*c*d^2)*x/d^2/e^4/(e*x^2+d)^(7/2)+2/105*(8*a^2*e^4-a*c*d^2*e^2+35*c^2*d^4
)*x/d^3/e^4/(e*x^2+d)^(5/2)-4/315*(-16*a^2*e^4+2*a*c*d^2*e^2+35*c^2*d^4)*x
/d^4/e^4/(e*x^2+d)^(3/2)+1/315*(128*a^2*e^4-16*a*c*d^2*e^2+35*c^2*d^4)*x/d
^5/e^4/(e*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.47

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{11/2}}\,dx = \frac{35c^2d^4x^9-2acd^2x^5(63d^2+36dex^2+8e^2x^4)+a^2(315d^4x+840d^3ex^3+1008d^2e^2x^5}{315d^5\,(d+ex^2)^{9/2}}$$

input
```
Integrate[(a - c*x^4)^2/(d + e*x^2)^(11/2),x]
```

output
```
(35*c^2*d^4*x^9 - 2*a*c*d^2*x^5*(63*d^2 + 36*d*e*x^2 + 8*e^2*x^4) + a^2*(3
15*d^4*x + 840*d^3*e*x^3 + 1008*d^2*e^2*x^5 + 576*d*e^3*x^7 + 128*e^4*x^9)
)/(315*d^5*(d + e*x^2)^(9/2))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1470, 2334, 27, 2090, 1587, 9, 27, 362, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{11/2}}\,dx$$

$$\Big\downarrow \text{1470}$$

$$\frac{\int \frac{x^2\left(8a^2e-d\left(2acx^2-c^2x^6\right)\right)}{(ex^2+d)^{11/2}}dx}{d}+\frac{a^2x}{d\,(d+ex^2)^{9/2}}$$

$$\Big\downarrow \text{2334}$$

$$\frac{\frac{\int \frac{3x^4\left(16a^2e^2-cd^2\left(2a-cx^4\right)\right)}{\left(ex^2+d\right)^{11/2}}dx}{3d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\,(d+ex^2)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{x^4\left(16a^2e^2-cd^2\left(2a-cx^4\right)\right)}{\left(ex^2+d\right)^{11/2}}dx}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\downarrow \text{2090}$$

$$\frac{\frac{\int \frac{x^4\left(c^2d^2x^4-2a\left(cd^2-8ae^2\right)\right)}{\left(ex^2+d\right)^{11/2}}dx}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\downarrow \text{1587}$$

$$\frac{\frac{-\frac{\int \frac{x^3\left(\left(\frac{5c^2d^4}{e^2}+8acd^2-64a^2e^2\right)x-\frac{9c^2d^3x^3}{e}\right)}{\left(ex^2+d\right)^{9/2}}dx}{9d}-\frac{x^5\left(-16a^2e^2+2acd^2-\frac{c^2d^4}{e^2}\right)}{9d\left(d+ex^2\right)^{9/2}}}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\downarrow \text{9}$$

$$\frac{\frac{-\frac{\int \frac{x^4\left(e\left(\frac{5c^2d^4}{e^2}+8acd^2-64a^2e^2\right)-9c^2d^3x^2\right)}{e\left(ex^2+d\right)^{9/2}}dx}{9d}-\frac{x^5\left(-16a^2e^2+2acd^2-\frac{c^2d^4}{e^2}\right)}{9d\left(d+ex^2\right)^{9/2}}}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\downarrow \text{27}$$

$$\frac{\frac{-\frac{\int \frac{x^4\left(\frac{5c^2d^4}{e}-9c^2x^2d^3+8aced^2-64a^2e^3\right)}{\left(ex^2+d\right)^{9/2}}dx}{9de}-\frac{x^5\left(-16a^2e^2+2acd^2-\frac{c^2d^4}{e^2}\right)}{9d\left(d+ex^2\right)^{9/2}}}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\downarrow \text{362}$$

$$\frac{\frac{-\frac{\frac{2x^5\left(-32a^2e^4+4acd^2e^2+7c^2d^4\right)}{7de\left(d+ex^2\right)^{7/2}}-\frac{\left(128a^2e^4-16acd^2e^2+35c^2d^4\right)\int \frac{x^4}{\left(ex^2+d\right)^{7/2}}dx}{7de}}{9de}-\frac{x^5\left(-16a^2e^2+2acd^2-\frac{c^2d^4}{e^2}\right)}{9d\left(d+ex^2\right)^{9/2}}}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\downarrow \text{242}$$

$$\frac{\frac{-\frac{x^5\left(-16a^2e^2+2acd^2-\frac{c^2d^4}{e^2}\right)}{9d\left(d+ex^2\right)^{9/2}}-\frac{\frac{2x^5\left(-32a^2e^4+4acd^2e^2+7c^2d^4\right)}{7de\left(d+ex^2\right)^{7/2}}-\frac{x^5\left(128a^2e^4-16acd^2e^2+35c^2d^4\right)}{35d^2e\left(d+ex^2\right)^{5/2}}}{9de}}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

input

```
Int[(a - c*x^4)^2/(d + e*x^2)^(11/2),x]
```

output

```
(a^2*x)/(d*(d + e*x^2)^(9/2)) + ((8*a^2*e*x^3)/(3*d*(d + e*x^2)^(9/2)) + (
-1/9*((2*a*c*d^2 - (c^2*d^4)/e^2 - 16*a^2*e^2)*x^5)/(d*(d + e*x^2)^(9/2))
- ((2*(7*c^2*d^4 + 4*a*c*d^2*e^2 - 32*a^2*e^4)*x^5)/(7*d*e*(d + e*x^2)^(7/
2)) - ((35*c^2*d^4 - 16*a*c*d^2*e^2 + 128*a^2*e^4)*x^5)/(35*d^2*e*(d + e*x
^2)^(5/2)))/(9*d*e))/d)/d
```

Defintions of rubi rules used

rule 9

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1470
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d + e*x^2)^q*(d*
PolynomialQuotient[(a + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q
+ 1/2, 0] && LtQ[4*p + 2*q + 1, 0]
```

rule 1587
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x]
, R = Coeff[PolynomialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[
(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d*f*(q + 1))), x] + Simp[f/(2*d*
(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*x*
Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, f}, x] && IGtQ[p,
 0] && LtQ[q, -1] && GtQ[m, 0]
```

rule 2090
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && BinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u
, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.43

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{9}c^{2}x^{8}-\frac{2}{5}ac\,x^{4}+a^{2}\right)d^{4}+\frac{8x^{2}\left(-\frac{3c\,x^{4}}{35}+a\right)ae\,d^{3}}{3}+\frac{16\left(-\frac{c\,x^{4}}{63}+a\right)x^{4}a\,e^{2}d^{2}}{5}+\frac{64a^{2}d\,e^{3}x^{6}}{35}+\frac{128a^{2}e^{4}x^{8}}{315}\right)}{\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
gosper	$\frac{x\left(128a^{2}e^{4}x^{8}-16ac\,d^{2}e^{2}x^{8}+35c^{2}d^{4}x^{8}+576a^{2}d\,e^{3}x^{6}-72ac\,d^{3}e\,x^{6}+1008a^{2}d^{2}e^{2}x^{4}-126ac\,d^{4}x^{4}+840a^{2}e\,x^{2}d^{3}+315a^{2}d^{4}\right)}{315\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
trager	$\frac{x\left(128a^{2}e^{4}x^{8}-16ac\,d^{2}e^{2}x^{8}+35c^{2}d^{4}x^{8}+576a^{2}d\,e^{3}x^{6}-72ac\,d^{3}e\,x^{6}+1008a^{2}d^{2}e^{2}x^{4}-126ac\,d^{4}x^{4}+840a^{2}e\,x^{2}d^{3}+315a^{2}d^{4}\right)}{315\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
orering	$\frac{x\left(128a^{2}e^{4}x^{8}-16ac\,d^{2}e^{2}x^{8}+35c^{2}d^{4}x^{8}+576a^{2}d\,e^{3}x^{6}-72ac\,d^{3}e\,x^{6}+1008a^{2}d^{2}e^{2}x^{4}-126ac\,d^{4}x^{4}+840a^{2}e\,x^{2}d^{3}+315a^{2}d^{4}\right)}{315\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
	$\left(\right.$

input
```
int((-c*x^4+a)^2/(e*x^2+d)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
x/(e*x^2+d)^(9/2)*((1/9*c^2*x^8-2/5*a*c*x^4+a^2)*d^4+8/3*x^2*(-3/35*c*x^4+
a)*a*e*d^3+16/5*(-1/63*c*x^4+a)*x^4*a*e^2*d^2+64/35*a^2*d*e^3*x^6+128/315*
a^2*e^4*x^8)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.72

$$\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{11/2}}\,dx=\frac{((35\,c^2d^4-16\,acd^2e^2+128\,a^2e^4)x^9+840\,a^2d^3ex^3-72\,(acd^3e-8\,a^2de^3)x^7+315\,a^2}{315\,(d^5e^5x^{10}+5\,d^6e^4x^8+10\,d^7e^3x^6+10\,d^8e^2x^4+5\,d^9}$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(11/2),x, algorithm="fricas")
```

output
```
1/315*((35*c^2*d^4 - 16*a*c*d^2*e^2 + 128*a^2*e^4)*x^9 + 840*a^2*d^3*e*x^3
 - 72*(a*c*d^3*e - 8*a^2*d*e^3)*x^7 + 315*a^2*d^4*x - 126*(a*c*d^4 - 8*a^2
*d^2*e^2)*x^5)*sqrt(e*x^2 + d)/(d^5*e^5*x^10 + 5*d^6*e^4*x^8 + 10*d^7*e^3*
x^6 + 10*d^8*e^2*x^4 + 5*d^9*e*x^2 + d^10)
```

Sympy [F]

$$\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{11/2}}\,dx=\int \frac{\left(-a+cx^4\right)^2}{\left(d+ex^2\right)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-c*x**4+a)**2/(e*x**2+d)**(11/2),x)
```

output
```
Integral((-a + c*x**4)**2/(d + e*x**2)**(11/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.60

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{11/2}}\,dx = -\frac{c^2x^7}{2\,(ex^2+d)^{\frac{9}{2}}e} - \frac{7\,c^2dx^5}{8\,(ex^2+d)^{\frac{9}{2}}e^2} - \frac{35\,c^2d^2x^3}{48\,(ex^2+d)^{\frac{9}{2}}e^3} + \frac{acx^3}{3\,(ex^2+d)^{\frac{9}{2}}e} + \frac{128\,a^2x}{315\,\sqrt{ex^2+d}d^5} + \frac{64\,a^2x}{315\,(ex^2+d)^{\frac{3}{2}}d^4} + \frac{16\,a^2x}{105\,(ex^2+d)^{\frac{5}{2}}d^3} + \frac{8\,a^2x}{63\,(ex^2+d)^{\frac{7}{2}}d^2} + \frac{a^2x}{9\,(ex^2+d)^{\frac{9}{2}}d} + \frac{c^2x}{18\,(ex^2+d)^{\frac{3}{2}}e^4} + \frac{c^2x}{9\,\sqrt{ex^2+d}de^4} + \frac{c^2dx}{24\,(ex^2+d)^{\frac{5}{2}}e^4} + \frac{5\,c^2d^2x}{144\,(ex^2+d)^{\frac{7}{2}}e^4} - \frac{35\,c^2d^3x}{144\,(ex^2+d)^{\frac{9}{2}}e^4} - \frac{acx}{63\,(ex^2+d)^{\frac{7}{2}}e^2} - \frac{16\,acx}{315\,\sqrt{ex^2+d}d^3e^2} - \frac{8\,acx}{315\,(ex^2+d)^{\frac{3}{2}}d^2e^2} - \frac{2\,acx}{105\,(ex^2+d)^{\frac{5}{2}}de^2} + \frac{acdx}{9\,(ex^2+d)^{\frac{9}{2}}e^2}$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(11/2),x, algorithm="maxima")
```

output
```
-1/2*c^2*x^7/((e*x^2 + d)^(9/2)*e) - 7/8*c^2*d*x^5/((e*x^2 + d)^(9/2)*e^2)
 - 35/48*c^2*d^2*x^3/((e*x^2 + d)^(9/2)*e^3) + 1/3*a*c*x^3/((e*x^2 + d)^(9
/2)*e) + 128/315*a^2*x/(sqrt(e*x^2 + d)*d^5) + 64/315*a^2*x/((e*x^2 + d)^(
3/2)*d^4) + 16/105*a^2*x/((e*x^2 + d)^(5/2)*d^3) + 8/63*a^2*x/((e*x^2 + d)
^(7/2)*d^2) + 1/9*a^2*x/((e*x^2 + d)^(9/2)*d) + 1/18*c^2*x/((e*x^2 + d)^(3
/2)*e^4) + 1/9*c^2*x/(sqrt(e*x^2 + d)*d*e^4) + 1/24*c^2*d*x/((e*x^2 + d)^(
5/2)*e^4) + 5/144*c^2*d^2*x/((e*x^2 + d)^(7/2)*e^4) - 35/144*c^2*d^3*x/((e
*x^2 + d)^(9/2)*e^4) - 1/63*a*c*x/((e*x^2 + d)^(7/2)*e^2) - 16/315*a*c*x/(
sqrt(e*x^2 + d)*d^3*e^2) - 8/315*a*c*x/((e*x^2 + d)^(3/2)*d^2*e^2) - 2/105
*a*c*x/((e*x^2 + d)^(5/2)*d*e^2) + 1/9*a*c*d*x/((e*x^2 + d)^(9/2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.61

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{11/2}}\,dx = \frac{\left(\left(\left(x^2\left(\frac{(35\,c^2d^4e^4-16\,acd^2e^6+128\,a^2e^8)x^2}{d^5e^4}-\frac{72\,(acd^3e^5-8\,a^2de^7)}{d^5e^4}\right)-\frac{126\,(acd^4e^4-8\,a^2d^2e^6)}{d^5e^4}\right)x^2+}{315\,(ex^2+d)^{\frac{9}{2}}}$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(11/2),x, algorithm="giac")
```

output
```
1/315*(((x^2*((35*c^2*d^4*e^4 - 16*a*c*d^2*e^6 + 128*a^2*e^8)*x^2/(d^5*e^4
) - 72*(a*c*d^3*e^5 - 8*a^2*d*e^7)/(d^5*e^4)) - 126*(a*c*d^4*e^4 - 8*a^2*d
^2*e^6)/(d^5*e^4))*x^2 + 840*a^2*e/d^2)*x^2 + 315*a^2/d)*x/(e*x^2 + d)^(9/
2)
```

Mupad [B] (verification not implemented)

Time = 17.60 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \frac{(a-cx^4)^2}{(d+ex^2)^{11/2}}\,dx &= \frac{x\left(\frac{a^2}{9\,d}+\frac{d^2\left(\frac{c^2\,d}{9\,e^2}-\frac{2\,a\,c}{9\,d}\right)}{e^2}\right)}{(e\,x^2+d)^{9/2}}\\ &\quad+\frac{x\left(\frac{8\,a^2\,e^4+2\,a\,c\,d^2\,e^2-c^2\,d^4}{63\,d^2\,e^4}-\frac{d\left(\frac{2\,c^2\,d}{7\,e^3}-\frac{c\,(2\,a\,e^2-c\,d^2)}{7\,d\,e^3}\right)}{e}\right)}{(e\,x^2+d)^{7/2}}\\ &\quad-\frac{x\left(\frac{c^2}{3\,e^4}+\frac{-64\,a^2\,e^4+8\,a\,c\,d^2\,e^2+35\,c^2\,d^4}{315\,d^4\,e^4}\right)}{(e\,x^2+d)^{3/2}}+\frac{x\left(\frac{3\,c^2\,d}{5\,e^4}+\frac{16\,a^2\,e^4-2\,a\,c\,d^2\,e^2+7\,c^2\,d^4}{105\,d^3\,e^4}\right)}{(e\,x^2+d)^{5/2}}\\ &\quad+\frac{x\,(128\,a^2\,e^4-16\,a\,c\,d^2\,e^2+35\,c^2\,d^4)}{315\,d^5\,e^4\,\sqrt{e\,x^2+d}}\end{aligned}$$

input
```
int((a - c*x^4)^2/(d + e*x^2)^(11/2),x)
```

output

```
(x*(a^2/(9*d) + (d^2*((c^2*d)/(9*e^2) - (2*a*c)/(9*d)))/e^2))/(d + e*x^2)^
(9/2) + (x*((8*a^2*e^4 - c^2*d^4 + 2*a*c*d^2*e^2)/(63*d^2*e^4) - (d*((2*c^
2*d)/(7*e^3) - (c*(2*a*e^2 - c*d^2))/(7*d*e^3)))/e))/(d + e*x^2)^(7/2) - (
x*(c^2/(3*e^4) + (35*c^2*d^4 - 64*a^2*e^4 + 8*a*c*d^2*e^2)/(315*d^4*e^4)))
/(d + e*x^2)^(3/2) + (x*((3*c^2*d)/(5*e^4) + (16*a^2*e^4 + 7*c^2*d^4 - 2*a
*c*d^2*e^2)/(105*d^3*e^4)))/(d + e*x^2)^(5/2) + (x*(128*a^2*e^4 + 35*c^2*d
^4 - 16*a*c*d^2*e^2))/(315*d^5*e^4*(d + e*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.22

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{11/2}}\,dx = \frac{-126\sqrt{e\,x^2+d}\,ac\,d^4e^5x^5-72\sqrt{e\,x^2+d}\,ac\,d^3e^6x^7-16\sqrt{e\,x^2+d}\,ac\,d^2e^7x^9+80\sqrt{e}\,a}{}$$

input

```
int((-c*x^4+a)^2/(e*x^2+d)^(11/2),x)
```

output

```
(315*sqrt(d + e*x**2)*a**2*d**4*e**5*x + 840*sqrt(d + e*x**2)*a**2*d**3*e*
*6*x**3 + 1008*sqrt(d + e*x**2)*a**2*d**2*e**7*x**5 + 576*sqrt(d + e*x**2)
*a**2*d*e**8*x**7 + 128*sqrt(d + e*x**2)*a**2*e**9*x**9 - 126*sqrt(d + e*x
**2)*a*c*d**4*e**5*x**5 - 72*sqrt(d + e*x**2)*a*c*d**3*e**6*x**7 - 16*sqrt
(d + e*x**2)*a*c*d**2*e**7*x**9 + 35*sqrt(d + e*x**2)*c**2*d**4*e**5*x**9
- 128*sqrt(e)*a**2*d**5*e**4 - 640*sqrt(e)*a**2*d**4*e**5*x**2 - 1280*sqrt
(e)*a**2*d**3*e**6*x**4 - 1280*sqrt(e)*a**2*d**2*e**7*x**6 - 640*sqrt(e)*a
**2*d*e**8*x**8 - 128*sqrt(e)*a**2*e**9*x**10 + 16*sqrt(e)*a*c*d**7*e**2 +
 80*sqrt(e)*a*c*d**6*e**3*x**2 + 160*sqrt(e)*a*c*d**5*e**4*x**4 + 160*sqrt
(e)*a*c*d**4*e**5*x**6 + 80*sqrt(e)*a*c*d**3*e**6*x**8 + 16*sqrt(e)*a*c*d*
*2*e**7*x**10 + 35*sqrt(e)*c**2*d**9 + 175*sqrt(e)*c**2*d**8*e*x**2 + 350*
sqrt(e)*c**2*d**7*e**2*x**4 + 350*sqrt(e)*c**2*d**6*e**3*x**6 + 175*sqrt(e
)*c**2*d**5*e**4*x**8 + 35*sqrt(e)*c**2*d**4*e**5*x**10)/(315*d**5*e**5*(d
**5 + 5*d**4*e*x**2 + 10*d**3*e**2*x**4 + 10*d**2*e**3*x**6 + 5*d*e**4*x**
8 + e**5*x**10))
```

3.340 $\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{13/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 280

$$\begin{aligned}\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{13/2}}\,dx &= \frac{\left(cd^2-ae^2\right)^2 x}{11de^4\left(d+ex^2\right)^{11/2}}-\frac{2\left(cd^2-ae^2\right)\left(17cd^2+5ae^2\right)x}{99d^2e^4\left(d+ex^2\right)^{9/2}}\\&+\frac{2\left(161c^2d^4-3acd^2e^2+40a^2e^4\right)x}{693d^3e^4\left(d+ex^2\right)^{7/2}}-\frac{4\left(70c^2d^4+3acd^2e^2-40a^2e^4\right)x}{1155d^4e^4\left(d+ex^2\right)^{5/2}}\\&+\frac{\left(35c^2d^4-48acd^2e^2+640a^2e^4\right)x}{3465d^5e^4\left(d+ex^2\right)^{3/2}}+\frac{2\left(35c^2d^4-48acd^2e^2+640a^2e^4\right)x}{3465d^6e^4\sqrt{d+ex^2}}\end{aligned}$$

output

```
1/11*(-a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^(11/2)-2/99*(-a*e^2+c*d^2)*(5*a*e^
2+17*c*d^2)*x/d^2/e^4/(e*x^2+d)^(9/2)+2/693*(40*a^2*e^4-3*a*c*d^2*e^2+161*
c^2*d^4)*x/d^3/e^4/(e*x^2+d)^(7/2)-4/1155*(-40*a^2*e^4+3*a*c*d^2*e^2+70*c^
2*d^4)*x/d^4/e^4/(e*x^2+d)^(5/2)+1/3465*(640*a^2*e^4-48*a*c*d^2*e^2+35*c^2
*d^4)*x/d^5/e^4/(e*x^2+d)^(3/2)+2/3465*(640*a^2*e^4-48*a*c*d^2*e^2+35*c^2*
d^4)*x/d^6/e^4/(e*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.51

$$\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{13/2}}\,dx=\frac{35c^2d^4x^9(11d+2ex^2)-6acd^2x^5(231d^3+198d^2ex^2+88de^2x^4+16e^3x^6)+5a^2(693}{3465d^6\left(d+ex^2\right)}$$

input

```
Integrate[(a - c*x^4)^2/(d + e*x^2)^(13/2),x]
```

output

```
(35*c^2*d^4*x^9*(11*d + 2*e*x^2) - 6*a*c*d^2*x^5*(231*d^3 + 198*d^2*e*x^2
+ 88*d*e^2*x^4 + 16*e^3*x^6) + 5*a^2*(693*d^5*x + 2310*d^4*e*x^3 + 3696*d^
3*e^2*x^5 + 3168*d^2*e^3*x^7 + 1408*d*e^4*x^9 + 256*e^5*x^11))/(3465*d^6*(
d + e*x^2)^(11/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1470, 2334, 2090, 1587, 9, 27, 362, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-cx^4\right)^2}{\left(d+ex^2\right)^{13/2}}\,dx$$

$$\Big\downarrow \text{1470}$$

$$\frac{\int \frac{x^2\left(10a^2e-d\left(2acx^2-c^2x^6\right)\right)}{\left(ex^2+d\right)^{13/2}}dx}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \text{2334}$$

$$\frac{\frac{\int \frac{x^4\left(80a^2e^2-3cd^2\left(2a-cx^4\right)\right)}{\left(ex^2+d\right)^{13/2}}dx}{3d}+\frac{10a^2ex^3}{3d(d+ex^2)^{11/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \text{2090}$$

$$\frac{\frac{\int \frac{x^4\left(3c^2d^2x^4-2a\left(3cd^2-40ae^2\right)\right)}{\left(ex^2+d\right)^{13/2}}dx}{3d}+\frac{10a^2ex^3}{3d(d+ex^2)^{11/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \text{1587}$$

$$\frac{\frac{-\frac{\int \frac{3x^3\left(\left(\frac{5c^2d^4}{e^2}+12acd^2-160a^2e^2\right)x-\frac{11c^2d^3x^3}{e}\right)}{\left(ex^2+d\right)^{11/2}}dx}{11d}-\frac{x^5\left(-80a^2e^2+6acd^2-\frac{3c^2d^4}{e^2}\right)}{11d\left(d+ex^2\right)^{11/2}}}{3d}+\frac{10a^2ex^3}{3d(d+ex^2)^{11/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \text{9}$$

$$\frac{\frac{-\frac{\int \frac{3x^4\left(e\left(\frac{5c^2d^4}{e^2}+12acd^2-160a^2e^2\right)-11c^2d^3x^2\right)}{e\left(ex^2+d\right)^{11/2}}dx}{11d}-\frac{x^5\left(-80a^2e^2+6acd^2-\frac{3c^2d^4}{e^2}\right)}{11d\left(d+ex^2\right)^{11/2}}}{3d}+\frac{10a^2ex^3}{3d(d+ex^2)^{11/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{3\int \frac{x^4\left(\frac{5c^2d^4}{e}-11c^2x^2d^3+12aced^2-160a^2e^3\right)}{\left(ex^2+d\right)^{11/2}}dx}{11de}-\frac{x^5\left(-80a^2e^2+6acd^2-\frac{3c^2d^4}{e^2}\right)}{11d\left(d+ex^2\right)^{11/2}}}{3d}+\frac{10a^2ex^3}{3d(d+ex^2)^{11/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{-\frac{3\left(\frac{4x^5\left(-40a^2e^4+3acd^2e^2+4c^2d^4\right)}{9de\left(d+ex^2\right)^{9/2}}-\frac{\left(640a^2e^4-48acd^2e^2+35c^2d^4\right)\int \frac{x^4}{\left(ex^2+d\right)^{9/2}}dx}{9de}\right)}{11de}-\frac{x^5\left(-80a^2e^2+6acd^2-\frac{3c^2d^4}{e^2}\right)}{11d\left(d+ex^2\right)^{11/2}}}{3d}+\frac{10a^2ex^3}{3d(d+ex^2)^{11/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{-\frac{3\left(\frac{4x^5\left(-40a^2e^4+3acd^2e^2+4c^2d^4\right)}{9de\left(d+ex^2\right)^{9/2}}-\frac{\left(640a^2e^4-48acd^2e^2+35c^2d^4\right)\left(\frac{2e\int\frac{x^6}{\left(ex^2+d\right)^{9/2}}dx}{5d}+\frac{x^5}{5d\left(d+ex^2\right)^{7/2}}\right)}{9de}\right)}{11de}-\frac{x^5\left(-80a^2e^2+6acd^2-\frac{3c^2d^4}{e^2}\right)}{11d\left(d+ex^2\right)^{11/2}}}{3d}+\frac{1}{3d(d}}{d}$$

$$\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \color{blue}{242}$$

$$\frac{\frac{-\frac{x^5\left(-80a^2e^2+6acd^2-\frac{3c^2d^4}{e^2}\right)}{11d\left(d+ex^2\right)^{11/2}}-\frac{3\left(\frac{4x^5\left(-40a^2e^4+3acd^2e^2+4c^2d^4\right)}{9de\left(d+ex^2\right)^{9/2}}-\frac{\left(\frac{2ex^7}{35d^2\left(d+ex^2\right)^{7/2}}+\frac{x^5}{5d\left(d+ex^2\right)^{7/2}}\right)\left(640a^2e^4-48acd^2e^2+35c^2d^4\right)}{9de}\right)}{11de}}{3d}+\frac{10a}{3d(d+e}}{d}$$

$$\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

input
```
Int[(a - c*x^4)^2/(d + e*x^2)^(13/2),x]
```

output
```
(a^2*x)/(d*(d + e*x^2)^(11/2)) + ((10*a^2*e*x^3)/(3*d*(d + e*x^2)^(11/2))
+ (-1/11*((6*a*c*d^2 - (3*c^2*d^4)/e^2 - 80*a^2*e^2)*x^5)/(d*(d + e*x^2)^(
11/2)) - (3*((4*(4*c^2*d^4 + 3*a*c*d^2*e^2 - 40*a^2*e^4)*x^5)/(9*d*e*(d +
e*x^2)^(9/2)) - ((35*c^2*d^4 - 48*a*c*d^2*e^2 + 640*a^2*e^4)*(x^5/(5*d*(d
+ e*x^2)^(7/2)) + (2*e*x^7)/(35*d^2*(d + e*x^2)^(7/2))))/(9*d*e)))/(11*d*e
))/(3*d))/d
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1470
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d + e*x^2)^q*(d*
PolynomialQuotient[(a + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q
+ 1/2, 0] && LtQ[4*p + 2*q + 1, 0]
```

rule 1587
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x]
, R = Coeff[PolynomialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[
(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d*f*(q + 1))), x] + Simp[f/(2*d*
(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*x*
Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, f}, x] && IGtQ[p,
 0] && LtQ[q, -1] && GtQ[m, 0]
```

rule 2090
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && BinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u
, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.46

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{9}c^{2}x^{8}-\frac{2}{5}ac\,x^{4}+a^{2}\right)d^{5}+\frac{10x^{2}\left(\frac{1}{165}c^{2}x^{8}-\frac{18}{175}ac\,x^{4}+a^{2}\right)e\,d^{4}}{3}+\frac{16x^{4}a\,e^{2}\left(-\frac{c\,x^{4}}{35}+a\right)d^{3}}{3}+\frac{32x^{6}a\left(-\frac{c\,x^{4}}{165}+a\right)e^{3}d^{2}}{7}+\frac{128a^{2}d\,e^{4}}{63}\right.}{\left(e\,x^{2}+d\right)^{\frac{11}{2}}d^{6}}$
gosper	$\frac{x\left(1280a^{2}e^{5}x^{10}-96ac\,d^{2}e^{3}x^{10}+70c^{2}d^{4}e\,x^{10}+7040a^{2}d\,e^{4}x^{8}-528ac\,d^{3}e^{2}x^{8}+385c^{2}d^{5}x^{8}+15840a^{2}d^{2}e^{3}x^{6}-1188ac\,d^{4}e\,x^{6}+\right.}{3465\left(e\,x^{2}+d\right)^{\frac{11}{2}}d^{6}}$
trager	$\frac{x\left(1280a^{2}e^{5}x^{10}-96ac\,d^{2}e^{3}x^{10}+70c^{2}d^{4}e\,x^{10}+7040a^{2}d\,e^{4}x^{8}-528ac\,d^{3}e^{2}x^{8}+385c^{2}d^{5}x^{8}+15840a^{2}d^{2}e^{3}x^{6}-1188ac\,d^{4}e\,x^{6}+\right.}{3465\left(e\,x^{2}+d\right)^{\frac{11}{2}}d^{6}}$
orering	$\frac{x\left(1280a^{2}e^{5}x^{10}-96ac\,d^{2}e^{3}x^{10}+70c^{2}d^{4}e\,x^{10}+7040a^{2}d\,e^{4}x^{8}-528ac\,d^{3}e^{2}x^{8}+385c^{2}d^{5}x^{8}+15840a^{2}d^{2}e^{3}x^{6}-1188ac\,d^{4}e\,x^{6}+\right.}{3465\left(e\,x^{2}+d\right)^{\frac{11}{2}}d^{6}}$

input
```
int((-c*x^4+a)^2/(e*x^2+d)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
x/(e*x^2+d)^(11/2)*((1/9*c^2*x^8-2/5*a*c*x^4+a^2)*d^5+10/3*x^2*(1/165*c^2*
x^8-18/175*a*c*x^4+a^2)*e*d^4+16/3*x^4*a*e^2*(-1/35*c*x^4+a)*d^3+32/7*x^6*
a*(-1/165*c*x^4+a)*e^3*d^2+128/63*a^2*d*e^4*x^8+256/693*a^2*e^5*x^10)/d^6
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.78

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{13/2}}\,dx = \frac{(2\,(35\,c^2d^4e-48\,acd^2e^3+640\,a^2e^5)x^{11}+11550\,a^2d^4ex^3+11\,(35\,c^2d^5-48\,acd^3e^2+}{3465\,(d^6e^6x^{12}+6\,d^7e^5x^{10}+15\,d^8e}$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(13/2),x, algorithm="fricas")
```

output
```
1/3465*(2*(35*c^2*d^4*e - 48*a*c*d^2*e^3 + 640*a^2*e^5)*x^11 + 11550*a^2*d
^4*e*x^3 + 11*(35*c^2*d^5 - 48*a*c*d^3*e^2 + 640*a^2*d*e^4)*x^9 + 3465*a^2
*d^5*x - 396*(3*a*c*d^4*e - 40*a^2*d^2*e^3)*x^7 - 462*(3*a*c*d^5 - 40*a^2*
d^3*e^2)*x^5)*sqrt(e*x^2 + d)/(d^6*e^6*x^12 + 6*d^7*e^5*x^10 + 15*d^8*e^4*
x^8 + 20*d^9*e^3*x^6 + 15*d^10*e^2*x^4 + 6*d^11*e*x^2 + d^12)
```

Sympy [F]

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{13/2}}\,dx = \int \frac{(-a+cx^4)^2}{(d+ex^2)^{\frac{13}{2}}}\,dx$$

input
```
integrate((-c*x**4+a)**2/(e*x**2+d)**(13/2),x)
```

output
```
Integral((-a + c*x**4)**2/(d + e*x**2)**(13/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.53

$$
\begin{aligned}
\int \frac{(a-cx^4)^2}{(d+ex^2)^{13/2}}\,dx = &-\frac{c^2x^7}{4\,(ex^2+d)^{\frac{11}{2}}e}-\frac{7\,c^2dx^5}{24\,(ex^2+d)^{\frac{11}{2}}e^2} \\
&-\frac{35\,c^2d^2x^3}{192\,(ex^2+d)^{\frac{11}{2}}e^3}+\frac{acx^3}{4\,(ex^2+d)^{\frac{11}{2}}e}+\frac{256\,a^2x}{693\,\sqrt{ex^2+d}d^6}+\frac{128\,a^2x}{693\,(ex^2+d)^{\frac{3}{2}}d^5} \\
&+\frac{32\,a^2x}{231\,(ex^2+d)^{\frac{5}{2}}d^4}+\frac{80\,a^2x}{693\,(ex^2+d)^{\frac{7}{2}}d^3}+\frac{10\,a^2x}{99\,(ex^2+d)^{\frac{9}{2}}d^2}+\frac{a^2x}{11\,(ex^2+d)^{\frac{11}{2}}d} \\
&+\frac{c^2x}{132\,(ex^2+d)^{\frac{5}{2}}e^4}+\frac{2\,c^2x}{99\,\sqrt{ex^2+d}d^2e^4}+\frac{c^2x}{99\,(ex^2+d)^{\frac{3}{2}}de^4} \\
&+\frac{5\,c^2dx}{792\,(ex^2+d)^{\frac{7}{2}}e^4}+\frac{35\,c^2d^2x}{6336\,(ex^2+d)^{\frac{9}{2}}e^4}-\frac{35\,c^2d^3x}{704\,(ex^2+d)^{\frac{11}{2}}e^4} \\
&-\frac{acx}{132\,(ex^2+d)^{\frac{9}{2}}e^2}-\frac{32\,acx}{1155\,\sqrt{ex^2+d}d^4e^2}-\frac{16\,acx}{1155\,(ex^2+d)^{\frac{3}{2}}d^3e^2} \\
&-\frac{4\,acx}{385\,(ex^2+d)^{\frac{5}{2}}d^2e^2}-\frac{2\,acx}{231\,(ex^2+d)^{\frac{7}{2}}de^2}+\frac{3\,acdx}{44\,(ex^2+d)^{\frac{11}{2}}e^2}
\end{aligned}
$$

input

```
integrate((-c*x^4+a)^2/(e*x^2+d)^(13/2),x, algorithm="maxima")
```

output

```
-1/4*c^2*x^7/((e*x^2 + d)^(11/2)*e) - 7/24*c^2*d*x^5/((e*x^2 + d)^(11/2)*e
^2) - 35/192*c^2*d^2*x^3/((e*x^2 + d)^(11/2)*e^3) + 1/4*a*c*x^3/((e*x^2 +
d)^(11/2)*e) + 256/693*a^2*x/(sqrt(e*x^2 + d)*d^6) + 128/693*a^2*x/((e*x^2
 + d)^(3/2)*d^5) + 32/231*a^2*x/((e*x^2 + d)^(5/2)*d^4) + 80/693*a^2*x/((e
*x^2 + d)^(7/2)*d^3) + 10/99*a^2*x/((e*x^2 + d)^(9/2)*d^2) + 1/11*a^2*x/((
e*x^2 + d)^(11/2)*d) + 1/132*c^2*x/((e*x^2 + d)^(5/2)*e^4) + 2/99*c^2*x/(s
qrt(e*x^2 + d)*d^2*e^4) + 1/99*c^2*x/((e*x^2 + d)^(3/2)*d*e^4) + 5/792*c^2
*d*x/((e*x^2 + d)^(7/2)*e^4) + 35/6336*c^2*d^2*x/((e*x^2 + d)^(9/2)*e^4) -
 35/704*c^2*d^3*x/((e*x^2 + d)^(11/2)*e^4) - 1/132*a*c*x/((e*x^2 + d)^(9/2
)*e^2) - 32/1155*a*c*x/(sqrt(e*x^2 + d)*d^4*e^2) - 16/1155*a*c*x/((e*x^2 +
 d)^(3/2)*d^3*e^2) - 4/385*a*c*x/((e*x^2 + d)^(5/2)*d^2*e^2) - 2/231*a*c*x
/((e*x^2 + d)^(7/2)*d*e^2) + 3/44*a*c*d*x/((e*x^2 + d)^(11/2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.68

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{13/2}}\,dx = \frac{\left(\left(\left(\left(x^2\left(\frac{2\,(35\,c^2d^4e^6-48\,acd^2e^8+640\,a^2e^{10})x^2}{d^6e^5}+\frac{11\,(35\,c^2d^5e^5-48\,acd^3e^7+640\,a^2de^9)}{d^6e^5}\right)-\frac{396\,(3\,acd^4e^6-}{d^6e^5}\right.\right.\right.\right.}{3465\,(ex^2+d)^{\frac{11}{2}}}$$

input
```
integrate((-c*x^4+a)^2/(e*x^2+d)^(13/2),x, algorithm="giac")
```

output
```
1/3465*((((x^2*(2*(35*c^2*d^4*e^6 - 48*a*c*d^2*e^8 + 640*a^2*e^10)*x^2/(d^
6*e^5) + 11*(35*c^2*d^5*e^5 - 48*a*c*d^3*e^7 + 640*a^2*d*e^9)/(d^6*e^5)) -
 396*(3*a*c*d^4*e^6 - 40*a^2*d^2*e^8)/(d^6*e^5))*x^2 - 462*(3*a*c*d^5*e^5
- 40*a^2*d^3*e^7)/(d^6*e^5))*x^2 + 11550*a^2*e/d^2)*x^2 + 3465*a^2/d)*x/(e
*x^2 + d)^(11/2)
```

Mupad [B] (verification not implemented)

Time = 17.67 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{(a-cx^4)^2}{(d+ex^2)^{13/2}}\,dx &= \frac{x\left(\frac{a^2}{11\,d}+\frac{d^2\left(\frac{c^2\,d}{11\,e^2}-\frac{2\,a\,c}{11\,d}\right)}{e^2}\right)}{\left(e\,x^2+d\right)^{11/2}} \\ &+ \frac{x\left(\frac{10\,a^2\,e^4+2\,a\,c\,d^2\,e^2-c^2\,d^4}{99\,d^2\,e^4}-\frac{d\left(\frac{2\,c^2\,d}{9\,e^3}-\frac{c\,\left(2\,a\,e^2-c\,d^2\right)}{9\,d\,e^3}\right)}{e}\right)}{\left(e\,x^2+d\right)^{9/2}} \\ &- \frac{x\left(\frac{c^2}{5\,e^4}+\frac{-160\,a^2\,e^4+12\,a\,c\,d^2\,e^2+49\,c^2\,d^4}{1155\,d^4\,e^4}\right)}{\left(e\,x^2+d\right)^{5/2}}+\frac{x\left(\frac{3\,c^2\,d}{7\,e^4}+\frac{80\,a^2\,e^4-6\,a\,c\,d^2\,e^2+25\,c^2\,d^4}{693\,d^3\,e^4}\right)}{\left(e\,x^2+d\right)^{7/2}} \\ &+ \frac{x\,\left(640\,a^2\,e^4-48\,a\,c\,d^2\,e^2+35\,c^2\,d^4\right)}{3465\,d^5\,e^4\,\left(e\,x^2+d\right)^{3/2}}+\frac{x\,\left(1280\,a^2\,e^4-96\,a\,c\,d^2\,e^2+70\,c^2\,d^4\right)}{3465\,d^6\,e^4\,\sqrt{e\,x^2+d}}\end{aligned}$$

input
```
int((a - c*x^4)^2/(d + e*x^2)^(13/2),x)
```

output

```
(x*(a^2/(11*d) + (d^2*((c^2*d)/(11*e^2) - (2*a*c)/(11*d)))/e^2))/(d + e*x^
2)^(11/2) + (x*((10*a^2*e^4 - c^2*d^4 + 2*a*c*d^2*e^2)/(99*d^2*e^4) - (d*(
(2*c^2*d)/(9*e^3) - (c*(2*a*e^2 - c*d^2))/(9*d*e^3)))/e))/(d + e*x^2)^(9/2
) - (x*(c^2/(5*e^4) + (49*c^2*d^4 - 160*a^2*e^4 + 12*a*c*d^2*e^2)/(1155*d^
4*e^4)))/(d + e*x^2)^(5/2) + (x*((3*c^2*d)/(7*e^4) + (80*a^2*e^4 + 25*c^2*
d^4 - 6*a*c*d^2*e^2)/(693*d^3*e^4)))/(d + e*x^2)^(7/2) + (x*(640*a^2*e^4 +
 35*c^2*d^4 - 48*a*c*d^2*e^2))/(3465*d^5*e^4*(d + e*x^2)^(3/2)) + (x*(1280
*a^2*e^4 + 70*c^2*d^4 - 96*a*c*d^2*e^2))/(3465*d^6*e^4*(d + e*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 636, normalized size of antiderivative = 2.27

$$\int \frac{(a-cx^4)^2}{(d+ex^2)^{13/2}}\,dx = \frac{-70\sqrt{e}\,c^2d^{10} + 3465\sqrt{e\,x^2+d}\,a^2d^5e^5x + 11550\sqrt{e\,x^2+d}\,a^2d^4e^6x^3 + 18480\sqrt{e\,x^2+}}{}$$

input

```
int((-c*x^4+a)^2/(e*x^2+d)^(13/2),x)
```

output

```
(3465*sqrt(d + e*x**2)*a**2*d**5*e**5*x + 11550*sqrt(d + e*x**2)*a**2*d**4
*e**6*x**3 + 18480*sqrt(d + e*x**2)*a**2*d**3*e**7*x**5 + 15840*sqrt(d + e
*x**2)*a**2*d**2*e**8*x**7 + 7040*sqrt(d + e*x**2)*a**2*d*e**9*x**9 + 1280
*sqrt(d + e*x**2)*a**2*e**10*x**11 - 1386*sqrt(d + e*x**2)*a*c*d**5*e**5*x
**5 - 1188*sqrt(d + e*x**2)*a*c*d**4*e**6*x**7 - 528*sqrt(d + e*x**2)*a*c*
d**3*e**7*x**9 - 96*sqrt(d + e*x**2)*a*c*d**2*e**8*x**11 + 385*sqrt(d + e*
x**2)*c**2*d**5*e**5*x**9 + 70*sqrt(d + e*x**2)*c**2*d**4*e**6*x**11 - 128
0*sqrt(e)*a**2*d**6*e**4 - 7680*sqrt(e)*a**2*d**5*e**5*x**2 - 19200*sqrt(e
)*a**2*d**4*e**6*x**4 - 25600*sqrt(e)*a**2*d**3*e**7*x**6 - 19200*sqrt(e)*
a**2*d**2*e**8*x**8 - 7680*sqrt(e)*a**2*d*e**9*x**10 - 1280*sqrt(e)*a**2*e
**10*x**12 + 96*sqrt(e)*a*c*d**8*e**2 + 576*sqrt(e)*a*c*d**7*e**3*x**2 + 1
440*sqrt(e)*a*c*d**6*e**4*x**4 + 1920*sqrt(e)*a*c*d**5*e**5*x**6 + 1440*sq
rt(e)*a*c*d**4*e**6*x**8 + 576*sqrt(e)*a*c*d**3*e**7*x**10 + 96*sqrt(e)*a*
c*d**2*e**8*x**12 - 70*sqrt(e)*c**2*d**10 - 420*sqrt(e)*c**2*d**9*e*x**2 -
 1050*sqrt(e)*c**2*d**8*e**2*x**4 - 1400*sqrt(e)*c**2*d**7*e**3*x**6 - 105
0*sqrt(e)*c**2*d**6*e**4*x**8 - 420*sqrt(e)*c**2*d**5*e**5*x**10 - 70*sqrt
(e)*c**2*d**4*e**6*x**12)/(3465*d**6*e**5*(d**6 + 6*d**5*e*x**2 + 15*d**4*
e**2*x**4 + 20*d**3*e**3*x**6 + 15*d**2*e**4*x**8 + 6*d*e**5*x**10 + e**6*
x**12))
```

3.341 $\int \frac{(d+ex^2)^{7/2}}{a-cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 232

$$\begin{aligned}\int \frac{(d+ex^2)^{7/2}}{a-cx^4}\,dx = {} & -\frac{13de^2x\sqrt{d+ex^2}}{8c}-\frac{e^3x^3\sqrt{d+ex^2}}{4c} \\ & +\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{7/2}\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}c^2}-\frac{e^{3/2}\left(35cd^2+8ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2} \\ & +\frac{\left(\sqrt{c}d+\sqrt{a}e\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}c^2}\end{aligned}$$

output

```
-13/8*d*e^2*x*(e*x^2+d)^(1/2)/c-1/4*e^3*x^3*(e*x^2+d)^(1/2)/c+1/2*(c^(1/2)
*d-a^(1/2)*e)^(7/2)*arctan((c^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)
^(1/2))/a^(3/4)/c^2-1/8*e^(3/2)*(8*a*e^2+35*c*d^2)*arctanh(e^(1/2)*x/(e*x^
2+d)^(1/2))/c^2+1/2*(c^(1/2)*d+a^(1/2)*e)^(7/2)*arctanh((c^(1/2)*d+a^(1/2)
*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(3/4)/c^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.00

$$\int \frac{(d+ex^2)^{7/2}}{a-cx^4}\,dx = \frac{e^{3/2}\Big(-c\sqrt{e}x\sqrt{d+ex^2}(13d+2ex^2)+(35cd^2+8ae^2)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)-16\mathrm{R}}{}$$

input
```
Integrate[(d + e*x^2)^(7/2)/(a - c*x^4),x]
```

output
```
(e^(3/2)*(-(c*Sqrt[e]*x*Sqrt[d + e*x^2]*(13*d + 2*e*x^2)) + (35*c*d^2 + 8*
a*e^2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]] - 16*RootSum[c*d^4 - 4*c*d^3*#1
 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (c^2*d^5*Log[d +
 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + a*c*d^3*e^2*Log[d + 2*e*x^2
 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - c^2*d^4*Log[d + 2*e*x^2 - 2*Sqrt[e]
*x*Sqrt[d + e*x^2] - #1]*#1 + 4*a*c*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*
Sqrt[d + e*x^2] - #1]*#1 + a^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d +
e*x^2] - #1]*#1 + c^2*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] -
#1]*#1^2 + a*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#
1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ]))/(8*c^2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 582 vs. 2(232) = 464.

Time = 1.59 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.51, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {1489, 27, 318, 25, 403, 27, 403, 25, 398, 224, 219, 291, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{7/2}}{a-cx^4}dx$$

$$\Big\downarrow \text{1489}$$

$$\frac{\sqrt{c}\int \frac{\left(ex^2+d\right)^{7/2}}{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}dx}{2\sqrt{a}}+\frac{\sqrt{c}\int \frac{\left(ex^2+d\right)^{7/2}}{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(ex^2+d\right)^{7/2}}{\sqrt{a}-\sqrt{c}x^2}dx}{2\sqrt{a}}+\frac{\int \frac{\left(ex^2+d\right)^{7/2}}{\sqrt{c}x^2+\sqrt{a}}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 318}$$

$$\frac{\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(e\left(11\sqrt{c}d-6\sqrt{a}e\right)x^2+d\left(6\sqrt{c}d-\sqrt{a}e\right)\right)}{\sqrt{c}x^2+\sqrt{a}}dx}{6\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{a}}+\frac{-\frac{\int -\frac{\left(ex^2+d\right)^{3/2}\left(e\left(11\sqrt{c}d+6\sqrt{a}e\right)x^2+d\left(6\sqrt{c}d+\sqrt{a}e\right)\right)}{\sqrt{a}-\sqrt{c}x^2}dx}{6\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(e\left(11\sqrt{c}d-6\sqrt{a}e\right)x^2+d\left(6\sqrt{c}d-\sqrt{a}e\right)\right)}{\sqrt{c}x^2+\sqrt{a}}dx}{6\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(e\left(11\sqrt{c}d+6\sqrt{a}e\right)x^2+d\left(6\sqrt{c}d+\sqrt{a}e\right)\right)}{\sqrt{a}-\sqrt{c}x^2}dx}{6\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\frac{\frac{\int \frac{3\sqrt{ex^2+d}\left(e\left(19cd^2-22\sqrt{a}\sqrt{c}ed+8ae^2\right)x^2+d\left(8cd^2-5\sqrt{a}\sqrt{c}ed+2ae^2\right)\right)}{\sqrt{c}x^2+\sqrt{a}}dx}{4\sqrt{c}}+\frac{1}{4}ex\left(d+ex^2\right)^{3/2}\left(11d-\frac{6\sqrt{a}e}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{-\frac{\int -\frac{3\sqrt{ex^2+d}\left(e\left(19cd^2+22\sqrt{a}\sqrt{c}ed+8ae^2\right)x^2+d\left(8cd^2+5\sqrt{a}\sqrt{c}ed+2ae^2\right)\right)}{\sqrt{a}-\sqrt{c}x^2}dx}{4\sqrt{c}}-\frac{1}{4}ex\left(d+ex^2\right)^{3/2}\left(\frac{6\sqrt{a}e}{\sqrt{c}}+11d\right)}{6\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{3\int \frac{\sqrt{ex^2+d}\left(e\left(19cd^2-22\sqrt{a}\sqrt{c}ed+8ae^2\right)x^2+d\left(8cd^2-5\sqrt{a}\sqrt{c}ed+2ae^2\right)\right)}{\sqrt{c}x^2+\sqrt{a}}dx}{4\sqrt{c}}+\frac{1}{4}ex\left(d+ex^2\right)^{3/2}\left(11d-\frac{6\sqrt{a}e}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{a}}+$$

$$\frac{\frac{\frac{3\int \frac{\sqrt{ex^2+d}\left(e\left(19cd^2+22\sqrt{a}\sqrt{c}ed+8ae^2\right)x^2+d\left(8cd^2+5\sqrt{a}\sqrt{c}ed+2ae^2\right)\right)}{\sqrt{a}-\sqrt{c}x^2}dx}{4\sqrt{c}}-\frac{1}{4}ex\left(d+ex^2\right)^{3/2}\left(\frac{6\sqrt{a}e}{\sqrt{c}}+11d\right)}{6\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \ 403$$

$$\frac{\frac{\frac{3\left(\frac{\int \frac{e\left(35c^{3/2}d^3-70\sqrt{a}ced^2+56a\sqrt{c}e^2d-16a^{3/2}e^3\right)x^2+d\left(16c^{3/2}d^3-29\sqrt{a}ced^2+26a\sqrt{c}e^2d-8a^{3/2}e^3\right)}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}\left(-22\sqrt{a}\sqrt{c}de+8ae^2+19cd^2\right)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{1}{4}ex(}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\frac{\frac{\frac{3\left(-\frac{\int -\frac{e\left(35c^{3/2}d^3+70\sqrt{a}ced^2+56a\sqrt{c}e^2d+16a^{3/2}e^3\right)x^2+d\left(16c^{3/2}d^3+29\sqrt{a}ced^2+26a\sqrt{c}e^2d+8a^{3/2}e^3\right)}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}\left(22\sqrt{a}\sqrt{c}de+8ae^2+19cd^2\right)}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{1}{4}ex}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{3\left(\frac{\int \frac{e\left(35c^{3/2}d^3-70\sqrt{a}ced^2+56a\sqrt{c}e^2d-16a^{3/2}e^3\right)x^2+d\left(16c^{3/2}d^3-29\sqrt{a}ced^2+26a\sqrt{c}e^2d-8a^{3/2}e^3\right)}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}\left(-22\sqrt{a}\sqrt{c}de+8ae^2+19cd^2\right)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{1}{4}ex(}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\frac{\frac{\frac{3\left(\frac{\int \frac{e\left(35c^{3/2}d^3+70\sqrt{a}ced^2+56a\sqrt{c}e^2d+16a^{3/2}e^3\right)x^2+d\left(16c^{3/2}d^3+29\sqrt{a}ced^2+26a\sqrt{c}e^2d+8a^{3/2}e^3\right)}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}\left(22\sqrt{a}\sqrt{c}de+8ae^2+19cd^2\right)}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{1}{4}ex(d}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \ 398$$

$$\frac{\frac{\frac{3\left(\frac{\frac{16\left(\sqrt{a}e+\sqrt{c}d\right)^4\int\frac{1}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-\frac{e\left(16a^{3/2}e^3+70\sqrt{a}cd^2e+56a\sqrt{c}de^2+35c^{3/2}d^3\right)\int\frac{1}{\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}\left(22\sqrt{a}\sqrt{c}de+8ae^2+19cd^2\right)}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{1}{4}e}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{e\left(-16a^{3/2}e^3-70\sqrt{a}cd^2e+56a\sqrt{c}de^2+35c^{3/2}d^3\right)\int\frac{1}{\sqrt{ex^2+d}}dx}{\sqrt{c}}+\frac{16\left(\sqrt{c}d-\sqrt{a}e\right)^4\int\frac{1}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}\left(-22\sqrt{a}\sqrt{c}de+8ae^2+19cd^2\right)}{2\sqrt{c}}\right)}{4\sqrt{c}}+}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{e\left(-16a^{3/2}e^3-70\sqrt{a}cd^2e+56a\sqrt{c}de^2+35c^{3/2}d^3\right)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}+\frac{16\left(\sqrt{c}d-\sqrt{a}e\right)^4\int\frac{1}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}\left(-22\sqrt{a}\sqrt{c}de+8ae^2+19\right.}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{16\left(\sqrt{a}e+\sqrt{c}d\right)^4\int\frac{1}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-\frac{e\left(16a^{3/2}e^3+70\sqrt{a}cd^2e+56a\sqrt{c}de^2+35c^{3/2}d^3\right)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}\left(22\sqrt{a}\sqrt{c}de+8ae^2+19cd^2\right.}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{16\left(\sqrt{a}e+\sqrt{c}d\right)^4\int\frac{1}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-\frac{\sqrt{e}\left(16a^{3/2}e^3+70\sqrt{a}cd^2e+56a\sqrt{c}de^2+35c^{3/2}d^3\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}\left(22\sqrt{a}\sqrt{c}de+8ae^2+19c\right.}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{16\left(\sqrt{c}d-\sqrt{a}e\right)^4\int\frac{1}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}+\frac{\sqrt{e}\left(-16a^{3/2}e^3-70\sqrt{a}cd^2e+56a\sqrt{c}de^2+35c^{3/2}d^3\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}\left(-22\sqrt{a}\sqrt{c}de+8ae^2+\right.}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{16\left(\sqrt{c}d-\sqrt{a}e\right)^4\int\frac{1}{\sqrt{a}-\frac{\left(\sqrt{a}e-\sqrt{c}d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}+\frac{\sqrt{e}\left(-16a^{3/2}e^3-70\sqrt{a}cd^2e+56a\sqrt{c}de^2+35c^{3/2}d^3\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}\left(-22\sqrt{a}\sqrt{c}de+\right.}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{16\left(\sqrt{a}e+\sqrt{c}d\right)^4\int\frac{1}{\sqrt{a}-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}-\frac{\sqrt{e}\left(16a^{3/2}e^3+70\sqrt{a}cd^2e+56a\sqrt{c}de^2+35c^{3/2}d^3\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}\left(22\sqrt{a}\sqrt{c}de+8ae\right.}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{16\left(\sqrt{a}e+\sqrt{c}d\right)^4\int\frac{1}{\sqrt{a}-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}-\frac{\sqrt{e}\left(16a^{3/2}e^3+70\sqrt{a}cd^2e+56a\sqrt{c}de^2+35c^{3/2}d^3\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}\left(22\sqrt{a}\sqrt{c}de+8ae\right.}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{\sqrt{e}\left(-16a^{3/2}e^3-70\sqrt{a}cd^2e+56a\sqrt{c}de^2+35c^{3/2}d^3\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}+\frac{16\left(\sqrt{c}d-\sqrt{a}e\right)^{7/2}\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{a}e}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{a}\sqrt{c}}}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}\left(-22\sqrt{a}\sqrt{c}de+8ae^2+\right.}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{\sqrt{e}\left(-16a^{3/2}e^3-70\sqrt{a}cd^2e+56a\sqrt{c}de^2+35c^{3/2}d^3\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}+\frac{16\left(\sqrt{c}d-\sqrt{a}e\right)^{7/2}\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{a}e}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{a}\sqrt{c}}}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}\left(-22\sqrt{a}\sqrt{c}de+8ae^2+\right.}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{a}}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{16\left(\sqrt{a}e+\sqrt{c}d\right)^{7/2}\operatorname{arctanh}\left(\frac{x\sqrt{\sqrt{a}e+\sqrt{c}d}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{a}\sqrt{c}}-\frac{\sqrt{e}\left(16a^{3/2}e^3+70\sqrt{a}cd^2e+56a\sqrt{c}de^2+35c^{3/2}d^3\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}\left(22\sqrt{a}\sqrt{c}de+8ae^2-\right.}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{a}}$$

input

```
Int[(d + e*x^2)^(7/2)/(a - c*x^4),x]
```

output

```
((e*x*(d + e*x^2)^(5/2))/(6*Sqrt[c]) + ((e*(11*d - (6*Sqrt[a]*e)/Sqrt[c])*
x*(d + e*x^2)^(3/2))/4 + (3*((e*(19*c*d^2 - 22*Sqrt[a]*Sqrt[c]*d*e + 8*a*e
^2)*x*Sqrt[d + e*x^2])/(2*Sqrt[c]) + ((16*(Sqrt[c]*d - Sqrt[a]*e)^(7/2)*Ar
cTan[(Sqrt[Sqrt[c]*d - Sqrt[a]*e]*x)/(a^(1/4)*Sqrt[d + e*x^2])])/(a^(1/4)*
Sqrt[c]) + (Sqrt[e]*(35*c^(3/2)*d^3 - 70*Sqrt[a]*c*d^2*e + 56*a*Sqrt[c]*d*
e^2 - 16*a^(3/2)*e^3)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[c])/(2*Sq
rt[c])))/(4*Sqrt[c]))/(6*Sqrt[c]))/(2*Sqrt[a]) + (-1/6*(e*x*(d + e*x^2)^(5
/2))/Sqrt[c] + (-1/4*(e*(11*d + (6*Sqrt[a]*e)/Sqrt[c])*x*(d + e*x^2)^(3/2)
) + (3*(-1/2*(e*(19*c*d^2 + 22*Sqrt[a]*Sqrt[c]*d*e + 8*a*e^2)*x*Sqrt[d + e
*x^2])/Sqrt[c] + (-((Sqrt[e]*(35*c^(3/2)*d^3 + 70*Sqrt[a]*c*d^2*e + 56*a*S
qrt[c]*d*e^2 + 16*a^(3/2)*e^3)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[
c]) + (16*(Sqrt[c]*d + Sqrt[a]*e)^(7/2)*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[a]*
e]*x)/(a^(1/4)*Sqrt[d + e*x^2])])/(a^(1/4)*Sqrt[c]))/(2*Sqrt[c])))/(4*Sqrt
[c]))/(6*Sqrt[c]))/(2*Sqrt[a])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 1489
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Simp[-c/(2*r)   Int[(d + e*x^2)^q/(r - c*x^2), x], x] - S
imp[c/(2*r)   Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e,
q}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(176) = 352$.

Time = 0.97 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.54

method	result
pseudoelliptic	$-\frac{2\sqrt{e}\left(\left(a^{2}e^{4}+6acd^{2}e^{2}+c^{2}d^{4}\right)\sqrt{d^{2}ac}-4a^{2}cd^{2}e^{3}-4c^{2}d^{4}ea\right)\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}\,\arctan\left(\frac{\sqrt{ex^{2}+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}}\right)+\sqrt{\left(-ae+\right.}}{}$
risch	$-\frac{e^{2}x\left(2ex^{2}+13d\right)\sqrt{ex^{2}+d}}{8c}-\frac{\frac{e^{\frac{3}{2}}\left(8ae^{2}+35cd^{2}\right)\ln\left(x\sqrt{e}+\sqrt{ex^{2}+d}\right)}{c}+\frac{4\left(4a^{\frac{3}{2}}\sqrt{c}\,de^{3}+4\sqrt{a}\,c^{\frac{3}{2}}d^{3}e-a^{2}e^{4}-6acd^{2}e^{2}-c^{2}d^{4}\right)\left(-\right.}{}}{}$
default	Expression too large to display

input
```
int((e*x^2+d)^(7/2)/(-c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)/e^(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1
/2)*(2*e^(1/2)*((a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)*(d^2*a*c)^(1/2)-4*a^2*c*d^
2*e^3-4*c^2*d^4*e*a)*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*arctan((e*x^2+d)^(1/2
)/x*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(
-2*((a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)*(d^2*a*c)^(1/2)+4*a^2*c*d^2*e^3+4*c^2*
d^4*e*a)*e^(1/2)*arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(d^2*a*c)^(1/2))*a)^(1/
2))+(d^2*a*c)^(1/2)*((4*e^4*a+35/2*c*d^2*e^2)*arctanh((e*x^2+d)^(1/2)/x/e^
(1/2))+x*(e*x^2+13/2*d)*e^(5/2)*c*(e*x^2+d)^(1/2))*((a*e+(d^2*a*c)^(1/2))*
a)^(1/2)))/(d^2*a*c)^(1/2)/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3366 vs. $2(176) = 352$.

Time = 52.55 (sec) , antiderivative size = 6742, normalized size of antiderivative = 29.06

$$\int \frac{\left(d+ex^2\right)^{7/2}}{a-cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\left(d+ex^2\right)^{7/2}}{a-cx^4}\,dx = -\int \frac{d^3\sqrt{d+ex^2}}{-a+cx^4}\,dx - \int \frac{e^3x^6\sqrt{d+ex^2}}{-a+cx^4}\,dx - \int \frac{3de^2x^4\sqrt{d+ex^2}}{-a+cx^4}\,dx - \int \frac{3d^2ex^2\sqrt{d+ex^2}}{-a+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(7/2)/(-c*x**4+a),x)
```

output
```
-Integral(d**3*sqrt(d + e*x**2)/(-a + c*x**4), x) - Integral(e**3*x**6*sqr
t(d + e*x**2)/(-a + c*x**4), x) - Integral(3*d*e**2*x**4*sqrt(d + e*x**2)/
(-a + c*x**4), x) - Integral(3*d**2*e*x**2*sqrt(d + e*x**2)/(-a + c*x**4),
 x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{7/2}}{a-cx^4}\,dx = \int -\frac{(ex^2+d)^{\frac{7}{2}}}{cx^4-a}\,dx$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 + d)^(7/2)/(c*x^4 - a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{7/2}}{a-cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{a-cx^4}\,dx=\int \frac{(e\,x^2+d)^{7/2}}{a-c\,x^4}\,dx$$

input

```
int((d + e*x^2)^(7/2)/(a - c*x^4),x)
```

output

```
int((d + e*x^2)^(7/2)/(a - c*x^4), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{7/2}}{a-cx^4}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}}{-c\,x^4+a}dx\right)d^3+\left(\int \frac{\sqrt{e\,x^2+d}\,x^6}{-c\,x^4+a}dx\right)e^3 +3\left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{-c\,x^4+a}dx\right)d\,e^2+3\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{-c\,x^4+a}dx\right)d^2e$$

input

```
int((e*x^2+d)^(7/2)/(-c*x^4+a),x)
```

output

```
int(sqrt(d + e*x**2)/(a - c*x**4),x)*d**3 + int((sqrt(d + e*x**2)*x**6)/(a
 - c*x**4),x)*e**3 + 3*int((sqrt(d + e*x**2)*x**4)/(a - c*x**4),x)*d*e**2
+ 3*int((sqrt(d + e*x**2)*x**2)/(a - c*x**4),x)*d**2*e
```

3.342 $\int \frac{(d+ex^2)^{5/2}}{a-cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 199

$$\int \frac{(d+ex^2)^{5/2}}{a-cx^4}\,dx = -\frac{e^2x\sqrt{d+ex^2}}{2c} + \frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}c^{3/2}} - \frac{5de^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}c^{3/2}}$$

output

```
-1/2*e^2*x*(e*x^2+d)^(1/2)/c+1/2*(c^(1/2)*d-a^(1/2)*e)^(5/2)*arctan((c^(1/
2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(3/4)/c^(3/2)-5/2*d*e^(
3/2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c+1/2*(c^(1/2)*d+a^(1/2)*e)^(5/2)*
arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(3/4)/c^(
3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.95

$$\int \frac{(d+ex^2)^{5/2}}{a-cx^4}\,dx =$$

$$-\frac{e^{3/2}\Big(\sqrt{e}x\sqrt{d+ex^2}-5d\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)+\mathrm{RootSum}\Big[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-}{}$$

input
```
Integrate[(d + e*x^2)^(5/2)/(a - c*x^4),x]
```

output
```
-1/2*(e^(3/2)*(Sqrt[e]*x*Sqrt[d + e*x^2] - 5*d*Log[-(Sqrt[e]*x) + Sqrt[d +
 e*x^2]] + RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c
*d*#1^3 + c*#1^4 & , (3*c*d^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2
] - #1] + a*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] -
2*c*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 10*a*d*e^
2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 3*c*d^2*Log[d +
 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + a*e^2*Log[d + 2*e*x^2
- 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1
 + 3*c*d*#1^2 - c*#1^3) & ]))/c
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 428 vs. 2(199) = 398.

Time = 1.13 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {1489, 27, 318, 25, 403, 25, 398, 224, 219, 291, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}}{a-cx^4}\,dx$$

$$\Big\downarrow \text{1489}$$

$$\frac{\sqrt{c}\int \frac{(ex^2+d)^{5/2}}{\sqrt{c}(\sqrt{a}-\sqrt{c}x^2)}dx}{2\sqrt{a}}+\frac{\sqrt{c}\int \frac{(ex^2+d)^{5/2}}{\sqrt{c}(\sqrt{c}x^2+\sqrt{a})}dx}{2\sqrt{a}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(ex^2+d)^{5/2}}{\sqrt{a}-\sqrt{c}x^2}dx}{2\sqrt{a}}+\frac{\int \frac{(ex^2+d)^{5/2}}{\sqrt{c}x^2+\sqrt{a}}dx}{2\sqrt{a}}$$

$$\Big\downarrow \; 318$$

$$\frac{\frac{\int \frac{\sqrt{ex^2+d}\left(e(7\sqrt{c}d-4\sqrt{a}e)x^2+d(4\sqrt{c}d-\sqrt{a}e)\right)}{\sqrt{c}x^2+\sqrt{a}}dx}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}+\frac{-\frac{\int -\frac{\sqrt{ex^2+d}\left(e(7\sqrt{c}d+4\sqrt{a}e)x^2+d(4\sqrt{c}d+\sqrt{a}e)\right)}{\sqrt{a}-\sqrt{c}x^2}dx}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\sqrt{ex^2+d}\left(e(7\sqrt{c}d-4\sqrt{a}e)x^2+d(4\sqrt{c}d-\sqrt{a}e)\right)}{\sqrt{c}x^2+\sqrt{a}}dx}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\int \frac{\sqrt{ex^2+d}\left(e(7\sqrt{c}d+4\sqrt{a}e)x^2+d(4\sqrt{c}d+\sqrt{a}e)\right)}{\sqrt{a}-\sqrt{c}x^2}dx}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\frac{\int \frac{e\left(15cd^2-20\sqrt{a}\sqrt{c}ed+8ae^2\right)x^2+d\left(8cd^2-9\sqrt{a}\sqrt{c}ed+4ae^2\right)}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\frac{4\sqrt{a}e}{\sqrt{c}}\right)}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{-\frac{\int -\frac{e\left(15cd^2+20\sqrt{a}\sqrt{c}ed+8ae^2\right)x^2+d\left(8cd^2+9\sqrt{a}\sqrt{c}ed+4ae^2\right)}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\frac{4\sqrt{a}e}{\sqrt{c}}+7d\right)}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{e\left(15cd^2-20\sqrt{a}\sqrt{c}ed+8ae^2\right)x^2+d\left(8cd^2-9\sqrt{a}\sqrt{c}ed+4ae^2\right)}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\frac{4\sqrt{a}e}{\sqrt{c}}\right)}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\frac{\int \frac{e\left(15cd^2+20\sqrt{a}\sqrt{c}ed+8ae^2\right)x^2+d\left(8cd^2+9\sqrt{a}\sqrt{c}ed+4ae^2\right)}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\frac{4\sqrt{a}e}{\sqrt{c}}+7d\right)}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 398$$

$$\frac{\frac{\frac{\frac{8\left(\sqrt{a}e+\sqrt{c}d\right)^3\int \frac{1}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-\frac{e\left(20\sqrt{a}\sqrt{c}de+8ae^2+15cd^2\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\frac{4\sqrt{a}e}{\sqrt{c}}+7d\right)}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\frac{\frac{e\left(-20\sqrt{a}\sqrt{c}de+8ae^2+15cd^2\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{\sqrt{c}}+\frac{8\left(\sqrt{c}d-\sqrt{a}e\right)^3\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\frac{4\sqrt{a}e}{\sqrt{c}}\right)}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{\frac{e\left(-20\sqrt{a}\sqrt{c}de+8ae^2+15cd^2\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}+\frac{8\left(\sqrt{c}d-\sqrt{a}e\right)^3\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\frac{4\sqrt{a}e}{\sqrt{c}}\right)}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\frac{\frac{8\left(\sqrt{a}e+\sqrt{c}d\right)^3\int \frac{1}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-\frac{e\left(20\sqrt{a}\sqrt{c}de+8ae^2+15cd^2\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\frac{4\sqrt{a}e}{\sqrt{c}}+7d\right)}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\frac{8\left(\sqrt{a}e+\sqrt{c}d\right)^3\int \frac{1}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-\frac{\sqrt{e}\left(20\sqrt{a}\sqrt{c}de+8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\frac{4\sqrt{a}e}{\sqrt{c}}+7d\right)}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\frac{\frac{8\left(\sqrt{c}d-\sqrt{a}e\right)^3\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}+\frac{\sqrt{e}\left(-20\sqrt{a}\sqrt{c}de+8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\frac{4\sqrt{a}e}{\sqrt{c}}\right)}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{\frac{\frac{8\left(\sqrt{c}d-\sqrt{a}e\right)^3\int \frac{1}{\sqrt{a}-\frac{\left(\sqrt{a}e-\sqrt{c}d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}+\frac{\sqrt{e}\left(-20\sqrt{a}\sqrt{c}de+8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\frac{4\sqrt{a}e}{\sqrt{c}}\right)}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}-$$

$$\frac{\frac{\frac{\frac{8\left(\sqrt{a}e+\sqrt{c}d\right)^3\int \frac{1}{\sqrt{a}-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}-\frac{\sqrt{e}\left(20\sqrt{a}\sqrt{c}de+8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\frac{4\sqrt{a}e}{\sqrt{c}}+7d\right)}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{8\left(\sqrt{a}e+\sqrt{c}d\right)^3\int \frac{1}{\sqrt{a}-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}-\frac{\sqrt{e}\left(20\sqrt{a}\sqrt{c}de+8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\frac{4\sqrt{a}e}{\sqrt{c}}+7d\right)}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}+$$

$$\frac{\frac{\frac{\frac{8\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{a}e}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{a}\sqrt{c}}+\frac{\sqrt{e}\left(-20\sqrt{a}\sqrt{c}de+8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\frac{4\sqrt{a}e}{\sqrt{c}}\right)}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\frac{8\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{a}e}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{a}\sqrt{c}}+\frac{\sqrt{e}\left(-20\sqrt{a}\sqrt{c}de+8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\frac{4\sqrt{a}e}{\sqrt{c}}\right)}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}+$$

$$\frac{\frac{\frac{\frac{8\left(\sqrt{a}e+\sqrt{c}d\right)^{5/2}\text{arctanh}\left(\frac{x\sqrt{\sqrt{a}e+\sqrt{c}d}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{a}\sqrt{c}}-\frac{\sqrt{e}\left(20\sqrt{a}\sqrt{c}de+8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\frac{4\sqrt{a}e}{\sqrt{c}}+7d\right)}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{a}}$$

input

```
Int[(d + e*x^2)^(5/2)/(a - c*x^4),x]
```

output
```
((e*x*(d + e*x^2)^(3/2))/(4*Sqrt[c]) + ((e*(7*d - (4*Sqrt[a]*e)/Sqrt[c])*x
*Sqrt[d + e*x^2])/2 + ((8*(Sqrt[c]*d - Sqrt[a]*e)^(5/2)*ArcTan[(Sqrt[Sqrt[
c]*d - Sqrt[a]*e]*x)/(a^(1/4)*Sqrt[d + e*x^2])])/(a^(1/4)*Sqrt[c]) + (Sqrt
[e]*(15*c*d^2 - 20*Sqrt[a]*Sqrt[c]*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt
[d + e*x^2]])/Sqrt[c])/(2*Sqrt[c]))/(4*Sqrt[c]))/(2*Sqrt[a]) + (-1/4*(e*x*
(d + e*x^2)^(3/2))/Sqrt[c] + (-1/2*(e*(7*d + (4*Sqrt[a]*e)/Sqrt[c])*x*Sqrt
[d + e*x^2]) + (-((Sqrt[e]*(15*c*d^2 + 20*Sqrt[a]*Sqrt[c]*d*e + 8*a*e^2)*A
rcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[c]) + (8*(Sqrt[c]*d + Sqrt[a]*e)
^(5/2)*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[a]*e]*x)/(a^(1/4)*Sqrt[d + e*x^2])])
/(a^(1/4)*Sqrt[c]))/(2*Sqrt[c]))/(4*Sqrt[c]))/(2*Sqrt[a])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 1489
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Simp[-c/(2*r)   Int[(d + e*x^2)^q/(r - c*x^2), x], x] - S
imp[c/(2*r)   Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e,
q}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(143) = 286.

Time = 0.58 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.48

method	result
pseudoelliptic	$\frac{\sqrt{\left(ae+\sqrt{d^2ac}\right)a}\,d\left(e^3a^2+3d^2eac-3\sqrt{d^2ac}\,a\,e^2-\sqrt{d^2ac}\,c\,d^2\right)\arctan\left(\frac{\sqrt{e\,x^2+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^2ac}\right)a}}\right)+\sqrt{\left(-ae+\sqrt{d^2ac}\right)a}\left(d\left((3a\right.\right.}{2\sqrt{d^2ac}\sqrt{\left(ae+\right.}}$
risch	$-\frac{e^2x\sqrt{e\,x^2+d}}{2c}-\frac{-\left(c^{\frac{3}{2}}d^3-a^{\frac{3}{2}}e^3+3a\sqrt{c}\,d\,e^2-3\sqrt{a}\,c\,d^2e\right)\left(-\frac{\ln\left(\frac{\frac{-2\sqrt{a}\sqrt{c}\,e+2cd}{c}+\frac{2e\sqrt{-\sqrt{a}\sqrt{c}}\left(x-\frac{\sqrt{-\sqrt{a}\sqrt{c}}}{\sqrt{c}}\right)}{\sqrt{c}}+2\sqrt{\frac{-\sqrt{a}\sqrt{c}}{c}}}{}\right.}{2\sqrt{-\sqrt{a}}}\right.}{}$
default	Expression too large to display

input
```
int((e*x^2+d)^(5/2)/(-c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/(d^2*a*c)^(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/((-a*e+(d^2*a*c)^(1/2)
)*a)^(1/2)*(((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*d*(e^3*a^2+3*d^2*e*a*c-3*(d^2*
a*c)^(1/2)*a*e^2-(d^2*a*c)^(1/2)*c*d^2)*arctan((e*x^2+d)^(1/2)/x*a/((-a*e+
(d^2*a*c)^(1/2))*a)^(1/2))+((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(d*((3*a*e^2+c
*d^2)*(d^2*a*c)^(1/2)+a*e*(a*e^2+3*c*d^2))*arctanh((e*x^2+d)^(1/2)/x*a/((a
*e+(d^2*a*c)^(1/2))*a)^(1/2))-((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(d^2*a*c)^(1
/2)*((e*x^2+d)^(1/2)*e^2*x+5*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))*d*e^(3/2))
))/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2299 vs. $2(143) = 286$.

Time = 12.90 (sec) , antiderivative size = 4605, normalized size of antiderivative = 23.14

$$\int \frac{(d+ex^2)^{5/2}}{a-cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(d+ex^2)^{5/2}}{a-cx^4}\,dx = -\int \frac{d^2\sqrt{d+ex^2}}{-a+cx^4}\,dx - \int \frac{e^2x^4\sqrt{d+ex^2}}{-a+cx^4}\,dx - \int \frac{2dex^2\sqrt{d+ex^2}}{-a+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)/(-c*x**4+a),x)
```

output
```
-Integral(d**2*sqrt(d + e*x**2)/(-a + c*x**4), x) - Integral(e**2*x**4*sqr
t(d + e*x**2)/(-a + c*x**4), x) - Integral(2*d*e*x**2*sqrt(d + e*x**2)/(-a
 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}}{a-cx^4}\,dx = \int -\frac{(ex^2+d)^{\frac{5}{2}}}{cx^4-a}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 + d)^(5/2)/(c*x^4 - a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}}{a-cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{a-cx^4}\,dx = \int \frac{(e\,x^2+d)^{5/2}}{a-c\,x^4}\,dx$$

input
```
int((d + e*x^2)^(5/2)/(a - c*x^4),x)
```

output
```
int((d + e*x^2)^(5/2)/(a - c*x^4), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{5/2}}{a-cx^4}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{-c\,x^4+a}dx\right)d^2 + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{-c\,x^4+a}dx\right)e^2 + 2\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{-c\,x^4+a}dx\right)de$$

input
```
int((e*x^2+d)^(5/2)/(-c*x^4+a),x)
```

output
```
int(sqrt(d + e*x**2)/(a - c*x**4),x)*d**2 + int((sqrt(d + e*x**2)*x**4)/(a
 - c*x**4),x)*e**2 + 2*int((sqrt(d + e*x**2)*x**2)/(a - c*x**4),x)*d*e
```

3.343 $\int \frac{(d+ex^2)^{3/2}}{a-cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 170

$$\int \frac{(d+ex^2)^{3/2}}{a-cx^4}\,dx = \frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}c} - \frac{e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}\text{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}c}$$

output

```
1/2*(c^(1/2)*d-a^(1/2)*e)^(3/2)*arctan((c^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/
4)/(e*x^2+d)^(1/2))/a^(3/4)/c-e^(3/2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c
+1/2*(c^(1/2)*d+a^(1/2)*e)^(3/2)*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(
1/4)/(e*x^2+d)^(1/2))/a^(3/4)/c
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex^2)^{3/2}}{a-cx^4}\,dx = \frac{e^{3/2}\Big(\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right) - \text{RootSum}\Big[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-}{}$$

input

```
Integrate[(d + e*x^2)^(3/2)/(a - c*x^4),x]
```

output

```
(e^(3/2)*(Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]] - RootSum[c*d^4 - 4*c*d^3*#1
 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (c*d^3*Log[d + 2
*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 2*a*e^2*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt
[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*
#1^3) & ]))/c
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.82, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1489, 27, 318, 25, 398, 224, 219, 291, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}}{a-cx^4}\,dx$$

$$\Big\downarrow \; 1489$$

$$\frac{\sqrt{c}\int \frac{(ex^2+d)^{3/2}}{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}dx}{2\sqrt{a}}+\frac{\sqrt{c}\int \frac{(ex^2+d)^{3/2}}{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}dx}{2\sqrt{a}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(ex^2+d\right)^{3/2}}{\sqrt{a}-\sqrt{c}x^2}dx}{2\sqrt{a}}+\frac{\int \frac{\left(ex^2+d\right)^{3/2}}{\sqrt{c}x^2+\sqrt{a}}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 318}$$

$$\frac{\frac{\int \frac{e\left(3\sqrt{c}d-2\sqrt{a}e\right)x^2+d\left(2\sqrt{c}d-\sqrt{a}e\right)}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}+\frac{-\frac{\int -\frac{e\left(3\sqrt{c}d+2\sqrt{a}e\right)x^2+d\left(2\sqrt{c}d+\sqrt{a}e\right)}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{e\left(3\sqrt{c}d-2\sqrt{a}e\right)x^2+d\left(2\sqrt{c}d-\sqrt{a}e\right)}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\int \frac{e\left(3\sqrt{c}d+2\sqrt{a}e\right)x^2+d\left(2\sqrt{c}d+\sqrt{a}e\right)}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\frac{\frac{2\left(\sqrt{a}e+\sqrt{c}d\right)^2\int \frac{1}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-e\left(\frac{2\sqrt{a}e}{\sqrt{c}}+3d\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\frac{2\left(\sqrt{c}d-\sqrt{a}e\right)^2\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}+e\left(3d-\frac{2\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\frac{2\left(\sqrt{c}d-\sqrt{a}e\right)^2\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}+e\left(3d-\frac{2\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\frac{2\left(\sqrt{a}e+\sqrt{c}d\right)^2\int \frac{1}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-e\left(\frac{2\sqrt{a}e}{\sqrt{c}}+3d\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\frac{2\left(\sqrt{a}e+\sqrt{c}d\right)^2\int \frac{1}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-\sqrt{e}\left(\frac{2\sqrt{a}e}{\sqrt{c}}+3d\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\frac{2\left(\sqrt{c}d-\sqrt{a}e\right)^2\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}+\sqrt{e}\left(3d-\frac{2\sqrt{a}e}{\sqrt{c}}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{\frac{2(\sqrt{c}d-\sqrt{a}e)^2\int \frac{1}{\sqrt{a}-\frac{(\sqrt{a}e-\sqrt{c}d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}+\sqrt{e}\left(3d-\frac{2\sqrt{a}e}{\sqrt{c}}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\frac{2(\sqrt{a}e+\sqrt{c}d)^2\int \frac{1}{\sqrt{a}-\frac{(\sqrt{c}d+\sqrt{a}e)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}-\sqrt{e}\left(\frac{2\sqrt{a}e}{\sqrt{c}}+3d\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2(\sqrt{a}e+\sqrt{c}d)^2\int \frac{1}{\sqrt{a}-\frac{(\sqrt{c}d+\sqrt{a}e)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}-\sqrt{e}\left(\frac{2\sqrt{a}e}{\sqrt{c}}+3d\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\frac{2(\sqrt{c}d-\sqrt{a}e)^{3/2}\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{a}e}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{a}\sqrt{c}}+\sqrt{e}\left(3d-\frac{2\sqrt{a}e}{\sqrt{c}}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{2(\sqrt{c}d-\sqrt{a}e)^{3/2}\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{a}e}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{a}\sqrt{c}}+\sqrt{e}\left(3d-\frac{2\sqrt{a}e}{\sqrt{c}}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\frac{2(\sqrt{a}e+\sqrt{c}d)^{3/2}\text{arctanh}\left(\frac{x\sqrt{\sqrt{a}e+\sqrt{c}d}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{a}\sqrt{c}}-\sqrt{e}\left(\frac{2\sqrt{a}e}{\sqrt{c}}+3d\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{a}}$$

input

```
Int[(d + e*x^2)^(3/2)/(a - c*x^4),x]
```

output

```
((e*x*Sqrt[d + e*x^2])/(2*Sqrt[c]) + ((2*(Sqrt[c]*d - Sqrt[a]*e)^(3/2)*Arc
Tan[(Sqrt[Sqrt[c]*d - Sqrt[a]*e]*x)/(a^(1/4)*Sqrt[d + e*x^2])])/(a^(1/4)*S
qrt[c]) + Sqrt[e]*(3*d - (2*Sqrt[a]*e)/Sqrt[c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]])/(2*Sqrt[c]))/(2*Sqrt[a]) + (-1/2*(e*x*Sqrt[d + e*x^2])/Sqrt[c]
 + (-(Sqrt[e]*(3*d + (2*Sqrt[a]*e)/Sqrt[c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e
*x^2]]) + (2*(Sqrt[c]*d + Sqrt[a]*e)^(3/2)*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[
a]*e]*x)/(a^(1/4)*Sqrt[d + e*x^2])])/(a^(1/4)*Sqrt[c]))/(2*Sqrt[c]))/(2*Sq
rt[a])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1489
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Simp[-c/(2*r)   Int[(d + e*x^2)^q/(r - c*x^2), x], x] - S
imp[c/(2*r)   Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e,
q}, x] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(124) = 248$.

Time = 0.51 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.51

method	result
pseudoelliptic	$-\frac{-\left(\frac{\left(-a\,e^{2}-c\,d^{2}\right)\sqrt{d^{2}ac}}{2}+d^{2}eac\right)\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}\,\arctan\left(\frac{\sqrt{e\,x^{2}+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}}\right)+\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}\left(\left(\frac{\left(-a\,e^{2}-c\,d^{2}\right)}{2}\right.\right.}{\sqrt{d^{2}ac}\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}\sqrt{\left(ae+\right.}}$
default	Expression too large to display

input
```
int((e*x^2+d)^(3/2)/(-c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/(d^2*a*c)^(1/2)*(-(1/2*(-a*e^2-c*d^2)*(d^2*a*c)^(1/2)+d^2*e*a*c)*((a*e+
(d^2*a*c)^(1/2))*a)^(1/2)*arctan((e*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c)^(1/2
))*a)^(1/2))+((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)*((1/2*(-a*e^2-c*d^2)*(d^2*a*
c)^(1/2)-d^2*e*a*c)*arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(d^2*a*c)^(1/2))*a)^
(1/2))+arctanh((e*x^2+d)^(1/2)/x/e^(1/2))*(d^2*a*c)^(1/2)*((a*e+(d^2*a*c)^
(1/2))*a)^(1/2)*e^(3/2)))/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)/((a*e+(d^2*a*c)
^(1/2))*a)^(1/2)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1393 vs. $2(124) = 248$.

Time = 1.23 (sec) , antiderivative size = 2793, normalized size of antiderivative = 16.43

$$\int \frac{(d+ex^2)^{3/2}}{a-cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(4*e^(3/2)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + c*sqrt((
3*c*d^2*e + a*e^3 + a*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(
a^3*c^3)))/(a*c^2))*log(-(c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4 + (a*c^3*d
^2 - a^2*c^2*e^2)*x^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*
c^3)) + 2*(c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 + 2*(a^2*c^3*x*sqrt(
(c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*c^3)) - (a*c^2*d^2*e + 3*a^
2*c*e^3)*x)*sqrt(e*x^2 + d)*sqrt((3*c*d^2*e + a*e^3 + a*c^2*sqrt((c^2*d^6
+ 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*c^3)))/(a*c^2)))/x^2) - c*sqrt((3*c*
d^2*e + a*e^3 + a*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*
c^3)))/(a*c^2))*log(-(c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4 + (a*c^3*d^2 -
 a^2*c^2*e^2)*x^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*c^3)
) + 2*(c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 - 2*(a^2*c^3*x*sqrt((c^2
*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*c^3)) - (a*c^2*d^2*e + 3*a^2*c*
e^3)*x)*sqrt(e*x^2 + d)*sqrt((3*c*d^2*e + a*e^3 + a*c^2*sqrt((c^2*d^6 + 6*
a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*c^3)))/(a*c^2)))/x^2) - c*sqrt((3*c*d^2*
e + a*e^3 - a*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*c^3)
))/(a*c^2))*log(-(c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4 - (a*c^3*d^2 - a^2
*c^2*e^2)*x^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*c^3)) +
2*(c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 + 2*(a^2*c^3*x*sqrt((c^2*d^6
 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*c^3)) + (a*c^2*d^2*e + 3*a^2*c*e...
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{a-cx^4}\,dx = -\int \frac{d\sqrt{d+ex^2}}{-a+cx^4}\,dx - \int \frac{ex^2\sqrt{d+ex^2}}{-a+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)/(-c*x**4+a),x)
```

output
```
-Integral(d*sqrt(d + e*x**2)/(-a + c*x**4), x) - Integral(e*x**2*sqrt(d +
e*x**2)/(-a + c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{a-cx^4}\,dx = \int -\frac{(ex^2+d)^{\frac{3}{2}}}{cx^4-a}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 + d)^(3/2)/(c*x^4 - a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}}{a-cx^4}\,dx = \int -\frac{(ex^2+d)^{\frac{3}{2}}}{cx^4-a}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{a-cx^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{a-c\,x^4}\,dx$$

input

```
int((d + e*x^2)^(3/2)/(a - c*x^4),x)
```

output

```
int((d + e*x^2)^(3/2)/(a - c*x^4), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}}{a-cx^4}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{-c\,x^4+a}dx\right)d + \left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{-c\,x^4+a}dx\right)e$$

input

```
int((e*x^2+d)^(3/2)/(-c*x^4+a),x)
```

output

```
int(sqrt(d + e*x**2)/(a - c*x**4),x)*d + int((sqrt(d + e*x**2)*x**2)/(a -
c*x**4),x)*e
```

3.344 $\int \frac{\sqrt{d+ex^2}}{a-cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int \frac{\sqrt{d+ex^2}}{a-cx^4}\,dx = \frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\sqrt{c}} + \frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\sqrt{c}}$$

output
```
1/2*(c^(1/2)*d-a^(1/2)*e)^(1/2)*arctan((c^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/
4)/(e*x^2+d)^(1/2))/a^(3/4)/c^(1/2)+1/2*(c^(1/2)*d+a^(1/2)*e)^(1/2)*arctan
h((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(3/4)/c^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{d+ex^2}}{a-cx^4}\,dx = -\frac{1}{2}e^{3/2}\text{RootSum}\Big[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-4cd\#1^3 + c\#1^4\&, \frac{d^2\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)+2d\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)\#1+}{cd^3-3cd^2\#1+8ae^2\#1+3cd\#1^2-c\#1^3}$$

input
```
Integrate[Sqrt[d + e*x^2]/(a - c*x^4),x]
```

output
```
-1/2*(e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 -
4*c*d*#1^3 + c*#1^4 & , (d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2]
 - #1] + 2*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + Log[
d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1
+ 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.65, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1489, 27, 301, 224, 219, 291, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{a-cx^4}dx$$

$$\Big\downarrow \text{1489}$$

$$\frac{\sqrt{c}\int \frac{\sqrt{ex^2+d}}{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}dx}{2\sqrt{a}}+\frac{\sqrt{c}\int \frac{\sqrt{ex^2+d}}{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{ex^2+d}}{\sqrt{a}-\sqrt{c}x^2}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{ex^2+d}}{\sqrt{c}x^2+\sqrt{a}}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{301}$$

$$\frac{\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx-\frac{e\int \frac{1}{\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{a}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx+\frac{e\int \frac{1}{\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx-\frac{e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}}{2\sqrt{a}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx+\frac{e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx-\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{a}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a}-\frac{\left(\sqrt{a}e-\sqrt{c}d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{a}}+\frac{\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{a}-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{a}-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{a}e}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{a}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{a}e}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{a}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{arctanh}\left(\frac{x\sqrt{\sqrt{a}e+\sqrt{c}d}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{a}\sqrt{\sqrt{a}e+\sqrt{c}d}}-\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{a}}$$

input
```
Int[Sqrt[d + e*x^2]/(a - c*x^4),x]
```

output
```
(((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[a]*e]*x)/(a^(1/4
)*Sqrt[d + e*x^2])])/(a^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) + (Sqrt[e]*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[c])/(2*Sqrt[a]) + (-((Sqrt[e]*ArcTa
nh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[c]) + ((d + (Sqrt[a]*e)/Sqrt[c])*Arc
Tanh[(Sqrt[Sqrt[c]*d + Sqrt[a]*e]*x)/(a^(1/4)*Sqrt[d + e*x^2])])/(a^(1/4)*
Sqrt[Sqrt[c]*d + Sqrt[a]*e]))/(2*Sqrt[a])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 1489
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Simp[-c/(2*r)   Int[(d + e*x^2)^q/(r - c*x^2), x], x] - S
imp[c/(2*r)   Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e,
q}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94

method	result	size
pseudoelliptic	$-\frac{d\left(\frac{\left(-ae+\sqrt{d^{2}ac}\right)\arctan\left(\frac{\sqrt{e\,x^{2}+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}}\right)}{\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}}-\frac{\left(ae+\sqrt{d^{2}ac}\right)\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}\,a}{x\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}}\right)}{\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}}\right)}{2\sqrt{d^{2}ac}}$	137
default	Expression too large to display	1626

input
```
int((e*x^2+d)^(1/2)/(-c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/(d^2*a*c)^(1/2)*((-a*e+(d^2*a*c)^(1/2))/((-a*e+(d^2*a*c)^(1/2))*a)^
(1/2)*arctan((e*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))-(a*e+(d
^2*a*c)^(1/2))/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*arctanh((e*x^2+d)^(1/2)/x*a
/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(101) = 202$.

Time = 0.19 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.36

$$\int \frac{\sqrt{d+ex^2}}{a-cx^4}\,dx$$

$$= \frac{1}{8}\sqrt{\frac{ac\sqrt{\frac{d^2}{a^3c}}+e}{ac}}\log\left(\frac{acdx^2\sqrt{\frac{d^2}{a^3c}}+2\sqrt{ex^2+d}a^2cx\sqrt{\frac{ac\sqrt{\frac{d^2}{a^3c}}+e}{ac}}\sqrt{\frac{d^2}{a^3c}}+2\,dex^2+d^2}{x^2}\right)$$

$$-\frac{1}{8}\sqrt{\frac{ac\sqrt{\frac{d^2}{a^3c}}+e}{ac}}\log\left(\frac{acdx^2\sqrt{\frac{d^2}{a^3c}}-2\sqrt{ex^2+d}a^2cx\sqrt{\frac{ac\sqrt{\frac{d^2}{a^3c}}+e}{ac}}\sqrt{\frac{d^2}{a^3c}}+2\,dex^2+d^2}{x^2}\right)$$

$$+\frac{1}{8}\sqrt{-\frac{ac\sqrt{\frac{d^2}{a^3c}}-e}{ac}}\log\left(-\frac{acdx^2\sqrt{\frac{d^2}{a^3c}}+2\sqrt{ex^2+d}a^2cx\sqrt{-\frac{ac\sqrt{\frac{d^2}{a^3c}}-e}{ac}}\sqrt{\frac{d^2}{a^3c}}-2\,dex^2-d^2}{x^2}\right)$$

$$-\frac{1}{8}\sqrt{-\frac{ac\sqrt{\frac{d^2}{a^3c}}-e}{ac}}\log\left(-\frac{acdx^2\sqrt{\frac{d^2}{a^3c}}-2\sqrt{ex^2+d}a^2cx\sqrt{-\frac{ac\sqrt{\frac{d^2}{a^3c}}-e}{ac}}\sqrt{\frac{d^2}{a^3c}}-2\,dex^2-d^2}{x^2}\right)$$

input

```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a),x, algorithm="fricas")
```

output

```
1/8*sqrt((a*c*sqrt(d^2/(a^3*c)) + e)/(a*c))*log((a*c*d*x^2*sqrt(d^2/(a^3*c
)) + 2*sqrt(e*x^2 + d)*a^2*c*x*sqrt((a*c*sqrt(d^2/(a^3*c)) + e)/(a*c))*sqr
t(d^2/(a^3*c)) + 2*d*e*x^2 + d^2)/x^2) - 1/8*sqrt((a*c*sqrt(d^2/(a^3*c)) +
 e)/(a*c))*log((a*c*d*x^2*sqrt(d^2/(a^3*c)) - 2*sqrt(e*x^2 + d)*a^2*c*x*sq
rt((a*c*sqrt(d^2/(a^3*c)) + e)/(a*c))*sqrt(d^2/(a^3*c)) + 2*d*e*x^2 + d^2)
/x^2) + 1/8*sqrt(-(a*c*sqrt(d^2/(a^3*c)) - e)/(a*c))*log(-(a*c*d*x^2*sqrt(
d^2/(a^3*c)) + 2*sqrt(e*x^2 + d)*a^2*c*x*sqrt(-(a*c*sqrt(d^2/(a^3*c)) - e)
/(a*c))*sqrt(d^2/(a^3*c)) - 2*d*e*x^2 - d^2)/x^2) - 1/8*sqrt(-(a*c*sqrt(d^
2/(a^3*c)) - e)/(a*c))*log(-(a*c*d*x^2*sqrt(d^2/(a^3*c)) - 2*sqrt(e*x^2 +
d)*a^2*c*x*sqrt(-(a*c*sqrt(d^2/(a^3*c)) - e)/(a*c))*sqrt(d^2/(a^3*c)) - 2*
d*e*x^2 - d^2)/x^2)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{a-cx^4}\,dx = -\int \frac{\sqrt{d+ex^2}}{-a+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(-c*x**4+a),x)
```

output
```
-Integral(sqrt(d + e*x**2)/(-a + c*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{a-cx^4}\,dx = \int -\frac{\sqrt{ex^2+d}}{cx^4-a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a),x, algorithm="maxima")
```

output
```
-integrate(sqrt(e*x^2 + d)/(c*x^4 - a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{a-cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{a-cx^4}\,dx=\int \frac{\sqrt{e\,x^2+d}}{a-c\,x^4}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a - c*x^4),x)
```

output
```
int((d + e*x^2)^(1/2)/(a - c*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}}{a-cx^4}\,dx=\int \frac{\sqrt{e\,x^2+d}}{-c\,x^4+a}dx$$

input
```
int((e*x^2+d)^(1/2)/(-c*x^4+a),x)
```

output
```
int(sqrt(d + e*x**2)/(a - c*x**4),x)
```

3.345 $\int \frac{1}{\sqrt{d+ex^2}(a-cx^4)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)} \, dx = \frac{\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\sqrt{\sqrt{c}d+\sqrt{a}e}}$$

output
```
1/2*arctan((c^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(3/4)/
(c^(1/2)*d-a^(1/2)*e)^(1/2)+1/2*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1
/4)/(e*x^2+d)^(1/2))/a^(3/4)/(c^(1/2)*d+a^(1/2)*e)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)} \, dx = 2e^{3/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-4cd\#1^3 + c\#1^4\&, \frac{\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)\#1}{-cd^3+3cd^2\#1-8ae^2\#1-3cd\#1^2+c\#1^3}\&\right]$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a - c*x^4)),x]
```

output
```
2*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*
d*#1^3 + c*#1^4 & , (Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#
1)/(-(c*d^3) + 3*c*d^2*#1 - 8*a*e^2*#1 - 3*c*d*#1^2 + c*#1^3) & ]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1489, 27, 291, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-cx^4\right)\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \; 1489$$

$$\frac{\sqrt{c}\int \frac{1}{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{a}}+\frac{\sqrt{c}\int \frac{1}{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{a}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{1}{\left(\sqrt{a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{a}}+\frac{\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{a}}$$

$$\Big\downarrow \; 291$$

$$\frac{\int \frac{1}{\sqrt{a}-\frac{\left(\sqrt{a}e-\sqrt{c}d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2\sqrt{a}}+\frac{\int \frac{1}{\sqrt{a}-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 218$$

$$\frac{\int \frac{1}{\sqrt{a}-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2\sqrt{a}}+\frac{\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{a}e}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}$$

$$\Big\downarrow \; 221$$

$$\frac{\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{a}e}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt{\sqrt{a}e+\sqrt{c}d}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\sqrt{\sqrt{a}e+\sqrt{c}d}}$$

input

```
Int[1/(Sqrt[d + e*x^2]*(a - c*x^4)),x]
```

output

```
ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[a]*e]*x)/(a^(1/4)*Sqrt[d + e*x^2])]/(2*a^(3/
4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) + ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[a]*e]*x)/
(a^(1/4)*Sqrt[d + e*x^2])]/(2*a^(3/4)*Sqrt[Sqrt[c]*d + Sqrt[a]*e])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 1489

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Simp[-c/(2*r)   Int[(d + e*x^2)^q/(r - c*x^2), x], x] - S
imp[c/(2*r)   Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e,
q}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{e\,x^{2}+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}}\right)}{2\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}\,a}{x\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}}\right)}{2\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}}$
default	$-\frac{\ln\left(\frac{\frac{-2\sqrt{a}\,\sqrt{c}\,e+2cd}{c}+\frac{2e\sqrt{-\sqrt{a}\,\sqrt{c}}\left(x-\frac{\sqrt{-\sqrt{a}\,\sqrt{c}}}{\sqrt{c}}\right)}{\sqrt{c}}+2\sqrt{\frac{-\sqrt{a}\,\sqrt{c}\,e+cd}{c}}\sqrt{\left(x-\frac{\sqrt{-\sqrt{a}\,\sqrt{c}}}{\sqrt{c}}\right)^{2}e+\frac{2e\sqrt{-\sqrt{a}\,\sqrt{c}}\left(x-\frac{\sqrt{-\sqrt{a}\,\sqrt{c}}}{\sqrt{c}}\right)}{\sqrt{c}}+-}}{x-\frac{\sqrt{-\sqrt{a}\,\sqrt{c}}}{\sqrt{c}}}\right)}{2\sqrt{-\sqrt{a}\,\sqrt{c}}\,\sqrt{\frac{-\sqrt{a}\,\sqrt{c}\,e+cd}{c}}}$

input
```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)*arctan((e*x^2+d)^(1/2)/x*a/((-a*e+(d
^2*a*c)^(1/2))*a)^(1/2))+1/2/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*arctanh((e*x^
2+d)^(1/2)/x*a/((a*e+(d^2*a*c)^(1/2))*a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1225 vs. $2(95) = 190$.

Time = 0.28 (sec) , antiderivative size = 1225, normalized size of antiderivative = 9.07

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a),x, algorithm="fricas")
```

output
```
1/8*sqrt(((a*c*d^2 - a^2*e^2)*sqrt(c*d^2/(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 +
a^5*e^4)) - e)/(a*c*d^2 - a^2*e^2))*log(((a*c*d^2 - a^2*e^2)*sqrt(c*d^2/(a
^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4))*x^2 + 2*e*x^2 + 2*(a*e*x + (a^2*c
*d^2 - a^3*e^2)*sqrt(c*d^2/(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4))*x)*s
qrt(e*x^2 + d)*sqrt(((a*c*d^2 - a^2*e^2)*sqrt(c*d^2/(a^3*c^2*d^4 - 2*a^4*c
*d^2*e^2 + a^5*e^4)) - e)/(a*c*d^2 - a^2*e^2)) + d)/x^2) - 1/8*sqrt(((a*c*
d^2 - a^2*e^2)*sqrt(c*d^2/(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)) - e)/
(a*c*d^2 - a^2*e^2))*log(((a*c*d^2 - a^2*e^2)*sqrt(c*d^2/(a^3*c^2*d^4 - 2*
a^4*c*d^2*e^2 + a^5*e^4))*x^2 + 2*e*x^2 - 2*(a*e*x + (a^2*c*d^2 - a^3*e^2)
*sqrt(c*d^2/(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4))*x)*sqrt(e*x^2 + d)*
sqrt(((a*c*d^2 - a^2*e^2)*sqrt(c*d^2/(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*
e^4)) - e)/(a*c*d^2 - a^2*e^2)) + d)/x^2) - 1/8*sqrt(-((a*c*d^2 - a^2*e^2)
*sqrt(c*d^2/(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)) + e)/(a*c*d^2 - a^2
*e^2))*log(-((a*c*d^2 - a^2*e^2)*sqrt(c*d^2/(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2
 + a^5*e^4))*x^2 - 2*e*x^2 + 2*(a*e*x - (a^2*c*d^2 - a^3*e^2)*sqrt(c*d^2/(
a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-((a*c*d
^2 - a^2*e^2)*sqrt(c*d^2/(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)) + e)/(
a*c*d^2 - a^2*e^2)) - d)/x^2) + 1/8*sqrt(-((a*c*d^2 - a^2*e^2)*sqrt(c*d^2/
(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)) + e)/(a*c*d^2 - a^2*e^2))*log(-
((a*c*d^2 - a^2*e^2)*sqrt(c*d^2/(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^...
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)}\,dx = -\int \frac{1}{-a\sqrt{d+ex^2}+cx^4\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(-c*x**4+a),x)
```

output
```
-Integral(1/(-a*sqrt(d + e*x**2) + c*x**4*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)}\,dx = \int -\frac{1}{(cx^4-a)\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a),x, algorithm="maxima")
```

output
```
-integrate(1/((c*x^4 - a)*sqrt(e*x^2 + d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)}\,dx = \int \frac{1}{(a-c\,x^4)\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a - c*x^4)*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a - c*x^4)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)}\,dx=\int \frac{1}{\sqrt{e\,x^2+d}\,a-\sqrt{e\,x^2+d}\,c\,x^4}dx$$

input

```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a),x)
```

output

```
int(1/(sqrt(d + e*x**2)*a - sqrt(d + e*x**2)*c*x**4),x)
```

3.346 $\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 179

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)}\,dx = -\frac{e^2x}{d(cd^2-ae^2)\sqrt{d+ex^2}} + \frac{\sqrt{c}\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}} + \frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}}$$

output

```
-e^2*x/d/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)+1/2*c^(1/2)*arctan((c^(1/2)*d-a^(1
/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(3/4)/(c^(1/2)*d-a^(1/2)*e)^(3/2
)+1/2*c^(1/2)*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2
))/a^(3/4)/(c^(1/2)*d+a^(1/2)*e)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.38

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)}\,dx =$$

$$-\frac{\frac{2e^2x}{\sqrt{d+ex^2}} - cde^{3/2}\text{RootSum}\Big[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-4cd\#1^3+c\#1^4\&, \frac{d^2\log\left(d+2ex^2-2\sqrt{e}x\sqrt{}\right.}{}}{2cd^3-2ade^2}$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(a - c*x^4)),x]
```

output
```
-(((2*e^2*x)/Sqrt[d + e*x^2] - c*d*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*
c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (d^2*Log[d + 2*e*x^2
- 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 6*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sq
rt[d + e*x^2] - #1]*#1 + Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #
1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(2*c
*d^3 - 2*a*d*e^2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1487, 208, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^4)(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1487}$$

$$\frac{c\int \frac{d-ex^2}{\sqrt{ex^2+d}(a-cx^4)}dx}{cd^2-ae^2}-\frac{e^2\int \frac{1}{(ex^2+d)^{3/2}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{c\int \frac{d-ex^2}{\sqrt{ex^2+d}(a-cx^4)}dx}{cd^2-ae^2}-\frac{e^2x}{d\sqrt{d+ex^2}\,(cd^2-ae^2)}$$

$$\Big\downarrow \textcolor{blue}{2257}$$

$$\frac{c\int \left(\frac{\sqrt{c}d-\sqrt{a}e}{2\sqrt{a}\sqrt{c}(\sqrt{a}-\sqrt{c}x^2)\sqrt{ex^2+d}}-\frac{-\sqrt{c}d-\sqrt{a}e}{2\sqrt{a}\sqrt{c}(\sqrt{c}x^2+\sqrt{a})\sqrt{ex^2+d}}\right)dx}{cd^2-ae^2}-\frac{e^2x}{d\sqrt{d+ex^2}\,(cd^2-ae^2)}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{c\left(\frac{(\sqrt{a}e+\sqrt{c}d)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{a}e}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{(\sqrt{c}d-\sqrt{a}e)\text{arctanh}\left(\frac{x\sqrt{\sqrt{a}e+\sqrt{c}d}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{cd^2-ae^2}-\frac{e^2x}{d\sqrt{d+ex^2}\,(cd^2-ae^2)}$$

input
```
Int[1/((d + e*x^2)^(3/2)*(a - c*x^4)),x]
```

output
```
-((e^2*x)/(d*(c*d^2 - a*e^2)*Sqrt[d + e*x^2])) + (c*(((Sqrt[c]*d + Sqrt[a]
*e)*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[a]*e]*x)/(a^(1/4)*Sqrt[d + e*x^2])])/(2*
a^(3/4)*Sqrt[c]*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) + ((Sqrt[c]*d - Sqrt[a]*e)*Ar
cTanh[(Sqrt[Sqrt[c]*d + Sqrt[a]*e]*x)/(a^(1/4)*Sqrt[d + e*x^2])])/(2*a^(3/
4)*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[a]*e])))/(c*d^2 - a*e^2)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 1487
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Simp[e^2
/(c*d^2 + a*e^2)   Int[(d + e*x^2)^q, x], x] + Simp[c/(c*d^2 + a*e^2)   Int
[(d + e*x^2)^(q + 1)*((d - e*x^2)/(a + c*x^4)), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.41

method	result
pseudoelliptic	$\frac{\frac{\left(ae+\sqrt{d^{2}ac}\right)d^{2}\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}\,c\sqrt{e\,x^{2}+d}\,\arctan\left(\frac{\sqrt{e\,x^{2}+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}}\right)}{2}+\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}\left(\frac{c\,d^{2}\sqrt{e\,x^{2}+d}\left(ae-\sqrt{d^{2}ac}\right)\arctan}{2}\right.}{\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}\,\sqrt{e\,x^{2}+d}\,\sqrt{d^{2}ac}\,\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}\,\left(a\,e^{2}-c\,d^{2}\right)d}$
default	Expression too large to display

input
```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/(e*x^2+d)^(1/2)/(d^2*a*c)^(1/2)*(1/2*(a*
e+(d^2*a*c)^(1/2))*d^2*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*c*(e*x^2+d)^(1/2)*a
rctan((e*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+((-a*e+(d^2*a*
c)^(1/2))*a)^(1/2)*(1/2*c*d^2*(e*x^2+d)^(1/2)*(a*e-(d^2*a*c)^(1/2))*arctan
h((e*x^2+d)^(1/2)/x*a/((a*e+(d^2*a*c)^(1/2))*a)^(1/2))+((a*e+(d^2*a*c)^(1/
2))*a)^(1/2)*(d^2*a*c)^(1/2)*e^2*x))/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)/(a*e
^2-c*d^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3813 vs. $2(133) = 266$.

Time = 11.03 (sec) , antiderivative size = 3813, normalized size of antiderivative = 21.30

$$\int \frac{1}{\left(d+ex^{2}\right)^{3/2}\left(a-cx^{4}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)}\,dx = -\int \frac{1}{-ad\sqrt{d+ex^2}-aex^2\sqrt{d+ex^2}+cdx^4\sqrt{d+ex^2}+cex^6\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(-c*x**4+a),x)
```

output
```
-Integral(1/(-a*d*sqrt(d + e*x**2) - a*e*x**2*sqrt(d + e*x**2) + c*d*x**4*
sqrt(d + e*x**2) + c*e*x**6*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)}\,dx = \int -\frac{1}{(cx^4-a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a),x, algorithm="maxima")
```

output
```
-integrate(1/((c*x^4 - a)*(e*x^2 + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)}\,dx = \int \frac{1}{(a-c\,x^4)\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int(1/((a - c*x^4)*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a - c*x^4)*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,ad+\sqrt{e\,x^2+d}\,ae\,x^2-\sqrt{e\,x^2+d}\,cd\,x^4-\sqrt{e\,x^2+d}\,ce\,x^6}dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a),x)
```

output
```
int(1/(sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2 - sqrt(d + e*x**2)
*c*d*x**4 - sqrt(d + e*x**2)*c*e*x**6),x)
```

3.347 $\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 222

$$\int \frac{1}{(d+ex^2)^{5/2}(a-cx^4)}\,dx = -\frac{e^2x}{3d(cd^2-ae^2)(d+ex^2)^{3/2}} - \frac{2e^2(4cd^2-ae^2)\,x}{3d^2(cd^2-ae^2)^2\sqrt{d+ex^2}} + \frac{c\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}} + \frac{c\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}}$$

output

```
-1/3*e^2*x/d/(-a*e^2+c*d^2)/(e*x^2+d)^(3/2)-2/3*e^2*(-a*e^2+4*c*d^2)*x/d^2
/(-a*e^2+c*d^2)^2/(e*x^2+d)^(1/2)+1/2*c*arctan((c^(1/2)*d-a^(1/2)*e)^(1/2)
*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(3/4)/(c^(1/2)*d-a^(1/2)*e)^(5/2)+1/2*c*arct
anh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(3/4)/(c^(1/2
)*d+a^(1/2)*e)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.53

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)}\,dx=-\frac{e^2x\left(9cd^3-3ade^2+8cd^2ex^2-2ae^3x^2\right)}{3d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^{3/2}}$$

$$+\frac{ce^{3/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\frac{cd^3\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)-}{}\right.}{\left(cd^2-\right.}$$

input

```
Integrate[1/((d + e*x^2)^(5/2)*(a - c*x^4)),x]
```

output

```
-1/3*(e^2*x*(9*c*d^3 - 3*a*d*e^2 + 8*c*d^2*e*x^2 - 2*a*e^3*x^2))/(d^2*(c*d
^2 - a*e^2)^2*(d + e*x^2)^(3/2)) + (c*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 +
 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (c*d^3*Log[d + 2*e
*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 4*c*d^2*Log[d + 2*e*x^2 - 2*Sqr
t[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 2*a*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sq
rt[d + e*x^2] - #1]*#1 + c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2]
 - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/
(c*d^2 - a*e^2)^2
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.65, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.227$, Rules used = {1487, 209, 208, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-cx^4\right)\left(d+ex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1487}$$

$$\frac{c\int \frac{d-ex^2}{\left(ex^2+d\right)^{3/2}\left(a-cx^4\right)}dx}{cd^2-ae^2}-\frac{e^2\int \frac{1}{\left(ex^2+d\right)^{5/2}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{c\int \frac{d-ex^2}{\left(ex^2+d\right)^{3/2}\left(a-cx^4\right)}dx}{cd^2-ae^2}-\frac{e^2\left(\frac{2\int \frac{1}{\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{x}{3d\left(d+ex^2\right)^{3/2}}\right)}{cd^2-ae^2}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{c\int \frac{d-ex^2}{\left(ex^2+d\right)^{3/2}\left(a-cx^4\right)}dx}{cd^2-ae^2}-\frac{e^2\left(\frac{2x}{3d^2\sqrt{d+ex^2}}+\frac{x}{3d\left(d+ex^2\right)^{3/2}}\right)}{cd^2-ae^2}$$

$$\Big\downarrow \text{ 2257}$$

$$\frac{c\int \left(\frac{\sqrt{c}d-\sqrt{a}e}{2\sqrt{a}\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)\left(ex^2+d\right)^{3/2}}-\frac{-\sqrt{c}d-\sqrt{a}e}{2\sqrt{a}\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)\left(ex^2+d\right)^{3/2}}\right)dx}{cd^2-ae^2}-\frac{e^2\left(\frac{2x}{3d^2\sqrt{d+ex^2}}+\frac{x}{3d\left(d+ex^2\right)^{3/2}}\right)}{cd^2-ae^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\left(\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{a}e}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\text{arctanh}\left(\frac{x\sqrt{\sqrt{a}e+\sqrt{c}d}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}}-\frac{ex\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{a}\sqrt{c}d\sqrt{d+ex^2}\left(\sqrt{c}d-\sqrt{a}e\right)}+\frac{ex\left(\sqrt{c}d-\sqrt{a}e\right.}{2\sqrt{a}\sqrt{c}d\sqrt{d+ex^2}\left(\sqrt{a}\right.}\right.}{cd^2-ae^2}-\frac{e^2\left(\frac{2x}{3d^2\sqrt{d+ex^2}}+\frac{x}{3d\left(d+ex^2\right)^{3/2}}\right)}{cd^2-ae^2}$$

input
```
Int[1/((d + e*x^2)^(5/2)*(a - c*x^4)),x]
```

output
```
-((e^2*(x/(3*d*(d + e*x^2)^(3/2)) + (2*x)/(3*d^2*Sqrt[d + e*x^2])))/(c*d^2
 - a*e^2)) + (c*((e*(Sqrt[c]*d - Sqrt[a]*e)*x)/(2*Sqrt[a]*Sqrt[c]*d*(Sqrt[
c]*d + Sqrt[a]*e)*Sqrt[d + e*x^2]) - (e*(Sqrt[c]*d + Sqrt[a]*e)*x)/(2*Sqrt
[a]*Sqrt[c]*d*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[d + e*x^2]) + ((Sqrt[c]*d + Sqr
t[a]*e)*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[a]*e]*x)/(a^(1/4)*Sqrt[d + e*x^2])])
/(2*a^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)^(3/2)) + ((Sqrt[c]*d - Sqrt[a]*e)*ArcT
anh[(Sqrt[Sqrt[c]*d + Sqrt[a]*e]*x)/(a^(1/4)*Sqrt[d + e*x^2])])/(2*a^(3/4)
*(Sqrt[c]*d + Sqrt[a]*e)^(3/2))))/(c*d^2 - a*e^2)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 1487
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Simp[e^2
/(c*d^2 + a*e^2)   Int[(d + e*x^2)^q, x], x] + Simp[c/(c*d^2 + a*e^2)   Int
[(d + e*x^2)^(q + 1)*((d - e*x^2)/(a + c*x^4)), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.42

method	result
pseudoelliptic	$-\frac{d^2\sqrt{\left(ae+\sqrt{d^2ac}\right)a}\left(\frac{\left(a\,e^2+c\,d^2\right)\sqrt{d^2ac}}{2}+d^2eac\right)c\left(e\,x^2+d\right)^{\frac{3}{2}}\arctan\left(\frac{\sqrt{e\,x^2+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^2ac}\right)a}}\right)+\sqrt{\left(-ae+\sqrt{d^2ac}\right)a}\left(d^2\left(\right.\right.}{\sqrt{\left(ae+\sqrt{d^2ac}\right)a\left(e\,x^2+d\right.}}$
default	Expression too large to display

input
```
int(1/(e*x^2+d)^(5/2)/(-c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-(d^2*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(1/2*(a*e^2+c*d^2)*(d^2*a*c)^(1/2)+d
^2*e*a*c)*c*(e*x^2+d)^(3/2)*arctan((e*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c)^(1
/2))*a)^(1/2))+((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(d^2*(1/2*(-a*e^2-c*d^2)*(
d^2*a*c)^(1/2)+d^2*e*a*c)*c*(e*x^2+d)^(3/2)*arctanh((e*x^2+d)^(1/2)/x*a/((
a*e+(d^2*a*c)^(1/2))*a)^(1/2))-x*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(e^2*(2/3
*e*x^2+d)*a-3*d^2*(8/9*e*x^2+d)*c)*e^2*(d^2*a*c)^(1/2)))/((a*e+(d^2*a*c)^(
1/2))*a)^(1/2)/(e*x^2+d)^(3/2)/(d^2*a*c)^(1/2)/((-a*e+(d^2*a*c)^(1/2))*a)^
(1/2)/(a*e^2-c*d^2)^2/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6571 vs. $2(174) = 348$.

Time = 45.75 (sec) , antiderivative size = 6571, normalized size of antiderivative = 29.60

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)}\,dx =$$
$$-\int \frac{1}{-ad^2\sqrt{d+ex^2}-2adex^2\sqrt{d+ex^2}-ae^2x^4\sqrt{d+ex^2}+cd^2x^4\sqrt{d+ex^2}+2cdex^6\sqrt{d+ex^2}+ce^2x^8\sqrt{d}}$$

input
```
integrate(1/(e*x**2+d)**(5/2)/(-c*x**4+a),x)
```

output
```
-Integral(1/(-a*d**2*sqrt(d + e*x**2) - 2*a*d*e*x**2*sqrt(d + e*x**2) - a*
e**2*x**4*sqrt(d + e*x**2) + c*d**2*x**4*sqrt(d + e*x**2) + 2*c*d*e*x**6*s
qrt(d + e*x**2) + c*e**2*x**8*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)}\,dx = \int -\frac{1}{\left(cx^4-a\right)\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a),x, algorithm="maxima")
```

output
```
-integrate(1/((c*x^4 - a)*(e*x^2 + d)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a-cx^4)}\,dx=\int \frac{1}{(a-c\,x^4)\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int(1/((a - c*x^4)*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a - c*x^4)*(d + e*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a-cx^4)}\,dx=\int \frac{1}{\sqrt{e\,x^2+d}\,a\,d^2+2\sqrt{e\,x^2+d}\,ade\,x^2+\sqrt{e\,x^2+d}\,a\,e^2x^4-\sqrt{e\,x^2+d}\,c\,d^2x$$

input
```
int(1/(e*x^2+d)^(5/2)/(-c*x^4+a),x)
```

output
```
int(1/(sqrt(d + e*x**2)*a*d**2 + 2*sqrt(d + e*x**2)*a*d*e*x**2 + sqrt(d +
e*x**2)*a*e**2*x**4 - sqrt(d + e*x**2)*c*d**2*x**4 - 2*sqrt(d + e*x**2)*c*
d*e*x**6 - sqrt(d + e*x**2)*c*e**2*x**8),x)
```

3.348 $\int \frac{1}{\left(d+ex^2\right)^{7/2}\left(a-cx^4\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 293

$$
\begin{aligned}
\int \frac{1}{\left(d+ex^2\right)^{7/2}\left(a-cx^4\right)}\,dx &= -\frac{e^2x}{5d\left(cd^2-ae^2\right)\left(d+ex^2\right)^{5/2}} \\
&\quad -\frac{2e^2\left(7cd^2-2ae^2\right)x}{15d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^{3/2}}-\frac{e^2\left(73c^2d^4-21acd^2e^2+8a^2e^4\right)x}{15d^3\left(cd^2-ae^2\right)^3\sqrt{d+ex^2}} \\
&\quad +\frac{c^{3/2}\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{7/2}}+\frac{c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)^{7/2}}
\end{aligned}
$$

output

```
-1/5*e^2*x/d/(-a*e^2+c*d^2)/(e*x^2+d)^(5/2)-2/15*e^2*(-2*a*e^2+7*c*d^2)*x/
d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)^(3/2)-1/15*e^2*(8*a^2*e^4-21*a*c*d^2*e^2+73
*c^2*d^4)*x/d^3/(-a*e^2+c*d^2)^3/(e*x^2+d)^(1/2)+1/2*c^(3/2)*arctan((c^(1/
2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(3/4)/(c^(1/2)*d-a^(1/2
)*e)^(7/2)+1/2*c^(3/2)*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^
2+d)^(1/2))/a^(3/4)/(c^(1/2)*d+a^(1/2)*e)^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.13 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.61

$$\int \frac{1}{(d+ex^2)^{7/2}\,(a-cx^4)}\,dx = \frac{e^{3/2}\Bigg(-\frac{2\sqrt{e}x(-3acd^2e^2(15d^2+20dex^2+7e^2x^4)+a^2e^4(15d^2+20dex^2+8e^2x^4)+c^2d^4(90d^2+160dex^2}{d^3(d+ex^2)^{5/2}}}{}$$

input
```
Integrate[1/((d + e*x^2)^(7/2)*(a - c*x^4)),x]
```

output
```
(e^(3/2)*((-2*Sqrt[e]*x*(-3*a*c*d^2*e^2*(15*d^2 + 20*d*e*x^2 + 7*e^2*x^4)
+ a^2*e^4*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4) + c^2*d^4*(90*d^2 + 160*d*e*x^
2 + 73*e^2*x^4)))/(d^3*(d + e*x^2)^(5/2)) + 15*c^2*RootSum[c*d^4 - 4*c*d^3
*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (3*c*d^4*Log[
d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + a*d^2*e^2*Log[d + 2*e*x^
2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 10*c*d^3*Log[d + 2*e*x^2 - 2*Sqrt[
e]*x*Sqrt[d + e*x^2] - #1]*#1 - 14*a*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*S
qrt[d + e*x^2] - #1]*#1 + 3*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e
*x^2] - #1]*#1^2 + a*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #
1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ]))/(30
*(c*d^2 - a*e^2)^3)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1487, 209, 209, 208, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^4)\,(d+ex^2)^{7/2}}\,dx$$

$$\Big\downarrow \; 1487$$

$$\frac{c\int \frac{d-ex^2}{\left(ex^2+d\right)^{5/2}\left(a-cx^4\right)}dx}{cd^2-ae^2}-\frac{e^2\int \frac{1}{\left(ex^2+d\right)^{7/2}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \; 209$$

$$\frac{c\int \frac{d-ex^2}{\left(ex^2+d\right)^{5/2}\left(a-cx^4\right)}dx}{cd^2-ae^2}-\frac{e^2\left(\frac{4\int \frac{1}{\left(ex^2+d\right)^{5/2}}dx}{5d}+\frac{x}{5d\left(d+ex^2\right)^{5/2}}\right)}{cd^2-ae^2}$$

$$\Big\downarrow \; 209$$

$$\frac{c\int \frac{d-ex^2}{\left(ex^2+d\right)^{5/2}\left(a-cx^4\right)}dx}{cd^2-ae^2}-\frac{e^2\left(\frac{4\left(\frac{2\int \frac{1}{\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{x}{3d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\frac{x}{5d\left(d+ex^2\right)^{5/2}}\right)}{cd^2-ae^2}$$

$$\Big\downarrow \; 208$$

$$\frac{c\int \frac{d-ex^2}{\left(ex^2+d\right)^{5/2}\left(a-cx^4\right)}dx}{cd^2-ae^2}-\frac{e^2\left(\frac{4\left(\frac{2x}{3d^2\sqrt{d+ex^2}}+\frac{x}{3d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\frac{x}{5d\left(d+ex^2\right)^{5/2}}\right)}{cd^2-ae^2}$$

$$\Big\downarrow \; 2257$$

$$\frac{c\int \left(\frac{\sqrt{c}d-\sqrt{a}e}{2\sqrt{a}\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)\left(ex^2+d\right)^{5/2}}-\frac{-\sqrt{c}d-\sqrt{a}e}{2\sqrt{a}\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)\left(ex^2+d\right)^{5/2}}\right)dx}{cd^2-ae^2}-\frac{e^2\left(\frac{4\left(\frac{2x}{3d^2\sqrt{d+ex^2}}+\frac{x}{3d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\frac{x}{5d\left(d+ex^2\right)^{5/2}}\right)}{cd^2-ae^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{c\left(\frac{\sqrt{c}(\sqrt{a}e+\sqrt{c}d)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{a}e}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}}+\frac{\sqrt{c}(\sqrt{c}d-\sqrt{a}e)\text{arctanh}\left(\frac{x\sqrt{\sqrt{a}e+\sqrt{c}d}}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{2a^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)^{5/2}}-\frac{ex\left(5\sqrt{c}d-2\sqrt{a}e\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{6\sqrt{a}\sqrt{c}d^2\sqrt{d+ex^2}\left(\sqrt{c}d-\sqrt{a}e\right)^2}+\right.}{cd^2-ae^2}$$

$$\frac{e^2\left(\frac{4\left(\frac{2x}{3d^2\sqrt{d+ex^2}}+\frac{x}{3d\left(d+ex^2\right)^{3/2}}\right)}{5d}+\frac{x}{5d(d+ex^2)^{5/2}}\right)}{cd^2-ae^2}$$

input
```
Int[1/((d + e*x^2)^(7/2)*(a - c*x^4)),x]
```

output
```
-((e^2*(x/(5*d*(d + e*x^2)^(5/2)) + (4*(x/(3*d*(d + e*x^2)^(3/2)) + (2*x)/
(3*d^2*Sqrt[d + e*x^2])))/(5*d)))/(c*d^2 - a*e^2)) + (c*((e*(Sqrt[c]*d - S
qrt[a]*e)*x)/(6*Sqrt[a]*Sqrt[c]*d*(Sqrt[c]*d + Sqrt[a]*e)*(d + e*x^2)^(3/2
)) - (e*(Sqrt[c]*d + Sqrt[a]*e)*x)/(6*Sqrt[a]*Sqrt[c]*d*(Sqrt[c]*d - Sqrt[
a]*e)*(d + e*x^2)^(3/2)) - (e*(5*Sqrt[c]*d - 2*Sqrt[a]*e)*(Sqrt[c]*d + Sqr
t[a]*e)*x)/(6*Sqrt[a]*Sqrt[c]*d^2*(Sqrt[c]*d - Sqrt[a]*e)^2*Sqrt[d + e*x^2
]) + (e*(Sqrt[c]*d - Sqrt[a]*e)*(5*Sqrt[c]*d + 2*Sqrt[a]*e)*x)/(6*Sqrt[a]*
Sqrt[c]*d^2*(Sqrt[c]*d + Sqrt[a]*e)^2*Sqrt[d + e*x^2]) + (Sqrt[c]*(Sqrt[c]
*d + Sqrt[a]*e)*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[a]*e]*x)/(a^(1/4)*Sqrt[d + e
*x^2])])/(2*a^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)^(5/2)) + (Sqrt[c]*(Sqrt[c]*d -
 Sqrt[a]*e)*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[a]*e]*x)/(a^(1/4)*Sqrt[d + e*x^
2])])/(2*a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)^(5/2))))/(c*d^2 - a*e^2)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 1487
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Simp[e^2
/(c*d^2 + a*e^2)   Int[(d + e*x^2)^q, x], x] + Simp[c/(c*d^2 + a*e^2)   Int
[(d + e*x^2)^(q + 1)*((d - e*x^2)/(a + c*x^4)), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$\frac{\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}\,c^{2}d^{4}\left(e\,x^{2}+d\right)^{\frac{5}{2}}\left(e^{3}a^{2}+3d^{2}eac+3\sqrt{d^{2}ac}\,a\,e^{2}+\sqrt{d^{2}ac}\,c\,d^{2}\right)\arctan\left(\frac{\sqrt{e\,x^{2}+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}}\right)+\sqrt{\left(-ae+\sqrt{d^{2}}}}{}$
default	Expression too large to display

input
```
int(1/(e*x^2+d)^(7/2)/(-c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)/(e*x^2+d)^(5/2)/(d^2*a*c)^(1/2)*(((a*
e+(d^2*a*c)^(1/2))*a)^(1/2)*c^2*d^4*(e*x^2+d)^(5/2)*(e^3*a^2+3*d^2*e*a*c+3
*(d^2*a*c)^(1/2)*a*e^2+(d^2*a*c)^(1/2)*c*d^2)*arctan((e*x^2+d)^(1/2)/x*a/(
(-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(c^2*d^4
*(e*x^2+d)^(5/2)*(e^3*a^2+3*d^2*e*a*c-3*(d^2*a*c)^(1/2)*a*e^2-(d^2*a*c)^(1
/2)*c*d^2)*arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(d^2*a*c)^(1/2))*a)^(1/2))+2*
(6*c^2*d^6+32/3*c^2*d^5*e*x^2-3*c*(-73/45*c*x^4+a)*e^2*d^4-4*a*c*d^3*e^3*x
^2+a*e^4*(-7/5*c*x^4+a)*d^2+4/3*a^2*d*e^5*x^2+8/15*a^2*e^6*x^4)*x*((a*e+(d
^2*a*c)^(1/2))*a)^(1/2)*(d^2*a*c)^(1/2)*e^2))/((a*e+(d^2*a*c)^(1/2))*a)^(1
/2)/(a*e^2-c*d^2)^3/d^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{7/2}(a-cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(7/2)/(-c*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^{7/2}(a-cx^4)}\,dx =$$
$$-\int \frac{1}{-ad^3\sqrt{d+ex^2}-3ad^2ex^2\sqrt{d+ex^2}-3ade^2x^4\sqrt{d+ex^2}-ae^3x^6\sqrt{d+ex^2}+cd^3x^4\sqrt{d+ex^2}+3cd^2e}$$

input
```
integrate(1/(e*x**2+d)**(7/2)/(-c*x**4+a),x)
```

output
```
-Integral(1/(-a*d**3*sqrt(d + e*x**2) - 3*a*d**2*e*x**2*sqrt(d + e*x**2) -
 3*a*d*e**2*x**4*sqrt(d + e*x**2) - a*e**3*x**6*sqrt(d + e*x**2) + c*d**3*
x**4*sqrt(d + e*x**2) + 3*c*d**2*e*x**6*sqrt(d + e*x**2) + 3*c*d*e**2*x**8
*sqrt(d + e*x**2) + c*e**3*x**10*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^{7/2}(a-cx^4)}\,dx = \int -\frac{1}{(cx^4-a)(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(7/2)/(-c*x^4+a),x, algorithm="maxima")
```

output
```
-integrate(1/((c*x^4 - a)*(e*x^2 + d)^(7/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{7/2}(a-cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(7/2)/(-c*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{7/2}(a-cx^4)}\,dx = \int \frac{1}{(a-c\,x^4)\,\left(e\,x^2+d\right)^{7/2}}\,dx$$

input
```
int(1/((a - c*x^4)*(d + e*x^2)^(7/2)),x)
```

output
```
int(1/((a - c*x^4)*(d + e*x^2)^(7/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^{7/2}(a-cx^4)}\,dx = \int \frac{}{\sqrt{e\,x^2+d}\,a\,d^3+3\sqrt{e\,x^2+d}\,a\,d^2e\,x^2+3\sqrt{e\,x^2+d}\,ad\,e^2x^4+\sqrt{e\,x^2+d}\,a}$$

input
```
int(1/(e*x^2+d)^(7/2)/(-c*x^4+a),x)
```

output
```
int(1/(sqrt(d + e*x**2)*a*d**3 + 3*sqrt(d + e*x**2)*a*d**2*e*x**2 + 3*sqrt
(d + e*x**2)*a*d*e**2*x**4 + sqrt(d + e*x**2)*a*e**3*x**6 - sqrt(d + e*x**
2)*c*d**3*x**4 - 3*sqrt(d + e*x**2)*c*d**2*e*x**6 - 3*sqrt(d + e*x**2)*c*d
*e**2*x**8 - sqrt(d + e*x**2)*c*e**3*x**10),x)
```

3.349 $\int \frac{\left(d+ex^2\right)^{9/2}}{\left(a-cx^4\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 327

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^{9/2}}{\left(a-cx^4\right)^2}\,dx &= \frac{3e^2\left(2cd^2+ae^2\right)x\sqrt{d+ex^2}}{4ac^2} \\ &+ \frac{de^3x^3\sqrt{d+ex^2}}{ac} + \frac{e^4x^5\sqrt{d+ex^2}}{4ac} + \frac{x\left(d+ex^2\right)^{9/2}}{4a\left(a-cx^4\right)} \\ &+ \frac{3\left(\sqrt{c}d-\sqrt{a}e\right)^{7/2}\left(\sqrt{c}d+2\sqrt{a}e\right)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}c^{5/2}} \\ &+ \frac{9de^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2} \\ &+ \frac{3\left(\sqrt{c}d-2\sqrt{a}e\right)\left(\sqrt{c}d+\sqrt{a}e\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}c^{5/2}}\end{aligned}$$

output
```
3/4*e^2*(a*e^2+2*c*d^2)*x*(e*x^2+d)^(1/2)/a/c^2+d*e^3*x^3*(e*x^2+d)^(1/2)/
a/c+1/4*e^4*x^5*(e*x^2+d)^(1/2)/a/c+1/4*x*(e*x^2+d)^(9/2)/a/(-c*x^4+a)+3/8
*(c^(1/2)*d-a^(1/2)*e)^(7/2)*(c^(1/2)*d+2*a^(1/2)*e)*arctan((c^(1/2)*d-a^(
1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(7/4)/c^(5/2)+9/2*d*e^(7/2)*arc
tanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c^2+3/8*(c^(1/2)*d-2*a^(1/2)*e)*(c^(1/2)*d
+a^(1/2)*e)^(7/2)*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^
(1/2))/a^(7/4)/c^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.04 (sec) , antiderivative size = 1004, normalized size of antiderivative = 3.07

$$\int \frac{\left(d+ex^2\right)^{9/2}}{\left(a-cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(d + e*x^2)^(9/2)/(a - c*x^4)^2,x]
```

output

```
((c*x*Sqrt[d + e*x^2]*(3*a^2*e^4 + c^2*d^3*(d + 4*e*x^2) + 2*a*c*e^2*(3*d^
2 + 2*d*e*x^2 - e^2*x^4)))/(a*(a - c*x^4)) - 18*c*d*e^(7/2)*Log[-(Sqrt[e]*
x) + Sqrt[d + e*x^2]] + 4*e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^
2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (45*c^2*d^4*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 81*a*c*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqr
t[e]*x*Sqrt[d + e*x^2] - #1] + 8*a^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqr
t[d + e*x^2] - #1] + 10*c^2*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x
^2] - #1]*#1 + 18*a*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2]
 - #1]*#1 + 5*c^2*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#
1^2 + a*c*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c
*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ] - (e^(3/2)*RootS
um[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4
 & , (3*c^3*d^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 168*
a*c^2*d^4*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 321*a^
2*c*d^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 32*a^3*e
^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 52*a*c^2*d^3*e^2*
Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 36*a^2*c*d*e^4*Lo
g[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 3*c^3*d^4*Log[d + 2
*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + 8*a*c^2*d^2*e^2*Log[d +
2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + a^2*c*e^4*Log[d + 2*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^2} \, dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^2}dx$$

input

```
Int[(d + e*x^2)^(9/2)/(a - c*x^4)^2,x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{3\left(\sqrt{\left(ae+\sqrt{d^2ac}\right)a}\,d\left(-c\,x^4+a\right)\left(\left(-\frac{7}{2}a^2e^4-ac\,d^2e^2+\frac{1}{2}c^2d^4\right)\sqrt{d^2ac}+ae\left(a^2e^4+4ac\,d^2e^2-c^2d^4\right)\right)\arctan\left(\frac{\sqrt{e\,x^2+}}{x\sqrt{\left(-ae+\sqrt{}\right.}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x^2+d)^(9/2)/(-c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/4/(d^2*a*c)^(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/((-a*e+(d^2*a*c)^(1/2
))*a)^(1/2)*(((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*d*(-c*x^4+a)*((-7/2*a^2*e^4-a
*c*d^2*e^2+1/2*c^2*d^4)*(d^2*a*c)^(1/2)+a*e*(a^2*e^4+4*a*c*d^2*e^2-c^2*d^4
))*arctan((e*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+(((7/2*a^2
*e^4+a*c*d^2*e^2-1/2*c^2*d^4)*(d^2*a*c)^(1/2)+a*e*(a^2*e^4+4*a*c*d^2*e^2-c
^2*d^4))*d*(-c*x^4+a)*arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(d^2*a*c)^(1/2))*a
)^(1/2))-((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(6*a*d*e^(7/2)*(-c*x^4+a)*arctanh
((e*x^2+d)^(1/2)/x/e^(1/2))+x*(e*x^2+d)^(1/2)*(a^2*e^4+2*(-1/3*e*x^2+d)*(e
*x^2+d)*c*e^2*a+1/3*c^2*d^3*(4*e*x^2+d)))*(d^2*a*c)^(1/2))*((-a*e+(d^2*a*c
)^(1/2))*a)^(1/2))/a/(-c*x^4+a)/c^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(9/2)/(-c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^2}\,dx = \int \frac{(d+ex^2)^{\frac{9}{2}}}{(-a+cx^4)^2}\,dx$$

input
```
integrate((e*x**2+d)**(9/2)/(-c*x**4+a)**2,x)
```

output
```
Integral((d + e*x**2)**(9/2)/(-a + c*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^2}\,dx = \int \frac{(ex^2+d)^{\frac{9}{2}}}{(cx^4-a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(9/2)/(-c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(9/2)/(c*x^4 - a)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^2}\,dx = \frac{\sqrt{ex^2+d}e^4x}{2\,c^2} - \frac{9\,de^{\frac{7}{2}}\log\left(\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2\right)}{4\,c^2}$$

$$+\frac{3\,(\sqrt{e}x-\sqrt{ex^2+d})^6c^2d^4e^{\frac{3}{2}}+8\,(\sqrt{e}x-\sqrt{ex^2+d})^6acd^2e^{\frac{7}{2}}+(\sqrt{e}x-\sqrt{ex^2+d})^6a^2e^{\frac{11}{2}}-6\,(\sqrt{e}x-}{(\,(\sqrt{e}x-\sqrt{ex^2+d})^8c-4\,(\sqrt{e}x-}$$

input
```
integrate((e*x^2+d)^(9/2)/(-c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/2*sqrt(e*x^2 + d)*e^4*x/c^2 - 9/4*d*e^(7/2)*log((sqrt(e)*x - sqrt(e*x^2
+ d))^2)/c^2 + (3*(sqrt(e)*x - sqrt(e*x^2 + d))^6*c^2*d^4*e^(3/2) + 8*(sqr
t(e)*x - sqrt(e*x^2 + d))^6*a*c*d^2*e^(7/2) + (sqrt(e)*x - sqrt(e*x^2 + d)
)^6*a^2*e^(11/2) - 6*(sqrt(e)*x - sqrt(e*x^2 + d))^4*c^2*d^5*e^(3/2) + 10*
(sqrt(e)*x - sqrt(e*x^2 + d))^4*a*c*d^3*e^(7/2) + 16*(sqrt(e)*x - sqrt(e*x
^2 + d))^4*a^2*d*e^(11/2) + 5*(sqrt(e)*x - sqrt(e*x^2 + d))^2*c^2*d^6*e^(3
/2) - (sqrt(e)*x - sqrt(e*x^2 + d))^2*a^2*d^2*e^(11/2) - 2*c^2*d^7*e^(3/2)
 - 2*a*c*d^5*e^(7/2))/(((sqrt(e)*x - sqrt(e*x^2 + d))^8*c - 4*(sqrt(e)*x -
 sqrt(e*x^2 + d))^6*c*d + 6*(sqrt(e)*x - sqrt(e*x^2 + d))^4*c*d^2 - 16*(sq
rt(e)*x - sqrt(e*x^2 + d))^4*a*e^2 - 4*(sqrt(e)*x - sqrt(e*x^2 + d))^2*c*d
^3 + c*d^4)*a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^2}\,dx = \int \frac{(e\,x^2+d)^{9/2}}{(a-c\,x^4)^2}\,dx$$

input
```
int((d + e*x^2)^(9/2)/(a - c*x^4)^2,x)
```

output
```
int((d + e*x^2)^(9/2)/(a - c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^2}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c^2x^8-2ac\,x^4+a^2}dx\right)d^4 + \left(\int \frac{\sqrt{e\,x^2+d}\,x^8}{c^2x^8-2ac\,x^4+a^2}dx\right)e^4 + 4\left(\int \frac{\sqrt{e\,x^2+d}\,x^6}{c^2x^8-2ac\,x^4+a^2}dx\right)d\,e^3 + 6\left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c^2x^8-2ac\,x^4+a^2}dx\right)d^2e^2 + 4\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c^2x^8-2ac\,x^4+a^2}dx\right)d^3e$$

input
```
int((e*x^2+d)^(9/2)/(-c*x^4+a)^2,x)
```

output
```
int(sqrt(d + e*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*d**4 + int((sqrt(d
 + e*x**2)*x**8)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*e**4 + 4*int((sqrt(d +
 e*x**2)*x**6)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*d*e**3 + 6*int((sqrt(d +
 e*x**2)*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*d**2*e**2 + 4*int((sqrt(
d + e*x**2)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*d**3*e
```

3.350 $\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 285

$$\begin{aligned}\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^2}\,dx &= \frac{3de^2x\sqrt{d+ex^2}}{4ac}+\frac{e^3x^3\sqrt{d+ex^2}}{4ac}+\frac{x(d+ex^2)^{7/2}}{4a\,(a-cx^4)} \\ &+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\left(3\sqrt{c}d+4\sqrt{a}e\right)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}c^2}+\frac{e^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2} \\ &+\frac{\left(3\sqrt{c}d-4\sqrt{a}e\right)\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}\text{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}c^2}\end{aligned}$$

output

```
3/4*d*e^2*x*(e*x^2+d)^(1/2)/a/c+1/4*e^3*x^3*(e*x^2+d)^(1/2)/a/c+1/4*x*(e*x
^2+d)^(7/2)/a/(-c*x^4+a)+1/8*(c^(1/2)*d-a^(1/2)*e)^(5/2)*(3*c^(1/2)*d+4*a^
(1/2)*e)*arctan((c^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(
7/4)/c^2+e^(7/2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c^2+1/8*(3*c^(1/2)*d-4
*a^(1/2)*e)*(c^(1/2)*d+a^(1/2)*e)^(5/2)*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2
)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(7/4)/c^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.78 (sec) , antiderivative size = 828, normalized size of antiderivative = 2.91

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^2}\,dx = \frac{\frac{2cx\sqrt{d+ex^2}\left(ae^2(3d+ex^2)+cd^2(d+3ex^2)\right)}{a(a-cx^4)} - 8e^{7/2}\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right) + 16e^{7/2}\text{RootSum}\Big[}{}$$

input
```
Integrate[(d + e*x^2)^(7/2)/(a - c*x^4)^2,x]
```

output
```
((2*c*x*Sqrt[d + e*x^2]*(a*e^2*(3*d + e*x^2) + c*d^2*(d + 3*e*x^2)))/(a*(a
 - c*x^4)) - 8*e^(7/2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]] + 16*e^(7/2)*Ro
otSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#
1^4 & , (17*c*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 16
*a*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 4*c*d^2*Log
[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 2*a*e^2*Log[d + 2*e*
x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*d*Log[d + 2*e*x^2 - 2*Sqrt[
e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*
#1^2 - c*#1^3) & ] - (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 -
16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (5*c^2*d^5*Log[d + 2*e*x^2 - 2*Sqr
t[e]*x*Sqrt[d + e*x^2] - #1] + 263*a*c*d^3*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]
*x*Sqrt[d + e*x^2] - #1] + 256*a^2*d*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqr
t[d + e*x^2] - #1] + 2*c^2*d^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1]*#1 + 70*a*c*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2]
 - #1]*#1 + 16*a^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]
*#1 + 5*c^2*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 +
 7*a*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*
d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/a)/(8*c^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a-cx^4\right)^2}dx$$
$$\Big\downarrow \ 1571$$
$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a-cx^4\right)^2}dx$$

input
```
Int[(d + e*x^2)^(7/2)/(a - c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$-\frac{9\Bigg(\sqrt{\left(ae+\sqrt{d^2ac}\right)a}\left(\frac{\left(-\frac{4}{3}a^2e^4-acd^2e^2+c^2d^4\right)\sqrt{d^2ac}}{3}+acd^2e\left(ae^2-\frac{5cd^2}{9}\right)\right)\left(-cx^4+a\right)\arctan\left(\frac{\sqrt{ex^2+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^2ac}\right)a}}\right)-}{}$
default	Expression too large to display

input
```
int((e*x^2+d)^(7/2)/(-c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-9/8/(d^2*a*c)^(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/((-a*e+(d^2*a*c)^(1/2
))*a)^(1/2)*(((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(1/3*(-4/3*a^2*e^4-a*c*d^2*e^
2+c^2*d^4)*(d^2*a*c)^(1/2)+a*c*d^2*e*(a*e^2-5/9*c*d^2))*(-c*x^4+a)*arctan(
(e*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+((1/3*(4/3*a^2*e^4+a
*c*d^2*e^2-c^2*d^4)*(d^2*a*c)^(1/2)+a*c*d^2*e*(a*e^2-5/9*c*d^2))*(-c*x^4+a
)*arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(d^2*a*c)^(1/2))*a)^(1/2))-8/9*(a*e^(7
/2)*(-c*x^4+a)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))+3/4*x*((d^2*e*x^2+1/3*d^
3)*c+e^2*(1/3*e*x^2+d)*a)*c*(e*x^2+d)^(1/2))*((a*e+(d^2*a*c)^(1/2))*a)^(1/
2)*(d^2*a*c)^(1/2))*((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))/a/(-c*x^4+a)/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3538 vs. $2(220) = 440$.

Time = 69.08 (sec) , antiderivative size = 7084, normalized size of antiderivative = 24.86

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^2}\,dx = \int \frac{(d+ex^2)^{\frac{7}{2}}}{(-a+cx^4)^2}\,dx$$

input
```
integrate((e*x**2+d)**(7/2)/(-c*x**4+a)**2,x)
```

output
```
Integral((d + e*x**2)**(7/2)/(-a + c*x**4)**2, x)
```

Maxima [F]

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a-cx^4\right)^2}\,dx=\int \frac{\left(ex^2+d\right)^{\frac{7}{2}}}{\left(cx^4-a\right)^2}\,dx$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(7/2)/(c*x^4 - a)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.36

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a-cx^4\right)^2}\,dx=-\frac{e^{\frac{7}{2}}\log\left(\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2\right)}{2\,c^2}+\frac{5\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^6c^2d^3e^{\frac{3}{2}}+7\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^6acde^{\frac{7}{2}}-9\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^4c^2d^4e^{\frac{3}{2}}+21\left(\sqrt{e}x-\right.}{2\left(\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^8c-4\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^6cd+6\left(\sqrt{e}\right.\right.}$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/2*e^(7/2)*log((sqrt(e)*x - sqrt(e*x^2 + d))^2)/c^2 + 1/2*(5*(sqrt(e)*x
- sqrt(e*x^2 + d))^6*c^2*d^3*e^(3/2) + 7*(sqrt(e)*x - sqrt(e*x^2 + d))^6*a
*c*d*e^(7/2) - 9*(sqrt(e)*x - sqrt(e*x^2 + d))^4*c^2*d^4*e^(3/2) + 21*(sqr
t(e)*x - sqrt(e*x^2 + d))^4*a*c*d^2*e^(7/2) + 8*(sqrt(e)*x - sqrt(e*x^2 +
d))^4*a^2*e^(11/2) + 7*(sqrt(e)*x - sqrt(e*x^2 + d))^2*c^2*d^5*e^(3/2) - 3
*(sqrt(e)*x - sqrt(e*x^2 + d))^2*a*c*d^3*e^(7/2) - 3*c^2*d^6*e^(3/2) - a*c
*d^4*e^(7/2))/(((sqrt(e)*x - sqrt(e*x^2 + d))^8*c - 4*(sqrt(e)*x - sqrt(e*
x^2 + d))^6*c*d + 6*(sqrt(e)*x - sqrt(e*x^2 + d))^4*c*d^2 - 16*(sqrt(e)*x
- sqrt(e*x^2 + d))^4*a*e^2 - 4*(sqrt(e)*x - sqrt(e*x^2 + d))^2*c*d^3 + c*d
^4)*a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^2}\,dx = \int \frac{(e\,x^2+d)^{7/2}}{(a-c\,x^4)^2}\,dx$$

input
```
int((d + e*x^2)^(7/2)/(a - c*x^4)^2,x)
```

output
```
int((d + e*x^2)^(7/2)/(a - c*x^4)^2, x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^2}\,dx &= \left(\int \frac{\sqrt{e\,x^2+d}}{c^2x^8-2ac\,x^4+a^2}dx\right)d^3\\&+\left(\int \frac{\sqrt{e\,x^2+d}\,x^6}{c^2x^8-2ac\,x^4+a^2}dx\right)e^3+3\left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c^2x^8-2ac\,x^4+a^2}dx\right)d\,e^2\\&+3\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c^2x^8-2ac\,x^4+a^2}dx\right)d^2e\end{aligned}$$

input
```
int((e*x^2+d)^(7/2)/(-c*x^4+a)^2,x)
```

output
```
int(sqrt(d + e*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*d**3 + int((sqrt(d
 + e*x**2)*x**6)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*e**3 + 3*int((sqrt(d +
 e*x**2)*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*d*e**2 + 3*int((sqrt(d +
 e*x**2)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*d**2*e
```

3.351 $\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 233

$$\begin{aligned}\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^2}\,dx &= \frac{e^2x\sqrt{d+ex^2}}{4ac}+\frac{x(d+ex^2)^{5/2}}{4a\,(a-cx^4)} \\ &+ \frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\left(3\sqrt{c}d+2\sqrt{a}e\right)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}c^{3/2}} \\ &+ \frac{\left(3\sqrt{c}d-2\sqrt{a}e\right)\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}c^{3/2}}\end{aligned}$$

output

```
1/4*e^2*x*(e*x^2+d)^(1/2)/a/c+1/4*x*(e*x^2+d)^(5/2)/a/(-c*x^4+a)+1/8*(c^(1
/2)*d-a^(1/2)*e)^(3/2)*(3*c^(1/2)*d+2*a^(1/2)*e)*arctan((c^(1/2)*d-a^(1/2)
*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(7/4)/c^(3/2)+1/8*(3*c^(1/2)*d-2*a^
(1/2)*e)*(c^(1/2)*d+a^(1/2)*e)^(3/2)*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x
/a^(1/4)/(e*x^2+d)^(1/2))/a^(7/4)/c^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 711, normalized size of antiderivative = 3.05

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^2}\,dx=\frac{\sqrt{d+ex^2}\left(-cd^2x-ae^2x-2cdex^3\right)}{4ac\left(-a+cx^4\right)}$$

$$+\frac{e^{7/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\ \frac{49cd^2\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)}{}\right.}{2c^2}$$

$$-\frac{e^{3/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\ \frac{2c^2d^4\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)}{}\right.}{}$$

input
```
Integrate[(d + e*x^2)^(5/2)/(a - c*x^4)^2,x]
```

output
```
(Sqrt[d + e*x^2]*(-(c*d^2*x) - a*e^2*x - 2*c*d*e*x^3))/(4*a*c*(-a + c*x^4)
) + (e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4
*c*d*#1^3 + c*#1^4 & , (49*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*
x^2] - #1] + 16*a*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]
+ 10*c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*Log[d
+ 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 +
8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(2*c^2) - (e^(3/2)*RootSum[c*d^4 -
4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (2*c^2
*d^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 97*a*c*d^2*e^2*
Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 32*a^2*e^4*Log[d + 2
*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 2*c^2*d^3*Log[d + 2*e*x^2 - 2
*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 20*a*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqr
t[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 2*c^2*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*
Sqrt[d + e*x^2] - #1]*#1^2 + a*c*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d
+ e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^
3) & ])/(4*a*c^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^2}dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^2}dx$$

input
```
Int[(d + e*x^2)^(5/2)/(a - c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$\frac{-d\sqrt{\left(ae+\sqrt{d^2ac}\right)a}\,\left(-c\,x^4+a\right)\left(\frac{\left(-a\,e^2+3c\,d^2\right)\sqrt{d^2ac}}{2}+ae\left(a\,e^2-2c\,d^2\right)\right)\arctan\left(\frac{\sqrt{e\,x^2+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^2ac}\right)a}}\right)+\sqrt{\left(-ae+\sqrt{d^2ac}\right.}}{4\sqrt{d^2ac}\,\sqrt{\left(-ae\right.}}$
default	Expression too large to display

input
```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/4/(d^2*a*c)^(1/2)*(-d*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(-c*x^4+a)*(1/2*(-
a*e^2+3*c*d^2)*(d^2*a*c)^(1/2)+a*e*(a*e^2-2*c*d^2))*arctan((e*x^2+d)^(1/2)
/x*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(-
d*(1/2*(a*e^2-3*c*d^2)*(d^2*a*c)^(1/2)+a*e*(a*e^2-2*c*d^2))*(-c*x^4+a)*arc
tanh((e*x^2+d)^(1/2)/x*a/((a*e+(d^2*a*c)^(1/2))*a)^(1/2))+x*(a*e^2+c*d*(2*
e*x^2+d))*(e*x^2+d)^(1/2)*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(d^2*a*c)^(1/2))
)/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/a/c/(-c
*x^4+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. $2(174) = 348$.

Time = 10.33 (sec) , antiderivative size = 1935, normalized size of antiderivative = 8.30

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^2,x, algorithm="fricas")
```

output

```
1/32*((a*c^2*x^4 - a^2*c)*sqrt((15*c^2*d^4*e - 15*a*c*d^2*e^3 + 4*a^2*e^5
+ a^3*c^3*sqrt((81*c^2*d^10 - 90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)))
/(a^3*c^3))*log(-(81*c^3*d^10 - 162*a*c^2*d^8*e^2 + 101*a^2*c*d^6*e^4 - 20
*a^3*d^4*e^6 + (9*a^3*c^4*d^5 - 13*a^4*c^3*d^3*e^2 + 4*a^5*c^2*d*e^4)*x^2*
sqrt((81*c^2*d^10 - 90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)) + 2*(81*c^
3*d^9*e - 162*a*c^2*d^7*e^3 + 101*a^2*c*d^5*e^5 - 20*a^3*d^3*e^7)*x^2 + 2*
sqrt(e*x^2 + d)*((3*a^5*c^4*d^2 - 2*a^6*c^3*e^2)*x*sqrt((81*c^2*d^10 - 90*
a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)) - (9*a^2*c^3*d^6*e - 5*a^3*c^2*d^
4*e^3)*x)*sqrt((15*c^2*d^4*e - 15*a*c*d^2*e^3 + 4*a^2*e^5 + a^3*c^3*sqrt((
81*c^2*d^10 - 90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)))/(a^3*c^3)))/x^2
) - (a*c^2*x^4 - a^2*c)*sqrt((15*c^2*d^4*e - 15*a*c*d^2*e^3 + 4*a^2*e^5 +
a^3*c^3*sqrt((81*c^2*d^10 - 90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)))/(
a^3*c^3))*log(-(81*c^3*d^10 - 162*a*c^2*d^8*e^2 + 101*a^2*c*d^6*e^4 - 20*a
^3*d^4*e^6 + (9*a^3*c^4*d^5 - 13*a^4*c^3*d^3*e^2 + 4*a^5*c^2*d*e^4)*x^2*sq
rt((81*c^2*d^10 - 90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)) + 2*(81*c^3*
d^9*e - 162*a*c^2*d^7*e^3 + 101*a^2*c*d^5*e^5 - 20*a^3*d^3*e^7)*x^2 - 2*sq
rt(e*x^2 + d)*((3*a^5*c^4*d^2 - 2*a^6*c^3*e^2)*x*sqrt((81*c^2*d^10 - 90*a*
c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)) - (9*a^2*c^3*d^6*e - 5*a^3*c^2*d^4*
e^3)*x)*sqrt((15*c^2*d^4*e - 15*a*c*d^2*e^3 + 4*a^2*e^5 + a^3*c^3*sqrt((81
*c^2*d^10 - 90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)))/(a^3*c^3)))/x^...
```

Sympy [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^2}\,dx = \int \frac{(d+ex^2)^{\frac{5}{2}}}{(-a+cx^4)^2}\,dx$$

input

```
integrate((e*x**2+d)**(5/2)/(-c*x**4+a)**2,x)
```

output

```
Integral((d + e*x**2)**(5/2)/(-a + c*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^2}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}}{(cx^4-a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(5/2)/(c*x^4 - a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^2}\,dx = \int \frac{(e\,x^2+d)^{5/2}}{(a-c\,x^4)^2}\,dx$$

input
```
int((d + e*x^2)^(5/2)/(a - c*x^4)^2,x)
```

output
```
int((d + e*x^2)^(5/2)/(a - c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{\left(d+e x^2\right)^{5/2}}{\left(a-c x^4\right)^2}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c^2 x^8-2 a c\,x^4+a^2}dx\right) d^2 + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c^2 x^8-2 a c\,x^4+a^2}dx\right) e^2 + 2\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c^2 x^8-2 a c\,x^4+a^2}dx\right) d e$$

input

```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^2,x)
```

output

```
int(sqrt(d + e*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*d**2 + int((sqrt(d
 + e*x**2)*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*e**2 + 2*int((sqrt(d +
 e*x**2)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*d*e
```

3.352 $\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 176

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^2}\,dx = \frac{x(d+ex^2)^{3/2}}{4a\,(a-cx^4)} + \frac{3d\sqrt{\sqrt{c}d-\sqrt{a}e}\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}\sqrt{c}} + \frac{3d\sqrt{\sqrt{c}d+\sqrt{a}e}\,\mathrm{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}\sqrt{c}}$$

output

```
1/4*x*(e*x^2+d)^(3/2)/a/(-c*x^4+a)+3/8*d*(c^(1/2)*d-a^(1/2)*e)^(1/2)*arcta
n((c^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(7/4)/c^(1/2)+3
/8*d*(c^(1/2)*d+a^(1/2)*e)^(1/2)*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(
1/4)/(e*x^2+d)^(1/2))/a^(7/4)/c^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.93

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^2}\,dx = -\frac{\sqrt{d+ex^2}(-dx-ex^3)}{4a\,(a-cx^4)}$$
$$+\frac{2e^{7/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\frac{8d\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)+}{cd^3-3cd^2\#1+8ae^2\#}\right.}{c}$$
$$-\frac{e^{3/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\frac{3cd^3\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)}{}\right.}{}$$

input
```
Integrate[(d + e*x^2)^(3/2)/(a - c*x^4)^2,x]
```

output
```
-1/4*(Sqrt[d + e*x^2]*(-(d*x) - e*x^3))/(a*(a - c*x^4)) + (2*e^(7/2)*RootS
um[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4
 & , (8*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + Log[d + 2*
e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^
2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/c - (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 +
 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (3*c*d^3*Log[d + 2
*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 128*a*d*e^2*Log[d + 2*e*x^2 -
 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 6*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x
*Sqrt[d + e*x^2] - #1]*#1 + 16*a*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d
+ e*x^2] - #1]*#1 + 3*c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] -
#1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(8*
a*c)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^2}\,dx$$

$$\Big\downarrow 1571$$

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a-cx^4\right)^2}dx$$

input
```
Int[(d + e*x^2)^(3/2)/(a - c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$-\frac{d^{2} \left(-\frac{x \left(e \,x^{2}+d \right)^{\frac{3}{2}}}{d^{2} \left(-c \,x^{4}+a \right)}+\frac{3 \left(-a e +\sqrt{d^{2} a c}\right) \arctan \left(\frac{\sqrt{e \,x^{2}+d}\, a}{x \sqrt{\left(-a e +\sqrt{d^{2} a c}\right) a}}\right)}{2 \sqrt{d^{2} a c}\, \sqrt{\left(-a e +\sqrt{d^{2} a c}\right) a}}-\frac{3 \left(a e +\sqrt{d^{2} a c}\right) \operatorname{arctanh}\left(\frac{\sqrt{e \,x^{2}+d}\, a}{x \sqrt{\left(a e +\sqrt{d^{2} a c}\right) a}}\right)}{2 \sqrt{d^{2} a c}\, \sqrt{\left(a e +\sqrt{d^{2} a c}\right) a}}\right)}{4 a}$	176
default	Expression too large to display	7828

input
```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*d^2/a*(-1/d^2/(-c*x^4+a)*x*(e*x^2+d)^(3/2)+3/2*(-a*e+(d^2*a*c)^(1/2))
/(d^2*a*c)^(1/2)/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)*arctan((e*x^2+d)^(1/2)/x
*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))-3/2*(a*e+(d^2*a*c)^(1/2))/(d^2*a*c)^(
1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(d^
2*a*c)^(1/2))*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(129) = 258$.

Time = 0.74 (sec) , antiderivative size = 641, normalized size of antiderivative = 3.64

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a-cx^4\right)^2}\,dx = \frac{3\left(acx^4-a^2\right)\sqrt{\frac{a^3c\sqrt{\frac{d^6}{a^7c}}+d^2e}{a^3c}}\log\left(\frac{27\left(a^3cd^2x^2\sqrt{\frac{d^6}{a^7c}}+2\sqrt{ex^2+d}a^5cx\sqrt{\frac{d^6}{a^7c}}\sqrt{\frac{a^3c\sqrt{\frac{d^6}{a^7c}}+d^2e}{a^3c}}+2d^4ex^2+d\right.}{x^2}\right.}{}$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/32*(3*(a*c*x^4 - a^2)*sqrt((a^3*c*sqrt(d^6/(a^7*c)) + d^2*e)/(a^3*c))*lo
g(27*(a^3*c*d^2*x^2*sqrt(d^6/(a^7*c)) + 2*sqrt(e*x^2 + d)*a^5*c*x*sqrt(d^6
/(a^7*c))*sqrt((a^3*c*sqrt(d^6/(a^7*c)) + d^2*e)/(a^3*c)) + 2*d^4*e*x^2 +
d^5)/x^2) - 3*(a*c*x^4 - a^2)*sqrt((a^3*c*sqrt(d^6/(a^7*c)) + d^2*e)/(a^3*
c))*log(27*(a^3*c*d^2*x^2*sqrt(d^6/(a^7*c)) - 2*sqrt(e*x^2 + d)*a^5*c*x*sq
rt(d^6/(a^7*c))*sqrt((a^3*c*sqrt(d^6/(a^7*c)) + d^2*e)/(a^3*c)) + 2*d^4*e*
x^2 + d^5)/x^2) + 3*(a*c*x^4 - a^2)*sqrt(-(a^3*c*sqrt(d^6/(a^7*c)) - d^2*e
)/(a^3*c))*log(-27*(a^3*c*d^2*x^2*sqrt(d^6/(a^7*c)) + 2*sqrt(e*x^2 + d)*a^
5*c*x*sqrt(d^6/(a^7*c))*sqrt(-(a^3*c*sqrt(d^6/(a^7*c)) - d^2*e)/(a^3*c)) -
 2*d^4*e*x^2 - d^5)/x^2) - 3*(a*c*x^4 - a^2)*sqrt(-(a^3*c*sqrt(d^6/(a^7*c)
) - d^2*e)/(a^3*c))*log(-27*(a^3*c*d^2*x^2*sqrt(d^6/(a^7*c)) - 2*sqrt(e*x^
2 + d)*a^5*c*x*sqrt(d^6/(a^7*c))*sqrt(-(a^3*c*sqrt(d^6/(a^7*c)) - d^2*e)/(
a^3*c)) - 2*d^4*e*x^2 - d^5)/x^2) - 8*(e*x^3 + d*x)*sqrt(e*x^2 + d))/(a*c*
x^4 - a^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a-cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)/(-c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{(cx^4-a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/(c*x^4 - a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^2}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{(a-c\,x^4)^2}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(a - c*x^4)^2,x)
```

output
```
int((d + e*x^2)^(3/2)/(a - c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^2}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c^2x^8-2ac\,x^4+a^2}dx\right)d+\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c^2x^8-2ac\,x^4+a^2}dx\right)e$$

input
```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^2,x)
```

output
```
int(sqrt(d + e*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*d + int((sqrt(d +
e*x**2)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*e
```

3.353 $\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 208

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^2} \, dx = \frac{x\sqrt{d+ex^2}}{4a(a-cx^4)} + \frac{\left(3\sqrt{c}d-2\sqrt{a}e\right)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{a}e}} + \frac{\left(3\sqrt{c}d+2\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{a}e}}$$

output
```
1/4*x*(e*x^2+d)^(1/2)/a/(-c*x^4+a)+1/8*(3*c^(1/2)*d-2*a^(1/2)*e)*arctan((c
^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(7/4)/c^(1/2)/(c^(1
/2)*d-a^(1/2)*e)^(1/2)+1/8*(3*c^(1/2)*d+2*a^(1/2)*e)*arctanh((c^(1/2)*d+a^
(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(7/4)/c^(1/2)/(c^(1/2)*d+a^(1/
2)*e)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^2}\,dx = \frac{x\sqrt{d+ex^2}}{4a\,(a-cx^4)} + \frac{8e^{7/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\ \frac{\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#\right.}{cd^3-3cd^2\#1+8ae^2\#1+3cd\#1^2}\right.}{c} - \frac{e^{3/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\ \frac{cd^2\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right.}{}\right.}{4}$$

input
```
Integrate[Sqrt[d + e*x^2]/(a - c*x^4)^2,x]
```

output
```
(x*Sqrt[d + e*x^2])/(4*a*(a - c*x^4)) + (8*e^(7/2)*RootSum[c*d^4 - 4*c*d^3
*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , Log[d + 2*e*x
^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 +
3*c*d*#1^2 - c*#1^3) & ])/c - (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^
2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (c*d^2*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 32*a*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x
*Sqrt[d + e*x^2] - #1] + 4*c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1]*#1 + c*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/
(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(4*a*c)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^2}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^2}dx$$

input

```
Int[Sqrt[d + e*x^2]/(a - c*x^4)^2,x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$-\frac{d\left(-\frac{\sqrt{e\,x^{2}+d}\,x}{d\left(-c\,x^{4}+a\right)}+\frac{\left(-2ae+3\sqrt{d^{2}ac}\right)\arctan\left(\frac{\sqrt{e\,x^{2}+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}}\right)}{2\sqrt{d^{2}ac}\,\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}}-\frac{\left(2ae+3\sqrt{d^{2}ac}\right)\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}\,a}{x\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}}\right)}{2\sqrt{d^{2}ac}\,\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}}\right)}{4a}$	179
default	Expression too large to display	4714

input

```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/4*d/a*(-(e*x^2+d)^(1/2)/d*x/(-c*x^4+a)+1/2*(-2*a*e+3*(d^2*a*c)^(1/2))/(
d^2*a*c)^(1/2)/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)*arctan((e*x^2+d)^(1/2)/x*a
/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))-1/2*(2*a*e+3*(d^2*a*c)^(1/2))/(d^2*a*c)
^(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(
d^2*a*c)^(1/2))*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2399 vs. $2(153) = 306$.

Time = 4.06 (sec) , antiderivative size = 2399, normalized size of antiderivative = 11.53

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^2,x, algorithm="fricas")
```

output

```
1/32*((a*c*x^4 - a^2)*sqrt((3*c*d^2*e - 4*a*e^3 + (a^3*c^2*d^2 - a^4*c*e^2
)*sqrt((81*c^2*d^6 - 144*a*c*d^4*e^2 + 64*a^2*d^2*e^4)/(a^7*c^3*d^4 - 2*a^
8*c^2*d^2*e^2 + a^9*c*e^4)))/(a^3*c^2*d^2 - a^4*c*e^2))*log((81*c^2*d^6 -
108*a*c*d^4*e^2 + 32*a^2*d^2*e^4 + (9*a^3*c^3*d^5 - 13*a^4*c^2*d^3*e^2 + 4
*a^5*c*d*e^4)*x^2*sqrt((81*c^2*d^6 - 144*a*c*d^4*e^2 + 64*a^2*d^2*e^4)/(a^
7*c^3*d^4 - 2*a^8*c^2*d^2*e^2 + a^9*c*e^4)) + 2*(81*c^2*d^5*e - 108*a*c*d^
3*e^3 + 32*a^2*d*e^5)*x^2 + 2*sqrt(e*x^2 + d)*((3*a^5*c^3*d^4 - 5*a^6*c^2*
d^2*e^2 + 2*a^7*c*e^4)*x*sqrt((81*c^2*d^6 - 144*a*c*d^4*e^2 + 64*a^2*d^2*e
^4)/(a^7*c^3*d^4 - 2*a^8*c^2*d^2*e^2 + a^9*c*e^4)) + (9*a^2*c^2*d^4*e - 8*
a^3*c*d^2*e^3)*x)*sqrt((3*c*d^2*e - 4*a*e^3 + (a^3*c^2*d^2 - a^4*c*e^2)*sq
rt((81*c^2*d^6 - 144*a*c*d^4*e^2 + 64*a^2*d^2*e^4)/(a^7*c^3*d^4 - 2*a^8*c^
2*d^2*e^2 + a^9*c*e^4)))/(a^3*c^2*d^2 - a^4*c*e^2)))/x^2) - (a*c*x^4 - a^2
)*sqrt((3*c*d^2*e - 4*a*e^3 + (a^3*c^2*d^2 - a^4*c*e^2)*sqrt((81*c^2*d^6 -
 144*a*c*d^4*e^2 + 64*a^2*d^2*e^4)/(a^7*c^3*d^4 - 2*a^8*c^2*d^2*e^2 + a^9*
c*e^4)))/(a^3*c^2*d^2 - a^4*c*e^2))*log((81*c^2*d^6 - 108*a*c*d^4*e^2 + 32
*a^2*d^2*e^4 + (9*a^3*c^3*d^5 - 13*a^4*c^2*d^3*e^2 + 4*a^5*c*d*e^4)*x^2*sq
rt((81*c^2*d^6 - 144*a*c*d^4*e^2 + 64*a^2*d^2*e^4)/(a^7*c^3*d^4 - 2*a^8*c^
2*d^2*e^2 + a^9*c*e^4)) + 2*(81*c^2*d^5*e - 108*a*c*d^3*e^3 + 32*a^2*d*e^5
)*x^2 - 2*sqrt(e*x^2 + d)*((3*a^5*c^3*d^4 - 5*a^6*c^2*d^2*e^2 + 2*a^7*c*e^
4)*x*sqrt((81*c^2*d^6 - 144*a*c*d^4*e^2 + 64*a^2*d^2*e^4)/(a^7*c^3*d^4 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(1/2)/(-c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^2}\,dx = \int \frac{\sqrt{ex^2+d}}{(cx^4-a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/(c*x^4 - a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^2}\,dx = \int \frac{\sqrt{e\,x^2+d}}{\left(a-c\,x^4\right)^2}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a - c*x^4)^2,x)
```

output
```
int((d + e*x^2)^(1/2)/(a - c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^2}\,dx = \int \frac{\sqrt{e\,x^2+d}}{c^2x^8-2ac\,x^4+a^2}dx$$

input
```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^2,x)
```

output
```
int(sqrt(d + e*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)
```

3.354 $\int \frac{1}{\sqrt{d+ex^2}(a-cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 221

$$\begin{aligned}\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^2}\,dx &= \frac{cx(d-ex^2)\sqrt{d+ex^2}}{4a\,(cd^2-ae^2)\,(a-cx^4)} \\ &\quad+ \frac{\left(3\sqrt{c}d-4\sqrt{a}e\right)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}} \\ &\quad+ \frac{\left(3\sqrt{c}d+4\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}}\end{aligned}$$

output

```
1/4*c*x*(-e*x^2+d)*(e*x^2+d)^(1/2)/a/(-a*e^2+c*d^2)/(-c*x^4+a)+1/8*(3*c^(1
/2)*d-4*a^(1/2)*e)*arctan((c^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^
(1/2))/a^(7/4)/(c^(1/2)*d-a^(1/2)*e)^(3/2)+1/8*(3*c^(1/2)*d+4*a^(1/2)*e)*a
rctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(7/4)/(c^(
1/2)*d+a^(1/2)*e)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^2}\,dx$$

$$= \frac{-2cx(d-ex^2)\sqrt{d+ex^2}+e^{3/2}(a-cx^4)\,\mathrm{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-4cd\#1^3+\right.}{}$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a - c*x^4)^2),x]
```

output
```
(-2*c*x*(d - e*x^2)*Sqrt[d + e*x^2] + e^(3/2)*(a - c*x^4)*RootSum[c*d^4 -
4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (c*d^3
*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 10*c*d^2*Log[d + 2*
e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 16*a*e^2*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*S
qrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 -
 c*#1^3) & ])/(8*a*(-(c*d^2) + a*e^2)*(a - c*x^4))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^4)^2\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{1}{(a-cx^4)^2\sqrt{d+ex^2}}dx$$

input
```
Int[1/(Sqrt[d + e*x^2]*(a - c*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$-\frac{\frac{\left(-c\,x^{4}+a\right)\left(\left(4a\,e^{2}-3c\,d^{2}\right)\sqrt{d^{2}ac}+d^{2}eac\right)\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}\,\arctan\left(\frac{\sqrt{e\,x^{2}+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}}\right)}{2}+\Bigl(\frac{\left(\left(-4a\,e^{2}+3c\,d^{2}\right)\sqrt{d^{2}ac}+d^{2}eac\right)\bigl(-}{}}{4\sqrt{d^{2}ac}\,\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}\,\sqrt{\left(ae+\sqrt{d^{2}}\right.}}$
default	Expression too large to display

input
```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(1/2*(-c*x^4+a)*((4*a*e^2-3*c*d^2)*(d^2*a*c)^(1/2)+d^2*e*a*c)*((a*e+(
d^2*a*c)^(1/2))*a)^(1/2)*arctan((e*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c)^(1/2)
)*a)^(1/2))+(1/2*((-4*a*e^2+3*c*d^2)*(d^2*a*c)^(1/2)+d^2*e*a*c)*(-c*x^4+a)
*arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(d^2*a*c)^(1/2))*a)^(1/2))+x*(-e*x^2+d)
*(e*x^2+d)^(1/2)*c*(d^2*a*c)^(1/2)*((a*e+(d^2*a*c)^(1/2))*a)^(1/2))*((-a*e
+(d^2*a*c)^(1/2))*a)^(1/2))/(d^2*a*c)^(1/2)/((-a*e+(d^2*a*c)^(1/2))*a)^(1/
2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/a/(a*e^2-c*d^2)/(-c*x^4+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4804 vs. $2(171) = 342$.

Time = 25.17 (sec) , antiderivative size = 4804, normalized size of antiderivative = 21.74

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(-c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^2}\,dx = \int \frac{1}{(cx^4-a)^2\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 - a)^2*sqrt(e*x^2 + d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^2}\,dx = \int \frac{1}{(a-c\,x^4)^2\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a - c*x^4)^2*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a - c*x^4)^2*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^2}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,a^2-2\sqrt{e\,x^2+d}\,ac\,x^4+\sqrt{e\,x^2+d}\,c^2x^8}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^2,x)
```

output
```
int(1/(sqrt(d + e*x**2)*a**2 - 2*sqrt(d + e*x**2)*a*c*x**4 + sqrt(d + e*x*
*2)*c**2*x**8),x)
```

3.355 $\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 280

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^2}\,dx = \frac{e^2(cd^2+2ae^2)\,x}{2ad\,(cd^2-ae^2)^2\sqrt{d+ex^2}} + \frac{cx(d-ex^2)}{4a\,(cd^2-ae^2)\sqrt{d+ex^2}\,(a-cx^4)} + \frac{3\sqrt{c}(\sqrt{c}d-2\sqrt{a}e)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}} + \frac{3\sqrt{c}(\sqrt{c}d+2\sqrt{a}e)\,\mathrm{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}}$$

output
```
1/2*e^2*(2*a*e^2+c*d^2)*x/a/d/(-a*e^2+c*d^2)^2/(e*x^2+d)^(1/2)+1/4*c*x*(-e
*x^2+d)/a/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)+3/8*c^(1/2)*(c^(1/2)*d
-2*a^(1/2)*e)*arctan((c^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2)
)/a^(7/4)/(c^(1/2)*d-a^(1/2)*e)^(5/2)+3/8*c^(1/2)*(c^(1/2)*d+2*a^(1/2)*e)*
arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(7/4)/(c^
(1/2)*d+a^(1/2)*e)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.75 (sec) , antiderivative size = 708, normalized size of antiderivative = 2.53

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^2}\,dx = \frac{x(-4a^2e^4-ace^2(d^2+dex^2-4e^2x^4)+c^2d^2(-d^2+dex^2+2e^2x^4))+2ade}{}$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(a - c*x^4)^2),x]
```

output
```
(x*(-4*a^2*e^4 - a*c*e^2*(d^2 + d*e*x^2 - 4*e^2*x^4) + c^2*d^2*(-d^2 + d*e
*x^2 + 2*e^2*x^4)) + 2*a*d*e^(7/2)*Sqrt[d + e*x^2]*(a - c*x^4)*RootSum[c*d
^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (
17*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 16*a*e^2*Lo
g[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 6*c*d*Log[d + 2*e*x^2
- 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*S
qrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 -
 c*#1^3) & ] + d*e^(3/2)*Sqrt[d + e*x^2]*(a - c*x^4)*RootSum[c*d^4 - 4*c*d
^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (-31*a*c*d^
2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 32*a^2*e^4*Log
[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 6*c^2*d^3*Log[d + 2*e*x
^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 12*a*c*d*e^2*Log[d + 2*e*x^2 -
 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + a*c*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e
]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#
1^2 - c*#1^3) & ])/(4*a*d*(c*d^2 - a*e^2)^2*Sqrt[d + e*x^2]*(-a + c*x^4))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^4)^2(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1571}$$

$$\int \frac{1}{\left(a-cx^4\right)^2 \left(d+ex^2\right)^{3/2}} dx$$

input
```
Int[1/((d + e*x^2)^(3/2)*(a - c*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.32

method	result
pseudoelliptic	$\frac{\frac{3 d^{2} \left(\frac{\left(3 a e^{2}-c d^{2}\right) \sqrt{d^{2} a c}}{2}+e^{3} a^{2}\right) \left(-c x^{4}+a \right) c \sqrt{\left(a e +\sqrt{d^{2} a c}\right) a}\, \sqrt{e x^{2}+d}\, \arctan \left(\frac{\sqrt{e x^{2}+d}\, a}{x \sqrt{\left(-a e +\sqrt{d^{2} a c}\right) a}}\right)}{4}+\frac{3 \sqrt{\left(-a e +\sqrt{d^{2} a c}\right) a}\, \left(d^{2} \left(}{}}{\left(-c x^{4}+a \right) \left(a e^{2}-c}$
default	Expression too large to display

input
```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/4/(e*x^2+d)^(1/2)/(d^2*a*c)^(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(d^2*(
1/2*(3*a*e^2-c*d^2)*(d^2*a*c)^(1/2)+e^3*a^2)*(-c*x^4+a)*c*((a*e+(d^2*a*c)^
(1/2))*a)^(1/2)*(e*x^2+d)^(1/2)*arctan((e*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c
)^(1/2))*a)^(1/2))+((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(d^2*(1/2*(-3*a*e^2+c*
d^2)*(d^2*a*c)^(1/2)+e^3*a^2)*(-c*x^4+a)*c*(e*x^2+d)^(1/2)*arctanh((e*x^2+
d)^(1/2)/x*a/((a*e+(d^2*a*c)^(1/2))*a)^(1/2))+4/3*x*(a^2*e^4+1/4*c*e^2*(-4
*e^2*x^4+d*e*x^2+d^2)*a+1/4*c^2*d^2*(e*x^2+d)*(-2*e*x^2+d))*((a*e+(d^2*a*c
)^(1/2))*a)^(1/2)*(d^2*a*c)^(1/2)))/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)/(-c*x
^4+a)/(a*e^2-c*d^2)^2/a/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7859 vs. $2(222) = 444$.

Time = 171.48 (sec) , antiderivative size = 7859, normalized size of antiderivative = 28.07

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(-c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^2}\,dx = \int \frac{1}{(cx^4-a)^2(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 - a)^2*(e*x^2 + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^2}\,dx = \int \frac{1}{(a-c\,x^4)^2\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(1/((a - c*x^4)^2*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a - c*x^4)^2*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a-cx^4)^2}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,a^2d+\sqrt{e\,x^2+d}\,a^2e\,x^2-2\sqrt{e\,x^2+d}\,acd\,x^4-2\sqrt{e\,x^2+d}\,ace}$$

input
```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^2,x)
```

output
```
int(1/(sqrt(d + e*x**2)*a**2*d + sqrt(d + e*x**2)*a**2*e*x**2 - 2*sqrt(d +
 e*x**2)*a*c*d*x**4 - 2*sqrt(d + e*x**2)*a*c*e*x**6 + sqrt(d + e*x**2)*c**
2*d*x**8 + sqrt(d + e*x**2)*c**2*e*x**10),x)
```

3.356 $\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 341

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)^2}\,dx = \frac{e^2\left(3cd^2+2ae^2\right)x}{6ad\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^{3/2}} + \frac{e^2\left(9c^2d^4+59acd^2e^2-8a^2e^4\right)x}{12ad^2\left(cd^2-ae^2\right)^3\sqrt{d+ex^2}} + \frac{cx\left(d-ex^2\right)}{4a\left(cd^2-ae^2\right)\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)} + \frac{c\left(3\sqrt{c}d-8\sqrt{a}e\right)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{7/2}} + \frac{c\left(3\sqrt{c}d+8\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{8a^{7/4}\left(\sqrt{c}d+\sqrt{a}e\right)^{7/2}}$$

output
```
1/6*e^2*(2*a*e^2+3*c*d^2)*x/a/d/(-a*e^2+c*d^2)^2/(e*x^2+d)^(3/2)+1/12*e^2*
(-8*a^2*e^4+59*a*c*d^2*e^2+9*c^2*d^4)*x/a/d^2/(-a*e^2+c*d^2)^3/(e*x^2+d)^(
1/2)+1/4*c*x*(-e*x^2+d)/a/(-a*e^2+c*d^2)/(e*x^2+d)^(3/2)/(-c*x^4+a)+1/8*c*
(3*c^(1/2)*d-8*a^(1/2)*e)*arctan((c^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*
x^2+d)^(1/2))/a^(7/4)/(c^(1/2)*d-a^(1/2)*e)^(7/2)+1/8*c*(3*c^(1/2)*d+8*a^(
1/2)*e)*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(
7/4)/(c^(1/2)*d+a^(1/2)*e)^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.58 (sec) , antiderivative size = 913, normalized size of antiderivative = 2.68

$$\int \frac{1}{(d+ex^2)^{5/2}(a-cx^4)^2}\,dx = \frac{-48ce^{7/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2-16ae^2\#1^2-4cd\#1^3+c\#\right.}{}$$

input
```
Integrate[1/((d + e*x^2)^(5/2)*(a - c*x^4)^2),x]
```

output
```
(-48*c*e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 -
 4*c*d*#1^3 + c*#1^4 & , (9*c*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e
*x^2] - #1] - 8*a*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1
] - 5*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - a*e^2
*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*d*Log[d + 2*e*
x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^
2*#1 + 3*c*d*#1^2 - c*#1^3) & ] + ((2*x*(-3*c^3*d^4*(d - 3*e*x^2)*(d + e*x
^2)^2 + 4*a^3*e^6*(3*d + 2*e*x^2) - 4*a^2*c*e^4*(15*d^3 + 14*d^2*e*x^2 + 3
*d*e^2*x^4 + 2*e^3*x^6) + a*c^2*d^2*e^2*(-9*d^3 - 15*d^2*e*x^2 + 57*d*e^2*
x^4 + 59*e^3*x^6)))/(d^2*(d + e*x^2)^(3/2)*(-a + c*x^4)) + 3*c*e^(3/2)*Roo
tSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1
^4 & , (c^2*d^5*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 123*
a*c*d^3*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 128*a^2*
d*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 14*c^2*d^4*Log
[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 22*a*c*d^2*e^2*Log[d
 + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 16*a^2*e^4*Log[d + 2*e
*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c^2*d^3*Log[d + 2*e*x^2 - 2*
Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 5*a*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqr
t[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*
d*#1^2 - c*#1^3) & ])/a)/(24*(c*d^2 - a*e^2)^3)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{tikzcd}
\int \frac{1}{(a-cx^4)^2 (d+ex^2)^{5/2}}dx \arrow[d, "1571"] \\
\int \frac{1}{(a-cx^4)^2 (d+ex^2)^{5/2}}dx
\end{tikzcd}$$

input
```
Int[1/((d + e*x^2)^(5/2)*(a - c*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.43

method	result
pseudoelliptic	$-\frac{21\left(\left(e\,x^2+d\right)^{\frac{3}{2}}d^2\left(\frac{\left(\frac{8}{3}a^2e^4+5ac\,d^2e^2-c^2d^4\right)\sqrt{d^2ac}}{7}+ac\,d^2e\left(a\,e^2-\frac{c\,d^2}{21}\right)\right)\sqrt{\left(ae+\sqrt{d^2ac}\right)a}\,\left(-c\,x^4+a\right)c\arctan\left(\frac{\sqrt{e}}{x\sqrt{\left(-a\right.}}\right.\right.\right.}{}$
default	Expression too large to display

input
```
int(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-21/8/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/(e*x^2+d)^(3/2)/(d^2*a*c)^(1/2)*((e*
x^2+d)^(3/2)*d^2*(1/7*(8/3*a^2*e^4+5*a*c*d^2*e^2-c^2*d^4)*(d^2*a*c)^(1/2)+
a*c*d^2*e*(a*e^2-1/21*c*d^2))*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(-c*x^4+a)*c
*arctan((e*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+((e*x^2+d)^(
3/2)*d^2*(1/7*(-8/3*a^2*e^4-5*a*c*d^2*e^2+c^2*d^4)*(d^2*a*c)^(1/2)+a*c*d^2
*e*(a*e^2-1/21*c*d^2))*(-c*x^4+a)*c*arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(d^2
*a*c)^(1/2))*a)^(1/2))-8/21*x*(e^6*(2/3*e*x^2+d)*a^3-5*(2/15*e^3*x^6+1/5*d
*e^2*x^4+14/15*d^2*e*x^2+d^3)*c*e^4*a^2-3/4*d^2*(-59/9*e^3*x^6-19/3*d*e^2*
x^4+5/3*d^2*e*x^2+d^3)*c^2*e^2*a-1/4*c^3*d^4*(-3*e*x^2+d)*(e*x^2+d)^2)*((a
*e+(d^2*a*c)^(1/2))*a)^(1/2)*(d^2*a*c)^(1/2))*((-a*e+(d^2*a*c)^(1/2))*a)^(
1/2))/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)/(-c*x^4+a)/(a*e^2-c*d^2)^3/a/d^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a-cx^4)^2}\,dx = \text{Timed out}$$

input

```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^2,x, algorithm="fricas")
```

output

```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a-cx^4)^2}\,dx = \text{Timed out}$$

input

```
integrate(1/(e*x**2+d)**(5/2)/(-c*x**4+a)**2,x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)^2}\,dx=\int \frac{1}{\left(cx^4-a\right)^2\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 - a)^2*(e*x^2 + d)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)^2}\,dx=\int \frac{1}{{\left(a-c\,x^4\right)}^2\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int(1/((a - c*x^4)^2*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a - c*x^4)^2*(d + e*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^{5/2}\left(a-cx^4\right)^2}\,dx = \int \frac{}{\sqrt{e\,x^2+d}\,a^2d^2+2\sqrt{e\,x^2+d}\,a^2de\,x^2+\sqrt{e\,x^2+d}\,a^2e^2x^4-2\sqrt{e\,x^2+d}}$$

input
```
int(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^2,x)
```

output
```
int(1/(sqrt(d + e*x**2)*a**2*d**2 + 2*sqrt(d + e*x**2)*a**2*d*e*x**2 + sqr
t(d + e*x**2)*a**2*e**2*x**4 - 2*sqrt(d + e*x**2)*a*c*d**2*x**4 - 4*sqrt(d
 + e*x**2)*a*c*d*e*x**6 - 2*sqrt(d + e*x**2)*a*c*e**2*x**8 + sqrt(d + e*x*
*2)*c**2*d**2*x**8 + 2*sqrt(d + e*x**2)*c**2*d*e*x**10 + sqrt(d + e*x**2)*
c**2*e**2*x**12),x)
```

3.357 $\int \frac{\left(d+ex^2\right)^{11/2}}{\left(a-cx^4\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 420

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^{11/2}}{\left(a-cx^4\right)^3}\,dx &= \frac{de^2\left(17cd^2-21ae^2\right)x\sqrt{d+ex^2}}{32a^2c^2}\\
&\quad+\frac{e^2\left(5cd^2-4ae^2\right)x\left(d+ex^2\right)^{3/2}}{16a^2c^2}+\frac{3de^2x\left(d+ex^2\right)^{5/2}}{32a^2c}\\
&\quad-\frac{e^2x\left(d+ex^2\right)^{7/2}}{8a^2c}+\frac{x\left(d+ex^2\right)^{11/2}}{8a\left(a-cx^4\right)^2}+\frac{x\left(7d-4ex^2\right)\left(d+ex^2\right)^{9/2}}{32a^2\left(a-cx^4\right)}\\
&\quad+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{7/2}\left(21cd^2+46\sqrt{a}\sqrt{c}de+32ae^2\right)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}c^3}\\
&\quad-\frac{e^{11/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3}\\
&\quad+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)^{7/2}\left(21cd^2-46\sqrt{a}\sqrt{c}de+32ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}c^3}\end{aligned}$$

output

```
1/32*d*e^2*(-21*a*e^2+17*c*d^2)*x*(e*x^2+d)^(1/2)/a^2/c^2+1/16*e^2*(-4*a*e
^2+5*c*d^2)*x*(e*x^2+d)^(3/2)/a^2/c^2+3/32*d*e^2*x*(e*x^2+d)^(5/2)/a^2/c-1
/8*e^2*x*(e*x^2+d)^(7/2)/a^2/c+1/8*x*(e*x^2+d)^(11/2)/a/(-c*x^4+a)^2+1/32*
x*(-4*e*x^2+7*d)*(e*x^2+d)^(9/2)/a^2/(-c*x^4+a)+1/64*(c^(1/2)*d-a^(1/2)*e)
^(7/2)*(21*c*d^2+46*a^(1/2)*c^(1/2)*d*e+32*a*e^2)*arctan((c^(1/2)*d-a^(1/2
)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(11/4)/c^3-e^(11/2)*arctanh(e^(1/2
)*x/(e*x^2+d)^(1/2))/c^3+1/64*(c^(1/2)*d+a^(1/2)*e)^(7/2)*(21*c*d^2-46*a^(
1/2)*c^(1/2)*d*e+32*a*e^2)*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(
e*x^2+d)^(1/2))/a^(11/4)/c^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.12 (sec) , antiderivative size = 2319, normalized size of antiderivative = 5.52

$$\int \frac{(d+ex^2)^{11/2}}{(a-cx^4)^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[(d + e*x^2)^(11/2)/(a - c*x^4)^3,x]
```

output

```
(-((c^3*x*Sqrt[d + e*x^2]*(a^3*e^4*(29*d + 8*e*x^2) + c^3*d^4*x^4*(7*d + 2
4*e*x^2) - a^2*c*e^2*(26*d^3 + 4*d^2*e*x^2 + 49*d*e^2*x^4 + 12*e^3*x^6) -
a*c^2*d^2*(11*d^3 + 44*d^2*e*x^2 + 14*d*e^2*x^4 + 36*e^3*x^6)))/(a^2*(a -
c*x^4)^2)) + 32*c^2*e^(11/2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]] - 32*c^2*
e^(11/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d
*#1^3 + c*#1^4 & , (163*c*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2
] - #1] + 96*a*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] +
 24*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 6*a*e^2
*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 3*c*d*Log[d + 2*
e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*
e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ] + (4*e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1
+ 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (5507*c^5*d^10*Lo
g[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 34770*a*c^4*d^8*e^2*Lo
g[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 166869*a^2*c^3*d^6*e^4
*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 158576*a^3*c^2*d^4*
e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 224000*a^4*c*d^2
*e^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 57344*a^5*e^10*
Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 3666*c^5*d^9*Log[d +
 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 38556*a*c^4*d^7*e^2*Log[
d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 11362*a^2*c^3*d^5*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{11/2}}{(a-cx^4)^3}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{(d+ex^2)^{11/2}}{(a-cx^4)^3}dx$$

input

```
Int[(d + e*x^2)^(11/2)/(a - c*x^4)^3,x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.28

method	result
pseudoelliptic	$\frac{41\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}\,\left(\frac{\left(-16a^{3}e^{6}-\frac{29}{2}a^{2}c\,d^{2}e^{4}+13a\,c^{2}d^{4}e^{2}-\frac{21}{2}c^{3}d^{6}\right)\sqrt{d^{2}ac}}{41}+ac\,d^{2}e\left(a^{2}e^{4}-\frac{32}{41}ac\,d^{2}e^{2}+\frac{19}{41}c^{2}d^{4}\right)\right)\left(-c\,x^{4}+a\right)^{2}\arctan}{32}$
default	Expression too large to display

input
```
int((e*x^2+d)^(11/2)/(-c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
41/32/(d^2*a*c)^(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/((-a*e+(d^2*a*c)^(1/
2))*a)^(1/2)*(((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(1/41*(-16*a^3*e^6-29/2*a^2*
c*d^2*e^4+13*a*c^2*d^4*e^2-21/2*c^3*d^6)*(d^2*a*c)^(1/2)+a*c*d^2*e*(a^2*e^
4-32/41*a*c*d^2*e^2+19/41*c^2*d^4))*(-c*x^4+a)^2*arctan((e*x^2+d)^(1/2)/x*
a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+((1/41*(16*a^3*e^6+29/2*a^2*c*d^2*e^4-
13*a*c^2*d^4*e^2+21/2*c^3*d^6)*(d^2*a*c)^(1/2)+a*c*d^2*e*(a^2*e^4-32/41*a*
c*d^2*e^2+19/41*c^2*d^4))*(-c*x^4+a)^2*arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(
d^2*a*c)^(1/2))*a)^(1/2))-32/41*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(d^2*a*c)^
(1/2)*(a^2*e^(11/2)*(-c*x^4+a)^2*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))+29/32*
x*(e^4*(8/29*e*x^2+d)*a^3-26/29*(6/13*e^3*x^6+49/26*d*e^2*x^4+2/13*d^2*e*x
^2+d^3)*c*e^2*a^2-11/29*(36/11*e^3*x^6+14/11*d*e^2*x^4+4*d^2*e*x^2+d^3)*d^
2*c^2*a+7/29*x^4*d^4*c^3*(24/7*e*x^2+d))*c*(e*x^2+d)^(1/2)))*((-a*e+(d^2*a
*c)^(1/2))*a)^(1/2))/(-c*x^4+a)^2/c^3/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{11/2}}{(a-cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(11/2)/(-c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{11/2}}{(a-cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(11/2)/(-c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{11/2}}{(a-cx^4)^3}\,dx = \int -\frac{(ex^2+d)^{\frac{11}{2}}}{(cx^4-a)^3}\,dx$$

input
```
integrate((e*x^2+d)^(11/2)/(-c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-integrate((e*x^2 + d)^(11/2)/(c*x^4 - a)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(344) = 688$.

Time = 0.22 (sec) , antiderivative size = 1035, normalized size of antiderivative = 2.46

$$\int \frac{(d+ex^2)^{11/2}}{(a-cx^4)^3}\,dx = \text{Too large to display}$$

input

```
integrate((e*x^2+d)^(11/2)/(-c*x^4+a)^3,x, algorithm="giac")
```

output

```
1/2*e^(11/2)*log((sqrt(e)*x - sqrt(e*x^2 + d))^2)/c^3 + 1/8*(19*(sqrt(e)*x
 - sqrt(e*x^2 + d))^14*c^4*d^5*e^(3/2) - 32*(sqrt(e)*x - sqrt(e*x^2 + d))^
14*a*c^3*d^3*e^(7/2) - 55*(sqrt(e)*x - sqrt(e*x^2 + d))^14*a^2*c^2*d*e^(11
/2) - 112*(sqrt(e)*x - sqrt(e*x^2 + d))^12*c^4*d^6*e^(3/2) + 198*(sqrt(e)*
x - sqrt(e*x^2 + d))^12*a*c^3*d^4*e^(7/2) - 66*(sqrt(e)*x - sqrt(e*x^2 + d
))^12*a^2*c^2*d^2*e^(11/2) - 64*(sqrt(e)*x - sqrt(e*x^2 + d))^12*a^3*c*e^(
15/2) + 287*(sqrt(e)*x - sqrt(e*x^2 + d))^10*c^4*d^7*e^(3/2) - 1040*(sqrt(
e)*x - sqrt(e*x^2 + d))^10*a*c^3*d^5*e^(7/2) + 397*(sqrt(e)*x - sqrt(e*x^2
 + d))^10*a^2*c^2*d^3*e^(11/2) + 784*(sqrt(e)*x - sqrt(e*x^2 + d))^10*a^3*
c*d*e^(15/2) - 420*(sqrt(e)*x - sqrt(e*x^2 + d))^8*c^4*d^8*e^(3/2) + 1782*
(sqrt(e)*x - sqrt(e*x^2 + d))^8*a*c^3*d^6*e^(7/2) - 3054*(sqrt(e)*x - sqrt
(e*x^2 + d))^8*a^2*c^2*d^4*e^(11/2) + 1728*(sqrt(e)*x - sqrt(e*x^2 + d))^8
*a^3*c*d^2*e^(15/2) + 768*(sqrt(e)*x - sqrt(e*x^2 + d))^8*a^4*e^(19/2) + 3
85*(sqrt(e)*x - sqrt(e*x^2 + d))^6*c^4*d^9*e^(3/2) - 1504*(sqrt(e)*x - sqr
t(e*x^2 + d))^6*a*c^3*d^7*e^(7/2) + 1571*(sqrt(e)*x - sqrt(e*x^2 + d))^6*a
^2*c^2*d^5*e^(11/2) - 16*(sqrt(e)*x - sqrt(e*x^2 + d))^6*a^3*c*d^3*e^(15/2
) - 224*(sqrt(e)*x - sqrt(e*x^2 + d))^4*c^4*d^10*e^(3/2) + 690*(sqrt(e)*x
- sqrt(e*x^2 + d))^4*a*c^3*d^8*e^(7/2) - 342*(sqrt(e)*x - sqrt(e*x^2 + d))
^4*a^2*c^2*d^6*e^(11/2) - 128*(sqrt(e)*x - sqrt(e*x^2 + d))^4*a^3*c*d^4*e^
(15/2) + 77*(sqrt(e)*x - sqrt(e*x^2 + d))^2*c^4*d^11*e^(3/2) - 112*(sqr...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{11/2}}{(a-cx^4)^3}\,dx=\int \frac{(e\,x^2+d)^{11/2}}{(a-c\,x^4)^3}\,dx$$

input
```
int((d + e*x^2)^(11/2)/(a - c*x^4)^3,x)
```

output
```
int((d + e*x^2)^(11/2)/(a - c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{11/2}}{(a-cx^4)^3}\,dx=\int \frac{(e\,x^2+d)^{\frac{11}{2}}}{(-c\,x^4+a)^3}dx$$

input
```
int((e*x^2+d)^(11/2)/(-c*x^4+a)^3,x)
```

output
```
int((e*x^2+d)^(11/2)/(-c*x^4+a)^3,x)
```

3.358 $\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 356

$$\begin{aligned}\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^3}\,dx &= \frac{3e^2(2cd^2-ae^2)\,x\sqrt{d+ex^2}}{16a^2c^2}+\frac{5de^2x(d+ex^2)^{3/2}}{32a^2c}\\ &\quad-\frac{e^2x(d+ex^2)^{5/2}}{16a^2c}+\frac{x(d+ex^2)^{9/2}}{8a\,(a-cx^4)^2}+\frac{x(7d-2ex^2)\,(d+ex^2)^{7/2}}{32a^2\,(a-cx^4)}\\ &\quad+\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\left(7cd^2+10\sqrt{a}\sqrt{c}de+4ae^2\right)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}c^{5/2}}\\ &\quad+\frac{3\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}\left(7cd^2-10\sqrt{a}\sqrt{c}de+4ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}c^{5/2}}\end{aligned}$$

output

```
3/16*e^2*(-a*e^2+2*c*d^2)*x*(e*x^2+d)^(1/2)/a^2/c^2+5/32*d*e^2*x*(e*x^2+d)
^(3/2)/a^2/c-1/16*e^2*x*(e*x^2+d)^(5/2)/a^2/c+1/8*x*(e*x^2+d)^(9/2)/a/(-c*
x^4+a)^2+1/32*x*(-2*e*x^2+7*d)*(e*x^2+d)^(7/2)/a^2/(-c*x^4+a)+3/64*(c^(1/2
)*d-a^(1/2)*e)^(5/2)*(7*c*d^2+10*a^(1/2)*c^(1/2)*d*e+4*a*e^2)*arctan((c^(1
/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(11/4)/c^(5/2)+3/64*(c
^(1/2)*d+a^(1/2)*e)^(5/2)*(7*c*d^2-10*a^(1/2)*c^(1/2)*d*e+4*a*e^2)*arctanh
((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(11/4)/c^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.95 (sec) , antiderivative size = 2220, normalized size of antiderivative = 6.24

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[(d + e*x^2)^(9/2)/(a - c*x^4)^3,x]
```

output

```
((-2*c^3*x*Sqrt[d + e*x^2]*(6*a^3*e^4 + c^3*d^3*x^4*(7*d + 19*e*x^2) - a^2
*c*e^2*(15*d^2 + d*e*x^2 + 10*e^2*x^4) - a*c^2*d*(11*d^3 + 35*d^2*e*x^2 +
9*d*e^2*x^4 + 15*e^3*x^6)))/(a^2*(a - c*x^4)^2) - 32*c^2*e^(11/2)*RootSum[
c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 &
, (161*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 32*a*e^
2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 18*c*d*Log[d + 2*e
*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*Log[d + 2*e*x^2 - 2*Sqrt[e
]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#
1^2 - c*#1^3) & ] + (32*e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2
- 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (1239*c^5*d^10*Log[d + 2*e*x^2 -
 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 4394*a*c^4*d^8*e^2*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 32487*a^2*c^3*d^6*e^4*Log[d + 2*e*x^2
- 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 4216*a^3*c^2*d^4*e^6*Log[d + 2*e*x^2
 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 28544*a^4*c*d^2*e^8*Log[d + 2*e*x^2
 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 2048*a^5*e^10*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1] - 840*c^5*d^9*Log[d + 2*e*x^2 - 2*Sqrt[e]*x
*Sqrt[d + e*x^2] - #1]*#1 - 6324*a*c^4*d^7*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]
*x*Sqrt[d + e*x^2] - #1]*#1 + 2160*a^2*c^3*d^5*e^4*Log[d + 2*e*x^2 - 2*Sqr
t[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 7360*a^3*c^2*d^3*e^6*Log[d + 2*e*x^2 - 2
*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 512*a^4*c*d*e^8*Log[d + 2*e*x^2 -...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{9/2}}{\left(a-cx^4\right)^3}dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{\left(d+ex^2\right)^{9/2}}{\left(a-cx^4\right)^3}dx$$

input
```
Int[(d + e*x^2)^(9/2)/(a - c*x^4)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.25

method	result
pseudoelliptic	$-\frac{3\left(-d\left(\left(-\frac{1}{2}a^2e^4+\frac{5}{4}ac\,d^2e^2-\frac{7}{4}c^2d^4\right)\sqrt{d^2ac}+ae\left(a^2e^4-\frac{11}{4}ac\,d^2e^2+\frac{11}{4}c^2d^4\right)\right)\sqrt{\left(ae+\sqrt{d^2ac}\right)a}\,\left(-c\,x^4+a\right)^2\arctan\left(\frac{}{x\sqrt{}}\right.\right.}{}$
default	Expression too large to display

input
```
int((e*x^2+d)^(9/2)/(-c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/16*(-d*((-1/2*a^2*e^4+5/4*a*c*d^2*e^2-7/4*c^2*d^4)*(d^2*a*c)^(1/2)+a*e*
(a^2*e^4-11/4*a*c*d^2*e^2+11/4*c^2*d^4))*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(
-c*x^4+a)^2*arctan((e*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+(
-d*((1/2*a^2*e^4-5/4*a*c*d^2*e^2+7/4*c^2*d^4)*(d^2*a*c)^(1/2)+a*e*(a^2*e^4
-11/4*a*c*d^2*e^2+11/4*c^2*d^4))*(-c*x^4+a)^2*arctanh((e*x^2+d)^(1/2)/x*a/
((a*e+(d^2*a*c)^(1/2))*a)^(1/2))+x*(e*x^2+d)^(1/2)*((a*e+(d^2*a*c)^(1/2))*
a)^(1/2)*(a^3*e^4-5/2*(2/3*e^2*x^4+1/15*d*e*x^2+d^2)*c*e^2*a^2-11/6*(15/11
*e^3*x^6+9/11*d*e^2*x^4+35/11*d^2*e*x^2+d^3)*d*c^2*a+7/6*x^4*d^3*(19/7*e*x
^2+d)*c^3)*(d^2*a*c)^(1/2))*((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))/(d^2*a*c)^(1
/2)/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/a^2/c
^2/(-c*x^4+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3392 vs. $2(286) = 572$.

Time = 143.23 (sec) , antiderivative size = 3392, normalized size of antiderivative = 9.53

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(9/2)/(-c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(9/2)/(-c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^3}\,dx = \int -\frac{(ex^2+d)^{\frac{9}{2}}}{(cx^4-a)^3}\,dx$$

input
```
integrate((e*x^2+d)^(9/2)/(-c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-integrate((e*x^2 + d)^(9/2)/(c*x^4 - a)^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(9/2)/(-c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^3}\,dx = \int \frac{(e\,x^2+d)^{9/2}}{(a-c\,x^4)^3}\,dx$$

input
```
int((d + e*x^2)^(9/2)/(a - c*x^4)^3,x)
```

output
```
int((d + e*x^2)^(9/2)/(a - c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^3}\,dx = \int \frac{(e\,x^2+d)^{\frac{9}{2}}}{(-c\,x^4+a)^3}dx$$

input

```
int((e*x^2+d)^(9/2)/(-c*x^4+a)^3,x)
```

output

```
int((e*x^2+d)^(9/2)/(-c*x^4+a)^3,x)
```

3.359 $\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 266

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^3}\,dx = \frac{7de^2x\sqrt{d+ex^2}}{32a^2c} + \frac{x(d+ex^2)^{7/2}}{8a\,(a-cx^4)^2} + \frac{7dx(d+ex^2)^{5/2}}{32a^2\,(a-cx^4)} + \frac{7d\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\left(3\sqrt{c}d+2\sqrt{a}e\right)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}c^{3/2}} + \frac{7d\left(3\sqrt{c}d-2\sqrt{a}e\right)\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}c^{3/2}}$$

output
```
7/32*d*e^2*x*(e*x^2+d)^(1/2)/a^2/c+1/8*x*(e*x^2+d)^(7/2)/a/(-c*x^4+a)^2+7/
32*d*x*(e*x^2+d)^(5/2)/a^2/(-c*x^4+a)+7/64*d*(c^(1/2)*d-a^(1/2)*e)^(3/2)*(
3*c^(1/2)*d+2*a^(1/2)*e)*arctan((c^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x
^2+d)^(1/2))/a^(11/4)/c^(3/2)+7/64*d*(3*c^(1/2)*d-2*a^(1/2)*e)*(c^(1/2)*d+
a^(1/2)*e)^(3/2)*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(
1/2))/a^(11/4)/c^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.35 (sec) , antiderivative size = 1818, normalized size of antiderivative = 6.83

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a-cx^4\right)^3}\,dx=\text{Too large to display}$$

input

```
Integrate[(d + e*x^2)^(7/2)/(a - c*x^4)^3,x]
```

output

```
(-((c^3*x*Sqrt[d + e*x^2]*(-7*a^2*d*e^2 + 7*c^2*d^2*x^4*(d + 2*e*x^2) - a*
c*(11*d^3 + 26*d^2*e*x^2 + 5*d*e^2*x^4 + 4*e^3*x^6)))/(a^2*(a - c*x^4)^2))
 - 64*c^2*e^(11/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1
^2 - 4*c*d*#1^3 + c*#1^4 & , (16*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d +
e*x^2] - #1] + Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1)/(c*
d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ] + (8*e^(7/2)*Root
Sum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^
4 & , (2205*c^4*d^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] +
2525*a*c^3*d^6*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 4
3992*a^2*c^2*d^4*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] +
 18304*a^3*c*d^2*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] +
 20480*a^4*e^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 1526*
c^4*d^7*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 7510*a*c^
3*d^5*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 6784*a^
2*c^2*d^3*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 512
0*a^3*c*d*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 413
*c^4*d^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + 2013*a
*c^3*d^4*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 16
24*a^2*c^2*d^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^
2 - 1280*a^3*c*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a-cx^4\right)^3}\,dx$$
$$\Big\downarrow \; \color{blue}{1571}$$
$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a-cx^4\right)^3}dx$$

input
```
Int[(d + e*x^2)^(7/2)/(a - c*x^4)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$-\frac{7\sqrt{\left(ae+\sqrt{d^{2}ac}\right)a}\,d^{2}\left(\frac{\left(-a\,e^{2}+3c\,d^{2}\right)\sqrt{d^{2}ac}}{2}+ae\left(a\,e^{2}-2c\,d^{2}\right)\right)\left(-c\,x^{4}+a\right)^{2}\arctan\left(\frac{\sqrt{e\,x^{2}+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^{2}ac}\right)a}}\right)}{32}+7\left(-d^{2}\left(\frac{\left(a\,e^{2}-3c\,d^{2}\right.}{2}\right.\right.$
default	Expression too large to display

input
```
int((e*x^2+d)^(7/2)/(-c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output

```
7/32/(d^2*a*c)^(1/2)*(-((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*d^2*(1/2*(-a*e^2+3*
c*d^2)*(d^2*a*c)^(1/2)+a*e*(a*e^2-2*c*d^2))*(-c*x^4+a)^2*arctan((e*x^2+d)^
(1/2)/x*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+(-d^2*(1/2*(a*e^2-3*c*d^2)*(d^
2*a*c)^(1/2)+a*e*(a*e^2-2*c*d^2))*(-c*x^4+a)^2*arctanh((e*x^2+d)^(1/2)/x*a
/((a*e+(d^2*a*c)^(1/2))*a)^(1/2))+((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*x*(e*x^2
+d)^(1/2)*(d*e^2*a^2+11/7*(4/11*e^3*x^6+5/11*d*e^2*x^4+26/11*d^2*e*x^2+d^3
)*c*a-c^2*d^2*x^4*(2*e*x^2+d))*(d^2*a*c)^(1/2))*((-a*e+(d^2*a*c)^(1/2))*a)
^(1/2))/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/a
^2/c/(-c*x^4+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2082 vs. $2(204) = 408$.

Time $= 28.79$ (sec) , antiderivative size $= 2082$, normalized size of antiderivative $= 7.83$

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^3}\,dx = \text{Too large to display}$$

input

```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a)^3,x, algorithm="fricas")
```

output

```
1/256*(7*(a^2*c^3*x^8 - 2*a^3*c^2*x^4 + a^4*c)*sqrt((15*c^2*d^6*e - 15*a*c
*d^4*e^3 + 4*a^2*d^2*e^5 + a^5*c^3*sqrt((81*c^2*d^14 - 90*a*c*d^12*e^2 + 2
5*a^2*d^10*e^4)/(a^11*c^3)))/(a^5*c^3))*log(-343*(81*c^3*d^13 - 162*a*c^2*
d^11*e^2 + 101*a^2*c*d^9*e^4 - 20*a^3*d^7*e^6 + (9*a^5*c^4*d^6 - 13*a^6*c^
3*d^4*e^2 + 4*a^7*c^2*d^2*e^4)*x^2*sqrt((81*c^2*d^14 - 90*a*c*d^12*e^2 + 2
5*a^2*d^10*e^4)/(a^11*c^3)) + 2*(81*c^3*d^12*e - 162*a*c^2*d^10*e^3 + 101*
a^2*c*d^8*e^5 - 20*a^3*d^6*e^7)*x^2 + 2*sqrt(e*x^2 + d)*((3*a^8*c^4*d^2 -
2*a^9*c^3*e^2)*x*sqrt((81*c^2*d^14 - 90*a*c*d^12*e^2 + 25*a^2*d^10*e^4)/(a
^11*c^3)) - (9*a^3*c^3*d^8*e - 5*a^4*c^2*d^6*e^3)*x)*sqrt((15*c^2*d^6*e -
15*a*c*d^4*e^3 + 4*a^2*d^2*e^5 + a^5*c^3*sqrt((81*c^2*d^14 - 90*a*c*d^12*e
^2 + 25*a^2*d^10*e^4)/(a^11*c^3)))/(a^5*c^3)))/x^2) - 7*(a^2*c^3*x^8 - 2*a
^3*c^2*x^4 + a^4*c)*sqrt((15*c^2*d^6*e - 15*a*c*d^4*e^3 + 4*a^2*d^2*e^5 +
a^5*c^3*sqrt((81*c^2*d^14 - 90*a*c*d^12*e^2 + 25*a^2*d^10*e^4)/(a^11*c^3))
)/(a^5*c^3))*log(-343*(81*c^3*d^13 - 162*a*c^2*d^11*e^2 + 101*a^2*c*d^9*e^
4 - 20*a^3*d^7*e^6 + (9*a^5*c^4*d^6 - 13*a^6*c^3*d^4*e^2 + 4*a^7*c^2*d^2*e
^4)*x^2*sqrt((81*c^2*d^14 - 90*a*c*d^12*e^2 + 25*a^2*d^10*e^4)/(a^11*c^3))
 + 2*(81*c^3*d^12*e - 162*a*c^2*d^10*e^3 + 101*a^2*c*d^8*e^5 - 20*a^3*d^6*
e^7)*x^2 - 2*sqrt(e*x^2 + d)*((3*a^8*c^4*d^2 - 2*a^9*c^3*e^2)*x*sqrt((81*c
^2*d^14 - 90*a*c*d^12*e^2 + 25*a^2*d^10*e^4)/(a^11*c^3)) - (9*a^3*c^3*d^8*
e - 5*a^4*c^2*d^6*e^3)*x)*sqrt((15*c^2*d^6*e - 15*a*c*d^4*e^3 + 4*a^2*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^3}\,dx = \text{Timed out}$$

input

```
integrate((e*x**2+d)**(7/2)/(-c*x**4+a)**3,x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^3}\,dx = \int -\frac{(ex^2+d)^{\frac{7}{2}}}{(cx^4-a)^3}\,dx$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-integrate((e*x^2 + d)^(7/2)/(c*x^4 - a)^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^3}\,dx = \int \frac{(e\,x^2+d)^{7/2}}{(a-c\,x^4)^3}\,dx$$

input
```
int((d + e*x^2)^(7/2)/(a - c*x^4)^3,x)
```

output
```
int((d + e*x^2)^(7/2)/(a - c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^3}\,dx = \int \frac{(e\,x^2+d)^{\frac{7}{2}}}{(-c\,x^4+a)^3}dx$$

input

```
int((e*x^2+d)^(7/2)/(-c*x^4+a)^3,x)
```

output

```
int((e*x^2+d)^(7/2)/(-c*x^4+a)^3,x)
```

3.360 $\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 292

$$\begin{aligned}\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^3}\,dx &= \frac{e^2x\sqrt{d+ex^2}}{16a^2c} + \frac{x(d+ex^2)^{5/2}}{8a(a-cx^4)^2} + \frac{x(d+ex^2)^{3/2}(7d+2ex^2)}{32a^2(a-cx^4)} \\ &+ \frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,(21cd^2-2\sqrt{a}\sqrt{c}de-4ae^2)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}c^{3/2}} \\ &+ \frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,(21cd^2+2\sqrt{a}\sqrt{c}de-4ae^2)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}c^{3/2}}\end{aligned}$$

output

```
1/16*e^2*x*(e*x^2+d)^(1/2)/a^2/c+1/8*x*(e*x^2+d)^(5/2)/a/(-c*x^4+a)^2+1/32
*x*(e*x^2+d)^(3/2)*(2*e*x^2+7*d)/a^2/(-c*x^4+a)+1/64*(c^(1/2)*d-a^(1/2)*e)
^(1/2)*(21*c*d^2-2*a^(1/2)*c^(1/2)*d*e-4*a*e^2)*arctan((c^(1/2)*d-a^(1/2)*
e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(11/4)/c^(3/2)+1/64*(c^(1/2)*d+a^(1/
2)*e)^(1/2)*(21*c*d^2+2*a^(1/2)*c^(1/2)*d*e-4*a*e^2)*arctanh((c^(1/2)*d+a^
(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(11/4)/c^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.19 (sec) , antiderivative size = 1766, normalized size of antiderivative = 6.05

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^3}\,dx = \text{Too large to display}$$

input

```
Integrate[(d + e*x^2)^(5/2)/(a - c*x^4)^3,x]
```

output

```
((-2*c^3*x*Sqrt[d + e*x^2]*(-2*a^2*e^2 + c^2*d*x^4*(7*d + 9*e*x^2) - a*c*(
11*d^2 + 17*d*e*x^2 + 2*e^2*x^4)))/(a^2*(a - c*x^4)^2) - 512*c^2*e^(11/2)*
RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c
*#1^4 & , Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]/(c*d^3 - 3*c
*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ] + (16*e^(7/2)*RootSum[c*d^
4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (1
934*c^4*d^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 1709*a*c
^3*d^6*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 24784*a^2
*c^2*d^4*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 20224*a
^3*c*d^2*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 4096*a^
4*e^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 1370*c^4*d^7*L
og[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 3724*a*c^3*d^5*e^2
*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 5504*a^2*c^2*d^3
*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 1024*a^3*c*d
*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 366*c^4*d^6*
Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + 1011*a*c^3*d^4*
e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 1360*a^2*c^
2*d^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 256*a
^3*c*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3
- 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(a*c*d^6 - a^2*d^...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^3}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^3}\,dx$$

input
```
Int[(d + e*x^2)^(5/2)/(a - c*x^4)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{-d\sqrt{\left(ae+\sqrt{d^2ac}\right)a}\,\left(\left(-\frac{a e^2}{2}+\frac{21 c d^2}{4}\right)\sqrt{d^2ac}+ae\left(a e^2-\frac{23 c d^2}{4}\right)\right)\left(-c x^4+a\right)^2 \arctan\left(\frac{\sqrt{e x^2+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^2ac}\right)a}}\right)+\Bigg(-\Bigg(\Bigg(\frac{a e^2}{2}}{}$
default	Expression too large to display

input
```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-d*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*((-1/2*a*e^2+21/4*c*d^2)*(d^2*a*c
)^(1/2)+a*e*(a*e^2-23/4*c*d^2))*(-c*x^4+a)^2*arctan((e*x^2+d)^(1/2)/x*a/((
-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+(-((1/2*a*e^2-21/4*c*d^2)*(d^2*a*c)^(1/2)+
a*e*(a*e^2-23/4*c*d^2))*d*(-c*x^4+a)^2*arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(
d^2*a*c)^(1/2))*a)^(1/2))+(a^2*e^2+11/2*(2/11*e^2*x^4+17/11*d*e*x^2+d^2)*c
*a-7/2*x^4*d*(9/7*e*x^2+d)*c^2)*x*(e*x^2+d)^(1/2)*((a*e+(d^2*a*c)^(1/2))*a
)^(1/2)*(d^2*a*c)^(1/2))*((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))/(d^2*a*c)^(1/2)
/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/a^2/c/(-
c*x^4+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2034 vs. $2(230) = 460$.

Time = 21.98 (sec) , antiderivative size = 2034, normalized size of antiderivative = 6.97

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^3,x, algorithm="fricas")
```

output

```
1/256*((a^2*c^3*x^8 - 2*a^3*c^2*x^4 + a^4*c)*sqrt((a^5*c^3*sqrt((194481*c^
2*d^10 - 70560*a*c*d^8*e^2 + 6400*a^2*d^6*e^4)/(a^11*c^3)) + 525*c^2*d^4*e
 - 180*a*c*d^2*e^3 + 16*a^2*e^5)/(a^5*c^3))*log(-(194481*c^3*d^10 - 111132
*a*c^2*d^8*e^2 + 20816*a^2*c*d^6*e^4 - 1280*a^3*d^4*e^6 + (441*a^5*c^4*d^5
 - 172*a^6*c^3*d^3*e^2 + 16*a^7*c^2*d*e^4)*x^2*sqrt((194481*c^2*d^10 - 705
60*a*c*d^8*e^2 + 6400*a^2*d^6*e^4)/(a^11*c^3)) + 2*(194481*c^3*d^9*e - 111
132*a*c^2*d^7*e^3 + 20816*a^2*c*d^5*e^5 - 1280*a^3*d^3*e^7)*x^2 + 2*sqrt(e
*x^2 + d)*((21*a^8*c^4*d^2 - 4*a^9*c^3*e^2)*x*sqrt((194481*c^2*d^10 - 7056
0*a*c*d^8*e^2 + 6400*a^2*d^6*e^4)/(a^11*c^3)) - 2*(441*a^3*c^3*d^6*e - 80*
a^4*c^2*d^4*e^3)*x)*sqrt((a^5*c^3*sqrt((194481*c^2*d^10 - 70560*a*c*d^8*e^
2 + 6400*a^2*d^6*e^4)/(a^11*c^3)) + 525*c^2*d^4*e - 180*a*c*d^2*e^3 + 16*a
^2*e^5)/(a^5*c^3)))/x^2) - (a^2*c^3*x^8 - 2*a^3*c^2*x^4 + a^4*c)*sqrt((a^5
*c^3*sqrt((194481*c^2*d^10 - 70560*a*c*d^8*e^2 + 6400*a^2*d^6*e^4)/(a^11*c
^3)) + 525*c^2*d^4*e - 180*a*c*d^2*e^3 + 16*a^2*e^5)/(a^5*c^3))*log(-(1944
81*c^3*d^10 - 111132*a*c^2*d^8*e^2 + 20816*a^2*c*d^6*e^4 - 1280*a^3*d^4*e^
6 + (441*a^5*c^4*d^5 - 172*a^6*c^3*d^3*e^2 + 16*a^7*c^2*d*e^4)*x^2*sqrt((1
94481*c^2*d^10 - 70560*a*c*d^8*e^2 + 6400*a^2*d^6*e^4)/(a^11*c^3)) + 2*(19
4481*c^3*d^9*e - 111132*a*c^2*d^7*e^3 + 20816*a^2*c*d^5*e^5 - 1280*a^3*d^3
*e^7)*x^2 - 2*sqrt(e*x^2 + d)*((21*a^8*c^4*d^2 - 4*a^9*c^3*e^2)*x*sqrt((19
4481*c^2*d^10 - 70560*a*c*d^8*e^2 + 6400*a^2*d^6*e^4)/(a^11*c^3)) - 2*(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^3}\,dx = \text{Timed out}$$

input

```
integrate((e*x**2+d)**(5/2)/(-c*x**4+a)**3,x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^3}\,dx=\int -\frac{\left(ex^2+d\right)^{\frac{5}{2}}}{\left(cx^4-a\right)^3}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-integrate((e*x^2 + d)^(5/2)/(c*x^4 - a)^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^3}\,dx=\int \frac{\left(e\,x^2+d\right)^{5/2}}{\left(a-c\,x^4\right)^3}\,dx$$

input
```
int((d + e*x^2)^(5/2)/(a - c*x^4)^3,x)
```

output
```
int((d + e*x^2)^(5/2)/(a - c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^3}\,dx=\int \frac{(e\,x^2+d)^{\frac{5}{2}}}{(-c\,x^4+a)^3}dx$$

input
```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^3,x)
```

output
```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^3,x)
```

3.361 $\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 249

$$\begin{aligned}\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^3}\,dx &= \frac{x(d+ex^2)^{3/2}}{8a\,(a-cx^4)^2}+\frac{x\sqrt{d+ex^2}(7d+4ex^2)}{32a^2\,(a-cx^4)}\\ &\quad+\frac{3d\big(7\sqrt{c}d-6\sqrt{a}e\big)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{a}e}}\\ &\quad+\frac{3d\big(7\sqrt{c}d+6\sqrt{a}e\big)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{a}e}}\end{aligned}$$

output

```
1/8*x*(e*x^2+d)^(3/2)/a/(-c*x^4+a)^2+1/32*x*(e*x^2+d)^(1/2)*(4*e*x^2+7*d)/
a^2/(-c*x^4+a)+3/64*d*(7*c^(1/2)*d-6*a^(1/2)*e)*arctan((c^(1/2)*d-a^(1/2)*
e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(11/4)/c^(1/2)/(c^(1/2)*d-a^(1/2)*e)
^(1/2)+3/64*d*(7*c^(1/2)*d+6*a^(1/2)*e)*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2
)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(11/4)/c^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.49 (sec) , antiderivative size = 1315, normalized size of antiderivative = 5.28

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^3}\,dx = \text{Too large to display}$$

input

```
Integrate[(d + e*x^2)^(3/2)/(a - c*x^4)^3,x]
```

output

```
((x*Sqrt[d + e*x^2]*(11*a*d + 8*a*e*x^2 - 7*c*d*x^4 - 4*c*e*x^6))/(a - c*x
^4)^2 + (4*a*e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*
#1^2 - 4*c*d*#1^3 + c*#1^4 & , (3329*c^3*d^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x
*Sqrt[d + e*x^2] - #1] - 8432*a*c^2*d^4*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*
Sqrt[d + e*x^2] - #1] - 19712*a^2*c*d^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*
Sqrt[d + e*x^2] - #1] + 24576*a^3*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d
 + e*x^2] - #1] - 2422*c^3*d^5*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1]*#1 - 2288*a*c^2*d^3*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*
x^2] - #1]*#1 + 6144*a^2*c*d*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*
x^2] - #1]*#1 + 641*c^3*d^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2]
- #1]*#1^2 + 656*a*c^2*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1]*#1^2 - 1536*a^2*c*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ]
)/(c^4*d^5 - a*c^3*d^3*e^2) + (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^
2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (9*c^4*d^8*Log[d + 2*e*x^
2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 13307*a*c^3*d^6*e^2*Log[d + 2*e*x^
2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 33728*a^2*c^2*d^4*e^4*Log[d + 2*e*
x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 78848*a^3*c*d^2*e^6*Log[d + 2*e*
x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 98304*a^4*e^8*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 24*c^4*d^7*Log[d + 2*e*x^2 - 2*Sqrt...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^3}\,dx$$
$$\Big\downarrow 1571$$
$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^3}dx$$

input
```
Int[(d + e*x^2)^(3/2)/(a - c*x^4)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{\frac{9d^2\left(ae-\frac{7\sqrt{d^2ac}}{6}\right)\sqrt{\left(ae+\sqrt{d^2ac}\right)a}\,\left(-cx^4+a\right)^2\arctan\left(\frac{\sqrt{ex^2+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^2ac}\right)a}}\right)}{32}+\frac{11\left(\frac{9\left(ae+\frac{7\sqrt{d^2ac}}{6}\right)d^2\left(-cx^4+a\right)^2\operatorname{arctanh}\left(\frac{\sqrt{}}{x\sqrt{(}}\right.}{11}\right.}{}}{a^2(-cx^4+a)^2\sqrt{d^2ac}\,\sqrt{\left(-ae+\sqrt{d^2ac}\right)a}}$
default	Expression too large to display

input
```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
11/32*(9/11*d^2*(a*e-7/6*(d^2*a*c)^(1/2))*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*
(-c*x^4+a)^2*arctan((e*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+
(9/11*(a*e+7/6*(d^2*a*c)^(1/2))*d^2*(-c*x^4+a)^2*arctanh((e*x^2+d)^(1/2)/x
*a/((a*e+(d^2*a*c)^(1/2))*a)^(1/2))+x*(e*x^2+d)^(1/2)*((a*e+(d^2*a*c)^(1/2
))*a)^(1/2)*((8/11*e*x^2+d)*a-7/11*x^4*c*(4/7*e*x^2+d))*(d^2*a*c)^(1/2))*(
(-a*e+(d^2*a*c)^(1/2))*a)^(1/2))/(d^2*a*c)^(1/2)/((-a*e+(d^2*a*c)^(1/2))*a
)^(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/a^2/(-c*x^4+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2526 vs. $2(191) = 382$.

Time = 11.70 (sec) , antiderivative size = 2526, normalized size of antiderivative = 10.14

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a-cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^3,x, algorithm="fricas")
```

output

```
1/256*(3*(a^2*c^2*x^8 - 2*a^3*c*x^4 + a^4)*sqrt((35*c*d^4*e - 36*a*d^2*e^3
 + (a^5*c^2*d^2 - a^6*c*e^2)*sqrt((2401*c^2*d^10 - 4704*a*c*d^8*e^2 + 2304
*a^2*d^6*e^4)/(a^11*c^3*d^4 - 2*a^12*c^2*d^2*e^2 + a^13*c*e^4)))/(a^5*c^2*
d^2 - a^6*c*e^2))*log(27*(2401*c^2*d^9 - 4116*a*c*d^7*e^2 + 1728*a^2*d^5*e
^4 + (49*a^5*c^3*d^6 - 85*a^6*c^2*d^4*e^2 + 36*a^7*c*d^2*e^4)*x^2*sqrt((24
01*c^2*d^10 - 4704*a*c*d^8*e^2 + 2304*a^2*d^6*e^4)/(a^11*c^3*d^4 - 2*a^12*
c^2*d^2*e^2 + a^13*c*e^4)) + 2*(2401*c^2*d^8*e - 4116*a*c*d^6*e^3 + 1728*a
^2*d^4*e^5)*x^2 + 2*sqrt(e*x^2 + d)*((7*a^8*c^3*d^4 - 13*a^9*c^2*d^2*e^2 +
 6*a^10*c*e^4)*x*sqrt((2401*c^2*d^10 - 4704*a*c*d^8*e^2 + 2304*a^2*d^6*e^4
)/(a^11*c^3*d^4 - 2*a^12*c^2*d^2*e^2 + a^13*c*e^4)) + (49*a^3*c^2*d^6*e -
48*a^4*c*d^4*e^3)*x)*sqrt((35*c*d^4*e - 36*a*d^2*e^3 + (a^5*c^2*d^2 - a^6*
c*e^2)*sqrt((2401*c^2*d^10 - 4704*a*c*d^8*e^2 + 2304*a^2*d^6*e^4)/(a^11*c^
3*d^4 - 2*a^12*c^2*d^2*e^2 + a^13*c*e^4)))/(a^5*c^2*d^2 - a^6*c*e^2)))/x^2
) - 3*(a^2*c^2*x^8 - 2*a^3*c*x^4 + a^4)*sqrt((35*c*d^4*e - 36*a*d^2*e^3 +
(a^5*c^2*d^2 - a^6*c*e^2)*sqrt((2401*c^2*d^10 - 4704*a*c*d^8*e^2 + 2304*a^
2*d^6*e^4)/(a^11*c^3*d^4 - 2*a^12*c^2*d^2*e^2 + a^13*c*e^4)))/(a^5*c^2*d^2
 - a^6*c*e^2))*log(27*(2401*c^2*d^9 - 4116*a*c*d^7*e^2 + 1728*a^2*d^5*e^4
+ (49*a^5*c^3*d^6 - 85*a^6*c^2*d^4*e^2 + 36*a^7*c*d^2*e^4)*x^2*sqrt((2401*
c^2*d^10 - 4704*a*c*d^8*e^2 + 2304*a^2*d^6*e^4)/(a^11*c^3*d^4 - 2*a^12*c^2
*d^2*e^2 + a^13*c*e^4)) + 2*(2401*c^2*d^8*e - 4116*a*c*d^6*e^3 + 1728*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^3}\,dx = \text{Timed out}$$

input

```
integrate((e*x**2+d)**(3/2)/(-c*x**4+a)**3,x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^3}\,dx = \int -\frac{(ex^2+d)^{\frac{3}{2}}}{(cx^4-a)^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-integrate((e*x^2 + d)^(3/2)/(c*x^4 - a)^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^3}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{(a-c\,x^4)^3}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(a - c*x^4)^3,x)
```

output
```
int((d + e*x^2)^(3/2)/(a - c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^3}\,dx=\int \frac{(e\,x^2+d)^{\frac{3}{2}}}{(-c\,x^4+a)^3}dx$$

input

```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^3,x)
```

output

```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^3,x)
```

3.362 $\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 292

$$\begin{aligned}\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^3}\,dx &= \frac{x\sqrt{d+ex^2}}{8a\,(a-cx^4)^2} + \frac{x\sqrt{d+ex^2}(7cd^2-6ae^2-cdex^2)}{32a^2\,(cd^2-ae^2)\,(a-cx^4)} \\ &\quad + \frac{(21cd^2-34\sqrt{a}\sqrt{c}de+12ae^2)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}\sqrt{c}\,\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}} \\ &\quad + \frac{(21cd^2+34\sqrt{a}\sqrt{c}de+12ae^2)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}\sqrt{c}\,\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}}\end{aligned}$$

output

```
1/8*x*(e*x^2+d)^(1/2)/a/(-c*x^4+a)^2+1/32*x*(e*x^2+d)^(1/2)*(-c*d*e*x^2-6*
a*e^2+7*c*d^2)/a^2/(-a*e^2+c*d^2)/(-c*x^4+a)+1/64*(21*c*d^2-34*a^(1/2)*c^(
1/2)*d*e+12*a*e^2)*arctan((c^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^
(1/2))/a^(11/4)/c^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(3/2)+1/64*(21*c*d^2+34*a^(1
/2)*c^(1/2)*d*e+12*a*e^2)*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(e
*x^2+d)^(1/2))/a^(11/4)/c^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.45 (sec) , antiderivative size = 1324, normalized size of antiderivative = 4.53

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^3}\,dx = \text{Too large to display}$$

input

```
Integrate[Sqrt[d + e*x^2]/(a - c*x^4)^3,x]
```

output

```
((2*x*Sqrt[d + e*x^2]*(10*a^2*e^2 + c^2*d*x^4*(7*d - e*x^2) + a*c*(-11*d^2
 + d*e*x^2 - 6*e^2*x^4)))/(a - c*x^4)^2 - (64*a*e^(7/2)*RootSum[c*d^4 - 4*
c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (349*c^3
*d^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 1324*a*c^2*d^4*
e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 64*a^2*c*d^2*e^4
*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 1024*a^3*e^6*Log[d
+ 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 262*c^3*d^5*Log[d + 2*e*x^
2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 96*a*c^2*d^3*e^2*Log[d + 2*e*x^
2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 256*a^2*c*d*e^4*Log[d + 2*e*x^2
 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 69*c^3*d^4*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 20*a*c^2*d^2*e^2*Log[d + 2*e*x^2 - 2
*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 64*a^2*c*e^4*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*
c*d*#1^2 - c*#1^3) & ])/(c^3*d^4) + (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 +
6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (13*c^4*d^8*Log[d +
 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 22324*a*c^3*d^6*e^2*Log[d +
 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 84736*a^2*c^2*d^4*e^4*Log[d
 + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 4096*a^3*c*d^2*e^6*Log[d
+ 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 65536*a^4*e^8*Log[d + 2*e*
x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 58*c^4*d^7*Log[d + 2*e*x^2 - ...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^3}\,dx$$
$$\Big\downarrow \; 1571$$
$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^3}dx$$

input
```
Int[Sqrt[d + e*x^2]/(a - c*x^4)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{\frac{3\left(\left(-\frac{11a\,e^2}{6}+\frac{7c\,d^2}{4}\right)\sqrt{d^2ac}+ae\left(a\,e^2-\frac{13c\,d^2}{12}\right)\right)\sqrt{\left(ae+\sqrt{d^2ac}\right)a}\,d\left(-c\,x^4+a\right)^2\arctan\left(\frac{\sqrt{e\,x^2+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^2ac}\right)a}}\right)}{16}+\frac{5\sqrt{\left(-ae+\sqrt{d^2ac}\right)a}}{}}{a^2(a\,e}$
default	Expression too large to display

input
```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output

```
5/16/(d^2*a*c)^(1/2)*(3/5*((-11/6*a*e^2+7/4*c*d^2)*(d^2*a*c)^(1/2)+a*e*(a*
e^2-13/12*c*d^2))*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*d*(-c*x^4+a)^2*arctan((e
*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+((-a*e+(d^2*a*c)^(1/2)
)*a)^(1/2)*(3/5*((11/6*a*e^2-7/4*c*d^2)*(d^2*a*c)^(1/2)+a*e*(a*e^2-13/12*c
*d^2))*d*(-c*x^4+a)^2*arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(d^2*a*c)^(1/2))*a
)^(1/2))+x*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(e*x^2+d)^(1/2)*(a^2*e^2-11/10*
(6/11*e^2*x^4-1/11*d*e*x^2+d^2)*c*a+7/10*x^4*d*(-1/7*e*x^2+d)*c^2)*(d^2*a*
c)^(1/2)))/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2
)/a^2/(a*e^2-c*d^2)/(-c*x^4+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6046 vs. $2(233) = 466$.

Time = 117.34 (sec) , antiderivative size = 6046, normalized size of antiderivative = 20.71

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^3}\,dx = \text{Too large to display}$$

input

```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^3,x, algorithm="fricas")
```

output

```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^3}\,dx = \text{Timed out}$$

input

```
integrate((e*x**2+d)**(1/2)/(-c*x**4+a)**3,x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^3}\,dx = \int -\frac{\sqrt{ex^2+d}}{(cx^4-a)^3}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-integrate(sqrt(e*x^2 + d)/(c*x^4 - a)^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^3}\,dx = \int \frac{\sqrt{e\,x^2+d}}{(a-c\,x^4)^3}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a - c*x^4)^3,x)
```

output
```
int((d + e*x^2)^(1/2)/(a - c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^3}\,dx=\int \frac{\sqrt{e\,x^2+d}}{(-c\,x^4+a)^3}dx$$

input
```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^3,x)
```

output
```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^3,x)
```

3.363 $\int \frac{1}{\sqrt{d+ex^2}(a-cx^4)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 319

$$\begin{aligned}\int \frac{1}{\sqrt{d+ex^2}\left(a-cx^4\right)^3}\,dx &= \frac{cx(d-ex^2)\sqrt{d+ex^2}}{8a\left(cd^2-ae^2\right)\left(a-cx^4\right)^2} \\ &\quad+\frac{cx\sqrt{d+ex^2}(d(7cd^2-13ae^2)-6e\left(cd^2-2ae^2\right)x^2)}{32a^2\left(cd^2-ae^2\right)^2\left(a-cx^4\right)} \\ &\quad+\frac{\left(21cd^2-50\sqrt{a}\sqrt{c}de+32ae^2\right)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}} \\ &\quad+\frac{\left(21cd^2+50\sqrt{a}\sqrt{c}de+32ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}}\end{aligned}$$

output

```
1/8*c*x*(-e*x^2+d)*(e*x^2+d)^(1/2)/a/(-a*e^2+c*d^2)/(-c*x^4+a)^2+1/32*c*x*
(e*x^2+d)^(1/2)*(d*(-13*a*e^2+7*c*d^2)-6*e*(-2*a*e^2+c*d^2)*x^2)/a^2/(-a*e
^2+c*d^2)^2/(-c*x^4+a)+1/64*(21*c*d^2-50*a^(1/2)*c^(1/2)*d*e+32*a*e^2)*arc
tan((c^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(11/4)/(c^(1/
2)*d-a^(1/2)*e)^(5/2)+1/64*(21*c*d^2+50*a^(1/2)*c^(1/2)*d*e+32*a*e^2)*arct
anh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+d)^(1/2))/a^(11/4)/(c^(1/
2)*d+a^(1/2)*e)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.53 (sec) , antiderivative size = 1214, normalized size of antiderivative = 3.81

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^3}\,dx = \text{Too large to display}$$

input

```
Integrate[1/(Sqrt[d + e*x^2]*(a - c*x^4)^3),x]
```

output

```
-1/32*((c^3*x*Sqrt[d + e*x^2]*(a^2*e^2*(17*d - 16*e*x^2) + c^2*d^2*x^4*(7*
d - 6*e*x^2) + a*c*(-11*d^3 + 10*d^2*e*x^2 - 13*d*e^2*x^4 + 12*e^3*x^6)))/
(a - c*x^4)^2 + (64*a*e^(7/2)*(-(c*d^2) + a*e^2)*RootSum[c*d^4 - 4*c*d^3*#
1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (141*c^2*d^4*Lo
g[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 656*a*c*d^2*e^2*Log[d
+ 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 512*a^2*e^4*Log[d + 2*e*x^
2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 110*c^2*d^3*Log[d + 2*e*x^2 - 2*Sq
rt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 128*a*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[
e]*x*Sqrt[d + e*x^2] - #1]*#1 + 29*c^2*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*S
qrt[d + e*x^2] - #1]*#1^2 - 32*a*c*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[
d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#
1^3) & ])/d^3 + (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 - 16*a*
e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (4*c^4*d^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*
x*Sqrt[d + e*x^2] - #1] + 9017*a*c^3*d^6*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x
*Sqrt[d + e*x^2] - #1] - 51008*a^2*c^2*d^4*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]
*x*Sqrt[d + e*x^2] - #1] + 74752*a^3*c*d^2*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]
*x*Sqrt[d + e*x^2] - #1] - 32768*a^4*e^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqr
t[d + e*x^2] - #1] + 34*c^4*d^7*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x
^2] - #1]*#1 - 7120*a*c^3*d^5*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e
*x^2] - #1]*#1 + 15296*a^2*c^2*d^3*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sq...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{1}{(a-cx^4)^3 \sqrt{d+ex^2}}dx \\ \Big\downarrow \textit{1571} \\ \int \frac{1}{(a-cx^4)^3 \sqrt{d+ex^2}}dx \end{array}$$

input

```
Int[1/(Sqrt[d + e*x^2]*(a - c*x^4)^3),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$-\frac{17\left(\frac{7\left(\left(\frac{16}{7}a^2e^4-\frac{47}{14}ac\,d^2e^2+\frac{3}{2}c^2d^4\right)\sqrt{d^2ac}+ac\,d^2e\left(a\,e^2-\frac{4c\,d^2}{7}\right)\right)\left(-c\,x^4+a\right)^2\sqrt{\left(ae+\sqrt{d^2ac}\right)a}\,\arctan\left(\frac{\sqrt{e\,x^2+d}\,a}{x\sqrt{\left(-ae+\sqrt{d^2ac}\right)a}}\right)}{17}\right.}{}$
default	Expression too large to display

input

```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-17/32/(a*e^2-c*d^2)^2*(7/17*((16/7*a^2*e^4-47/14*a*c*d^2*e^2+3/2*c^2*d^4)
*(d^2*a*c)^(1/2)+a*c*d^2*e*(a*e^2-4/7*c*d^2))*(-c*x^4+a)^2*((a*e+(d^2*a*c)
^(1/2))*a)^(1/2)*arctan((e*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/
2))+((-a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(7/17*((-16/7*a^2*e^4+47/14*a*c*d^2*e
^2-3/2*c^2*d^4)*(d^2*a*c)^(1/2)+a*c*d^2*e*(a*e^2-4/7*c*d^2))*(-c*x^4+a)^2*
arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(d^2*a*c)^(1/2))*a)^(1/2))+x*(e*x^2+d)^(
1/2)*(e^2*(-16/17*e*x^2+d)*a^2-11/17*(-12/11*e^3*x^6+13/11*d*e^2*x^4-10/11
*d^2*e*x^2+d^3)*c*a+7/17*x^4*(-6/7*e*x^2+d)*d^2*c^2)*c*(d^2*a*c)^(1/2)*((a
*e+(d^2*a*c)^(1/2))*a)^(1/2)))/(d^2*a*c)^(1/2)/((-a*e+(d^2*a*c)^(1/2))*a)^
(1/2)/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/a^2/(-c*x^4+a)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(-c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^3}\,dx = \int -\frac{1}{(cx^4-a)^3\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-integrate(1/((c*x^4 - a)^3*sqrt(e*x^2 + d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^3}\,dx = \int \frac{1}{(a-c\,x^4)^3\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a - c*x^4)^3*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a - c*x^4)^3*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^3}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,(-c\,x^4+a)^3}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^3,x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^3,x)
```

3.364 $\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 396

$$
\begin{aligned}
\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^3}\,dx &= \frac{e^2(11c^2d^4-39acd^2e^2-32a^2e^4)\,x}{32a^2d\,(cd^2-ae^2)^3\sqrt{d+ex^2}} \\
&+ \frac{cx(d-ex^2)}{8a\,(cd^2-ae^2)\sqrt{d+ex^2}\,(a-cx^4)^2} \\
&+ \frac{cx(d(7cd^2-17ae^2)-2e(2cd^2-7ae^2)\,x^2)}{32a^2\,(cd^2-ae^2)^2\sqrt{d+ex^2}\,(a-cx^4)} \\
&+ \frac{3\sqrt{c}\left(7cd^2-22\sqrt{a}\sqrt{c}de+20ae^2\right)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{7/2}} \\
&+ \frac{3\sqrt{c}\left(7cd^2+22\sqrt{a}\sqrt{c}de+20ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,x}{\sqrt[4]{a}\sqrt{d+ex^2}}\right)}{64a^{11/4}\left(\sqrt{c}d+\sqrt{a}e\right)^{7/2}}
\end{aligned}
$$

output

```
1/32*e^2*(-32*a^2*e^4-39*a*c*d^2*e^2+11*c^2*d^4)*x/a^2/d/(-a*e^2+c*d^2)^3/
(e*x^2+d)^(1/2)+1/8*c*x*(-e*x^2+d)/a/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^
4+a)^2+1/32*c*x*(d*(-17*a*e^2+7*c*d^2)-2*e*(-7*a*e^2+2*c*d^2)*x^2)/a^2/(-a
*e^2+c*d^2)^2/(e*x^2+d)^(1/2)/(-c*x^4+a)+3/64*c^(1/2)*(7*c*d^2-22*a^(1/2)*
c^(1/2)*d*e+20*a*e^2)*arctan((c^(1/2)*d-a^(1/2)*e)^(1/2)*x/a^(1/4)/(e*x^2+
d)^(1/2))/a^(11/4)/(c^(1/2)*d-a^(1/2)*e)^(7/2)+3/64*c^(1/2)*(7*c*d^2+22*a^
(1/2)*c^(1/2)*d*e+20*a*e^2)*arctanh((c^(1/2)*d+a^(1/2)*e)^(1/2)*x/a^(1/4)/
(e*x^2+d)^(1/2))/a^(11/4)/(c^(1/2)*d+a^(1/2)*e)^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.53 (sec) , antiderivative size = 2037, normalized size of antiderivative = 5.14

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[1/((d + e*x^2)^(3/2)*(a - c*x^4)^3),x]
```

output
```
((-2*x*(32*a^4*e^6 + 2*a^3*c*e^4*(9*d^2 + 9*d*e*x^2 - 32*e^2*x^4) + c^4*d^
4*x^4*(7*d^2 - 4*d*e*x^2 - 11*e^2*x^4) + a^2*c^2*e^2*(21*d^4 - 26*d^3*e*x^
2 - 61*d^2*e^2*x^4 - 14*d*e^3*x^6 + 32*e^4*x^8) + a*c^3*d^2*(-11*d^4 + 8*d
^3*e*x^2 - 2*d^2*e^2*x^4 + 18*d*e^3*x^6 + 39*e^4*x^8)))/(a^2*d*(c*d^2 - a*
e^2)^3*Sqrt[d + e*x^2]*(a - c*x^4)^2) + (32*e^(11/2)*RootSum[c*d^4 - 4*c*d
^3*#1 + 6*c*d^2*#1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (17*c*d^2*L
og[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 16*a*e^2*Log[d + 2*e*
x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 6*c*d*Log[d + 2*e*x^2 - 2*Sqrt[e
]*x*Sqrt[d + e*x^2] - #1]*#1 + c*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*
x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 + 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) &
 ])/(c*d^2 - a*e^2)^3 - (16*e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#
1^2 - 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (860*c^4*d^8*Log[d + 2*e*x^2
 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 6091*a*c^3*d^6*e^2*Log[d + 2*e*x^2
- 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 13680*a^2*c^2*d^4*e^4*Log[d + 2*e*x^
2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 12544*a^3*c*d^2*e^6*Log[d + 2*e*x^
2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 4096*a^4*e^8*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1] - 706*c^4*d^7*Log[d + 2*e*x^2 - 2*Sqrt[e]*x
*Sqrt[d + e*x^2] - #1]*#1 + 2364*a*c^3*d^5*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]
*x*Sqrt[d + e*x^2] - #1]*#1 - 2688*a^2*c^2*d^3*e^4*Log[d + 2*e*x^2 - 2*Sqr
t[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 1024*a^3*c*d*e^6*Log[d + 2*e*x^2 - 2*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-cx^4\right)^3 \left(d+ex^2\right)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 1571}$$
$$\int \frac{1}{\left(a-cx^4\right)^3 \left(d+ex^2\right)^{3/2}}dx$$

input
```
Int[1/((d + e*x^2)^(3/2)*(a - c*x^4)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.38

method	result
pseudoelliptic	$\frac{15 d^2 \sqrt{\left(a e+\sqrt{d^2 a c}\right) a}\, \left(\left(\frac{19}{10} a^2 e^4-\frac{5}{4} a c\, d^2 e^2+\frac{7}{20} c^2 d^4\right) \sqrt{d^2 a c}+a^3 e^5+\frac{a^2 c\, d^2 e^3}{20}-\frac{c^2 d^4 e a}{20}\right) \left(-c\, x^4+a\right)^2 c \sqrt{e\, x^2+d}\ \operatorname{arctan}\left(\frac{\sqrt{e\, x^2+}}{x \sqrt{\left(-a e+\right.}}\right.}{16}$
default	Expression too large to display

input
```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
15/16/(e*x^2+d)^(1/2)/(d^2*a*c)^(1/2)*(d^2*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)
*((19/10*a^2*e^4-5/4*a*c*d^2*e^2+7/20*c^2*d^4)*(d^2*a*c)^(1/2)+a^3*e^5+1/2
0*a^2*c*d^2*e^3-1/20*c^2*d^4*e*a)*(-c*x^4+a)^2*c*(e*x^2+d)^(1/2)*arctan((e
*x^2+d)^(1/2)/x*a/((-a*e+(d^2*a*c)^(1/2))*a)^(1/2))+((-a*e+(d^2*a*c)^(1/2)
)*a)^(1/2)*(d^2*((-19/10*a^2*e^4+5/4*a*c*d^2*e^2-7/20*c^2*d^4)*(d^2*a*c)^(
1/2)+a^3*e^5+1/20*a^2*c*d^2*e^3-1/20*c^2*d^4*e*a)*(-c*x^4+a)^2*c*(e*x^2+d)
^(1/2)*arctanh((e*x^2+d)^(1/2)/x*a/((a*e+(d^2*a*c)^(1/2))*a)^(1/2))+16/15*
x*(a^4*e^6+9/16*(-32/9*e^2*x^4+d*e*x^2+d^2)*c*e^4*a^3+21/32*(32/21*e^4*x^8
-2/3*d*e^3*x^6-61/21*d^2*e^2*x^4-26/21*d^3*e*x^2+d^4)*c^2*e^2*a^2-11/32*(-
39/11*e^3*x^6+21/11*d*e^2*x^4-19/11*d^2*e*x^2+d^3)*d^2*(e*x^2+d)*c^3*a+7/3
2*x^4*d^4*(e*x^2+d)*c^4*(-11/7*e*x^2+d))*((a*e+(d^2*a*c)^(1/2))*a)^(1/2)*(
d^2*a*c)^(1/2)))/((a*e+(d^2*a*c)^(1/2))*a)^(1/2)/((-a*e+(d^2*a*c)^(1/2))*a
)^(1/2)/d/(a*e^2-c*d^2)^3/a^2/(-c*x^4+a)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(-c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^3}\,dx=\int -\frac{1}{\left(cx^4-a\right)^3\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-integrate(1/((c*x^4 - a)^3*(e*x^2 + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^3}\,dx = \int \frac{1}{(a-c\,x^4)^3\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(1/((a - c*x^4)^3*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a - c*x^4)^3*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^3}\,dx = \int \frac{1}{(e\,x^2+d)^{\frac{3}{2}}\,(-c\,x^4+a)^3}dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^3,x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^3,x)
```

3.365 $\int (d+ex^2)^{3/2} (a+cx^4)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 144

$$\int (d+ex^2)^{3/2} (a+cx^4)\, dx = \frac{3}{128} d\left(16a+\frac{cd^2}{e^2}\right) x\sqrt{d+ex^2} + \frac{1}{64}\left(16a+\frac{cd^2}{e^2}\right) x\left(d+ex^2\right)^{3/2} - \frac{cdx(d+ex^2)^{5/2}}{16e^2} + \frac{cx^3(d+ex^2)^{5/2}}{8e} + \frac{3d^2(cd^2+16ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{128e^{5/2}}$$

output
```
3/128*d*(16*a+c*d^2/e^2)*x*(e*x^2+d)^(1/2)+1/64*(16*a+c*d^2/e^2)*x*(e*x^2+
d)^(3/2)-1/16*c*d*x*(e*x^2+d)^(5/2)/e^2+1/8*c*x^3*(e*x^2+d)^(5/2)/e+3/128*
d^2*(16*a*e^2+c*d^2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.82

$$\int (d+ex^2)^{3/2} (a+cx^4)\, dx = \frac{\sqrt{d+ex^2}(-3cd^3x+80ade^2x+2cd^2ex^3+32ae^3x^3+24cde^2x^5+16ce^3x^7)}{128e^2} - \frac{3(cd^4+16ad^2e^2)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{128e^{5/2}}$$

input
```
Integrate[(d + e*x^2)^(3/2)*(a + c*x^4),x]
```

output
```
(Sqrt[d + e*x^2]*(-3*c*d^3*x + 80*a*d*e^2*x + 2*c*d^2*e*x^3 + 32*a*e^3*x^3
 + 24*c*d*e^2*x^5 + 16*c*e^3*x^7))/(128*e^2) - (3*(c*d^4 + 16*a*d^2*e^2)*L
og[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(128*e^(5/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1474, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right) \left(d+ex^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1474}$$

$$\frac{\int \left(8ae-3cdx^2\right) \left(ex^2+d\right)^{3/2}dx}{8e}+\frac{cx^3 \left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\left(16ae^2+cd^2\right) \int \left(ex^2+d\right)^{3/2}dx}{2e}-\frac{cdx \left(d+ex^2\right)^{5/2}}{2e}}{8e}+\frac{cx^3 \left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\left(16ae^2+cd^2\right) \left(\frac{3}{4} d \int \sqrt{ex^2+d}dx+\frac{1}{4} x \left(d+ex^2\right)^{3/2}\right)}{2e}-\frac{cdx \left(d+ex^2\right)^{5/2}}{2e}}{8e}+\frac{cx^3 \left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\left(16ae^2+cd^2\right) \left(\frac{3}{4} d \left(\frac{1}{2} d \int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2} x \sqrt{d+ex^2}\right)+\frac{1}{4} x \left(d+ex^2\right)^{3/2}\right)}{2e}-\frac{cdx \left(d+ex^2\right)^{5/2}}{2e}}{8e}+\frac{cx^3 \left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{(16ae^2+cd^2)\left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{2e}-\frac{cdx\left(d+ex^2\right)^{5/2}}{2e}}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow 219$$

$$\frac{\frac{(16ae^2+cd^2)\left(\frac{3}{4}d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{2e}-\frac{cdx\left(d+ex^2\right)^{5/2}}{2e}}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}}{8e}$$

input

```
Int[(d + e*x^2)^(3/2)*(a + c*x^4),x]
```

output

```
(c*x^3*(d + e*x^2)^(5/2))/(8*e) + (-1/2*(c*d*x*(d + e*x^2)^(5/2))/e + ((c*
d^2 + 16*a*e^2)*((x*(d + e*x^2)^(3/2))/4 + (3*d*((x*Sqrt[d + e*x^2])/2 + (
d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/4))/(2*e))/(8*e)
```

Defintions of rubi rules used

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\frac{5\left(\frac{3}{5} a\, d^{2} e^{2}+\frac{3}{80} d^{4} c\right) \operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x \sqrt{e}}\right)}{8}+\frac{5 x \left(d \left(\frac{3 c\,x^{4}}{10}+a\right) e^{\frac{5}{2}}+\frac{2 x^{2} \left(\frac{c\,x^{4}}{2}+a\right) e^{\frac{7}{2}}}{5}-\frac{3 d^{2} c \left(-\frac{2 e^{\frac{3}{2}} x^{2}}{3}+\sqrt{e}\, d\right)}{80}\right) \sqrt{e\,x^{2}+d}}{8}}{e^{\frac{5}{2}}}$
risch	$\frac{x \left(16 e^{3} c\,x^{6}+24 c d\,e^{2} x^{4}+32 a\,e^{3} x^{2}+2 c\,d^{2} e\,x^{2}+80 d\,e^{2} a-3 d^{3} c\right) \sqrt{e\,x^{2}+d}}{128 e^{2}}+\frac{3 d^{2} \left(16 a\,e^{2}+c\,d^{2}\right) \ln\left(x \sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{128 e^{\frac{5}{2}}}$
default	$a \left(\frac{x \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{4}+\frac{3 d \left(\frac{x \sqrt{e\,x^{2}+d}}{2}+\frac{d \ln\left(x \sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2 \sqrt{e}}\right)}{4}\right)+c \left(\frac{x^{3} \left(e\,x^{2}+d\right)^{\frac{5}{2}}}{8 e}-\frac{3 d \left(\frac{x \left(e\,x^{2}+d\right)^{\frac{5}{2}}}{6 e}-\frac{d \left(\frac{x \left(e\,x^{2}+d\right)}{4}\right.}{}\right.}{}\right.$

input
```
int((e*x^2+d)^(3/2)*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
5/8/e^(5/2)*((3/5*a*d^2*e^2+3/80*d^4*c)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))
+x*(d*(3/10*c*x^4+a)*e^(5/2)+2/5*x^2*(1/2*c*x^4+a)*e^(7/2)-3/80*d^2*c*(-2/
3*e^(3/2)*x^2+e^(1/2)*d))*(e*x^2+d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.64

$$\int \left(d+ex^2\right)^{3/2} \left(a +cx^4\right)\,dx = \left[\frac{3\left(cd^4+16\,ad^2e^2\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\left(16\,ce^4x^7+24\,cde^3x^5+2\left(cd^2e\right.\right.}{256\,e^3}\right.$$

$$-\frac{3\left(cd^4+16\,ad^2e^2\right)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\left(16\,ce^4x^7+24\,cde^3x^5+2\left(cd^2e^2+16\,ae^4\right)x^3-\left(3\,cd^3e-80\,a\right.\right.}{128\,e^3}$$

input
```
integrate((e*x^2+d)^(3/2)*(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/256*(3*(c*d^4 + 16*a*d^2*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*
sqrt(e)*x - d) + 2*(16*c*e^4*x^7 + 24*c*d*e^3*x^5 + 2*(c*d^2*e^2 + 16*a*e^
4)*x^3 - (3*c*d^3*e - 80*a*d*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/128*(3*(c*d^
4 + 16*a*d^2*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (16*c*e^4*
x^7 + 24*c*d*e^3*x^5 + 2*(c*d^2*e^2 + 16*a*e^4)*x^3 - (3*c*d^3*e - 80*a*d*
e^3)*x)*sqrt(e*x^2 + d))/e^3]
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.18

$$\int \left(d+ex^2\right)^{3/2} \left(a +cx^4\right)\,dx = \begin{cases} \sqrt{d+ex^2}\cdot\left(\frac{3cdx^5}{16}+\frac{cex^7}{8}+\frac{x^3\left(ae^2+\frac{cd^2}{16}\right)}{4e}+\frac{x\left(2ade-\frac{3d\left(ae^2+\frac{cd^2}{16}\right)}{4e}\right)}{2e}\right)+\left(ad^2-\frac{d\left(2ade-\frac{3d\left(ae^2+\frac{cd^2}{16}\right)}{4e}\right)}{2e}\right. \\ d^{\frac{3}{2}}\left(ax+\frac{cx^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**(3/2)*(c*x**4+a),x)
```

output
```
Piecewise((sqrt(d + e*x**2)*(3*c*d*x**5/16 + c*e*x**7/8 + x**3*(a*e**2 + c
*d**2/16)/(4*e) + x*(2*a*d*e - 3*d*(a*e**2 + c*d**2/16)/(4*e))/(2*e)) + (a
*d**2 - d*(2*a*d*e - 3*d*(a*e**2 + c*d**2/16)/(4*e))/(2*e))*Piecewise((log
(2*sqrt(e)*sqrt(d + e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqrt(e*
x**2), True)), Ne(e, 0)), (d**(3/2)*(a*x + c*x**5/5), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2}\left(a+cx^4\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.81

$$\int \left(d+ex^2\right)^{3/2}\left(a+cx^4\right)\,dx = \frac{1}{128}\left(2\left(4\left(2\,cex^2+3\,cd\right)x^2+\frac{cd^2e^5+16\,ae^7}{e^6}\right)x^2-\frac{3\,cd^3e^4-80\,ade^6}{e^6}\right)\sqrt{ex^2+d}\,x - \frac{3\left(cd^4+16\,ad^2e^2\right)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{128\,e^{\frac{5}{2}}}$$

input
```
integrate((e*x^2+d)^(3/2)*(c*x^4+a),x, algorithm="giac")
```

output
```
1/128*(2*(4*(2*c*e*x^2 + 3*c*d)*x^2 + (c*d^2*e^5 + 16*a*e^7)/e^6)*x^2 - (3
*c*d^3*e^4 - 80*a*d*e^6)/e^6)*sqrt(e*x^2 + d)*x - 3/128*(c*d^4 + 16*a*d^2*
e^2)*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2}\left(a+cx^4\right)\,dx=\int \left(c\,x^4+a\right)\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int((a + c*x^4)*(d + e*x^2)^(3/2),x)
```

output
```
int((a + c*x^4)*(d + e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.16

$$\int \left(d+ex^2\right)^{3/2}\left(a+cx^4\right)\,dx=\frac{80\sqrt{e\,x^2+d}\,a\,d\,e^3x+32\sqrt{e\,x^2+d}\,a\,e^4x^3-3\sqrt{e\,x^2+d}\,c\,d^3ex+2\sqrt{e\,x^2+d}\,c\,d^2e^2x^3+24\sqrt{e\,x^2}}{12}$$

input
```
int((e*x^2+d)^(3/2)*(c*x^4+a),x)
```

output
```
(80*sqrt(d + e*x**2)*a*d*e**3*x + 32*sqrt(d + e*x**2)*a*e**4*x**3 - 3*sqrt
(d + e*x**2)*c*d**3*e*x + 2*sqrt(d + e*x**2)*c*d**2*e**2*x**3 + 24*sqrt(d
+ e*x**2)*c*d*e**3*x**5 + 16*sqrt(d + e*x**2)*c*e**4*x**7 + 48*sqrt(e)*log
((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d**2*e**2 + 3*sqrt(e)*log((sqrt
(d + e*x**2) + sqrt(e)*x)/sqrt(d))*c*d**4)/(128*e**3)
```

3.366 $\int \sqrt{d+ex^2}\left(a+cx^4\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)\,dx = \frac{1}{16}\left(8a+\frac{cd^2}{e^2}\right)x\sqrt{d+ex^2}-\frac{cdx\left(d+ex^2\right)^{3/2}}{8e^2}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}+\frac{d\left(cd^2+8ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16e^{5/2}}$$

output

```
1/16*(8*a+c*d^2/e^2)*x*(e*x^2+d)^(1/2)-1/8*c*d*x*(e*x^2+d)^(3/2)/e^2+1/6*c
*x^3*(e*x^2+d)^(3/2)/e+1/16*d*(8*a*e^2+c*d^2)*arctanh(e^(1/2)*x/(e*x^2+d)^
(1/2))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)\,dx = \frac{x\sqrt{d+ex^2}\left(-3cd^2+24ae^2+2cdex^2+8ce^2x^4\right)}{48e^2}-\frac{d\left(cd^2+8ae^2\right)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{16e^{5/2}}$$

input

```
Integrate[Sqrt[d + e*x^2]*(a + c*x^4),x]
```

output
```
(x*Sqrt[d + e*x^2]*(-3*c*d^2 + 24*a*e^2 + 2*c*d*e*x^2 + 8*c*e^2*x^4))/(48*
e^2) - (d*(c*d^2 + 8*a*e^2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(16*e^(5/
2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1474, 27, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)\sqrt{d+ex^2}\,dx$$

$$\Big\downarrow \text{1474}$$

$$\frac{\int 3\left(2ae-cdx^2\right)\sqrt{ex^2+d}dx}{6e}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \left(2ae-cdx^2\right)\sqrt{ex^2+d}dx}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\left(8ae^2+cd^2\right)\int \sqrt{ex^2+d}dx}{4e}-\frac{cdx\left(d+ex^2\right)^{3/2}}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\left(8ae^2+cd^2\right)\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}-\frac{cdx\left(d+ex^2\right)^{3/2}}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\left(8ae^2+cd^2\right)\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}-\frac{cdx\left(d+ex^2\right)^{3/2}}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(8ae^2+cd^2)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}-\frac{cdx(d+ex^2)^{3/2}}{4e}}{2e}+\frac{cx^3(d+ex^2)^{3/2}}{6e}$$

input

```
Int[Sqrt[d + e*x^2]*(a + c*x^4),x]
```

output

```
(c*x^3*(d + e*x^2)^(3/2))/(6*e) + (-1/4*(c*d*x*(d + e*x^2)^(3/2))/e + ((c*
d^2 + 8*a*e^2)*((x*Sqrt[d + e*x^2])/2 + (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*
x^2]])/(2*Sqrt[e])))/(4*e))/(2*e)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{d\left(a e^2+\frac{c d^2}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{e x^2+d}}{x\sqrt{e}}\right)+x\sqrt{e x^2+d}\left(\left(\frac{c x^4}{3}+a\right)e^{\frac{5}{2}}-\frac{dc\left(-\frac{2e^{\frac{3}{2}}x^2}{3}+\sqrt{e}\,d\right)}{8}\right)}{2e^{\frac{5}{2}}}$
risch	$\frac{x\left(8c x^4 e^2+2de x^2 c+24a e^2-3c d^2\right)\sqrt{e x^2+d}}{48e^2}+\frac{d\left(8a e^2+c d^2\right)\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{16e^{\frac{5}{2}}}$
default	$a\left(\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}\right)+c\left(\frac{x^3\left(e x^2+d\right)^{\frac{3}{2}}}{6e}-\frac{d\left(\frac{x\left(e x^2+d\right)^{\frac{3}{2}}}{4e}-\frac{d\left(\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}\right.}{4e}\right.}{2e}\right.$

input
```
int((e*x^2+d)^(1/2)*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*(a*e^2+1/8*c*d^2)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))+x*(e*x^2+d)^(1
/2)*((1/3*c*x^4+a)*e^(5/2)-1/8*d*c*(-2/3*e^(3/2)*x^2+e^(1/2)*d)))/e^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.66

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)\,dx$$

$$= \left[\frac{3\,(cd^3+8\,ade^2)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\,(8\,ce^3x^5+2\,cde^2x^3-3\,(cd^2e-8\,ae^3)x)\sqrt{e}}{96\,e^3}\right.$$

$$\left.-\frac{3\,(cd^3+8\,ade^2)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(8\,ce^3x^5+2\,cde^2x^3-3\,(cd^2e-8\,ae^3)x)\sqrt{ex^2+d}}{48\,e^3}\right]$$

input
```
integrate((e*x^2+d)^(1/2)*(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/96*(3*(c*d^3 + 8*a*d*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt
(e)*x - d) + 2*(8*c*e^3*x^5 + 2*c*d*e^2*x^3 - 3*(c*d^2*e - 8*a*e^3)*x)*sqr
t(e*x^2 + d))/e^3, -1/48*(3*(c*d^3 + 8*a*d*e^2)*sqrt(-e)*arctan(sqrt(-e)*x
/sqrt(e*x^2 + d)) - (8*c*e^3*x^5 + 2*c*d*e^2*x^3 - 3*(c*d^2*e - 8*a*e^3)*x
)*sqrt(e*x^2 + d))/e^3]
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)\,dx$$

$$= \begin{cases} \sqrt{d+ex^2}\left(\frac{cdx^3}{24e}+\frac{cx^5}{6}+\frac{x\left(ae-\frac{cd^2}{8e}\right)}{2e}\right)+\left(ad-\frac{d\left(ae-\frac{cd^2}{8e}\right)}{2e}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{e}\sqrt{d+ex^2}+2ex\right)}{\sqrt{e}} & \text{for}\ d\neq 0\\ \frac{x\log(x)}{\sqrt{ex^2}} & \text{otherwise}\end{cases}\right) & \text{for}\ e \\ \sqrt{d}\left(ax+\frac{cx^5}{5}\right) & \text{other} \end{cases}$$

input
```
integrate((e*x**2+d)**(1/2)*(c*x**4+a),x)
```

output
```
Piecewise((sqrt(d + e*x**2)*(c*d*x**3/(24*e) + c*x**5/6 + x*(a*e - c*d**2/
(8*e))/(2*e)) + (a*d - d*(a*e - c*d**2/(8*e))/(2*e))*Piecewise((log(2*sqrt
(e)*sqrt(d + e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqrt(e*x**2),
True)), Ne(e, 0)), (sqrt(d)*(a*x + c*x**5/5), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)\,dx=\frac{1}{48}\left(2\left(4\,cx^2+\frac{cd}{e}\right)x^2-\frac{3\left(cd^2e^2-8\,ae^4\right)}{e^4}\right)\sqrt{ex^2+d}x-\frac{\left(cd^3+8\,ade^2\right)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{16\,e^{\frac{5}{2}}}$$

input
```
integrate((e*x^2+d)^(1/2)*(c*x^4+a),x, algorithm="giac")
```

output
```
1/48*(2*(4*c*x^2 + c*d/e)*x^2 - 3*(c*d^2*e^2 - 8*a*e^4)/e^4)*sqrt(e*x^2 +
d)*x - 1/16*(c*d^3 + 8*a*d*e^2)*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(
5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)\,dx=\int \left(c\,x^4+a\right)\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a + c*x^4)*(d + e*x^2)^(1/2),x)
```

output
```
int((a + c*x^4)*(d + e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)\,dx$$
$$=\frac{24\sqrt{e\,x^2+d}\,a\,e^3x-3\sqrt{e\,x^2+d}\,c\,d^2ex+2\sqrt{e\,x^2+d}\,cd\,e^2x^3+8\sqrt{e\,x^2+d}\,c\,e^3x^5+24\sqrt{e}\log\Bigg(\frac{\sqrt{e\,x^2+d}+\sqrt{e}}{\sqrt{d}}}{48e^3}$$

input
```
int((e*x^2+d)^(1/2)*(c*x^4+a),x)
```

output
```
(24*sqrt(d + e*x**2)*a*e**3*x - 3*sqrt(d + e*x**2)*c*d**2*e*x + 2*sqrt(d +
 e*x**2)*c*d*e**2*x**3 + 8*sqrt(d + e*x**2)*c*e**3*x**5 + 24*sqrt(e)*log((
sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d*e**2 + 3*sqrt(e)*log((sqrt(d +
e*x**2) + sqrt(e)*x)/sqrt(d))*c*d**3)/(48*e**3)
```

3.367 $\int \frac{a+cx^4}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \frac{a+cx^4}{\sqrt{d+ex^2}}\,dx=-\frac{3cdx\sqrt{d+ex^2}}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}+\frac{(3cd^2+8ae^2)\,\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{5/2}}$$

output

```
-3/8*c*d*x*(e*x^2+d)^(1/2)/e^2+1/4*c*x^3*(e*x^2+d)^(1/2)/e+1/8*(8*a*e^2+3*
c*d^2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{a+cx^4}{\sqrt{d+ex^2}}\,dx=\frac{c\sqrt{d+ex^2}(-3dx+2ex^3)}{8e^2}+\frac{(3cd^2+8ae^2)\,\operatorname{arctanh}\left(\frac{\sqrt{e}x}{-\sqrt{d}+\sqrt{d+ex^2}}\right)}{4e^{5/2}}$$

input

```
Integrate[(a + c*x^4)/Sqrt[d + e*x^2],x]
```

output

```
(c*Sqrt[d + e*x^2]*(-3*d*x + 2*e*x^3))/(8*e^2) + ((3*c*d^2 + 8*a*e^2)*ArcT
anh[(Sqrt[e]*x)/(-Sqrt[d] + Sqrt[d + e*x^2])])/(4*e^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1474, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \; {\color{blue}1474}$$

$$\frac{\int \frac{4ae-3cdx^2}{\sqrt{ex^2+d}}dx}{4e}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

$$\Big\downarrow \; {\color{blue}299}$$

$$\frac{\frac{(8ae^2+3cd^2)\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}-\frac{3cdx\sqrt{d+ex^2}}{2e}}{4e}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

$$\Big\downarrow \; {\color{blue}224}$$

$$\frac{\frac{(8ae^2+3cd^2)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}-\frac{3cdx\sqrt{d+ex^2}}{2e}}{4e}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

$$\Big\downarrow \; {\color{blue}219}$$

$$\frac{\frac{(8ae^2+3cd^2)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}-\frac{3cdx\sqrt{d+ex^2}}{2e}}{4e}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

input

```
Int[(a + c*x^4)/Sqrt[d + e*x^2],x]
```

output

```
(c*x^3*Sqrt[d + e*x^2])/(4*e) + ((-3*c*d*x*Sqrt[d + e*x^2])/(2*e) + ((3*c*
d^2 + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(3/2)))/(4*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{cx\left(-2e\,x^{2}+3d\right)\sqrt{e\,x^{2}+d}}{8e^{2}}+\frac{\left(8a\,e^{2}+3c\,d^{2}\right)\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{8e^{\frac{5}{2}}}$	62
default	$\frac{a\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{\sqrt{e}}+c\left(\frac{x^{3}\sqrt{e\,x^{2}+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)$	87
pseudoelliptic	$\frac{2c\,e^{\frac{3}{2}}x^{3}\sqrt{e\,x^{2}+d}-3cdx\sqrt{e\,x^{2}+d}\,\sqrt{e}+8\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)a\,e^{2}+3\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)c\,d^{2}}{8e^{\frac{5}{2}}}$	88

input
```
int((c*x^4+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*c*x*(-2*e*x^2+3*d)/e^2*(e*x^2+d)^(1/2)+1/8*(8*a*e^2+3*c*d^2)/e^(5/2)*
ln(x*e^(1/2)+(e*x^2+d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.72

$$\int \frac{a+cx^4}{\sqrt{d+ex^2}}\,dx = \left[\frac{(3\,cd^2+8\,ae^2)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\,(2\,ce^2x^3-3\,cdex)\sqrt{ex^2+d}}{16\,e^3}, -\frac{(3\,cd^2+8\,ae^2)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(2\,ce^2x^3-3\,cdex)\sqrt{ex^2+d}}{8\,e^3}\right]$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((3*c*d^2 + 8*a*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e
)*x - d) + 2*(2*c*e^2*x^3 - 3*c*d*e*x)*sqrt(e*x^2 + d))/e^3, -1/8*((3*c*d^
2 + 8*a*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*c*e^2*x^3 -
3*c*d*e*x)*sqrt(e*x^2 + d))/e^3]
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int \frac{a+cx^4}{\sqrt{d+ex^2}}\,dx = \begin{cases} \left(a+\frac{3cd^2}{8e^2}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{e}\sqrt{d+ex^2}+2ex\right)}{\sqrt{e}} & \text{for}\ d\neq 0\\ \frac{x\log(x)}{\sqrt{ex^2}} & \text{otherwise}\end{cases}\right)+\sqrt{d+ex^2}\left(-\frac{3cdx}{8e^2}+\frac{cx^3}{4e}\right) & \text{for}\ e\neq 0\\ \frac{ax+\frac{cx^5}{5}}{\sqrt{d}} & \text{otherwise}\end{cases}$$

input
```
integrate((c*x**4+a)/(e*x**2+d)**(1/2),x)
```

output
```
Piecewise(((a + 3*c*d**2/(8*e**2))*Piecewise((log(2*sqrt(e)*sqrt(d + e*x**
2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqrt(e*x**2), True)) + sqrt(d +
e*x**2)*(-3*c*d*x/(8*e**2) + c*x**3/(4*e)), Ne(e, 0)), ((a*x + c*x**5/5)/s
qrt(d), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int \frac{a+cx^4}{\sqrt{d+ex^2}}\,dx = \frac{1}{8}\,\sqrt{ex^2+d}\left(\frac{2\,cx^2}{e}-\frac{3\,cd}{e^2}\right)x-\frac{(3\,cd^2+8\,ae^2)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{8\,e^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(e*x^2 + d)*(2*c*x^2/e - 3*c*d/e^2)*x - 1/8*(3*c*d^2 + 8*a*e^2)*lo
g(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+cx^4}{\sqrt{d+ex^2}}\,dx=\int \frac{c\,x^4+a}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + c*x^4)/(d + e*x^2)^(1/2),x)
```

output
```
int((a + c*x^4)/(d + e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07

$$\int \frac{a+cx^4}{\sqrt{d+ex^2}}\,dx = \frac{-3\sqrt{e\,x^2+d}\,cdex+2\sqrt{e\,x^2+d}\,c\,e^2x^3+8\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,e^2+3\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)c\,d^2}{8e^3}$$

input
```
int((c*x^4+a)/(e*x^2+d)^(1/2),x)
```

output
```
( - 3*sqrt(d + e*x**2)*c*d*e*x + 2*sqrt(d + e*x**2)*c*e**2*x**3 + 8*sqrt(e
)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*e**2 + 3*sqrt(e)*log((sqrt
(d + e*x**2) + sqrt(e)*x)/sqrt(d))*c*d**2)/(8*e**3)
```

3.368 $\int \frac{a+cx^4}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{a+cx^4}{(d+ex^2)^{3/2}}\,dx=\frac{\left(a+\frac{cd^2}{e^2}\right)x}{d\sqrt{d+ex^2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}-\frac{3cd\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}$$

output
```
(a+c*d^2/e^2)*x/d/(e*x^2+d)^(1/2)+1/2*c*x*(e*x^2+d)^(1/2)/e^2-3/2*c*d*arct
anh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{a+cx^4}{(d+ex^2)^{3/2}}\,dx=\frac{x(3cd^2+2ae^2+cdex^2)}{2de^2\sqrt{d+ex^2}}+\frac{3cd\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{2e^{5/2}}$$

input
```
Integrate[(a + c*x^4)/(d + e*x^2)^(3/2),x]
```

output
```
(x*(3*c*d^2 + 2*a*e^2 + c*d*e*x^2))/(2*d*e^2*Sqrt[d + e*x^2]) + (3*c*d*Log
[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(2*e^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1472, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{d\sqrt{d+ex^2}}-\frac{\int \frac{cd\left(d-ex^2\right)}{e^2\sqrt{ex^2+d}}dx}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{d\sqrt{d+ex^2}}-\frac{c\int \frac{d-ex^2}{\sqrt{ex^2+d}}dx}{e^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{d\sqrt{d+ex^2}}-\frac{c\left(\frac{3}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{1}{2}x\sqrt{d+ex^2}\right)}{e^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{d\sqrt{d+ex^2}}-\frac{c\left(\frac{3}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{1}{2}x\sqrt{d+ex^2}\right)}{e^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{d\sqrt{d+ex^2}}-\frac{c\left(\frac{3d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}-\frac{1}{2}x\sqrt{d+ex^2}\right)}{e^2}$$

input

```
Int[(a + c*x^4)/(d + e*x^2)^(3/2),x]
```

output
```
((a + (c*d^2)/e^2)*x)/(d*Sqrt[d + e*x^2]) - (c*(-1/2*(x*Sqrt[d + e*x^2]) +
 (3*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/e^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{cx\sqrt{e\,x^{2}+d}}{2e^{2}}+\frac{cdx}{e^{2}\sqrt{e\,x^{2}+d}}+\frac{ax}{d\sqrt{e\,x^{2}+d}}-\frac{3cd\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2e^{\frac{5}{2}}}$	72
pseudoelliptic	$\frac{-\frac{3\sqrt{e\,x^{2}+d}\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)c\,d^{2}}{2}+x\left(\frac{1}{2}de\,x^{2}c+a\,e^{2}+\frac{3}{2}c\,d^{2}\right)\sqrt{e}}{\sqrt{e\,x^{2}+d}\,d\,e^{\frac{5}{2}}}$	75
default	$\frac{ax}{d\sqrt{e\,x^{2}+d}}+c\left(\frac{x^{3}}{2e\sqrt{e\,x^{2}+d}}-\frac{3d\left(-\frac{x}{e\sqrt{e\,x^{2}+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{e^{\frac{3}{2}}}\right)}{2e}\right)$	79

input
```
int((c*x^4+a)/(e*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*c*x*(e*x^2+d)^(1/2)/e^2+1/e^2*c*d*x/(e*x^2+d)^(1/2)+a*x/d/(e*x^2+d)^(1
/2)-3/2/e^(5/2)*c*d*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.61

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{3/2}}\,dx = \Bigg[\frac{3\,(cd^2ex^2+cd^3)\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\,(cde^2x^3+(3\,cd^2e+2\,ae}{4\,(de^4x^2+d^2e^3)}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(c*d^2*e*x^2 + c*d^3)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqr
t(e)*x - d) + 2*(c*d*e^2*x^3 + (3*c*d^2*e + 2*a*e^3)*x)*sqrt(e*x^2 + d))/(
d*e^4*x^2 + d^2*e^3), 1/2*(3*(c*d^2*e*x^2 + c*d^3)*sqrt(-e)*arctan(sqrt(-e
)*x/sqrt(e*x^2 + d)) + (c*d*e^2*x^3 + (3*c*d^2*e + 2*a*e^3)*x)*sqrt(e*x^2
 + d))/(d*e^4*x^2 + d^2*e^3)]
```

Sympy [A] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22

$$\int \frac{a+cx^4}{(d+ex^2)^{3/2}}\,dx = \frac{ax}{d^{\frac{3}{2}}\sqrt{1+\frac{ex^2}{d}}}+c\left(\frac{3\sqrt{d}x}{2e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{3d\,\mathrm{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{d}e\sqrt{1+\frac{ex^2}{d}}}\right)$$

input
```
integrate((c*x**4+a)/(e*x**2+d)**(3/2),x)
```

output
```
a*x/(d**(3/2)*sqrt(1 + e*x**2/d)) + c*(3*sqrt(d)*x/(2*e**2*sqrt(1 + e*x**2
/d)) - 3*d*asinh(sqrt(e)*x/sqrt(d))/(2*e**(5/2)) + x**3/(2*sqrt(d)*e*sqrt(
1 + e*x**2/d)))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time $= 0.29$ (sec) , antiderivative size $= 68$, normalized size of antiderivative $= 0.88$

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{\left(\frac{cx^2}{e}+\frac{3\,cd^2e+2\,ae^3}{de^3}\right)x}{2\,\sqrt{ex^2+d}}+\frac{3\,cd\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{2\,e^{\frac{5}{2}}}$$

input

```
integrate((c*x^4+a)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output

```
1/2*(c*x^2/e + (3*c*d^2*e + 2*a*e^3)/(d*e^3))*x/sqrt(e*x^2 + d) + 3/2*c*d*
log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{c\,x^4+a}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

input

```
int((a + c*x^4)/(d + e*x^2)^(3/2),x)
```

output

```
int((a + c*x^4)/(d + e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time $= 0.19$ (sec) , antiderivative size $= 165$, normalized size of antiderivative $= 2.14$

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{8\sqrt{e\,x^2+d}\,a\,e^3x+12\sqrt{e\,x^2+d}\,c\,d^2ex+4\sqrt{e\,x^2+d}\,cd\,e^2x^3-12\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right.}{8d\,e}$$

input

```
int((c*x^4+a)/(e*x^2+d)^(3/2),x)
```

output

```
(8*sqrt(d + e*x**2)*a*e**3*x + 12*sqrt(d + e*x**2)*c*d**2*e*x + 4*sqrt(d +
 e*x**2)*c*d*e**2*x**3 - 12*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqr
t(d))*c*d**3 - 12*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*c*d*
*2*e*x**2 + 8*sqrt(e)*a*d*e**2 + 8*sqrt(e)*a*e**3*x**2 + 9*sqrt(e)*c*d**3
+ 9*sqrt(e)*c*d**2*e*x**2)/(8*d*e**3*(d + e*x**2))
```

3.369 $\int \frac{a+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx=\frac{\left(a+\frac{cd^2}{e^2}\right)x}{3d\left(d+ex^2\right)^{3/2}}+\frac{2\left(\frac{a}{d^2}-\frac{2c}{e^2}\right)x}{3\sqrt{d+ex^2}}+\frac{c\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

output
```
1/3*(a+c*d^2/e^2)*x/d/(e*x^2+d)^(3/2)+2/3*(a/d^2-2*c/e^2)*x/(e*x^2+d)^(1/2
)+c*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx=\frac{-3cd^3x+3ade^2x-4cd^2ex^3+2ae^3x^3}{3d^2e^2\left(d+ex^2\right)^{3/2}}-\frac{c\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{e^{5/2}}$$

input
```
Integrate[(a + c*x^4)/(d + e*x^2)^(5/2),x]
```

output
```
(-3*c*d^3*x + 3*a*d*e^2*x - 4*c*d^2*e*x^3 + 2*a*e^3*x^3)/(3*d^2*e^2*(d + e
*x^2)^(3/2)) - (c*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/e^(5/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1472, 25, 25, 27, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 1472$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{3cdx^2+\left(2a-\frac{cd^2}{e^2}\right)e}{e\left(ex^2+d\right)^{3/2}}dx}{3d}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{\frac{cd^2}{e}-3cx^2d-2ae}{e\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\int \frac{\frac{cd^2}{e}-3cx^2d-2ae}{e\left(ex^2+d\right)^{3/2}}dx}{3d}$$

$$\Big\downarrow \ 27$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\int \frac{\frac{cd^2}{e}-3cx^2d-2ae}{\left(ex^2+d\right)^{3/2}}dx}{3de}$$

$$\Big\downarrow \ 298$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\frac{2x\left(\frac{2cd}{e}-\frac{ae}{d}\right)}{\sqrt{d+ex^2}}-\frac{3cd\int \frac{1}{\sqrt{ex^2+d}}dx}{e}}{3de}$$

$$\Big\downarrow \ 224$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\frac{2x\left(\frac{2cd}{e}-\frac{ae}{d}\right)}{\sqrt{d+ex^2}}-\frac{3cd\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}}{3de}$$

$$\Big\downarrow 219$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\frac{2x\left(\frac{2cd}{e}-\frac{ae}{d}\right)}{\sqrt{d+ex^2}}-\frac{3cd\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}}{3de}$$

input
```
Int[(a + c*x^4)/(d + e*x^2)^(5/2),x]
```

output
```
((a + (c*d^2)/e^2)*x)/(3*d*(d + e*x^2)^(3/2)) - ((2*((2*c*d)/e - (a*e)/d)*
x)/Sqrt[d + e*x^2] - (3*c*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(3/2))
/(3*d*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], 
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05

method	result	size
pseudoelliptic	$\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} c\,d^{2} \operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)+x\left(-\frac{4c\,d^{2} e^{\frac{3}{2}} x^{2}}{3}-c\,d^{3}\sqrt{e}+\frac{2a\,e^{\frac{7}{2}} x^{2}}{3}+a d\,e^{\frac{5}{2}}\right)}{e^{\frac{5}{2}} \left(e\,x^{2}+d\right)^{\frac{3}{2}} d^{2}}$	88
default	$a\left(\frac{x}{3d(e\,x^{2}+d)^{\frac{3}{2}}}+\frac{2x}{3d^{2}\sqrt{e\,x^{2}+d}}\right)+c\left(-\frac{x^{3}}{3e(e\,x^{2}+d)^{\frac{3}{2}}}+\frac{-\frac{x}{e\sqrt{e\,x^{2}+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{e^{\frac{3}{2}}}}{e}\right)$	95

input
```
int((c*x^4+a)/(e*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x^2+d)^(3/2)*c*d^2*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))+x*(-4/3*c*d^2*e^
(3/2)*x^2-c*d^3*e^(1/2)+2/3*a*e^(7/2)*x^2+a*d*e^(5/2)))/e^(5/2)/(e*x^2+d)^
(3/2)/d^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.30

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx = \left[\frac{3\left(cd^2e^2x^4+2\,cd^3ex^2+cd^4\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-2\left(2\left(2\,cd^2e^2-\right.\right.}{6\left(d^2e^5x^4+2\,d^3e^4x^2+d^4e^3\right)}\right.$$

$$\left.-\frac{3\left(cd^2e^2x^4+2\,cd^3ex^2+cd^4\right)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+\left(2\left(2\,cd^2e^2-ae^4\right)x^3+3\left(cd^3e-ade^3\right)x\right)\sqrt{ex^2+d}}{3\left(d^2e^5x^4+2\,d^3e^4x^2+d^4e^3\right)}\right]$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(e)*log(-2*e*x^2 - 2*s
qrt(e*x^2 + d)*sqrt(e)*x - d) - 2*(2*(2*c*d^2*e^2 - a*e^4)*x^3 + 3*(c*d^3*
e - a*d*e^3)*x)*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 + d^4*e^3),
-1/3*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(-e)*arctan(sqrt(-e)*x
/sqrt(e*x^2 + d)) + (2*(2*c*d^2*e^2 - a*e^4)*x^3 + 3*(c*d^3*e - a*d*e^3)*x
)*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 + d^4*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(80) = 160$.

Time = 4.83 (sec) , antiderivative size = 403, normalized size of antiderivative = 4.80

$$\int \frac{a+cx^4}{(d+ex^2)^{5/2}}\,dx = a\left(\frac{3dx}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}} + \frac{2ex^3}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}\right) + c\left(\frac{3d^{\frac{39}{2}}e^{11}\sqrt{1+\frac{ex^2}{d}}\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}} + \frac{3d^{\frac{37}{2}}e^{12}x^2\sqrt{1+\frac{ex^2}{d}}\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}} - \frac{3d^{19}e^{\frac{23}{2}}x}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}} - \frac{4d^{18}e^{\frac{25}{2}}x^3}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}}\right)$$

input
```
integrate((c*x**4+a)/(e*x**2+d)**(5/2),x)
```

output
```
a*(3*d*x/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 + e*x**
2/d)) + 2*e*x**3/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1
 + e*x**2/d))) + c*(3*d**(39/2)*e**11*sqrt(1 + e*x**2/d)*asinh(sqrt(e)*x/s
qrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*
x**2*sqrt(1 + e*x**2/d)) + 3*d**(37/2)*e**12*x**2*sqrt(1 + e*x**2/d)*asinh
(sqrt(e)*x/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2
)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) - 3*d**19*e**(23/2)*x/(3*d**(39/2)*e*
*(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)
) - 4*d**18*e**(25/2)*x**3/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d
**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{x\left(\frac{2\,(2\,cd^2e^2-ae^4)x^2}{d^2e^3}+\frac{3\,(cd^3e-ade^3)}{d^2e^3}\right)}{3\,(ex^2+d)^{\frac{3}{2}}}-\frac{c\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{e^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
-1/3*x*(2*(2*c*d^2*e^2 - a*e^4)*x^2/(d^2*e^3) + 3*(c*d^3*e - a*d*e^3)/(d^2
*e^3))/(e*x^2 + d)^(3/2) - c*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+cx^4}{(d+ex^2)^{5/2}}\,dx = \int \frac{c\,x^4+a}{{\left(e\,x^2+d\right)}^{5/2}} \,dx$$

input
```
int((a + c*x^4)/(d + e*x^2)^(5/2),x)
```

output
```
int((a + c*x^4)/(d + e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.65

$$\int \frac{a+cx^4}{(d+ex^2)^{5/2}}\,dx = \frac{3\sqrt{e\,x^2+d}\,a d\,e^3x + 2\sqrt{e\,x^2+d}\,a\,e^4x^3 - 3\sqrt{e\,x^2+d}\,c\,d^3ex - 4\sqrt{e\,x^2+d}\,c\,d^2e^2x^3 + 3}{}$$

input
```
int((c*x^4+a)/(e*x^2+d)^(5/2),x)
```

output
```
(3*sqrt(d + e*x**2)*a*d*e**3*x + 2*sqrt(d + e*x**2)*a*e**4*x**3 - 3*sqrt(d
 + e*x**2)*c*d**3*e*x - 4*sqrt(d + e*x**2)*c*d**2*e**2*x**3 + 3*sqrt(e)*lo
g((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*c*d**4 + 6*sqrt(e)*log((sqrt(d +
 e*x**2) + sqrt(e)*x)/sqrt(d))*c*d**3*e*x**2 + 3*sqrt(e)*log((sqrt(d + e*x
**2) + sqrt(e)*x)/sqrt(d))*c*d**2*e**2*x**4 - 2*sqrt(e)*a*d**2*e**2 - 4*sq
rt(e)*a*d*e**3*x**2 - 2*sqrt(e)*a*e**4*x**4)/(3*d**2*e**3*(d**2 + 2*d*e*x*
*2 + e**2*x**4))
```

3.370 $\int \frac{a+cx^4}{\left(d+ex^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{7/2}}\,dx=\frac{\left(a+\frac{cd^2}{e^2}\right)x}{5d\left(d+ex^2\right)^{5/2}}+\frac{2\left(\frac{2a}{d^2}-\frac{3c}{e^2}\right)x}{15\left(d+ex^2\right)^{3/2}}+\frac{\left(\frac{8a}{d^2}+\frac{3c}{e^2}\right)x}{15d\sqrt{d+ex^2}}$$

output
```
1/5*(a+c*d^2/e^2)*x/d/(e*x^2+d)^(5/2)+2/15*(2*a/d^2-3*c/e^2)*x/(e*x^2+d)^(
3/2)+1/15*(8*a/d^2+3*c/e^2)*x/d/(e*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{7/2}}\,dx=\frac{15ad^2x+20adex^3+3cd^2x^5+8ae^2x^5}{15d^3\left(d+ex^2\right)^{5/2}}$$

input
```
Integrate[(a + c*x^4)/(d + e*x^2)^(7/2),x]
```

output
```
(15*a*d^2*x + 20*a*d*e*x^3 + 3*c*d^2*x^5 + 8*a*e^2*x^5)/(15*d^3*(d + e*x^2
)^(5/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1470, 362, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{1470}$$

$$\frac{\int \frac{x^2\left(cdx^2+4ae\right)}{\left(ex^2+d\right)^{7/2}}dx}{d}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{1}{5}\left(\frac{8ae}{d}+\frac{3cd}{e}\right)\int \frac{x^2}{\left(ex^2+d\right)^{5/2}}dx-\frac{x^3\left(\frac{cd}{e}-\frac{4ae}{d}\right)}{5\left(d+ex^2\right)^{5/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{\frac{x^3\left(\frac{8ae}{d}+\frac{3cd}{e}\right)}{15d\left(d+ex^2\right)^{3/2}}-\frac{x^3\left(\frac{cd}{e}-\frac{4ae}{d}\right)}{5\left(d+ex^2\right)^{5/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

input
```
Int[(a + c*x^4)/(d + e*x^2)^(7/2),x]
```

output
```
(a*x)/(d*(d + e*x^2)^(5/2)) + (-1/5*(((c*d)/e - (4*a*e)/d)*x^3)/(d + e*x^2
)^(5/2) + (((3*c*d)/e + (8*a*e)/d)*x^3)/(15*d*(d + e*x^2)^(3/2)))/d
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1470
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d + e*x^2)^q*(d*
PolynomialQuotient[(a + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q
+ 1/2, 0] && LtQ[4*p + 2*q + 1, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.54

method	result
gosper	$\frac{x\left(8a\,e^{2}x^{4}+3c\,d^{2}x^{4}+20ade\,x^{2}+15a\,d^{2}\right)}{15\left(e\,x^{2}+d\right)^{\frac{5}{2}}d^{3}}$
trager	$\frac{x\left(8a\,e^{2}x^{4}+3c\,d^{2}x^{4}+20ade\,x^{2}+15a\,d^{2}\right)}{15\left(e\,x^{2}+d\right)^{\frac{5}{2}}d^{3}}$
pseudoelliptic	$\frac{x\left(8a\,e^{2}x^{4}+3c\,d^{2}x^{4}+20ade\,x^{2}+15a\,d^{2}\right)}{15\left(e\,x^{2}+d\right)^{\frac{5}{2}}d^{3}}$
orering	$\frac{x\left(8a\,e^{2}x^{4}+3c\,d^{2}x^{4}+20ade\,x^{2}+15a\,d^{2}\right)}{15\left(e\,x^{2}+d\right)^{\frac{5}{2}}d^{3}}$
default	$a\left(\frac{x}{5d(e\,x^{2}+d)^{\frac{5}{2}}}+\frac{\frac{4x}{15d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^{2}\sqrt{e\,x^{2}+d}}}{d}\right)+c\left(-\frac{x^{3}}{2e(e\,x^{2}+d)^{\frac{5}{2}}}+\frac{3d\left(-\frac{x}{4e\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{d\left(\frac{x}{5d\left(e\,x^{2}+d\right)^{\frac{5}{2}}}\right.}{}\right.}{2e}\right.$

input
```
int((c*x^4+a)/(e*x^2+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(8*a*e^2*x^4+3*c*d^2*x^4+20*a*d*e*x^2+15*a*d^2)/(e*x^2+d)^(5/2)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int \frac{a+cx^4}{(d+ex^2)^{7/2}}\,dx=\frac{(20\,adex^3+(3\,cd^2+8\,ae^2)x^5+15\,ad^2x)\sqrt{ex^2+d}}{15\,(d^3e^3x^6+3\,d^4e^2x^4+3\,d^5ex^2+d^6)}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(20*a*d*e*x^3 + (3*c*d^2 + 8*a*e^2)*x^5 + 15*a*d^2*x)*sqrt(e*x^2 + d)
/(d^3*e^3*x^6 + 3*d^4*e^2*x^4 + 3*d^5*e*x^2 + d^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(83) = 166$.

Time = 9.67 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.36

$$
\begin{aligned}
\int \frac{a+cx^4}{(d+ex^2)^{7/2}}\,dx = {} & a\left(\frac{15d^5x}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}\right. \\
& +\frac{35d^4ex^3}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}} \\
& +\frac{28d^3e^2x^5}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}} \\
& \left.+\frac{8d^2e^3x^7}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}\right) \\
& +\frac{cx^5}{5d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+10d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+5d^{\frac{3}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}}
\end{aligned}
$$

input
```
integrate((c*x**4+a)/(e*x**2+d)**(7/2),x)
```

output
```
a*(15*d**5*x/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1
 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e*
*3*x**6*sqrt(1 + e*x**2/d)) + 35*d**4*e*x**3/(15*d**(17/2)*sqrt(1 + e*x**2
/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt
(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 28*d**3*e**2
*x**5/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x*
*2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6
*sqrt(1 + e*x**2/d)) + 8*d**2*e**3*x**7/(15*d**(17/2)*sqrt(1 + e*x**2/d) +
 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 +
e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d))) + c*x**5/(5*d**(7/
2)*sqrt(1 + e*x**2/d) + 10*d**(5/2)*e*x**2*sqrt(1 + e*x**2/d) + 5*d**(3/2)
*e**2*x**4*sqrt(1 + e*x**2/d))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.31

$$\int \frac{a+cx^4}{(d+ex^2)^{7/2}}\,dx = -\frac{cx^3}{2\,(ex^2+d)^{\frac{5}{2}}e} + \frac{8\,ax}{15\,\sqrt{ex^2+d}d^3} + \frac{4\,ax}{15\,(ex^2+d)^{\frac{3}{2}}d^2} + \frac{ax}{5\,(ex^2+d)^{\frac{5}{2}}d} + \frac{cx}{10\,(ex^2+d)^{\frac{3}{2}}e^2} + \frac{cx}{5\,\sqrt{ex^2+d}de^2} - \frac{3\,cdx}{10\,(ex^2+d)^{\frac{5}{2}}e^2}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
-1/2*c*x^3/((e*x^2 + d)^(5/2)*e) + 8/15*a*x/(sqrt(e*x^2 + d)*d^3) + 4/15*a
*x/((e*x^2 + d)^(3/2)*d^2) + 1/5*a*x/((e*x^2 + d)^(5/2)*d) + 1/10*c*x/((e*
x^2 + d)^(3/2)*e^2) + 1/5*c*x/(sqrt(e*x^2 + d)*d*e^2) - 3/10*c*d*x/((e*x^2
 + d)^(5/2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

$$\int \frac{a+cx^4}{(d+ex^2)^{7/2}}\,dx = \frac{\left(x^2\left(\frac{20\,ae}{d^2} + \frac{(3\,cd^2e^2+8\,ae^4)x^2}{d^3e^2}\right) + \frac{15\,a}{d}\right)x}{15\,(ex^2+d)^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
1/15*(x^2*(20*a*e/d^2 + (3*c*d^2*e^2 + 8*a*e^4)*x^2/(d^3*e^2)) + 15*a/d)*x
/(e*x^2 + d)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 17.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^4}{(d+ex^2)^{7/2}}\,dx = \frac{3\,c\,d^4\,x-6\,c\,d^3\,x\,\left(e\,x^2+d\right)+8\,a\,e^2\,x\,\left(e\,x^2+d\right)^2+3\,c\,d^2\,x\,\left(e\,x^2+d\right)^2+3\,a\,d^2\,e^2\,x+}{15\,d^3\,e^2\,\left(e\,x^2+d\right)^{5/2}}$$

input
```
int((a + c*x^4)/(d + e*x^2)^(7/2),x)
```

output
```
(3*c*d^4*x - 6*c*d^3*x*(d + e*x^2) + 8*a*e^2*x*(d + e*x^2)^2 + 3*c*d^2*x*(
d + e*x^2)^2 + 3*a*d^2*e^2*x + 4*a*d*e^2*x*(d + e*x^2))/(15*d^3*e^2*(d + e
*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.30

$$\int \frac{a+cx^4}{(d+ex^2)^{7/2}}\,dx = \frac{15\sqrt{e\,x^2+d}\,a\,d^2e^3x+20\sqrt{e\,x^2+d}\,a d\,e^4x^3+8\sqrt{e\,x^2+d}\,a\,e^5x^5+3\sqrt{e\,x^2+d}\,c\,d^2e^3x}{15}$$

input
```
int((c*x^4+a)/(e*x^2+d)^(7/2),x)
```

output
```
(15*sqrt(d + e*x**2)*a*d**2*e**3*x + 20*sqrt(d + e*x**2)*a*d*e**4*x**3 + 8
*sqrt(d + e*x**2)*a*e**5*x**5 + 3*sqrt(d + e*x**2)*c*d**2*e**3*x**5 - 8*sq
rt(e)*a*d**3*e**2 - 24*sqrt(e)*a*d**2*e**3*x**2 - 24*sqrt(e)*a*d*e**4*x**4
 - 8*sqrt(e)*a*e**5*x**6 + 3*sqrt(e)*c*d**5 + 9*sqrt(e)*c*d**4*e*x**2 + 9*
sqrt(e)*c*d**3*e**2*x**4 + 3*sqrt(e)*c*d**2*e**3*x**6)/(15*d**3*e**3*(d**3
 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6))
```

3.371 $\int \frac{a+cx^4}{\left(d+ex^2\right)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{9/2}}\,dx = \frac{\left(a+\frac{cd^2}{e^2}\right)x}{7d\left(d+ex^2\right)^{7/2}} + \frac{2\left(\frac{3a}{d^2}-\frac{4c}{e^2}\right)x}{35\left(d+ex^2\right)^{5/2}} + \frac{\left(\frac{8a}{d^2}+\frac{c}{e^2}\right)x}{35d\left(d+ex^2\right)^{3/2}} + \frac{2(cd^2+8ae^2)\,x}{35d^4e^2\sqrt{d+ex^2}}$$

output
```
1/7*(a+c*d^2/e^2)*x/d/(e*x^2+d)^(7/2)+2/35*(3*a/d^2-4*c/e^2)*x/(e*x^2+d)^(
5/2)+1/35*(8*a/d^2+c/e^2)*x/d/(e*x^2+d)^(3/2)+2/35*(8*a*e^2+c*d^2)*x/d^4/e
^2/(e*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.60

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{9/2}}\,dx = \frac{35ad^3x+70ad^2ex^3+7cd^3x^5+56ade^2x^5+2cd^2ex^7+16ae^3x^7}{35d^4\left(d+ex^2\right)^{7/2}}$$

input
```
Integrate[(a + c*x^4)/(d + e*x^2)^(9/2),x]
```

output
```
(35*a*d^3*x + 70*a*d^2*e*x^3 + 7*c*d^3*x^5 + 56*a*d*e^2*x^5 + 2*c*d^2*e*x^
7 + 16*a*e^3*x^7)/(35*d^4*(d + e*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1470, 362, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 1470}$$

$$\frac{\int \frac{x^2\left(cdx^2+6ae\right)}{\left(ex^2+d\right)^{9/2}}dx}{d}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 362}$$

$$\frac{\frac{3}{7}\left(\frac{8ae}{d}+\frac{cd}{e}\right)\int \frac{x^2}{\left(ex^2+d\right)^{7/2}}dx-\frac{x^3\left(\frac{cd}{e}-\frac{6ae}{d}\right)}{7\left(d+ex^2\right)^{7/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 245}$$

$$\frac{\frac{3}{7}\left(\frac{8ae}{d}+\frac{cd}{e}\right)\left(\frac{2e\int \frac{x^4}{\left(ex^2+d\right)^{7/2}}dx}{3d}+\frac{x^3}{3d\left(d+ex^2\right)^{5/2}}\right)-\frac{x^3\left(\frac{cd}{e}-\frac{6ae}{d}\right)}{7\left(d+ex^2\right)^{7/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{\frac{3}{7}\left(\frac{2ex^5}{15d^2\left(d+ex^2\right)^{5/2}}+\frac{x^3}{3d\left(d+ex^2\right)^{5/2}}\right)\left(\frac{8ae}{d}+\frac{cd}{e}\right)-\frac{x^3\left(\frac{cd}{e}-\frac{6ae}{d}\right)}{7\left(d+ex^2\right)^{7/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

input
```
Int[(a + c*x^4)/(d + e*x^2)^(9/2),x]
```

output
```
(a*x)/(d*(d + e*x^2)^(7/2)) + (-1/7*(((c*d)/e - (6*a*e)/d)*x^3)/(d + e*x^2
)^(7/2) + (3*((c*d)/e + (8*a*e)/d)*(x^3/(3*d*(d + e*x^2)^(5/2)) + (2*e*x^5
)/(15*d^2*(d + e*x^2)^(5/2))))/7)/d
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1470
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d + e*x^2)^q*(d*
PolynomialQuotient[(a + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q
+ 1/2, 0] && LtQ[4*p + 2*q + 1, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{c\,x^{4}}{5}+a\right)d^{3}+2x^{2}\left(\frac{c\,x^{4}}{35}+a\right)e\,d^{2}+\frac{8ad\,e^{2}x^{4}}{5}+\frac{16a\,e^{3}x^{6}}{35}\right)}{\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$
gosper	$\frac{x\left(16a\,e^{3}x^{6}+2c\,d^{2}e\,x^{6}+56ad\,e^{2}x^{4}+7c\,d^{3}x^{4}+70a\,d^{2}e\,x^{2}+35a\,d^{3}\right)}{35\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$
trager	$\frac{x\left(16a\,e^{3}x^{6}+2c\,d^{2}e\,x^{6}+56ad\,e^{2}x^{4}+7c\,d^{3}x^{4}+70a\,d^{2}e\,x^{2}+35a\,d^{3}\right)}{35\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$
orering	$\frac{x\left(16a\,e^{3}x^{6}+2c\,d^{2}e\,x^{6}+56ad\,e^{2}x^{4}+7c\,d^{3}x^{4}+70a\,d^{2}e\,x^{2}+35a\,d^{3}\right)}{35\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$
default	$a\left(\frac{x}{7d\left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35d\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^{2}\sqrt{e\,x^{2}+d}}\right)}{7d}}{d}\right)+c\left(-\frac{x^{3}}{4e\left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{3d\left(-\frac{x}{6e\left(e\,x^{2}+d\right)}\right.}{}\right.$

input

```
int((c*x^4+a)/(e*x^2+d)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
x/(e*x^2+d)^(7/2)*((1/5*c*x^4+a)*d^3+2*x^2*(1/35*c*x^4+a)*e*d^2+8/5*a*d*e^
2*x^4+16/35*a*e^3*x^6)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{a+cx^4}{(d+ex^2)^{9/2}}\,dx = \frac{(2\,(cd^2e+8\,ae^3)x^7+70\,ad^2ex^3+7\,(cd^3+8\,ade^2)x^5+35\,ad^3x)\sqrt{ex^2+d}}{35\,(d^4e^4x^8+4\,d^5e^3x^6+6\,d^6e^2x^4+4\,d^7ex^2+d^8)}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(2*(c*d^2*e + 8*a*e^3)*x^7 + 70*a*d^2*e*x^3 + 7*(c*d^3 + 8*a*d*e^2)*x
^5 + 35*a*d^3*x)*sqrt(e*x^2 + d)/(d^4*e^4*x^8 + 4*d^5*e^3*x^6 + 6*d^6*e^2*
x^4 + 4*d^7*e*x^2 + d^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(119) = 238$.

Time = 22.09 (sec) , antiderivative size = 1469, normalized size of antiderivative = 11.85

$$\int \frac{a+cx^4}{(d+ex^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x**4+a)/(e*x**2+d)**(9/2),x)
```

output

```
a*(35*d**14*x/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt
(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2
)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d
) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*
sqrt(1 + e*x**2/d)) + 175*d**13*e*x**3/(35*d**(37/2)*sqrt(1 + e*x**2/d) +
210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 +
 e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e*
*4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) +
 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 371*d**12*e**2*x**5/(35*d**
(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*
d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 +
e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**
5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) +
 429*d**11*e**3*x**7/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x*
*2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d
**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e
*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6
*x**12*sqrt(1 + e*x**2/d)) + 286*d**10*e**4*x**9/(35*d**(37/2)*sqrt(1 + e*
x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**
4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \frac{a+cx^4}{(d+ex^2)^{9/2}}\,dx = &-\frac{cx^3}{4\,(ex^2+d)^{\frac{7}{2}}e}+\frac{16\,ax}{35\,\sqrt{ex^2+d}d^4}+\frac{8\,ax}{35\,(ex^2+d)^{\frac{3}{2}}d^3}\\&+\frac{6\,ax}{35\,(ex^2+d)^{\frac{5}{2}}d^2}+\frac{ax}{7\,(ex^2+d)^{\frac{7}{2}}d}+\frac{3\,cx}{140\,(ex^2+d)^{\frac{5}{2}}e^2}\\&+\frac{2\,cx}{35\,\sqrt{ex^2+d}d^2e^2}+\frac{cx}{35\,(ex^2+d)^{\frac{3}{2}}de^2}-\frac{3\,cdx}{28\,(ex^2+d)^{\frac{7}{2}}e^2}\end{aligned}$$

input

```
integrate((c*x^4+a)/(e*x^2+d)^(9/2),x, algorithm="maxima")
```

output

```
-1/4*c*x^3/((e*x^2 + d)^(7/2)*e) + 16/35*a*x/(sqrt(e*x^2 + d)*d^4) + 8/35*
a*x/((e*x^2 + d)^(3/2)*d^3) + 6/35*a*x/((e*x^2 + d)^(5/2)*d^2) + 1/7*a*x/(
(e*x^2 + d)^(7/2)*d) + 3/140*c*x/((e*x^2 + d)^(5/2)*e^2) + 2/35*c*x/(sqrt(
e*x^2 + d)*d^2*e^2) + 1/35*c*x/((e*x^2 + d)^(3/2)*d*e^2) - 3/28*c*d*x/((e*
x^2 + d)^(7/2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

$$\int \frac{a+cx^4}{(d+ex^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{2\,(cd^2e^4+8\,ae^6)x^2}{d^4e^3}+\frac{7\,(cd^3e^3+8\,ade^5)}{d^4e^3}\right)+\frac{70\,ae}{d^2}\right)x^2+\frac{35\,a}{d}\right)x}{35\,(ex^2+d)^{\frac{7}{2}}}$$

input

```
integrate((c*x^4+a)/(e*x^2+d)^(9/2),x, algorithm="giac")
```

output

```
1/35*((x^2*(2*(c*d^2*e^4 + 8*a*e^6)*x^2/(d^4*e^3) + 7*(c*d^3*e^3 + 8*a*d*e
^5)/(d^4*e^3)) + 70*a*e/d^2)*x^2 + 35*a/d)*x/(e*x^2 + d)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 17.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{a+cx^4}{(d+ex^2)^{9/2}}\,dx = \frac{x\left(\frac{a}{7\,d}+\frac{c\,d}{7\,e^2}\right)}{\left(e\,x^2+d\right)^{7/2}}-\frac{x\left(\frac{c}{5\,e^2}-\frac{6\,a\,e^2-c\,d^2}{35\,d^2\,e^2}\right)}{\left(e\,x^2+d\right)^{5/2}} + \frac{x\,\left(c\,d^2+8\,a\,e^2\right)}{35\,d^3\,e^2\,\left(e\,x^2+d\right)^{3/2}}+\frac{x\,\left(2\,c\,d^2+16\,a\,e^2\right)}{35\,d^4\,e^2\,\sqrt{e\,x^2+d}}$$

input

```
int((a + c*x^4)/(d + e*x^2)^(9/2),x)
```

output

```
(x*(a/(7*d) + (c*d)/(7*e^2)))/(d + e*x^2)^(7/2) - (x*(c/(5*e^2) - (6*a*e^2
 - c*d^2)/(35*d^2*e^2)))/(d + e*x^2)^(5/2) + (x*(8*a*e^2 + c*d^2))/(35*d^3
*e^2*(d + e*x^2)^(3/2)) + (x*(16*a*e^2 + 2*c*d^2))/(35*d^4*e^2*(d + e*x^2)
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.32

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{9/2}}\,dx = \frac{35\sqrt{e\,x^2+d}\,a\,d^3e^3x+70\sqrt{e\,x^2+d}\,a\,d^2e^4x^3+56\sqrt{e\,x^2+d}\,ad\,e^5x^5+16\sqrt{e\,x^2+d}\,a\,e}{}$$

input
```
int((c*x^4+a)/(e*x^2+d)^(9/2),x)
```

output
```
(35*sqrt(d + e*x**2)*a*d**3*e**3*x + 70*sqrt(d + e*x**2)*a*d**2*e**4*x**3
+ 56*sqrt(d + e*x**2)*a*d*e**5*x**5 + 16*sqrt(d + e*x**2)*a*e**6*x**7 + 7*
sqrt(d + e*x**2)*c*d**3*e**3*x**5 + 2*sqrt(d + e*x**2)*c*d**2*e**4*x**7 -
16*sqrt(e)*a*d**4*e**2 - 64*sqrt(e)*a*d**3*e**3*x**2 - 96*sqrt(e)*a*d**2*e
**4*x**4 - 64*sqrt(e)*a*d*e**5*x**6 - 16*sqrt(e)*a*e**6*x**8 - 2*sqrt(e)*c
*d**6 - 8*sqrt(e)*c*d**5*e*x**2 - 12*sqrt(e)*c*d**4*e**2*x**4 - 8*sqrt(e)*
c*d**3*e**3*x**6 - 2*sqrt(e)*c*d**2*e**4*x**8)/(35*d**4*e**3*(d**4 + 4*d**
3*e*x**2 + 6*d**2*e**2*x**4 + 4*d*e**3*x**6 + e**4*x**8))
```

3.372 $\int \frac{a+cx^4}{\left(d+ex^2\right)^{11/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 158

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{11/2}}\,dx=\frac{\left(a+\frac{cd^2}{e^2}\right)x}{9d\left(d+ex^2\right)^{9/2}}+\frac{2\left(\frac{4a}{d^2}-\frac{5c}{e^2}\right)x}{63\left(d+ex^2\right)^{7/2}}+\frac{\left(\frac{16a}{d^2}+\frac{c}{e^2}\right)x}{105d\left(d+ex^2\right)^{5/2}}+\frac{4(cd^2+16ae^2)\,x}{315d^4e^2\left(d+ex^2\right)^{3/2}}+\frac{8(cd^2+16ae^2)\,x}{315d^5e^2\sqrt{d+ex^2}}$$

output

```
1/9*(a+c*d^2/e^2)*x/d/(e*x^2+d)^(9/2)+2/63*(4*a/d^2-5*c/e^2)*x/(e*x^2+d)^(
7/2)+1/105*(16*a/d^2+c/e^2)*x/d/(e*x^2+d)^(5/2)+4/315*(16*a*e^2+c*d^2)*x/d
^4/e^2/(e*x^2+d)^(3/2)+8/315*(16*a*e^2+c*d^2)*x/d^5/e^2/(e*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.59

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{11/2}}\,dx=\frac{cd^2x^5(63d^2+36dex^2+8e^2x^4)+a(315d^4x+840d^3ex^3+1008d^2e^2x^5+576de^3x^7+1}{315d^5\left(d+ex^2\right)^{9/2}}$$

input

```
Integrate[(a + c*x^4)/(d + e*x^2)^(11/2),x]
```

output

```
(c*d^2*x^5*(63*d^2 + 36*d*e*x^2 + 8*e^2*x^4) + a*(315*d^4*x + 840*d^3*e*x^
3 + 1008*d^2*e^2*x^5 + 576*d*e^3*x^7 + 128*e^4*x^9))/(315*d^5*(d + e*x^2)^
(9/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1470, 362, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{11/2}}dx$$

$$\Big\downarrow \text{1470}$$

$$\frac{\int \frac{x^2\left(cdx^2+8ae\right)}{\left(ex^2+d\right)^{11/2}}dx}{d}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{1}{3}\left(\frac{16ae}{d}+\frac{cd}{e}\right)\int \frac{x^2}{\left(ex^2+d\right)^{9/2}}dx-\frac{x^3\left(\frac{cd}{e}-\frac{8ae}{d}\right)}{9\left(d+ex^2\right)^{9/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{1}{3}\left(\frac{16ae}{d}+\frac{cd}{e}\right)\left(\frac{4e\int \frac{x^4}{\left(ex^2+d\right)^{9/2}}dx}{3d}+\frac{x^3}{3d\left(d+ex^2\right)^{7/2}}\right)-\frac{x^3\left(\frac{cd}{e}-\frac{8ae}{d}\right)}{9\left(d+ex^2\right)^{9/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{1}{3}\left(\frac{16ae}{d}+\frac{cd}{e}\right)\left(\frac{4e\left(\frac{2e\int \frac{x^6}{\left(ex^2+d\right)^{9/2}}dx}{5d}+\frac{x^5}{5d\left(d+ex^2\right)^{7/2}}\right)}{3d}+\frac{x^3}{3d(d+ex^2)^{7/2}}\right)-\frac{x^3\left(\frac{cd}{e}-\frac{8ae}{d}\right)}{9(d+ex^2)^{9/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{\frac{1}{3}\left(\frac{4e\left(\frac{2ex^7}{35d^2\left(d+ex^2\right)^{7/2}}+\frac{x^5}{5d\left(d+ex^2\right)^{7/2}}\right)}{3d}+\frac{x^3}{3d(d+ex^2)^{7/2}}\right)\left(\frac{16ae}{d}+\frac{cd}{e}\right)-\frac{x^3\left(\frac{cd}{e}-\frac{8ae}{d}\right)}{9(d+ex^2)^{9/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

input
```
Int[(a + c*x^4)/(d + e*x^2)^(11/2),x]
```

output
```
(a*x)/(d*(d + e*x^2)^(9/2)) + (-1/9*(((c*d)/e - (8*a*e)/d)*x^3)/(d + e*x^2
)^(9/2) + (((c*d)/e + (16*a*e)/d)*(x^3/(3*d*(d + e*x^2)^(7/2)) + (4*e*(x^5
/(5*d*(d + e*x^2)^(7/2)) + (2*e*x^7)/(35*d^2*(d + e*x^2)^(7/2))))/(3*d)))/
3)/d
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1470
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d + e*x^2)^q*(d*
PolynomialQuotient[(a + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q
+ 1/2, 0] && LtQ[4*p + 2*q + 1, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.53

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{c\,x^{4}}{5}+a\right)d^{4}+\frac{8\left(\frac{3c\,x^{4}}{70}+a\right)x^{2}e\,d^{3}}{3}+\frac{16x^{4}\left(\frac{c\,x^{4}}{126}+a\right)e^{2}d^{2}}{5}+\frac{64ad\,e^{3}x^{6}}{35}+\frac{128a\,e^{4}x^{8}}{315}\right)}{\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
gosper	$\frac{x\left(128a\,e^{4}x^{8}+8c\,d^{2}e^{2}x^{8}+576ad\,e^{3}x^{6}+36c\,d^{3}e\,x^{6}+1008a\,d^{2}e^{2}x^{4}+63c\,d^{4}x^{4}+840a\,d^{3}e\,x^{2}+315d^{4}a\right)}{315\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
trager	$\frac{x\left(128a\,e^{4}x^{8}+8c\,d^{2}e^{2}x^{8}+576ad\,e^{3}x^{6}+36c\,d^{3}e\,x^{6}+1008a\,d^{2}e^{2}x^{4}+63c\,d^{4}x^{4}+840a\,d^{3}e\,x^{2}+315d^{4}a\right)}{315\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
orering	$\frac{x\left(128a\,e^{4}x^{8}+8c\,d^{2}e^{2}x^{8}+576ad\,e^{3}x^{6}+36c\,d^{3}e\,x^{6}+1008a\,d^{2}e^{2}x^{4}+63c\,d^{4}x^{4}+840a\,d^{3}e\,x^{2}+315d^{4}a\right)}{315\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
default	$a\left(\frac{x}{9d\left(e\,x^{2}+d\right)^{\frac{9}{2}}}+\frac{\frac{8x}{63d\left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35d\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^{2}\sqrt{e\,x^{2}+d}}\right)}{7d}\right)}{9d}}{d}\right)+c\left(-\frac{x^{3}}{6e\left(e\,x^{2}+d\right)^{\frac{9}{2}}}\right.$

input
```
int((c*x^4+a)/(e*x^2+d)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
x/(e*x^2+d)^(9/2)*((1/5*c*x^4+a)*d^4+8/3*(3/70*c*x^4+a)*x^2*e*d^3+16/5*x^4
*(1/126*c*x^4+a)*e^2*d^2+64/35*a*d*e^3*x^6+128/315*a*e^4*x^8)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{11/2}}\,dx = \frac{(8\,(cd^2e^2+16\,ae^4)x^9+840\,ad^3ex^3+36\,(cd^3e+16\,ade^3)x^7+315\,ad^4x+63\,(cd^4+1}{315\,(d^5e^5x^{10}+5\,d^6e^4x^8+10\,d^7e^3x^6+10\,d^8e^2x^4+5\,d^9ex^2+d^{10}}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(11/2),x, algorithm="fricas")
```

output
```
1/315*(8*(c*d^2*e^2 + 16*a*e^4)*x^9 + 840*a*d^3*e*x^3 + 36*(c*d^3*e + 16*a
*d*e^3)*x^7 + 315*a*d^4*x + 63*(c*d^4 + 16*a*d^2*e^2)*x^5)*sqrt(e*x^2 + d)
/(d^5*e^5*x^10 + 5*d^6*e^4*x^8 + 10*d^7*e^3*x^6 + 10*d^8*e^2*x^4 + 5*d^9*e
*x^2 + d^10)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3738 vs. 2(155) = 310.

Time = 47.79 (sec) , antiderivative size = 3738, normalized size of antiderivative = 23.66

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x**4+a)/(e*x**2+d)**(11/2),x)
```

output

```
a*(315*d**30*x/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e*x**2*s
qrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 37800*d
**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*sqrt(1 +
 e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*d**(59/
2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(1 + e*x
**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(53/2)*e*
*9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*x**2/d)
) + 2730*d**29*e*x**3/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e
*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) +
37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*s
qrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*
d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(
1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(5
3/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*
x**2/d)) + 10773*d**28*e**2*x**5/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*
d**(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e
*x**2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*
e**4*x**8*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/
d) + 66150*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*
x**14*sqrt(1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.20

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^{11/2}}\,dx = -\frac{cx^3}{6\,(ex^2+d)^{\frac{9}{2}}e} + \frac{128\,ax}{315\,\sqrt{ex^2+d}d^5} + \frac{64\,ax}{315\,(ex^2+d)^{\frac{3}{2}}d^4} + \frac{16\,ax}{105\,(ex^2+d)^{\frac{5}{2}}d^3} + \frac{8\,ax}{63\,(ex^2+d)^{\frac{7}{2}}d^2} + \frac{ax}{9\,(ex^2+d)^{\frac{9}{2}}d} + \frac{cx}{126\,(ex^2+d)^{\frac{7}{2}}e^2} + \frac{8\,cx}{315\,\sqrt{ex^2+d}d^3e^2} + \frac{4\,cx}{315\,(ex^2+d)^{\frac{3}{2}}d^2e^2} + \frac{cx}{105\,(ex^2+d)^{\frac{5}{2}}de^2} - \frac{cdx}{18\,(ex^2+d)^{\frac{9}{2}}e^2}$$

input

```
integrate((c*x^4+a)/(e*x^2+d)^(11/2),x, algorithm="maxima")
```

output
```
-1/6*c*x^3/((e*x^2 + d)^(9/2)*e) + 128/315*a*x/(sqrt(e*x^2 + d)*d^5) + 64/
315*a*x/((e*x^2 + d)^(3/2)*d^4) + 16/105*a*x/((e*x^2 + d)^(5/2)*d^3) + 8/6
3*a*x/((e*x^2 + d)^(7/2)*d^2) + 1/9*a*x/((e*x^2 + d)^(9/2)*d) + 1/126*c*x/
((e*x^2 + d)^(7/2)*e^2) + 8/315*c*x/(sqrt(e*x^2 + d)*d^3*e^2) + 4/315*c*x/
((e*x^2 + d)^(3/2)*d^2*e^2) + 1/105*c*x/((e*x^2 + d)^(5/2)*d*e^2) - 1/18*c
*d*x/((e*x^2 + d)^(9/2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.75

$$\int \frac{a+cx^4}{(d+ex^2)^{11/2}}\,dx = \frac{\left(\left(\left(4\,x^2\left(\frac{2\,(cd^2e^6+16\,ae^8)x^2}{d^5e^4}+\frac{9\,(cd^3e^5+16\,ade^7)}{d^5e^4}\right)+\frac{63\,(cd^4e^4+16\,ad^2e^6)}{d^5e^4}\right)x^2+\frac{840\,ae}{d^2}\right)x^2+\frac{315\,a}{d}}{315\,(ex^2+d)^{\frac{9}{2}}}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^(11/2),x, algorithm="giac")
```

output
```
1/315*(((4*x^2*(2*(c*d^2*e^6 + 16*a*e^8)*x^2/(d^5*e^4) + 9*(c*d^3*e^5 + 16
*a*d*e^7)/(d^5*e^4)) + 63*(c*d^4*e^4 + 16*a*d^2*e^6)/(d^5*e^4))*x^2 + 840*
a*e/d^2)*x^2 + 315*a/d)*x/(e*x^2 + d)^(9/2)
```

Mupad [B] (verification not implemented)

Time = 17.51 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{a+cx^4}{(d+ex^2)^{11/2}}\,dx &= \frac{x\left(\frac{a}{9\,d}+\frac{c\,d}{9\,e^2}\right)}{\left(e\,x^2+d\right)^{9/2}}-\frac{x\left(\frac{c}{7\,e^2}-\frac{8\,a\,e^2-c\,d^2}{63\,d^2\,e^2}\right)}{\left(e\,x^2+d\right)^{7/2}}\\&\quad+\frac{x\,(c\,d^2+16\,a\,e^2)}{105\,d^3\,e^2\,\left(e\,x^2+d\right)^{5/2}}+\frac{x\,(4\,c\,d^2+64\,a\,e^2)}{315\,d^4\,e^2\,\left(e\,x^2+d\right)^{3/2}}+\frac{x\,(8\,c\,d^2+128\,a\,e^2)}{315\,d^5\,e^2\,\sqrt{e\,x^2+d}}\end{aligned}$$

input
```
int((a + c*x^4)/(d + e*x^2)^(11/2),x)
```

output
```
(x*(a/(9*d) + (c*d)/(9*e^2)))/(d + e*x^2)^(9/2) - (x*(c/(7*e^2) - (8*a*e^2
 - c*d^2)/(63*d^2*e^2)))/(d + e*x^2)^(7/2) + (x*(16*a*e^2 + c*d^2))/(105*d
^3*e^2*(d + e*x^2)^(5/2)) + (x*(64*a*e^2 + 4*c*d^2))/(315*d^4*e^2*(d + e*x
^2)^(3/2)) + (x*(128*a*e^2 + 8*c*d^2))/(315*d^5*e^2*(d + e*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.32

$$\int \frac{a+cx^4}{(d+ex^2)^{11/2}}\,dx = \frac{315\sqrt{e\,x^2+d}\,a\,d^4e^3x+840\sqrt{e\,x^2+d}\,a\,d^3e^4x^3+1008\sqrt{e\,x^2+d}\,a\,d^2e^5x^5+576\sqrt{e\,x^2}}{}$$

input
```
int((c*x^4+a)/(e*x^2+d)^(11/2),x)
```

output
```
(315*sqrt(d + e*x**2)*a*d**4*e**3*x + 840*sqrt(d + e*x**2)*a*d**3*e**4*x**
3 + 1008*sqrt(d + e*x**2)*a*d**2*e**5*x**5 + 576*sqrt(d + e*x**2)*a*d*e**6
*x**7 + 128*sqrt(d + e*x**2)*a*e**7*x**9 + 63*sqrt(d + e*x**2)*c*d**4*e**3
*x**5 + 36*sqrt(d + e*x**2)*c*d**3*e**4*x**7 + 8*sqrt(d + e*x**2)*c*d**2*e
**5*x**9 - 128*sqrt(e)*a*d**5*e**2 - 640*sqrt(e)*a*d**4*e**3*x**2 - 1280*s
qrt(e)*a*d**3*e**4*x**4 - 1280*sqrt(e)*a*d**2*e**5*x**6 - 640*sqrt(e)*a*d*
e**6*x**8 - 128*sqrt(e)*a*e**7*x**10 - 8*sqrt(e)*c*d**7 - 40*sqrt(e)*c*d**
6*e*x**2 - 80*sqrt(e)*c*d**5*e**2*x**4 - 80*sqrt(e)*c*d**4*e**3*x**6 - 40*
sqrt(e)*c*d**3*e**4*x**8 - 8*sqrt(e)*c*d**2*e**5*x**10)/(315*d**5*e**3*(d*
*5 + 5*d**4*e*x**2 + 10*d**3*e**2*x**4 + 10*d**2*e**3*x**6 + 5*d*e**4*x**8
 + e**5*x**10))
```

3.373 $\int (d+ex^2)^{3/2} (a+cx^4)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 270

$$\int (d+ex^2)^{3/2} (a+cx^4)^2 \, dx = \frac{d(7c^2d^4+48acd^2e^2+384a^2e^4)\,x\sqrt{d+ex^2}}{1024e^4} + \frac{(7c^2d^4+48acd^2e^2+384a^2e^4)\,x(d+ex^2)^{3/2}}{1536e^4} - \frac{cd(7cd^2+48ae^2)\,x(d+ex^2)^{5/2}}{384e^4} + \frac{c(7cd^2+48ae^2)\,x^3(d+ex^2)^{5/2}}{192e^3} - \frac{7c^2dx^5(d+ex^2)^{5/2}}{120e^2} + \frac{c^2x^7(d+ex^2)^{5/2}}{12e} + \frac{d^2(7c^2d^4+48acd^2e^2+384a^2e^4)\,\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{1024e^{9/2}}$$

output

```
1/1024*d*(384*a^2*e^4+48*a*c*d^2*e^2+7*c^2*d^4)*x*(e*x^2+d)^(1/2)/e^4+1/15
36*(384*a^2*e^4+48*a*c*d^2*e^2+7*c^2*d^4)*x*(e*x^2+d)^(3/2)/e^4-1/384*c*d*
(48*a*e^2+7*c*d^2)*x*(e*x^2+d)^(5/2)/e^4+1/192*c*(48*a*e^2+7*c*d^2)*x^3*(e
*x^2+d)^(5/2)/e^3-7/120*c^2*d*x^5*(e*x^2+d)^(5/2)/e^2+1/12*c^2*x^7*(e*x^2+
d)^(5/2)/e+1/1024*d^2*(384*a^2*e^4+48*a*c*d^2*e^2+7*c^2*d^4)*arctanh(e^(1/
2)*x/(e*x^2+d)^(1/2))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.73

$$\int \left(d+ex^2\right)^{3/2}\left(a+cx^4\right)^2\,dx=\frac{\sqrt{e}x\sqrt{d+ex^2}\left(1920a^2e^4(5d+2ex^2)+240ace^2(-3d^3+2d^2ex^2+24de^2x^4+16e^3x^6)+c^2(-1\right.}{}$$

input
```
Integrate[(d + e*x^2)^(3/2)*(a + c*x^4)^2,x]
```

output
```
(Sqrt[e]*x*Sqrt[d + e*x^2]*(1920*a^2*e^4*(5*d + 2*e*x^2) + 240*a*c*e^2*(-3
*d^3 + 2*d^2*e*x^2 + 24*d*e^2*x^4 + 16*e^3*x^6) + c^2*(-105*d^5 + 70*d^4*e
*x^2 - 56*d^3*e^2*x^4 + 48*d^2*e^3*x^6 + 1664*d*e^4*x^8 + 1280*e^5*x^10))
- 15*(7*c^2*d^6 + 48*a*c*d^4*e^2 + 384*a^2*d^2*e^4)*Log[-(Sqrt[e]*x) + Sqr
t[d + e*x^2]])/(15360*e^(9/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {1474, 2346, 27, 1474, 27, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^2\left(d+ex^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1474}$$

$$\frac{\int \left(ex^2+d\right)^{3/2}\left(-7c^2dx^6+24acex^4+12a^2e\right)dx}{12e}+\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{\int 5\left(ex^2+d\right)^{3/2}\left(c\left(7cd^2+48ae^2\right)x^4+24a^2e^2\right)dx}{10e}-\frac{7c^2dx^5\left(d+ex^2\right)^{5/2}}{10e}}{12e}+\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \left(ex^2+d\right)^{3/2}\left(c\left(7cd^2+48ae^2\right)x^4+24a^2e^2\right)dx}{2e}-\frac{7c^2dx^5\left(d+ex^2\right)^{5/2}}{10e}}{12e}+\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

$$\Big\downarrow \text{1474}$$

$$\frac{\frac{\frac{\int 3\left(ex^2+d\right)^{3/2}\left(64a^2e^3-cd\left(7cd^2+48ae^2\right)x^2\right)dx}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}\left(48ae^2+7cd^2\right)}{8e}}{2e}-\frac{7c^2dx^5\left(d+ex^2\right)^{5/2}}{10e}}{12e}+\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \left(ex^2+d\right)^{3/2}\left(64a^2e^3-cd\left(7cd^2+48ae^2\right)x^2\right)dx}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}\left(48ae^2+7cd^2\right)}{8e}}{2e}-\frac{7c^2dx^5\left(d+ex^2\right)^{5/2}}{10e}}{12e}+\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\frac{3\left(\frac{\left(384a^2e^4+48acd^2e^2+7c^2d^4\right)\int \left(ex^2+d\right)^{3/2}dx}{6e}-\frac{cdx\left(d+ex^2\right)^{5/2}\left(48ae^2+7cd^2\right)}{6e}\right)}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}\left(48ae^2+7cd^2\right)}{8e}}{2e}-\frac{7c^2dx^5\left(d+ex^2\right)^{5/2}}{10e}}{12e}+\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\frac{3\left(\frac{\left(384a^2e^4+48acd^2e^2+7c^2d^4\right)\left(\frac{3}{4}d\int \sqrt{ex^2+d}dx+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{6e}-\frac{cdx\left(d+ex^2\right)^{5/2}\left(48ae^2+7cd^2\right)}{6e}\right)}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}\left(48ae^2+7cd^2\right)}{8e}}{2e}-\frac{7c^2dx^5(d+ex}{10e}}{12e}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\frac{3\left(\frac{\left(384a^2e^4+48acd^2e^2+7c^2d^4\right)\left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{6e}-\frac{cdx\left(d+ex^2\right)^{5/2}\left(48ae^2+7cd^2\right)}{6e}\right)}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}\left(48ae^2+7cd^2\right)}{8e}}{2e}}{12e}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

$$\Big\downarrow \, {\color{blue}224}$$

$$\frac{\frac{\frac{3\left(\frac{\left(384a^2e^4+48acd^2e^2+7c^2d^4\right)\left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{6e}-\frac{cdx\left(d+ex^2\right)^{5/2}\left(48ae^2+7cd^2\right)}{6e}\right)}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}\big(48}{8e}}{2e}}{12e}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

$$\Big\downarrow \, {\color{blue}219}$$

$$\frac{\frac{\frac{3\left(\frac{\left(384a^2e^4+48acd^2e^2+7c^2d^4\right)\left(\frac{3}{4}d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{6e}-\frac{cdx\left(d+ex^2\right)^{5/2}\left(48ae^2+7cd^2\right)}{6e}\right)}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}\big(48a}{8e}}{2e}}{12e}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{5/2}}{12e}$$

input
```
Int[(d + e*x^2)^(3/2)*(a + c*x^4)^2,x]
```

output
```
(c^2*x^7*(d + e*x^2)^(5/2))/(12*e) + ((-7*c^2*d*x^5*(d + e*x^2)^(5/2))/(10
*e) + ((c*(7*c*d^2 + 48*a*e^2)*x^3*(d + e*x^2)^(5/2))/(8*e) + (3*(-1/6*(c*
d*(7*c*d^2 + 48*a*e^2)*x*(d + e*x^2)^(5/2))/e + ((7*c^2*d^4 + 48*a*c*d^2*e
^2 + 384*a^2*e^4)*((x*(d + e*x^2)^(3/2))/4 + (3*d*((x*Sqrt[d + e*x^2])/2 +
 (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/4))/(6*e)))/(8*e))
/(2*e))/(12*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{\frac{3\left(a^{2} e^{4}+\frac{1}{8} a c \,d^{2} e^{2}+\frac{7}{384} c^{2} d^{4}\right) d^{2} \operatorname{arctanh}\left(\frac{\sqrt{e \,x^{2}+d}}{x \sqrt{e}}\right)}{8}+\frac{5 x \left(\frac{2 x^{2} \left(\frac{1}{3} c^{2} x^{8}+a c \,x^{4}+a^{2}\right) e^{\frac{11}{2}}}{5}+\left(\left(\frac{13}{75} c^{2} x^{8}+\frac{3}{5} a c \,x^{4}+a^{2}\right) e^{\frac{9}{2}}-\frac{3 \left(d \left(\frac{7 c \,x^{4}}{90}\right.\right.}{\right.\right.}{8}}{e^{\frac{9}{2}}}$
risch	$\frac{x \left(1280 e^{5} c^{2} x^{10}+1664 d \,c^{2} e^{4} x^{8}+3840 a c \,e^{5} x^{6}+48 c^{2} d^{2} e^{3} x^{6}+5760 a c d \,e^{4} x^{4}-56 c^{2} d^{3} e^{2} x^{4}+3840 a^{2} e^{5} x^{2}+480 a c \,d^{2} e^{3} x^{2}+70 c^{2}\right.}{15360 e^{4}}$
default	$a^{2} \left(\frac{x \left(e \,x^{2}+d \right)^{\frac{3}{2}}}{4}+\frac{3 d \left(\frac{x \sqrt{e \,x^{2}+d}}{2}+\frac{d \ln \left(x \sqrt{e}+\sqrt{e \,x^{2}+d}\right)}{2 \sqrt{e}}\right)}{4}\right)+c^{2} \left(\frac{x^{7} \left(e \,x^{2}+d \right)^{\frac{5}{2}}}{12 e}-\frac{7 d \left(\frac{x^{5} \left(e \,x^{2}+d \right)^{\frac{5}{2}}}{10 e}-\frac{d \left(\frac{x^{3} \left(e \,x^{2}\right.}{8}\right.}{\right.}\right.}{\right.}$

input
```
int((e*x^2+d)^(3/2)*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
5/8*(3/5*(a^2*e^4+1/8*a*c*d^2*e^2+7/384*c^2*d^4)*d^2*arctanh((e*x^2+d)^(1/
2)/x/e^(1/2))+x*(2/5*x^2*(1/3*c^2*x^8+a*c*x^4+a^2)*e^(11/2)+((13/75*c^2*x^
8+3/5*a*c*x^4+a^2)*e^(9/2)-3/40*(d*(7/90*c*x^4+a)*e^(5/2)-2/3*x^2*(1/10*c*
x^4+a)*e^(7/2)+7/48*d^2*c*(-2/3*e^(3/2)*x^2+e^(1/2)*d))*d*c)*d)*(e*x^2+d)^
(1/2))/e^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.59

$$\int \left(d+ex^2\right)^{3/2} \left(a +cx^4\right)^2 \,dx = \left[\frac{15\left(7\,c^2d^6+48\,acd^4e^2+384\,a^2d^2e^4\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\left(1280\,c^2e^6x^1\right.}{}\right.$$

$$-\frac{15\left(7\,c^2d^6+48\,acd^4e^2+384\,a^2d^2e^4\right)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\left(1280\,c^2e^6x^{11}+1664\,c^2de^5x^9+48\left(c^2d^2e^4+\right.\right.}{}$$

input
```
integrate((e*x^2+d)^(3/2)*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/30720*(15*(7*c^2*d^6 + 48*a*c*d^4*e^2 + 384*a^2*d^2*e^4)*sqrt(e)*log(-2
*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(1280*c^2*e^6*x^11 + 1664*c^
2*d*e^5*x^9 + 48*(c^2*d^2*e^4 + 80*a*c*e^6)*x^7 - 8*(7*c^2*d^3*e^3 - 720*a
*c*d*e^5)*x^5 + 10*(7*c^2*d^4*e^2 + 48*a*c*d^2*e^4 + 384*a^2*e^6)*x^3 - 15
*(7*c^2*d^5*e + 48*a*c*d^3*e^3 - 640*a^2*d*e^5)*x)*sqrt(e*x^2 + d))/e^5, -
1/15360*(15*(7*c^2*d^6 + 48*a*c*d^4*e^2 + 384*a^2*d^2*e^4)*sqrt(-e)*arctan
(sqrt(-e)*x/sqrt(e*x^2 + d)) - (1280*c^2*e^6*x^11 + 1664*c^2*d*e^5*x^9 + 4
8*(c^2*d^2*e^4 + 80*a*c*e^6)*x^7 - 8*(7*c^2*d^3*e^3 - 720*a*c*d*e^5)*x^5 +
 10*(7*c^2*d^4*e^2 + 48*a*c*d^2*e^4 + 384*a^2*e^6)*x^3 - 15*(7*c^2*d^5*e +
 48*a*c*d^3*e^3 - 640*a^2*d*e^5)*x)*sqrt(e*x^2 + d))/e^5]
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.41

$$\int \left(d+ex^2\right)^{3/2} \left(a+cx^4\right)^2 dx = \begin{cases} \sqrt{d+ex^2} \cdot \left(\frac{13c^2dx^9}{120} + \frac{c^2ex^{11}}{12} + \frac{x^7 \cdot \left(2ace^2+\frac{c^2d^2}{40}\right)}{8e} + \frac{x^5 \cdot \left(4acde-\frac{7d\left(2ace^2+\frac{c^2d^2}{40}\right)}{8e}\right)}{6e} + \frac{x^3\left(a^2e^2+2acd^2-\frac{5d}{}\right.}{} \right. \\ d^{\frac{3}{2}}\left(a^2x+\frac{2acx^5}{5}+\frac{c^2x^9}{9}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**(3/2)*(c*x**4+a)**2,x)
```

output
```
Piecewise((sqrt(d + e*x**2)*(13*c**2*d*x**9/120 + c**2*e*x**11/12 + x**7*(
2*a*c*e**2 + c**2*d**2/40)/(8*e) + x**5*(4*a*c*d*e - 7*d*(2*a*c*e**2 + c**
2*d**2/40)/(8*e))/(6*e) + x**3*(a**2*e**2 + 2*a*c*d**2 - 5*d*(4*a*c*d*e -
7*d*(2*a*c*e**2 + c**2*d**2/40)/(8*e))/(6*e))/(4*e) + x*(2*a**2*d*e - 3*d*
(a**2*e**2 + 2*a*c*d**2 - 5*d*(4*a*c*d*e - 7*d*(2*a*c*e**2 + c**2*d**2/40)
/(8*e))/(6*e))/(4*e))/(2*e)) + (a**2*d**2 - d*(2*a**2*d*e - 3*d*(a**2*e**2
 + 2*a*c*d**2 - 5*d*(4*a*c*d*e - 7*d*(2*a*c*e**2 + c**2*d**2/40)/(8*e))/(6
*e))/(4*e))/(2*e))*Piecewise((log(2*sqrt(e)*sqrt(d + e*x**2) + 2*e*x)/sqrt
(e), Ne(d, 0)), (x*log(x)/sqrt(e*x**2), True)), Ne(e, 0)), (d**(3/2)*(a**2
*x + 2*a*c*x**5/5 + c**2*x**9/9), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2}\left(a+cx^4\right)^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.83

$$\int \left(d+ex^2\right)^{3/2}(a +cx^4)^2\,dx = \frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,c^2ex^2+13\,c^2d\right)x^2+\frac{3\left(c^2d^2e^9+80\,ace^{11}\right)}{e^{10}}\right)x^2-\frac{7\,c^2d^3e^8-720\,acde^{1}}{e^{10}}\right.\right.\right.\right. - \frac{\left(7\,c^2d^6+48\,acd^4e^2+384\,a^2d^2e^4\right)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{1024\,e^{\frac{9}{2}}}$$

input
```
integrate((e*x^2+d)^(3/2)*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/15360*(2*(4*(2*(8*(10*c^2*e*x^2 + 13*c^2*d)*x^2 + 3*(c^2*d^2*e^9 + 80*a*
c*e^11)/e^10)*x^2 - (7*c^2*d^3*e^8 - 720*a*c*d*e^10)/e^10)*x^2 + 5*(7*c^2*
d^4*e^7 + 48*a*c*d^2*e^9 + 384*a^2*e^11)/e^10)*x^2 - 15*(7*c^2*d^5*e^6 + 4
8*a*c*d^3*e^8 - 640*a^2*d*e^10)/e^10)*sqrt(e*x^2 + d)*x - 1/1024*(7*c^2*d^
6 + 48*a*c*d^4*e^2 + 384*a^2*d^2*e^4)*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)
))/e^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2}\left(a+cx^4\right)^2\,dx=\int \left(c\,x^4+a\right)^2\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int((a + c*x^4)^2*(d + e*x^2)^(3/2),x)
```

output
```
int((a + c*x^4)^2*(d + e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.24

$$\int \left(d+ex^2\right)^{3/2}(a+cx^4)^2\,dx=\frac{9600\sqrt{e\,x^2+d}\,a^2d\,e^5x+3840\sqrt{e\,x^2+d}\,a^2e^6x^3-720\sqrt{e\,x^2+d}\,ac\,d^3e^3x+480\sqrt{e\,x^2+d}\,ac\,d^{}}{}$$

input
```
int((e*x^2+d)^(3/2)*(c*x^4+a)^2,x)
```

output
```
(9600*sqrt(d + e*x**2)*a**2*d*e**5*x + 3840*sqrt(d + e*x**2)*a**2*e**6*x**
3 - 720*sqrt(d + e*x**2)*a*c*d**3*e**3*x + 480*sqrt(d + e*x**2)*a*c*d**2*e
**4*x**3 + 5760*sqrt(d + e*x**2)*a*c*d*e**5*x**5 + 3840*sqrt(d + e*x**2)*a
*c*e**6*x**7 - 105*sqrt(d + e*x**2)*c**2*d**5*e*x + 70*sqrt(d + e*x**2)*c*
*2*d**4*e**2*x**3 - 56*sqrt(d + e*x**2)*c**2*d**3*e**3*x**5 + 48*sqrt(d +
e*x**2)*c**2*d**2*e**4*x**7 + 1664*sqrt(d + e*x**2)*c**2*d*e**5*x**9 + 128
0*sqrt(d + e*x**2)*c**2*e**6*x**11 + 5760*sqrt(e)*log((sqrt(d + e*x**2) +
sqrt(e)*x)/sqrt(d))*a**2*d**2*e**4 + 720*sqrt(e)*log((sqrt(d + e*x**2) + s
qrt(e)*x)/sqrt(d))*a*c*d**4*e**2 + 105*sqrt(e)*log((sqrt(d + e*x**2) + sqr
t(e)*x)/sqrt(d))*c**2*d**6)/(15360*e**5)
```

3.374 $\int \sqrt{d+ex^2}\left(a+cx^4\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 221

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)^2\,dx = \frac{\left(7c^2d^4+32acd^2e^2+128a^2e^4\right)x\sqrt{d+ex^2}}{256e^4} - \frac{cd\left(7cd^2+32ae^2\right)x\left(d+ex^2\right)^{3/2}}{128e^4} + \frac{c\left(7cd^2+32ae^2\right)x^3\left(d+ex^2\right)^{3/2}}{96e^3} - \frac{7c^2dx^5\left(d+ex^2\right)^{3/2}}{80e^2} + \frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e} + \frac{d\left(7c^2d^4+32acd^2e^2+128a^2e^4\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{256e^{9/2}}$$

output
```
1/256*(128*a^2*e^4+32*a*c*d^2*e^2+7*c^2*d^4)*x*(e*x^2+d)^(1/2)/e^4-1/128*c
*d*(32*a*e^2+7*c*d^2)*x*(e*x^2+d)^(3/2)/e^4+1/96*c*(32*a*e^2+7*c*d^2)*x^3*
(e*x^2+d)^(3/2)/e^3-7/80*c^2*d*x^5*(e*x^2+d)^(3/2)/e^2+1/10*c^2*x^7*(e*x^2
+d)^(3/2)/e+1/256*d*(128*a^2*e^4+32*a*c*d^2*e^2+7*c^2*d^4)*arctanh(e^(1/2)
*x/(e*x^2+d)^(1/2))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.74

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)^2\,dx$$
$$=\frac{\sqrt{e}x\sqrt{d+ex^2}(1920a^2e^4+160ace^2(-3d^2+2dex^2+8e^2x^4)+c^2(-105d^4+70d^3ex^2-56d^2e^2x^4+48de^3}{3840e^{9/2}}$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + c*x^4)^2,x]
```

output
```
(Sqrt[e]*x*Sqrt[d + e*x^2]*(1920*a^2*e^4 + 160*a*c*e^2*(-3*d^2 + 2*d*e*x^2
 + 8*e^2*x^4) + c^2*(-105*d^4 + 70*d^3*e*x^2 - 56*d^2*e^2*x^4 + 48*d*e^3*x
^6 + 384*e^4*x^8)) - 15*(7*c^2*d^5 + 32*a*c*d^3*e^2 + 128*a^2*d*e^4)*Log[-
(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(3840*e^(9/2))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1474, 2346, 27, 1474, 27, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^2\sqrt{d+ex^2}\,dx$$
$$\Big\downarrow \text{1474}$$
$$\frac{\int \sqrt{ex^2+d}\left(-7c^2dx^6+20acex^4+10a^2e\right)dx}{10e}+\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$
$$\Big\downarrow \text{2346}$$
$$\frac{\frac{\int 5\sqrt{ex^2+d}\left(c\left(7cd^2+32ae^2\right)x^4+16a^2e^2\right)dx}{8e}-\frac{7c^2dx^5\left(d+ex^2\right)^{3/2}}{8e}}{10e}+\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$
$$\Big\downarrow \text{27}$$

$$\frac{\frac{5\int \sqrt{ex^2+d}\left(c\left(7cd^2+32ae^2\right)x^4+16a^2e^2\right)dx}{8e}-\frac{7c^2dx^5\left(d+ex^2\right)^{3/2}}{8e}}{10e}+\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

$$\Big\downarrow \ 1474$$

$$\frac{\frac{5\left(\frac{\int 3\sqrt{ex^2+d}\left(32a^2e^3-cd\left(7cd^2+32ae^2\right)x^2\right)dx}{6e}+\frac{cx^3\left(d+ex^2\right)^{3/2}\left(32ae^2+7cd^2\right)}{6e}\right)}{8e}-\frac{7c^2dx^5\left(d+ex^2\right)^{3/2}}{8e}}{10e}+\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{5\left(\frac{\int \sqrt{ex^2+d}\left(32a^2e^3-cd\left(7cd^2+32ae^2\right)x^2\right)dx}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}\left(32ae^2+7cd^2\right)}{6e}\right)}{8e}-\frac{7c^2dx^5\left(d+ex^2\right)^{3/2}}{8e}}{10e}+\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{5\left(\frac{\frac{\left(128a^2e^4+32acd^2e^2+7c^2d^4\right)\int \sqrt{ex^2+d}dx}{4e}-\frac{cdx\left(d+ex^2\right)^{3/2}\left(32ae^2+7cd^2\right)}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}\left(32ae^2+7cd^2\right)}{6e}\right)}{8e}-\frac{7c^2dx^5\left(d+ex^2\right)^{3/2}}{8e}}{10e}+\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{5\left(\frac{\frac{\left(128a^2e^4+32acd^2e^2+7c^2d^4\right)\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}-\frac{cdx\left(d+ex^2\right)^{3/2}\left(32ae^2+7cd^2\right)}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}\left(32ae^2+7cd^2\right)}{6e}\right)}{8e}-\frac{7c^2dx^5\left(d+ex^2\right)}{8e}}{10e}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{5\left(\frac{\frac{\left(128a^2e^4+32acd^2e^2+7c^2d^4\right)\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}-\frac{cdx\left(d+ex^2\right)^{3/2}\left(32ae^2+7cd^2\right)}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}\left(32ae^2+7cd^2\right)}{6e}\right)}{8e}-\frac{7c^2dx}{}}{10e}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

$$\Big\downarrow 219$$

$$\frac{\frac{5\left(\frac{\frac{\left(128a^2e^4+32acd^2e^2+7c^2d^4\right)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}-\frac{cdx\left(d+ex^2\right)^{3/2}\left(32ae^2+7cd^2\right)}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}\left(32ae^2+7cd^2\right)}{6e}\right)}{8e}-\frac{7c^2dx^5}{}}{10e}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{3/2}}{10e}$$

input

```
Int[Sqrt[d + e*x^2]*(a + c*x^4)^2,x]
```

output

```
(c^2*x^7*(d + e*x^2)^(3/2))/(10*e) + ((-7*c^2*d*x^5*(d + e*x^2)^(3/2))/(8*
e) + (5*((c*(7*c*d^2 + 32*a*e^2)*x^3*(d + e*x^2)^(3/2))/(6*e) + (-1/4*(c*d
*(7*c*d^2 + 32*a*e^2)*x*(d + e*x^2)^(3/2))/e + ((7*c^2*d^4 + 32*a*c*d^2*e^
2 + 128*a^2*e^4)*((x*Sqrt[d + e*x^2])/2 + (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
e*x^2]])/(2*Sqrt[e])))/(4*e))/(2*e)))/(8*e))/(10*e)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{d\left(a^{2} e^{4}+\frac{1}{4} a c\, d^{2} e^{2}+\frac{7}{128} c^{2} d^{4}\right) \operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x \sqrt{e}}\right)+x \left(\left(\frac{1}{5} c^{2} x^{8}+\frac{2}{3} a c\, x^{4}+a^{2}\right) e^{\frac{9}{2}}-\frac{\left(d \left(\frac{7 c\,x^{4}}{60}+a\right) e^{\frac{5}{2}}-\frac{2 x^{2} \left(\frac{3 c\,x^{4}}{20}+a\right) e^{\frac{7}{2}}}{3}+\frac{7 d^{2}}{}\right.}{4}\right.}{2 e^{\frac{9}{2}}}$
risch	$\frac{x \left(384 c^{2} e^{4} x^{8}+48 d\, e^{3} x^{6} c^{2}+1280 a c\, e^{4} x^{4}-56 c^{2} d^{2} e^{2} x^{4}+320 a c d\, e^{3} x^{2}+70 c^{2} d^{3} e\, x^{2}+1920 a^{2} e^{4}-480 a c\, d^{2} e^{2}-105 c^{2} d^{4}\right) \sqrt{e\,x^{2}+}}{3840 e^{4}}$
default	$a^{2} \left(\frac{x \sqrt{e\,x^{2}+d}}{2}+\frac{d \ln\left(x \sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2 \sqrt{e}}\right)+c^{2} \left(\frac{x^{7} \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{10 e}-\frac{7 d \left(\frac{x^{5} \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{8 e}-\frac{5 d \left(\frac{x^{3} \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{6 e}-\frac{d \left(\frac{x \left(e\,x^{2}\right.}{4e}\right.}{}\right.}{}\right.}{1}\right.$

input
```
int((e*x^2+d)^(1/2)*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/e^(9/2)*(d*(a^2*e^4+1/4*a*c*d^2*e^2+7/128*c^2*d^4)*arctanh((e*x^2+d)^(
1/2)/x/e^(1/2))+x*((1/5*c^2*x^8+2/3*a*c*x^4+a^2)*e^(9/2)-1/4*(d*(7/60*c*x^
4+a)*e^(5/2)-2/3*x^2*(3/20*c*x^4+a)*e^(7/2)+7/32*d^2*c*(-2/3*e^(3/2)*x^2+e
^(1/2)*d))*d*c)*(e*x^2+d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.60

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)^2\,dx$$

$$= \left[\frac{15\left(7\,c^2d^5+32\,acd^3e^2+128\,a^2de^4\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\left(384\,c^2e^5x^9+48\,c^2de^4x^7\right.}{7}\right.$$

$$-\frac{15\left(7\,c^2d^5+32\,acd^3e^2+128\,a^2de^4\right)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\left(384\,c^2e^5x^9+48\,c^2de^4x^7-8\left(7\,c^2d^2e^3-1\right.\right.}{3840\,e^5}$$

input
```
integrate((e*x^2+d)^(1/2)*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/7680*(15*(7*c^2*d^5 + 32*a*c*d^3*e^2 + 128*a^2*d*e^4)*sqrt(e)*log(-2*e*
x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(384*c^2*e^5*x^9 + 48*c^2*d*e^4
*x^7 - 8*(7*c^2*d^2*e^3 - 160*a*c*e^5)*x^5 + 10*(7*c^2*d^3*e^2 + 32*a*c*d*
e^4)*x^3 - 15*(7*c^2*d^4*e + 32*a*c*d^2*e^3 - 128*a^2*e^5)*x)*sqrt(e*x^2 +
 d))/e^5, -1/3840*(15*(7*c^2*d^5 + 32*a*c*d^3*e^2 + 128*a^2*d*e^4)*sqrt(-e
)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (384*c^2*e^5*x^9 + 48*c^2*d*e^4*x^7
 - 8*(7*c^2*d^2*e^3 - 160*a*c*e^5)*x^5 + 10*(7*c^2*d^3*e^2 + 32*a*c*d*e^4)
*x^3 - 15*(7*c^2*d^4*e + 32*a*c*d^2*e^3 - 128*a^2*e^5)*x)*sqrt(e*x^2 + d))
/e^5]
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.22

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)^2\,dx$$
$$= \begin{cases} \sqrt{d+ex^2}\left(\frac{c^2dx^7}{80e}+\frac{c^2x^9}{10}+\frac{x^5\cdot\left(2ace-\frac{7c^2d^2}{80e}\right)}{6e}+\frac{x^3\cdot\left(2acd-\frac{5d\left(2ace-\frac{7c^2d^2}{80e}\right)}{6e}\right)}{4e}+\frac{x\left(a^2e-\frac{3d\left(2acd-\frac{5d\left(2ace-\frac{7c^2d^2}{80e}\right)}{6e}\right)}{4e}\right)}{2e}\right)+ \\ \sqrt{d}\left(a^2x+\frac{2acx^5}{5}+\frac{c^2x^9}{9}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**(1/2)*(c*x**4+a)**2,x)
```

output
```
Piecewise((sqrt(d + e*x**2)*(c**2*d*x**7/(80*e) + c**2*x**9/10 + x**5*(2*a
*c*e - 7*c**2*d**2/(80*e))/(6*e) + x**3*(2*a*c*d - 5*d*(2*a*c*e - 7*c**2*d
**2/(80*e))/(6*e))/(4*e) + x*(a**2*e - 3*d*(2*a*c*d - 5*d*(2*a*c*e - 7*c**
2*d**2/(80*e))/(6*e))/(4*e))/(2*e)) + (a**2*d - d*(a**2*e - 3*d*(2*a*c*d -
 5*d*(2*a*c*e - 7*c**2*d**2/(80*e))/(6*e))/(4*e))/(2*e))*Piecewise((log(2*
sqrt(e)*sqrt(d + e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqrt(e*x**
2), True)), Ne(e, 0)), (sqrt(d)*(a**2*x + 2*a*c*x**5/5 + c**2*x**9/9), Tru
e))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.83

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)^2\,dx$$

$$= \frac{1}{3840}\left(2\left(4\left(6\left(8\,c^2x^2+\frac{c^2d}{e}\right)x^2-\frac{7\,c^2d^2e^6-160\,ace^8}{e^8}\right)x^2+\frac{5\left(7\,c^2d^3e^5+32\,acde^7\right)}{e^8}\right)x^2-15\left(7\,c^2d^4e\right.\right.$$

$$-\frac{\left(7\,c^2d^5+32\,acd^3e^2+128\,a^2de^4\right)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{256\,e^{\frac{9}{2}}}$$

input
```
integrate((e*x^2+d)^(1/2)*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*c^2*x^2 + c^2*d/e)*x^2 - (7*c^2*d^2*e^6 - 160*a*c*e^8)/
e^8)*x^2 + 5*(7*c^2*d^3*e^5 + 32*a*c*d*e^7)/e^8)*x^2 - 15*(7*c^2*d^4*e^4 +
 32*a*c*d^2*e^6 - 128*a^2*e^8)/e^8)*sqrt(e*x^2 + d)*x - 1/256*(7*c^2*d^5 +
 32*a*c*d^3*e^2 + 128*a^2*d*e^4)*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^
(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)^2\,dx=\int {\left(c\,x^4+a\right)}^2\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a + c*x^4)^2*(d + e*x^2)^(1/2),x)
```

output
```
int((a + c*x^4)^2*(d + e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.22

$$\int \sqrt{d+ex^2}\left(a+cx^4\right)^2\,dx$$

$$= \frac{1920\sqrt{e\,x^2+d}\,a^2e^5x-480\sqrt{e\,x^2+d}\,ac\,d^2e^3x+320\sqrt{e\,x^2+d}\,acd\,e^4x^3+1280\sqrt{e\,x^2+d}\,ac\,e^5x^5-105}{}$$

input

```
int((e*x^2+d)^(1/2)*(c*x^4+a)^2,x)
```

output

```
(1920*sqrt(d + e*x**2)*a**2*e**5*x - 480*sqrt(d + e*x**2)*a*c*d**2*e**3*x
+ 320*sqrt(d + e*x**2)*a*c*d*e**4*x**3 + 1280*sqrt(d + e*x**2)*a*c*e**5*x*
*5 - 105*sqrt(d + e*x**2)*c**2*d**4*e*x + 70*sqrt(d + e*x**2)*c**2*d**3*e*
*2*x**3 - 56*sqrt(d + e*x**2)*c**2*d**2*e**3*x**5 + 48*sqrt(d + e*x**2)*c*
*2*d*e**4*x**7 + 384*sqrt(d + e*x**2)*c**2*e**5*x**9 + 1920*sqrt(e)*log((s
qrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a**2*d*e**4 + 480*sqrt(e)*log((sqrt(
d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*c*d**3*e**2 + 105*sqrt(e)*log((sqrt(d
+ e*x**2) + sqrt(e)*x)/sqrt(d))*c**2*d**5)/(3840*e**5)
```

3.375 $\int \frac{(a+cx^4)^2}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\begin{aligned}\int \frac{(a+cx^4)^2}{\sqrt{d+ex^2}}\,dx =& -\frac{cd(35cd^2+96ae^2)\,x\sqrt{d+ex^2}}{128e^4}+\frac{c(35cd^2+96ae^2)\,x^3\sqrt{d+ex^2}}{192e^3}\\ &-\frac{7c^2dx^5\sqrt{d+ex^2}}{48e^2}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}\\ &+\frac{(35c^2d^4+96acd^2e^2+128a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{128e^{9/2}}\end{aligned}$$

output

```
-1/128*c*d*(96*a*e^2+35*c*d^2)*x*(e*x^2+d)^(1/2)/e^4+1/192*c*(96*a*e^2+35*
c*d^2)*x^3*(e*x^2+d)^(1/2)/e^3-7/48*c^2*d*x^5*(e*x^2+d)^(1/2)/e^2+1/8*c^2*
x^7*(e*x^2+d)^(1/2)/e+1/128*(128*a^2*e^4+96*a*c*d^2*e^2+35*c^2*d^4)*arctan
h(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.75

$$\int \frac{(a+cx^4)^2}{\sqrt{d+ex^2}}\,dx = \frac{cx\sqrt{d+ex^2}(-105cd^3-288ade^2+70cd^2ex^2+192ae^3x^2-56cde^2x^4+48ce^3x^6)}{384e^4} + \frac{(-35c^2d^4-96acd^2e^2-128a^2e^4)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{128e^{9/2}}$$

input

```
Integrate[(a + c*x^4)^2/Sqrt[d + e*x^2],x]
```

output

```
(c*x*Sqrt[d + e*x^2]*(-105*c*d^3 - 288*a*d*e^2 + 70*c*d^2*e*x^2 + 192*a*e^
3*x^2 - 56*c*d*e^2*x^4 + 48*c*e^3*x^6))/(384*e^4) + ((-35*c^2*d^4 - 96*a*c
*d^2*e^2 - 128*a^2*e^4)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(128*e^(9/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1474, 2346, 1474, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{1474}$$

$$\frac{\int \frac{-7c^2dx^6+16acex^4+8a^2e}{\sqrt{ex^2+d}}dx}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{\int \frac{c\left(35cd^2+96ae^2\right)x^4+48a^2e^2}{\sqrt{ex^2+d}}dx}{6e}-\frac{7c^2dx^5\sqrt{d+ex^2}}{6e}}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

$$\Big\downarrow \text{ 1474}$$

$$\frac{\frac{\frac{\int \frac{3\left(64a^2e^3-cd\left(35cd^2+96ae^2\right)x^2\right)}{\sqrt{ex^2+d}}dx}{4e}+\frac{cx^3\sqrt{d+ex^2}\left(96ae^2+35cd^2\right)}{4e}}{6e}-\frac{7c^2dx^5\sqrt{d+ex^2}}{6e}}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{3\int \frac{64a^2e^3-cd\left(35cd^2+96ae^2\right)x^2}{\sqrt{ex^2+d}}dx}{4e}+\frac{cx^3\sqrt{d+ex^2}\left(96ae^2+35cd^2\right)}{4e}}{6e}-\frac{7c^2dx^5\sqrt{d+ex^2}}{6e}}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{\frac{3\left(\frac{\left(128a^2e^4+96acd^2e^2+35c^2d^4\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}-\frac{cdx\sqrt{d+ex^2}\left(96ae^2+35cd^2\right)}{2e}\right)}{4e}+\frac{cx^3\sqrt{d+ex^2}\left(96ae^2+35cd^2\right)}{4e}}{6e}-\frac{7c^2dx^5\sqrt{d+ex^2}}{6e}}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\frac{3\left(\frac{\left(128a^2e^4+96acd^2e^2+35c^2d^4\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}-\frac{cdx\sqrt{d+ex^2}\left(96ae^2+35cd^2\right)}{2e}\right)}{4e}+\frac{cx^3\sqrt{d+ex^2}\left(96ae^2+35cd^2\right)}{4e}}{6e}-\frac{7c^2dx^5\sqrt{d+ex^2}}{6e}}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\frac{3\left(\frac{\left(128a^2e^4+96acd^2e^2+35c^2d^4\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}-\frac{cdx\sqrt{d+ex^2}\left(96ae^2+35cd^2\right)}{2e}\right)}{4e}+\frac{cx^3\sqrt{d+ex^2}\left(96ae^2+35cd^2\right)}{4e}}{6e}-\frac{7c^2dx^5\sqrt{d+ex^2}}{6e}}{8e}+\frac{c^2x^7\sqrt{d+ex^2}}{8e}$$

input

```
Int[(a + c*x^4)^2/Sqrt[d + e*x^2],x]
```

output
```
(c^2*x^7*Sqrt[d + e*x^2])/(8*e) + ((-7*c^2*d*x^5*Sqrt[d + e*x^2])/(6*e) +
((c*(35*c*d^2 + 96*a*e^2)*x^3*Sqrt[d + e*x^2])/(4*e) + (3*(-1/2*(c*d*(35*c
*d^2 + 96*a*e^2)*x*Sqrt[d + e*x^2])/e + ((35*c^2*d^4 + 96*a*c*d^2*e^2 + 12
8*a^2*e^4)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(3/2))))/(4*e))/(6*e
))/(8*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{\left(a^{2} e^{4}+\frac{3}{4} a c \,d^{2} e^{2}+\frac{35}{128} c^{2} d^{4}\right) \operatorname{arctanh}\left(\frac{\sqrt{e \,x^{2}+d}}{x \sqrt{e}}\right)-\frac{3 x \left(d \left(\frac{7 c \,x^{4}}{36}+a \right) e^{\frac{5}{2}}-\frac{2 x^{2} \left(\frac{c \,x^{4}}{4}+a \right) e^{\frac{7}{2}}}{3}+\frac{35 d^{2} c \left(-\frac{2 e^{\frac{3}{2}} x^{2}}{3}+\sqrt{e}\, d \right)}{96}\right) \sqrt{e \,x^{2}+}}{4}}{e^{\frac{9}{2}}}$
risch	$-\frac{c x \left(-48 e^{3} c \,x^{6}+56 c d \,e^{2} x^{4}-192 a \,e^{3} x^{2}-70 c \,d^{2} e \,x^{2}+288 d \,e^{2} a +105 d^{3} c \right) \sqrt{e \,x^{2}+d}}{384 e^{4}}+\frac{\left(128 a^{2} e^{4}+96 a c \,d^{2} e^{2}+35 c^{2} d^{4}\right) \ln \left(x\right.}{128 e^{\frac{9}{2}}}$
default	$\frac{a^{2} \ln \left(x \sqrt{e}+\sqrt{e \,x^{2}+d}\right)}{\sqrt{e}}+c^{2} \left(\frac{x^{7} \sqrt{e \,x^{2}+d}}{8 e}-\frac{7 d \left(\frac{x^{5} \sqrt{e \,x^{2}+d}}{6 e}-\frac{5 d \left(\frac{x^{3} \sqrt{e \,x^{2}+d}}{4 e}-\frac{3 d \left(\frac{x \sqrt{e \,x^{2}+d}}{2 e}-\frac{d \ln \left(x \sqrt{e}+\sqrt{e \,x^{2}+d}\right)}{2 e^{\frac{3}{2}}}\right)}{4 e}\right.}{6 e}\right.}{8 e}\right.$

input
```
int((c*x^4+a)^2/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^(9/2)*((a^2*e^4+3/4*a*c*d^2*e^2+35/128*c^2*d^4)*arctanh((e*x^2+d)^(1/2
)/x/e^(1/2))-3/4*x*(d*(7/36*c*x^4+a)*e^(5/2)-2/3*x^2*(1/4*c*x^4+a)*e^(7/2)
+35/96*d^2*c*(-2/3*e^(3/2)*x^2+e^(1/2)*d))*(e*x^2+d)^(1/2)*c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.62

$$\int \frac{(a+cx^4)^2}{\sqrt{d+ex^2}}\,dx$$

$$= \left[\frac{3\,(35\,c^2d^4+96\,acd^2e^2+128\,a^2e^4)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\,(48\,c^2e^4x^7-56\,c^2de^3x^5+}{768\,e^5}\right.$$

$$-\frac{3\,(35\,c^2d^4+96\,acd^2e^2+128\,a^2e^4)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(48\,c^2e^4x^7-56\,c^2de^3x^5+2\,(35\,c^2d^2e^2+96}{384\,e^5}$$

input

```
integrate((c*x^4+a)^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output

```
[1/768*(3*(35*c^2*d^4 + 96*a*c*d^2*e^2 + 128*a^2*e^4)*sqrt(e)*log(-2*e*x^2
 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(48*c^2*e^4*x^7 - 56*c^2*d*e^3*x^5
 + 2*(35*c^2*d^2*e^2 + 96*a*c*e^4)*x^3 - 3*(35*c^2*d^3*e + 96*a*c*d*e^3)*x
)*sqrt(e*x^2 + d))/e^5, -1/384*(3*(35*c^2*d^4 + 96*a*c*d^2*e^2 + 128*a^2*e
^4)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (48*c^2*e^4*x^7 - 56*c^2
*d*e^3*x^5 + 2*(35*c^2*d^2*e^2 + 96*a*c*e^4)*x^3 - 3*(35*c^2*d^3*e + 96*a*
c*d*e^3)*x)*sqrt(e*x^2 + d))/e^5]
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\int \frac{(a+cx^4)^2}{\sqrt{d+ex^2}}\,dx$$

$$= \begin{cases} \left(a^2+\frac{3d^2\cdot\left(2ac+\frac{35c^2d^2}{48e^2}\right)}{8e^2}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{e}\sqrt{d+ex^2}+2ex\right)}{\sqrt{e}} & \text{for}\ d\neq 0\\ \frac{x\log(x)}{\sqrt{ex^2}} & \text{otherwise}\end{cases}\right)+\sqrt{d+ex^2}\left(-\frac{7c^2dx^5}{48e^2}+\frac{c^2x^7}{8e}-\frac{3dx\left(2ac+\frac{35c}{48}\right.}{8e^2}\right. \\ \frac{a^2x+\frac{2acx^5}{5}+\frac{c^2x^9}{9}}{\sqrt{d}} \end{cases}$$

input

```
integrate((c*x**4+a)**2/(e*x**2+d)**(1/2),x)
```

output

```
Piecewise(((a**2 + 3*d**2*(2*a*c + 35*c**2*d**2/(48*e**2))/(8*e**2))*Piece
wise((log(2*sqrt(e)*sqrt(d + e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x
)/sqrt(e*x**2), True)) + sqrt(d + e*x**2)*(-7*c**2*d*x**5/(48*e**2) + c**2
*x**7/(8*e) - 3*d*x*(2*a*c + 35*c**2*d**2/(48*e**2))/(8*e**2) + x**3*(2*a*
c + 35*c**2*d**2/(48*e**2))/(4*e)), Ne(e, 0)), ((a**2*x + 2*a*c*x**5/5 + c
**2*x**9/9)/sqrt(d), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((c*x^4+a)^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.83

$$\begin{aligned}&\int \frac{(a+cx^4)^2}{\sqrt{d+ex^2}}\,dx\\&=\frac{1}{384}\left(2\left(4\left(\frac{6\,c^2x^2}{e}-\frac{7\,c^2d}{e^2}\right)x^2+\frac{35\,c^2d^2e^4+96\,ace^6}{e^7}\right)x^2-\frac{3\,(35\,c^2d^3e^3+96\,acde^5)}{e^7}\right)\sqrt{ex^2+d}x\\&\quad-\frac{(35\,c^2d^4+96\,acd^2e^2+128\,a^2e^4)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{128\,e^{\frac{9}{2}}}\end{aligned}$$

input

```
integrate((c*x^4+a)^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*c^2*x^2/e - 7*c^2*d/e^2)*x^2 + (35*c^2*d^2*e^4 + 96*a*c*e^6
)/e^7)*x^2 - 3*(35*c^2*d^3*e^3 + 96*a*c*d*e^5)/e^7)*sqrt(e*x^2 + d)*x - 1/
128*(35*c^2*d^4 + 96*a*c*d^2*e^2 + 128*a^2*e^4)*log(abs(-sqrt(e)*x + sqrt(
e*x^2 + d)))/e^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^2}{\sqrt{d+ex^2}}\,dx = \int \frac{(c\,x^4+a)^2}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^(1/2),x)
```

output
```
int((a + c*x^4)^2/(d + e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.20

$$\int \frac{(a+cx^4)^2}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{-288\sqrt{e\,x^2+d}\,acd\,e^3x+192\sqrt{e\,x^2+d}\,ac\,e^4x^3-105\sqrt{e\,x^2+d}\,c^2d^3ex+70\sqrt{e\,x^2+d}\,c^2d^2e^2x^3-56\sqrt{e}}{}$$

input
```
int((c*x^4+a)^2/(e*x^2+d)^(1/2),x)
```

output
```
( - 288*sqrt(d + e*x**2)*a*c*d*e**3*x + 192*sqrt(d + e*x**2)*a*c*e**4*x**3
 - 105*sqrt(d + e*x**2)*c**2*d**3*e*x + 70*sqrt(d + e*x**2)*c**2*d**2*e**2
*x**3 - 56*sqrt(d + e*x**2)*c**2*d*e**3*x**5 + 48*sqrt(d + e*x**2)*c**2*e*
*4*x**7 + 384*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a**2*e**
4 + 288*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*c*d**2*e**2
+ 105*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*c**2*d**4)/(384*
e**5)
```

3.376 $\int \frac{(a+cx^4)^2}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \frac{(cd^2+ae^2)^2\,x}{de^4\sqrt{d+ex^2}} + \frac{c(19cd^2+16ae^2)\,x\sqrt{d+ex^2}}{16e^4} - \frac{11c^2dx^3\sqrt{d+ex^2}}{24e^3} + \frac{c^2x^5\sqrt{d+ex^2}}{6e^2} - \frac{cd(35cd^2+48ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16e^{9/2}}$$

output
```
(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^(1/2)+1/16*c*(16*a*e^2+19*c*d^2)*x*(e*x^
2+d)^(1/2)/e^4-11/24*c^2*d*x^3*(e*x^2+d)^(1/2)/e^3+1/6*c^2*x^5*(e*x^2+d)^(
1/2)/e^2-1/16*c*d*(48*a*e^2+35*c*d^2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/e
^(9/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \frac{x(48a^2e^4+48acde^2(3d+ex^2)+c^2d(105d^3+35d^2ex^2-14de^2x^4+8e^3x^6))}{48de^4\sqrt{d+ex^2}} + \frac{cd(35cd^2+48ae^2)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{16e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^(3/2),x]
```

output
```
(x*(48*a^2*e^4 + 48*a*c*d*e^2*(3*d + e*x^2) + c^2*d*(105*d^3 + 35*d^2*e*x^
2 - 14*d*e^2*x^4 + 8*e^3*x^6)))/(48*d*e^4*Sqrt[d + e*x^2]) + (c*d*(35*c*d^
2 + 48*a*e^2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(16*e^(9/2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1472, 2346, 1473, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(ae^2+cd^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\int \frac{-\frac{c^2dx^6}{e}+\frac{c^2d^2x^4}{e^2}-\frac{cd\left(cd^2+2ae^2\right)x^2}{e^3}+\frac{cd^2\left(cd^2+2ae^2\right)}{e^4}}{\sqrt{ex^2+d}}dx}{d}$$

$$\Big\downarrow \text{2346}$$

$$\frac{x\left(ae^2+cd^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\frac{\int \frac{\frac{11c^2d^2x^4}{e}-6cd\left(\frac{cd^2}{e^2}+2a\right)x^2+\frac{6cd^2\left(cd^2+2ae^2\right)}{e^3}}{\sqrt{ex^2+d}}dx}{6e}-\frac{c^2dx^5\sqrt{d+ex^2}}{6e^2}}{d}$$

$$\Big\downarrow \text{1473}$$

$$\frac{x\left(ae^2+cd^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\frac{\frac{\int \frac{3cd\left(8d\left(\frac{cd^2}{e^2}+2a\right)-\left(\frac{19cd^2}{e}+16ae\right)x^2\right)}{\sqrt{ex^2+d}}dx}{4e}+\frac{11c^2d^2x^3\sqrt{d+ex^2}}{4e^2}}{6e}-\frac{c^2dx^5\sqrt{d+ex^2}}{6e^2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\left(ae^2+cd^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\frac{\frac{3cd\int\frac{8d\left(\frac{cd^2}{e^2}+2a\right)-\left(\frac{19cd^2}{e}+16ae\right)x^2}{\sqrt{ex^2+d}}dx}{4e}+\frac{11c^2d^2x^3\sqrt{d+ex^2}}{4e^2}}{6e}-\frac{c^2dx^5\sqrt{d+ex^2}}{6e^2}}{d}$$

$$\Big\downarrow \; 299$$

$$\frac{x\left(ae^2+cd^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\frac{\frac{3cd\left(\frac{1}{2}d\left(48a+\frac{35cd^2}{e^2}\right)\int\frac{1}{\sqrt{ex^2+d}}dx-\frac{1}{2}x\sqrt{d+ex^2}\left(16a+\frac{19cd^2}{e^2}\right)\right)}{4e}+\frac{11c^2d^2x^3\sqrt{d+ex^2}}{4e^2}}{6e}-\frac{c^2dx^5\sqrt{d+ex^2}}{6e^2}}{d}$$

$$\Big\downarrow \; 224$$

$$\frac{x\left(ae^2+cd^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\frac{\frac{3cd\left(\frac{1}{2}d\left(48a+\frac{35cd^2}{e^2}\right)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{1}{2}x\sqrt{d+ex^2}\left(16a+\frac{19cd^2}{e^2}\right)\right)}{4e}+\frac{11c^2d^2x^3\sqrt{d+ex^2}}{4e^2}}{6e}-\frac{c^2dx^5\sqrt{d+ex^2}}{6e^2}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{x\left(ae^2+cd^2\right)^2}{de^4\sqrt{d+ex^2}}-\frac{\frac{\frac{3cd\left(\frac{d\left(48a+\frac{35cd^2}{e^2}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}-\frac{1}{2}x\sqrt{d+ex^2}\left(16a+\frac{19cd^2}{e^2}\right)\right)}{4e}+\frac{11c^2d^2x^3\sqrt{d+ex^2}}{4e^2}}{6e}-\frac{c^2dx^5\sqrt{d+ex^2}}{6e^2}}{d}$$

input

```
Int[(a + c*x^4)^2/(d + e*x^2)^(3/2),x]
```

output

```
((c*d^2 + a*e^2)^2*x)/(d*e^4*Sqrt[d + e*x^2]) - (-1/6*(c^2*d*x^5*Sqrt[d +
e*x^2])/e^2 + ((11*c^2*d^2*x^3*Sqrt[d + e*x^2])/(4*e^2) + (3*c*d*(-1/2*((1
6*a + (19*c*d^2)/e^2)*x*Sqrt[d + e*x^2]) + (d*(48*a + (35*c*d^2)/e^2)*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/(4*e))/(6*e))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-3d^{2}\sqrt{e\,x^{2}+d}\,(a\,e^{2}+\frac{35c\,d^{2}}{48})c\,\operatorname{arctanh}(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}})+x(3d^{2}(-\frac{7c\,x^{4}}{72}+a)c\,e^{\frac{5}{2}}+cd\,x^{2}(\frac{c\,x^{4}}{6}+a)e^{\frac{7}{2}}+\frac{35e^{\frac{3}{2}}c^{2}d^{3}x^{2}}{48}+\frac{35\sqrt{e}\,c^{2}}{16}}{e^{\frac{9}{2}}\sqrt{e\,x^{2}+d}\,d}$
risch	$\frac{cx(8c\,x^{4}e^{2}-22de\,x^{2}c+48a\,e^{2}+57c\,d^{2})\sqrt{e\,x^{2}+d}}{48e^{4}}+\frac{2cdxa}{e^{2}\sqrt{e\,x^{2}+d}}+\frac{c^{2}d^{3}x}{e^{4}\sqrt{e\,x^{2}+d}}-\frac{3cd\ln(x\sqrt{e}+\sqrt{e\,x^{2}+d})a}{e^{\frac{5}{2}}}-35c^{2}d^{3}$
default	$\frac{a^{2}x}{d\sqrt{e\,x^{2}+d}}+c^{2}\left(\frac{x^{7}}{6e\sqrt{e\,x^{2}+d}}-\frac{7d\left(\frac{x^{5}}{4e\sqrt{e\,x^{2}+d}}-\frac{5d\left(\frac{x^{3}}{2e\sqrt{e\,x^{2}+d}}-\frac{3d\left(-\frac{x}{e\sqrt{e\,x^{2}+d}}+\frac{\ln(x\sqrt{e}+\sqrt{e\,x^{2}+d})}{e^{\frac{3}{2}}}\right)}{2e}\right)}{4e}\right)}{6e}\right)+2$

input
```
int((c*x^4+a)^2/(e*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^(9/2)/(e*x^2+d)^(1/2)*(-3*d^2*(e*x^2+d)^(1/2)*(a*e^2+35/48*c*d^2)*c*ar
ctanh((e*x^2+d)^(1/2)/x/e^(1/2))+x*(3*d^2*(-7/72*c*x^4+a)*c*e^(5/2)+c*d*x^
2*(1/6*c*x^4+a)*e^(7/2)+35/48*e^(3/2)*c^2*d^3*x^2+35/16*e^(1/2)*c^2*d^4+e^
(9/2)*a^2))/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.34

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \left[\frac{3\,(35\,c^2d^5+48\,acd^3e^2+(35\,c^2d^4e+48\,acd^2e^3)x^2)\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x\right.}{}\right.$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(35*c^2*d^5 + 48*a*c*d^3*e^2 + (35*c^2*d^4*e + 48*a*c*d^2*e^3)*x^
2)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(8*c^2*d*e^
4*x^7 - 14*c^2*d^2*e^3*x^5 + (35*c^2*d^3*e^2 + 48*a*c*d*e^4)*x^3 + 3*(35*c
^2*d^4*e + 48*a*c*d^2*e^3 + 16*a^2*e^5)*x)*sqrt(e*x^2 + d))/(d*e^6*x^2 + d
^2*e^5), 1/48*(3*(35*c^2*d^5 + 48*a*c*d^3*e^2 + (35*c^2*d^4*e + 48*a*c*d^2
*e^3)*x^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (8*c^2*d*e^4*x^7
- 14*c^2*d^2*e^3*x^5 + (35*c^2*d^3*e^2 + 48*a*c*d*e^4)*x^3 + 3*(35*c^2*d^4
*e + 48*a*c*d^2*e^3 + 16*a^2*e^5)*x)*sqrt(e*x^2 + d))/(d*e^6*x^2 + d^2*e^5
)]
```

Sympy [F]

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(a+cx^4)^2}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**4+a)**2/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + c*x**4)**2/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((c*x^4+a)^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.95

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \frac{\left(\left(2\left(\frac{4\,c^2x^2}{e}-\frac{7\,c^2d}{e^2}\right)x^2+\frac{35\,c^2d^3e^4+48\,acde^6}{de^7}\right)x^2+\frac{3\,(35\,c^2d^4e^3+48\,acd^2e^5+16\,a^2e^7)}{de^7}\right)x}{48\,\sqrt{ex^2+d}} + \frac{(35\,c^2d^3+48\,acde^2)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{16\,e^{\frac{9}{2}}}$$

input

```
integrate((c*x^4+a)^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output

```
1/48*((2*(4*c^2*x^2/e - 7*c^2*d/e^2)*x^2 + (35*c^2*d^3*e^4 + 48*a*c*d*e^6)
/(d*e^7))*x^2 + 3*(35*c^2*d^4*e^3 + 48*a*c*d^2*e^5 + 16*a^2*e^7)/(d*e^7))*
x/sqrt(e*x^2 + d) + 1/16*(35*c^2*d^3 + 48*a*c*d*e^2)*log(abs(-sqrt(e)*x +
sqrt(e*x^2 + d)))/e^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(c\,x^4+a)^2}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^(3/2),x)
```

output
```
int((a + c*x^4)^2/(d + e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.26

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{3/2}}\,dx = \frac{384\sqrt{e\,x^2+d}\,a^2e^5x+1152\sqrt{e\,x^2+d}\,ac\,d^2e^3x+384\sqrt{e\,x^2+d}\,acd\,e^4x^3+840\sqrt{e\,x^2+}{}$$

input
```
int((c*x^4+a)^2/(e*x^2+d)^(3/2),x)
```

output
```
(384*sqrt(d + e*x**2)*a**2*e**5*x + 1152*sqrt(d + e*x**2)*a*c*d**2*e**3*x
+ 384*sqrt(d + e*x**2)*a*c*d*e**4*x**3 + 840*sqrt(d + e*x**2)*c**2*d**4*e*
x + 280*sqrt(d + e*x**2)*c**2*d**3*e**2*x**3 - 112*sqrt(d + e*x**2)*c**2*d
**2*e**3*x**5 + 64*sqrt(d + e*x**2)*c**2*d*e**4*x**7 - 1152*sqrt(e)*log((s
qrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*c*d**3*e**2 - 1152*sqrt(e)*log((sq
rt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*c*d**2*e**3*x**2 - 840*sqrt(e)*log(
(sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*c**2*d**5 - 840*sqrt(e)*log((sqrt(
d + e*x**2) + sqrt(e)*x)/sqrt(d))*c**2*d**4*e*x**2 + 384*sqrt(e)*a**2*d*e*
*4 + 384*sqrt(e)*a**2*e**5*x**2 + 864*sqrt(e)*a*c*d**3*e**2 + 864*sqrt(e)*
a*c*d**2*e**3*x**2 + 525*sqrt(e)*c**2*d**5 + 525*sqrt(e)*c**2*d**4*e*x**2)
/(384*d*e**5*(d + e*x**2))
```

3.377 $\int \frac{(a+cx^4)^2}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 171

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{5/2}}\,dx=\frac{(cd^2+ae^2)^2x}{3de^4(d+ex^2)^{3/2}}-\frac{2(5cd^2-ae^2)(cd^2+ae^2)x}{3d^2e^4\sqrt{d+ex^2}}-\frac{11c^2dx\sqrt{d+ex^2}}{8e^4}+\frac{c^2x^3\sqrt{d+ex^2}}{4e^3}+\frac{c(35cd^2+16ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{9/2}}$$

output
```
1/3*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^(3/2)-2/3*(-a*e^2+5*c*d^2)*(a*e^2+c*
d^2)*x/d^2/e^4/(e*x^2+d)^(1/2)-11/8*c^2*d*x*(e*x^2+d)^(1/2)/e^4+1/4*c^2*x^
3*(e*x^2+d)^(1/2)/e^3+1/8*c*(16*a*e^2+35*c*d^2)*arctanh(e^(1/2)*x/(e*x^2+d
)^(1/2))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{5/2}}\,dx=\frac{x(8a^2e^4(3d+2ex^2)-16acd^2e^2(3d+4ex^2)-c^2d^2(105d^3+140d^2ex^2+21de^2x^4-6e}{24d^2e^4(d+ex^2)^{3/2}}-\frac{c(35cd^2+16ae^2)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{8e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^(5/2),x]
```

output
```
(x*(8*a^2*e^4*(3*d + 2*e*x^2) - 16*a*c*d^2*e^2*(3*d + 4*e*x^2) - c^2*d^2*(
105*d^3 + 140*d^2*e*x^2 + 21*d*e^2*x^4 - 6*e^3*x^6)))/(24*d^2*e^4*(d + e*x
^2)^(3/2)) - (c*(35*c*d^2 + 16*a*e^2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])
/(8*e^(9/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1472, 25, 2345, 27, 1473, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(ae^2+cd^2\right)^2}{3de^4\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{\frac{3c^2dx^6}{e}-\frac{3c^2d^2x^4}{e^2}+\frac{3cd\left(cd^2+2ae^2\right)x^2}{e^3}+2a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{\left(ex^2+d\right)^{3/2}}dx}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{3c^2dx^6}{e}-\frac{3c^2d^2x^4}{e^2}+\frac{3cd\left(cd^2+2ae^2\right)x^2}{e^3}+2a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{x\left(ae^2+cd^2\right)^2}{3de^4\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{2x\left(a^2-\frac{4acd^2}{e^2}-\frac{5c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}-\frac{\int -\frac{3\left(\frac{c^2d^2x^4}{e^2}-\frac{2c^2d^3x^2}{e^3}+\frac{cd^2\left(3cd^2+2ae^2\right)}{e^4}\right)}{\sqrt{ex^2+d}}dx}{d}}{3d}+\frac{x\left(ae^2+cd^2\right)^2}{3de^4\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{\frac{c^2d^2x^4}{e^2}-\frac{2c^2d^3x^2}{e^3}+\frac{cd^2\left(3cd^2+2ae^2\right)}{e^4}}{\sqrt{ex^2+d}}dx}{d}+\frac{2x\left(a^2-\frac{4acd^2}{e^2}-\frac{5c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}}{3d}+\frac{x\left(ae^2+cd^2\right)^2}{3de^4\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{3\left(\frac{\int \frac{cd^2\left(4\left(3cd^2+2ae^2\right)-11cdex^2\right)}{e^3\sqrt{ex^2+d}}dx}{4e}+\frac{c^2d^2x^3\sqrt{d+ex^2}}{4e^3}\right)}{d}+\frac{2x\left(a^2-\frac{4acd^2}{e^2}-\frac{5c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}}{3d}+\frac{x\left(ae^2+cd^2\right)^2}{3de^4\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\left(\frac{cd^2\int \frac{4\left(3cd^2+2ae^2\right)-11cdex^2}{\sqrt{ex^2+d}}dx}{4e^4}+\frac{c^2d^2x^3\sqrt{d+ex^2}}{4e^3}\right)}{d}+\frac{2x\left(a^2-\frac{4acd^2}{e^2}-\frac{5c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}}{3d}+\frac{x\left(ae^2+cd^2\right)^2}{3de^4\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{3\left(\frac{cd^2\left(\frac{1}{2}\left(16ae^2+35cd^2\right)\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{11}{2}cdx\sqrt{d+ex^2}\right)}{4e^4}+\frac{c^2d^2x^3\sqrt{d+ex^2}}{4e^3}\right)}{d}+\frac{2x\left(a^2-\frac{4acd^2}{e^2}-\frac{5c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}}{3d}+\frac{x\left(ae^2+cd^2\right)^2}{3de^4\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3\left(\frac{cd^2\left(\frac{1}{2}\left(16ae^2+35cd^2\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{11}{2}cdx\sqrt{d+ex^2}\right)}{4e^4}+\frac{c^2d^2x^3\sqrt{d+ex^2}}{4e^3}\right)}{d}+\frac{2x\left(a^2-\frac{4acd^2}{e^2}-\frac{5c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}}{3d}+\frac{x\left(ae^2+cd^2\right)^2}{3de^4\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2x\left(a^2-\frac{4acd^2}{e^2}-\frac{5c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}+\frac{3\left(\frac{cd^2\left(\frac{\left(16ae^2+35cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}-\frac{11}{2}cdx\sqrt{d+ex^2}\right)}{4e^4}+\frac{c^2d^2x^3\sqrt{d+ex^2}}{4e^3}\right)}{d}}{3d}+\frac{x\left(ae^2+cd^2\right)^2}{3de^4\left(d+ex^2\right)^{3/2}}$$

input

```
Int[(a + c*x^4)^2/(d + e*x^2)^(5/2),x]
```

output

```
((c*d^2 + a*e^2)^2*x)/(3*d*e^4*(d + e*x^2)^(3/2)) + ((2*(a^2 - (5*c^2*d^4)
/e^4 - (4*a*c*d^2)/e^2)*x)/(d*Sqrt[d + e*x^2]) + (3*((c^2*d^2*x^3*Sqrt[d +
 e*x^2])/(4*e^3) + (c*d^2*((-11*c*d*x*Sqrt[d + e*x^2])/2 + ((35*c*d^2 + 16
*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/(4*e^4)))/d)/(
3*d)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1472

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1473

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2345

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{2\left(a e^{2}+\frac{35 c d^{2}}{16}\right) d^{2} (e x^{2}+d)^{\frac{3}{2}} c \operatorname{arctanh}\left(\frac{\sqrt{e x^{2}+d}}{x\sqrt{e}}\right)+\frac{2x(-\frac{105 c^{2} d^{5}\sqrt{e}}{16}+(-\frac{35 c^{2} d^{4} x^{2}}{4}-3\left(\frac{7 c x^{4}}{16}+a\right) c e d^{3}-4 x^{2}\left(-\frac{3 c x^{4}}{32}+a\right) c e^{2}}{3}}{e^{\frac{9}{2}} (e x^{2}+d)^{\frac{3}{2}} d^{2}}$
default	$a^{2}\left(\frac{x}{3d(e x^{2}+d)^{\frac{3}{2}}}+\frac{2x}{3d^{2}\sqrt{e x^{2}+d}}\right)+c^{2}(\frac{x^{7}}{4e(e x^{2}+d)^{\frac{3}{2}}}-\frac{7d(\frac{x^{5}}{2e(e x^{2}+d)^{\frac{3}{2}}}-\frac{5d(-\frac{x^{3}}{3e(e x^{2}+d)^{\frac{3}{2}}}+\frac{-\frac{x}{e\sqrt{e x^{2}+d}}+\ln}{}}{2e}}{4e}$
risch	$-\frac{c^{2}x(-2e x^{2}+11d)\sqrt{e x^{2}+d}}{8e^{4}}+\frac{\frac{c(16a e^{2}+35 c d^{2})\ln(x\sqrt{e}+\sqrt{e x^{2}+d})}{\sqrt{e}}-\frac{2(a^{2}e^{4}+2acd^{2}e^{2}+c^{2}d^{4})(-\frac{\sqrt{(x-\frac{\sqrt{-de}}{e})^{2}e+2\sqrt{-de}}}{3\sqrt{-de}(x-\frac{\sqrt{-d}}{e}}}{e}}{}$

input
```
int((c*x^4+a)^2/(e*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(3*(a*e^2+35/16*c*d^2)*d^2*(e*x^2+d)^(3/2)*c*arctanh((e*x^2+d)^(1/2)/x
/e^(1/2))+x*(-105/16*c^2*d^5*e^(1/2)+(-35/4*c^2*d^4*x^2-3*(7/16*c*x^4+a)*c
*e*d^3-4*x^2*(-3/32*c*x^4+a)*c*e^2*d^2+3/2*e^3*a^2*d+e^4*x^2*a^2)*e^(3/2))
)/(e*x^2+d)^(3/2)/e^(9/2)/d^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.80

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{5/2}}\,dx = \left[\frac{3\,(35\,c^2d^6+16\,acd^4e^2+(35\,c^2d^4e^2+16\,acd^2e^4)x^4+2\,(35\,c^2d^5e+16\,acd^3e^3)x^2)\sqrt{e}}{}\right.$$

$$-\frac{3\,(35\,c^2d^6+16\,acd^4e^2+(35\,c^2d^4e^2+16\,acd^2e^4)x^4+2\,(35\,c^2d^5e+16\,acd^3e^3)x^2)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-}{24\,(d^2e^7x^4}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(35*c^2*d^6 + 16*a*c*d^4*e^2 + (35*c^2*d^4*e^2 + 16*a*c*d^2*e^4)*
x^4 + 2*(35*c^2*d^5*e + 16*a*c*d^3*e^3)*x^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt
(e*x^2 + d)*sqrt(e)*x - d) + 2*(6*c^2*d^2*e^4*x^7 - 21*c^2*d^3*e^3*x^5 - 4
*(35*c^2*d^4*e^2 + 16*a*c*d^2*e^4 - 4*a^2*e^6)*x^3 - 3*(35*c^2*d^5*e + 16*
a*c*d^3*e^3 - 8*a^2*d*e^5)*x)*sqrt(e*x^2 + d))/(d^2*e^7*x^4 + 2*d^3*e^6*x^
2 + d^4*e^5), -1/24*(3*(35*c^2*d^6 + 16*a*c*d^4*e^2 + (35*c^2*d^4*e^2 + 16
*a*c*d^2*e^4)*x^4 + 2*(35*c^2*d^5*e + 16*a*c*d^3*e^3)*x^2)*sqrt(-e)*arctan
(sqrt(-e)*x/sqrt(e*x^2 + d)) - (6*c^2*d^2*e^4*x^7 - 21*c^2*d^3*e^3*x^5 - 4
*(35*c^2*d^4*e^2 + 16*a*c*d^2*e^4 - 4*a^2*e^6)*x^3 - 3*(35*c^2*d^5*e + 16*
a*c*d^3*e^3 - 8*a^2*d*e^5)*x)*sqrt(e*x^2 + d))/(d^2*e^7*x^4 + 2*d^3*e^6*x^
2 + d^4*e^5)]
```

Sympy [F]

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(a+cx^4)^2}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x**4+a)**2/(e*x**2+d)**(5/2),x)
```

output
```
Integral((a + c*x**4)**2/(d + e*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((c*x^4+a)^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{5/2}}\,dx = \frac{\left(\left(3\left(\frac{2\,c^2x^2}{e}-\frac{7\,c^2d}{e^2}\right)x^2-\frac{4\,(35\,c^2d^4e^4+16\,acd^2e^6-4\,a^2e^8)}{d^2e^7}\right)x^2-\frac{3\,(35\,c^2d^5e^3+16\,acd^3e^5-8\,a^2de^7)}{d^2e^7}\right)x}{24\,(ex^2+d)^{\frac{3}{2}}} - \frac{(35\,c^2d^2+16\,ace^2)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{8\,e^{\frac{9}{2}}}$$

input

```
integrate((c*x^4+a)^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output

```
1/24*((3*(2*c^2*x^2/e - 7*c^2*d/e^2)*x^2 - 4*(35*c^2*d^4*e^4 + 16*a*c*d^2*
e^6 - 4*a^2*e^8)/(d^2*e^7))*x^2 - 3*(35*c^2*d^5*e^3 + 16*a*c*d^3*e^5 - 8*a
^2*d*e^7)/(d^2*e^7))*x/(e*x^2 + d)^(3/2) - 1/8*(35*c^2*d^2 + 16*a*c*e^2)*l
og(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(c\,x^4+a)^2}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^(5/2),x)
```

output
```
int((a + c*x^4)^2/(d + e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.73

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{5/2}}\,dx = \frac{192\sqrt{e\,x^2+d}\,a^2 d\,e^5 x + 128\sqrt{e\,x^2+d}\,a^2 e^6 x^3 - 384\sqrt{e\,x^2+d}\,ac\,d^3 e^3 x - 512\sqrt{e\,x^2+}}{}$$

input
```
int((c*x^4+a)^2/(e*x^2+d)^(5/2),x)
```

output
```
(192*sqrt(d + e*x**2)*a**2*d*e**5*x + 128*sqrt(d + e*x**2)*a**2*e**6*x**3
- 384*sqrt(d + e*x**2)*a*c*d**3*e**3*x - 512*sqrt(d + e*x**2)*a*c*d**2*e**
4*x**3 - 840*sqrt(d + e*x**2)*c**2*d**5*e*x - 1120*sqrt(d + e*x**2)*c**2*d
**4*e**2*x**3 - 168*sqrt(d + e*x**2)*c**2*d**3*e**3*x**5 + 48*sqrt(d + e*x
**2)*c**2*d**2*e**4*x**7 + 384*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/
sqrt(d))*a*c*d**4*e**2 + 768*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sq
rt(d))*a*c*d**3*e**3*x**2 + 384*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)
/sqrt(d))*a*c*d**2*e**4*x**4 + 840*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)
*x)/sqrt(d))*c**2*d**6 + 1680*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/s
qrt(d))*c**2*d**5*e*x**2 + 840*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/
sqrt(d))*c**2*d**4*e**2*x**4 - 128*sqrt(e)*a**2*d**2*e**4 - 256*sqrt(e)*a*
*2*d*e**5*x**2 - 128*sqrt(e)*a**2*e**6*x**4 + 175*sqrt(e)*c**2*d**6 + 350*
sqrt(e)*c**2*d**5*e*x**2 + 175*sqrt(e)*c**2*d**4*e**2*x**4)/(192*d**2*e**5
*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.378 $\int \frac{(a+cx^4)^2}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 185

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{7/2}}\,dx=\frac{(cd^2+ae^2)^2x}{5de^4(d+ex^2)^{5/2}}-\frac{4(4cd^2-ae^2)(cd^2+ae^2)x}{15d^2e^4(d+ex^2)^{3/2}}+\frac{2(29c^2d^4+3acd^2e^2+4a^2e^4)x}{15d^3e^4\sqrt{d+ex^2}}+\frac{c^2x\sqrt{d+ex^2}}{2e^4}-\frac{7c^2d\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{9/2}}$$

output
```
1/5*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^(5/2)-4/15*(-a*e^2+4*c*d^2)*(a*e^2+c
*d^2)*x/d^2/e^4/(e*x^2+d)^(3/2)+2/15*(4*a^2*e^4+3*a*c*d^2*e^2+29*c^2*d^4)*
x/d^3/e^4/(e*x^2+d)^(1/2)+1/2*c^2*x*(e*x^2+d)^(1/2)/e^4-7/2*c^2*d*arctanh(
e^(1/2)*x/(e*x^2+d)^(1/2))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.75

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{7/2}}\,dx=\frac{x(12acd^2e^4x^4+2a^2e^4(15d^2+20dex^2+8e^2x^4)+c^2d^3(105d^3+245d^2ex^2+161de^2x^4}{30d^3e^4(d+ex^2)^{5/2}}+\frac{7c^2d\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{2e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^(7/2),x]
```

output
```
(x*(12*a*c*d^2*e^4*x^4 + 2*a^2*e^4*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4) + c^2
*d^3*(105*d^3 + 245*d^2*e*x^2 + 161*d*e^2*x^4 + 15*e^3*x^6)))/(30*d^3*e^4*
(d + e*x^2)^(5/2)) + (7*c^2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(2*e^(9
/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1472, 25, 2345, 25, 1471, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(ae^2+cd^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}-\frac{\int -\frac{\frac{5c^2dx^6}{e}-\frac{5c^2d^2x^4}{e^2}+\frac{5cd\left(cd^2+2ae^2\right)x^2}{e^3}+4a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{\left(ex^2+d\right)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{5c^2dx^6}{e}-\frac{5c^2d^2x^4}{e^2}+\frac{5cd\left(cd^2+2ae^2\right)x^2}{e^3}+4a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{\left(ex^2+d\right)^{5/2}}dx}{5d}+\frac{x\left(ae^2+cd^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{4x\left(a^2-\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}-\frac{\int -\frac{\frac{13c^2d^4}{e^4}-\frac{30c^2x^2d^3}{e^3}+\frac{15c^2x^4d^2}{e^2}+\frac{6acd^2}{e^2}+8a^2}{\left(ex^2+d\right)^{3/2}}dx}{3d}}{5d}+\frac{x\left(ae^2+cd^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\frac{13c^2d^4}{e^4}-\frac{30c^2x^2d^3}{e^3}+\frac{15c^2x^4d^2}{e^2}+\frac{6acd^2}{e^2}+8a^2}{\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{4x\left(a^2-\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}}{5d}+\frac{x\left(ae^2+cd^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \ 1471$$

$$\frac{\frac{\frac{2x\left(4a^2+\frac{3acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}-\frac{\int \frac{15c^2d^3\left(3d-ex^2\right)}{e^4\sqrt{ex^2+d}}dx}{d}}{3d}+\frac{4x\left(a^2-\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}}{5d}+\frac{x\left(ae^2+cd^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{2x\left(4a^2+\frac{3acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}-\frac{15c^2d^2\int \frac{3d-ex^2}{\sqrt{ex^2+d}}dx}{e^4}}{3d}+\frac{4x\left(a^2-\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}}{5d}+\frac{x\left(ae^2+cd^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{\frac{2x\left(4a^2+\frac{3acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}-\frac{15c^2d^2\left(\frac{7}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{1}{2}x\sqrt{d+ex^2}\right)}{e^4}}{3d}+\frac{4x\left(a^2-\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}}{5d}+\frac{x\left(ae^2+cd^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{\frac{2x\left(4a^2+\frac{3acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}-\frac{15c^2d^2\left(\frac{7}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{1}{2}x\sqrt{d+ex^2}\right)}{e^4}}{3d}+\frac{4x\left(a^2-\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}}{5d}+\frac{x\left(ae^2+cd^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{2x\left(4a^2+\frac{3acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{d\sqrt{d+ex^2}}-\frac{15c^2d^2\left(\frac{7d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}-\frac{1}{2}x\sqrt{d+ex^2}\right)}{e^4}}{3d}+\frac{4x\left(a^2-\frac{3acd^2}{e^2}-\frac{4c^2d^4}{e^4}\right)}{3d(d+ex^2)^{3/2}}}{5d}+\frac{x\left(ae^2+cd^2\right)^2}{5de^4\left(d+ex^2\right)^{5/2}}$$

input

```
Int[(a + c*x^4)^2/(d + e*x^2)^(7/2),x]
```

output
```
((c*d^2 + a*e^2)^2*x)/(5*d*e^4*(d + e*x^2)^(5/2)) + ((4*(a^2 - (4*c^2*d^4)
/e^4 - (3*a*c*d^2)/e^2)*x)/(3*d*(d + e*x^2)^(3/2)) + ((2*(4*a^2 + (29*c^2*
d^4)/e^4 + (3*a*c*d^2)/e^2)*x)/(d*Sqrt[d + e*x^2]) - (15*c^2*d^2*(-1/2*(x*
Sqrt[d + e*x^2]) + (7*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e]))
)/e^4)/(3*d))/(5*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{-\frac{7\left(e x^{2}+d\right)^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{e x^{2}+d}}{x \sqrt{e}}\right) c^{2} d^{4}}{2}+x\left(a d^{2}\left(\frac{2 c x^{4}}{5}+a\right) e^{\frac{9}{2}}+\frac{4 e^{\frac{11}{2}} a^{2} d x^{2}}{3}+\frac{8 e^{\frac{13}{2}} a^{2} x^{4}}{15}+\frac{161 d^{3}\left(\frac{15 x^{6} e^{\frac{7}{2}}}{161}+e^{\frac{5}{2}} d x^{4}+\frac{35 e^{\frac{3}{2}} d^{2} x^{2}}{23}\right.}{30}\right.}{e^{\frac{9}{2}}\left(e x^{2}+d\right)^{\frac{5}{2}} d^{3}}$
default	$a^{2}\left(\frac{x}{5 d \left(e x^{2}+d\right)^{\frac{5}{2}}}+\frac{\frac{4 x}{15 d \left(e x^{2}+d\right)^{\frac{3}{2}}}+\frac{8 x}{15 d^{2} \sqrt{e x^{2}+d}}}{d}\right)+c^{2}\left(\frac{x^{7}}{2 e \left(e x^{2}+d\right)^{\frac{5}{2}}}-\frac{7 d \left(-\frac{x^{5}}{5 e \left(e x^{2}+d\right)^{\frac{5}{2}}}+\frac{-\frac{x^{3}}{3 e \left(e x^{2}+d\right)^{\frac{3}{2}}}+}{}\right.}{2 e}\right.$
risch	Expression too large to display

input
```
int((c*x^4+a)^2/(e*x^2+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^(9/2)*(-7/2*(e*x^2+d)^(5/2)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))*c^2*d^4
+x*(a*d^2*(2/5*c*x^4+a)*e^(9/2)+4/3*e^(11/2)*a^2*d*x^2+8/15*e^(13/2)*a^2*x
^4+161/30*d^3*(15/161*x^6*e^(7/2)+e^(5/2)*d*x^4+35/23*e^(3/2)*d^2*x^2+15/2
3*e^(1/2)*d^3)*c^2))/(e*x^2+d)^(5/2)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.48

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{7/2}}\,dx = \left[\frac{105\left(c^2d^4e^3x^6+3\,c^2d^5e^2x^4+3\,c^2d^6ex^2+c^2d^7\right)\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-\right.}{6}\right.$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
[1/60*(105*(c^2*d^4*e^3*x^6 + 3*c^2*d^5*e^2*x^4 + 3*c^2*d^6*e*x^2 + c^2*d^
7)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(15*c^2*d^3
*e^4*x^7 + (161*c^2*d^4*e^3 + 12*a*c*d^2*e^5 + 16*a^2*e^7)*x^5 + 5*(49*c^2
*d^5*e^2 + 8*a^2*d*e^6)*x^3 + 15*(7*c^2*d^6*e + 2*a^2*d^2*e^5)*x)*sqrt(e*x
^2 + d))/(d^3*e^8*x^6 + 3*d^4*e^7*x^4 + 3*d^5*e^6*x^2 + d^6*e^5), 1/30*(10
5*(c^2*d^4*e^3*x^6 + 3*c^2*d^5*e^2*x^4 + 3*c^2*d^6*e*x^2 + c^2*d^7)*sqrt(-
e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (15*c^2*d^3*e^4*x^7 + (161*c^2*d^4
*e^3 + 12*a*c*d^2*e^5 + 16*a^2*e^7)*x^5 + 5*(49*c^2*d^5*e^2 + 8*a^2*d*e^6)
*x^3 + 15*(7*c^2*d^6*e + 2*a^2*d^2*e^5)*x)*sqrt(e*x^2 + d))/(d^3*e^8*x^6 +
 3*d^4*e^7*x^4 + 3*d^5*e^6*x^2 + d^6*e^5)]
```

Sympy [F]

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{7/2}}\,dx = \int \frac{(a+cx^4)^2}{(d+ex^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x**4+a)**2/(e*x**2+d)**(7/2),x)
```

output
```
Integral((a + c*x**4)**2/(d + e*x**2)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{7/2}}\,dx = \frac{\left(\left(\left(\frac{15\,c^2x^2}{e} + \frac{161\,c^2d^4e^5+12\,acd^2e^7+16\,a^2e^9}{d^3e^7}\right)x^2 + \frac{5\,(49\,c^2d^5e^4+8\,a^2de^8)}{d^3e^7}\right)x^2 + \frac{15\,(7\,c^2d^6e^3+2\,a^2d^2e^7)}{d^3e^7}\right.}{30\,(ex^2+d)^{\frac{5}{2}}} + \frac{7\,c^2d\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{2\,e^{\frac{9}{2}}}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
1/30*(((15*c^2*x^2/e + (161*c^2*d^4*e^5 + 12*a*c*d^2*e^7 + 16*a^2*e^9)/(d^
3*e^7))*x^2 + 5*(49*c^2*d^5*e^4 + 8*a^2*d*e^8)/(d^3*e^7))*x^2 + 15*(7*c^2*
d^6*e^3 + 2*a^2*d^2*e^7)/(d^3*e^7))*x/(e*x^2 + d)^(5/2) + 7/2*c^2*d*log(ab
s(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{7/2}}\,dx = \int \frac{(c\,x^4+a)^2}{(e\,x^2+d)^{7/2}}\,dx$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^(7/2),x)
```

output
```
int((a + c*x^4)^2/(d + e*x^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.73

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{7/2}}\,dx = \frac{120\sqrt{e\,x^2+d}\,a^2d^2e^5x+160\sqrt{e\,x^2+d}\,a^2d\,e^6x^3+64\sqrt{e\,x^2+d}\,a^2e^7x^5+48\sqrt{e\,x^2+d}}{}$$

input
```
int((c*x^4+a)^2/(e*x^2+d)^(7/2),x)
```

output
```
(120*sqrt(d + e*x**2)*a**2*d**2*e**5*x + 160*sqrt(d + e*x**2)*a**2*d*e**6*
x**3 + 64*sqrt(d + e*x**2)*a**2*e**7*x**5 + 48*sqrt(d + e*x**2)*a*c*d**2*e
**5*x**5 + 420*sqrt(d + e*x**2)*c**2*d**6*e*x + 980*sqrt(d + e*x**2)*c**2*
d**5*e**2*x**3 + 644*sqrt(d + e*x**2)*c**2*d**4*e**3*x**5 + 60*sqrt(d + e*
x**2)*c**2*d**3*e**4*x**7 - 420*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)
/sqrt(d))*c**2*d**7 - 1260*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt
(d))*c**2*d**6*e*x**2 - 1260*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sq
rt(d))*c**2*d**5*e**2*x**4 - 420*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x
)/sqrt(d))*c**2*d**4*e**3*x**6 - 64*sqrt(e)*a**2*d**3*e**4 - 192*sqrt(e)*a
**2*d**2*e**5*x**2 - 192*sqrt(e)*a**2*d*e**6*x**4 - 64*sqrt(e)*a**2*e**7*x
**6 + 48*sqrt(e)*a*c*d**5*e**2 + 144*sqrt(e)*a*c*d**4*e**3*x**2 + 144*sqrt
(e)*a*c*d**3*e**4*x**4 + 48*sqrt(e)*a*c*d**2*e**5*x**6 - 203*sqrt(e)*c**2*
d**7 - 609*sqrt(e)*c**2*d**6*e*x**2 - 609*sqrt(e)*c**2*d**5*e**2*x**4 - 20
3*sqrt(e)*c**2*d**4*e**3*x**6)/(120*d**3*e**5*(d**3 + 3*d**2*e*x**2 + 3*d*
e**2*x**4 + e**3*x**6))
```

3.379 $\int \frac{(a+cx^4)^2}{(d+ex^2)^{9/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 208

$$\begin{aligned}\int \frac{(a+cx^4)^2}{(d+ex^2)^{9/2}}\,dx &= \frac{(cd^2+ae^2)^2\,x}{7de^4\,(d+ex^2)^{7/2}} \\ &\quad - \frac{2(11cd^2-3ae^2)\,(cd^2+ae^2)\,x}{35d^2e^4\,(d+ex^2)^{5/2}} + \frac{2(61c^2d^4+3acd^2e^2+12a^2e^4)\,x}{105d^3e^4\,(d+ex^2)^{3/2}} \\ &\quad - \frac{4(44c^2d^4-3acd^2e^2-12a^2e^4)\,x}{105d^4e^4\sqrt{d+ex^2}} + \frac{c^2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{9/2}}\end{aligned}$$

output

```
1/7*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^(7/2)-2/35*(-3*a*e^2+11*c*d^2)*(a*e^
2+c*d^2)*x/d^2/e^4/(e*x^2+d)^(5/2)+2/105*(12*a^2*e^4+3*a*c*d^2*e^2+61*c^2*
d^4)*x/d^3/e^4/(e*x^2+d)^(3/2)-4/105*(-12*a^2*e^4-3*a*c*d^2*e^2+44*c^2*d^4
)*x/d^4/e^4/(e*x^2+d)^(1/2)+c^2*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.76

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{9/2}}\,dx = \frac{x(6acd^2e^4x^4(7d+2ex^2)+3a^2e^4(35d^3+70d^2ex^2+56de^2x^4+16e^3x^6)-c^2d^4(105d^3}{105d^4e^4\left(d+ex^2\right)^{7/2}}$$
$$-\frac{c^2\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^(9/2),x]
```

output
```
(x*(6*a*c*d^2*e^4*x^4*(7*d + 2*e*x^2) + 3*a^2*e^4*(35*d^3 + 70*d^2*e*x^2 +
 56*d*e^2*x^4 + 16*e^3*x^6) - c^2*d^4*(105*d^3 + 350*d^2*e*x^2 + 406*d*e^2
*x^4 + 176*e^3*x^6)))/(105*d^4*e^4*(d + e*x^2)^(7/2)) - (c^2*Log[-(Sqrt[e]
*x) + Sqrt[d + e*x^2]])/e^(9/2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {1472, 25, 2345, 25, 1471, 25, 25, 27, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{9/2}}\,dx$$
$$\Big\downarrow \text{1472}$$
$$\frac{x\left(ae^2+cd^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}-\frac{\int -\frac{\frac{7c^2dx^6}{e}-\frac{7c^2d^2x^4}{e^2}+\frac{7cd\left(cd^2+2ae^2\right)x^2}{e^3}+6a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{(ex^2+d)^{7/2}}dx}{7d}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{\frac{7c^2dx^6}{e}-\frac{7c^2d^2x^4}{e^2}+\frac{7cd\left(cd^2+2ae^2\right)x^2}{e^3}+6a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{(ex^2+d)^{7/2}}dx}{7d}+\frac{x\left(ae^2+cd^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow 2345$$

$$\frac{\frac{2x\left(3a^2-\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}-\frac{\int -\frac{\frac{17c^2d^4}{e^4}-\frac{70c^2x^2d^3}{e^3}+\frac{35c^2x^4d^2}{e^2}+\frac{6acd^2}{e^2}+24a^2}{\left(ex^2+d\right)^{5/2}}dx}{5d}}{7d}+\frac{x\left(ae^2+cd^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow 25$$

$$\frac{\frac{\int \frac{\frac{17c^2d^4}{e^4}-\frac{70c^2x^2d^3}{e^3}+\frac{35c^2x^4d^2}{e^2}+\frac{6acd^2}{e^2}+24a^2}{\left(ex^2+d\right)^{5/2}}dx}{5d}+\frac{2x\left(3a^2-\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(ae^2+cd^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow 1471$$

$$\frac{\frac{\frac{2x\left(12a^2+\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\int -\frac{105c^2x^2d^3+\left(-\frac{71c^2d^4}{e^4}+\frac{12acd^2}{e^2}+48a^2\right)e^3}{e^3\left(ex^2+d\right)^{3/2}}dx}{3d}}{5d}+\frac{2x\left(3a^2-\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(ae^2+cd^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow 25$$

$$\frac{\frac{\frac{\int -\frac{\frac{71c^2d^4}{e}-105c^2x^2d^3-12aced^2-48a^2e^3}{e^3\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{2x\left(12a^2+\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}}{5d}+\frac{2x\left(3a^2-\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(ae^2+cd^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow 25$$

$$\frac{\frac{\frac{2x\left(12a^2+\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\int \frac{\frac{71c^2d^4}{e}-105c^2x^2d^3-12aced^2-48a^2e^3}{e^3\left(ex^2+d\right)^{3/2}}dx}{3d}}{5d}+\frac{2x\left(3a^2-\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(ae^2+cd^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{2x\left(12a^2+\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\int\frac{\frac{71c^2d^4}{e}-105c^2x^2d^3-12aced^2-48a^2e^3}{\left(ex^2+d\right)^{3/2}}dx}{3de^3}}{5d}+\frac{2x\left(3a^2-\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(ae^2+cd^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{\frac{2x\left(12a^2+\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\frac{4x\left(-12a^2e^4-3acd^2e^2+44c^2d^4\right)}{de\sqrt{d+ex^2}}-\frac{105c^2d^3\int\frac{1}{\sqrt{ex^2+d}}dx}{e}}{3de^3}}{5d}+\frac{2x\left(3a^2-\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(ae^2+cd^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{2x\left(12a^2+\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\frac{4x\left(-12a^2e^4-3acd^2e^2+44c^2d^4\right)}{de\sqrt{d+ex^2}}-\frac{105c^2d^3\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}}{3de^3}}{5d}+\frac{2x\left(3a^2-\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(ae^2+cd^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{2x\left(12a^2+\frac{3acd^2}{e^2}+\frac{61c^2d^4}{e^4}\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{\frac{4x\left(-12a^2e^4-3acd^2e^2+44c^2d^4\right)}{de\sqrt{d+ex^2}}-\frac{105c^2d^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}}{3de^3}}{5d}+\frac{2x\left(3a^2-\frac{8acd^2}{e^2}-\frac{11c^2d^4}{e^4}\right)}{5d(d+ex^2)^{5/2}}}{7d}+\frac{x\left(ae^2+cd^2\right)^2}{7de^4\left(d+ex^2\right)^{7/2}}$$

input
```
Int[(a + c*x^4)^2/(d + e*x^2)^(9/2),x]
```

output

```
((c*d^2 + a*e^2)^2*x)/(7*d*e^4*(d + e*x^2)^(7/2)) + ((2*(3*a^2 - (11*c^2*d
^4)/e^4 - (8*a*c*d^2)/e^2)*x)/(5*d*(d + e*x^2)^(5/2)) + ((2*(12*a^2 + (61*
c^2*d^4)/e^4 + (3*a*c*d^2)/e^2)*x)/(3*d*(d + e*x^2)^(3/2)) - ((4*(44*c^2*d
^4 - 3*a*c*d^2*e^2 - 12*a^2*e^4)*x)/(d*e*Sqrt[d + e*x^2]) - (105*c^2*d^3*A
rcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(3/2))/(3*d*e^3))/(5*d))/(7*d)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{c^2 d^4 \operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}}{x\sqrt{e}}\right) e^4 \left(e\,x^2+d\right)^{\frac{7}{2}}+\frac{x\,e^{\frac{9}{2}} \left(48a^2e^7x^6+12ac\,d^2e^5x^6-176c^2d^4e^3x^6+168a^2d\,e^6x^4+42ac\,d^3e^4x^4-406c^2d^5e^2x^4+\right.}{105}}{\left(e\,x^2+d\right)^{\frac{7}{2}} e^{\frac{17}{2}} d^4}$
default	$a^2\left(\frac{x}{7d(e\,x^2+d)^{\frac{7}{2}}}+\frac{\frac{6x}{35d\left(e\,x^2+d\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d\left(e\,x^2+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^2\sqrt{e\,x^2+d}}\right)}{7d}}{d}\right)+c^2\left(-\frac{x^7}{7e(e\,x^2+d)^{\frac{7}{2}}}+\frac{-\frac{x^5}{5e\left(e\,x^2+d\right)^{\frac{5}{2}}}}{}\right.$

input

```
int((c*x^4+a)^2/(e*x^2+d)^(9/2),x,method=_RETURNVERBOSE)
```

output

```
(c^2*d^4*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))*e^4*(e*x^2+d)^(7/2)+1/105*x*e^
(9/2)*(48*a^2*e^7*x^6+12*a*c*d^2*e^5*x^6-176*c^2*d^4*e^3*x^6+168*a^2*d*e^6
*x^4+42*a*c*d^3*e^4*x^4-406*c^2*d^5*e^2*x^4+210*a^2*d^2*e^5*x^2-350*c^2*d^
6*e*x^2+105*a^2*d^3*e^4-105*c^2*d^7))/(e*x^2+d)^(7/2)/e^(17/2)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.68

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{9/2}}\,dx = \Bigg[\frac{105\,(c^2d^4e^4x^8+4\,c^2d^5e^3x^6+6\,c^2d^6e^2x^4+4\,c^2d^7ex^2+c^2d^8)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex}\right.}{}$$

$$-\frac{105\,(c^2d^4e^4x^8+4\,c^2d^5e^3x^6+6\,c^2d^6e^2x^4+4\,c^2d^7ex^2+c^2d^8)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+(4\,(44\,c^2d^4e^4-3\,ac}{105\,(d^4e^9x^8+4\,d^5}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(c^2*d^4*e^4*x^8 + 4*c^2*d^5*e^3*x^6 + 6*c^2*d^6*e^2*x^4 + 4*c
^2*d^7*e*x^2 + c^2*d^8)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x
 - d) - 2*(4*(44*c^2*d^4*e^4 - 3*a*c*d^2*e^6 - 12*a^2*e^8)*x^7 + 14*(29*c^
2*d^5*e^3 - 3*a*c*d^3*e^5 - 12*a^2*d*e^7)*x^5 + 70*(5*c^2*d^6*e^2 - 3*a^2*
d^2*e^6)*x^3 + 105*(c^2*d^7*e - a^2*d^3*e^5)*x)*sqrt(e*x^2 + d))/(d^4*e^9*
x^8 + 4*d^5*e^8*x^6 + 6*d^6*e^7*x^4 + 4*d^7*e^6*x^2 + d^8*e^5), -1/105*(10
5*(c^2*d^4*e^4*x^8 + 4*c^2*d^5*e^3*x^6 + 6*c^2*d^6*e^2*x^4 + 4*c^2*d^7*e*x
^2 + c^2*d^8)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (4*(44*c^2*d^4
*e^4 - 3*a*c*d^2*e^6 - 12*a^2*e^8)*x^7 + 14*(29*c^2*d^5*e^3 - 3*a*c*d^3*e^
5 - 12*a^2*d*e^7)*x^5 + 70*(5*c^2*d^6*e^2 - 3*a^2*d^2*e^6)*x^3 + 105*(c^2*
d^7*e - a^2*d^3*e^5)*x)*sqrt(e*x^2 + d))/(d^4*e^9*x^8 + 4*d^5*e^8*x^6 + 6*
d^6*e^7*x^4 + 4*d^7*e^6*x^2 + d^8*e^5)]
```

Sympy [F]

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{9/2}}\,dx = \int \frac{(a+cx^4)^2}{(d+ex^2)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x**4+a)**2/(e*x**2+d)**(9/2),x)
```

output
```
Integral((a + c*x**4)**2/(d + e*x**2)**(9/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^(9/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.92

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{9/2}}\,dx = -\frac{\left(2\left(x^2\left(\frac{2\,(44\,c^2d^4e^6-3\,acd^2e^8-12\,a^2e^{10})x^2}{d^4e^7}+\frac{7\,(29\,c^2d^5e^5-3\,acd^3e^7-12\,a^2de^9)}{d^4e^7}\right)+\frac{35\,(5\,c^2d^6e^4-3\,a^2d^2e^8)}{d^4e^7}\right)x^2+\frac{105\,(c^2d^7e^3-a^2}{d^4e^7}}{105\,(ex^2+d)^{\frac{7}{2}}} - \frac{c^2\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{e^{\frac{9}{2}}}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^(9/2),x, algorithm="giac")
```

output
```
-1/105*(2*(x^2*(2*(44*c^2*d^4*e^6 - 3*a*c*d^2*e^8 - 12*a^2*e^10)*x^2/(d^4*
e^7) + 7*(29*c^2*d^5*e^5 - 3*a*c*d^3*e^7 - 12*a^2*d*e^9)/(d^4*e^7)) + 35*(
5*c^2*d^6*e^4 - 3*a^2*d^2*e^8)/(d^4*e^7))*x^2 + 105*(c^2*d^7*e^3 - a^2*d^3
*e^7)/(d^4*e^7))*x/(e*x^2 + d)^(7/2) - c^2*log(abs(-sqrt(e)*x + sqrt(e*x^2
 + d)))/e^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{9/2}}\,dx = \int \frac{(c\,x^4+a)^2}{(e\,x^2+d)^{9/2}}\,dx$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^(9/2),x)
```

output
```
int((a + c*x^4)^2/(d + e*x^2)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 641, normalized size of antiderivative = 3.08

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{9/2}}\,dx = \frac{56\sqrt{e}\,c^2d^8+42\sqrt{e\,x^2+d}\,ac\,d^3e^5x^5+12\sqrt{e\,x^2+d}\,ac\,d^2e^6x^7+420\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}}{\sqrt{d}}\right.}{}$$

input
```
int((c*x^4+a)^2/(e*x^2+d)^(9/2),x)
```

output

```
(105*sqrt(d + e*x**2)*a**2*d**3*e**5*x + 210*sqrt(d + e*x**2)*a**2*d**2*e*
*6*x**3 + 168*sqrt(d + e*x**2)*a**2*d*e**7*x**5 + 48*sqrt(d + e*x**2)*a**2
*e**8*x**7 + 42*sqrt(d + e*x**2)*a*c*d**3*e**5*x**5 + 12*sqrt(d + e*x**2)*
a*c*d**2*e**6*x**7 - 105*sqrt(d + e*x**2)*c**2*d**7*e*x - 350*sqrt(d + e*x
**2)*c**2*d**6*e**2*x**3 - 406*sqrt(d + e*x**2)*c**2*d**5*e**3*x**5 - 176*
sqrt(d + e*x**2)*c**2*d**4*e**4*x**7 + 105*sqrt(e)*log((sqrt(d + e*x**2) +
 sqrt(e)*x)/sqrt(d))*c**2*d**8 + 420*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(
e)*x)/sqrt(d))*c**2*d**7*e*x**2 + 630*sqrt(e)*log((sqrt(d + e*x**2) + sqrt
(e)*x)/sqrt(d))*c**2*d**6*e**2*x**4 + 420*sqrt(e)*log((sqrt(d + e*x**2) +
sqrt(e)*x)/sqrt(d))*c**2*d**5*e**3*x**6 + 105*sqrt(e)*log((sqrt(d + e*x**2
) + sqrt(e)*x)/sqrt(d))*c**2*d**4*e**4*x**8 - 48*sqrt(e)*a**2*d**4*e**4 -
192*sqrt(e)*a**2*d**3*e**5*x**2 - 288*sqrt(e)*a**2*d**2*e**6*x**4 - 192*sq
rt(e)*a**2*d*e**7*x**6 - 48*sqrt(e)*a**2*e**8*x**8 - 12*sqrt(e)*a*c*d**6*e
**2 - 48*sqrt(e)*a*c*d**5*e**3*x**2 - 72*sqrt(e)*a*c*d**4*e**4*x**4 - 48*s
qrt(e)*a*c*d**3*e**5*x**6 - 12*sqrt(e)*a*c*d**2*e**6*x**8 + 56*sqrt(e)*c**
2*d**8 + 224*sqrt(e)*c**2*d**7*e*x**2 + 336*sqrt(e)*c**2*d**6*e**2*x**4 +
224*sqrt(e)*c**2*d**5*e**3*x**6 + 56*sqrt(e)*c**2*d**4*e**4*x**8)/(105*d**
4*e**5*(d**4 + 4*d**3*e*x**2 + 6*d**2*e**2*x**4 + 4*d*e**3*x**6 + e**4*x**
8))
```

3.380 $\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^{11/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 228

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^{11/2}}\,dx &= \frac{\left(cd^2+ae^2\right)^2 x}{9de^4\left(d+ex^2\right)^{9/2}}\\ &- \frac{4(7cd^2-2ae^2)\left(cd^2+ae^2\right)x}{63d^2e^4\left(d+ex^2\right)^{7/2}} + \frac{2(35c^2d^4+acd^2e^2+8a^2e^4)\,x}{105d^3e^4\left(d+ex^2\right)^{5/2}}\\ &- \frac{4(35c^2d^4-2acd^2e^2-16a^2e^4)\,x}{315d^4e^4\left(d+ex^2\right)^{3/2}} + \frac{(35c^2d^4+16acd^2e^2+128a^2e^4)\,x}{315d^5e^4\sqrt{d+ex^2}}\end{aligned}$$

output

```
1/9*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^(9/2)-4/63*(-2*a*e^2+7*c*d^2)*(a*e^2
+c*d^2)*x/d^2/e^4/(e*x^2+d)^(7/2)+2/105*(8*a^2*e^4+a*c*d^2*e^2+35*c^2*d^4)
*x/d^3/e^4/(e*x^2+d)^(5/2)-4/315*(-16*a^2*e^4-2*a*c*d^2*e^2+35*c^2*d^4)*x/
d^4/e^4/(e*x^2+d)^(3/2)+1/315*(128*a^2*e^4+16*a*c*d^2*e^2+35*c^2*d^4)*x/d^
5/e^4/(e*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.48

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^{11/2}}\,dx = \frac{35c^2d^4x^9+2acd^2x^5\left(63d^2+36dex^2+8e^2x^4\right)+a^2\left(315d^4x+840d^3ex^3+1008d^2e^2x^5\right.}{315d^5\left(d+ex^2\right)^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^(11/2),x]
```

output
```
(35*c^2*d^4*x^9 + 2*a*c*d^2*x^5*(63*d^2 + 36*d*e*x^2 + 8*e^2*x^4) + a^2*(3
15*d^4*x + 840*d^3*e*x^3 + 1008*d^2*e^2*x^5 + 576*d*e^3*x^7 + 128*e^4*x^9)
)/(315*d^5*(d + e*x^2)^(9/2))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {1470, 2334, 27, 2090, 1587, 9, 25, 25, 27, 362, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^{11/2}}\,dx$$

$$\Big\downarrow \text{1470}$$

$$\frac{\int \frac{x^2\left(8ea^2+d\left(c^2x^6+2acx^2\right)\right)}{\left(ex^2+d\right)^{11/2}}dx}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \text{2334}$$

$$\frac{\frac{\int \frac{3x^4\left(c\left(cx^4+2a\right)d^2+16a^2e^2\right)}{\left(ex^2+d\right)^{11/2}}dx}{3d}+\frac{8a^2ex^3}{3d\left(d+ex^2\right)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{x^4\left(c\left(cx^4+2a\right)d^2+16a^2e^2\right)}{\left(ex^2+d\right)^{11/2}}dx}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \; 2090$$

$$\frac{\frac{\int \frac{x^4\left(c^2d^2x^4+2a\left(cd^2+8ae^2\right)\right)}{\left(ex^2+d\right)^{11/2}}dx}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \; 1587$$

$$\frac{\frac{\frac{x^5\left(16a^2e^2+2acd^2+\frac{c^2d^4}{e^2}\right)}{9d\left(d+ex^2\right)^{9/2}}-\frac{\int -\frac{x^3\left(\frac{9c^2d^3x^3}{e}+\left(-\frac{5c^2d^4}{e^2}+8acd^2+64a^2e^2\right)x\right)}{\left(ex^2+d\right)^{9/2}}dx}{9d}}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \; 9$$

$$\frac{\frac{\frac{x^5\left(16a^2e^2+2acd^2+\frac{c^2d^4}{e^2}\right)}{9d\left(d+ex^2\right)^{9/2}}-\frac{\int -\frac{x^4\left(9c^2x^2d^3+e\left(-\frac{5c^2d^4}{e^2}+8acd^2+64a^2e^2\right)\right)}{e\left(ex^2+d\right)^{9/2}}dx}{9d}}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -\frac{x^4\left(\frac{5c^2d^4}{e}-9c^2x^2d^3-8aced^2-64a^2e^3\right)}{e\left(ex^2+d\right)^{9/2}}dx}{9d}+\frac{x^5\left(16a^2e^2+2acd^2+\frac{c^2d^4}{e^2}\right)}{9d\left(d+ex^2\right)^{9/2}}}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{x^5\left(16a^2e^2+2acd^2+\frac{c^2d^4}{e^2}\right)}{9d\left(d+ex^2\right)^{9/2}}-\frac{\int \frac{x^4\left(\frac{5c^2d^4}{e}-9c^2x^2d^3-8aced^2-64a^2e^3\right)}{e\left(ex^2+d\right)^{9/2}}dx}{9d}}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{x^5\left(16a^2e^2+2acd^2+\frac{c^2d^4}{e^2}\right)}{9d\left(d+ex^2\right)^{9/2}}-\frac{\int\frac{x^4\left(\frac{5c^2d^4}{e}-9c^2x^2d^3-8aced^2-64a^2e^3\right)}{\left(ex^2+d\right)^{9/2}}dx}{9de}}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{\frac{x^5\left(16a^2e^2+2acd^2+\frac{c^2d^4}{e^2}\right)}{9d\left(d+ex^2\right)^{9/2}}-\frac{\frac{2x^5\left(-32a^2e^4-4acd^2e^2+7c^2d^4\right)}{7de\left(d+ex^2\right)^{7/2}}-\frac{\left(128a^2e^4+16acd^2e^2+35c^2d^4\right)\int\frac{x^4}{\left(ex^2+d\right)^{7/2}}dx}{7de}}{9de}}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{\frac{\frac{x^5\left(16a^2e^2+2acd^2+\frac{c^2d^4}{e^2}\right)}{9d\left(d+ex^2\right)^{9/2}}-\frac{\frac{2x^5\left(-32a^2e^4-4acd^2e^2+7c^2d^4\right)}{7de\left(d+ex^2\right)^{7/2}}-\frac{x^5\left(128a^2e^4+16acd^2e^2+35c^2d^4\right)}{35d^2e\left(d+ex^2\right)^{5/2}}}{9de}}{d}+\frac{8a^2ex^3}{3d(d+ex^2)^{9/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{9/2}}$$

input

```
Int[(a + c*x^4)^2/(d + e*x^2)^(11/2),x]
```

output

```
(a^2*x)/(d*(d + e*x^2)^(9/2)) + ((8*a^2*e*x^3)/(3*d*(d + e*x^2)^(9/2)) + (
((2*a*c*d^2 + (c^2*d^4)/e^2 + 16*a^2*e^2)*x^5)/(9*d*(d + e*x^2)^(9/2)) - (
(2*(7*c^2*d^4 - 4*a*c*d^2*e^2 - 32*a^2*e^4)*x^5)/(7*d*e*(d + e*x^2)^(7/2))
 - ((35*c^2*d^4 + 16*a*c*d^2*e^2 + 128*a^2*e^4)*x^5)/(35*d^2*e*(d + e*x^2)
^(5/2)))/(9*d*e))/d)/d
```

Defintions of rubi rules used

rule 9

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 362

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1470

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d + e*x^2)^q*(d*
PolynomialQuotient[(a + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q
+ 1/2, 0] && LtQ[4*p + 2*q + 1, 0]
```

rule 1587
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x]
, R = Coeff[PolynomialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[
(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d*f*(q + 1))), x] + Simp[f/(2*d*
(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*x*
Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, f}, x] && IGtQ[p,
 0] && LtQ[q, -1] && GtQ[m, 0]
```

rule 2090
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && BinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u
, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.44

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{9}c^{2}x^{8}+\frac{2}{5}ac\,x^{4}+a^{2}\right)d^{4}+\frac{8x^{2}a\left(\frac{3c\,x^{4}}{35}+a\right)e\,d^{3}}{3}+\frac{16x^{4}a\left(\frac{c\,x^{4}}{63}+a\right)e^{2}d^{2}}{5}+\frac{64a^{2}d\,e^{3}x^{6}}{35}+\frac{128a^{2}e^{4}x^{8}}{315}\right)}{\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
gosper	$\frac{x\left(128a^{2}e^{4}x^{8}+16ac\,d^{2}e^{2}x^{8}+35c^{2}d^{4}x^{8}+576a^{2}d\,e^{3}x^{6}+72ac\,d^{3}e\,x^{6}+1008a^{2}d^{2}e^{2}x^{4}+126ac\,d^{4}x^{4}+840a^{2}e\,x^{2}d^{3}+315a^{2}d^{4}\right)}{315\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
trager	$\frac{x\left(128a^{2}e^{4}x^{8}+16ac\,d^{2}e^{2}x^{8}+35c^{2}d^{4}x^{8}+576a^{2}d\,e^{3}x^{6}+72ac\,d^{3}e\,x^{6}+1008a^{2}d^{2}e^{2}x^{4}+126ac\,d^{4}x^{4}+840a^{2}e\,x^{2}d^{3}+315a^{2}d^{4}\right)}{315\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
orering	$\frac{x\left(128a^{2}e^{4}x^{8}+16ac\,d^{2}e^{2}x^{8}+35c^{2}d^{4}x^{8}+576a^{2}d\,e^{3}x^{6}+72ac\,d^{3}e\,x^{6}+1008a^{2}d^{2}e^{2}x^{4}+126ac\,d^{4}x^{4}+840a^{2}e\,x^{2}d^{3}+315a^{2}d^{4}\right)}{315\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$

input
```
int((c*x^4+a)^2/(e*x^2+d)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
x/(e*x^2+d)^(9/2)*((1/9*c^2*x^8+2/5*a*c*x^4+a^2)*d^4+8/3*x^2*a*(3/35*c*x^4
+a)*e*d^3+16/5*x^4*a*(1/63*c*x^4+a)*e^2*d^2+64/35*a^2*d*e^3*x^6+128/315*a^
2*e^4*x^8)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.73

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{11/2}}\,dx = \frac{((35\,c^2d^4+16\,acd^2e^2+128\,a^2e^4)x^9+840\,a^2d^3ex^3+72\,(acd^3e+8\,a^2de^3)x^7+315\,a^2}{315\,(d^5e^5x^{10}+5\,d^6e^4x^8+10\,d^7e^3x^6+10\,d^8e^2x^4+5\,d^9}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^(11/2),x, algorithm="fricas")
```

output
```
1/315*((35*c^2*d^4 + 16*a*c*d^2*e^2 + 128*a^2*e^4)*x^9 + 840*a^2*d^3*e*x^3
 + 72*(a*c*d^3*e + 8*a^2*d*e^3)*x^7 + 315*a^2*d^4*x + 126*(a*c*d^4 + 8*a^2
*d^2*e^2)*x^5)*sqrt(e*x^2 + d)/(d^5*e^5*x^10 + 5*d^6*e^4*x^8 + 10*d^7*e^3*
x^6 + 10*d^8*e^2*x^4 + 5*d^9*e*x^2 + d^10)
```

Sympy [F]

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{11/2}}\,dx = \int \frac{(a+cx^4)^2}{(d+ex^2)^{\frac{11}{2}}}\,dx$$

input
```
integrate((c*x**4+a)**2/(e*x**2+d)**(11/2),x)
```

output
```
Integral((a + c*x**4)**2/(d + e*x**2)**(11/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.62

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{11/2}}\,dx = -\frac{c^2x^7}{2\,(ex^2+d)^{\frac{9}{2}}e} - \frac{7\,c^2dx^5}{8\,(ex^2+d)^{\frac{9}{2}}e^2} - \frac{35\,c^2d^2x^3}{48\,(ex^2+d)^{\frac{9}{2}}e^3} - \frac{acx^3}{3\,(ex^2+d)^{\frac{9}{2}}e} + \frac{128\,a^2x}{315\,\sqrt{ex^2+d}d^5} + \frac{64\,a^2x}{315\,(ex^2+d)^{\frac{3}{2}}d^4} + \frac{16\,a^2x}{105\,(ex^2+d)^{\frac{5}{2}}d^3} + \frac{8\,a^2x}{63\,(ex^2+d)^{\frac{7}{2}}d^2} + \frac{a^2x}{9\,(ex^2+d)^{\frac{9}{2}}d} + \frac{c^2x}{18\,(ex^2+d)^{\frac{3}{2}}e^4} + \frac{c^2x}{9\,\sqrt{ex^2+d}de^4} + \frac{c^2dx}{24\,(ex^2+d)^{\frac{5}{2}}e^4} + \frac{5\,c^2d^2x}{144\,(ex^2+d)^{\frac{7}{2}}e^4} - \frac{35\,c^2d^3x}{144\,(ex^2+d)^{\frac{9}{2}}e^4} + \frac{acx}{63\,(ex^2+d)^{\frac{7}{2}}e^2} + \frac{16\,acx}{315\,\sqrt{ex^2+d}d^3e^2} + \frac{8\,acx}{315\,(ex^2+d)^{\frac{3}{2}}d^2e^2} + \frac{2\,acx}{105\,(ex^2+d)^{\frac{5}{2}}de^2} - \frac{acdx}{9\,(ex^2+d)^{\frac{9}{2}}e^2}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^(11/2),x, algorithm="maxima")
```

output
```
-1/2*c^2*x^7/((e*x^2 + d)^(9/2)*e) - 7/8*c^2*d*x^5/((e*x^2 + d)^(9/2)*e^2)
 - 35/48*c^2*d^2*x^3/((e*x^2 + d)^(9/2)*e^3) - 1/3*a*c*x^3/((e*x^2 + d)^(9
/2)*e) + 128/315*a^2*x/(sqrt(e*x^2 + d)*d^5) + 64/315*a^2*x/((e*x^2 + d)^(
3/2)*d^4) + 16/105*a^2*x/((e*x^2 + d)^(5/2)*d^3) + 8/63*a^2*x/((e*x^2 + d)
^(7/2)*d^2) + 1/9*a^2*x/((e*x^2 + d)^(9/2)*d) + 1/18*c^2*x/((e*x^2 + d)^(3
/2)*e^4) + 1/9*c^2*x/(sqrt(e*x^2 + d)*d*e^4) + 1/24*c^2*d*x/((e*x^2 + d)^(
5/2)*e^4) + 5/144*c^2*d^2*x/((e*x^2 + d)^(7/2)*e^4) - 35/144*c^2*d^3*x/((e
*x^2 + d)^(9/2)*e^4) + 1/63*a*c*x/((e*x^2 + d)^(7/2)*e^2) + 16/315*a*c*x/(
sqrt(e*x^2 + d)*d^3*e^2) + 8/315*a*c*x/((e*x^2 + d)^(3/2)*d^2*e^2) + 2/105
*a*c*x/((e*x^2 + d)^(5/2)*d*e^2) - 1/9*a*c*d*x/((e*x^2 + d)^(9/2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.62

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{11/2}}\,dx = \frac{\left(\left(\left(x^2\left(\frac{(35\,c^2d^4e^4+16\,acd^2e^6+128\,a^2e^8)x^2}{d^5e^4} + \frac{72\,(acd^3e^5+8\,a^2de^7)}{d^5e^4}\right) + \frac{126\,(acd^4e^4+8\,a^2d^2e^6)}{d^5e^4}\right)x^2 + 84}{315\,(ex^2+d)^{\frac{9}{2}}}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^(11/2),x, algorithm="giac")
```

output
```
1/315*(((x^2*((35*c^2*d^4*e^4 + 16*a*c*d^2*e^6 + 128*a^2*e^8)*x^2/(d^5*e^4
) + 72*(a*c*d^3*e^5 + 8*a^2*d*e^7)/(d^5*e^4)) + 126*(a*c*d^4*e^4 + 8*a^2*d
^2*e^6)/(d^5*e^4))*x^2 + 840*a^2*e/d^2)*x^2 + 315*a^2/d)*x/(e*x^2 + d)^(9/
2)
```

Mupad [B] (verification not implemented)

Time = 17.43 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.25

$$
\begin{aligned}
\int \frac{(a+cx^4)^2}{(d+ex^2)^{11/2}}\,dx ={}& \frac{x\left(\frac{a^2}{9\,d}+\frac{d^2\left(\frac{c^2\,d}{9\,e^2}+\frac{2\,a\,c}{9\,d}\right)}{e^2}\right)}{{\left(e\,x^2+d\right)}^{9/2}} \\
& -\frac{x\left(\frac{-8\,a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4}{63\,d^2\,e^4}+\frac{d\left(\frac{2\,c^2\,d}{7\,e^3}+\frac{c\left(c\,d^2+2\,a\,e^2\right)}{7\,d\,e^3}\right)}{e}\right)}{{\left(e\,x^2+d\right)}^{7/2}} \\
& -\frac{x\left(\frac{c^2}{3\,e^4}-\frac{64\,a^2\,e^4+8\,a\,c\,d^2\,e^2-35\,c^2\,d^4}{315\,d^4\,e^4}\right)}{{\left(e\,x^2+d\right)}^{3/2}}+\frac{x\left(\frac{3\,c^2\,d}{5\,e^4}+\frac{16\,a^2\,e^4+2\,a\,c\,d^2\,e^2+7\,c^2\,d^4}{105\,d^3\,e^4}\right)}{{\left(e\,x^2+d\right)}^{5/2}} \\
& +\frac{x\left(128\,a^2\,e^4+16\,a\,c\,d^2\,e^2+35\,c^2\,d^4\right)}{315\,d^5\,e^4\,\sqrt{e\,x^2+d}}
\end{aligned}
$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^(11/2),x)
```

output
```
(x*(a^2/(9*d) + (d^2*((c^2*d)/(9*e^2) + (2*a*c)/(9*d)))/e^2))/(d + e*x^2)^
(9/2) - (x*((c^2*d^4 - 8*a^2*e^4 + 2*a*c*d^2*e^2)/(63*d^2*e^4) + (d*((2*c^
2*d)/(7*e^3) + (c*(2*a*e^2 + c*d^2))/(7*d*e^3)))/e))/(d + e*x^2)^(7/2) - (
x*(c^2/(3*e^4) - (64*a^2*e^4 - 35*c^2*d^4 + 8*a*c*d^2*e^2)/(315*d^4*e^4)))
/(d + e*x^2)^(3/2) + (x*((3*c^2*d)/(5*e^4) + (16*a^2*e^4 + 7*c^2*d^4 + 2*a
*c*d^2*e^2)/(105*d^3*e^4)))/(d + e*x^2)^(5/2) + (x*(128*a^2*e^4 + 35*c^2*d
^4 + 16*a*c*d^2*e^2))/(315*d^5*e^4*(d + e*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.25

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{11/2}}\,dx = \frac{126\sqrt{e\,x^2+d}\,ac\,d^4e^5x^5+72\sqrt{e\,x^2+d}\,ac\,d^3e^6x^7+16\sqrt{e\,x^2+d}\,ac\,d^2e^7x^9-80\sqrt{e}\,ac}{}$$

input
```
int((c*x^4+a)^2/(e*x^2+d)^(11/2),x)
```

output
```
(315*sqrt(d + e*x**2)*a**2*d**4*e**5*x + 840*sqrt(d + e*x**2)*a**2*d**3*e*
*6*x**3 + 1008*sqrt(d + e*x**2)*a**2*d**2*e**7*x**5 + 576*sqrt(d + e*x**2)
*a**2*d*e**8*x**7 + 128*sqrt(d + e*x**2)*a**2*e**9*x**9 + 126*sqrt(d + e*x
**2)*a*c*d**4*e**5*x**5 + 72*sqrt(d + e*x**2)*a*c*d**3*e**6*x**7 + 16*sqrt
(d + e*x**2)*a*c*d**2*e**7*x**9 + 35*sqrt(d + e*x**2)*c**2*d**4*e**5*x**9
 - 128*sqrt(e)*a**2*d**5*e**4 - 640*sqrt(e)*a**2*d**4*e**5*x**2 - 1280*sqrt
(e)*a**2*d**3*e**6*x**4 - 1280*sqrt(e)*a**2*d**2*e**7*x**6 - 640*sqrt(e)*a
**2*d*e**8*x**8 - 128*sqrt(e)*a**2*e**9*x**10 - 16*sqrt(e)*a*c*d**7*e**2 -
 80*sqrt(e)*a*c*d**6*e**3*x**2 - 160*sqrt(e)*a*c*d**5*e**4*x**4 - 160*sqrt
(e)*a*c*d**4*e**5*x**6 - 80*sqrt(e)*a*c*d**3*e**6*x**8 - 16*sqrt(e)*a*c*d*
*2*e**7*x**10 + 35*sqrt(e)*c**2*d**9 + 175*sqrt(e)*c**2*d**8*e*x**2 + 350*
sqrt(e)*c**2*d**7*e**2*x**4 + 350*sqrt(e)*c**2*d**6*e**3*x**6 + 175*sqrt(e
)*c**2*d**5*e**4*x**8 + 35*sqrt(e)*c**2*d**4*e**5*x**10)/(315*d**5*e**5*(d
**5 + 5*d**4*e*x**2 + 10*d**3*e**2*x**4 + 10*d**2*e**3*x**6 + 5*d*e**4*x**
8 + e**5*x**10))
```

3.381 $\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^{13/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 278

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^{13/2}}\,dx &= \frac{\left(cd^2+ae^2\right)^2x}{11de^4\left(d+ex^2\right)^{11/2}}-\frac{2(17cd^2-5ae^2)\left(cd^2+ae^2\right)x}{99d^2e^4\left(d+ex^2\right)^{9/2}}\\ &+\frac{2(161c^2d^4+3acd^2e^2+40a^2e^4)\,x}{693d^3e^4\left(d+ex^2\right)^{7/2}}-\frac{4(70c^2d^4-3acd^2e^2-40a^2e^4)\,x}{1155d^4e^4\left(d+ex^2\right)^{5/2}}\\ &+\frac{(35c^2d^4+48acd^2e^2+640a^2e^4)\,x}{3465d^5e^4\left(d+ex^2\right)^{3/2}}+\frac{2(35c^2d^4+48acd^2e^2+640a^2e^4)\,x}{3465d^6e^4\sqrt{d+ex^2}}\end{aligned}$$

output

```
1/11*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^(11/2)-2/99*(-5*a*e^2+17*c*d^2)*(a*
e^2+c*d^2)*x/d^2/e^4/(e*x^2+d)^(9/2)+2/693*(40*a^2*e^4+3*a*c*d^2*e^2+161*c
^2*d^4)*x/d^3/e^4/(e*x^2+d)^(7/2)-4/1155*(-40*a^2*e^4-3*a*c*d^2*e^2+70*c^2
*d^4)*x/d^4/e^4/(e*x^2+d)^(5/2)+1/3465*(640*a^2*e^4+48*a*c*d^2*e^2+35*c^2*
d^4)*x/d^5/e^4/(e*x^2+d)^(3/2)+2/3465*(640*a^2*e^4+48*a*c*d^2*e^2+35*c^2*d
^4)*x/d^6/e^4/(e*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.51

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{13/2}}\,dx = \frac{35c^2d^4x^9(11d+2ex^2)+6acd^2x^5(231d^3+198d^2ex^2+88de^2x^4+16e^3x^6)+5a^2(693}{3465d^6\left(d+ex^2\right)}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^(13/2),x]
```

output
```
(35*c^2*d^4*x^9*(11*d + 2*e*x^2) + 6*a*c*d^2*x^5*(231*d^3 + 198*d^2*e*x^2
+ 88*d*e^2*x^4 + 16*e^3*x^6) + 5*a^2*(693*d^5*x + 2310*d^4*e*x^3 + 3696*d^
3*e^2*x^5 + 3168*d^2*e^3*x^7 + 1408*d*e^4*x^9 + 256*e^5*x^11))/(3465*d^6*(
d + e*x^2)^(11/2))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {1470, 2334, 2090, 1587, 9, 27, 25, 362, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{13/2}}\,dx$$

$$\Big\downarrow \text{1470}$$

$$\frac{\int \frac{x^2\left(10ea^2+d\left(c^2x^6+2acx^2\right)\right)}{(ex^2+d)^{13/2}}dx}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \text{2334}$$

$$\frac{\frac{\int \frac{x^4\left(3c\left(cx^4+2a\right)d^2+80a^2e^2\right)}{\left(ex^2+d\right)^{13/2}}dx}{3d}+\frac{10a^2ex^3}{3d(d+ex^2)^{11/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \text{2090}$$

$$\frac{\frac{\int \frac{x^4\left(3c^2d^2x^4+2a\left(3cd^2+40ae^2\right)\right)}{\left(ex^2+d\right)^{13/2}}dx}{3d}+\frac{10a^2ex^3}{3d(d+ex^2)^{11/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \, {\color{blue}1587}$$

$$\frac{\frac{\frac{x^5\left(80a^2e^2+6acd^2+\frac{3c^2d^4}{e^2}\right)}{11d\left(d+ex^2\right)^{11/2}}-\frac{\int -\frac{3x^3\left(\frac{11c^2d^3x^3}{e}+\left(-\frac{5c^2d^4}{e^2}+12acd^2+160a^2e^2\right)x\right)}{\left(ex^2+d\right)^{11/2}}dx}{11d}}{3d}+\frac{10a^2ex^3}{3d(d+ex^2)^{11/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \, {\color{blue}9}$$

$$\frac{\frac{\frac{x^5\left(80a^2e^2+6acd^2+\frac{3c^2d^4}{e^2}\right)}{11d\left(d+ex^2\right)^{11/2}}-\frac{\int -\frac{3x^4\left(11c^2x^2d^3+e\left(-\frac{5c^2d^4}{e^2}+12acd^2+160a^2e^2\right)\right)}{e\left(ex^2+d\right)^{11/2}}dx}{11d}}{3d}+\frac{10a^2ex^3}{3d(d+ex^2)^{11/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \, {\color{blue}27}$$

$$\frac{\frac{\frac{3\int -\frac{x^4\left(\frac{5c^2d^4}{e}-11c^2x^2d^3-12aced^2-160a^2e^3\right)}{\left(ex^2+d\right)^{11/2}}dx}{11de}+\frac{x^5\left(80a^2e^2+6acd^2+\frac{3c^2d^4}{e^2}\right)}{11d\left(d+ex^2\right)^{11/2}}}{3d}+\frac{10a^2ex^3}{3d(d+ex^2)^{11/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \, {\color{blue}25}$$

$$\frac{\frac{\frac{x^5\left(80a^2e^2+6acd^2+\frac{3c^2d^4}{e^2}\right)}{11d\left(d+ex^2\right)^{11/2}}-\frac{3\int \frac{x^4\left(\frac{5c^2d^4}{e}-11c^2x^2d^3-12aced^2-160a^2e^3\right)}{\left(ex^2+d\right)^{11/2}}dx}{11de}}{3d}+\frac{10a^2ex^3}{3d(d+ex^2)^{11/2}}}{d}+\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \, {\color{blue}362}$$

$$\frac{\frac{\frac{x^5\left(80a^2e^2+6acd^2+\frac{3c^2d^4}{e^2}\right)}{11d\left(d+ex^2\right)^{11/2}}-\frac{3\left(\frac{4x^5\left(-40a^2e^4-3acd^2e^2+4c^2d^4\right)}{9de\left(d+ex^2\right)^{9/2}}-\frac{\left(640a^2e^4+48acd^2e^2+35c^2d^4\right)\int \frac{x^4}{\left(ex^2+d\right)^{9/2}}dx}{9de}\right)}{11de}}{3d}+\frac{10a^2ex^3}{3d(d+ex^2)^{11/2}}}{d}+$$

$$\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow 245$$

$$\frac{\frac{\frac{x^5\left(80a^2e^2+6acd^2+\frac{3c^2d^4}{e^2}\right)}{11d\left(d+ex^2\right)^{11/2}}-\frac{3\left(\frac{4x^5\left(-40a^2e^4-3acd^2e^2+4c^2d^4\right)}{9de\left(d+ex^2\right)^{9/2}}-\frac{\left(640a^2e^4+48acd^2e^2+35c^2d^4\right)\left(\frac{2e\int \frac{x^6}{\left(ex^2+d\right)^{9/2}}dx}{5d}+\frac{x^5}{5d\left(d+ex^2\right)^{7/2}}\right)}{9de}\right)}{11de}}{3d}+\frac{10a^2}{3d(d+e}}{d}$$

$$\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow 242$$

$$\frac{\frac{\frac{x^5\left(80a^2e^2+6acd^2+\frac{3c^2d^4}{e^2}\right)}{11d\left(d+ex^2\right)^{11/2}}-\frac{3\left(\frac{4x^5\left(-40a^2e^4-3acd^2e^2+4c^2d^4\right)}{9de\left(d+ex^2\right)^{9/2}}-\frac{\left(\frac{2ex^7}{35d^2\left(d+ex^2\right)^{7/2}}+\frac{x^5}{5d\left(d+ex^2\right)^{7/2}}\right)\left(640a^2e^4+48acd^2e^2+35c^2d^4\right)}{9de}\right)}{11de}}{3d}+\frac{10a^2ex}{3d(d+ex^2)}}{d}$$

$$\frac{a^2x}{d\left(d+ex^2\right)^{11/2}}$$

input
```
Int[(a + c*x^4)^2/(d + e*x^2)^(13/2),x]
```

output
```
(a^2*x)/(d*(d + e*x^2)^(11/2)) + ((10*a^2*e*x^3)/(3*d*(d + e*x^2)^(11/2))
+ (((6*a*c*d^2 + (3*c^2*d^4)/e^2 + 80*a^2*e^2)*x^5)/(11*d*(d + e*x^2)^(11/
2)) - (3*((4*(4*c^2*d^4 - 3*a*c*d^2*e^2 - 40*a^2*e^4)*x^5)/(9*d*e*(d + e*x
^2)^(9/2)) - ((35*c^2*d^4 + 48*a*c*d^2*e^2 + 640*a^2*e^4)*(x^5/(5*d*(d + e
*x^2)^(7/2)) + (2*e*x^7)/(35*d^2*(d + e*x^2)^(7/2))))/(9*d*e)))/(11*d*e))/
(3*d))/d
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1470
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d + e*x^2)^q*(d*
PolynomialQuotient[(a + c*x^4)^p - a^p, x^2, x] - e*a^p*(2*q + 3)), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q
+ 1/2, 0] && LtQ[4*p + 2*q + 1, 0]
```

rule 1587
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x]
, R = Coeff[PolynomialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[
(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d*f*(q + 1))), x] + Simp[f/(2*d*
(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*x*
Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, f}, x] && IGtQ[p,
 0] && LtQ[q, -1] && GtQ[m, 0]
```

rule 2090
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && BinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u
, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.47

method	result
pseudoelliptic	$\frac{x\Bigg(\left(\frac{1}{9}c^{2}x^{8}+\frac{2}{5}ac\,x^{4}+a^{2}\right)d^{5}+\frac{10x^{2}\left(\frac{1}{165}c^{2}x^{8}+\frac{18}{175}ac\,x^{4}+a^{2}\right)e\,d^{4}}{3}+\frac{16x^{4}\left(\frac{c\,x^{4}}{35}+a\right)a\,e^{2}d^{3}}{3}+\frac{32x^{6}a\left(\frac{c\,x^{4}}{165}+a\right)e^{3}d^{2}}{7}+\frac{128a^{2}d\,e^{4}x^{8}}{63}+}{(e\,x^{2}+d)^{\frac{11}{2}}d^{6}}$
gosper	$\frac{x(1280a^{2}e^{5}x^{10}+96ac\,d^{2}e^{3}x^{10}+70c^{2}d^{4}e\,x^{10}+7040a^{2}d\,e^{4}x^{8}+528ac\,d^{3}e^{2}x^{8}+385c^{2}d^{5}x^{8}+15840a^{2}d^{2}e^{3}x^{6}+1188ac\,d^{4}e\,x^{6}+}{3465(e\,x^{2}+d)^{\frac{11}{2}}d^{6}}$
trager	$\frac{x(1280a^{2}e^{5}x^{10}+96ac\,d^{2}e^{3}x^{10}+70c^{2}d^{4}e\,x^{10}+7040a^{2}d\,e^{4}x^{8}+528ac\,d^{3}e^{2}x^{8}+385c^{2}d^{5}x^{8}+15840a^{2}d^{2}e^{3}x^{6}+1188ac\,d^{4}e\,x^{6}+}{3465(e\,x^{2}+d)^{\frac{11}{2}}d^{6}}$
orering	$\frac{x(1280a^{2}e^{5}x^{10}+96ac\,d^{2}e^{3}x^{10}+70c^{2}d^{4}e\,x^{10}+7040a^{2}d\,e^{4}x^{8}+528ac\,d^{3}e^{2}x^{8}+385c^{2}d^{5}x^{8}+15840a^{2}d^{2}e^{3}x^{6}+1188ac\,d^{4}e\,x^{6}+}{3465(e\,x^{2}+d)^{\frac{11}{2}}d^{6}}$

input

```
int((c*x^4+a)^2/(e*x^2+d)^(13/2),x,method=_RETURNVERBOSE)
```

output

```
x/(e*x^2+d)^(11/2)*((1/9*c^2*x^8+2/5*a*c*x^4+a^2)*d^5+10/3*x^2*(1/165*c^2*
x^8+18/175*a*c*x^4+a^2)*e*d^4+16/3*x^4*(1/35*c*x^4+a)*a*e^2*d^3+32/7*x^6*a
*(1/165*c*x^4+a)*e^3*d^2+128/63*a^2*d*e^4*x^8+256/693*a^2*e^5*x^10)/d^6
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.78

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{13/2}}\,dx = \frac{(2\,(35\,c^2d^4e+48\,acd^2e^3+640\,a^2e^5)x^{11}+11550\,a^2d^4ex^3+11\,(35\,c^2d^5+48\,acd^3e^2+}{3465\,(d^6e^6x^{12}+6\,d^7e^5x^{10}+15\,d^8e}$$

input

```
integrate((c*x^4+a)^2/(e*x^2+d)^(13/2),x, algorithm="fricas")
```

output

```
1/3465*(2*(35*c^2*d^4*e + 48*a*c*d^2*e^3 + 640*a^2*e^5)*x^11 + 11550*a^2*d
^4*e*x^3 + 11*(35*c^2*d^5 + 48*a*c*d^3*e^2 + 640*a^2*d*e^4)*x^9 + 3465*a^2
*d^5*x + 396*(3*a*c*d^4*e + 40*a^2*d^2*e^3)*x^7 + 462*(3*a*c*d^5 + 40*a^2*
d^3*e^2)*x^5)*sqrt(e*x^2 + d)/(d^6*e^6*x^12 + 6*d^7*e^5*x^10 + 15*d^8*e^4*
x^8 + 20*d^9*e^3*x^6 + 15*d^10*e^2*x^4 + 6*d^11*e*x^2 + d^12)
```

Sympy [F]

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{13/2}}\,dx = \int \frac{(a+cx^4)^2}{(d+ex^2)^{\frac{13}{2}}}\,dx$$

input

```
integrate((c*x**4+a)**2/(e*x**2+d)**(13/2),x)
```

output

```
Integral((a + c*x**4)**2/(d + e*x**2)**(13/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.54

$$
\begin{aligned}
\int \frac{(a+cx^4)^2}{(d+ex^2)^{13/2}}\,dx = &-\frac{c^2x^7}{4\,(ex^2+d)^{\frac{11}{2}}e} - \frac{7\,c^2dx^5}{24\,(ex^2+d)^{\frac{11}{2}}e^2} \\
&- \frac{35\,c^2d^2x^3}{192\,(ex^2+d)^{\frac{11}{2}}e^3} - \frac{acx^3}{4\,(ex^2+d)^{\frac{11}{2}}e} + \frac{256\,a^2x}{693\,\sqrt{ex^2+d}d^6} + \frac{128\,a^2x}{693\,(ex^2+d)^{\frac{3}{2}}d^5} \\
&+ \frac{32\,a^2x}{231\,(ex^2+d)^{\frac{5}{2}}d^4} + \frac{80\,a^2x}{693\,(ex^2+d)^{\frac{7}{2}}d^3} + \frac{10\,a^2x}{99\,(ex^2+d)^{\frac{9}{2}}d^2} + \frac{a^2x}{11\,(ex^2+d)^{\frac{11}{2}}d} \\
&+ \frac{c^2x}{132\,(ex^2+d)^{\frac{5}{2}}e^4} + \frac{2\,c^2x}{99\,\sqrt{ex^2+d}d^2e^4} + \frac{c^2x}{99\,(ex^2+d)^{\frac{3}{2}}de^4} \\
&+ \frac{5\,c^2dx}{792\,(ex^2+d)^{\frac{7}{2}}e^4} + \frac{35\,c^2d^2x}{6336\,(ex^2+d)^{\frac{9}{2}}e^4} - \frac{35\,c^2d^3x}{704\,(ex^2+d)^{\frac{11}{2}}e^4} \\
&+ \frac{acx}{132\,(ex^2+d)^{\frac{9}{2}}e^2} + \frac{32\,acx}{1155\,\sqrt{ex^2+d}d^4e^2} + \frac{16\,acx}{1155\,(ex^2+d)^{\frac{3}{2}}d^3e^2} \\
&+ \frac{4\,acx}{385\,(ex^2+d)^{\frac{5}{2}}d^2e^2} + \frac{2\,acx}{231\,(ex^2+d)^{\frac{7}{2}}de^2} - \frac{3\,acdx}{44\,(ex^2+d)^{\frac{11}{2}}e^2}
\end{aligned}
$$

input

```
integrate((c*x^4+a)^2/(e*x^2+d)^(13/2),x, algorithm="maxima")
```

output

```
-1/4*c^2*x^7/((e*x^2 + d)^(11/2)*e) - 7/24*c^2*d*x^5/((e*x^2 + d)^(11/2)*e
^2) - 35/192*c^2*d^2*x^3/((e*x^2 + d)^(11/2)*e^3) - 1/4*a*c*x^3/((e*x^2 +
d)^(11/2)*e) + 256/693*a^2*x/(sqrt(e*x^2 + d)*d^6) + 128/693*a^2*x/((e*x^2
 + d)^(3/2)*d^5) + 32/231*a^2*x/((e*x^2 + d)^(5/2)*d^4) + 80/693*a^2*x/((e
*x^2 + d)^(7/2)*d^3) + 10/99*a^2*x/((e*x^2 + d)^(9/2)*d^2) + 1/11*a^2*x/((
e*x^2 + d)^(11/2)*d) + 1/132*c^2*x/((e*x^2 + d)^(5/2)*e^4) + 2/99*c^2*x/(s
qrt(e*x^2 + d)*d^2*e^4) + 1/99*c^2*x/((e*x^2 + d)^(3/2)*d*e^4) + 5/792*c^2
*d*x/((e*x^2 + d)^(7/2)*e^4) + 35/6336*c^2*d^2*x/((e*x^2 + d)^(9/2)*e^4) -
 35/704*c^2*d^3*x/((e*x^2 + d)^(11/2)*e^4) + 1/132*a*c*x/((e*x^2 + d)^(9/2
)*e^2) + 32/1155*a*c*x/(sqrt(e*x^2 + d)*d^4*e^2) + 16/1155*a*c*x/((e*x^2 +
 d)^(3/2)*d^3*e^2) + 4/385*a*c*x/((e*x^2 + d)^(5/2)*d^2*e^2) + 2/231*a*c*x
/((e*x^2 + d)^(7/2)*d*e^2) - 3/44*a*c*d*x/((e*x^2 + d)^(11/2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.68

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{13/2}}\,dx = \frac{\left(\left(\left(\left(x^2\left(\frac{2\,(35\,c^2d^4e^6+48\,acd^2e^8+640\,a^2e^{10})x^2}{d^6e^5}+\frac{11\,(35\,c^2d^5e^5+48\,acd^3e^7+640\,a^2de^9)}{d^6e^5}\right)+\frac{396\,(3\,acd^4e^6+}{d^6e^5}\right.\right.\right.\right.}{3465\,(ex^2+d)^{\frac{11}{2}}}$$

input

```
integrate((c*x^4+a)^2/(e*x^2+d)^(13/2),x, algorithm="giac")
```

output

```
1/3465*((((x^2*(2*(35*c^2*d^4*e^6 + 48*a*c*d^2*e^8 + 640*a^2*e^10)*x^2/(d^
6*e^5) + 11*(35*c^2*d^5*e^5 + 48*a*c*d^3*e^7 + 640*a^2*d*e^9)/(d^6*e^5)) +
 396*(3*a*c*d^4*e^6 + 40*a^2*d^2*e^8)/(d^6*e^5))*x^2 + 462*(3*a*c*d^5*e^5
+ 40*a^2*d^3*e^7)/(d^6*e^5))*x^2 + 11550*a^2*e/d^2)*x^2 + 3465*a^2/d)*x/(e
*x^2 + d)^(11/2)
```

Mupad [B] (verification not implemented)

Time = 17.70 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{(a+cx^4)^2}{(d+ex^2)^{13/2}}\,dx &= \frac{x\left(\frac{a^2}{11\,d}+\frac{d^2\left(\frac{c^2\,d}{11\,e^2}+\frac{2\,a\,c}{11\,d}\right)}{e^2}\right)}{(e\,x^2+d)^{11/2}} \\ &\quad - \frac{x\left(\frac{-10\,a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4}{99\,d^2\,e^4}+\frac{d\left(\frac{2\,c^2\,d}{9\,e^3}+\frac{c\,(c\,d^2+2\,a\,e^2)}{9\,d\,e^3}\right)}{e}\right)}{(e\,x^2+d)^{9/2}} \\ &\quad - \frac{x\left(\frac{c^2}{5\,e^4}-\frac{160\,a^2\,e^4+12\,a\,c\,d^2\,e^2-49\,c^2\,d^4}{1155\,d^4\,e^4}\right)}{(e\,x^2+d)^{5/2}} + \frac{x\left(\frac{3\,c^2\,d}{7\,e^4}+\frac{80\,a^2\,e^4+6\,a\,c\,d^2\,e^2+25\,c^2\,d^4}{693\,d^3\,e^4}\right)}{(e\,x^2+d)^{7/2}} \\ &\quad + \frac{x\,(640\,a^2\,e^4+48\,a\,c\,d^2\,e^2+35\,c^2\,d^4)}{3465\,d^5\,e^4\,(e\,x^2+d)^{3/2}} + \frac{x\,(1280\,a^2\,e^4+96\,a\,c\,d^2\,e^2+70\,c^2\,d^4)}{3465\,d^6\,e^4\,\sqrt{e\,x^2+d}}\end{aligned}$$

input

```
int((a + c*x^4)^2/(d + e*x^2)^(13/2),x)
```

output
```
(x*(a^2/(11*d) + (d^2*((c^2*d)/(11*e^2) + (2*a*c)/(11*d)))/e^2))/(d + e*x^
2)^(11/2) - (x*((c^2*d^4 - 10*a^2*e^4 + 2*a*c*d^2*e^2)/(99*d^2*e^4) + (d*(
(2*c^2*d)/(9*e^3) + (c*(2*a*e^2 + c*d^2))/(9*d*e^3)))/e))/(d + e*x^2)^(9/2
) - (x*(c^2/(5*e^4) - (160*a^2*e^4 - 49*c^2*d^4 + 12*a*c*d^2*e^2)/(1155*d^
4*e^4)))/(d + e*x^2)^(5/2) + (x*((3*c^2*d)/(7*e^4) + (80*a^2*e^4 + 25*c^2*
d^4 + 6*a*c*d^2*e^2)/(693*d^3*e^4)))/(d + e*x^2)^(7/2) + (x*(640*a^2*e^4 +
 35*c^2*d^4 + 48*a*c*d^2*e^2))/(3465*d^5*e^4*(d + e*x^2)^(3/2)) + (x*(1280
*a^2*e^4 + 70*c^2*d^4 + 96*a*c*d^2*e^2))/(3465*d^6*e^4*(d + e*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 636, normalized size of antiderivative = 2.29

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^{13/2}}\,dx = \frac{-70\sqrt{e}\,c^2d^{10}+3465\sqrt{e\,x^2+d}\,a^2d^5e^5x+11550\sqrt{e\,x^2+d}\,a^2d^4e^6x^3+18480\sqrt{e\,x^2+}}{}$$

input
```
int((c*x^4+a)^2/(e*x^2+d)^(13/2),x)
```

output
```
(3465*sqrt(d + e*x**2)*a**2*d**5*e**5*x + 11550*sqrt(d + e*x**2)*a**2*d**4
*e**6*x**3 + 18480*sqrt(d + e*x**2)*a**2*d**3*e**7*x**5 + 15840*sqrt(d + e
*x**2)*a**2*d**2*e**8*x**7 + 7040*sqrt(d + e*x**2)*a**2*d*e**9*x**9 + 1280
*sqrt(d + e*x**2)*a**2*e**10*x**11 + 1386*sqrt(d + e*x**2)*a*c*d**5*e**5*x
**5 + 1188*sqrt(d + e*x**2)*a*c*d**4*e**6*x**7 + 528*sqrt(d + e*x**2)*a*c*
d**3*e**7*x**9 + 96*sqrt(d + e*x**2)*a*c*d**2*e**8*x**11 + 385*sqrt(d + e*
x**2)*c**2*d**5*e**5*x**9 + 70*sqrt(d + e*x**2)*c**2*d**4*e**6*x**11 - 128
0*sqrt(e)*a**2*d**6*e**4 - 7680*sqrt(e)*a**2*d**5*e**5*x**2 - 19200*sqrt(e
)*a**2*d**4*e**6*x**4 - 25600*sqrt(e)*a**2*d**3*e**7*x**6 - 19200*sqrt(e)*
a**2*d**2*e**8*x**8 - 7680*sqrt(e)*a**2*d*e**9*x**10 - 1280*sqrt(e)*a**2*e
**10*x**12 - 96*sqrt(e)*a*c*d**8*e**2 - 576*sqrt(e)*a*c*d**7*e**3*x**2 - 1
440*sqrt(e)*a*c*d**6*e**4*x**4 - 1920*sqrt(e)*a*c*d**5*e**5*x**6 - 1440*sq
rt(e)*a*c*d**4*e**6*x**8 - 576*sqrt(e)*a*c*d**3*e**7*x**10 - 96*sqrt(e)*a*
c*d**2*e**8*x**12 - 70*sqrt(e)*c**2*d**10 - 420*sqrt(e)*c**2*d**9*e*x**2 -
 1050*sqrt(e)*c**2*d**8*e**2*x**4 - 1400*sqrt(e)*c**2*d**7*e**3*x**6 - 105
0*sqrt(e)*c**2*d**6*e**4*x**8 - 420*sqrt(e)*c**2*d**5*e**5*x**10 - 70*sqrt
(e)*c**2*d**4*e**6*x**12)/(3465*d**6*e**5*(d**6 + 6*d**5*e*x**2 + 15*d**4*
e**2*x**4 + 20*d**3*e**3*x**6 + 15*d**2*e**4*x**8 + 6*d*e**5*x**10 + e**6*
x**12))
```

3.382 $\int \frac{(d+ex^2)^{7/2}}{a+cx^4} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 555

$$\int \frac{(d+ex^2)^{7/2}}{a+cx^4} \, dx = \frac{13de^2x\sqrt{d+ex^2}}{8c} + \frac{e^3x^3\sqrt{d+ex^2}}{4c}$$

$$+ \frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(4cd^2e(cd^2-ae^2)-(c^2d^4-6acd^2e^2+a^2e^4)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}}}{\sqrt{a}\left(\sqrt{a}e+\right.}\right.}{2\sqrt{2}\sqrt[4]{a}c^{5/2}d\sqrt{cd^2+ae^2}}$$

$$+ \frac{e^{3/2}(35cd^2-8ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2}$$

$$+ \frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(4cd^2e(cd^2-ae^2)-(c^2d^4-6acd^2e^2+a^2e^4)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{}}{\sqrt{a}\left(\sqrt{}\right.}\right.}{2\sqrt{2}\sqrt[4]{a}c^{5/2}d\sqrt{cd^2+ae^2}}$$

output

```
13/8*d*e^2*x*(e*x^2+d)^(1/2)/c+1/4*e^3*x^3*(e*x^2+d)^(1/2)/c+1/4*(a^(1/2)*
e+(a*e^2+c*d^2)^(1/2))^(1/2)*(4*c*d^2*e*(-a*e^2+c*d^2)-(a^2*e^4-6*a*c*d^2*
e^2+c^2*d^4)*(e-(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctan(2^(1/2)*a^(1/4)*c^(1/
2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/
2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(1/4)/c^(5/2)/d/(a*e^2+c*d^2
)^(1/2)+1/8*e^(3/2)*(-8*a*e^2+35*c*d^2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))
/c^2+1/4*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(4*c*d^2*e*(-a*e^2+c*d^2)-
(a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)*(e+(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctanh(2
^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^
(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(1/4)/c
^(5/2)/d/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)^{7/2}}{a+cx^4}\,dx = \frac{e^{3/2}\Bigl(c\sqrt{e}x\sqrt{d+ex^2}(13d+2ex^2)+(-35cd^2+8ae^2)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)+16\mathrm{R}}{}$$

input

```
Integrate[(d + e*x^2)^(7/2)/(a + c*x^4),x]
```

output

```
(e^(3/2)*(c*Sqrt[e]*x*Sqrt[d + e*x^2]*(13*d + 2*e*x^2) + (-35*c*d^2 + 8*a*
e^2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]] + 16*RootSum[c*d^4 - 4*c*d^3*#1 +
 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (c^2*d^5*Log[d + 2
*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - a*c*d^3*e^2*Log[d + 2*e*x^2 -
 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - c^2*d^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x
*Sqrt[d + e*x^2] - #1]*#1 - 4*a*c*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sq
rt[d + e*x^2] - #1]*#1 + a^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*
x^2] - #1]*#1 + c^2*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1
]*#1^2 - a*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^
2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ]))/(8*c^2)
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.34, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1489, 27, 318, 25, 403, 27, 403, 25, 398, 224, 219, 291, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{7/2}}{a+cx^4}dx$$

$$\Big\downarrow \ 1489$$

$$-\frac{\sqrt{c}\int \frac{\left(ex^2+d\right)^{7/2}}{\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x^2\right)}dx}{2\sqrt{-a}}-\frac{\sqrt{c}\int \frac{\left(ex^2+d\right)^{7/2}}{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{-a}\right)}dx}{2\sqrt{-a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{\left(ex^2+d\right)^{7/2}}{\sqrt{-a}-\sqrt{c}x^2}dx}{2\sqrt{-a}}-\frac{\int \frac{\left(ex^2+d\right)^{7/2}}{\sqrt{c}x^2+\sqrt{-a}}dx}{2\sqrt{-a}}$$

$$\Big\downarrow \ 318$$

$$-\frac{\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(e\left(11\sqrt{c}d-6\sqrt{-a}e\right)x^2+d\left(6\sqrt{c}d-\sqrt{-a}e\right)\right)}{\sqrt{c}x^2+\sqrt{-a}}dx}{6\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{-a}}-\frac{-\frac{\int -\frac{\left(ex^2+d\right)^{3/2}\left(e\left(11\sqrt{c}d+6\sqrt{-a}e\right)x^2+d\left(6\sqrt{c}d+\sqrt{-a}e\right)\right)}{\sqrt{-a}-\sqrt{c}x^2}dx}{6\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(e\left(11\sqrt{c}d-6\sqrt{-a}e\right)x^2+d\left(6\sqrt{c}d-\sqrt{-a}e\right)\right)}{\sqrt{c}x^2+\sqrt{-a}}dx}{6\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(e\left(11\sqrt{c}d+6\sqrt{-a}e\right)x^2+d\left(6\sqrt{c}d+\sqrt{-a}e\right)\right)}{\sqrt{-a}-\sqrt{c}x^2}dx}{6\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \ 403$$

$$-\frac{\frac{\frac{\int \frac{3\sqrt{ex^2+d}\left(e\left(19cd^2-22\sqrt{-a}\sqrt{c}ed-8ae^2\right)x^2+d\left(8cd^2-5\sqrt{-a}\sqrt{c}ed-2ae^2\right)\right)}{\sqrt{c}x^2+\sqrt{-a}}dx}{4\sqrt{c}}+\frac{1}{4}ex\left(d+ex^2\right)^{3/2}\left(11d-\frac{6\sqrt{-a}e}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{-a}}-\frac{-\frac{\frac{\int -\frac{3\sqrt{ex^2+d}\left(e\left(19cd^2+22\sqrt{-a}\sqrt{c}ed-8ae^2\right)x^2+d\left(8cd^2+5\sqrt{-a}\sqrt{c}ed-2ae^2\right)\right)}{\sqrt{-a}-\sqrt{c}x^2}dx}{4\sqrt{c}}-\frac{1}{4}ex\left(d+ex^2\right)^{3/2}\left(\frac{6\sqrt{-a}e}{\sqrt{c}}+11d\right)}{6\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{3\int \frac{\sqrt{ex^2+d}\left(e\left(19cd^2-22\sqrt{-a}\sqrt{c}ed-8ae^2\right)x^2+d\left(8cd^2-5\sqrt{-a}\sqrt{c}ed-2ae^2\right)\right)}{\sqrt{c}x^2+\sqrt{-a}}dx}{4\sqrt{c}}+\frac{1}{4}ex\left(d+ex^2\right)^{3/2}\left(11d-\frac{6\sqrt{-a}e}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{3\int \frac{\sqrt{ex^2+d}\left(e\left(19cd^2+22\sqrt{-a}\sqrt{c}ed-8ae^2\right)x^2+d\left(8cd^2+5\sqrt{-a}\sqrt{c}ed-2ae^2\right)\right)}{\sqrt{-a}-\sqrt{c}x^2}dx}{4\sqrt{c}}-\frac{1}{4}ex\left(d+ex^2\right)^{3/2}\left(\frac{6\sqrt{-a}e}{\sqrt{c}}+11d\right)}{6\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{5/2}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \; 403$$

$$-\frac{\frac{\frac{3\left(-\frac{\int -\frac{e\left(35c^{3/2}d^3+70\sqrt{-a}ced^2-56a\sqrt{c}e^2d+16(-a)^{3/2}e^3\right)x^2+d\left(16c^{3/2}d^3+29\sqrt{-a}ced^2-26a\sqrt{c}e^2d-8\sqrt{-a}ae^3\right)}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}\left(22\sqrt{-a}\sqrt{c}de-8ae^2+19\right.}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\frac{\frac{3\left(\frac{\int \frac{e\left(35c^{3/2}d^3-70\sqrt{-a}ced^2-56a\sqrt{c}e^2d+16\sqrt{-a}ae^3\right)x^2+d\left(16c^{3/2}d^3-29\sqrt{-a}ced^2-26a\sqrt{c}e^2d+8\sqrt{-a}ae^3\right)}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}\left(-22\sqrt{-a}\sqrt{c}de-8ae^2+19cd^2\right)}{2\sqrt{c}}\right)}{4\sqrt{c}}}{6\sqrt{c}}$$

$$2\sqrt{-a}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{3\left(\frac{\int \frac{e\left(35c^{3/2}d^3+70\sqrt{-a}ced^2-56a\sqrt{c}e^2d+16(-a)^{3/2}e^3\right)x^2+d\left(16c^{3/2}d^3+29\sqrt{-a}ced^2-26a\sqrt{c}e^2d+8(-a)^{3/2}e^3\right)}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}\left(22\sqrt{-a}\sqrt{c}de-8ae^2+19cd\right.}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\frac{\frac{\frac{3\left(\frac{\int \frac{e\left(35c^{3/2}d^3-70\sqrt{-a}ced^2-56a\sqrt{c}e^2d+16\sqrt{-a}ae^3\right)x^2+d\left(16c^{3/2}d^3-29\sqrt{-a}ced^2-26a\sqrt{c}e^2d+8\sqrt{-a}ae^3\right)}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}\left(-22\sqrt{-a}\sqrt{c}de-8ae^2+19cd^2\right)}{2\sqrt{c}}\right)}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\frac{\frac{3\left(\frac{\frac{16\left(a^2e^4+4\sqrt{-a}c^{3/2}d^3e-6acd^2e^2+4(-a)^{3/2}\sqrt{c}de^3+c^2d^4\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}-\frac{e\left(70\sqrt{-a}cd^2e-56a\sqrt{c}de^2+16(-a)^{3/2}e^3+35c^{3/2}d^3\right)\int \frac{1}{\sqrt{ex^2}}}{\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{16\left(a^2e^4-4\sqrt{-a}c^{3/2}d^3e-6acd^2e^2+4\sqrt{-a}a\sqrt{c}de^3+c^2d^4\right)\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}+\frac{e\left(-70\sqrt{-a}cd^2e-56a\sqrt{c}de^2+16\sqrt{-a}ae^3+35c^{3/2}d^3\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{\frac{3\left(\frac{\frac{16\left(a^2e^4+4\sqrt{-a}c^{3/2}d^3e-6acd^2e^2+4(-a)^{3/2}\sqrt{c}de^3+c^2d^4\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}-\frac{e\left(70\sqrt{-a}cd^2e-56a\sqrt{c}de^2+16(-a)^{3/2}e^3+35c^{3/2}d^3\right)\int \frac{1}{1-\frac{ex}{ex^2}}}{\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{16\left(a^2e^4-4\sqrt{-a}c^{3/2}d^3e-6acd^2e^2+4\sqrt{-a}a\sqrt{c}de^3+c^2d^4\right)\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}+\frac{e\left(-70\sqrt{-a}cd^2e-56a\sqrt{c}de^2+16\sqrt{-a}ae^3+35c^{3/2}d^3\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d}{\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\frac{3\left(\frac{\frac{16\left(a^2e^4+4\sqrt{-a}c^{3/2}d^3e-6acd^2e^2+4(-a)^{3/2}\sqrt{c}de^3+c^2d^4\right)\int\frac{1}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}-\frac{\sqrt{e}\left(70\sqrt{-a}cd^2e-56a\sqrt{c}de^2+16(-a)^{3/2}e^3+35c^{3/2}d^3\right)\text{arcta}}{\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{16\left(a^2e^4-4\sqrt{-a}c^{3/2}d^3e-6acd^2e^2+4\sqrt{-a}a\sqrt{c}de^3+c^2d^4\right)\int\frac{1}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}+\frac{\sqrt{e}\left(-70\sqrt{-a}cd^2e-56a\sqrt{c}de^2+16\sqrt{-a}ae^3+35c^{3/2}d^3\right)\text{arctanh}\left(\right.}{\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \; 291$$

$$-\frac{\frac{\frac{3\left(\frac{\frac{16\left(a^2e^4-4\sqrt{-a}c^{3/2}d^3e-6acd^2e^2+4\sqrt{-a}a\sqrt{c}de^3+c^2d^4\right)\int\frac{1}{\sqrt{-a}-\frac{\left(\sqrt{-a}e-\sqrt{c}d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}}{2\sqrt{c}}+\frac{\sqrt{e}\left(-70\sqrt{-a}cd^2e-56a\sqrt{c}de^2+16\sqrt{-a}ae^3+35c^{3/2}d^3\right)\text{a}}{\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{16\left(a^2e^4+4\sqrt{-a}c^{3/2}d^3e-6acd^2e^2+4(-a)^{3/2}\sqrt{c}de^3+c^2d^4\right)\int\frac{1}{\sqrt{-a}-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}}{2\sqrt{c}}-\frac{\sqrt{e}\left(70\sqrt{-a}cd^2e-56a\sqrt{c}de^2+16(-a)^{3/2}e^3+35c^{3/2}d^3\right)\text{a}}{\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{3\left(\frac{\frac{16\left(a^2e^4+4\sqrt{-a}c^{3/2}d^3e-6acd^2e^2+4(-a)^{3/2}\sqrt{c}de^3+c^2d^4\right)\int \frac{1}{\sqrt{-a}-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}-\frac{\sqrt{e}\left(70\sqrt{-a}cd^2e-56a\sqrt{c}de^2+16(-a)^{3/2}e^3+35c^{3/2}d^3\right.}{\sqrt{c}}}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\frac{\frac{3\left(\frac{\frac{16\left(a^2e^4-4\sqrt{-a}c^{3/2}d^3e-6acd^2e^2+4\sqrt{-a}a\sqrt{c}de^3+c^2d^4\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}}+\frac{\sqrt{e}\left(-70\sqrt{-a}cd^2e-56a\sqrt{c}de^2+16\sqrt{-a}ae^3+35c^{3/2}d^3\right)\text{arctanh}\left(\right.}{\sqrt{c}}}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{3\left(\frac{\frac{16\left(a^2e^4-4\sqrt{-a}c^{3/2}d^3e-6acd^2e^2+4\sqrt{-a}a\sqrt{c}de^3+c^2d^4\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}}+\frac{\sqrt{e}\left(-70\sqrt{-a}cd^2e-56a\sqrt{c}de^2+16\sqrt{-a}ae^3+35c^{3/2}d^3\right)\text{arctanh}}{\sqrt{c}}}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

$$\frac{\frac{3\left(\frac{\frac{16\left(a^2e^4+4\sqrt{-a}c^{3/2}d^3e-6acd^2e^2+4(-a)^{3/2}\sqrt{c}de^3+c^2d^4\right)\text{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{-a}\sqrt{c}\sqrt{\sqrt{-a}e+\sqrt{c}d}}-\frac{\sqrt{e}\left(70\sqrt{-a}cd^2e-56a\sqrt{c}de^2+16(-a)^{3/2}e^3+35c^{3/2}d^3\right)\text{arcta}}{\sqrt{c}}}{2\sqrt{c}}\right.}{4\sqrt{c}}}{6\sqrt{c}}}{2\sqrt{-a}}$$

input

```
Int[(d + e*x^2)^(7/2)/(a + c*x^4),x]
```

output

```
-1/2*((e*x*(d + e*x^2)^(5/2))/(6*Sqrt[c]) + ((e*(11*d - (6*Sqrt[-a]*e)/Sqr
t[c])*x*(d + e*x^2)^(3/2))/4 + (3*((e*(19*c*d^2 - 22*Sqrt[-a]*Sqrt[c]*d*e
- 8*a*e^2)*x*Sqrt[d + e*x^2])/(2*Sqrt[c]) + ((16*(c^2*d^4 - 4*Sqrt[-a]*c^(
3/2)*d^3*e - 6*a*c*d^2*e^2 + 4*Sqrt[-a]*a*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[
(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])])/((-a)^(1/4
)*Sqrt[c]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]) + (Sqrt[e]*(35*c^(3/2)*d^3 - 70*Sq
rt[-a]*c*d^2*e - 56*a*Sqrt[c]*d*e^2 + 16*Sqrt[-a]*a*e^3)*ArcTanh[(Sqrt[e]*
x)/Sqrt[d + e*x^2]])/Sqrt[c])/(2*Sqrt[c])))/(4*Sqrt[c]))/(6*Sqrt[c]))/Sqrt
[-a] - (-1/6*(e*x*(d + e*x^2)^(5/2))/Sqrt[c] + (-1/4*(e*(11*d + (6*Sqrt[-a
]*e)/Sqrt[c])*x*(d + e*x^2)^(3/2)) + (3*(-1/2*(e*(19*c*d^2 + 22*Sqrt[-a]*S
qrt[c]*d*e - 8*a*e^2)*x*Sqrt[d + e*x^2])/Sqrt[c] + (-((Sqrt[e]*(35*c^(3/2)
*d^3 + 70*Sqrt[-a]*c*d^2*e - 56*a*Sqrt[c]*d*e^2 + 16*(-a)^(3/2)*e^3)*ArcTa
nh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[c]) + (16*(c^2*d^4 + 4*Sqrt[-a]*c^(3
/2)*d^3*e - 6*a*c*d^2*e^2 + 4*(-a)^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTanh[
(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])])/((-a)^(1/4
)*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]))/(2*Sqrt[c])))/(4*Sqrt[c]))/(6*Sqr
t[c]))/(2*Sqrt[-a])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 1489
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Simp[-c/(2*r)   Int[(d + e*x^2)^q/(r - c*x^2), x], x] - S
imp[c/(2*r)   Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e,
q}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. 2(459) = 918.

Time = 3.10 (sec) , antiderivative size = 990, normalized size of antiderivative = 1.78

method	result
pseudoelliptic	$-\frac{-\frac{\sqrt{2\sqrt{a\left(a\,e^{2}+c\,d^{2}\right)}+2ae}\,\sqrt{4\sqrt{a\,e^{2}+c\,d^{2}}\,\sqrt{a}-2\sqrt{a\left(a\,e^{2}+c\,d^{2}\right)}-2ae}\,(((\left(3e^{\frac{3}{2}}\,c\,d^{2}\sqrt{a}-e^{\frac{7}{2}}\,a^{\frac{3}{2}}\right)\sqrt{a\,e^{2}+c\,d^{2}}-\sqrt{e}\,c^{2}d^{4}+6e^{\frac{5}{2}}\,c\,d^{2}a-e}{}}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x^2+d)^(7/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^(3/2)*(-1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2
)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(((3*e^(3/2)*c*d^2*
a^(1/2)-e^(7/2)*a^(3/2))*(a*e^2+c*d^2)^(1/2)-e^(1/2)*c^2*d^4+6*e^(5/2)*c*d
^2*a-e^(9/2)*a^2)*(a*(a*e^2+c*d^2))^(1/2)+(-3*e^(5/2)*c*d^2*a^(3/2)+e^(9/2
)*a^(5/2))*(a*e^2+c*d^2)^(1/2)+a*(e^(3/2)*c^2*d^4-6*a*e^(7/2)*c*d^2+a^2*e^
(11/2)))*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+
2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+1/4*(2*(a*(a*e^2+c*d^2))^(1/2
)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*
a*e)^(1/2)*(((3*e^(3/2)*c*d^2*a^(1/2)-e^(7/2)*a^(3/2))*(a*e^2+c*d^2)^(1/2)
-e^(1/2)*c^2*d^4+6*e^(5/2)*c*d^2*a-e^(9/2)*a^2)*(a*(a*e^2+c*d^2))^(1/2)+(-
3*e^(5/2)*c*d^2*a^(3/2)+e^(9/2)*a^(5/2))*(a*e^2+c*d^2)^(1/2)+a*(e^(3/2)*c^
2*d^4-6*a*e^(7/2)*c*d^2+a^2*e^(11/2)))*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/
2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)
+(((2*a^(5/2)*e^4-35/4*a^(3/2)*c*d^2*e^2)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2
))-13/4*x*a^(3/2)*(e*x^2+d)^(1/2)*(2/13*e^(7/2)*x^2+d*e^(5/2))*c)*(4*(a*e^
2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)+(arctan((2*a
^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*
e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arctan(((
2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(
a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)))*((3...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3394 vs. $2(461) = 922$.

Time = 53.72 (sec) , antiderivative size = 6799, normalized size of antiderivative = 12.25

$$\int \frac{(d+ex^2)^{7/2}}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(7/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(d+ex^2)^{7/2}}{a+cx^4}\,dx = \int \frac{(d+ex^2)^{\frac{7}{2}}}{a+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(7/2)/(c*x**4+a),x)
```

output
```
Integral((d + e*x**2)**(7/2)/(a + c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{7/2}}{a+cx^4}\,dx = \int \frac{(ex^2+d)^{\frac{7}{2}}}{cx^4+a}\,dx$$

input
```
integrate((e*x^2+d)^(7/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(7/2)/(c*x^4 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{7/2}}{a+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(7/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{a+cx^4}\,dx = \int \frac{(e\,x^2+d)^{7/2}}{c\,x^4+a}\,dx$$

input
```
int((d + e*x^2)^(7/2)/(a + c*x^4),x)
```

output
```
int((d + e*x^2)^(7/2)/(a + c*x^4), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{7/2}}{a+cx^4}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+a}dx\right)d^3+\left(\int \frac{\sqrt{e\,x^2+d}\,x^6}{c\,x^4+a}dx\right)e^3 + 3\left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c\,x^4+a}dx\right)d\,e^2+3\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c\,x^4+a}dx\right)d^2e$$

input
```
int((e*x^2+d)^(7/2)/(c*x^4+a),x)
```

output
```
int(sqrt(d + e*x**2)/(a + c*x**4),x)*d**3 + int((sqrt(d + e*x**2)*x**6)/(a
 + c*x**4),x)*e**3 + 3*int((sqrt(d + e*x**2)*x**4)/(a + c*x**4),x)*d*e**2
+ 3*int((sqrt(d + e*x**2)*x**2)/(a + c*x**4),x)*d**2*e
```

3.383 $\int \frac{(d+ex^2)^{5/2}}{a+cx^4} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 474

$$\int \frac{(d+ex^2)^{5/2}}{a+cx^4}\,dx = \frac{e^2x\sqrt{d+ex^2}}{2c}$$

$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(2\sqrt{a}cd^2e+2a^{3/2}e^3+(cd^2-3ae^2)\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d-}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cdx^2}\right.}{2\sqrt{2}a^{3/4}c^{3/2}\sqrt{cd^2+ae^2}}$$

$$+\frac{5de^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c}$$

$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(2\sqrt{a}cd^2e+2a^{3/2}e^3-(cd^2-3ae^2)\sqrt{cd^2+ae^2}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-c}\right.}{2\sqrt{2}a^{3/4}c^{3/2}\sqrt{cd^2+ae^2}}$$

output
```
1/2*e^2*x*(e*x^2+d)^(1/2)/c+1/4*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(2*a
^(1/2)*c*d^2*e+2*a^(3/2)*e^3+(-3*a*e^2+c*d^2)*(a*e^2+c*d^2)^(1/2))*arctan(
2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^
(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(3/4)/c
^(3/2)/(a*e^2+c*d^2)^(1/2)+5/2*d*e^(3/2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2)
)/c+1/4*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(2*a^(1/2)*c*d^2*e+2*a^(3/2
)*e^3-(-3*a*e^2+c*d^2)*(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/2)*a^(1/4)*c^(1/2
)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/
2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(3/4)/c^(3/2)/(a*e^2+c*d^2)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex^2)^{5/2}}{a+cx^4}\,dx = \frac{e^{3/2}\Big(\sqrt{e}x\sqrt{d+ex^2}-5d\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)+\text{RootSum}\Big[cd^4-4cd^3\#1+6cd^2}{}$$

input
```
Integrate[(d + e*x^2)^(5/2)/(a + c*x^4),x]
```

output
```
(e^(3/2)*(Sqrt[e]*x*Sqrt[d + e*x^2] - 5*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^
2]] + RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1
^3 + c*#1^4 & , (3*c*d^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #
1] - a*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 2*c*d
^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 10*a*d*e^2*Log
[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 3*c*d^2*Log[d + 2*e*
x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - a*e^2*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*
c*d*#1^2 - c*#1^3) & ]))/(2*c)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1489, 27, 318, 25, 403, 25, 398, 224, 219, 291, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{5/2}}{a+cx^4} \, dx$$

$$\Big\downarrow \text{1489}$$

$$-\frac{\sqrt{c}\int \frac{\left(ex^2+d\right)^{5/2}}{\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x^2\right)}dx}{2\sqrt{-a}}-\frac{\sqrt{c}\int \frac{\left(ex^2+d\right)^{5/2}}{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{-a}\right)}dx}{2\sqrt{-a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(ex^2+d\right)^{5/2}}{\sqrt{-a}-\sqrt{c}x^2}dx}{2\sqrt{-a}}-\frac{\int \frac{\left(ex^2+d\right)^{5/2}}{\sqrt{c}x^2+\sqrt{-a}}dx}{2\sqrt{-a}}$$

$$\Big\downarrow \text{318}$$

$$-\frac{\frac{\int \frac{\sqrt{ex^2+d}\left(e\left(7\sqrt{c}d-4\sqrt{-a}e\right)x^2+d\left(4\sqrt{c}d-\sqrt{-a}e\right)\right)}{\sqrt{c}x^2+\sqrt{-a}}dx}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{-a}}-\frac{-\frac{\int -\frac{\sqrt{ex^2+d}\left(e\left(7\sqrt{c}d+4\sqrt{-a}e\right)x^2+d\left(4\sqrt{c}d+\sqrt{-a}e\right)\right)}{\sqrt{-a}-\sqrt{c}x^2}dx}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{\sqrt{ex^2+d}\left(e\left(7\sqrt{c}d-4\sqrt{-a}e\right)x^2+d\left(4\sqrt{c}d-\sqrt{-a}e\right)\right)}{\sqrt{c}x^2+\sqrt{-a}}dx}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt{ex^2+d}\left(e\left(7\sqrt{c}d+4\sqrt{-a}e\right)x^2+d\left(4\sqrt{c}d+\sqrt{-a}e\right)\right)}{\sqrt{-a}-\sqrt{c}x^2}dx}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \text{403}$$

$$-\frac{\frac{\frac{\int \frac{e\left(15cd^2-20\sqrt{-a}\sqrt{c}ed-8ae^2\right)x^2+d\left(8cd^2-9\sqrt{-a}\sqrt{c}ed-4ae^2\right)}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\frac{4\sqrt{-a}e}{\sqrt{c}}\right)}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{-\frac{\int -\frac{e\left(15cd^2+20\sqrt{-a}\sqrt{c}ed-8ae^2\right)x^2+d\left(8cd^2+9\sqrt{-a}\sqrt{c}ed-4ae^2\right)}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\frac{4\sqrt{-a}e}{\sqrt{c}}+7d\right)}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{\int \frac{e\left(15cd^2-20\sqrt{-a}\sqrt{c}ed-8ae^2\right)x^2+d\left(8cd^2-9\sqrt{-a}\sqrt{c}ed-4ae^2\right)}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\frac{4\sqrt{-a}e}{\sqrt{c}}\right)}{4\sqrt{c}}+\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{\int \frac{e\left(15cd^2+20\sqrt{-a}\sqrt{c}ed-8ae^2\right)x^2+d\left(8cd^2+9\sqrt{-a}\sqrt{c}ed-4ae^2\right)}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\frac{4\sqrt{-a}e}{\sqrt{c}}+7d\right)}{4\sqrt{c}}-\frac{ex\left(d+ex^2\right)^{3/2}}{4\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \; 398$$

$$-\frac{\frac{\frac{\frac{8\left(3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+(-a)^{3/2}e^3+c^{3/2}d^3\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-\frac{e\left(20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\frac{4\sqrt{-a}e}{\sqrt{c}}+7d\right)}{4\sqrt{c}}-}{2\sqrt{-a}}$$

$$\frac{\frac{\frac{\frac{8\left(-3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+\sqrt{-a}ae^3+c^{3/2}d^3\right)\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}+\frac{e\left(-20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\frac{4\sqrt{-a}e}{\sqrt{c}}\right)}{4\sqrt{c}}+}{2\sqrt{-a}}$$

$$\Big\downarrow \; 224$$

$$-\frac{\frac{\frac{\frac{8\left(-3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+\sqrt{-a}ae^3+c^{3/2}d^3\right)\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}+\frac{e\left(-20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\right.}{4\sqrt{c}}}{2\sqrt{-a}}$$

$$\frac{\frac{\frac{\frac{8\left(3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+(-a)^{3/2}e^3+c^{3/2}d^3\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-\frac{e\left(20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\frac{4\sqrt{-a}e}{\sqrt{c}}+\right.}{4\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\frac{\frac{8\left(3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+(-a)^{3/2}e^3+c^{3/2}d^3\right)\int\frac{1}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-\frac{\sqrt{e}\left(20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\right.}{4\sqrt{c}}}{2\sqrt{-a}}$$

$$\frac{\frac{\frac{\frac{8\left(-3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+\sqrt{-a}ae^3+c^{3/2}d^3\right)\int\frac{1}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}+\frac{\sqrt{e}\left(-20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\right.}{4\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \ 291$$

$$-\frac{\frac{\frac{\frac{8\left(-3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+\sqrt{-a}ae^3+c^{3/2}d^3\right)\int\frac{1}{\sqrt{-a}-\frac{\left(\sqrt{-a}e-\sqrt{c}d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}+\frac{\sqrt{e}\left(-20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+e}}{4\sqrt{c}}}{2\sqrt{-a}}$$

$$\frac{\frac{\frac{\frac{8\left(3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+(-a)^{3/2}e^3+c^{3/2}d^3\right)\int\frac{1}{\sqrt{-a}-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}-\frac{\sqrt{e}\left(20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\right.}{4\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \ 218$$

$$-\frac{\frac{\frac{\frac{8\left(3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+(-a)^{3/2}e^3+c^{3/2}d^3\right)\int\frac{1}{\sqrt{-a}-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}-\frac{\sqrt{e}\left(20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex}}{4\sqrt{c}}}{2\sqrt{-a}}$$

$$\frac{\frac{\frac{\frac{8\left(-3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+\sqrt{-a}ae^3+c^{3/2}d^3\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}}+\frac{\sqrt{e}\left(-20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\right.}{4\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \ 221$$

$$-\frac{\frac{\frac{\frac{8\left(-3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+\sqrt{-a}ae^3+c^{3/2}d^3\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}}+\frac{\sqrt{e}\left(-20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}+\frac{1}{2}ex\sqrt{d+ex^2}\left(7d-\right.}{4\sqrt{c}}}{2\sqrt{-a}}$$

$$\frac{\frac{\frac{\frac{8\left(3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+(-a)^{3/2}e^3+c^{3/2}d^3\right)\text{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{-a}\sqrt{c}\sqrt{\sqrt{-a}e+\sqrt{c}d}}-\frac{\sqrt{e}\left(20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{c}}-\frac{1}{2}ex\sqrt{d+ex^2}\left(\frac{4\sqrt{-}}{\sqrt{}}\right.}{4\sqrt{c}}}{2\sqrt{-a}}$$

input

```
Int[(d + e*x^2)^(5/2)/(a + c*x^4),x]
```

output

```
-1/2*((e*x*(d + e*x^2)^(3/2))/(4*Sqrt[c]) + ((e*(7*d - (4*Sqrt[-a]*e)/Sqrt
[c])*x*Sqrt[d + e*x^2])/2 + ((8*(c^(3/2)*d^3 - 3*Sqrt[-a]*c*d^2*e - 3*a*Sq
rt[c]*d*e^2 + Sqrt[-a]*a*e^3)*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*x)/((-a
)^(1/4)*Sqrt[d + e*x^2])])/((-a)^(1/4)*Sqrt[c]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e
]) + (Sqrt[e]*(15*c*d^2 - 20*Sqrt[-a]*Sqrt[c]*d*e - 8*a*e^2)*ArcTanh[(Sqrt
[e]*x)/Sqrt[d + e*x^2]])/Sqrt[c])/(2*Sqrt[c]))/(4*Sqrt[c]))/Sqrt[-a] - (-1
/4*(e*x*(d + e*x^2)^(3/2))/Sqrt[c] + (-1/2*(e*(7*d + (4*Sqrt[-a]*e)/Sqrt[c
])*x*Sqrt[d + e*x^2]) + (-((Sqrt[e]*(15*c*d^2 + 20*Sqrt[-a]*Sqrt[c]*d*e -
8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[c]) + (8*(c^(3/2)*d^3
+ 3*Sqrt[-a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + (-a)^(3/2)*e^3)*ArcTanh[(Sqrt[S
qrt[c]*d + Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])])/((-a)^(1/4)*Sqrt[
c]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]))/(2*Sqrt[c]))/(4*Sqrt[c]))/(2*Sqrt[-a])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 1489
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Simp[-c/(2*r)   Int[(d + e*x^2)^q/(r - c*x^2), x], x] - S
imp[c/(2*r)   Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e,
q}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(378) = 756$.

Time = 1.05 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.74

method	result
pseudoelliptic	$-\frac{3\left(\frac{\sqrt{2\sqrt{a\left(a e^{2}+c d^{2}\right)}+2ae}\left(\left(\frac{\sqrt{e}\,c\,d^{2}}{3}-\frac{2\sqrt{a e^{2}+c d^{2}}\,e^{\frac{3}{2}}\sqrt{a}}{3}-e^{\frac{5}{2}}a\right)\sqrt{a\left(a e^{2}+c d^{2}\right)}-\frac{e^{\frac{3}{2}}ac\,d^{2}}{3}+a^{2}e^{\frac{7}{2}}+\frac{2\sqrt{a e^{2}+c d^{2}}\,e^{\frac{5}{2}}a^{\frac{3}{2}}}{3}\right)\sqrt{4}}{4}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x^2+d)^(5/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-3/2*(1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*((1/3*e^(1/2)*c*d^2-2/3*
(a*e^2+c*d^2)^(1/2)*e^(3/2)*a^(1/2)-e^(5/2)*a)*(a*(a*e^2+c*d^2))^(1/2)-1/3
*e^(3/2)*a*c*d^2+a^2*e^(7/2)+2/3*(a*e^2+c*d^2)^(1/2)*e^(5/2)*a^(3/2))*(4*(
a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*ln((a^(1
/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a
*e^2+c*d^2)^(1/2)*x^2)/x^2)-1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*((
1/3*e^(1/2)*c*d^2-2/3*(a*e^2+c*d^2)^(1/2)*e^(3/2)*a^(1/2)-e^(5/2)*a)*(a*(a
*e^2+c*d^2))^(1/2)-1/3*e^(3/2)*a*c*d^2+a^2*e^(7/2)+2/3*(a*e^2+c*d^2)^(1/2)
*e^(5/2)*a^(3/2))*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)
-2*a*e)^(1/2)*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(
1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+(-1/3*a^(3/2)*(5*arctanh
((e*x^2+d)^(1/2)/x/e^(1/2))*d*e^2+(e*x^2+d)^(1/2)*e^(5/2)*x)*(4*(a*e^2+c*d
^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)+d*(arctan(((2*(a*
(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2
+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arctan((2*a^
(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e
^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)))*(-1/3*e^(
1/2)*a*c*d^2+a^2*e^(5/2)-2/3*(a*e^2+c*d^2)^(1/2)*e^(3/2)*a^(3/2)))*d*c)/a^
(3/2)/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2
)/e^(1/2)/d/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2327 vs. 2(380) = 760.

Time = 13.10 (sec) , antiderivative size = 4661, normalized size of antiderivative = 9.83

$$\int \frac{\left(d+ex^2\right)^{5/2}}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(5/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(d+ex^2)^{5/2}}{a+cx^4}\,dx=\int \frac{(d+ex^2)^{\frac{5}{2}}}{a+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)/(c*x**4+a),x)
```

output
```
Integral((d + e*x**2)**(5/2)/(a + c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}}{a+cx^4}\,dx=\int \frac{(ex^2+d)^{\frac{5}{2}}}{cx^4+a}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(5/2)/(c*x^4 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}}{a+cx^4}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(5/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{a+cx^4}\,dx = \int \frac{(e\,x^2+d)^{5/2}}{c\,x^4+a}\,dx$$

input
```
int((d + e*x^2)^(5/2)/(a + c*x^4),x)
```

output
```
int((d + e*x^2)^(5/2)/(a + c*x^4), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{5/2}}{a+cx^4}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+a}dx\right)d^2 + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c\,x^4+a}dx\right)e^2 + 2\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c\,x^4+a}dx\right)de$$

input
```
int((e*x^2+d)^(5/2)/(c*x^4+a),x)
```

output
```
int(sqrt(d + e*x**2)/(a + c*x**4),x)*d**2 + int((sqrt(d + e*x**2)*x**4)/(a
 + c*x**4),x)*e**2 + 2*int((sqrt(d + e*x**2)*x**2)/(a + c*x**4),x)*d*e
```

3.384 $\int \frac{\left(d+ex^2\right)^{3/2}}{a+cx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 442

$$\int \frac{\left(d+ex^2\right)^{3/2}}{a+cx^4}\,dx = \frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(2cd^2e-\left(cd^2-ae^2\right)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{c}}}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+}\right.}\right.}{2\sqrt{2}\sqrt[4]{a}c^{3/2}d\sqrt{cd^2+ae^2}} + \frac{e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c} + \frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(2cd^2e-\left(cd^2-ae^2\right)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{2\sqrt{2}\sqrt[4]{a}c^{3/2}d\sqrt{cd^2+ae^2}}$$

output

```
1/4*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(2*c*d^2*e-(-a*e^2+c*d^2)*(e-(a*
e^2+c*d^2)^(1/2)/a^(1/2)))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^
2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^
(1/2))-c*d*x^2))*2^(1/2)/a^(1/4)/c^(3/2)/d/(a*e^2+c*d^2)^(1/2)+e^(3/2)*arc
tanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c+1/4*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/
2)*(2*c*d^2*e-(-a*e^2+c*d^2)*(e+(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctanh(2^(1
/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/
2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(1/4)/c^(3
/2)/d/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.55

$$\int \frac{\left(d+ex^2\right)^{3/2}}{a+cx^4}\,dx = \frac{e^{3/2}\Big(-\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)+\text{RootSum}\Big[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2\#1^2}{}$$

input
```
Integrate[(d + e*x^2)^(3/2)/(a + c*x^4),x]
```

output
```
(e^(3/2)*(-Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]] + RootSum[c*d^4 - 4*c*d^3*#
1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (c*d^3*Log[d +
2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 2*a*e^2*Log[d + 2*e*x^2 - 2*
Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqr
t[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c
*#1^3) & ]))/c
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {1489, 27, 318, 25, 398, 224, 219, 291, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}}{a+cx^4}dx$$

$$\Big\downarrow \text{1489}$$

$$-\frac{\sqrt{c}\int \frac{\left(ex^2+d\right)^{3/2}}{\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x^2\right)}dx}{2\sqrt{-a}}-\frac{\sqrt{c}\int \frac{\left(ex^2+d\right)^{3/2}}{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{-a}\right)}dx}{2\sqrt{-a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(ex^2+d\right)^{3/2}}{\sqrt{-a}-\sqrt{c}x^2}dx}{2\sqrt{-a}}-\frac{\int \frac{\left(ex^2+d\right)^{3/2}}{\sqrt{c}x^2+\sqrt{-a}}dx}{2\sqrt{-a}}$$

$$\Big\downarrow 318$$

$$-\frac{\frac{\int \frac{e(3\sqrt{c}d-2\sqrt{-a}e)x^2+d(2\sqrt{c}d-\sqrt{-a}e)}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}-\frac{-\frac{\int -\frac{e(3\sqrt{c}d+2\sqrt{-a}e)x^2+d(2\sqrt{c}d+\sqrt{-a}e)}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{\int \frac{e(3\sqrt{c}d-2\sqrt{-a}e)x^2+d(2\sqrt{c}d-\sqrt{-a}e)}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{e(3\sqrt{c}d+2\sqrt{-a}e)x^2+d(2\sqrt{c}d+\sqrt{-a}e)}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow 398$$

$$-\frac{\frac{\frac{2\left(2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-e\left(\frac{2\sqrt{-a}e}{\sqrt{c}}+3d\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{2\left(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}+e\left(3d-\frac{2\sqrt{-a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow 224$$

$$-\frac{\frac{\frac{2\left(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}+e\left(3d-\frac{2\sqrt{-a}e}{\sqrt{c}}\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{2\left(2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-e\left(\frac{2\sqrt{-a}e}{\sqrt{c}}+3d\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow 219$$

$$-\frac{\frac{\frac{2\left(2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}-\sqrt{e}\left(\frac{2\sqrt{-a}e}{\sqrt{c}}+3d\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{2\left(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{\sqrt{c}}+\sqrt{e}\left(3d-\frac{2\sqrt{-a}e}{\sqrt{c}}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow 291$$

$$-\frac{\frac{\frac{2\left(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\int \frac{1}{\sqrt{-a}-\frac{\left(\sqrt{-a}e-\sqrt{c}d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}+\sqrt{e}\left(3d-\frac{2\sqrt{-a}e}{\sqrt{c}}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{2\left(2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\int \frac{1}{\sqrt{-a}-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}-\sqrt{e}\left(\frac{2\sqrt{-a}e}{\sqrt{c}}+3d\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow 218$$

$$-\frac{\frac{\frac{2\left(2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\int \frac{1}{\sqrt{-a}-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}-\sqrt{e}\left(\frac{2\sqrt{-a}e}{\sqrt{c}}+3d\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{2\left(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}}+\sqrt{e}\left(3d-\frac{2\sqrt{-a}e}{\sqrt{c}}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow 221$$

$$-\frac{\frac{\frac{2\left(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}}+\sqrt{e}\left(3d-\frac{2\sqrt{-a}e}{\sqrt{c}}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}+\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{2\left(2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\text{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{-a}\sqrt{c}\sqrt{\sqrt{-a}e+\sqrt{c}d}}-\sqrt{e}\left(\frac{2\sqrt{-a}e}{\sqrt{c}}+3d\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{c}}-\frac{ex\sqrt{d+ex^2}}{2\sqrt{c}}}{2\sqrt{-a}}$$

input

```
Int[(d + e*x^2)^(3/2)/(a + c*x^4),x]
```

output
```
-1/2*((e*x*Sqrt[d + e*x^2])/(2*Sqrt[c]) + ((2*(c*d^2 - 2*Sqrt[-a]*Sqrt[c]*
d*e - a*e^2)*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d +
e*x^2])])/((-a)^(1/4)*Sqrt[c]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]) + Sqrt[e]*(3*d
 - (2*Sqrt[-a]*e)/Sqrt[c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[c
]))/Sqrt[-a] - (-1/2*(e*x*Sqrt[d + e*x^2])/Sqrt[c] + (-(Sqrt[e]*(3*d + (2*
Sqrt[-a]*e)/Sqrt[c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]) + (2*(c*d^2 + 2
*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*x)/((
-a)^(1/4)*Sqrt[d + e*x^2])])/((-a)^(1/4)*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[-a]
*e]))/(2*Sqrt[c]))/(2*Sqrt[-a])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1489
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Simp[-c/(2*r)   Int[(d + e*x^2)^q/(r - c*x^2), x], x] - S
imp[c/(2*r)   Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e,
q}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(356) = 712$.

Time = 0.78 (sec) , antiderivative size = 783, normalized size of antiderivative = 1.77

method	result
pseudoelliptic	$-\frac{\frac{\sqrt{4\sqrt{a e^2+c d^2}\,\sqrt{a}-2\sqrt{a\left(a e^2+c d^2\right)}-2ae}\,\sqrt{2\sqrt{a\left(a e^2+c d^2\right)}+2ae}\,\left(\left(-a e^2-e\sqrt{a e^2+c d^2}\,\sqrt{a}+c d^2\right)\sqrt{a\left(a e^2+c d^2\right)}+e\left(a^2 e^2-d^2 a\right.\right.}{4}}{}$
default	Expression too large to display

input
```
int((e*x^2+d)^(3/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(1/4*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^
(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*((-a*e^2-e*(a*e^2+c*d^2)^(1/
2)*a^(1/2)+c*d^2)*(a*(a*e^2+c*d^2))^(1/2)+e*(a^2*e^2-d^2*a*c+a^(3/2)*(a*e^
2+c*d^2)^(1/2)*e))*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^
2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)-1/4*(4*(a*e^2+c*d^2
)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(2*(a*(a*e^2+c*d^2)
)^(1/2)+2*a*e)^(1/2)*((-a*e^2-e*(a*e^2+c*d^2)^(1/2)*a^(1/2)+c*d^2)*(a*(a*e
^2+c*d^2))^(1/2)+e*(a^2*e^2-d^2*a*c+a^(3/2)*(a*e^2+c*d^2)^(1/2)*e))*ln((a^
(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+
(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+d^2*c*(-2*e^(3/2)*arctanh((e*x^2+d)^(1/2)/x/
e^(1/2))*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(
1/2)*a^(3/2)+(a^2*e^2-d^2*a*c-a^(3/2)*(a*e^2+c*d^2)^(1/2)*e)*(arctan(((2*(
a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e
^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arctan((2*
a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a
*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)))))/a^(3/
2)/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)/d
^2/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. $2(358) = 716$.

Time = 1.30 (sec) , antiderivative size = 2849, normalized size of antiderivative = 6.45

$$\int \frac{\left(d+ex^2\right)^{3/2}}{a+cx^4}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^2+d)^(3/2)/(c*x^4+a),x, algorithm="fricas")
```

output

```
[1/8*(4*e^(3/2)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + c*sqrt(-
(3*c*d^2*e - a*e^3 + a*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)
/(a^3*c^3)))/(a*c^2))*log(-(c^2*d^5 - 2*a*c*d^3*e^2 - 3*a^2*d*e^4 + (a*c^3
*d^2 + a^2*c^2*e^2)*x^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a
^3*c^3)) + 2*(c^2*d^4*e - 2*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 + 2*(a^2*c^3*x*sq
rt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*c^3)) - (a*c^2*d^2*e -
3*a^2*c*e^3)*x)*sqrt(e*x^2 + d)*sqrt(-(3*c*d^2*e - a*e^3 + a*c^2*sqrt(-(c^
2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*c^3)))/(a*c^2)))/x^2) - c*sqrt
(-(3*c*d^2*e - a*e^3 + a*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^
4)/(a^3*c^3)))/(a*c^2))*log(-(c^2*d^5 - 2*a*c*d^3*e^2 - 3*a^2*d*e^4 + (a*c
^3*d^2 + a^2*c^2*e^2)*x^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/
(a^3*c^3)) + 2*(c^2*d^4*e - 2*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 - 2*(a^2*c^3*x*
sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*c^3)) - (a*c^2*d^2*e
- 3*a^2*c*e^3)*x)*sqrt(e*x^2 + d)*sqrt(-(3*c*d^2*e - a*e^3 + a*c^2*sqrt(-(
c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*c^3)))/(a*c^2)))/x^2) - c*sq
rt(-(3*c*d^2*e - a*e^3 - a*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*
e^4)/(a^3*c^3)))/(a*c^2))*log(-(c^2*d^5 - 2*a*c*d^3*e^2 - 3*a^2*d*e^4 - (a
*c^3*d^2 + a^2*c^2*e^2)*x^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4
)/(a^3*c^3)) + 2*(c^2*d^4*e - 2*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 + 2*(a^2*c^3*
x*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^3*c^3)) + (a*c^2*d...
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{a+cx^4}\,dx=\int \frac{(d+ex^2)^{\frac{3}{2}}}{a+cx^4}\,dx$$

input

```
integrate((e*x**2+d)**(3/2)/(c*x**4+a),x)
```

output

```
Integral((d + e*x**2)**(3/2)/(a + c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{a+cx^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{cx^4+a}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/(c*x^4 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}}{a+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(3/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{a+cx^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{c\,x^4+a}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(a + c*x^4),x)
```

output
```
int((d + e*x^2)^(3/2)/(a + c*x^4), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}}{a+cx^4}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+a}dx\right)d + \left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c\,x^4+a}dx\right)e$$

input

```
int((e*x^2+d)^(3/2)/(c*x^4+a),x)
```

output

```
int(sqrt(d + e*x**2)/(a + c*x**4),x)*d + int((sqrt(d + e*x**2)*x**2)/(a +
c*x**4),x)*e
```

3.385 $\int \frac{\sqrt{d+ex^2}}{a+cx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 287

$$\int \frac{\sqrt{d+ex^2}}{a+cx^4}\,dx = \frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\,x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{2\sqrt{2}a^{3/4}\sqrt{c}} - \frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\,x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

output

```
1/4*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(
a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e
+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(3/4)/c^(1/2)-1/4*(-a^(1/2)*e+(a
*e^2+c*d^2)^(1/2))^(1/2)*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^
2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^
(1/2))-c*d*x^2))*2^(1/2)/a^(3/4)/c^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{d+ex^2}}{a+cx^4}\,dx=\frac{1}{2}e^{3/2}\text{RootSum}\Big[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2\#1^2-4cd\#1^3$$

$$+c\#1^4\&,\frac{d^2\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)+2d\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)\#1+}{cd^3-3cd^2\#1-8ae^2\#1+3cd\#1^2-c\#1^3}$$

input

```
Integrate[Sqrt[d + e*x^2]/(a + c*x^4),x]
```

output

```
(e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d
*#1^3 + c*#1^4 & , (d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1
] + 2*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + Log[d + 2
*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a
*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/2
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1489, 27, 301, 224, 219, 291, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{a+cx^4}\,dx$$

$$\Big\downarrow \text{1489}$$

$$-\frac{\sqrt{c}\int \frac{\sqrt{ex^2+d}}{\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x^2\right)}dx}{2\sqrt{-a}}-\frac{\sqrt{c}\int \frac{\sqrt{ex^2+d}}{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{-a}\right)}dx}{2\sqrt{-a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{ex^2+d}}{\sqrt{-a}-\sqrt{c}x^2}dx}{2\sqrt{-a}}-\frac{\int \frac{\sqrt{ex^2+d}}{\sqrt{c}x^2+\sqrt{-a}}dx}{2\sqrt{-a}}$$

$$\Big\downarrow \ 301$$

$$-\frac{\left(\frac{\sqrt{-a}e}{\sqrt{c}}+d\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx-\frac{e\int \frac{1}{\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{-a}}-\frac{\left(d-\frac{\sqrt{-a}e}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx+\frac{e\int \frac{1}{\sqrt{ex^2+d}}dx}{\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \ 224$$

$$-\frac{\left(\frac{\sqrt{-a}e}{\sqrt{c}}+d\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx-\frac{e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}}{2\sqrt{-a}}-\frac{\left(d-\frac{\sqrt{-a}e}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx+\frac{e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\left(\frac{\sqrt{-a}e}{\sqrt{c}}+d\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx-\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{-a}}-\frac{\left(d-\frac{\sqrt{-a}e}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \ 291$$

$$-\frac{\left(d-\frac{\sqrt{-a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-a}-\frac{\left(\sqrt{-a}e-\sqrt{c}d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{-a}}-\frac{\left(\frac{\sqrt{-a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{-a}-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \ 218$$

$$-\frac{\left(\frac{\sqrt{-a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{-a}-\frac{(\sqrt{c}d+\sqrt{-a}e)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{\left(d-\frac{\sqrt{-a}e}{\sqrt{c}}\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{-a}}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{\left(d-\frac{\sqrt{-a}e}{\sqrt{c}}\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{-a}}-\frac{\frac{\left(\frac{\sqrt{-a}e}{\sqrt{c}}+d\right)\text{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{\sqrt[4]{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}}-\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c}}}{2\sqrt{-a}}$$

input

```
Int[Sqrt[d + e*x^2]/(a + c*x^4),x]
```

output

```
-1/2*(((d - (Sqrt[-a]*e)/Sqrt[c])*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*x)/
((-a)^(1/4)*Sqrt[d + e*x^2])])/((-a)^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]) +
 (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[c])/Sqrt[-a] - (-((Sq
rt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[c]) + ((d + (Sqrt[-a]*e)/
Sqrt[c])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x
^2])])/((-a)^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]))/(2*Sqrt[-a])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 1489
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Simp[-c/(2*r)   Int[(d + e*x^2)^q/(r - c*x^2), x], x] - S
imp[c/(2*r)   Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e,
q}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(219) = 438.

Time = 1.11 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.92

method	result
pseudoelliptic	$-\frac{\left(ae-\sqrt{a(a e^2+c d^2)}\right)\left(-\frac{\ln\left(\frac{\sqrt{e x^2+d}\,\sqrt{2\sqrt{a\left(a e^2+c d^2\right)}+2ae}\,x-\sqrt{a e^2+c d^2}\,x^2-\sqrt{a}\,\left(e x^2+d\right)}{x^2}\right)\sqrt{2\sqrt{a\left(a e^2+c d^2\right)}+2ae}\,\sqrt{4\sqrt{}}}{4}\right.}{}$
default	Expression too large to display

input
```
int((e*x^2+d)^(1/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^(3/2)/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*
e)^(1/2)*(a*e-(a*(a*e^2+c*d^2))^(1/2))*(-1/4*ln(((e*x^2+d)^(1/2)*(2*(a*(a*
e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-(a*e^2+c*d^2)^(1/2)*x^2-a^(1/2)*(e*x^2+d)
)/x^2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1
/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)+1/4*ln((a^(1/2)*(e*x^2+d)+(e*x^
2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x
^2)/x^2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^
(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)+(a*e+(a*(a*e^2+c*d^2))^(1/2))
*(arctan(((2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1
/2))/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/
2))-arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1
/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(
1/2))))/d/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(221) = 442$.

Time = 0.21 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{d+ex^2}}{a+cx^4}\,dx$$

$$\begin{aligned}
&= \frac{1}{8}\sqrt{-\frac{ac\sqrt{-\frac{d^2}{a^3c}}+e}{ac}}\log\left(\frac{acdx^2\sqrt{-\frac{d^2}{a^3c}}+2\sqrt{ex^2+d}\,a^2cx\sqrt{-\frac{ac\sqrt{-\frac{d^2}{a^3c}}+e}{ac}}\sqrt{-\frac{d^2}{a^3c}}+2\,dex^2+d^2}{x^2}\right) \\
&\quad -\frac{1}{8}\sqrt{-\frac{ac\sqrt{-\frac{d^2}{a^3c}}+e}{ac}}\log\left(\frac{acdx^2\sqrt{-\frac{d^2}{a^3c}}-2\sqrt{ex^2+d}\,a^2cx\sqrt{-\frac{ac\sqrt{-\frac{d^2}{a^3c}}+e}{ac}}\sqrt{-\frac{d^2}{a^3c}}+2\,dex^2+d^2}{x^2}\right) \\
&\quad +\frac{1}{8}\sqrt{\frac{ac\sqrt{-\frac{d^2}{a^3c}}-e}{ac}}\log\left(-\frac{acdx^2\sqrt{-\frac{d^2}{a^3c}}+2\sqrt{ex^2+d}\,a^2cx\sqrt{\frac{ac\sqrt{-\frac{d^2}{a^3c}}-e}{ac}}\sqrt{-\frac{d^2}{a^3c}}-2\,dex^2-d^2}{x^2}\right) \\
&\quad -\frac{1}{8}\sqrt{\frac{ac\sqrt{-\frac{d^2}{a^3c}}-e}{ac}}\log\left(-\frac{acdx^2\sqrt{-\frac{d^2}{a^3c}}-2\sqrt{ex^2+d}\,a^2cx\sqrt{\frac{ac\sqrt{-\frac{d^2}{a^3c}}-e}{ac}}\sqrt{-\frac{d^2}{a^3c}}-2\,dex^2-d^2}{x^2}\right)
\end{aligned}$$

input

```
integrate((e*x^2+d)^(1/2)/(c*x^4+a),x, algorithm="fricas")
```

output

```
1/8*sqrt(-(a*c*sqrt(-d^2/(a^3*c)) + e)/(a*c))*log((a*c*d*x^2*sqrt(-d^2/(a^
3*c)) + 2*sqrt(e*x^2 + d)*a^2*c*x*sqrt(-(a*c*sqrt(-d^2/(a^3*c)) + e)/(a*c)
)*sqrt(-d^2/(a^3*c)) + 2*d*e*x^2 + d^2)/x^2) - 1/8*sqrt(-(a*c*sqrt(-d^2/(a
^3*c)) + e)/(a*c))*log((a*c*d*x^2*sqrt(-d^2/(a^3*c)) - 2*sqrt(e*x^2 + d)*a
^2*c*x*sqrt(-(a*c*sqrt(-d^2/(a^3*c)) + e)/(a*c))*sqrt(-d^2/(a^3*c)) + 2*d*
e*x^2 + d^2)/x^2) + 1/8*sqrt((a*c*sqrt(-d^2/(a^3*c)) - e)/(a*c))*log(-(a*c
*d*x^2*sqrt(-d^2/(a^3*c)) + 2*sqrt(e*x^2 + d)*a^2*c*x*sqrt((a*c*sqrt(-d^2/
(a^3*c)) - e)/(a*c))*sqrt(-d^2/(a^3*c)) - 2*d*e*x^2 - d^2)/x^2) - 1/8*sqrt
((a*c*sqrt(-d^2/(a^3*c)) - e)/(a*c))*log(-(a*c*d*x^2*sqrt(-d^2/(a^3*c)) -
2*sqrt(e*x^2 + d)*a^2*c*x*sqrt((a*c*sqrt(-d^2/(a^3*c)) - e)/(a*c))*sqrt(-d
^2/(a^3*c)) - 2*d*e*x^2 - d^2)/x^2)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{a+cx^4}\,dx = \int \frac{\sqrt{d+ex^2}}{a+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(c*x**4+a),x)
```

output
```
Integral(sqrt(d + e*x**2)/(a + c*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{a+cx^4}\,dx = \int \frac{\sqrt{ex^2+d}}{cx^4+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/(c*x^4 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{a+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{a+cx^4}\,dx=\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+a}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a + c*x^4),x)
```

output
```
int((d + e*x^2)^(1/2)/(a + c*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}}{a+cx^4}\,dx=\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+a}dx$$

input
```
int((e*x^2+d)^(1/2)/(c*x^4+a),x)
```

output
```
int(sqrt(d + e*x**2)/(a + c*x**4),x)
```

3.386 $\int \frac{1}{\sqrt{d+ex^2}(a+cx^4)} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 370

$$\begin{aligned}\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)}\,dx &= -\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}d\sqrt{cd^2+ae^2}} \\ &\quad -\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{2\sqrt{2}a^{3/4}\sqrt{c}d\sqrt{cd^2+ae^2}}\end{aligned}$$

output
```
-1/4*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(e-(a*e^2+c*d^2)^(1/2)/a^(1/2))
*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e
*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a
^(1/4)/c^(1/2)/d/(a*e^2+c*d^2)^(1/2)-1/4*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^
(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^
(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(
a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(3/4)/c^(1/2)/d/(a*e^2+c*d^2)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.36

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)}\,dx = -2e^{3/2}\text{RootSum}\Bigg[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\frac{\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)\#1}{-cd^3+3cd^2\#1+8ae^2\#1-3cd\#1^2+c\#1^3}\&\Bigg]$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a + c*x^4)),x]
```

output
```
-2*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c
*d*#1^3 + c*#1^4 & , (Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*
#1)/(-(c*d^3) + 3*c*d^2*#1 + 8*a*e^2*#1 - 3*c*d*#1^2 + c*#1^3) & ]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.41, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1489, 27, 291, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{1489}$$

$$-\frac{\sqrt{c}\int \frac{1}{\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{-a}}-\frac{\sqrt{c}\int \frac{1}{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{-a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x^2\right)\sqrt{ex^2+d}}dx}{2\sqrt{-a}}-\frac{\int \frac{1}{\left(\sqrt{c}x^2+\sqrt{-a}\right)\sqrt{ex^2+d}}dx}{2\sqrt{-a}}$$

$$\Big\downarrow 291$$

$$-\frac{\int \frac{1}{\sqrt{-a}-\frac{(\sqrt{-a}e-\sqrt{c}d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2\sqrt{-a}}-\frac{\int \frac{1}{\sqrt{-a}-\frac{(\sqrt{c}d+\sqrt{-a}e)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2\sqrt{-a}}$$

$$\Big\downarrow 218$$

$$-\frac{\int \frac{1}{\sqrt{-a}-\frac{(\sqrt{c}d+\sqrt{-a}e)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2\sqrt{-a}}-\frac{\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\sqrt{\sqrt{c}d-\sqrt{-a}e}}$$

$$\Big\downarrow 221$$

$$-\frac{\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{\text{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\sqrt{\sqrt{-a}e+\sqrt{c}d}}$$

input

```
Int[1/(Sqrt[d + e*x^2]*(a + c*x^4)),x]
```

output

```
-1/2*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])]
/((-a)^(3/4)*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]) - ArcTanh[(Sqrt[Sqrt[c]*d + Sqr
t[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])]/(2*(-a)^(3/4)*Sqrt[Sqrt[c]*d + S
qrt[-a]*e])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 1489
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Simp[-c/(2*r)   Int[(d + e*x^2)^q/(r - c*x^2), x], x] - S
imp[c/(2*r)   Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e,
q}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. 2(290) = 580.

Time = 0.90 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.82

method	result
pseudoelliptic	$-\frac{-\frac{\left(\left(-e\sqrt{a}-\sqrt{a e^2+c d^2}\right)\sqrt{a\left(a e^2+c d^2\right)}+a e\sqrt{a e^2+c d^2}+a^{\frac{3}{2}} e^2\right)\sqrt{2\sqrt{a\left(a e^2+c d^2\right)}+2 a e}\,\sqrt{4\sqrt{a e^2+c d^2}\,\sqrt{a}-2\sqrt{a\left(a e^2+c d^2\right)}-2a}}{4}}{}$
default	$-\frac{\ln\left(\frac{\frac{2\sqrt{-a}\,\sqrt{c}\,e+2cd}{c}+\frac{2e\sqrt{\sqrt{-a}\,\sqrt{c}}\left(x-\frac{\sqrt{\sqrt{-a}\,\sqrt{c}}}{\sqrt{c}}\right)}{\sqrt{c}}+2\sqrt{\frac{\sqrt{-a}\,\sqrt{c}\,e+cd}{c}}\sqrt{\left(x-\frac{\sqrt{\sqrt{-a}\,\sqrt{c}}}{\sqrt{c}}\right)^2 e+\frac{2e\sqrt{\sqrt{-a}\,\sqrt{c}}\left(x-\frac{\sqrt{\sqrt{-a}\,\sqrt{c}}}{\sqrt{c}}\right)}{\sqrt{c}}+}}{x-\frac{\sqrt{\sqrt{-a}\,\sqrt{c}}}{\sqrt{c}}}\right)}{2\sqrt{\sqrt{-a}\,\sqrt{c}}\,\sqrt{\frac{\sqrt{-a}\,\sqrt{c}\,e+cd}{c}}}$

input
```
int(1/(e*x^2+d)^(1/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^(3/2)/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*
e)^(1/2)*(-1/4*((-e*a^(1/2)-(a*e^2+c*d^2)^(1/2))*(a*(a*e^2+c*d^2))^(1/2)+a
*e*(a*e^2+c*d^2)^(1/2)+a^(3/2)*e^2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2
)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*ln
((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2
)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+1/4*((-e*a^(1/2)-(a*e^2+c*d^2)^(1/2))*(a
*(a*e^2+c*d^2))^(1/2)+a*e*(a*e^2+c*d^2)^(1/2)+a^(3/2)*e^2)*(2*(a*(a*e^2+c*
d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2)
)^(1/2)-2*a*e)^(1/2)*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*
d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+d^2*(arctan((2*a^
(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e
^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arctan(((2
*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a
*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)))*(a*(a*e
^2+c*d^2)^(1/2)-a^(3/2)*e)*c)/(a*e^2+c*d^2)^(1/2)/c/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1217 vs. $2(292) = 584$.

Time = 0.29 (sec) , antiderivative size = 1217, normalized size of antiderivative = 3.29

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(c*x^4+a),x, algorithm="fricas")
```

output

```
1/8*sqrt(-((a*c*d^2 + a^2*e^2)*sqrt(-c*d^2/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2
+ a^5*e^4)) - e)/(a*c*d^2 + a^2*e^2))*log(((a*c*d^2 + a^2*e^2)*sqrt(-c*d^2
/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*x^2 + 2*e*x^2 + 2*(a*e*x + (a^
2*c*d^2 + a^3*e^2)*sqrt(-c*d^2/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*
x)*sqrt(e*x^2 + d)*sqrt(-((a*c*d^2 + a^2*e^2)*sqrt(-c*d^2/(a^3*c^2*d^4 + 2
*a^4*c*d^2*e^2 + a^5*e^4)) - e)/(a*c*d^2 + a^2*e^2)) + d)/x^2) - 1/8*sqrt(
-((a*c*d^2 + a^2*e^2)*sqrt(-c*d^2/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4
)) - e)/(a*c*d^2 + a^2*e^2))*log(((a*c*d^2 + a^2*e^2)*sqrt(-c*d^2/(a^3*c^2
*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*x^2 + 2*e*x^2 - 2*(a*e*x + (a^2*c*d^2 +
 a^3*e^2)*sqrt(-c*d^2/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*x)*sqrt(e
*x^2 + d)*sqrt(-((a*c*d^2 + a^2*e^2)*sqrt(-c*d^2/(a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2 + a^5*e^4)) - e)/(a*c*d^2 + a^2*e^2)) + d)/x^2) - 1/8*sqrt(((a*c*d^2
 + a^2*e^2)*sqrt(-c*d^2/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)) + e)/(a
*c*d^2 + a^2*e^2))*log(-((a*c*d^2 + a^2*e^2)*sqrt(-c*d^2/(a^3*c^2*d^4 + 2*
a^4*c*d^2*e^2 + a^5*e^4))*x^2 - 2*e*x^2 + 2*(a*e*x - (a^2*c*d^2 + a^3*e^2)
*sqrt(-c*d^2/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*x)*sqrt(e*x^2 + d)
*sqrt(((a*c*d^2 + a^2*e^2)*sqrt(-c*d^2/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^
5*e^4)) + e)/(a*c*d^2 + a^2*e^2)) - d)/x^2) + 1/8*sqrt(((a*c*d^2 + a^2*e^2
)*sqrt(-c*d^2/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)) + e)/(a*c*d^2 + a
^2*e^2))*log(-((a*c*d^2 + a^2*e^2)*sqrt(-c*d^2/(a^3*c^2*d^4 + 2*a^4*c*d...
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)}\,dx = \int \frac{1}{(a+cx^4)\,\sqrt{d+ex^2}}\,dx$$

input

```
integrate(1/(e*x**2+d)**(1/2)/(c*x**4+a),x)
```

output

```
Integral(1/((a + c*x**4)*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)}\,dx = \int \frac{1}{(cx^4+a)\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)}\,dx = \int \frac{1}{(c\,x^4+a)\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a + c*x^4)*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a + c*x^4)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)}\,dx=\int \frac{1}{\sqrt{e\,x^2+d}\,a+\sqrt{e\,x^2+d}\,c\,x^4}dx$$

input

```
int(1/(e*x^2+d)^(1/2)/(c*x^4+a),x)
```

output

```
int(1/(sqrt(d + e*x**2)*a + sqrt(d + e*x**2)*c*x**4),x)
```

3.387 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+cx^4\right)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 399

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+cx^4\right)}\,dx=\frac{e^2x}{d\left(cd^2+ae^2\right)\sqrt{d+ex^2}}-\frac{\sqrt{c}\left(2\sqrt{a}e-\sqrt{cd^2+ae^2}\right)\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^{3/2}}-\frac{\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(2\sqrt{a}e+\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^{3/2}}$$

output

```
e^2*x/d/(a*e^2+c*d^2)/(e*x^2+d)^(1/2)-1/4*c^(1/2)*(2*a^(1/2)*e-(a*e^2+c*d^
2)^(1/2))*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*arctan(2^(1/2)*a^(1/4)*c^(
1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(
1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(3/4)/(a*e^2+c*d^2)^(3/2)-
1/4*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(2*a^(1/2)*e+(a*e^2+c*d
^2)^(1/2))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2)
)^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2
))*2^(1/2)/a^(3/4)/(a*e^2+c*d^2)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+cx^4\right)}\,dx = \frac{\frac{2e^2x}{\sqrt{d+ex^2}} - cde^{3/2}\text{RootSum}\left[cd^4 - 4cd^3\#1 + 6cd^2\#1^2 + 16ae^2\#1^2 - 4cd\#1^3\right.}{}$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(a + c*x^4)),x]
```

output
```
((2*e^2*x)/Sqrt[d + e*x^2] - c*d*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*
d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (d^2*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 6*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt
[d + e*x^2] - #1]*#1 + Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]
*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(2*c*d
^3 + 2*a*d*e^2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1487, 208, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+cx^4\right)\left(d+ex^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1487}$$

$$\frac{e^2\int \frac{1}{\left(ex^2+d\right)^{3/2}}dx}{ae^2+cd^2}+\frac{c\int \frac{d-ex^2}{\sqrt{ex^2+d}\left(cx^4+a\right)}dx}{ae^2+cd^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{c\int \frac{d-ex^2}{\sqrt{ex^2+d}(cx^4+a)}dx}{ae^2+cd^2}+\frac{e^2x}{d\sqrt{d+ex^2}\,(ae^2+cd^2)}$$

$$\Big\downarrow \textcolor{blue}{2257}$$

$$\frac{c\int \left(\frac{-\sqrt{c}d-\sqrt{-a}e}{2\sqrt{-a}\sqrt{c}(\sqrt{c}x^2+\sqrt{-a})\sqrt{ex^2+d}}-\frac{\sqrt{c}d-\sqrt{-a}e}{2\sqrt{-a}\sqrt{c}(\sqrt{-a}-\sqrt{c}x^2)\sqrt{ex^2+d}}\right)dx}{ae^2+cd^2}+\frac{e^2x}{d\sqrt{d+ex^2}\,(ae^2+cd^2)}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{c\left(-\frac{(\sqrt{-a}e+\sqrt{c}d)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{(\sqrt{c}d-\sqrt{-a}e)\text{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\sqrt{c}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{ae^2+cd^2}+\frac{e^2x}{d\sqrt{d+ex^2}\,(ae^2+cd^2)}$$

input
```
Int[1/((d + e*x^2)^(3/2)*(a + c*x^4)),x]
```

output
```
(e^2*x)/(d*(c*d^2 + a*e^2)*Sqrt[d + e*x^2]) + (c*(-1/2*((Sqrt[c]*d + Sqrt[
-a]*e)*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2]
)])/((-a)^(3/4)*Sqrt[c]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]) - ((Sqrt[c]*d - Sqrt
[-a]*e)*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^
2])])/(2*(-a)^(3/4)*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e])))/(c*d^2 + a*e^2
)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 1487
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Simp[e^2
/(c*d^2 + a*e^2)   Int[(d + e*x^2)^q, x], x] + Simp[c/(c*d^2 + a*e^2)   Int
[(d + e*x^2)^(q + 1)*((d - e*x^2)/(a + c*x^4)), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. 2(317) = 634.

Time = 1.01 (sec) , antiderivative size = 773, normalized size of antiderivative = 1.94

method	result
pseudoelliptic	$-\frac{-\frac{\sqrt{2\sqrt{a\left(a e^{2}+c d^{2}\right)}+2 a e}\,\sqrt{4\sqrt{a e^{2}+c d^{2}}\,\sqrt{a}-2\sqrt{a\left(a e^{2}+c d^{2}\right)}-2 a e}\,\left(\left(-2 e\sqrt{a}-\sqrt{a e^{2}+c d^{2}}\right)\sqrt{a\left(a e^{2}+c d^{2}\right)}+a e\sqrt{a e^{2}+c d^{2}}+2 a^{\frac{3}{2}}}{4}}{}$
default	Expression too large to display

input
```
int(1/(e*x^2+d)^(3/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*e^2+c*d^2)^(3/2)*(-1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*
(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*((-2*e*
a^(1/2)-(a*e^2+c*d^2)^(1/2))*(a*(a*e^2+c*d^2))^(1/2)+a*e*(a*e^2+c*d^2)^(1/
2)+2*a^(3/2)*e^2)*(e*x^2+d)^(1/2)*ln(((e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))
^(1/2)+2*a*e)^(1/2)*x-(a*e^2+c*d^2)^(1/2)*x^2-a^(1/2)*(e*x^2+d))/x^2)+1/4*
(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(
a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*((-2*e*a^(1/2)-(a*e^2+c*d^2)^(1/2))*(a
*(a*e^2+c*d^2))^(1/2)+a*e*(a*e^2+c*d^2)^(1/2)+2*a^(3/2)*e^2)*(e*x^2+d)^(1/
2)*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)
^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)-2*a^(3/2)*e^2*x*(4*(a*e^2+c*d^2)^(1
/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(a*e^2+c*d^2)^(1/2)+d^2
*(a*(a*e^2+c*d^2)^(1/2)-2*a^(3/2)*e)*(arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2
*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-
2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arctan(((2*(a*(a*e^2+c*d^2))^(1/2)
+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2
)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)))*c*(e*x^2+d)^(1/2))/a^(3/2)/(4*(
a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)/(e*x^2+d
)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3803 vs. $2(319) = 638$.

Time = 10.93 (sec) , antiderivative size = 3803, normalized size of antiderivative = 9.53

$$\int \frac{1}{(d+ex^2)^{3/2}(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(d+e x^2\right)^{3/2} \left(a+c x^4\right)} \, dx=\int \frac{1}{\left(a+c x^{4}\right) \left(d+e x^{2}\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(c*x**4+a),x)
```

output
```
Integral(1/((a + c*x**4)*(d + e*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+e x^2\right)^{3/2} \left(a+c x^4\right)} \, dx=\int { \frac{1}{{\left(c x^{4} + a\right)} {\left(e x^{2} + d\right)}^{\frac{3}{2}}} \,d x }$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + a)*(e*x^2 + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+e x^2\right)^{3/2} \left(a+c x^4\right)} \, dx=\text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+cx^4\right)}\,dx=\int \frac{1}{\left(c\,x^4+a\right)\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int(1/((a + c*x^4)*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a + c*x^4)*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+cx^4\right)}\,dx=\int \frac{1}{\sqrt{e\,x^2+d}\,ad+\sqrt{e\,x^2+d}\,ae\,x^2+\sqrt{e\,x^2+d}\,cd\,x^4+\sqrt{e\,x^2+d}\,ce\,x^6}dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(c*x^4+a),x)
```

output
```
int(1/(sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2 + sqrt(d + e*x**2)
*c*d*x**4 + sqrt(d + e*x**2)*c*e*x**6),x)
```

3.388 $\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+cx^4\right)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 494

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+cx^4\right)}\,dx=\frac{e^2x}{3d\left(cd^2+ae^2\right)\left(d+ex^2\right)^{3/2}}+\frac{2e^2\left(4cd^2+ae^2\right)x}{3d^2\left(cd^2+ae^2\right)^2\sqrt{d+ex^2}}$$
$$-\frac{\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(2cd^2e+\left(cd^2-ae^2\right)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{2\sqrt{2}\sqrt[4]{a}d\left(cd^2+ae^2\right)^{5/2}}$$
$$-\frac{\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(2cd^2e+\left(cd^2-ae^2\right)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-cdx^2}\right.}{2\sqrt{2}\sqrt[4]{a}d\left(cd^2+ae^2\right)^{5/2}}$$

output
```
1/3*e^2*x/d/(a*e^2+c*d^2)/(e*x^2+d)^(3/2)+2/3*e^2*(a*e^2+4*c*d^2)*x/d^2/(a
*e^2+c*d^2)^2/(e*x^2+d)^(1/2)-1/4*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^
(1/2)*(2*c*d^2*e+(-a*e^2+c*d^2)*(e-(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctan(2^
(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1
/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(1/4)/d/(
a*e^2+c*d^2)^(5/2)-1/4*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(2*c
*d^2*e+(-a*e^2+c*d^2)*(e+(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctanh(2^(1/2)*a^(
1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(
1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(1/4)/d/(a*e^2+c*
d^2)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.68

$$\int \frac{1}{(d+ex^2)^{5/2}(a+cx^4)}\,dx = \frac{ae^4x(3d+2ex^2)+cd^2e^2x(9d+8ex^2)-3cd^2e^{3/2}(d+ex^2)^{3/2}\,\mathrm{RootSum}\Big[cd^4}{}$$

input
```
Integrate[1/((d + e*x^2)^(5/2)*(a + c*x^4)),x]
```

output
```
(a*e^4*x*(3*d + 2*e*x^2) + c*d^2*e^2*x*(9*d + 8*e*x^2) - 3*c*d^2*e^(3/2)*(
d + e*x^2)^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2
 - 4*c*d*#1^3 + c*#1^4 & , (c*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e
*x^2] - #1] - 4*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*
#1 + 2*a*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*d*
Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2
*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(3*(c*d^3 + a*d*e^2)^2*(d + e
*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1487, 209, 208, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{1487}$$

$$\frac{e^2\int \frac{1}{(ex^2+d)^{5/2}}dx}{ae^2+cd^2}+\frac{c\int \frac{d-ex^2}{(ex^2+d)^{3/2}(cx^4+a)}dx}{ae^2+cd^2}$$

$$\Big\downarrow \text{209}$$

$$\frac{e^2\left(\frac{2\int \frac{1}{\left(ex^2+d\right)^{3/2}}dx}{3d}+\frac{x}{3d(d+ex^2)^{3/2}}\right)}{ae^2+cd^2}+\frac{c\int \frac{d-ex^2}{(ex^2+d)^{3/2}(cx^4+a)}dx}{ae^2+cd^2}$$

$$\Big\downarrow \ 208$$

$$\frac{c\int \frac{d-ex^2}{(ex^2+d)^{3/2}(cx^4+a)}dx}{ae^2+cd^2}+\frac{e^2\left(\frac{2x}{3d^2\sqrt{d+ex^2}}+\frac{x}{3d(d+ex^2)^{3/2}}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \ 2257$$

$$\frac{c\int \left(\frac{-\sqrt{c}d-\sqrt{-a}e}{2\sqrt{-a}\sqrt{c}\left(\sqrt{c}x^2+\sqrt{-a}\right)(ex^2+d)^{3/2}}-\frac{\sqrt{c}d-\sqrt{-a}e}{2\sqrt{-a}\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x^2\right)(ex^2+d)^{3/2}}\right)dx}{ae^2+cd^2}+\frac{e^2\left(\frac{2x}{3d^2\sqrt{d+ex^2}}+\frac{x}{3d(d+ex^2)^{3/2}}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \ 2009$$

$$c\left(-\frac{\left(\sqrt{-a}e+\sqrt{c}d\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}}-\frac{\left(\sqrt{c}d-\sqrt{-a}e\right)\text{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}}-\frac{ex\left(\sqrt{c}d-\sqrt{-a}e\right)}{2\sqrt{c}d\sqrt{d+ex^2}\left(\sqrt{-a}\sqrt{c}d-ae\right)}+$$

$$ae^2+cd^2$$

$$\frac{e^2\left(\frac{2x}{3d^2\sqrt{d+ex^2}}+\frac{x}{3d(d+ex^2)^{3/2}}\right)}{ae^2+cd^2}$$

input

```
Int[1/((d + e*x^2)^(5/2)*(a + c*x^4)),x]
```

output

```
(e^2*(x/(3*d*(d + e*x^2)^(3/2)) + (2*x)/(3*d^2*Sqrt[d + e*x^2])))/(c*d^2 +
 a*e^2) + (c*(-1/2*(e*(Sqrt[c]*d - Sqrt[-a]*e)*x)/(Sqrt[c]*d*(Sqrt[-a]*Sqr
t[c]*d - a*e)*Sqrt[d + e*x^2]) + (e*(Sqrt[c]*d + Sqrt[-a]*e)*x)/(2*Sqrt[c]
*d*(Sqrt[-a]*Sqrt[c]*d + a*e)*Sqrt[d + e*x^2]) - ((Sqrt[c]*d + Sqrt[-a]*e)
*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])])/(2
*(-a)^(3/4)*(Sqrt[c]*d - Sqrt[-a]*e)^(3/2)) - ((Sqrt[c]*d - Sqrt[-a]*e)*Ar
cTanh[(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])])/(2*(
-a)^(3/4)*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2))))/(c*d^2 + a*e^2)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 1487
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Simp[e^2
/(c*d^2 + a*e^2)   Int[(d + e*x^2)^q, x], x] + Simp[c/(c*d^2 + a*e^2)   Int
[(d + e*x^2)^(q + 1)*((d - e*x^2)/(a + c*x^4)), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(406) = 812$.

Time = 1.00 (sec) , antiderivative size = 924, normalized size of antiderivative = 1.87

method	result
pseudoelliptic	$-\frac{\frac{\left(\left(\left(-a e^{2}+c d^{2}\right) \sqrt{a e^{2}+c d^{2}}+3 \sqrt{a}\, c d^{2} e-a^{\frac{3}{2}} e^{3}\right) \sqrt{a \left(a e^{2}+c d^{2}\right)}+\left(\left(a^{2} e^{2}-d^{2} a c\right) \sqrt{a e^{2}+c d^{2}}-3 c d^{2} e a^{\frac{3}{2}}+e^{3} a^{\frac{5}{2}}\right) e\right) \left(e x^{2}+d\right)^{\frac{3}{2}}}{}}{}$
default	Expression too large to display

input
```
int(1/(e*x^2+d)^(5/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^(3/2)/(a*e^2+c*d^2)^(5/2)/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^
2+c*d^2))^(1/2)-2*a*e)^(1/2)*(1/4*(((-a*e^2+c*d^2)*(a*e^2+c*d^2)^(1/2)+3*a
^(1/2)*c*d^2*e-a^(3/2)*e^3)*(a*(a*e^2+c*d^2))^(1/2)+((a^2*e^2-a*c*d^2)*(a*
e^2+c*d^2)^(1/2)-3*c*d^2*e*a^(3/2)+e^3*a^(5/2))*e)*(e*x^2+d)^(3/2)*(4*(a*e
^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(2*(a*(a*e^
2+c*d^2))^(1/2)+2*a*e)^(1/2)*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(
a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)-1/4*(((-a
*e^2+c*d^2)*(a*e^2+c*d^2)^(1/2)+3*a^(1/2)*c*d^2*e-a^(3/2)*e^3)*(a*(a*e^2+c
*d^2))^(1/2)+((a^2*e^2-a*c*d^2)*(a*e^2+c*d^2)^(1/2)-3*c*d^2*e*a^(3/2)+e^3*
a^(5/2))*e)*(e*x^2+d)^(3/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d
^2))^(1/2)-2*a*e)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*ln((a^(1/2
)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e
^2+c*d^2)^(1/2)*x^2)/x^2)-6*x*(a*e^2+c*d^2)^(1/2)*(d^2*(8/9*e*x^2+d)*c*a^(
3/2)+1/3*(2/3*e*x^2+d)*e^2*a^(5/2))*e^2*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(
a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)+(arctan(((2*(a*(a*e^2+c*d^2))^(1/2)+2*
a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2
*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(
2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)
-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)))*d^2*c*(e*x^2+d)^(3/2)*((a^2*e^2-
a*c*d^2)*(a*e^2+c*d^2)^(1/2)+3*c*d^2*e*a^(3/2)-e^3*a^(5/2)))/(e*x^2+d)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6557 vs. $2(408) = 816$.

Time = 46.72 (sec) , antiderivative size = 6557, normalized size of antiderivative = 13.27

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(d+e x^2\right)^{5/2} \left(a+c x^4\right)}\,dx=\int \frac{1}{\left(a+c x^4\right) \left(d+e x^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(5/2)/(c*x**4+a),x)
```

output
```
Integral(1/((a + c*x**4)*(d + e*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+e x^2\right)^{5/2} \left(a+c x^4\right)}\,dx=\int \frac{1}{{\left(c x^{4} + a\right)} {\left(e x^{2} + d\right)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + a)*(e*x^2 + d)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+e x^2\right)^{5/2} \left(a+c x^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+cx^4)}\,dx=\int \frac{1}{(c\,x^4+a)\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input

```
int(1/((a + c*x^4)*(d + e*x^2)^(5/2)),x)
```

output

```
int(1/((a + c*x^4)*(d + e*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+cx^4)}\,dx=\int \frac{1}{\sqrt{e\,x^2+d}\,a\,d^2+2\sqrt{e\,x^2+d}\,ade\,x^2+\sqrt{e\,x^2+d}\,a\,e^2x^4+\sqrt{e\,x^2+d}\,c\,d^2x}$$

input

```
int(1/(e*x^2+d)^(5/2)/(c*x^4+a),x)
```

output

```
int(1/(sqrt(d + e*x**2)*a*d**2 + 2*sqrt(d + e*x**2)*a*d*e*x**2 + sqrt(d +
e*x**2)*a*e**2*x**4 + sqrt(d + e*x**2)*c*d**2*x**4 + 2*sqrt(d + e*x**2)*c*
d*e*x**6 + sqrt(d + e*x**2)*c*e**2*x**8),x)
```

3.389 $\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 627

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^2} \, dx = -\frac{3e^2(2cd^2-ae^2)\,x\sqrt{d+ex^2}}{4ac^2}$$

$$-\frac{de^3x^3\sqrt{d+ex^2}}{ac}-\frac{e^4x^5\sqrt{d+ex^2}}{4ac}+\frac{x(d+ex^2)^{9/2}}{4a\,(a+cx^4)}$$

$$+\frac{3\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(2c^2d^4e+8acd^2e^3-2a^2e^5-(c^2d^4+2acd^2e^2-7a^2e^4)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\arctan\Bigg(}{8\sqrt{2}a^{5/4}c^{5/2}\sqrt{cd^2+ae^2}}$$

$$+\frac{9de^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2}$$

$$+\frac{3\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(2c^2d^4e+8acd^2e^3-2a^2e^5-(c^2d^4+2acd^2e^2-7a^2e^4)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\operatorname{arctanh}}{8\sqrt{2}a^{5/4}c^{5/2}\sqrt{cd^2+ae^2}}$$

output
```
-3/4*e^2*(-a*e^2+2*c*d^2)*x*(e*x^2+d)^(1/2)/a/c^2-d*e^3*x^3*(e*x^2+d)^(1/2
)/a/c-1/4*e^4*x^5*(e*x^2+d)^(1/2)/a/c+1/4*x*(e*x^2+d)^(9/2)/a/(c*x^4+a)+3/
16*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(2*c^2*d^4*e+8*a*c*d^2*e^3-2*a^2*
e^5-(-7*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*(e-(a*e^2+c*d^2)^(1/2)/a^(1/2)))*ar
ctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^
2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(5
/4)/c^(5/2)/(a*e^2+c*d^2)^(1/2)+9/2*d*e^(7/2)*arctanh(e^(1/2)*x/(e*x^2+d)^
(1/2))/c^2+3/16*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(2*c^2*d^4*e+8*a*c*
d^2*e^3-2*a^2*e^5-(-7*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*(e+(a*e^2+c*d^2)^(1/2
)/a^(1/2)))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2
))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^
2))*2^(1/2)/a^(5/4)/c^(5/2)/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.95 (sec) , antiderivative size = 1002, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(d + e*x^2)^(9/2)/(a + c*x^4)^2,x]
```

output
```
((c*x*Sqrt[d + e*x^2]*(3*a^2*e^4 + c^2*d^3*(d + 4*e*x^2) + 2*a*c*e^2*(-3*d
^2 - 2*d*e*x^2 + e^2*x^4)))/(a*(a + c*x^4)) - 18*c*d*e^(7/2)*Log[-(Sqrt[e]
*x) + Sqrt[d + e*x^2]] + 4*e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1
^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (45*c^2*d^4*Log[d + 2*e*x^2 -
 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 81*a*c*d^2*e^2*Log[d + 2*e*x^2 - 2*Sq
rt[e]*x*Sqrt[d + e*x^2] - #1] + 8*a^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sq
rt[d + e*x^2] - #1] + 10*c^2*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*
x^2] - #1]*#1 - 18*a*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2]
 - #1]*#1 + 5*c^2*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*
#1^2 - a*c*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(
c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ] + (e^(3/2)*Root
Sum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^
4 & , (3*c^3*d^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 168
*a*c^2*d^4*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 321*a
^2*c*d^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 32*a^3*
e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 52*a*c^2*d^3*e^2
*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 36*a^2*c*d*e^4*L
og[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 3*c^3*d^4*Log[d +
2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 8*a*c^2*d^2*e^2*Log[d +
 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + a^2*c*e^4*Log[d + 2...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^2}dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^2}dx$$

input
```
Int[(d + e*x^2)^(9/2)/(a + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. 2(525) = 1050.

Time = 1.66 (sec) , antiderivative size = 1175, normalized size of antiderivative = 1.87

method	result	size
pseudoelliptic	Expression too large to display	1175
risch	Expression too large to display	18791
default	Expression too large to display	26470

input
```
int((e*x^2+d)^(9/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-15/8/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2
)/a^(9/2)*(1/4*((-e*(a^(9/2)*e^2+1/5*c*((5*e^2*x^4+d^2)*a^(7/2)+a^(5/2)*c*
d^2*x^4))*(a*e^2+c*d^2)^(1/2)-7/5*(a^2*e^4-2/7*a*c*d^2*e^2-1/7*c^2*d^4)*a^
2*(c*x^4+a))*(a*(a*e^2+c*d^2))^(1/2)+((1/5*c*(5*e^2*x^4+d^2)*a^(9/2)+a^(11
/2)*e^2+1/5*a^(7/2)*c^2*d^2*x^4)*e*(a*e^2+c*d^2)^(1/2)+7/5*(a^2*e^4-2/7*a*
c*d^2*e^2-1/7*c^2*d^4)*a^3*(c*x^4+a))*e)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)
^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/
2)*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)
^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)-1/4*((-e*(a^(9/2)*e^2+1/5*c*((5*e^2
*x^4+d^2)*a^(7/2)+a^(5/2)*c*d^2*x^4))*(a*e^2+c*d^2)^(1/2)-7/5*(a^2*e^4-2/7
*a*c*d^2*e^2-1/7*c^2*d^4)*a^2*(c*x^4+a))*(a*(a*e^2+c*d^2))^(1/2)+((1/5*c*(
5*e^2*x^4+d^2)*a^(9/2)+a^(11/2)*e^2+1/5*a^(7/2)*c^2*d^2*x^4)*e*(a*e^2+c*d^
2)^(1/2)+7/5*(a^2*e^4-2/7*a*c*d^2*e^2-1/7*c^2*d^4)*a^3*(c*x^4+a))*e)*(2*(a
*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*
e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a
*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+d*c*(2/
5*(-6*d*e^(7/2)*(a^(11/2)+a^(9/2)*c*x^4)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2)
)-x*(e*x^2+d)^(1/2)*(-2*(e*x^2+d)*(-1/3*e*x^2+d)*c*e^2*a^(9/2)+1/3*c^2*d^3
*(4*e*x^2+d)*a^(7/2)+a^(11/2)*e^4))*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a
*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)+(arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(9/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^2}\,dx = \int \frac{(d+ex^2)^{\frac{9}{2}}}{(a+cx^4)^2}\,dx$$

input
```
integrate((e*x**2+d)**(9/2)/(c*x**4+a)**2,x)
```

output
```
Integral((d + e*x**2)**(9/2)/(a + c*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^2}\,dx = \int \frac{(ex^2+d)^{\frac{9}{2}}}{(cx^4+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(9/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(9/2)/(c*x^4 + a)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^2}\,dx = \frac{\sqrt{ex^2+d}\,e^4x}{2\,c^2} - \frac{9\,de^{\frac{7}{2}}\log\left(\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2\right)}{4\,c^2}$$

$$-\frac{3\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^6c^2d^4e^{\frac{3}{2}}-8\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^6acd^2e^{\frac{7}{2}}+\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^6a^2e^{\frac{11}{2}}-6\left(\sqrt{e}x-\sqrt{e}\right.}{\left(\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^8c-4\left(\sqrt{e}x-\sqrt{ex^2}\right.\right.}$$

input
```
integrate((e*x^2+d)^(9/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/2*sqrt(e*x^2 + d)*e^4*x/c^2 - 9/4*d*e^(7/2)*log((sqrt(e)*x - sqrt(e*x^2
+ d))^2)/c^2 - (3*(sqrt(e)*x - sqrt(e*x^2 + d))^6*c^2*d^4*e^(3/2) - 8*(sqr
t(e)*x - sqrt(e*x^2 + d))^6*a*c*d^2*e^(7/2) + (sqrt(e)*x - sqrt(e*x^2 + d)
)^6*a^2*e^(11/2) - 6*(sqrt(e)*x - sqrt(e*x^2 + d))^4*c^2*d^5*e^(3/2) - 10*
(sqrt(e)*x - sqrt(e*x^2 + d))^4*a*c*d^3*e^(7/2) + 16*(sqrt(e)*x - sqrt(e*x
^2 + d))^4*a^2*d*e^(11/2) + 5*(sqrt(e)*x - sqrt(e*x^2 + d))^2*c^2*d^6*e^(3
/2) - (sqrt(e)*x - sqrt(e*x^2 + d))^2*a^2*d^2*e^(11/2) - 2*c^2*d^7*e^(3/2)
 + 2*a*c*d^5*e^(7/2))/(((sqrt(e)*x - sqrt(e*x^2 + d))^8*c - 4*(sqrt(e)*x -
 sqrt(e*x^2 + d))^6*c*d + 6*(sqrt(e)*x - sqrt(e*x^2 + d))^4*c*d^2 + 16*(sq
rt(e)*x - sqrt(e*x^2 + d))^4*a*e^2 - 4*(sqrt(e)*x - sqrt(e*x^2 + d))^2*c*d
^3 + c*d^4)*a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^2}\,dx = \int \frac{{\left(e\,x^2+d\right)}^{9/2}}{{\left(c\,x^4+a\right)}^2}\,dx$$

input
```
int((d + e*x^2)^(9/2)/(a + c*x^4)^2,x)
```

output
```
int((d + e*x^2)^(9/2)/(a + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^2}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c^2x^8+2ac\,x^4+a^2}dx\right)d^4 + \left(\int \frac{\sqrt{e\,x^2+d}\,x^8}{c^2x^8+2ac\,x^4+a^2}dx\right)e^4 + 4\left(\int \frac{\sqrt{e\,x^2+d}\,x^6}{c^2x^8+2ac\,x^4+a^2}dx\right)d\,e^3 + 6\left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c^2x^8+2ac\,x^4+a^2}dx\right)d^2e^2 + 4\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c^2x^8+2ac\,x^4+a^2}dx\right)d^3e$$

input
```
int((e*x^2+d)^(9/2)/(c*x^4+a)^2,x)
```

output
```
int(sqrt(d + e*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*d**4 + int((sqrt(d
 + e*x**2)*x**8)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*e**4 + 4*int((sqrt(d +
 e*x**2)*x**6)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*d*e**3 + 6*int((sqrt(d +
 e*x**2)*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*d**2*e**2 + 4*int((sqrt(
d + e*x**2)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*d**3*e
```

3.390 $\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 573

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^2}\,dx = -\frac{3de^2x\sqrt{d+ex^2}}{4ac}-\frac{e^3x^3\sqrt{d+ex^2}}{4ac}+\frac{x(d+ex^2)^{7/2}}{4a\,(a+cx^4)}$$

$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(cd^2e(5cd^2+9ae^2)+\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(4a^2e^4-3cd^2(cd^2+ae^2))\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}}{\sqrt{a}\left(\right.}\right.}{8\sqrt{2}a^{5/4}c^{5/2}d\sqrt{cd^2+ae^2}}$$

$$+\frac{e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}$$

$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(cd^2e(5cd^2+9ae^2)+\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(4a^2e^4-3cd^2(cd^2+ae^2))\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{}}\right.}{8\sqrt{2}a^{5/4}c^{5/2}d\sqrt{cd^2+ae^2}}$$

output
```
-3/4*d*e^2*x*(e*x^2+d)^(1/2)/a/c-1/4*e^3*x^3*(e*x^2+d)^(1/2)/a/c+1/4*x*(e*
x^2+d)^(7/2)/a/(c*x^4+a)+1/16*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(c*d^2
*e*(9*a*e^2+5*c*d^2)+(e-(a*e^2+c*d^2)^(1/2)/a^(1/2))*(4*a^2*e^4-3*c*d^2*(a
*e^2+c*d^2)))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2
))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^
2))*2^(1/2)/a^(5/4)/c^(5/2)/d/(a*e^2+c*d^2)^(1/2)+e^(7/2)*arctanh(e^(1/2)*
x/(e*x^2+d)^(1/2))/c^2+1/16*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(c*d^2*
e*(9*a*e^2+5*c*d^2)+(e+(a*e^2+c*d^2)^(1/2)/a^(1/2))*(4*a^2*e^4-3*c*d^2*(a*
e^2+c*d^2)))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/
2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x
^2))*2^(1/2)/a^(5/4)/c^(5/2)/d/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.70 (sec) , antiderivative size = 827, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^2}\,dx = \frac{\frac{2cx\sqrt{d+ex^2}\left(-ae^2\left(3d+ex^2\right)+cd^2\left(d+3ex^2\right)\right)}{a(a+cx^4)} - 8e^{7/2}\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right) + 16e^{7/2}\text{RootSum}}{}$$

input
```
Integrate[(d + e*x^2)^(7/2)/(a + c*x^4)^2,x]
```

output
```
((2*c*x*Sqrt[d + e*x^2]*(-(a*e^2*(3*d + e*x^2)) + c*d^2*(d + 3*e*x^2)))/(a
*(a + c*x^4)) - 8*e^(7/2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]] + 16*e^(7/2)
*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 +
c*#1^4 & , (17*c*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] -
 16*a*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 4*c*d^2*
Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 2*a*e^2*Log[d + 2
*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*d*Log[d + 2*e*x^2 - 2*Sq
rt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c
*d*#1^2 - c*#1^3) & ] + (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2
 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (5*c^2*d^5*Log[d + 2*e*x^2 - 2*
Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 263*a*c*d^3*e^2*Log[d + 2*e*x^2 - 2*Sqrt
[e]*x*Sqrt[d + e*x^2] - #1] + 256*a^2*d*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*
Sqrt[d + e*x^2] - #1] + 2*c^2*d^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e
*x^2] - #1]*#1 - 70*a*c*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x
^2] - #1]*#1 + 16*a^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] -
#1]*#1 + 5*c^2*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^
2 - 7*a*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/
(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/a)/(8*c^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a+cx^4\right)^2}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a+cx^4\right)^2}dx$$

input
```
Int[(d + e*x^2)^(7/2)/(a + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1154 vs. 2(475) = 950.

Time = 1.46 (sec) , antiderivative size = 1155, normalized size of antiderivative = 2.02

method	result	size
pseudoelliptic	Expression too large to display	1155
default	Expression too large to display	19398

input
```
int((e*x^2+d)^(7/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)/
a^(9/2)*(-1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/
2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*((-(a^(9/2)*e^2+1/2*((2*
e^2*x^4+d^2)*a^(7/2)+a^(5/2)*c*d^2*x^4)*c)*e*(a*e^2+c*d^2)^(1/2)-a^2*(c*x^
4+a)*(a^2*e^4-3/4*a*c*d^2*e^2-3/4*c^2*d^4))*(a*(a*e^2+c*d^2))^(1/2)+((1/2*
c*(2*e^2*x^4+d^2)*a^(9/2)+a^(11/2)*e^2+1/2*a^(7/2)*c^2*d^2*x^4)*e*(a*e^2+c
*d^2)^(1/2)+a^3*(c*x^4+a)*(a^2*e^4-3/4*a*c*d^2*e^2-3/4*c^2*d^4))*e)*ln((a^
(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+
(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*
(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*((-(
a^(9/2)*e^2+1/2*((2*e^2*x^4+d^2)*a^(7/2)+a^(5/2)*c*d^2*x^4)*c)*e*(a*e^2+c*
d^2)^(1/2)-a^2*(c*x^4+a)*(a^2*e^4-3/4*a*c*d^2*e^2-3/4*c^2*d^4))*(a*(a*e^2+
c*d^2))^(1/2)+((1/2*c*(2*e^2*x^4+d^2)*a^(9/2)+a^(11/2)*e^2+1/2*a^(7/2)*c^2
*d^2*x^4)*e*(a*e^2+c*d^2)^(1/2)+a^3*(c*x^4+a)*(a^2*e^4-3/4*a*c*d^2*e^2-3/4
*c^2*d^4))*e)*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(
1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+d^2*((2*e^(7/2)*(a^(11/2
)+a^(9/2)*c*x^4)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))-3/2*x*(e*x^2+d)^(1/2)*
(e^2*(1/3*e*x^2+d)*a^(9/2)-1/3*a^(7/2)*c*d^2*(3*e*x^2+d))*c)*(4*(a*e^2+c*d
^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)+((1/2*c*(2*e^2*x^
4+d^2)*a^(9/2)+a^(11/2)*e^2+1/2*a^(7/2)*c^2*d^2*x^4)*e*(a*e^2+c*d^2)^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3557 vs. $2(477) = 954$.

Time = 68.68 (sec) , antiderivative size = 7122, normalized size of antiderivative = 12.43

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(7/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^2}\,dx = \int \frac{(d+ex^2)^{\frac{7}{2}}}{(a+cx^4)^2}\,dx$$

input
```
integrate((e*x**2+d)**(7/2)/(c*x**4+a)**2,x)
```

output
```
Integral((d + e*x**2)**(7/2)/(a + c*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^2}\,dx = \int \frac{(ex^2+d)^{\frac{7}{2}}}{(cx^4+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(7/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(7/2)/(c*x^4 + a)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^2}\,dx = -\frac{e^{\frac{7}{2}}\log\left(\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2\right)}{2\,c^2}$$

$$-\frac{5\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^6c^2d^3e^{\frac{3}{2}}-7\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^6acde^{\frac{7}{2}}-9\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^4c^2d^4e^{\frac{3}{2}}-21\left(\sqrt{e}x-\right.}{2\left(\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^8c-4\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^6cd+6\left(\sqrt{}\right.\right.}$$

input
```
integrate((e*x^2+d)^(7/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/2*e^(7/2)*log((sqrt(e)*x - sqrt(e*x^2 + d))^2)/c^2 - 1/2*(5*(sqrt(e)*x
- sqrt(e*x^2 + d))^6*c^2*d^3*e^(3/2) - 7*(sqrt(e)*x - sqrt(e*x^2 + d))^6*a
*c*d*e^(7/2) - 9*(sqrt(e)*x - sqrt(e*x^2 + d))^4*c^2*d^4*e^(3/2) - 21*(sqr
t(e)*x - sqrt(e*x^2 + d))^4*a*c*d^2*e^(7/2) + 8*(sqrt(e)*x - sqrt(e*x^2 +
d))^4*a^2*e^(11/2) + 7*(sqrt(e)*x - sqrt(e*x^2 + d))^2*c^2*d^5*e^(3/2) + 3
*(sqrt(e)*x - sqrt(e*x^2 + d))^2*a*c*d^3*e^(7/2) - 3*c^2*d^6*e^(3/2) + a*c
*d^4*e^(7/2))/(((sqrt(e)*x - sqrt(e*x^2 + d))^8*c - 4*(sqrt(e)*x - sqrt(e*
x^2 + d))^6*c*d + 6*(sqrt(e)*x - sqrt(e*x^2 + d))^4*c*d^2 + 16*(sqrt(e)*x
- sqrt(e*x^2 + d))^4*a*e^2 - 4*(sqrt(e)*x - sqrt(e*x^2 + d))^2*c*d^3 + c*d
^4)*a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^2}\,dx = \int \frac{\left(e\,x^2+d\right)^{7/2}}{\left(c\,x^4+a\right)^2}\,dx$$

input
```
int((d + e*x^2)^(7/2)/(a + c*x^4)^2,x)
```

output
```
int((d + e*x^2)^(7/2)/(a + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^2}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c^2x^8+2ac\,x^4+a^2}dx\right)d^3 + \left(\int \frac{\sqrt{e\,x^2+d}\,x^6}{c^2x^8+2ac\,x^4+a^2}dx\right)e^3 + 3\left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c^2x^8+2ac\,x^4+a^2}dx\right)d\,e^2 + 3\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c^2x^8+2ac\,x^4+a^2}dx\right)d^2e$$

input
```
int((e*x^2+d)^(7/2)/(c*x^4+a)^2,x)
```

output
```
int(sqrt(d + e*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*d**3 + int((sqrt(d
 + e*x**2)*x**6)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*e**3 + 3*int((sqrt(d +
 e*x**2)*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*d*e**2 + 3*int((sqrt(d +
 e*x**2)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*d**2*e
```

3.391 $\int \frac{(d+ex^2)^{5/2}}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 473

$$\int \frac{(d+ex^2)^{5/2}}{(a+cx^4)^2}\,dx = -\frac{e^2x\sqrt{d+ex^2}}{4ac}+\frac{x(d+ex^2)^{5/2}}{4a(a+cx^4)}$$

$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(4cd^2e+2ae^3-(3cd^2+ae^2)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+e}}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cdx^2}\right.}{8\sqrt{2}a^{5/4}c^{3/2}\sqrt{cd^2+ae^2}}$$

$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(4cd^2e+2ae^3-(3cd^2+ae^2)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}x}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-cd}\right.}{8\sqrt{2}a^{5/4}c^{3/2}\sqrt{cd^2+ae^2}}$$

output

```
-1/4*e^2*x*(e*x^2+d)^(1/2)/a/c+1/4*x*(e*x^2+d)^(5/2)/a/(c*x^4+a)+1/16*(a^(
1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(4*c*d^2*e+2*a*e^3-(a*e^2+3*c*d^2)*(e-(a
*e^2+c*d^2)^(1/2)/a^(1/2)))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e
^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)
^(1/2))-c*d*x^2))*2^(1/2)/a^(5/4)/c^(3/2)/(a*e^2+c*d^2)^(1/2)+1/16*(-a^(1/
2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(4*c*d^2*e+2*a*e^3-(a*e^2+3*c*d^2)*(e+(a*e
^2+c*d^2)^(1/2)/a^(1/2)))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e
^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)
^(1/2))-c*d*x^2))*2^(1/2)/a^(5/4)/c^(3/2)/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.57 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex^2)^{5/2}}{(a+cx^4)^2}\,dx = \frac{x\sqrt{d+ex^2}(cd^2-ae^2+2cdex^2)}{4ac\,(a+cx^4)}$$

$$+\frac{e^{7/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\frac{49cd^2\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)}{}\right.}{2c^2}$$

$$+\frac{e^{3/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\frac{2c^2d^4\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)}{}\right.}{}$$

input
```
Integrate[(d + e*x^2)^(5/2)/(a + c*x^4)^2,x]
```

output
```
(x*Sqrt[d + e*x^2]*(c*d^2 - a*e^2 + 2*c*d*e*x^2))/(4*a*c*(a + c*x^4)) + (e
^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#
1^3 + c*#1^4 & , (49*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] -
 #1] - 16*a*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 10*c
*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*Log[d + 2*e*
x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^
2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(2*c^2) + (e^(3/2)*RootSum[c*d^4 - 4*c*d^
3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (2*c^2*d^4*L
og[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 97*a*c*d^2*e^2*Log[d
+ 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 32*a^2*e^4*Log[d + 2*e*x^2
 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 2*c^2*d^3*Log[d + 2*e*x^2 - 2*Sqrt[
e]*x*Sqrt[d + e*x^2] - #1]*#1 - 20*a*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x
*Sqrt[d + e*x^2] - #1]*#1 + 2*c^2*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d
 + e*x^2] - #1]*#1^2 - a*c*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ]
)/(4*a*c^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \text{1571}$$
$$\int \frac{(d+ex^2)^{5/2}}{(a+cx^4)^2}\,dx$$

input
```
Int[(d + e*x^2)^(5/2)/(a + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 870 vs. 2(385) = 770.

Time = 1.17 (sec) , antiderivative size = 871, normalized size of antiderivative = 1.84

method	result
	$\frac{d^2\left(\left(\left(a^{\frac{9}{2}}+c\,x^4 a^{\frac{7}{2}}\right)e\sqrt{a\,e^2+c\,d^2}+a^3\left(c\,x^4+a\right)\left(a\,e^2+3c\,d^2\right)\right)c\arctan\left(\frac{2\sqrt{a}\,\sqrt{e\,x^2+d}+\sqrt{2\sqrt{a\left(a\,e^2+c\,d^2\right)}+2ae}\,x}{x\sqrt{4\sqrt{a\,e^2+c\,d^2}\,\sqrt{a}-2\sqrt{a\left(a\,e^2+c\,d^2\right)}-2ae}}\right)}{2} - d^2\left(\left(a^{\frac{9}{2}}+c\right.\right.$
pseudoelliptic	$-$
default	Expression too large to display

input
```
int((e*x^2+d)^(5/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^(9/2)/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*
e)^(1/2)*(1/2*d^2*((a^(9/2)+c*x^4*a^(7/2))*e*(a*e^2+c*d^2)^(1/2)+a^3*(c*x^
4+a)*(a*e^2+3*c*d^2))*c*arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d
^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*
d^2))^(1/2)-2*a*e)^(1/2))-1/2*d^2*((a^(9/2)+c*x^4*a^(7/2))*e*(a*e^2+c*d^2)
^(1/2)+a^3*(c*x^4+a)*(a*e^2+3*c*d^2))*c*arctan(((2*(a*(a*e^2+c*d^2))^(1/2)
+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2
)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))+(1/8*((-e*(c*x^4*a^(5/2)+a^(7/2)
)*(a*e^2+c*d^2)^(1/2)+a^2*(c*x^4+a)*(a*e^2+3*c*d^2))*(a*(a*e^2+c*d^2))^(1/
2)+e*((a^(9/2)+c*x^4*a^(7/2))*e*(a*e^2+c*d^2)^(1/2)-a^3*(c*x^4+a)*(a*e^2+3
*c*d^2)))*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*ln((a^(1/2)*(e*x^2+d)-(e
*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2
)*x^2)/x^2)-1/8*((-e*(c*x^4*a^(5/2)+a^(7/2))*(a*e^2+c*d^2)^(1/2)+a^2*(c*x^
4+a)*(a*e^2+3*c*d^2))*(a*(a*e^2+c*d^2))^(1/2)+e*((a^(9/2)+c*x^4*a^(7/2))*e
*(a*e^2+c*d^2)^(1/2)-a^3*(c*x^4+a)*(a*e^2+3*c*d^2)))*(2*(a*(a*e^2+c*d^2))^
(1/2)+2*a*e)^(1/2)*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^
2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+(-c*d*(2*e*x^2+d)*a
^(7/2)+a^(9/2)*e^2)*x*d*c*(e*x^2+d)^(1/2))*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-
2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))/d/c^2/(c*x^4+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1951 vs. $2(387) = 774$.

Time = 10.82 (sec) , antiderivative size = 1951, normalized size of antiderivative = 4.12

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(5/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output

```
1/32*((a*c^2*x^4 + a^2*c)*sqrt(-(15*c^2*d^4*e + 15*a*c*d^2*e^3 + 4*a^2*e^5
 + a^3*c^3*sqrt(-(81*c^2*d^10 + 90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)
))/(a^3*c^3))*log((81*c^3*d^10 + 162*a*c^2*d^8*e^2 + 101*a^2*c*d^6*e^4 + 2
0*a^3*d^4*e^6 + (9*a^3*c^4*d^5 + 13*a^4*c^3*d^3*e^2 + 4*a^5*c^2*d*e^4)*x^2
*sqrt(-(81*c^2*d^10 + 90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)) + 2*(81*
c^3*d^9*e + 162*a*c^2*d^7*e^3 + 101*a^2*c*d^5*e^5 + 20*a^3*d^3*e^7)*x^2 +
2*sqrt(e*x^2 + d)*((3*a^5*c^4*d^2 + 2*a^6*c^3*e^2)*x*sqrt(-(81*c^2*d^10 +
90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)) - (9*a^2*c^3*d^6*e + 5*a^3*c^2
*d^4*e^3)*x)*sqrt(-(15*c^2*d^4*e + 15*a*c*d^2*e^3 + 4*a^2*e^5 + a^3*c^3*sq
rt(-(81*c^2*d^10 + 90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)))/(a^3*c^3))
)/x^2) - (a*c^2*x^4 + a^2*c)*sqrt(-(15*c^2*d^4*e + 15*a*c*d^2*e^3 + 4*a^2*
e^5 + a^3*c^3*sqrt(-(81*c^2*d^10 + 90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c
^3)))/(a^3*c^3))*log((81*c^3*d^10 + 162*a*c^2*d^8*e^2 + 101*a^2*c*d^6*e^4
+ 20*a^3*d^4*e^6 + (9*a^3*c^4*d^5 + 13*a^4*c^3*d^3*e^2 + 4*a^5*c^2*d*e^4)*
x^2*sqrt(-(81*c^2*d^10 + 90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)) + 2*(
81*c^3*d^9*e + 162*a*c^2*d^7*e^3 + 101*a^2*c*d^5*e^5 + 20*a^3*d^3*e^7)*x^2
 - 2*sqrt(e*x^2 + d)*((3*a^5*c^4*d^2 + 2*a^6*c^3*e^2)*x*sqrt(-(81*c^2*d^10
 + 90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)) - (9*a^2*c^3*d^6*e + 5*a^3*
c^2*d^4*e^3)*x)*sqrt(-(15*c^2*d^4*e + 15*a*c*d^2*e^3 + 4*a^2*e^5 + a^3*c^3
*sqrt(-(81*c^2*d^10 + 90*a*c*d^8*e^2 + 25*a^2*d^6*e^4)/(a^7*c^3)))/(a^3...
```

Sympy [**F**]

$$\int \frac{(d+ex^2)^{5/2}}{(a+cx^4)^2}\,dx = \int \frac{(d+ex^2)^{\frac{5}{2}}}{(a+cx^4)^2}\,dx$$

input

```
integrate((e*x**2+d)**(5/2)/(c*x**4+a)**2,x)
```

output

```
Integral((d + e*x**2)**(5/2)/(a + c*x**4)**2, x)
```

Maxima [F]

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a+cx^4\right)^2}\,dx=\int \frac{\left(ex^2+d\right)^{\frac{5}{2}}}{\left(cx^4+a\right)^2}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(5/2)/(c*x^4 + a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x^2+d)^(5/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a+cx^4\right)^2}\,dx=\int \frac{{\left(e\,x^2+d\right)}^{5/2}}{{\left(c\,x^4+a\right)}^2}\,dx$$

input
```
int((d + e*x^2)^(5/2)/(a + c*x^4)^2,x)
```

output
```
int((d + e*x^2)^(5/2)/(a + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a+cx^4)^2}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c^2x^8+2ac\,x^4+a^2}dx\right)d^2 + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c^2x^8+2ac\,x^4+a^2}dx\right)e^2 + 2\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c^2x^8+2ac\,x^4+a^2}dx\right)de$$

input

```
int((e*x^2+d)^(5/2)/(c*x^4+a)^2,x)
```

output

```
int(sqrt(d + e*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*d**2 + int((sqrt(d
 + e*x**2)*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*e**2 + 2*int((sqrt(d +
 e*x**2)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*d*e
```

3.392 $\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 317

$$\int \frac{(d+ex^2)^{3/2}}{(a+cx^4)^2}\,dx = \frac{x(d+ex^2)^{3/2}}{4a\left(a+cx^4\right)} + \frac{3d\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{8\sqrt{2}a^{7/4}\sqrt{c}} - \frac{3d\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{8\sqrt{2}a^{7/4}\sqrt{c}}$$

output

```
1/4*x*(e*x^2+d)^(3/2)/a/(c*x^4+a)+3/16*d*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(
1/2)*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*
x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/
2)/a^(7/4)/c^(1/2)-3/16*d*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*arctanh(2
^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^
(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(7/4)/c
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex^2)^{3/2}}{(a+cx^4)^2}\,dx = \frac{2e^{7/2}\mathrm{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\frac{8d\log\left(d+2ex\right.}{}\right.}{c} + \frac{\frac{2x(d+ex^2)^{3/2}}{a+cx^4}+\frac{e^{3/2}\mathrm{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\frac{3cd^3\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)-128ad}{}\right.}{}}{}$$

input
```
Integrate[(d + e*x^2)^(3/2)/(a + c*x^4)^2,x]
```

output
```
(2*e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c
*d*#1^3 + c*#1^4 & , (8*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] -
#1] + Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1)/(c*d^3 - 3*c
*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/c + ((2*x*(d + e*x^2)^(3/
2))/(a + c*x^4) + (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*
a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (3*c*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*
x*Sqrt[d + e*x^2] - #1] - 128*a*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d
 + e*x^2] - #1] + 6*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] -
#1]*#1 - 16*a*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 +
 3*c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 -
3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/c)/(8*a)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \ 1571$$

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a+cx^4\right)^2}dx$$

input
```
Int[(d + e*x^2)^(3/2)/(a + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(245) = 490.

Time = 0.88 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.74

method	result
pseudoelliptic	$\frac{\frac{3\sqrt{4\sqrt{a e^2+c d^2}\,\sqrt{a}-2\sqrt{a\left(a e^2+c d^2\right)}-2ae}\,a\left(\ln\left(\frac{\sqrt{a}\left(e x^2+d\right)-\sqrt{e x^2+d}\,\sqrt{2\sqrt{a\left(a e^2+c d^2\right)}+2ae}\,x+\sqrt{a e^2+c d^2}\,x^2}{x^2}\right)-\ln\left(\frac{\sqrt{a}\left(e x^2\right.}{}\right.\right.}{32}}{}$
default	Expression too large to display

input
```
int((e*x^2+d)^(3/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/32/a^(7/2)*((4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a
*e)^(1/2)*a*(ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1
/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)-ln((a^(1/2)*(e*x^2+d)+(e*
x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)
*x^2)/x^2))*(c*x^4+a)*(a*e-(a*(a*e^2+c*d^2))^(1/2))*(2*(a*(a*e^2+c*d^2))^(
1/2)+2*a*e)^(1/2)+8/3*(x*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2)
)^(1/2)-2*a*e)^(1/2)*a^(5/2)*(e*x^2+d)^(3/2)-3/2*(arctan((2*a^(1/2)*(e*x^2
+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1
/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arctan(((2*(a*(a*e^2+c
*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^
(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)))*d^2*a^2*(c*x^4+a))*
c)/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)/(
c*x^4+a)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(247) = 494.

Time = 0.75 (sec) , antiderivative size = 647, normalized size of antiderivative = 2.04

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a+cx^4\right)^2}\,dx=\frac{3\left(acx^4+a^2\right)\sqrt{-\frac{a^3c\sqrt{-\frac{d^6}{a^7c}}+d^2e}{a^3c}}\log\left(\frac{27\left(a^3cd^2x^2\sqrt{-\frac{d^6}{a^7c}}+2\sqrt{ex^2+d}a^5cx\sqrt{-\frac{d^6}{a^7c}}\sqrt{-\frac{a^3c\sqrt{-\frac{d^6}{a^7c}}+d^2e}{a^3c}}+\right.}{x^2}\right.}{}$$

input
```
integrate((e*x^2+d)^(3/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/32*(3*(a*c*x^4 + a^2)*sqrt(-(a^3*c*sqrt(-d^6/(a^7*c)) + d^2*e)/(a^3*c))*
log(27*(a^3*c*d^2*x^2*sqrt(-d^6/(a^7*c)) + 2*sqrt(e*x^2 + d)*a^5*c*x*sqrt(
-d^6/(a^7*c))*sqrt(-(a^3*c*sqrt(-d^6/(a^7*c)) + d^2*e)/(a^3*c)) + 2*d^4*e*
x^2 + d^5)/x^2) - 3*(a*c*x^4 + a^2)*sqrt(-(a^3*c*sqrt(-d^6/(a^7*c)) + d^2*
e)/(a^3*c))*log(27*(a^3*c*d^2*x^2*sqrt(-d^6/(a^7*c)) - 2*sqrt(e*x^2 + d)*a
^5*c*x*sqrt(-d^6/(a^7*c))*sqrt(-(a^3*c*sqrt(-d^6/(a^7*c)) + d^2*e)/(a^3*c)
) + 2*d^4*e*x^2 + d^5)/x^2) + 3*(a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-d^6/(a^7
*c)) - d^2*e)/(a^3*c))*log(-27*(a^3*c*d^2*x^2*sqrt(-d^6/(a^7*c)) + 2*sqrt(
e*x^2 + d)*a^5*c*x*sqrt(-d^6/(a^7*c))*sqrt((a^3*c*sqrt(-d^6/(a^7*c)) - d^2
*e)/(a^3*c)) - 2*d^4*e*x^2 - d^5)/x^2) - 3*(a*c*x^4 + a^2)*sqrt((a^3*c*sqr
t(-d^6/(a^7*c)) - d^2*e)/(a^3*c))*log(-27*(a^3*c*d^2*x^2*sqrt(-d^6/(a^7*c)
) - 2*sqrt(e*x^2 + d)*a^5*c*x*sqrt(-d^6/(a^7*c))*sqrt((a^3*c*sqrt(-d^6/(a^
7*c)) - d^2*e)/(a^3*c)) - 2*d^4*e*x^2 - d^5)/x^2) + 8*(e*x^3 + d*x)*sqrt(e
*x^2 + d))/(a*c*x^4 + a^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a+cx^4\right)^2}\,dx=\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{\left(cx^4+a\right)^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/(c*x^4 + a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{(a+cx^4)^2}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{(c\,x^4+a)^2}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(a + c*x^4)^2,x)
```

output
```
int((d + e*x^2)^(3/2)/(a + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a+cx^4)^2}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c^2x^8+2ac\,x^4+a^2}dx\right)d+\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c^2x^8+2ac\,x^4+a^2}dx\right)e$$

input
```
int((e*x^2+d)^(3/2)/(c*x^4+a)^2,x)
```

output
```
int(sqrt(d + e*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*d + int((sqrt(d +
e*x**2)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*e
```

3.393 $\int \frac{\sqrt{d+ex^2}}{\left(a+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 395

$$\int \frac{\sqrt{d+ex^2}}{\left(a+cx^4\right)^2}\,dx = \frac{x\sqrt{d+ex^2}}{4a\left(a+cx^4\right)} - \frac{\left(\sqrt{a}e-3\sqrt{cd^2+ae^2}\right)\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{8\sqrt{2}a^{7/4}\sqrt{c}\sqrt{cd^2+ae^2}} - \frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(\sqrt{a}e+3\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{8\sqrt{2}a^{7/4}\sqrt{c}\sqrt{cd^2+ae^2}}$$

output
```
1/4*x*(e*x^2+d)^(1/2)/a/(c*x^4+a)-1/16*(a^(1/2)*e-3*(a*e^2+c*d^2)^(1/2))*(
a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/
2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e
^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(7/4)/c^(1/2)/(a*e^2+c*d^2)^(1/2)-1/1
6*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(a^(1/2)*e+3*(a*e^2+c*d^2)^(1/2))
*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*
(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)
/a^(7/4)/c^(1/2)/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^2}\,dx=\frac{x\sqrt{d+ex^2}}{4a(a+cx^4)}$$

$$+\frac{8e^{7/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\frac{\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#\right.}{cd^3-3cd^2\#1-8ae^2\#1+3cd\#1^2}\right.}{c}$$

$$+\frac{e^{3/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\frac{cd^2\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right.}{}\right.}{4}$$

input
```
Integrate[Sqrt[d + e*x^2]/(a + c*x^4)^2,x]
```

output
```
(x*Sqrt[d + e*x^2])/(4*a*(a + c*x^4)) + (8*e^(7/2)*RootSum[c*d^4 - 4*c*d^3
*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , Log[d + 2*e*x
^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 +
3*c*d*#1^2 - c*#1^3) & ])/c + (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^
2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (c*d^2*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 32*a*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x
*Sqrt[d + e*x^2] - #1] + 4*c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1]*#1 + c*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/
(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(4*a*c)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 1571}$$

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^2}dx$$

input
```
Int[Sqrt[d + e*x^2]/(a + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 838 vs. 2(311) = 622.

Time = 1.40 (sec) , antiderivative size = 839, normalized size of antiderivative = 2.12

method	result
pseudoelliptic	$\frac{\frac{3\left(\left(\left(-c x^{4}-a\right)\sqrt{a e^{2}+c d^{2}}-\frac{e\left(c x^{4}\sqrt{a}+a^{\frac{3}{2}}\right)}{3}\right)\sqrt{a\left(a e^{2}+c d^{2}\right)}+\left(a\left(c x^{4}+a\right)\sqrt{a e^{2}+c d^{2}}+\frac{e\left(c x^{4} a^{\frac{3}{2}}+a^{\frac{5}{2}}\right)}{3}\right)e\right)\sqrt{2\sqrt{a\left(a e^{2}+c d^{2}\right)}+}{32}}{}$
default	Expression too large to display

input
```
int((e*x^2+d)^(1/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/8/(a*e^2+c*d^2)^(1/2)/a^(5/2)/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2
+c*d^2))^(1/2)-2*a*e)^(1/2)*(1/4*(((-c*x^4-a)*(a*e^2+c*d^2)^(1/2)-1/3*e*(c
*x^4*a^(1/2)+a^(3/2)))*(a*(a*e^2+c*d^2))^(1/2)+(a*(c*x^4+a)*(a*e^2+c*d^2)^
(1/2)+1/3*e*(c*x^4*a^(3/2)+a^(5/2)))*e)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^
(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2
)*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^
(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)-1/4*(((-c*x^4-a)*(a*e^2+c*d^2)^(1/2)
-1/3*e*(c*x^4*a^(1/2)+a^(3/2)))*(a*(a*e^2+c*d^2))^(1/2)+(a*(c*x^4+a)*(a*e^
2+c*d^2)^(1/2)+1/3*e*(c*x^4*a^(3/2)+a^(5/2)))*e)*(2*(a*(a*e^2+c*d^2))^(1/2
)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*
a*e)^(1/2)*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2
)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+d*(2/3*a^(3/2)*x*(a*e^2+c*d
^2)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^
(1/2)*(e*x^2+d)^(1/2)+d*(a*(c*x^4+a)*(a*e^2+c*d^2)^(1/2)-1/3*e*(c*x^4*a^(3
/2)+a^(5/2)))*(arctan(((2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)
*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/
2)-2*a*e)^(1/2))-arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1
/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(
1/2)-2*a*e)^(1/2))))*c)/d/c/(c*x^4+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2397 vs. $2(313) = 626$.

Time = 3.66 (sec) , antiderivative size = 2397, normalized size of antiderivative = 6.07

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(1/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output

```
1/32*((a*c*x^4 + a^2)*sqrt(-(3*c*d^2*e + 4*a*e^3 + (a^3*c^2*d^2 + a^4*c*e^
2)*sqrt(-(81*c^2*d^6 + 144*a*c*d^4*e^2 + 64*a^2*d^2*e^4)/(a^7*c^3*d^4 + 2*
a^8*c^2*d^2*e^2 + a^9*c*e^4)))/(a^3*c^2*d^2 + a^4*c*e^2))*log((81*c^2*d^6
 + 108*a*c*d^4*e^2 + 32*a^2*d^2*e^4 + (9*a^3*c^3*d^5 + 13*a^4*c^2*d^3*e^2 +
 4*a^5*c*d*e^4)*x^2*sqrt(-(81*c^2*d^6 + 144*a*c*d^4*e^2 + 64*a^2*d^2*e^4)/
(a^7*c^3*d^4 + 2*a^8*c^2*d^2*e^2 + a^9*c*e^4)) + 2*(81*c^2*d^5*e + 108*a*c
*d^3*e^3 + 32*a^2*d*e^5)*x^2 + 2*sqrt(e*x^2 + d)*((3*a^5*c^3*d^4 + 5*a^6*c
^2*d^2*e^2 + 2*a^7*c*e^4)*x*sqrt(-(81*c^2*d^6 + 144*a*c*d^4*e^2 + 64*a^2*d
^2*e^4)/(a^7*c^3*d^4 + 2*a^8*c^2*d^2*e^2 + a^9*c*e^4)) + (9*a^2*c^2*d^4*e
 + 8*a^3*c*d^2*e^3)*x)*sqrt(-(3*c*d^2*e + 4*a*e^3 + (a^3*c^2*d^2 + a^4*c*e^
2)*sqrt(-(81*c^2*d^6 + 144*a*c*d^4*e^2 + 64*a^2*d^2*e^4)/(a^7*c^3*d^4 + 2*
a^8*c^2*d^2*e^2 + a^9*c*e^4)))/(a^3*c^2*d^2 + a^4*c*e^2)))/x^2) - (a*c*x^4
 + a^2)*sqrt(-(3*c*d^2*e + 4*a*e^3 + (a^3*c^2*d^2 + a^4*c*e^2)*sqrt(-(81*c
^2*d^6 + 144*a*c*d^4*e^2 + 64*a^2*d^2*e^4)/(a^7*c^3*d^4 + 2*a^8*c^2*d^2*e^
2 + a^9*c*e^4)))/(a^3*c^2*d^2 + a^4*c*e^2))*log((81*c^2*d^6 + 108*a*c*d^4*
e^2 + 32*a^2*d^2*e^4 + (9*a^3*c^3*d^5 + 13*a^4*c^2*d^3*e^2 + 4*a^5*c*d*e^4
)*x^2*sqrt(-(81*c^2*d^6 + 144*a*c*d^4*e^2 + 64*a^2*d^2*e^4)/(a^7*c^3*d^4 +
 2*a^8*c^2*d^2*e^2 + a^9*c*e^4)) + 2*(81*c^2*d^5*e + 108*a*c*d^3*e^3 + 32*
a^2*d*e^5)*x^2 - 2*sqrt(e*x^2 + d)*((3*a^5*c^3*d^4 + 5*a^6*c^2*d^2*e^2 + 2
*a^7*c*e^4)*x*sqrt(-(81*c^2*d^6 + 144*a*c*d^4*e^2 + 64*a^2*d^2*e^4)/(a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input

```
integrate((e*x**2+d)**(1/2)/(c*x**4+a)**2,x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^2}\,dx=\int \frac{\sqrt{ex^2+d}}{(cx^4+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/(c*x^4 + a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^2}\,dx=\int \frac{\sqrt{e\,x^2+d}}{(c\,x^4+a)^2}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a + c*x^4)^2,x)
```

output
```
int((d + e*x^2)^(1/2)/(a + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^2}\,dx = \int \frac{\sqrt{e\,x^2+d}}{c^2x^8+2ac\,x^4+a^2}dx$$

input

```
int((e*x^2+d)^(1/2)/(c*x^4+a)^2,x)
```

output

```
int(sqrt(d + e*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)
```

3.394 $\int \frac{1}{\sqrt{d+ex^2}\left(a+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 464

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+cx^4\right)^2}\,dx = \frac{cx(d-ex^2)\sqrt{d+ex^2}}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(cd^2e-\left(3cd^2+4ae^2\right)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{8\sqrt{2}a^{5/4}\sqrt{c}d\left(cd^2+ae^2\right)^{3/2}} + \frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(cd^2e-\left(3cd^2+4ae^2\right)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+e}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-cdx^2}\right.}{8\sqrt{2}a^{5/4}\sqrt{c}d\left(cd^2+ae^2\right)^{3/2}}$$

output

```
1/4*c*x*(-e*x^2+d)*(e*x^2+d)^(1/2)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/16*(a^(1/2)
*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(c*d^2*e-(4*a*e^2+3*c*d^2)*(e-(a*e^2+c*d^2)^
(1/2)/a^(1/2)))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1
/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*
x^2))*2^(1/2)/a^(5/4)/c^(1/2)/d/(a*e^2+c*d^2)^(3/2)+1/16*(-a^(1/2)*e+(a*e^
2+c*d^2)^(1/2))^(1/2)*(c*d^2*e-(4*a*e^2+3*c*d^2)*(e+(a*e^2+c*d^2)^(1/2)/a^
(1/2)))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(
1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*
2^(1/2)/a^(5/4)/c^(1/2)/d/(a*e^2+c*d^2)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx$$
$$= \frac{2cx(d-ex^2)\sqrt{d+ex^2}+e^{3/2}(a+cx^4)\,\mathrm{RootSum}\Big[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2\#1^2-4cd\#1^3+c\#}{}$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a + c*x^4)^2),x]
```

output
```
(2*c*x*(d - e*x^2)*Sqrt[d + e*x^2] + e^(3/2)*(a + c*x^4)*RootSum[c*d^4 - 4
*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (c*d^3*
Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 10*c*d^2*Log[d + 2*e
*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 16*a*e^2*Log[d + 2*e*x^2 - 2
*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sq
rt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 -
c*#1^3) & ])/(8*a*(c*d^2 + a*e^2)*(a + c*x^4))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow 1571$$
$$\int \frac{1}{(a+cx^4)^2\sqrt{d+ex^2}}dx$$

input
```
Int[1/(Sqrt[d + e*x^2]*(a + c*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. 2(380) = 760.

Time = 1.32 (sec) , antiderivative size = 1041, normalized size of antiderivative = 2.24

method	result	size
pseudoelliptic	Expression too large to display	1041
default	Expression too large to display	2506

input
```
int(1/(e*x^2+d)^(1/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-1/4*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)
^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*((-(a*e^2+3/4*c*d^2)*(c*x^4
+a)*(a*e^2+c*d^2)^(1/2)-1/2*(c*(2*e^2*x^4+d^2)*a^(3/2)+c^2*d^2*x^4*a^(1/2)
+2*e^2*a^(5/2))*e)*(a*(a*e^2+c*d^2))^(1/2)+((a*e^2+3/4*c*d^2)*a*(c*x^4+a)*
(a*e^2+c*d^2)^(1/2)+(1/2*c^2*d^2*x^4*a^(3/2)+1/2*c*(2*e^2*x^4+d^2)*a^(5/2)
+a^(7/2)*e^2)*e)*e)*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d
^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+1/4*(4*(a*e^2+c*d^
2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(2*(a*(a*e^2+c*d^2
))^(1/2)+2*a*e)^(1/2)*((-(a*e^2+3/4*c*d^2)*(c*x^4+a)*(a*e^2+c*d^2)^(1/2)-1
/2*(c*(2*e^2*x^4+d^2)*a^(3/2)+c^2*d^2*x^4*a^(1/2)+2*e^2*a^(5/2))*e)*(a*(a*
e^2+c*d^2))^(1/2)+((a*e^2+3/4*c*d^2)*a*(c*x^4+a)*(a*e^2+c*d^2)^(1/2)+(1/2*
c^2*d^2*x^4*a^(3/2)+1/2*c*(2*e^2*x^4+d^2)*a^(5/2)+a^(7/2)*e^2)*e)*e)*ln((a
^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x
+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+d^2*(-1/2*a^(3/2)*c*x*(e*x^2+d)^(1/2)*(a*e^
2+c*d^2)^(1/2)*(-e*x^2+d)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2
))^(1/2)-2*a*e)^(1/2)+(arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^
2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d
^2))^(1/2)-2*a*e)^(1/2))-arctan(((2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x
-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c
*d^2))^(1/2)-2*a*e)^(1/2)))*((a*e^2+3/4*c*d^2)*a*(c*x^4+a)*(a*e^2+c*d^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4777 vs. $2(383) = 766$.

Time = 18.12 (sec) , antiderivative size = 4777, normalized size of antiderivative = 10.30

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx = \int \frac{1}{(cx^4+a)^2\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + a)^2*sqrt(e*x^2 + d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx = \int \frac{1}{\left(c\,x^4+a\right)^2\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a + c*x^4)^2*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a + c*x^4)^2*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,a^2+2\sqrt{e\,x^2+d}\,ac\,x^4+\sqrt{e\,x^2+d}\,c^2x^8}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(c*x^4+a)^2,x)
```

output
```
int(1/(sqrt(d + e*x**2)*a**2 + 2*sqrt(d + e*x**2)*a*c*x**4 + sqrt(d + e*x*
*2)*c**2*x**8),x)
```

3.395 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 504

$$\int \frac{1}{(d+ex^2)^{3/2}(a+cx^4)^2}\,dx = -\frac{e^2(cd^2-2ae^2)\,x}{2ad\,(cd^2+ae^2)^2\sqrt{d+ex^2}} + \frac{cx(d-ex^2)}{4a\,(cd^2+ae^2)\sqrt{d+ex^2}\,(a+cx^4)} - \frac{3\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(2ae^3+(cd^2+3ae^2)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{8\sqrt{2}a^{5/4}\,(cd^2+ae^2)^{5/2}} - \frac{3\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(2ae^3+(cd^2+3ae^2)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+e}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-cdx^2}\right.}{8\sqrt{2}a^{5/4}\,(cd^2+ae^2)^{5/2}}$$

output
```
-1/2*e^2*(-2*a*e^2+c*d^2)*x/a/d/(a*e^2+c*d^2)^2/(e*x^2+d)^(1/2)+1/4*c*x*(-
e*x^2+d)/a/(a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(c*x^4+a)-3/16*c^(1/2)*(a^(1/2)*e
+(a*e^2+c*d^2)^(1/2))^(1/2)*(2*a*e^3+(3*a*e^2+c*d^2)*(e-(a*e^2+c*d^2)^(1/2
)/a^(1/2)))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))
^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2)
)*2^(1/2)/a^(5/4)/(a*e^2+c*d^2)^(5/2)-3/16*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^
2)^(1/2))^(1/2)*(2*a*e^3+(3*a*e^2+c*d^2)*(e+(a*e^2+c*d^2)^(1/2)/a^(1/2)))*
arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(
e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/
a^(5/4)/(a*e^2+c*d^2)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.89 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+cx^4\right)^2}\,dx=$$

$$-\frac{2e^{7/2}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\frac{17cd^2\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#\right.}{}\right.}{}$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(a + c*x^4)^2),x]
```

output

```
-1/4*(2*e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2
- 4*c*d*#1^3 + c*#1^4 & , (17*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d +
 e*x^2] - #1] + 16*a*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #
1] - 6*c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*Log[
d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1
- 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ] + ((x*(-4*a^2*e^4 + a*c*e^2*(d^2 +
 d*e*x^2 - 4*e^2*x^4) + c^2*d^2*(-d^2 + d*e*x^2 + 2*e^2*x^4)))/(d*Sqrt[d +
 e*x^2]*(a + c*x^4)) + e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 +
 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (31*a*c*d^2*e^2*Log[d + 2*e*x^2 -
 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 32*a^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e
]*x*Sqrt[d + e*x^2] - #1] + 6*c^2*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d
 + e*x^2] - #1]*#1 + 12*a*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e
*x^2] - #1]*#1 - a*c*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #
1]*#1^2)/(-(c*d^3) + 3*c*d^2*#1 + 8*a*e^2*#1 - 3*c*d*#1^2 + c*#1^3) & ])/a
)/(c*d^2 + a*e^2)^2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2 (d+ex^2)^{3/2}} \, dx$$
$$\Big\downarrow \; 1571$$
$$\int \frac{1}{(a+cx^4)^2 (d+ex^2)^{3/2}} dx$$

input

```
Int[1/((d + e*x^2)^(3/2)*(a + c*x^4)^2),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. 2(416) = 832.

Time = 1.95 (sec) , antiderivative size = 1116, normalized size of antiderivative = 2.21

method	result	size
pseudoelliptic	Expression too large to display	1116
default	Expression too large to display	5390

input
```
int(1/(e*x^2+d)^(3/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-9/8/(e*x^2+d)^(1/2)/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1
/2)-2*a*e)^(1/2)/a^(5/2)/(a*e^2+c*d^2)^(5/2)*(-1/4*(4*(a*e^2+c*d^2)^(1/2)*
a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(e*x^2+d)^(1/2)*(2*(a*(a*e^
2+c*d^2))^(1/2)+2*a*e)^(1/2)*((-(a*e^2+1/3*c*d^2)*(c*x^4+a)*(a*e^2+c*d^2)^
(1/2)-1/3*(c*(5*e^2*x^4+d^2)*a^(3/2)+c^2*d^2*x^4*a^(1/2)+5*e^2*a^(5/2))*e)
*(a*(a*e^2+c*d^2))^(1/2)+((a*e^2+1/3*c*d^2)*a*(c*x^4+a)*(a*e^2+c*d^2)^(1/2
)+5/3*e*(1/5*c*(5*e^2*x^4+d^2)*a^(5/2)+1/5*c^2*d^2*x^4*a^(3/2)+a^(7/2)*e^2
))*e)*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a
*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+1/4*(4*(a*e^2+c*d^2)^(1/2)*a^(1/
2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d
^2))^(1/2)+2*a*e)^(1/2)*((-(a*e^2+1/3*c*d^2)*(c*x^4+a)*(a*e^2+c*d^2)^(1/2)
-1/3*(c*(5*e^2*x^4+d^2)*a^(3/2)+c^2*d^2*x^4*a^(1/2)+5*e^2*a^(5/2))*e)*(a*(
a*e^2+c*d^2))^(1/2)+((a*e^2+1/3*c*d^2)*a*(c*x^4+a)*(a*e^2+c*d^2)^(1/2)+5/3
*e*(1/5*c*(5*e^2*x^4+d^2)*a^(5/2)+1/5*c^2*d^2*x^4*a^(3/2)+a^(7/2)*e^2))*e)
*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(
1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)-8/9*x*(a*e^2+c*d^2)^(1/2)*(1/4*c^2*d^
2*(e*x^2+d)*(-2*e*x^2+d)*a^(3/2)+(-1/4*c*(-4*e^2*x^4+d*e*x^2+d^2)*a^(5/2)+
a^(7/2)*e^2)*e^2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)
-2*a*e)^(1/2)+(arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/2
)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7841 vs. $2(419) = 838$.

Time = 170.30 (sec) , antiderivative size = 7841, normalized size of antiderivative = 15.56

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+cx^4)^2}\,dx = \int \frac{1}{(cx^4+a)^2(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + a)^2*(e*x^2 + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+cx^4\right)^2}\,dx=\int \frac{1}{\left(c\,x^4+a\right)^2\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int(1/((a + c*x^4)^2*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a + c*x^4)^2*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+cx^4\right)^2}\,dx=\int \frac{1}{\sqrt{e\,x^2+d}\,a^2d+\sqrt{e\,x^2+d}\,a^2e\,x^2+2\sqrt{e\,x^2+d}\,acd\,x^4+2\sqrt{e\,x^2+d}\,ace}$$

input
```
int(1/(e*x^2+d)^(3/2)/(c*x^4+a)^2,x)
```

output
```
int(1/(sqrt(d + e*x**2)*a**2*d + sqrt(d + e*x**2)*a**2*e*x**2 + 2*sqrt(d +
 e*x**2)*a*c*d*x**4 + 2*sqrt(d + e*x**2)*a*c*e*x**6 + sqrt(d + e*x**2)*c**
2*d*x**8 + sqrt(d + e*x**2)*c**2*e*x**10),x)
```

3.396 $\int \frac{1}{(d+ex^2)^{5/2}(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 630

$$\int \frac{1}{(d+ex^2)^{5/2}(a+cx^4)^2}\,dx = -\frac{e^2(3cd^2-2ae^2)\,x}{6ad\,(cd^2+ae^2)^2\,(d+ex^2)^{3/2}}$$
$$-\frac{e^2(9c^2d^4-59acd^2e^2-8a^2e^4)\,x}{12ad^2\,(cd^2+ae^2)^3\,\sqrt{d+ex^2}}+\frac{cx(d-ex^2)}{4a\,(cd^2+ae^2)\,(d+ex^2)^{3/2}\,(a+cx^4)}$$
$$-\frac{\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(cd^2e(cd^2+21ae^2)+(3c^2d^4+15acd^2e^2-8a^2e^4)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\operatorname{arctan}\left(\frac{\sqrt{2}}{}\right.}{8\sqrt{2}a^{5/4}d\,(cd^2+ae^2)^{7/2}}$$
$$-\frac{\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(cd^2e(cd^2+21ae^2)+(3c^2d^4+15acd^2e^2-8a^2e^4)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\operatorname{arctanh}\left(\right.}{8\sqrt{2}a^{5/4}d\,(cd^2+ae^2)^{7/2}}$$

output
```
-1/6*e^2*(-2*a*e^2+3*c*d^2)*x/a/d/(a*e^2+c*d^2)^2/(e*x^2+d)^(3/2)-1/12*e^2
*(-8*a^2*e^4-59*a*c*d^2*e^2+9*c^2*d^4)*x/a/d^2/(a*e^2+c*d^2)^3/(e*x^2+d)^(
1/2)+1/4*c*x*(-e*x^2+d)/a/(a*e^2+c*d^2)/(e*x^2+d)^(3/2)/(c*x^4+a)-1/16*c^(
1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(c*d^2*e*(21*a*e^2+c*d^2)+(-8*a
^2*e^4+15*a*c*d^2*e^2+3*c^2*d^4)*(e-(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctan(2
^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(
1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(5/4)/d/
(a*e^2+c*d^2)^(7/2)-1/16*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(c
*d^2*e*(21*a*e^2+c*d^2)+(-8*a^2*e^4+15*a*c*d^2*e^2+3*c^2*d^4)*(e+(a*e^2+c*
d^2)^(1/2)/a^(1/2)))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*
d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2
))-c*d*x^2))*2^(1/2)/a^(5/4)/d/(a*e^2+c*d^2)^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.36 (sec) , antiderivative size = 910, normalized size of antiderivative = 1.44

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+cx^4)^2}\,dx = \frac{\frac{2x\Big(3c^3d^4(d-3ex^2)\,(d+ex^2)^2+4a^3e^6\,(3d+2ex^2)+4a^2ce^4\,(15d^3+14d^2ex^2+3de^2x^4+2e^3x^6)+ac^2d^2e^2\,(}{ad^2(d+ex^2)^{3/2}(a+cx^4)}}{}$$

input
```
Integrate[1/((d + e*x^2)^(5/2)*(a + c*x^4)^2),x]
```

output
```
((2*x*(3*c^3*d^4*(d - 3*e*x^2)*(d + e*x^2)^2 + 4*a^3*e^6*(3*d + 2*e*x^2) +
 4*a^2*c*e^4*(15*d^3 + 14*d^2*e*x^2 + 3*d*e^2*x^4 + 2*e^3*x^6) + a*c^2*d^2
*e^2*(-9*d^3 - 15*d^2*e*x^2 + 57*d*e^2*x^4 + 59*e^3*x^6)))/(a*d^2*(d + e*x
^2)^(3/2)*(a + c*x^4)) - 48*c*e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2
*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (9*c*d^3*Log[d + 2*e*x^2 -
 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 8*a*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]
*x*Sqrt[d + e*x^2] - #1] - 5*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d +
e*x^2] - #1]*#1 + a*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1
]*#1 + c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^
3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ] - (3*c*e^(3/2)*Root
Sum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^
4 & , (c^2*d^5*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 123*a
*c*d^3*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 128*a^2*d
*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 14*c^2*d^4*Log[
d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 22*a*c*d^2*e^2*Log[d
+ 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 16*a^2*e^4*Log[d + 2*e*
x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c^2*d^3*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + 5*a*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt
[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d
*#1^2 - c*#1^3) & ])/a)/(24*(c*d^2 + a*e^2)^3)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2 (d+ex^2)^{5/2}} \, dx$$
$$\Big\downarrow \; 1571$$
$$\int \frac{1}{(a+cx^4)^2 (d+ex^2)^{5/2}} dx$$

input
```
Int[1/((d + e*x^2)^(5/2)*(a + c*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1381 vs. $2(538) = 1076$.

Time = 2.85 (sec) , antiderivative size = 1382, normalized size of antiderivative = 2.19

method	result	size
pseudoelliptic	Expression too large to display	1382
default	Expression too large to display	9378

input
```
int(1/(e*x^2+d)^(5/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)/a^
(5/2)/(a*e^2+c*d^2)^(7/2)/(e*x^2+d)^(3/2)*(-1/4*(2*(a*(a*e^2+c*d^2))^(1/2)
+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a
*e)^(1/2)*((-(c*x^4+a)*(a^2*e^4-15/8*a*c*d^2*e^2-3/8*c^2*d^4)*(a*e^2+c*d^2
)^(1/2)-((c*e^4*x^4-9/2*c*d^2*e^2)*a^(5/2)+a^(7/2)*e^4-1/2*d^2*((9*e^2*x^4
+d^2)*a^(3/2)+c*d^2*x^4*a^(1/2))*c^2)*e)*(a*(a*e^2+c*d^2))^(1/2)+(a*(c*x^4
+a)*(a^2*e^4-15/8*a*c*d^2*e^2-3/8*c^2*d^4)*(a*e^2+c*d^2)^(1/2)-9/2*(1/9*c^
2*d^2*(9*e^2*x^4+d^2)*a^(5/2)+c*e^2*(-2/9*e^2*x^4+d^2)*a^(7/2)+1/9*c^3*d^4
*x^4*a^(3/2)-2/9*a^(9/2)*e^4)*e)*e)*(e*x^2+d)^(3/2)*ln(((e*x^2+d)^(1/2)*(2
*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-(a*e^2+c*d^2)^(1/2)*x^2-a^(1/2)*(e
*x^2+d))/x^2)+1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)
^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*((-(c*x^4+a)*(a^2*e^
4-15/8*a*c*d^2*e^2-3/8*c^2*d^4)*(a*e^2+c*d^2)^(1/2)-((c*e^4*x^4-9/2*c*d^2*
e^2)*a^(5/2)+a^(7/2)*e^4-1/2*d^2*((9*e^2*x^4+d^2)*a^(3/2)+c*d^2*x^4*a^(1/2
))*c^2)*e)*(a*(a*e^2+c*d^2))^(1/2)+(a*(c*x^4+a)*(a^2*e^4-15/8*a*c*d^2*e^2-
3/8*c^2*d^4)*(a*e^2+c*d^2)^(1/2)-9/2*(1/9*c^2*d^2*(9*e^2*x^4+d^2)*a^(5/2)+
c*e^2*(-2/9*e^2*x^4+d^2)*a^(7/2)+1/9*c^3*d^4*x^4*a^(3/2)-2/9*a^(9/2)*e^4)*
e)*e)*(e*x^2+d)^(3/2)*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c
*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+5*x*(-3/20*d^2*c
^2*e^2*(-59/9*e^3*x^6-19/3*d*e^2*x^4+5/3*d^2*e*x^2+d^3)*a^(5/2)+c*e^4*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**(5/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+cx^4\right)^2}\,dx=\int \frac{1}{\left(cx^4+a\right)^2\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + a)^2*(e*x^2 + d)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+cx^4)^2}\,dx = \int \frac{1}{\left(c\,x^4+a\right)^2\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int(1/((a + c*x^4)^2*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a + c*x^4)^2*(d + e*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+cx^4)^2}\,dx = \int \frac{}{\sqrt{e\,x^2+d}\,a^2d^2+2\sqrt{e\,x^2+d}\,a^2de\,x^2+\sqrt{e\,x^2+d}\,a^2e^2x^4+2\sqrt{e\,x^2+d}\,a$$

input
```
int(1/(e*x^2+d)^(5/2)/(c*x^4+a)^2,x)
```

output
```
int(1/(sqrt(d + e*x**2)*a**2*d**2 + 2*sqrt(d + e*x**2)*a**2*d*e*x**2 + sqr
t(d + e*x**2)*a**2*e**2*x**4 + 2*sqrt(d + e*x**2)*a*c*d**2*x**4 + 4*sqrt(d
 + e*x**2)*a*c*d*e*x**6 + 2*sqrt(d + e*x**2)*a*c*e**2*x**8 + sqrt(d + e*x*
*2)*c**2*d**2*x**8 + 2*sqrt(d + e*x**2)*c**2*d*e*x**10 + sqrt(d + e*x**2)*
c**2*e**2*x**12),x)
```

3.397 $\int \frac{(d+ex^2)^{11/2}}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 748

$$\int \frac{(d+ex^2)^{11/2}}{(a+cx^4)^3}\,dx = -\frac{de^2(17cd^2+21ae^2)\,x\sqrt{d+ex^2}}{32a^2c^2} - \frac{e^2(5cd^2+4ae^2)\,x(d+ex^2)^{3/2}}{16a^2c^2}$$

$$-\frac{3de^2x(d+ex^2)^{5/2}}{32a^2c} + \frac{e^2x(d+ex^2)^{7/2}}{8a^2c} + \frac{x(d+ex^2)^{11/2}}{8a\,(a+cx^4)^2} + \frac{x(7d-4ex^2)\,(d+ex^2)^{9/2}}{32a^2\,(a+cx^4)}$$

$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\bigg(2cd^2e(19c^2d^4+32acd^2e^2+41a^2e^4)-(21c^3d^6+26ac^2d^4e^2+29a^2cd^2e^4-32a^3e^6)}{64\sqrt{2}a^{9/4}c^{7/2}d\sqrt{cd^2+ae^2}}$$

$$+\frac{e^{11/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3}$$

$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\bigg(2cd^2e(19c^2d^4+32acd^2e^2+41a^2e^4)-(21c^3d^6+26ac^2d^4e^2+29a^2cd^2e^4-32a^3e}{64\sqrt{2}a^{9/4}c^{7/2}d\sqrt{cd^2+ae^2}}$$

output
```
-1/32*d*e^2*(21*a*e^2+17*c*d^2)*x*(e*x^2+d)^(1/2)/a^2/c^2-1/16*e^2*(4*a*e^
2+5*c*d^2)*x*(e*x^2+d)^(3/2)/a^2/c^2-3/32*d*e^2*x*(e*x^2+d)^(5/2)/a^2/c+1/
8*e^2*x*(e*x^2+d)^(7/2)/a^2/c+1/8*x*(e*x^2+d)^(11/2)/a/(c*x^4+a)^2+1/32*x*
(-4*e*x^2+7*d)*(e*x^2+d)^(9/2)/a^2/(c*x^4+a)+1/128*(a^(1/2)*e+(a*e^2+c*d^2
)^(1/2))^(1/2)*(2*c*d^2*e*(41*a^2*e^4+32*a*c*d^2*e^2+19*c^2*d^4)-(-32*a^3*
e^6+29*a^2*c*d^2*e^4+26*a*c^2*d^4*e^2+21*c^3*d^6)*(e-(a*e^2+c*d^2)^(1/2)/a
^(1/2)))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1
/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2
^(1/2)/a^(9/4)/c^(7/2)/d/(a*e^2+c*d^2)^(1/2)+e^(11/2)*arctanh(e^(1/2)*x/(e
*x^2+d)^(1/2))/c^3+1/128*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(2*c*d^2*e
*(41*a^2*e^4+32*a*c*d^2*e^2+19*c^2*d^4)-(-32*a^3*e^6+29*a^2*c*d^2*e^4+26*a
*c^2*d^4*e^2+21*c^3*d^6)*(e+(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctanh(2^(1/2)*
a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(
a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(9/4)/c^(7/2)/
d/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.29 (sec) , antiderivative size = 2315, normalized size of antiderivative = 3.09

$$\int \frac{(d+ex^2)^{11/2}}{(a+cx^4)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + e*x^2)^(11/2)/(a + c*x^4)^3,x]
```

output
```
((c^3*x*Sqrt[d + e*x^2]*(-(a^3*e^4*(29*d + 8*e*x^2)) + c^3*d^4*x^4*(7*d +
24*e*x^2) - a^2*c*e^2*(26*d^3 + 4*d^2*e*x^2 + 49*d*e^2*x^4 + 12*e^3*x^6) +
 a*c^2*d^2*(11*d^3 + 44*d^2*e*x^2 + 14*d*e^2*x^4 + 36*e^3*x^6)))/(a^2*(a +
 c*x^4)^2) - 32*c^2*e^(11/2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]] + 32*c^2*
e^(11/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d
*#1^3 + c*#1^4 & , (163*c*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2
] - #1] - 96*a*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] +
 24*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 6*a*e^2
*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 3*c*d*Log[d + 2*
e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*
e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ] + (4*e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1
+ 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (5507*c^5*d^10*Lo
g[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 34770*a*c^4*d^8*e^2*Lo
g[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 166869*a^2*c^3*d^6*e^4
*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 158576*a^3*c^2*d^4*
e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 224000*a^4*c*d^2
*e^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 57344*a^5*e^10*
Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 3666*c^5*d^9*Log[d +
 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 38556*a*c^4*d^7*e^2*Log[
d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 11362*a^2*c^3*d^5*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{11/2}}{\left(a+cx^4\right)^3}dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{\left(d+ex^2\right)^{11/2}}{\left(a+cx^4\right)^3}dx$$

input
```
Int[(d + e*x^2)^(11/2)/(a + c*x^4)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1516 vs. $2(638) = 1276$.

Time = 2.52 (sec) , antiderivative size = 1517, normalized size of antiderivative = 2.03

method	result	size
pseudoelliptic	Expression too large to display	1517
default	Expression too large to display	69866

input

```
int((e*x^2+d)^(11/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/2*(1/4*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^
(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(c*x^4+a)^2*((-1/32*a^(7/2)*
(32*a^2*e^4+21*a*c*d^2*e^2+17*c^2*d^4)*e*(a*e^2+c*d^2)^(1/2)-a^3*(a^3*e^6-
29/32*a^2*c*d^2*e^4-13/16*a*c^2*d^4*e^2-21/32*c^3*d^6))*(a*(a*e^2+c*d^2))^
(1/2)+(1/32*a^(9/2)*(32*a^2*e^4+21*a*c*d^2*e^2+17*c^2*d^4)*e*(a*e^2+c*d^2)
^(1/2)+a^4*(a^3*e^6-29/32*a^2*c*d^2*e^4-13/16*a*c^2*d^4*e^2-21/32*c^3*d^6)
)*e)*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*
e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)-1/4*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2
)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)
^(1/2)*(c*x^4+a)^2*((-1/32*a^(7/2)*(32*a^2*e^4+21*a*c*d^2*e^2+17*c^2*d^4)*
e*(a*e^2+c*d^2)^(1/2)-a^3*(a^3*e^6-29/32*a^2*c*d^2*e^4-13/16*a*c^2*d^4*e^2
-21/32*c^3*d^6))*(a*(a*e^2+c*d^2))^(1/2)+(1/32*a^(9/2)*(32*a^2*e^4+21*a*c*
d^2*e^2+17*c^2*d^4)*e*(a*e^2+c*d^2)^(1/2)+a^4*(a^3*e^6-29/32*a^2*c*d^2*e^4
-13/16*a*c^2*d^4*e^2-21/32*c^3*d^6))*e)*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1
/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2
)+d^2*c*(-2*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e
)^(1/2)*(c*x^4+a)^2*a^(13/2)*e^(11/2)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))+1
/32*(c*x^4+a)^2*a^(9/2)*(32*a^2*e^4+21*a*c*d^2*e^2+17*c^2*d^4)*(arctan((2*
a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a
*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arcta...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{11/2}}{\left(a+cx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((e*x^2+d)^(11/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{11/2}}{\left(a+cx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((e*x**2+d)**(11/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(d+ex^2\right)^{11/2}}{\left(a+cx^4\right)^3}\,dx=\int \frac{\left(ex^2+d\right)^{\frac{11}{2}}}{\left(cx^4+a\right)^3}\,dx$$

input
```
integrate((e*x^2+d)^(11/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(11/2)/(c*x^4 + a)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1035, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex^2)^{11/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input

```
integrate((e*x^2+d)^(11/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output

```
-1/2*e^(11/2)*log((sqrt(e)*x - sqrt(e*x^2 + d))^2)/c^3 - 1/8*(19*(sqrt(e)*
x - sqrt(e*x^2 + d))^14*c^4*d^5*e^(3/2) + 32*(sqrt(e)*x - sqrt(e*x^2 + d))
^14*a*c^3*d^3*e^(7/2) - 55*(sqrt(e)*x - sqrt(e*x^2 + d))^14*a^2*c^2*d*e^(1
1/2) - 112*(sqrt(e)*x - sqrt(e*x^2 + d))^12*c^4*d^6*e^(3/2) - 198*(sqrt(e)
*x - sqrt(e*x^2 + d))^12*a*c^3*d^4*e^(7/2) - 66*(sqrt(e)*x - sqrt(e*x^2 +
d))^12*a^2*c^2*d^2*e^(11/2) + 64*(sqrt(e)*x - sqrt(e*x^2 + d))^12*a^3*c*e^
(15/2) + 287*(sqrt(e)*x - sqrt(e*x^2 + d))^10*c^4*d^7*e^(3/2) + 1040*(sqrt
(e)*x - sqrt(e*x^2 + d))^10*a*c^3*d^5*e^(7/2) + 397*(sqrt(e)*x - sqrt(e*x^
2 + d))^10*a^2*c^2*d^3*e^(11/2) - 784*(sqrt(e)*x - sqrt(e*x^2 + d))^10*a^3
*c*d*e^(15/2) - 420*(sqrt(e)*x - sqrt(e*x^2 + d))^8*c^4*d^8*e^(3/2) - 1782
*(sqrt(e)*x - sqrt(e*x^2 + d))^8*a*c^3*d^6*e^(7/2) - 3054*(sqrt(e)*x - sqr
t(e*x^2 + d))^8*a^2*c^2*d^4*e^(11/2) - 1728*(sqrt(e)*x - sqrt(e*x^2 + d))^
8*a^3*c*d^2*e^(15/2) + 768*(sqrt(e)*x - sqrt(e*x^2 + d))^8*a^4*e^(19/2) +
385*(sqrt(e)*x - sqrt(e*x^2 + d))^6*c^4*d^9*e^(3/2) + 1504*(sqrt(e)*x - sq
rt(e*x^2 + d))^6*a*c^3*d^7*e^(7/2) + 1571*(sqrt(e)*x - sqrt(e*x^2 + d))^6*
a^2*c^2*d^5*e^(11/2) + 16*(sqrt(e)*x - sqrt(e*x^2 + d))^6*a^3*c*d^3*e^(15/
2) - 224*(sqrt(e)*x - sqrt(e*x^2 + d))^4*c^4*d^10*e^(3/2) - 690*(sqrt(e)*x
 - sqrt(e*x^2 + d))^4*a*c^3*d^8*e^(7/2) - 342*(sqrt(e)*x - sqrt(e*x^2 + d)
)^4*a^2*c^2*d^6*e^(11/2) + 128*(sqrt(e)*x - sqrt(e*x^2 + d))^4*a^3*c*d^4*e
^(15/2) + 77*(sqrt(e)*x - sqrt(e*x^2 + d))^2*c^4*d^11*e^(3/2) + 112*(sq...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{11/2}}{(a+cx^4)^3}\,dx = \int \frac{{\left(e\,x^2+d\right)}^{11/2}}{{\left(c\,x^4+a\right)}^3}\,dx$$

input

```
int((d + e*x^2)^(11/2)/(a + c*x^4)^3,x)
```

output
```
int((d + e*x^2)^(11/2)/(a + c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{11/2}}{(a+cx^4)^3}\,dx=\int \frac{(e\,x^2+d)^{\frac{11}{2}}}{(c\,x^4+a)^3}dx$$

input
```
int((e*x^2+d)^(11/2)/(c*x^4+a)^3,x)
```

output
```
int((e*x^2+d)^(11/2)/(c*x^4+a)^3,x)
```

3.398 $\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 632

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^3}\,dx = -\frac{3e^2(2cd^2+ae^2)\,x\sqrt{d+ex^2}}{16a^2c^2} - \frac{5de^2x(d+ex^2)^{3/2}}{32a^2c}$$

$$+\frac{e^2x(d+ex^2)^{5/2}}{16a^2c} + \frac{x(d+ex^2)^{9/2}}{8a\,(a+cx^4)^2} + \frac{x(7d-2ex^2)\,(d+ex^2)^{7/2}}{32a^2\,(a+cx^4)}$$

$$+\frac{3\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(11c^2d^4e+11acd^2e^3+4a^2e^5-(7c^2d^4+5acd^2e^2+2a^2e^4)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\text{arctan}}{64\sqrt{2}a^{9/4}c^{5/2}\sqrt{cd^2+ae^2}}$$

$$+\frac{3\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(11c^2d^4e+11acd^2e^3+4a^2e^5-(7c^2d^4+5acd^2e^2+2a^2e^4)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\text{arcta}}{64\sqrt{2}a^{9/4}c^{5/2}\sqrt{cd^2+ae^2}}$$

output

```
-3/16*e^2*(a*e^2+2*c*d^2)*x*(e*x^2+d)^(1/2)/a^2/c^2-5/32*d*e^2*x*(e*x^2+d)
^(3/2)/a^2/c+1/16*e^2*x*(e*x^2+d)^(5/2)/a^2/c+1/8*x*(e*x^2+d)^(9/2)/a/(c*x
^4+a)^2+1/32*x*(-2*e*x^2+7*d)*(e*x^2+d)^(7/2)/a^2/(c*x^4+a)+3/128*(a^(1/2)
*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(11*c^2*d^4*e+11*a*c*d^2*e^3+4*a^2*e^5-(2*a^
2*e^4+5*a*c*d^2*e^2+7*c^2*d^4)*(e-(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctan(2^(
1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/
2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(9/4)/c^(5
/2)/(a*e^2+c*d^2)^(1/2)+3/128*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(11*c
^2*d^4*e+11*a*c*d^2*e^3+4*a^2*e^5-(2*a^2*e^4+5*a*c*d^2*e^2+7*c^2*d^4)*(e+(
a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(
a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d
^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(9/4)/c^(5/2)/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.09 (sec) , antiderivative size = 2216, normalized size of antiderivative = 3.51

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[(d + e*x^2)^(9/2)/(a + c*x^4)^3,x]
```

output
```
((2*c^3*x*Sqrt[d + e*x^2]*(-6*a^3*e^4 + c^3*d^3*x^4*(7*d + 19*e*x^2) - a^2
*c*e^2*(15*d^2 + d*e*x^2 + 10*e^2*x^4) + a*c^2*d*(11*d^3 + 35*d^2*e*x^2 +
9*d*e^2*x^4 + 15*e^3*x^6)))/(a^2*(a + c*x^4)^2) + 32*c^2*e^(11/2)*RootSum[
c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 &
, (161*c*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 32*a*e^
2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 18*c*d*Log[d + 2*e
*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*Log[d + 2*e*x^2 - 2*Sqrt[e
]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#
1^2 - c*#1^3) & ] + (32*e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2
+ 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (1239*c^5*d^10*Log[d + 2*e*x^2 -
 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 4394*a*c^4*d^8*e^2*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 32487*a^2*c^3*d^6*e^4*Log[d + 2*e*x^2
- 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 4216*a^3*c^2*d^4*e^6*Log[d + 2*e*x^2
 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 28544*a^4*c*d^2*e^8*Log[d + 2*e*x^2
 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 2048*a^5*e^10*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1] - 840*c^5*d^9*Log[d + 2*e*x^2 - 2*Sqrt[e]*x
*Sqrt[d + e*x^2] - #1]*#1 + 6324*a*c^4*d^7*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]
*x*Sqrt[d + e*x^2] - #1]*#1 + 2160*a^2*c^3*d^5*e^4*Log[d + 2*e*x^2 - 2*Sqr
t[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 7360*a^3*c^2*d^3*e^6*Log[d + 2*e*x^2 - 2
*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 512*a^4*c*d*e^8*Log[d + 2*e*x^2 -...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{9/2}}{\left(a+cx^4\right)^3} \, dx$$
$$\Big\downarrow \; 1571$$
$$\int \frac{\left(d+ex^2\right)^{9/2}}{\left(a+cx^4\right)^3}dx$$

input
```
Int[(d + e*x^2)^(9/2)/(a + c*x^4)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1243 vs. 2(532) = 1064.

Time = 1.94 (sec) , antiderivative size = 1244, normalized size of antiderivative = 1.97

method	result	size
pseudoelliptic	Expression too large to display	1244
default	Expression too large to display	53338

input
```
int((e*x^2+d)^(9/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/32*(d^2*c*(c*x^4+a)^2*(a^(9/2)*(a*e^2+2*c*d^2)*e*(a*e^2+c*d^2)^(1/2)+a^
4*(a^2*e^4+5/2*a*c*d^2*e^2+7/2*c^2*d^4))*arctan((2*a^(1/2)*(e*x^2+d)^(1/2)
+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/
2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))+1/4*(c*x^4+a)^2*((-a^(7/2)*(a*e
^2+2*c*d^2)*e*(a*e^2+c*d^2)^(1/2)+a^3*(a^2*e^4+5/2*a*c*d^2*e^2+7/2*c^2*d^4
))*(a*(a*e^2+c*d^2))^(1/2)+(a^(9/2)*(a*e^2+2*c*d^2)*e*(a*e^2+c*d^2)^(1/2)-
a^4*(a^2*e^4+5/2*a*c*d^2*e^2+7/2*c^2*d^4))*e)*(2*(a*(a*e^2+c*d^2))^(1/2)+2
*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e
)^(1/2)*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2
*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)-1/4*(c*x^4+a)^2*((-a^(7/2)*(a*
e^2+2*c*d^2)*e*(a*e^2+c*d^2)^(1/2)+a^3*(a^2*e^4+5/2*a*c*d^2*e^2+7/2*c^2*d^
4))*(a*(a*e^2+c*d^2))^(1/2)+(a^(9/2)*(a*e^2+2*c*d^2)*e*(a*e^2+c*d^2)^(1/2)
-a^4*(a^2*e^4+5/2*a*c*d^2*e^2+7/2*c^2*d^4))*e)*(2*(a*(a*e^2+c*d^2))^(1/2)+
2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*
e)^(1/2)*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+
2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+d*(-arctan(((2*(a*(a*e^2+c*d^
2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/
2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))*d*a^(9/2)*(c*x^4+a)^2*(
a*e^2+2*c*d^2)*e*(a*e^2+c*d^2)^(1/2)+1/3*(-15*a*c^2*d*e^3*x^6-19*c^3*d^3*e
*x^6+10*a^2*c*e^4*x^4-9*a*c^2*d^2*e^2*x^4-7*c^3*d^4*x^4+a^2*c*d*e^3*x^2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(9/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(9/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^3}\,dx = \int \frac{(ex^2+d)^{\frac{9}{2}}}{(cx^4+a)^3}\,dx$$

input
```
integrate((e*x^2+d)^(9/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(9/2)/(c*x^4 + a)^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^3}\,dx=\text{Timed out}$$

input
```
integrate((e*x^2+d)^(9/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^3}\,dx=\int \frac{{\left(e\,x^2+d\right)}^{9/2}}{{\left(c\,x^4+a\right)}^3}\,dx$$

input
```
int((d + e*x^2)^(9/2)/(a + c*x^4)^3,x)
```

output
```
int((d + e*x^2)^(9/2)/(a + c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a+cx^4)^3}\,dx=\int \frac{\left(e\,x^{2}+d\right)^{\frac{9}{2}}}{\left(c\,x^{4}+a\right)^{3}}d x$$

input
```
int((e*x^2+d)^(9/2)/(c*x^4+a)^3,x)
```

output
```
int((e*x^2+d)^(9/2)/(c*x^4+a)^3,x)
```

3.399 $\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 505

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^3} \, dx = -\frac{7de^2x\sqrt{d+ex^2}}{32a^2c} + \frac{x(d+ex^2)^{7/2}}{8a\,(a+cx^4)^2} + \frac{7dx(d+ex^2)^{5/2}}{32a^2\,(a+cx^4)}$$
$$+\frac{7d\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(4cd^2e+2ae^3-(3cd^2+ae^2)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}x}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cdx^2}\right.}{64\sqrt{2}a^{9/4}c^{3/2}\sqrt{cd^2+ae^2}}$$
$$+\frac{7d\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(4cd^2e+2ae^3-(3cd^2+ae^2)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-}\right.}{64\sqrt{2}a^{9/4}c^{3/2}\sqrt{cd^2+ae^2}}$$

output
```
-7/32*d*e^2*x*(e*x^2+d)^(1/2)/a^2/c+1/8*x*(e*x^2+d)^(7/2)/a/(c*x^4+a)^2+7/
32*d*x*(e*x^2+d)^(5/2)/a^2/(c*x^4+a)+7/128*d*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2
))^(1/2)*(4*c*d^2*e+2*a*e^3-(a*e^2+3*c*d^2)*(e-(a*e^2+c*d^2)^(1/2)/a^(1/2)
))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*
(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)
/a^(9/4)/c^(3/2)/(a*e^2+c*d^2)^(1/2)+7/128*d*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/
2))^(1/2)*(4*c*d^2*e+2*a*e^3-(a*e^2+3*c*d^2)*(e+(a*e^2+c*d^2)^(1/2)/a^(1/2
)))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)
*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1
/2)/a^(9/4)/c^(3/2)/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.42 (sec) , antiderivative size = 1814, normalized size of antiderivative = 3.59

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(d + e*x^2)^(7/2)/(a + c*x^4)^3,x]
```

output
```
((c^3*Sqrt[d + e*x^2]*(-7*a^2*d*e^2*x + 7*c^2*d^2*x^5*(d + 2*e*x^2) + a*c*
x*(11*d^3 + 26*d^2*e*x^2 + 5*d*e^2*x^4 + 4*e^3*x^6)))/(a^2*(a + c*x^4)^2)
+ 64*c^2*e^(11/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^
2 - 4*c*d*#1^3 + c*#1^4 & , (16*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e
*x^2] - #1] + Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1)/(c*d
^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ] + (8*e^(7/2)*RootS
um[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4
 & , (2205*c^4*d^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 2
525*a*c^3*d^6*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 43
992*a^2*c^2*d^4*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] -
18304*a^3*c*d^2*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] +
20480*a^4*e^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 1526*c
^4*d^7*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 7510*a*c^3
*d^5*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 6784*a^2
*c^2*d^3*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 5120
*a^3*c*d*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 413*
c^4*d^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 2013*a*
c^3*d^4*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 162
4*a^2*c^2*d^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2
 + 1280*a^3*c*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a+cx^4\right)^3}dx$$

$$\Big\downarrow \ 1571$$

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a+cx^4\right)^3}dx$$

input
```
Int[(d + e*x^2)^(7/2)/(a + c*x^4)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(413) = 826.

Time = 1.43 (sec) , antiderivative size = 993, normalized size of antiderivative = 1.97

method	result
pseudoelliptic	$-\ 7\ (-\ \sqrt{2\sqrt{a(a e^2+c d^2)}+2ae}\sqrt{4\sqrt{a e^2+c d^2}\sqrt{a}-2\sqrt{a(a e^2+c d^2)}-2ae}\ ((e(c^2x^8\sqrt{a}+2c x^4 a^{\frac{3}{2}}+a^{\frac{5}{2}})\sqrt{a e^2+c d^2}-(c x^4+a)^2(a e$
default	Expression too large to display

input
```
int((e*x^2+d)^(7/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-7/64*(-1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)
*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*((e*(c^2*x^8*a^(1/2)+2*c*x
^4*a^(3/2)+a^(5/2))*(a*e^2+c*d^2)^(1/2)-(c*x^4+a)^2*(a*e^2+3*c*d^2))*(a*(a
*e^2+c*d^2))^(1/2)+(-e*(c^2*x^8*a^(3/2)+2*c*x^4*a^(5/2)+a^(7/2))*(a*e^2+c*
d^2)^(1/2)+a*(c*x^4+a)^2*(a*e^2+3*c*d^2))*e)*ln((a^(1/2)*(e*x^2+d)-(e*x^2+
d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2
)/x^2)+1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*
a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*((e*(c^2*x^8*a^(1/2)+2*c*x^
4*a^(3/2)+a^(5/2))*(a*e^2+c*d^2)^(1/2)-(c*x^4+a)^2*(a*e^2+3*c*d^2))*(a*(a*
e^2+c*d^2))^(1/2)+(-e*(c^2*x^8*a^(3/2)+2*c*x^4*a^(5/2)+a^(7/2))*(a*e^2+c*d
^2)^(1/2)+a*(c*x^4+a)^2*(a*e^2+3*c*d^2))*e)*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d
)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)
/x^2)+(2*x*(e*x^2+d)^(1/2)*(-c^2*d^2*x^4*(2*e*x^2+d)*a^(3/2)-11/7*(4/11*e^
3*x^6+5/11*d*e^2*x^4+26/11*d^2*e*x^2+d^3)*c*a^(5/2)+a^(7/2)*d*e^2)*(4*(a*e
^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)+(e*(c^2*x^8
*a^(3/2)+2*c*x^4*a^(5/2)+a^(7/2))*(a*e^2+c*d^2)^(1/2)+a*(c*x^4+a)^2*(a*e^2
+3*c*d^2))*d^2*(arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/
2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1
/2)-2*a*e)^(1/2))-arctan(((2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1
/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2105 vs. $2(415) = 830$.

Time = 28.54 (sec) , antiderivative size = 2105, normalized size of antiderivative = 4.17

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(7/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/256*(7*(a^2*c^3*x^8 + 2*a^3*c^2*x^4 + a^4*c)*sqrt(-(15*c^2*d^6*e + 15*a*
c*d^4*e^3 + 4*a^2*d^2*e^5 + a^5*c^3*sqrt(-(81*c^2*d^14 + 90*a*c*d^12*e^2 +
 25*a^2*d^10*e^4)/(a^11*c^3)))/(a^5*c^3))*log(343*(81*c^3*d^13 + 162*a*c^2
*d^11*e^2 + 101*a^2*c*d^9*e^4 + 20*a^3*d^7*e^6 + (9*a^5*c^4*d^6 + 13*a^6*c
^3*d^4*e^2 + 4*a^7*c^2*d^2*e^4)*x^2*sqrt(-(81*c^2*d^14 + 90*a*c*d^12*e^2 +
 25*a^2*d^10*e^4)/(a^11*c^3)) + 2*(81*c^3*d^12*e + 162*a*c^2*d^10*e^3 + 10
1*a^2*c*d^8*e^5 + 20*a^3*d^6*e^7)*x^2 + 2*sqrt(e*x^2 + d)*((3*a^8*c^4*d^2
+ 2*a^9*c^3*e^2)*x*sqrt(-(81*c^2*d^14 + 90*a*c*d^12*e^2 + 25*a^2*d^10*e^4)
/(a^11*c^3)) - (9*a^3*c^3*d^8*e + 5*a^4*c^2*d^6*e^3)*x)*sqrt(-(15*c^2*d^6*
e + 15*a*c*d^4*e^3 + 4*a^2*d^2*e^5 + a^5*c^3*sqrt(-(81*c^2*d^14 + 90*a*c*d
^12*e^2 + 25*a^2*d^10*e^4)/(a^11*c^3)))/(a^5*c^3)))/x^2) - 7*(a^2*c^3*x^8
+ 2*a^3*c^2*x^4 + a^4*c)*sqrt(-(15*c^2*d^6*e + 15*a*c*d^4*e^3 + 4*a^2*d^2*
e^5 + a^5*c^3*sqrt(-(81*c^2*d^14 + 90*a*c*d^12*e^2 + 25*a^2*d^10*e^4)/(a^1
1*c^3)))/(a^5*c^3))*log(343*(81*c^3*d^13 + 162*a*c^2*d^11*e^2 + 101*a^2*c*
d^9*e^4 + 20*a^3*d^7*e^6 + (9*a^5*c^4*d^6 + 13*a^6*c^3*d^4*e^2 + 4*a^7*c^2
*d^2*e^4)*x^2*sqrt(-(81*c^2*d^14 + 90*a*c*d^12*e^2 + 25*a^2*d^10*e^4)/(a^1
1*c^3)) + 2*(81*c^3*d^12*e + 162*a*c^2*d^10*e^3 + 101*a^2*c*d^8*e^5 + 20*a
^3*d^6*e^7)*x^2 - 2*sqrt(e*x^2 + d)*((3*a^8*c^4*d^2 + 2*a^9*c^3*e^2)*x*sqr
t(-(81*c^2*d^14 + 90*a*c*d^12*e^2 + 25*a^2*d^10*e^4)/(a^11*c^3)) - (9*a^3*
c^3*d^8*e + 5*a^4*c^2*d^6*e^3)*x)*sqrt(-(15*c^2*d^6*e + 15*a*c*d^4*e^3 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(7/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^3}\,dx = \int \frac{(ex^2+d)^{\frac{7}{2}}}{(cx^4+a)^3}\,dx$$

input
```
integrate((e*x^2+d)^(7/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(7/2)/(c*x^4 + a)^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(7/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^3}\,dx = \int \frac{(e\,x^2+d)^{7/2}}{(c\,x^4+a)^3}\,dx$$

input
```
int((d + e*x^2)^(7/2)/(a + c*x^4)^3,x)
```

output
```
int((d + e*x^2)^(7/2)/(a + c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{7/2}}{(a+cx^4)^3}\,dx = \int \frac{(e\,x^2+d)^{\frac{7}{2}}}{(c\,x^4+a)^3}dx$$

input

```
int((e*x^2+d)^(7/2)/(c*x^4+a)^3,x)
```

output

```
int((e*x^2+d)^(7/2)/(c*x^4+a)^3,x)
```

3.400 $\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a+cx^4\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 512

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a+cx^4\right)^3}\,dx = -\frac{e^2x\sqrt{d+ex^2}}{16a^2c}+\frac{x\left(d+ex^2\right)^{5/2}}{8a\left(a+cx^4\right)^2}+\frac{x\left(d+ex^2\right)^{3/2}\left(7d+2ex^2\right)}{32a^2\left(a+cx^4\right)}$$
$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(2\sqrt{a}cd^2e+2a^{3/2}e^3+\sqrt{cd^2+ae^2}\left(21cd^2+2ae^2\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}x}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cd}\right.}{64\sqrt{2}a^{11/4}c^{3/2}\sqrt{cd^2+ae^2}}$$
$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(23cd^2e+4ae^3-\left(21cd^2+2ae^2\right)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+}}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)}\right.}{64\sqrt{2}a^{9/4}c^{3/2}\sqrt{cd^2+ae^2}}$$

output

```
-1/16*e^2*x*(e*x^2+d)^(1/2)/a^2/c+1/8*x*(e*x^2+d)^(5/2)/a/(c*x^4+a)^2+1/32
*x*(e*x^2+d)^(3/2)*(2*e*x^2+7*d)/a^2/(c*x^4+a)+1/128*(a^(1/2)*e+(a*e^2+c*d
^2)^(1/2))^(1/2)*(2*a^(1/2)*c*d^2*e+2*a^(3/2)*e^3+(a*e^2+c*d^2)^(1/2)*(2*a
*e^2+21*c*d^2))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1
/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*
x^2))*2^(1/2)/a^(11/4)/c^(3/2)/(a*e^2+c*d^2)^(1/2)+1/128*(-a^(1/2)*e+(a*e^
2+c*d^2)^(1/2))^(1/2)*(23*c*d^2*e+4*a*e^3-(2*a*e^2+21*c*d^2)*(e+(a*e^2+c*d
^2)^(1/2)/a^(1/2)))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d
^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2)
)-c*d*x^2))*2^(1/2)/a^(9/4)/c^(3/2)/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.27 (sec) , antiderivative size = 1763, normalized size of antiderivative = 3.44

$$\int \frac{(d+ex^2)^{5/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input

```
Integrate[(d + e*x^2)^(5/2)/(a + c*x^4)^3,x]
```

output

```
((2*c^3*Sqrt[d + e*x^2]*(-2*a^2*e^2*x + c^2*d*x^5*(7*d + 9*e*x^2) + a*c*x*
(11*d^2 + 17*d*e*x^2 + 2*e^2*x^4)))/(a^2*(a + c*x^4)^2) + 512*c^2*e^(11/2)
*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 +
c*#1^4 & , Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]/(c*d^3 - 3*
c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ] + (16*e^(7/2)*RootSum[c*d
^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (
1934*c^4*d^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 1709*a*
c^3*d^6*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 24784*a^
2*c^2*d^4*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 20224*
a^3*c*d^2*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 4096*a
^4*e^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 1370*c^4*d^7*
Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 3724*a*c^3*d^5*e^
2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 5504*a^2*c^2*d^
3*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 1024*a^3*c*
d*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 366*c^4*d^6
*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 1011*a*c^3*d^4
*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 1360*a^2*c
^2*d^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + 256*
a^3*c*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3
 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(a*c*d^6 + a^2*d...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{\left(d+ex^2\right)^{5/2}}{\left(a+cx^4\right)^3}dx \\ \Big\downarrow \text{1571} \\ \int \frac{\left(d+ex^2\right)^{5/2}}{\left(a+cx^4\right)^3}dx \end{array}$$

input
```
Int[(d + e*x^2)^(5/2)/(a + c*x^4)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 981 vs. 2(418) = 836.

Time = 1.41 (sec) , antiderivative size = 982, normalized size of antiderivative = 1.92

method	result
pseudoelliptic	$\sqrt{2\sqrt{a(ae^2+cd^2)}+2ae}\,\sqrt{4\sqrt{ae^2+cd^2}\,\sqrt{a}-2\sqrt{a(ae^2+cd^2)}-2ae}\,((e(c^2x^8\sqrt{a}+2cx^4a^{\frac{3}{2}}+a^{\frac{5}{2}})\sqrt{ae^2+cd^2}-(ae^2+\frac{21cd^2}{2})(cx^4+$
default	Expression too large to display

input
```
int((e*x^2+d)^(5/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32/a^(7/2)/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*
e)^(1/2)*(1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/
2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*((e*(c^2*x^8*a^(1/2)+2*c
*x^4*a^(3/2)+a^(5/2))*(a*e^2+c*d^2)^(1/2)-(a*e^2+21/2*c*d^2)*(c*x^4+a)^2)*
(a*(a*e^2+c*d^2))^(1/2)+(-e*(c^2*x^8*a^(3/2)+2*c*x^4*a^(5/2)+a^(7/2))*(a*e
^2+c*d^2)^(1/2)+(a*e^2+21/2*c*d^2)*a*(c*x^4+a)^2)*e)*ln(((e*x^2+d)^(1/2)*(
2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-(a*e^2+c*d^2)^(1/2)*x^2-a^(1/2)*(
e*x^2+d))/x^2)-1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2
)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*((e*(c^2*x^8*a^(1/2
)+2*c*x^4*a^(3/2)+a^(5/2))*(a*e^2+c*d^2)^(1/2)-(a*e^2+21/2*c*d^2)*(c*x^4+a
)^2)*(a*(a*e^2+c*d^2))^(1/2)+(-e*(c^2*x^8*a^(3/2)+2*c*x^4*a^(5/2)+a^(7/2))
*(a*e^2+c*d^2)^(1/2)+(a*e^2+21/2*c*d^2)*a*(c*x^4+a)^2)*e)*ln((a^(1/2)*(e*x
^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d
^2)^(1/2)*x^2)/x^2)+d*(-2*x*(e*x^2+d)^(1/2)*(-11/2*(2/11*e^2*x^4+17/11*d*e
*x^2+d^2)*c*a^(5/2)-7/2*(9/7*e*x^2+d)*x^4*d*c^2*a^(3/2)+a^(7/2)*e^2)*(4*(a
*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)+(e*(c^2*x
^8*a^(3/2)+2*c*x^4*a^(5/2)+a^(7/2))*(a*e^2+c*d^2)^(1/2)+(a*e^2+21/2*c*d^2)
*a*(c*x^4+a)^2)*d*(arctan(((2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(
1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))
^(1/2)-2*a*e)^(1/2))-arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2045 vs. $2(420) = 840$.

Time = 21.00 (sec) , antiderivative size = 2045, normalized size of antiderivative = 3.99

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(5/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output

```
1/256*((a^2*c^3*x^8 + 2*a^3*c^2*x^4 + a^4*c)*sqrt(-(a^5*c^3*sqrt(-(194481*
c^2*d^10 + 70560*a*c*d^8*e^2 + 6400*a^2*d^6*e^4)/(a^11*c^3)) + 525*c^2*d^4
*e + 180*a*c*d^2*e^3 + 16*a^2*e^5)/(a^5*c^3))*log((194481*c^3*d^10 + 11113
2*a*c^2*d^8*e^2 + 20816*a^2*c*d^6*e^4 + 1280*a^3*d^4*e^6 + (441*a^5*c^4*d^
5 + 172*a^6*c^3*d^3*e^2 + 16*a^7*c^2*d*e^4)*x^2*sqrt(-(194481*c^2*d^10 + 7
0560*a*c*d^8*e^2 + 6400*a^2*d^6*e^4)/(a^11*c^3)) + 2*(194481*c^3*d^9*e + 1
11132*a*c^2*d^7*e^3 + 20816*a^2*c*d^5*e^5 + 1280*a^3*d^3*e^7)*x^2 + 2*sqrt
(e*x^2 + d)*((21*a^8*c^4*d^2 + 4*a^9*c^3*e^2)*x*sqrt(-(194481*c^2*d^10 + 7
0560*a*c*d^8*e^2 + 6400*a^2*d^6*e^4)/(a^11*c^3)) - 2*(441*a^3*c^3*d^6*e +
80*a^4*c^2*d^4*e^3)*x)*sqrt(-(a^5*c^3*sqrt(-(194481*c^2*d^10 + 70560*a*c*d
^8*e^2 + 6400*a^2*d^6*e^4)/(a^11*c^3)) + 525*c^2*d^4*e + 180*a*c*d^2*e^3 +
 16*a^2*e^5)/(a^5*c^3)))/x^2) - (a^2*c^3*x^8 + 2*a^3*c^2*x^4 + a^4*c)*sqrt
(-(a^5*c^3*sqrt(-(194481*c^2*d^10 + 70560*a*c*d^8*e^2 + 6400*a^2*d^6*e^4)/
(a^11*c^3)) + 525*c^2*d^4*e + 180*a*c*d^2*e^3 + 16*a^2*e^5)/(a^5*c^3))*log
((194481*c^3*d^10 + 111132*a*c^2*d^8*e^2 + 20816*a^2*c*d^6*e^4 + 1280*a^3*
d^4*e^6 + (441*a^5*c^4*d^5 + 172*a^6*c^3*d^3*e^2 + 16*a^7*c^2*d*e^4)*x^2*s
qrt(-(194481*c^2*d^10 + 70560*a*c*d^8*e^2 + 6400*a^2*d^6*e^4)/(a^11*c^3))
+ 2*(194481*c^3*d^9*e + 111132*a*c^2*d^7*e^3 + 20816*a^2*c*d^5*e^5 + 1280*
a^3*d^3*e^7)*x^2 - 2*sqrt(e*x^2 + d)*((21*a^8*c^4*d^2 + 4*a^9*c^3*e^2)*x*s
qrt(-(194481*c^2*d^10 + 70560*a*c*d^8*e^2 + 6400*a^2*d^6*e^4)/(a^11*c^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input

```
integrate((e*x**2+d)**(5/2)/(c*x**4+a)**3,x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a+cx^4)^3}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}}{(cx^4+a)^3}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(5/2)/(c*x^4 + a)^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(5/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{(a+cx^4)^3}\,dx = \int \frac{(e\,x^2+d)^{5/2}}{(c\,x^4+a)^3}\,dx$$

input
```
int((d + e*x^2)^(5/2)/(a + c*x^4)^3,x)
```

output
```
int((d + e*x^2)^(5/2)/(a + c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a+cx^4)^3}\,dx = \int \frac{(e\,x^2+d)^{\frac{5}{2}}}{(c\,x^4+a)^3}dx$$

input

```
int((e*x^2+d)^(5/2)/(c*x^4+a)^3,x)
```

output

```
int((e*x^2+d)^(5/2)/(c*x^4+a)^3,x)
```

3.401 $\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a+cx^4\right)^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 435

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a+cx^4\right)^3} \, dx = \frac{x\left(d+ex^2\right)^{3/2}}{8a\left(a+cx^4\right)^2} + \frac{x\sqrt{d+ex^2}\left(7d+4ex^2\right)}{32a^2\left(a+cx^4\right)} - \frac{3d\left(\sqrt{a}e-7\sqrt{cd^2+ae^2}\right)\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{64\sqrt{2}a^{11/4}\sqrt{c}\sqrt{cd^2+ae^2}} - \frac{3d\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(\sqrt{a}e+7\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{64\sqrt{2}a^{11/4}\sqrt{c}\sqrt{cd^2+ae^2}}$$

output

```
1/8*x*(e*x^2+d)^(3/2)/a/(c*x^4+a)^2+1/32*x*(e*x^2+d)^(1/2)*(4*e*x^2+7*d)/a
^2/(c*x^4+a)-3/128*d*(a^(1/2)*e-7*(a*e^2+c*d^2)^(1/2))*(a^(1/2)*e+(a*e^2+c
*d^2)^(1/2))^(1/2)*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)
^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c
*d*x^2))*2^(1/2)/a^(11/4)/c^(1/2)/(a*e^2+c*d^2)^(1/2)-3/128*d*(-a^(1/2)*e+
(a*e^2+c*d^2)^(1/2))^(1/2)*(a^(1/2)*e+7*(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/
2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2
)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(11/4)/c^(1
/2)/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.59 (sec) , antiderivative size = 1310, normalized size of antiderivative = 3.01

$$\int \frac{(d+ex^2)^{3/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(d + e*x^2)^(3/2)/(a + c*x^4)^3,x]
```

output
```
((x*Sqrt[d + e*x^2]*(11*a*d + 8*a*e*x^2 + 7*c*d*x^4 + 4*c*e*x^6))/(a + c*x
^4)^2 + (4*a*e^(7/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*
#1^2 - 4*c*d*#1^3 + c*#1^4 & , (3329*c^3*d^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x
*Sqrt[d + e*x^2] - #1] + 8432*a*c^2*d^4*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*
Sqrt[d + e*x^2] - #1] - 19712*a^2*c*d^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*
Sqrt[d + e*x^2] - #1] - 24576*a^3*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d
 + e*x^2] - #1] - 2422*c^3*d^5*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1]*#1 + 2288*a*c^2*d^3*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*
x^2] - #1]*#1 + 6144*a^2*c*d*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*
x^2] - #1]*#1 + 641*c^3*d^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2]
- #1]*#1^2 - 656*a*c^2*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1]*#1^2 - 1536*a^2*c*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ]
)/(c^3*d^3*(c*d^2 + a*e^2)) + (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^
2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (9*c^4*d^8*Log[d + 2*e*x^
2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 13307*a*c^3*d^6*e^2*Log[d + 2*e*x^
2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 33728*a^2*c^2*d^4*e^4*Log[d + 2*e*
x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 78848*a^3*c*d^2*e^6*Log[d + 2*e*
x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 98304*a^4*e^8*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 24*c^4*d^7*Log[d + 2*e*x^2 - 2*Sqrt...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a+cx^4\right)^3}dx$$
$$\Big\downarrow \text{ 1571}$$
$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a+cx^4\right)^3}dx$$

input
```
Int[(d + e*x^2)^(3/2)/(a + c*x^4)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(347) = 694.

Time = 1.64 (sec) , antiderivative size = 925, normalized size of antiderivative = 2.13

method	result
pseudoelliptic	$-\frac{21\left(-\frac{\sqrt{4\sqrt{a e^{2}+c d^{2}}\,\sqrt{a}-2\sqrt{a\left(a e^{2}+c d^{2}\right)}-2ae}\,\left(\left(-\left(c x^{4}+a\right)^{2}\sqrt{a e^{2}+c d^{2}}-\frac{e\left(c^{2}x^{8}\sqrt{a}+2c x^{4}a^{\frac{3}{2}}+a^{\frac{5}{2}}\right)}{7}\right)\sqrt{a\left(a e^{2}+c d^{2}\right)}+\left(a\left(\right.\right.\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((e*x^2+d)^(3/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-21/64*(-1/4*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*
e)^(1/2)*((-(c*x^4+a)^2*(a*e^2+c*d^2)^(1/2)-1/7*e*(c^2*x^8*a^(1/2)+2*c*x^4
*a^(3/2)+a^(5/2)))*(a*(a*e^2+c*d^2))^(1/2)+(a*(c*x^4+a)^2*(a*e^2+c*d^2)^(1
/2)+1/7*e*(c^2*x^8*a^(3/2)+2*c*x^4*a^(5/2)+a^(7/2)))*e)*(2*(a*(a*e^2+c*d^2
))^(1/2)+2*a*e)^(1/2)*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c
*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+1/4*(4*(a*e^2+c*
d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*((-(c*x^4+a)^2*(
a*e^2+c*d^2)^(1/2)-1/7*e*(c^2*x^8*a^(1/2)+2*c*x^4*a^(3/2)+a^(5/2)))*(a*(a*
e^2+c*d^2))^(1/2)+(a*(c*x^4+a)^2*(a*e^2+c*d^2)^(1/2)+1/7*e*(c^2*x^8*a^(3/2
)+2*c*x^4*a^(5/2)+a^(7/2)))*e)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*ln(
(a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)
*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+(-2/3*x*(e*x^2+d)^(1/2)*(a*e^2+c*d^2)^(1/
2)*(x^4*(4/7*e*x^2+d)*c*a^(3/2)+11/7*(8/11*e*x^2+d)*a^(5/2))*(4*(a*e^2+c*d
^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)+(a*(c*x^4+a)^2*(a
*e^2+c*d^2)^(1/2)-1/7*e*(c^2*x^8*a^(3/2)+2*c*x^4*a^(5/2)+a^(7/2)))*d^2*(ar
ctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)
/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-
arctan(((2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2
))/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)
)))*c)/a^(7/2)/(a*e^2+c*d^2)^(1/2)/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2534 vs. $2(349) = 698$.

Time = 11.14 (sec) , antiderivative size = 2534, normalized size of antiderivative = 5.83

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(3/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output

```
1/256*(3*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*sqrt(-(35*c*d^4*e + 36*a*d^2*e^
3 + (a^5*c^2*d^2 + a^6*c*e^2)*sqrt(-(2401*c^2*d^10 + 4704*a*c*d^8*e^2 + 23
04*a^2*d^6*e^4)/(a^11*c^3*d^4 + 2*a^12*c^2*d^2*e^2 + a^13*c*e^4)))/(a^5*c^
2*d^2 + a^6*c*e^2))*log(27*(2401*c^2*d^9 + 4116*a*c*d^7*e^2 + 1728*a^2*d^5
*e^4 + (49*a^5*c^3*d^6 + 85*a^6*c^2*d^4*e^2 + 36*a^7*c*d^2*e^4)*x^2*sqrt(-
(2401*c^2*d^10 + 4704*a*c*d^8*e^2 + 2304*a^2*d^6*e^4)/(a^11*c^3*d^4 + 2*a^
12*c^2*d^2*e^2 + a^13*c*e^4)) + 2*(2401*c^2*d^8*e + 4116*a*c*d^6*e^3 + 172
8*a^2*d^4*e^5)*x^2 + 2*sqrt(e*x^2 + d)*((7*a^8*c^3*d^4 + 13*a^9*c^2*d^2*e^
2 + 6*a^10*c*e^4)*x*sqrt(-(2401*c^2*d^10 + 4704*a*c*d^8*e^2 + 2304*a^2*d^6
*e^4)/(a^11*c^3*d^4 + 2*a^12*c^2*d^2*e^2 + a^13*c*e^4)) + (49*a^3*c^2*d^6*
e + 48*a^4*c*d^4*e^3)*x)*sqrt(-(35*c*d^4*e + 36*a*d^2*e^3 + (a^5*c^2*d^2 +
 a^6*c*e^2)*sqrt(-(2401*c^2*d^10 + 4704*a*c*d^8*e^2 + 2304*a^2*d^6*e^4)/(a
^11*c^3*d^4 + 2*a^12*c^2*d^2*e^2 + a^13*c*e^4)))/(a^5*c^2*d^2 + a^6*c*e^2)
))/x^2) - 3*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*sqrt(-(35*c*d^4*e + 36*a*d^2
*e^3 + (a^5*c^2*d^2 + a^6*c*e^2)*sqrt(-(2401*c^2*d^10 + 4704*a*c*d^8*e^2 +
 2304*a^2*d^6*e^4)/(a^11*c^3*d^4 + 2*a^12*c^2*d^2*e^2 + a^13*c*e^4)))/(a^5
*c^2*d^2 + a^6*c*e^2))*log(27*(2401*c^2*d^9 + 4116*a*c*d^7*e^2 + 1728*a^2*
d^5*e^4 + (49*a^5*c^3*d^6 + 85*a^6*c^2*d^4*e^2 + 36*a^7*c*d^2*e^4)*x^2*sqr
t(-(2401*c^2*d^10 + 4704*a*c*d^8*e^2 + 2304*a^2*d^6*e^4)/(a^11*c^3*d^4 + 2
*a^12*c^2*d^2*e^2 + a^13*c*e^4)) + 2*(2401*c^2*d^8*e + 4116*a*c*d^6*e^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{(a+cx^4)^3}\,dx=\text{Timed out}$$

input

```
integrate((e*x**2+d)**(3/2)/(c*x**4+a)**3,x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a+cx^4\right)^3}\,dx=\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{\left(cx^4+a\right)^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/(c*x^4 + a)^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a+cx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a+cx^4\right)^3}\,dx=\int \frac{\left(e\,x^2+d\right)^{3/2}}{\left(c\,x^4+a\right)^3}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(a + c*x^4)^3,x)
```

output
```
int((d + e*x^2)^(3/2)/(a + c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a+cx^4)^3}\,dx=\int \frac{(e\,x^2+d)^{\frac{3}{2}}}{(c\,x^4+a)^3}dx$$

input
```
int((e*x^2+d)^(3/2)/(c*x^4+a)^3,x)
```

output
```
int((e*x^2+d)^(3/2)/(c*x^4+a)^3,x)
```

3.402 $\int \frac{\sqrt{d+ex^2}}{\left(a+cx^4\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 512

$$\int \frac{\sqrt{d+ex^2}}{\left(a+cx^4\right)^3}\,dx = \frac{x\sqrt{d+ex^2}}{8a\left(a+cx^4\right)^2}+\frac{x\sqrt{d+ex^2}\left(7cd^2+6ae^2-cdex^2\right)}{32a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}$$
$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(13cd^2e+12ae^3-\left(21cd^2+22ae^2\right)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2}}}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae}\right.}\right.}{64\sqrt{2}a^{9/4}\sqrt{c}\left(cd^2+ae^2\right)^{3/2}}$$
$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(13cd^2e+12ae^3-\left(21cd^2+22ae^2\right)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd}\right.}\right.}{64\sqrt{2}a^{9/4}\sqrt{c}\left(cd^2+ae^2\right)^{3/2}}$$

output
```
1/8*x*(e*x^2+d)^(1/2)/a/(c*x^4+a)^2+1/32*x*(e*x^2+d)^(1/2)*(-c*d*e*x^2+6*a
*e^2+7*c*d^2)/a^2/(a*e^2+c*d^2)/(c*x^4+a)+1/128*(a^(1/2)*e+(a*e^2+c*d^2)^(
1/2))^(1/2)*(13*c*d^2*e+12*a*e^3-(22*a*e^2+21*c*d^2)*(e-(a*e^2+c*d^2)^(1/2
)/a^(1/2)))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))
^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2)
)*2^(1/2)/a^(9/4)/c^(1/2)/(a*e^2+c*d^2)^(3/2)+1/128*(-a^(1/2)*e+(a*e^2+c*d
^2)^(1/2))^(1/2)*(13*c*d^2*e+12*a*e^3-(22*a*e^2+21*c*d^2)*(e+(a*e^2+c*d^2)
^(1/2)/a^(1/2)))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)
^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c
*d*x^2))*2^(1/2)/a^(9/4)/c^(1/2)/(a*e^2+c*d^2)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.56 (sec) , antiderivative size = 1323, normalized size of antiderivative = 2.58

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[d + e*x^2]/(a + c*x^4)^3,x]
```

output
```
((2*x*Sqrt[d + e*x^2]*(10*a^2*e^2 + c^2*d*x^4*(7*d - e*x^2) + a*c*(11*d^2
- d*e*x^2 + 6*e^2*x^4)))/(a + c*x^4)^2 + (64*a*e^(7/2)*RootSum[c*d^4 - 4*c
*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (349*c^3*
d^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 1324*a*c^2*d^4*e
^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 64*a^2*c*d^2*e^4*
Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 1024*a^3*e^6*Log[d +
 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 262*c^3*d^5*Log[d + 2*e*x^2
 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 96*a*c^2*d^3*e^2*Log[d + 2*e*x^2
 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 256*a^2*c*d*e^4*Log[d + 2*e*x^2
- 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 69*c^3*d^4*Log[d + 2*e*x^2 - 2*Sq
rt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + 20*a*c^2*d^2*e^2*Log[d + 2*e*x^2 - 2*
Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 64*a^2*c*e^4*Log[d + 2*e*x^2 - 2*Sq
rt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c
*d*#1^2 - c*#1^3) & ])/(c^3*d^4) + (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6
*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (13*c^4*d^8*Log[d +
2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 22324*a*c^3*d^6*e^2*Log[d +
2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 84736*a^2*c^2*d^4*e^4*Log[d
+ 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 4096*a^3*c*d^2*e^6*Log[d +
 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 65536*a^4*e^8*Log[d + 2*e*x
^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 58*c^4*d^7*Log[d + 2*e*x^2 - 2...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^3} \, dx$$
$$\Big\downarrow \; 1571$$
$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^3} dx$$

input
```
Int[Sqrt[d + e*x^2]/(a + c*x^4)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1209 vs. $2(424) = 848$.

Time = 2.10 (sec) , antiderivative size = 1210, normalized size of antiderivative = 2.36

method	result	size
pseudoelliptic	Expression too large to display	1210
default	Expression too large to display	11062

input
```
int((e*x^2+d)^(1/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
11/32/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2
)/a^(7/2)*(1/4*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*
a*e)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*((-(a*e^2+21/22*c*d^2)*
(c*x^4+a)^2*(a*e^2+c*d^2)^(1/2)-5/11*(c^2*(x^8*e^2+8/5*d^2*x^4)*a^(3/2)+2*
c*(e^2*x^4+2/5*d^2)*a^(5/2)+4/5*c^3*d^2*x^8*a^(1/2)+a^(7/2)*e^2)*e)*(a*(a*
e^2+c*d^2))^(1/2)+(a*(a*e^2+21/22*c*d^2)*(c*x^4+a)^2*(a*e^2+c*d^2)^(1/2)+4
/11*(2*x^4*(5/8*e^2*x^4+d^2)*c^2*a^(5/2)+c*(5/2*e^2*x^4+d^2)*a^(7/2)+c^3*d
^2*x^8*a^(3/2)+5/4*a^(9/2)*e^2)*e)*e)*ln(((e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d
^2))^(1/2)+2*a*e)^(1/2)*x-(a*e^2+c*d^2)^(1/2)*x^2-a^(1/2)*(e*x^2+d))/x^2)-
1/4*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*
(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*((-(a*e^2+21/22*c*d^2)*(c*x^4+a)^2
*(a*e^2+c*d^2)^(1/2)-5/11*(c^2*(x^8*e^2+8/5*d^2*x^4)*a^(3/2)+2*c*(e^2*x^4+
2/5*d^2)*a^(5/2)+4/5*c^3*d^2*x^8*a^(1/2)+a^(7/2)*e^2)*e)*(a*(a*e^2+c*d^2))
^(1/2)+(a*(a*e^2+21/22*c*d^2)*(c*x^4+a)^2*(a*e^2+c*d^2)^(1/2)+4/11*(2*x^4*
(5/8*e^2*x^4+d^2)*c^2*a^(5/2)+c*(5/2*e^2*x^4+d^2)*a^(7/2)+c^3*d^2*x^8*a^(3
/2)+5/4*a^(9/2)*e^2)*e)*e)*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*
e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+d*(10/11*(1
1/10*(6/11*e^2*x^4-1/11*d*e*x^2+d^2)*c*a^(5/2)+a^(7/2)*e^2+7/10*x^4*a^(3/2
)*d*(-1/7*e*x^2+d)*c^2)*x*(a*e^2+c*d^2)^(1/2)*(e*x^2+d)^(1/2)*(4*(a*e^2+c*
d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)+(arctan(((2*(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6035 vs. $2(425) = 850$.

Time = 107.95 (sec) , antiderivative size = 6035, normalized size of antiderivative = 11.79

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(1/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(1/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^3}\,dx = \int \frac{\sqrt{ex^2+d}}{(cx^4+a)^3}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/(c*x^4 + a)^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^3}\,dx=\int \frac{\sqrt{e\,x^2+d}}{{\left(c\,x^4+a\right)}^3}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a + c*x^4)^3,x)
```

output
```
int((d + e*x^2)^(1/2)/(a + c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}}{(a+cx^4)^3}\,dx=\int \frac{\sqrt{e\,x^2+d}}{\left(c\,x^4+a\right)^3}dx$$

input
```
int((e*x^2+d)^(1/2)/(c*x^4+a)^3,x)
```

output
```
int((e*x^2+d)^(1/2)/(c*x^4+a)^3,x)
```

3.403 $\int \frac{1}{\sqrt{d+ex^2}(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 596

$$\begin{aligned}
&\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)^3}\,dx \\
&= \frac{cx(d-ex^2)\sqrt{d+ex^2}}{8a\,(cd^2+ae^2)\,(a+cx^4)^2} + \frac{cx\sqrt{d+ex^2}(d(7cd^2+13ae^2)-6e(cd^2+2ae^2)\,x^2)}{32a^2\,(cd^2+ae^2)^2\,(a+cx^4)} \\
&\quad + \frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\Big(2cd^2e(4cd^2+7ae^2)-(21c^2d^4+47acd^2e^2+32a^2e^4)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\Big)\arctan\Big(}{64\sqrt{2}a^{9/4}\sqrt{c}d\,(cd^2+ae^2)^{5/2}} \\
&\quad + \frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\Big(2cd^2e(4cd^2+7ae^2)-(21c^2d^4+47acd^2e^2+32a^2e^4)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\Big)\operatorname{arctanh}}{64\sqrt{2}a^{9/4}\sqrt{c}d\,(cd^2+ae^2)^{5/2}}
\end{aligned}$$

output

```
1/8*c*x*(-e*x^2+d)*(e*x^2+d)^(1/2)/a/(a*e^2+c*d^2)/(c*x^4+a)^2+1/32*c*x*(e
*x^2+d)^(1/2)*(d*(13*a*e^2+7*c*d^2)-6*e*(2*a*e^2+c*d^2)*x^2)/a^2/(a*e^2+c*
d^2)^2/(c*x^4+a)+1/128*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(2*c*d^2*e*(7
*a*e^2+4*c*d^2)-(32*a^2*e^4+47*a*c*d^2*e^2+21*c^2*d^4)*(e-(a*e^2+c*d^2)^(1
/2)/a^(1/2)))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2
))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^
2))*2^(1/2)/a^(9/4)/c^(1/2)/d/(a*e^2+c*d^2)^(5/2)+1/128*(-a^(1/2)*e+(a*e^2
+c*d^2)^(1/2))^(1/2)*(2*c*d^2*e*(7*a*e^2+4*c*d^2)-(32*a^2*e^4+47*a*c*d^2*e
^2+21*c^2*d^4)*(e+(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctanh(2^(1/2)*a^(1/4)*c^
(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a
^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(9/4)/c^(1/2)/d/(a*e^2+c
*d^2)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.55 (sec) , antiderivative size = 1211, normalized size of antiderivative = 2.03

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)^3}\,dx = \text{Too large to display}$$

input

```
Integrate[1/(Sqrt[d + e*x^2]*(a + c*x^4)^3),x]
```

output

```
((c^3*x*Sqrt[d + e*x^2]*(a^2*e^2*(17*d - 16*e*x^2) + c^2*d^2*x^4*(7*d - 6*
e*x^2) + a*c*(11*d^3 - 10*d^2*e*x^2 + 13*d*e^2*x^4 - 12*e^3*x^6)))/(a + c*
x^4)^2 + (64*a*e^(7/2)*(c*d^2 + a*e^2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^
2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (141*c^2*d^4*Log[d + 2*e*
x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 656*a*c*d^2*e^2*Log[d + 2*e*x^2
- 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 512*a^2*e^4*Log[d + 2*e*x^2 - 2*Sqrt
[e]*x*Sqrt[d + e*x^2] - #1] - 110*c^2*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sq
rt[d + e*x^2] - #1]*#1 - 128*a*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[
d + e*x^2] - #1]*#1 + 29*c^2*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*
x^2] - #1]*#1^2 + 32*a*c*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2]
 - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/
d^3 + (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 -
 4*c*d*#1^3 + c*#1^4 & , (4*c^4*d^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d +
 e*x^2] - #1] - 9017*a*c^3*d^6*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d +
e*x^2] - #1] - 51008*a^2*c^2*d^4*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d
+ e*x^2] - #1] - 74752*a^3*c*d^2*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d
+ e*x^2] - #1] - 32768*a^4*e^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1] + 34*c^4*d^7*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*
#1 + 7120*a*c^3*d^5*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1
]*#1 + 15296*a^2*c^2*d^3*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+cx^4\right)^3 \sqrt{d+ex^2}} \, dx$$

$$\Big\downarrow \text{ 1571}$$

$$\int \frac{1}{\left(a+cx^4\right)^3 \sqrt{d+ex^2}}dx$$

input

```
Int[1/(Sqrt[d + e*x^2]*(a + c*x^4)^3),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1842 vs. 2(508) = 1016.

Time = 2.40 (sec) , antiderivative size = 1843, normalized size of antiderivative = 3.09

method	result	size
pseudoelliptic	Expression too large to display	1843
default	Expression too large to display	5410

input
```
int(1/(e*x^2+d)^(1/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/64/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2
)*(-8*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2
)*(c*x^4+a)^2*((-a^(17/2)*e^11-129/32*d^2*a^(15/2)*c*e^9-(a*e^2+c*d^2)^(3/
2)*a^7*e^8-13/2*d^4*a^(13/2)*c^2*e^7-111/32*d^2*(a*e^2+c*d^2)^(3/2)*a^6*c*
e^6-85/16*d^6*a^(11/2)*c^3*e^5-147/32*d^4*(a*e^2+c*d^2)^(3/2)*a^5*c^2*e^4-
9/4*d^8*a^(9/2)*c^4*e^3-89/32*d^6*(a*e^2+c*d^2)^(3/2)*a^4*c^3*e^2-13/32*d^
10*a^(7/2)*c^5*e-21/32*d^8*(a*e^2+c*d^2)^(3/2)*a^3*c^4)*(a*(a*e^2+c*d^2))^
(1/2)+(a^(19/2)*e^11+129/32*d^2*a^(17/2)*c*e^9+(a*e^2+c*d^2)^(3/2)*a^8*e^8
+13/2*d^4*a^(15/2)*c^2*e^7+111/32*d^2*(a*e^2+c*d^2)^(3/2)*a^7*c*e^6+85/16*
d^6*a^(13/2)*c^3*e^5+147/32*d^4*(a*e^2+c*d^2)^(3/2)*a^6*c^2*e^4+9/4*d^8*a^
(11/2)*c^4*e^3+89/32*d^6*(a*e^2+c*d^2)^(3/2)*a^5*c^3*e^2+13/32*d^10*a^(9/2
)*c^5*e+21/32*d^8*(a*e^2+c*d^2)^(3/2)*a^4*c^4)*e)*(2*(a*(a*e^2+c*d^2))^(1/
2)+2*a*e)^(1/2)*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))
^(1/2)+2*a*e)^(1/2)*x+(a*e^2+c*d^2)^(1/2)*x^2)/x^2)+8*(4*(a*e^2+c*d^2)^(1/
2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(c*x^4+a)^2*((-a^(17/2)*
e^11-129/32*d^2*a^(15/2)*c*e^9-(a*e^2+c*d^2)^(3/2)*a^7*e^8-13/2*d^4*a^(13/
2)*c^2*e^7-111/32*d^2*(a*e^2+c*d^2)^(3/2)*a^6*c*e^6-85/16*d^6*a^(11/2)*c^3
*e^5-147/32*d^4*(a*e^2+c*d^2)^(3/2)*a^5*c^2*e^4-9/4*d^8*a^(9/2)*c^4*e^3-89
/32*d^6*(a*e^2+c*d^2)^(3/2)*a^4*c^3*e^2-13/32*d^10*a^(7/2)*c^5*e-21/32*d^8
*(a*e^2+c*d^2)^(3/2)*a^3*c^4)*(a*(a*e^2+c*d^2))^(1/2)+(a^(19/2)*e^11+12...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+cx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+cx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+cx^4\right)^3}\,dx=\int \frac{1}{\left(cx^4+a\right)^3\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + a)^3*sqrt(e*x^2 + d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)^3}\,dx = \int \frac{1}{(c\,x^4+a)^3\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a + c*x^4)^3*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a + c*x^4)^3*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+cx^4)^3}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,(c\,x^4+a)^3}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(c*x^4+a)^3,x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(c*x^4+a)^3,x)
```

3.404 $\int \frac{1}{(d+ex^2)^{3/2}(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 668

$$\int \frac{1}{(d+ex^2)^{3/2}(a+cx^4)^3}\,dx = -\frac{e^2(11c^2d^4+39acd^2e^2-32a^2e^4)\,x}{32a^2d\,(cd^2+ae^2)^3\sqrt{d+ex^2}}$$

$$+\frac{cx(d-ex^2)}{8a\,(cd^2+ae^2)\sqrt{d+ex^2}\,(a+cx^4)^2}+\frac{cx(d(7cd^2+17ae^2)-2e(2cd^2+7ae^2)\,x^2)}{32a^2\,(cd^2+ae^2)^2\sqrt{d+ex^2}\,(a+cx^4)}$$

$$+\frac{3\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(e(cd^2-4ae^2)\,(cd^2+5ae^2)-(7c^2d^4+25acd^2e^2+38a^2e^4)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\mathrm{arc}}{64\sqrt{2}a^{9/4}\,(cd^2+ae^2)^{7/2}}$$

$$+\frac{3\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(e(cd^2-4ae^2)\,(cd^2+5ae^2)-(7c^2d^4+25acd^2e^2+38a^2e^4)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)a}{64\sqrt{2}a^{9/4}\,(cd^2+ae^2)^{7/2}}$$

output

```
-1/32*e^2*(-32*a^2*e^4+39*a*c*d^2*e^2+11*c^2*d^4)*x/a^2/d/(a*e^2+c*d^2)^3/
(e*x^2+d)^(1/2)+1/8*c*x*(-e*x^2+d)/a/(a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(c*x^4+
a)^2+1/32*c*x*(d*(17*a*e^2+7*c*d^2)-2*e*(7*a*e^2+2*c*d^2)*x^2)/a^2/(a*e^2+
c*d^2)^2/(e*x^2+d)^(1/2)/(c*x^4+a)+3/128*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^
(1/2))^(1/2)*(e*(-4*a*e^2+c*d^2)*(5*a*e^2+c*d^2)-(38*a^2*e^4+25*a*c*d^2*e^
2+7*c^2*d^4)*(e-(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctan(2^(1/2)*a^(1/4)*c^(1/
2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/
2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(9/4)/(a*e^2+c*d^2)^(7/2)+3/
128*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*(-4*a*e^2+c*d^2)*(5*
a*e^2+c*d^2)-(38*a^2*e^4+25*a*c*d^2*e^2+7*c^2*d^4)*(e+(a*e^2+c*d^2)^(1/2)/
a^(1/2)))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))
^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2)
)*2^(1/2)/a^(9/4)/(a*e^2+c*d^2)^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.42 (sec) , antiderivative size = 1993, normalized size of antiderivative = 2.98

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+cx^4\right)^3}\,dx = \text{Too large to display}$$

input

```
Integrate[1/((d + e*x^2)^(3/2)*(a + c*x^4)^3),x]
```

output

```
((2*x*(32*a^4*e^6 + c^4*d^4*x^4*(7*d^2 - 4*d*e*x^2 - 11*e^2*x^4) + 2*a^3*c
*e^4*(-9*d^2 - 9*d*e*x^2 + 32*e^2*x^4) + a*c^3*d^2*(11*d^4 - 8*d^3*e*x^2 +
 2*d^2*e^2*x^4 - 18*d*e^3*x^6 - 39*e^4*x^8) + a^2*c^2*e^2*(21*d^4 - 26*d^3
*e*x^2 - 61*d^2*e^2*x^4 - 14*d*e^3*x^6 + 32*e^4*x^8)))/(a^2*d*Sqrt[d + e*x
^2]*(a + c*x^4)^2) - 32*e^(11/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2
 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (17*c*d^2*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1] + 16*a*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sq
rt[d + e*x^2] - #1] - 6*c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2]
- #1]*#1 + c*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*
d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ] + (16*e^(7/2)*Roo
tSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1
^4 & , (860*c^4*d^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] +
6091*a*c^3*d^6*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 1
3680*a^2*c^2*d^4*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] +
 12544*a^3*c*d^2*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] +
 4096*a^4*e^8*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 706*c^
4*d^7*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 2364*a*c^3*
d^5*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 2688*a^2*
c^2*d^3*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 1024*
a^3*c*d*e^6*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 18...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^3 (d+ex^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{1}{(a+cx^4)^3 (d+ex^2)^{3/2}} dx$$

input

```
Int[1/((d + e*x^2)^(3/2)*(a + c*x^4)^3),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1597 vs. 2(576) = 1152.

Time = 2.96 (sec) , antiderivative size = 1598, normalized size of antiderivative = 2.39

method	result	size
pseudoelliptic	Expression too large to display	1598
default	Expression too large to display	11178

input
```
int(1/(e*x^2+d)^(3/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-57/32*(-1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(e*x^2+d)^(1/2)*((-(c
*x^4+a)^2*(a^2*e^4+25/38*a*c*d^2*e^2+7/38*c^2*d^4)*(a*e^2+c*d^2)^(1/2)-12/
19*((29/6*e^2*x^4+d^2)*c*e^2*a^(7/2)+1/4*c^2*(29/3*e^4*x^8+8*d^2*e^2*x^4+d
^4)*a^(5/2)+29/12*a^(9/2)*e^4+1/4*x^4*d^2*((4*e^2*x^4+2*d^2)*a^(3/2)+c*d^2
*x^4*a^(1/2))*c^3)*e)*(a*(a*e^2+c*d^2))^(1/2)+(a*(c*x^4+a)^2*(a^2*e^4+25/3
8*a*c*d^2*e^2+7/38*c^2*d^4)*(a*e^2+c*d^2)^(1/2)+3/19*(c^2*(29/3*e^4*x^8+8*
d^2*e^2*x^4+d^4)*a^(7/2)+2*c^3*d^2*x^4*(2*e^2*x^4+d^2)*a^(5/2)+4*(29/6*e^2
*x^4+d^2)*c*e^2*a^(9/2)+a^(3/2)*c^4*d^4*x^8+29/3*a^(11/2)*e^4)*e)*e)*(4*(a
*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*ln((a^(1/
2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+(a*
e^2+c*d^2)^(1/2)*x^2)/x^2)+1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(e*
x^2+d)^(1/2)*((-(c*x^4+a)^2*(a^2*e^4+25/38*a*c*d^2*e^2+7/38*c^2*d^4)*(a*e^
2+c*d^2)^(1/2)-12/19*((29/6*e^2*x^4+d^2)*c*e^2*a^(7/2)+1/4*c^2*(29/3*e^4*x
^8+8*d^2*e^2*x^4+d^4)*a^(5/2)+29/12*a^(9/2)*e^4+1/4*x^4*d^2*((4*e^2*x^4+2*
d^2)*a^(3/2)+c*d^2*x^4*a^(1/2))*c^3)*e)*(a*(a*e^2+c*d^2))^(1/2)+(a*(c*x^4+
a)^2*(a^2*e^4+25/38*a*c*d^2*e^2+7/38*c^2*d^4)*(a*e^2+c*d^2)^(1/2)+3/19*(c^
2*(29/3*e^4*x^8+8*d^2*e^2*x^4+d^4)*a^(7/2)+2*c^3*d^2*x^4*(2*e^2*x^4+d^2)*a
^(5/2)+4*(29/6*e^2*x^4+d^2)*c*e^2*a^(9/2)+a^(3/2)*c^4*d^4*x^8+29/3*a^(11/2
)*e^4)*e)*e)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*
e)^(1/2)*ln((a^(1/2)*(e*x^2+d)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}\left(a+cx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}\left(a+cx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\left(a+cx^4\right)^3}\,dx=\int \frac{1}{(cx^4+a)^3(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + a)^3*(e*x^2 + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}(a+cx^4)^3}\,dx = \int \frac{1}{(c\,x^4+a)^3\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(1/((a + c*x^4)^3*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a + c*x^4)^3*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(a+cx^4)^3}\,dx = \int \frac{1}{(e\,x^2+d)^{\frac{3}{2}}\,(c\,x^4+a)^3}dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(c*x^4+a)^3,x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(c*x^4+a)^3,x)
```

3.405 $\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 213

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx = -\frac{de^2x\sqrt{a-cx^4}}{c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{3a^{3/4}e(5cd^2+ae^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(5\sqrt{c}d(cd^2+ae^2)-3\sqrt{a}e(5cd^2+ae^2)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{7/4}\sqrt{a-cx^4}}$$

output
```
-d*e^2*x*(-c*x^4+a)^(1/2)/c-1/5*e^3*x^3*(-c*x^4+a)^(1/2)/c+3/5*a^(3/4)*e*(
a*e^2+5*c*d^2)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(7/4)/(-
c*x^4+a)^(1/2)+1/5*a^(1/4)*(5*c^(1/2)*d*(a*e^2+c*d^2)-3*a^(1/2)*e*(a*e^2+5
*c*d^2))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(7/4)/(-c*x^4+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx = \frac{5d(cd^2+ae^2)\,x\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+ex\Big(e(5d+ex^2)\,(-a+cx^4)+(5cd^2+ae^2}{5c\sqrt{a-cx^4}}$$

input
```
Integrate[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```

output
```
(5*d*(c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4
, (c*x^4)/a] + e*x*(e*(5*d + e*x^2)*(-a + c*x^4) + (5*c*d^2 + a*e^2)*x^2*S
qrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(5*c*Sqrt
[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1519, 25, 2427, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx$$

$$\Big\downarrow \text{1519}$$

$$-\frac{\int -\frac{15cde^2x^4+3e(5cd^2+ae^2)x^2+5cd^3}{\sqrt{a-cx^4}}dx}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{15cde^2x^4+3e\left(5cd^2+ae^2\right)x^2+5cd^3}{\sqrt{a-cx^4}}dx}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\Big\downarrow \ 2427$$

$$\frac{-\frac{\int -\frac{3c\left(3e\left(5cd^2+ae^2\right)x^2+5d\left(cd^2+ae^2\right)\right)}{\sqrt{a-cx^4}}dx}{3c}-5de^2x\sqrt{a-cx^4}}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3e\left(5cd^2+ae^2\right)x^2+5d\left(cd^2+ae^2\right)}{\sqrt{a-cx^4}}dx-5de^2x\sqrt{a-cx^4}}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\Big\downarrow \ 1513$$

$$\frac{\left(5d\left(ae^2+cd^2\right)-\frac{3\sqrt{a}e\left(ae^2+5cd^2\right)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{3\sqrt{a}e\left(ae^2+5cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}-5de^2x\sqrt{a-cx^4}}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(5d\left(ae^2+cd^2\right)-\frac{3\sqrt{a}e\left(ae^2+5cd^2\right)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{3e\left(ae^2+5cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}-5de^2x\sqrt{a-cx^4}}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\Big\downarrow \ 765$$

$$\frac{\frac{\sqrt{1-\frac{cx^4}{a}}\left(5d\left(ae^2+cd^2\right)-\frac{3\sqrt{a}e\left(ae^2+5cd^2\right)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+\frac{3e\left(ae^2+5cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}-5de^2x\sqrt{a-cx^4}}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\Big\downarrow \ 762$$

$$\frac{\frac{3e\left(ae^2+5cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5d\left(ae^2+cd^2\right)-\frac{3\sqrt{a}e\left(ae^2+5cd^2\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-5de^2x\sqrt{a-cx^4}}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\Big\downarrow \ 1390$$

$$\frac{\frac{3e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+5cd^2\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5d\left(ae^2+cd^2\right)-\frac{3\sqrt{a}e\left(ae^2+5cd^2\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-5de^2x\sqrt{a-}}{5c}$$

$$\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{\frac{3\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+5cd^2\right)\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5d\left(ae^2+cd^2\right)-\frac{3\sqrt{a}e\left(ae^2+5cd^2\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-5de^2x\sqrt{a}}{5c}$$

$$\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+5cd^2\right)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5d\left(ae^2+cd^2\right)-\frac{3\sqrt{a}e\left(ae^2+5cd^2\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-}{5c}$$

$$\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

input
```
Int[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```

output
```
-1/5*(e^3*x^3*Sqrt[a - c*x^4])/c + (-5*d*e^2*x*Sqrt[a - c*x^4] + (3*a^(3/4
)*e*(5*c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(
1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(5*d*(c*d^2 + a*e^2) - (3
*Sqrt[a]*e*(5*c*d^2 + a*e^2))/Sqrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4]))/(5*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.04

method	result
elliptic	$-\frac{e^{3} x^{3} \sqrt{-c\,x^{4}+a}}{5 c}-\frac{d\,e^{2} x \sqrt{-c\,x^{4}+a}}{c}+\frac{\left(d^{3}+\frac{a d\,e^{2}}{c}\right) \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}}-\frac{\left(3 d^{2} e+\frac{3 a\,e^{3}}{5 c}\right) \sqrt{a}\, \sqrt{1-}}{}$
risch	$-\frac{x\,e^{2} \left(e\,x^{2}+5 d\right) \sqrt{-c\,x^{4}+a}}{5 c}+\frac{-\frac{3 e \left(a\,e^{2}+5 c\,d^{2}\right) \sqrt{a}\, \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}\, \sqrt{c}}+\frac{5 d^{3} c \sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{a}}}}{}}{5 c}$
default	$\frac{d^{3} \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}}+e^{3} \left(-\frac{x^{3} \sqrt{-c\,x^{4}+a}}{5 c}-\frac{3 a^{\frac{3}{2}} \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)-\right.}{5 c^{\frac{3}{2}} \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}}\right.$

input
```
int((e*x^2+d)^3/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/5*e^3*x^3*(-c*x^4+a)^(1/2)/c-d*e^2*x*(-c*x^4+a)^(1/2)/c+(d^3+a/c*d*e^2)
/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)
-(3*d^2*e+3/5*a/c*e^3)*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(E
llipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1
/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a-cx^4}}\,dx = -\frac{3\left(5\,acd^2e+a^2e^3\right)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-\left(5\,c^2d^3+15\,acd^2e+5\,acde^2+3\,a^2e^3\right)\sqrt{-c}x\left(\right.}{5\,ac^2x}$$

input

```
integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output

```
-1/5*(3*(5*a*c*d^2*e + a^2*e^3)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_e(arcsin((
a/c)^(1/4)/x), -1) - (5*c^2*d^3 + 15*a*c*d^2*e + 5*a*c*d*e^2 + 3*a^2*e^3)*
sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + (a*c*e^3*x^
4 + 5*a*c*d*e^2*x^2 + 15*a*c*d^2*e + 3*a^2*e^3)*sqrt(-c*x^4 + a))/(a*c^2*x
)
```

Sympy [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.85

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a-cx^4}}\,dx = \frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{3d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}+\frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
```

output
```
d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4
*sqrt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,),
c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5
/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma
(9/4)) + e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(
2*I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a-cx^4}}\,dx=\int \frac{\left(e\,x^2+d\right)^3}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int((d + e*x^2)^3/(a - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^3/(a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a-cx^4}}\,dx$$
$$=\frac{-5\sqrt{-c\,x^4+a}\,d\,e^2x-\sqrt{-c\,x^4+a}\,e^3x^3+5\left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)a d\,e^2+5\left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)c\,d^3+3\left(\int \frac{\sqrt{-c\,x^4+}}{-c\,x^4+}\right.}{5c}$$

input
```
int((e*x^2+d)^3/(-c*x^4+a)^(1/2),x)
```

output
```
( - 5*sqrt(a - c*x**4)*d*e**2*x - sqrt(a - c*x**4)*e**3*x**3 + 5*int(sqrt(
a - c*x**4)/(a - c*x**4),x)*a*d*e**2 + 5*int(sqrt(a - c*x**4)/(a - c*x**4)
,x)*c*d**3 + 3*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a*e**3 + 15*int
((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*c*d**2*e)/(5*c)
```

3.406 $\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 162

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{3c}+\frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(3cd^2-6\sqrt{a}\sqrt{c}de+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}\sqrt{a-cx^4}}$$

output
```
-1/3*e^2*x*(-c*x^4+a)^(1/2)/c+2*a^(3/4)*d*e*(1-c*x^4/a)^(1/2)*EllipticE(c^
(1/4)*x/a^(1/4),I)/c^(3/4)/(-c*x^4+a)^(1/2)+1/3*a^(1/4)*(3*c*d^2-6*a^(1/2)
*c^(1/2)*d*e+a*e^2)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(5/
4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx$$
$$= \frac{(3cd^2+ae^2)\,x\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+ex\left(-ae+cex^4+2cdx^2\sqrt{1-\frac{cx^4}{a}}\,\text{Hyperge}\right.}{3c\sqrt{a-cx^4}}$$

input

```
Integrate[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```

output

```
((3*c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4,
(c*x^4)/a] + e*x*(-(a*e) + c*e*x^4 + 2*c*d*x^2*Sqrt[1 - (c*x^4)/a]*Hyperge
ometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*c*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1519, 25, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx$$
$$\Big\downarrow \text{ 1519}$$
$$-\frac{\int -\frac{3cd^2+6cex^2d+ae^2}{\sqrt{a-cx^4}}dx}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$
$$\Big\downarrow \text{ 25}$$
$$\frac{\int \frac{3cd^2+6cex^2d+ae^2}{\sqrt{a-cx^4}}dx}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \ 1513$$

$$\frac{\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{1}{\sqrt{a-cx^4}}dx+6\sqrt{a}\sqrt{c}de\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{1}{\sqrt{a-cx^4}}dx+6\sqrt{c}de\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \ 765$$

$$\frac{\frac{\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+6\sqrt{c}de\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \ 762$$

$$\frac{6\sqrt{c}de\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \ 1390$$

$$\frac{\frac{6\sqrt{c}de\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \ 1389$$

$$\frac{\frac{6\sqrt{a}\sqrt{c}de\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \ 327$$

$$\frac{\frac{6a^{3/4}\sqrt[4]{c}de\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

input

```
Int[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```

output

```
-1/3*(e^2*x*Sqrt[a - c*x^4])/c + ((6*a^(3/4)*c^(1/4)*d*e*Sqrt[1 - (c*x^4)/
a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - c*x^4] + (a^(1/4)*
(3*c*d^2 - 6*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[Ar
cSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4]))/(3*c)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.15

method	result
elliptic	$-\frac{e^{2} x \sqrt{-c\,x^{4}+a}}{3 c}+\frac{\left(d^{2}+\frac{a\,e^{2}}{3 c}\right) \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{2 d e \sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{}\right.\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}\,\sqrt{}}$
default	$\frac{d^{2}\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+e^{2}\left(-\frac{x\sqrt{-c\,x^{4}+a}}{3 c}+\frac{a\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3 c\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}\right)-$
risch	$-\frac{e^{2} x \sqrt{-c\,x^{4}+a}}{3 c}+\frac{\frac{a\,e^{2}\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+\frac{3 c\,d^{2}\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{6 e\sqrt{c}\,d\sqrt{a}\,\sqrt{1-}}{}}{3 c}$

input
```
int((e*x^2+d)^2/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*e^2*x*(-c*x^4+a)^(1/2)/c+(d^2+1/3*a*e^2/c)/(1/a^(1/2)*c^(1/2))^(1/2)*
(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)
^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-2*d*e*a^(1/2)/(1/a^(1/2)*c
^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-Ell
ipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = -\frac{6\,a\sqrt{-c}dex\left(\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-(3\,cd^2+6\,ade+ae^2)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+}{3\,acx}$$

input
```
integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(6*a*sqrt(-c)*d*e*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1)
 - (3*c*d^2 + 6*a*d*e + a*e^2)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a
/c)^(1/4)/x), -1) + (a*e^2*x^2 + 6*a*d*e)*sqrt(-c*x^4 + a))/(a*c*x)
```

Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)}+\frac{e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

output
```
d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4
*sqrt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**
4*exp_polar(2*I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper
((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx=\int \frac{(ex^2+d)^2}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx=\int \frac{(ex^2+d)^2}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = \int \frac{(e\,x^2+d)^2}{\sqrt{a-c\,x^4}}\,dx$$

input

```
int((d + e*x^2)^2/(a - c*x^4)^(1/2),x)
```

output

```
int((d + e*x^2)^2/(a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = \frac{-\sqrt{-c\,x^4+a}\,e^2x+\left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)a\,e^2+3\left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)c\,d^2+6\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-c\,x^4+a}dx\right)cde}{3c}$$

input

```
int((e*x^2+d)^2/(-c*x^4+a)^(1/2),x)
```

output

```
( - sqrt(a - c*x**4)*e**2*x + int(sqrt(a - c*x**4)/(a - c*x**4),x)*a*e**2
+ 3*int(sqrt(a - c*x**4)/(a - c*x**4),x)*c*d**2 + 6*int((sqrt(a - c*x**4)*
x**2)/(a - c*x**4),x)*c*d*e)/(3*c)
```

3.407 $\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 123

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\ \text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

output
```
a^(3/4)*e*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(3/4)/(-c*x^4
+a)^(1/2)+a^(1/4)*(d-a^(1/2)*e/c^(1/2))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4
)*x/a^(1/4),I)/c^(1/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.63

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{a-cx^4}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

output
```
(Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] +
e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}dx$$

$$\Big\downarrow \text{1513}$$

$$\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{762}$$

$$\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

$$\Big\downarrow\ 1389$$

$$\frac{\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

$$\Big\downarrow\ 327$$

$$\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

input

```
Int[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

output

```
(a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(d - (Sqrt[a]*e)/Sqrt[c])*Sqrt[1 - (
c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*
x^4])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.25

method	result
default	$\frac{d\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}\,\sqrt{c}}$
elliptic	$\frac{d\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}\,\sqrt{c}}$

input

```
int((e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I
)-e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1
/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/
2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.74

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = -\frac{a\sqrt{-c}ex\left(\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-(cd+ae)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+\sqrt{-cx^4+a}ae}{acx}$$

input

```
integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output

```
-(a*sqrt(-c)*e*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - (c*d
+ a*e)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + sqrt
(-c*x^4 + a)*a*e)/(a*c*x)
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input

```
integrate((e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp
_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int((d + e*x^2)/(a - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx=\left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)d+\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-c\,x^4+a}dx\right)e$$

input
```
int((e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a - c*x**4),x)*d + int((sqrt(a - c*x**4)*x**2)/(a -
c*x**4),x)*e
```

3.408 $\int \frac{1}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 53

$$\int \frac{1}{\sqrt{a-cx^4}}\,dx = \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

output
```
a^(1/4)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(1/4)/(-c*x^4+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{a-cx^4}}\,dx = -\frac{i\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{a-cx^4}}$$

input
```
Integrate[1/Sqrt[a - c*x^4],x]
```

output
```
((-I)*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x],
 -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^4}}dx$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

input
```
Int[1/Sqrt[a - c*x^4],x]
```

output
```
(a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(
c^(1/4)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

method	result	size
default	$\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\ \operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}$	64
elliptic	$\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\ \operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}$	64

input
```
int(1/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{a-cx^4}}\,dx = \frac{\sqrt{a}\left(\frac{c}{a}\right)^{\frac{3}{4}} F\left(\arcsin\left(x\left(\frac{c}{a}\right)^{\frac{1}{4}}\right) \middle| -1\right)}{c}$$

input
```
integrate(1/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a)*(c/a)^(3/4)*elliptic_f(arcsin(x*(c/a)^(1/4)), -1)/c
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a-cx^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-c*x**4+a)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt
(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate(1/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate(1/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-c*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 17.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{a-cx^4}}\,dx=\frac{x\,\sqrt{1-\frac{c\,x^4}{a}}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ \frac{c\,x^4}{a}\right)}{\sqrt{a-c\,x^4}}$$

input
```
int(1/(a - c*x^4)^(1/2),x)
```

output
```
(x*(1 - (c*x^4)/a)^(1/2)*hypergeom([1/4, 1/2], 5/4, (c*x^4)/a))/(a - c*x^4
)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a-cx^4}}\,dx=\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx$$

input
```
int(1/(-c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a - c*x**4),x)
```

3.409 $\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}$$

output

```
a^(1/4)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/
d,I)/c^(1/4)/d/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = -\frac{i\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},i\,\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}d\sqrt{a-cx^4}}$$

input

```
Integrate[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh
[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[a - c*
x^4])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)}dx$$

$$\Big\downarrow \text{ 1543}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{(ex^2+d)\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 1542}$$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}$$

input
```
Int[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin
[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}$	97
elliptic	$\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}$	97

input
```
int(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2
),-a^(1/2)*e/c^(1/2)/d,(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2
))
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 + a)/(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx=\int \frac{\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx$$

input
```
int(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)
```

3.410 $\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 299

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx &= -\frac{e^2x\sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} - \frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d(cd^2-ae^2)\sqrt{a-cx^4}} \\ &\quad - \frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2d\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{a-cx^4}} \\ &\quad + \frac{\sqrt[4]{a}(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{c}d^2(cd^2-ae^2)\sqrt{a-cx^4}}\end{aligned}$$

output

```
-1/2*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)-1/2*a^(3/4)*c^(1/4)
*e*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/d/(-a*e^2+c*d^2)/(-c*x
^4+a)^(1/2)-1/2*a^(1/4)*c^(1/4)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1
/4),I)/d/(c^(1/2)*d+a^(1/2)*e)/(-c*x^4+a)^(1/2)+1/2*a^(1/4)*(-a*e^2+3*c*d^
2)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/d,I)/
c^(1/4)/d^2/(-a*e^2+c*d^2)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.73 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.70

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx$$
$$= \frac{-a\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}de^2x+\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cde^2x^5+i\sqrt{a}\sqrt{c}de(d+ex^2)\sqrt{1-\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-i\sqrt{c}d(-}{}$$

input

```
Integrate[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

output

```
(-(a*Sqrt[-(Sqrt[c]/Sqrt[a])]*d*e^2*x) + Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d*e^2*
x^5 + I*Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[I*Ar
cSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*Sqrt[c]*d*(-(Sqrt[c]*d) + Sqrt[
a]*e)*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/S
qrt[a])]*x], -1] - (3*I)*c*d^3*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e
)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*a*d*e^2*Sqr
t[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(S
qrt[c]/Sqrt[a])]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 - (c*x^4)/a]*EllipticP
i[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] +
 I*a*e^3*x^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*
ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(2*Sqrt[-(Sqrt[c]/Sqrt[a])]*d^2*
(c*d^2 - a*e^2)*(d + e*x^2)*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {1552, 2235, 27, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^4}\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 1552}$$

$$\frac{\int \frac{-ce^2x^4-2cdex^2+2cd^2-ae^2}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 2235}$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-\frac{\int \frac{ce^2\left(ex^2+d\right)}{\sqrt{a-cx^4}}dx}{e^2}}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\int \frac{ex^2+d}{\sqrt{a-cx^4}}dx}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1513}$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx-c\left(\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)}{2d\left(cd^2-ae^2\right)}-$$

$$\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 1390$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx-c\left(\frac{e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)}{2d\left(cd^2-ae^2\right)}-$$

$$\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 1389$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx-c\left(\frac{\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)}{2d\left(cd^2-ae^2\right)}-$$

$$\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 327$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx-c\Bigg(\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\Bigg)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{2d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 1543$$

$$\frac{\frac{\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-c\left(\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}}{\sqrt[4]{}}\right.\right.}{\sqrt[4]{c}\sqrt{a-cx^4}}\right.}{2d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1542}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}-c\left(\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)}{\sqrt[4]{}}\right.}{2d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

output
```
-1/2*(e^2*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e*x^2)) + (-(c*((a^(3
/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(
3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(d - (Sqrt[a]*e)/Sqrt[c])*Sqrt[1 - (c*x^4
)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4])
)) + (a^(1/4)*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*
e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[a - c*x
^4]))/(2*d*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 1552
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sq
rt[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
 + c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && ILtQ[q, -1]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(245) = 490$.

Time = 0.65 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.75

method	result
default	$\frac{e^{2} x \sqrt{-c\,x^{4}+a}}{2 d \left(a\,e^{2}-c\,d^{2}\right) \left(e\,x^{2}+d\right)}+\frac{c \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{2 \left(a\,e^{2}-c\,d^{2}\right) \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}}-\frac{\sqrt{c}\, e \sqrt{a}\, \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right.}{2 \left(a\,e^{2}-c\,d^{2}\right) d \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}}$
elliptic	$\frac{e^{2} x \sqrt{-c\,x^{4}+a}}{2 d \left(a\,e^{2}-c\,d^{2}\right) \left(e\,x^{2}+d\right)}+\frac{c \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{2 \left(a\,e^{2}-c\,d^{2}\right) \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}}-\frac{\sqrt{c}\, e \sqrt{a}\, \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right.}{2 \left(a\,e^{2}-c\,d^{2}\right) d \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}}$

input
```
int(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*e^2/d/(a*e^2-c*d^2)*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/2*c/(a*e^2-c*d^2)/(
1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1
/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-1
/2*c^(1/2)*e/(a*e^2-c*d^2)/d*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elli
pticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+1/2*c^(1/2)*e/(a*e^2-c*d^2)/d*a^(1/2)
/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)
+1/2/(a*e^2-c*d^2)/d^2*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(
1/a^(1/2)*c^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-1/a^(1/2)*c^(1/2))^(1/2)/(
1/a^(1/2)*c^(1/2))^(1/2))*a-3/2/(a*e^2-c*d^2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1
-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(
1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-1/a^(1/
2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\left(d+ex^2\right)^2}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a-cx^4}}\,dx=\int \frac{\sqrt{-c\,x^4+a}}{-c\,e^2x^8-2cde\,x^6+a\,e^2x^4-c\,d^2x^4+2ade\,x^2+a\,d^2}dx$$

input
```
int(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 -
2*c*d*e*x**6 - c*e**2*x**8),x)
```

3.411 $\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 425

$$\begin{aligned}
&\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx \\
&= -\frac{e^2x\sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2}-\frac{3e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} \\
&\quad-\frac{3a^{3/4}\sqrt[4]{c}e(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2(cd^2-ae^2)^2\sqrt{a-cx^4}} \\
&\quad-\frac{\sqrt[4]{a}\sqrt[4]{c}(7cd^2-2\sqrt{a}\sqrt{c}de-3ae^2)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8d^2(\sqrt{c}d+\sqrt{a}e)(cd^2-ae^2)\sqrt{a-cx^4}} \\
&\quad+\frac{3\sqrt[4]{a}(5c^2d^4-2acd^2e^2+a^2e^4)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{c}d^3(cd^2-ae^2)^2\sqrt{a-cx^4}}
\end{aligned}$$

output
```
-1/4*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^2-3/8*e^2*(-a*e^2+3
*c*d^2)*x*(-c*x^4+a)^(1/2)/d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)-3/8*a^(3/4)*c^(1
/4)*e*(-a*e^2+3*c*d^2)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/d^
2/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)-1/8*a^(1/4)*c^(1/4)*(7*c*d^2-2*a^(1/2)
*c^(1/2)*d*e-3*a*e^2)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/d^2
/(c^(1/2)*d+a^(1/2)*e)/(-a*e^2+c*d^2)/(-c*x^4+a)^(1/2)+3/8*a^(1/4)*(a^2*e^
4-2*a*c*d^2*e^2+5*c^2*d^4)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),
-a^(1/2)*e/c^(1/2)/d,I)/c^(1/4)/d^3/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.96 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx$$

$$= \frac{\frac{de^2x(a-cx^4)(ae^2(5d+3ex^2)-cd^2(11d+9ex^2))}{(d+ex^2)^2} - \frac{i\sqrt{1-\frac{cx^4}{a}}\Big(3\sqrt{a}\sqrt{c}de(-3cd^2+ae^2)E\Big(i\operatorname{arcsinh}\Big(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\Big)\Big|-1\Big)+(-7c^2d^4+9\sqrt{a}c^{3/2}}{}}{8d^3\,(cd^2-ae}$$

input
```
Integrate[1/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```

output
```
((d*e^2*x*(a - c*x^4)*(a*e^2*(5*d + 3*e*x^2) - c*d^2*(11*d + 9*e*x^2)))/(d
 + e*x^2)^2 - (I*Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*Sqrt[c]*d*e*(-3*c*d^2 + a*
e^2)*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (-7*c^2*d^4 +
9*Sqrt[a]*c^(3/2)*d^3*e + a*c*d^2*e^2 - 3*a^(3/2)*Sqrt[c]*d*e^3)*EllipticF
[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + 3*(5*c^2*d^4 - 2*a*c*d^2*e^2
 + a^2*e^4)*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c
]/Sqrt[a])]*x], -1]))/Sqrt[-(Sqrt[c]/Sqrt[a])])/(8*d^3*(c*d^2 - a*e^2)^2*S
qrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {1552, 2211, 2235, 27, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^3}dx$$

$$\Big\downarrow \text{ 1552}$$

$$\frac{\int \frac{ce^2x^4-4cdex^2+4cd^2-3ae^2}{(ex^2+d)^2\sqrt{a-cx^4}}dx}{4d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 2211}$$

$$\frac{\frac{\int \frac{8c^2d^4-5ace^2d^2-4ce\left(4cd^2-ae^2\right)x^2d+3a^2e^4-3ce^2\left(3cd^2-ae^2\right)x^4}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{2d(cd^2-ae^2)}-\frac{3e^2x\sqrt{a-cx^4}(3cd^2-ae^2)}{2d(d+ex^2)(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 2235}$$

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-\frac{\int \frac{ce^2\left(3e\left(3cd^2-ae^2\right)x^2+d\left(7cd^2-ae^2\right)\right)}{\sqrt{a-cx^4}}dx}{e^2}}{2d(cd^2-ae^2)}-\frac{3e^2x\sqrt{a-cx^4}(3cd^2-ae^2)}{2d(d+ex^2)(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\int \frac{3e\left(3cd^2-ae^2\right)x^2+d\left(7cd^2-ae^2\right)}{\sqrt{a-cx^4}}dx}{2d(cd^2-ae^2)}-\frac{3e^2x\sqrt{a-cx^4}(3cd^2-ae^2)}{2d(d+ex^2)(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1513}$$

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{(\sqrt{c}d-\sqrt{a}e)\left(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2\right)\int \frac{1}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{3\sqrt{a}e\left(3cd^2-ae^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{2d(cd^2-ae^2)}-\frac{3e^2}{2}}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{(\sqrt{c}d-\sqrt{a}e)\left(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2\right)\int \frac{1}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{3e\left(3cd^2-ae^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{2d(cd^2-ae^2)}-\frac{3e^2x\sqrt{c}}{2d(d+}}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 765$$

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{\sqrt{1-\frac{cx^4}{a}}(\sqrt{c}d-\sqrt{a}e)\left(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{3e\left(3cd^2-ae^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{2d(cd^2-ae^2)}-}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 762$$

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3e\left(3cd^2-ae^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(\sqrt{c}d-\sqrt{a}e)\left(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2\right)\text{EllipticF}\left(\text{arcsi}\right.}{c^{3/4}\sqrt{a-cx^4}}\right.}{2d(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1390$$

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\left(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2\right)\text{Ellipt}}{c^{3/4}\sqrt{a-cx^4}}\right.}{2d(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\left(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2\right)\text{E}}{c^{3/4}\sqrt{a-cx^4}}\right.}{2d(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\left(-2\sqrt{a}\sqrt{c}de-3ae\right.}{c^{3/4}\sqrt{a}}\right.}{2d(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1543}$$

$$\frac{\frac{\frac{3\sqrt{1-\frac{cx^4}{a}}\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-c\left(\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\left(-2\sqrt{a}\sqrt{c}\right.}{}\right.}{2d(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1542}$$

$$\frac{\frac{\frac{3\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}-c\left(\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}}{}\right.}{2d(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```

output
```
-1/4*(e^2*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e*x^2)^2) + ((-3*e^2*
(3*c*d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(2*d*(c*d^2 - a*e^2)*(d + e*x^2)) + (
-(c*((3*a^(3/4)*e*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(
c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(Sqrt[c]*d
 - Sqrt[a]*e)*(7*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Sqrt[1 - (c*x^4)/
a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4])))
 + (3*a^(1/4)*(5*c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*Sqrt[1 - (c*x^4)/a]*El
lipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^
(1/4)*d*Sqrt[a - c*x^4]))/(2*d*(c*d^2 - a*e^2)))/(4*d*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 1552
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sq
rt[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
 + c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && ILtQ[q, -1]
```

rule 2211
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))
Int[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(
2*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(
C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 960 vs. 2(363) = 726.

Time = 1.09 (sec) , antiderivative size = 961, normalized size of antiderivative = 2.26

method	result	size
default	Expression too large to display	961
elliptic	Expression too large to display	961

input
```
int(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/4*e^2/d/(a*e^2-c*d^2)*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+3/8*e^2*(a*e^2-3*c*
d^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/8*c/d/(a*e^2-c*d^2
)^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2)
,I)*a*e^2-7/8*c^2*d/(a*e^2-c*d^2)^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)
*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Ellip
ticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d^2*a^
(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/
2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/
2),I)+9/8*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1
/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/
2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^
2/d^2*a^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1
/2))^(1/2),I)-9/8*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1
/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^
4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+3/8/(a*e^2-c*d^2)^2/d^
3*e^4/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/
2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1
/2),-a^(1/2)*e/c^(1/2)/d,(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^3 \sqrt{a-cx^4}}\, dx = \text{Timed out}$$

input

```
integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output

```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^3}\,dx$$

input
```
integrate(1/(e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**3), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^3}\,dx$$

input
```
integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^3}\,dx$$

input
```
integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{a-c\,x^4}\,(e\,x^2+d)^3}\,dx$$

input
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^3),x)
```

output
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx$$
$$=\int \frac{\sqrt{-c\,x^4+a}}{-c\,e^3x^{10}-3cd\,e^2x^8+a\,e^3x^6-3c\,d^2e\,x^6+3ad\,e^2x^4-c\,d^3x^4+3a\,d^2e\,x^2+a\,d^3}dx$$

input
```
int(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*
x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)
```

3.412 $\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 563

$$
\begin{aligned}
\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx &= -\frac{e^2x\sqrt{a-cx^4}}{6d\,(cd^2-ae^2)\,(d+ex^2)^3} \\
&- \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\,(cd^2-ae^2)^2\,(d+ex^2)^2} - \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)\,x\sqrt{a-cx^4}}{16d^3\,(cd^2-ae^2)^3\,(d+ex^2)} \\
&- \frac{a^{3/4}\sqrt[4]{c}e(29c^2d^4-14acd^2e^2+5a^2e^4)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{16d^3\,(cd^2-ae^2)^3\sqrt{a-cx^4}} \\
&- \frac{\sqrt[4]{a}\sqrt[4]{c}(57c^2d^4-30\sqrt{a}c^{3/2}d^3e-32acd^2e^2+10a^{3/2}\sqrt{c}de^3+15a^2e^4)\sqrt{1-\frac{cx^4}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{48d^3\,(\sqrt{c}d-\sqrt{a}e)^2\,(\sqrt{c}d+\sqrt{a}e)^3\sqrt{a-cx^4}} \\
&+ \frac{\sqrt[4]{a}(35c^3d^6-7ac^2d^4e^2+17a^2cd^2e^4-5a^3e^6)\sqrt{1-\frac{cx^4}{a}}\,\mathrm{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{16\sqrt[4]{c}d^4\,(cd^2-ae^2)^3\sqrt{a-cx^4}}
\end{aligned}
$$

output

```
-1/6*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^3-5/24*e^2*(-a*e^2+
3*c*d^2)*x*(-c*x^4+a)^(1/2)/d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)^2-1/16*e^2*(5*a
^2*e^4-14*a*c*d^2*e^2+29*c^2*d^4)*x*(-c*x^4+a)^(1/2)/d^3/(-a*e^2+c*d^2)^3/
(e*x^2+d)-1/16*a^(3/4)*c^(1/4)*e*(5*a^2*e^4-14*a*c*d^2*e^2+29*c^2*d^4)*(1-
c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/d^3/(-a*e^2+c*d^2)^3/(-c*x^4
+a)^(1/2)-1/48*a^(1/4)*c^(1/4)*(57*c^2*d^4-30*a^(1/2)*c^(3/2)*d^3*e-32*a*c
*d^2*e^2+10*a^(3/2)*c^(1/2)*d*e^3+15*a^2*e^4)*(1-c*x^4/a)^(1/2)*EllipticF(
c^(1/4)*x/a^(1/4),I)/d^3/(c^(1/2)*d-a^(1/2)*e)^2/(c^(1/2)*d+a^(1/2)*e)^3/(
-c*x^4+a)^(1/2)+1/16*a^(1/4)*(-5*a^3*e^6+17*a^2*c*d^2*e^4-7*a*c^2*d^4*e^2+
35*c^3*d^6)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1
/2)/d,I)/c^(1/4)/d^4/(-a*e^2+c*d^2)^3/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.40 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.81

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx$$

$$= \frac{-\frac{de^2x(a-cx^4)\left(8(cd^3-ade^2)^2+10d(cd^2-ae^2)(3cd^2-ae^2)(d+ex^2)+3(29c^2d^4-14acd^2e^2+5a^2e^4)(d+ex^2)^2\right)}{(cd^2-ae^2)^3(d+ex^2)^3} - \frac{i\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\sqrt{c}de(29c}{}}{}$$

input

```
Integrate[1/((d + e*x^2)^4*Sqrt[a - c*x^4]),x]
```

output

```
(-((d*e^2*x*(a - c*x^4)*(8*(c*d^3 - a*d*e^2)^2 + 10*d*(c*d^2 - a*e^2)*(3*c
*d^2 - a*e^2)*(d + e*x^2) + 3*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*(d
 + e*x^2)^2))/((c*d^2 - a*e^2)^3*(d + e*x^2)^3)) - (I*Sqrt[1 - (c*x^4)/a]*
(3*Sqrt[a]*Sqrt[c]*d*e*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*EllipticE
[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + Sqrt[c]*d*(57*c^(5/2)*d^5 -
87*Sqrt[a]*c^2*d^4*e - 2*a*c^(3/2)*d^3*e^2 + 42*a^(3/2)*c*d^2*e^3 + 5*a^2*
Sqrt[c]*d*e^4 - 15*a^(5/2)*e^5)*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a]
)]*x], -1] + 3*(-35*c^3*d^6 + 7*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 + 5*a^3*e
^6)*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a
])]*x], -1]))/(Sqrt[-(Sqrt[c]/Sqrt[a])]*(-(c*d^2) + a*e^2)^3))/(48*d^4*Sqr
t[a - c*x^4])
```

Rubi [A] (verified)

Time = 3.19 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {1552, 2211, 2211, 2235, 27, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^4}\left(d+ex^2\right)^4}dx$$

$$\Big\downarrow \text{1552}$$

$$\frac{\int \frac{3ce^2x^4-6cdex^2+6cd^2-5ae^2}{\left(ex^2+d\right)^3\sqrt{a-cx^4}}dx}{6d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{2211}$$

$$\frac{\frac{\int \frac{24c^2d^4-29ace^2d^2-8ce\left(6cd^2-ae^2\right)x^2d+15a^2e^4+5ce^2\left(3cd^2-ae^2\right)x^4}{\left(ex^2+d\right)^2\sqrt{a-cx^4}}dx}{4d(cd^2-ae^2)}-\frac{5e^2x\sqrt{a-cx^4}(3cd^2-ae^2)}{4d(d+ex^2)^2(cd^2-ae^2)}}{6d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{2211}$$

$$\frac{\frac{\frac{\int \frac{48c^3d^6-19ac^2e^2d^4+46a^2ce^4d^2-4ce\left(36c^2d^4-11ace^2d^2+5a^2e^4\right)x^2d-15a^3e^6-3ce^2\left(29c^2d^4-14ace^2d^2+5a^2e^4\right)x^4}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{2d\left(cd^2-ae^2\right)}-\frac{3e^2x\sqrt{a-cx^4}\left(5a^2e^4-14acd^2e^2+29\right.}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}}{4d(cd^2-ae^2)}}{6d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{2235}$$

$$\frac{\frac{\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-\frac{\int \frac{ce^2\left(3e\left(29c^2d^4-14ace^2d^2+5a^2e^4\right)x^2+d\left(57c^2d^4-2ace^2d^2+5a^2e^4\right)\right)}{\sqrt{a-cx^4}}dx}{e^2}}{2d\left(cd^2-ae^2\right)}-\frac{3e^2x\sqrt{a-cx}}{2d}}{4d(cd^2-ae^2)}}{6d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\int\frac{3e\left(29c^2d^4-14ace^2d^2+5a^2e^4\right)x^2+d\left(57c^2d^4-2ace^2d^2+5a^2e^4\right)}{\sqrt{a-cx^4}}dx}{2d\left(cd^2-ae^2\right)}-\frac{3e^2x\sqrt{a-cx^4}\left(5a\right.}{2d\left(d+e\right.}$$

$$4d(cd^2-ae^2)$$

$$6d\left(cd^2-ae^2\right)$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 1513$$

$$\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}+\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+29c^2\right.}{\sqrt{c}}\right.\right.}{2d\left(cd^2-ae^2\right)}$$

$$4d(cd^2-ae^2)$$

$$6d\left(cd^2-ae^2\right)$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)}{\sqrt{c}}\right.\right.}{2d\left(cd^2-ae^2\right)}$$

$$4d(cd^2-ae^2)$$

$$6d\left(cd^2-ae^2\right)$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 765$$

$$\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+\right.}{\sqrt{c}}\right.}{}\right.}{2d\left(cd^2-ae^2\right)}$$

$$4d(cd^2-ae^2)$$

$$6d\left(cd^2-ae^2\right)$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 762$$

$$\frac{\frac{\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd\right.}{\sqrt{c}}\right.}{}\right.}{2d\left(cd^2-ae^2\right)}}{4d(cd^2-ae^2)}}{6d\left(cd^2\right.}$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1390}$$

$$\frac{\frac{\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3e\sqrt{1-\frac{cx^4}{a}}\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(5a^2\right.}{}\right.}{}\right.}{2d\left(cd^2-ae^2\right)}}{4d(cd^2-ae^2)}}{6d\left(c\right.}$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{\frac{\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(\right.}{}\right.}{}\right.}{2d\left(cd^2-ae^2\right)}}{4d(cd^2-ae^2)}}{6d\left(\right.}$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)}{\sqrt{c}}+5a^2de^4-2acd^3e^2+57c^2d^5\right)\text{Ellip}}{\sqrt[4]{c}\sqrt{a-cx^4}}\right.}{2d\left(cd^2-ae^2\right)}}{4d(cd^2-ae^2}$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1543}$$

$$\frac{\frac{\frac{3\sqrt{1-\frac{cx^4}{a}}\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)}{\sqrt{c}}+5a^2de^4-2acd^3e^2+57c\right.}{\sqrt[4]{c}\sqrt{a-cx^4}}\right.}{2d\left(cd^2-ae^2\right)}}{4d(cd^2}$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1542}$$

$$\frac{\frac{3\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}-c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)}{\sqrt{c}}+\right.}{\sqrt[4]{}}\right.}{2d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

input

```
Int[1/((d + e*x^2)^4*Sqrt[a - c*x^4]),x]
```

output

```
-1/6*(e^2*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e*x^2)^3) + ((-5*e^2*
(3*c*d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(4*d*(c*d^2 - a*e^2)*(d + e*x^2)^2) +
 ((-3*e^2*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[a - c*x^4])/(2*
d*(c*d^2 - a*e^2)*(d + e*x^2)) + (-(c*((3*a^(3/4)*e*(29*c^2*d^4 - 14*a*c*d
^2*e^2 + 5*a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/
4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(57*c^2*d^5 - 2*a*c*d^3*e^2
 + 5*a^2*d*e^4 - (3*Sqrt[a]*e*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4))/S
qrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c
^(1/4)*Sqrt[a - c*x^4]))) + (3*a^(1/4)*(35*c^3*d^6 - 7*a*c^2*d^4*e^2 + 17*
a^2*c*d^2*e^4 - 5*a^3*e^6)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(S
qrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[a - c*x^4]))
/(2*d*(c*d^2 - a*e^2)))/(4*d*(c*d^2 - a*e^2)))/(6*d*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 1552

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sq
rt[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
 + c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && ILtQ[q, -1]
```

rule 2211

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))
Int[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(
2*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(
C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1419 vs. $2(489) = 978$.

Time = 1.59 (sec) , antiderivative size = 1420, normalized size of antiderivative = 2.52

method	result	size
default	Expression too large to display	1420
elliptic	Expression too large to display	1420

input
```
int(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/6*e^2/d/(a*e^2-c*d^2)*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^3+5/24*e^2*(a*e^2-3*c
*d^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+1/16*e^2*(5*a^2*e
^4-14*a*c*d^2*e^2+29*c^2*d^4)/(a*e^2-c*d^2)^3/d^3*x*(-c*x^4+a)^(1/2)/(e*x^
2+d)+19/16*c^3*d^2/(a*e^2-c*d^2)^3/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*
c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Ellipt
icF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-35/16/(a*e^2-c*d^2)^3*d^2/(1/a^(1/2)*c^
(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-a^(1/2)*e/c^(1
/2)/d,(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c^3+5/48*c/d^2
/(a*e^2-c*d^2)^3/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)
*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c
^(1/2))^(1/2),I)*a^2*e^4-1/24*c^2/(a*e^2-c*d^2)^3/(1/a^(1/2)*c^(1/2))^(1/2
)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+
a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*a*e^2-5/16*c^(1/2)*e^5/(
a*e^2-c*d^2)^3/d^3*a^(5/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1
/a^(1/2)*c^(1/2))^(1/2),I)+7/8*c^(3/2)*e^3/(a*e^2-c*d^2)^3/d*a^(3/2)/(1/a^
(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*
x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-29/16
*c^(5/2)*e/(a*e^2-c*d^2)^3*d*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^4\sqrt{a-cx^4}}\,dx=\text{Timed out}$$

input

```
integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output

```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^4}\,dx$$

input
```
integrate(1/(e*x**2+d)**4/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**4), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^4}\,dx$$

input
```
integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^4), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^4}\,dx$$

input
```
integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^4 \sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)^4}\,dx$$

input
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^4),x)
```

output
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^4), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+ex^2\right)^4 \sqrt{a-cx^4}}\,dx$$
$$= \int \frac{\sqrt{-c\,x^4+a}}{-c\,e^4x^{12}-4cd\,e^3x^{10}+a\,e^4x^8-6c\,d^2e^2x^8+4ad\,e^3x^6-4c\,d^3e\,x^6+6a\,d^2e^2x^4-c\,d^4x^4+4a\,d^3e\,x^2+}$$

input
```
int(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a*d**4 + 4*a*d**3*e*x**2 + 6*a*d**2*e**2*x**4 + 4*a*
d*e**3*x**6 + a*e**4*x**8 - c*d**4*x**4 - 4*c*d**3*e*x**6 - 6*c*d**2*e**2*
x**8 - 4*c*d*e**3*x**10 - c*e**4*x**12),x)
```

3.413 $\int \frac{(d+ex^2)^3}{(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 226

$$\int \frac{(d+ex^2)^3}{(a-cx^4)^{3/2}}\,dx = \frac{x\left(cd\left(d^2+\frac{3ae^2}{c}\right)+e(3cd^2+ae^2)\,x^2\right)}{2ac\sqrt{a-cx^4}} - \frac{3e(cd^2+ae^2)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}c^{7/4}\sqrt{a-cx^4}} + \frac{\left(c^{3/2}d^3+3\sqrt{a}cd^2e-3a\sqrt{c}de^2+3a^{3/2}e^3\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}c^{7/4}\sqrt{a-cx^4}}$$

output

```
1/2*x*(c*d*(d^2+3*a*e^2/c)+e*(a*e^2+3*c*d^2)*x^2)/a/c/(-c*x^4+a)^(1/2)-3/2
*e*(a*e^2+c*d^2)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(1/4)/
c^(7/4)/(-c*x^4+a)^(1/2)+1/2*(c^(3/2)*d^3+3*a^(1/2)*c*d^2*e-3*a*c^(1/2)*d*
e^2+3*a^(3/2)*e^3)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(3/4
)/c^(7/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex^2)^3}{(a-cx^4)^{3/2}}\,dx = \frac{cd^3x+ae^2x(3d-2ex^2)+d(cd^2-3ae^2)\,x\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right.}{2ac\sqrt{a-cx^4}}$$

input
```
Integrate[(d + e*x^2)^3/(a - c*x^4)^(3/2),x]
```

output
```
(c*d^3*x + a*e^2*x*(3*d - 2*e*x^2) + d*(c*d^2 - 3*a*e^2)*x*Sqrt[1 - (c*x^4
)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + 2*e*(c*d^2 + a*e^2)*x^3
*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, (c*x^4)/a])/(2*a*c*S
qrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1519, 25, 2397, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{(a-cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1519}$$

$$-\frac{\int -\frac{3cde^2x^4+3e(cd^2+ae^2)x^2+cd^3}{(a-cx^4)^{3/2}}dx}{c}-\frac{e^3x^3}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3cde^2x^4+3e(cd^2+ae^2)x^2+cd^3}{(a-cx^4)^{3/2}}dx}{c}-\frac{e^3x^3}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \ 2397$$

$$\frac{\frac{\int \frac{c\left(d\left(cd^2-3ae^2\right)-3e\left(cd^2+ae^2\right)x^2\right)}{\sqrt{a-cx^4}}dx}{2ac}+\frac{x(3ex^2(ae^2+cd^2)+d(3ae^2+cd^2))}{2a\sqrt{a-cx^4}}}{c}-\frac{e^3x^3}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{d\left(cd^2-3ae^2\right)-3e\left(cd^2+ae^2\right)x^2}{\sqrt{a-cx^4}}dx}{2a}+\frac{x(3ex^2(ae^2+cd^2)+d(3ae^2+cd^2))}{2a\sqrt{a-cx^4}}}{c}-\frac{e^3x^3}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \ 1513$$

$$\frac{\frac{\frac{\left(3a^{3/2}e^3+3\sqrt{a}cd^2e-3a\sqrt{c}de^2+c^{3/2}d^3\right)\int \frac{1}{\sqrt{a-cx^4}}dx}{\sqrt{c}}-\frac{3\sqrt{a}e\left(ae^2+cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x(3ex^2(ae^2+cd^2)+d(3ae^2+cd^2))}{2a\sqrt{a-cx^4}}}{c}-\frac{e^3x^3}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\left(3a^{3/2}e^3+3\sqrt{a}cd^2e-3a\sqrt{c}de^2+c^{3/2}d^3\right)\int \frac{1}{\sqrt{a-cx^4}}dx}{\sqrt{c}}-\frac{3e\left(ae^2+cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x(3ex^2(ae^2+cd^2)+d(3ae^2+cd^2))}{2a\sqrt{a-cx^4}}}{c}-\frac{e^3x^3}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \ 765$$

$$\frac{\frac{\frac{\sqrt{1-\frac{cx^4}{a}}\left(3a^{3/2}e^3+3\sqrt{a}cd^2e-3a\sqrt{c}de^2+c^{3/2}d^3\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}-\frac{3e\left(ae^2+cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x(3ex^2(ae^2+cd^2)+d(3ae^2+cd^2))}{2a\sqrt{a-cx^4}}}{c}-\frac{e^3x^3}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \ 762$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3a^{3/2}e^3+3\sqrt{a}cd^2e-3a\sqrt{c}de^2+c^{3/2}d^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{3e\left(ae^2+cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x(3ex^2(ae^2+cd^2)+d(3ae^2+cd}{2a\sqrt{a-cx^4}}}{c}$$

$$\frac{e^3x^3}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3a^{3/2}e^3+3\sqrt{a}cd^2e-3a\sqrt{c}de^2+c^{3/2}d^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{3e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+cd^2\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}}{2a}+\frac{x\left(3ex^2\left(ae^2+cd^2\right)+d(\right.}{2a\sqrt{a-cx^4}}}{c}$$

$$\frac{e^3x^3}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3a^{3/2}e^3+3\sqrt{a}cd^2e-3a\sqrt{c}de^2+c^{3/2}d^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{3\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+cd^2\right)\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}}{2a}+\frac{x\left(3ex^2\left(ae^2+cd^2\right)-\right.}{2a\sqrt{a-}}}{c}$$

$$\frac{e^3x^3}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3a^{3/2}e^3+3\sqrt{a}cd^2e-3a\sqrt{c}de^2+c^{3/2}d^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+cd^2\right)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}}{2a}+\frac{x\left(3ex^2\left(\right.\right.}{}}{c}$$

$$\frac{e^3x^3}{c\sqrt{a-cx^4}}$$

input

```
Int[(d + e*x^2)^3/(a - c*x^4)^(3/2),x]
```

output

```
-((e^3*x^3)/(c*Sqrt[a - c*x^4])) + ((x*(d*(c*d^2 + 3*a*e^2) + 3*e*(c*d^2 +
 a*e^2)*x^2))/(2*a*Sqrt[a - c*x^4]) + ((-3*a^(3/4)*e*(c*d^2 + a*e^2)*Sqrt[
1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a
 - c*x^4]) + (a^(1/4)*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2
 + 3*a^(3/2)*e^3)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)
], -1])/(c^(3/4)*Sqrt[a - c*x^4]))/(2*a))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.23

method	result
elliptic	$\frac{2c\left(\frac{e\left(a\,e^{2}+3c\,d^{2}\right)x^{3}}{4c^{2}a}+\frac{d\left(3a\,e^{2}+c\,d^{2}\right)x}{4c^{2}a}\right)}{\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}+\frac{\left(-\frac{3d\,e^{2}}{c}+\frac{d\left(3a\,e^{2}+c\,d^{2}\right)}{2ac}\right)\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-$
default	$d^{3}\left(\frac{x}{2a\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}\right)+e^{3}\left(\frac{x^{3}}{2c\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}+\right.$

input
```
int((e*x^2+d)^3/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(1/4*e*(a*e^2+3*c*d^2)/c^2/a*x^3+1/4*d*(3*a*e^2+c*d^2)/c^2/a*x)/(-(x^4
-1/c*a)*c)^(1/2)+(-3*d*e^2/c+1/2*d*(3*a*e^2+c*d^2)/a/c)/(1/a^(1/2)*c^(1/2)
)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-
c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-(-e^3/c-1/2*e*(a*e
^2+3*c*d^2)/a/c)*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^
2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(Ellipti
cF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex^2)^3}{(a-cx^4)^{3/2}}\,dx = \frac{3\,((ac^2d^2e+a^2ce^3)x^5-(a^2cd^2e+a^3e^3)x)\sqrt{-c}\left(\frac{a}{c}\right)^{\frac{3}{4}} E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+((c^3d^3}{}$$

input
```
integrate((e*x^2+d)^3/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(3*((a*c^2*d^2*e + a^2*c*e^3)*x^5 - (a^2*c*d^2*e + a^3*e^3)*x)*sqrt(-c
)*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) + ((c^3*d^3 - 3*a*c^2*
d^2*e - 3*a*c^2*d*e^2 - 3*a^2*c*e^3)*x^5 - (a*c^2*d^3 - 3*a^2*c*d^2*e - 3*
a^2*c*d*e^2 - 3*a^3*e^3)*x)*sqrt(-c)*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(
1/4)/x), -1) + (2*a^2*c*e^3*x^4 - 3*a^2*c*d^2*e - 3*a^3*e^3 - (a*c^2*d^3 +
 3*a^2*c*d*e^2)*x^2)*sqrt(-c*x^4 + a))/(a^2*c^3*x^5 - a^3*c^2*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^3}{(a-cx^4)^{3/2}}\,dx = \int \frac{(d+ex^2)^3}{(a-cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**3/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((d + e*x**2)**3/(a - c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{(a-cx^4)^{3/2}}\,dx=\int \frac{(ex^2+d)^3}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^3/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^3/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3}{(a-cx^4)^{3/2}}\,dx=\int \frac{(ex^2+d)^3}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^3/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{(a-cx^4)^{3/2}}\,dx=\int \frac{(e\,x^2+d)^3}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int((d + e*x^2)^3/(a - c*x^4)^(3/2),x)
```

output
```
int((d + e*x^2)^3/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^3}{(a-cx^4)^{3/2}}\,dx = \frac{3\sqrt{-c\,x^4+a}\,d\,e^2x-\sqrt{-c\,x^4+a}\,e^3x^3-3\left(\int \frac{\sqrt{-c\,x^4+a}}{c^2x^8-2ac\,x^4+a^2}dx\right)a^2d\,e^2+\left(\int \frac{\sqrt{-c\,x^4+a}}{c^2x^8-2ac\,x^4}\right.}{}$$

input
```
int((e*x^2+d)^3/(-c*x^4+a)^(3/2),x)
```

output
```
(3*sqrt(a - c*x**4)*d*e**2*x - sqrt(a - c*x**4)*e**3*x**3 - 3*int(sqrt(a -
 c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a**2*d*e**2 + int(sqrt(a - c*x
**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*c*d**3 + 3*int(sqrt(a - c*x**4)/
(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*c*d*e**2*x**4 - int(sqrt(a - c*x**4)/
(a**2 - 2*a*c*x**4 + c**2*x**8),x)*c**2*d**3*x**4 + 3*int((sqrt(a - c*x**4
)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a**2*e**3 + 3*int((sqrt(a - c*x
**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*c*d**2*e - 3*int((sqrt(a -
 c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*c*e**3*x**4 - 3*int((s
qrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*c**2*d**2*e*x**4)
/(c*(a - c*x**4))
```

3.414 $\int \frac{\left(d+ex^2\right)^2}{\left(a-cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 178

$$\int \frac{\left(d+ex^2\right)^2}{\left(a-cx^4\right)^{3/2}}\,dx = \frac{x\left(d^2+\frac{ae^2}{c}+2dex^2\right)}{2a\sqrt{a-cx^4}} - \frac{de\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}c^{3/4}\sqrt{a-cx^4}} + \frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}c^{5/4}\sqrt{a-cx^4}}$$

output

```
1/2*x*(d^2+a*e^2/c+2*d*e*x^2)/a/(-c*x^4+a)^(1/2)-d*e*(1-c*x^4/a)^(1/2)*Ell
ipticE(c^(1/4)*x/a^(1/4),I)/a^(1/4)/c^(3/4)/(-c*x^4+a)^(1/2)+1/2*(c*d^2+2*
a^(1/2)*c^(1/2)*d*e-a*e^2)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I
)/a^(3/4)/c^(5/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex^2)^2}{(a-cx^4)^{3/2}}\,dx = \frac{3(cd^2+ae^2)\,x+3(cd^2-ae^2)\,x\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+4cdex^3}{6ac\sqrt{a-cx^4}}$$

input
```
Integrate[(d + e*x^2)^2/(a - c*x^4)^(3/2),x]
```

output
```
(3*(c*d^2 + a*e^2)*x + 3*(c*d^2 - a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + 4*c*d*e*x^3*Sqrt[1 - (c*x^4)/a]*Hyperge
ometric2F1[3/4, 3/2, 7/4, (c*x^4)/a])/(6*a*c*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1519, 1493, 25, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{(a-cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 1519}$$

$$\frac{\int \frac{cd^2+2cex^2d-ae^2}{(a-cx^4)^{3/2}}dx}{c}+\frac{e^2x}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 1493}$$

$$\frac{\frac{x(-ae^2+cd^2+2cdex^2)}{2a\sqrt{a-cx^4}}-\frac{\int -\frac{cd^2-2cex^2d-ae^2}{\sqrt{a-cx^4}}dx}{2a}}{c}+\frac{e^2x}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{cd^2-2cex^2d-ae^2}{\sqrt{a-cx^4}}dx}{2a}+\frac{x\left(-ae^2+cd^2+2cdex^2\right)}{2a\sqrt{a-cx^4}}}{c}+\frac{e^2x}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \ \textbf{1513}$$

$$\frac{\frac{\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\int \frac{1}{\sqrt{a-cx^4}}dx-2\sqrt{a}\sqrt{c}de\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{2a}+\frac{x\left(-ae^2+cd^2+2cdex^2\right)}{2a\sqrt{a-cx^4}}}{c}+\frac{e^2x}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{\frac{\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\int \frac{1}{\sqrt{a-cx^4}}dx-2\sqrt{c}de\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{2a}+\frac{x\left(-ae^2+cd^2+2cdex^2\right)}{2a\sqrt{a-cx^4}}}{c}+\frac{e^2x}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \ \textbf{765}$$

$$\frac{\frac{\frac{\sqrt{1-\frac{cx^4}{a}}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-2\sqrt{c}de\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{2a}+\frac{x\left(-ae^2+cd^2+2cdex^2\right)}{2a\sqrt{a-cx^4}}}{c}+\frac{e^2x}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \ \textbf{762}$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-2\sqrt{c}de\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{2a}+\frac{x\left(-ae^2+cd^2+2cdex^2\right)}{2a\sqrt{a-cx^4}}}{c}+\frac{e^2x}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \ \textbf{1390}$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{2\sqrt{c}de\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}}{2a}+\frac{x\left(-ae^2+cd^2+2cdex^2\right)}{2a\sqrt{a-cx^4}}}{c}+\frac{e^2x}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \ \textbf{1389}$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{2\sqrt{a}\sqrt{c}de\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{a-cx^4}}}{2a}+\frac{x\left(-ae^2+cd^2+2cdex^2\right)}{2a\sqrt{a-cx^4}}}{c}+\frac{e^2x}{c\sqrt{a-cx^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{2a^{3/4}\sqrt[4]{c}de\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-cx^4}}}{2a}+\frac{x\left(-ae^2+cd^2+2cdex^2\right)}{2a\sqrt{a-cx^4}}}{c}+\frac{e^2x}{c\sqrt{a-cx^4}}$$

input

```
Int[(d + e*x^2)^2/(a - c*x^4)^(3/2),x]
```

output

```
(e^2*x)/(c*Sqrt[a - c*x^4]) + ((x*(c*d^2 - a*e^2 + 2*c*d*e*x^2))/(2*a*Sqrt
[a - c*x^4]) + ((-2*a^(3/4)*c^(1/4)*d*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcS
in[(c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - c*x^4] + (a^(1/4)*(c*d^2 + 2*Sqrt[a
]*Sqrt[c]*d*e - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^
(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4]))/(2*a))/c
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.31

method	result
elliptic	$\frac{2c\left(\frac{ed\,x^3}{2ac}+\frac{\left(a\,e^2+c\,d^2\right)x}{4a\,c^2}\right)}{\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\left(-\frac{e^2}{c}+\frac{a\,e^2+c\,d^2}{2ac}\right)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}+\frac{ed\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\text{Elli}\right.}{\sqrt{a}\,\sqrt{}}$
default	$d^2\left(\frac{x}{2a\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)+e^2\left(\frac{x}{2c\sqrt{-\left(x^4-\frac{a}{c}\right)c}}-\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{E}}{2c\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-}}\right.$

input
```
int((e*x^2+d)^2/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(1/2/a*e/c*d*x^3+1/4*(a*e^2+c*d^2)/a/c^2*x)/(-(x^4-1/c*a)*c)^(1/2)+(-e
^2/c+1/2*(a*e^2+c*d^2)/a/c)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)
*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(
1/a^(1/2)*c^(1/2))^(1/2),I)+1/a^(1/2)*e*d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a
^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)
/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*
c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^2}{(a-cx^4)^{3/2}}\,dx =$$

$$-\frac{2\,(c^2dex^4-acde)\sqrt{a}\left(\frac{c}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(\frac{c}{a}\right)^{\frac{1}{4}}\right)\,|-1)-((c^2d^2+2\,c^2de-ace^2)x^4-acd^2-2\,acde+a^2e^2)}{2\,(ac^3x^4-a^2c^2)}$$

input
```
integrate((e*x^2+d)^2/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*(c^2*d*e*x^4 - a*c*d*e)*sqrt(a)*(c/a)^(3/4)*elliptic_e(arcsin(x*(c
/a)^(1/4)), -1) - ((c^2*d^2 + 2*c^2*d*e - a*c*e^2)*x^4 - a*c*d^2 - 2*a*c*d
*e + a^2*e^2)*sqrt(a)*(c/a)^(3/4)*elliptic_f(arcsin(x*(c/a)^(1/4)), -1) +
(2*c^2*d*e*x^3 + (c^2*d^2 + a*c*e^2)*x)*sqrt(-c*x^4 + a))/(a*c^3*x^4 - a^2
*c^2)
```

Sympy [F]

$$\int \frac{\left(d+ex^2\right)^2}{\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{\left(d+ex^2\right)^2}{\left(a-cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**2/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((d + e*x**2)**2/(a - c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{\left(d+ex^2\right)^2}{\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{\left(ex^2+d\right)^2}{\left(-cx^4+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{(a-cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)^2}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{(a-cx^4)^{3/2}}\,dx = \int \frac{(e\,x^2+d)^2}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int((d + e*x^2)^2/(a - c*x^4)^(3/2),x)
```

output
```
int((d + e*x^2)^2/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{(a-cx^4)^{3/2}}\,dx = \frac{\sqrt{-c\,x^4+a}\,e^2x - \left(\int \frac{\sqrt{-c\,x^4+a}}{c^2x^8-2ac\,x^4+a^2}dx\right)a^2e^2 + \left(\int \frac{\sqrt{-c\,x^4+a}}{c^2x^8-2ac\,x^4+a^2}dx\right)ac\,d^2 +}{}$$

input
```
int((e*x^2+d)^2/(-c*x^4+a)^(3/2),x)
```

output
```
(sqrt(a - c*x**4)*e**2*x - int(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*
x**8),x)*a**2*e**2 + int(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),
x)*a*c*d**2 + int(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*c*
e**2*x**4 - int(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*c**2*d
**2*x**4 + 2*int((sqrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x
)*a*c*d*e - 2*int((sqrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),
x)*c**2*d*e*x**4)/(c*(a - c*x**4))
```

3.415 $\int \frac{d+ex^2}{(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \frac{d+ex^2}{(a-cx^4)^{3/2}}\,dx = \frac{x(d+ex^2)}{2a\sqrt{a-cx^4}} - \frac{e\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a-cx^4}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}c^{3/4}\sqrt{a-cx^4}}$$

output

```
1/2*x*(e*x^2+d)/a/(-c*x^4+a)^(1/2)-1/2*e*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/
4)*x/a^(1/4),I)/a^(1/4)/c^(3/4)/(-c*x^4+a)^(1/2)+1/2*(c^(1/2)*d+a^(1/2)*e)
*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(3/4)/c^(3/4)/(-c*x^4+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.64

$$\int \frac{d+ex^2}{(a-cx^4)^{3/2}}\,dx = \frac{3dx+3dx\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+2ex^3\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeomet}}{6a\sqrt{a-cx^4}}$$

input
```
Integrate[(d + e*x^2)/(a - c*x^4)^(3/2),x]
```

output
```
(3*d*x + 3*d*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4
)/a] + 2*e*x^3*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, (c*x^4
)/a])/(6*a*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1493, 25, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a-cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1493}$$

$$\frac{x(d+ex^2)}{2a\sqrt{a-cx^4}}-\frac{\int -\frac{d-ex^2}{\sqrt{a-cx^4}}dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d-ex^2}{\sqrt{a-cx^4}}dx}{2a}+\frac{x(d+ex^2)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1513}$$

$$\frac{\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{a-cx^4}}dx-\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{a-cx^4}}dx-\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\frac{\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}}{2a}+\frac{x\left(d+ex^2\right)}{2a\sqrt{a-cx^4}}$$

input

```
Int[(d + e*x^2)/(a - c*x^4)^(3/2),x]
```

output
```
(x*(d + e*x^2))/(2*a*Sqrt[a - c*x^4]) + (-((a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*
EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4])) + (
a^(1/4)*(d + (Sqrt[a]*e)/Sqrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^
(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4]))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.28

method	result
elliptic	$\frac{2c\left(\frac{e\,x^{3}}{4ac}+\frac{dx}{4ac}\right)}{\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}+\frac{d\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+\frac{e\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}(}{2\sqrt{a}\,\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}\,\sqrt{c}}$
default	$d\left(\frac{x}{2a\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}\right)+e\,(\frac{x^{3}}{2a\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,(\mathrm{El}}{2\sqrt{a}}$

input
```
int((e*x^2+d)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(1/4/a*e/c*x^3+1/4*d/a/c*x)/(-(x^4-1/c*a)*c)^(1/2)+1/2*d/a/(1/a^(1/2)*
c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(
1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+1/2/a^(1/2)
*e/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2
))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.77

$$\int \frac{d+ex^2}{\left(a-cx^4\right)^{3/2}}\,dx =$$

$$-\frac{(cex^4-ae)\sqrt{a}\left(\frac{c}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(\frac{c}{a}\right)^{\frac{1}{4}}\right)|-1\right)-((cd+ce)x^4-ad-ae)\sqrt{a}\left(\frac{c}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(\frac{c}{a}\right)^{\frac{1}{4}}\right)|-1\right)}{2\left(ac^2x^4-a^2c\right)}$$

input

```
integrate((e*x^2+d)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output

```
-1/2*((c*e*x^4 - a*e)*sqrt(a)*(c/a)^(3/4)*elliptic_e(arcsin(x*(c/a)^(1/4))
, -1) - ((c*d + c*e)*x^4 - a*d - a*e)*sqrt(a)*(c/a)^(3/4)*elliptic_f(arcsi
n(x*(c/a)^(1/4)), -1) + (c*e*x^3 + c*d*x)*sqrt(-c*x^4 + a))/(a*c^2*x^4 - a
^2*c)
```

Sympy [A] (verification not implemented)

Time = 3.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

$$\int \frac{d+ex^2}{\left(a-cx^4\right)^{3/2}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input

```
integrate((e*x**2+d)/(-c*x**4+a)**(3/2),x)
```

output

```
d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*a*
*(3/2)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), c*x**4*ex
p_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{(a-cx^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(a-cx^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a-cx^4)^{3/2}}\,dx = \int \frac{e\,x^2+d}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int((d + e*x^2)/(a - c*x^4)^(3/2),x)
```

output
```
int((d + e*x^2)/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(a-cx^4)^{3/2}}\,dx = \left(\int \frac{\sqrt{-c\,x^4+a}}{c^2x^8-2ac\,x^4+a^2}dx\right)d+\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{c^2x^8-2ac\,x^4+a^2}dx\right)e$$

input
```
int((e*x^2+d)/(-c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*d + int((sqrt(a -
c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*e
```

3.416 $\int \frac{1}{(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{1}{(a-cx^4)^{3/2}}\,dx=\frac{x}{2a\sqrt{a-cx^4}}+\frac{\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a-cx^4}}$$

output
```
1/2*x/a/(-c*x^4+a)^(1/2)+1/2*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4)
,I)/a^(3/4)/c^(1/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a-cx^4)^{3/2}}\,dx=\frac{x+x\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)}{2a\sqrt{a-cx^4}}$$

input
```
Integrate[(a - c*x^4)^(-3/2),x]
```

output
```
(x + x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a])/(2
*a*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {749, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{749}$$

$$\frac{\int \frac{1}{\sqrt{a-cx^4}}dx}{2a}+\frac{x}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{2a\sqrt{a-cx^4}}+\frac{x}{2a\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a-cx^4}}+\frac{x}{2a\sqrt{a-cx^4}}$$

input
```
Int[(a - c*x^4)^(-3/2),x]
```

output
```
x/(2*a*Sqrt[a - c*x^4]) + (Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x
)/a^(1/4)], -1])/(2*a^(3/4)*c^(1/4)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{x}{2 a \sqrt{-\left(x^{4}-\frac{a}{c}\right) c}}+\frac{\sqrt{1-\frac{\sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{2 a \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\, x^{4}+a}}$	90
elliptic	$\frac{x}{2 a \sqrt{-\left(x^{4}-\frac{a}{c}\right) c}}+\frac{\sqrt{1-\frac{\sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{2 a \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\, x^{4}+a}}$	90

input
```
int(1/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a/(-(x^4-1/c*a)*c)^(1/2)+1/2/a/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elli
pticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a-cx^4)^{3/2}}\,dx = \frac{(cx^4-a)\sqrt{a}\left(\frac{c}{a}\right)^{\frac{3}{4}} F\left(\arcsin\left(x\left(\frac{c}{a}\right)^{\frac{1}{4}}\right)|-1\right) - \sqrt{-cx^4+a}\,acx}{2\left(ac^2x^4-a^2c\right)}$$

input

```
integrate(1/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output

```
1/2*((c*x^4 - a)*sqrt(a)*(c/a)^(3/4)*elliptic_f(arcsin(x*(c/a)^(1/4)), -1)
 - sqrt(-c*x^4 + a)*c*x)/(a*c^2*x^4 - a^2*c)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a-cx^4)^{3/2}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate(1/(-c*x**4+a)**(3/2),x)
```

output

```
x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*a**(
3/2)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(a-cx^4)^{3/2}}\,dx = \int \frac{1}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input

```
integrate(1/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a-cx^4)^{3/2}}\,dx=\int \frac{1}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 17.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a-cx^4)^{3/2}}\,dx=\frac{x\left(1-\frac{c\,x^4}{a}\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ \frac{c\,x^4}{a}\right)}{(a-c\,x^4)^{3/2}}$$

input
```
int(1/(a - c*x^4)^(3/2),x)
```

output
```
(x*(1 - (c*x^4)/a)^(3/2)*hypergeom([1/4, 3/2], 5/4, (c*x^4)/a))/(a - c*x^4
)^(3/2)
```

Reduce [F]

$$\int \frac{1}{(a-cx^4)^{3/2}}\,dx=\int \frac{\sqrt{-c\,x^4+a}}{c^2x^8-2ac\,x^4+a^2}dx$$

input
```
int(1/(-c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)
```

3.417 $\int \frac{1}{(d+ex^2)(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 278

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a-cx^4)^{3/2}}\,dx = {} & \frac{cx(d-ex^2)}{2a(cd^2-ae^2)\sqrt{a-cx^4}} \\ & + \frac{\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}(cd^2-ae^2)\sqrt{a-cx^4}} \\ & + \frac{\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{a-cx^4}} \\ & - \frac{\sqrt[4]{a}e^2\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d(cd^2-ae^2)\sqrt{a-cx^4}}\end{aligned}$$

output
```
1/2*c*x*(-e*x^2+d)/a/(-a*e^2+c*d^2)/(-c*x^4+a)^(1/2)+1/2*c^(1/4)*e*(1-c*x^
4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(1/4)/(-a*e^2+c*d^2)/(-c*x^4+a
)^(1/2)+1/2*c^(1/4)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(3/
4)/(c^(1/2)*d+a^(1/2)*e)/(-c*x^4+a)^(1/2)-a^(1/4)*e^2*(1-c*x^4/a)^(1/2)*El
lipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/d,I)/c^(1/4)/d/(-a*e^2+c*d^2
)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.38 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex^2)\left(a-cx^4\right)^{3/2}}\,dx = \frac{-\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cd^2x+\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cdex^3+i\sqrt{a}\sqrt{c}de\sqrt{1-\frac{cx^4}{a}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\right|}{}$$

input
```
Integrate[1/((d + e*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(-(Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^2*x) + Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d*e*x^3
+ I*Sqrt[a]*Sqrt[c]*d*e*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqr
t[c]/Sqrt[a])]*x], -1] - I*Sqrt[c]*d*(-(Sqrt[c]*d) + Sqrt[a]*e)*Sqrt[1 - (
c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (2*I)*a*e
^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sq
rt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(2*a*Sqrt[-(Sqrt[c]/Sqrt[a])]*d*(-(c*d^2)
+ a*e^2)*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {1550, 25, 27, 1493, 25, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-cx^4\right)^{3/2}(d+ex^2)}dx$$

$$\Big\downarrow \text{1550}$$

$$-\frac{e^2\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{cd^2-ae^2}-\frac{\int -\frac{c(d-ex^2)}{(a-cx^4)^{3/2}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{c\left(d-ex^2\right)}{\left(a-cx^4\right)^{3/2}}dx}{cd^2-ae^2}-\frac{e^2\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{d-ex^2}{\left(a-cx^4\right)^{3/2}}dx}{cd^2-ae^2}-\frac{e^2\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \; 1493$$

$$\frac{c\left(\frac{x\left(d-ex^2\right)}{2a\sqrt{a-cx^4}}-\frac{\int -\frac{ex^2+d}{\sqrt{a-cx^4}}dx}{2a}\right)}{cd^2-ae^2}-\frac{e^2\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \; 25$$

$$\frac{c\left(\frac{\int \frac{ex^2+d}{\sqrt{a-cx^4}}dx}{2a}+\frac{x\left(d-ex^2\right)}{2a\sqrt{a-cx^4}}\right)}{cd^2-ae^2}-\frac{e^2\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \; 1513$$

$$\frac{c\left(\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x\left(d-ex^2\right)}{2a\sqrt{a-cx^4}}\right)}{cd^2-ae^2}-\frac{e^2\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \; 27$$

$$\frac{c\left(\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x\left(d-ex^2\right)}{2a\sqrt{a-cx^4}}\right)}{cd^2-ae^2}-\frac{e^2\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \; 765$$

$$\frac{c\left(\frac{\frac{\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{2a}+\frac{x\left(d-ex^2\right)}{2a\sqrt{a-cx^4}}\right)}{cd^2-ae^2}-\frac{e^2\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \; 762$$

$$\frac{c\left(\frac{\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{2a}+\frac{x(d-ex^2)}{2a\sqrt{a-cx^4}}\right)}{cd^2-ae^2}-\frac{e^2\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \; 1390$$

$$\frac{c\left(\frac{\frac{e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{2a}+\frac{x(d-ex^2)}{2a\sqrt{a-cx^4}}\right)}{cd^2-ae^2}-\frac{e^2\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \; 1389$$

$$\frac{c\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{2a}+\frac{x(d-ex^2)}{2a\sqrt{a-cx^4}}\right)}{cd^2-ae^2}-\frac{e^2\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \; 327$$

$$\frac{c\left(\frac{\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{2a}+\frac{x(d-ex^2)}{2a\sqrt{a-cx^4}}\right)}{cd^2-ae^2}-\frac{e^2\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{cd^2-ae^2}$$

$$\Big\downarrow \; 1543$$

$$\frac{c\left(\frac{\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{2a}+\frac{x(d-ex^2)}{2a\sqrt{a-cx^4}}\right)}{cd^2-ae^2}-\frac{e^2\sqrt{1-\frac{cx^4}{a}}\int\frac{1}{(ex^2+d)\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}\,(cd^2-ae^2)}$$

$$\Big\downarrow \text{1542}$$

$$\frac{c\left(\frac{\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{2a}+\frac{x(d-ex^2)}{2a\sqrt{a-cx^4}}\right)}{cd^2-ae^2}-\frac{\sqrt[4]{a}e^2\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}\,(cd^2-ae^2)}$$

input

```
Int[1/((d + e*x^2)*(a - c*x^4)^(3/2)),x]
```

output

```
(c*((x*(d - e*x^2))/(2*a*Sqrt[a - c*x^4]) + ((a^(3/4)*e*Sqrt[1 - (c*x^4)/a
]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) +
(a^(1/4)*(d - (Sqrt[a]*e)/Sqrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c
^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4]))/(2*a)))/(c*d^2 - a*e^2
) - (a^(1/4)*e^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d))
, ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*(c*d^2 - a*e^2)*Sqrt[a - c*
x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 1550
```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[(c
*d^2 + a*e^2)^(p + 1/2)/e^(2*p + 1)   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]),
x], x] + Simp[(c*d^2 + a*e^2)^(p + 1/2)   Int[(a + c*x^4)^p*ExpandToSum[((c
*d^2 + a*e^2)^(-p - 1/2) - e^(-2*p - 1)*(a + c*x^4)^(-p - 1/2))/(d + e*x^2)
, x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[p +
 1/2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.55

method	result
default	$\frac{2c\left(\frac{e\,x^{3}}{4a\left(a\,e^{2}-c\,d^{2}\right)}-\frac{d x}{4a\left(a\,e^{2}-c\,d^{2}\right)}\right)}{\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}-\frac{c d\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a(a\,e^{2}-c\,d^{2})\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+\frac{\sqrt{c}\,e\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\Big(}{2\sqrt{a}\,(a\,e^{2}-c\,d^{2})\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}}}$
elliptic	$\frac{2c\left(\frac{e\,x^{3}}{4a\left(a\,e^{2}-c\,d^{2}\right)}-\frac{d x}{4a\left(a\,e^{2}-c\,d^{2}\right)}\right)}{\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}-\frac{c d\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a(a\,e^{2}-c\,d^{2})\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+\frac{\sqrt{c}\,e\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\Big(}{2\sqrt{a}\,(a\,e^{2}-c\,d^{2})\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}}}$

input
```
int(1/(e*x^2+d)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2*c*(1/4/a*e/(a*e^2-c*d^2)*x^3-1/4*d/a/(a*e^2-c*d^2)*x)/(-(x^4-1/c*a)*c)^(
1/2)-1/2*c/a*d/(a*e^2-c*d^2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2
)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*
(1/a^(1/2)*c^(1/2))^(1/2),I)+1/2*c^(1/2)/a^(1/2)*e/(a*e^2-c*d^2)/(1/a^(1/2
)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)
^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-1/2*c^(1/
2)/a^(1/2)*e/(a*e^2-c*d^2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1
/a^(1/2)*c^(1/2))^(1/2),I)+1/(a*e^2-c*d^2)*e^2/d/(1/a^(1/2)*c^(1/2))^(1/2)
*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a
)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-1/a^
(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)(a-cx^4)^{3/2}}\,dx=\int \frac{1}{(-cx^4+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

input

```
integrate(1/(e*x^2+d)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output

```
integral(sqrt(-c*x^4 + a)/(c^2*e*x^10 + c^2*d*x^8 - 2*a*c*e*x^6 - 2*a*c*d*
x^4 + a^2*e*x^2 + a^2*d), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)(a-cx^4)^{3/2}}\,dx=\int \frac{1}{(a-cx^4)^{\frac{3}{2}}(d+ex^2)}\,dx$$

input

```
integrate(1/(e*x**2+d)/(-c*x**4+a)**(3/2),x)
```

output

```
Integral(1/((a - c*x**4)**(3/2)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{1}{(-cx^4+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{1}{(-cx^4+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(a-c\,x^4\right)^{3/2}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)),x)
```

output
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)\left(a-cx^4\right)^{3/2}}\,dx = \int \frac{\sqrt{-c\,x^4+a}}{c^2e\,x^{10}+c^2d\,x^8-2ace\,x^6-2acd\,x^4+a^2e\,x^2+a^2d}dx$$

input

```
int(1/(e*x^2+d)/(-c*x^4+a)^(3/2),x)
```

output

```
int(sqrt(a - c*x**4)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 +
 c**2*d*x**8 + c**2*e*x**10),x)
```

3.418 $\int \frac{1}{\left(d+ex^2\right)^2\left(a-cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 420

$$
\begin{aligned}
\int \frac{1}{\left(d+ex^2\right)^2\left(a-cx^4\right)^{3/2}}\,dx &= \frac{cx(d-ex^2)}{2a\left(cd^2-ae^2\right)\left(d+ex^2\right)\sqrt{a-cx^4}} \\
&+ \frac{e^2(2cd^2+ae^2)\,x\sqrt{a-cx^4}}{2ad\left(cd^2-ae^2\right)^2\left(d+ex^2\right)} \\
&+ \frac{\sqrt[4]{c}\,e(2cd^2+ae^2)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}\,d\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}} \\
&+ \frac{\sqrt[4]{c}\left(cd^2-\sqrt{a}\sqrt{c}de+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}d\left(\sqrt{c}d+\sqrt{a}e\right)\left(cd^2-ae^2\right)\sqrt{a-cx^4}} \\
&- \frac{\sqrt[4]{a}\,e^2(7cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{c}\,d^2\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}}
\end{aligned}
$$

output
```
1/2*c*x*(-e*x^2+d)/a/(-a*e^2+c*d^2)/(e*x^2+d)/(-c*x^4+a)^(1/2)+1/2*e^2*(a*
e^2+2*c*d^2)*x*(-c*x^4+a)^(1/2)/a/d/(-a*e^2+c*d^2)^2/(e*x^2+d)+1/2*c^(1/4)
*e*(a*e^2+2*c*d^2)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(1/4
)/d/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)+1/2*c^(1/4)*(c*d^2-a^(1/2)*c^(1/2)*d
*e+a*e^2)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(3/4)/d/(c^(1
/2)*d+a^(1/2)*e)/(-a*e^2+c*d^2)/(-c*x^4+a)^(1/2)-1/2*a^(1/4)*e^2*(-a*e^2+7
*c*d^2)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/
d,I)/c^(1/4)/d^2/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.64 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.79

$$\int \frac{1}{(d+ex^2)^2\,(a-cx^4)^{3/2}}\,dx = \frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}d(ae^4x(a-cx^4)+cdx(d+ex^2)\left(ae^2+cd(d-2ex^2)\right))-i(d+ex^2)}{}$$

input
```
Integrate[1/((d + e*x^2)^2*(a - c*x^4)^(3/2)),x]
```

output
```
(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*(a*e^4*x*(a - c*x^4) + c*d*x*(d + e*x^2)*(a*e^
2 + c*d*(d - 2*e*x^2))) - I*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*(Sqrt[a]*Sqrt[
c]*d*e*(2*c*d^2 + a*e^2)*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x],
-1] + Sqrt[c]*d*(c^(3/2)*d^3 - 2*Sqrt[a]*c*d^2*e + 2*a*Sqrt[c]*d*e^2 - a^(
3/2)*e^3)*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + a*e^2*(-7
*c*d^2 + a*e^2)*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sq
rt[c]/Sqrt[a])]*x], -1]))/(2*a*Sqrt[-(Sqrt[c]/Sqrt[a])]*(c*d^3 - a*d*e^2)^
2*(d + e*x^2)*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1557, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-cx^4\right)^{3/2}\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{1557}$$

$$\int \left(-\frac{2cde^2}{\sqrt{a-cx^4}\left(d+ex^2\right)\left(ae^2-cd^2\right)^2}+\frac{e^2}{\sqrt{a-cx^4}\left(d+ex^2\right)^2\left(ae^2-cd^2\right)}-\frac{c\left(-ae^2-cd^2+2cdex^2\right)}{\left(a-cx^4\right)^{3/2}\left(ae^2-cd^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^{3/4}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}\sqrt{a-cx^4}\left(\sqrt{a}e+\sqrt{c}d\right)^2}+\frac{a^{3/4}\sqrt[4]{c}e^3\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}+\frac{c^{5/4}de\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}-\frac{2\sqrt[4]{a}c^{3/4}e^2\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}+\frac{\sqrt[4]{a}\sqrt[4]{c}e^2\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2d\sqrt{a-cx^4}\left(\sqrt{a}e+\sqrt{c}d\right)\left(cd^2-ae^2\right)}-\frac{\sqrt[4]{a}e^2\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{c}d^2\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}+\frac{cx\left(ae^2+cd^2-2cdex^2\right)}{2a\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}+\frac{e^4x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)^2}$$

input
```
Int[1/((d + e*x^2)^2*(a - c*x^4)^(3/2)),x]
```

output
```
(c*x*(c*d^2 + a*e^2 - 2*c*d*e*x^2))/(2*a*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]
) + (e^4*x*Sqrt[a - c*x^4])/(2*d*(c*d^2 - a*e^2)^2*(d + e*x^2)) + (c^(5/4)
*d*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(a^(1
/4)*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) + (a^(3/4)*c^(1/4)*e^3*Sqrt[1 - (c*
x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*(c*d^2 - a*e^2)^2
*Sqrt[a - c*x^4]) + (c^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)
*x)/a^(1/4)], -1])/(2*a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)^2*Sqrt[a - c*x^4]) +
 (a^(1/4)*c^(1/4)*e^2*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(
1/4)], -1])/(2*d*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 - a*e^2)*Sqrt[a - c*x^4])
- (2*a^(1/4)*c^(3/4)*e^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqr
t[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/((c*d^2 - a*e^2)^2*Sqrt[a - c*
x^4]) - (a^(1/4)*e^2*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((S
qrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*c^(1/4)*d^2*(
c*d^2 - a*e^2)^2*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 1557
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Mod
ule[{aa, cc}, Int[ExpandIntegrand[1/Sqrt[aa + cc*x^4], (d + e*x^2)^q*(aa +
cc*x^4)^(p + 1/2), x] /. {aa -> a, cc -> c}, x]] /; FreeQ[{a, c, d, e}, x]
&& NeQ[c*d^2 + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 864 vs. 2(358) = 716.

Time = 0.69 (sec) , antiderivative size = 865, normalized size of antiderivative = 2.06

method	result
default	$\frac{2c\left(-\frac{ecd\,x^{3}}{2a\left(a\,e^{2}-c\,d^{2}\right)^{2}}+\frac{\left(a\,e^{2}+c\,d^{2}\right)x}{4a\left(a\,e^{2}-c\,d^{2}\right)^{2}}\right)}{\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}+\frac{e^{4}x\sqrt{-c\,x^{4}+a}}{2d(a\,e^{2}-c\,d^{2})^{2}(e\,x^{2}+d)}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)e^{2}c}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}\,(a\,e^{2}-c\,d^{2})^{2}}+$
elliptic	$\frac{2c\left(-\frac{ecd\,x^{3}}{2a\left(a\,e^{2}-c\,d^{2}\right)^{2}}+\frac{\left(a\,e^{2}+c\,d^{2}\right)x}{4a\left(a\,e^{2}-c\,d^{2}\right)^{2}}\right)}{\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}+\frac{e^{4}x\sqrt{-c\,x^{4}+a}}{2d(a\,e^{2}-c\,d^{2})^{2}(e\,x^{2}+d)}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)e^{2}c}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}\,(a\,e^{2}-c\,d^{2})^{2}}+$

input

```
int(1/(e*x^2+d)^2/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2*c*(-1/2/a*e*c*d/(a*e^2-c*d^2)^2*x^3+1/4*(a*e^2+c*d^2)/a/(a*e^2-c*d^2)^2*
x)/(-(x^4-1/c*a)*c)^(1/2)+1/2*e^4/d/(a*e^2-c*d^2)^2*x*(-c*x^4+a)^(1/2)/(e*
x^2+d)+1/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^
(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^
(1/2),I)*e^2*c/(a*e^2-c*d^2)^2+1/2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*
c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Ellipt
icF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*c^2/a/(a*e^2-c*d^2)^2*d^2-1/a^(1/2)/(1/
a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2
)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*c^(3/2)*e*d/(a*e^2-c*d^2)^2*EllipticF(x*(1/a
^(1/2)*c^(1/2))^(1/2),I)+1/a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*
c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*c^(3/2
)*e*d/(a*e^2-c*d^2)^2*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-1/2*a^(1/2)
/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d*EllipticF(
x*(1/a^(1/2)*c^(1/2))^(1/2),I)+1/2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/
a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2
)*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+1
/2*e^4/(a*e^2-c*d^2)^2/d^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(
1/a^(1/2)*c^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-1/a^(1/2)*c^(1/2))^(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2 (a-cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^2 (a-cx^4)^{3/2}}\,dx = \int \frac{1}{(a-cx^4)^{\frac{3}{2}} (d+ex^2)^2}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(-c*x**4+a)**(3/2),x)
```

output
```
Integral(1/((a - c*x**4)**(3/2)*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^2 (a-cx^4)^{3/2}}\,dx = \int \frac{1}{(-cx^4+a)^{\frac{3}{2}} (ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex^2\right)^2 \left(a-cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-cx^4+a\right)^{\frac{3}{2}} \left(ex^2+d\right)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2 \left(a-cx^4\right)^{3/2}}\,dx=\int \frac{1}{{\left(a-c\,x^4\right)}^{3/2}\,{\left(e\,x^2+d\right)}^2}\,dx$$

input
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+ex^2\right)^2 \left(a-cx^4\right)^{3/2}}\,dx=\int \frac{\sqrt{-c\,x^4+a}}{c^2e^2x^{12}+2c^2de\,x^{10}-2ac\,e^2x^8+c^2d^2x^8-4acde\,x^6+a^2e^2x^4-2ac\,d^2x^4$$

input
```
int(1/(e*x^2+d)^2/(-c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a**2*d**2 + 2*a**2*d*e*x**2 + a**2*e**2*x**4 - 2*a*c
*d**2*x**4 - 4*a*c*d*e*x**6 - 2*a*c*e**2*x**8 + c**2*d**2*x**8 + 2*c**2*d*
e*x**10 + c**2*e**2*x**12),x)
```

3.419 $\int \frac{1}{(d+ex^2)^3 (a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 562

$$\int \frac{1}{(d+ex^2)^3 (a-cx^4)^{3/2}}\,dx = \frac{cx(d-ex^2)}{2a(cd^2-ae^2)(d+ex^2)^2\sqrt{a-cx^4}}$$

$$+\frac{e^2(4cd^2+ae^2)x\sqrt{a-cx^4}}{4ad(cd^2-ae^2)^2(d+ex^2)^2}+\frac{3e^2(4c^2d^4+7acd^2e^2-a^2e^4)x\sqrt{a-cx^4}}{8ad^2(cd^2-ae^2)^3(d+ex^2)}$$

$$+\frac{3\sqrt[4]{c}e(4c^2d^4+7acd^2e^2-a^2e^4)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{a}d^2(cd^2-ae^2)^3\sqrt{a-cx^4}}$$

$$+\frac{\sqrt[4]{c}\left(4c^2d^4-8\sqrt{a}c^{3/2}d^3e+19acd^2e^2-2a^{3/2}\sqrt{c}de^3-3a^2e^4\right)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8a^{3/4}d^2\left(\sqrt{c}d-\sqrt{a}e\right)^2\left(\sqrt{c}d+\sqrt{a}e\right)^3\sqrt{a-cx^4}}$$

$$-\frac{3\sqrt[4]{a}e^2(21c^2d^4-2acd^2e^2+a^2e^4)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{c}d^3(cd^2-ae^2)^3\sqrt{a-cx^4}}$$

output

```
1/2*c*x*(-e*x^2+d)/a/(-a*e^2+c*d^2)/(e*x^2+d)^2/(-c*x^4+a)^(1/2)+1/4*e^2*(
a*e^2+4*c*d^2)*x*(-c*x^4+a)^(1/2)/a/d/(-a*e^2+c*d^2)^2/(e*x^2+d)^2+3/8*e^2
*(-a^2*e^4+7*a*c*d^2*e^2+4*c^2*d^4)*x*(-c*x^4+a)^(1/2)/a/d^2/(-a*e^2+c*d^2
)^3/(e*x^2+d)+3/8*c^(1/4)*e*(-a^2*e^4+7*a*c*d^2*e^2+4*c^2*d^4)*(1-c*x^4/a)
^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(1/4)/d^2/(-a*e^2+c*d^2)^3/(-c*x^4
+a)^(1/2)+1/8*c^(1/4)*(4*c^2*d^4-8*a^(1/2)*c^(3/2)*d^3*e+19*a*c*d^2*e^2-2*
a^(3/2)*c^(1/2)*d*e^3-3*a^2*e^4)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(
1/4),I)/a^(3/4)/d^2/(c^(1/2)*d-a^(1/2)*e)^2/(c^(1/2)*d+a^(1/2)*e)^3/(-c*x^
4+a)^(1/2)-3/8*a^(1/4)*e^2*(a^2*e^4-2*a*c*d^2*e^2+21*c^2*d^4)*(1-c*x^4/a)^
(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/d,I)/c^(1/4)/d^3/(-a
*e^2+c*d^2)^3/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.10 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.82

$$\int \frac{1}{(d+ex^2)^3 (a-cx^4)^{3/2}}\,dx = \frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}dx\bigl(2ade^4(cd^2-ae^2)(a-cx^4)+ae^4(17cd^2-3ae^2)(d+ex^2)(a-}{}$$

input

```
Integrate[1/((d + e*x^2)^3*(a - c*x^4)^(3/2)),x]
```

output

```
(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*x*(2*a*d*e^4*(c*d^2 - a*e^2)*(a - c*x^4) + a*e
^4*(17*c*d^2 - 3*a*e^2)*(d + e*x^2)*(a - c*x^4) + 4*c^2*d^2*(d + e*x^2)^2*
(c*d^2*(d - 3*e*x^2) + a*e^2*(3*d - e*x^2))) + I*(d + e*x^2)^2*Sqrt[1 - (c
*x^4)/a]*(3*Sqrt[a]*Sqrt[c]*d*e*(-4*c^2*d^4 - 7*a*c*d^2*e^2 + a^2*e^4)*Ell
ipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (-4*c^3*d^6 + 12*Sqrt[
a]*c^(5/2)*d^5*e - 27*a*c^2*d^4*e^2 + 21*a^(3/2)*c^(3/2)*d^3*e^3 + a^2*c*d
^2*e^4 - 3*a^(5/2)*Sqrt[c]*d*e^5)*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[
a])]*x], -1] + 3*a*e^2*(21*c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*EllipticPi[-
((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/(8
*a*Sqrt[-(Sqrt[c]/Sqrt[a])]*(c*d^3 - a*d*e^2)^3*(d + e*x^2)^2*Sqrt[a - c*x
^4])
```

Rubi [A] (verified)

Time = 3.29 (sec) , antiderivative size = 1066, normalized size of antiderivative = 1.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1557, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^4)^{3/2}(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{1557}$$

$$\int \Bigg(\frac{c^2\left(d\left(-3ae^2-cd^2\right)-ex^2\left(-ae^2-3cd^2\right)\right)}{(a-cx^4)^{3/2}(ae^2-cd^2)^3} - \frac{ce^2\left(-ae^2-3cd^2\right)}{\sqrt{a-cx^4}\,(d+ex^2)(ae^2-cd^2)^3} - \frac{2cde^2}{\sqrt{a-cx^4}\,(d+ex^2)^2\,(ae^2 -}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}e^4}{8d^2\left(cd^2-ae^2\right)^3\left(ex^2+d\right)}+\frac{cx\sqrt{a-cx^4}e^4}{\left(cd^2-ae^2\right)^3\left(ex^2+d\right)}+\frac{x\sqrt{a-cx^4}e^4}{4d\left(cd^2-ae^2\right)^2\left(ex^2+d\right)^2}+$$
$$\frac{3a^{3/4}\sqrt[4]{c}\left(3cd^2-ae^2\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)e^3}{8d^2\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}+$$
$$\frac{a^{3/4}c^{5/4}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)e^3}{\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}+$$
$$\frac{\sqrt[4]{a}\sqrt[4]{c}\left(7cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2\right)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{8d^2\left(\sqrt{c}d-\sqrt{a}e\right)^2\left(\sqrt{c}d+\sqrt{a}e\right)^3\sqrt{a-cx^4}}+$$
$$\frac{\sqrt[4]{a}c^{5/4}\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{\left(\sqrt{c}d+\sqrt{a}e\right)\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}}-$$
$$\frac{\sqrt[4]{a}c^{3/4}\left(3cd^2-ae^2\right)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{d\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}-$$
$$\frac{\sqrt[4]{a}c^{3/4}\left(3cd^2+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{d\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}-$$
$$\frac{3\sqrt[4]{a}\left(5c^2d^4-2ace^2d^2+a^2e^4\right)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{8\sqrt[4]{c}d^3\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}+$$
$$\frac{c^{5/4}\left(3cd^2+ae^2\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)e}{2\sqrt[4]{a}\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}+$$
$$\frac{c^{5/4}\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)^3\sqrt{a-cx^4}}+\frac{c^2x\left(d\left(cd^2+3ae^2\right)-e\left(3cd^2+ae^2\right)x^2\right)}{2a\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}$$

input

```
Int[1/((d + e*x^2)^3*(a - c*x^4)^(3/2)),x]
```

output

```
(c^2*x*(d*(c*d^2 + 3*a*e^2) - e*(3*c*d^2 + a*e^2)*x^2))/(2*a*(c*d^2 - a*e^
2)^3*Sqrt[a - c*x^4]) + (e^4*x*Sqrt[a - c*x^4])/(4*d*(c*d^2 - a*e^2)^2*(d
+ e*x^2)^2) + (c*e^4*x*Sqrt[a - c*x^4])/((c*d^2 - a*e^2)^3*(d + e*x^2)) +
(3*e^4*(3*c*d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(8*d^2*(c*d^2 - a*e^2)^3*(d +
e*x^2)) + (a^(3/4)*c^(5/4)*e^3*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/
4)*x)/a^(1/4)], -1])/((c*d^2 - a*e^2)^3*Sqrt[a - c*x^4]) + (3*a^(3/4)*c^(1
/4)*e^3*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)
/a^(1/4)], -1])/(8*d^2*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4]) + (c^(5/4)*e*(3*
c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)],
-1])/(2*a^(1/4)*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4]) + (c^(5/4)*Sqrt[1 - (c*
x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*(Sqrt[c]*d
+ Sqrt[a]*e)^3*Sqrt[a - c*x^4]) + (a^(1/4)*c^(1/4)*e^2*(7*c*d^2 - 2*Sqrt[a
]*Sqrt[c]*d*e - 3*a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/
a^(1/4)], -1])/(8*d^2*(Sqrt[c]*d - Sqrt[a]*e)^2*(Sqrt[c]*d + Sqrt[a]*e)^3*
Sqrt[a - c*x^4]) + (a^(1/4)*c^(5/4)*e^2*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcS
in[(c^(1/4)*x)/a^(1/4)], -1])/((Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 - a*e^2)^2*S
qrt[a - c*x^4]) - (a^(1/4)*c^(3/4)*e^2*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/
a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1]
)/(d*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4]) - (a^(1/4)*c^(3/4)*e^2*(3*c*d^2 +
a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSi...
```

Defintions of rubi rules used

rule 1557

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Mod
ule[{aa, cc}, Int[ExpandIntegrand[1/Sqrt[aa + cc*x^4], (d + e*x^2)^q*(aa +
cc*x^4)^(p + 1/2), x] /. {aa -> a, cc -> c}, x]] /; FreeQ[{a, c, d, e}, x]
&& NeQ[c*d^2 + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1323 vs. 2(488) = 976.

Time = 1.20 (sec) , antiderivative size = 1324, normalized size of antiderivative = 2.36

method	result	size
default	Expression too large to display	1324
elliptic	Expression too large to display	1324

input

```
int(1/(e*x^2+d)^3/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2*c*(1/4*c*e*(a*e^2+3*c*d^2)/a/(a*e^2-c*d^2)^3*x^3-1/4*c*d*(3*a*e^2+c*d^2)
/a/(a*e^2-c*d^2)^3*x)/(-(x^4-1/c*a)*c)^(1/2)+1/4*e^4/d/(a*e^2-c*d^2)^2*x*(
-c*x^4+a)^(1/2)/(e*x^2+d)^2+1/8*e^4*(3*a*e^2-17*c*d^2)/(a*e^2-c*d^2)^3/d^2
*x*(-c*x^4+a)^(1/2)/(e*x^2+d)-27/8/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*
c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Ellipt
icF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*c^2*d/(a*e^2-c*d^2)^3*e^2-1/2/(1/a^(1/2
)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)
^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*c^3*d^3/a
/(a*e^2-c*d^2)^3+1/8/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(
1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/
2)*c^(1/2))^(1/2),I)*e^4*c/d/(a*e^2-c*d^2)^3*a+21/8*a^(1/2)/(1/a^(1/2)*c^(
1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(-c*x^4+a)^(1/2)*c^(3/2)*e^3/(a*e^2-c*d^2)^3*EllipticF(x*(1/a^(1/2)*c^(1
/2))^(1/2),I)-21/8*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*c^(3/2)*e^3/(a
*e^2-c*d^2)^3*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+3/2/a^(1/2)/(1/a^(1
/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^
2)^(1/2)/(-c*x^4+a)^(1/2)*c^(5/2)*e/(a*e^2-c*d^2)^3*d^2*EllipticF(x*(1/a^(
1/2)*c^(1/2))^(1/2),I)-3/2/a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*
c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*c^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^3 (a-cx^4)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^3 (a-cx^4)^{3/2}} \, dx = \int \frac{1}{(a-cx^4)^{\frac{3}{2}} (d+ex^2)^3} \, dx$$

input
```
integrate(1/(e*x**2+d)**3/(-c*x**4+a)**(3/2),x)
```

output
```
Integral(1/((a - c*x**4)**(3/2)*(d + e*x**2)**3), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^3 (a-cx^4)^{3/2}} \, dx = \int \frac{1}{(-cx^4+a)^{\frac{3}{2}} (ex^2+d)^3} \, dx$$

input
```
integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^3\,(a-cx^4)^{3/2}}\,dx = \int \frac{1}{(-cx^4+a)^{\frac{3}{2}}(ex^2+d)^3}\,dx$$

input
```
integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^3\,(a-cx^4)^{3/2}}\,dx = \int \frac{1}{(a-c\,x^4)^{3/2}\,(e\,x^2+d)^3}\,dx$$

input
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)^3),x)
```

output
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^3\,(a-cx^4)^{3/2}}\,dx = \int \frac{\sqrt{-c\,x^4+}}{c^2e^3x^{14}+3c^2d\,e^2x^{12}-2ac\,e^3x^{10}+3c^2d^2e\,x^{10}-6acd\,e^2x^8+c^2d^3x^8+a^2}$$

input
```
int(1/(e*x^2+d)^3/(-c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a**2*d**3 + 3*a**2*d**2*e*x**2 + 3*a**2*d*e**2*x**4
+ a**2*e**3*x**6 - 2*a*c*d**3*x**4 - 6*a*c*d**2*e*x**6 - 6*a*c*d*e**2*x**8
 - 2*a*c*e**3*x**10 + c**2*d**3*x**8 + 3*c**2*d**2*e*x**10 + 3*c**2*d*e**2
*x**12 + c**2*e**3*x**14),x)
```

3.420 $\int \frac{1}{(d+ex^2)^4(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 713

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^4(a-cx^4)^{3/2}}\,dx ={}& \frac{cx(d-ex^2)}{2a(cd^2-ae^2)(d+ex^2)^3\sqrt{a-cx^4}} \\
&+ \frac{e^2(6cd^2+ae^2)x\sqrt{a-cx^4}}{6ad(cd^2-ae^2)^2(d+ex^2)^3} + \frac{e^2(36c^2d^4+39acd^2e^2-5a^2e^4)x\sqrt{a-cx^4}}{24ad^2(cd^2-ae^2)^3(d+ex^2)^2} \\
&+ \frac{e^2(32c^3d^6+121ac^2d^4e^2-18a^2cd^2e^4+5a^3e^6)x\sqrt{a-cx^4}}{16ad^3(cd^2-ae^2)^4(d+ex^2)} \\
&+ \frac{\sqrt[4]{c}e(32c^3d^6+121ac^2d^4e^2-18a^2cd^2e^4+5a^3e^6)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{16\sqrt[4]{a}d^3(cd^2-ae^2)^4\sqrt{a-cx^4}} \\
&+ \frac{\sqrt[4]{c}\left(24c^3d^6-72\sqrt{a}c^{5/2}d^5e+285ac^2d^4e^2-78a^{3/2}c^{3/2}d^3e^3-44a^2cd^2e^4+10a^{5/2}\sqrt{c}de^5+15a^3e^6\right)\sqrt{1-}}{48a^{3/4}d^3\left(\sqrt{c}d-\sqrt{a}e\right)^3\left(\sqrt{c}d+\sqrt{a}e\right)^4\sqrt{a-cx^4}} \\
&- \frac{\sqrt[4]{a}e^2(231c^3d^6+33ac^2d^4e^2+21a^2cd^2e^4-5a^3e^6)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{16\sqrt[4]{c}d^4(cd^2-ae^2)^4\sqrt{a-cx^4}}
\end{aligned}$$

output
```
1/2*c*x*(-e*x^2+d)/a/(-a*e^2+c*d^2)/(e*x^2+d)^3/(-c*x^4+a)^(1/2)+1/6*e^2*(
a*e^2+6*c*d^2)*x*(-c*x^4+a)^(1/2)/a/d/(-a*e^2+c*d^2)^2/(e*x^2+d)^3+1/24*e^
2*(-5*a^2*e^4+39*a*c*d^2*e^2+36*c^2*d^4)*x*(-c*x^4+a)^(1/2)/a/d^2/(-a*e^2+
c*d^2)^3/(e*x^2+d)^2+1/16*e^2*(5*a^3*e^6-18*a^2*c*d^2*e^4+121*a*c^2*d^4*e^
2+32*c^3*d^6)*x*(-c*x^4+a)^(1/2)/a/d^3/(-a*e^2+c*d^2)^4/(e*x^2+d)+1/16*c^(
1/4)*e*(5*a^3*e^6-18*a^2*c*d^2*e^4+121*a*c^2*d^4*e^2+32*c^3*d^6)*(1-c*x^4/
a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(1/4)/d^3/(-a*e^2+c*d^2)^4/(-c*x
^4+a)^(1/2)+1/48*c^(1/4)*(24*c^3*d^6-72*a^(1/2)*c^(5/2)*d^5*e+285*a*c^2*d^
4*e^2-78*a^(3/2)*c^(3/2)*d^3*e^3-44*a^2*c*d^2*e^4+10*a^(5/2)*c^(1/2)*d*e^5
+15*a^3*e^6)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(3/4)/d^3/
(c^(1/2)*d-a^(1/2)*e)^3/(c^(1/2)*d+a^(1/2)*e)^4/(-c*x^4+a)^(1/2)-1/16*a^(1
/4)*e^2*(-5*a^3*e^6+21*a^2*c*d^2*e^4+33*a*c^2*d^4*e^2+231*c^3*d^6)*(1-c*x^
4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/d,I)/c^(1/4)/d^
4/(-a*e^2+c*d^2)^4/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.55 (sec) , antiderivative size = 602, normalized size of antiderivative = 0.84

$$\int \frac{1}{(d+ex^2)^4 (a-cx^4)^{3/2}}\,dx = \frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}dx\Big(8ae^4(cd^3-ade^2)^2(a-cx^4)+2ade^4(-cd^2+ae^2)(-27cd^2+5a}{}$$

input
```
Integrate[1/((d + e*x^2)^4*(a - c*x^4)^(3/2)),x]
```

output

```
(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*x*(8*a*e^4*(c*d^3 - a*d*e^2)^2*(a - c*x^4) + 2
*a*d*e^4*(-(c*d^2) + a*e^2)*(-27*c*d^2 + 5*a*e^2)*(d + e*x^2)*(a - c*x^4)
+ 3*a*e^4*(89*c^2*d^4 - 18*a*c*d^2*e^2 + 5*a^2*e^4)*(d + e*x^2)^2*(a - c*x
^4) + 24*c^2*d^3*(d + e*x^2)^3*(a^2*e^4 + c^2*d^3*(d - 4*e*x^2) + 2*a*c*d*
e^2*(3*d - 2*e*x^2))) - I*(d + e*x^2)^3*Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*Sqr
t[c]*d*e*(32*c^3*d^6 + 121*a*c^2*d^4*e^2 - 18*a^2*c*d^2*e^4 + 5*a^3*e^6)*E
llipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + Sqrt[c]*d*(24*c^(7/2
)*d^7 - 96*Sqrt[a]*c^3*d^6*e + 357*a*c^(5/2)*d^5*e^2 - 363*a^(3/2)*c^2*d^4
*e^3 + 34*a^2*c^(3/2)*d^3*e^4 + 54*a^(5/2)*c*d^2*e^5 + 5*a^3*Sqrt[c]*d*e^6
 - 15*a^(7/2)*e^7)*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] +
3*a*e^2*(-231*c^3*d^6 - 33*a*c^2*d^4*e^2 - 21*a^2*c*d^2*e^4 + 5*a^3*e^6)*E
llipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x
], -1]))/(48*a*Sqrt[-(Sqrt[c]/Sqrt[a])]*(c*d^3 - a*d*e^2)^4*(d + e*x^2)^3*
Sqrt[a - c*x^4])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1700 vs. $2(713) = 1426$.

Time = 5.70 (sec) , antiderivative size = 1700, normalized size of antiderivative = 2.38, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1557, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^4)^{3/2}(d+ex^2)^4} \, dx$$

$$\Big\downarrow \text{1557}$$

$$\int \left(\frac{c^2(a^2e^4+4cdex^2(-ae^2-cd^2)+6acd^2e^2+c^2d^4)}{(a-cx^4)^{3/2}(ae^2-cd^2)^4} + \frac{4c^2de^2(-ae^2-cd^2)}{\sqrt{a-cx^4}(d+ex^2)(ae^2-cd^2)^4} - \frac{ce^2(-ae^2 -}{\sqrt{a-cx^4}(d+ex^2)} \right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3c\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}e^4}{4d\left(cd^2-ae^2\right)^4\left(ex^2+d\right)}+\frac{c\left(3cd^2+ae^2\right)x\sqrt{a-cx^4}e^4}{2d\left(cd^2-ae^2\right)^4\left(ex^2+d\right)}+$$

$$\frac{\left(29c^2d^4-14ace^2d^2+5a^2e^4\right)x\sqrt{a-cx^4}e^4}{16d^3\left(cd^2-ae^2\right)^4\left(ex^2+d\right)}+\frac{5\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}e^4}{24d^2\left(cd^2-ae^2\right)^3\left(ex^2+d\right)^2}+$$

$$\frac{cx\sqrt{a-cx^4}e^4}{2\left(cd^2-ae^2\right)^3\left(ex^2+d\right)^2}+\frac{x\sqrt{a-cx^4}e^4}{6d\left(cd^2-ae^2\right)^2\left(ex^2+d\right)^3}+$$

$$\frac{3a^{3/4}c^{5/4}\left(3cd^2-ae^2\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)e^3}{4d\left(cd^2-ae^2\right)^4\sqrt{a-cx^4}}+$$

$$\frac{a^{3/4}c^{5/4}\left(3cd^2+ae^2\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)e^3}{2d\left(cd^2-ae^2\right)^4\sqrt{a-cx^4}}+$$

$$\frac{a^{3/4}\sqrt[4]{c}\left(29c^2d^4-14ace^2d^2+5a^2e^4\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)e^3}{16d^3\left(cd^2-ae^2\right)^4\sqrt{a-cx^4}}+$$

$$\frac{\sqrt[4]{a}c^{5/4}\left(7cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{4d\left(\sqrt{c}d-\sqrt{a}e\right)^3\left(\sqrt{c}d+\sqrt{a}e\right)^4\sqrt{a-cx^4}}+$$

$$\frac{\sqrt[4]{a}c^{5/4}\left(3cd^2+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{2d\left(\sqrt{c}d+\sqrt{a}e\right)\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}+$$

$$\frac{\sqrt[4]{a}\sqrt[4]{c}\left(57c^2d^4-30\sqrt{a}c^{3/2}ed^3-32ace^2d^2+10a^{3/2}\sqrt{c}e^3d+15a^2e^4\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e}{48d^3\left(\sqrt{c}d-\sqrt{a}e\right)^3\left(\sqrt{c}d+\sqrt{a}e\right)^4\sqrt{a-cx^4}}$$

$$\frac{4\sqrt[4]{a}c^{7/4}\left(cd^2+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{\left(cd^2-ae^2\right)^4\sqrt{a-cx^4}}-$$

$$\frac{\sqrt[4]{a}c^{3/4}\left(9c^2d^4-a^2e^4\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{2d^2\left(cd^2-ae^2\right)^4\sqrt{a-cx^4}}-$$

$$\frac{3\sqrt[4]{a}c^{3/4}\left(5c^2d^4-2ace^2d^2+a^2e^4\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{4d^2\left(cd^2-ae^2\right)^4\sqrt{a-cx^4}}-$$

$$\frac{\sqrt[4]{a}\left(35c^3d^6-7ac^2e^2d^4+17a^2ce^4d^2-5a^3e^6\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{16\sqrt[4]{c}d^4\left(cd^2-ae^2\right)^4\sqrt{a-cx^4}}+$$

$$\frac{2c^{9/4}d\left(cd^2+ae^2\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)e}{\sqrt[4]{a}\left(cd^2-ae^2\right)^4\sqrt{a-cx^4}}+$$

$$\frac{c^{7/4}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)^4\sqrt{a-cx^4}}+$$

$$\frac{c^2x\left(c^2d^4+6ace^2d^2-4ce\left(cd^2+ae^2\right)x^2d+a^2e^4\right)}{2a\left(cd^2-ae^2\right)^4\sqrt{a-cx^4}}$$

input
```
Int[1/((d + e*x^2)^4*(a - c*x^4)^(3/2)),x]
```

output
```
(c^2*x*(c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*c*d*e*(c*d^2 + a*e^2)*x^2))/
(2*a*(c*d^2 - a*e^2)^4*Sqrt[a - c*x^4]) + (e^4*x*Sqrt[a - c*x^4])/(6*d*(c*
d^2 - a*e^2)^2*(d + e*x^2)^3) + (c*e^4*x*Sqrt[a - c*x^4])/(2*(c*d^2 - a*e^
2)^3*(d + e*x^2)^2) + (5*e^4*(3*c*d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(24*d^2*
(c*d^2 - a*e^2)^3*(d + e*x^2)^2) + (3*c*e^4*(3*c*d^2 - a*e^2)*x*Sqrt[a - c
*x^4])/(4*d*(c*d^2 - a*e^2)^4*(d + e*x^2)) + (c*e^4*(3*c*d^2 + a*e^2)*x*Sq
rt[a - c*x^4])/(2*d*(c*d^2 - a*e^2)^4*(d + e*x^2)) + (e^4*(29*c^2*d^4 - 14
*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[a - c*x^4])/(16*d^3*(c*d^2 - a*e^2)^4*(d
+ e*x^2)) + (3*a^(3/4)*c^(5/4)*e^3*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*E
llipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(4*d*(c*d^2 - a*e^2)^4*Sqrt[a -
 c*x^4]) + (2*c^(9/4)*d*e*(c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[Ar
cSin[(c^(1/4)*x)/a^(1/4)], -1])/(a^(1/4)*(c*d^2 - a*e^2)^4*Sqrt[a - c*x^4]
) + (a^(3/4)*c^(5/4)*e^3*(3*c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[A
rcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*(c*d^2 - a*e^2)^4*Sqrt[a - c*x^4]) +
 (a^(3/4)*c^(1/4)*e^3*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*Sqrt[1 - (
c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(16*d^3*(c*d^2 - a*e
^2)^4*Sqrt[a - c*x^4]) + (c^(7/4)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^
(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)^4*Sqrt[a - c*x^
4]) + (a^(1/4)*c^(5/4)*e^2*(7*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Sqr
t[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(4*d*(Sqrt...
```

Defintions of rubi rules used

rule 1557
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Mod
ule[{aa, cc}, Int[ExpandIntegrand[1/Sqrt[aa + cc*x^4], (d + e*x^2)^q*(aa +
cc*x^4)^(p + 1/2), x] /. {aa -> a, cc -> c}, x]] /; FreeQ[{a, c, d, e}, x]
&& NeQ[c*d^2 + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1796 vs. 2(631) = 1262.

Time = 1.56 (sec) , antiderivative size = 1797, normalized size of antiderivative = 2.52

method	result	size
default	Expression too large to display	1797
elliptic	Expression too large to display	1797

input

```
int(1/(e*x^2+d)^4/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2*c*(-c^2*e*d*(a*e^2+c*d^2)/a/(a*e^2-c*d^2)^4*x^3+1/4*c/a*(a^2*e^4+6*a*c*d
^2*e^2+c^2*d^4)/(a*e^2-c*d^2)^4*x)/(-(x^4-1/c*a)*c)^(1/2)+1/6*e^4/d/(a*e^2
-c*d^2)^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^3+1/24*e^4*(5*a*e^2-27*c*d^2)/(a*e^
2-c*d^2)^3/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+1/16*e^4*(5*a^2*e^4-18*a*c*d
^2*e^2+89*c^2*d^4)/(a*e^2-c*d^2)^4/d^3*x*(-c*x^4+a)^(1/2)/(e*x^2+d)-231/16
*e^2/(a*e^2-c*d^2)^4*d^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^
2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/
a^(1/2)*c^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-1/a^(1/2)*c^(1/2))^(1/2)/(1/
a^(1/2)*c^(1/2))^(1/2))*c^3+1/2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(
1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF
(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*c^4/a/(a*e^2-c*d^2)^4*d^4+5/16*e^8/(a*e^2-
c*d^2)^4/d^4/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1
/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1
/2))^(1/2))*a^3-33/16*e^4/(a*e^2-c*d^2)^4/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a
^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)
*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-1/a^(1/2)*c
^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a*c^2+5/48/(1/a^(1/2)*c^(1/2))^(1
/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^
4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*e^6*c/d^2/(a*e^2-c*...
```

Fricas [F]

$$\int \frac{1}{\left(d+e x^2\right)^4 \left(a-c x^4\right)^{3/2}} \, dx=\int \frac{1}{\left(-c x^4+a\right)^{\frac{3}{2}} \left(e x^2+d\right)^4} \, dx$$

input
```
integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*x^4 + a)/(c^2*e^4*x^16 + 4*c^2*d*e^3*x^14 + 2*(3*c^2*d^2*
e^2 - a*c*e^4)*x^12 + 4*(c^2*d^3*e - 2*a*c*d*e^3)*x^10 + (c^2*d^4 - 12*a*c
*d^2*e^2 + a^2*e^4)*x^8 + 4*a^2*d^3*e*x^2 - 4*(2*a*c*d^3*e - a^2*d*e^3)*x^
6 + a^2*d^4 - 2*(a*c*d^4 - 3*a^2*d^2*e^2)*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+e x^2\right)^4 \left(a-c x^4\right)^{3/2}} \, dx=\text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**4/(-c*x**4+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(d+e x^2\right)^4 \left(a-c x^4\right)^{3/2}} \, dx=\int \frac{1}{\left(-c x^4+a\right)^{\frac{3}{2}} \left(e x^2+d\right)^4} \, dx$$

input
```
integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^4), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^4\,(a-cx^4)^{3/2}}\,dx = \int \frac{1}{(-cx^4+a)^{\frac{3}{2}}(ex^2+d)^4}\,dx$$

input
```
integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^4\,(a-cx^4)^{3/2}}\,dx = \int \frac{1}{(a-c\,x^4)^{3/2}\,(e\,x^2+d)^4}\,dx$$

input
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)^4),x)
```

output
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)^4), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^4\,(a-cx^4)^{3/2}}\,dx = \int \frac{}{c^2e^4x^{16}+4c^2d\,e^3x^{14}-2ac\,e^4x^{12}+6c^2d^2e^2x^{12}-8acd\,e^3x^{10}+4c^2d^3e\,x^{10}}$$

input
```
int(1/(e*x^2+d)^4/(-c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a**2*d**4 + 4*a**2*d**3*e*x**2 + 6*a**2*d**2*e**2*x*
*4 + 4*a**2*d*e**3*x**6 + a**2*e**4*x**8 - 2*a*c*d**4*x**4 - 8*a*c*d**3*e*
x**6 - 12*a*c*d**2*e**2*x**8 - 8*a*c*d*e**3*x**10 - 2*a*c*e**4*x**12 + c**
2*d**4*x**8 + 4*c**2*d**3*e*x**10 + 6*c**2*d**2*e**2*x**12 + 4*c**2*d*e**3
*x**14 + c**2*e**4*x**16),x)
```

3.421 $\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 386

$$\begin{aligned}\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx &= \frac{e^2(42cd^2-5ae^2)\,x\sqrt{a+cx^4}}{21c^2}+\frac{4de^3x^3\sqrt{a+cx^4}}{5c}\\ &+\frac{e^4x^5\sqrt{a+cx^4}}{7c}+\frac{4de(5cd^2-3ae^2)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}\\ &-\frac{4\sqrt[4]{a}de(5cd^2-3ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}}\\ &+\frac{\left(84\sqrt{a}\sqrt{c}de(5cd^2-3ae^2)+5(21c^2d^4-42acd^2e^2+5a^2e^4)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\Bigg(2\,\mathrm{a}}{210\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}}\end{aligned}$$

output
```
1/21*e^2*(-5*a*e^2+42*c*d^2)*x*(c*x^4+a)^(1/2)/c^2+4/5*d*e^3*x^3*(c*x^4+a)
^(1/2)/c+1/7*e^4*x^5*(c*x^4+a)^(1/2)/c+4/5*d*e*(-3*a*e^2+5*c*d^2)*x*(c*x^4
+a)^(1/2)/c^(3/2)/(a^(1/2)+c^(1/2)*x^2)-4/5*a^(1/4)*d*e*(-3*a*e^2+5*c*d^2)
*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE
(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(7/4)/(c*x^4+a)^(1/2)+1/2
10*(84*a^(1/2)*c^(1/2)*d*e*(-3*a*e^2+5*c*d^2)+25*a^2*e^4-210*a*c*d^2*e^2+1
05*c^2*d^4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2
)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(9/4)
/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.47

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \frac{5(21c^2d^4-42acd^2e^2+5a^2e^4)\,x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex\Bigg(-e(a+cx^4)\,(25ae^2}{105c^2\sqrt{a}}$$

input
```
Integrate[(d + e*x^2)^4/Sqrt[a + c*x^4],x]
```

output
```
(5*(21*c^2*d^4 - 42*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[1 + (c*x^4)/a]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + e*x*(-(e*(a + c*x^4)*(25*a*e^2 -
 3*c*(70*d^2 + 28*d*e*x^2 + 5*e^2*x^4))) + 28*c*d*(5*c*d^2 - 3*a*e^2)*x^2*
Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(105*
c^2*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1519, 2427, 2427, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx$$

$$\Big\downarrow \; \color{blue}{1519}$$

$$\frac{\int \frac{28cde^3x^6+e^2(42cd^2-5ae^2)x^4+28cd^3ex^2+7cd^4}{\sqrt{cx^4+a}}dx}{7c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

$$\Big\downarrow \; \color{blue}{2427}$$

$$\frac{\frac{\int \frac{35c^2d^4+28ce\left(5cd^2-3ae^2\right)x^2d+5ce^2\left(42cd^2-5ae^2\right)x^4}{\sqrt{cx^4+a}}dx}{5c}+\frac{28}{5}de^3x^3\sqrt{a+cx^4}}{7c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

$$\Big\downarrow \; 2427$$

$$\frac{\frac{\frac{\int \frac{c\left(84cde\left(5cd^2-3ae^2\right)x^2+5\left(21c^2d^4-42ace^2d^2+5a^2e^4\right)\right)}{\sqrt{cx^4+a}}dx}{3c}+\frac{5}{3}e^2x\sqrt{a+cx^4}(42cd^2-5ae^2)}{5c}+\frac{28}{5}de^3x^3\sqrt{a+cx^4}}{7c}+$$

$$\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{3}\int \frac{84cde\left(5cd^2-3ae^2\right)x^2+5\left(21c^2d^4-42ace^2d^2+5a^2e^4\right)}{\sqrt{cx^4+a}}dx+\frac{5}{3}e^2x\sqrt{a+cx^4}(42cd^2-5ae^2)}{5c}+\frac{28}{5}de^3x^3\sqrt{a+cx^4}}{7c}+$$

$$\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

$$\Big\downarrow \; 1512$$

$$\frac{\frac{\frac{1}{3}\left(\left(-252a^{3/2}\sqrt{c}de^3+25a^2e^4+420\sqrt{a}c^{3/2}d^3e-210acd^2e^2+105c^2d^4\right)\int \frac{1}{\sqrt{cx^4+a}}dx-84\sqrt{a}\sqrt{c}de\left(5cd^2-3ae^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx\right)+\frac{5}{3}e^2x\sqrt{a+cx^4}(4}{5c}}{7c}$$

$$\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{3}\left(\left(-252a^{3/2}\sqrt{c}de^3+25a^2e^4+420\sqrt{a}c^{3/2}d^3e-210acd^2e^2+105c^2d^4\right)\int \frac{1}{\sqrt{cx^4+a}}dx-84\sqrt{c}de\left(5cd^2-3ae^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx\right)+\frac{5}{3}e^2x\sqrt{a+cx^4}(42cd^2}{5c}}{7c}$$

$$\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{\frac{1}{3}\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-252a^{3/2}\sqrt{c}de^3+25a^2e^4+420\sqrt{a}c^{3/2}d^3e-210acd^2e^2+105c^2d^4\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-84\sqrt{c}de(5cd^2-3ae^2)\right.}{5c}}{7c}$$

$$\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

$$\Big\downarrow \; 1510$$

$$\frac{\frac{\frac{1}{3}\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-252a^{3/2}\sqrt{c}de^3+25a^2e^4+420\sqrt{a}c^{3/2}d^3e-210acd^2e^2+105c^2d^4\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-84\sqrt{c}de\left(5cd^2-3ae^2\right)\right.}{5c}}{7c}$$

$$\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

input
```
Int[(d + e*x^2)^4/Sqrt[a + c*x^4],x]
```

output
```
(e^4*x^5*Sqrt[a + c*x^4])/(7*c) + ((28*d*e^3*x^3*Sqrt[a + c*x^4])/5 + ((5*
e^2*(42*c*d^2 - 5*a*e^2)*x*Sqrt[a + c*x^4])/3 + (-84*Sqrt[c]*d*e*(5*c*d^2
- 3*a*e^2)*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])) + ((105*c^
2*d^4 + 420*Sqrt[a]*c^(3/2)*d^3*e - 210*a*c*d^2*e^2 - 252*a^(3/2)*Sqrt[c]*
d*e^3 + 25*a^2*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^
(1/4)*Sqrt[a + c*x^4]))/3)/(5*c))/(7*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.94 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.74

method	result
elliptic	$\frac{e^4x^5\sqrt{c\,x^4+a}}{7c}+\frac{4d\,e^3x^3\sqrt{c\,x^4+a}}{5c}+\frac{\left(6d^2e^2-\frac{5a\,e^4}{7c}\right)x\sqrt{c\,x^4+a}}{3c}+\frac{\left(d^4-\frac{a\left(6d^2e^2-\frac{5a\,e^4}{7c}\right)}{3c}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$
risch	$-\frac{x\,e^2\left(-15c\,x^4e^2-84de\,x^2c+25a\,e^2-210c\,d^2\right)\sqrt{c\,x^4+a}}{105c^2}+\frac{-\frac{84ie\sqrt{c}\,d\left(3a\,e^2-5c\,d^2\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\big(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}}{}$
default	$\frac{d^4\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+e^4\Bigg(\frac{x^5\sqrt{c\,x^4+a}}{7c}-\frac{5ax\sqrt{c\,x^4+a}}{21c^2}+\frac{5a^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{Elliptic}}{21c^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$

input
```
int((e*x^2+d)^4/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*e^4*x^5*(c*x^4+a)^(1/2)/c+4/5*d*e^3*x^3*(c*x^4+a)^(1/2)/c+1/3*(6*d^2*e
^2-5/7*a/c*e^4)/c*x*(c*x^4+a)^(1/2)+(d^4-1/3*a/c*(6*d^2*e^2-5/7*a/c*e^4))/
(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I
*(4*d^3*e-12/5*a/c*d*e^3)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*
(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^
(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \frac{84\,(5\,acd^3e-3\,a^2de^3)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\middle|\,-1\right)+(105\,c^2d^4-420\,acd^3e-210\,acd^2e^2+252\,a}{}$$

input
```
integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(84*(5*a*c*d^3*e - 3*a^2*d*e^3)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_e(ar
csin((-a/c)^(1/4)/x), -1) + (105*c^2*d^4 - 420*a*c*d^3*e - 210*a*c*d^2*e^2
 + 252*a^2*d*e^3 + 25*a^2*e^4)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-
a/c)^(1/4)/x), -1) + (15*a*c*e^4*x^6 + 84*a*c*d*e^3*x^4 + 420*a*c*d^3*e -
252*a^2*d*e^3 + 5*(42*a*c*d^2*e^2 - 5*a^2*e^4)*x^2)*sqrt(c*x^4 + a))/(a*c^
2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \frac{d^4x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{d^3ex^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3d^2e^2x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{de^3x^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(\frac{11}{4}\right)} + \frac{e^4x^9\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((e*x**2+d)**4/(c*x**4+a)**(1/2),x)
```

output
```
d**4*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(5/4)) + d**3*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x*
*4*exp_polar(I*pi)/a)/(sqrt(a)*gamma(7/4)) + 3*d**2*e**2*x**5*gamma(5/4)*h
yper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(9/4))
+ d*e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)
/a)/(sqrt(a)*gamma(11/4)) + e**4*x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,)
, c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(13/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^4}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^4/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^4}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^4/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \int \frac{(e\,x^2+d)^4}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x^2)^4/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^4/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \frac{-25\sqrt{c\,x^4+a}\,a\,e^4x+210\sqrt{c\,x^4+a}\,c\,d^2e^2x+84\sqrt{c\,x^4+a}\,cd\,e^3x^3+15\sqrt{c\,x^4+a}\,c\,e^4x^5+25\Bigg(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}}{}$$

input
```
int((e*x^2+d)^4/(c*x^4+a)^(1/2),x)
```

output

```
( - 25*sqrt(a + c*x**4)*a*e**4*x + 210*sqrt(a + c*x**4)*c*d**2*e**2*x + 84
*sqrt(a + c*x**4)*c*d*e**3*x**3 + 15*sqrt(a + c*x**4)*c*e**4*x**5 + 25*int
(sqrt(a + c*x**4)/(a + c*x**4),x)*a**2*e**4 - 210*int(sqrt(a + c*x**4)/(a
+ c*x**4),x)*a*c*d**2*e**2 + 105*int(sqrt(a + c*x**4)/(a + c*x**4),x)*c**2
*d**4 - 252*int((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)*a*c*d*e**3 + 420*i
nt((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)*c**2*d**3*e)/(105*c**2)
```

3.422 $\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 329

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+cx^4}}\,dx = \frac{de^2x\sqrt{a+cx^4}}{c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}+\frac{3e(5cd^2-ae^2)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{3\sqrt[4]{a}e(5cd^2-ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}} + \frac{\left(5\sqrt{c}d(cd^2-ae^2)+3\sqrt{a}e(5cd^2-ae^2)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10\sqrt[4]{a}c^{7/4}\sqrt{a+cx^4}}$$

output

```
d*e^2*x*(c*x^4+a)^(1/2)/c+1/5*e^3*x^3*(c*x^4+a)^(1/2)/c+3/5*e*(-a*e^2+5*c*
d^2)*x*(c*x^4+a)^(1/2)/c^(3/2)/(a^(1/2)+c^(1/2)*x^2)-3/5*a^(1/4)*e*(-a*e^2
+5*c*d^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*
EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(7/4)/(c*x^4+a)^
(1/2)+1/10*(5*c^(1/2)*d*(-a*e^2+c*d^2)+3*a^(1/2)*e*(-a*e^2+5*c*d^2))*(a^(1
/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM
(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(7/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.43

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{5d(cd^2-ae^2)\,x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex\Big(e(5d+ex^2)\,(a+cx^4)+(5cd^2-ae}{5c\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```

output
```
(5*d*(c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4
, -((c*x^4)/a)] + e*x*(e*(5*d + e*x^2)*(a + c*x^4) + (5*c*d^2 - a*e^2)*x^2
*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(5*c
*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1519, 2427, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$$

$$\Big\downarrow \text{1519}$$

$$\frac{\int \frac{15cde^2x^4+3e(5cd^2-ae^2)x^2+5cd^3}{\sqrt{cx^4+a}}dx}{5c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

$$\Big\downarrow \text{2427}$$

$$\frac{\frac{\int \frac{3c\left(3e\left(5cd^2-ae^2\right)x^2+5d\left(cd^2-ae^2\right)\right)}{\sqrt{cx^4+a}}dx}{3c}+5de^2x\sqrt{a+cx^4}}{5c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3e(5cd^2-ae^2)x^2+5d(cd^2-ae^2)}{\sqrt{cx^4+a}}dx+5de^2x\sqrt{a+cx^4}}{5c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

$$\Big\downarrow \ 1512$$

$$\frac{\frac{\left(-3a^{3/2}e^3+15\sqrt{a}cd^2e-5a\sqrt{c}de^2+5c^{3/2}d^3\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{3\sqrt{a}e\left(5cd^2-ae^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}+5de^2x\sqrt{a+cx^4}}{5c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\left(-3a^{3/2}e^3+15\sqrt{a}cd^2e-5a\sqrt{c}de^2+5c^{3/2}d^3\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{3e\left(5cd^2-ae^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}+5de^2x\sqrt{a+cx^4}}{5c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

$$\Big\downarrow \ 761$$

$$\frac{-\frac{3e\left(5cd^2-ae^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-3a^{3/2}e^3+15\sqrt{a}cd^2e-5a\sqrt{c}de^2+5c^{3/2}d^3\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}-}{5c}$$
$$\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

$$\Big\downarrow \ 1510$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-3a^{3/2}e^3+15\sqrt{a}cd^2e-5a\sqrt{c}de^2+5c^{3/2}d^3\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}-\frac{3e\left(5cd^2-ae^2\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)}{}\right.}{}}{5c}$$
$$\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

input
```
Int[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```

output

```
(e^3*x^3*Sqrt[a + c*x^4])/(5*c) + (5*d*e^2*x*Sqrt[a + c*x^4] - (3*e*(5*c*d
^2 - a*e^2)*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/Sqrt[c]
 + ((5*c^(3/2)*d^3 + 15*Sqrt[a]*c*d^2*e - 5*a*Sqrt[c]*d*e^2 - 3*a^(3/2)*e^3
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(3/4)*Sqrt[a + c*x
^4]))/(5*c)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1519

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.58 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.71

method	result
elliptic	$\frac{e^3x^3\sqrt{cx^4+a}}{5c}+\frac{de^2x\sqrt{cx^4+a}}{c}+\frac{\left(d^3-\frac{ade^2}{c}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}}+\frac{i\left(3d^2e-\frac{3ae^3}{5c}\right)\sqrt{a}\sqrt{1-\frac{i\sqrt{c}}{\sqrt{a}}}}{}$
risch	$\frac{xe^2\left(ex^2+5d\right)\sqrt{cx^4+a}}{5c}-\frac{\frac{3ie\left(ae^2-5cd^2\right)\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}\sqrt{c}}-\frac{5d^3c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{}}{5c}$
default	$\frac{d^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}}+e^3\left(\frac{x^3\sqrt{cx^4+a}}{5c}-\frac{3ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\right.}{5c^{\frac{3}{2}}\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}}\right.$

input
```
int((e*x^2+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*e^3*x^3*(c*x^4+a)^(1/2)/c+d*e^2*x*(c*x^4+a)^(1/2)/c+(d^3-a/c*d*e^2)/(I
/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I*(
3*d^2*e-3/5*a/c*e^3)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(Elli
pticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2)
,I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.50

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \frac{3\left(5\,acd^2e-a^2e^3\right)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+\left(5\,c^2d^3-15\,acd^2e-5\,acde^2+3\,a^2e^3\right)\sqrt{c}x\left(-\frac{a}{c}\right.}{5\,ac^2x}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(3*(5*a*c*d^2*e - a^2*e^3)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_e(arcsin((-
a/c)^(1/4)/x), -1) + (5*c^2*d^3 - 15*a*c*d^2*e - 5*a*c*d*e^2 + 3*a^2*e^3)*
sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + (a*c*e^3*x
^4 + 5*a*c*d*e^2*x^2 + 15*a*c*d^2*e - 3*a^2*e^3)*sqrt(c*x^4 + a))/(a*c^2*x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{3d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

output

```
d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*
x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5/4)*
hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
 + e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/
a)/(4*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx=\int \frac{(ex^2+d)^3}{\sqrt{cx^4+a}}\,dx$$

input

```
integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output

```
integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx=\int \frac{(ex^2+d)^3}{\sqrt{cx^4+a}}\,dx$$

input

```
integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output

```
integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{{\left(e\,x^2+d\right)}^3}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x^2)^3/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^3/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{5\sqrt{c\,x^4+a}\,d\,e^2x+\sqrt{c\,x^4+a}\,e^3x^3-5\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)a d\,e^2+5\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)c\,d^3-3\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}dx\right)a}{5c}$$

input
```
int((e*x^2+d)^3/(c*x^4+a)^(1/2),x)
```

output
```
(5*sqrt(a + c*x**4)*d*e**2*x + sqrt(a + c*x**4)*e**3*x**3 - 5*int(sqrt(a +
 c*x**4)/(a + c*x**4),x)*a*d*e**2 + 5*int(sqrt(a + c*x**4)/(a + c*x**4),x)
*c*d**3 - 3*int((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)*a*e**3 + 15*int((s
qrt(a + c*x**4)*x**2)/(a + c*x**4),x)*c*d**2*e)/(5*c)
```

3.423 $\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 264

$$\begin{aligned}\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = {} & \frac{e^2x\sqrt{a+cx^4}}{3c}+\frac{2dex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\ & -\frac{2\sqrt[4]{a}de\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} \\ & +\frac{\left(3cd^2+6\sqrt{a}\sqrt{c}de-ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}\end{aligned}$$

output

```
1/3*e^2*x*(c*x^4+a)^(1/2)/c+2*d*e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/
2)*x^2)-2*a^(1/4)*d*e*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^
2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4
)/(c*x^4+a)^(1/2)+1/6*(3*c*d^2+6*a^(1/2)*c^(1/2)*d*e-a*e^2)*(a^(1/2)+c^(1/
2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan
(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(5/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.45

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{(3cd^2-ae^2)\,x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex\Big(e(a+cx^4)+2cdx^2\sqrt{1+\frac{cx^4}{a}}\,\text{Hyperg}}{3c\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

output
```
((3*c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4,
-((c*x^4)/a)] + e*x*(e*(a + c*x^4) + 2*c*d*x^2*Sqrt[1 + (c*x^4)/a]*Hyperge
ometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*c*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1519, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$$
$$\Big\downarrow \text{1519}$$
$$\frac{\int \frac{3cd^2+6cex^2d-ae^2}{\sqrt{cx^4+a}}dx}{3c}+\frac{e^2x\sqrt{a+cx^4}}{3c}$$
$$\Big\downarrow \text{1512}$$
$$\frac{\left(6\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)\int \frac{1}{\sqrt{cx^4+a}}dx-6\sqrt{a}\sqrt{c}de\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{3c}+\frac{e^2x\sqrt{a+cx^4}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(6\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)\int \frac{1}{\sqrt{cx^4+a}}dx-6\sqrt{c}de\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{3c}+\frac{e^2x\sqrt{a+cx^4}}{3c}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-6\sqrt{c}de\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{3c}+\frac{e^2x\sqrt{a+cx^4}}{3c}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-6\sqrt{c}de\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\,\text{arcta}\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}\right.}{3c}$$

$$\frac{e^2x\sqrt{a+cx^4}}{3c}$$

input
```
Int[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

output
```
(e^2*x*Sqrt[a + c*x^4])/(3*c) + (-6*Sqrt[c]*d*e*(-((x*Sqrt[a + c*x^4])/(Sq
rt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(c^(1/4)*Sqrt[a + c*x^4])) + ((3*c*d^2 + 6*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])
)/(3*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.76

method	result
elliptic	$\frac{e^{2} x \sqrt{c\,x^{4}+a}}{3 c}+\frac{\left(d^{2}-\frac{a\,e^{2}}{3 c}\right) \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}+\frac{2 i d e \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i}{}}\right.\right.}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}\, \sqrt{c}}$
default	$\frac{d^{2} \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}+e^{2} \left(\frac{x \sqrt{c\,x^{4}+a}}{3 c}-\frac{a \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{3 c \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}\right)+$
risch	$\frac{e^{2} x \sqrt{c\,x^{4}+a}}{3 c}-\frac{\frac{a\,e^{2} \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}-\frac{3 c\,d^{2} \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}-\frac{6 i e \sqrt{c}\,d \sqrt{a}\, \sqrt{1-}{}}{3 c}$

input
```
int((e*x^2+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*e^2*x*(c*x^4+a)^(1/2)/c+(d^2-1/3*a*e^2/c)/(I/a^(1/2)*c^(1/2))^(1/2)*(1
-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+2*I*d*e*a^(1/2)/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-Ellip
ticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.43

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \frac{6\,a\sqrt{c}dex\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+(3\,cd^2-6\,ade-ae^2)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+}{3\,acx}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(6*a*sqrt(c)*d*e*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1)
 + (3*c*d^2 - 6*a*d*e - a*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-
a/c)^(1/4)/x), -1) + (a*e^2*x^2 + 6*a*d*e)*sqrt(c*x^4 + a))/(a*c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.47

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

output
```
d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*
exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1/
2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx=\int \frac{(ex^2+d)^2}{\sqrt{cx^4+a}}\,dx$$

input

```
integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output

```
integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx=\int \frac{(e\,x^2+d)^2}{\sqrt{c\,x^4+a}}\,dx$$

input

```
int((d + e*x^2)^2/(a + c*x^4)^(1/2),x)
```

output

```
int((d + e*x^2)^2/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \frac{\sqrt{c\,x^4+a}\,e^2x-\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)a\,e^2+3\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)c\,d^2+6\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}dx\right)cde}{3c}$$

input

```
int((e*x^2+d)^2/(c*x^4+a)^(1/2),x)
```

output

```
(sqrt(a + c*x**4)*e**2*x - int(sqrt(a + c*x**4)/(a + c*x**4),x)*a*e**2 + 3
*int(sqrt(a + c*x**4)/(a + c*x**4),x)*c*d**2 + 6*int((sqrt(a + c*x**4)*x**
2)/(a + c*x**4),x)*c*d*e)/(3*c)
```

3.424 $\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 227

$$
\begin{aligned}
\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx
&= \frac{ex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} \\
&\quad + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}
\end{aligned}
$$

output

```
e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)-a^(1/4)*e*(a^(1/2)+c^(1/
2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c
^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)+1/2*(c^(1/2)*d+a^(
1/2)*e)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*In
verseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(3/4)/(c*
x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.34

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \frac{\sqrt{1+\frac{cx^4}{a}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)\right)}{3\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

output
```
(Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
 + e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[a + c*x^
4])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx$$

$$\Big\downarrow \; 1512$$

$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{\sqrt{a}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \; 27$$

$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{e\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{e\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

input

```
Int[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

output

```
-((e*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/Sqrt[c]) + ((d
+ (Sqrt[a]*e)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)
*c^(1/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.74

method	result
default	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\Bigl(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\Bigl(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$

input

```
int((e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
+I*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/
2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.39

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \frac{a\sqrt{c}ex\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1) + (cd-ae)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1) + \sqrt{cx^4+a}ae}{acx}$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
(a*sqrt(c)*e*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) + (c*d
 - a*e)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + sqr
t(c*x^4 + a)*a*e)/(a*c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.34

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(c*x**4+a)**(1/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_p
olar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x^2)/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)d+\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}dx\right)e$$

input

```
int((e*x^2+d)/(c*x^4+a)^(1/2),x)
```

output

```
int(sqrt(a + c*x**4)/(a + c*x**4),x)*d + int((sqrt(a + c*x**4)*x**2)/(a +
c*x**4),x)*e
```

3.425 $\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 336

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \frac{\sqrt{e}\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{cd^2+ae^2}} + \frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}c^{3/4}d\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{a+cx^4}}$$

output
```
1/2*e^(1/2)*arctan((a*e^2+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2))/
d^(1/2)/(a*e^2+c*d^2)^(1/2)+1/2*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(
a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),
1/2*2^(1/2))/a^(1/4)/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+a)^(1/2)-1/4*(c^(1/2)*d+
a^(1/2)*e)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)
*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(1/2)*d-a^(1/2)*e)^2/
a^(1/2)/c^(1/2)/d/e,1/2*2^(1/2))/a^(1/4)/c^(3/4)/d/(d-a^(1/2)*e/c^(1/2))/(
c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.28

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx=-\frac{i\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d},i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}d\sqrt{a+cx^4}}$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[a
+ c*x^4])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1541, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}\,(d+ex^2)}dx$$

$$\Big\downarrow \text{1541}$$

$$\frac{\sqrt{c}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{e\int\frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \textcolor{blue}{2221}$$

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{e\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2+cd^2}}\right)}{\sqrt{c}d-\sqrt{a}e}$$

input

```
Int[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output

```
(c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d -
Sqrt[a]*e)*Sqrt[a + c*x^4]) - (e*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sq
rt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(Sqrt[d]*Sqrt[e]*
Sqrt[c*d^2 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[
c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + c*x^4])))/(Sqrt[c]*d - Sqrt[a]*e)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.32

method	result	size
default	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	107
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	107

input
```
int(1/(e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2)
,I*a^(1/2)/c^(1/2)/d*e,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2
))
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + a)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{1}{\sqrt{a+cx^4}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{1}{\sqrt{c\,x^4+a}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((a + c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int(1/((a + c*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{\sqrt{c\,x^4+a}}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx$$

input
```
int(1/(e*x^2+d)/(c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a*d + a*e*x**2 + c*d*x**4 + c*e*x**6),x)
```

3.426 $\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 581

$$
\begin{aligned}
\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = & -\frac{\sqrt{c}ex\sqrt{a+cx^4}}{2d(cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
& + \frac{e^2x\sqrt{a+cx^4}}{2d(cd^2+ae^2)(d+ex^2)} + \frac{\sqrt{e}(3cd^2+ae^2)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}(cd^2+ae^2)^{3/2}} \\
& + \frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d(cd^2+ae^2)\sqrt{a+cx^4}} \\
& + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}} \\
& - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)(3cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)\sqrt{a+cx^4}}
\end{aligned}
$$

output

```
-1/2*c^(1/2)*e*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(a^(1/2)+c^(1/2)*x^2)+1/2
*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)+1/4*e^(1/2)*(a*e^2+3*c*d^
2)*arctan((a*e^2+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2))/d^(3/2)/(
a*e^2+c*d^2)^(3/2)+1/2*a^(1/4)*c^(1/4)*e*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/
(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),
1/2*2^(1/2))/d/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)+1/2*c^(1/4)*(a^(1/2)+c^(1/2)*
x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^
(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/d/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+a)^(1
/2)-1/8*(c^(1/2)*d+a^(1/2)*e)*(a*e^2+3*c*d^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^
4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1
/4))),-1/4*(c^(1/2)*d-a^(1/2)*e)^2/a^(1/2)/c^(1/2)/d/e,1/2*2^(1/2))/a^(1/4
)/c^(1/4)/d^2/(c^(1/2)*d-a^(1/2)*e)/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.60 (sec) , antiderivative size = 522, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \frac{a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}de^2x+\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}cde^2x^5-\sqrt{a}\sqrt{c}de(d+ex^2)\sqrt{1+\frac{cx^4}{a}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{c}d(i\sqrt{c}d+}{}$$

input

```
Integrate[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

output
```
(a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e^2*x + Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d*e^2*x
^5 - Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[c]*d*(I*Sqrt[c]*d + Sqrt[a]*e)
*(d + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt
[a]]*x], -1] - (3*I)*c*d^3*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)
/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a*d*e^2*Sqrt
[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[a]]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 + (c*x^4)/a]*Ellipti
cPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x],
-1] - I*a*e^3*x^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]
*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(2*Sqrt[(I*Sqrt[c])/Sqrt
[a]]*d^2*(c*d^2 + a*e^2)*(d + e*x^2)*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 569, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1552, 25, 2233, 27, 1510, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{1552}$$

$$\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}-\frac{\int -\frac{-ce^2x^4-2cdex^2+2cd^2+ae^2}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{2d\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-ce^2x^4-2cdex^2+2cd^2+ae^2}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{2d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{2233}$$

$$\frac{\frac{\int \frac{ce\left(2cd^2-\sqrt{a}\sqrt{c}ed+ae^2-\sqrt{c}e\left(\sqrt{c}d+\sqrt{a}e\right)x^2\right)}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{ce}+\sqrt{a}\sqrt{c}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{2d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2cd^2-\sqrt{a}\sqrt{c}ed+ae^2-\sqrt{c}e\left(\sqrt{c}d+\sqrt{a}e\right)x^2}{(ex^2+d)\sqrt{cx^4+a}}dx+\sqrt{c}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{2d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 1510$$

$$\frac{\int \frac{2cd^2-\sqrt{a}\sqrt{c}ed+ae^2-\sqrt{c}e\left(\sqrt{c}d+\sqrt{a}e\right)x^2}{(ex^2+d)\sqrt{cx^4+a}}dx+\sqrt{c}e\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{2d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 2227$$

$$\frac{\frac{2\sqrt{c}\left(ae^2+cd^2\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\sqrt{a}e\left(ae^2+3cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\sqrt{c}e\Bigg(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}}{2d\left(ae^2+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\sqrt{c}\left(ae^2+cd^2\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{e\left(ae^2+3cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\sqrt{c}e\Bigg(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-}{2d\left(ae^2+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 761$$

$$\frac{-\frac{e\left(ae^2+3cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^2+cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}+\sqrt{c}e\Bigg(\sqrt[4]{a}\left(\sqrt{a}+\right.}{2d\left(ae^2+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 2221$$

$$\frac{\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(ae^2+cd^2)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}(\sqrt{c}d-\sqrt{a}e)} - \frac{e(ae^2+3cd^2)\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(\sqrt{a}e+\sqrt{c}d)\operatorname{EllipticPi}}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{}}\right.}{}}{}$$

$$\frac{e^2x\sqrt{a+cx^4}}{2d\,(d+ex^2)\,(ae^2+cd^2)}$$

input
```
Int[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

output
```
(e^2*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (Sqrt[c]*e*(-(
(x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])) + (c^(1/4)*(c*d^2 + a*e^
2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*(Sqrt[c]*d - Sqrt[a]*
e)*Sqrt[a + c*x^4]) - (e*(3*c*d^2 + a*e^2)*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*
ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(Sqrt[d
]*Sqrt[e]*Sqrt[c*d^2 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[
a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + c*x^4])))/(Sqrt[c]*d - Sqrt[a]*e
))/(2*d*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1552

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sq
rt[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
 + c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && ILtQ[q, -1]
```

rule 2221

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 556, normalized size of antiderivative = 0.96

method	result
default	$\frac{e^2 x\sqrt{c x^4+a}}{2d(a e^2+c d^2)(e x^2+d)}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\ \operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2(a e^2+c d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}}-\frac{i\sqrt{c}\,e\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\ \operatorname{EllipticF}\left(x\right.}{2d(a e^2+c d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}}$
elliptic	$\frac{e^2 x\sqrt{c x^4+a}}{2d(a e^2+c d^2)(e x^2+d)}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\ \operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2(a e^2+c d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}}-\frac{i\sqrt{c}\,e\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\ \operatorname{EllipticF}\left(x\right.}{2d(a e^2+c d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}}$

input
```
int(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)-1/2*c/(a*e^2+c*d^2)/(I
/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/2
*I*c^(1/2)*e/d/(a*e^2+c*d^2)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellip
ticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*I*c^(1/2)*e/d/(a*e^2+c*d^2)*a^(1/2
)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
+1/2/d^2/(a*e^2+c*d^2)*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I
/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)/d*e,(-I/a^(1/2)*c^(1/2))^(1/2)/(
I/a^(1/2)*c^(1/2))^(1/2))*a+3/2/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1
-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1
/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)/d*e,(-I/a^(1/
2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex^2)^2}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,(e\,x^2+d)^2}\,dx$$

input
```
int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+e x^2\right)^2 \sqrt{a+c x^4}} \, dx = \int \frac{\sqrt{c\,x^4+a}}{c\,e^2x^8+2cde\,x^6+a\,e^2x^4+c\,d^2x^4+2ade\,x^2+a\,d^2}dx$$

input

```
int(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x)
```

output

```
int(sqrt(a + c*x**4)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 + c*d**2*x**4 +
2*c*d*e*x**6 + c*e**2*x**8),x)
```

3.427 $\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 729

$$\begin{aligned}\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx &= -\frac{3\sqrt{c}e(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\,(cd^2+ae^2)^2\,\left(\sqrt{a}+\sqrt{c}x^2\right)} \\ &+ \frac{e^2x\sqrt{a+cx^4}}{4d\,(cd^2+ae^2)\,(d+ex^2)^2} + \frac{3e^2(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\,(cd^2+ae^2)^2\,(d+ex^2)} \\ &+ \frac{3\sqrt{e}(5c^2d^4+2acd^2e^2+a^2e^4)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{16d^{5/2}\,(cd^2+ae^2)^{5/2}} \\ &+ \frac{3\sqrt[4]{a}\sqrt[4]{c}e(3cd^2+ae^2)\,\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8d^2\,(cd^2+ae^2)^2\,\sqrt{a+cx^4}} \\ &+ \frac{\sqrt[4]{c}\left(4cd^2-\sqrt{a}\sqrt{c}de+3ae^2\right)\,\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}d^2\,\left(\sqrt{c}d-\sqrt{a}e\right)\,(cd^2+ae^2)\sqrt{a+cx^4}} \\ &- \frac{3\left(\sqrt{c}d+\sqrt{a}e\right)\,(5c^2d^4+2acd^2e^2+a^2e^4)\,\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\right.}{32\sqrt[4]{a}\sqrt[4]{c}d^3\,\left(\sqrt{c}d-\sqrt{a}e\right)\,(cd^2+ae^2)^2\sqrt{a+cx^4}}\end{aligned}$$

output
```
-3/8*c^(1/2)*e*(a*e^2+3*c*d^2)*x*(c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(a^(1
/2)+c^(1/2)*x^2)+1/4*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2+3/8
*e^2*(a*e^2+3*c*d^2)*x*(c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)+3/16*
e^(1/2)*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctan((a*e^2+c*d^2)^(1/2)*x/d^(
1/2)/e^(1/2)/(c*x^4+a)^(1/2))/d^(5/2)/(a*e^2+c*d^2)^(5/2)+3/8*a^(1/4)*c^(1
/4)*e*(a*e^2+3*c*d^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^
2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/d^2/(a
*e^2+c*d^2)^2/(c*x^4+a)^(1/2)+1/8*c^(1/4)*(4*c*d^2-a^(1/2)*c^(1/2)*d*e+3*a
*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Inve
rseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/d^2/(c^(1/2)*
d-a^(1/2)*e)/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)-3/32*(c^(1/2)*d+a^(1/2)*e)*(a^2
*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^
(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(1
/2)*d-a^(1/2)*e)^2/a^(1/2)/c^(1/2)/d/e,1/2*2^(1/2))/a^(1/4)/c^(1/4)/d^3/(c
^(1/2)*d-a^(1/2)*e)/(a*e^2+c*d^2)^2/(c*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.88 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.46

$$\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx = \frac{\frac{de^2x(a+cx^4)(ae^2(5d+3ex^2)+cd^2(11d+9ex^2))}{(d+ex^2)^2}+\frac{\sqrt{1+\frac{cx^4}{a}}\left(-3\sqrt{a}\sqrt{c}de(3cd^2+ae^2)E\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+i\left(\sqrt{c}d(7c^{3/2}d^3-9i\right.\right.}{}}{8d^3(cd^2+ae}$$

input
```
Integrate[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

output

```
((d*e^2*x*(a + c*x^4)*(a*e^2*(5*d + 3*e*x^2) + c*d^2*(11*d + 9*e*x^2)))/(d
 + e*x^2)^2 + (Sqrt[1 + (c*x^4)/a]*(-3*Sqrt[a]*Sqrt[c]*d*e*(3*c*d^2 + a*e^
2)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*(Sqrt[c]*d*(7
*c^(3/2)*d^3 - (9*I)*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - (3*I)*a^(3/2)*e^3
)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - 3*(5*c^2*d^4 + 2
*a*c*d^2*e^2 + a^2*e^4)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh
[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])))/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(8*d^3*(c
*d^2 + a*e^2)^2*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 2.80 (sec) , antiderivative size = 700, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {1552, 25, 2211, 25, 2233, 27, 1510, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{1552}$$

$$\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}-\frac{\int -\frac{ce^2x^4-4cdex^2+4cd^2+3ae^2}{\left(ex^2+d\right)^2\sqrt{cx^4+a}}dx}{4d\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ce^2x^4-4cdex^2+4cd^2+3ae^2}{\left(ex^2+d\right)^2\sqrt{cx^4+a}}dx}{4d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{2211}$$

$$\frac{\frac{3e^2x\sqrt{a+cx^4}\left(ae^2+3cd^2\right)}{2d(d+ex^2)(ae^2+cd^2)}-\frac{\int -\frac{8c^2d^4+5ace^2d^2-4ce\left(4cd^2+ae^2\right)x^2d+3a^2e^4-3ce^2\left(3cd^2+ae^2\right)x^4}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{2d(ae^2+cd^2)}}{4d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{8c^2d^4+5ace^2d^2-4ce\left(4cd^2+ae^2\right)x^2d+3a^2e^4-3ce^2\left(3cd^2+ae^2\right)x^4}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{2d(ae^2+cd^2)}+\frac{3e^2x\sqrt{a+cx^4}(ae^2+3cd^2)}{2d(d+ex^2)(ae^2+cd^2)}}{4d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \ 2233$$

$$\frac{\frac{\frac{\int \frac{ce\left(8c^2d^4+5ace^2d^2-3\sqrt{a}\sqrt{c}e\left(3cd^2+ae^2\right)d+3a^2e^4-\sqrt{c}e\left(7c^{3/2}d^3+9\sqrt{a}ced^2+a\sqrt{c}e^2d+3a^{3/2}e^3\right)x^2\right)}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{ce}+3\sqrt{a}\sqrt{c}e(ae^2+3cd^2)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{2d(ae^2+cd^2)}+\frac{3e^2x}{2d(}}{4d\left(ae^2+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{8c^2d^4+5ace^2d^2-3\sqrt{a}\sqrt{c}e\left(3cd^2+ae^2\right)d+3a^2e^4-\sqrt{c}e\left(7c^{3/2}d^3+9\sqrt{a}ced^2+a\sqrt{c}e^2d+3a^{3/2}e^3\right)x^2}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx+3\sqrt{c}e(ae^2+3cd^2)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{2d(ae^2+cd^2)}+\frac{3e^2x\sqrt{a+cx^4}}{2d(d+ex^2)(}}{4d\left(ae^2+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \ 1510$$

$$\frac{\frac{\int \frac{8c^2d^4+5ace^2d^2-3\sqrt{a}\sqrt{c}e\left(3cd^2+ae^2\right)d+3a^2e^4-\sqrt{c}e\left(7c^{3/2}d^3+9\sqrt{a}ced^2+a\sqrt{c}e^2d+3a^{3/2}e^3\right)x^2}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx+3\sqrt{c}e(ae^2+3cd^2)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)}}}{\sqrt[4]{c}\sqrt{a+}}\right.}{2d(ae^2+cd^2)}}{4d\left(ae^2+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \ 2227$$

$$-\frac{3\sqrt{a}e\Big(a^2e^4+2acd^2e^2+5c^2d^4\Big)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\Big(ex^2+d\Big)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{2\sqrt{c}\Big(ae^2+cd^2\Big)\Big(-\sqrt{a}\sqrt{c}de+3ae^2+4cd^2\Big)\int\frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+3\sqrt{c}e(ae^2+3cd^2)\Bigg(\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)$$

$$2d(ae^2+cd^2)$$

$$4d\left(ae^2+cd^2\right)$$

$$\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow\ 27$$

$$-\frac{3e\Big(a^2e^4+2acd^2e^2+5c^2d^4\Big)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\Big(ex^2+d\Big)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{2\sqrt{c}\Big(ae^2+cd^2\Big)\Big(-\sqrt{a}\sqrt{c}de+3ae^2+4cd^2\Big)\int\frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+3\sqrt{c}e(ae^2+3cd^2)\Bigg(\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)$$

$$2d(ae^2+cd^2)$$

$$4d\left(ae^2+cd^2\right)$$

$$\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow\ 761$$

$$-\frac{3e\Big(a^2e^4+2acd^2e^2+5c^2d^4\Big)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\Big(ex^2+d\Big)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\Big(\sqrt{a}+\sqrt{c}x^2\Big)^2}}\Big(ae^2+cd^2\Big)\Big(-\sqrt{a}\sqrt{c}de+3ae^2+4cd^2\Big)\operatorname{EllipticF}\Bigg(2\arctan\Bigg(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\Bigg),\frac{1}{2}}{\sqrt[4]{a}\sqrt{a+cx^4}(\sqrt{c}d-\sqrt{a}e)}$$

$$2d(ae^2+cd^2)$$

$$4d\left(ae^2+cd^2\right)$$

$$\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow\ 2221$$

$$-\frac{3e\Big(a^2e^4+2acd^2e^2+5c^2d^4\Big)\Bigg(\frac{\Big(\sqrt{a}+\sqrt{c}x^2\Big)\sqrt{\frac{a+cx^4}{\Big(\sqrt{a}+\sqrt{c}x^2\Big)^2}}(\sqrt{a}e+\sqrt{c}d)\operatorname{EllipticPi}\Bigg(-\frac{\sqrt{a}\Big(\frac{\sqrt{c}d}{\sqrt{a}}-e\Big)^2}{4\sqrt{c}de},2\arctan\Bigg(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\Bigg),\frac{1}{2}\Bigg)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+cx^4}}-\frac{(\sqrt{c}d-\sqrt{a}e)\arctan\Bigg(\frac{x\sqrt{ae^2+}}{\sqrt{d}\sqrt{e}\sqrt{a-}}}{2\sqrt{d}\sqrt{e}\sqrt{ae^2+cd^2}}}{\sqrt{c}d-\sqrt{a}e}$$

$$\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

input

```
Int[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

output

```
(e^2*x*Sqrt[a + c*x^4])/(4*d*(c*d^2 + a*e^2)*(d + e*x^2)^2) + ((3*e^2*(3*c
*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (3*Sq
rt[c]*e*(3*c*d^2 + a*e^2)*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2))
 + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^
4])) + (c^(1/4)*(c*d^2 + a*e^2)*(4*c*d^2 - Sqrt[a]*Sqrt[c]*d*e + 3*a*e^2)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*
Sqrt[a + c*x^4]) - (3*e*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(-1/2*((Sqrt
[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a
 + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*
e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticPi[-1/4*(Sqrt[a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan
[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + c*x^4])))/(Sq
rt[c]*d - Sqrt[a]*e))/(2*d*(c*d^2 + a*e^2)))/(4*d*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1552
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sq
rt[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
 + c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && ILtQ[q, -1]
```

rule 2211
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))
Int[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(
2*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(
C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 1018, normalized size of antiderivative = 1.40

method	result	size
default	Expression too large to display	1018
elliptic	Expression too large to display	1018

input
```
int(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d
^2)*x*(c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)-1/8*c/d/(a*e^2+c*d^2)^
2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
*a*e^2-7/8*c^2*d/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I
/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/8
*I*c^(1/2)*e^3/d^2/(a*e^2+c*d^2)^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/
a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)
*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-3/8*I*c^(1/2)*e^3/d^2/(a*e^2+c*d
^2)^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/
2))^(1/2),I)+9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(
1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^
4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/8/d^3/(a*e^2+c*d^2)^
2*e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/
2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/
2),I*a^(1/2)/c^(1/2)/d*e,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^3 \sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex^2)^3}\,dx$$

input
```
integrate(1/(e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**3), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^3 \sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^3}\,dx$$

input
```
integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^3 \sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^3}\,dx$$

input
```
integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx=\int \frac{1}{\sqrt{c\,x^4+a}\,{\left(e\,x^2+d\right)}^3}\,dx$$

input
```
int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^3),x)
```

output
```
int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^3), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx\\&=\int \frac{\sqrt{c\,x^4+a}}{c\,e^3x^{10}+3cd\,e^2x^8+a\,e^3x^6+3c\,d^2e\,x^6+3ad\,e^2x^4+c\,d^3x^4+3a\,d^2e\,x^2+a\,d^3}dx\end{aligned}$$

input
```
int(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*
x**6 + c*d**3*x**4 + 3*c*d**2*e*x**6 + 3*c*d*e**2*x**8 + c*e**3*x**10),x)
```

3.428 $\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = \frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{-a+cx^4}} + \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\ \mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{-a+cx^4}}$$

output
```
a^(3/4)*e*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(3/4)/(c*x^4-
a)^(1/2)+a^(1/4)*(d-a^(1/2)*e/c^(1/2))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)
*x/a^(1/4),I)/c^(1/4)/(c*x^4-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.62

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\left(3dx\ \mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+ex^3\ \mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{-a+cx^4}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

output
```
(Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] +
e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{cx^4-a}}dx$$

$$\Big\downarrow \text{1513}$$

$$\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4-a}}dx+\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{cx^4-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4-a}}dx+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{cx^4-a}}+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{762}$$

$$\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{cx^4-a}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{cx^4-a}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{327}$$

$$\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

input

```
Int[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

output

```
(a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(c^(3/4)*Sqrt[-a + c*x^4]) + (a^(1/4)*(d - (Sqrt[a]*e)/Sqrt[c])*Sqrt[1 -
(c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[-a +
c*x^4])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.28

method	result
default	$\frac{d\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}+\frac{e\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,(\operatorname{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}\,\sqrt{c}}$
elliptic	$\frac{d\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}+\frac{e\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,(\operatorname{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}\,\sqrt{c}}$

input
```
int((e*x^2+d)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),
I)+e*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-1/a^(
1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = \frac{a\sqrt{c}ex\left(\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-(cd+ae)\sqrt{c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+\sqrt{cx^4-a}ae}{acx}$$

input
```
integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
(a*sqrt(c)*e*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - (c*d +
a*e)*sqrt(c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + sqrt(c*
x^4 - a)*a*e)/(a*c*x)
```

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = -\frac{idx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}-\frac{iex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(c*x**4-a)**(1/2),x)
```

output
```
-I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt(a)*gamma(5/4
)) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*ga
mma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{cx^4-a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{cx^4-a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{c\,x^4-a}}\,dx$$

input
```
int((d + e*x^2)/(c*x^4 - a)^(1/2),x)
```

output
```
int((d + e*x^2)/(c*x^4 - a)^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = -\left(\int \frac{\sqrt{c\,x^4-a}}{-c\,x^4+a}dx\right)d-\left(\int \frac{\sqrt{c\,x^4-a}\,x^2}{-c\,x^4+a}dx\right)e$$

input

```
int((e*x^2+d)/(c*x^4-a)^(1/2),x)
```

output

```
 - (int(sqrt( - a + c*x**4)/(a - c*x**4),x)*d + int((sqrt( - a + c*x**4)*x
**2)/(a - c*x**4),x)*e)
```

3.429 $\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx=\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{-a+cx^4}}$$

output
```
a^(1/4)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/
d,I)/c^(1/4)/d/(c*x^4-a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx=-\frac{i\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},i\operatorname{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}d\sqrt{-a+cx^4}}$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

output
```
((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh
[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[-a + c
*x^4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx^4-a}\,(d+ex^2)}\,dx$$

$$\Big\downarrow \text{ 1543}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{(ex^2+d)\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{ 1542}$$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{cx^4-a}}$$

input
```
Int[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

output
```
(a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin
[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[-a + c*x^4])
```

Defintions of rubi rules used

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

method	result	size
default	$\frac{\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}$	99
elliptic	$\frac{\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}$	99

input
```
int(1/(e*x^2+d)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2
)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticPi(x*(-1/a^(1/2)*c^(1/2))^(1/
2),a^(1/2)*e/c^(1/2)/d,(1/a^(1/2)*c^(1/2))^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2
))
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 - a)/(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \int \frac{1}{\sqrt{-a+cx^4}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(c*x**4-a)**(1/2),x)
```

output
```
Integral(1/(sqrt(-a + c*x**4)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx=\int \frac{1}{\sqrt{c\,x^4-a}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((c*x^4 - a)^(1/2)*(d + e*x^2)),x)
```

output
```
int(1/((c*x^4 - a)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx=-\left(\int \frac{\sqrt{c\,x^4-a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)$$

input
```
int(1/(e*x^2+d)/(c*x^4-a)^(1/2),x)
```

output
```
- int(sqrt( - a + c*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)
```

3.430 $\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 237

$$\begin{aligned}\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx &= -\frac{ex\sqrt{-a-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{-a-cx^4}}\\ &\quad + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{-a-cx^4}}\end{aligned}$$

output

```
-e*x*(-c*x^4-a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)-a^(1/4)*e*(a^(1/2)+c^(
1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan
(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4)/(-c*x^4-a)^(1/2)+1/2*(c^(1/2)*d+
a^(1/2)*e)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)
*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(3/4)/
(-c*x^4-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.34

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = \frac{\sqrt{1+\frac{cx^4}{a}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)\right)}{3\sqrt{-a-cx^4}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[-a - c*x^4],x]
```

output
```
(Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
 + e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[-a - c*x
^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx$$

$$\Big\downarrow \text{1512}$$

$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{-cx^4-a}}dx-\frac{\sqrt{a}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{-cx^4-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{-cx^4-a}}dx-\frac{e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{-cx^4-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \color{blue}{761}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{-a-cx^4}}-\frac{e\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{-cx^4-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \color{blue}{1510}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{-a-cx^4}}-\frac{e\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{-a-cx^4}}+\frac{x\sqrt{-a-cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

input

```
Int[(d + e*x^2)/Sqrt[-a - c*x^4],x]
```

output

```
-((e*((x*Sqrt[-a - c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) + (a^(1/4)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[-a - c*x^4])))/Sqrt[c]) + ((d +
 (Sqrt[a]*e)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*
c^(1/4)*Sqrt[-a - c*x^4])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.74

method	result
default	$\frac{d\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}}-\frac{ie\sqrt{a}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{-}\right.\right.}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}\,\sqrt{c}}$
elliptic	$\frac{d\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}}-\frac{ie\sqrt{a}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{-}\right.\right.}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}\,\sqrt{c}}$

input

```
int((e*x^2+d)/(-c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
d/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)*EllipticF(x*(-I/a^(1/2)*c^(1/2))^(1/2)
,I)-I*e*a^(1/2)/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)
*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-I
/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.41

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = -\frac{a\sqrt{-c}ex\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)+(cd-ae)\sqrt{-c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)+\sqrt{-cx^4}}{acx}$$

input
```
integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
-(a*sqrt(-c)*e*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) + (c*
d - a*e)*sqrt(-c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) +
sqrt(-c*x^4 - a)*a*e)/(a*c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.35

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = -\frac{idx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}-\frac{iex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

output
```
-I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(5/4)) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*
exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{-cx^4-a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{-cx^4-a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{-c\,x^4-a}}\,dx$$

input
```
int((d + e*x^2)/(- a - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(- a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx=-i\left(\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)d+\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}dx\right)e\right)$$

input

```
int((e*x^2+d)/(-c*x^4-a)^(1/2),x)
```

output

```
 - i*(int(sqrt(a + c*x**4)/(a + c*x**4),x)*d + int((sqrt(a + c*x**4)*x**2)
/(a + c*x**4),x)*e)
```

3.431 $\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 345

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\sqrt{cd^2+ae^2}} + \frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{-a-cx^4}} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}c^{3/4}d\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{-a-cx^4}}$$

output

```
1/2*e^(1/2)*arctanh((a*e^2+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(-c*x^4-a)^(1/2)
)/d^(1/2)/(a*e^2+c*d^2)^(1/2)+1/2*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)
/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)
),1/2*2^(1/2))/a^(1/4)/(c^(1/2)*d-a^(1/2)*e)/(-c*x^4-a)^(1/2)-1/4*(c^(1/2)
*d+a^(1/2)*e)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1
/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(1/2)*d-a^(1/2)*e)
^2/a^(1/2)/c^(1/2)/d/e,1/2*2^(1/2))/a^(1/4)/c^(3/4)/d/(d-a^(1/2)*e/c^(1/2)
)/(-c*x^4-a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.28

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = -\frac{i\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d}, i\,\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right), -1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}d\sqrt{-a-cx^4}}$$

input

```
Integrate[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]
```

output

```
((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[-a
 - c*x^4])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1541, 27, 761, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-a-cx^4}\,(d+ex^2)}dx$$

$$\Big\downarrow \text{ 1541}$$

$$\frac{\sqrt{c}\int \frac{1}{\sqrt{-cx^4-a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(ex^2+d)\sqrt{-cx^4-a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{c}\int \frac{1}{\sqrt{-cx^4-a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{-cx^4-a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{-a-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{e\int\frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{-cx^4-a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \text{2223}$$

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{-a-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{e\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{-a-cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2+cd^2}}\right)}{\sqrt{c}d-\sqrt{a}e}$$

input

```
Int[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]
```

output

```
(c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d -
Sqrt[a]*e)*Sqrt[-a - c*x^4]) - (e*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTanh[(
Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a - c*x^4])])/(Sqrt[d]*Sqrt[
e]*Sqrt[c*d^2 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sq
rt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[-a - c*x^4])))/(Sqrt[c]*d - Sqrt[a]*e)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.32

method	result	size
default	$\frac{\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}}$	110
elliptic	$\frac{\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}}$	110

input
```
int(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)*EllipticPi(x*(-I/a^(1/2)*c^(1/2))^(1
/2),-I*a^(1/2)/c^(1/2)/d*e,(I/a^(1/2)*c^(1/2))^(1/2)/(-I/a^(1/2)*c^(1/2))^
(1/2))
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 - a)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-a-cx^4}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

output
```
Integral(1/(sqrt(-a - c*x**4)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-c\,x^4-a}\,(e\,x^2+d)}\,dx$$

input
```
int(1/((- a - c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int(1/((- a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = -\left(\int \frac{\sqrt{c\,x^4+a}}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)i$$

input
```
int(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x)
```

output
```
 - int(sqrt(a + c*x**4)/(a*d + a*e*x**2 + c*d*x**4 + c*e*x**6),x)*i
```

3.432 $\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \frac{\text{EllipticPi}\left(-\frac{2b}{\sqrt{5}a},\arcsin\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

output
```
1/10*EllipticPi(1/2*5^(1/4)*x*2^(1/2),-2/5*b*5^(1/2)/a,I)*2^(1/2)*5^(3/4)/
a
```

Mathematica [A] (verified)

Time = 10.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \frac{\text{EllipticPi}\left(-\frac{2b}{\sqrt{5}a},\arcsin\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

input
```
Integrate[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
```

output
```
EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5
^(1/4)*a)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4-5x^4}\,(a+bx^2)}dx$$
$$\Big\downarrow \textcolor{blue}{1537}$$
$$\sqrt{5}\int \frac{1}{\sqrt{2\sqrt{5}-5x^2}\sqrt{5x^2+2\sqrt{5}}\,(bx^2+a)}dx$$
$$\Big\downarrow \textcolor{blue}{412}$$
$$\frac{\text{EllipticPi}\left(-\frac{2b}{\sqrt{5}a},\arcsin\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

input
```
Int[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
```

output
```
EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5
^(1/4)*a)
```

Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(32) = 64$.

Time = 0.59 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.98

method	result	size
default	$\frac{\sqrt{2}\,5^{\frac{3}{4}}\sqrt{1-\frac{\sqrt{5}\,x^{2}}{2}}\,\sqrt{1+\frac{\sqrt{5}\,x^{2}}{2}}\,\operatorname{EllipticPi}\left(\frac{5^{\frac{1}{4}}x\sqrt{2}}{2}, -\frac{2b\sqrt{5}}{5a}, \frac{\sqrt{-\frac{\sqrt{5}}{2}}\,\sqrt{2}\,5^{\frac{3}{4}}}{5}\right)}{5a\sqrt{-5x^{4}+4}}$	79
elliptic	$\frac{\sqrt{2}\,5^{\frac{3}{4}}\sqrt{1-\frac{\sqrt{5}\,x^{2}}{2}}\,\sqrt{1+\frac{\sqrt{5}\,x^{2}}{2}}\,\operatorname{EllipticPi}\left(\frac{5^{\frac{1}{4}}x\sqrt{2}}{2}, -\frac{2b\sqrt{5}}{5a}, \frac{\sqrt{-\frac{\sqrt{5}}{2}}\,\sqrt{2}\,5^{\frac{3}{4}}}{5}\right)}{5a\sqrt{-5x^{4}+4}}$	79

input
```
int(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/a*2^(1/2)*5^(3/4)*(1-1/2*5^(1/2)*x^2)^(1/2)*(1+1/2*5^(1/2)*x^2)^(1/2)/
(-5*x^4+4)^(1/2)*EllipticPi(1/2*5^(1/4)*x*2^(1/2),-2/5*b*5^(1/2)/a,1/5*(-1
/2*5^(1/2))^(1/2)*2^(1/2)*5^(3/4))
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)\,\sqrt{4-5x^4}}\,dx = \int \frac{1}{\sqrt{-5\,x^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 - 4*b*x^2 - 4*a), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{\sqrt{4-5x^4}\,(a+bx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)/(-5*x**4+4)**(1/2),x)
```

output
```
Integral(1/(sqrt(4 - 5*x**4)*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{\sqrt{-5\,x^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{\sqrt{-5\,x^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{4-5\,x^4}}\,dx$$

input
```
int(1/((a + b*x^2)*(4 - 5*x^4)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(4 - 5*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = -\left(\int \frac{\sqrt{-5x^4+4}}{5b\,x^6+5a\,x^4-4b\,x^2-4a}dx\right)$$

input
```
int(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x)
```

output
```
- int(sqrt( - 5*x**4 + 4)/(5*a*x**4 - 4*a + 5*b*x**6 - 4*b*x**2),x)
```

3.433 $\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 299

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{5a^2+4b^2}x}{\sqrt{a}\sqrt{b}\sqrt{4+5x^4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}} + \frac{5^{3/4}\left(2+\sqrt{5}x^2\right)\sqrt{\frac{4+5x^4}{\left(2+\sqrt{5}x^2\right)^2}}\,\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\left(5a-2\sqrt{5}b\right)\sqrt{4+5x^4}} - \frac{\sqrt[4]{5}\left(\sqrt{5}a+2b\right)\left(2+\sqrt{5}x^2\right)\sqrt{\frac{4+5x^4}{\left(2+\sqrt{5}x^2\right)^2}}\,\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{5}a-2b\right)^2}{8\sqrt{5}ab},2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{4\sqrt{2}a\left(5a-2\sqrt{5}b\right)\sqrt{4+5x^4}}$$

output

```
1/2*b^(1/2)*arctan((5*a^2+4*b^2)^(1/2)*x/a^(1/2)/b^(1/2)/(5*x^4+4)^(1/2))/
a^(1/2)/(5*a^2+4*b^2)^(1/2)+1/4*5^(3/4)*(2+5^(1/2)*x^2)*((5*x^4+4)/(2+5^(1
/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*5^(1/4)*x*2^(1/2)),1/2*2^(1
/2))*2^(1/2)/(5*a-2*b*5^(1/2))/(5*x^4+4)^(1/2)-1/8*5^(1/4)*(5^(1/2)*a+2*b)
*(2+5^(1/2)*x^2)*((5*x^4+4)/(2+5^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arct
an(1/2*5^(1/4)*x*2^(1/2))),-1/40*(5^(1/2)*a-2*b)^2*5^(1/2)/a/b,1/2*2^(1/2)
)*2^(1/2)/a/(5*a-2*b*5^(1/2))/(5*x^4+4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.17

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\text{EllipticPi}\left(-\frac{2ib}{\sqrt{5}a}, i\text{arcsinh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{5}x\right),-1\right)}{\sqrt[4]{5}a}$$

input

```
Integrate[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
```

output

```
((-1/2 - I/2)*EllipticPi[((-2*I)*b)/(Sqrt[5]*a), I*ArcSinh[(1/2 + I/2)*5^(
1/4)*x], -1])/(5^(1/4)*a)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1541, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{5x^4+4}\,(a+bx^2)}dx$$

$$\Big\downarrow \text{1541}$$

$$\frac{\left(5a+2\sqrt{5}b\right)\int \frac{1}{\sqrt{5x^4+4}}dx}{5a^2-4b^2}-\frac{2b\left(\sqrt{5}a+2b\right)\int \frac{\sqrt{5}x^2+2}{2(bx^2+a)\sqrt{5x^4+4}}dx}{5a^2-4b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(5a+2\sqrt{5}b\right)\int \frac{1}{\sqrt{5x^4+4}}dx}{5a^2-4b^2}-\frac{b\left(\sqrt{5}a+2b\right)\int \frac{\sqrt{5}x^2+2}{(bx^2+a)\sqrt{5x^4+4}}dx}{5a^2-4b^2}$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(5a+2\sqrt{5}b\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt[4]{5}\sqrt{5x^4+4}\left(5a^2-4b^2\right)}-\frac{b\left(\sqrt{5}a+2b\right)\int\frac{\sqrt{5}x^2+2}{(bx^2+a)\sqrt{5x^4+4}}dx}{5a^2-4b^2}$$

$$\Big\downarrow \text{2221}$$

$$\frac{\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(5a+2\sqrt{5}b\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt[4]{5}\sqrt{5x^4+4}\left(5a^2-4b^2\right)}-$$

$$\frac{b\left(\sqrt{5}a+2b\right)\Bigg(\frac{\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(\sqrt{5}a+2b\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{5}a-2b\right)^2}{8\sqrt{5}ab},2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt[4]{5}ab\sqrt{5x^4+4}}-\frac{\left(\sqrt{5}a-2b\right)\arctan\left(\frac{x\sqrt{5a^2+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{5x^4+4}}\right)}{2\sqrt{a}\sqrt{b}\sqrt{5a^2+4b^2}}}{5a^2-4b^2}$$

input

```
Int[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
```

output

```
((5*a + 2*Sqrt[5]*b)*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^
2]*EllipticF[2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2])/(2*Sqrt[2]*5^(1/4)*(5*a^
2 - 4*b^2)*Sqrt[4 + 5*x^4]) - (b*(Sqrt[5]*a + 2*b)*(-1/2*((Sqrt[5]*a - 2*b
)*ArcTan[(Sqrt[5*a^2 + 4*b^2]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + 5*x^4])])/(Sqrt
[a]*Sqrt[b]*Sqrt[5*a^2 + 4*b^2]) + ((Sqrt[5]*a + 2*b)*(2 + Sqrt[5]*x^2)*Sq
rt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticPi[-1/8*(Sqrt[5]*a - 2*b)^2/(S
qrt[5]*a*b), 2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2])/(4*Sqrt[2]*5^(1/4)*a*b*S
qrt[4 + 5*x^4])))/(5*a^2 - 4*b^2)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1541

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{5}\,x^{2}}{2}}\,\sqrt{1+\frac{i\sqrt{5}\,x^{2}}{2}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{5}}\,x}{2},\frac{2i\sqrt{5}\,b}{5a},\frac{\sqrt{-\frac{i\sqrt{5}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{5}}}\right)}{a\sqrt{i\sqrt{5}}\,\sqrt{5x^{4}+4}}$	86
elliptic	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{5}\,x^{2}}{2}}\,\sqrt{1+\frac{i\sqrt{5}\,x^{2}}{2}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{5}}\,x}{2},\frac{2i\sqrt{5}\,b}{5a},\frac{\sqrt{-\frac{i\sqrt{5}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{5}}}\right)}{a\sqrt{i\sqrt{5}}\,\sqrt{5x^{4}+4}}$	86

input

```
int(1/(b*x^2+a)/(5*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/a/(1/2*I*5^(1/2))^(1/2)*(1-1/2*I*5^(1/2)*x^2)^(1/2)*(1+1/2*I*5^(1/2)*x^2
)^(1/2)/(5*x^4+4)^(1/2)*EllipticPi((1/2*I*5^(1/2))^(1/2)*x,2/5*I*5^(1/2)/a
*b,(-1/2*I*5^(1/2))^(1/2)/(1/2*I*5^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{\sqrt{5\,x^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 + 4*b*x^2 + 4*a), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{5x^4+4}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(5*x**4+4)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)*sqrt(5*x**4 + 4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{\sqrt{5\,x^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{\sqrt{5\,x^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{5\,x^4+4}}\,dx$$

input
```
int(1/((a + b*x^2)*(5*x^4 + 4)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(5*x^4 + 4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{\sqrt{5x^4+4}}{5b\,x^6+5a\,x^4+4b\,x^2+4a}dx$$

input
```
int(1/(b*x^2+a)/(5*x^4+4)^(1/2),x)
```

output
```
int(sqrt(5*x**4 + 4)/(5*a*x**4 + 4*a + 5*b*x**6 + 4*b*x**2),x)
```

3.434 $\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx=\frac{\operatorname{EllipticPi}\left(-\frac{2b}{a\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

output

```
1/2*EllipticPi(1/2*d^(1/4)*x*2^(1/2),-2*b/a/d^(1/2),I)*2^(1/2)/a/d^(1/4)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx=-\frac{i\operatorname{EllipticPi}\left(-\frac{2b}{a\sqrt{d}},i\operatorname{arcsinh}\left(\frac{\sqrt{-\sqrt{d}}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}a\sqrt{-\sqrt{d}}}$$

input

```
Integrate[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

output

```
((-I)*EllipticPi[(-2*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[-Sqrt[d]]*x)/Sqrt[2]]
, -1])/(Sqrt[2]*a*Sqrt[-Sqrt[d]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{1542\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4-dx^4}\left(a+bx^2\right)} \, dx$$

$$\Big\downarrow \text{1542}$$

$$\frac{\text{EllipticPi}\left(-\frac{2b}{a\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

input
```
Int[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

output
```
EllipticPi[(-2*b)/(a*Sqrt[d]), ArcSin[(d^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*a
*d^(1/4))
```

Defintions of rubi rules used

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(32) = 64.

Time = 0.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1-\frac{\sqrt{d}\,x^{2}}{2}}\,\sqrt{1+\frac{\sqrt{d}\,x^{2}}{2}}\,\mathrm{EllipticPi}\left(\frac{d^{\frac{1}{4}} x \sqrt{2}}{2}, -\frac{2b}{a\sqrt{d}}, \frac{\sqrt{-\frac{\sqrt{d}}{2}}\,\sqrt{2}}{d^{\frac{1}{4}}}\right)}{a\,d^{\frac{1}{4}}\sqrt{-d\,x^{4}+4}}$	78
elliptic	$\frac{\sqrt{2}\,\sqrt{1-\frac{\sqrt{d}\,x^{2}}{2}}\,\sqrt{1+\frac{\sqrt{d}\,x^{2}}{2}}\,\mathrm{EllipticPi}\left(\frac{d^{\frac{1}{4}} x \sqrt{2}}{2}, -\frac{2b}{a\sqrt{d}}, \frac{\sqrt{-\frac{\sqrt{d}}{2}}\,\sqrt{2}}{d^{\frac{1}{4}}}\right)}{a\,d^{\frac{1}{4}}\sqrt{-d\,x^{4}+4}}$	78

input

```
int(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/a*2^(1/2)/d^(1/4)*(1-1/2*d^(1/2)*x^2)^(1/2)*(1+1/2*d^(1/2)*x^2)^(1/2)/(-
d*x^4+4)^(1/2)*EllipticPi(1/2*d^(1/4)*x*2^(1/2),-2*b/a/d^(1/2),(-1/2*d^(1/
2))^(1/2)*2^(1/2)/d^(1/4))
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)\,\sqrt{4-dx^4}}\,dx = \int \frac{1}{\sqrt{-dx^4+4}(bx^2+a)}\,dx$$

input

```
integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="fricas")
```

output

```
integral(-sqrt(-d*x^4 + 4)/(b*d*x^6 + a*d*x^4 - 4*b*x^2 - 4*a), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)\,\sqrt{4-dx^4}}\,dx = \int \frac{1}{(a+bx^2)\,\sqrt{-dx^4+4}}\,dx$$

input

```
integrate(1/(b*x**2+a)/(-d*x**4+4)**(1/2),x)
```

output

```
Integral(1/((a + b*x**2)*sqrt(-d*x**4 + 4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = \int \frac{1}{\sqrt{-dx^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = \int \frac{1}{\sqrt{-dx^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{4-d\,x^4}}\,dx$$

input
```
int(1/((a + b*x^2)*(4 - d*x^4)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(4 - d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = -\left(\int \frac{\sqrt{-d\,x^4+4}}{bd\,x^6+ad\,x^4-4b\,x^2-4a}dx\right)$$

input

```
int(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x)
```

output

```
 - int(sqrt( - d*x**4 + 4)/(a*d*x**4 - 4*a + b*d*x**6 - 4*b*x**2),x)
```

3.435 $\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 300

$$\begin{aligned}\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx &= \frac{\sqrt{b}\arctan\left(\frac{\sqrt{4b^2+a^2d}\,x}{\sqrt{a}\sqrt{b}\sqrt{4+dx^4}}\right)}{2\sqrt{a}\sqrt{4b^2+a^2d}} \\ &\quad - \frac{\sqrt[4]{d}\left(2+\sqrt{d}x^2\right)\sqrt{\frac{4+dx^4}{\left(2+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\left(2b-a\sqrt{d}\right)\sqrt{4+dx^4}} \\ &\quad + \frac{\left(2b+a\sqrt{d}\right)\left(2+\sqrt{d}x^2\right)\sqrt{\frac{4+dx^4}{\left(2+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(2b-a\sqrt{d}\right)^2}{8ab\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{4\sqrt{2}a\left(2b-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{4+dx^4}}\end{aligned}$$

output
```
1/2*b^(1/2)*arctan((a^2*d+4*b^2)^(1/2)*x/a^(1/2)/b^(1/2)/(d*x^4+4)^(1/2))/
a^(1/2)/(a^2*d+4*b^2)^(1/2)-1/4*d^(1/4)*(2+d^(1/2)*x^2)*((d*x^4+4)/(2+d^(1
/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*d^(1/4)*x*2^(1/2)),1/2*2^(1
/2))*2^(1/2)/(2*b-a*d^(1/2))/(d*x^4+4)^(1/2)+1/8*(2*b+a*d^(1/2))*(2+d^(1/2
)*x^2)*((d*x^4+4)/(2+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(1/2*d^(
1/4)*x*2^(1/2))),-1/8*(2*b-a*d^(1/2))^2/a/b/d^(1/2),1/2*2^(1/2))*2^(1/2)/a
/(2*b-a*d^(1/2))/d^(1/4)/(d*x^4+4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.22

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = -\frac{i\,\text{EllipticPi}\left(-\frac{2ib}{a\sqrt{d}}, i\text{arcsinh}\left(\frac{\sqrt{i\sqrt{d}}x}{\sqrt{2}}\right), -1\right)}{\sqrt{2}a\sqrt{i\sqrt{d}}}$$

input
```
Integrate[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
```

output
```
((-I)*EllipticPi[((-2*I)*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[I*Sqrt[d]]*x)/Sqr
t[2]], -1])/(Sqrt[2]*a*Sqrt[I*Sqrt[d]])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1541, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx^4+4}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \text{1541}$$

$$\frac{2b\int \frac{\sqrt{d}x^2+2}{2(bx^2+a)\sqrt{dx^4+4}}dx}{2b-a\sqrt{d}} - \frac{\sqrt{d}\int \frac{1}{\sqrt{dx^4+4}}dx}{2b-a\sqrt{d}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\sqrt{d}x^2+2}{(bx^2+a)\sqrt{dx^4+4}}dx}{2b-a\sqrt{d}} - \frac{\sqrt{d}\int \frac{1}{\sqrt{dx^4+4}}dx}{2b-a\sqrt{d}}$$

$$\Big\downarrow \text{761}$$

$$\frac{b\int \frac{\sqrt{d}x^2+2}{(bx^2+a)\sqrt{dx^4+4}}dx}{2b-a\sqrt{d}}-\frac{\sqrt[4]{d}\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}$$

$$\Big\downarrow \text{2221}$$

$$\frac{b\left(\frac{\left(2b-a\sqrt{d}\right)\arctan\left(\frac{x\sqrt{a^2d+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{dx^4+4}}\right)}{2\sqrt{a}\sqrt{b}\sqrt{a^2d+4b^2}}+\frac{\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\left(a\sqrt{d}+2b\right)\text{EllipticPi}\left(-\frac{\left(2b-a\sqrt{d}\right)^2}{8ab\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{4\sqrt{2}ab\sqrt[4]{d}\sqrt{dx^4+4}}\right)}{2b-a\sqrt{d}}-\frac{\sqrt[4]{d}\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}$$

input

```
Int[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
```

output

```
-1/2*(d^(1/4)*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])/(Sqrt[2]*(2*b - a*Sqrt[d])*Sqrt
[4 + d*x^4]) + (b*(((2*b - a*Sqrt[d])*ArcTan[(Sqrt[4*b^2 + a^2*d]*x)/(Sqrt
[a]*Sqrt[b]*Sqrt[4 + d*x^4])])/(2*Sqrt[a]*Sqrt[b]*Sqrt[4*b^2 + a^2*d]) + (
(2*b + a*Sqrt[d])*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*
EllipticPi[-1/8*(2*b - a*Sqrt[d])^2/(a*b*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/Sq
rt[2]], 1/2])/(4*Sqrt[2]*a*b*d^(1/4)*Sqrt[4 + d*x^4])))/(2*b - a*Sqrt[d])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1541

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{2}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{2}}\,\operatorname{EllipticPi}\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{d}}\,x}{2},\frac{2ib}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{d}}}\right)}{a\sqrt{i\sqrt{d}}\,\sqrt{d\,x^{4}+4}}$	86
elliptic	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{2}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{2}}\,\operatorname{EllipticPi}\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{d}}\,x}{2},\frac{2ib}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{d}}}\right)}{a\sqrt{i\sqrt{d}}\,\sqrt{d\,x^{4}+4}}$	86

input

```
int(1/(b*x^2+a)/(d*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/a/(1/2*I*d^(1/2))^(1/2)*(1-1/2*I*d^(1/2)*x^2)^(1/2)*(1+1/2*I*d^(1/2)*x^2
)^(1/2)/(d*x^4+4)^(1/2)*EllipticPi((1/2*I*d^(1/2))^(1/2)*x,2*I/d^(1/2)/a*b
,(-1/2*I*d^(1/2))^(1/2)/(1/2*I*d^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = \int \frac{1}{\sqrt{dx^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + 4)/(b*d*x^6 + a*d*x^4 + 4*b*x^2 + 4*a), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{dx^4+4}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**4+4)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)*sqrt(d*x**4 + 4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = \int \frac{1}{\sqrt{dx^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx=\int \frac{1}{\sqrt{dx^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx=\int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^4+4}}\,dx$$

input
```
int(1/((a + b*x^2)*(d*x^4 + 4)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(d*x^4 + 4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx=\int \frac{\sqrt{d\,x^4+4}}{bd\,x^6+ad\,x^4+4b\,x^2+4a}dx$$

input
```
int(1/(b*x^2+a)/(d*x^4+4)^(1/2),x)
```

output
```
int(sqrt(d*x**4 + 4)/(a*d*x**4 + 4*a + b*d*x**6 + 4*b*x**2),x)
```

3.436 $\int (d+ex^2)^{3/2}\sqrt{a-cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 567

$$\begin{aligned}\int (d+ex^2)^{3/2}\sqrt{a-cx^4}\,dx &= \frac{\left(\frac{3d^2}{e}-\frac{8ae}{c}\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}{48x}\\
&+\frac{7}{24}dx\sqrt{d+ex^2}\sqrt{a-cx^4}+\frac{1}{6}ex^3\sqrt{d+ex^2}\sqrt{a-cx^4}\\
&+\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(3cd^2-8ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{48e\sqrt{d+ex^2}\sqrt{a-cx^4}}\\
&+\frac{\sqrt{a}(31cd^2+8ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{48\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}}\\
&+\frac{d(cd^2+12ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{16e\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output
```
1/48*(3*d^2/e-8*a*e/c)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x+7/24*d*x*(e*x^2+
d)^(1/2)*(-c*x^4+a)^(1/2)+1/6*e*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)+1/48*
(d+a^(1/2)*e/c^(1/2))*(-8*a*e^2+3*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e
*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/
x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1
/2)/(-c*x^4+a)^(1/2)+1/48*a^(1/2)*(8*a*e^2+31*c*d^2)*(1-a/c/x^4)^(1/2)*x^3
*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1
/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^
(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/16*d*(12*a*e^2+c*d^2)*(1-a/c/x^4)
^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(
1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2
)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx$$

input
```
Integrate[(d + e*x^2)^(3/2)*Sqrt[a - c*x^4],x]
```

output
```
Integrate[(d + e*x^2)^(3/2)*Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-cx^4}\left(d+ex^2\right)^{3/2}\,dx$$
$$\Big\downarrow \; 1571$$
$$\int \sqrt{a-cx^4}\left(d+ex^2\right)^{3/2}dx$$

input
```
Int[(d + e*x^2)^(3/2)*Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)^{\frac{3}{2}} \sqrt{-c\,x^{4}+a}dx$$

input
```
int((e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x)
```

output
```
int((e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2} \sqrt{a-cx^4}\,dx=\text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \sqrt{a-cx^4}\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(-c*x**4+a)**(1/2),x)
```

output
```
Integral(sqrt(a - c*x**4)*(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input

```
int((a - c*x^4)^(1/2)*(d + e*x^2)^(3/2),x)
```

output

```
int((a - c*x^4)^(1/2)*(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\frac{7\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,dx}{24}+\frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,e\,x^3}{6}+\frac{\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)a\,e^2}{3}-\frac{\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)c\,d^2}{8}+\frac{11\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^2}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)ade}{12}+\frac{17\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)a\,d^2}{24}$$

input

```
int((e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x)
```

output

```
(7*sqrt(d + e*x**2)*sqrt(a - c*x**4)*d*x + 4*sqrt(d + e*x**2)*sqrt(a - c*x
**4)*e*x**3 + 8*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x*
*2 - c*d*x**4 - c*e*x**6),x)*a*e**2 - 3*int((sqrt(d + e*x**2)*sqrt(a - c*x
**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c*d**2 + 22*int((sqrt
(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6)
,x)*a*d*e + 17*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c
*d*x**4 - c*e*x**6),x)*a*d**2)/24
```

3.437 $\int \sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 499

$$\begin{aligned}
&\int \sqrt{d+ex^2}\sqrt{a-cx^4}\,dx \\
&= \frac{d\sqrt{d+ex^2}\sqrt{a-cx^4}}{8ex}+\frac{1}{4}x\sqrt{d+ex^2}\sqrt{a-cx^4} \\
&\quad+\frac{cd\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&\quad+\frac{5\sqrt{a}\sqrt{c}d\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&\quad+\frac{(cd^2+4ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}$$

output

```
1/8*d*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e/x+1/4*x*(e*x^2+d)^(1/2)*(-c*x^4+a
)^(1/2)+1/8*c*d*(d+a^(1/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^
2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2
)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1/2)
/(-c*x^4+a)^(1/2)+5/8*a^(1/2)*c^(1/2)*d*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*
x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x
^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)
/(-c*x^4+a)^(1/2)+1/8*(4*a*e^2+c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^
2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^
2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1
/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int \sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$$

input

```
Integrate[Sqrt[d + e*x^2]*Sqrt[a - c*x^4],x]
```

output

```
Integrate[Sqrt[d + e*x^2]*Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-cx^4}\sqrt{d+ex^2}\,dx$$

$$\Big\downarrow \text{ 1571}$$

$$\int \sqrt{a-cx^4}\sqrt{d+ex^2}dx$$

input

```
Int[Sqrt[d + e*x^2]*Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}dx$$

input
```
int((e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

output
```
int((e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}\sqrt{ex^2+d}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int \sqrt{d+ex^2}\sqrt{a-cx^4}\,dx = \int \sqrt{a-cx^4}\sqrt{d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(-c*x**4+a)**(1/2),x)
```

output
```
Integral(sqrt(a - c*x**4)*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \sqrt{d+ex^2}\sqrt{a-cx^4}\,dx = \int \sqrt{-cx^4+a}\sqrt{ex^2+d}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \sqrt{d+ex^2}\sqrt{a-cx^4}\,dx = \int \sqrt{-cx^4+a}\sqrt{ex^2+d}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int \sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2),x)
```

output
```
int((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x}{4}-\frac{\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)cd}{4}+\frac{\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^2}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)ae}{2}+\frac{3\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)ad}{4}$$

input
```
int((e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*x - int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c*d + 2*int((sqrt(d +
e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a
*e + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4
- c*e*x**6),x)*a*d)/4
```

3.438 $\int \frac{\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 458

$$
\begin{aligned}
\int \frac{\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx
&= \frac{\sqrt{d+ex^2}\sqrt{a-cx^4}}{2ex} \\
&+ \frac{c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2e\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{cd\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2e\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}
$$

output
```
1/2*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e/x+1/2*c*(d+a^(1/2)*e/c^(1/2))*(1-a/
c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ellip
ticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(
1/2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*a^(1/2)*c^(1/2)*(1-a/
c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ellip
ticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(
1/2)))^(1/2))/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*c*d*(1-a/c/x^4)^(1/2)*x
^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a
^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2
))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[Sqrt[a - c*x^4]/Sqrt[d + e*x^2],x]
```

output
```
Integrate[Sqrt[a - c*x^4]/Sqrt[d + e*x^2], x]
```

Rubi [A] (warning: unable to verify)

Time = 2.00 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.13, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1566, 1803, 628, 2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}}{\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{1566}$$

$$\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\int \frac{\sqrt{\frac{a}{x^4}-cx}}{\sqrt{\frac{d}{x^2}+e}}dx}{x\sqrt{\frac{a}{x^4}-c}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{1803}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\int \frac{\sqrt{\frac{a}{x^4}-cx^4}}{\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}}{2x\sqrt{\frac{a}{x^4}-c}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{628}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\left(\frac{1}{2}\int \frac{\left(\frac{2a}{x^2}+\frac{da}{ex^4}+\frac{cd}{e}\right)x^2}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}-\frac{x^2\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}{e}\right)}{2x\sqrt{\frac{a}{x^4}-c}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{2351}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\left(\frac{1}{2}\left(\int \frac{\frac{da}{ex^2}+2a}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}+\frac{cd\int \frac{x^2}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}}{e}\right)-\frac{x^2\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}{e}\right)}{2x\sqrt{\frac{a}{x^4}-c}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{600}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\left(\frac{1}{2}\left(a\int \frac{1}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}+\frac{a\int \frac{\sqrt{\frac{d}{x^2}+e}}{\sqrt{\frac{a}{x^4}-c}}d\frac{1}{x^2}}{e}+\frac{cd\int \frac{x^2}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}}{e}\right)-\frac{x^2\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}{e}\right)}{2x\sqrt{\frac{a}{x^4}-c}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{509}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\left(\frac{1}{2}\left(a\int \frac{1}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}+\frac{a\sqrt{1-\frac{a}{cx^4}}\int \frac{\sqrt{\frac{d}{x^2}+e}}{\sqrt{1-\frac{a}{cx^4}}}d\frac{1}{x^2}}{e\sqrt{\frac{a}{x^4}-c}}+\frac{cd\int \frac{x^2}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}}{e}\right)-\frac{x^2\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}{e}\right.}{2x\sqrt{\frac{a}{x^4}-c}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{508}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\left(\frac{1}{2}\left(-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{d}{x^2}+e}\int \frac{\sqrt{1-\frac{2d}{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)x^4}}}{\sqrt{1-\frac{1}{x^4}}}d\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}}{e\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{\sqrt{a}\left(\frac{d}{x^2}+e\right)}{\sqrt{a}e+\sqrt{c}d}}}+a\int \frac{1}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}+\frac{cd\int \frac{x^2}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}}}}{e}\right.\right.}{2x\sqrt{\frac{a}{x^4}-c}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\left(\frac{1}{2}\left(a\int \frac{1}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}+\frac{cd\int \frac{x^2}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{d}{x^2}+e}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\right.}{e\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{\sqrt{a}\left(\frac{d}{x^2}+e\right)}{\sqrt{a}e+\sqrt{c}d}}}\right.\right.}{2x\sqrt{\frac{a}{x^4}-c}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{512}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\left(\frac{1}{2}\left(\frac{a\sqrt{1-\frac{a}{cx^4}}\int \frac{1}{\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}}{\sqrt{\frac{a}{x^4}-c}}+\frac{cd\int \frac{x^2}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{d}{x^2}+e}E\left(\arcsin\left(\right.\right.}{e\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{\sqrt{a}\left(\frac{d}{x^2}+e\right)}{\sqrt{a}e+\sqrt{c}d}}}\right.\right.}{2x\sqrt{\frac{a}{x^4}-c}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{511}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\left(\frac{1}{2}\left(-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{\sqrt{a}\left(\frac{d}{x^2}+e\right)}{\sqrt{a}e+\sqrt{c}d}}\int \frac{1}{\sqrt{1-\frac{1}{x^4}}\sqrt{1-\frac{2d}{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)x^4}}}d\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}+\frac{cd\int \frac{x^2}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}}{e}-\right.\right.}{2x\sqrt{\frac{a}{x^4}-c}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{321}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\Bigg(\frac{1}{2}\Bigg(\frac{cd\int\frac{x^2}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{\sqrt{a}\left(\frac{d}{x^2}+e\right)}{\sqrt{a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}-2\sqrt{a}}{2x\sqrt{\frac{a}{x^4}-c}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 633}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\Bigg(\frac{1}{2}\Bigg(\frac{cd\sqrt{1-\frac{a}{cx^4}}\int\frac{x^2}{\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}}{e\sqrt{\frac{a}{x^4}-c}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{\sqrt{a}\left(\frac{d}{x^2}+e\right)}{\sqrt{a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right.}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}}{2x\sqrt{\frac{a}{x^4}-c}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 632}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\Bigg(\frac{1}{2}\Bigg(\frac{cd\sqrt{1-\frac{a}{cx^4}}\int\frac{x^2}{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}\sqrt{\frac{\sqrt{a}}{\sqrt{c}x^2}+1}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}}{e\sqrt{\frac{a}{x^4}-c}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{\sqrt{a}\left(\frac{d}{x^2}+e\right)}{\sqrt{a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right.\right.}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}}{2x\sqrt{\frac{a}{x^4}-c}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 186}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\Bigg(\frac{1}{2}\Bigg(-\frac{2cd\sqrt{1-\frac{a}{cx^4}}\int\frac{1}{\left(1-\frac{1}{x^4}\right)\sqrt{2-\frac{1}{x^4}}\sqrt{\frac{\sqrt{c}d}{\sqrt{a}}-\frac{\sqrt{c}d}{\sqrt{a}x^4}+e}}d\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{e\sqrt{\frac{a}{x^4}-c}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{\sqrt{a}\left(\frac{d}{x^2}+e\right)}{\sqrt{a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\text{ar}\right.}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}}{2x\sqrt{\frac{a}{x^4}-c}\sqrt{d+e}}$$

$$\Big\downarrow \text{ 413}$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\left(\frac{1}{2}\left(-\frac{2cd\sqrt{1-\frac{a}{cx^4}}\sqrt{1-\frac{\sqrt{c}d}{x^4(\sqrt{a}e+\sqrt{c}d)}}\int\frac{1}{\left(1-\frac{1}{x^4}\right)\sqrt{2-\frac{1}{x^4}}\sqrt{1-\frac{\sqrt{c}d}{(\sqrt{c}d+\sqrt{a}e)x^4}}}d\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{e\sqrt{\frac{a}{x^4}-c}\sqrt{-\frac{\sqrt{c}d}{\sqrt{a}x^4}+\frac{\sqrt{c}d}{\sqrt{a}}+e}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{\sqrt{a}(}{\sqrt{a}}}}{}\right.\right.}{2x\sqrt{\frac{a}{x^4}-}}$$

$$\Big\downarrow \; 412$$

$$-\frac{\sqrt{a-cx^4}\sqrt{\frac{d}{x^2}+e}\left(\frac{1}{2}\left(-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{\sqrt{a}\left(\frac{d}{x^2}+e\right)}{\sqrt{a}e+\sqrt{c}d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{d}{x^2}+e}E\left(\text{ar}\right.}{e\sqrt{\frac{a}{x^4}-c}\sqrt{}}\right.\right.}{2x\sqrt{\frac{a}{x^4}-}}$$

input

```
Int[Sqrt[a - c*x^4]/Sqrt[d + e*x^2],x]
```

output

```
-1/2*(Sqrt[e + d/x^2]*Sqrt[a - c*x^4]*(-((Sqrt[-c + a/x^4]*Sqrt[e + d/x^2]
*x^2)/e) + ((-2*Sqrt[a]*Sqrt[c]*Sqrt[1 - a/(c*x^4)]*Sqrt[e + d/x^2]*Ellipt
icE[ArcSin[Sqrt[1 - Sqrt[a]/(Sqrt[c]*x^2)]/Sqrt[2]], (2*d)/(d + (Sqrt[a]*e
)/Sqrt[c])])/(e*Sqrt[-c + a/x^4]*Sqrt[(Sqrt[a]*(e + d/x^2))/(Sqrt[c]*d + S
qrt[a]*e)]) - (2*Sqrt[a]*Sqrt[c]*Sqrt[1 - a/(c*x^4)]*Sqrt[(Sqrt[a]*(e + d/
x^2))/(Sqrt[c]*d + Sqrt[a]*e)]*EllipticF[ArcSin[Sqrt[1 - Sqrt[a]/(Sqrt[c]*
x^2)]/Sqrt[2]], (2*d)/(d + (Sqrt[a]*e)/Sqrt[c])])/(Sqrt[-c + a/x^4]*Sqrt[e
 + d/x^2]) - (2*c*d*Sqrt[1 - a/(c*x^4)]*Sqrt[1 - (Sqrt[c]*d)/((Sqrt[c]*d +
 Sqrt[a]*e)*x^4)]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[a]/(Sqrt[c]*x^2)]/Sqr
t[2]], (2*Sqrt[c]*d)/(Sqrt[c]*d + Sqrt[a]*e)])/(e*Sqrt[-c + a/x^4]*Sqrt[(S
qrt[c]*d)/Sqrt[a] + e - (Sqrt[c]*d)/(Sqrt[a]*x^4)]))/2))/(Sqrt[-c + a/x^4]
*x*Sqrt[d + e*x^2])
```

Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 628
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*Sqrt[(a_) + (b_.)*(x_)^2], x
_Symbol] :> Simp[c^(n - 1/2)*(e*x)^(m + 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(
e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*
Sqrt[a + b*x^2]))*ExpandToSum[(2*a*c^(n + 1/2)*(m + 1) + a*c^(n - 1/2)*d*(2
*m + 3)*x + 2*b*c^(n + 1/2)*(m + 2)*x^2 + b*c^(n - 1/2)*d*(2*m + 5)*x^3 - 2
*a*(m + 1)*(c + d*x)^(n + 1/2) - 2*b*(m + 1)*x^2*(c + d*x)^(n + 1/2))/x, x]
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n + 3/2, 0] && LtQ[m, -1] &&
IntegerQ[2*m]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633

```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 1566

```
Int[Sqrt[(a_) + (c_.)*(x_)^4]/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[e + d/x^2]*(Sqrt[a + c*x^4]/(x*Sqrt[d + e*x^2]*Sqrt[c + a/x^4]))   Int
[(x*Sqrt[c + a/x^4])/Sqrt[e + d/x^2], x], x] /; FreeQ[{a, c, d, e}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

rule 1803

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2351

```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int \frac{\sqrt{-c\,x^{4}+a}}{\sqrt{e\,x^{2}+d}}dx$$

input

```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output

```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{-cx^4+a}}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*x^4 + a)/sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((-c*x**4+a)**(1/2)/(e*x**2+d)**(1/2),x)
```

output
```
Integral(sqrt(a - c*x**4)/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{-cx^4+a}}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)/sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{-cx^4+a}}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{a-c\,x^4}}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a - c*x^4)^(1/2)/(d + e*x^2)^(1/2),x)
```

output
```
int((a - c*x^4)^(1/2)/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{e\,x^2+d}\,dx$$

input
```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(d + e*x**2),x)
```

3.439 $\int \frac{\sqrt{a-cx^4}}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 490

$$\begin{aligned}\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx &= \frac{x\sqrt{a-cx^4}}{d\sqrt{d+ex^2}}-\frac{\sqrt{d+ex^2}\sqrt{a-cx^4}}{dex}\\
&-\frac{\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{de\sqrt{d+ex^2}\sqrt{a-cx^4}}\\
&+\frac{\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{d\sqrt{d+ex^2}\sqrt{a-cx^4}}\\
&-\frac{c\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{e\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output

```
x*(-c*x^4+a)^(1/2)/d/(e*x^2+d)^(1/2)-(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/d/e/
x-c^(1/2)*(c^(1/2)*d+a^(1/2)*e)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(
c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2
)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d/e/(e*x^2+d)^(1/2)/(-c
*x^4+a)^(1/2)+a^(1/2)*c^(1/2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^
(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*
2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d/(e*x^2+d)^(1/2)/(-c*x^4
+a)^(1/2)-c*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)
/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)
*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx=\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx$$

input

```
Integrate[Sqrt[a - c*x^4]/(d + e*x^2)^(3/2),x]
```

output

```
Integrate[Sqrt[a - c*x^4]/(d + e*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}dx$$

input

```
Int[Sqrt[a - c*x^4]/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\sqrt{-c\,x^{4}+a}}{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

output
```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*x**4+a)**(1/2)/(e*x**2+d)**(3/2),x)
```

output
```
Integral(sqrt(a - c*x**4)/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)/(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{a-c\,x^4}}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a - c*x^4)^(1/2)/(d + e*x^2)^(3/2),x)
```

output
```
int((a - c*x^4)^(1/2)/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{e^2x^4+2de\,x^2+d^2}dx$$

input
```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(d**2 + 2*d*e*x**2 + e**2*x**4),x)
```

3.440 $\int \frac{\sqrt{a-cx^4}}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 382

$$\begin{aligned}\int \frac{\sqrt{a-cx^4}}{\left(d+ex^2\right)^{5/2}}\,dx &= \frac{x\sqrt{a-cx^4}}{3d\left(d+ex^2\right)^{3/2}}+\frac{2ae\sqrt{a-cx^4}}{3d\left(cd^2-ae^2\right)x\sqrt{d+ex^2}}\\
&+\frac{2a\sqrt{c}e\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3d^2\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}\\
&+\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3d^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output

```
1/3*x*(-c*x^4+a)^(1/2)/d/(e*x^2+d)^(3/2)+2/3*a*e*(-c*x^4+a)^(1/2)/d/(-a*e^
2+c*d^2)/x/(e*x^2+d)^(1/2)+2/3*a*c^(1/2)*e*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*
(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2
)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^2/(c^(1/2)
*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+2/3*a^(1/2)*c^(1/2)*(1-a/c/
x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ellipti
cF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/
2)))^(1/2))/d^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx=\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[a - c*x^4]/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[Sqrt[a - c*x^4]/(d + e*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}dx$$

input
```
Int[Sqrt[a - c*x^4]/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\sqrt{-c\,x^{4}+a}}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2),x)
```

output
```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx=\int \frac{\sqrt{-cx^{4}+a}}{(ex^{2}+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e
*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx=\int \frac{\sqrt{a-cx^{4}}}{(d+ex^{2})^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*x**4+a)**(1/2)/(e*x**2+d)**(5/2),x)
```

output
```
Integral(sqrt(a - c*x**4)/(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx=\int \frac{\sqrt{-cx^4+a}}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx=\int \frac{\sqrt{-cx^4+a}}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx=\int \frac{\sqrt{a-c\,x^4}}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a - c*x^4)^(1/2)/(d + e*x^2)^(5/2),x)
```

output
```
int((a - c*x^4)^(1/2)/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx$$

input
```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x
**4 + e**3*x**6),x)
```

3.441 $\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 506

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx = \frac{x\sqrt{a-cx^4}}{5d\,(d+ex^2)^{5/2}}$$

$$+\frac{2(cd^2-2ae^2)\,x\sqrt{a-cx^4}}{15d^2\,(cd^2-ae^2)\,(d+ex^2)^{3/2}}+\frac{8ae(2cd^2-ae^2)\,\sqrt{a-cx^4}}{15d^2\,(cd^2-ae^2)^2\,x\sqrt{d+ex^2}}$$

$$+\frac{8a\sqrt{c}e(2cd^2-ae^2)\,\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{15d^3\,(\sqrt{c}d-\sqrt{a}e)\,(cd^2-ae^2)\,\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{2\sqrt{a}\sqrt{c}(5cd^2-4ae^2)\,\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{15d^3\,(cd^2-ae^2)\,\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/5*x*(-c*x^4+a)^(1/2)/d/(e*x^2+d)^(5/2)+2/15*(-2*a*e^2+c*d^2)*x*(-c*x^4+a
)^(1/2)/d^2/(-a*e^2+c*d^2)/(e*x^2+d)^(3/2)+8/15*a*e*(-a*e^2+2*c*d^2)*(-c*x
^4+a)^(1/2)/d^2/(-a*e^2+c*d^2)^2/x/(e*x^2+d)^(1/2)+8/15*a*c^(1/2)*e*(-a*e^
2+2*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/
x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/
(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^3/(c^(1/2)*d-a^(1/2)*e)/(-a*e^2+c*d^2)/(e*
x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+2/15*a^(1/2)*c^(1/2)*(-4*a*e^2+5*c*d^2)*(1-a
/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Elli
pticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^
(1/2)))^(1/2))/d^3/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx=\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx$$

input
```
Integrate[Sqrt[a - c*x^4]/(d + e*x^2)^(7/2),x]
```

output
```
Integrate[Sqrt[a - c*x^4]/(d + e*x^2)^(7/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx$$
$$\Big\downarrow \text{1571}$$
$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}dx$$

input
```
Int[Sqrt[a - c*x^4]/(d + e*x^2)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\sqrt{-c\,x^{4}+a}}{\left(e\,x^{2}+d\right)^{\frac{7}{2}}}dx$$

input
```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2),x)
```

output
```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx = \int \frac{\sqrt{-cx^4+a}}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(e^4*x^8 + 4*d*e^3*x^6 + 6*d^2*e
^2*x^4 + 4*d^3*e*x^2 + d^4), x)
```

Sympy [F]

$$\int \frac{\sqrt{a-cx^4}}{\left(d+ex^2\right)^{7/2}}\,dx=\int \frac{\sqrt{a-cx^4}}{\left(d+ex^2\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-c*x**4+a)**(1/2)/(e*x**2+d)**(7/2),x)
```

output
```
Integral(sqrt(a - c*x**4)/(d + e*x**2)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-cx^4}}{\left(d+ex^2\right)^{7/2}}\,dx=\int \frac{\sqrt{-cx^4+a}}{\left(ex^2+d\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)/(e*x^2 + d)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a-cx^4}}{\left(d+ex^2\right)^{7/2}}\,dx=\int \frac{\sqrt{-cx^4+a}}{\left(ex^2+d\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)/(e*x^2 + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx=\int \frac{\sqrt{a-c\,x^4}}{{\left(e\,x^2+d\right)}^{7/2}}\,dx$$

input
```
int((a - c*x^4)^(1/2)/(d + e*x^2)^(7/2),x)
```

output
```
int((a - c*x^4)^(1/2)/(d + e*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx=\text{Too large to display}$$

input
```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2),x)
```

output

```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*x**5 - 2*int((sqrt(d + e*x**2)*sqr
t(a - c*x**4)*x**10)/(a*d**4 + 4*a*d**3*e*x**2 + 6*a*d**2*e**2*x**4 + 4*a*
d*e**3*x**6 + a*e**4*x**8 - c*d**4*x**4 - 4*c*d**3*e*x**6 - 6*c*d**2*e**2*
x**8 - 4*c*d*e**3*x**10 - c*e**4*x**12),x)*c**2*d**3*e - 6*int((sqrt(d + e
*x**2)*sqrt(a - c*x**4)*x**10)/(a*d**4 + 4*a*d**3*e*x**2 + 6*a*d**2*e**2*x
**4 + 4*a*d*e**3*x**6 + a*e**4*x**8 - c*d**4*x**4 - 4*c*d**3*e*x**6 - 6*c*
d**2*e**2*x**8 - 4*c*d*e**3*x**10 - c*e**4*x**12),x)*c**2*d**2*e**2*x**2 -
 6*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**10)/(a*d**4 + 4*a*d**3*e*x**2
 + 6*a*d**2*e**2*x**4 + 4*a*d*e**3*x**6 + a*e**4*x**8 - c*d**4*x**4 - 4*c*
d**3*e*x**6 - 6*c*d**2*e**2*x**8 - 4*c*d*e**3*x**10 - c*e**4*x**12),x)*c**
2*d*e**3*x**4 - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**10)/(a*d**4 +
4*a*d**3*e*x**2 + 6*a*d**2*e**2*x**4 + 4*a*d*e**3*x**6 + a*e**4*x**8 - c*d
**4*x**4 - 4*c*d**3*e*x**6 - 6*c*d**2*e**2*x**8 - 4*c*d*e**3*x**10 - c*e**
4*x**12),x)*c**2*e**4*x**6 - 7*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**8
)/(a*d**4 + 4*a*d**3*e*x**2 + 6*a*d**2*e**2*x**4 + 4*a*d*e**3*x**6 + a*e**
4*x**8 - c*d**4*x**4 - 4*c*d**3*e*x**6 - 6*c*d**2*e**2*x**8 - 4*c*d*e**3*x
**10 - c*e**4*x**12),x)*c**2*d**4 - 21*int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*x**8)/(a*d**4 + 4*a*d**3*e*x**2 + 6*a*d**2*e**2*x**4 + 4*a*d*e**3*x**6
 + a*e**4*x**8 - c*d**4*x**4 - 4*c*d**3*e*x**6 - 6*c*d**2*e**2*x**8 - 4*c*
d*e**3*x**10 - c*e**4*x**12),x)*c**2*d**3*e*x**2 - 21*int((sqrt(d + e*x...
```

3.442 $\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 623

$$\begin{aligned}\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx &= \frac{x\sqrt{a-cx^4}}{7d\,(d+ex^2)^{7/2}}+\frac{2(2cd^2-3ae^2)\,x\sqrt{a-cx^4}}{35d^2\,(cd^2-ae^2)\,(d+ex^2)^{5/2}}\\
&+\frac{8(c^2d^4-6acd^2e^2+3a^2e^4)\,x\sqrt{a-cx^4}}{105d^3\,(cd^2-ae^2)^2\,(d+ex^2)^{3/2}}+\frac{2ae(77c^2d^4-69acd^2e^2+24a^2e^4)\sqrt{a-cx^4}}{105d^3\,(cd^2-ae^2)^3\,x\sqrt{d+ex^2}}\\
&+\frac{2a\sqrt{c}e(77c^2d^4-69acd^2e^2+24a^2e^4)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{105d^4\left(\sqrt{c}d-\sqrt{a}e\right)^3\left(\sqrt{c}d+\sqrt{a}e\right)^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\\
&+\frac{2\sqrt{a}\sqrt{c}(35c^2d^4-51acd^2e^2+24a^2e^4)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{105d^4\,(cd^2-ae^2)^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output
```
1/7*x*(-c*x^4+a)^(1/2)/d/(e*x^2+d)^(7/2)+2/35*(-3*a*e^2+2*c*d^2)*x*(-c*x^4
+a)^(1/2)/d^2/(-a*e^2+c*d^2)/(e*x^2+d)^(5/2)+8/105*(3*a^2*e^4-6*a*c*d^2*e^
2+c^2*d^4)*x*(-c*x^4+a)^(1/2)/d^3/(-a*e^2+c*d^2)^2/(e*x^2+d)^(3/2)+2/105*a
*e*(24*a^2*e^4-69*a*c*d^2*e^2+77*c^2*d^4)*(-c*x^4+a)^(1/2)/d^3/(-a*e^2+c*d
^2)^3/x/(e*x^2+d)^(1/2)+2/105*a*c^(1/2)*e*(24*a^2*e^4-69*a*c*d^2*e^2+77*c^
2*d^4)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)
^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a
^(1/2)*e/c^(1/2)))^(1/2))/d^4/(c^(1/2)*d-a^(1/2)*e)^3/(c^(1/2)*d+a^(1/2)*e
)^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+2/105*a^(1/2)*c^(1/2)*(24*a^2*e^4-51*
a*c*d^2*e^2+35*c^2*d^4)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*
d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2
),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^4/(-a*e^2+c*d^2)^2/(e*x^2+d)^
(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx=\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx$$

input
```
Integrate[Sqrt[a - c*x^4]/(d + e*x^2)^(9/2),x]
```

output
```
Integrate[Sqrt[a - c*x^4]/(d + e*x^2)^(9/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx$$
$$\Big\downarrow \text{1571}$$
$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}dx$$

input
```
Int[Sqrt[a - c*x^4]/(d + e*x^2)^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\sqrt{-c\,x^{4}+a}}{\left(e\,x^{2}+d\right)^{\frac{9}{2}}}d x$$

input
```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(9/2),x)
```

output
```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(9/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx=\int \frac{\sqrt{-cx^4+a}}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(9/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(e^5*x^10 + 5*d*e^4*x^8 + 10*d^2
*e^3*x^6 + 10*d^3*e^2*x^4 + 5*d^4*e*x^2 + d^5), x)
```

Sympy [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx = \int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-c*x**4+a)**(1/2)/(e*x**2+d)**(9/2),x)
```

output
```
Integral(sqrt(a - c*x**4)/(d + e*x**2)**(9/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx = \int \frac{\sqrt{-cx^4+a}}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)/(e*x^2 + d)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx = \int \frac{\sqrt{-cx^4+a}}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)/(e*x^2+d)^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)/(e*x^2 + d)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx=\int \frac{\sqrt{a-c\,x^4}}{{\left(e\,x^2+d\right)}^{9/2}} \,d x$$

input
```
int((a - c*x^4)^(1/2)/(d + e*x^2)^(9/2),x)
```

output
```
int((a - c*x^4)^(1/2)/(d + e*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx=\int \frac{\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}}{e^{5} x^{10}+5 d\,e^{4} x^{8}+10 d^{2} e^{3} x^{6}+10 d^{3} e^{2} x^{4}+5 d^{4} e\,x^{2}+d^{5}}d x$$

input
```
int((-c*x^4+a)^(1/2)/(e*x^2+d)^(9/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(d**5 + 5*d**4*e*x**2 + 10*d**3*e*
*2*x**4 + 10*d**2*e**3*x**6 + 5*d*e**4*x**8 + e**5*x**10),x)
```

3.443 $\int (d+ex^2)^{3/2} (a-cx^4)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 716

$$\begin{aligned}
\int (d+ex^2)^{3/2} (a-cx^4)^{3/2} \, dx &= -\frac{(5cd^2-32ae^2)(3cd^2-4ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{1280ce^3x} \\
&+ \frac{1}{640} d \left(236a+\frac{5cd^2}{e^2}\right) x\sqrt{d+ex^2}\sqrt{a-cx^4} - \frac{(cd^2-32ae^2)x^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{160e} \\
&- \frac{11}{80} cdx^5\sqrt{d+ex^2}\sqrt{a-cx^4} - \frac{1}{10} cex^7\sqrt{d+ex^2}\sqrt{a-cx^4} \\
&- \frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(5cd^2-32ae^2)(3cd^2-4ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{1280e^3\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{\sqrt{a}(5c^2d^4+692acd^2e^2+128a^2e^4)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{1280\sqrt{c}e^2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&- \frac{3d(cd^2-12ae^2)(cd^2+4ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{256e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}$$

output
```
-1/1280*(-32*a*e^2+5*c*d^2)*(-4*a*e^2+3*c*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a)^
(1/2)/c/e^3/x+1/640*d*(236*a+5*c*d^2/e^2)*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/
2)-1/160*(-32*a*e^2+c*d^2)*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e-11/80*c*
d*x^5*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)-1/10*c*e*x^7*(e*x^2+d)^(1/2)*(-c*x^
4+a)^(1/2)-1/1280*(d+a^(1/2)*e/c^(1/2))*(-32*a*e^2+5*c*d^2)*(-4*a*e^2+3*c*
d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(
1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(
1/2)*e/c^(1/2)))^(1/2))/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/1280*a^(1/2
)*(128*a^2*e^4+692*a*c*d^2*e^2+5*c^2*d^4)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(
e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)
/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/e^2/(
e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-3/256*d*(-12*a*e^2+c*d^2)*(4*a*e^2+c*d^2)*
(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*
EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/
2)*e/c^(1/2)))^(1/2))/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}\,dx$$

input
```
Integrate[(d + e*x^2)^(3/2)*(a - c*x^4)^(3/2),x]
```

output
```
Integrate[(d + e*x^2)^(3/2)*(a - c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)^{3/2}\left(d+ex^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 1571}$$

$$\int \left(a-cx^4\right)^{3/2} \left(d+ex^2\right)^{3/2} \, dx$$

input
```
Int[(d + e*x^2)^(3/2)*(a - c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \left(e \,x^{2}+d \right)^{\frac{3}{2}} \left(-c \,x^{4}+a \right)^{\frac{3}{2}}d x$$

input
```
int((e*x^2+d)^(3/2)*(-c*x^4+a)^(3/2),x)
```

output
```
int((e*x^2+d)^(3/2)*(-c*x^4+a)^(3/2),x)
```

Fricas [F]

$$\int \left(d+ex^2\right)^{3/2} \left(a-cx^4\right)^{3/2} \, dx=\int \left(-cx^{4}+a\right)^{\frac{3}{2}} \left(ex^{2}+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d)*sqrt(-c*x^4 + a)*sqrt(e*x^2
+ d), x)
```

Sympy [F]

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(a-cx^4\right)^{\frac{3}{2}}\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(-c*x**4+a)**(3/2),x)
```

output
```
Integral((a - c*x**4)**(3/2)*(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(-cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 + a)^(3/2)*(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(-cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)*(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(a-c\,x^4\right)^{3/2}\left(e\,x^2+d\right)^{3/2}dx$$

input
```
int((a - c*x^4)^(3/2)*(d + e*x^2)^(3/2),x)
```

output
```
int((a - c*x^4)^(3/2)*(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}\,dx=\frac{236\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,a\,d\,e^2x+128\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,a\,e^3x^3+5\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+}}{}$$

input
```
int((e*x^2+d)^(3/2)*(-c*x^4+a)^(3/2),x)
```

output
```
(236*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*d*e**2*x + 128*sqrt(d + e*x**2)*s
qrt(a - c*x**4)*a*e**3*x**3 + 5*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d**3*x
 - 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d**2*e*x**3 - 88*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*c*d*e**2*x**5 - 64*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*e
**3*x**7 + 128*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**
2 - c*d*x**4 - c*e*x**6),x)*a**2*e**4 - 116*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*d**2*e**2 + 1
5*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4
- c*e*x**6),x)*c**2*d**4 + 424*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2
)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*d*e**3 + 2*int((sqrt(d +
e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a
*c*d**3*e + 404*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 -
c*d*x**4 - c*e*x**6),x)*a**2*d**2*e**2 - 5*int((sqrt(d + e*x**2)*sqrt(a -
c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*d**4)/(640*e**2)
```

3.444 $\int \sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 634

$$\begin{aligned}
\int \sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}\,dx &= -\frac{d\left(15cd^2-68ae^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}{384e^3x} \\
&+ \frac{5}{192}\left(12a+\frac{cd^2}{e^2}\right)x\sqrt{d+ex^2}\sqrt{a-cx^4} \\
&- \frac{cdx^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{48e} - \frac{1}{8}cx^5\sqrt{d+ex^2}\sqrt{a-cx^4} \\
&- \frac{cd\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(15cd^2-68ae^2\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{384e^3\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{\sqrt{a}\sqrt{c}d\left(5cd^2+196ae^2\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{384e^2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&- \frac{\left(5c^2d^4-24acd^2e^2-48a^2e^4\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{128e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}$$

output
```
-1/384*d*(-68*a*e^2+15*c*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e^3/x+5/192
*(12*a+c*d^2/e^2)*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)-1/48*c*d*x^3*(e*x^2+d
)^(1/2)*(-c*x^4+a)^(1/2)/e-1/8*c*x^5*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)-1/38
4*c*d*(d+a^(1/2)*e/c^(1/2))*(-68*a*e^2+15*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^
(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/
c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^3/(e
*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/384*a^(1/2)*c^(1/2)*d*(196*a*e^2+5*c*d^2)
*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)
*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)
*e/c^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/128*(-48*a^2*e^
4-24*a*c*d^2*e^2+5*c^2*d^4)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1
/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2
^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^3/(e*x^2+d)^(1/2)/(-c*
x^4+a)^(1/2)
```

Mathematica [F]

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}\,dx=\int \sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a - c*x^4)^(3/2),x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a - c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)^{3/2}\sqrt{d+ex^2}\,dx$$
$$\Big\downarrow \text{ 1571}$$
$$\int \left(a-cx^4\right)^{3/2}\sqrt{d+ex^2}dx$$

input
```
Int[Sqrt[d + e*x^2]*(a - c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \sqrt{e\,x^{2}+d}\,\left(-c\,x^{4}+a\right)^{\frac{3}{2}}d x$$

input
```
int((e*x^2+d)^(1/2)*(-c*x^4+a)^(3/2),x)
```

output
```
int((e*x^2+d)^(1/2)*(-c*x^4+a)^(3/2),x)
```

Fricas [F]

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(-cx^4+a\right)^{\frac{3}{2}}\sqrt{ex^2+d}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((-c*x^4 + a)^(3/2)*sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(a-cx^4\right)^{\frac{3}{2}}\sqrt{d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(-c*x**4+a)**(3/2),x)
```

output
```
Integral((a - c*x**4)**(3/2)*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(-cx^4+a\right)^{\frac{3}{2}}\sqrt{ex^2+d}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 + a)^(3/2)*sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(-cx^4+a\right)^{\frac{3}{2}}\sqrt{ex^2+d}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)*sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}\,dx = \int \left(a-c\,x^4\right)^{3/2}\sqrt{e\,x^2+d}\,dx$$

input
```
int((a - c*x^4)^(3/2)*(d + e*x^2)^(1/2),x)
```

output
```
int((a - c*x^4)^(3/2)*(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+ex^2}(a-cx^4)^{3/2}\,dx = \frac{60\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,a\,e^2x+5\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,c\,d^2x-4\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,c\,d\,e}{}$$

input
```
int((e*x^2+d)^(1/2)*(-c*x^4+a)^(3/2),x)
```

output
```
(60*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*e**2*x + 5*sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*c*d**2*x - 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d*e*x**3 - 24*
sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*e**2*x**5 - 68*int((sqrt(d + e*x**2)*s
qrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*d*e**2
 + 15*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x
**4 - c*e*x**6),x)*c**2*d**3 + 72*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x
**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*e**3 + 2*int((sqrt(d +
 e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*
a*c*d**2*e + 132*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 -
 c*d*x**4 - c*e*x**6),x)*a**2*d*e**2 - 5*int((sqrt(d + e*x**2)*sqrt(a - c*
x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*d**3)/(192*e**2)
```

3.445 $\int \frac{(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 580

$$\begin{aligned}\int \frac{(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx &= -\frac{(15cd^2-32ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{48e^3x} \\ &+ \frac{5cdx\sqrt{d+ex^2}\sqrt{a-cx^4}}{24e^2} - \frac{cx^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{6e} \\ &- \frac{c\left(d+\frac{\sqrt{ae}}{\sqrt{c}}\right)(15cd^2-32ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{48e^3\sqrt{d+ex^2}\sqrt{a-cx^4}} \\ &+ \frac{\sqrt{a}\sqrt{c}(5cd^2+16ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{48e^2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\ &- \frac{cd(5cd^2-12ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{16e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output

```
-1/48*(-32*a*e^2+15*c*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e^3/x+5/24*c*d
*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e^2-1/6*c*x^3*(e*x^2+d)^(1/2)*(-c*x^4+
a)^(1/2)/e-1/48*c*(d+a^(1/2)*e/c^(1/2))*(-32*a*e^2+15*c*d^2)*(1-a/c/x^4)^(
1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2
*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(
1/2))/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/48*a^(1/2)*c^(1/2)*(16*a*e^2+
5*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^
2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d
+a^(1/2)*e/c^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/16*c*d*
(-12*a*e^2+5*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^
(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2
,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(
1/2)
```

Mathematica [F]

$$\int \frac{(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx=\int \frac{(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx$$

input

```
Integrate[(a - c*x^4)^(3/2)/Sqrt[d + e*x^2],x]
```

output

```
Integrate[(a - c*x^4)^(3/2)/Sqrt[d + e*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a - c*x^4)^(3/2)/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\left(-c\,x^{4}+a\right)^{\frac{3}{2}}}{\sqrt{e\,x^{2}+d}}dx$$

input
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(1/2),x)
```

output
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx=\text{Timed out}$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \int \frac{(a-cx^4)^{\frac{3}{2}}}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((-c*x**4+a)**(3/2)/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a - c*x**4)**(3/2)/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \int \frac{(-cx^4+a)^{\frac{3}{2}}}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 + a)^(3/2)/sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \int \frac{(-cx^4+a)^{\frac{3}{2}}}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx=\int \frac{{\left(a-c\,x^4\right)}^{3/2}}{\sqrt{e\,x^2+d}} \,d x$$

input

```
int((a - c*x^4)^(3/2)/(d + e*x^2)^(1/2),x)
```

output

```
int((a - c*x^4)^(3/2)/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx=\frac{5\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,c d x-4\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,c e\,x^3-32\bigg(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-c e\,x^6-c d\,x^4+a e\,x^2+a}}{}$$

input

```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(1/2),x)
```

output

```
(5*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d*x - 4*sqrt(d + e*x**2)*sqrt(a - c
*x**4)*c*e*x**3 - 32*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a
*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*e**2 + 15*int((sqrt(d + e*x**2)*sqrt
(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c**2*d**2 + 2
*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 -
 c*e*x**6),x)*a*c*d*e + 24*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d +
a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*e**2 - 5*int((sqrt(d + e*x**2)*sqr
t(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*d**2)/(24*e**
2)
```

3.446 $\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 597

$$
\begin{aligned}
\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx &= \frac{\left(a-\frac{cd^2}{e^2}\right)x\sqrt{a-cx^4}}{d\sqrt{d+ex^2}} \\
&+ \frac{(15cd^2-8ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{8de^3x} - \frac{cx\sqrt{d+ex^2}\sqrt{a-cx^4}}{4e^2} \\
&+ \frac{\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)(15cd^2-8ae^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8de^3\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&- \frac{\sqrt{a}\sqrt{c}(5cd^2-8ae^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8de^2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{3c(5cd^2-4ae^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}
$$

output
```
(a-c*d^2/e^2)*x*(-c*x^4+a)^(1/2)/d/(e*x^2+d)^(1/2)+1/8*(-8*a*e^2+15*c*d^2)
*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/d/e^3/x-1/4*c*x*(e*x^2+d)^(1/2)*(-c*x^4+
a)^(1/2)/e^2+1/8*c^(1/2)*(c^(1/2)*d+a^(1/2)*e)*(-8*a*e^2+15*c*d^2)*(1-a/c/
x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ellipti
cE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/
2)))^(1/2))/d/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/8*a^(1/2)*c^(1/2)*(-8
*a*e^2+5*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2
)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2
)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+
3/8*c*(-4*a*e^2+5*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)
*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1
/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^3/(e*x^2+d)^(1/2)/(-c*x^4
+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx=\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(a - c*x^4)^(3/2)/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(a - c*x^4)^(3/2)/(d + e*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}dx$$

input
```
Int[(a - c*x^4)^(3/2)/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\left(-c\,x^{4}+a\right)^{\frac{3}{2}}}{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}d x$$

input
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(3/2),x)
```

output
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx=\int \frac{(a-cx^4)^{\frac{3}{2}}}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*x**4+a)**(3/2)/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a - c*x**4)**(3/2)/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 + a)^(3/2)/(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(a-c\,x^4)^{3/2}}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a - c*x^4)^(3/2)/(d + e*x^2)^(3/2),x)
```

output
```
int((a - c*x^4)^(3/2)/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \frac{2\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,aex-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,cd\,x^3+4\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-}}{-c\,e^2x^8-2cde\,x^6+a\,e^2x^4-}\right.}{}$$

input
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(3/2),x)
```

output
```
(2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*e*x - sqrt(d + e*x**2)*sqrt(a - c*x
**4)*c*d*x**3 + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2
*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a
*c*d*e**2 + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d
*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*c*e
**3*x**2 - 5*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*
e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*c**2*d
**3 - 5*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**
2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*c**2*d**2*e
*x**2 + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d**2 + 2*a*d*e*x
**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*c*d**3
 + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d**2 + 2*a*d*e*x**2 +
a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*c*d**2*e*x**2
 + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2 + 2*a*d*e*x**2 + a*e*
*2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*d**2*e + 2*int
((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4
- c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*d*e**2*x**2)/(4*d*e*(d
 + e*x**2))
```

3.447 $\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 598

$$\begin{aligned}
\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx &= \frac{\left(a-\frac{cd^2}{e^2}\right)x\sqrt{a-cx^4}}{3d\,(d+ex^2)^{3/2}} \\
&+ \frac{2\left(\frac{a}{d^2}+\frac{3c}{e^2}\right)x\sqrt{a-cx^4}}{3\sqrt{d+ex^2}} - \frac{(15cd^2+4ae^2)\,\sqrt{d+ex^2}\sqrt{a-cx^4}}{6d^2e^3x} \\
&- \frac{\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)(15cd^2+4ae^2)\,\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{6d^2e^3\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{\sqrt{a}\sqrt{c}(5cd^2+4ae^2)\,\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{6d^2e^2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&- \frac{5c^2d\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}$$

output

```
1/3*(a-c*d^2/e^2)*x*(-c*x^4+a)^(1/2)/d/(e*x^2+d)^(3/2)+2/3*(a/d^2+3*c/e^2)
*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2)-1/6*(4*a*e^2+15*c*d^2)*(e*x^2+d)^(1/2)
*(-c*x^4+a)^(1/2)/d^2/e^3/x-1/6*c^(1/2)*(c^(1/2)*d+a^(1/2)*e)*(4*a*e^2+15*
c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)
^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a
^(1/2)*e/c^(1/2)))^(1/2))/d^2/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/6*a^(
1/2)*c^(1/2)*(4*a*e^2+5*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c
^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)
*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^2/e^2/(e*x^2+d)^(1/2)/
(-c*x^4+a)^(1/2)-5/2*c^2*d*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/
2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^
(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^3/(e*x^2+d)^(1/2)/(-c*x
^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx=\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx$$

input

```
Integrate[(a - c*x^4)^(3/2)/(d + e*x^2)^(5/2),x]
```

output

```
Integrate[(a - c*x^4)^(3/2)/(d + e*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}dx$$

input
```
Int[(a - c*x^4)^(3/2)/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\left(-c\,x^{4}+a \right)^{\frac{3}{2}}}{\left(e\,x^{2}+d \right)^{\frac{5}{2}}}d x$$

input
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(5/2),x)
```

output
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \int \frac{(-cx^4+a)^{\frac{3}{2}}}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((-c*x^4 + a)^(3/2)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2
*e*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx=\int \frac{(a-cx^4)^{\frac{3}{2}}}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*x**4+a)**(3/2)/(e*x**2+d)**(5/2),x)
```

output
```
Integral((a - c*x**4)**(3/2)/(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 + a)^(3/2)/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx=\int \frac{(a-c\,x^4)^{3/2}}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a - c*x^4)^(3/2)/(d + e*x^2)^(5/2),x)
```

output
```
int((a - c*x^4)^(3/2)/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx=-\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)c+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)a$$

input
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(5/2),x)
```

output
```
 - int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(d**3 + 3*d**2*e*x**2 + 3*
d*e**2*x**4 + e**3*x**6),x)*c + int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(d
**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*a
```

3.448 $\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 639

$$\begin{aligned}
\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx &= \frac{\left(a-\frac{cd^2}{e^2}\right)x\sqrt{a-cx^4}}{5d\,(d+ex^2)^{5/2}} \\
&+ \frac{4\left(\frac{a}{d^2}+\frac{2c}{e^2}\right)x\sqrt{a-cx^4}}{15\,(d+ex^2)^{3/2}} + \frac{(15c^2d^4-11acd^2e^2+8a^2e^4)\sqrt{a-cx^4}}{15d^2e^3\,(cd^2-ae^2)\,x\sqrt{d+ex^2}} \\
&+ \frac{\sqrt{c}(15c^2d^4-11acd^2e^2+8a^2e^4)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{15d^3e^3\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&- \frac{\sqrt{a}\sqrt{c}(5cd^2-8ae^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{15d^3e^2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{c^2\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}$$

output

```
1/5*(a-c*d^2/e^2)*x*(-c*x^4+a)^(1/2)/d/(e*x^2+d)^(5/2)+4/15*(a/d^2+2*c/e^2
)*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2)+1/15*(8*a^2*e^4-11*a*c*d^2*e^2+15*c^2
*d^4)*(-c*x^4+a)^(1/2)/d^2/e^3/(-a*e^2+c*d^2)/x/(e*x^2+d)^(1/2)+1/15*c^(1/
2)*(8*a^2*e^4-11*a*c*d^2*e^2+15*c^2*d^4)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e
*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/
x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^3/e^3/(c^(1/
2)*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/15*a^(1/2)*c^(1/2)*(-8*
a*e^2+5*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)
*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)
*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^3/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
+c^2*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(
1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+
a^(1/2)*e/c^(1/2)))^(1/2))/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}\,dx=\int \frac{\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}\,dx$$

input

```
Integrate[(a - c*x^4)^(3/2)/(d + e*x^2)^(7/2),x]
```

output

```
Integrate[(a - c*x^4)^(3/2)/(d + e*x^2)^(7/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}dx$$

input
```
Int[(a - c*x^4)^(3/2)/(d + e*x^2)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{(-c\,x^{4}+a)^{\frac{3}{2}}}{(e\,x^{2}+d)^{\frac{7}{2}}}dx$$

input
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(7/2),x)
```

output
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(7/2),x)
```

Fricas [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
integral((-c*x^4 + a)^(3/2)*sqrt(e*x^2 + d)/(e^4*x^8 + 4*d*e^3*x^6 + 6*d^2
*e^2*x^4 + 4*d^3*e*x^2 + d^4), x)
```

Sympy [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\int \frac{(a-cx^4)^{\frac{3}{2}}}{(d+ex^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-c*x**4+a)**(3/2)/(e*x**2+d)**(7/2),x)
```

output
```
Integral((a - c*x**4)**(3/2)/(d + e*x**2)**(7/2), x)
```

Maxima [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 + a)^(3/2)/(e*x^2 + d)^(7/2), x)
```

Giac [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)/(e*x^2 + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\int \frac{{\left(a-c\,x^4\right)}^{3/2}}{{\left(e\,x^2+d\right)}^{7/2}} \,d x$$

input
```
int((a - c*x^4)^(3/2)/(d + e*x^2)^(7/2),x)
```

output
```
int((a - c*x^4)^(3/2)/(d + e*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\text{too large to display}$$

input
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(7/2),x)
```

output

```
(10*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d*x + 4*sqrt(d + e*x**2)*sqrt(a
- c*x**4)*a*c*e*x**3 + 16*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**8)/(4*
a**2*d**4*e**2 + 16*a**2*d**3*e**3*x**2 + 24*a**2*d**2*e**4*x**4 + 16*a**2
*d*e**5*x**6 + 4*a**2*e**6*x**8 + 15*a*c*d**6 + 60*a*c*d**5*e*x**2 + 86*a*
c*d**4*e**2*x**4 + 44*a*c*d**3*e**3*x**6 - 9*a*c*d**2*e**4*x**8 - 16*a*c*d
*e**5*x**10 - 4*a*c*e**6*x**12 - 15*c**2*d**6*x**4 - 60*c**2*d**5*e*x**6 -
 90*c**2*d**4*e**2*x**8 - 60*c**2*d**3*e**3*x**10 - 15*c**2*d**2*e**4*x**1
2),x)*a**2*c**2*d**3*e**4 + 48*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**8
)/(4*a**2*d**4*e**2 + 16*a**2*d**3*e**3*x**2 + 24*a**2*d**2*e**4*x**4 + 16
*a**2*d*e**5*x**6 + 4*a**2*e**6*x**8 + 15*a*c*d**6 + 60*a*c*d**5*e*x**2 +
86*a*c*d**4*e**2*x**4 + 44*a*c*d**3*e**3*x**6 - 9*a*c*d**2*e**4*x**8 - 16*
a*c*d*e**5*x**10 - 4*a*c*e**6*x**12 - 15*c**2*d**6*x**4 - 60*c**2*d**5*e*x
**6 - 90*c**2*d**4*e**2*x**8 - 60*c**2*d**3*e**3*x**10 - 15*c**2*d**2*e**4
*x**12),x)*a**2*c**2*d**2*e**5*x**2 + 48*int((sqrt(d + e*x**2)*sqrt(a - c*
x**4)*x**8)/(4*a**2*d**4*e**2 + 16*a**2*d**3*e**3*x**2 + 24*a**2*d**2*e**4
*x**4 + 16*a**2*d*e**5*x**6 + 4*a**2*e**6*x**8 + 15*a*c*d**6 + 60*a*c*d**5
*e*x**2 + 86*a*c*d**4*e**2*x**4 + 44*a*c*d**3*e**3*x**6 - 9*a*c*d**2*e**4*
x**8 - 16*a*c*d*e**5*x**10 - 4*a*c*e**6*x**12 - 15*c**2*d**6*x**4 - 60*c**
2*d**5*e*x**6 - 90*c**2*d**4*e**2*x**8 - 60*c**2*d**3*e**3*x**10 - 15*c**2
*d**2*e**4*x**12),x)*a**2*c**2*d*e**6*x**4 + 16*int((sqrt(d + e*x**2)*s...
```

3.449 $\int \frac{\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 580

$$\int \frac{\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{9/2}}\,dx = \frac{\left(a-\frac{cd^2}{e^2}\right)x\sqrt{a-cx^4}}{7d\left(d+ex^2\right)^{7/2}}+\frac{2\left(\frac{3a}{d^2}+\frac{5c}{e^2}\right)x\sqrt{a-cx^4}}{35\left(d+ex^2\right)^{5/2}}$$

$$-\frac{\left(5c^2d^4-9acd^2e^2+8a^2e^4\right)x\sqrt{a-cx^4}}{35d^3e^2\left(cd^2-ae^2\right)\left(d+ex^2\right)^{3/2}}+\frac{16a^2e\left(2cd^2-ae^2\right)\sqrt{a-cx^4}}{35d^3\left(cd^2-ae^2\right)^2x\sqrt{d+ex^2}}$$

$$+\frac{16a^2\sqrt{c}e\left(2cd^2-ae^2\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{35d^4\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-ae^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{4a^{3/2}\sqrt{c}\left(5cd^2-4ae^2\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{35d^4\left(cd^2-ae^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/7*(a-c*d^2/e^2)*x*(-c*x^4+a)^(1/2)/d/(e*x^2+d)^(7/2)+2/35*(3*a/d^2+5*c/e
^2)*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2)-1/35*(8*a^2*e^4-9*a*c*d^2*e^2+5*c^2
*d^4)*x*(-c*x^4+a)^(1/2)/d^3/e^2/(-a*e^2+c*d^2)/(e*x^2+d)^(3/2)+16/35*a^2*
e*(-a*e^2+2*c*d^2)*(-c*x^4+a)^(1/2)/d^3/(-a*e^2+c*d^2)^2/x/(e*x^2+d)^(1/2)
+16/35*a^2*c^(1/2)*e*(-a*e^2+2*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^
2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2
)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^4/(c^(1/2)*d-a^
(1/2)*e)/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+4/35*a^(3/2)*c^(1
/2)*(-4*a*e^2+5*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d
+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2)
,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^4/(-a*e^2+c*d^2)/(e*x^2+d)^(1/
2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx$$

input
```
Integrate[(a - c*x^4)^(3/2)/(d + e*x^2)^(9/2),x]
```

output
```
Integrate[(a - c*x^4)^(3/2)/(d + e*x^2)^(9/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}dx$$

input
```
Int[(a - c*x^4)^(3/2)/(d + e*x^2)^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\left(-c\,x^{4}+a\right)^{\frac{3}{2}}}{\left(e\,x^{2}+d\right)^{\frac{9}{2}}}d x$$

input
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(9/2),x)
```

output
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(9/2),x)
```

Fricas [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(9/2),x, algorithm="fricas")
```

output
```
integral((-c*x^4 + a)^(3/2)*sqrt(e*x^2 + d)/(e^5*x^10 + 5*d*e^4*x^8 + 10*d
^2*e^3*x^6 + 10*d^3*e^2*x^4 + 5*d^4*e*x^2 + d^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate((-c*x**4+a)**(3/2)/(e*x**2+d)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{9/2}}\,dx=\int \frac{\left(-cx^4+a\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(9/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 + a)^(3/2)/(e*x^2 + d)^(9/2), x)
```

Giac [F]

$$\int \frac{\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{9/2}}\,dx=\int \frac{\left(-cx^4+a\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(9/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)/(e*x^2 + d)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\int \frac{{\left(a-c\,x^4\right)}^{3/2}}{{\left(e\,x^2+d\right)}^{9/2}}\,dx$$

input
```
int((a - c*x^4)^(3/2)/(d + e*x^2)^(9/2),x)
```

output
```
int((a - c*x^4)^(3/2)/(d + e*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\text{too large to display}$$

input
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(9/2),x)
```

output

```
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d*x + 2*sqrt(d + e*x**2)*sqrt(a - c
*x**4)*a*c*e*x**3 - 3*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c**2*d*x**5 - 96*i
nt((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**2*d**5*e**2 + 20*a**2*d*
*4*e**3*x**2 + 40*a**2*d**3*e**4*x**4 + 40*a**2*d**2*e**5*x**6 + 20*a**2*d
*e**6*x**8 + 4*a**2*e**7*x**10 + 21*a*c*d**7 + 105*a*c*d**6*e*x**2 + 206*a
*c*d**5*e**2*x**4 + 190*a*c*d**4*e**3*x**6 + 65*a*c*d**3*e**4*x**8 - 19*a*
c*d**2*e**5*x**10 - 20*a*c*d*e**6*x**12 - 4*a*c*e**7*x**14 - 21*c**2*d**7*
x**4 - 105*c**2*d**6*e*x**6 - 210*c**2*d**5*e**2*x**8 - 210*c**2*d**4*e**3
*x**10 - 105*c**2*d**3*e**4*x**12 - 21*c**2*d**2*e**5*x**14),x)*a**2*c**2*
d**6*e**2 - 384*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**2*d**5*
e**2 + 20*a**2*d**4*e**3*x**2 + 40*a**2*d**3*e**4*x**4 + 40*a**2*d**2*e**5
*x**6 + 20*a**2*d*e**6*x**8 + 4*a**2*e**7*x**10 + 21*a*c*d**7 + 105*a*c*d*
*6*e*x**2 + 206*a*c*d**5*e**2*x**4 + 190*a*c*d**4*e**3*x**6 + 65*a*c*d**3*
e**4*x**8 - 19*a*c*d**2*e**5*x**10 - 20*a*c*d*e**6*x**12 - 4*a*c*e**7*x**1
4 - 21*c**2*d**7*x**4 - 105*c**2*d**6*e*x**6 - 210*c**2*d**5*e**2*x**8 - 2
10*c**2*d**4*e**3*x**10 - 105*c**2*d**3*e**4*x**12 - 21*c**2*d**2*e**5*x**
14),x)*a**2*c**2*d**5*e**3*x**2 - 576*int((sqrt(d + e*x**2)*sqrt(a - c*x**
4)*x**4)/(4*a**2*d**5*e**2 + 20*a**2*d**4*e**3*x**2 + 40*a**2*d**3*e**4*x*
*4 + 40*a**2*d**2*e**5*x**6 + 20*a**2*d*e**6*x**8 + 4*a**2*e**7*x**10 + 21
*a*c*d**7 + 105*a*c*d**6*e*x**2 + 206*a*c*d**5*e**2*x**4 + 190*a*c*d**4...
```

3.450 $\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 714

$$\begin{aligned}
\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx &= \frac{\left(a-\frac{cd^2}{e^2}\right)x\sqrt{a-cx^4}}{9d\,(d+ex^2)^{9/2}} \\
&+ \frac{4\left(\frac{2a}{d^2}+\frac{3c}{e^2}\right)x\sqrt{a-cx^4}}{63\,(d+ex^2)^{7/2}} - \frac{(5c^2d^4-17acd^2e^2+16a^2e^4)\,x\sqrt{a-cx^4}}{105d^3e^2\,(cd^2-ae^2)\,(d+ex^2)^{5/2}} \\
&- \frac{2(5c^3d^6-22ac^2d^4e^2+65a^2cd^2e^4-32a^3e^6)\,x\sqrt{a-cx^4}}{315d^4e^2\,(cd^2-ae^2)^2\,(d+ex^2)^{3/2}} \\
&+ \frac{4a^2e(93c^2d^4-93acd^2e^2+32a^2e^4)\sqrt{a-cx^4}}{315d^4\,(cd^2-ae^2)^3\,x\sqrt{d+ex^2}} \\
&+ \frac{4a^2\sqrt{c}e(93c^2d^4-93acd^2e^2+32a^2e^4)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{315d^5\left(\sqrt{c}d-\sqrt{a}e\right)^3\left(\sqrt{c}d+\sqrt{a}e\right)^2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{4a^{3/2}\sqrt{c}(45c^2d^4-69acd^2e^2+32a^2e^4)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{315d^5\,(cd^2-ae^2)^2\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}$$

output
```
1/9*(a-c*d^2/e^2)*x*(-c*x^4+a)^(1/2)/d/(e*x^2+d)^(9/2)+4/63*(2*a/d^2+3*c/e
^2)*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2)-1/105*(16*a^2*e^4-17*a*c*d^2*e^2+5*
c^2*d^4)*x*(-c*x^4+a)^(1/2)/d^3/e^2/(-a*e^2+c*d^2)/(e*x^2+d)^(5/2)-2/315*(
-32*a^3*e^6+65*a^2*c*d^2*e^4-22*a*c^2*d^4*e^2+5*c^3*d^6)*x*(-c*x^4+a)^(1/2
)/d^4/e^2/(-a*e^2+c*d^2)^2/(e*x^2+d)^(3/2)+4/315*a^2*e*(32*a^2*e^4-93*a*c*
d^2*e^2+93*c^2*d^4)*(-c*x^4+a)^(1/2)/d^4/(-a*e^2+c*d^2)^3/x/(e*x^2+d)^(1/2
)+4/315*a^2*c^(1/2)*e*(32*a^2*e^4-93*a*c*d^2*e^2+93*c^2*d^4)*(1-a/c/x^4)^(
1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2
*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(
1/2))/d^5/(c^(1/2)*d-a^(1/2)*e)^3/(c^(1/2)*d+a^(1/2)*e)^2/(e*x^2+d)^(1/2)/
(-c*x^4+a)^(1/2)+4/315*a^(3/2)*c^(1/2)*(32*a^2*e^4-69*a*c*d^2*e^2+45*c^2*d
^4)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1
/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1
/2)*e/c^(1/2)))^(1/2))/d^5/(-a*e^2+c*d^2)^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/
2)
```

Mathematica [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx=\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx$$

input
```
Integrate[(a - c*x^4)^(3/2)/(d + e*x^2)^(11/2),x]
```

output
```
Integrate[(a - c*x^4)^(3/2)/(d + e*x^2)^(11/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx$$

input
```
Int[(a - c*x^4)^(3/2)/(d + e*x^2)^(11/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\left(-c\,x^{4}+a\right)^{\frac{3}{2}}}{\left(e\,x^{2}+d\right)^{\frac{11}{2}}}dx$$

input
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(11/2),x)
```

output
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(11/2),x)
```

Fricas [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx = \int \frac{(-cx^4+a)^{\frac{3}{2}}}{(ex^2+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(11/2),x, algorithm="fricas")
```

output
```
integral((-c*x^4 + a)^(3/2)*sqrt(e*x^2 + d)/(e^6*x^12 + 6*d*e^5*x^10 + 15*
d^2*e^4*x^8 + 20*d^3*e^3*x^6 + 15*d^4*e^2*x^4 + 6*d^5*e*x^2 + d^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{11/2}}\,dx=\text{Timed out}$$

input
```
integrate((-c*x**4+a)**(3/2)/(e*x**2+d)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{11/2}}\,dx=\int \frac{\left(-cx^4+a\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(11/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^4 + a)^(3/2)/(e*x^2 + d)^(11/2), x)
```

Giac [F]

$$\int \frac{\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{11/2}}\,dx=\int \frac{\left(-cx^4+a\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)/(e*x^2+d)^(11/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)/(e*x^2 + d)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx=\int \frac{(a-c\,x^4)^{3/2}}{(e\,x^2+d)^{11/2}}\,dx$$

input
```
int((a - c*x^4)^(3/2)/(d + e*x^2)^(11/2),x)
```

output
```
int((a - c*x^4)^(3/2)/(d + e*x^2)^(11/2), x)
```

Reduce [F]

$$\int \frac{(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx=\text{too large to display}$$

input
```
int((-c*x^4+a)^(3/2)/(e*x^2+d)^(11/2),x)
```

output

```
(3*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d*x + 8*sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*a*c*e*x**3 - 9*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c**2*d*x**5 - 2*
sqrt(d + e*x**2)*sqrt(a - c*x**4)*c**2*e*x**7 - 96*int((sqrt(d + e*x**2)*s
qrt(a - c*x**4)*x**4)/(4*a**2*d**6*e**2 + 24*a**2*d**5*e**3*x**2 + 60*a**2
*d**4*e**4*x**4 + 80*a**2*d**3*e**5*x**6 + 60*a**2*d**2*e**6*x**8 + 24*a**
2*d*e**7*x**10 + 4*a**2*e**8*x**12 + 7*a*c*d**8 + 42*a*c*d**7*e*x**2 + 101
*a*c*d**6*e**2*x**4 + 116*a*c*d**5*e**3*x**6 + 45*a*c*d**4*e**4*x**8 - 38*
a*c*d**3*e**5*x**10 - 53*a*c*d**2*e**6*x**12 - 24*a*c*d*e**7*x**14 - 4*a*c
*e**8*x**16 - 7*c**2*d**8*x**4 - 42*c**2*d**7*e*x**6 - 105*c**2*d**6*e**2*
x**8 - 140*c**2*d**5*e**3*x**10 - 105*c**2*d**4*e**4*x**12 - 42*c**2*d**3*
e**5*x**14 - 7*c**2*d**2*e**6*x**16),x)*a**3*c*d**5*e**4 - 480*int((sqrt(d
 + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**2*d**6*e**2 + 24*a**2*d**5*e**3*x*
*2 + 60*a**2*d**4*e**4*x**4 + 80*a**2*d**3*e**5*x**6 + 60*a**2*d**2*e**6*x
**8 + 24*a**2*d*e**7*x**10 + 4*a**2*e**8*x**12 + 7*a*c*d**8 + 42*a*c*d**7*
e*x**2 + 101*a*c*d**6*e**2*x**4 + 116*a*c*d**5*e**3*x**6 + 45*a*c*d**4*e**
4*x**8 - 38*a*c*d**3*e**5*x**10 - 53*a*c*d**2*e**6*x**12 - 24*a*c*d*e**7*x
**14 - 4*a*c*e**8*x**16 - 7*c**2*d**8*x**4 - 42*c**2*d**7*e*x**6 - 105*c**
2*d**6*e**2*x**8 - 140*c**2*d**5*e**3*x**10 - 105*c**2*d**4*e**4*x**12 - 4
2*c**2*d**3*e**5*x**14 - 7*c**2*d**2*e**6*x**16),x)*a**3*c*d**4*e**5*x**2
- 960*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**2*d**6*e**2 + ...
```

3.451 $\int \frac{(d+ex^2)^{5/2}}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 518

$$\begin{aligned}\int \frac{(d+ex^2)^{5/2}}{\sqrt{a-cx^4}}\,dx &= -\frac{9de\sqrt{d+ex^2}\sqrt{a-cx^4}}{8cx} - \frac{e^2x\sqrt{d+ex^2}\sqrt{a-cx^4}}{4c} \\ &- \frac{9de\left(d+\frac{\sqrt{ae}}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{8\sqrt{d+ex^2}\sqrt{a-cx^4}} \\ &+ \frac{d(8cd^2+11ae^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{8\sqrt{a}\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}} \\ &+ \frac{e(15cd^2+4ae^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{8c\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output
```
-9/8*d*e*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/x-1/4*e^2*x*(e*x^2+d)^(1/2)*(-
c*x^4+a)^(1/2)/c-9/8*d*e*(d+a^(1/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1
/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^
(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/(e*x^2+d
)^(1/2)/(-c*x^4+a)^(1/2)+1/8*d*(11*a*e^2+8*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a
^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)
/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/
2)/c^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/8*e*(4*a*e^2+15*c*d^2)*(1-a/
c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ellip
ticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/
c^(1/2)))^(1/2))/c/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{a-cx^4}}\,dx=\int \frac{(d+ex^2)^{5/2}}{\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(d + e*x^2)^(5/2)/Sqrt[a - c*x^4],x]
```

output
```
Integrate[(d + e*x^2)^(5/2)/Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{a-cx^4}}\,dx$$
$$\Big\downarrow \; 1571$$
$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{a-cx^4}}dx$$

input
```
Int[(d + e*x^2)^(5/2)/Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{5}{2}}}{\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{a-cx^4}}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*
x^4 - a), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{a-cx^4}}\,dx = \int \frac{(d+ex^2)^{\frac{5}{2}}}{\sqrt{a-cx^4}}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((d + e*x**2)**(5/2)/sqrt(a - c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{a-cx^4}}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(5/2)/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{a-cx^4}}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(5/2)/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{a-cx^4}}\,dx = \int \frac{{\left(e\,x^2+d\right)}^{5/2}}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int((d + e*x^2)^(5/2)/(a - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^(5/2)/(a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{5/2}}{\sqrt{a-cx^4}}\,dx = \frac{-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,e^2x+9\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)cd\,e^2+2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+}}{-ce\,x^6-cd\,x^4+ae\,x}\right.}{}$$

input
```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*e**2*x + 9*int((sqrt(d + e*x**2)*sqr
t(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c*d*e**2 + 2
*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 -
 c*e*x**6),x)*a*e**3 + 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*
d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c*d**2*e + int((sqrt(d + e*x**2)*sq
rt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*d*e**2 + 4*int
((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6
),x)*c*d**3)/(4*c)
```

3.452 $\int \frac{\left(d+ex^2\right)^{3/2}}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 465

$$\begin{aligned}
\int \frac{\left(d+ex^2\right)^{3/2}}{\sqrt{a-cx^4}}\,dx &= -\frac{e\sqrt{d+ex^2}\sqrt{a-cx^4}}{2cx} \\
&- \frac{e\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{(2cd^2+ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{3de\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}$$

output

```
-1/2*e*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/x-1/2*e*(d+a^(1/2)*e/c^(1/2))*(1
-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*El
lipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/
c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*(a*e^2+2*c*d^2)*(1-a
/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Elli
pticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^
(1/2)))^(1/2))/a^(1/2)/c^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+3/2*d*e*(1
-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*El
lipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)
*e/c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx=\int \frac{(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx$$

input

```
Integrate[(d + e*x^2)^(3/2)/Sqrt[a - c*x^4],x]
```

output

```
Integrate[(d + e*x^2)^(3/2)/Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}dx$$

input

```
Int[(d + e*x^2)^(3/2)/Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)/(c*x^4 - a), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx=\int \frac{(d+ex^2)^{\frac{3}{2}}}{\sqrt{a-cx^4}}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((d + e*x**2)**(3/2)/sqrt(a - c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx=\int \frac{(e\,x^2+d)^{3/2}}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(a - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^(3/2)/(a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^2}{-c\,x^4+a}dx\right)e+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)d$$

input
```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*e + int((sqrt
(d + e*x**2)*sqrt(a - c*x**4))/(a - c*x**4),x)*d
```

3.453 $\int \frac{\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 268

$$\begin{aligned}\int \frac{\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx &= \frac{\sqrt{c}d\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{a}\sqrt{d+ex^2}\sqrt{a-cx^4}}\\ &+ \frac{e\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output

```
c^(1/2)*d*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x
^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(
d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+e*(1
-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*El
lipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)
*e/c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx=\int \frac{\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]/Sqrt[a - c*x^4],x]
```

output
```
Integrate[Sqrt[d + e*x^2]/Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \displaystyle\int \frac{\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx \\ \Big\downarrow \ 1571 \\ \displaystyle\int \frac{\sqrt{d+ex^2}}{\sqrt{a-cx^4}}dx \end{array}$$

input
```
Int[Sqrt[d + e*x^2]/Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\sqrt{e\,x^{2}+d}}{\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx=\int \frac{\sqrt{ex^2+d}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*x^4 - a), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx=\int \frac{\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x**2)/sqrt(a - c*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx=\int \frac{\sqrt{ex^2+d}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx=\int \frac{\sqrt{ex^2+d}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx=\int \frac{\sqrt{e\,x^2+d}}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^(1/2)/(a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx$$

input
```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a - c*x**4),x)
```

3.454 $\int \frac{1}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 137

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \frac{\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{a}\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output

```
c^(1/2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2
)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+
a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input

```
Integrate[1/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[1/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]), x]
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1562, 1799, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{1562}$$

$$\frac{x^3\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}\int \frac{1}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}x^3}dx}{\sqrt{a-cx^4}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{1799}$$

$$-\frac{x^3\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}\int \frac{1}{\sqrt{\frac{a}{x^4}-c}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}}{2\sqrt{a-cx^4}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{512}$$

$$-\frac{x^3\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{d}{x^2}+e}\int \frac{1}{\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{d}{x^2}+e}}d\frac{1}{x^2}}{2\sqrt{a-cx^4}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{511}$$

$$\frac{\sqrt{c}x^3\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{\sqrt{a}\left(\frac{d}{x^2}+e\right)}{\sqrt{a}e+\sqrt{c}d}}\int \frac{1}{\sqrt{1-\frac{1}{x^4}}\sqrt{1-\frac{2d}{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)x^4}}}d\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}}{\sqrt{a}\sqrt{a-cx^4}\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{c}x^3\sqrt{1-\frac{a}{cx^4}}\sqrt{\frac{\sqrt{a}\left(\frac{d}{x^2}+e\right)}{\sqrt{a}e+\sqrt{c}d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{a}\sqrt{a-cx^4}\sqrt{d+ex^2}}$$

input
```
Int[1/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
(Sqrt[c]*Sqrt[1 - a/(c*x^4)]*Sqrt[(Sqrt[a]*(e + d/x^2))/(Sqrt[c]*d + Sqrt[
a]*e)]*x^3*EllipticF[ArcSin[Sqrt[1 - Sqrt[a]/(Sqrt[c]*x^2)]/Sqrt[2]], (2*d
)/(d + (Sqrt[a]*e)/Sqrt[c])])/(Sqrt[a]*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 1562
```
Int[1/(Sqrt[(d_) + (e_.)*(x_)^2]*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> S
imp[x^3*Sqrt[e + d/x^2]*(Sqrt[c + a/x^4]/(Sqrt[d + e*x^2]*Sqrt[a + c*x^4]))
   Int[1/(x^3*Sqrt[e + d/x^2]*Sqrt[c + a/x^4]), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

Maple [F]

$$\int \frac{1}{\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*e*x^6 + c*d*x^4 - a*e*x^2 -
a*d), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - c*x**4)*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx$$

input

```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output

```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x
**6),x)
```

3.455 $\int \frac{1}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 337

$$\begin{aligned}\int \frac{1}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx &= \frac{e\sqrt{a-cx^4}}{(cd^2-ae^2)\,x\sqrt{d+ex^2}} \\ &+ \frac{\sqrt{c}e\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{d+ex^2}\sqrt{a-cx^4}} \\ &+ \frac{\sqrt{c}\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{a}d\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output
```
e*(-c*x^4+a)^(1/2)/(-a*e^2+c*d^2)/x/(e*x^2+d)^(1/2)+c^(1/2)*e*(1-a/c/x^4)^
(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/
2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^
(1/2))/d/(c^(1/2)*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+c^(1/2)*(1
-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*El
lipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/
c^(1/2)))^(1/2))/a^(1/2)/d/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{1}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[1/((d + e*x^2)^(3/2)*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^{3/2}}dx$$
$$\Big\downarrow \; 1571$$
$$\int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^{3/2}}dx$$

input
```
Int[1/((d + e*x^2)^(3/2)*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{1}{(e\,x^{2}+d)^{\frac{3}{2}}\sqrt{-c\,x^{4}+a}}dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*e^2*x^8 + 2*c*d*e*x^6 - 2*a*
d*e*x^2 + (c*d^2 - a*e^2)*x^4 - a*d^2), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-c\,x^4}\,{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-c\,e^2x^8-2cde\,x^6+a\,e^2x^4-c\,d^2x^4+2ade\,x^2+a\,d^2}dx$$

input

```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output

```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x*
*4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)
```

3.456 $\int \frac{1}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 464

$$\begin{aligned}\int \frac{1}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx &= -\frac{e^2x\sqrt{a-cx^4}}{3d\,(cd^2-ae^2)\,(d+ex^2)^{3/2}} \\ &+ \frac{2e(3cd^2-ae^2)\,\sqrt{a-cx^4}}{3d\,(cd^2-ae^2)^2\,x\sqrt{d+ex^2}} + \frac{2\sqrt{c}e(3cd^2-ae^2)\,\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3d^2\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2-ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}} \\ &+ \frac{\sqrt{c}(3cd^2-2ae^2)\,\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3\sqrt{a}d^2\,(cd^2-ae^2)\,\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output
```
-1/3*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^(3/2)+2/3*e*(-a*e^2
+3*c*d^2)*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)^2/x/(e*x^2+d)^(1/2)+2/3*c^(1/2
)*e*(-a*e^2+3*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a
^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2
^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^2/(c^(1/2)*d-a^(1/2)*e)/(-a*e^2+
c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/3*c^(1/2)*(-2*a*e^2+3*c*d^2)*(1-
a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ell
ipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c
^(1/2)))^(1/2))/a^(1/2)/d^2/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2
)
```

Mathematica [F]

$$\int \frac{1}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx=\int \frac{1}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(5/2)*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[1/((d + e*x^2)^(5/2)*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 1571}$$
$$\int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^{5/2}}dx$$

input
```
Int[1/((d + e*x^2)^(5/2)*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{5}{2}} \sqrt{-c\,x^{4}+a}}d x$$

input
```
int(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*e^3*x^10 + 3*c*d*e^2*x^8 + (
3*c*d^2*e - a*e^3)*x^6 - 3*a*d^2*e*x^2 + (c*d^3 - 3*a*d*e^2)*x^4 - a*d^3),
 x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(5/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\sqrt{a-cx^4}}\,dx=\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-c\,e^3x^{10}-3cd\,e^2x^8+a\,e^3x^6-3c\,d^2e\,x^6+3ad\,e^2x^4-c\,d^3x^4+3a\,d^2e\,x^2}$$

input
```
int(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*
e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8
- c*e**3*x**10),x)
```

3.457 $\int \frac{1}{(d+ex^2)^{7/2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 570

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^{7/2}\sqrt{a-cx^4}}\,dx &= -\frac{e^2x\sqrt{a-cx^4}}{5d\,(cd^2-ae^2)\,(d+ex^2)^{5/2}} \\
&- \frac{4e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{15d^2\,(cd^2-ae^2)^2\,(d+ex^2)^{3/2}} + \frac{e(45c^2d^4-21acd^2e^2+8a^2e^4)\sqrt{a-cx^4}}{15d^2\,(cd^2-ae^2)^3\,x\sqrt{d+ex^2}} \\
&+ \frac{\sqrt{c}e(45c^2d^4-21acd^2e^2+8a^2e^4)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{15d^3\left(\sqrt{c}d-\sqrt{a}e\right)^3\left(\sqrt{c}d+\sqrt{a}e\right)^2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{\sqrt{c}(15c^2d^4-15acd^2e^2+8a^2e^4)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{15\sqrt{a}d^3\,(cd^2-ae^2)^2\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}$$

output

```
-1/5*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^(5/2)-4/15*e^2*(-a*
e^2+3*c*d^2)*x*(-c*x^4+a)^(1/2)/d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)^(3/2)+1/15*
e*(8*a^2*e^4-21*a*c*d^2*e^2+45*c^2*d^4)*(-c*x^4+a)^(1/2)/d^2/(-a*e^2+c*d^2
)^3/x/(e*x^2+d)^(1/2)+1/15*c^(1/2)*e*(8*a^2*e^4-21*a*c*d^2*e^2+45*c^2*d^4)
*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)
*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)
*e/c^(1/2)))^(1/2))/d^3/(c^(1/2)*d-a^(1/2)*e)^3/(c^(1/2)*d+a^(1/2)*e)^2/(e
*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/15*c^(1/2)*(8*a^2*e^4-15*a*c*d^2*e^2+15*c
^2*d^4)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2
)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+
a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)/d^3/(-a*e^2+c*d^2)^2/(e*x^2+d)^(1/2)/(-
c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\left(d+ex^2\right)^{7/2}\sqrt{a-cx^4}}\,dx=\int \frac{1}{\left(d+ex^2\right)^{7/2}\sqrt{a-cx^4}}\,dx$$

input

```
Integrate[1/((d + e*x^2)^(7/2)*Sqrt[a - c*x^4]),x]
```

output

```
Integrate[1/((d + e*x^2)^(7/2)*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^4}\left(d+ex^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{1}{\sqrt{a-cx^4}\left(d+ex^2\right)^{7/2}}dx$$

input

```
Int[1/((d + e*x^2)^(7/2)*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{7}{2}}\sqrt{-c\,x^{4}+a}}d x$$

input
```
int(1/(e*x^2+d)^(7/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int(1/(e*x^2+d)^(7/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)^{7/2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(7/2)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*e^4*x^12 + 4*c*d*e^3*x^10 +
(6*c*d^2*e^2 - a*e^4)*x^8 - 4*a*d^3*e*x^2 + 4*(c*d^3*e - a*d*e^3)*x^6 - a*
d^4 + (c*d^4 - 6*a*d^2*e^2)*x^4), x)
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^{7/2}\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{a-cx^4}\left(d+ex^2\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(7/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**(7/2)), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^{7/2}\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(7/2)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex^2\right)^{7/2}\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(7/2)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{7/2}\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-c\,x^4}\,(e\,x^2+d)^{7/2}}\,dx$$

input
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^(7/2)),x)
```

output
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^(7/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^{7/2}\sqrt{a-cx^4}}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-c\,e^4x^{12}-4cd\,e^3x^{10}+a\,e^4x^8-6c\,d^2e^2x^8+4ad\,e^3x^6-4c\,d^3e\,x^6+6a\,d^2}$$

input
```
int(1/(e*x^2+d)^(7/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**4 + 4*a*d**3*e*x**2 + 6*a*d*
*2*e**2*x**4 + 4*a*d*e**3*x**6 + a*e**4*x**8 - c*d**4*x**4 - 4*c*d**3*e*x*
*6 - 6*c*d**2*e**2*x**8 - 4*c*d*e**3*x**10 - c*e**4*x**12),x)
```

3.458 $\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 627

$$
\begin{aligned}
\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{3/2}}\,dx &= \frac{x(d+ex^2)^{7/2}}{2a\sqrt{a-cx^4}} + \frac{e(3cd^2+2ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{2ac^2x} \\
&+ \frac{3de^2x\sqrt{d+ex^2}\sqrt{a-cx^4}}{2ac} + \frac{e^3x^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{2ac} \\
&+ \frac{e\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(3cd^2+2ae^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2ac\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{(c^2d^4-6acd^2e^2-2a^2e^4)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a^{3/2}c^{3/2}\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&- \frac{7de^3\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2c\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}
$$

output
```
1/2*x*(e*x^2+d)^(7/2)/a/(-c*x^4+a)^(1/2)+1/2*e*(2*a*e^2+3*c*d^2)*(e*x^2+d)
^(1/2)*(-c*x^4+a)^(1/2)/a/c^2/x+3/2*d*e^2*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/
2)/a/c+1/2*e^3*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/c+1/2*e*(d+a^(1/2)*e
/c^(1/2))*(2*a*e^2+3*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1
/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^
(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/c/(e*x^2+d)^(1/2)/(-c*x^4
+a)^(1/2)+1/2*(-2*a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)*(1-a/c/x^4)^(1/2)*x^3*(a^
(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/
c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2
)/c^(3/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-7/2*d*e^3*(1-a/c/x^4)^(1/2)*x^3
*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(
1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))
/c/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{3/2}}\,dx$$

input
```
Integrate[(d + e*x^2)^(7/2)/(a - c*x^4)^(3/2),x]
```

output
```
Integrate[(d + e*x^2)^(7/2)/(a - c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{3/2}}dx$$

input
```
Int[(d + e*x^2)^(7/2)/(a - c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{7}{2}}}{\left(-c\,x^{4}+a\right)^{\frac{3}{2}}}d x$$

input
```
int((e*x^2+d)^(7/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int((e*x^2+d)^(7/2)/(-c*x^4+a)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{\left(d+ex^2\right)^{\frac{7}{2}}}{\left(a-cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**(7/2)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((d + e*x**2)**(7/2)/(a - c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{\left(ex^2+d\right)^{\frac{7}{2}}}{\left(-cx^4+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(7/2)/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{\left(ex^2+d\right)^{\frac{7}{2}}}{\left(-cx^4+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(7/2)/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(e\,x^2+d)^{7/2}}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int((d + e*x^2)^(7/2)/(a - c*x^4)^(3/2),x)
```

output
```
int((d + e*x^2)^(7/2)/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x^2+d)^(7/2)/(-c*x^4+a)^(3/2),x)
```

output

```
(16*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*e**4*x + 6*sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*c*d**2*e**2*x - 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d*e**3*x*
*3 - 7*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**8)/(a**2*d + a**2*e*x**2
- 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a*c**2*d*e*
*4 + 7*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**8)/(a**2*d + a**2*e*x**2
- 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*c**3*d*e**4
*x**4 - 32*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2*d + a**2*e*x
**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a**3*e*
*5 + 32*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2*d + a**2*e*x**2
 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a**2*c*e**
5*x**4 + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2*d + a**2*e*x
**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a*c**2*
d**4*e - 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2*d + a**2*e*x
**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*c**3*d*
*4*e*x**4 - 16*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**2*d + a**2*e*x*
*2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a**3*d*e
**4 - 6*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**2*d + a**2*e*x**2 - 2*
a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a**2*c*d**3*e**
2 + 16*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**2*d + a**2*e*x**2 - 2*a
*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a**2*c*d*e**4...
```

3.459 $\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 538

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx = \frac{x(d+ex^2)^{5/2}}{2a\sqrt{a-cx^4}} + \frac{de\sqrt{d+ex^2}\sqrt{a-cx^4}}{acx} + \frac{e^2x\sqrt{d+ex^2}\sqrt{a-cx^4}}{2ac}$$
$$+ \frac{de\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{a\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{d(cd^2-3ae^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a^{3/2}\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{e^3\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{c\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/2*x*(e*x^2+d)^(5/2)/a/(-c*x^4+a)^(1/2)+d*e*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1
/2)/a/c/x+1/2*e^2*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/c+d*e*(d+a^(1/2)*e/
c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^
2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d
+a^(1/2)*e/c^(1/2)))^(1/2))/a/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*d*(-3*a
*e^2+c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)
/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d
/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/c^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^
(1/2)-e^3*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x
^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(
d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^{3/2}}\,dx$$

input
```
Integrate[(d + e*x^2)^(5/2)/(a - c*x^4)^(3/2),x]
```

output
```
Integrate[(d + e*x^2)^(5/2)/(a - c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1571}$$

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^{3/2}}dx$$

input
```
Int[(d + e*x^2)^(5/2)/(a - c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{(e\,x^{2}+d)^{\frac{5}{2}}}{(-c\,x^{4}+a)^{\frac{3}{2}}}dx$$

input
```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{(ex^2+d)^{\frac{5}{2}}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c^2
*x^8 - 2*a*c*x^4 + a^2), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{(d+ex^2)^{\frac{5}{2}}}{(a-cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((d + e*x**2)**(5/2)/(a - c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{(ex^2+d)^{\frac{5}{2}}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(5/2)/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{(ex^2+d)^{\frac{5}{2}}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(5/2)/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(e\,x^2+d)^{5/2}}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int((d + e*x^2)^(5/2)/(a - c*x^4)^(3/2),x)
```

output
```
int((d + e*x^2)^(5/2)/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx = \frac{3\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,e^2x+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^6}{c^2e\,x^{10}+c^2d\,x^8-2ace\,x^6-2acd\,x^4+a^2e\,x^2+a^2d}dx\right)ac\,e^3-\left(\int \right.}{}$$

input
```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x)
```

output

```
(3*sqrt(d + e*x**2)*sqrt(a - c*x**4)*e**2*x + int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*x**6)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**
2*d*x**8 + c**2*e*x**10),x)*a*c*e**3 - int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*x**6)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**
8 + c**2*e*x**10),x)*c**2*e**3*x**4 - 6*int((sqrt(d + e*x**2)*sqrt(a - c*x
**4)*x**2)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x*
*8 + c**2*e*x**10),x)*a**2*e**3 + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)
*x**2)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 +
 c**2*e*x**10),x)*a*c*d**2*e + 6*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x*
*2)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c*
*2*e*x**10),x)*a*c*e**3*x**4 - 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x*
*2)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c*
*2*e*x**10),x)*c**2*d**2*e*x**4 - 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)
)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2
*e*x**10),x)*a**2*d*e**2 + int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**2*d
 + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10)
,x)*a*c*d**3 + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**2*d + a**2*e*
x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a*c*d*
e**2*x**4 - int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**2*d + a**2*e*x**2
- 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*c**2*d**...
```

3.460 $\int \frac{\left(d+e x^2\right)^{3/2}}{\left(a-c x^4\right)^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 369

$$\int \frac{\left(d+e x^2\right)^{3/2}}{\left(a-c x^4\right)^{3/2}} \, dx = \frac{x\left(d+e x^2\right)^{3/2}}{2a\sqrt{a-c x^4}}+\frac{e\sqrt{d+e x^2}\sqrt{a-c x^4}}{2acx} + \frac{e\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{c x^4}}x^3\sqrt{\frac{\sqrt{a}(d+e x^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a\sqrt{d+e x^2}\sqrt{a-c x^4}} + \frac{\left(c d^2-a e^2\right)\sqrt{1-\frac{a}{c x^4}}x^3\sqrt{\frac{\sqrt{a}(d+e x^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a^{3/2}\sqrt{c}\sqrt{d+e x^2}\sqrt{a-c x^4}}$$

output
```
1/2*x*(e*x^2+d)^(3/2)/a/(-c*x^4+a)^(1/2)+1/2*e*(e*x^2+d)^(1/2)*(-c*x^4+a)^
(1/2)/a/c/x+1/2*e*(d+a^(1/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*
x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x
^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/(e*x^2+d)^(1/
2)/(-c*x^4+a)^(1/2)+1/2*(-a*e^2+c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x
^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^
2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/c^(1/2)/
(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx$$

input
```
Integrate[(d + e*x^2)^(3/2)/(a - c*x^4)^(3/2),x]
```

output
```
Integrate[(d + e*x^2)^(3/2)/(a - c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx$$
$$\Big\downarrow \text{1571}$$
$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}dx$$

input
```
Int[(d + e*x^2)^(3/2)/(a - c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{(e\,x^{2}+d)^{\frac{3}{2}}}{(-c\,x^{4}+a)^{\frac{3}{2}}}dx$$

input
```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)/(c^2*x^8 - 2*a*c*x^4 + a^2), x
)
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{(d+ex^2)^{\frac{3}{2}}}{(a-cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((d + e*x**2)**(3/2)/(a - c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(a - c*x^4)^(3/2),x)
```

output
```
int((d + e*x^2)^(3/2)/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,dx + \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{c^2e\,x^{10}+c^2d\,x^8-2ace\,x^6-2acd\,x^4+a^2e\,x^2+a^2d}dx\right) a^2e^2 - \left(\int \frac{}{c^2e\,x}\right.}{}$$

input
```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*d*x + int((sqrt(d + e*x**2)*sqrt(a - c*
x**4)*x**4)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x
**8 + c**2*e*x**10),x)*a**2*e**2 - int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*
x**4)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 +
c**2*e*x**10),x)*a*c*d**2 - int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(
a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*
x**10),x)*a*c*e**2*x**4 + int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*
*2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x*
*10),x)*c**2*d**2*x**4)/(a*(a - c*x**4))
```

3.461 $\int \frac{\sqrt{d+ex^2}}{\left(a-cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{\sqrt{d+ex^2}}{\left(a-cx^4\right)^{3/2}}\,dx=\frac{x\sqrt{d+ex^2}}{2a\sqrt{a-cx^4}}+\frac{\sqrt{c}d\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{\left(\sqrt{c}d+\sqrt{a}e\right)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a^{3/2}\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output

```
1/2*x*(e*x^2+d)^(1/2)/a/(-c*x^4+a)^(1/2)+1/2*c^(1/2)*d*(1-a/c/x^4)^(1/2)*x
^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^
(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/
a^(3/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]/(a - c*x^4)^(3/2),x]
```

output
```
Integrate[Sqrt[d + e*x^2]/(a - c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}dx$$

input
```
Int[Sqrt[d + e*x^2]/(a - c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\sqrt{e\,x^{2}+d}}{\left(-c\,x^{4}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{\sqrt{ex^2+d}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c^2*x^8 - 2*a*c*x^4 + a^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral(sqrt(d + e*x**2)/(a - c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{\sqrt{ex^2+d}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{\sqrt{ex^2+d}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{\sqrt{e\,x^2+d}}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a - c*x^4)^(3/2),x)
```

output
```
int((d + e*x^2)^(1/2)/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}}{\left(a-cx^4\right)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)ad-\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)}{2a\left(-c\,x^4+a\right)}$$

input

```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x)
```

output

```
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*x + int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*d - int((sqrt(d + e*x**2)
*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c*d*x**4)/(2*
a*(a - c*x**4))
```

3.462 $\int \frac{1}{\sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 390

$$
\begin{aligned}
\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx = &-\frac{e\sqrt{d+ex^2}}{2\,(cd^2-ae^2)\,x\sqrt{a-cx^4}}+\frac{cdx\sqrt{d+ex^2}}{2a\,(cd^2-ae^2)\,\sqrt{a-cx^4}}\\
&-\frac{\sqrt{c}e\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}\\
&+\frac{\sqrt{c}\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a^{3/2}\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}
$$

output
```
-1/2*e*(e*x^2+d)^(1/2)/(-a*e^2+c*d^2)/x/(-c*x^4+a)^(1/2)+1/2*c*d*x*(e*x^2+
d)^(1/2)/a/(-a*e^2+c*d^2)/(-c*x^4+a)^(1/2)-1/2*c^(1/2)*e*(1-a/c/x^4)^(1/2)
*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-
a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2)
)/a/(c^(1/2)*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*c^(1/2)*(1-
a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ell
ipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c
^(1/2)))^(1/2))/a^(3/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx = \int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a - c*x^4)^(3/2)),x]
```

output
```
Integrate[1/(Sqrt[d + e*x^2]*(a - c*x^4)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^4)^{3/2}\sqrt{d+ex^2}}dx$$
$$\Big\downarrow \text{1571}$$
$$\int \frac{1}{(a-cx^4)^{3/2}\sqrt{d+ex^2}}dx$$

input
```
Int[1/(Sqrt[d + e*x^2]*(a - c*x^4)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{e\,x^{2}+d}\,\left(-c\,x^{4}+a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx=\int \frac{1}{\left(-cx^{4}+a\right)^{\frac{3}{2}}\sqrt{ex^{2}+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c^2*e*x^10 + c^2*d*x^8 - 2*a*c*
e*x^6 - 2*a*c*d*x^4 + a^2*e*x^2 + a^2*d), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx=\int \frac{1}{\left(a-cx^{4}\right)^{\frac{3}{2}}\sqrt{d+ex^{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral(1/((a - c*x**4)**(3/2)*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx=\int \frac{1}{(-cx^4+a)^{\frac{3}{2}}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx=\int \frac{1}{(-cx^4+a)^{\frac{3}{2}}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx=\int \frac{1}{{\left(a-c\,x^4\right)}^{3/2}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{c^2e\,x^{10}+c^2d\,x^8-2ace\,x^6-2acd\,x^4+a^2e\,x^2+a^2d}dx$$

input

```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x)
```

output

```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**2*d + a**2*e*x**2 - 2*a*c*d*x*
*4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)
```

3.463 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 464

$$
\begin{aligned}
\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}}\,dx &= \frac{cx(d-ex^2)}{2a\left(cd^2-ae^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&- \frac{e(cd^2+ae^2)\sqrt{a-cx^4}}{a\left(cd^2-ae^2\right)^2 x\sqrt{d+ex^2}} \\
&- \frac{\sqrt{c}e(cd^2+ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{ad\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-ae^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{\sqrt{c}(cd^2-2ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a^{3/2}d\left(cd^2-ae^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}
$$

output

```
1/2*c*x*(-e*x^2+d)/a/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-e*(a*
e^2+c*d^2)*(-c*x^4+a)^(1/2)/a/(-a*e^2+c*d^2)^2/x/(e*x^2+d)^(1/2)-c^(1/2)*e
*(a*e^2+c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)
*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)
*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/d/(c^(1/2)*d-a^(1/2)*e)/(-a*e^2+c*d^2)
/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*c^(1/2)*(-2*a*e^2+c*d^2)*(1-a/c/x^4)
^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1
/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))
^(1/2))/a^(3/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^{3/2}}\,dx = \int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^{3/2}}\,dx$$

input

```
Integrate[1/((d + e*x^2)^(3/2)*(a - c*x^4)^(3/2)),x]
```

output

```
Integrate[1/((d + e*x^2)^(3/2)*(a - c*x^4)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^4)^{3/2}(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{1}{(a-cx^4)^{3/2}(d+ex^2)^{3/2}}dx$$

input

```
Int[1/((d + e*x^2)^(3/2)*(a - c*x^4)^(3/2)),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(-c\,x^{4}+a\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a-cx^4)^{3/2}}\,dx = \int \frac{1}{(-cx^4+a)^{\frac{3}{2}}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c^2*e^2*x^12 + 2*c^2*d*e*x^10 -
 4*a*c*d*e*x^6 + (c^2*d^2 - 2*a*c*e^2)*x^8 + 2*a^2*d*e*x^2 - (2*a*c*d^2 -
a^2*e^2)*x^4 + a^2*d^2), x)
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(a-cx^4\right)^{\frac{3}{2}}\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral(1/((a - c*x**4)**(3/2)*(d + e*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(a-c\,x^4\right)^{3/2}\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{c^2e^2x^{12}+2c^2de\,x^{10}-2ac\,e^2x^8+c^2d^2x^8-4acde\,x^6+a^2e^2x^4-2ac\,d^2}$$

input
```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**2*d**2 + 2*a**2*d*e*x**2 + a**
2*e**2*x**4 - 2*a*c*d**2*x**4 - 4*a*c*d*e*x**6 - 2*a*c*e**2*x**8 + c**2*d*
*2*x**8 + 2*c**2*d*e*x**10 + c**2*e**2*x**12),x)
```

3.464 $\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 584

$$\begin{aligned}\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)^{3/2}}\,dx &= \frac{cx(d-ex^2)}{2a\left(cd^2-ae^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}} \\
&+ \frac{e^2(3cd^2+ae^2)\,x\sqrt{a-cx^4}}{3ad\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^{3/2}} - \frac{e(9c^2d^4+27acd^2e^2-4a^2e^4)\sqrt{a-cx^4}}{6ad\left(cd^2-ae^2\right)^3 x\sqrt{d+ex^2}} \\
&- \frac{\sqrt{c}e(9c^2d^4+27acd^2e^2-4a^2e^4)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{6ad^2\left(\sqrt{c}d-\sqrt{a}e\right)^3\left(\sqrt{c}d+\sqrt{a}e\right)^2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{\sqrt{c}(3c^2d^4-15acd^2e^2+4a^2e^4)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{6a^{3/2}d^2\left(cd^2-ae^2\right)^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output
```
1/2*c*x*(-e*x^2+d)/a/(-a*e^2+c*d^2)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2)+1/3*e
^2*(a*e^2+3*c*d^2)*x*(-c*x^4+a)^(1/2)/a/d/(-a*e^2+c*d^2)^2/(e*x^2+d)^(3/2)
-1/6*e*(-4*a^2*e^4+27*a*c*d^2*e^2+9*c^2*d^4)*(-c*x^4+a)^(1/2)/a/d/(-a*e^2+
c*d^2)^3/x/(e*x^2+d)^(1/2)-1/6*c^(1/2)*e*(-4*a^2*e^4+27*a*c*d^2*e^2+9*c^2*
d^4)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(
1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(
1/2)*e/c^(1/2)))^(1/2))/a/d^2/(c^(1/2)*d-a^(1/2)*e)^3/(c^(1/2)*d+a^(1/2)*e
)^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/6*c^(1/2)*(4*a^2*e^4-15*a*c*d^2*e^2
+3*c^2*d^4)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)
/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d
/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)^(1/2
)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a-cx^4)^{3/2}}\,dx=\int \frac{1}{(d+ex^2)^{5/2}\,(a-cx^4)^{3/2}}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(5/2)*(a - c*x^4)^(3/2)),x]
```

output
```
Integrate[1/((d + e*x^2)^(5/2)*(a - c*x^4)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^4)^{3/2}\,(d+ex^2)^{5/2}}\,dx$$
$$\Big\downarrow \ 1571$$
$$\int \frac{1}{(a-cx^4)^{3/2}\,(d+ex^2)^{5/2}}dx$$

input
```
Int[1/((d + e*x^2)^(5/2)*(a - c*x^4)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{5}{2}} \left(-c\,x^{4}+a\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a-cx^4)^{3/2}}\,dx = \int \frac{1}{(-cx^4+a)^{\frac{3}{2}}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c^2*e^3*x^14 + 3*c^2*d*e^2*x^12
 + (3*c^2*d^2*e - 2*a*c*e^3)*x^10 + (c^2*d^3 - 6*a*c*d*e^2)*x^8 + 3*a^2*d^
2*e*x^2 - (6*a*c*d^2*e - a^2*e^3)*x^6 + a^2*d^3 - (2*a*c*d^3 - 3*a^2*d*e^2
)*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**(5/2)/(-c*x**4+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{5/2}(a-cx^4)^{3/2}}\,dx = \int \frac{1}{(a-c\,x^4)^{3/2}\,(e\,x^2+d)^{5/2}}\,dx$$

input
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a - c*x^4)^(3/2)*(d + e*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex^2)^{5/2}(a-cx^4)^{3/2}}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c}}{c^2e^3x^{14}+3c^2d\,e^2x^{12}-2ac\,e^3x^{10}+3c^2d^2e\,x^{10}-6acd\,e^2x^8+c^2d^3x^8+}$$

input
```
int(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**2*d**3 + 3*a**2*d**2*e*x**2 +
3*a**2*d*e**2*x**4 + a**2*e**3*x**6 - 2*a*c*d**3*x**4 - 6*a*c*d**2*e*x**6
- 6*a*c*d*e**2*x**8 - 2*a*c*e**3*x**10 + c**2*d**3*x**8 + 3*c**2*d**2*e*x*
*10 + 3*c**2*d*e**2*x**12 + c**2*e**3*x**14),x)
```

3.465 $\int \frac{\left(d+ex^2\right)^{9/2}}{\left(a-cx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 719

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)^{9/2}}{\left(a-cx^4\right)^{5/2}}\,dx &= \frac{x\left(d+ex^2\right)^{9/2}}{6a\left(a-cx^4\right)^{3/2}}+\frac{x\left(5d-4ex^2\right)\left(d+ex^2\right)^{7/2}}{12a^2\sqrt{a-cx^4}} \\
&+\frac{de\left(11cd^2-15ae^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}{12a^2c^2x}+\frac{e^2\left(cd^2-2ae^2\right)x\sqrt{d+ex^2}\sqrt{a-cx^4}}{4a^2c^2} \\
&-\frac{7de^3x^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{12a^2c}-\frac{e^4x^5\sqrt{d+ex^2}\sqrt{a-cx^4}}{3a^2c} \\
&+\frac{de\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(11cd^2-15ae^2\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{12a^2c\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+\frac{d\left(5c^2d^4-14acd^2e^2+21a^2e^4\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{12a^{5/2}c^{3/2}\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+\frac{e^5\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{c^2\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}
$$

output
```
1/6*x*(e*x^2+d)^(9/2)/a/(-c*x^4+a)^(3/2)+1/12*x*(-4*e*x^2+5*d)*(e*x^2+d)^(
7/2)/a^2/(-c*x^4+a)^(1/2)+1/12*d*e*(-15*a*e^2+11*c*d^2)*(e*x^2+d)^(1/2)*(-
c*x^4+a)^(1/2)/a^2/c^2/x+1/4*e^2*(-2*a*e^2+c*d^2)*x*(e*x^2+d)^(1/2)*(-c*x^
4+a)^(1/2)/a^2/c^2-7/12*d*e^3*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/c-1
/3*e^4*x^5*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/c+1/12*d*e*(d+a^(1/2)*e/c^
(1/2))*(-15*a*e^2+11*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1
/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^
(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/c/(e*x^2+d)^(1/2)/(-c*x
^4+a)^(1/2)+1/12*d*(21*a^2*e^4-14*a*c*d^2*e^2+5*c^2*d^4)*(1-a/c/x^4)^(1/2)
*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-
a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2)
)/a^(5/2)/c^(3/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+e^5*(1-a/c/x^4)^(1/2)*x
^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a
^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2
))/c^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^{5/2}}\,dx=\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^{5/2}}\,dx$$

input
```
Integrate[(d + e*x^2)^(9/2)/(a - c*x^4)^(5/2),x]
```

output
```
Integrate[(d + e*x^2)^(9/2)/(a - c*x^4)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^{5/2}}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^{5/2}}dx$$

input
```
Int[(d + e*x^2)^(9/2)/(a - c*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{9}{2}}}{(-c\,x^4+a)^{\frac{5}{2}}}dx$$

input
```
int((e*x^2+d)^(9/2)/(-c*x^4+a)^(5/2),x)
```

output
```
int((e*x^2+d)^(9/2)/(-c*x^4+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(ex^2+d)^{\frac{9}{2}}}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(9/2)/(-c*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(-(e^4*x^8 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4 + 4*d^3*e*x^2 + d^4)*sqrt
(-c*x^4 + a)*sqrt(e*x^2 + d)/(c^3*x^12 - 3*a*c^2*x^8 + 3*a^2*c*x^4 - a^3),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(9/2)/(-c*x**4+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(ex^2+d)^{\frac{9}{2}}}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(9/2)/(-c*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(9/2)/(-c*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(ex^2+d)^{\frac{9}{2}}}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(9/2)/(-c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(9/2)/(-c*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^{5/2}}\,dx=\int \frac{(e\,x^2+d)^{9/2}}{(a-c\,x^4)^{5/2}}\,dx$$

input
```
int((d + e*x^2)^(9/2)/(a - c*x^4)^(5/2),x)
```

output
```
int((d + e*x^2)^(9/2)/(a - c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{9/2}}{(a-cx^4)^{5/2}}\,dx=\text{too large to display}$$

input
```
int((e*x^2+d)^(9/2)/(-c*x^4+a)^(5/2),x)
```

output

```
( - 120*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*e**6*x + 135*sqrt(d + e*x**
2)*sqrt(a - c*x**4)*a*c*d**2*e**4*x + 50*sqrt(d + e*x**2)*sqrt(a - c*x**4)
*a*c*d*e**5*x**3 + 80*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*e**6*x**5 - 30
*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c**2*d**4*e**2*x - 10*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*c**2*d**3*e**3*x**3 - 55*sqrt(d + e*x**2)*sqrt(a - c*x**4
)*c**2*d**2*e**4*x**5 + 432*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**10)/
(12*a**4*d*e**2 + 12*a**4*e**3*x**2 - 5*a**3*c*d**3 - 5*a**3*c*d**2*e*x**2
 - 36*a**3*c*d*e**2*x**4 - 36*a**3*c*e**3*x**6 + 15*a**2*c**2*d**3*x**4 +
15*a**2*c**2*d**2*e*x**6 + 36*a**2*c**2*d*e**2*x**8 + 36*a**2*c**2*e**3*x*
*10 - 15*a*c**3*d**3*x**8 - 15*a*c**3*d**2*e*x**10 - 12*a*c**3*d*e**2*x**1
2 - 12*a*c**3*e**3*x**14 + 5*c**4*d**3*x**12 + 5*c**4*d**2*e*x**14),x)*a**
4*c**2*e**9 - 360*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**10)/(12*a**4*d
*e**2 + 12*a**4*e**3*x**2 - 5*a**3*c*d**3 - 5*a**3*c*d**2*e*x**2 - 36*a**3
*c*d*e**2*x**4 - 36*a**3*c*e**3*x**6 + 15*a**2*c**2*d**3*x**4 + 15*a**2*c*
*2*d**2*e*x**6 + 36*a**2*c**2*d*e**2*x**8 + 36*a**2*c**2*e**3*x**10 - 15*a
*c**3*d**3*x**8 - 15*a*c**3*d**2*e*x**10 - 12*a*c**3*d*e**2*x**12 - 12*a*c
**3*e**3*x**14 + 5*c**4*d**3*x**12 + 5*c**4*d**2*e*x**14),x)*a**3*c**3*d**
2*e**7 - 864*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**10)/(12*a**4*d*e**2
 + 12*a**4*e**3*x**2 - 5*a**3*c*d**3 - 5*a**3*c*d**2*e*x**2 - 36*a**3*c*d*
e**2*x**4 - 36*a**3*c*e**3*x**6 + 15*a**2*c**2*d**3*x**4 + 15*a**2*c**2...
```

3.466 $\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 531

$$\begin{aligned}\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{5/2}}\,dx &= \frac{x(d+ex^2)^{7/2}}{6a\,(a-cx^4)^{3/2}} + \frac{x(5d-2ex^2)\,(d+ex^2)^{5/2}}{12a^2\sqrt{a-cx^4}} \\ &+ \frac{e(2cd^2-ae^2)\,\sqrt{d+ex^2}\sqrt{a-cx^4}}{3a^2c^2x} + \frac{de^2x\sqrt{d+ex^2}\sqrt{a-cx^4}}{12a^2c} - \frac{e^3x^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{6a^2c} \\ &+ \frac{e\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(2cd^2-ae^2)\,\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3a^2c\sqrt{d+ex^2}\sqrt{a-cx^4}} \\ &+ \frac{(5c^2d^4-9acd^2e^2+4a^2e^4)\,\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{12a^{5/2}c^{3/2}\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output
```
1/6*x*(e*x^2+d)^(7/2)/a/(-c*x^4+a)^(3/2)+1/12*x*(-2*e*x^2+5*d)*(e*x^2+d)^(
5/2)/a^2/(-c*x^4+a)^(1/2)+1/3*e*(-a*e^2+2*c*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a
)^(1/2)/a^2/c^2/x+1/12*d*e^2*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/c-1/6*
e^3*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/c+1/3*e*(d+a^(1/2)*e/c^(1/2))
*(-a*e^2+2*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1
/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1
/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/c/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2
)+1/12*(4*a^2*e^4-9*a*c*d^2*e^2+5*c^2*d^4)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*
(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2
)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(5/2)/c^(3
/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{5/2}}\,dx$$

input
```
Integrate[(d + e*x^2)^(7/2)/(a - c*x^4)^(5/2),x]
```

output
```
Integrate[(d + e*x^2)^(7/2)/(a - c*x^4)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{5/2}}dx$$

input
```
Int[(d + e*x^2)^(7/2)/(a - c*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{(e\,x^{2}+d)^{\frac{7}{2}}}{(-c\,x^{4}+a)^{\frac{5}{2}}}dx$$

input
```
int((e*x^2+d)^(7/2)/(-c*x^4+a)^(5/2),x)
```

output
```
int((e*x^2+d)^(7/2)/(-c*x^4+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{5/2}}\,dx=\int \frac{(ex^2+d)^{\frac{7}{2}}}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(-(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)*sqrt(-c*x^4 + a)*sqr
t(e*x^2 + d)/(c^3*x^12 - 3*a*c^2*x^8 + 3*a^2*c*x^4 - a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a-cx^4\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(7/2)/(-c*x**4+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a-cx^4\right)^{5/2}}\,dx = \int \frac{\left(ex^2+d\right)^{\frac{7}{2}}}{\left(-cx^4+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(7/2)/(-c*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\left(d+ex^2\right)^{7/2}}{\left(a-cx^4\right)^{5/2}}\,dx = \int \frac{\left(ex^2+d\right)^{\frac{7}{2}}}{\left(-cx^4+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(7/2)/(-c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(7/2)/(-c*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(e\,x^2+d)^{7/2}}{(a-c\,x^4)^{5/2}}\,dx$$

input
```
int((d + e*x^2)^(7/2)/(a - c*x^4)^(5/2),x)
```

output
```
int((d + e*x^2)^(7/2)/(a - c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{7/2}}{(a-cx^4)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((e*x^2+d)^(7/2)/(-c*x^4+a)^(5/2),x)
```

output

```
( - 9*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*d*e**4*x + 6*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*a**2*e**5*x**3 + 30*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*
c*d**3*e**2*x - 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**2*e**3*x**3 + 1
1*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d*e**4*x**5 - 6*sqrt(d + e*x**2)*s
qrt(a - c*x**4)*c**2*d**5*x + 8*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c**2*d**
4*e*x**3 - 16*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c**2*d**3*e**2*x**5 - 96*i
nt((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**4*d*e**2 + 4*a**4*e**3*x
**2 + a**3*c*d**3 + a**3*c*d**2*e*x**2 - 12*a**3*c*d*e**2*x**4 - 12*a**3*c
*e**3*x**6 - 3*a**2*c**2*d**3*x**4 - 3*a**2*c**2*d**2*e*x**6 + 12*a**2*c**
2*d*e**2*x**8 + 12*a**2*c**2*e**3*x**10 + 3*a*c**3*d**3*x**8 + 3*a*c**3*d*
*2*e*x**10 - 4*a*c**3*d*e**2*x**12 - 4*a*c**3*e**3*x**14 - c**4*d**3*x**12
 - c**4*d**2*e*x**14),x)*a**6*e**8 + 288*int((sqrt(d + e*x**2)*sqrt(a - c*
x**4)*x**4)/(4*a**4*d*e**2 + 4*a**4*e**3*x**2 + a**3*c*d**3 + a**3*c*d**2*
e*x**2 - 12*a**3*c*d*e**2*x**4 - 12*a**3*c*e**3*x**6 - 3*a**2*c**2*d**3*x*
*4 - 3*a**2*c**2*d**2*e*x**6 + 12*a**2*c**2*d*e**2*x**8 + 12*a**2*c**2*e**
3*x**10 + 3*a*c**3*d**3*x**8 + 3*a*c**3*d**2*e*x**10 - 4*a*c**3*d*e**2*x**
12 - 4*a*c**3*e**3*x**14 - c**4*d**3*x**12 - c**4*d**2*e*x**14),x)*a**5*c*
d**2*e**6 + 192*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**4*d*e**
2 + 4*a**4*e**3*x**2 + a**3*c*d**3 + a**3*c*d**2*e*x**2 - 12*a**3*c*d*e**2
*x**4 - 12*a**3*c*e**3*x**6 - 3*a**2*c**2*d**3*x**4 - 3*a**2*c**2*d**2*...
```

3.467 $\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 404

$$\int \frac{\left(d+ex^2\right)^{5/2}}{\left(a-cx^4\right)^{5/2}}\,dx=\frac{x\left(d+ex^2\right)^{5/2}}{6a\left(a-cx^4\right)^{3/2}}+\frac{5dx\left(d+ex^2\right)^{3/2}}{12a^2\sqrt{a-cx^4}}+\frac{5de\sqrt{d+ex^2}\sqrt{a-cx^4}}{12a^2cx}$$
$$+\frac{5de\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{12a^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{5d\left(cd^2-ae^2\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{12a^{5/2}\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/6*x*(e*x^2+d)^(5/2)/a/(-c*x^4+a)^(3/2)+5/12*d*x*(e*x^2+d)^(3/2)/a^2/(-c*
x^4+a)^(1/2)+5/12*d*e*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/c/x+5/12*d*e*(d
+a^(1/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^
(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^
(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2
)+5/12*d*(-a*e^2+c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*
d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2
),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(5/2)/c^(1/2)/(e*x^2+d)^(1/2)
/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{5/2}}\,dx$$

input
```
Integrate[(d + e*x^2)^(5/2)/(a - c*x^4)^(5/2),x]
```

output
```
Integrate[(d + e*x^2)^(5/2)/(a - c*x^4)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}\displaystyle \int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{5/2}}\,dx \\ \Big\downarrow \text{1571} \\ \displaystyle \int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{5/2}}dx\end{array}$$

input
```
Int[(d + e*x^2)^(5/2)/(a - c*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{5}{2}}}{\left(-c\,x^{4}+a\right)^{\frac{5}{2}}}dx$$

input
```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^(5/2),x)
```

output
```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(-(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c^
3*x^12 - 3*a*c^2*x^8 + 3*a^2*c*x^4 - a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(5/2)/(-c*x**4+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(5/2)/(-c*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(-c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(5/2)/(-c*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{5/2}}\,dx=\int \frac{(e\,x^2+d)^{5/2}}{(a-c\,x^4)^{5/2}}\,dx$$

input
```
int((d + e*x^2)^(5/2)/(a - c*x^4)^(5/2),x)
```

output
```
int((d + e*x^2)^(5/2)/(a - c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{5/2}}{(a-cx^4)^{5/2}}\,dx=\text{too large to display}$$

input
```
int((e*x^2+d)^(5/2)/(-c*x^4+a)^(5/2),x)
```

output

```
(39*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*d**2*e**2*x - 2*sqrt(d + e*x**2)*s
qrt(a - c*x**4)*a*d*e**3*x**3 + 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*e**4
*x**5 - 9*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d**4*x + 12*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*c*d**3*e*x**3 - 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d
**2*e**2*x**5 + 240*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**4*d
*e**2 + 4*a**4*e**3*x**2 + a**3*c*d**3 + a**3*c*d**2*e*x**2 - 12*a**3*c*d*
e**2*x**4 - 12*a**3*c*e**3*x**6 - 3*a**2*c**2*d**3*x**4 - 3*a**2*c**2*d**2
*e*x**6 + 12*a**2*c**2*d*e**2*x**8 + 12*a**2*c**2*e**3*x**10 + 3*a*c**3*d*
*3*x**8 + 3*a*c**3*d**2*e*x**10 - 4*a*c**3*d*e**2*x**12 - 4*a*c**3*e**3*x*
*14 - c**4*d**3*x**12 - c**4*d**2*e*x**14),x)*a**5*d*e**6 - 360*int((sqrt(
d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**4*d*e**2 + 4*a**4*e**3*x**2 + a**
3*c*d**3 + a**3*c*d**2*e*x**2 - 12*a**3*c*d*e**2*x**4 - 12*a**3*c*e**3*x**
6 - 3*a**2*c**2*d**3*x**4 - 3*a**2*c**2*d**2*e*x**6 + 12*a**2*c**2*d*e**2*
x**8 + 12*a**2*c**2*e**3*x**10 + 3*a*c**3*d**3*x**8 + 3*a*c**3*d**2*e*x**1
0 - 4*a*c**3*d*e**2*x**12 - 4*a*c**3*e**3*x**14 - c**4*d**3*x**12 - c**4*d
**2*e*x**14),x)*a**4*c*d**3*e**4 - 480*int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*x**4)/(4*a**4*d*e**2 + 4*a**4*e**3*x**2 + a**3*c*d**3 + a**3*c*d**2*e*
x**2 - 12*a**3*c*d*e**2*x**4 - 12*a**3*c*e**3*x**6 - 3*a**2*c**2*d**3*x**4
 - 3*a**2*c**2*d**2*e*x**6 + 12*a**2*c**2*d*e**2*x**8 + 12*a**2*c**2*e**3*
x**10 + 3*a*c**3*d**3*x**8 + 3*a*c**3*d**2*e*x**10 - 4*a*c**3*d*e**2*x*...
```

3.468 $\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a-cx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 411

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)^{3/2}}{\left(a-cx^4\right)^{5/2}}\,dx &= \frac{x\left(d+ex^2\right)^{3/2}}{6a\left(a-cx^4\right)^{3/2}} \\
&+ \frac{x\sqrt{d+ex^2}\left(5d+2ex^2\right)}{12a^2\sqrt{a-cx^4}} + \frac{e\sqrt{d+ex^2}\sqrt{a-cx^4}}{6a^2cx} \\
&+ \frac{e\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{6a^2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{\left(5cd^2-2ae^2\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{12a^{5/2}\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}
$$

output
```
1/6*x*(e*x^2+d)^(3/2)/a/(-c*x^4+a)^(3/2)+1/12*x*(e*x^2+d)^(1/2)*(2*e*x^2+5
*d)/a^2/(-c*x^4+a)^(1/2)+1/6*e*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/c/x+1/
6*e*(d+a^(1/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2
)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1
/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/(e*x^2+d)^(1/2)/(-c*x^4+a
)^(1/2)+1/12*(-2*a*e^2+5*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(
c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2
)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(5/2)/c^(1/2)/(e*x^2+
d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{5/2}}\,dx$$

input
```
Integrate[(d + e*x^2)^(3/2)/(a - c*x^4)^(5/2),x]
```

output
```
Integrate[(d + e*x^2)^(3/2)/(a - c*x^4)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{5/2}}\,dx$$

$$\Big\downarrow \; 1571$$

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{5/2}}dx$$

input
```
Int[(d + e*x^2)^(3/2)/(a - c*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{(e\,x^{2}+d)^{\frac{3}{2}}}{(-c\,x^{4}+a)^{\frac{5}{2}}}dx$$

input
```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^(5/2),x)
```

output
```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{5/2}}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)/(c^3*x^12 - 3*a*c^2*x^8 + 3*a
^2*c*x^4 - a^3), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(d+ex^2)^{\frac{3}{2}}}{(a-cx^4)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)/(-c*x**4+a)**(5/2),x)
```

output
```
Integral((d + e*x**2)**(3/2)/(a - c*x**4)**(5/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/(-c*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)/(-c*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{(a-c\,x^4)^{5/2}}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(a - c*x^4)^(5/2),x)
```

output
```
int((d + e*x^2)^(3/2)/(a - c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}}{(a-cx^4)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((e*x^2+d)^(3/2)/(-c*x^4+a)^(5/2),x)
```

output

```
(12*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*d*e**2*x - 3*sqrt(d + e*x**2)*sqrt
(a - c*x**4)*c*d**3*x + 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d**2*e*x**3
- 8*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d*e**2*x**5 + 48*int((sqrt(d + e*x
**2)*sqrt(a - c*x**4)*x**4)/(4*a**4*d*e**2 + 4*a**4*e**3*x**2 + a**3*c*d**
3 + a**3*c*d**2*e*x**2 - 12*a**3*c*d*e**2*x**4 - 12*a**3*c*e**3*x**6 - 3*a
**2*c**2*d**3*x**4 - 3*a**2*c**2*d**2*e*x**6 + 12*a**2*c**2*d*e**2*x**8 +
12*a**2*c**2*e**3*x**10 + 3*a*c**3*d**3*x**8 + 3*a*c**3*d**2*e*x**10 - 4*a
*c**3*d*e**2*x**12 - 4*a*c**3*e**3*x**14 - c**4*d**3*x**12 - c**4*d**2*e*x
**14),x)*a**5*e**6 - 120*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a
**4*d*e**2 + 4*a**4*e**3*x**2 + a**3*c*d**3 + a**3*c*d**2*e*x**2 - 12*a**3
*c*d*e**2*x**4 - 12*a**3*c*e**3*x**6 - 3*a**2*c**2*d**3*x**4 - 3*a**2*c**2
*d**2*e*x**6 + 12*a**2*c**2*d*e**2*x**8 + 12*a**2*c**2*e**3*x**10 + 3*a*c*
*3*d**3*x**8 + 3*a*c**3*d**2*e*x**10 - 4*a*c**3*d*e**2*x**12 - 4*a*c**3*e*
*3*x**14 - c**4*d**3*x**12 - c**4*d**2*e*x**14),x)*a**4*c*d**2*e**4 - 96*i
nt((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**4*d*e**2 + 4*a**4*e**3*x
**2 + a**3*c*d**3 + a**3*c*d**2*e*x**2 - 12*a**3*c*d*e**2*x**4 - 12*a**3*c
*e**3*x**6 - 3*a**2*c**2*d**3*x**4 - 3*a**2*c**2*d**2*e*x**6 + 12*a**2*c**
2*d*e**2*x**8 + 12*a**2*c**2*e**3*x**10 + 3*a*c**3*d**3*x**8 + 3*a*c**3*d*
*2*e*x**10 - 4*a*c**3*d*e**2*x**12 - 4*a*c**3*e**3*x**14 - c**4*d**3*x**12
 - c**4*d**2*e*x**14),x)*a**4*c*e**6*x**4 + 27*int((sqrt(d + e*x**2)*sq...
```

3.469 $\int \frac{\sqrt{d+ex^2}}{\left(a-cx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 438

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}}{\left(a-cx^4\right)^{5/2}}\,dx = {} & \frac{x\sqrt{d+ex^2}}{6a\left(a-cx^4\right)^{3/2}} \\
& - \frac{de\sqrt{d+ex^2}}{12a\left(cd^2-ae^2\right)x\sqrt{a-cx^4}} + \frac{\left(5cd^2-4ae^2\right)x\sqrt{d+ex^2}}{12a^2\left(cd^2-ae^2\right)\sqrt{a-cx^4}} \\
& - \frac{\sqrt{c}de\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{12a^2\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
& + \frac{5\sqrt{c}d\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{12a^{5/2}\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}$$

output

```
1/6*x*(e*x^2+d)^(1/2)/a/(-c*x^4+a)^(3/2)-1/12*d*e*(e*x^2+d)^(1/2)/a/(-a*e^
2+c*d^2)/x/(-c*x^4+a)^(1/2)+1/12*(-4*a*e^2+5*c*d^2)*x*(e*x^2+d)^(1/2)/a^2/
(-a*e^2+c*d^2)/(-c*x^4+a)^(1/2)-1/12*c^(1/2)*d*e*(1-a/c/x^4)^(1/2)*x^3*(a^
(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/
c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/(c
^(1/2)*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+5/12*c^(1/2)*d*(1-a/c
/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ellipt
icF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1
/2)))^(1/2))/a^(5/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{d+ex^2}}{\left(a-cx^4\right)^{5/2}}\,dx=\int \frac{\sqrt{d+ex^2}}{\left(a-cx^4\right)^{5/2}}\,dx$$

input

```
Integrate[Sqrt[d + e*x^2]/(a - c*x^4)^(5/2),x]
```

output

```
Integrate[Sqrt[d + e*x^2]/(a - c*x^4)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{\left(a-cx^4\right)^{5/2}}\,dx$$

$$\Big\downarrow \ 1571$$

$$\int \frac{\sqrt{d+ex^2}}{\left(a-cx^4\right)^{5/2}}dx$$

input

```
Int[Sqrt[d + e*x^2]/(a - c*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\sqrt{e\,x^{2}+d}}{\left(-c\,x^{4}+a\right)^{\frac{5}{2}}}dx$$

input
```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^(5/2),x)
```

output
```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{5/2}}\,dx=\int \frac{\sqrt{ex^2+d}}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c^3*x^12 - 3*a*c^2*x^8 + 3*a^2
*c*x^4 - a^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(-c*x**4+a)**(5/2),x)
```

output
```
Integral(sqrt(d + e*x**2)/(a - c*x**4)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{\sqrt{ex^2+d}}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/(-c*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{\sqrt{ex^2+d}}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)/(-c*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{\sqrt{e\,x^2+d}}{(a-c\,x^4)^{5/2}}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a - c*x^4)^(5/2),x)
```

output
```
int((d + e*x^2)^(1/2)/(a - c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}}{(a-cx^4)^{5/2}}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-c^3x^{12}+3a\,c^2x^8-3a^2c\,x^4+a^3}dx$$

input
```
int((e*x^2+d)^(1/2)/(-c*x^4+a)^(5/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x
**8 - c**3*x**12),x)
```

3.470 $\int \frac{1}{\sqrt{d+ex^2}\left(a-cx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 525

$$\begin{aligned}\int \frac{1}{\sqrt{d+ex^2}\left(a-cx^4\right)^{5/2}}\,dx &= \frac{cx(d-ex^2)\sqrt{d+ex^2}}{6a\left(cd^2-ae^2\right)\left(a-cx^4\right)^{3/2}} \\ &- \frac{e(cd^2-2ae^2)\sqrt{d+ex^2}}{3a\left(cd^2-ae^2\right)^2x\sqrt{a-cx^4}} + \frac{cd(5cd^2-9ae^2)\,x\sqrt{d+ex^2}}{12a^2\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}} \\ &- \frac{\sqrt{c}e(cd^2-2ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3a^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-ae^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}} \\ &+ \frac{\sqrt{c}(5cd^2-4ae^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{12a^{5/2}\left(cd^2-ae^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output
```
1/6*c*x*(-e*x^2+d)*(e*x^2+d)^(1/2)/a/(-a*e^2+c*d^2)/(-c*x^4+a)^(3/2)-1/3*e
*(-2*a*e^2+c*d^2)*(e*x^2+d)^(1/2)/a/(-a*e^2+c*d^2)^2/x/(-c*x^4+a)^(1/2)+1/
12*c*d*(-9*a*e^2+5*c*d^2)*x*(e*x^2+d)^(1/2)/a^2/(-a*e^2+c*d^2)^2/(-c*x^4+a
)^(1/2)-1/3*c^(1/2)*e*(-2*a*e^2+c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x
^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^
2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/(c^(1/2)*d-a
^(1/2)*e)/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/12*c^(1/2)*(-4
*a*e^2+5*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2
)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2
)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(5/2)/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/
(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{5/2}}\,dx = \int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{5/2}}\,dx$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a - c*x^4)^(5/2)),x]
```

output
```
Integrate[1/(Sqrt[d + e*x^2]*(a - c*x^4)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^4)^{5/2}\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{1571}$$
$$\int \frac{1}{(a-cx^4)^{5/2}\sqrt{d+ex^2}}dx$$

input
```
Int[1/(Sqrt[d + e*x^2]*(a - c*x^4)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{e\,x^{2}+d}\,\left(-c\,x^{4}+a\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(5/2),x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{5/2}}\,dx=\int \frac{1}{(-cx^4+a)^{\frac{5}{2}}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c^3*e*x^14 + c^3*d*x^12 - 3*a*
c^2*e*x^10 - 3*a*c^2*d*x^8 + 3*a^2*c*e*x^6 + 3*a^2*c*d*x^4 - a^3*e*x^2 - a
^3*d), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{5/2}}\,dx = \int \frac{1}{(a-cx^4)^{\frac{5}{2}}\sqrt{d+ex^2}}\,dx$$

input

```
integrate(1/(e*x**2+d)**(1/2)/(-c*x**4+a)**(5/2),x)
```

output

```
Integral(1/((a - c*x**4)**(5/2)*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{5/2}}\,dx = \int \frac{1}{(-cx^4+a)^{\frac{5}{2}}\sqrt{ex^2+d}}\,dx$$

input

```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(5/2),x, algorithm="maxima")
```

output

```
integrate(1/((-c*x^4 + a)^(5/2)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{5/2}}\,dx = \int \frac{1}{(-cx^4+a)^{\frac{5}{2}}\sqrt{ex^2+d}}\,dx$$

input

```
integrate(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(5/2),x, algorithm="giac")
```

output

```
integrate(1/((-c*x^4 + a)^(5/2)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{5/2}}\,dx = \int \frac{1}{\left(a-c\,x^4\right)^{5/2}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a - c*x^4)^(5/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a - c*x^4)^(5/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a-cx^4)^{5/2}}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-c^3e\,x^{14}-c^3d\,x^{12}+3a\,c^2e\,x^{10}+3a\,c^2d\,x^8-3a^2ce\,x^6-3a^2cd\,x^4+a^3e\,x}$$

input
```
int(1/(e*x^2+d)^(1/2)/(-c*x^4+a)^(5/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**3*d + a**3*e*x**2 - 3*a**2*c*d
*x**4 - 3*a**2*c*e*x**6 + 3*a*c**2*d*x**8 + 3*a*c**2*e*x**10 - c**3*d*x**1
2 - c**3*e*x**14),x)
```

3.471 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 657

$$
\begin{aligned}
\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^{5/2}}\,dx &= \frac{cx(d-ex^2)}{6a\left(cd^2-ae^2\right)\sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}} \\
&+ \frac{e^2(cd^2+3ae^2)\,x}{3ad\left(cd^2-ae^2\right)^2\sqrt{d+ex^2}\sqrt{a-cx^4}} - \frac{e(7c^2d^4-27acd^2e^2-12a^2e^4)\sqrt{d+ex^2}}{12ad\left(cd^2-ae^2\right)^3 x\sqrt{a-cx^4}} \\
&+ \frac{c(5c^2d^4-15acd^2e^2-22a^2e^4)\,x\sqrt{d+ex^2}}{12a^2\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}} \\
&- \frac{\sqrt{c}e(7c^2d^4-27acd^2e^2-12a^2e^4)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{12a^2d\left(\sqrt{c}d-\sqrt{a}e\right)^3\left(\sqrt{c}d+\sqrt{a}e\right)^2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{\sqrt{c}(5c^2d^4-9acd^2e^2+12a^2e^4)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{12a^{5/2}d\left(cd^2-ae^2\right)^2\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}
$$

output

```
1/6*c*x*(-e*x^2+d)/a/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2)+1/3*e
^2*(3*a*e^2+c*d^2)*x/a/d/(-a*e^2+c*d^2)^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
-1/12*e*(-12*a^2*e^4-27*a*c*d^2*e^2+7*c^2*d^4)*(e*x^2+d)^(1/2)/a/d/(-a*e^2
+c*d^2)^3/x/(-c*x^4+a)^(1/2)+1/12*c*(-22*a^2*e^4-15*a*c*d^2*e^2+5*c^2*d^4)
*x*(e*x^2+d)^(1/2)/a^2/(-a*e^2+c*d^2)^3/(-c*x^4+a)^(1/2)-1/12*c^(1/2)*e*(-
12*a^2*e^4-27*a*c*d^2*e^2+7*c^2*d^4)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2
+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)
^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/d/(c^(1/2)*d-a
^(1/2)*e)^3/(c^(1/2)*d+a^(1/2)*e)^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/12*
c^(1/2)*(12*a^2*e^4-9*a*c*d^2*e^2+5*c^2*d^4)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2
)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1
/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(5/2)/d/
(-a*e^2+c*d^2)^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^{5/2}}\,dx = \int \frac{1}{(d+ex^2)^{3/2}(a-cx^4)^{5/2}}\,dx$$

input

```
Integrate[1/((d + e*x^2)^(3/2)*(a - c*x^4)^(5/2)),x]
```

output

```
Integrate[1/((d + e*x^2)^(3/2)*(a - c*x^4)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^4)^{5/2}(d+ex^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{1571}$$
$$\int \frac{1}{(a-cx^4)^{5/2}(d+ex^2)^{3/2}}dx$$

input

```
Int[1/((d + e*x^2)^(3/2)*(a - c*x^4)^(5/2)),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(-c\,x^{4}+a\right)^{\frac{5}{2}}}dx$$

input

```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(5/2),x)
```

output

```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a-cx^4)^{5/2}}\,dx=\int \frac{1}{(-cx^4+a)^{\frac{5}{2}}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input

```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(5/2),x, algorithm="fricas")
```

output

```
integral(-sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c^3*e^2*x^16 + 2*c^3*d*e*x^14
- 6*a*c^2*d*e*x^10 + (c^3*d^2 - 3*a*c^2*e^2)*x^12 + 6*a^2*c*d*e*x^6 - 3*(a
*c^2*d^2 - a^2*c*e^2)*x^8 - 2*a^3*d*e*x^2 - a^3*d^2 + (3*a^2*c*d^2 - a^3*e
^2)*x^4), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a-cx^4)^{5/2}}\,dx=\int \frac{1}{(a-cx^4)^{\frac{5}{2}}\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(-c*x**4+a)**(5/2),x)
```

output
```
Integral(1/((a - c*x**4)**(5/2)*(d + e*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a-cx^4)^{5/2}}\,dx=\int \frac{1}{(-cx^4+a)^{\frac{5}{2}}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c*x^4 + a)^(5/2)*(e*x^2 + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a-cx^4)^{5/2}}\,dx=\int \frac{1}{(-cx^4+a)^{\frac{5}{2}}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-c*x^4 + a)^(5/2)*(e*x^2 + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{5/2}}\,dx=\int \frac{1}{\left(a-c\,x^4\right)^{5/2}\,\left(e\,x^2+d\right)^{3/2}} \,d x$$

input
```
int(1/((a - c*x^4)^(5/2)*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a - c*x^4)^(5/2)*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{5/2}}\,dx=\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-}}{-c^3e^2x^{16}-2c^3de\,x^{14}+3a\,c^2e^2x^{12}-c^3d^2x^{12}+6a\,c^2de\,x^{10}-3a^2c\,e^2x^8}$$

input
```
int(1/(e*x^2+d)^(3/2)/(-c*x^4+a)^(5/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**3*d**2 + 2*a**3*d*e*x**2 + a**
3*e**2*x**4 - 3*a**2*c*d**2*x**4 - 6*a**2*c*d*e*x**6 - 3*a**2*c*e**2*x**8
+ 3*a*c**2*d**2*x**8 + 6*a*c**2*d*e*x**10 + 3*a*c**2*e**2*x**12 - c**3*d**
2*x**12 - 2*c**3*d*e*x**14 - c**3*e**2*x**16),x)
```

3.472 $\int \frac{1}{(d+ex^2)^{5/2}(a-cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 793

$$\int \frac{1}{(d+ex^2)^{5/2}(a-cx^4)^{5/2}}\,dx = \frac{cx(d-ex^2)}{6a(cd^2-ae^2)(d+ex^2)^{3/2}(a-cx^4)^{3/2}}$$
$$+\frac{e^2(cd^2+ae^2)x}{3ad(cd^2-ae^2)^2(d+ex^2)^{3/2}\sqrt{a-cx^4}}+\frac{e^2(3c^2d^4+33acd^2e^2-4a^2e^4)x}{6ad^2(cd^2-ae^2)^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{e(5c^3d^6-30ac^2d^4e^2-43a^2cd^2e^4+4a^3e^6)\sqrt{d+ex^2}}{6ad^2(cd^2-ae^2)^4x\sqrt{a-cx^4}}$$
$$+\frac{c(5c^3d^6-22ac^2d^4e^2-115a^2cd^2e^4+4a^3e^6)x\sqrt{d+ex^2}}{12a^2d(cd^2-ae^2)^4\sqrt{a-cx^4}}$$
$$-\frac{\sqrt{c}e(5c^3d^6-30ac^2d^4e^2-43a^2cd^2e^4+4a^3e^6)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{6a^2d^2(\sqrt{c}d-\sqrt{a}e)^4(\sqrt{c}d+\sqrt{a}e)^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{\sqrt{c}(5c^3d^6-15ac^2d^4e^2+50a^2cd^2e^4-8a^3e^6)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\right.}{12a^{5/2}d^2(cd^2-ae^2)^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output

```
1/6*c*x*(-e*x^2+d)/a/(-a*e^2+c*d^2)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2)+1/3*e
^2*(a*e^2+c*d^2)*x/a/d/(-a*e^2+c*d^2)^2/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2)+1
/6*e^2*(-4*a^2*e^4+33*a*c*d^2*e^2+3*c^2*d^4)*x/a/d^2/(-a*e^2+c*d^2)^3/(e*x
^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/6*e*(4*a^3*e^6-43*a^2*c*d^2*e^4-30*a*c^2*d^
4*e^2+5*c^3*d^6)*(e*x^2+d)^(1/2)/a/d^2/(-a*e^2+c*d^2)^4/x/(-c*x^4+a)^(1/2)
+1/12*c*(4*a^3*e^6-115*a^2*c*d^2*e^4-22*a*c^2*d^4*e^2+5*c^3*d^6)*x*(e*x^2+
d)^(1/2)/a^2/d/(-a*e^2+c*d^2)^4/(-c*x^4+a)^(1/2)-1/6*c^(1/2)*e*(4*a^3*e^6-
43*a^2*c*d^2*e^4-30*a*c^2*d^4*e^2+5*c^3*d^6)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2
)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1
/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/d^2/(c
^(1/2)*d-a^(1/2)*e)^4/(c^(1/2)*d+a^(1/2)*e)^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(
1/2)+1/12*c^(1/2)*(-8*a^3*e^6+50*a^2*c*d^2*e^4-15*a*c^2*d^4*e^2+5*c^3*d^6)
*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)
*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)
*e/c^(1/2)))^(1/2))/a^(5/2)/d^2/(-a*e^2+c*d^2)^3/(e*x^2+d)^(1/2)/(-c*x^4+a
)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(d+ex^2)^{5/2}(a-cx^4)^{5/2}}\,dx=\int \frac{1}{(d+ex^2)^{5/2}(a-cx^4)^{5/2}}\,dx$$

input

```
Integrate[1/((d + e*x^2)^(5/2)*(a - c*x^4)^(5/2)),x]
```

output

```
Integrate[1/((d + e*x^2)^(5/2)*(a - c*x^4)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^4)^{5/2}(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{1}{(a-cx^4)^{5/2}(d+ex^2)^{5/2}}dx$$

input
```
Int[1/((d + e*x^2)^(5/2)*(a - c*x^4)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}(-c\,x^4+a)^{\frac{5}{2}}}dx$$

input
```
int(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(5/2),x)
```

output
```
int(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)^{5/2}(a-cx^4)^{5/2}}\,dx=\int \frac{1}{(-cx^4+a)^{\frac{5}{2}}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(5/2),x, algorithm="fricas")
```

output

```
integral(-sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c^3*e^3*x^18 + 3*c^3*d*e^2*x^1
6 + 3*(c^3*d^2*e - a*c^2*e^3)*x^14 + (c^3*d^3 - 9*a*c^2*d*e^2)*x^12 - 3*(3
*a*c^2*d^2*e - a^2*c*e^3)*x^10 - 3*(a*c^2*d^3 - 3*a^2*c*d*e^2)*x^8 - 3*a^3
*d^2*e*x^2 + (9*a^2*c*d^2*e - a^3*e^3)*x^6 - a^3*d^3 + 3*(a^2*c*d^3 - a^3*
d*e^2)*x^4), x)
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)^{5/2}}\,dx=\int \frac{1}{\left(a-cx^4\right)^{\frac{5}{2}}\left(d+ex^2\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(1/(e*x**2+d)**(5/2)/(-c*x**4+a)**(5/2),x)
```

output

```
Integral(1/((a - c*x**4)**(5/2)*(d + e*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a-cx^4\right)^{5/2}}\,dx=\int \frac{1}{\left(-cx^4+a\right)^{\frac{5}{2}}\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(5/2),x, algorithm="maxima")
```

output

```
integrate(1/((-c*x^4 + a)^(5/2)*(e*x^2 + d)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex^2\right)^{5/2} \left(a-cx^4\right)^{5/2}}\,dx = \int \frac{1}{\left(-cx^4+a\right)^{\frac{5}{2}}\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-c*x^4 + a)^(5/2)*(e*x^2 + d)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{5/2} \left(a-cx^4\right)^{5/2}}\,dx = \int \frac{1}{\left(a-c\,x^4\right)^{5/2}\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int(1/((a - c*x^4)^(5/2)*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a - c*x^4)^(5/2)*(d + e*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+ex^2\right)^{5/2} \left(a-cx^4\right)^{5/2}}\,dx = \int \frac{}{-c^3e^3x^{18}-3c^3d\,e^2x^{16}+3a\,c^2e^3x^{14}-3c^3d^2e\,x^{14}+9a\,c^2d\,e^2x^{12}-c^3d^3}$$

input
```
int(1/(e*x^2+d)^(5/2)/(-c*x^4+a)^(5/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**3*d**3 + 3*a**3*d**2*e*x**2 +
3*a**3*d*e**2*x**4 + a**3*e**3*x**6 - 3*a**2*c*d**3*x**4 - 9*a**2*c*d**2*e
*x**6 - 9*a**2*c*d*e**2*x**8 - 3*a**2*c*e**3*x**10 + 3*a*c**2*d**3*x**8 +
9*a*c**2*d**2*e*x**10 + 9*a*c**2*d*e**2*x**12 + 3*a*c**2*e**3*x**14 - c**3
*d**3*x**12 - 3*c**3*d**2*e*x**14 - 3*c**3*d*e**2*x**16 - c**3*e**3*x**18)
,x)
```

3.473 $\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 172

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \frac{a\sqrt{1-\frac{1}{x^4}}\,x^3\sqrt{\frac{a+bx^2}{(a+b)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{1}{x^2}}}{\sqrt{2}}\right),\frac{2a}{a+b}\right)}{\sqrt{a+bx^2}\sqrt{1-x^4}} + \frac{b\sqrt{1-\frac{1}{x^4}}\,x^3\sqrt{\frac{a+bx^2}{(a+b)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{1}{x^2}}}{\sqrt{2}}\right),\frac{2a}{a+b}\right)}{\sqrt{a+bx^2}\sqrt{1-x^4}}$$

output

```
a*(1-1/x^4)^(1/2)*x^3*((b*x^2+a)/(a+b)/x^2)^(1/2)*EllipticF(1/2*(1-1/x^2)^
(1/2)*2^(1/2),2^(1/2)*(a/(a+b))^(1/2))/(b*x^2+a)^(1/2)/(-x^4+1)^(1/2)+b*(1
-1/x^4)^(1/2)*x^3*((b*x^2+a)/(a+b)/x^2)^(1/2)*EllipticPi(1/2*(1-1/x^2)^(1/
2)*2^(1/2),2,2^(1/2)*(a/(a+b))^(1/2))/(b*x^2+a)^(1/2)/(-x^4+1)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]
```

output
```
Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$
$$\Big\downarrow \; 1571$$
$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}dx$$

input
```
Int[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\sqrt{-x^{4}+1}}dx$$

input

```
int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
```

output

```
int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-x^4+1}}\,dx$$

input

```
integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output

```
integral(-sqrt(-x^4 + 1)*sqrt(b*x^2 + a)/(x^4 - 1), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{-\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}}\,dx$$

input

```
integrate((b*x**2+a)**(1/2)/(-x**4+1)**(1/2),x)
```

output

```
Integral(sqrt(a + b*x**2)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{1-x^4}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(1 - x^4)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(1 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = -\left(\int \frac{\sqrt{b\,x^2+a}\,\sqrt{-x^4+1}}{x^4-1}dx\right)$$

input

```
int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
```

output

```
 - int((sqrt(a + b*x**2)*sqrt( - x**4 + 1))/(x**4 - 1),x)
```

3.474 $\int (d+ex^2)^q (a+cx^4)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 321

$$\int (d+ex^2)^q (a+cx^4)^2 \, dx = -\frac{3cd(35cd^2+2ae^2(63+32q+4q^2))\, x(d+ex^2)^{1+q}}{e^4(3+2q)(5+2q)(7+2q)(9+2q)} + \frac{c(35cd^2+2ae^2(63+32q+4q^2))\, x^3(d+ex^2)^{1+q}}{e^3(5+2q)(7+2q)(9+2q)} - \frac{7c^2dx^5(d+ex^2)^{1+q}}{e^2(7+2q)(9+2q)} + \frac{c^2x^7(d+ex^2)^{1+q}}{e(9+2q)} + \frac{\left(a^2e^4(63+32q+4q^2)+\frac{3cd^2(35cd^2+2ae^2(63+32q+4q^2))}{(3+2q)(5+2q)}\right) x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} \text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{e^4(7+2q)(9+2q)}$$

output

```
-3*c*d*(35*c*d^2+2*a*e^2*(4*q^2+32*q+63))*x*(e*x^2+d)^(1+q)/e^4/(3+2*q)/(5
+2*q)/(7+2*q)/(9+2*q)+c*(35*c*d^2+2*a*e^2*(4*q^2+32*q+63))*x^3*(e*x^2+d)^(
1+q)/e^3/(5+2*q)/(7+2*q)/(9+2*q)-7*c^2*d*x^5*(e*x^2+d)^(1+q)/e^2/(7+2*q)/(
9+2*q)+c^2*x^7*(e*x^2+d)^(1+q)/e/(9+2*q)+(a^2*e^4*(4*q^2+32*q+63)+3*c*d^2*
(35*c*d^2+2*a*e^2*(4*q^2+32*q+63))/(3+2*q)/(5+2*q))*x*(e*x^2+d)^q*hypergeo
m([1/2, -q],[3/2],-e*x^2/d)/e^4/(7+2*q)/(9+2*q)/((1+e*x^2/d)^q)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.33

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right)^2 \,dx = \frac{1}{45}x\left(d+ex^2\right)^q \left(1+\frac{ex^2}{d}\right)^{-q} \left(45a^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)+cx^4\left(18a\,\text{Hypergeometric2F1}\left(\frac{5}{2},-q,\frac{7}{2},-\frac{ex^2}{d}\right)+5cx^4\,\text{Hypergeometric2F1}\left(\frac{9}{2},-q,\frac{11}{2},-\frac{ex^2}{d}\right)\right)\right)$$

input
```
Integrate[(d + e*x^2)^q*(a + c*x^4)^2,x]
```

output
```
(x*(d + e*x^2)^q*(45*a^2*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)] + c
*x^4*(18*a*Hypergeometric2F1[5/2, -q, 7/2, -((e*x^2)/d)] + 5*c*x^4*Hyperge
ometric2F1[9/2, -q, 11/2, -((e*x^2)/d)])))/(45*(1 + (e*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1474, 2346, 1474, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^2 \left(d+ex^2\right)^q \,dx$$

$$\Big\downarrow \text{1474}$$

$$\frac{\int \left(ex^2+d\right)^q \left(-7c^2dx^6+2ace(2q+9)x^4+a^2e(2q+9)\right)dx}{e(2q+9)}+\frac{c^2x^7\left(d+ex^2\right)^{q+1}}{e(2q+9)}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{\int \left(ex^2+d\right)^q \left(c\left(35cd^2+2ae^2\left(4q^2+32q+63\right)\right)x^4+a^2e^2\left(4q^2+32q+63\right)\right)dx}{e(2q+7)}-\frac{7c^2dx^5\left(d+ex^2\right)^{q+1}}{e(2q+7)}}{e(2q+9)}+\frac{c^2x^7\left(d+ex^2\right)^{q+1}}{e(2q+9)}$$

$$\Big\downarrow \; 1474$$

$$\frac{\frac{\frac{\int \left(ex^2+d\right)^q \left(a^2e^3\left(8q^3+84q^2+286q+315\right)-3cd\left(35cd^2+2ae^2\left(4q^2+32q+63\right)\right)x^2\right)dx}{e(2q+5)}+\frac{cx^3\left(d+ex^2\right)^{q+1}\left(2ae^2\left(4q^2+32q+63\right)+35cd^2\right)}{e(2q+5)}}{e(2q+7)}-\frac{7c^2dx^5\left(d+ex^2\right)^{q+1}}{e(2q+7)}}{e(2q+9)}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{q+1}}{e(2q+9)}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{\frac{\frac{\left(a^2e^4\left(8q^3+84q^2+286q+315\right)+\frac{3cd^2\left(2ae^2\left(4q^2+32q+63\right)+35cd^2\right)}{2q+3}\right)\int \left(ex^2+d\right)^q dx}{e}-\frac{3cdx\left(d+ex^2\right)^{q+1}\left(2ae^2\left(4q^2+32q+63\right)+35cd^2\right)}{e(2q+3)}}{e(2q+5)}+\frac{cx^3\left(d+ex^2\right)^{q+1}(2ae}{e(}}{e(2q+7)}}{e(2q+9)}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{q+1}}{e(2q+9)}$$

$$\Big\downarrow \; 238$$

$$\frac{\frac{\frac{\frac{\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(a^2e^4\left(8q^3+84q^2+286q+315\right)+\frac{3cd^2\left(2ae^2\left(4q^2+32q+63\right)+35cd^2\right)}{2q+3}\right)\int \left(\frac{ex^2}{d}+1\right)^q dx}{e}-\frac{3cdx\left(d+ex^2\right)^{q+1}\left(2ae^2\left(4q^2+32q+63\right)+35cd^2\right)}{e(2q+3)}}{e(2q+5)}}{e(2q+7)}}{e(2q+9)}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{q+1}}{e(2q+9)}$$

$$\Big\downarrow \; 237$$

$$\frac{\frac{\frac{\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(a^2e^4\left(8q^3+84q^2+286q+315\right)+\frac{3cd^2\left(2ae^2\left(4q^2+32q+63\right)+35cd^2\right)}{2q+3}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{e}-\frac{3cdx\left(d+ex^2\right)^{q+1}(2ae^2}{e(2q}}{e(2q+5)}}{e(2q+7)}}{e(2q+9)}$$

$$\frac{c^2x^7\left(d+ex^2\right)^{q+1}}{e(2q+9)}$$

input

```
Int[(d + e*x^2)^q*(a + c*x^4)^2,x]
```

output

```
(c^2*x^7*(d + e*x^2)^(1 + q))/(e*(9 + 2*q)) + ((-7*c^2*d*x^5*(d + e*x^2)^(
1 + q))/(e*(7 + 2*q)) + ((c*(35*c*d^2 + 2*a*e^2*(63 + 32*q + 4*q^2))*x^3*(
d + e*x^2)^(1 + q))/(e*(5 + 2*q)) + ((-3*c*d*(35*c*d^2 + 2*a*e^2*(63 + 32*
q + 4*q^2))*x*(d + e*x^2)^(1 + q))/(e*(3 + 2*q)) + ((a^2*e^4*(315 + 286*q
+ 84*q^2 + 8*q^3) + (3*c*d^2*(35*c*d^2 + 2*a*e^2*(63 + 32*q + 4*q^2)))/(3
+ 2*q))*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(e*
(1 + (e*x^2)/d)^q))/(e*(5 + 2*q)))/(e*(7 + 2*q)))/(e*(9 + 2*q))
```

Defintions of rubi rules used

rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)^{q} \left(c\,x^{4}+a\right)^{2}dx$$

input
```
int((e*x^2+d)^q*(c*x^4+a)^2,x)
```

output
```
int((e*x^2+d)^q*(c*x^4+a)^2,x)
```

Fricas [F]

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right)^2\,dx=\int \left(cx^4+a\right)^2\left(ex^2+d\right)^q\,dx$$

input
```
integrate((e*x^2+d)^q*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
integral((c^2*x^8 + 2*a*c*x^4 + a^2)*(e*x^2 + d)^q, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 24.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.27

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right)^2 \, dx = a^2 d^q x {}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-q\\ \frac{3}{2}\end{matrix}\right|\frac{ex^2 e^{i\pi}}{d}\right) + \frac{2acd^q x^5 {}_2F_1\left(\left.\begin{matrix}\frac{5}{2},-q\\ \frac{7}{2}\end{matrix}\right|\frac{ex^2 e^{i\pi}}{d}\right)}{5} + \frac{c^2 d^q x^9 {}_2F_1\left(\left.\begin{matrix}\frac{9}{2},-q\\ \frac{11}{2}\end{matrix}\right|\frac{ex^2 e^{i\pi}}{d}\right)}{9}$$

input
```
integrate((e*x**2+d)**q*(c*x**4+a)**2,x)
```

output
```
a**2*d**q*x*hyper((1/2, -q), (3/2,), e*x**2*exp_polar(I*pi)/d) + 2*a*c*d**
q*x**5*hyper((5/2, -q), (7/2,), e*x**2*exp_polar(I*pi)/d)/5 + c**2*d**q*x*
*9*hyper((9/2, -q), (11/2,), e*x**2*exp_polar(I*pi)/d)/9
```

Maxima [F]

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right)^2 \, dx = \int \left(cx^4+a\right)^2 \left(ex^2+d\right)^q \, dx$$

input
```
integrate((e*x^2+d)^q*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^2*(e*x^2 + d)^q, x)
```

Giac [F]

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right)^2 \, dx=\int \left(cx^4+a\right)^2\left(ex^2+d\right)^q \, dx$$

input
```
integrate((e*x^2+d)^q*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^2*(e*x^2 + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right)^2 \, dx=\int \left(c\,x^4+a\right)^2\,\left(e\,x^2+d\right)^q \,dx$$

input
```
int((a + c*x^4)^2*(d + e*x^2)^q,x)
```

output
```
int((a + c*x^4)^2*(d + e*x^2)^q, x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right)^2 \, dx=\text{too large to display}$$

input
```
int((e*x^2+d)^q*(c*x^4+a)^2,x)
```

output

```
(16*(d + e*x**2)**q*a**2*e**4*q**4*x + 192*(d + e*x**2)**q*a**2*e**4*q**3*
x + 824*(d + e*x**2)**q*a**2*e**4*q**2*x + 1488*(d + e*x**2)**q*a**2*e**4*
q*x + 945*(d + e*x**2)**q*a**2*e**4*x - 48*(d + e*x**2)**q*a*c*d**2*e**2*q
**3*x - 384*(d + e*x**2)**q*a*c*d**2*e**2*q**2*x - 756*(d + e*x**2)**q*a*c
*d**2*e**2*q*x + 32*(d + e*x**2)**q*a*c*d*e**3*q**4*x**3 + 272*(d + e*x**2
)**q*a*c*d*e**3*q**3*x**3 + 632*(d + e*x**2)**q*a*c*d*e**3*q**2*x**3 + 252
*(d + e*x**2)**q*a*c*d*e**3*q*x**3 + 32*(d + e*x**2)**q*a*c*e**4*q**4*x**5
 + 320*(d + e*x**2)**q*a*c*e**4*q**3*x**5 + 1040*(d + e*x**2)**q*a*c*e**4*
q**2*x**5 + 1200*(d + e*x**2)**q*a*c*e**4*q*x**5 + 378*(d + e*x**2)**q*a*c
*e**4*x**5 - 210*(d + e*x**2)**q*c**2*d**4*q*x + 140*(d + e*x**2)**q*c**2*
d**3*e*q**2*x**3 + 70*(d + e*x**2)**q*c**2*d**3*e*q*x**3 - 56*(d + e*x**2)
**q*c**2*d**2*e**2*q**3*x**5 - 112*(d + e*x**2)**q*c**2*d**2*e**2*q**2*x**
5 - 42*(d + e*x**2)**q*c**2*d**2*e**2*q*x**5 + 16*(d + e*x**2)**q*c**2*d*e
**3*q**4*x**7 + 72*(d + e*x**2)**q*c**2*d*e**3*q**3*x**7 + 92*(d + e*x**2)
**q*c**2*d*e**3*q**2*x**7 + 30*(d + e*x**2)**q*c**2*d*e**3*q*x**7 + 16*(d
+ e*x**2)**q*c**2*e**4*q**4*x**9 + 128*(d + e*x**2)**q*c**2*e**4*q**3*x**9
 + 344*(d + e*x**2)**q*c**2*e**4*q**2*x**9 + 352*(d + e*x**2)**q*c**2*e**4
*q*x**9 + 105*(d + e*x**2)**q*c**2*e**4*x**9 + 1024*int((d + e*x**2)**q/(3
2*d*q**5 + 400*d*q**4 + 1840*d*q**3 + 3800*d*q**2 + 3378*d*q + 945*d + 32*
e*q**5*x**2 + 400*e*q**4*x**2 + 1840*e*q**3*x**2 + 3800*e*q**2*x**2 + 3...
```

3.475 $\int (d+ex^2)^q (a+cx^4)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 143

$$\int (d+ex^2)^q (a+cx^4)\,dx = -\frac{3cdx(d+ex^2)^{1+q}}{e^2(3+2q)(5+2q)} + \frac{cx^3(d+ex^2)^{1+q}}{e(5+2q)} + \frac{(3cd^2+ae^2(15+16q+4q^2))\,x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} \text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{e^2(3+2q)(5+2q)}$$

output
```
-3*c*d*x*(e*x^2+d)^(1+q)/e^2/(3+2*q)/(5+2*q)+c*x^3*(e*x^2+d)^(1+q)/e/(5+2*
q)+(3*c*d^2+a*e^2*(4*q^2+16*q+15))*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2]
,-e*x^2/d)/e^2/(3+2*q)/(5+2*q)/((1+e*x^2/d)^q)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.52

$$\int (d+ex^2)^q (a+cx^4)\,dx = \frac{1}{5}x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} \left(5a\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right) + cx^4\,\text{Hypergeometric2F1}\left(\frac{5}{2},-q,\frac{7}{2},-\frac{ex^2}{d}\right)\right)$$

input

```
Integrate[(d + e*x^2)^q*(a + c*x^4),x]
```

output

```
(x*(d + e*x^2)^q*(5*a*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)] + c*x^
4*Hypergeometric2F1[5/2, -q, 7/2, -((e*x^2)/d)]))/(5*(1 + (e*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1474, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)\left(d+ex^2\right)^q\,dx$$

$$\Big\downarrow \text{1474}$$

$$\frac{\int \left(ae(2q+5)-3cdx^2\right)\left(ex^2+d\right)^q dx}{e(2q+5)}+\frac{cx^3\left(d+ex^2\right)^{q+1}}{e(2q+5)}$$

$$\Big\downarrow \text{299}$$

$$\frac{\left(ae(2q+5)+\frac{3cd^2}{2eq+3e}\right)\int \left(ex^2+d\right)^q dx-\frac{3cdx\left(d+ex^2\right)^{q+1}}{e(2q+3)}}{e(2q+5)}+\frac{cx^3\left(d+ex^2\right)^{q+1}}{e(2q+5)}$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(ae(2q+5)+\frac{3cd^2}{2eq+3e}\right)\int \left(\frac{ex^2}{d}+1\right)^q dx-\frac{3cdx\left(d+ex^2\right)^{q+1}}{e(2q+3)}}{e(2q+5)}+\frac{cx^3\left(d+ex^2\right)^{q+1}}{e(2q+5)}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(ae(2q+5)+\frac{3cd^2}{2eq+3e}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)-\frac{3cdx\left(d+ex^2\right)^{q+1}}{e(2q+3)}}{e(2q+5)}+\frac{cx^3\left(d+ex^2\right)^{q+1}}{e(2q+5)}$$

input
```
Int[(d + e*x^2)^q*(a + c*x^4),x]
```

output
```
(c*x^3*(d + e*x^2)^(1 + q))/(e*(5 + 2*q)) + ((-3*c*d*x*(d + e*x^2)^(1 + q)
)/(e*(3 + 2*q)) + ((a*e*(5 + 2*q) + (3*c*d^2)/(3*e + 2*e*q))*x*(d + e*x^2)
^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(1 + (e*x^2)/d)^q)/(e*(5
 + 2*q))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1474
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Si
mp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1))), x] + Simp[1/(
e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + c
*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p), x],
x], x] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
&&  !LtQ[q, -1]
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)^{q} \left(c\,x^{4}+a\right)dx$$

input
```
int((e*x^2+d)^q*(c*x^4+a),x)
```

output
```
int((e*x^2+d)^q*(c*x^4+a),x)
```

Fricas [F]

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right)\,dx=\int \left(cx^{4}+a\right)\left(ex^{2}+d\right)^{q}\,dx$$

input
```
integrate((e*x^2+d)^q*(c*x^4+a),x, algorithm="fricas")
```

output
```
integral((c*x^4 + a)*(e*x^2 + d)^q, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.85 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.37

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right)\,dx=ad^{q}x{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},-q\\ \frac{3}{2}\end{matrix}\right|\frac{ex^{2}e^{i\pi}}{d}\right)+\frac{cd^{q}x^{5}{}_{2}F_{1}\left(\left.\begin{matrix}\frac{5}{2},-q\\ \frac{7}{2}\end{matrix}\right|\frac{ex^{2}e^{i\pi}}{d}\right)}{5}$$

input
```
integrate((e*x**2+d)**q*(c*x**4+a),x)
```

output
```
a*d**q*x*hyper((1/2, -q), (3/2,), e*x**2*exp_polar(I*pi)/d) + c*d**q*x**5*
hyper((5/2, -q), (7/2,), e*x**2*exp_polar(I*pi)/d)/5
```

Maxima [F]

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right) \, dx = \int \left(cx^4+a\right)\left(ex^2+d\right)^q \, dx$$

input
```
integrate((e*x^2+d)^q*(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)*(e*x^2 + d)^q, x)
```

Giac [F]

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right) \, dx = \int \left(cx^4+a\right)\left(ex^2+d\right)^q \, dx$$

input
```
integrate((e*x^2+d)^q*(c*x^4+a),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)*(e*x^2 + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right) \, dx = \int \left(c\,x^4+a\right)\,\left(e\,x^2+d\right)^q \, dx$$

input
```
int((a + c*x^4)*(d + e*x^2)^q,x)
```

output
```
int((a + c*x^4)*(d + e*x^2)^q, x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right) \, dx = \text{Too large to display}$$

input

```
int((e*x^2+d)^q*(c*x^4+a),x)
```

output

```
(4*(d + e*x**2)**q*a*e**2*q**2*x + 16*(d + e*x**2)**q*a*e**2*q*x + 15*(d +
 e*x**2)**q*a*e**2*x - 6*(d + e*x**2)**q*c*d**2*q*x + 4*(d + e*x**2)**q*c*
d*e*q**2*x**3 + 2*(d + e*x**2)**q*c*d*e*q*x**3 + 4*(d + e*x**2)**q*c*e**2*
q**2*x**5 + 8*(d + e*x**2)**q*c*e**2*q*x**5 + 3*(d + e*x**2)**q*c*e**2*x**
5 + 64*int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q**
3*x**2 + 36*e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**2*q**6 + 544*
int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q**3*x**2
+ 36*e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**2*q**5 + 1760*int((d
 + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q**3*x**2 + 36*e
*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**2*q**4 + 2672*int((d + e*x
**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q**3*x**2 + 36*e*q**2*
x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**2*q**3 + 1860*int((d + e*x**2)**
q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q**3*x**2 + 36*e*q**2*x**2 +
 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**2*q**2 + 450*int((d + e*x**2)**q/(8*d*
q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q**3*x**2 + 36*e*q**2*x**2 + 46*e*q
*x**2 + 15*e*x**2),x)*a*d*e**2*q + 48*int((d + e*x**2)**q/(8*d*q**3 + 36*d
*q**2 + 46*d*q + 15*d + 8*e*q**3*x**2 + 36*e*q**2*x**2 + 46*e*q*x**2 + 15*
e*x**2),x)*c*d**3*q**4 + 216*int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 4
6*d*q + 15*d + 8*e*q**3*x**2 + 36*e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x
)*c*d**3*q**3 + 276*int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q ...
```

3.476 $\int (d+ex^2)^q \, dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (d+ex^2)^q \, dx = x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} \text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)$$

output
```
x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/((1+e*x^2/d)^q)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^q \, dx = x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} \text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)$$

input
```
Integrate[(d + e*x^2)^q,x]
```

output
```
(x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(1 + (e*x^
2)/d)^q
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)^q\,dx$$
$$\Big\downarrow \text{238}$$
$$(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\int\left(\frac{ex^2}{d}+1\right)^q dx$$
$$\Big\downarrow \text{237}$$
$$x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)$$

input
```
Int[(d + e*x^2)^q,x]
```

output
```
(x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(1 + (e*x^
2)/d)^q
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)^{q}dx$$

input
```
int((e*x^2+d)^q,x)
```

output
```
int((e*x^2+d)^q,x)
```

Fricas [F]

$$\int \left(d+ex^{2}\right)^{q}\,dx=\int \left(ex^{2}+d\right)^{q}dx$$

input
```
integrate((e*x^2+d)^q,x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \left(d+ex^{2}\right)^{q}\,dx=d^{q}x\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},-q\\ \frac{3}{2}\end{matrix}\right|\frac{ex^{2}e^{i\pi}}{d}\right)$$

input
```
integrate((e*x**2+d)**q,x)
```

output
```
d**q*x*hyper((1/2, -q), (3/2,), e*x**2*exp_polar(I*pi)/d)
```

Maxima [F]

$$\int \left(d+ex^2\right)^q\,dx=\int \left(ex^2+d\right)^q\,dx$$

input
```
integrate((e*x^2+d)^q,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q, x)
```

Giac [F]

$$\int \left(d+ex^2\right)^q\,dx=\int \left(ex^2+d\right)^q\,dx$$

input
```
integrate((e*x^2+d)^q,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q, x)
```

Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \left(d+ex^2\right)^q\,dx=\frac{x\,\left(e\,x^2+d\right)^q\,{}_2\mathrm{F}_1\left(\frac{1}{2},-q;\ \frac{3}{2};\ -\frac{e\,x^2}{d}\right)}{{\left(\frac{e\,x^2}{d}+1\right)}^q}$$

input
```
int((d + e*x^2)^q,x)
```

output
```
(x*(d + e*x^2)^q*hypergeom([1/2, -q], 3/2, -(e*x^2)/d))/((e*x^2)/d + 1)^q
```

Reduce [F]

$$\int \left(d+ex^2\right)^q\,dx = \frac{\left(e\,x^2+d\right)^q x+4\left(\int \frac{\left(e\,x^2+d\right)^q}{2eq\,x^2+e\,x^2+2dq+d}dx\right)d\,q^2+2\left(\int \frac{\left(e\,x^2+d\right)^q}{2eq\,x^2+e\,x^2+2dq+d}dx\right)dq}{2q+1}$$

input
```
int((e*x^2+d)^q,x)
```

output
```
((d + e*x**2)**q*x + 4*int((d + e*x**2)**q/(2*d*q + d + 2*e*q*x**2 + e*x**
2),x)*d*q**2 + 2*int((d + e*x**2)**q/(2*d*q + d + 2*e*q*x**2 + e*x**2),x)*
d*q)/(2*q + 1)
```

3.477 $\int \frac{(d+ex^2)^q}{a+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\int \frac{(d+ex^2)^q}{a+cx^4}\,dx = \frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-q,1,\frac{3}{2},-\frac{ex^2}{d},-\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)}{2a} + \frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-q,1,\frac{3}{2},-\frac{ex^2}{d},\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)}{2a}$$

output
```
1/2*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-c^(1/2)*x^2/(-a)^(1/2),-e*x^2/d)/
a/((1+e*x^2/d)^q)+1/2*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,c^(1/2)*x^2/(-a)
^(1/2),-e*x^2/d)/a/((1+e*x^2/d)^q)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^q}{a+cx^4}\,dx = \int \frac{(d+ex^2)^q}{a+cx^4}\,dx$$

input
```
Integrate[(d + e*x^2)^q/(a + c*x^4),x]
```

output
```
Integrate[(d + e*x^2)^q/(a + c*x^4), x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1489, 27, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^q}{a+cx^4}dx$$

$$\Big\downarrow \text{1489}$$

$$-\frac{\sqrt{c}\int \frac{(ex^2+d)^q}{\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x^2\right)}dx}{2\sqrt{-a}}-\frac{\sqrt{c}\int \frac{(ex^2+d)^q}{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{-a}\right)}dx}{2\sqrt{-a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(ex^2+d)^q}{\sqrt{-a}-\sqrt{c}x^2}dx}{2\sqrt{-a}}-\frac{\int \frac{(ex^2+d)^q}{\sqrt{c}x^2+\sqrt{-a}}dx}{2\sqrt{-a}}$$

$$\Big\downarrow \text{334}$$

$$-\frac{(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\int \frac{\left(\frac{ex^2}{d}+1\right)^q}{\sqrt{-a}-\sqrt{c}x^2}dx}{2\sqrt{-a}}-\frac{(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\int \frac{\left(\frac{ex^2}{d}+1\right)^q}{\sqrt{c}x^2+\sqrt{-a}}dx}{2\sqrt{-a}}$$

$$\Big\downarrow \text{333}$$

$$\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2a}+\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2a}$$

input

```
Int[(d + e*x^2)^q/(a + c*x^4),x]
```

output

```
(x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, -((Sqrt[c]*x^2)/Sqrt[-a]), -((e
*x^2)/d)])/(2*a*(1 + (e*x^2)/d)^q) + (x*(d + e*x^2)^q*AppellF1[1/2, 1, -q,
 3/2, (Sqrt[c]*x^2)/Sqrt[-a], -((e*x^2)/d)])/(2*a*(1 + (e*x^2)/d)^q)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1489
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Simp[-c/(2*r)   Int[(d + e*x^2)^q/(r - c*x^2), x], x] - S
imp[c/(2*r)   Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e,
q}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \frac{(e\,x^{2}+d)^{q}}{c\,x^{4}+a}dx$$

input
```
int((e*x^2+d)^q/(c*x^4+a),x)
```

output
```
int((e*x^2+d)^q/(c*x^4+a),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^q}{a+cx^4}\,dx = \int \frac{(ex^2+d)^q}{cx^4+a}\,dx$$

input
```
integrate((e*x^2+d)^q/(c*x^4+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q/(c*x^4 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{a+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**q/(c*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^q}{a+cx^4}\,dx = \int \frac{(ex^2+d)^q}{cx^4+a}\,dx$$

input
```
integrate((e*x^2+d)^q/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q/(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^q}{a+cx^4}\,dx = \int \frac{(ex^2+d)^q}{cx^4+a}\,dx$$

input
```
integrate((e*x^2+d)^q/(c*x^4+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q/(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{a+cx^4}\,dx = \int \frac{(e\,x^2+d)^q}{c\,x^4+a}\,dx$$

input
```
int((d + e*x^2)^q/(a + c*x^4),x)
```

output
```
int((d + e*x^2)^q/(a + c*x^4), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^q}{a+cx^4}\,dx = \int \frac{(e\,x^2+d)^q}{c\,x^4+a}dx$$

input
```
int((e*x^2+d)^q/(c*x^4+a),x)
```

output
```
int((d + e*x**2)**q/(a + c*x**4),x)
```

3.478 $\int \frac{(d+ex^2)^q}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 351

$$\begin{aligned}\int \frac{(d+ex^2)^q}{(a+cx^4)^2}\,dx &= \frac{cx(d-ex^2)\left(d+ex^2\right)^{1+q}}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}\\
&+\frac{\left(3cd^2+ae^2(3-2q)+2\sqrt{-a}\sqrt{c}deq\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},-q,1,\frac{3}{2},-\frac{ex^2}{d},-\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)}{8a^2\left(cd^2+ae^2\right)}\\
&+\frac{\left(3cd^2+ae^2(3-2q)-2\sqrt{-a}\sqrt{c}deq\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},-q,1,\frac{3}{2},-\frac{ex^2}{d},\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)}{8a^2\left(cd^2+ae^2\right)}\\
&+\frac{e^2(1+2q)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{4a\left(cd^2+ae^2\right)}\end{aligned}$$

output

```
1/4*c*x*(-e*x^2+d)*(e*x^2+d)^(1+q)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/8*(3*c*d^2+
a*e^2*(3-2*q)+2*(-a)^(1/2)*c^(1/2)*d*e*q)*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,
3/2,-c^(1/2)*x^2/(-a)^(1/2),-e*x^2/d)/a^2/(a*e^2+c*d^2)/((1+e*x^2/d)^q)+1/
8*(3*c*d^2+a*e^2*(3-2*q)-2*(-a)^(1/2)*c^(1/2)*d*e*q)*x*(e*x^2+d)^q*AppellF
1(1/2,1,-q,3/2,c^(1/2)*x^2/(-a)^(1/2),-e*x^2/d)/a^2/(a*e^2+c*d^2)/((1+e*x^
2/d)^q)+1/4*e^2*(1+2*q)*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/
a/(a*e^2+c*d^2)/((1+e*x^2/d)^q)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^2}\,dx=\int \frac{(d+ex^2)^q}{(a+cx^4)^2}\,dx$$

input

```
Integrate[(d + e*x^2)^q/(a + c*x^4)^2,x]
```

output

```
Integrate[(d + e*x^2)^q/(a + c*x^4)^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^2}dx$$

input

```
Int[(d + e*x^2)^q/(a + c*x^4)^2,x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{(e\,x^{2}+d)^{q}}{(c\,x^{4}+a)^{2}}dx$$

input
```
int((e*x^2+d)^q/(c*x^4+a)^2,x)
```

output
```
int((e*x^2+d)^q/(c*x^4+a)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^2}\,dx=\int \frac{(ex^2+d)^q}{(cx^4+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^q/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q/(c^2*x^8 + 2*a*c*x^4 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x**2+d)**q/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^2}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^q/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q/(c*x^4 + a)^2, x)
```

Giac [F]

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^2}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^q/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q/(c*x^4 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^2}\,dx = \int \frac{(e\,x^2+d)^q}{(c\,x^4+a)^2}\,dx$$

input
```
int((d + e*x^2)^q/(a + c*x^4)^2,x)
```

output
```
int((d + e*x^2)^q/(a + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^2}\,dx = \int \frac{(e\,x^2+d)^q}{c^2x^8+2ac\,x^4+a^2}dx$$

input
```
int((e*x^2+d)^q/(c*x^4+a)^2,x)
```

output
```
int((d + e*x**2)**q/(a**2 + 2*a*c*x**4 + c**2*x**8),x)
```

3.479 $\int \frac{(d+ex^2)^q}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 553

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^3}\,dx = \frac{cx(d-ex^2)\,(d+ex^2)^{1+q}}{8a\,(cd^2+ae^2)\,(a+cx^4)^2}$$
$$+\frac{cx(d+ex^2)^{1+q}\,\left(d(7cd^2+ae^2(11-4q))-e(ae^2(11-2q)+cd^2(7+2q))\,x^2\right)}{32a^2\,(cd^2+ae^2)^2\,(a+cx^4)}$$
$$+\frac{\left(21c^2d^4+8\sqrt{-a}\sqrt{c}de(2cd^2+ae^2(3-q))\,q+2acd^2e^2(21-6q-2q^2)+a^2e^4(21-20q+4q^2)\right)x(d+}{64a^3\,(cd^2+ae^2)^2}$$
$$+\frac{\left(21c^2d^4-8\sqrt{-a}\sqrt{c}de(2cd^2+ae^2(3-q))\,q+2acd^2e^2(21-6q-2q^2)+a^2e^4(21-20q+4q^2)\right)x(d+}{64a^3\,(cd^2+ae^2)^2}$$
$$+\frac{e^2(1+2q)\,(ae^2(11-2q)+cd^2(7+2q))\,x(d+ex^2)^q\,\left(1+\frac{ex^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{32a^2\,(cd^2+ae^2)^2}$$

output
```
1/8*c*x*(-e*x^2+d)*(e*x^2+d)^(1+q)/a/(a*e^2+c*d^2)/(c*x^4+a)^2+1/32*c*x*(e
*x^2+d)^(1+q)*(d*(7*c*d^2+a*e^2*(11-4*q))-e*(a*e^2*(11-2*q)+c*d^2*(7+2*q))
*x^2)/a^2/(a*e^2+c*d^2)^2/(c*x^4+a)+1/64*(21*c^2*d^4+8*(-a)^(1/2)*c^(1/2)*
d*e*(2*c*d^2+a*e^2*(3-q))*q+2*a*c*d^2*e^2*(-2*q^2-6*q+21)+a^2*e^4*(4*q^2-2
0*q+21))*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-c^(1/2)*x^2/(-a)^(1/2),-e*x^
2/d)/a^3/(a*e^2+c*d^2)^2/((1+e*x^2/d)^q)+1/64*(21*c^2*d^4-8*(-a)^(1/2)*c^(
1/2)*d*e*(2*c*d^2+a*e^2*(3-q))*q+2*a*c*d^2*e^2*(-2*q^2-6*q+21)+a^2*e^4*(4*
q^2-20*q+21))*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,c^(1/2)*x^2/(-a)^(1/2),-
e*x^2/d)/a^3/(a*e^2+c*d^2)^2/((1+e*x^2/d)^q)+1/32*e^2*(1+2*q)*(a*e^2*(11-2
*q)+c*d^2*(7+2*q))*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/a^2/(
a*e^2+c*d^2)^2/((1+e*x^2/d)^q)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^3}\,dx = \int \frac{(d+ex^2)^q}{(a+cx^4)^3}\,dx$$

input
```
Integrate[(d + e*x^2)^q/(a + c*x^4)^3,x]
```

output
```
Integrate[(d + e*x^2)^q/(a + c*x^4)^3, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \text{ 1571}$$

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^3}dx$$

input
```
Int[(d + e*x^2)^q/(a + c*x^4)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{q}}{\left(c\,x^{4}+a\right)^{3}}dx$$

input
```
int((e*x^2+d)^q/(c*x^4+a)^3,x)
```

output
```
int((e*x^2+d)^q/(c*x^4+a)^3,x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^3}\,dx=\int \frac{(ex^2+d)^q}{(cx^4+a)^3}\,dx$$

input
```
integrate((e*x^2+d)^q/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q/(c^3*x^12 + 3*a*c^2*x^8 + 3*a^2*c*x^4 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**q/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^3}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+a)^3}\,dx$$

input
```
integrate((e*x^2+d)^q/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q/(c*x^4 + a)^3, x)
```

Giac [F]

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^3}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+a)^3}\,dx$$

input
```
integrate((e*x^2+d)^q/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q/(c*x^4 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^3}\,dx = \int \frac{(e\,x^2+d)^q}{(c\,x^4+a)^3}\,dx$$

input
```
int((d + e*x^2)^q/(a + c*x^4)^3,x)
```

output
```
int((d + e*x^2)^q/(a + c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^q}{(a+cx^4)^3}\,dx = \int \frac{(e\,x^2+d)^q}{c^3x^{12}+3a\,c^2x^8+3a^2c\,x^4+a^3}dx$$

input
```
int((e*x^2+d)^q/(c*x^4+a)^3,x)
```

output
```
int((d + e*x**2)**q/(a**3 + 3*a**2*c*x**4 + 3*a*c**2*x**8 + c**3*x**12),x)
```

3.480 $\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx = \mathrm{Int}\left(\left(c+ex^2\right)^q \left(a+bx^4\right)^p, x\right)$$

output
```
Defer(Int)((e*x^2+c)^q*(b*x^4+a)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx = \int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx$$

input
```
Integrate[(c + e*x^2)^q*(a + b*x^4)^p,x]
```

output
```
Integrate[(c + e*x^2)^q*(a + b*x^4)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1571}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^p \left(c+ex^2\right)^q \, dx$$

$$\Big\downarrow \ 1571$$

$$\int \left(a+bx^4\right)^p \left(c+ex^2\right)^q dx$$

input
```
Int[(c + e*x^2)^q*(a + b*x^4)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(e\,x^{2}+c\right)^{q} \left(b\,x^{4}+a\right)^{p} dx$$

input
```
int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

output
```
int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx = \int \left(bx^4+a\right)^p \left(ex^2+c\right)^q \, dx$$

input
```
integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x**2+c)**q*(b*x**4+a)**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx = \int \left(bx^4+a\right)^p \left(ex^2+c\right)^q \, dx$$

input
```
integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx = \int (bx^4+a)^p (ex^2+c)^q \, dx$$

input
```
integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

Mupad [N/A]

Not integrable

Time = 17.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx = \int \left(b\,x^4+a\right)^p\,\left(e\,x^2+c\right)^q \,dx$$

input
```
int((a + b*x^4)^p*(c + e*x^2)^q,x)
```

output
```
int((a + b*x^4)^p*(c + e*x^2)^q, x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 1249, normalized size of antiderivative = 65.74

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p\,dx = \text{Too large to display}$$

input

```
int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

output

```
((c + e*x**2)**q*(a + b*x**4)**p*x + 8*int(((c + e*x**2)**q*(a + b*x**4)**
p*x**4)/(4*a*c*p + 2*a*c*q + a*c + 4*a*e*p*x**2 + 2*a*e*q*x**2 + a*e*x**2
+ 4*b*c*p*x**4 + 2*b*c*q*x**4 + b*c*x**4 + 4*b*e*p*x**6 + 2*b*e*q*x**6 + b
*e*x**6),x)*b*c*p*q + 4*int(((c + e*x**2)**q*(a + b*x**4)**p*x**4)/(4*a*c*
p + 2*a*c*q + a*c + 4*a*e*p*x**2 + 2*a*e*q*x**2 + a*e*x**2 + 4*b*c*p*x**4
+ 2*b*c*q*x**4 + b*c*x**4 + 4*b*e*p*x**6 + 2*b*e*q*x**6 + b*e*x**6),x)*b*c
*q**2 + 2*int(((c + e*x**2)**q*(a + b*x**4)**p*x**4)/(4*a*c*p + 2*a*c*q +
a*c + 4*a*e*p*x**2 + 2*a*e*q*x**2 + a*e*x**2 + 4*b*c*p*x**4 + 2*b*c*q*x**4
 + b*c*x**4 + 4*b*e*p*x**6 + 2*b*e*q*x**6 + b*e*x**6),x)*b*c*q + 16*int(((
c + e*x**2)**q*(a + b*x**4)**p*x**2)/(4*a*c*p + 2*a*c*q + a*c + 4*a*e*p*x*
*2 + 2*a*e*q*x**2 + a*e*x**2 + 4*b*c*p*x**4 + 2*b*c*q*x**4 + b*c*x**4 + 4*
b*e*p*x**6 + 2*b*e*q*x**6 + b*e*x**6),x)*a*e*p**2 + 8*int(((c + e*x**2)**q
*(a + b*x**4)**p*x**2)/(4*a*c*p + 2*a*c*q + a*c + 4*a*e*p*x**2 + 2*a*e*q*x
**2 + a*e*x**2 + 4*b*c*p*x**4 + 2*b*c*q*x**4 + b*c*x**4 + 4*b*e*p*x**6 + 2
*b*e*q*x**6 + b*e*x**6),x)*a*e*p*q + 4*int(((c + e*x**2)**q*(a + b*x**4)**
p*x**2)/(4*a*c*p + 2*a*c*q + a*c + 4*a*e*p*x**2 + 2*a*e*q*x**2 + a*e*x**2
+ 4*b*c*p*x**4 + 2*b*c*q*x**4 + b*c*x**4 + 4*b*e*p*x**6 + 2*b*e*q*x**6 + b
*e*x**6),x)*a*e*p + 16*int(((c + e*x**2)**q*(a + b*x**4)**p)/(4*a*c*p + 2*
a*c*q + a*c + 4*a*e*p*x**2 + 2*a*e*q*x**2 + a*e*x**2 + 4*b*c*p*x**4 + 2*b*
c*q*x**4 + b*c*x**4 + 4*b*e*p*x**6 + 2*b*e*q*x**6 + b*e*x**6),x)*a*c*p*...
```

3.481 $\int (c+dx^2)^3 (a+bx^4)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 189

$$
\begin{aligned}
\int (c+dx^2)^3 (a+bx^4)^p \, dx = {} & \frac{3cd^2x(a+bx^4)^{1+p}}{b(5+4p)} + \frac{d^3x^3(a+bx^4)^{1+p}}{b(7+4p)} \\
& + c\left(c^2 - \frac{3ad^2}{5b+4bp}\right) x (a+bx^4)^p \left(1 \right. \\
& \left. + \frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -\frac{bx^4}{a}\right) \\
& + d\left(c^2 - \frac{ad^2}{7b+4bp}\right) x^3 (a+bx^4)^p \left(1 \right. \\
& \left. + \frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4}, -p, \frac{7}{4}, -\frac{bx^4}{a}\right)
\end{aligned}
$$

output

```
3*c*d^2*x*(b*x^4+a)^(p+1)/b/(5+4*p)+d^3*x^3*(b*x^4+a)^(p+1)/b/(7+4*p)+c*(c
^2-3*a*d^2/(4*b*p+5*b))*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/
((1+b*x^4/a)^p)+d*(c^2-a*d^2/(4*b*p+7*b))*x^3*(b*x^4+a)^p*hypergeom([3/4,
-p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.72

$$\int \left(c+dx^2\right)^3 \left(a+bx^4\right)^p \, dx$$
$$=\frac{1}{35}x\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(35c^3 \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right) + dx^2\left(35c^2 \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right) +dx^2\left(21c \text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)+5dx^2 \text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{bx^4}{a}\right)\right)\right)\right)$$

input
```
Integrate[(c + d*x^2)^3*(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*(35*c^3*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + d
*x^2*(35*c^2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + d*x^2*(21*c*H
ypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 5*d*x^2*Hypergeometric2F1[7
/4, -p, 11/4, -((b*x^4)/a)])))/(35*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1519, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^2\right)^3 \left(a+bx^4\right)^p \, dx$$
$$\Big\downarrow \text{1519}$$
$$\frac{\int \left(bx^4+a\right)^p \left(3bcd^2(4p+7)x^4-3d\left(ad^2-bc^2(4p+7)\right)x^2+bc^3(4p+7)\right)dx}{b(4p+7)}+\frac{d^3x^3\left(a+bx^4\right)^{p+1}}{b(4p+7)}$$

$$\Big\downarrow \; 2432$$

$$\frac{\int \left(3bcd^2(4p+7)x^4\left(bx^4+a\right)^p+3d\left(bc^2(4p+7)-ad^2\right)x^2\left(bx^4+a\right)^p+bc^3(4p+7)\left(bx^4+a\right)^p\right)dx}{b(4p+7)}+ \frac{d^3x^3\left(a+bx^4\right)^{p+1}}{b(4p+7)}$$

$$\Big\downarrow \; 2009$$

$$\frac{bc^3(4p+7)x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)-dx^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(ad^2-\right.}{}$$

$$\frac{d^3x^3\left(a+bx^4\right)^{p+1}}{b(4p+7)}$$

input
```
Int[(c + d*x^2)^3*(a + b*x^4)^p,x]
```

output
```
(d^3*x^3*(a + b*x^4)^(1 + p))/(b*(7 + 4*p)) + ((b*c^3*(7 + 4*p)*x*(a + b*x
^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p - (
d*(a*d^2 - b*c^2*(7 + 4*p))*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7
/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (3*b*c*d^2*(7 + 4*p)*x^5*(a + b*x^4
)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p))/
(b*(7 + 4*p))
```

Defintions of rubi rules used

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \left(d\,x^{2}+c\right)^{3} \left(b\,x^{4}+a\right)^{p}d x$$

input
```
int((d*x^2+c)^3*(b*x^4+a)^p,x)
```

output
```
int((d*x^2+c)^3*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int (c+dx^2)^3 (a+bx^4)^p \, dx = \int (dx^2+c)^3 (bx^4+a)^p \, dx$$

input
```
integrate((d*x^2+c)^3*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 58.62 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.88

$$\int \left(c+dx^2\right)^3 \left(a+bx^4\right)^p \, dx = \frac{a^p c^3 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{3a^p c^2 d x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{3a^p c d^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, -p \\ \frac{9}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{a^p d^3 x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} \frac{7}{4}, -p \\ \frac{11}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((d*x**2+c)**3*(b*x**4+a)**p,x)
```

output
```
a**p*c**3*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4)) + 3*a**p*c**2*d*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*
x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + 3*a**p*c*d**2*x**5*gamma(5/4)*hyp
er((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + a**p*d**3
*x**7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*ga
mma(11/4))
```

Maxima [F]

$$\int \left(c+dx^2\right)^3 \left(a+bx^4\right)^p \, dx = \int \left(dx^2+c\right)^3 \left(bx^4+a\right)^p \, dx$$

input
```
integrate((d*x^2+c)^3*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^3*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(c+dx^2\right)^3 \left(a+bx^4\right)^p \, dx = \int \left(dx^2+c\right)^3 \left(bx^4+a\right)^p \, dx$$

input
```
integrate((d*x^2+c)^3*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^3*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+dx^2\right)^3 \left(a+bx^4\right)^p \, dx = \int \left(b\,x^4+a\right)^p \left(d\,x^2+c\right)^3 \, dx$$

input
```
int((a + b*x^4)^p*(c + d*x^2)^3,x)
```

output
```
int((a + b*x^4)^p*(c + d*x^2)^3, x)
```

Reduce [F]

$$\int \left(c+dx^2\right)^3 \left(a+bx^4\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x^2+c)^3*(b*x^4+a)^p,x)
```

output

```
(192*(a + b*x**4)**p*a*c*d**2*p**3*x + 480*(a + b*x**4)**p*a*c*d**2*p**2*x
 + 252*(a + b*x**4)**p*a*c*d**2*p*x + 64*(a + b*x**4)**p*a*d**3*p**3*x**3
+ 96*(a + b*x**4)**p*a*d**3*p**2*x**3 + 20*(a + b*x**4)**p*a*d**3*p*x**3 +
 64*(a + b*x**4)**p*b*c**3*p**3*x + 240*(a + b*x**4)**p*b*c**3*p**2*x + 28
4*(a + b*x**4)**p*b*c**3*p*x + 105*(a + b*x**4)**p*b*c**3*x + 192*(a + b*x
**4)**p*b*c**2*d*p**3*x**3 + 624*(a + b*x**4)**p*b*c**2*d*p**2*x**3 + 564*
(a + b*x**4)**p*b*c**2*d*p*x**3 + 105*(a + b*x**4)**p*b*c**2*d*x**3 + 192*
(a + b*x**4)**p*b*c*d**2*p**3*x**5 + 528*(a + b*x**4)**p*b*c*d**2*p**2*x**
5 + 372*(a + b*x**4)**p*b*c*d**2*p*x**5 + 63*(a + b*x**4)**p*b*c*d**2*x**5
 + 64*(a + b*x**4)**p*b*d**3*p**3*x**7 + 144*(a + b*x**4)**p*b*d**3*p**2*x
**7 + 92*(a + b*x**4)**p*b*d**3*p*x**7 + 15*(a + b*x**4)**p*b*d**3*x**7 -
49152*int((a + b*x**4)**p/(256*a*p**4 + 1024*a*p**3 + 1376*a*p**2 + 704*a*
p + 105*a + 256*b*p**4*x**4 + 1024*b*p**3*x**4 + 1376*b*p**2*x**4 + 704*b*
p*x**4 + 105*b*x**4),x)*a**2*c*d**2*p**7 - 319488*int((a + b*x**4)**p/(256
*a*p**4 + 1024*a*p**3 + 1376*a*p**2 + 704*a*p + 105*a + 256*b*p**4*x**4 +
1024*b*p**3*x**4 + 1376*b*p**2*x**4 + 704*b*p*x**4 + 105*b*x**4),x)*a**2*c
*d**2*p**6 - 820224*int((a + b*x**4)**p/(256*a*p**4 + 1024*a*p**3 + 1376*a
*p**2 + 704*a*p + 105*a + 256*b*p**4*x**4 + 1024*b*p**3*x**4 + 1376*b*p**2
*x**4 + 704*b*p*x**4 + 105*b*x**4),x)*a**2*c*d**2*p**5 - 1053696*int((a +
b*x**4)**p/(256*a*p**4 + 1024*a*p**3 + 1376*a*p**2 + 704*a*p + 105*a + ...
```

3.482 $\int \left(c+dx^2\right)^2 \left(a+bx^4\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\int \left(c+dx^2\right)^2 \left(a+bx^4\right)^p \, dx = \frac{d^2x\left(a+bx^4\right)^{1+p}}{b(5+4p)} + \left(c^2-\frac{ad^2}{5b+4bp}\right)x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right) + \frac{2}{3}cdx^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)$$

output

```
d^2*x*(b*x^4+a)^(p+1)/b/(5+4*p)+(c^2-a*d^2/(4*b*p+5*b))*x*(b*x^4+a)^p*hype
rgeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)+2/3*c*d*x^3*(b*x^4+a)^p*hy
pergeom([3/4, -p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

$$\int \left(c+dx^2\right)^2 \left(a+bx^4\right)^p \,dx = \frac{1}{15}x\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(15c^2 \operatorname{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+dx^2\left(10c \operatorname{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)+3dx^2 \operatorname{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)\right)\right)$$

input

```
Integrate[(c + d*x^2)^2*(a + b*x^4)^p,x]
```

output

```
(x*(a + b*x^4)^p*(15*c^2*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + d
*x^2*(10*c*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + 3*d*x^2*Hyperge
ometric2F1[5/4, -p, 9/4, -((b*x^4)/a)])))/(15*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1519, 25, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^2\right)^2 \left(a+bx^4\right)^p \,dx$$

$$\Big\downarrow \text{1519}$$

$$\frac{\int -\left(\left(-b(4p+5)c^2-2bd(4p+5)x^2c+ad^2\right)\left(bx^4+a\right)^p\right)dx}{b(4p+5)}+\frac{d^2x\left(a+bx^4\right)^{p+1}}{b(4p+5)}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2x\left(a+bx^4\right)^{p+1}}{b(4p+5)}-\frac{\int \left(-b(4p+5)c^2-2bd(4p+5)x^2c+ad^2\right)\left(bx^4+a\right)^p dx}{b(4p+5)}$$

$$\Big\downarrow \text{1516}$$

$$\frac{d^2x\left(a+bx^4\right)^{p+1}}{b(4p+5)}-\frac{\int \left(ad^2\left(1-\frac{bc^2(4p+5)}{ad^2}\right)\left(bx^4+a\right)^p-2bcd(4p+5)x^2\left(bx^4+a\right)^p\right)dx}{b(4p+5)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2x\left(a+bx^4\right)^{p+1}}{b(4p+5)}-$$

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(ad^2-bc^2(4p+5)\right)\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)-\frac{2}{3}bcd(4p+5)x^3\left(a+bx^4\right)^p}{b(4p+5)}$$

input

```
Int[(c + d*x^2)^2*(a + b*x^4)^p,x]
```

output

```
(d^2*x*(a + b*x^4)^(1 + p))/(b*(5 + 4*p)) - (((a*d^2 - b*c^2*(5 + 4*p))*x*
(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/
a)^p - (2*b*c*d*(5 + 4*p)*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4
, -((b*x^4)/a)])/(3*(1 + (b*x^4)/a)^p))/(b*(5 + 4*p))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1516

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 1519

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(d\,x^{2}+c\right)^{2} \left(b\,x^{4}+a\right)^{p}dx$$

input
```
int((d*x^2+c)^2*(b*x^4+a)^p,x)
```

output
```
int((d*x^2+c)^2*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int \left(c+dx^2\right)^2 \left(a+bx^4\right)^p\,dx=\int \left(dx^{2}+c\right)^{2}\left(bx^{4}+a\right)^{p}dx$$

input
```
integrate((d*x^2+c)^2*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^4 + 2*c*d*x^2 + c^2)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 32.91 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

$$\int \left(c+dx^2\right)^2 \left(a+bx^4\right)^p \, dx = \frac{a^p c^2 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, -p\\ \frac{5}{4}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^p c d x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, -p\\ \frac{7}{4}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{a^p d^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{5}{4}, -p\\ \frac{9}{4}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**2+c)**2*(b*x**4+a)**p,x)
```

output
```
a**p*c**2*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4)) + a**p*c*d*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*
exp_polar(I*pi)/a)/(2*gamma(7/4)) + a**p*d**2*x**5*gamma(5/4)*hyper((5/4,
-p), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int \left(c+dx^2\right)^2 \left(a+bx^4\right)^p \, dx = \int \left(dx^2+c\right)^2 \left(bx^4+a\right)^p \, dx$$

input
```
integrate((d*x^2+c)^2*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^2*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(c+dx^2\right)^2 \left(a+bx^4\right)^p \, dx=\int \left(dx^2+c\right)^2 \left(bx^4+a\right)^p \, dx$$

input
```
integrate((d*x^2+c)^2*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^2*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+dx^2\right)^2 \left(a+bx^4\right)^p \, dx=\int {\left(b\,x^4+a\right)}^p\,{\left(d\,x^2+c\right)}^2 \,d x$$

input
```
int((a + b*x^4)^p*(c + d*x^2)^2,x)
```

output
```
int((a + b*x^4)^p*(c + d*x^2)^2, x)
```

Reduce [F]

$$\int \left(c+dx^2\right)^2 \left(a+bx^4\right)^p \, dx=\text{Too large to display}$$

input
```
int((d*x^2+c)^2*(b*x^4+a)^p,x)
```

output

```
(16*(a + b*x**4)**p*a*d**2*p**2*x + 12*(a + b*x**4)**p*a*d**2*p*x + 16*(a
+ b*x**4)**p*b*c**2*p**2*x + 32*(a + b*x**4)**p*b*c**2*p*x + 15*(a + b*x**
4)**p*b*c**2*x + 32*(a + b*x**4)**p*b*c*d*p**2*x**3 + 48*(a + b*x**4)**p*b
*c*d*p*x**3 + 10*(a + b*x**4)**p*b*c*d*x**3 + 16*(a + b*x**4)**p*b*d**2*p*
*2*x**5 + 16*(a + b*x**4)**p*b*d**2*p*x**5 + 3*(a + b*x**4)**p*b*d**2*x**5
 - 1024*int((a + b*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b
*p**3*x**4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4),x)*a**2*d**2*p**5
- 3072*int((a + b*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*
p**3*x**4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4),x)*a**2*d**2*p**4 -
 3200*int((a + b*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p
**3*x**4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4),x)*a**2*d**2*p**3 -
1344*int((a + b*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p*
*3*x**4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4),x)*a**2*d**2*p**2 - 1
80*int((a + b*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3
*x**4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4),x)*a**2*d**2*p + 4096*i
nt((a + b*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**
4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4),x)*a*b*c**2*p**6 + 17408*in
t((a + b*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**4
 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4),x)*a*b*c**2*p**5 + 28160*int
((a + b*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x*...
```

3.483 $\int (c+dx^2)(a+bx^4)^p \, dx$

Optimal result

Integrand size $=17$, antiderivative size $=96$

$$\int (c+dx^2)(a+bx^4)^p \, dx = cx(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{1}{3}dx^3(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)$$

output
```
c*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)+1/3*d*
x^3*(b*x^4+a)^p*hypergeom([3/4, -p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time $=0.44$ (sec) , antiderivative size $=75$, normalized size of antiderivative $=0.78$

$$\int (c+dx^2)(a+bx^4)^p \, dx = \frac{1}{3}x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(3c\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+dx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)\right)$$

input
```
Integrate[(c + d*x^2)*(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*(3*c*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + d*x^
2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)]))/(3*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^2)\left(a+bx^4\right)^p\,dx$$
$$\Big\downarrow \text{1516}$$
$$\int \left(c\left(a+bx^4\right)^p+dx^2\left(a+bx^4\right)^p\right)dx$$
$$\Big\downarrow \text{2009}$$
$$cx\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{1}{3}dx^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)$$

input
```
Int[(c + d*x^2)*(a + b*x^4)^p,x]
```

output
```
(c*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*
x^4)/a)^p + (d*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)
/a)])/(3*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(d\,x^{2}+c\right) \left(b\,x^{4}+a\right)^{p}d x$$

input
```
int((d*x^2+c)*(b*x^4+a)^p,x)
```

output
```
int((d*x^2+c)*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int \left(c+dx^2\right) \left(a+bx^4\right)^p \, dx = \int \left(dx^{2}+c\right){\left(bx^{4}+a\right)}^{p}\,{d x}$$

input
```
integrate((d*x^2+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x^2 + c)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int \left(c+dx^2\right)\left(a+bx^4\right)^p\,dx=\frac{a^pcx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^pdx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**2+c)*(b*x**4+a)**p,x)
```

output
```
a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(5/4)) + a**p*d*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(7/4))
```

Maxima [F]

$$\int \left(c+dx^2\right)\left(a+bx^4\right)^p\,dx=\int \left(dx^2+c\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((d*x^2+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(c+dx^2\right)\left(a+bx^4\right)^p\,dx=\int \left(dx^2+c\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((d*x^2+c)*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+dx^2\right)\left(a+bx^4\right)^p\,dx=\int \left(b\,x^4+a\right)^p\,\left(d\,x^2+c\right)\,dx$$

input
```
int((a + b*x^4)^p*(c + d*x^2),x)
```

output
```
int((a + b*x^4)^p*(c + d*x^2), x)
```

Reduce [F]

$$\int \left(c+dx^2\right)\left(a+bx^4\right)^p\,dx$$
$$=\frac{4(b\,x^4+a)^p\,cpx+3(b\,x^4+a)^p\,cx+4(b\,x^4+a)^p\,dp\,x^3+(b\,x^4+a)^p\,d\,x^3+256\bigg(\int \frac{(b\,x^4+a)^p}{16b\,p^2x^4+16bp\,x^4+3b\,x^4+16a}}{}$$

input
```
int((d*x^2+c)*(b*x^4+a)^p,x)
```

output

```
(4*(a + b*x**4)**p*c*p*x + 3*(a + b*x**4)**p*c*x + 4*(a + b*x**4)**p*d*p*x
**3 + (a + b*x**4)**p*d*x**3 + 256*int((a + b*x**4)**p/(16*a*p**2 + 16*a*p
 + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4),x)*a*c*p**4 + 448*int((a
 + b*x**4)**p/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4 + 3
*b*x**4),x)*a*c*p**3 + 240*int((a + b*x**4)**p/(16*a*p**2 + 16*a*p + 3*a +
 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4),x)*a*c*p**2 + 36*int((a + b*x**4
)**p/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4),
x)*a*c*p + 256*int(((a + b*x**4)**p*x**2)/(16*a*p**2 + 16*a*p + 3*a + 16*b
*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4),x)*a*d*p**4 + 320*int(((a + b*x**4)**
p*x**2)/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**
4),x)*a*d*p**3 + 112*int(((a + b*x**4)**p*x**2)/(16*a*p**2 + 16*a*p + 3*a
+ 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4),x)*a*d*p**2 + 12*int(((a + b*x*
*4)**p*x**2)/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4 + 3*
b*x**4),x)*a*d*p)/(16*p**2 + 16*p + 3)
```

3.484 $\int (a+bx^4)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^4)^p \, dx = x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

output
```
x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^p \, dx = x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

input
```
Integrate[(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^
4)/a)^p
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^p\,dx$$

$$\Big\downarrow \text{779}$$

$$(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\int\left(\frac{bx^4}{a}+1\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

input

```
Int[(a + b*x^4)^p,x]
```

output

```
(x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^
4)/a)^p
```

Defintions of rubi rules used

rule 778

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{p}dx$$

input
```
int((b*x^4+a)^p,x)
```

output
```
int((b*x^4+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^p\,dx=\int \left(bx^{4}+a\right)^{p}dx$$

input
```
integrate((b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+bx^4\right)^p\,dx=\frac{a^{p}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+a)**p,x)
```

output
```
a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*ga
mma(5/4))
```

Maxima [F]

$$\int \left(a+bx^4\right)^p\,dx=\int \left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^4\right)^p\,dx=\int \left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 17.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \left(a+bx^4\right)^p\,dx=\frac{x\,\left(b\,x^4+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{{\left(\frac{b\,x^4}{a}+1\right)}^p}$$

input
```
int((a + b*x^4)^p,x)
```

output
```
(x*(a + b*x^4)^p*hypergeom([1/4, -p], 5/4, -(b*x^4)/a))/((b*x^4)/a + 1)^p
```

Reduce [F]

$$
\int \left(a+bx^4\right)^p\,dx
=\frac{\left(b\,x^{4}+a\right)^{p} x+16\left(\int \frac{\left(b\,x^{4}+a\right)^{p}}{4bp\,x^{4}+b\,x^{4}+4ap+a}d x\right) a\,p^{2}+4\left(\int \frac{\left(b\,x^{4}+a\right)^{p}}{4bp\,x^{4}+b\,x^{4}+4ap+a}d x\right) a p}{4p+1}
$$

input

```
int((b*x^4+a)^p,x)
```

output

```
((a + b*x**4)**p*x + 16*int((a + b*x**4)**p/(4*a*p + a + 4*b*p*x**4 + b*x*
*4),x)*a*p**2 + 4*int((a + b*x**4)**p/(4*a*p + a + 4*b*p*x**4 + b*x**4),x)
*a*p)/(4*p + 1)
```

3.485 $\int \frac{(a+bx^4)^p}{c+dx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{(a+bx^4)^p}{c+dx^2}\,dx = \frac{x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^4}{a},\frac{d^2x^4}{c^2}\right)}{c} - \frac{dx^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-\frac{bx^4}{a},\frac{d^2x^4}{c^2}\right)}{3c^2}$$

output
```
x*(b*x^4+a)^p*AppellF1(1/4,1,-p,5/4,d^2*x^4/c^2,-b*x^4/a)/c/((1+b*x^4/a)^p
)-1/3*d*x^3*(b*x^4+a)^p*AppellF1(3/4,1,-p,7/4,d^2*x^4/c^2,-b*x^4/a)/c^2/((
1+b*x^4/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^4)^p}{c+dx^2}\,dx = \int \frac{(a+bx^4)^p}{c+dx^2}\,dx$$

input
```
Integrate[(a + b*x^4)^p/(c + d*x^2),x]
```

output
```
Integrate[(a + b*x^4)^p/(c + d*x^2), x]
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^p}{c+dx^2}\,dx$$

$$\Big\downarrow \text{1569}$$

$$\int \left(\frac{c(a+bx^4)^p}{c^2-d^2x^4}+\frac{dx^2(a+bx^4)^p}{d^2x^4-c^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^4}{a},\frac{d^2x^4}{c^2}\right)}{c}-\frac{dx^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-\frac{bx^4}{a},\frac{d^2x^4}{c^2}\right)}{3c^2}$$

input

```
Int[(a + b*x^4)^p/(c + d*x^2),x]
```

output

```
(x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), (d^2*x^4)/c^2])/(
c*(1 + (b*x^4)/a)^p) - (d*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 1, 7/4, -((b
*x^4)/a), (d^2*x^4)/c^2])/(3*c^2*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 1569

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^p}{d\,x^2+c}dx$$

input
```
int((b*x^4+a)^p/(d*x^2+c),x)
```

output
```
int((b*x^4+a)^p/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^p}{c+dx^2}\,dx = \int \frac{(bx^4+a)^p}{dx^2+c}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p/(d*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+a)**p/(d*x**2+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^4)^p}{c+dx^2}\,dx=\int \frac{(bx^4+a)^p}{dx^2+c}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^4)^p}{c+dx^2}\,dx=\int \frac{(bx^4+a)^p}{dx^2+c}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{c+dx^2}\,dx=\int \frac{(b\,x^4+a)^p}{d\,x^2+c}\,dx$$

input
```
int((a + b*x^4)^p/(c + d*x^2),x)
```

output
```
int((a + b*x^4)^p/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^p}{c+dx^2}\,dx = \int \frac{(b\,x^4+a)^p}{d\,x^2+c}dx$$

input

```
int((b*x^4+a)^p/(d*x^2+c),x)
```

output

```
int((a + b*x**4)**p/(c + d*x**2),x)
```

3.486 $\int \frac{(a+bx^4)^p}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 189

$$\begin{aligned}\int \frac{(a+bx^4)^p}{(c+dx^2)^2}\,dx &= \frac{x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^4}{a},\frac{d^2x^4}{c^2}\right)}{c^2}\\&\quad-\frac{2dx^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{4},-p,2,\frac{7}{4},-\frac{bx^4}{a},\frac{d^2x^4}{c^2}\right)}{3c^3}\\&\quad+\frac{d^2x^5(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{4},-p,2,\frac{9}{4},-\frac{bx^4}{a},\frac{d^2x^4}{c^2}\right)}{5c^4}\end{aligned}$$

output

```
x*(b*x^4+a)^p*AppellF1(1/4,2,-p,5/4,d^2*x^4/c^2,-b*x^4/a)/c^2/((1+b*x^4/a)
^p)-2/3*d*x^3*(b*x^4+a)^p*AppellF1(3/4,2,-p,7/4,d^2*x^4/c^2,-b*x^4/a)/c^3/
((1+b*x^4/a)^p)+1/5*d^2*x^5*(b*x^4+a)^p*AppellF1(5/4,2,-p,9/4,d^2*x^4/c^2,
-b*x^4/a)/c^4/((1+b*x^4/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^4)^p}{(c+dx^2)^2}\,dx = \int \frac{(a+bx^4)^p}{(c+dx^2)^2}\,dx$$

input
```
Integrate[(a + b*x^4)^p/(c + d*x^2)^2,x]
```

output
```
Integrate[(a + b*x^4)^p/(c + d*x^2)^2, x]
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^p}{(c+dx^2)^2}\,dx$$

$$\Big\downarrow \; 1569$$

$$\int \left(\frac{c^2(a+bx^4)^p}{(c^2-d^2x^4)^2}+\frac{d^2x^4(a+bx^4)^p}{(d^2x^4-c^2)^2}-\frac{2cdx^2(a+bx^4)^p}{(c^2-d^2x^4)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^4}{a},\frac{d^2x^4}{c^2}\right)}{c^2}+\frac{d^2x^5(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{4},-p,2,\frac{9}{4},-\frac{bx^4}{a},\frac{d^2x^4}{c^2}\right)}{5c^4}-\frac{2dx^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{4},-p,2,\frac{7}{4},-\frac{bx^4}{a},\frac{d^2x^4}{c^2}\right)}{3c^3}$$

input
```
Int[(a + b*x^4)^p/(c + d*x^2)^2,x]
```

output
```
(x*(a + b*x^4)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), (d^2*x^4)/c^2])/(
c^2*(1 + (b*x^4)/a)^p) - (2*d*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 2, 7/4,
-((b*x^4)/a), (d^2*x^4)/c^2])/(3*c^3*(1 + (b*x^4)/a)^p) + (d^2*x^5*(a + b*
x^4)^p*AppellF1[5/4, -p, 2, 9/4, -((b*x^4)/a), (d^2*x^4)/c^2])/(5*c^4*(1 +
 (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 1569
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^p}{(d\,x^2+c)^2}dx$$

input
```
int((b*x^4+a)^p/(d*x^2+c)^2,x)
```

output
```
int((b*x^4+a)^p/(d*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^p}{(c+dx^2)^2}\,dx = \int \frac{(bx^4+a)^p}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+a)**p/(d*x**2+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^4)^p}{(c+dx^2)^2}\,dx = \int \frac{(bx^4+a)^p}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p/(d*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^p}{(c+dx^2)^2}\,dx = \int \frac{(bx^4+a)^p}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p/(d*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{(c+dx^2)^2}\,dx=\int \frac{(b\,x^4+a)^p}{(d\,x^2+c)^2}\,dx$$

input
```
int((a + b*x^4)^p/(c + d*x^2)^2,x)
```

output
```
int((a + b*x^4)^p/(c + d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^p}{(c+dx^2)^2}\,dx=\int \frac{(b\,x^4+a)^p}{d^2x^4+2cd\,x^2+c^2}dx$$

input
```
int((b*x^4+a)^p/(d*x^2+c)^2,x)
```

output
```
int((a + b*x**4)**p/(c**2 + 2*c*d*x**2 + d**2*x**4),x)
```

3.487 $\int (1-x^2)^3 (1+bx^4)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int (1-x^2)^3 (1+bx^4)^p \, dx = \frac{3x(1+bx^4)^{1+p}}{b(5+4p)} - \frac{x^3(1+bx^4)^{1+p}}{b(7+4p)} + \left(1 - \frac{3}{5b+4bp}\right) x \, \text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -bx^4\right) - \left(1 - \frac{1}{7b+4bp}\right) x^3 \, \text{Hypergeometric2F1}\left(\frac{3}{4}, -p, \frac{7}{4}, -bx^4\right)$$

output
```
3*x*(b*x^4+1)^(p+1)/b/(5+4*p)-x^3*(b*x^4+1)^(p+1)/b/(7+4*p)+(1-3/(4*b*p+5*
b))*x*hypergeom([1/4, -p],[5/4],-b*x^4)-(1-1/(4*b*p+7*b))*x^3*hypergeom([3
/4, -p],[7/4],-b*x^4)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.73

$$\begin{aligned}\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx = {} & x \,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) \\ & - x^3 \,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right) \\ & + \frac{3}{5}x^5 \,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-bx^4\right) \\ & - \frac{1}{7}x^7 \,\text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-bx^4\right)\end{aligned}$$

input

```
Integrate[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

output

```
x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - x^3*Hypergeometric2F1[3/4, -
p, 7/4, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5 -
(x^7*Hypergeometric2F1[7/4, -p, 11/4, -(b*x^4)])/7
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1519, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-x^2\right)^3 \left(bx^4+1\right)^p \, dx$$

$$\Big\downarrow \text{1519}$$

$$\frac{\int \left(bx^4+1\right)^p \left(3b(4p+7)x^4+3(1-b(4p+7))x^2+b(4p+7)\right) dx}{b(4p+7)} - \frac{x^3 \left(bx^4+1\right)^{p+1}}{b(4p+7)}$$

$$\Big\downarrow \text{2432}$$

$$\frac{\int \left(3b(4p+7)x^4\left(bx^4+1\right)^p+3(1-b(4p+7))x^2\left(bx^4+1\right)^p+b(4p+7)\left(bx^4+1\right)^p\right)dx}{b(4p+7)}-\frac{x^3\left(bx^4+1\right)^{p+1}}{b(4p+7)}$$

$$\Big\downarrow 2009$$

$$\frac{b(4p+7)x\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)+\frac{3}{5}b(4p+7)x^5\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-bx^4\right)+x^3(1-}{b(4p+7)}$$

$$\frac{x^3\left(bx^4+1\right)^{p+1}}{b(4p+7)}$$

input

```
Int[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

output

```
-((x^3*(1 + b*x^4)^(1 + p))/(b*(7 + 4*p))) + (b*(7 + 4*p)*x*Hypergeometric
2F1[1/4, -p, 5/4, -(b*x^4)] + (1 - b*(7 + 4*p))*x^3*Hypergeometric2F1[3/4,
 -p, 7/4, -(b*x^4)] + (3*b*(7 + 4*p)*x^5*Hypergeometric2F1[5/4, -p, 9/4, -
(b*x^4)])/5)/(b*(7 + 4*p))
```

Defintions of rubi rules used

rule 1519

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

method	result
meijerg	$-\frac{x^{7} \operatorname{hypergeom}\left(\left[\frac{7}{4},-p\right],\left[\frac{11}{4}\right],-b\,x^{4}\right)}{7}+\frac{3 x^{5} \operatorname{hypergeom}\left(\left[\frac{5}{4},-p\right],\left[\frac{9}{4}\right],-b\,x^{4}\right)}{5}-x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4},-p\right],\left[\frac{7}{4}\right],-b\,x^{4}$

input
```
int((-x^2+1)^3*(b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/7*x^7*hypergeom([7/4,-p],[11/4],-b*x^4)+3/5*x^5*hypergeom([5/4,-p],[9/4
],-b*x^4)-x^3*hypergeom([3/4,-p],[7/4],-b*x^4)+x*hypergeom([1/4,-p],[5/4],
-b*x^4)
```

Fricas [F]

$$\int \left(1-x^{2}\right)^{3}\left(1+bx^{4}\right)^{p}\,dx=\int -\left(x^{2}-1\right)^{3}\left(bx^{4}+1\right)^{p}\,dx$$

input
```
integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="fricas")
```

output
```
integral(-(x^6 - 3*x^4 + 3*x^2 - 1)*(b*x^4 + 1)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 51.95 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.09

$$\int (1-x^2)^3 (1+bx^4)^p \, dx = -\frac{x^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\left.\begin{matrix} \frac{7}{4}, -p \\ \frac{11}{4} \end{matrix}\right| bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{3x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\left.\begin{matrix} \frac{5}{4}, -p \\ \frac{9}{4} \end{matrix}\right| bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{3x^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix}\right| bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\right| bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input

```
integrate((-x**2+1)**3*(b*x**4+1)**p,x)
```

output

```
-x**7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I*pi))/(4*gamm
a(11/4)) + 3*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*p
i))/(4*gamma(9/4)) - 3*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp
_polar(I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**
4*exp_polar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx = \int -\left(x^2-1\right)^3 \left(bx^4+1\right)^p \, dx$$

input
```
integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

Giac [F]

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx = \int -\left(x^2-1\right)^3 \left(bx^4+1\right)^p \, dx$$

input
```
integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx = -\int \left(x^2-1\right)^3 \left(b\,x^4+1\right)^p \, dx$$

input
```
int(-(x^2 - 1)^3*(b*x^4 + 1)^p,x)
```

output
```
-int((x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

Reduce [F]

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx = \text{too large to display}$$

input

```
int((-x^2+1)^3*(b*x^4+1)^p,x)
```

output

```
( - 64*(b*x**4 + 1)**p*b*p**3*x**7 + 192*(b*x**4 + 1)**p*b*p**3*x**5 - 192
*(b*x**4 + 1)**p*b*p**3*x**3 + 64*(b*x**4 + 1)**p*b*p**3*x - 144*(b*x**4 +
 1)**p*b*p**2*x**7 + 528*(b*x**4 + 1)**p*b*p**2*x**5 - 624*(b*x**4 + 1)**p
*b*p**2*x**3 + 240*(b*x**4 + 1)**p*b*p**2*x - 92*(b*x**4 + 1)**p*b*p*x**7
+ 372*(b*x**4 + 1)**p*b*p*x**5 - 564*(b*x**4 + 1)**p*b*p*x**3 + 284*(b*x**
4 + 1)**p*b*p*x - 15*(b*x**4 + 1)**p*b*x**7 + 63*(b*x**4 + 1)**p*b*x**5 -
105*(b*x**4 + 1)**p*b*x**3 + 105*(b*x**4 + 1)**p*b*x - 64*(b*x**4 + 1)**p*
p**3*x**3 + 192*(b*x**4 + 1)**p*p**3*x - 96*(b*x**4 + 1)**p*p**2*x**3 + 48
0*(b*x**4 + 1)**p*p**2*x - 20*(b*x**4 + 1)**p*p*x**3 + 252*(b*x**4 + 1)**p
*p*x + 65536*int((b*x**4 + 1)**p/(256*b*p**4*x**4 + 1024*b*p**3*x**4 + 137
6*b*p**2*x**4 + 704*b*p*x**4 + 105*b*x**4 + 256*p**4 + 1024*p**3 + 1376*p*
*2 + 704*p + 105),x)*b*p**8 + 507904*int((b*x**4 + 1)**p/(256*b*p**4*x**4
+ 1024*b*p**3*x**4 + 1376*b*p**2*x**4 + 704*b*p*x**4 + 105*b*x**4 + 256*p*
*4 + 1024*p**3 + 1376*p**2 + 704*p + 105),x)*b*p**7 + 1626112*int((b*x**4
+ 1)**p/(256*b*p**4*x**4 + 1024*b*p**3*x**4 + 1376*b*p**2*x**4 + 704*b*p*x
**4 + 105*b*x**4 + 256*p**4 + 1024*p**3 + 1376*p**2 + 704*p + 105),x)*b*p*
*6 + 2771968*int((b*x**4 + 1)**p/(256*b*p**4*x**4 + 1024*b*p**3*x**4 + 137
6*b*p**2*x**4 + 704*b*p*x**4 + 105*b*x**4 + 256*p**4 + 1024*p**3 + 1376*p*
*2 + 704*p + 105),x)*b*p**5 + 2695936*int((b*x**4 + 1)**p/(256*b*p**4*x**4
 + 1024*b*p**3*x**4 + 1376*b*p**2*x**4 + 704*b*p*x**4 + 105*b*x**4 + 25...
```

3.488 $\int (1-x^2)^2 (1+bx^4)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = \frac{x(1+bx^4)^{1+p}}{b(5+4p)} + \left(1-\frac{1}{5b+4bp}\right) x \,\text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -bx^4\right) - \frac{2}{3}x^3 \,\text{Hypergeometric2F1}\left(\frac{3}{4}, -p, \frac{7}{4}, -bx^4\right)$$

output

```
x*(b*x^4+1)^(p+1)/b/(5+4*p)+(1-1/(4*b*p+5*b))*x*hypergeom([1/4, -p],[5/4],
-b*x^4)-2/3*x^3*hypergeom([3/4, -p],[7/4],-b*x^4)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = x \,\text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -bx^4\right) - \frac{2}{3}x^3 \,\text{Hypergeometric2F1}\left(\frac{3}{4}, -p, \frac{7}{4}, -bx^4\right) + \frac{1}{5}x^5 \,\text{Hypergeometric2F1}\left(\frac{5}{4}, -p, \frac{9}{4}, -bx^4\right)$$

input
```
Integrate[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

output
```
x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometric2F1[3/4
, -p, 7/4, -(b*x^4)])/3 + (x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/
5
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1519, 25, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-x^2\right)^2 \left(bx^4+1\right)^p \, dx$$

$$\Big\downarrow \text{1519}$$

$$\frac{\int -\left(\left(2b(4p+5)x^2-b(4p+5)+1\right)\left(bx^4+1\right)^p\right)dx}{b(4p+5)}+\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}-\frac{\int \left(2b(4p+5)x^2-5b-4bp+1\right)\left(bx^4+1\right)^p dx}{b(4p+5)}$$

$$\Big\downarrow \text{1516}$$

$$\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}-\frac{\int \left(2b(4p+5)x^2\left(bx^4+1\right)^p+(1-b(4p+5))\left(bx^4+1\right)^p\right)dx}{b(4p+5)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}-\frac{x(1-b(4p+5))\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)+\frac{2}{3}b(4p+5)x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)}{b(4p+5)}$$

input
```
Int[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

output
```
(x*(1 + b*x^4)^(1 + p))/(b*(5 + 4*p)) - ((1 - b*(5 + 4*p))*x*Hypergeometri
c2F1[1/4, -p, 5/4, -(b*x^4)] + (2*b*(5 + 4*p)*x^3*Hypergeometric2F1[3/4, -
p, 7/4, -(b*x^4)])/3)/(b*(5 + 4*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

method	result
meijerg	$\frac{x^{5} \operatorname{hypergeom}\left(\left[\frac{5}{4},-p\right],\left[\frac{9}{4}\right],-b\,x^{4}\right)}{5}-\frac{2 x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4},-p\right],\left[\frac{7}{4}\right],-b\,x^{4}\right)}{3}+x \operatorname{hypergeom}\left(\left[\frac{1}{4},-p\right],\left[\frac{5}{4}\right],-b\,x^{4}\right)$

input
```
int((-x^2+1)^2*(b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*hypergeom([5/4,-p],[9/4],-b*x^4)-2/3*x^3*hypergeom([3/4,-p],[7/4],
-b*x^4)+x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

Fricas [F]

$$\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = \int \left(x^2-1\right)^2 \left(bx^4+1\right)^p \, dx$$

input
```
integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="fricas")
```

output
```
integral((x^4 - 2*x^2 + 1)*(b*x^4 + 1)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = \frac{x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix}\frac{5}{4}, -p \\ \frac{9}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{x^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}\frac{3}{4}, -p \\ \frac{7}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4}, -p \\ \frac{5}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-x**2+1)**2*(b*x**4+1)**p,x)
```

output
```
x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi))/(4*gamma(
9/4)) - x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(
2*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*p
i))/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = \int \left(x^2-1\right)^2 \left(bx^4+1\right)^p \, dx$$

input
```
integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="maxima")
```

output
```
integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

Giac [F]

$$\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = \int \left(x^2-1\right)^2 \left(bx^4+1\right)^p \, dx$$

input
```
integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="giac")
```

output
```
integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = \int \left(x^2-1\right)^2 \left(b\,x^4+1\right)^p \, dx$$

input
```
int((x^2 - 1)^2*(b*x^4 + 1)^p,x)
```

output
```
int((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

Reduce [F]

$$\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = \text{Too large to display}$$

input

```
int((-x^2+1)^2*(b*x^4+1)^p,x)
```

output

```
(16*(b*x**4 + 1)**p*b*p**2*x**5 - 32*(b*x**4 + 1)**p*b*p**2*x**3 + 16*(b*x
**4 + 1)**p*b*p**2*x + 16*(b*x**4 + 1)**p*b*p*x**5 - 48*(b*x**4 + 1)**p*b*
p*x**3 + 32*(b*x**4 + 1)**p*b*p*x + 3*(b*x**4 + 1)**p*b*x**5 - 10*(b*x**4
+ 1)**p*b*x**3 + 15*(b*x**4 + 1)**p*b*x + 16*(b*x**4 + 1)**p*p**2*x + 12*(
b*x**4 + 1)**p*p*x + 4096*int((b*x**4 + 1)**p/(64*b*p**3*x**4 + 144*b*p**2
*x**4 + 92*b*p*x**4 + 15*b*x**4 + 64*p**3 + 144*p**2 + 92*p + 15),x)*b*p**
6 + 17408*int((b*x**4 + 1)**p/(64*b*p**3*x**4 + 144*b*p**2*x**4 + 92*b*p*x
**4 + 15*b*x**4 + 64*p**3 + 144*p**2 + 92*p + 15),x)*b*p**5 + 28160*int((b
*x**4 + 1)**p/(64*b*p**3*x**4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4
+ 64*p**3 + 144*p**2 + 92*p + 15),x)*b*p**4 + 21376*int((b*x**4 + 1)**p/(6
4*b*p**3*x**4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4 + 64*p**3 + 144*
p**2 + 92*p + 15),x)*b*p**3 + 7440*int((b*x**4 + 1)**p/(64*b*p**3*x**4 + 1
44*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4 + 64*p**3 + 144*p**2 + 92*p + 15)
,x)*b*p**2 + 900*int((b*x**4 + 1)**p/(64*b*p**3*x**4 + 144*b*p**2*x**4 + 9
2*b*p*x**4 + 15*b*x**4 + 64*p**3 + 144*p**2 + 92*p + 15),x)*b*p - 1024*int
((b*x**4 + 1)**p/(64*b*p**3*x**4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x*
*4 + 64*p**3 + 144*p**2 + 92*p + 15),x)*p**5 - 3072*int((b*x**4 + 1)**p/(6
4*b*p**3*x**4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4 + 64*p**3 + 144*
p**2 + 92*p + 15),x)*p**4 - 3200*int((b*x**4 + 1)**p/(64*b*p**3*x**4 + 144
*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4 + 64*p**3 + 144*p**2 + 92*p + 15...
```

3.489 $\int (1-x^2)(1+bx^4)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int (1-x^2)(1+bx^4)^p\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) - \frac{1}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)$$

output

```
x*hypergeom([1/4, -p],[5/4],-b*x^4)-1/3*x^3*hypergeom([3/4, -p],[7/4],-b*x
^4)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (1-x^2)(1+bx^4)^p\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) - \frac{1}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)$$

input

```
Integrate[(1 - x^2)*(1 + b*x^4)^p,x]
```

output
```
x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4,
-p, 7/4, -(b*x^4)])/3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x^2)\left(bx^4+1\right)^p\,dx$$

$$\Big\downarrow \text{1516}$$

$$\int \left(\left(bx^4+1\right)^p - x^2\left(bx^4+1\right)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)-\frac{1}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)$$

input
```
Int[(1 - x^2)*(1 + b*x^4)^p,x]
```

output
```
x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4,
-p, 7/4, -(b*x^4)])/3
```

Defintions of rubi rules used

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
meijerg	$x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, -p\right], \left[\frac{5}{4}\right], -b\,x^4\right) - \frac{x^3 \,\mathrm{hypergeom}\left(\left[\frac{3}{4}, -p\right], \left[\frac{7}{4}\right], -b\,x^4\right)}{3}$	37

input
```
int((-x^2+1)*(b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,-p],[5/4],-b*x^4)-1/3*x^3*hypergeom([3/4,-p],[7/4],-b*x^4
)
```

Fricas [F]

$$\int (1-x^2)\,(1+bx^4)^p\,dx = \int -(x^2-1)\,(bx^4+1)^p\,dx$$

input
```
integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="fricas")
```

output
```
integral(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.81 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int (1-x^2)\,(1+bx^4)^p\,dx = -\frac{x^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-x**2+1)*(b*x**4+1)**p,x)
```

output
```
-x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(4*gamma
(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*
gamma(5/4))
```

Maxima [F]

$$\int (1-x^2)(1+bx^4)^p\,dx = \int -(x^2-1)(bx^4+1)^p\,dx$$

input
```
integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)*(b*x^4 + 1)^p, x)
```

Giac [F]

$$\int (1-x^2)(1+bx^4)^p\,dx = \int -(x^2-1)(bx^4+1)^p\,dx$$

input
```
integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-x^2)(1+bx^4)^p\,dx = -\int (x^2-1)\,(b\,x^4+1)^p\,dx$$

input
```
int(-(x^2 - 1)*(b*x^4 + 1)^p,x)
```

output
```
-int((x^2 - 1)*(b*x^4 + 1)^p, x)
```

Reduce [F]

$$\int (1-x^2)\left(1+bx^4\right)^p\,dx$$
$$= \frac{-4(b\,x^4+1)^p\,p\,x^3+4(b\,x^4+1)^p\,px-(b\,x^4+1)^p\,x^3+3(b\,x^4+1)^p\,x+256\Big(\int\frac{(b\,x^4+1)^p}{16b\,p^2x^4+16bp\,x^4+3b\,x^4+16p^2+16}}{}$$

input
```
int((-x^2+1)*(b*x^4+1)^p,x)
```

output
```
( - 4*(b*x**4 + 1)**p*p*x**3 + 4*(b*x**4 + 1)**p*p*x - (b*x**4 + 1)**p*x**
3 + 3*(b*x**4 + 1)**p*x + 256*int((b*x**4 + 1)**p/(16*b*p**2*x**4 + 16*b*p
*x**4 + 3*b*x**4 + 16*p**2 + 16*p + 3),x)*p**4 + 448*int((b*x**4 + 1)**p/(
16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4 + 16*p**2 + 16*p + 3),x)*p**3 + 24
0*int((b*x**4 + 1)**p/(16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4 + 16*p**2 +
 16*p + 3),x)*p**2 + 36*int((b*x**4 + 1)**p/(16*b*p**2*x**4 + 16*b*p*x**4
+ 3*b*x**4 + 16*p**2 + 16*p + 3),x)*p - 256*int(((b*x**4 + 1)**p*x**2)/(16
*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4 + 16*p**2 + 16*p + 3),x)*p**4 - 320*
int(((b*x**4 + 1)**p*x**2)/(16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4 + 16*p
**2 + 16*p + 3),x)*p**3 - 112*int(((b*x**4 + 1)**p*x**2)/(16*b*p**2*x**4 +
 16*b*p*x**4 + 3*b*x**4 + 16*p**2 + 16*p + 3),x)*p**2 - 12*int(((b*x**4 +
1)**p*x**2)/(16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4 + 16*p**2 + 16*p + 3)
,x)*p)/(16*p**2 + 16*p + 3)
```

3.490 $\int (1+bx^4)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int (1+bx^4)^p \, dx = x \,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)$$

output

```
x*hypergeom([1/4, -p],[5/4],-b*x^4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (1+bx^4)^p \, dx = x \,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)$$

input

```
Integrate[(1 + b*x^4)^p,x]
```

output

```
x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^4+1)^p\,dx$$

$$\Big\downarrow \color{blue}{778}$$

$$x\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)$$

input
```
Int[(1 + b*x^4)^p,x]
```

output
```
x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
meijerg	$x\,\text{hypergeom}\left(\left[\frac{1}{4},-p\right],\left[\frac{5}{4}\right],-b\,x^4\right)$	17

input
```
int((b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

Fricas [F]

$$\int \left(1+bx^4\right)^p\,dx = \int \left(bx^4+1\right)^p\,dx$$

input
```
integrate((b*x^4+1)^p,x, algorithm="fricas")
```

output
```
integral((b*x^4 + 1)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \left(1+bx^4\right)^p\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+1)**p,x)
```

output
```
x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*gamma(5/4
))
```

Maxima [F]

$$\int (1+bx^4)^p \, dx = \int (bx^4+1)^p \, dx$$

input
```
integrate((b*x^4+1)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + 1)^p, x)
```

Giac [F]

$$\int (1+bx^4)^p \, dx = \int (bx^4+1)^p \, dx$$

input
```
integrate((b*x^4+1)^p,x, algorithm="giac")
```

output
```
integrate((b*x^4 + 1)^p, x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int (1+bx^4)^p \, dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{4}, -p;\ \frac{5}{4};\ -b\,x^4\right)$$

input
```
int((b*x^4 + 1)^p,x)
```

output
```
x*hypergeom([1/4, -p], 5/4, -b*x^4)
```

Reduce [F]

$$\int \left(1+bx^4\right)^p \,dx = \frac{\left(b\,x^4+1\right)^p x+16\left(\int \frac{\left(b\,x^4+1\right)^p}{4bp\,x^4+b\,x^4+4p+1}dx\right)p^2+4\left(\int \frac{\left(b\,x^4+1\right)^p}{4bp\,x^4+b\,x^4+4p+1}dx\right)p}{4p+1}$$

input
```
int((b*x^4+1)^p,x)
```

output
```
((b*x**4 + 1)**p*x + 16*int((b*x**4 + 1)**p/(4*b*p*x**4 + b*x**4 + 4*p + 1
),x)*p**2 + 4*int((b*x**4 + 1)**p/(4*b*p*x**4 + b*x**4 + 4*p + 1),x)*p)/(4
*p + 1)
```

3.491 $\int \frac{(1+bx^4)^p}{1-x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = x\operatorname{AppellF1}\left(\frac{1}{4},1,-p,\frac{5}{4},x^4,-bx^4\right) + \frac{1}{3}x^3\operatorname{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-bx^4,x^4\right)$$

output
```
x*AppellF1(1/4,1,-p,5/4,x^4,-b*x^4)+1/3*x^3*AppellF1(3/4,1,-p,7/4,x^4,-b*x
^4)
```

Mathematica [F]

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = \int \frac{(1+bx^4)^p}{1-x^2}\,dx$$

input
```
Integrate[(1 + b*x^4)^p/(1 - x^2),x]
```

output
```
Integrate[(1 + b*x^4)^p/(1 - x^2), x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^4+1)^p}{1-x^2} \, dx$$

$$\Big\downarrow \text{1569}$$

$$\int \left(\frac{(bx^4+1)^p}{1-x^4}-\frac{x^2(bx^4+1)^p}{x^4-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x \operatorname{AppellF1}\left(\frac{1}{4},1,-p,\frac{5}{4},x^4,-bx^4\right)+\frac{1}{3}x^3 \operatorname{AppellF1}\left(\frac{3}{4},1,-p,\frac{7}{4},x^4,-bx^4\right)$$

input

```
Int[(1 + b*x^4)^p/(1 - x^2),x]
```

output

```
x*AppellF1[1/4, 1, -p, 5/4, x^4, -(b*x^4)] + (x^3*AppellF1[3/4, 1, -p, 7/4
, x^4, -(b*x^4)])/3
```

Defintions of rubi rules used

rule 1569

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^{4}+1)^{p}}{-x^{2}+1}dx$$

input
```
int((b*x^4+1)^p/(-x^2+1),x)
```

output
```
int((b*x^4+1)^p/(-x^2+1),x)
```

Fricas [F]

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx=\int -\frac{(bx^4+1)^p}{x^2-1}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="fricas")
```

output
```
integral(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x**4+1)**p/(-x**2+1),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = \int -\frac{(bx^4+1)^p}{x^2-1}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="maxima")
```

output
```
-integrate((b*x^4 + 1)^p/(x^2 - 1), x)
```

Giac [F]

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = \int -\frac{(bx^4+1)^p}{x^2-1}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="giac")
```

output
```
integrate(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = -\int \frac{(b\,x^4+1)^p}{x^2-1}\,dx$$

input
```
int(-(b*x^4 + 1)^p/(x^2 - 1),x)
```

output
```
-int((b*x^4 + 1)^p/(x^2 - 1), x)
```

Reduce [F]

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = -\left(\int \frac{(b\,x^4+1)^p}{x^2-1}dx\right)$$

input

```
int((b*x^4+1)^p/(-x^2+1),x)
```

output

```
 - int((b*x**4 + 1)**p/(x**2 - 1),x)
```

3.492 $\int \frac{(1+bx^4)^p}{(1-x^2)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\begin{aligned}\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx &= x\,\text{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-bx^4,x^4\right)\\ &\quad+\frac{2}{3}x^3\,\text{AppellF1}\left(\frac{3}{4},-p,2,\frac{7}{4},-bx^4,x^4\right)\\ &\quad+\frac{1}{5}x^5\,\text{AppellF1}\left(\frac{5}{4},-p,2,\frac{9}{4},-bx^4,x^4\right)\end{aligned}$$

output
```
x*AppellF1(1/4,2,-p,5/4,x^4,-b*x^4)+2/3*x^3*AppellF1(3/4,2,-p,7/4,x^4,-b*x
^4)+1/5*x^5*AppellF1(5/4,2,-p,9/4,x^4,-b*x^4)
```

Mathematica [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx$$

input
```
Integrate[(1 + b*x^4)^p/(1 - x^2)^2,x]
```

output
```
Integrate[(1 + b*x^4)^p/(1 - x^2)^2, x]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^4+1)^p}{(1-x^2)^2}\,dx$$

$$\Big\downarrow \text{1569}$$

$$\int \left(\frac{x^4(bx^4+1)^p}{(x^4-1)^2}+\frac{(bx^4+1)^p}{(x^4-1)^2}+\frac{2x^2(bx^4+1)^p}{(x^4-1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x\,\text{AppellF1}\left(\frac{1}{4},2,-p,\frac{5}{4},x^4,-bx^4\right)+\frac{1}{5}x^5\,\text{AppellF1}\left(\frac{5}{4},2,-p,\frac{9}{4},x^4,-bx^4\right)+\frac{2}{3}x^3\,\text{AppellF1}\left(\frac{3}{4},2,-p,\frac{7}{4},x^4,-bx^4\right)$$

input
```
Int[(1 + b*x^4)^p/(1 - x^2)^2,x]
```

output
```
x*AppellF1[1/4, 2, -p, 5/4, x^4, -(b*x^4)] + (2*x^3*AppellF1[3/4, 2, -p, 7
/4, x^4, -(b*x^4)])/3 + (x^5*AppellF1[5/4, 2, -p, 9/4, x^4, -(b*x^4)])/5
```

Defintions of rubi rules used

rule 1569
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{\left(b\,x^{4}+1\right)^{p}}{\left(-x^{2}+1\right)^{2}}dx$$

input
```
int((b*x^4+1)^p/(-x^2+1)^2,x)
```

output
```
int((b*x^4+1)^p/(-x^2+1)^2,x)
```

Fricas [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx=\int \frac{(bx^4+1)^p}{(x^2-1)^2}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4 + 1)^p/(x^4 - 2*x^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+1)**p/(-x**2+1)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \int \frac{(bx^4+1)^p}{(x^2-1)^2}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

Giac [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \int \frac{(bx^4+1)^p}{(x^2-1)^2}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \int \frac{(b\,x^4+1)^p}{(x^2-1)^2}\,dx$$

input
```
int((b*x^4 + 1)^p/(x^2 - 1)^2,x)
```

output
```
int((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

Reduce [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \int \frac{(b\,x^4+1)^p}{x^4-2x^2+1}dx$$

input
```
int((b*x^4+1)^p/(-x^2+1)^2,x)
```

output
```
int((b*x**4 + 1)**p/(x**4 - 2*x**2 + 1),x)
```

3.493 $\int \frac{(1+bx^4)^p}{(1-x^2)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\begin{aligned}\int \frac{(1+bx^4)^p}{(1-x^2)^3} \, dx &= x \operatorname{AppellF1}\left(\frac{1}{4},-p,3,\frac{5}{4},-bx^4,x^4\right) \\ &\quad + x^3 \operatorname{AppellF1}\left(\frac{3}{4},-p,3,\frac{7}{4},-bx^4,x^4\right) \\ &\quad + \frac{3}{5}x^5 \operatorname{AppellF1}\left(\frac{5}{4},-p,3,\frac{9}{4},-bx^4,x^4\right) \\ &\quad + \frac{1}{7}x^7 \operatorname{AppellF1}\left(\frac{7}{4},-p,3,\frac{11}{4},-bx^4,x^4\right)\end{aligned}$$

output

```
x*AppellF1(1/4,3,-p,5/4,x^4,-b*x^4)+x^3*AppellF1(3/4,3,-p,7/4,x^4,-b*x^4)+
3/5*x^5*AppellF1(5/4,3,-p,9/4,x^4,-b*x^4)+1/7*x^7*AppellF1(7/4,3,-p,11/4,x
^4,-b*x^4)
```

Mathematica [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx$$

input
```
Integrate[(1 + b*x^4)^p/(1 - x^2)^3,x]
```

output
```
Integrate[(1 + b*x^4)^p/(1 - x^2)^3, x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^4+1)^p}{(1-x^2)^3}\,dx$$

$$\Big\downarrow \text{1569}$$

$$\int \left(-\frac{3x^4(bx^4+1)^p}{(x^4-1)^3}-\frac{(bx^4+1)^p}{(x^4-1)^3}-\frac{x^6(bx^4+1)^p}{(x^4-1)^3}-\frac{3x^2(bx^4+1)^p}{(x^4-1)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x\operatorname{AppellF1}\left(\frac{1}{4},3,-p,\frac{5}{4},x^4,-bx^4\right)+\frac{1}{7}x^7\operatorname{AppellF1}\left(\frac{7}{4},3,-p,\frac{11}{4},x^4,-bx^4\right)+\frac{3}{5}x^5\operatorname{AppellF1}\left(\frac{5}{4},3,-p,\frac{9}{4},x^4,-bx^4\right)+x^3\operatorname{AppellF1}\left(\frac{3}{4},3,-p,\frac{7}{4},x^4,-bx^4\right)$$

input
```
Int[(1 + b*x^4)^p/(1 - x^2)^3,x]
```

output
```
x*AppellF1[1/4, 3, -p, 5/4, x^4, -(b*x^4)] + x^3*AppellF1[3/4, 3, -p, 7/4,
 x^4, -(b*x^4)] + (3*x^5*AppellF1[5/4, 3, -p, 9/4, x^4, -(b*x^4)])/5 + (x^
7*AppellF1[7/4, 3, -p, 11/4, x^4, -(b*x^4)])/7
```

Defintions of rubi rules used

rule 1569
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^4+1)^p}{(-x^2+1)^3}dx$$

input
```
int((b*x^4+1)^p/(-x^2+1)^3,x)
```

output
```
int((b*x^4+1)^p/(-x^2+1)^3,x)
```

Fricas [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \int -\frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="fricas")
```

output
```
integral(-(b*x^4 + 1)^p/(x^6 - 3*x^4 + 3*x^2 - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+1)**p/(-x**2+1)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \int -\frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="maxima")
```

output
```
-integrate((b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

Giac [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \int -\frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-(b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \int -\frac{(b\,x^4+1)^p}{(x^2-1)^3}\,dx$$

input
```
int(-(b*x^4 + 1)^p/(x^2 - 1)^3,x)
```

output
```
int(-(b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

Reduce [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = -\left(\int \frac{(b\,x^4+1)^p}{x^6-3x^4+3x^2-1}dx\right)$$

input
```
int((b*x^4+1)^p/(-x^2+1)^3,x)
```

output
```
- int((b*x**4 + 1)**p/(x**6 - 3*x**4 + 3*x**2 - 1),x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != −1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != −1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file